

Optimizing the Transportation cost Using Mixed integer linear
programming under constraints for fizzy drink industry



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SEPTEMBER, 2017**

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MS Design and Manufacturing Engineering

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Declaration

It is certify that this research thesis titled “*Optimizing the Transportation cost Using Mixed integer linear programing under constraints for fizzy drink industry*” is my own work. The work has not been presented elsewhere for assessment. The material that has been used from other sources it has been properly referred.

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and support made it all possible.*

Abstract

In the recent years, companies are being faced with an environment full of competition and pressure to minimize their sourcing and procurement cost, production cost, inventory management cost and logistics system cost meanwhile achieving the commendable levels of customer satisfaction. Warehousing and Distribution System is one of the most important echelons in the supply chain of any company. Therefore in the process of optimizing supply chains, Distribution echelon should be given a lot of attention as it is one of the highest money consuming echelons in the supply chain of any company.

Supply chain network (SCN) design having multiple echelons and multiple products for a fizzy drink industry is considered in this research thesis. The SCN consists of plant/plants where the products are manufactured as well as a number of first marketing warehouses (FMW) at different locations where the final product is stored and then transported to distribution centers (DC) all over the region from where the product is made available to a number of customer zones. Based on different decisions to be determined like number of warehouses, location and capacity of warehouses, the number of distribution centers, location and capacity of the distribution centers and the method of transportation of products; the entire system is modeled as a mixed integer linear programming optimization problem.

The objective is to minimize the total annual cost of transporting products from one echelon to another and also to develop a graphical user interface so anyone can solve even the most complex and large-scale problems by just plugging in data values and obtaining the results. A well designed Distribution Network can result in a company winning a competitive advantage over its competitors.

Key Words: *Supply chain design, MILP, Optimization, Cost minimization, Transportation problem, GUI, Multi echelon network, logistics*

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CHAPTER 1: INTRODUCTION

In today's world supply chain network design is of high importance and it influences the total cost of any given product throughout its lifecycle in a major way and also plays a major role in customer satisfaction. Supply chain network is comprised of many interrelated activities such as manufacturing of the product in the production plant, warehousing the products and customer service [1, 2]. The supply chain starts with the suppliers providing raw material which is converted into final products after a number of operations in the manufacturing plant and then stored in a number of first marketing warehouses at different locations which is then supplied to different distribution centers according to the demand and from there the products reach the retail shops and are finally available for the customers to purchase. All the activities mentioned above are coordinated and integrated into a smooth and efficient process by means of the supply chain network.

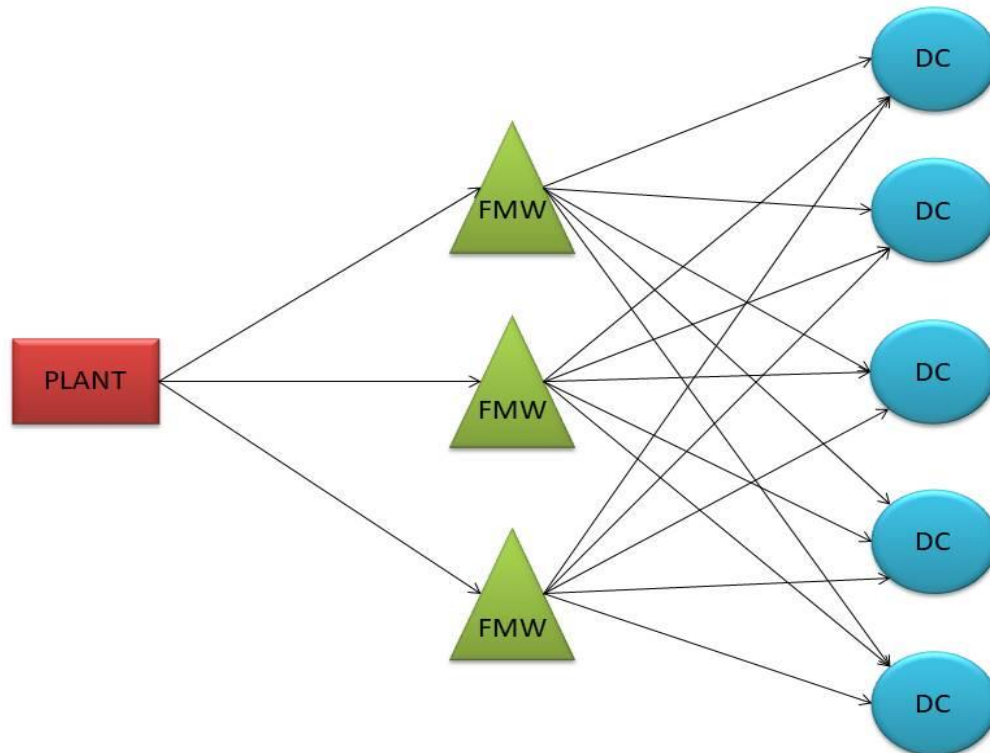


Figure 4.1: Supply Chain Distribution Network

A major advantage of an efficient supply chain network is to provide a company with a competitive edge over the other companies. In any supply chain network one of the prior objectives is to minimize the costs associated with the transportation of products between different echelons along with customer satisfaction and providing global competition. Supply chain management is considered and practiced as a major and preferred solution for minimization of transportation cost and increase in the company's profit [3, 4].

In recent research and studies attention has been focused upon multi-echelon, multi-products, multi-facilities and multi-period systems and networks. Many different programs and algorithms have been developed for the optimization and solution of SCN related problems by first modeling the problem mathematically and then solving it using linear programming, mixed integer linear programming, nonlinear programming, integer programming and other different techniques and methods. But before going into the details of different optimization algorithms and SCN related problems it is necessary to have the basic knowledge and know how related to the transportation system, supply system and distribution system.

1.1. Transportation System

Transportation system also known as logistics system refers to the effective transportation of products from suppliers to the manufacturing plants to the warehouses to the distribution centers to the customers in the most efficient way and bearing minimum cost with excellent customer services. So basically transportation system can be shown as the sum of sourcing system, material handling system and distribution system.

Transportation system = Sourcing System + Material Handling System + Distribution System

The figure given below explains the flow of materials and information between different echelons of a company in forward as well as in reverse directions of supply chain:

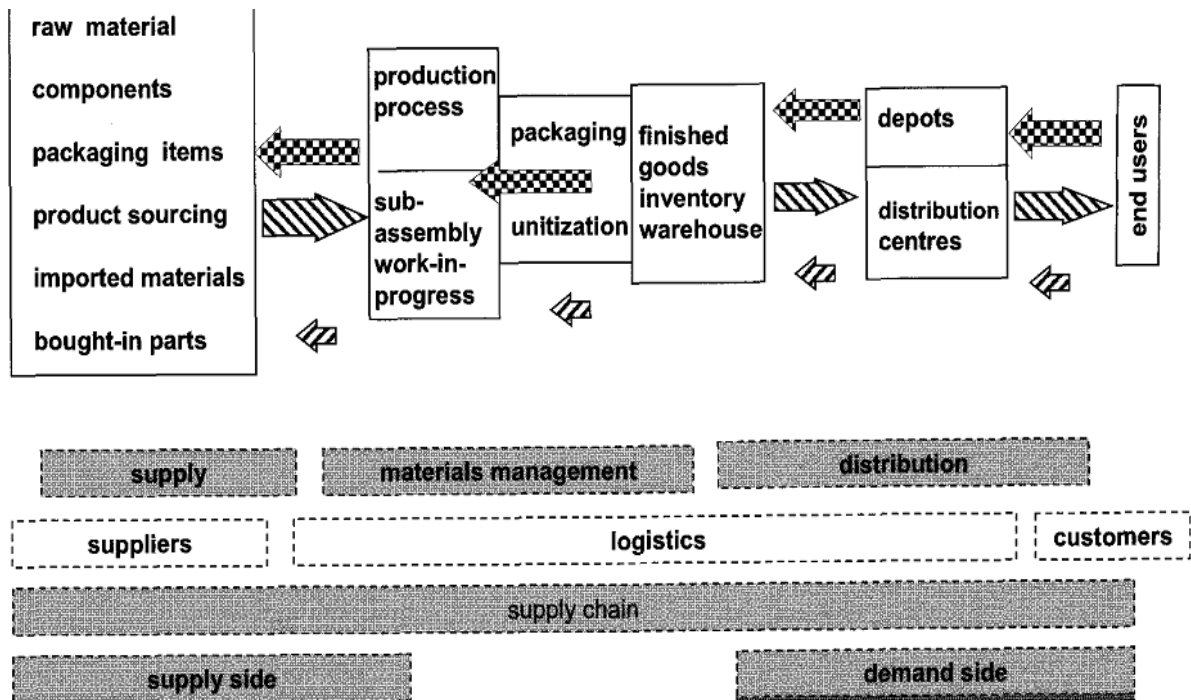


Figure 1.2: Configuration of transportation system of a company operating in FMCG category, illustrating functions, directions of flow of information as well as of material in supply chain

1.2. The Evolution of Transportation System:

It is known from studies that during different decades the functions of transportation management and distribution network management were already in practice along with the concept of manufacturing, production and warehousing of finished or raw goods. However, due to the rapid evolution in business, social, cultural and economic conditions in last few decades the concept of transportation and distribution have been evolved as vital functions for any company.

Now, transportation play a key role for gaining competitive advantage over the competitors, as companies have started to design the structure and strategy of their transportation network based upon their requirements. The basic principles and methodologies remain the same for developing and upgrading the concepts and theories of transportation management.

The evolution and development of transportation management and distribution network system has been summarized in the following time frames:

- 1.2.1. Transportation systems during 1950-1960:** During This period, no organization had a specific model for transportation of material and information between different echelons. Activities were executed as they were encountered. There were no synergy between Manufacturers, Distributors, and retailers.
- 1.2.2. Transportation systems during 1961-1970:** During this time period, efforts were made to develop a connection between the material's supply with demand, manufacturing with distribution network and by the end of this decade the benefits obtained from this interconnection were realized.
- 1.2.3. Transportation systems during 1971-1979:** This time period marked a significant development in the re-structuring, or organizing and optimizing of the distribution network system. An effective way to control the management activities associated with the distribution network system was included in the functional level of management in the hierarchy of the organization. Business relationships were developed between retailers and manufacturers and as a result cost reduction and customer satisfaction levels increased.
- 1.2.4. Transportation systems during 1980-1989:** Once the benefits associated with the interconnections between different echelons of supply chain distribution network were realized companies started developing different models in order to optimize their transportation systems. In this manner the concepts of centralized distribution strategy, decentralized distribution network and integrated distribution network came into being. Effort was made to optimize inventories at each distribution node. Companies also took advantage of the increasing use of computers in order to store information and also exchange information with partners. Other concepts like using third party logistics and warehousing companies was also practiced.
- 1.2.5. Transportation systems during the Early 1990s:** Due to the increasing use of computers and the invention of the personal computers it became more easier for companies to make use of information technology to efficiently manage information regarding the incoming and outgoing materials and also to maintain records.
- 1.2.6. Transportation systems during the Late 1990s:** Once the significance of interdepartmental relationships was realized, integration of operations with

independent business partners was started by companies. Resulting in highlighting the concept of supply chain management.

1.2.7. Transportation systems during the 20th Century: In this period companies started to take advantage of the globalization, the importance of supply chain distribution networks increased. In the ever changing economic and social conditions of the world, the supply chain disruption concept is most eminent; which can be prevented and taken care of by designing the supply chain to be more agile.

1.3. Integrated Transportation System:

Integrated transportation systems mean the coordination of different departments with the transportation functions. The following are some of the ways or techniques for integrated transportation systems:

1.3.1. Distribution Requirement Planning (DRP): DRP deals with the utilization of Manufacturing Resource Planning, efficient control of the inventory and material and their connection with warehousing and transportation scheduling system.

1.3.2. Just In Time (JIT): Just In time is a Japanese management philosophy that states sourcing the right goods in right quantity can result in reduction in waste, also producing the right goods in right quantity and also distributing the right goods at right time at right place so that inventory carrying cost can be minimized and also hassles can be avoided.

1.4. Competitive advantage due to Transportation system:

Many recent studies have proved that a well-designed transportation and distribution system can result in a competitive advantage of a company over other companies as it results in the reduction of cost and increase in profits of the company. Also it contributes to the increased level of customer satisfaction.

Competitive advantage can be gained by any company either through service leadership or through cost leadership. An optimum transportation and distribution network can make a company a service leader or a cost leader.

The transportation and distribution network design is different for both service leadership and cost leadership. So, transportation network should be designed by each company in

accordance with their business strategy. The following points should be considered for different competitive positions regarding transportation:

Service Leadership:

1. Tailored service
2. Tailored Distribution Strategy
3. Reliability
4. Responsiveness
5. Information Flexibility

Cost Leadership:

1. Capacity Utilization
2. Asset turn
3. Low inventory
4. Low waste

1.5. Components of Transportation System:

The following are the key components of the transportation system:

- Transportation
- Warehousing
- Inventory
- Information

1.6. Distribution System:

Distribution Network system is defined as “Series of activities, tasks and processes involved in physically moving a bunch of products from the point of where they are manufactured to the end customer”.

Distribution network is composed of First marketing warehouses and Distribution centers

1.7. Distribution Network Anatomy:

1.7.1. Supply Points: Supply points could be manufacturing facilities, Intermediate DCs or warehouse, raw material suppliers, retail stores or pick up points.

1.7.2. Demand Points: Demand Points could be end customers, retail stores, and pick up locations or even manufacturers and DC/warehouses.

1.7.3. Transshipment Points: These are the warehouses or cross docking points for moving goods from plants to customers.

1.8. Structures of Distribution Network:

1.8.1. Two stage Distribution Network- Direct Delivery: There is only one Supply Point and demand Point in this type of Distribution Network.

1.8.2. Three Stage Distribution Network: Other than supply and demand point there is an intermediate point in the distribution network.

1.8.3. Multistage Distribution Network: There is more than one transshipment facility along entire distribution network.

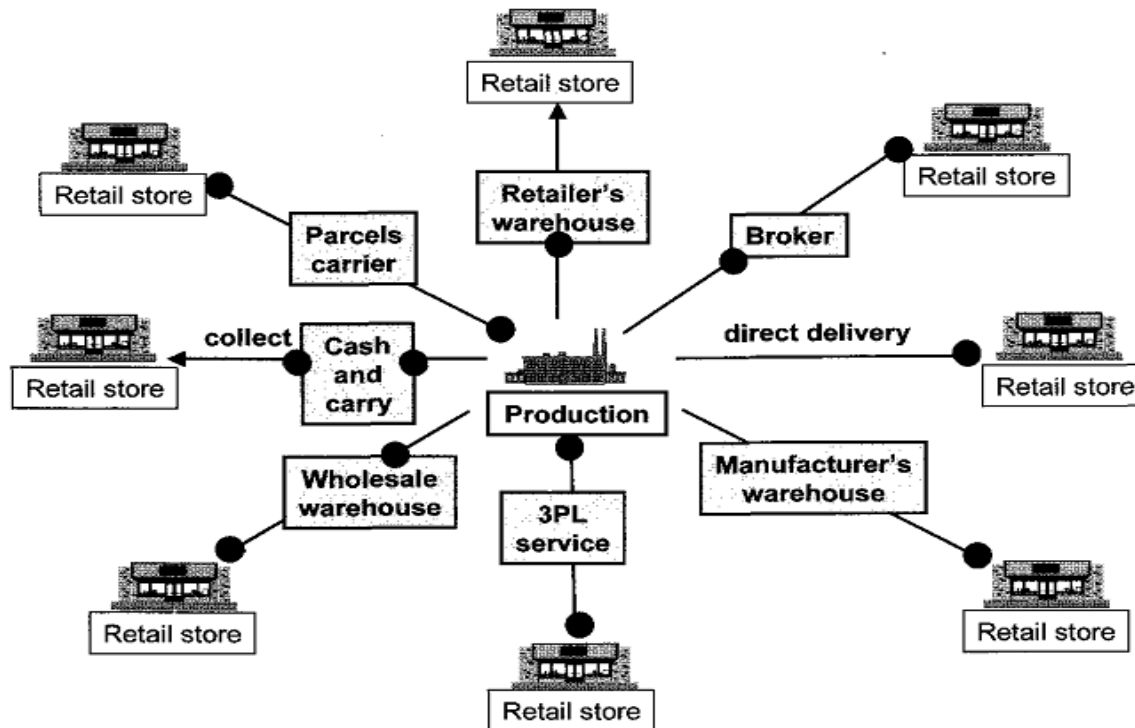


Fig 1.3: Alternative distribution channels for consumer products to retail outlets

1.9. Distribution Channel:

Distribution centers are the facilities or locations where batch of products arrive in bulk from one or more than one places and then they are distributed to many different locations depending upon the demand.

1.10. Functions of Distribution Channel in Distribution Network:

- Minimizing transportation and total costs.
- Increasing the sales.
- Making the right product available at the right market for right customers.
- Protecting against uncertainty of demand.
- Providing some value adding activities like postponement, Accumulation, Sorting, Allocation, and Assembly at the distribution center.

- Supporting Economy of Scale in Production.
- Promoting transportation economies.

1.11. Transportation in Pakistan

Transportation sector is one of the key sectors regarding the economy of Pakistan. It contributes about 10% in the GDP of Pakistan. A large number of Pakistani national are employed in this sector; about 6% of the population of Pakistan is employed with this sector.

In comparison to India, Malaysia and Thailand, the road infrastructure is far better but the railway infrastructure of Pakistan has a lot of room for improvement by using effective management techniques. Also the quality and infrastructure of the airways and ships is very good but it can be made even better. The following table shows the comparison of quality of infrastructure for different Asian Countries:

	Road	Rail	Port	Air
Pakistan	72	55	73	81
India	90	23	83	71
China	53	27	67	79
Bangladesh	100	71	107	117
Indonesia	84	56	96	69
Malaysia	21	20	19	29
Thailand	36	57	43	28

Table 1.1: Quality of infrastructure

1.11.1. Different Transportation Modes in Pakistan:

- **Road and Trucking:**

At the end of 1996, the length of road was increased by 13%, which constitutes 259000 kilometers, of which most of the length is of high type. Out of the total road network of Pakistan the National Highways and motorways has a share of 4.2% and also more than 85% of Pakistan's total traffic is handled by it.

- **Trucking Services:**

In accordance with the report presented by the Government of Pakistan in 2010, Pakistan's transportation system mostly depends upon the trucking industry which makes up about 96% of the total freight traffic. There are about 216,043 registered trucks out of which 196,850 trucks are on the operating on the road. About 70% out of the total registered trucks are single or double axel trucks mostly owned by the small truck operators. National Logistics Cell (NLC) is largest truck operating company whose

market share in Pakistan is 10%. Operators are willing to purchase multi-axel trucks to increase the efficiency of transportation but they encounter many problems such as high taxes and low freight rate in domestic market.

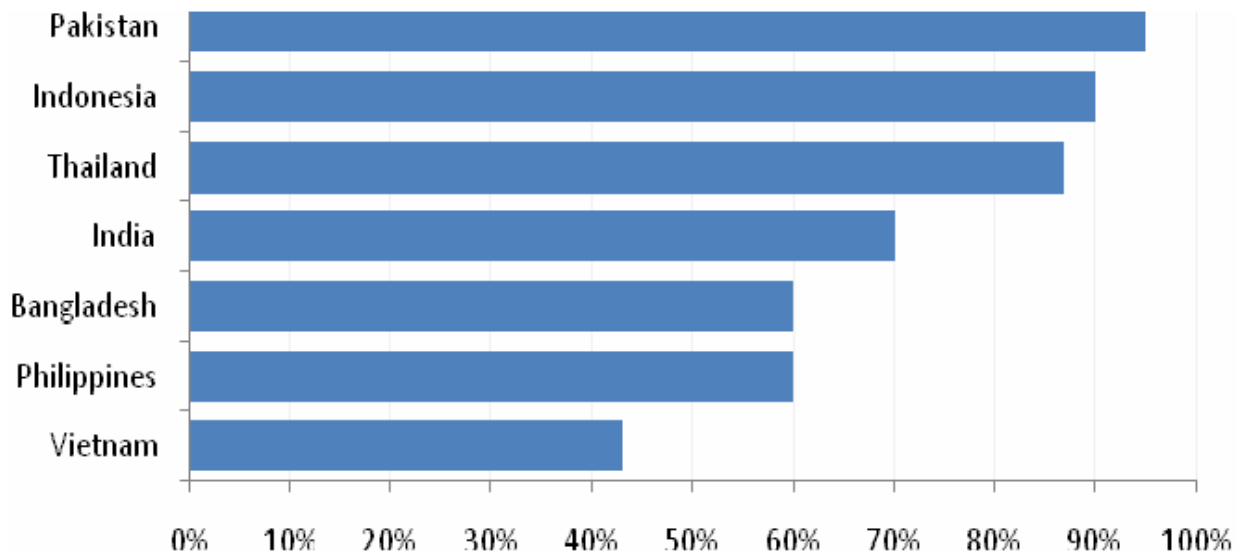


Fig 1.4: Percentage Share of road transport in freight Activity in Selected Asian Countries

- **Railways:**

Railway was the primary mode for the transportation of large batches of goods and also passengers in the 70s. However, the current condition of Pakistan railways has gone from bad to worse since the Government of Pakistan had shifted its attention from rail to roads. There is drastic difference between the budget for railway and roads. According to the 2010 budget, an allocation of 45 billion rupees was made for the railways while a budget of 155 billion was allocated for national highway. Currently the freight carried by railways is 4.77 million tons in 2016 which was 3.3 million tons in 2015 while it carried 52.2 million passengers in 2016 which were 50 million in 2015. So it can be said that there has been gradual improvements in Pakistan railways.

- **Airways:**

As of now, Pakistan has a total of 27 airports which are actively operating, of which there are 9 international airports, 13 domestic airports and 1 is for general and private aviation. In Pakistan airways handle about 17-18 million passenger and 7 billion tons of cargo a year. Jinnah International Airport is the busiest of all the airports in Pakistan and

is located in Karachi. Also two Major airports are located in Lahore and Islamabad which handle both domestic and international flights. The highest market share in terms of passenger traffic is handled by Pakistan International Airline (PIA) which is about 73% and also handles nearly all the freight forwarding business through airline. Pakistani airports are directly connected with UK, USA and Middle East countries. Average time for a direct flight to USA is eighteen hours while connecting flights takes normally more than 24 hours. The direct flights to other countries like Germany and other European countries have become frequent and commence more than once in a week. The air travelling cost per kilometer is very expensive in Pakistan.

1.12. Optimization

Optimization is basically a mathematical technique or application through which we can minimize or maximize a real function by choosing or selecting best input values from an allowed set of values. The function may have a single variable or multiple variables and also when we are maximizing or minimizing the real function we may have to subject the function to certain constraints or limitations. An optimization problem can be formulated as a linear programming problem or a nonlinear programming problem but here we are only concerned with the linear programming problem.

1.12.1. Linear Programming

Linear programming is a widely implemented technique which is aimed at finding out the minimum and maximum values of the function over a system of inequalities in which each variable involved is a real number and with each of those inequalities denoting a constraint or a restriction. Once the graph of the function is made and the vertices of the solution set also known as the feasible region are substituted into the real function, the minimum and maximum values of the function can be determined [5, 6]. A linear programming problem consists of:

- Variables x_1, \dots, x_n .
- m linear inequalities in these variables (maybe equalities).
E.g., $8x_1 - 5x_2 \leq 6, 0 \leq x_1 \leq 4$, etc.
- We may also have a linear objective function. E.g., $3x_1 + 5x_2 + 7x_3$.

Goal is to find the values of the variables that maximize or minimize the objective function.

1.12.2. Mixed Integer Linear Programming

Mixed integer linear programming problems come into being when some decision variables in the mathematically formulated model have real values while some decision variables have integer values. Therefore, the model is called “mixed”. It is known to be a mixed integer linear program if the objective functions and the constraints are all linear. In context, mixed integer linear programming is similar to linear programming but it simply doesn’t work if integer value is treated as a real one.

In this research thesis the mixed integer linear programming under constraints has been used to optimize the transportation cost for a fizzy drink industry.

CHAPTER 2: LITERATURE REVIEW

2.1. Background

In the area of supply chain management, transportation and distribution system a lot of research has been carried out considering the transportation of various different products. Also the different company policies have been taken into consideration. Different echelons of the supply chain distribution network have been accounted for and various different techniques have been developed for the optimal design of the network. Also different software and algorithms have been developed and implemented for the optimization of supply chain distribution networks. Different cost effective strategies have been studied new strategies have been developed for the maximization of the company's profits. Studies regarding different risk factors involved in supply chain management and how to overcome them were also taken into consideration. Comparison of different optimization techniques were also discussed in different studies.

Also transportation system and Distribution Network Management has become an important part of any supply chain. In the distribution network system, the strategic level of decisions involved are deciding the number of distribution Channels, deciding the transshipment schedule, transshipment cycle time, appropriate level of inventory at each place, while keeping the transportation cost, warehousing cost and inventory holding cost at minimum level. Calculating these factors while balancing against cost is the key for minimizing the distribution Network cost.

2.2. Literature Review on Supply Chain Distribution Cost Optimization

In recent research and studies attention has been focused upon multi-echelon, multi-products, multi-facilities and multi-period systems and networks. Many different programs and algorithms have been developed for the optimization and solution of SCN related problems by first modeling the problem mathematically and then solving it using linear programming, mixed integer linear programming, nonlinear programming, integer programming and other different techniques and methods [7, 8].

Extensive research has been carried out in the field of supply chain distribution network design and optimization. Various techniques have been developed by the researchers to design the supply chain distribution network including techniques such as the cost benefit

analysis and complex mathematical modeling. In this portion of the thesis, I am going to present the research work done in this sector and link it to my work.

Current et al. [9] reviewed and focused on the optimization and design on supply chain distribution network and then formulated a mathematical model consisting of four objective functions i.e 1) minimization of the supply chain distribution network cost, 2) fulfillment of the customer's demand 3) Maximization of the company's profit 4) minimization of the risk factors imposed on the environment. The number of manufacturing plants and the constraints on the capacity of their production were determined by the decision variables

Edward A Morash [10] emphasized on the various capabilities of transportation in SCM such as customization, flexibility, JIT delivery, IT support, reliability, time compression and standardization. Also, the integration of transportation was emphasized in order to reduce the total cost and to increase the level of customer satisfaction.

Vaidyanathan [11] developed a mixed integer programming model in order to minimize the transportation/distribution design costs by considering the effects of modes of transportation, the location of the facilities and inventory related issues so as to provide a more realistic insight into the design of the distribution networks.

Linnet Ozdamar and Tulin Yazgac [12] studied the mode of transportation for factories which transport goods to warehouses that are geographically at a very large distance and proposed a hierarchical production-distribution planning approach. This approach aims at solving the problem optimally by taking an aggregate of the different time periods and product families while ignoring the capacity consumption by setup.

J Reeb and S Leavengood [13] studied different techniques, approaches and methods for solving transportation problems and concluded that although problems involving transportation of goods from one place to another can be solved using the simplex method but it takes a lot of time and also there is a high chance of committing a mistake while the problem is being solved by hand. On the other hand if the same problems are solved using transportation method it takes less time and can be easily solved by hand and also computer algorithms take less time to solve the problems modeled as transportation problems.

Syarif et al. [14] worked on the formulation of a mathematical model using mixed integer

linear programming for a multi echelon distribution network, minimization of the total distribution network cost was the objective, the opening and closing of the warehouses as well as the flow of products from the plant to the warehouses and from the warehouses to the distribution centers were determined by the decision variables. He used spanning tree genetic algorithm to solve the model and then he compared the results obtained with other models.

Hokey Min [15] thoroughly studied supply chain management and design of distribution networks along with various techniques for solving transportation problems and concluded that refining and reinventing the traditional analytical tools and techniques do not tackle many different issues like forecasting, communication, resistance to change, customer relations, information sharing and many others. He proposed diversification of techniques and tools for supply chain problems and also proposed the much-needed lines of research for better supply chain management in the future.

Wang, Sun, and Yang [16] formulated a mathematical model for multi echelon supply chain distribution network by considering the level of inventory in different distribution centers as well as the facility location model along with the transshipment schedule.

Nikolaos V Sahinidis [17] proposed many different modeling frameworks and techniques in order to optimize different problems under uncertainty and also developed and implemented different algorithms in many different scenarios and concluded that exact solutions of deterministic equivalents are difficult to obtain and it needs advanced computer architectures to be brought into use.

Luis Contesse [18] developed and presented a large mixed integer linear programming model for a local natural gas distribution company in order to take into account all the difficulties in the purchasing and transporting of gas from one place to another and to assist in optimizing the daily decisions made by the company while there are no local storage facilities.

Esteban Lopez Milan [19] formulated a mathematical model for integrating different transportation modes such as rail and road transport systems in order to minimize transportation cost, automatically scheduling daily transport plans and also to control the freshness of the sugar cane by considering minimum supply through direct transport to the sugar mill.

Elhedhli and Gzara [20] focused on the multi echelon supply chain distribution network and made an effort to identify the problems faced in the supply chain. After that an effort was made to formulate a mathematical model to minimize the supply chain distribution network cost by analyzing and considering location of plants warehouses and the capacity of production of plants and the warehouse capacity. In view of the developed model an optimized transshipment schedule was calculated as a result of which transportation cost from the warehouses to the distributors was reduced.

Rabbani et al [21] formulated a mathematical model for the optimization of the multi-product supply chain distribution network. The supply chain system consisted of suppliers, manufacturing plants, warehouses, distributors and customers. A distribution strategy was formulated based on the outcome of the model.

Turan Paksoy [22] presented a mixed integer linear programming model for a supply chain network having multiple objectives and echelons. The model optimizes the supply chain network problem by taking into consideration three objectives functions which are transportation cost minimization, minimizing holding and ordering costs in distribution centers and minimization of unused capacity of distributions centers and plants.

Josefa Mula [23] reviewed various mathematical modeling and programming methods and techniques used to optimize supply chain related problems in order provide beginners with the basic know how about mathematical programming in supply chain distribution network and transportation problems by identifying current and future work in the field of supply chain management and transportation planning.

Ozlem Akgul [24] presented mixed integer linear programming mathematical models for reducing the total supply chain costs by optimally designing a bioethanol supply chain which aims to optimize the location and scales of bioethanol production plants, flow of bioethanol from one place to another, biomass cultivation rate determination and number of transport units required for the transportation of bioethanol between different locations.

Hadi Basirzadeh [25] introduced a parametric method which involved ranking of fuzzy numbers in order to solve a fuzzy transportation problem. Also, the method presented is a systematic procedure and can be easily applied to all kinds of transportation problems

whether they have minimizing or maximizing objective functions.

Begona Vitoriano [26] studied the different aspects of disaster management and humanitarian aid distribution problem and developed a new approach for optimizing the multi-criteria optimization model by taking into account all these aspects i.e response time, security of routes and reliability.

Pawel Sitek and Jaroslaw Wikarek [27] developed a MILP model for optimization of supply chain costs by making use of mathematical programming tools present in “LINGO” package. It optimizes the supply chain costs by finding out the flow of distribution for the supply chain network under consideration which in turn results in minimization of cost and satisfying customer demand. Also, other SCM decision support possibilities are presented including determination of the sensitivity analysis of the solution and also determining the importance of quality and range of different parameters on structure and cost of supply chain network.

Pradip Kundu and Samarjit Kar [28] investigated a multi-objective solid transportation problem taking into account many different uncertain environments. Random, fuzzy and hybrid uncertainties were taken into account while other parameters like resources, demands, transportation capacities and budget were formulated as fuzzy using appropriate techniques.

Mollah Mesbahuddin Ahmed [29] presented a new approach for determining the initial basic feasible solution for supply chain models and transportation models named allocation table method. Also, the allocation table method was applied to several problems and it was found to be more efficient in minimizing transportation costs.

Bartosz Sawik [30] developed and solved a formulation having multiple objectives for transportation problem and travelling salesman problem for a Spanish company. Minimization of total distances and CO₂ emission was the goal and it was achieved by minimizing the distance covered by trucks during the deliveries by finding the best possible route that minimizes the total cost.

CHAPTER 3: METHODOLOGY

In this portion of the research thesis, different steps taken in the optimization of the supply chain distribution network supply cost for a fizzy drink industry are discussed. To begin with first of all a survey was carried out and a fizzy drink manufacturing and bottling plant was selected located in Peshawar, Pakistan.

The flow chart representing all the steps involved in the optimization of transportation cost for the fizzy drink industry all given below:

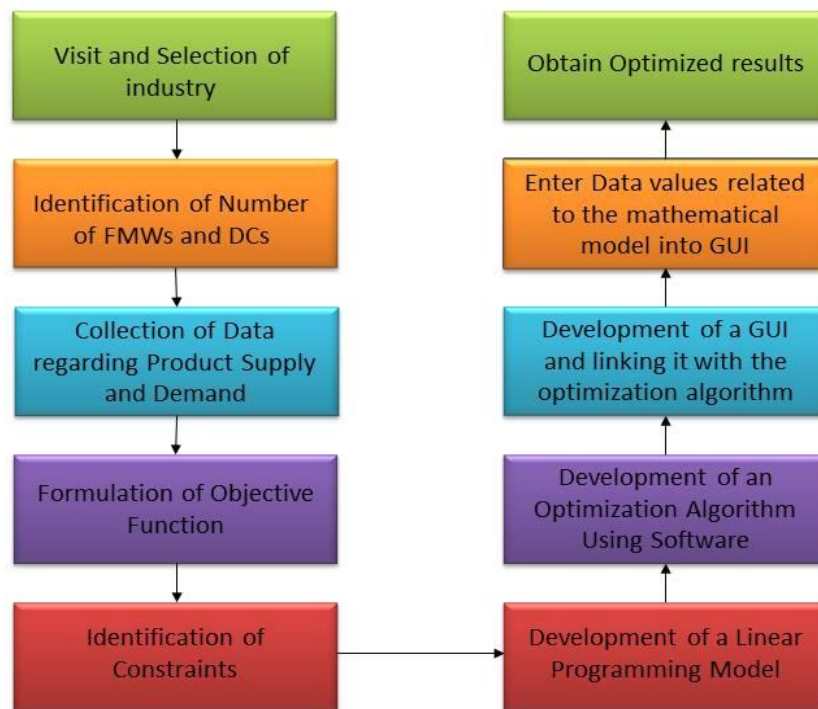


Fig 5.1: Steps involved in the Optimization of transportation cost for fizzy drink industry

3.1 Introduction to the Company

The selected company is one of the top food and beverage companies in the world deals in manufacturing of foods and beverages and their distribution to over 200 countries. The food products manufactured include potato chips, dairy-based products, breakfast cereals, rice, whole wheat pasta, and many flavored snacks. The beverages produced include fizzy drinks, fruit juices, ice tea and coffee, energy drinks, and mineral water. The Headquarter of this company is situated in New York and it being a giant this company employs more than 274000 people across the globe.

This company owns a large number of packaged goods trademarks; 9 out of 40 in the United States. Also many different brands are owned by the company and the most famous 22 of them generate more than \$1 billion revenue.

This company has a dominant reputation in the food and beverage industry and follows a highly diverse business model. While the competitors focus mainly on carbonated soft drinks, this company gains a competitive edge over the competing companies due to its strong intimidating presence in the snack and food industry, as it not only produces various carbonated beverages but also manufactures mouthwatering snacks and other food based cook able products. In 2013, this company's beverage business accounted for 48% and food business accounted for 52% of its \$66.5 billion profit.

This company is one of the market giants among food and beverage companies with an intimidating presence all over the world. The products manufactured by the company reach the market through three channels: customer warehouses, direct delivery to the stores also known as DSD and through third party logistics and supply companies. The distribution channel is chosen based on the characteristics of the products, local market practices and the requirement of the customer.

3.2. Case Study

In this research thesis the mathematical model formulated represents a single product multi echelon supply chain network for the company's fizzy drink manufacturing plant located in Peshawar, Pakistan. It is responsible for distributing carbonated soft drinks as well as mineral water to the whole province of Khyber Pakhtunkhwa. It consists of a single manufacturing plant where the fizzy drink is produced after undergoing a series of processes and then it is bottled and supplied to three first marketing warehouses located in Peshawar, Kohat and Chakdara where it

is stored in the form of inventory and from there the product is supplied to 29 distribution centers according to customer demand. It is desired to design the supply chain distribution network in such a way that will satisfy all supply capacities and demand requirements for the product that are imposed by the customers. The supply chain network design problem is formulated as a mixed integer linear programming model. The main objective of this model is to minimize the total cost of transporting the products throughout the supply chain. The following assumptions have been used in this problem (1) since there is only a single manufacturing plant so the cost for transporting products from the plant to the FMWs is fixed and only transportation of products from the FMWs to the distribution centers is considered in the formulation of the model (2) the supply capacity of the FMWs and demands of the distribution centers are known (3) for the sake of simplicity of formulation 250ml is considered as a single unit so average number of units in one case and also average number of units per truckload are calculated as:

Since one truckload consists of 1000 cases and they are divided as 500 cases of 1.5L and 1 case carries 6 bottles of 1.5L, 250 cases of 2.25L and 1 case carries 4 bottles of 2.25L, 50 cases of 250ml and 1 case carries 24 bottles of 250ml, 50 cases of 1L and 1 case carries 6 bottles of 1L, 150 cases of 500ml and 1 case carries 12 bottles of 500ml so one 250ml bottle represents 1 unit, one 1.5L bottle represents 6 units, one 2.25L bottle represents 9 units, one 1L bottle represents 4 units and one 500ml bottle represents 2 units.

Average units/case = [(total units in 1.5L bottles) +(total units in to 2.25L bottles) +(total units in to 250ml bottles) +(total units in 1L bottles) +(total units in 500ml bottles)]/Total cases per truckload

$$\text{Average units/case} = (500*6*6) + (250*9*4) + (50*24) + (50*4*6) + (150*2*12)/1000$$

$$\text{Average units/case} = (18000+9000+1200+1200+3600)/1000$$

$$\text{Average units/case} = 33000/1000$$

$$\text{Average units per case} = 33 \text{ units}$$

$$\text{Average units per truckload} = 33000 \text{ unit}$$

$$\text{Transportation cost per unit} = \text{Transportation cost per trip} / \text{Average units per truckload}$$

3.2.1. Transportation Cost Data

The transportation cost per kilometer is fixed as PKR 80/km and all the distances between all the warehouses and the distribution centers are known and shown in the tables below along with the transportation cost per trip and transportation cost per unit:

D.C	Distance	Transportation Cost	TC per unit
Peshawar	09km	720	0.0218
Noshehra	40km	3200	0.0969
Akora Khatak	51km	4080	0.1236
Pabbi	20km	1600	0.0485
Chamkani	11km	880	0.0267
Khyber agency	32km	2560	0.0776
Darra Adamkhel	48km	3840	0.1164
Swat	248km	19840	0.6012
Batkhela	99km	7920	0.24
Buner	136km	10880	0.3297
Bajaur Agency	173km	13840	0.4194
Lower Dir	181km	14480	0.4388
Upper Dir	212km	16960	0.5139
Chitral	370km	29600	0.8969
Mardan	62km	4960	0.1503
Swabi	101km	8080	0.2448
Charsadda	31km	2480	0.0752
DI khan	309km	24720	0.7491
Wana	405km	32400	0.9818
Tank	314km	25120	0.7612
Kohat	79km	6320	0.1915
Hangu	114km	9120	0.2764
Karak	143km	11440	0.3467
Parachinar	252km	20160	0.6109
Kurram agency	221km	17680	0.5358
Orakzai agency	110km	8800	0.2667
Bannu	219km	17520	0.5309
Lakki marwat	215km	17200	0.5212
North waziristan	262km	20960	0.6352

Table 3.1: Distance, Transportation cost per trip and Transportation cost per unit from Peshawar FMW to all DCs

The above table shows the transportation costs per trip and transportation costs per unit from the Peshawar first marketing warehouse to all the distribution centers across Khyber Pakhtunkhwa.

D.C	Distance	Transportation cost	TC per unit
Peshawar	71km	5680	0.1721
Noshehra	111km	8880	0.2691
Akora Khatak	120km	9600	0.2909
Pabbi	92km	7360	0.2230
Chamkani	73km	5840	0.1769
Khyber agency	91km	7280	0.2206
Darra adamkhel	29km	2320	0.0703
Swat	316km	25280	0.7661
Batkhela	163km	13040	0.3952
Buner	209km	16720	0.5067
Bajaur agency	237km	18960	0.5745
Lower dir	244km	19520	0.5915
Upper dir	274km	21920	0.6642
Chitral	421km	33680	1.0206
Mardan	126km	10080	0.3055
Swabi	169km	13520	0.4097
Charsadda	98km	7840	0.2376
DI khan	236km	18880	0.5721
Wana	334km	26720	0.8097
Tank	217km	17360	0.5261
Kohat	05km	400	0.0121
Hangu	40km	3200	0.0969
Karak	76km	6080	0.1842
Parachinar	180km	14400	0.4364
Kurram agency	147km	11760	0.3564
Orakzai agency	35km	2800	0.0848
Bannu	151km	12080	0.3661
Lakki marwat	144km	11520	0.3491
North waziristan	185km	14800	0.4485

Table 6.2: Distance, Transportation cost per trip and Transportation cost per unit from Kohat FMW to all DCs

The above table shows the transportation costs per trip and transportation costs per unit from the Kohat first marketing warehouse to all the distribution centers across Khyber Pakhtunkhwa.

D.C	Distance	Transportation cost	TC per Unit
Peshawar	127km	10160	0.3079
Noshehra	90km	7200	0.2182
Akora Khatak	97km	7760	0.2352
Pabbi	108km	8640	0.2618
Chamkani	123km	9840	0.2982
Khyber Agency	163km	13040	0.3952
Darra Adamkhel	166km	13280	0.4024
Swat	143km	11440	0.3467
Batkhela	14km	1120	0.0339
Buner	103km	8640	0.2618
Bajaur agency	82km	6560	0.1988
Lower Dir	50km	4000	0.1212
Upper Dir	99km	7920	0.24
Chitral	223km	17840	0.5406
Mardan	69km	5520	0.1673
Swabi	128km	10240	0.3103
Charsadda	92km	7360	0.2230
DI khan	422km	33760	1.0230
Wana	520km	41600	1.2606
Tank	389km	31120	0.9430
Kohat	194km	15520	0.4703
Hangu	232km	18560	0.5624
Karak	261km	20880	0.6327
Parachinar	371km	29680	0.8994
Kurram agency	338km	27040	0.8194
Orakzai agency	226km	18080	0.5479
Bannu	337km	26960	0.8169
Lakki Marwat	330km	26400	0.8000
North Waziristan	371km	29680	0.8994

Table 3.3: Distance, Transportation cost per trip and Transportation cost per unit from Chakdara FMW to all DCs

The above table shows the transportation costs per trip and transportation costs per unit from the Chakdara first marketing warehouse to all the distribution centers across Khyber Pakhtunkhwa. Based on this cost data elaborate from/to table has been constructed incorporating the supply capacity of first marketing warehouses and the demand for each distribution center. This table facilitates the formulation of the mathematical model and also gives us a clear picture of the entire material flow going on in the supply chain distribution network. The table is shown below:

	Peshawar	Kohat	Chakdara	Demand
Peshawar	0.0218	0.1721	0.3079	144729915
Noshehra	0.0969	0.2691	0.2182	12080409
Akora Khatak	0.1236	0.2909	0.2352	1298550
Pabbi	0.0485	0.223	0.2618	1347885
Chamkani	0.0267	0.1769	0.2982	12093180
Khyber Agency	0.0776	0.2206	0.3952	16779807
Darra Adamkhel	0.1164	0.0703	0.4024	9015369
Swat	0.6012	0.7661	0.3467	30515991
Batkhela	0.24	0.3952	0.0339	12545445
Buner	0.3297	0.5067	0.2618	11203005
Bajaur Agency	0.1419	0.5745	0.1988	9049854
Lower Dir	0.4388	0.5915	0.1212	30442401
Upper Dir	0.5139	0.6642	0.24	16253193
Chitral	0.8969	1.0206	0.5406	21471318
Mardan	0.1503	0.3055	0.1673	29855661
Swabi	0.2448	0.4097	0.3103	8079324
Charsadda	0.0752	0.2376	0.223	26370135
Di Khan	0.7491	0.5721	1.023	16622925
Wana	0.9818	0.8097	1.2606	22625724
Tank	0.7612	0.5261	0.943	5754705
Kohat	0.1915	0.0121	0.4703	19556460
Hangu	0.2764	0.0969	0.5624	7597194
Karak	0.3467	0.1842	0.6325	7942883
Parachinar	0.6109	0.4364	0.8994	8602902
Kurram Agency	0.5358	0.3564	0.8194	7894425
Orakzai Agency	0.2667	0.0848	0.5479	7819746
Bannu	0.5309	0.3661	0.8169	4078193
Lakki Marwat	0.212	0.3491	0.8	15309261
North Waziristan	0.6352	0.4485	0.8994	1709625
Supply	267300000	213840000	588060000	

Table 3.4: Elaboration of transportation cost per item for transportation between all the supply and demand points and also shows supply capacity of each FMW as well as the demand of each distribution center

CHAPTER 4: PROBLEM FORMULATION AND SOLUTION

In this part of the thesis, a mathematical model is formulated keeping in view the all the data related to the supply capacity of the first marketing warehouses, demand of each distribution center, distances between different first marketing warehouses and distribution centers, transportation cost per trip between first marketing warehouses and distribution centers and transportation cost per item between first marketing warehouses.

4.1. Problem statement

In this problem a single manufacturing plant is involved, from which the product is transported to 3 first marketing warehouses and then to 29 distribution centers all over the province of Khyber Pakhtunkhwa. The supply capacities and demands are known. Transportation costs for transporting products from the plant to first marketing warehouses are known. Transportation costs for transporting products from the first marketing warehouses to distribution centers are known. Objective is to minimize the transportation cost between first marketing warehouses to distribution centers since the transportation cost between the plant and the first marketing warehouses is fixed.

4.2. Mathematical Modeling

In any optimization problem, the formulation or mathematical modeling depends on the following:

- Decision variables
- Objective function
- Constraints
- Non Negativity constraints

Since, we are dealing with a supply chain distribution network problem so here we shall consider many different parameters including the number of first marketing warehouses, the number of distribution centers, the quantity transported from first marketing warehouses to distribution centers and the transportation cost per unit from first marketing warehouses to DC.

4.2.1. Parameter Definition

i = representation of the number of FMWs ($i=1, 2, 3$)

j = representation of the number of distribution centers ($j=1, 2, 3 \dots 29$)

x_{ij} = quantity supplied from FMW to distribution center

c_{ij} = Transportation cost per unit from FMW to distribution center

4.2.2. Decision Variables

x_{ij} = Quantity supplied from FMWs to distribution centers

4.2.3. Objective Function

Objective function is basically a mathematical representation of the objective of the problem. Objective function maybe to minimize cost, time and losses or to maximize profits, productivity and efficiency depending upon different scenarios. In our case the objective is to minimize the transportation cost “ C_{ij} ” for transporting “ x_{ij} ” units from FMW to distribution centers where “ i ” is the number of FMWs and “ j ” is the number of distribution centers. So the objective function can be mathematically modeled as:

$$Z = \text{Min} \sum_i^n \sum_j^n C_{ij} X_{ij}$$

But since there are 3 first marketing warehouses and 29 distribution centers so it can be written modeled as:

$$Z = \text{Min} \sum_{i=1}^3 \sum_{j=1}^{29} C_{ij} X_{ij}$$

4.2.4. Constraints

Constraints are known as the limitations or restrictions on the optimization problem which need to be satisfied. In our case as it is a transportation problem so we have supply and demand constraints. The problem under consideration involves 3 supply constraints and 29 demand constraints. The less than or equal to sign in supply constraints shows that the supply capacity of each first marketing warehouse may be less than or equal to the amount shown but never greater than it. The greater than or equal to sign in demand constraints shows that the demand of each distribution center maybe greater than or equal to the amount shown but never less. The following table shows the supply and demand constraints:

Supply Constraints	Demand Constraints		
$\sum_{j=1}^{29} X_{1,j} \leq 267300000$	$\sum_{i=1}^3 X_{i,1} \geq 144729915$	$\sum_{i=1}^3 X_{i,11} \geq 9049854$	$\sum_{i=1}^3 X_{i,21} \geq 19556460$
	$\sum_{i=1}^3 X_{i,2} \geq 12080409$	$\sum_{i=1}^3 X_{i,12} \geq 30442401$	$\sum_{i=1}^3 X_{i,22} \geq 7597194$
	$\sum_{i=1}^3 X_{i,3} \geq 1298550$	$\sum_{i=1}^3 X_{i,13} \geq 16253193$	$\sum_{i=1}^3 X_{i,23} \geq 7942883$
$\sum_{j=1}^{29} X_{2,j} \leq 213840000$	$\sum_{i=1}^3 X_{i,4} \geq 1347885$	$\sum_{i=1}^3 X_{i,14} \geq 21471318$	$\sum_{i=1}^3 X_{i,24} \geq 8602902$
	$\sum_{i=1}^3 X_{i,5} \geq 12093180$	$\sum_{i=1}^3 X_{i,15} \geq 29855661$	$\sum_{i=1}^3 X_{i,25} \geq 7894425$
	$\sum_{i=1}^3 X_{i,6} \geq 16779807$	$\sum_{i=1}^3 X_{i,16} \geq 8079324$	$\sum_{i=1}^3 X_{i,26} \geq 7819746$
	$\sum_{i=1}^3 X_{i,7} \geq 9015369$	$\sum_{i=1}^3 X_{i,17} \geq 26370135$	$\sum_{i=1}^3 X_{i,27} \geq 4076193$
$\sum_{j=1}^{29} X_{3,j} \leq 588060000$	$\sum_{i=1}^3 X_{i,8} \geq 30515991$	$\sum_{i=1}^3 X_{i,18} \geq 16622925$	$\sum_{i=1}^3 X_{i,28} \geq 15309261$
	$\sum_{i=1}^3 X_{i,9} \geq 12545445$	$\sum_{i=1}^3 X_{i,19} \geq 22625724$	$\sum_{i=1}^3 X_{i,29} \geq 1709625$
	$\sum_{i=1}^3 X_{i,10} \geq 11203005$	$\sum_{i=1}^3 X_{i,20} \geq 5754705$	

Table 4.7: Supply and Demand Constraints

Also, there will be a non-negativity constraint associated with the model showing that the value of the decision variable cannot be negative $X_{ij} \geq 0$.

4.3. Technique used to find solution

Mathematical programming technique known as mixed integer linear programming (MILP) was used since it provides exact transportation Schedule along with minimum distribution cost, tells which warehouse to open and which to close and also provides minimum distribution cost precisely.

4.4. Software Used

In order to find a solution for this optimization problem using mixed integer linear programming under constraints the software package used is MATLAB optimization toolbox because of its flexibility, efficiency and ease of implementation and also due to the presence of many different commands to facilitate problem solving. MATLAB is based on design processes and analyzing behavior of different models by observing iterations and it is operated by providing programming language code to achieve desired results. In our case the programming pseudo code is given below:

```
function pivot(int p, int q)
{
  for (int i = 0; i <= M; i++)
  for (int j = 0; j <= M + N; j++)
    if (i != p && j != q)
a[i][j] -= a[p][j] * a[i][q] / a[p][q];
  for (int i = 0; i <= M; i++)
    if (i != p) a[i][q] = 0.0;
  for (int j = 0; j <= M + N; j++)
    if (j != q) a[p][j] /= a[p][q];
  a[p][q] = 1.0;
}
function Simplex(double[][] A, double[] b, double[] c)
{
  M = b.length;
  N = c.length;
```

```

a = new double[M+1][M+N+1];
for (int i = 0; i < M; i++)
    for (int j = 0; j < N; j++)
        a[i][j] = A[i][j];
for (int j = N; j < M + N; j++)
a[j-N][j] = 1.0;
for (int j = 0; j < N; j++)
a[M][j] = c[j];
for (int i = 0; i < M; i++)
a[i][M+N] = b[i];
}
function solve()
{
while (true)
{
int p, q;
for (q = 0; q < M + N; q++)
if (a[M][q] > 0)
break;
if (q >= M + N)
break;
for (p = 0; p < M; p++)
if (a[p][q] > 0)
break
for (int i = p+1; i < M; i++)
if (a[i][q] > 0)
if (a[i][M+N] / a[i][q] < a[p][M+N] / a[p][q])
p = i;
pivot(p, q);
}}

```

Figure 4.1 Programming Pseudo code

4.5. Graphical User Interface

A graphical user interface (GUI) is developed using MATLAB Graphical User Interface Development Environment (GUIDE) so that anyone can easily determine a solution for any transportation optimization problem ranging from small scale to very large scale. GUIDE (GUI development environment) provides tools for the design of user interfaces for different apps. GUIDE Layout Editor can be utilized to graphically design any user interface. MATLAB code is automatically generated by GUIDE for the construction of the user interface, which can be changed or modified to program the behavior of the user interface being developed. In our case the Graphical User Interface operates through a sequence of steps wherein each step a certain input is required. These steps are diagrammatically shown below:

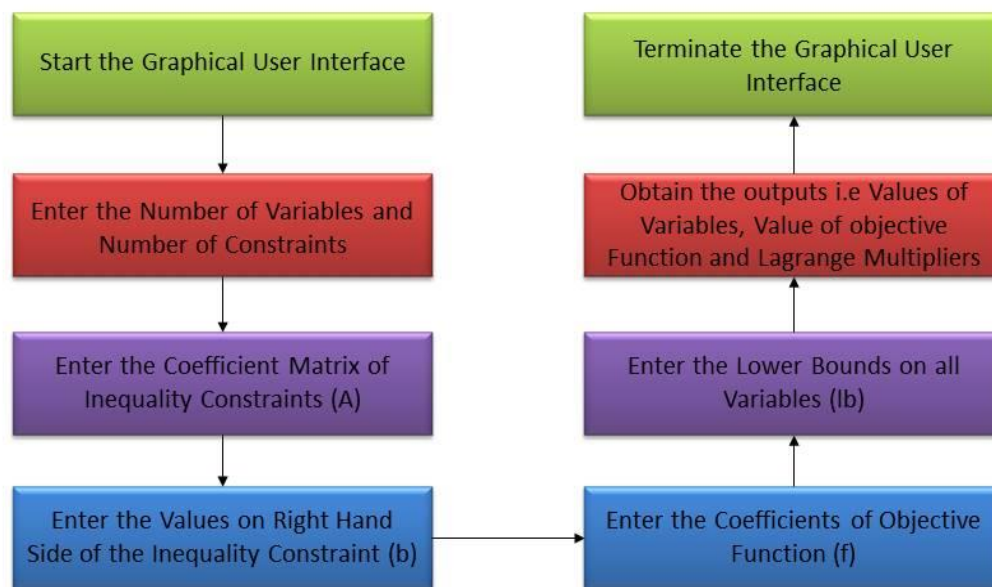


Figure 4.2: Steps involved in the operation of the Graphical User Interface (GUI)

As shown in the above figure, The GUI requires the number of decision variables and constraints to be entered at the start as shown in the figure below:

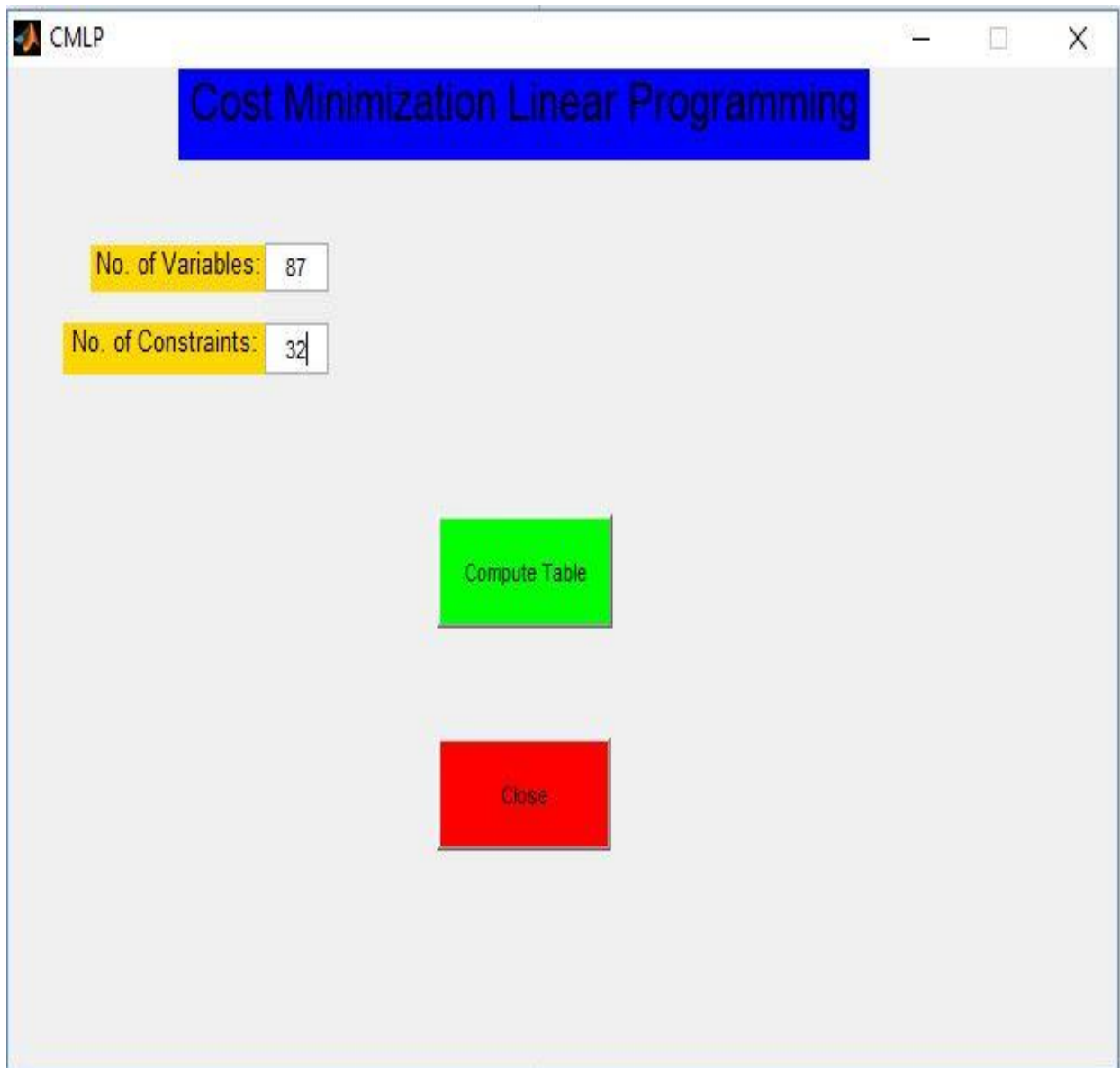


Figure 4.3: Input Window Requiring Number of Variables and Constraints

As a result of entering the number of variables and constraints it generates an input matrix which requires the coefficients of linear inequality constraints (A) to be entered as shown:

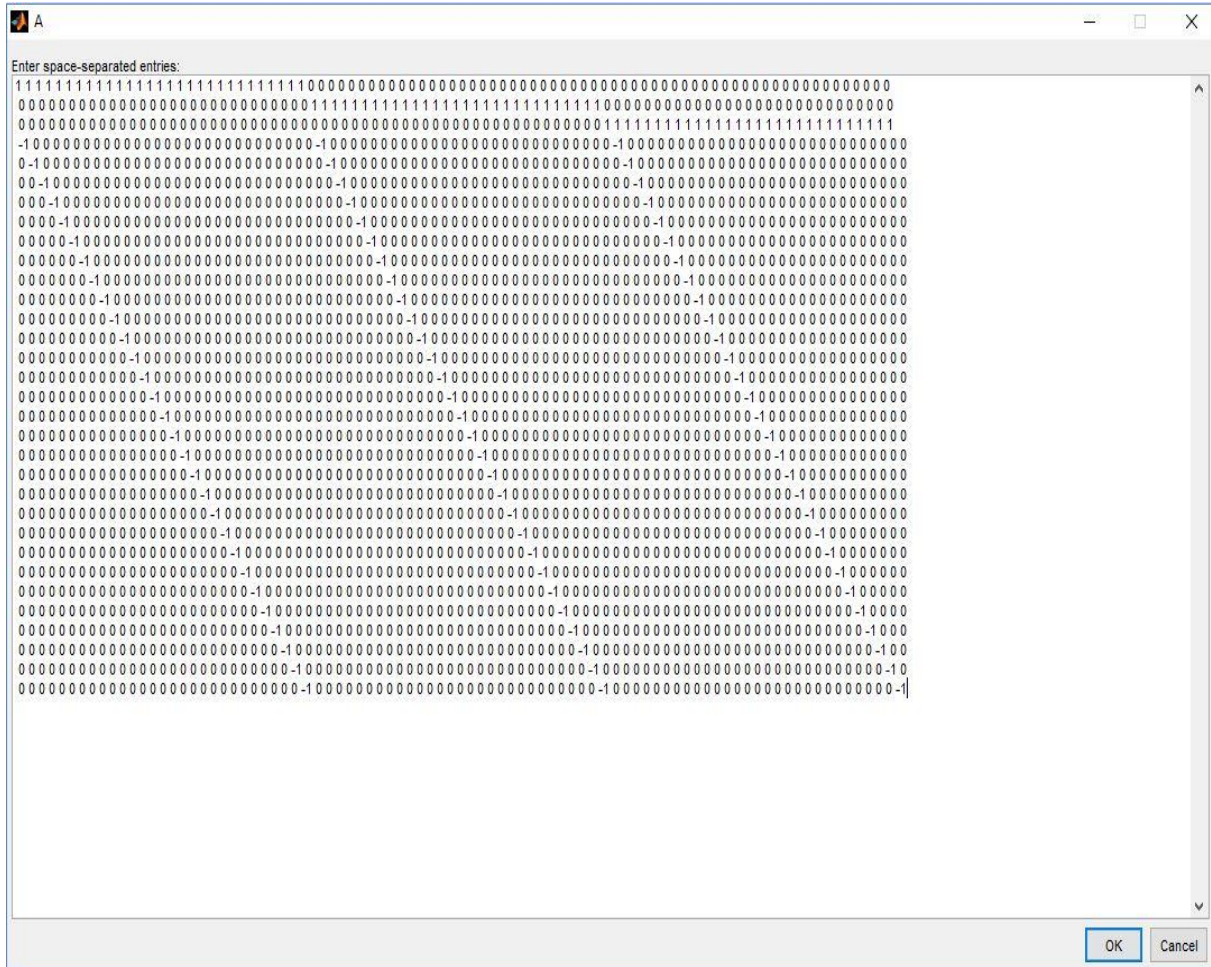


Figure 4.4: Input window requiring Coefficient Matrix of inequality Constraints (A)

After entering the coefficients of constraints, the GUI generates the input vector which requires values on the right hand side of the inequality constraints (b) as shown below:

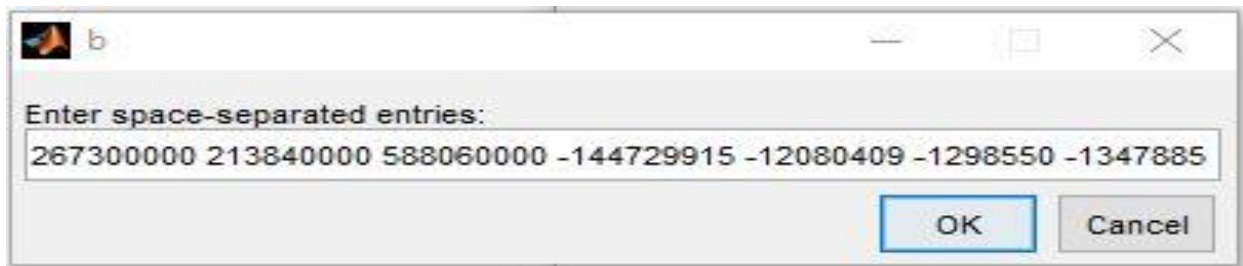


Figure 4.5: Input window requiring values on right hand side of inequality constraints (b)

After that the input vector requiring the coefficients of objective function is generated as shown in the figure below:

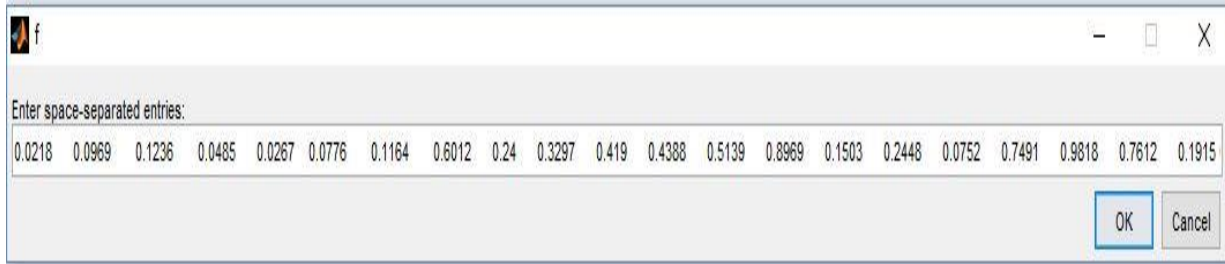


Figure 4.6: Input window requiring coefficients of the objective function (f)

After this finally an input vector requiring the lower bounds on the decision variables (lb) is generated as shown in the figure below:

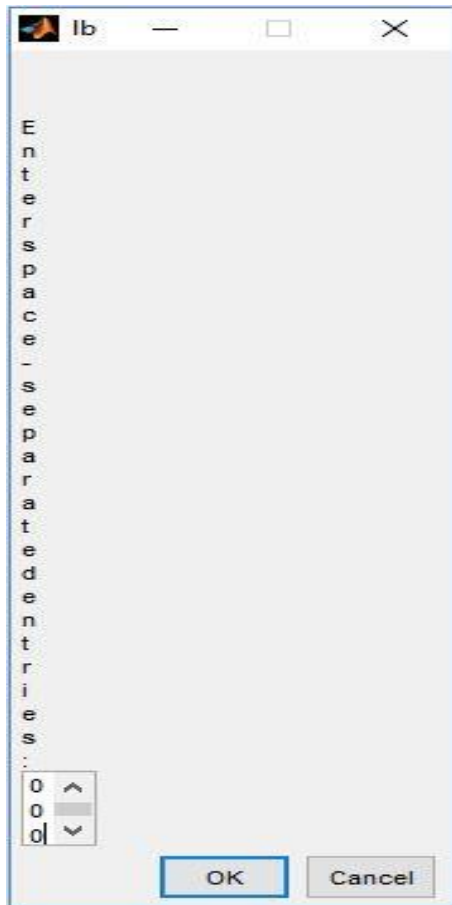


Figure 4.7: Input window requiring lower bounds on all variables (lb)

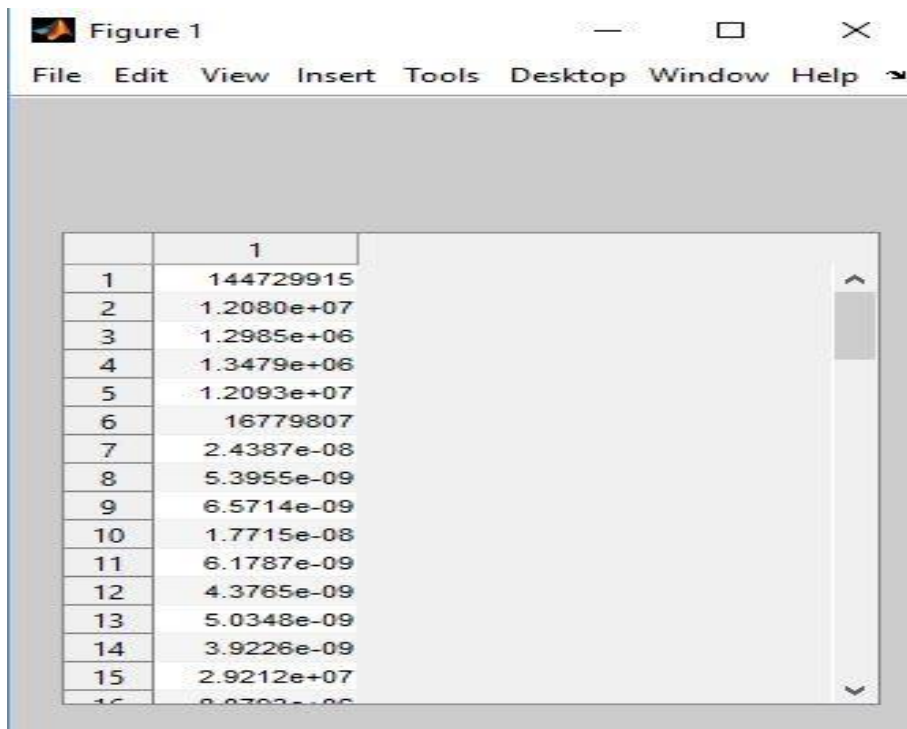
As a result of entering these values the output is obtained in the form of values of decision variables, final value of the objective function and lambda (inequalities and equalities).

CHAPTER 5: RESULTS AND CONCLUSIONS

This chapter comprises of two portions, one deals with the results obtained by solving the mathematical model formulated for fizzy drink industry are discussed, as a result of all the data entered into the GUI the optimal values of the decision variables are achieved which shows the optimal quantities of units to be supplied from FMWs to DCs while choosing the routes having the minimum distance between the sources and destinations, The second portion concludes the research work and shows the progressions made from previous research as well as it also shows the research potential in this sector and the future work that can be done in this field.

5.1. Results

Once the graphical user interface takes all the values from the user, it instantly displays results if there are no entry errors made by the user such as the imbalance between the number of rows in the matrix “A” and number of columns in vector “b” or the imbalance between number of columns of matrix “A” and vector “f”. The results include the values of the decision variables (x) which are basically the number of units transported from first marketing warehouses to the distribution centers as shown below:



	1
1	144729915
2	1.2080e+07
3	1.2985e+06
4	1.3479e+06
5	1.2093e+07
6	16779807
7	2.4387e-08
8	5.3955e-09
9	6.5714e-09
10	1.7715e-08
11	6.1787e-09
12	4.3765e-09
13	5.0348e-09
14	3.9226e-09
15	2.9212e+07
16	0.0703e-06

Figure 5.1: Results showing values of the decision

Also, the optimized value of the objective function is achieved due to the determination of the optimal values of the decision variables which shows the minimum transportation cost for the whole distribution network (fval) as shown below:

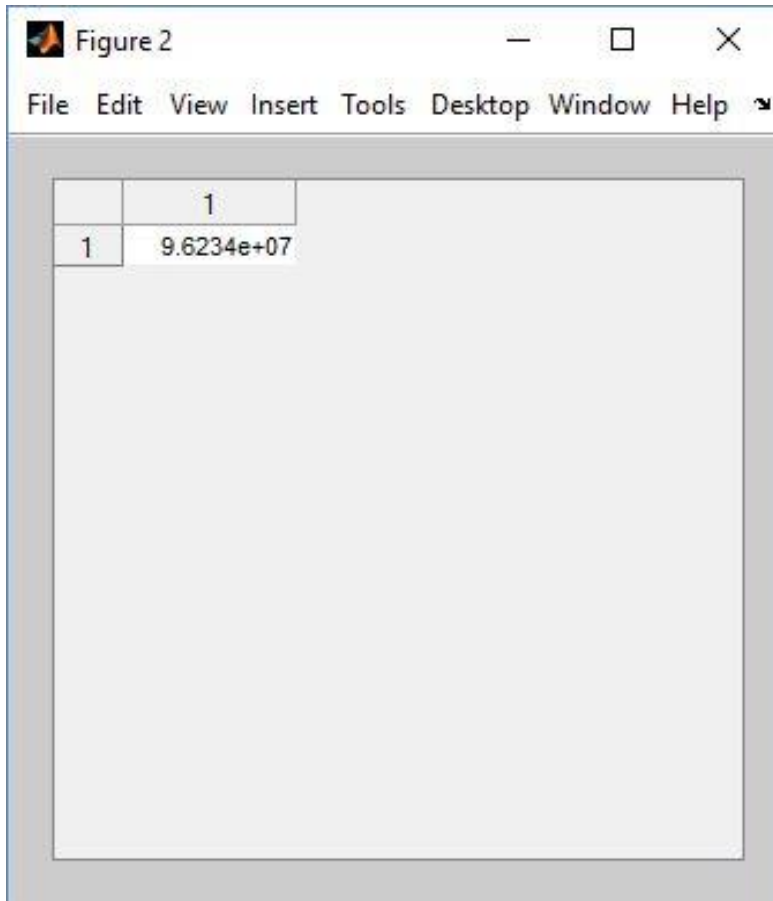


Figure 5.2: Results showing Optimized value of the Objective Function

Also, it outputs lambda which are the Lagrange multipliers at the function which basically show the local minima and maxima of the function which is subjected to inequality constraints which basically means that it graphs all the constraints on the objective function and shows the boundaries of the feasible region beyond which the solution becomes infeasible. It may be valid for equality constraints too but since in this case there is no equality constraint so there are no values associated with the equality constraints are displayed in the output. The langrage multipliers subjected to inequality constraints are shown below:

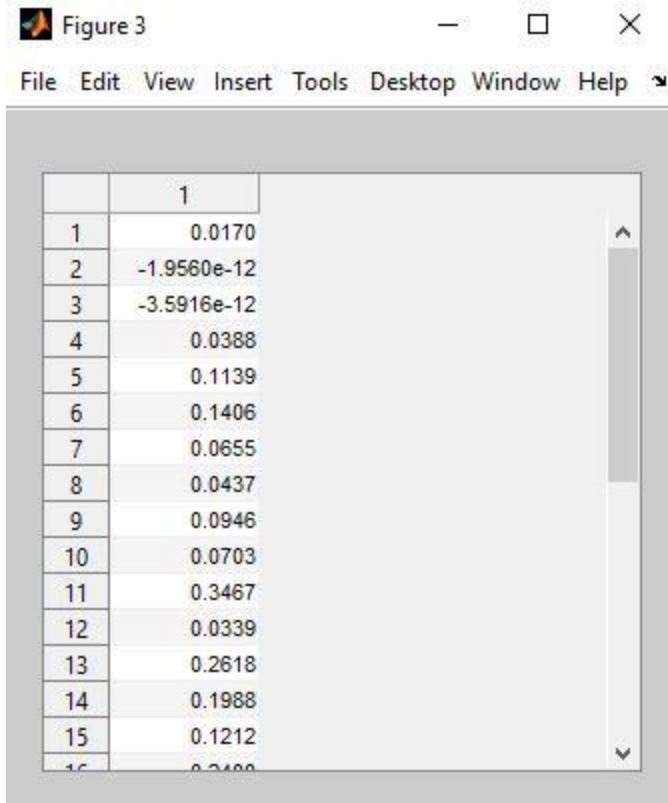


Figure 5.3: Results showing Lagrange Multipliers of a function subjected to Inequality Constraints

In this way the results for any linear mathematical model can be obtained with least knowledge of the software. This graphical user interface can be used for efficiently solving linear mathematical models having 100+ variables and saves a lot of time and effort.

5.2. Conclusions

In this research paper, a mathematical programming approach in the form of mixed integer linear programming has been used to minimize the transportation cost involved in the transportation of certain quantities of units from different sources to different destinations. A case study involving a fizzy drink industry is considered in which a single manufacturing plant produces items which are then transported to 3 first marketing warehouses from where a third party logistics company transports the items to distributions centers all across Khyber Pakhtunkhwa, Pakistan. A mathematical model was formulated involving cost per item, quantity of items to be transported and supply and demand constraints. Then this linear mathematical model was solved using “linprog” solver in MATLAB optimization toolbox. Using this technique the transportation cost of the fizzy drink industry under concentration was significantly decreased. According to the data provided by the company the transportation cost for fiscal year 2016 was known as follows:

Transportation cost before optimization: PKR. 105279517/- for fiscal year 2016

Transportation cost after optimization: PKR. 96233562/- for fiscal year 2016

Difference = $105279517 - 96233562 = 9045955$

Percentage reduction in transportation cost = 9.4%

Also, a graphical user interface was developed using MATLAB so that any linear programming transportation problem can be easily solved by simply plugging in the values in accordance with the mathematical model developed. This approach can be used to simply and efficiently solve transportation problems in less time and with very little chances of mistakes.

5.3: Recommendations for Future Work

In this research thesis, a detailed study was done on the distribution network responsible for the transportation of products from source to destination within the supply chain and also on the mathematical modeling of the entire transportation problem and optimization of the total transportation cost.

It has been concluded that although extensive work has been done in this sector still some areas have a lot of room for improvement and research. One of these areas is the stochastic modeling of the supply chain distribution network since very few papers have addressed the stochastic modeling of supply chain distribution networks having multiple channels.

An algorithm and Graphical User Interface considering multiple objectives like minimization of time taken, environmental constraints, unique organizational values of the firm and bureaucratic decision structures can be developed since nearly no paper has addressed this.

Effort can be made to decrease the amount of carbon emissions and design of a green supply chain network.

Very few papers focus on selecting the transportation models in the supply chain distribution model and also the effect of different supply modes on the supply chain distribution network.

One of the most neglected areas in the supply chain distribution network modeling and optimization is to research on different ways to integrate the forward and reverse flow of material and information and to develop different models and algorithms for it.

A Graphical User Interface for graphical solution of optimization problem can be developed showing the complete graphs with clear indication of the feasible region.

At last, effort can be made and research can be carried out for further refinement in the solving algorithms such as less time taken to find optimum solution of complex problems.

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