Path Planning of Robot Motion using Vector Pursuit Method

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ISLAMABAD

August, 2020

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A thesis submitted in partial fulfillment of the requirements for the degree of

MS Robotics and Intelligent Machine Engineering

Thesis Supervisor:

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August, 2020

National University of Sciences and Technology

MASTER THESIS WORK

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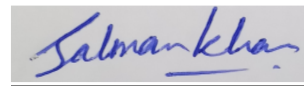

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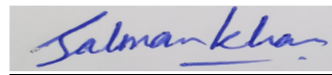

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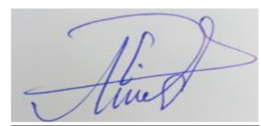

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Copyright Statement

Acknowledgements

I am thankful to the most merciful and mighty Allah Subhana-Watala to have guided me throughout this work at every step and for every new thought which He put in my mind and improve it. Indeed, I could have done nothing without Your priceless help and guidance. Whosoever helped me throughout the course of my thesis, whether my parents or any other individual was Your will, so indeed none be worthy of praise but You.

I am profusely thankful to my beloved parents and my elder brothers who raised me when I was not capable of walking and continued to support me throughout in every department of my life.

I would also like to express special thanks to my supervisor Dr. Mian Ashfaq Ali and our Head of Department Dr. Yasar Ayaz for their help throughout my thesis and for teaching me different courses.

I would also like to pay special thanks to Dr. Muhammad Jawad Khan and Dr. Kashif Javed for their tremendous support and cooperation. I appreciate their patience and guidance throughout the whole thesis.

Finally, I would like to express my gratitude to all the individuals who have rendered valuable assistance to my study.

Dedicated to my exceptional parents and adored siblings whose tremendous support and cooperation led me to this wonderful accomplishment.

Abstract

Ground robots are classified by locomotion into three major categories i.e. wheeled, tracked and Legged robots. Typically, wheeled robots are energy efficient and simple to control. Autonomous navigation requires integration of position and orientation sensing, path planning, obstacle avoidance and vehicle control. Autonomous navigation can be divided in to four sub tasks i.e. perceiving and modeling the environment, localizing the vehicle, plan and decide desired motion and execute the desired motion. This thesis mainly focuses on decide desired motion and its execution using Vector pursuit method. Vector pursuit is a geometric path tracking technique based on Screw theory developed by Sir Robert Ball. It is robust with large tracking error. This technique determines desired turning radius from vehicle current position and orientation to position and orientation at look ahead point. By using turning radius wheel velocities of vehicle were determined. The simulation results show that the vehicle satisfy its constraint and follow the waypoints accurately.

Key Words: *Autonomous navigation, Pioneer-3DX robot, path planning, Vector Pursuit, Geometrical path tracking, AGV, Screw theory.*

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LIST OF ABBREVIATIONS

AGV	Autonomous ground vehicles
RADAR	Radio detection and ranging
LIDAR	Light detection and ranging
DARPA	Defense advance research project agency
HRI	Human Robot Interaction
GPS	Global positioning service
UAV	Unmanned aerial vehicle
ROS	Robot operating system
TCP	Transmission Control Protocol
P-3DX	Pioneer-3DX
FRMLCs	Fuzzy reference model learning controllers
NTV	Navigation test vehicle
UTM	Universal Transverse Mercator
NI	National Instruments
V-REP	Virtual Robot Experimentation Platform

CHAPTER 1

INTRODUCTION

In near future autonomous vehicles may bring massive changes to our current traffic infrastructures in daily life. Several automotive companies, information technology service providers and electric manufactures are interested in this technology. The DARPA Urban Challenge [1], [2] in 2007 showed that their vehicles were able to drive autonomously in urban environments. Different path tracking techniques for Autonomous ground vehicles were developed. Autonomous ground vehicles are able for autonomous navigation which means that they may operate and regulates according to the environment needs without any outside control. This process may be further divided into four steps. These are perceiving and modeling of the environment, localizing robot in environment, planning and deciding robot motion and executing robot desired motion. This dissertation focuses mainly on deciding of robot desired motion and execution of this desired motion.

For deciding of robot motion mainly three different geometric techniques are used for real time applications. They are: follow the carrot, pure pursuit algorithm and vector pursuit algorithm. Follow the carrot [3] is based on to acquire a goal point and then aim the robot to achieve the desired point. In Pure Pursuit technique [4] curvature is finding out which will make the vehicle to move from its current position to goal or desired position then, a circle is defined which passes through current vehicle position and desired position. Finally, a control algorithm is used which chooses a steering angle to achieve this circle. A new path tracking method known as Vector Pursuit [6], which based on Screw theory. Vector pursuit calculate two instantaneous screws that is translational screw and rotational screw. Then both screws are added to find out the desired instantaneous motion. Uses this information a turning radius is being find out which will take the vehicle to move from current to goal position.

1.1 Background, Scope and Motivation

Vector pursuit [6] uses screws theory, it is just like other geometric technique which uses look-ahead distance. Lookahead distance is from current to next goal point. Geometry is used for determination of desired motion. This method is different from the previous geometric path tracking technique like follow the carrot or pure pursuit [3], where at look ahead point desired orientation is not used. Proportional path tracking [12] is another geometric method which also uses vehicle desired orientation at lookahead point. In proportional path tracking technique, current position error times gain with some gain and add with current orientation error times some gain, but this technique becomes meaningless because terms with different units were added.

In this dissertation, to follow waypoints by mobile robot we used vector pursuit geometric path tracking technique. This technique basically finds out turning radius for given path then from turning radius we find out the wheel velocities i.e. left wheel and right wheel. So, to overcome this problem two tasks are considered, First is development of an algorithm which will eventually find out desired motion of vehicle to track a given path. While second task is to design an efficient control technique so that robot achieve the way points accurately.

The scope of our work is too much in mobile robotics. There are various important and prominent areas of applications of chosen research filed. They are:

I. Autonomous car:

   Autonomous car is a vehicle which may guide itself without interaction of human. They can detect environment using a variety of motion sensors, cameras, Radar, Lidar, GPS and computer vision. They are used for transportation purposes. It has been observed that autonomous vehicle can also be used for food and parcel deliveries, trash collections and pick drop services etc.

II. Mobile Robots in industry

   Industries are being automatized due to revolution in mobile robotics. There are many mobile robots working side by side and transport different stuff with human being and playing an important role in our production sectors.

III. Mobile Robots in military purposes:

   Due to advancement in technology in defense purposes, people are continuously working on mobile robots for military use. Autonomous mobile robots are design to search, transport and rescue purposes. Like ANDROS F6 is a mine rescue robot and UAV's (unmanned aerial vehicle's) etc.

IV. Mobile Robots in space technologies:

   Usually in space technology mobile robot are used for exploration, automatic navigation and transportation. So very advance mobile robots are used for these purposes.

V. Mobile Robots as Wheel chair:

Wheel chair is a mobile robot which is used in hospitals, paralyzed and elderly homes, these wheel chairs are being used as for transportation purposes in daily routine life. Here again locomotion is involved, and mobile robots must directly interact with people, so, we must have to give guarantee of a safe, harm less and effective locomotion and interaction and all these things fall in mobile robotics and Human Robot Interaction (HRI) research field.

As stated in introduction, basically there are two tasks considered here. First task is developing of an algorithm which will finds the current desired motion of mobile robot to track a given path. Second task is to development of control technique which will execute desired motion for AGV. We have two main motivations here; first motivation is the ability to operate Pioneer-3DX in various surfaces or grounds. Second motivation is tuned the parameter in the algorithm so that it will be optimized and will require less amount of time to reach its goal point.

1.2 Thesis Contribution

There are various techniques exist which are based on geometry of lookahead point on robot coordinate system. System stability is very much affected by lookahead point. There is tradeoff between look ahead distance and system stability. For example, if we take look-ahead distance large the system will be relatively more stable, while for accurate tracking lookahead distance will be chosen smaller and this is desirable. So, lookahead distance is basically a tuning parameter. There is another tuning parameter in this algorithm 'k'. 'k' is basically the ratio between time to translate (t_t) and time to rotate (t_r). Here we have tuned these two parameters. Our second contribution is that to check the response of the robot in virtual experimentation platform i.e. CoppeliaSim and their results are compared and analyzed.

1.3 Types of Robots

Robots are categorized three different types. These are: mobile robots, humanoid robots, and aerial robots. Each type of them has its own limitations. Their roles vary according to its application and uses. In path planning, we usually use either mobile robots or humanoid robots or aerial robots or sometimes combination of these but nowadays two passible ways in which robots are divided. First, robots' types are divided by applications and second is robots' types are divided by locomotion and kinematics. According to application point of view different tasks in several fields are performed by different robots they are as follow domestic robot, industrial robots, medical robots, military robots and space robots etc.

Similarly, robots are also categorized by locomotion and kinematics. That is because of robot classified by application does not show enough information like when we generally talking about industrial robots, we think of stationary robots that perform specific tasks like pick and place etc. but there are also automatic guided vehicles (AGV) in industries. So, here is the classification by locomotion & kinematics like wheeled robots, swimming robots, stationary robots, legged robots, flying robots, Swarm robots and snake like robots etc. after some time they will be more useful and common.

For path planning and tracking Wheeled robots are commonly used in robotics. Some of mobile robot used commonly wheeled robot having less than four wheels are shown in Figure 1.1, four wheeled mobile robots are shown in Figure 1.2 and while six wheeled mobile robots are shown in Figure 1.3 below.

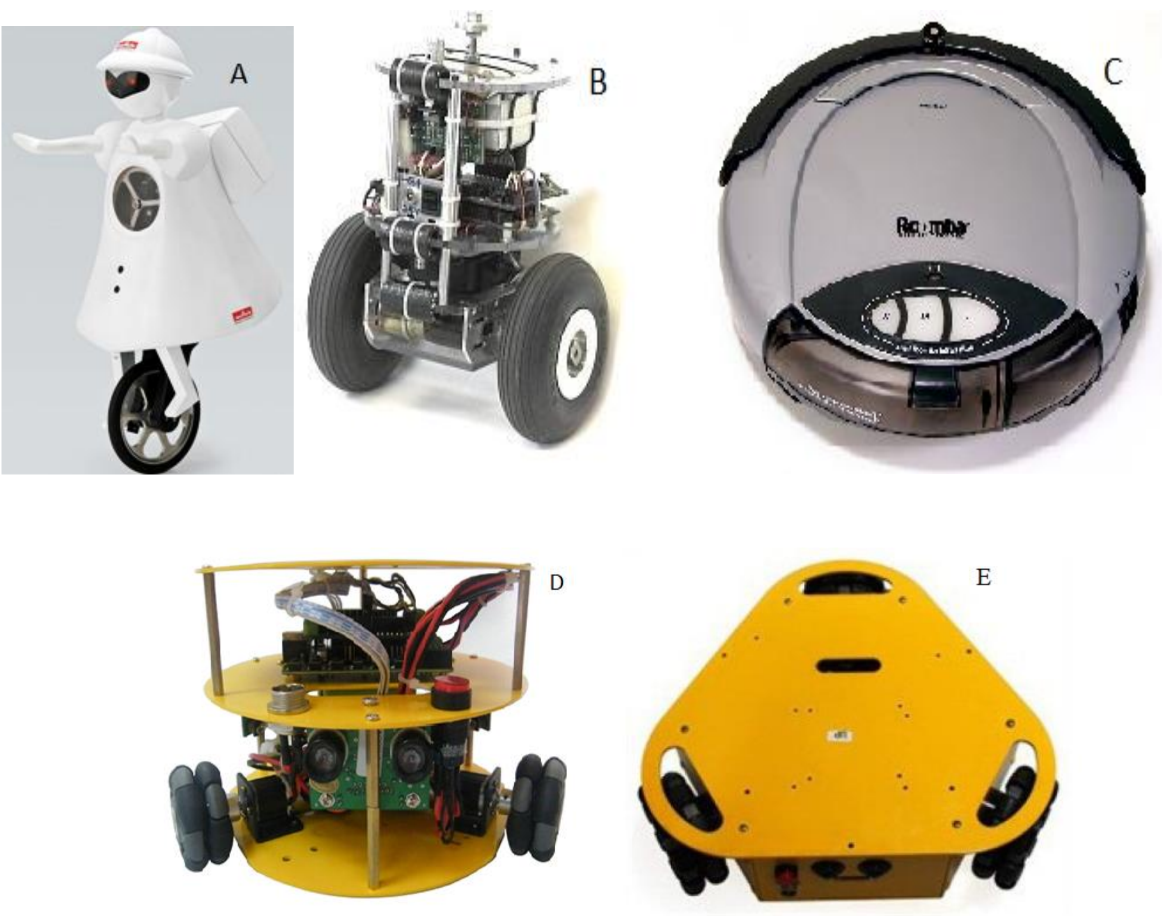


Figure 1.1: Commonly used mobile robots having one, two and three wheels.

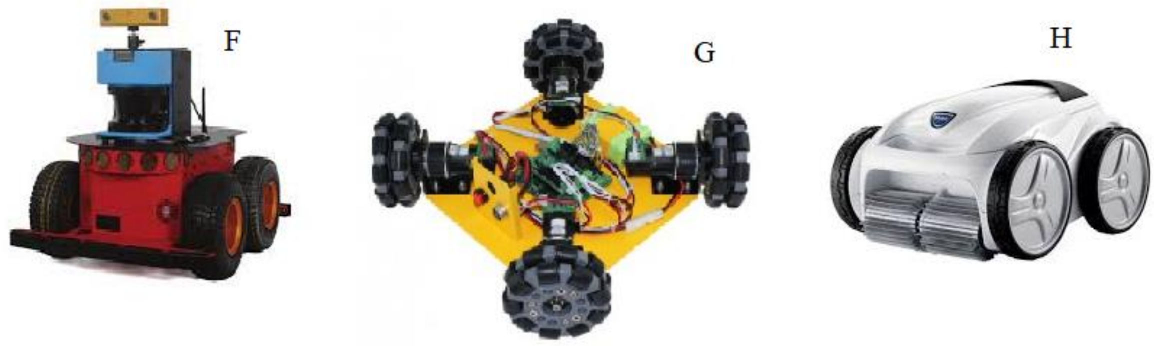


Figure 1.2: Four wheeled mobile robots

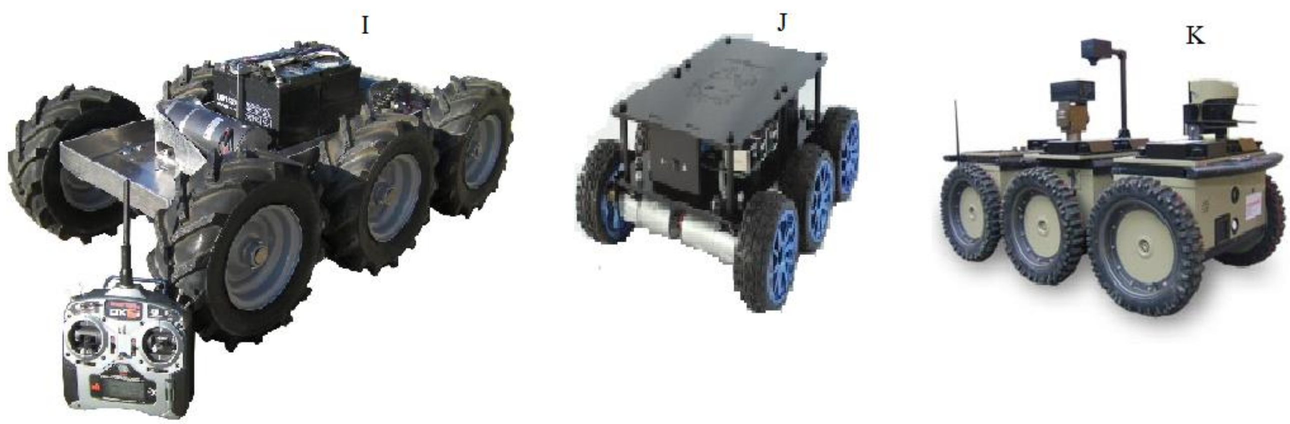


Figure 1.3: Six wheeled mobile robots

Table 1-1: Robots along with their names, given in above Figures 1.1, 1.2,1.3and 1.4.

Figure Number	Alphabet Allotted	Robot's Name
1.1	A	Seiko
1.1	B	2-wheel nbot
1.1	C	Roomba (vacuum cleaner)
1.1	D	3WD
1.1	E	3WD Triangular
1.2	F	Pioneer 3-AT
1.2	G	4WD Omni Wheels
1.2	H	Polarize 4WD (pool cleaner)
1.3	I	6WD (Heavy duty Robot)

1.3	J	6 WHEEL Piborg
1.3	K	RobuRoc 6
1.4	L	Pioneer-3DX

These different mobile robots categorized according to wheels of robot. Name of these mobile robots are mentioned here above in the table 1-1.

Selection of robot

Selection of robot is very important for path planning according to control point of view. It is observed that three wheeled mobile robots are more stable than mobile robot having less than three wheels. So, we consider Pioneer-3DX robot shown in figure 1.4 for our work. Pioneer is a family of two wheel and four-wheel mobile robot that are Pioneer 1, AT, Pioneer 2-DX, Pioneer 2-DX8/Dx8 plus, Pioneer 3-AT, 3-DX etc. they all share a common architecture and software.

L

Figure. 1.4: Pioneer-3DX

We used Pioneer-3DX mobile robot here, which is basically two wheeled two motors differential drive mobile robot with third wheel as a free castor wheel. Its configuration is represented by equation (A) below:

$$q = [x \;\; y \;\; \Theta_r]^T \qquad \text{(A)}$$

where (x, y) is the position of robot and Θ_r is the orientation of robot, there are two control inputs i.e. ω is an angular velocity while v is linear velocity control input. The overall control system can be written as in following below equation (B).

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta}_r \end{bmatrix} = \begin{bmatrix} \cos\theta_r \\ \sin\theta_r \\ 0 \end{bmatrix} \nu + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \omega \qquad \text{(B)}$$

The robot is two wheeled two motor differential drive robot and third wheel as a castor free wheel. So, orthogonal motion is impossible to achieve and nonholonomic constraint can be achieved using the below equation (C).

$$\dot{x} \sin\dot{\theta}_V - \dot{y} \cos\theta_V = 0 \qquad \text{(C)}$$

1.4 Thesis Outline

Chapter 2 of our thesis consist of literature review which state of art technique being done in field of geometrical path tracking using wheeled robots. It discusses and differentiate the technique used here and previously used technique.

Chapter 3 discusses about Screw theory, Vector pursuit technique and its methods.

Chapter 4 is the discussion about system architecture of Pioneer-3DX and justification of selecting Pioneer-3DX robot, LABVIEW, CoppeliaSim (Virtual Robotics experimentation platform) and results.

While **chapter 5** will conclude this thesis.

CHAPTER 2

LITERATURE REVIEW

In the past few decades much research has been done in the area autonomous navigation. The main reason behind this interest is that advancement of technologies like sensors technology and computing technology. Sensors gives information about its environment and the current position of mobile robot in it, they are also reliable. Computer runs more complicated programs.

In mobile robotics autonomous navigation includes techniques like path tracking & obstacle avoidance. Path tracking is process used to determine steering angles or wheels velocities for the mobile robot while following series of way points from a desired path by using positional information from GPS or vehicle's odometery. A path can be defined in different ways for path tracking, Path is basically set of two or more waypoints from start to goal. In static environment, waypoints produce collision free path but in dynamic environment moving obstacle may appear between waypoints. In the dynamic case mobile robot needs to avoid the obstacle safely.

There several path tracking techniques for UGV's were developed in the past couple of decades. Geometric path tracking method [3] uses the lookahead point. Lookahead point is just like a point as driver look at fixed distance point while driving. There are three different geometric path tracking methods.

- **I. Follow the Carrot**
- **II. Pure Pursuit**
- **III. Vector Pursuit.**

Follow the Carrot obtain goal point and then achieve that point. Basically, a line is perpendicularly drawn from origin of vehicle coordinate system to path. Goal or Carrot point on path is point which is lookahead distance away from the intersection point of this line. In this technique orientation error is most important parameter, which is angle from vehicle's heading to line drawn from vehicle coordinate system to the goal point. Proportional control law minimizes this orientation error. This technique is simple for implementation but has some drawbacks as well. First is that vehicle has tendency to cut corners. Second is vehicle could oscillate about path. It is used for educational purposes to compare with other algorithms.

To show better results and less oscillation Pure pursuit algorithm [1], [4] is same in manner to the previous approach follow the carrot, in pure pursuit a circle is defined which passes through current vehicle position and goal point. Finally, a control algorithm is used to control wheel velocities or steering angle of the vehicle to achieve the circle. Mobile robot pushes the goal point forward while changing its curvature using repeatedly fitting circular arcs. To follow path quick, smooth and accurately adaptive pure pursuit controller [5] is used. Pure pursuit gives the target wheel velocities which is based on location of robot and path it wants to follow which make it robust for example if parameters are not tuned or robot initial point is wrong place but robot will be still able to correct itself in following the path and reach the goal.

Autonomous vehicle navigation requires path planning, orientation sensing, positional sensing obstacle avoidance and vehicle control. So, this process is broken into four steps these are as follow:

i) Perceive and model the environment
ii) Localize the vehicle
iii) Plan and decide desired motion of vehicle
iv) Execute the desired motion

Pure pursuit and follow the carrot does not use desired orientation at look ahead side while new geometric path tracking technique that is vector pursuit uses position as well as desired orientation at look ahead point. Screw theory is used by vector pursuit. which is developed in 1900 by Sir Robert Ball. In ground vehicle path tracking [6] mainly focuses on control of non-holonomic vehicles to track given waypoints and results of vector pursuit is also compared to the previous geometrical methods i.e. Pure Pursuit & Follow the Carrot. From these comparisons it is being concluded that vector pursuit is more robust and accurate technique.

Path tracking mainly focuses on deciding and execution of vehicle's desired motion. So, Jeffrey S. Wit [7] uses two FRMLCs (fuzzy reference model learning controllers) for execution the vehicle desired speed and turning rate. It is designed in such a way that it depends on vehicle characteristics like maximum speed and maximum turning rate. They have implemented their technique on Navigation test vehicle (NTV). From this experiment it is been concluded that in case of disturbances vector pursuit is more robust as compared to other geometric path tracking techniques.

Tae-Kyeong Yeu [8] proposed vector pursuit on Soft cohesive soil for tracked vehicle to drive. He proposed that due to larger contact area of track, tracked vehicle are generally better than wheeled vehicles. which provide good traction and floatation at different grounds. Basically, carried out the experiment on deep sea soft cohesive soil. Here vector pursuit is used for generation of vehicles motion to follow a specified path and proposed control method based on relation between track slip and traction force. Tracking controller should be robust with large tracking error, [9] determine look ahead distance in presence of constraints as well. These constraints are limits on angular velocity and reference velocity and it has been shown that while following a path this technique satisfies these given constraints. The obstacle avoidance for this technique is being proposed by Dong-Hyung Kim and Ji-Yeong Lee [10]. They use the potential field which attract the autonomous vehicle to the goal and keep it away from the obstacles or other robots. But rather than producing workspace in vector here a turning radius is being produced for satisfying nonholonomic constraints. One of a main issue in path planning and tracking is weight variation of AGVs when a load is transported slipping and skidding is induced. So, to deal these problems three control technique are being compared [11], here these three techniques are Vector Pursuit, fuzzy and flatness-based control at industrial applications for satisfaction of robustness.

CHAPTER 03:
METHODOLOGY

Process concerned with how to find out wheel velocities or steering configurations at every instant for robots is known as Path tracking. Here in this section, a new geometric path-tracking with nonholonomic constraints for navigation of automated ground vehicles (AGVs) is presented which is based on theory of screw. It is introduced in 1900 AD by Sir Robert S. Ball. Geometric path tracking technique is one of the most power full technique used in robotics path planning. Screw theory is basically used for the description of instantaneous motion of a moving rigid body relative to some coordinate system. Let us discuss a concise overview of screw theory in section 3.1 here below.

3.1 Screw theory:

Screw consists of a centerline and a pitch. Centerline of a screw is defined in world coordinate system while pitch is the distance covered by screw when it is rotated for one complete rotation. At any instant motion of a rigid body can be shown as if the body was accompanied to the screw, and rotates around that screw with angular velocity.

There are different ways for defining centerlines. One of them is Plücker line coordinates. Let assume two points shown by $\underline{r}_1$ and $\underline{r}_2$ in a given coordinate system. which defines a line as shown in Figure:3.1. Unit vector of this centerline is $\underline{S}$, which is in the direction of the line and moment vector $\underline{S}_0$ of the line around the origin. Unit vector and moment vector are given below in equation (1) and (2) respectively:

$$\underline{S} = \frac{\underline{r}_2 - \underline{r}_1}{|\underline{r}_2 - \underline{r}_1|} \tag{1}$$

and,

$$\underline{S}_0 = \underline{r} \times \underline{S} \tag{2}$$

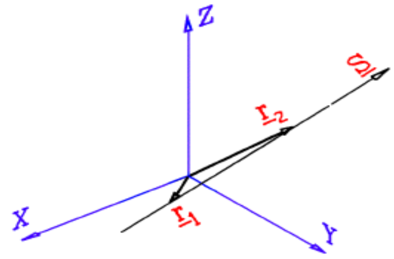


Figure 3.1: Defined Line

Here $\underline{r}$ means either $\underline{r_1}$ or $\underline{r_2}$ and the Plücker line coordinates of this line is ($\underline{S}$; $\underline{S_0}$) vector. Let us first define $\underline{S} = \begin{bmatrix} L \\ M \\ N \end{bmatrix}$ and $\underline{S_0} = \begin{bmatrix} P \\ Q \\ R \end{bmatrix}$ while $\underline{r_1} = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}$ coordinates and $\underline{r_2} = \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}$ coordinates in three dimension, from the above equations we find out that:

$$L = \frac{x_2 - x_1}{\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}} \tag{3}$$

$$M = \frac{y_2 - y_1}{\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}} \tag{4}$$

$$N = \frac{z_2 - z_1}{\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}} \tag{5}$$

Where as

$$P = y_1 N - z_1 M \tag{6}$$

$$Q = z_1 L - x_1 N \tag{7}$$

$$R = x_1 M - y_1 L \tag{8}$$

In Figure: 3.2, Rigid body instantaneous motion is shown having a centerline defined by ($\underline{S}$; $\underline{S_0}$) with a pitch 'h'. Which rotates with some angular velocity 'ω' about screw '$\underline{\$}$'. The velocity of this rigid body at any instant is equal to the sum of rotational velocity and translational velocity. Translational velocity is due to the pitch of the screw. Pitch is equals to distance covered by screw when screw is rotate about on 360 degree. The velocity of the rigid body can be calculated by:

$$\omega\underline{\$} = (\omega\underline{S}; \omega\underline{S_{0h}}) \tag{9}$$

where as

$$\underline{S_{0h}} = \underline{S_0} + h\underline{S} \tag{10}$$

By putting equation no (2) in (10) we get that

$$\underline{S}_{0h} = \underline{r} \times \underline{S} + h\underline{S} \tag{11}$$

While $\underline{r}$ is a vector defined from the origin to the screw's centerline. Point which is coincident with the origin of the rigid body its instantaneous velocity in coordinate system is given by '$\omega\underline{S}_{0h}$'.

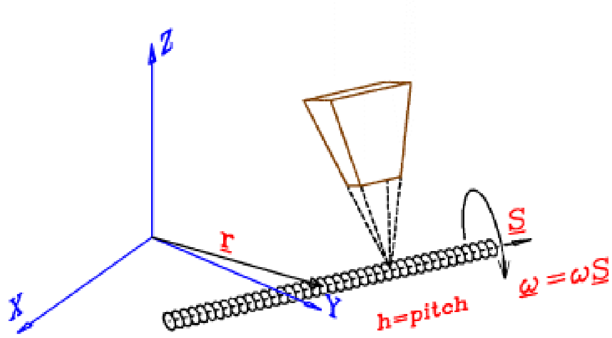


Figure: 3.2 Instantaneous motion of Screw

Here two screws are used to develop the path tracking algorithm, first is translation screw while second is rotation screw. The pure translation motion of a body in a screw will have an infinite pitch model at some velocity 'v' in the direction $\underline{S}$. So, as the pitch goes to infinity:

$$v\underline{\$} = (0; v\underline{S}) \tag{12}$$

On contrarily, the motion of a rigid body about screw having pitch is equal to zero model's pure rotational. By putting a pitch, 'h' equal to zero in equation (9) we get:

$$\omega\underline{\$} = (\omega\underline{S}; \omega\underline{S}_{0h}) \tag{13}$$

One important property of these rotation and translation screws is that proves that they are additive. because units of translational screw and rotational screw are the same.

3.2 Vector pursuit

In geometric path tracking technique Vector pursuit is new technique that is based on theory of screws. To define current goal point it also uses look-ahead distance just as other previously used geometric path tracking methods. Then for determination of desired motion geometry is being used. On contrarily, this technique is different from current geometric path-tracking methods like follow-the-carrot and pure pursuit. Because these methods do not consider the orientation at the goal/lookahead point while in vector pursuit look ahead orientation is also used. In previously used techniques only Proportional path tracking also uses the orientation at the look-ahead point but, this technique is geometrically meaningless as terms with different units are added. As it sums up both i.e. current positional error multiplied by some gain to the current orientational error multiplied by some gain. So in geometrical based methods only Vector pursuit uses both the position and orientation of current and lookahead point.

There are two different variation in motion translational and rotational. So, vector pursuit finds out two instantaneous screws i.e. translational screw ‘$\underline{\$}_t$’ and rotational screw ‘$\underline{\$}_r$’. These screws accounts for translation as well as rotation from the vehicle current position and orientation to the desired position and orientation at the look-ahead point respectively. Then by using additive property desired instantaneous screw ‘$\underline{\$}_d$’ is equals to the sum of ‘$\underline{\$}_r$’ ‘and $\underline{\$}_t$’. There are two different methods are used to calculate these screws, ‘$\underline{\$}_r$’ and ‘$\underline{\$}_t$’. In first method nonholonomic constraints of the vehicle are initially ignored and calculate ‘$\underline{\$}_t$’ and ‘$\underline{\$}_r$’ and finds out the instantaneous desired screw and then deals with the constraints of robot. While, in second method nonholonomic constraints are not ignored and calculate ‘$\underline{\$}_t$’ and ‘$\underline{\$}_r$’. The final step is to calculate a desired turning radius from ‘$\underline{\$}_d$’.

First, we must define three coordinate systems. First one is global or world coordinate system to be defined in such a way that its x-axis points to the north, while y-axis points in east and finally z-axis points down to form a right-hand coordinate system. First assumption is that the desired path is given in world coordinate system and its origin will be determined using conversion from a geodetic coordinate system to a UTM coordinate system. Figure 3.3: shows the world coordinate system as follow.

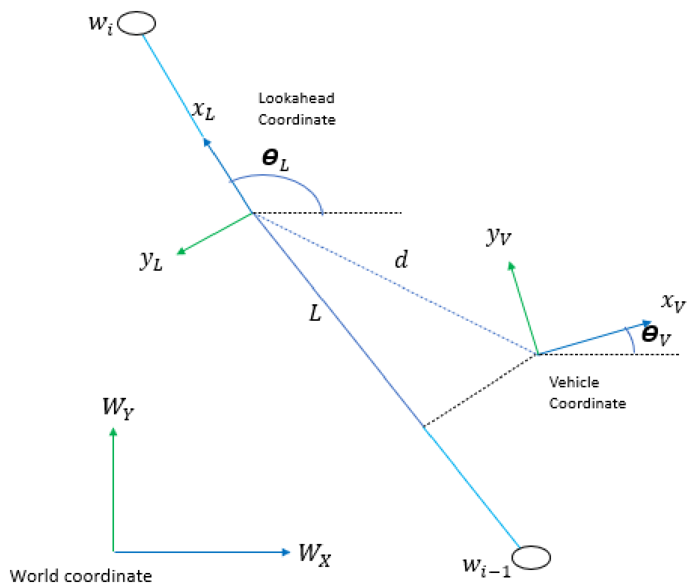


Figure 3.3: Defined Co-ordinate systems

After defining world coordinate system, lookahead coordinate system or a moving coordinate system is to be defined as shown in above Figure: 3.3 (lookahead coordinate system). A moving coordinate system is defined in such a way that its origin is on the planned path while lookahead point is in front of the orthogonal projection of the vehicle's position onto the planned path. A planned path consists of series of waypoints i.e. w_1, w_2, w_3,…, w_i. X-axis of lookahead coordinate system is oriented in direction of these planned path, i.e. direction from previous waypoint 'w_{i-1}' to the current waypoint 'w_i'. Z-axis is directed downward while Y-axis is defined to form a right-hand coordinate system. As the origin of lookahead coordinate system is situated at some lookahead point. So, we call this coordinate system as lookahead coordinate system. While 'L' distance is directly proportional to the distance 'd' between any two waypoints.

Now one more coordinate system needs to be defined which will show the vehicle position and orientation which will be called as the vehicle coordinate system. X-axis in vehicle coordinate system is in the forward direction of mobile robot or vehicle. Z-axis is directed downward while Y-axis is directed in such a way that forms right hand coordinate system. Origin of vehicle coordinate system is defined in such a way that for nonholonomic vehicles that decouples the controlling of angular and linear velocities. Vector pursuit is presented as follow using these three coordinate systems.

As here more than one coordinate system needs to be defined so, coordinate system is referred as a vector with a leading superscript which will indicate specific coordinate system. These superscripts are W for world coordinate system, L for look ahead coordinate system and while V for vehicle coordinate system. As we discussed earlier that two methods have been used to develop vector pursuit algorithm. So, we will be discussed both methods here one by one.

3.2.1 Method I:

In first method nonholonomic constraints for vehicle are initially ignored. Then by using equation (12), we get '$\underline{\$}_t$' as:

$$w_{\underline{\$}_t} = k_t\left(\ 0,0,0;\ \frac{w_{x_L}-w_{x_V}}{d}, \frac{w_{y_L}-w_{y_V}}{d}, 0\right) \tag{14}$$

Where 'd' denote distance from vehicle position to look-ahead point, 'k_t' is a weighting factor, Coordinates of lookahead point in world coordinate system is (w_{x_L}, w_{y_L}) and vehicles position in world coordinate is (w_{x_V}, w_{y_V}). Now to find rotational screw '$\underline{\$}_r$'equation (13) is used:

$$w_{\underline{\$}_r} = k_r(\ 0,0,1;\ w_{y_V}, -w_{x_V}, 0) \tag{15}$$

In the above equation (15) 'k_r' is weighting factor. Origin of vehicle coordinate system is chosen as rotational axis because then there will be no translation linked with '$\underline{\$}_r$'. To calculate the desired instantaneous screw '$\underline{\$}_d$' in world coordinate system we use additive property that is as below:

$$w_{\underline{\$}_d} = w_{\underline{\$}_t} + w_{\underline{\$}_r}$$

$$w_{\underline{\$}_d} = \left(\ 0,0,k_r; k_r(w_{y_V}) +\ k_t\left(\frac{w_{x_L}-w_{x_V}}{d}\right), -k_r(w_{x_V}) + k_t\left(\frac{w_{y_L}-w_{y_V}}{d}\right), 0\right) \tag{16}$$

'k_r' and 'k_t' denotes weighting factors. To control the desired screw these weighting factors are used i.e. how much it is being influenced by '$\underline{\$}_r$' and '$\underline{\$}_t$'. '$k_r$' and '$k_t$' are weighting factors for angular and linear velocities respectively. Let us assume that vehicle moves on screw defined by '$\underline{\$}_t$' with some linear velocity $k_t = v$, then time taken by vehicle to reach its lookahead point will be:

$$t_t = \frac{d}{V} \tag{17}$$

Similarly, if the vehicle rotates on the screw defined by '$\underline{\$}_r$' with an angular velocity $k_t = \omega$. Then time required to completes its rotation is:

$$t_r = \frac{\theta_L - \theta_V}{\omega} \tag{18}$$

Here θ_V is the angle from the X-axis of world coordinate system going clockwise till X-axis of vehicle coordinate system, θ_V is the angle from the x-axis of world coordinate system going clockwise to x-axis of lookahead coordinate system. The difference $\theta_L - \theta_V$ must lies in between (-π,π]. The relationship between t_r and $t_t$is to be defined as:

$$t_r = kt_t \tag{19}$$

The weighting factors k_t and k_r can be determined as follow:

$$k_t = v \tag{20}$$

and

$$k_r = \omega = \frac{v(\theta_L - \theta_V)}{kd} \tag{21}$$

where *k* in equation no. (19) is some positive constant. So, now *k* can be calculated in such a way that $\theta_L - \theta_V$ must be in the interval (-π,π]. Next step is to find out the centerline of screw in world coordinate system and then transform this position of centerline to vehicle coordinate system. As shown in the below equations i.e. (22) and (23):

$$w_{x_{\underline{\$}_d}} = w_{x_V} - \frac{k_t}{k_r}\left(\frac{w_{y_L} - w_{y_V}}{d}\right) = w_{x_V} - k\left(\frac{w_{y_L} - w_{y_V}}{\theta_L - \theta_V}\right) \tag{22}$$

&

$$w_{y_{\underline{\$}_d}} = w_{y_V} - \frac{k_t}{k_r}\left(\frac{w_{x_L} - w_{x_V}}{d}\right) = w_{y_V} - k\left(\frac{w_{x_L} - w_{x_V}}{\theta_L - \theta_V}\right) \tag{23}$$

Equations (23) and (24) will be valid only if $\theta_L - \theta_V \neq 0$ and $k_r \neq 0$. Now transform these to vehicle coordinate system we get:

$$V_{x_{\$_d}} = w_{x_V} \cos\theta_V + w_{y_V} \sin\theta_V - \left(w_{x_{\underline{\$}_d}} \cos\theta_V + w_{y_{\underline{\$}_d}} \sin\theta_V\right) \quad (24)$$

&

$$V_{y_{\$_d}} = -w_{x_V} \sin\theta_V + w_{y_V} \cos\theta_V - \left(-w_{x_{\underline{\$}_d}} \sin\theta_V + w_{y_{\underline{\$}_d}} \cos\theta_V\right) \quad (25)$$

If we have a situation where $k_r = 0$ then we will reduce these equation no. (16) to equation (14). Which shows that it connects the vehicle position and look ahead point by a centerline which is located at infinity. By finding out the desired centerline position we can now be able to determine the required motion for vehicle. For example, the figure 3.4: shows the desired motion of vehicle and it is clearly seen from the graph that its rotation is clock wise and translational movement is along negative y-axis which is impossible due to nonholonomic constraints. Therefore, it must be rotating counter clockwise which is opposite of the defined instantaneous rotational screw. When motion is orthogonal i.e. motion of vehicle parallel to x-axis in our case nonholonomic constraint exists. So, velocity at y-axis becomes equal to zero shown in the below equation (26).

$$w_{x^{\cdot}} \sin\theta_V - w_{y^{\cdot}} \cos\theta_V = 0 \quad (26)$$

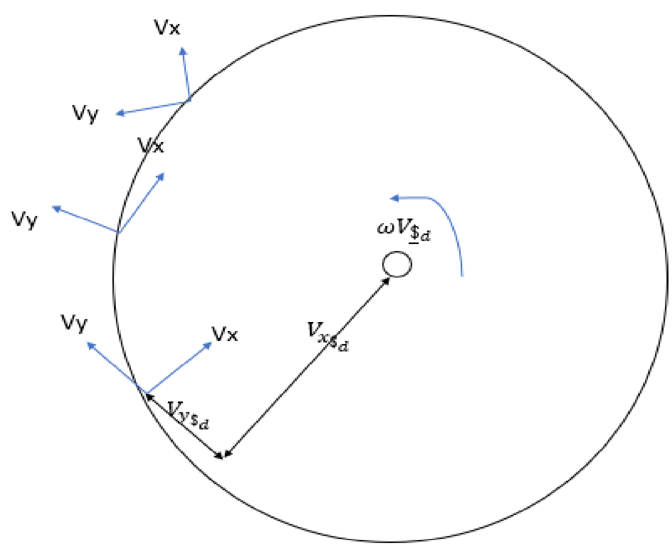


Figure 3.4: Vehicle's motion

To overcome this issue a new desired screw $\underline{\$}_d{}'$ needs to be defined. Which will be based upon previously desired screw $\underline{\$}_d$, It is obtained from new look ahead point. This new desired lookahead point which is distance L from vehicle position along an arc defined by $\underline{\$}_d$, new circle is obtain which is tangent to the vehicle direction while passing through vehicle position and new lookahead position as shown in figure 3.5:

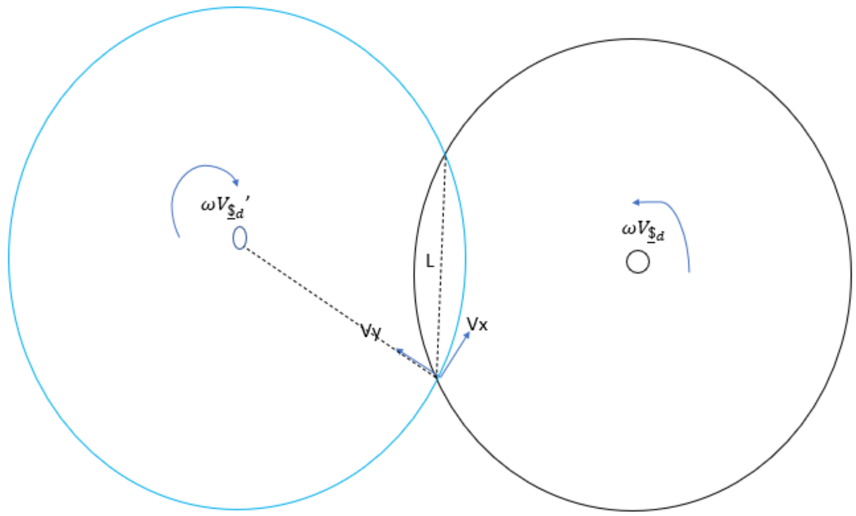


Figure 3.5: Desired screw and new desired screw

Now there is a restriction on distance L from vehicle to centerline which is that $\underline{\$}_d \geq \frac{1}{2}L$. The position new desired screw $\underline{\$}_d{}'$ in vehicle coordinate as follow:

When desired screw centerline direction is positive z direction

$$V_{x_{\$_{d}'}} = 0 \tag{27}$$

and

$$V_{y_{\$_{d}'}} = r_{min} \tag{28}$$

Similarly, when desired screw centerline direction is negative z direction.

$$V_{x_{\$_{d}'}} = 0 \tag{29}$$

and

$$V_{y_{\$_{d}'}} = -r_{min} \tag{30}$$

If the restriction on distance of desired screw $\underline{\$}_d \geq \frac{1}{2}L$ is satisfied then there exists two points on a circle. Their center is on centerline of $\underline{\$}_d$ and whose radius is distance to vehicle position that are 'L' distance away from the position of vehicle as shown in figure 3.6: Angle from X-axis of vehicle coordinate to centerline of $\underline{\$}_d$is calculated as:

$$\alpha = \text{atan2}\left(V_{y_{\$_d}}, V_{x_{\$_d}}\right) \tag{31}$$

There exists two lookahead points from vehicle position i.e. p_1 and p_2. These points form a symmetry in angles which is an angle from vehicle position line to p_1 and line from vehicle position to centerline of $\underline{\$}_d$ form same angle as formed from vehicle position line to p_2 and line from vehicle position to centerline of $\underline{\$}_d$. This angle β can be calculated as:

$$\beta = \text{acos}\frac{L}{2\sqrt{V_{x_{\$_d}}{}^2 + V_{y_{\$_d}}{}^2}} \tag{32}$$

Angle from X-axis to $p_1$and p_2 of vehicle coordinate system can be define as γ and can be calculated as:

$$\gamma = \alpha \pm \beta \tag{33}$$

From p_1 and p_2 only one point is being used as lookahead point. $\underline{\$}_d$ direction is being used for selection of this point. If $\underline{\$}_d$ centerline has positive z-direction of vehicle reference frame then:

$$\gamma = \alpha + \beta \tag{34}$$

Similarly, if $\underline{\$}_d$ centerline has negative z-direction of vehicle coordinate system then:

$$\gamma = \alpha - \beta \tag{35}$$

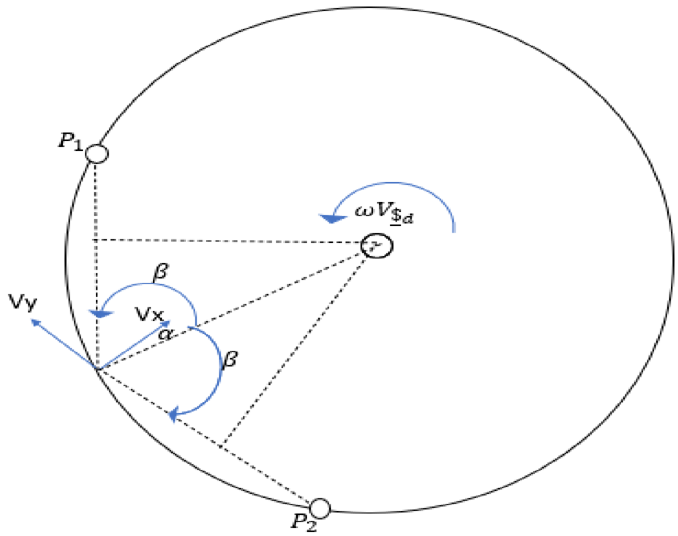


Figure 3.6: P_1 and P_2 as possible look ahead points

After γ is being calculated next step is to calculate new look ahead point position in vehicle coordinate system which is as follow:

$$V_{x_L} = L\,cos\,\gamma \tag{36}$$

&

$$V_{y_L} = \ L\,sin\,\gamma \tag{37}$$

Let us suppose that $p = p_1$or $p = p_2$ it is clearly seen in Figure: 3.7 that the new desired screw's $\underline{\$}_d{}'$ centerline location is on the vehicle's Y-axis at some distance $'R'$ from the X-axis. This R may be calculated as follow. First, we know that

$$a^2 + V_{x_p{}^2} = R^2 \tag{38}$$

and

$$a = R - V_{y_p} \tag{39}$$

$$V_{x_{p1}{}^2} = L^2 - V_{y_p}{}^2 \tag{40}$$

Simplify it to get R

$$R = \frac{L^2}{2V_{y_p}} \tag{41}$$

As shown from the figure 3.7: The new desired screw $\underline{\$}_d{}'$centerline is situated in vehicle coordinate system as:

$$V_{x_{\$_{d'}}} = 0 \tag{42}$$

$$V_{y_{\$_{d'}}} = \frac{L^2}{2V_{y_p}} \tag{43}$$

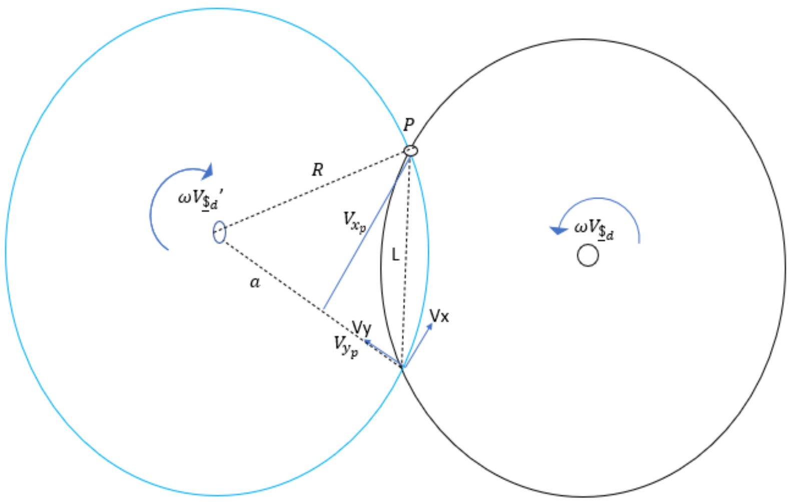


Figure 3.7: $V_{y_{\$_{d'}}}$'s centerline

From new look ahead point we can determine the direction of the new desired screw's $\underline{\$}_d{}'$ centerline. Detail about direction is being shown in the table 3-1 below:

$\boldsymbol{V_{x_p}}$	$\boldsymbol{V_{y_p}}$	**$\underline{\$}_d{}'$ direction of centerline direction along Z-axis**
Positive ($\boldsymbol{V_{x_p}}$>0)	Positive (V_{y_p}**>0)**	Positive
Positive ($\boldsymbol{V_{x_p}}$>0)	Negative **(**V_{y_p}**<0)**	Negative
Negative ($\boldsymbol{V_{x_p}}$<0)	Positive **(**V_{y_p}**>0)**	Negative
Negative ($\boldsymbol{V_{x_p}}$<0)	Negative **(**V_{y_p}**<0)**	Positive

Table 3-1: Direction for new desired screw centerline.

As shown from the table that when V_{x_p} is negative then vehicle velocity will be negative which means that the vehicle $2V_{y_p}$ direction will have to be changed from forward to reverse. So, this x-value of new lookahead point must be larger than zero, else the vehicle would be commanded as simply to turn around. In this specific situation again, these equations are being used i.e. (27) and (28) or (29) and (30) for the determination of location of screw centerline. Selection of constant k and the look-ahead distance L must be selected according to optimize or to tune performance of vehicle.

3.2.2 Method II:

In this approach we consider the nonholonomic constraints of vehicle for determination of $\underline{\$}_r$ and $\underline{\$}_t$. For the satisfaction of constraints, we consider that the instantaneous screws centerlines must be on y-axis of the vehicle while distance from the x-axis is greater than or equal to minimum turning radius of vehicle but, this requirement is ignored initially. Because in some vehicles like differentially driven vehicles have no minimum turning radius. So, we have only one constraint in the current scenario which is that the instantaneous screws centerlines must be on y-axis of the vehicle.

Translational screw $\underline{\$}_t$ will be selected as center of circle that will pass through origin of vehicle coordinate system and while look-ahead coordinate system and will be tangent to current orientation of vehicle. So, $\underline{\$}_t$ is to be defined as:

$$w_{\underline{\$}_t} = k_t\left(\ 0, 0, 0; w_{y_V} + \frac{d^2}{2V_{y_L}}\cos\theta_v\,, -w_{x_V} + \frac{d^2}{2V_{y_L}}\sin\theta_v\,, 0\right) \qquad (44)$$

Here d denote distance between origin of vehicle coordinate system and origin of look-ahead coordinate system, while (V_{x_L}, V_{y_L}) are coordinates of origin of look-ahead coordinate system in the vehicle coordinate system and similarly (W_{x_V}, W_{y_V}) are the coordinates of vehicle's position in world coordinate system. The term θ_v denotes the angle that is from X-axis of world coordinate till X-axis of vehicle coordinate. While k_t denotes weighting factor for translational screw. Above equation (44) is valid only if and only if the term $V_{y_L} \neq 0$, else $\underline{\$}_t$ will be determine from this equation:

$$w_{\underline{\$}_t} = k_t\left(\;0, 0, 0; \frac{w_{x_L} - w_{x_V}}{d}, \frac{w_{y_L} - w_{y_V}}{d}, 0\right) \qquad (45)$$

While instantaneous rotational screw is to be determine from this equation (46):

$$w_{\underline{\$}_r} = k_r\left(0, 0, 1; w_{y_V}, -w_{x_V}, 0\right) \qquad (46)$$

As we know that desired screw i.e. $w_{\underline{\$}_d} = w_{\underline{\$}_t} + w_{\underline{\$}_r}$ so, by putting above two equations i.e. (45) and (46) in this we get that:

$$w_{\underline{\$}_d} = \left(0,0, k_r;\; k_r w_{y_V} + k_t(w_{y_V} + \frac{d^2}{2V_{y_L}}\cos\theta_v), -k_r w_{x_V} + k_t(-w_{x_V} + \frac{d^2}{2V_{y_L}}\sin\theta_v),0\right) \quad (47)$$

And if $V_{y_L} == 0$, then desired instantaneous screw will become:

$$w_{\underline{\$}_d} = \left(\;0, 0, k_r; k_r\left(w_{y_V}\right) + \; k_t\left(\frac{w_{x_L} - w_{x_V}}{d}\right), -k_r\left(w_{x_V}\right) + k_t\left(\frac{w_{y_L} - w_{y_V}}{d}\right), 0\right) \qquad (48)$$

As we know that from previous approach as well that k_t and k_r are used here to control the influence on $\underline{\$}_d$ by $\underline{\$}_t$ and $\underline{\$}_r$ respectively. Now here we assume that if $V_{y_L} \neq 0$, Rotation $\emptyset$ will be determined from:

$$\emptyset = \text{atan}\,2\left(\left(2\,{V_{y_L}}^2 - d^2\right), \left(2V_{x_L}V_{y_L}\right)\right) - \text{atan2}\left(\left(\frac{d^2}{2V_{y_L}}\right), 0\right) \qquad (49)$$

From the above equation it is clearly seen that its last part i.e $\text{atan2}\left(\left(\frac{d^2}{2V_{y_L}}\right), 0\right)$ will always be $\pm\pi/2$ radian depends upon sign of V_{y_L}. In which $\emptyset$ is in the interval of $(0,2\pi]$ radians. Therefore, assuming $k_t = \omega_t$, this time some angular velocity. The required time to translate t_t from vehicle to lookahead position will find from this equation:

$$t_t = \frac{\emptyset}{\omega_t} \qquad (50)$$

Similarly, we can find out the rotational time t_r required that will be from vehicle's current orientation to required orientation at the look-ahead point:

$$t_r = \frac{(\theta_L - \theta_V) - \emptyset}{\omega_r} \tag{51}$$

Here we have taken two assumptions first is that $k_r = \omega_r$ and second assumption is that $t_r = kt_t$, where k is some positive constant. Weighting factors will be determined from:

$$k_t = \omega_t \tag{52}$$

and,

$$k_r = \frac{(\theta_L - \theta_V) - \emptyset}{t_r} = \frac{(\theta_L - \theta_V) - \emptyset}{kt_t} = \frac{\omega_t((\theta_L - \theta_V) - \emptyset)}{k\emptyset} \tag{53}$$

Now to find the centerline position in world coordinate system we have use equation (47) and find $w_{x_{\underline{\$}d}}$ and $w_{y_{\underline{\$}d}}$ as follow:

$$w_{x_{\underline{\$}d}} = w_{x_V} - \frac{k_t}{k_t + k_r}\left(\frac{d^2}{2V_{y_L}} \cos\theta_v\right) = w_{x_V} - \frac{k\emptyset}{(k-1)\emptyset + (\theta_L - \theta_V)}\left(\frac{d^2}{2V_{y_L}} \cos\theta_v\right) \tag{54}$$

and,

$$w_{y_{\underline{\$}d}} = w_{y_V} + \frac{k_t}{k_t + k_r}\left(\frac{d^2}{2V_{y_L}} \sin\theta_v\right) = w_{y_V} + \frac{k\emptyset}{(k-1)\emptyset + (\theta_L - \theta_V)}\left(\frac{d^2}{2V_{y_L}} \sin\theta_v\right) \tag{55}$$

Above derivations are for the assumption of $V_{y_L} \neq 0$, but if we $V_{y_L} = 0$ then weighting factor k_t becomes the linear velocity. Which will result in that $k_t = v$, so time to translate t_t can be determined as follow:

$$t_t = \frac{d}{v} \tag{56}$$

As $\emptyset = 0$ then $k_r = \omega$ and while time required for the rotation of vehicle's orientation to desired look ahead orientation will be determined as follow:

$$t_r = \frac{\theta_L - \theta_V}{\omega} \tag{57}$$

Weighting factors i.e. k_t and k_r will be defined as:

$$k_t = v \tag{58}$$

and

$$k_r = \omega \tag{59}$$

While ω becomes this

$$\omega = \frac{\theta_L - \theta_V}{t_r} = \frac{\theta_L - \theta_V}{k t_t} = \frac{v(\theta_L - \theta_V)}{kd} \tag{60}$$

Desired screw centerline position i.e. $(w_{x_{\underline{\$}_d}}, w_{y_{\underline{\$}_d}})$ in world coordinate will be calculated as follow:

$$w_{x_{\underline{\$}_d}} = w_{x_V} - \frac{k_t}{k_r}\left(\frac{w_{y_L} - w_{y_V}}{d}\right) = w_{x_V} - k\left(\frac{w_{y_L} - w_{y_V}}{\theta_L - \theta_V}\right) \tag{61}$$

and,

$$w_{y_{\underline{\$}_d}} = w_{y_V} - \frac{k_t}{k_r}\left(\frac{w_{x_L} - w_{x_V}}{d}\right) = w_{y_V} - k\left(\frac{w_{x_L} - w_{x_V}}{\theta_L - \theta_V}\right) \tag{62}$$

Final step is to use these two equations i.e. (24) and (25) and find out the desired screw's centerline can in the vehicle coordinate system for the determination of the vehicle's desired motion. Previously, we have considered that vehicle's nonholonomic constraints while calculating $\underline{\$}_r$ and $\underline{\$}_t$ but here one thing is being ignore that is the minimum turning radius. In this approach always $V_{x_{\$_d}} == 0$, which satisfy nonholonomic constraints of vehicle. The magnitude of $V_{y_{\$_d}} \geq r_{min}$, where r_{min} shows the minimum turning radius. Else-if $V_{y_{\$_d}}$ is less than the r_{min}, then equations (27) and (28) or (29) and (30) are used again for the determination desired screw centerline position in vehicle coordinate system.

$\boldsymbol{V_{x_p}}$	$\boldsymbol{V_{y_p}}$	$\underline{\$}_d$ **direction of centerline along Z-axis**
Positive	Positive	Positive
Positive	Negative	Negative
Negative	Positive	Negative
Negative	Negative	Positive

Table 3-2: $\underline{\$}_d$ centerline direction along z-axis

For the direction of centerline of desired screw above table 3-2. Similar to the first approach, when the x-value in look-ahead point is negative, Vehicle's direction will have to be changed from forward to reverse. To overcome this issue, x-value in lookahead point will be more than zero, else the vehicle needs to be turn around. To calculate the position of screw centerline we use following equations i.e. (27) and (28) or (29) and (30):

Similar to first approach the parameters k and L are free choices here too. While the programmer needs to selects parameter in order to optimize or tune performance of the vehicle.

3.3 Wheels Velocities:

The final step is to calculate the desired velocities for wheels so that vehicle may achieve the required waypoints accurately. In application problem vehicle should be able to making the desired turning radius. If lateral direction slippage is ignored then we can fine the desired velocities for wheels in terms of the desired turning radius and its velocity. Here we deal with a vehicle which has two differential wheels. So, v_L for left and v_R right wheel desired velocities in terms of R are as follow respectively:

$$v_L = v_{ref}\left(1 \pm \frac{w}{2R}\right) \tag{63}$$

$$v_R = v_{ref}\left(1 \mp \frac{w}{2R}\right) \tag{64}$$

Here v_{ref} is the linear velocity of vehicle, R is the radius and w is the width of vehicle. While v_L and v_R are linear velocities. By using these linear velocities, we control the actual wheels angular velocities in rad/s.

Here in this dissertation, we use Pioneer-3DX as a mobile robot. It is a differential drive robot with two motor control two front wheels and third wheel as a rear castor wheel.

Wheels will cover a distance equal of $r\theta$. For example, if we assume $v = 1m/s$, it means that $v_{wheel} = 10.5263$ rad/s. the radius of wheels is $r = 0.095m$.

CHAPTER 04:
SIMULATION AND RESULTS

The new geometrics path tracking algorithm vector pursuit is developed here. This technique uses a look-ahead point L ahead of the orthogonal projection onto the path from the position of vehicle. Here selection of L is very much important as because increasing of L leads to dampen the system and also tends to cut corners of a path. So, smaller look ahead distance is desirable in order for accurately navigation of path.

Generally, Vehicle speed is considered while choosing 'L'. Because when vehicle speed increases L needs to be increased also. So, to track a given path vector pursuit would allow smaller L values. In this chapter, Simulations is done to determine vector pursuit's ability to track waypoints accurately in two different software's. These software's are LabVIEW and CoppeliaSim. CoppeliaSim is a virtual robotics experimentation platform.

4.1 Pioneer-3DX

Pioneer-3DX is a light weight mobile robot having two wheeled two motors differential drive with third wheel as a castor wheel. Pioneer robots are the world most intelligent mobile robots for research and education purposes. Their reliability and versatility have made them preferred mobile robots. So, Pioneer-3DX is used as mobile robot here to track the waypoint. Fig.4.1 shows the mobile robot here below.

Pioneer-3DX is light weight of 9kg, it's operating payload is 17 kg. It's minimum turning radius is 0 cm. Pioneer-3DX maximum forward and backward speed is 1.2 m/s, while maximum rotational speed is 300^0/sec. Pioneer-3DX is used in various configurations like mapping & vision, Gripping & Manipulation and audio speech purposes etc.

Fig 4.1: Pioneer-3DX

4.2 LabVIEW:

Laboratory Virtual Instrument Engineering Workbench is the abbreviation of LabVIEW is creation of Natural Instrument (NI). LabVIEW is graphical based programming language. The graphical language is called G, which was released for Apple. LabVIEW is mostly used for instrument control, data acquisition and industrial automation. Extension of code files for LabVIEW are ".vi", its abbreviation is virtual instrument. A VI file has three main parts namely front panel, block diagram and connector pane.

Front panel interact with users consists of controls, indicators and graphs. Block diagram contains the main script while in connector pane method of connection with VI's are being shown. Figure: 4.2 (A), (B) and (C) shows front panel, block diagram and connector pane respectively.

4.2.1 Advantages of LabVIEW:

Automate and validate signals requires less effort as compared to traditional programming environment. Main advantage of LabVIEW is its extensive support in accessing instrumentation hardware. Drivers for various types of buses and instruments are either available or included. In LabVIEW dataflow nature is parallel processing. For example, if you want to do multiple tasks like acquiring fast pressure data while monitoring for safety conditions LabVIEW is able to do these tasks easily by dropping parallel multiple loops on block diagram.

NI (National Instrument) ecosystem is very much flexible while interfacing with actuators and sensors. In same environment real time applications and desktop applications can be developed. Series of plug-ins and add-ons helps to get job done faster.

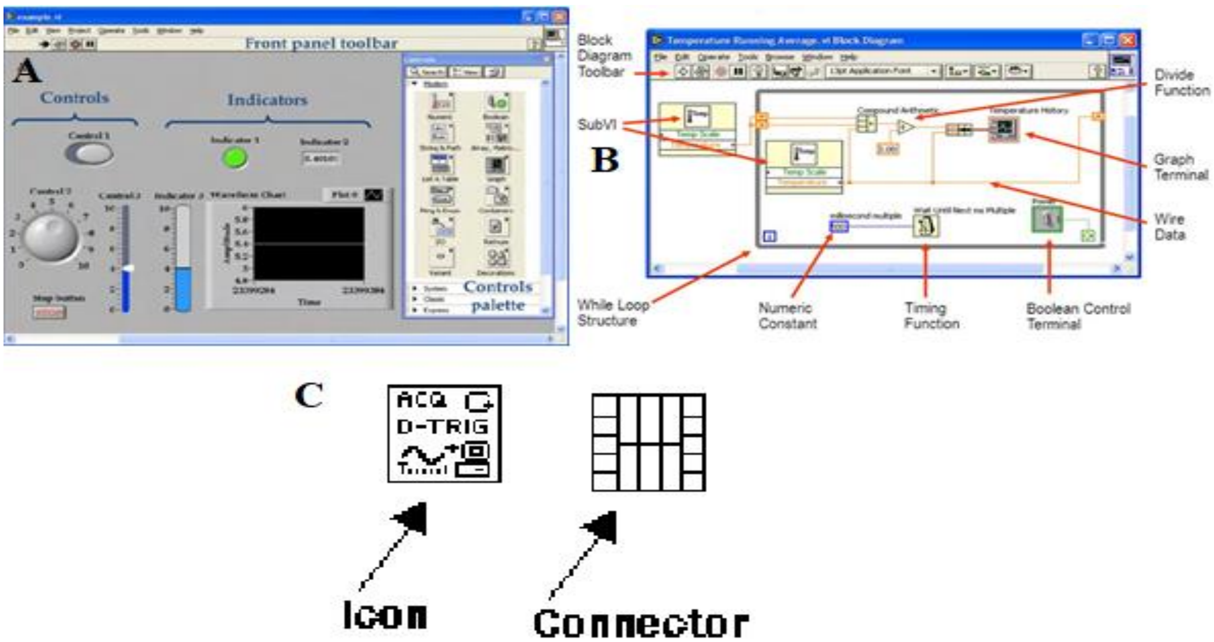


Figure:4.2 (A) front panel, (B) Block diagram and (C) Connectors pane

4.2.2 Results

Scenario 1:

Way points are chosen as following [(0, 0,0^0),(2, 1,30^0),(4, 3,45^0),(5, 6, 60^0),(2, 8, 150^0)] and initially Robot position and orientation is (0, 0,0^0) while the reference velocity is chosen as $V_{ref} = 1\ m/s$.

LabVIEW is used as simulator. Figure: 4.3 shows the front panel of LabVIEW during simulation while figure: 4.4 (a) shows the variation of x-position with respect to time, it can be clearly seen from the figure: 4.4(a) that robot tries to achieve the x-position of the look ahead waypoints accurately. Similarly, in figure: 4.4(b): shows the variation of y-position with respect to time, it can be clearly seen that robot tries to achieve the y-position of the look ahead waypoints accurately.

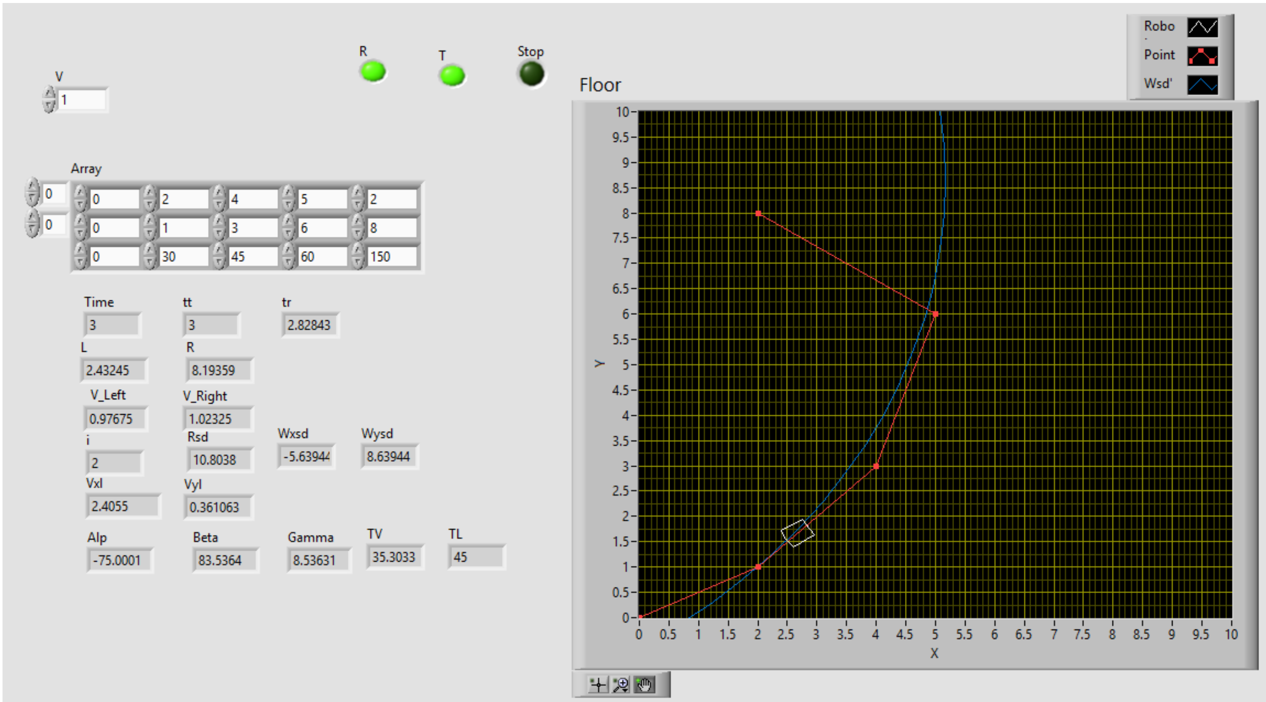


Figure: 4.3 Front Panel of LabVIEW

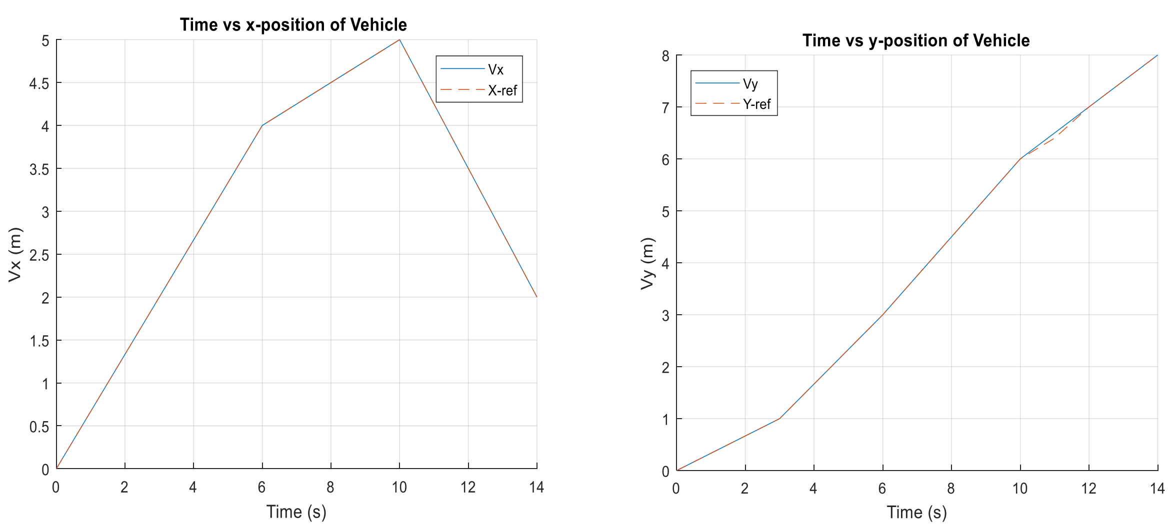


Figure: 4.4 (a) X-variation with respect to time **(b)** Y-variation with respect to time

In Figure:4.4 shows the vehicle reference velocity vs vehicles' velocity here below.

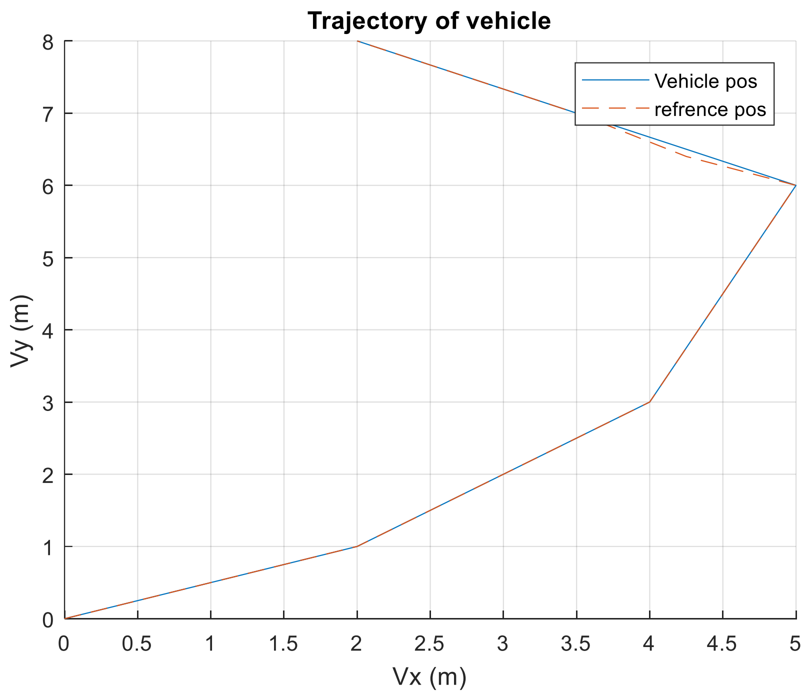
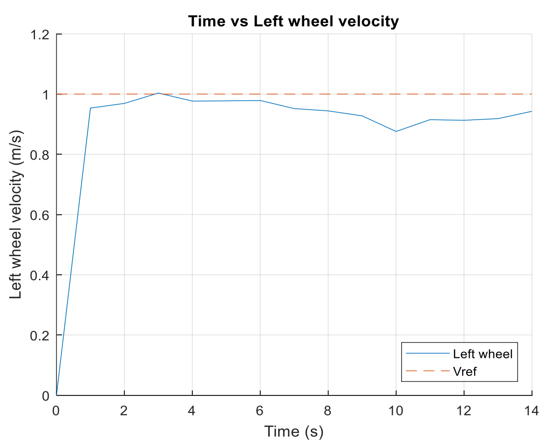


Figure: 4.5 Vehicle position vs reference position



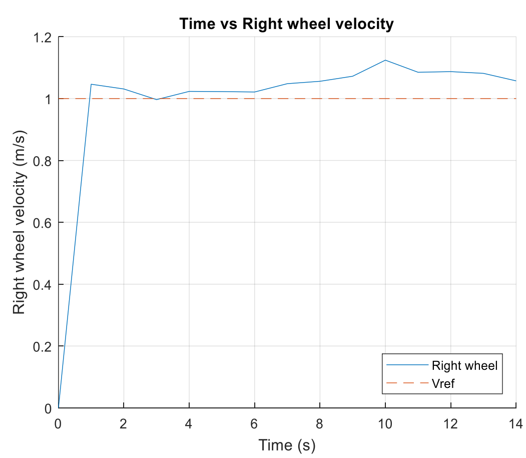


Figure: 4.6 (a) Left wheel velocity **(b)** Right wheel velocity

Scenario 2:

Way points are chosen as following [(5, 1,90^0), (1, 5,135^0), (5, 9,45^0), (9, 5, 315^0), (5, 1, 225^0)] and initially Robot position and orientation is (5, 1,90^0) while the reference velocity is chosen as $V_{ref} = 1\, m/s$.

In figure: 4.7 (a) shows the variation of x-position with respect to time, it can be clearly seen from the figure: 4.7(a) that robot tries to achieve the x-position of the look ahead waypoints accurately. Similarly, in figure: 4.7(b): shows the variation of y-position with respect to time, it can be clearly seen that robot tries to achieve the y-position of the look ahead waypoints accurately.

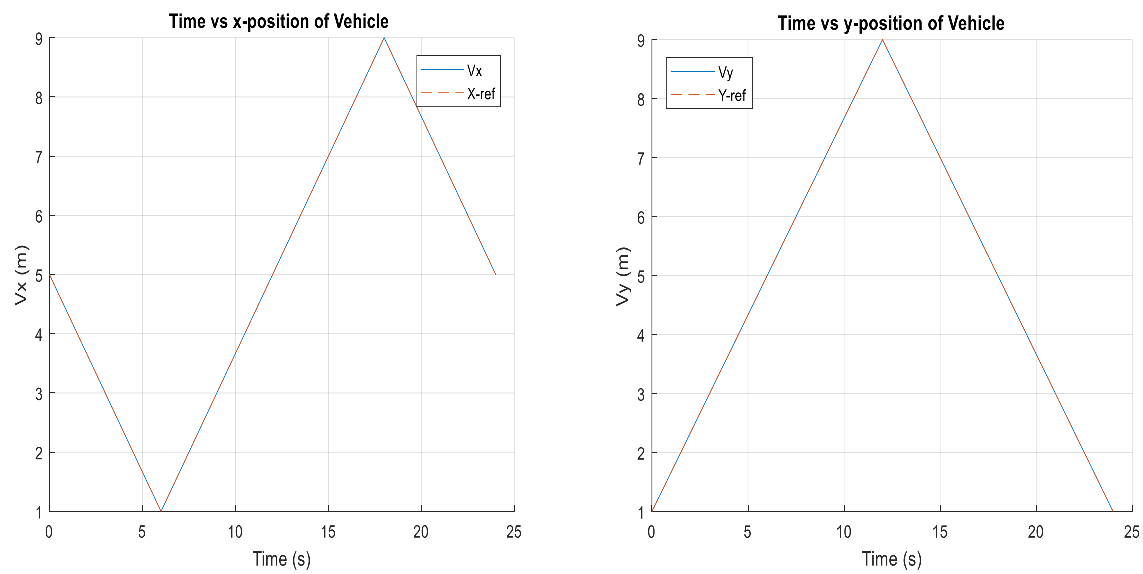


Figure: 4.7 (a) X-Variation with respect to Time **(b)** Y-variation with respect to time

In Figure:4.8 shows the Vehicles' position vs reference position here below.

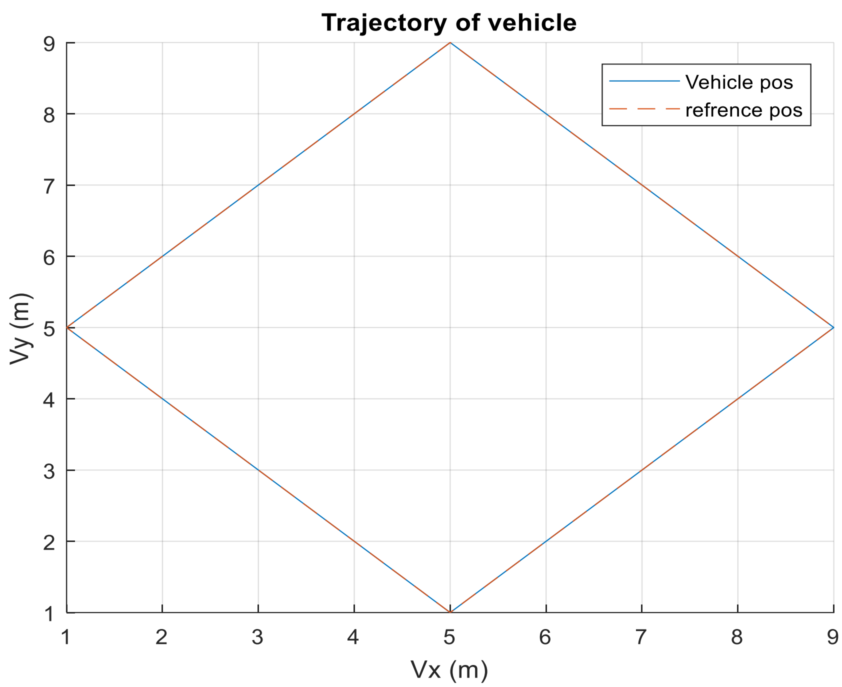
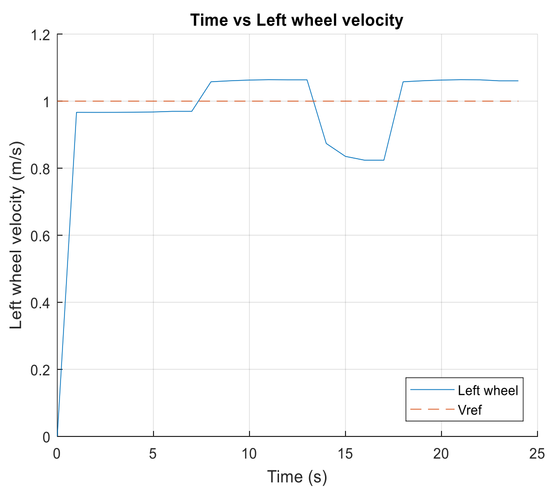


Figure: 4.8 Vehicle position vs reference position

Finally, Figure:4.9 (a) and 4.9 (b) show the variation wheels velocities with respect to time of left and right wheel respectively. It can be clearly seen from figure 4.9 that when robot tries to turn right so right wheel velocity is a little bit lesser then the right wheel velocity and vice versa. While the dotted line shows the reference velocity.



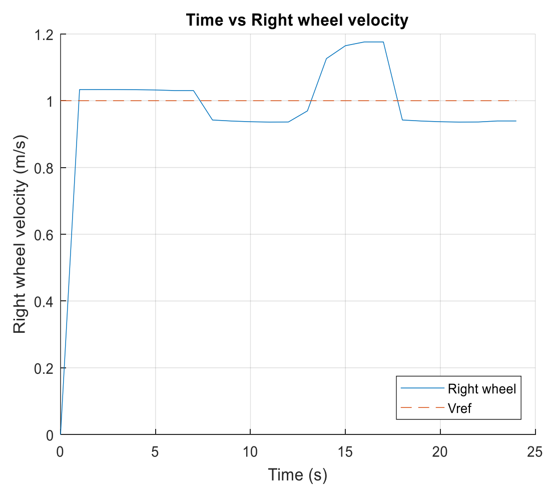


Figure: 4.9 (a) Left wheel velocity **(b)** Right wheel velocity

4.3 CoppeliaSim:

CoppeliaSim is the successor of V-REP (Virtual Robot Experimentation Platform). CoppeliaSim is a 3D robot simulator which runs faster with more feature then V-REP. CoppeliaSim is based on distributed control architecture where each object is controlled via script, ROS or BlueZero node, a plugin, remote API client etc. CoppeliaSim is ideal and very versatile for multiple robot applications.

In application point of view, CoppeliaSim is mainly used in fast prototyping, factory automation, verification, robotics related scenarios, remoted monitoring etc. Figure: 4.10 show the front view CoppeliaSim scenario. In which mobile robot Pioneer-3DX is on 10m by 10m of floor. Where the blue lines show the path to be followed and its edges shows the waypoints. One square box is of 0.5m by 0.5m.

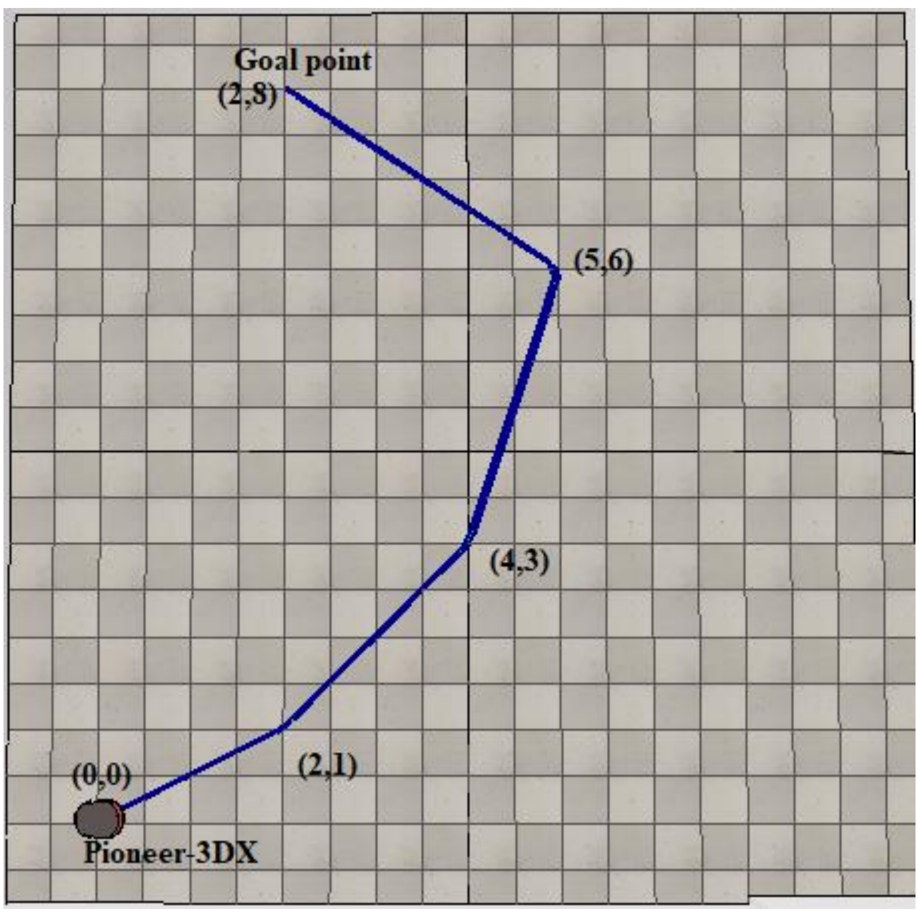


Figure:4.10 Pioneer-3DX in 10 x 10 floor

4.3.1 Results

Scenario 1:

Different scenarios have been tested. For example, in first scenario way points are chosen as following [(0, 0,0^0),(2, 1,30^0),(4, 3,45^0),(5, 6, 60^0),(2, 8, 150^0)] and initially Robot position and orientation is (0, 0,0^0) while the reference velocity is chosen as $V_{ref} = 1\ m/s$. Mobile robot pioneer 3DX is used as test platform. In fig.4.11: part(a) shows the variation of x-position with respect to time, it can be clearly seen from the part(a) that robot tries to achieve the x-position of the look ahead waypoints accurately. Similarly, in part(b) fig.4.2: shows the variation of y-position with respect to time, it can be clearly seen that robot tries to achieve the y-position of the look ahead waypoints accurately.

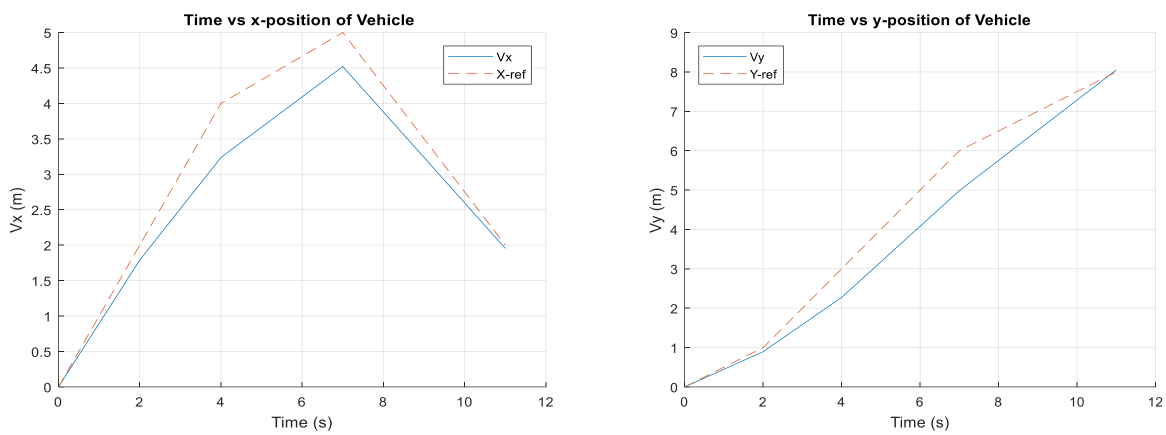


Figure: 4.11 (a) X-variation with time (b) Y-variation with time

In figure:4.12 shows the combined effect of vehicle's x-variation and in y-variation here below.

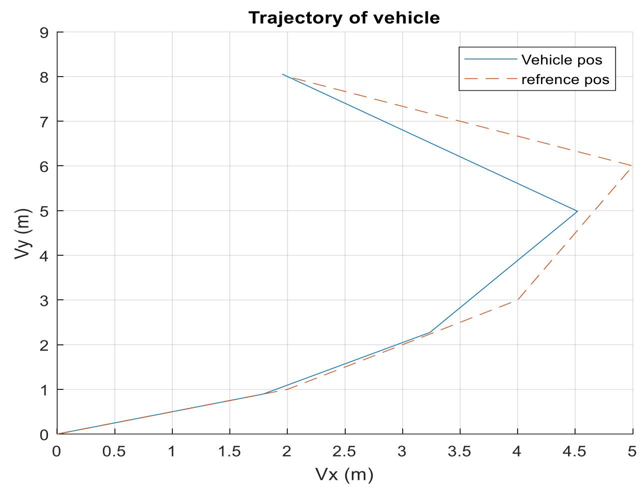


Figure:4.12 Vehicle's position vs reference position

Similarly, figure:4.13 shows the variation in orientation with respect to time here robot tries to achieve orientation of the look ahead orientation.

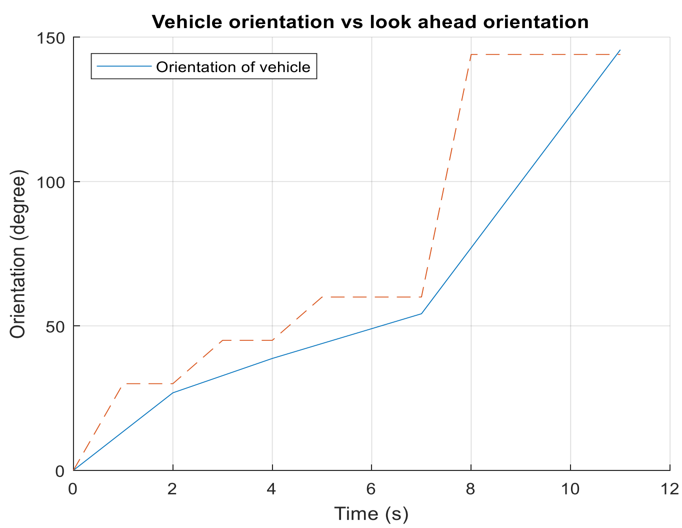


Figure:4.13 Vehicle Orientation

Finally, Figure:4.14 (a) and (b) show the variation wheels velocities with respect to time of left and right wheel respectively. It can be clearly seen from figure 4.14 that robot tries to turn left so left wheel velocity is a little bit lesser then the right wheel velocity. While the dotted line shows the reference velocity.

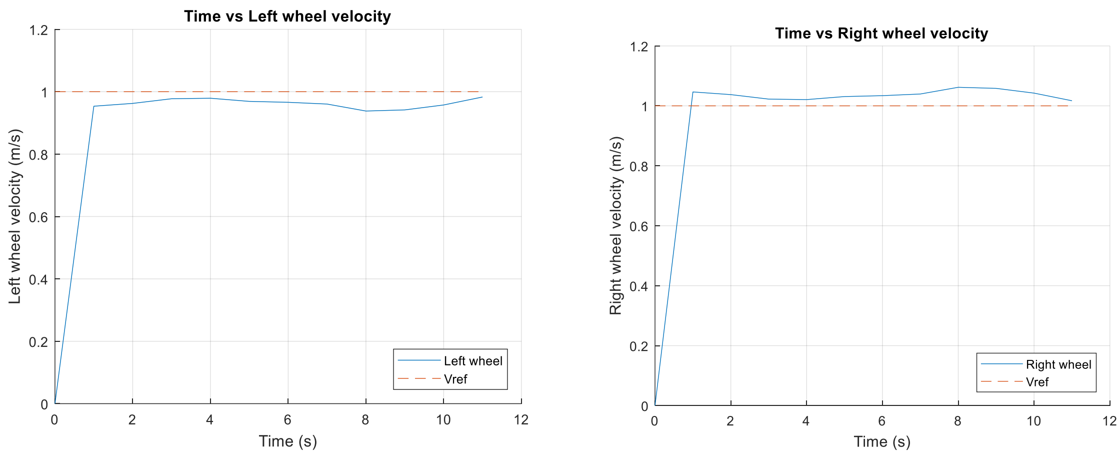


Figure: 4.14 (a) Left wheel velocity (b) Right wheel velocity

From these results it is clearly seen that during simulation of LabVIEW desired vs actual result are almost equals while in experiment from CoppeliaSim and its results it is observed that error in actual and desired motion exist. Here, Vector Pursuit is implemented for forward motion. This technique is also able for backward motion too.

CHAPTER 05:
CONCLUSION AND FUTURE WORK

Ground robots are classified by locomotion into three major categories i.e. wheeled, tracked and Legged robots. Typically, wheeled robots are energy efficient and simple to control. Here in this research main task was path following. So, vector pursuit is implemented and modified techniques by tuning its parameters that are ‘k’ and look ahead distance. This technique determines desired turning radius from vehicle current position and orientation to position and orientation at look ahead point. By using turning radius wheel velocities of vehicle were determined. For simulation perspective we have used two software’s LabVIEW and CoppeliaSim. The simulation results show that the vehicle satisfy its constraint and follow the waypoints accurately. Possible future research area is that it may be tested on high speed mobile robots

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