Development of Material Model for Assessment of Brittle Cracking Behavior of Plexiglass

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In the name of Allah, the most Beneficent and the most Merciful

Declaration

I hereby affirm that I have developed this thesis wholly on the basis of my personal hard work under the valuable guidance of my supervisor Dr Hasan Aftab Saeed. The Contents have not been plagiarized and sources used are cited. No part of work presented in thesis has been submitted in favour of any application of other degree of qualification to this or any other university or institute of learning.

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Ali Javed Khan

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Ali Javed Khan, 2014

Dedication

I dedicate my thesis to my family and friends. A special feeling of gratitude to my loving parents, whose words of encouragement were always there in my moral support.

I would place sincere thanks to Dr. Wasim Akram for his guidance and support throughout this study, and specially his confidence in me.

I pray ALLAH, for always being kind by way His countless blessings.

Abstract

The objective of this research is to study the brittle cracking behavior of poly methyl methacrylate (PMMA) material when subjected to indentation loading. Experiments were conducted on modified Vickers testing machine to acquire the experimental data in form of load-displacement curves. The mechanical properties like hardness, yield stress and fracture toughness have been determined by analyzing the geometrical dimensions of the indent and from the analysis of a load-displacement curve. In addition, stress strain graph is generated from the load-displacement data. After getting the stress strain graph, a mathematical model is also created for this graph by doing curve fitting. Furthermore, numerical simulation based study was carried out by using a simple elastic model and brittle cracking model in order to simulate the brittle cracking in Plexiglas plate when subjected to indentation loading. These simulations have been performed using the FE solver ABAQUS. Two separate Comparisons are made between the load displacement curves obtain from simple elastic model and brittle cracking model with experimental load displacement curve to validate the FEA procedures and accuracy of predictions. The brittle cracking model in ABAQUS/Explicit is proposed which determine the required force and displacement to produce crack in Plexiglas. The numerical predictions of load-displacement curve and crack initiation and direction were remarkably consistent with experimental results. Total energy absorbed by a Plexiglas plate before fracture is also calculated from both experimental results and by finite element analysis.

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CHAPTER 1

INTRODUCTION

1.1 Background

Engineering materials are classified into five categories: alloys and metal, intermetallics, glasses and ceramics, polymers, and composites. The behavior of these materials can be generally classified as brittle and ductile. Ductile materials generally withstand considerable plastic deformation prior to the specimen ruptures. Whereas a little or no plastic deformation occurs in brittle materials.

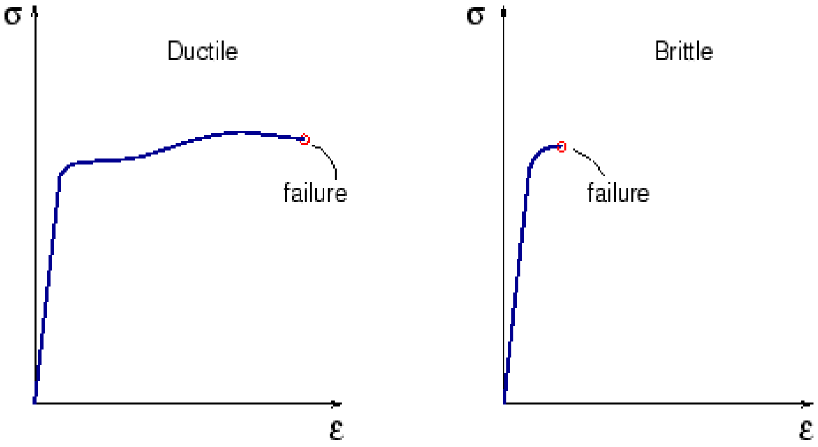


Figure 1: Ductile and brittle material behavior

Microscopically the plastic deformation is because of the movement of dislocations and is not directly associated with an appearance of crack or other discontinuities. The two main types of dislocations, edge dislocation and screw dislocation are shown in Figure 2.

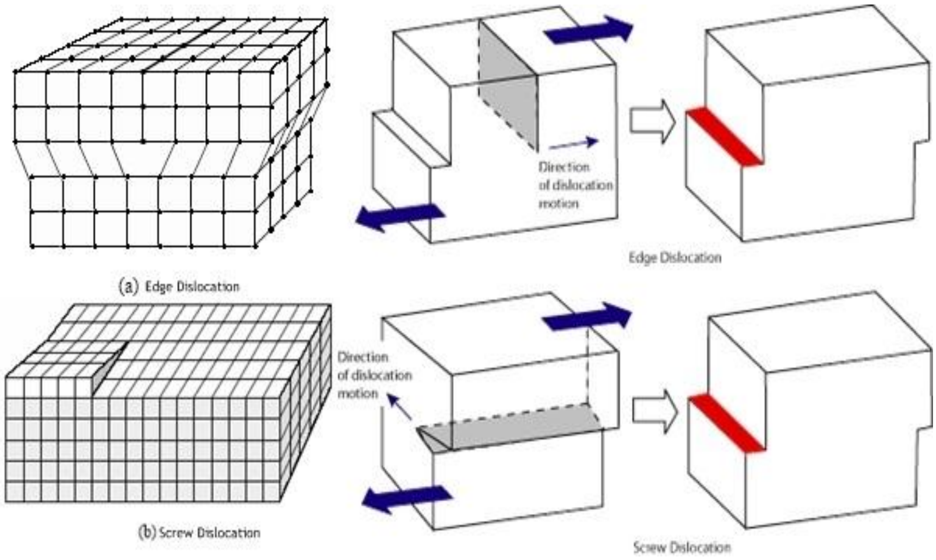


Figure 2: Edge and Screw Dislocation

On the other hand Brittle behavior is accompanied with nucleation and growth of one or numerous cracks. In broader aspect one should say that no absolutely brittle or ductile materials exist. Based on the loading conditions and on stress condition, the material response of almost all solids can be brittle or ductile. Many ductile materials become brittle when the temperature is reduced.

Different experimental techniques have been used for testing the materials. With the help of these techniques we are able to understand the mechanical behavior of materials under different conditions. The design of the experimental technique depends upon the information we wants to extract from the experiment. Unfortunately we can’t afford to do experiments all the time due to certain limitation i.e cost constraint, time consuming, resources limitation etc.

1.2 Modeling & Simulation

The use of numerical simulations for understanding material response under different conditions are becoming common since it is cheaper, safer and more flexible than doing many series of experiments. These simulations are performed using different commercial softwares like Ansys, Nastran, Abaqus etc. These simulations need a material model to describe the response of material under different conditions. Material models have constants which are determined from the acquired experimental data and used in the constitutive equations to predict the behavior of specific material. The accuracy of experimental data is essential feature of successful numerical simulations.

The development of materials modeling has experienced a huge growth in the last few decades, although it is not a new area or topic for research. It has been noted that some material descriptions accepted and used today were actually proposed in the eighteenth century. The role and importance of materials modeling has long been established by the works of Tresca, 1864; von Mises, 1913; and Hencky, 1924. Materials modeling are as vast as the types of existing materials.

For decades, emphasis has been placed on developing the constitutive models for metals. These well established models are used to predict plastic behavior of metals. Some of these commonly used material models include Power-law, Johnson– Cook model [1], Symonds-Cowper model [2], Zerilli–Armstrong model [3] etc.

Nonmetals are difficult to model and characterize because damage and fracture mechanisms in these materials are not easily definable. So unlike numerous available material models for metals, well established models for nonmetals for dynamic loading are not very common. The models available today for non metals are either completely new or in modified form of the existing material models that are used for metals. Establishing a constitutive model for non metal is currently a new area of research at the global level. There are quite few universities which are working in this area through research groups. Some well established material models available today are RDA ceramic failure model [4], Johnson-Holmquist ceramic models [5], Elastic-plastic-cracking model [6] etc.

1.3 Scope of the Thesis

1.3.1 Goal

The goal of this research is to study the brittle cracking behavior of Plexiglas material when subjected to indentation loading. For this purpose modified Vickers testing apparatus is used to ascertained different mechanical properties of Plexiglas and to investigate the force required to initiate a crack in Plexiglas. We will also determine the amount of energy absorbed by Plexiglas sample before fracture. These tests will base on technology that allows for either load-controlled or depth-controlled experiments. Our resulting data will be in the form of load depth curves. Thus from this modified Vickers indentation technique we will acquire the data in form of load displacement curve.

The experimental data will be then used to extract parameters of brittle cracking model for prediction of brittle cracking behavior of Plexiglas. The model will be used in commercial software ABAQUS for simulation of this phenomenon using the FEA procedures. From simulation we acquire the required force and displacement to produce crack in Plexiglas. Finally a comparison of simulation results will be made to the experimental data to validate the FEA procedures and accuracy of predictions.

The final outcome of this work will be in the form of experimental method based on modified Vickers indentation test, a brittle cracking model for Plexiglas material and FEA procedures for brittle cracking simulation using ABAQUS software.

1.3.2 Thesis Contributions

Contributions of this research include:

1. Conduct experiments using Modified Vickers testing apparatus to acquire the experimental data.
2. Calculate different mechanical properties of Plexiglas.
3. Create a mathematical model of Plexiglas for its brittle cracking behaviour during indentation loading.
4. Propose a model and develop a simulation procedure in Abaqus for assessment of brittle cracking behavior of Plexiglas when subjected to indentation loading.
5. Estimate the total energy observed by Plexiglas sample before fracture by using numerical method and finite element analysis.

1.3.3 Outline of the Thesis

The thesis is prepared as follows:

1. In Chapter 2, we talk about the Plexiglas significance and its application. In addition we present a literature survey of some work done on Plexiglas in order to understand its behavior under different mechanical and environmental conditions.
2. In Chapter 3, fracture mechanics approach is discussed in details. Two fundamental approaches to fracture analysis i.e. energy release rate and stress intensity factor are also discussed. In the end of this chapter different classes of fracture mechanics have been laid.
3. In Chapter 4, different indentation techniques are discussed in detail. Various types of indenter tips with recommended applications are also presented. A brief introduction to Indentation Fracture Mechanics has been given at the end of the chapter.

4. In Chapter 5, a detail related to experiments performed in this research is explained. The mechanical properties such as hardness, yield stress and fracture toughness have been determined from the experimental results. In addition stress strain graph is generated. The energy absorbed by Plexiglas sample before fracture is also determined. After getting the stress strain graph, a mathematical model is also created for this graph by doing curve fitting technique.
5. In Chapter 6, a numerical simulation is done in commercial software Abaqus to determine cracking in Plexiglas plate when subjected to indentation loading. The crack prediction in Plexiglas plate by simulation of the experimental data is achieved successfully and a good correlation is observed between experiments and simulations results.
6. In Chapter 7, we summarize and concluded our discussion and also presented recommendation for future work.

CHAPTER 2

PLEXIGLAS

2.1 Background

Plexiglas is the common name for a type of polymer that is a cross between plastic and glass. It is a commercial name of PMMA (poly-methyl-methacrylate). It is a transparent thermoplastic which has many properties of glass. It was first introduced in market in 1933 by the Rohm and Hass Company. It is also sometimes called acrylic glass.

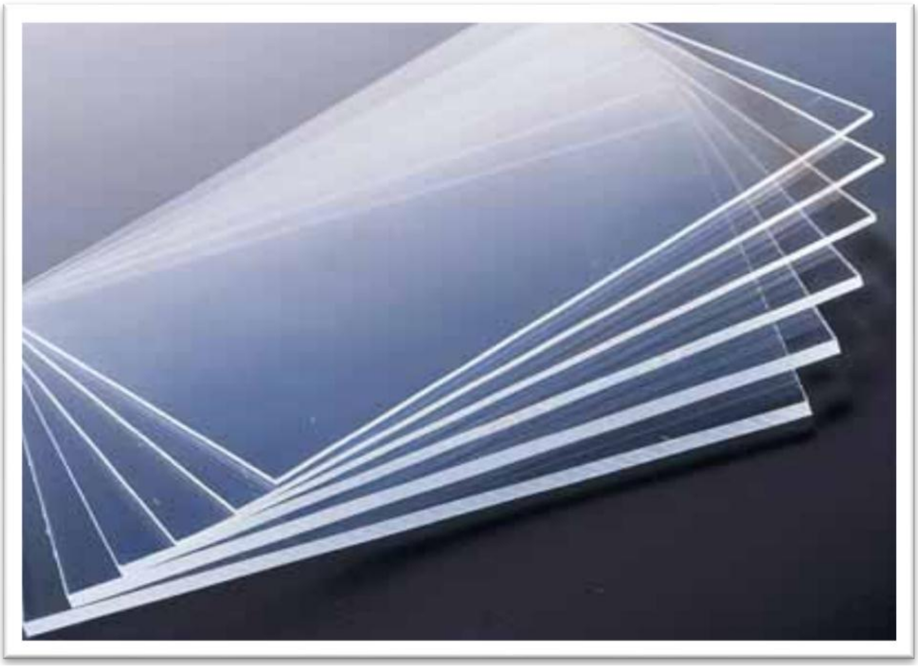


Figure 3: Plexiglas sheets

2.2 Significance

Plexiglas is an extremely useful product and offers many distinct advantages over other materials. Due to its higher impact strength than glass, it outperforms traditional glass in many respects. Also Plexiglas is of great engineering interest due to its wide range of application.

2.3 Application

Plexiglas plates have been widely used in defense, commercial as well as in medical applications due to its transparency and high toughness.

2.3.1 Defense

Some of the common defense applications of Plexiglas are as below

- Bullet proof windows
- Bullet-proof armored vehicles
- Police shields
- Police vehicles for riot control

Figure 4: Defense application of Plexiglas

2.3.2 Commercial

Due to its higher impact strength than glass, it outperforms traditional glass in many commercial applications; few of them are as under

- Motorcycle Helmet
- Automobile windshield And Headlights
- Train locomotive windows
- Aircraft windows
- Aquariums

Figure 5: Commercial application of Plexiglas

2.3.3 Medical

Plexiglas has medical uses as well. The body responds well to PMMA. Some of the common Medical applications of Plexiglas are as below

- Contact lenses
- Artificial fingernails
- Acrylic dentures

Figure 6: Medical application of Plexiglas

2.4 Literature Review

Various researchers and investigators have studied the behaviour of a Plexiglas and other brittle materials under different mechanical and environmental conditions. Tim J. Holmquist et al. [7] presented experimental data for a glass material for different strain rates. The test data are used to obtain constitutive model constants for the JH-2 model. The model is then used to perform numerical simulation of the various tests. Computational comparisons to the impact tests show good correlation.

Rikards et al. [8] developed a numerical and experimental method to determine elastoplastic properties i.e. yield stress, tangential modulus and hardness of different polymers when subjected to Vickers indentation.

Richard J. Anton et al. [9] carried out static and dynamic Vickers indentations on zirconia ceramics and pyrex glass to examine the effect of strain rate on hardness and fracture toughness. It was concluded in their research that the hardness in these brittle material increases with dynamic indentations as compared to static indentations. Also they observed that the fracture toughness of zirconia ceramics increases whereas the fracture toughness of pyrex glass decreases during dynamic indentations.

Zhouhua li et al. [10] investigated the thermo-mechanical behavior of polymethyl methacrylate under compressive dynamic loading. It was found that the material behaviour changes from ductile to brittle as compressive strain rate increases.

W.Zhang et al. [6] developed an elastic-plastic-cracking model for tensile cracking and compressive yielding in brittle materials. The model was then implemented in Abaqus in order to simulate the fracture characteristics of brittle materials during Vickers indentation loading.

Gordon R. Johnson et al. [11] developed new computational model for brittle materials under high pressures and strain rates. They performed a variety of impact computations analysis and computed results show good correlation with experimental results.

N. M. Ames et al. [12] implemented a constitutive model in commercial software ABAQUS/Explicit, to predict the load versus indentation depth (P-h) response of amorphous polymeric solid (PMMA) subjected to micro-indentation.

Pelletier et al. [13] Presented experimental/numerical technique that enables full characterization of the stress-strain behavior of glassy polymers i.e. polycarbonate and polystyrene, when subjected to flat-tip indentation. The elasto-viscoplastic constitutive model is capable of giving good qualitative descriptions of the behavior observed experimentally.

Sai Sarva et al. [14] carried out experimental and analytical investigation to understand the high rate deformation and projectile impact behavior of polycarbonate and polymethylmethacrylate and develop a new constitutive model. This constitutive model was then implemented into a commercial software Abaqus/Explicit. These simulation shows that these polymers have significantly contrasting deformation and failure behavior.

Xin Sun et al. [15] used experimental and fem approach to understand the damage mechanisms of glass ply when subjected to impact loading. The intention of his study is to develop a simulation procedure and to determine the critical stress parameters of continuum damage model during impact loading.

Based on a chain of experiments and by using theoretical model (ZWT), Tao Suo et al. [16] investigate the temperature and strain rate effect on acrylic polymer during quasi-static and dynamic loading. It was established that the yield stress and Young’s modulus decreases with increasing temperature at low strain rate.

S. Swaddiwudhipong et al. [17] implemented a Govaert’s constitutive model as user subroutines in Abaqus. Numerical simulations of indentations are performed to describe the viscoelastic–plastic behavior of glassy polymers. It was found out that the hardness of polymers increases with decreasing indentation depth.

Mulliken et al. [18] performed a combined experimental and analytical investigation to understand the mechanical response of polycarbonate and polymethyl methacrylate at varying strain rates. The constitutive model used in his research is capable to predict the behavior of these materials under various strain rates.

Jewan Ismail et al. [19] implemented a constitutive model into a commercial software to perform the simulations of a glass plate subjected to spherical indentation. The purpose of his study is to predict the initiation and propagation of crack in brittle materials by using a finite element (FE) analysis. Predicted directions were found in good correlation with the experimentally results obtained from the literature.

C. G. Fountzoulas et al. [20] carried out a numerical simulation of ballistic impact on polycarbonate and polymethylmethacrylate by using commercial software Autodyn. The crack reproduction by simulation of the experimental data was achieved successfully by modifying the existing failure models of polymer in the AUTODYN materials library.

Tusit Weerasooriya et al. [21] have studied the fracture toughness of PMMA at different loading rates. Variation of the loading rates reveals that the dynamic fracture toughness is considerably higher than the quasi-static toughness.

G. Subhash et al. [22] presented the deformation behavior and fracture patterns evolved in ceramics during high velocity impact and dynamic indentation loading. Several formation of micro-cracking has been observed under dynamic indentation loading.

N.K. Naik et al. [23] have carried out experimental studies and presented an analytical method to understand the effect of strain rate on tensile properties of acrylic. It was found that the tensile strength and Young's modulus increases, whereas the ultimate tensile strain decreases at high strain rate loading as compared with that at quasi-static strain rate loading.

By using both methods experimental and simulation through commercial software Abaqus, Arjun Tekalur et al. [24] have studied the failure mechanism of PMMA plate when subjected to low velocity impact response. It was observed that the higher velocity took less time to initiate crack.

Alessandro et al. [25] presented a detailed investigation on the consistency of numerical simulations on soda lime glass when subjected to sharp indentations. It was concluded that median and radial cracks are formed at the loading phase when subjected to Vickers lndentation loading. While in the case of Berkovich indentations, the the Palmqvist cracks are formed which were justified by the prediction of stress intensity factor.

Basim Mohammed Fadhel et al. [26] presented simulation-based fem in order to study the behaviour of polycarbonate under impact loading. It was observed in the results that the velocity of impact, the thickness of plate and projectile nose plays a vital part.

Ajmer et al. [27] have conducted serious of experiments on polycarbonate at varying deformation rates and elevated temperatures. Data from the experiments were used to calculate parameters for the JC model and ZA strength model, to investigate the basic mechanical response of PC. It was found that strain rate and temperature were important factors that influence the material behavior.

2.4 Motivation

From the literature, it is ascertained that different numerical, experimental and analytical investigations are carried out in order to study the behavior of Plexiglas and many other non metals under different mechanical and environmental conditions. However, in order to simulate the brittle cracking behavior of Plexiglas, no appropriate model is available in literature. The study of brittle cracking in Plexiglas is of great interest due to its wide range of application as discussed above.

CHAPTER 3

FRACTION MECHANICS

3.1 Introduction

Fracture is one of the most fundamental concepts in Engineering and Materials Science. It can be expressed as separation of a body into pieces by a compelled stress. Fracture mechanics is a technical approach that has been developed to analyze bodies containing cracks. Pre existing cracks as well as newly formed cracks are of crucial because these cracks reduce the strength or load capacity of the material. The field of fracture mechanics was created to develop a fundamental understanding of crack propagation problems. From engineering point of view, fracture mechanics are used as a basis for predicting

a) Critical crack size.
b) Strength of a structure as a function of crack length.
c) Crack growth rates as a function of cycles of load or time under load.
d) Inspection requirement pertaining to size of an admissible crack and the period of time between inspections.

When cracks are indentified in structures or components during service, they must be evaluated to determine suitability for continued operation.

3.2 Ductile vs. Brittle Behavior

Fracture occurs with the initiation and propagation of the crack and mainly classified as brittle and ductile. Generally the main difference between brittle and ductile fracture depends on the amount of plastic deformation that the material undergoes prior to fracture. Ductile materials undergo sufficient plastic deformation prior to fracture, whereas brittle materials show little or no plastic deformation prior to fracture. The stress-strain behaviors for ductile and brittle types of failure are shown in Figure 7.

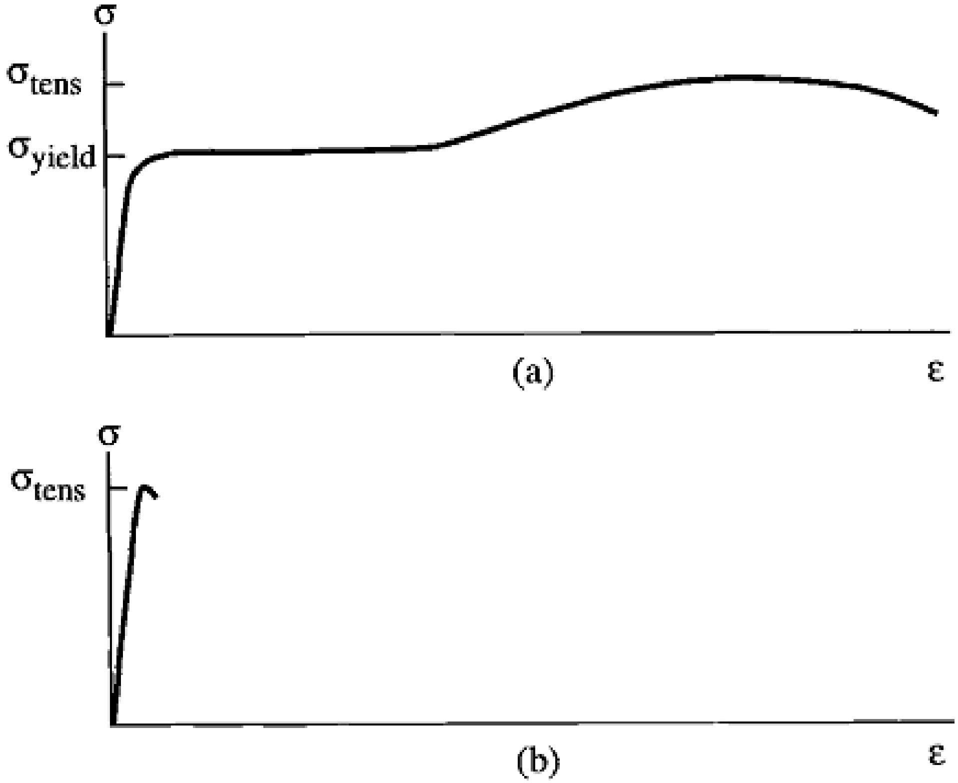


Figure 7: Stress strain curve (a) ductile material (b) brittle material

Ductile fractures normally are characterized by large shear lips with an overall dull fracture surface as shown in Figure 8a. In contrast, brittle fractures are usually distinguished by a flat bright/shiny fracture surface as shown in Figure 8b.

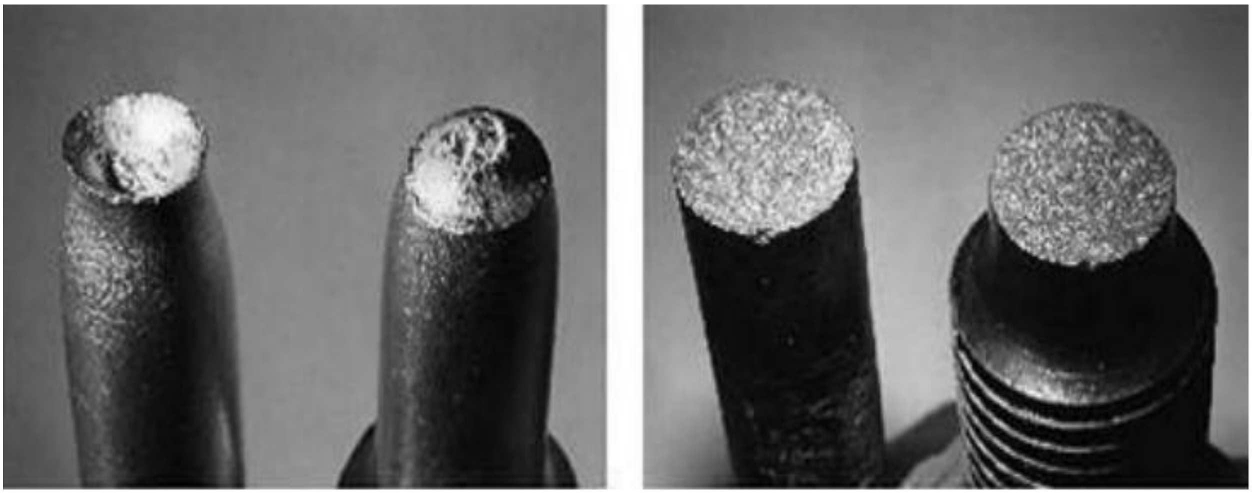


Figure 8: (a) cup-and-cone type fracture in ductile material (b) flat cleavage fracture in brittle material

As we know that crack initiation and propagation leads to fracture. The mode of fracture usually depends on how the crack propagates within a material. The crack propagates steadily and turns out large amount of plastic deformation prior to fracture in case of ductile fracture. Without increasing the applied stress, these cracks will usually not propagate further. Conversely in brittle fracture, the crack propagates quickly with little or no plastic deformation. Once the crack in brittle material started it will grow continuously. One more major characteristic of crack propagation is the way that crack travels within a material. The material experience transgranular fracture when crack passes through the grains of the material. On the other hand material experience intergranular fracture when the crack propagates along the grain boundaries of material. Most metals fail by transgranular fracture. Under high temperatures and by intrusive environment intergranular type of fracture is usually occurs. Figure 9 shows a transgranular and intergranular fracture.

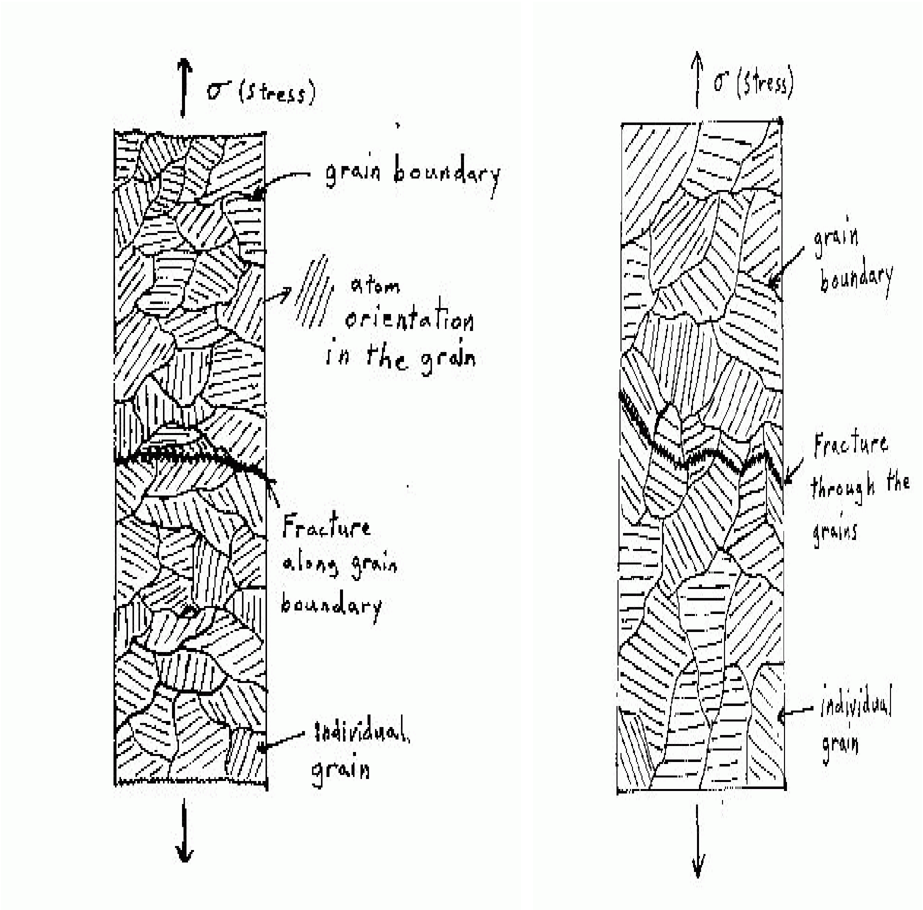


Figure 9:(a) transgranular fracture *(b) intergranular fracture*

In majority of conditions, the ideal material are those that exhibits ductile fracture due to a number of reasons; one of them is that brittle fracture take place very rapidly and disastrously with no word of warning. Ductile materials undergo plastic deformation before material fails, in this way it slow down the process of fracture and we have plenty of time to solve the problem. Secondly additional strain energy is required to cause ductile fracture. For this reason, the engineering applications which involve safety issue the understandable choice are the ductile materials. In material design with the understanding of fracture mechanics of materials, the engineer will develop more secure and reliable products.

3.2.1 Ductile to Brittle Fracture Transition

Sometimes even ductile materials may not show any plastic deformation before fracture. In these cases, the stress-strain curve of ductile material look like as shown in Figure 7b. In engineering practice brittle fracture provides evidence to be one of the most threatening failures occurs in history. Relying on temperature many metallic materials also show evidence of brittle properties. A critical temperature exists known as ductile-brittle transition temperature beneath which the material behave as brittle, while the material behave as ductile above this temperature. Generally the yield strength of material is lesser at high temperatures. On the other hand, the yield strength of material is greater at low temperatures. The vibrations of atoms in the material are larger when the temperature rises. The increase in vibration allows the atoms to break the bonds under stress and build new places with different atoms in the material. As a result of this slippage of atoms plastic deformation occurs, a ordinary characteristic of ductile fracture. The exact opposite is true when temperature decreases. Atom vibration in the material reduces when temperature decreases and therefore the atoms don’t skid to new positions in the material, results in the rapture of atomic bond. This causes little plastic deformation prior to fracture occurs and form a brittle type fracture. Offensive application of a material underneath this temperature can have disastrous consequences for example, the sinking of the Titanic.

Another factor is dislocation density that determines the type of fracture that take place in a material. The materials that have greater dislocation density generate brittle fracture in the material. As we know that that plastic deformation occurs due to the movement of dislocations. Therefore when the stress applied the dislocations increases in a material and it becomes hard for the dislocations to shift because they stack into each other. So a material with high dislocation density can deform a little before it breaks in a brittle way.

The last factor is grain size of material. The fracture becomes further brittle as grains get smaller in a material. This occurrence is due to the fact that in smaller grains, before they strike grain dislocations have less space to move boundary. Therefore the material fracture is more brittle.

3.3 Fracture Analysis

The fundamental principle in any fracture mechanics analysis is to avoid failure. With the intention of achieving it, the resistance of material to crack should be higher than the crack driving force as shown in the figure 10. The material resistance and the crack driving force rely on the fracture mechanics system. The flaw size at which the crack driving force equals the material resistance at a given set of stresses is called the critical flaw size. Generally a safety factor is applied to the material resistance in association with the crack driving force to determine the acceptable flaw size for a component.

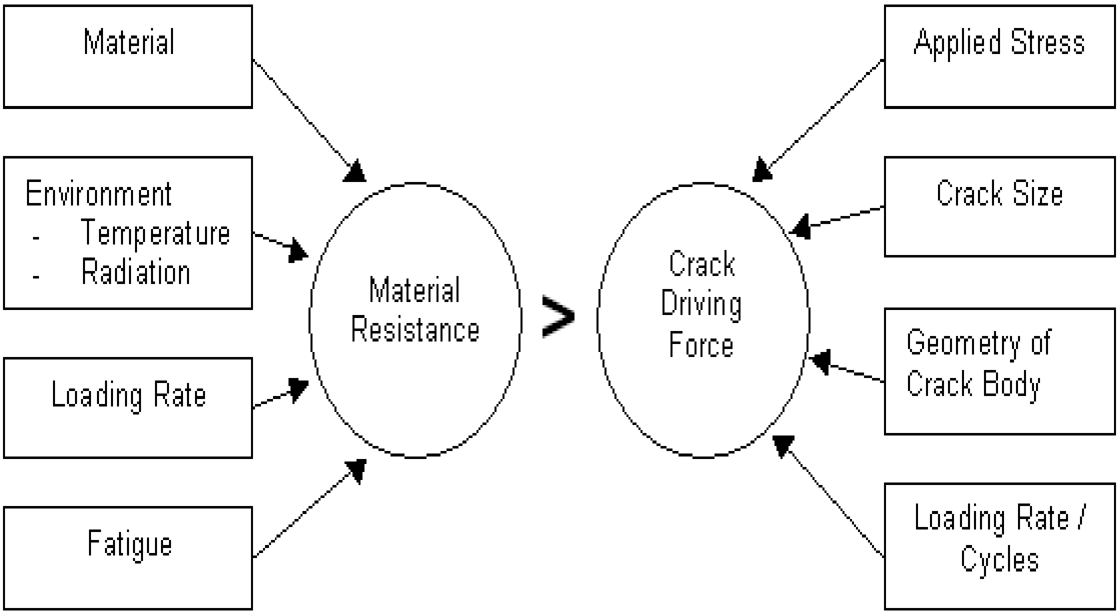


Figure 10: Factors influencing crack driving force and material resistance

The two different approaches to fracture analysis are

- The energy criterion
- The stress intensity approach

These two approaches are equivalent in certain situation. Both of them are discussed briefly below.

3.3.1 The energy criterion

This approach states that when the energy available for crack growth is as much as necessary to conquer the material resistance the crack starts to propagates in a material. The material resistance includes plastic work, surface energy or other energy dissipation related to crack propagation.

Griffith [28] was the pioneer in suggesting the energy criterion for fracture, but Irwin [29] plays the most important role for establishing the present version of this approach i.e. the energy release rate "G". The crack propagates in a material and the fracture occurs, if the energy release rate is greater than the critical energy release rate also known as fracture toughness i.e. $G>G_c$.

When tensile stress is applied to an infinite plate containing a crack of length 2a as shown in Figure 11, the energy release rate is given by

$$\mathcal{G} = \frac{\pi\sigma^2 a}{E} \qquad (3.1)$$

Where

E is modulus of Elasticity
σ is the remotely applied stress
a is the half-crack length

Equation (3.2) expresses the critical combinations of stress and crack size for failure

$$\mathcal{G}_c = \frac{\pi \sigma_f^2 a_c}{E} \tag{3.2}$$

The value the failure stress σ_f varies with $1/\sqrt{(a)}$ for a constant G_c . The energy release rate G is the force that causes fracture, whereas G_c is the resistance to matrial fracture.

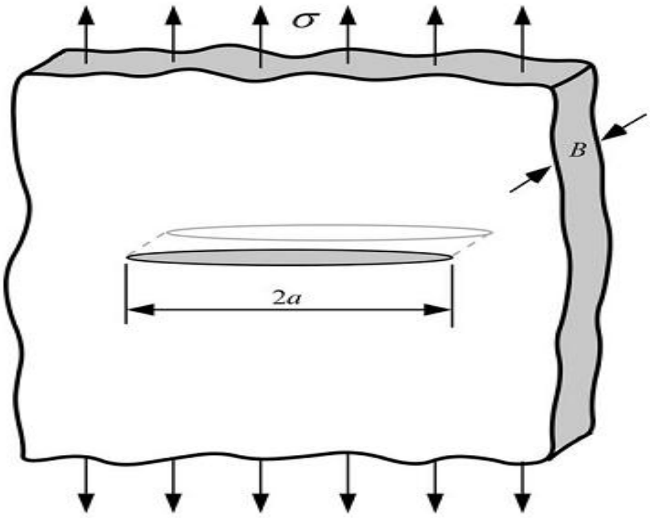


Figure 11: Crack in an infinite plate subject to tensile stress.

3.3.2 The stress intensity approach

In fracture mechanics the stress intensity factor (k_I) is utilized to estimate the stress intensity near the crack tip. When stress turns out to be significant a small crack initiates result in failure of material.

The magnitude of SIF “k_I” depends on:

- Sample geometry
- Size and position of the crack
- extent of load
- division of load

For the plate shown in Figure 11, the stress-intensity factor is given by

$$K_I = \sigma\sqrt{\pi a} \tag{3.3}$$

Where

σ is the stress applied,
a is the crack size,

3.3.2.1 Fracture Toughness

Engineers are more often concerned about the brittle fracture because it brings most shocking accidents happened around the world. When the stress at the crack tip reaches a significant value the brittle fractures take place in material. The fracture toughness at a critical stress situation can be defined as

$$K_{Ic} = Y\,\sigma\sqrt{\pi\, a} \tag{3.4}$$

Where

Y depends on the specimen as well as crack geometry.

3.3.3 Relationship between G and K_I

By going through literature we come to know that many researchers prefer using strain energy release rate more than thestress intensity factor. By comparing equation (3.1) and equation (3.3) the resultionship between K_I and G is established.

$$\mathcal{G} = \frac{K_I^2}{E} \tag{3.5}$$

This same relationship is also used for G_c and K_{IC}. Thus, for linear elastic materials the energy and stress-intensity approaches are basically same.

3.4 Classification of Fracture Mechanics

Fracture mechanics problems are usually categorize as linear elastic fracture mechanics, elastic plastic fracture mechanics and time-dependent fracture mechanics. We can classify fracture mechanics into the following regimes depending upon the material properties. Figure 12 shows a family tree of fracture mechanics field.

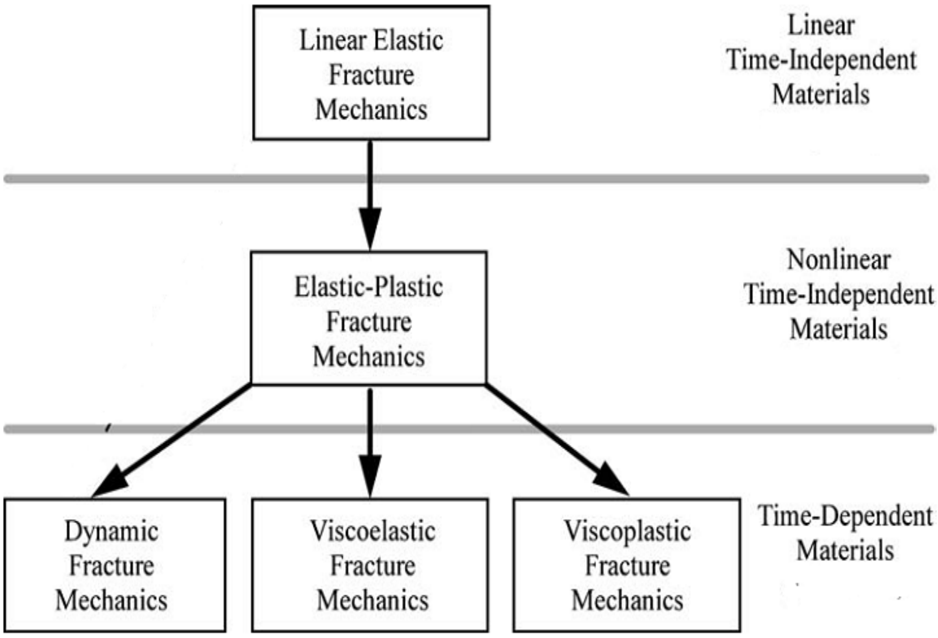


Figure 12: Simplified family tree of fracture mechanics

3.4.1 Linear Elastic Fracture Mechanics (LEFM)

LEFM is the fundamental bone of the majority fracture mechanics analysis. Materials with relatively low fracture resistance fail below their collapse strength and can be analyzed with the use of linear elastic fracture mechanics or we can say that this concept is only applicable to the materials that obey Hook’s law [30]. Linear elastic fracture mechanics is used if the crack tip is sharp and there is only a little amount of plastic deformation close to the crack tip.

3.4.2 Elastic Plastic Fracture Mechanics (EPFM)

EPFM can be applied when the fracture occurrence is larger than that allowable in LEFM. When the plasticity at a crack tip is adequately large it reflects the nonlinear load-deflection behaviour of the material. Mainly it applies to materials that show time independent nonlinear behaviour i.e. large plastic deformation. It is used when the crack tip is not sharp enough.

3.4.3 Dynamic Fracture Mechanics (DFM)

DFM incorporates nonlinear, time dependent material behaviour. It contains three complicating features that are not present in LEFM and EPFM i.e. inertia forces, rate dependent material behaviour and reflected stress waves. Inertia effects are vital when the load changes suddenly or the crack grows quickly because a portion of the work that is related to the specimen is transformed into kinetic energy. Most metals are not sensitive to reasonable variation in strain rate, but the flow stress can increase substantially when strain rate increases by numerous orders of magnitude. The consequence of rapid loading is even more obvious in rate sensitive materials such as polymers. When the load changes rapidly or the crack grows quickly, stress waves transmit through the specimen and reflect off from free surfaces such as material boundaries and the crack plane. Reflecting stress wave affect the fracture behaviour.

The dynamic version of LEFM is termed as elasto-dynamic fracture mechanics, where nonlinear material behaviour is ignored, but inertia forces and reflected stress waves are incorporated when necessary [30].

3.4.4 Viscoelastic Fracture Mechanics

Polymeric materials have seen increasing service in structural applications in recent years. Consequently, the fracture resistance of these materials has become an important consideration. Much of the fracture mechanics methodology that was developed for metals is not directly transferable to polymers because they behave in a viscoelastic manner.

Viscoelastic fracture mechanics used to model materials exhibits a gradual increase in the strain when loaded at constant stress. This shows hysteresis during cyclic loading e.g. polymer based composites [30].

3.4.5 Viscoplastic Fracture Mechanics

Rate independent metal plasticity is used to model permanent deformation in metal loaded above their yield point. Viscoplastic fracture mechanics is similar in nature to metal plasticity but accounts for the tendency of the flow stress of a material to increase when deform at high strain rates [30].

3.5 Effect of material properties on Fracture

The property of the material primarily decides which type or class of fracture mechanics must be applied to a certain problem. Figure 13 shows a plot of fracture toughness versus failure stress. As we can see in the plot that for low toughness values the critical stress varies linearly with K_{IC} and the brittle fracture is the leading failure process. LEFM is no longer valid at very high toughness values, and the failure is administrate by flow properties of the material. There is a conversion between brittle fracture under linear elastic conditions and ductile excess at the intermediate toughness levels. If toughness value is small, LEFM is valid for the problem. The fracture mechanics come to an end for a problem that has high toughness value, because failure stress is insensitive to toughness. Then a straightforward limit load study is mandatory to forecast failure stress in a material.

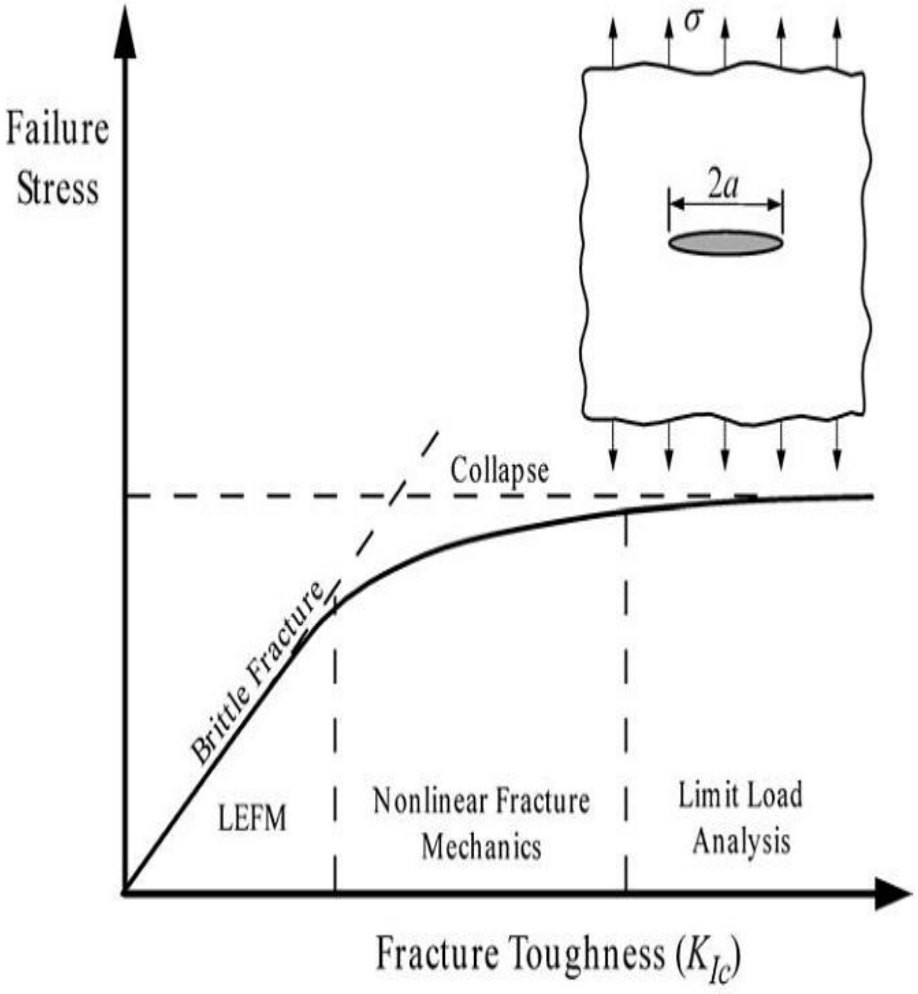


Figure 13: Effect of fracture toughness on the failure mechanism.

A list of different materials in concert with usual fracture regime for each material is shown in Table 1. Here chapter 3 finishes with a complete explanation for a need of fracture mechanics. Fracture mechanics helps us to save any big and unpleasant failure before it occurs.

Table 1

List of different materials with typical fracture regime

Materials	Typical Fracture Behaviour
High Strength steel	Linear elastic
Low/Medium Strength steel	Elastic-Plastic/fully plastic
Austenitic stainless steel	Fully plastic
Metals at high strain rates	Dynamic Viscoplastic
Metals at high temperature	Viscoplastic
Monolithic ceramics	Linear elastic
Ceramics at high temperature	Viscoplastic
Polymers	Linear Elastic/viscoelastic

CHAPTER 4

INDENTATION FRACTURE MECHANICS

4.1 Indentation:

The general principle of indentation test consists of pressing a hard indenter into a flat surface of a material under a specific load for a definite interval of time. Indentation tests have been commonly employed because they do not require elaborate sample preparation and straightforward to execute. Initially they were used for determining the hardness of a material. With the help of computer controlled testing, we are able to find out hardness, the elastic modulus, the yield strength and the energy occupied in the indentation process.

Now a day's indentation techniques have also been used to find hardness, fracture toughness and other mechanical properties of material [31-33]. The mechanical properties are determined by analyzing the geometrical dimensions of the residual indent or from the examination of a load-depth curve.

In indentation tests, the size and shape of the indenter and the magnitude of the load applied are selected in accordance with the

- Purpose of the test
- The structural properties of the material
- The state of its surface and
- The size of the part or specimen

Based on the magnitude of the load applied, the indentation test are generally classified as follow

- Macro Indentation
- Micro indentation

The surface of a material to be indented relies on the type of indentation test performed and the magnitude of load applied. Usually a milled surface is acceptable for macro indentation tests whereas a refined or polished surface is required for micro indentation tests. Both tests are discussed in brief.

4.1.1 Macro Indentation

Macro-indentation tests can be applied with heavier loads than micro-indentation tests. The magnitude of applied test load should be 1 kgf or more. Macro indentation is a fast and straightforward method for acquiring mechanical properties of the material. It is also extensively used for the quality manage of surface treatments processes.

4.1.2 Micro Indentation

In micro-indentation test, a diamond indenter of definite geometry is indented into the surface of the material by means of a notable applied load, ranges from 1gf to 1000 gf. Generally micro-indentation tests have forces of approximately 200 gf and produce tiny indentations. The size of indentation formed is small enough that a powerful microscope is necessary to evaluate the size of indentation. This technique is used for testing very thin materials i.e. up to 0.01 mm thickness.

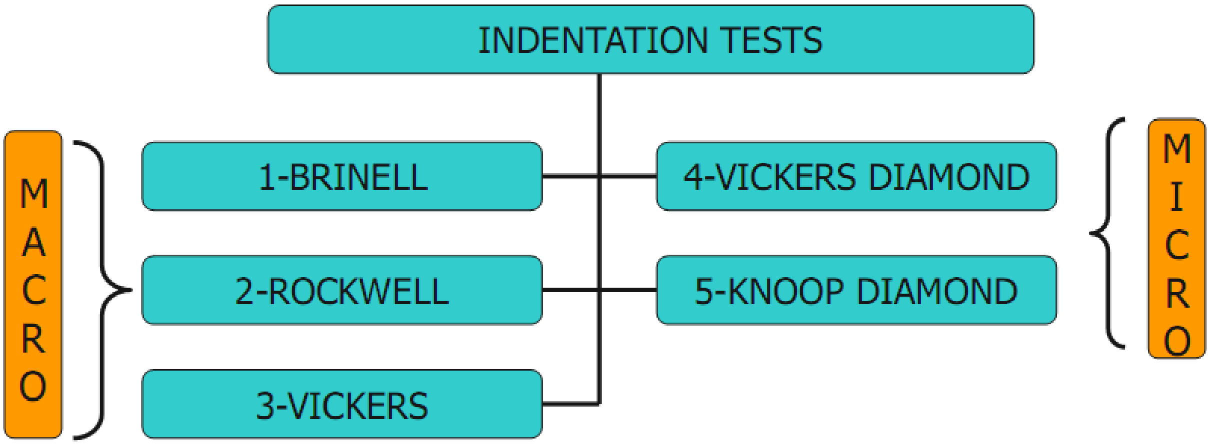


Figure 14: Types of indentation Tests

4.2 Indentation Tests Methods

The most common indentation test methods are briefly discussed below

4.2.1 Rockwell Test

Rockwell test method is used for production control. This method is used to determine the indentation depth produced by the indenter. Initially with the help of conical indenter, a minor load is applied to a material known as preload. Depending on the range, the conical indenter is made of either sintered carbide or steel. This load stands for zero or reference position and it reduces the effects of surface finish. After this preload, a major load is applied on the specimen up to a certain limit. This load is applied for fixed amount of time known as dwell time. Then this load is then removed and the total depth of the indentation is measured from the position derived from the preload to the major load. Rockwell tests are of basically two types depending on the range of test load. Regular Rockwell test have minor load usually equals to 10 kgf whereas the major loads are equal to 70, 100 or 150 kgf. Superficial Rockwell test have minor load equals to 3 kgf and major loads are equal to 15, 30 or 50 kgf.

Rockwell can be used for the majority of materials but usually it can be used for larger sized materials because of the high indentation loads.

4.2.2 Brinell Test

The Brinell test method consists of pressing a carbide ball of predetermined diameter into the specimen for a fixed time period and then removed. These tests often use a very high test load i.e.3000 kgf and a 10mm wide indenter. For soft materials the load is reduced to 1500 or 500 kg. Brinell test is appropriate for metals which contain coarse structural elements. These tests are restricted to larger specimens because of high loads and indenters used.

4.2.3 Vickers Test

The Vickers test method is typically used for thin specimens. The Vickers testing is similar to the Brinell test, but a diamond indenter is used for testing instead of using the steel ball type indenter. In Vickers indentation test usually an indenter of a load ranges from 10 gf to 100 kgf is indented into the surface of the specimen. The complete load is generally applied for 10 to 15 seconds. The indenter used in this test is a square-based pyramid and the angle among the two opposite sides is equal to 136^0 as shown in Figure 15. Vickers test is the most adaptable method which can be used for almost all materials and numerous applications.

4.2.4 Knoop Test

knoop test method was mainly developed to overcome cracking in brittle materials. The indenter used in this test is a pyramid diamond which is rectangular in shape, with the angle between two of the opposite faces is approximately 170^o and the angle between the two other remaining sides is approximately 130^o as shown in Figure 15. The test load used in knoop test ranges from 10 gf to 1 kgf. Knoop test is more delicate to surface grounding and have less loads range as compared to Vickers test method. It is in appropriate for ceramics which needs a good surface finish.

4.3 Select the Correct Indenter Tip

The indenters used in these above tests have different geometrical shapes such as spherical, conical or pyramidal etc. Select the correct tip is essential for our application. There are four main types of indenter tips as shown in Figure 15, each have different geometry for a range of applications.

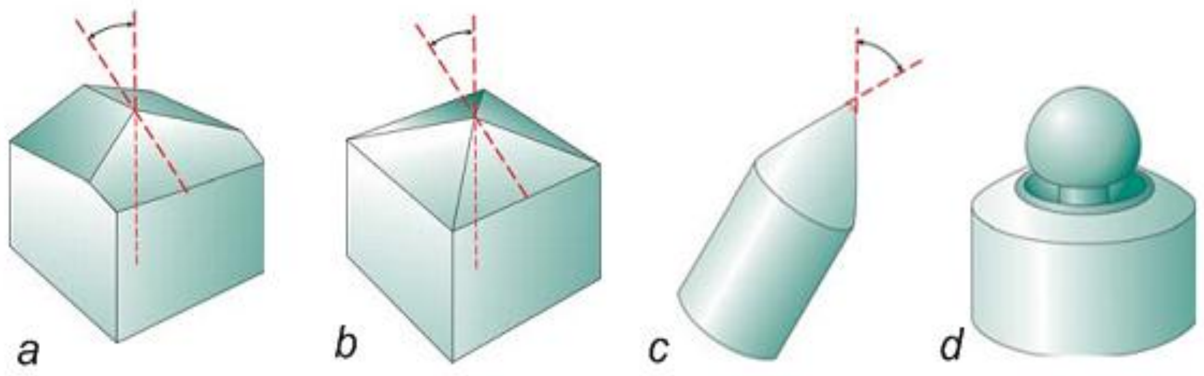


Figure 15: (a) Berkovich indenter tip (b) Vickers indenter tip (c) Cone indenter tip (d) Sphere indenter tip.

4.3.1 Berkovich Recommended Applications

The indenter tip of Berkovich has an angle of 142.3° in order to lessen the affect of friction. Some applications appropriate for the Berkovich indenter tips include:

- Bulk Materials
- Thin Films
- Polymers (E > 1GPa)
- Scratch Testing
- Wear Testing

4.3.2 Vickers Recommended Applications

The Vickers indenter tip is a four-sided pyramid with angle among the two opposite sides is equal to 136^0. Some suitable applications for the Vickers indenter tips include:

- Bulk Materials
- Films and Foils
- Scratch Testing
- Wear Testing
- Fracture toughness

4.3.3 Cone Recommended Applications

The cone indenter has a sharp tip with self-similar geometry. Some appropriate applications for the Cone indenter tips include:

- Fracture Toughness
- Wear Testing
- Micro-electromechanical Systems (MEMS)
- Scratch Testing

4.3.4 Sphere Recommended Applications

When using a spherical indenter tip stresses develop differently during indentation as compared to a Berkovich or Vickers tip. For spherical indenters, the contact stresses are small in the beginning and generate only elastic deformation. After spherical indenter is indented more into the surface of sample, a conversion from elastic to plastic deformation occurs, which can be used to study yielding and work hardening. In general, Sphere indenter tip is utilized for MEMS applications.

4.4 Introduction to Indentation Fracture Mechanics

Indentation fracture mechanics approach is most widely used to estimate the fracture characteristics of brittle materials. Experiments and Simulations have indicated that when brittle materials are subjected to indentation various form of cracking is observed [19, 22, 25, 34, 35, 36] including median, radial, lateral cracking etc (shown in Figure 16). The nature of cracking generally depends on the shape of the indenter and on the properties of the material. Fracture occurs with the initiation and propagation of these cracks. Therefore a qualitative study should be made in order to understand the phenomena of this cracking, to determine the influencing factors and to find the most effective means of reducing or preventing such damage.

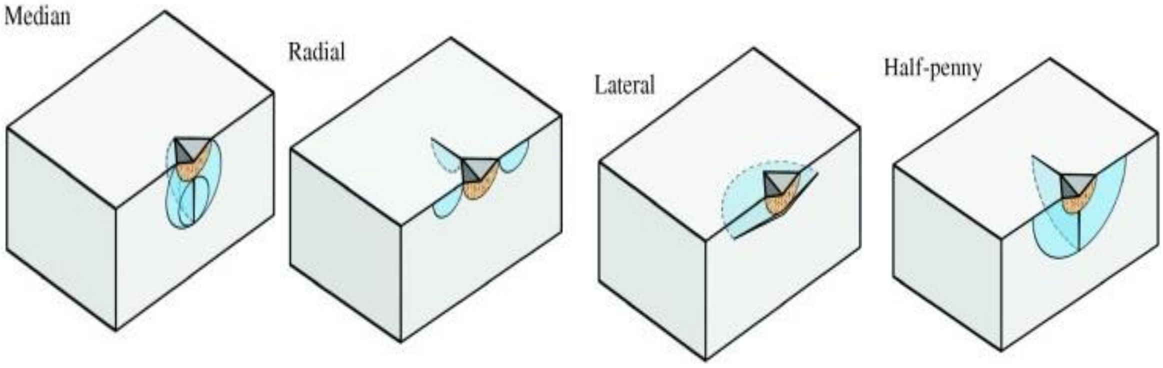


Figure 16: Common cracking patterns observed in brittle materials with sharp indentation

A necessary condition for crack to occur for the duration of the loading phase when subjected to sharp tip indentation loading is that the materials show evidence of high strain hardening, i.e. a comparatively high hardness “H” to Young’s modulus “E” ratio.

The fundamental equation used to estimate the material fracture toughness Kc by indentation is:

$$K_c = \xi \left(\frac{E}{H} \right)^{1/2} \frac{P}{c_0^{3/2}}, \quad c_0 \gg a \tag{4.1}$$

Where P is the maximum applied load, C_0 the surface crack length, α is the dimension of the impression and ξ is a constant which depend on the shape of the indenter.

4.5 Vickers Indentation Fracture Test

The most common method to examine the fracture characteristics of brittle materials is Vicker indentation test [6, 9, 37, 38]. This is use to explore the rate effects in hardness and fracture toughness of brittle material. Generally in Vickers indentation, an indenter is forced into the surface of a specimen using a load ranges from 10 gf to 100 kgf. The complete load is applied for 10 to 15 seconds normally. The indenter is a square-based diamond pyramid amoung the angle between the two opposite sides is equal to 136^0 as shown in Figure 17. From the literature it has been well recognized that radial and median cracks grow normal to the surface of specimen for the duration of the loading phase and lateral cracks grow parallel to the surface for the duration of the unloading phase. Now a day's depth-sensing indentation techniques have been used extensively with Vickers indentation test. This technique gives an indentation load-depth (P-h) curve which can be used to find out different mechanical properties of brittle materials for example hardness, modulus and fracture toughness.

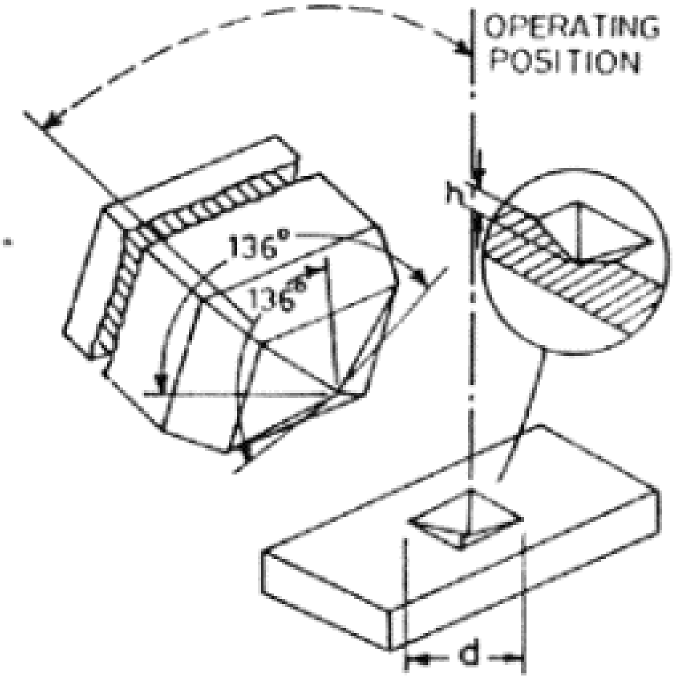


Figure 17: Vickers indentation

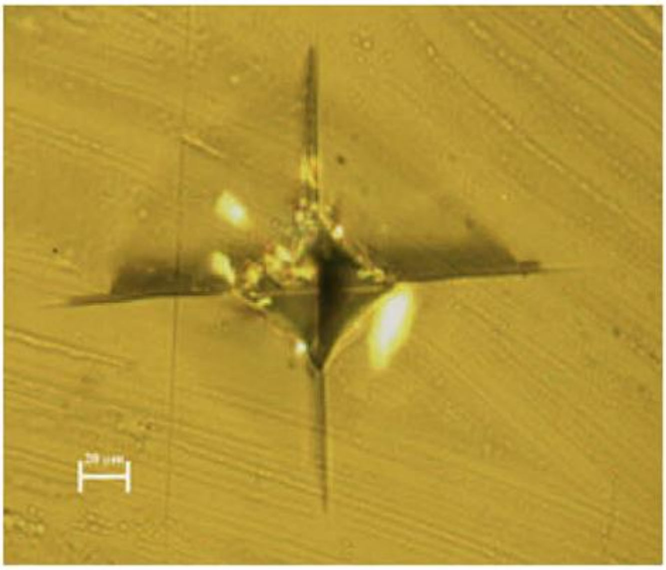


Figure 18: Vickers indentations on glass showing the radial cracks

CHAPTER 5

EXPERIMENTAL PROCEDURE

We Performed hybrid sort of experiments in which a hard indenter was pressed into a flat surface of Plexiglas under a specific load for a definite period of time. The indenter employed in the test was conical with self-similar geometry as shown in Figure 19. The Plexiglas specimen has 25.4 mm diameter and 3 mm thickness as shown in Figure 20. The time was few seconds to reach the maximum load regardless of its volume. These tests were based on technology that allows for either load-controlled or depth-controlled experiments. During these tests, we continuously measured the applied load, penetration depth and cycle time. The mechanical properties were determined by analyzing the geometrical dimensions of the indent or from the study of a load-displacement curve. All the experiments were performed at room temperature. Our resulting data is in the form of load depth curves. In our final output, we have both the plots and the pictures of cracking. The same tests were repeated thrice, and similar results were obtained repeatedly.

Figure 19: Cone indenter tip

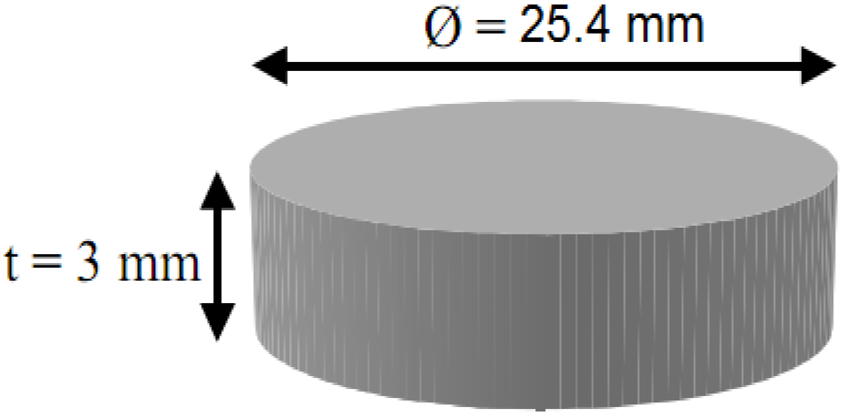


Figure 20: Dimensions of sample

5.1 Results and discussion

5.1.1 Indentation cracking in Plexiglas plate

Figure 21 shows an image after a conical indenter forced into the surface of Plexiglas. Two cracks were initiated from the egde of the indent. In literature, these kinds of cracks are referred to as radial cracks [6, 25, 39, 40, 41]. Therefore, it can be seen in the experiments that when a conical indenter was indented into Plexiglas materials, radial cracks developed normal to the surface after a critical load has been reached.

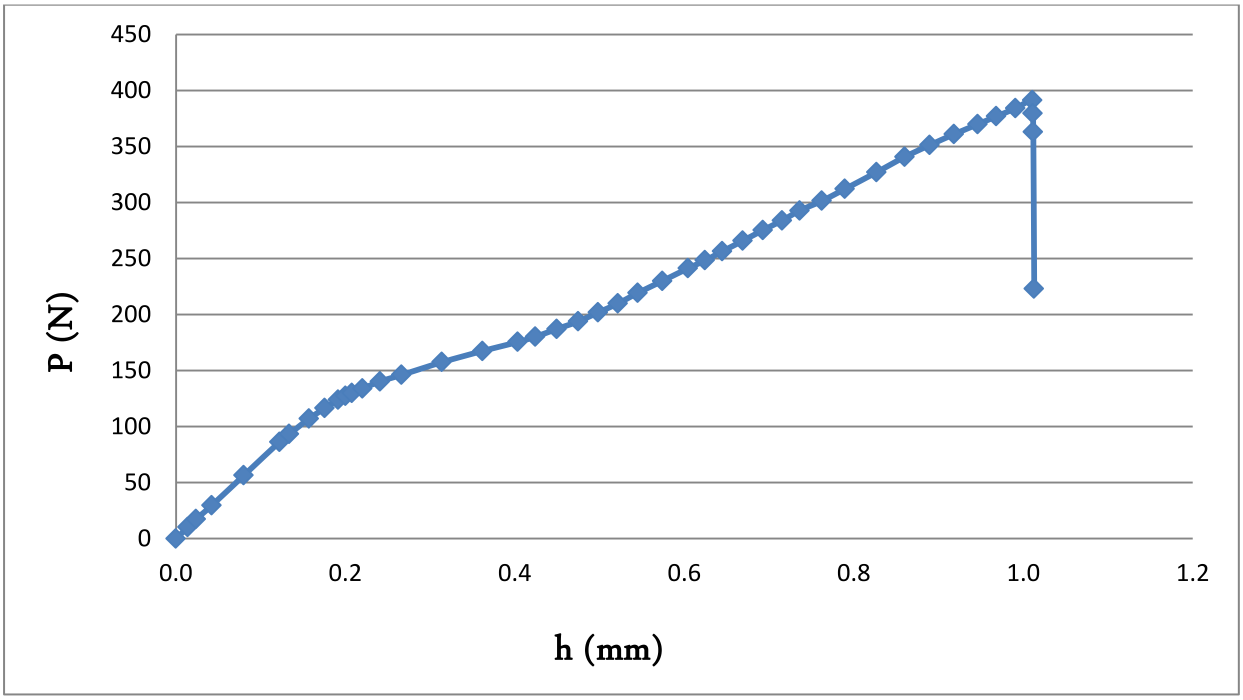
Figure 21: Indentation in Plexiglas with the conical indenter showing well defined radial cracks

After indentation test size of the existing radial cracks can be measured. This parameter combined with the elastic modulus of the specimen, hardness and maximum indenter force gives a possibility to calculate crack toughness parameter.

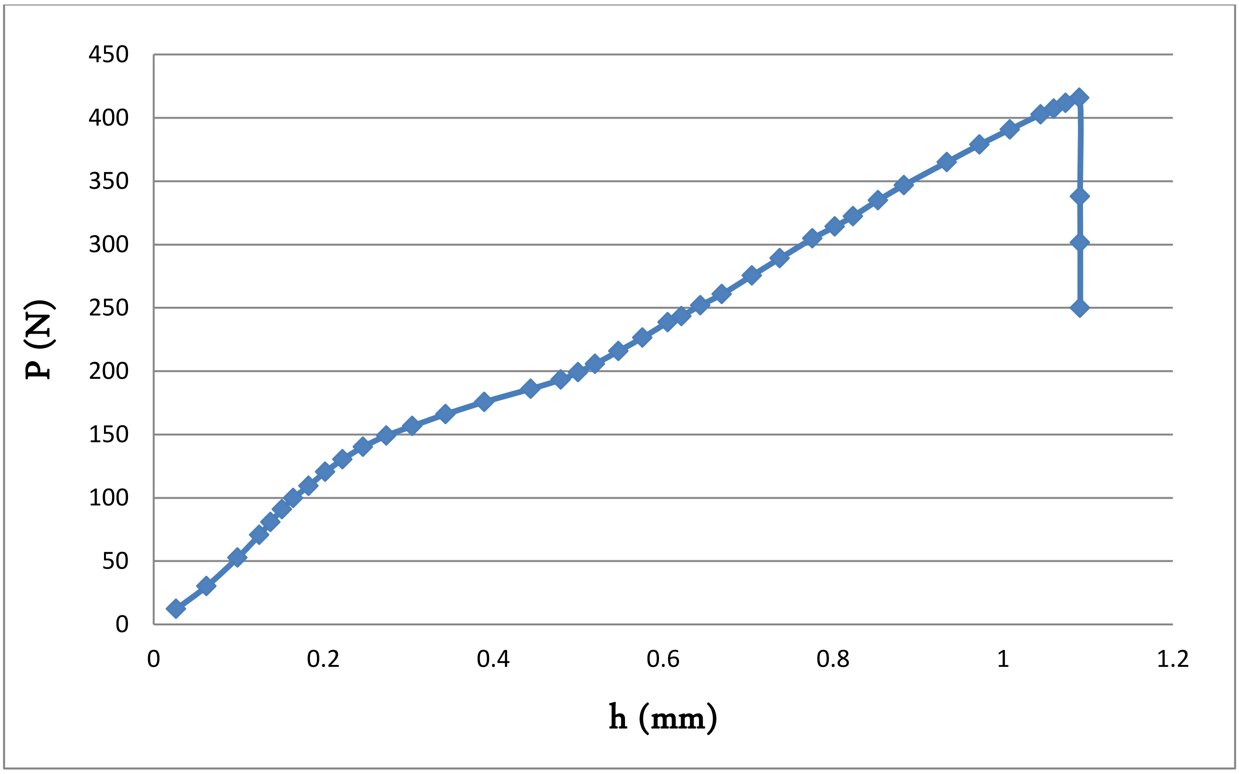
5.1.2 Analysis of indentation load displacment curve

Load displacement (P-h) curve is the straightforward characterization of indentation results, which could be precisely obtained by instrumented indentation equipment. The data in the form of load-displacement acquired from the indentation process may contain information about elastic deformation, plastic deformation, hardness, fracture, maximum crack load and creep of materials. Primary information about an investigated material in this test was described by the load-displacement curve.

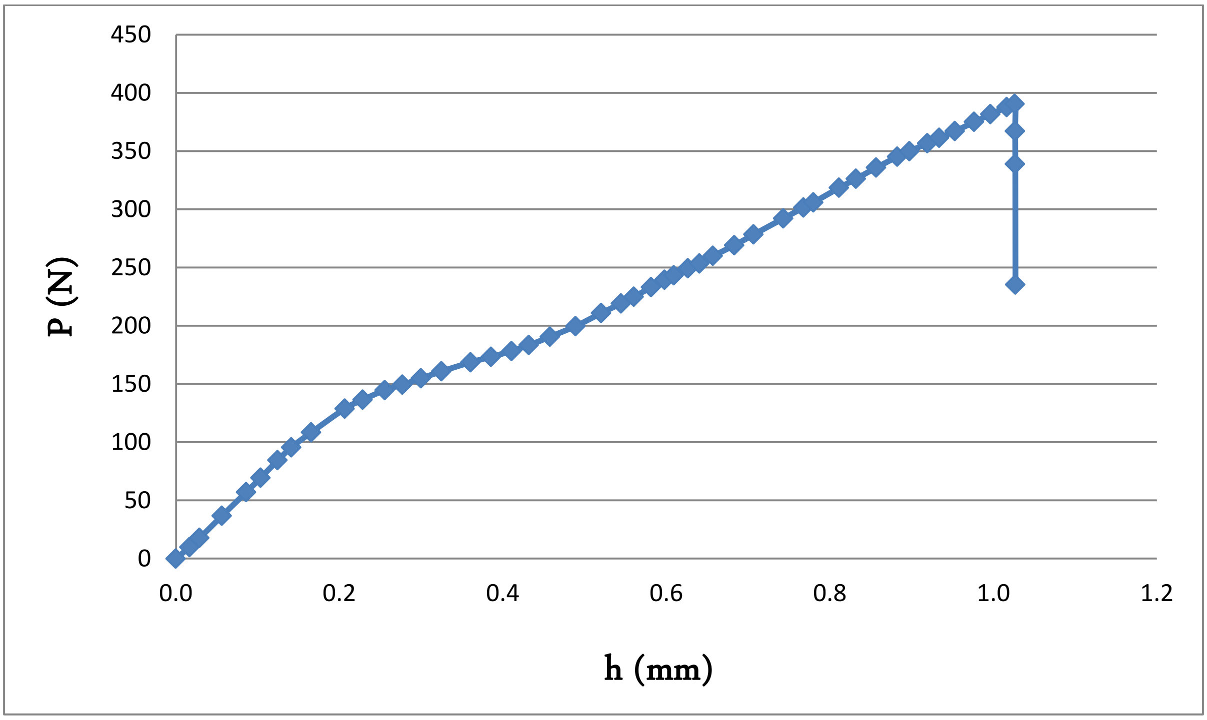
Figure 22 displays the load–deflection curves obtained for the Plexiglas material used in this study.



22(a)



22(b)



22(c)

Figure 22: load–deflection curves

Figure 23 shows the average force displacement curve of the above three experimental P-h curves.

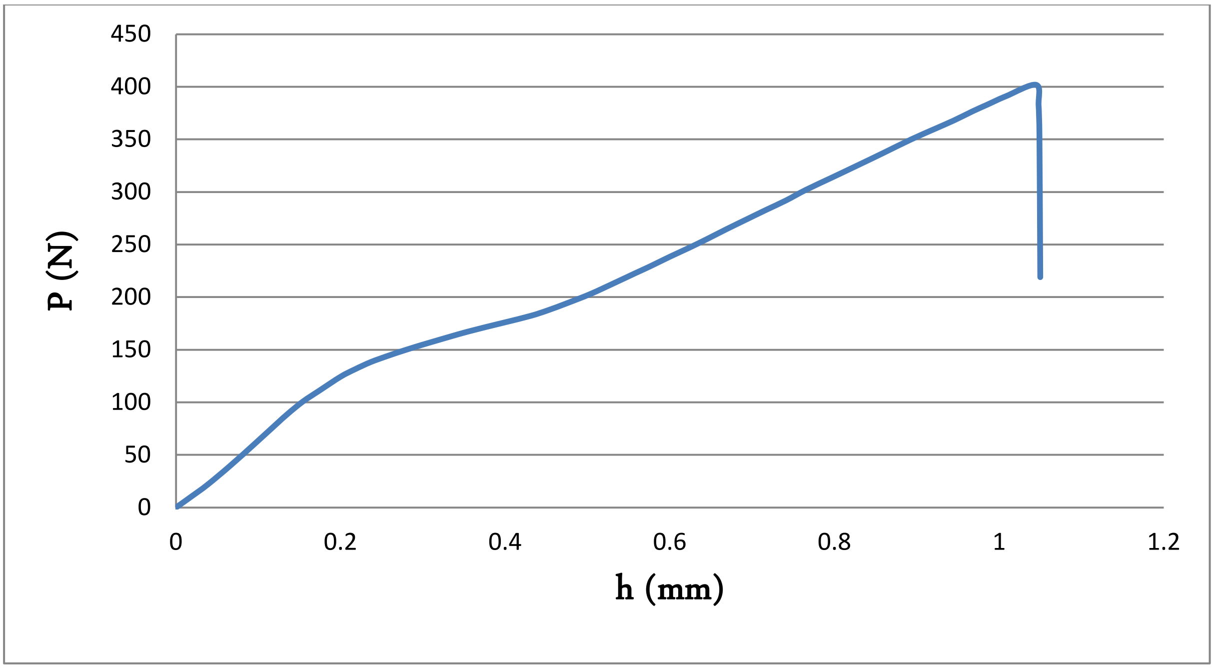


Figure 23: Average load–deflection curve

As shown in figure 23, when the indentation force reaches to value of 400 N, the crack initiated in Plexiglas material. With the help of this P-h curve we can estimate different mechanical properties like the hardness and fracture toughness of material.

5.1.3 Measurement of hardness and yield stress

Hardness is an important property of a material that permits it to oppose deformation, typically by penetration of a hard indenter. The indentation hardness was calculated by dividing the peak indentation load by the projected contact area, defined as.

$$H=P_{max}/A_p \tag{5.1}$$

For conical indenter, the projected contact area can be determined by

$$A_p = \pi h_c^2 \tan^2 \theta \tag{5.2}$$

Where the contact depth h_c can be estimated as

$$h_c=(2/\pi)h \tag{5.3}$$

Here h is the maximum indentation depth shown in Figure 24.

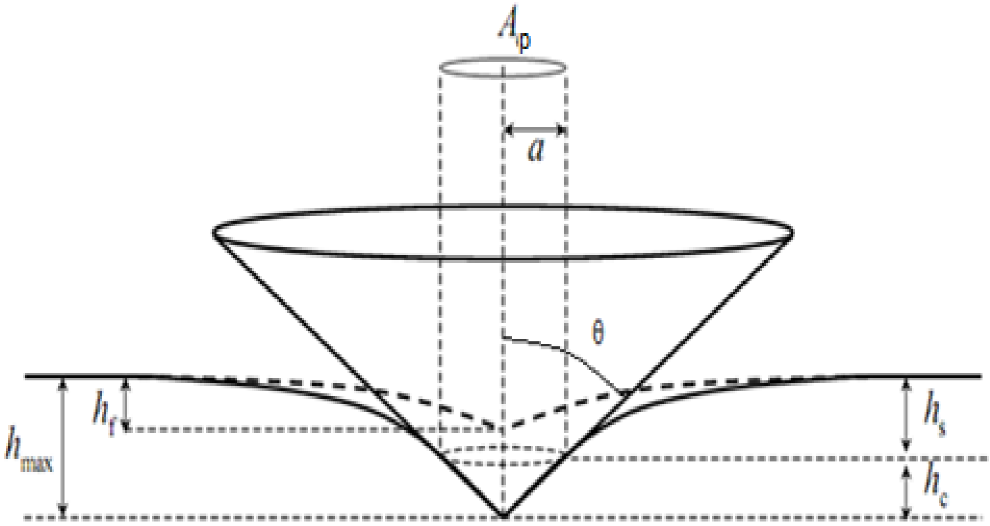


Figure 24: Geometry to characterize Indentation

The specimen's yield stress, σ_Y, is directly proportional to the hardness and can expressed as

$$H \approx C.\sigma_Y \tag{5.4}$$

Where C ≈ 3 for materials with a large ratio of E/ σ_Y such as metals, and C ≈ 1.5 for materials with a low ratio of E/ σ_Y such as polymers, glass etc.

By substituting the values in equations (5.1) and (5.4), the indentation hardness and yield stress can be calculated as shown in Table 2.

Table 2

Properties of materials used in measurement of indentation hardness and yield stress

Material	h_c (mm)	Ap (mm^2)	P (N)	H (MPa)	σ_Y (MPa)
Plexiglass	0.664	4.155	400	96	64

5.1.4 Estimation of fracture toughness

Measuring the fracture toughness of brittle materials can often be challenging. Different indentation techniques could be used to find out the fracture toughness of brittle materials. Evaluating fracture toughness from the measurements of cracks formed by using a sharp tip indenter is an attractive substitute to traditional fracture toughness testing techniques. As is evident from figure 21, when a sharp tip such as conical indenter is pressed into brittle materials, radial cracking generally occurs after a significant load is reached, which allow us to determine fracture toughness of material.

The fracture toughness is based on the peak indentation load and the critical crack length. The equation used to evaluate the material fracture toughness Kc by indentation is

$$K_C = \alpha\left[\frac{E}{H}\right]^{1/2} + \left[\frac{P}{C_0^{3/2}}\right] \tag{5.5}$$

Where P is the indentation load, H is the hardness, and c is the average radial crack length as shown in Figure 25. α is a constant and its is equal to 0.04 for sharp tip indentation geometries.

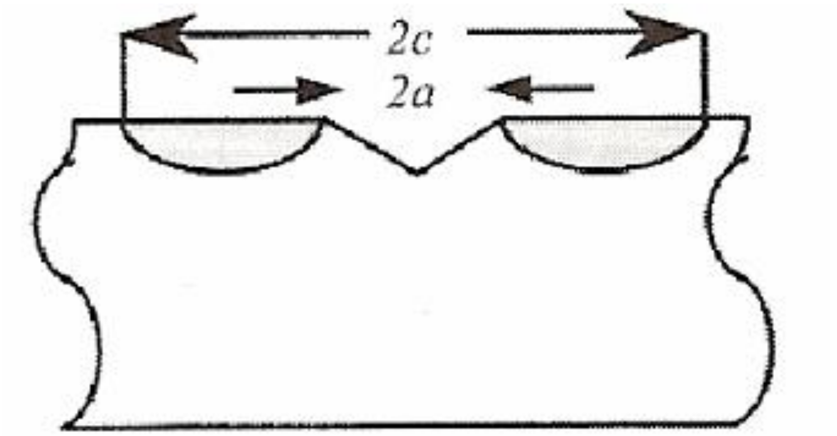


Figure 25: Radial crack (side view)

By substituting the values of elastic modulus, E, hardness, H, crack length and maximum indentation load in equation (5.5), we get the fracture toughness value as shown in Table 3.

Table 3

Properties of materials used in indentation cracking measurement of fracture toughness

Material	E (MPa)	H (MPa)	P (N)	C_o (mm)	K_c (MPa.mm$^{1/2)}$
Plexiglas	3300	96	400	2.4	0.797

5.2 Stress Strain graph for Conical Indentation

We calculated the stress and strain values from the force displacement data by following calculation. The average stress can be estimated by dividing the indentation force, P, by the true contact area A, as follow

$$\text{Stress} = \text{Force/Contact Area}$$

$$\sigma = P/A \qquad (5.6)$$

For simple conical indenter shown in fig 26, the true contact area A at indentation depth h can be calculated as

$$A = \pi h^2 \tan^2\alpha \qquad (5.7)$$

where

$$r = h \tan \alpha \qquad (5.8)$$

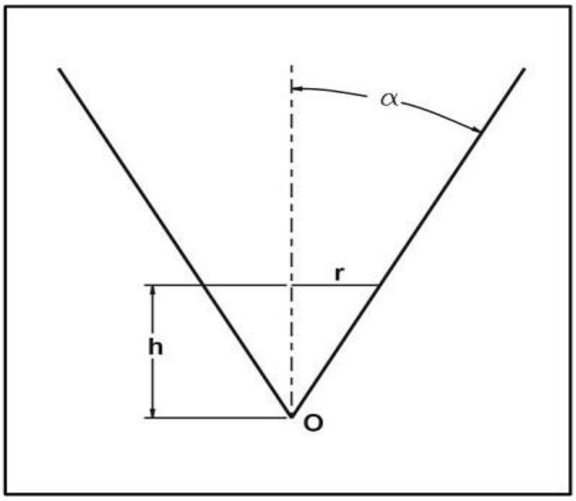


Figure 26 *Cone indenter*

The average strain in the direction of the indentation load can be found by dividing the change in depth, h, by the initial depth h_o.

Strain = change in depth/ initial depth

$$\varepsilon = h/h_o$$

After calculating the stress strain values from the load displacement data we generated the stress strain graph as shown in figure 27.

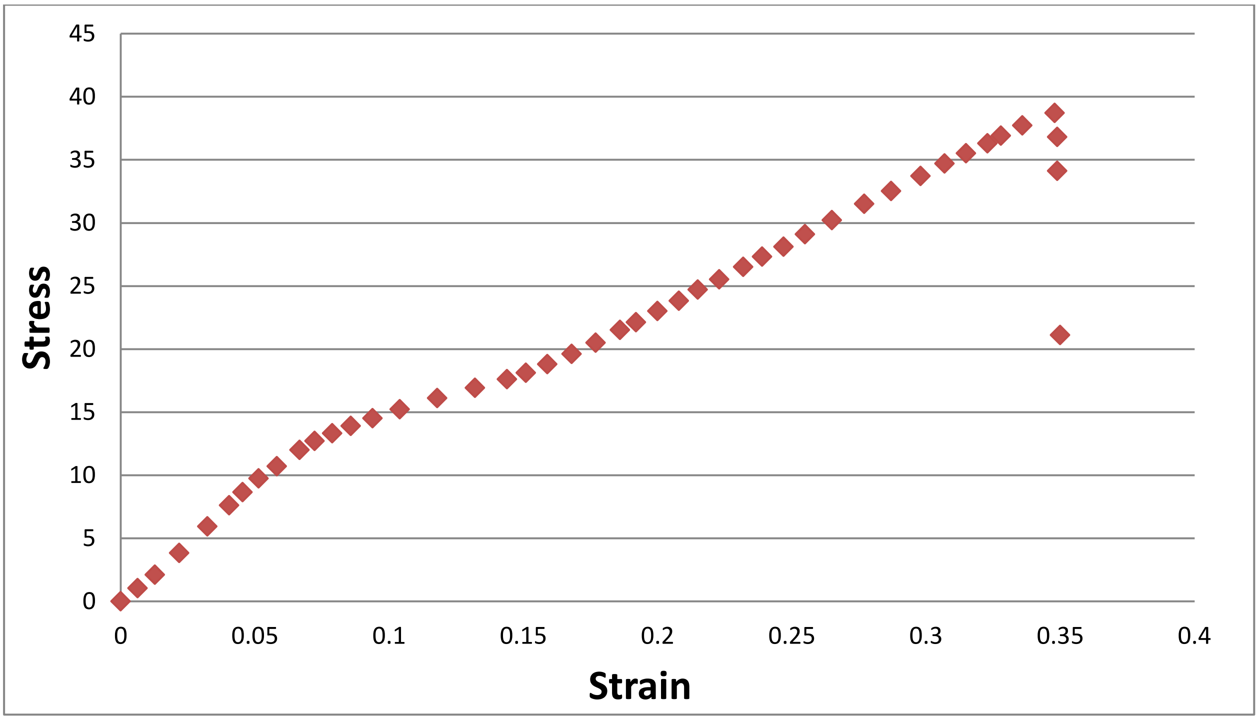


Figure 27: *Stress strain curve*

5.3 Absorbed Energy Evaluation with Trapezoidal Rule

Energy can be determined by calculating the area under the stress-strain curve. Therefore we employed a polynomial function to the stress-strain curve as shown in figure 28 and the area under the curve is evaluated in order to estimate the energy absorbed by Plexiglas sample at fracture. The applied polynomial function is given as

$$F(x) = -8998.x^4 + 7165.x^3 - 1889.x^2 + 281.1x - 0.784 \qquad (5.9)$$

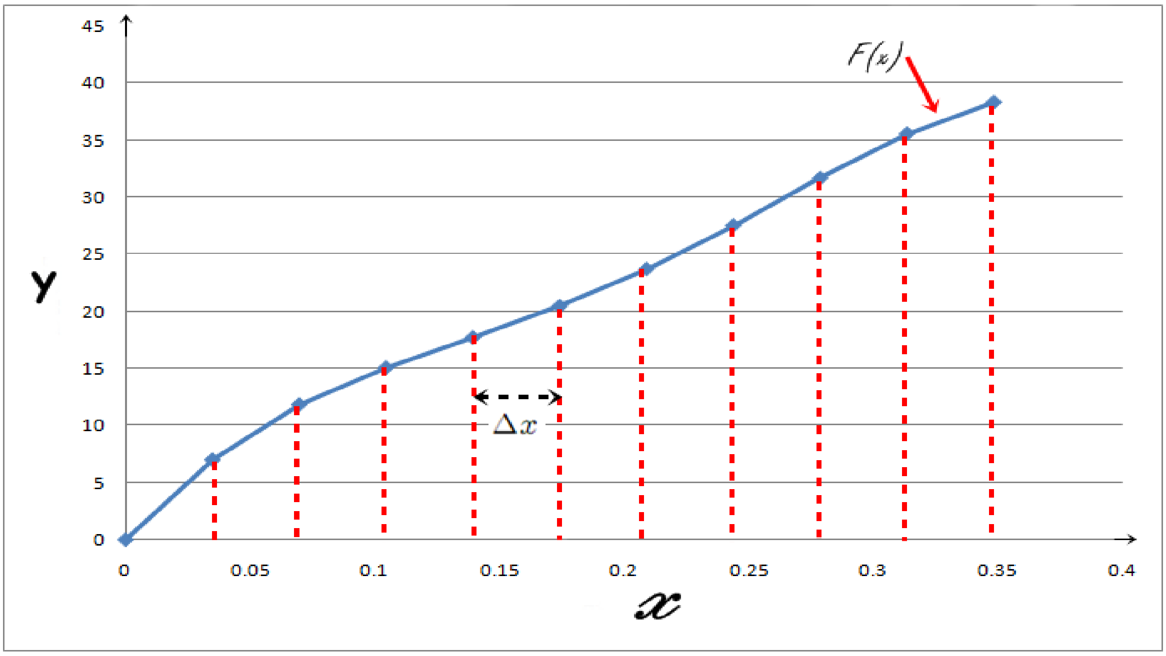


Figure 28: Depiction of Area under the Stress- Strain Curve

The area under the curve for the above polynomial function of figure 28 from the starting point to the maximum point where the finite interval [a, b] = [0, 0.348] is given as

$$A = \int_0^{0.348} (-8998.x^4 + 7165.x^3 - 1889.x^2 + 281.1x - 0.784)\, dx \qquad (5.10)$$

We determined this area using numerical method i.e. trapezoidal rule. The rule is based on approximating the region underneath the graph as a trapezoid and calculating its area.

The Trapezoidal rule states that

$$A = \int_a^b f(x)dx \tag{5.11}$$

$$A = \int_{x0}^{x1} f(x)dx + \int_{x1}^{x2} f(x)dx + ... + \int_{xn-1}^{xn} f(x)dx \tag{5.12}$$

$$A = \Delta x \frac{f(x_0) + f(x_1)}{2} + \Delta \mathrm{x} \frac{\mathrm{f}(\mathrm{x}_1) + \mathrm{f}(\mathrm{x}_2)}{2} + \ldots + \Delta x \frac{f(x_{n-1}) + f(x_n)}{2} \tag{5.13}$$

Here Δ**x** is the segment width and can be calculated as

$$\Delta x = \frac{b-a}{n}$$

Where

n = subintervals of equal length

We approximated the above polynomial curve by using n trapezoids formed by straight line segments as shown in the figure 28. Table 4 is generated by dividing the curve shown in figure 3 into n subintervals of equal length.

Table 4

Width of n subintervals

X_0	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	Δ**X**
0	0.035	0.070	0.104	0.139	0.174	0.209	0.244	0.279	0.313	0.348	0.034

The area under curve calculated by plugging in the values of table 4 into equation 5.13, gives the energy absorbed by material before fracture. Hence the amount of energy absorbed per unit volume of Plexiglas plate can be estimated as 7.28 J/mm^3. In order to calculate the entire amount of energy absorbed before the fracture of material we can multiply the area under the graph by volume of the material and can be estimated as 11098 J for our Plexiglas plate.

5.4 Model created using curve fitting technique

After getting the stress strain graph, we created a mathematical model of Plexiglas for its brittle cracking behaviour during the indentation loading. This model can be used for data apparition to gather values of a function where no data are obtainable and to sum up the relationships among two or more variables. The mathematical model was created by using Fourier fit to the data points in curve fitting toolbox of Matlab as shown in figure 29. The mathematical model is expressed as

$$\sigma = a0 + a1\times\cos(\varepsilon\times w) + b1\times\sin(\varepsilon\times w) + a2\times\cos(2\times\varepsilon\times w) + b2\times\sin(2\times\varepsilon\times w) + a3\times\cos(3\times\varepsilon\times w) + b3\times\sin(3\times\varepsilon\times w) + a4\times\cos(4\times\varepsilon\times w) + b4\times\sin(4\times\varepsilon\times w)$$

The values of the model coefficients are shown in Table 3.

Table 5

Coefficients of mathematical model values

Coefficients	Values	Coefficients	Values
a0	1.491e+6	a3	3.703e+5
a1	-1.78e+6	b3	-7.152e+5
b1	-2.294e+6	a4	-1.025e+5
a2	2.101e+4	b4	8.268e+4
b2	2.055e+6	w	1.664

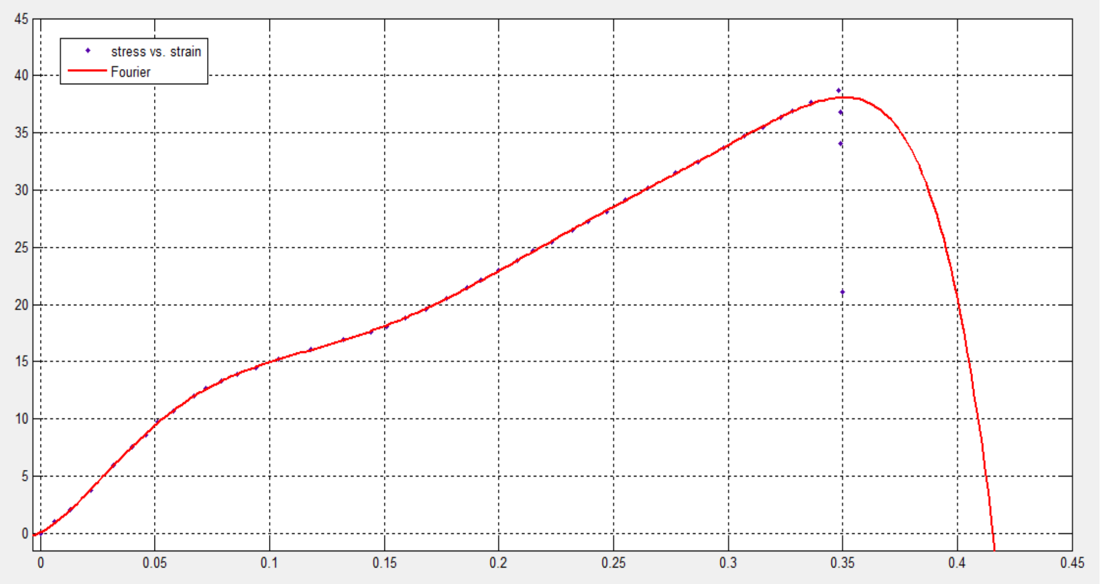


Figure 29: Stress strain curve with Fourier fit

CHAPTER 6

FINITE ELEMENT MODELING

Finite element method (FEM) is the powerful numerical technique for analyzing structural and mechanical problems and has become one of the most important and useful tools for scientists and engineers in recent years. The consistency of the numerical simulation results mainly depends on the accurate definition of the problem and by vigilant control of the vital parameters. From the numerical results of the simulation, background knowledge of processes and physical understanding of the simulation region can be obtained. The use of numerical simulations for identifying mechanical response of materials is gradually becoming more and more common because it’s a time saving and cost effective substitute. Simulations are performed using a number of commercial codes like LS-DYNA, ANSYS, NASTRAN, ABAQUS etc.

Finite element modeling plays an significant role in the study of indentation behaviour of materials. With an appropriate formulation and discretization of the indentation problem, finite element method has been effectively used to solve various indentation problems with sufficient accuracy, for example, indentations with different shapes of indenters, e.g. axisymmetric (2D) and pyramidal (3D), indentations on different materials, e.g. bulk coatings, materials and gradient materials, and indentations with different boundary conditions, e.g. contacts with friction or without friction, etc. Finite element methods have also been used to examine the indentation fracture characteristics of brittle material.

6.1 Introduction to the Abaqus software

Abaqus has been developed by Karrisson, Hibbitt and Sorensoen, Inc. It was the first finite element program to introduce a gateway for researchers to add elements and materials models. Based on the finite element method, Abaqus is an influential engineering simulation programs which is known for its quality, high performance and ability to solve more kinds of challenging simulations than any other software. This software is not only capable of performing simple linear analysis but also capable to perform large amount of complex nonlinear simulations.

Abaqus CAE contains very influential options to mesh difficult geometries and authenticate the resulting analysis model. Defining the material properties to the geometry, boundary conditions, load applications and submitting the completed model for a job analysis can be done using Abaqus CAE. Abaqus contain two major analysis modules i.e. Abaqus/Standard and Abaqus/Explicit. Abaqus/Standard is an analysis module that can resolve a diversity of problems covering all linear and nonlinear problems, also maintaining the accuracy and the reliability of the results. Abaqus/Explicit is an analysis module that uses dynamic finite element formulation, which is applied to deal with the problems of transient and dynamic in nature.

6.2 Numerical model and simulation

We used a commercial FE package ABAQUS 6.13, to simulate the brittle cracking in Plexiglas and then validate our numerical results by comparing it with the experimental results. In our simulation we did static analysis in Abaqus/Standard to generate the force displacement graph and then we used brittle cracking model in Abaqus/Explicit to initiate cracking in the Plexiglas.

Modeling of a conical indentation experiment can be accomplishd by mean of an axi-symmetric model as the system is symmetric about an axis. So we used an axi-symmetric 2D model in the FE studies to simulate the experimental load-displacement curve using a conical indenter. The numerical model consists of two parts created separately: a plate and an indenter. Since the indenter is much stiffer than the plate, the indenter is considered to be perfectly rigid and modeled as analytical rigid surface. It is time-effective because translations and rotations are the only variables related with a rigid surface on a particular node known as the rigid body reference node. In our case, reference point is assigned to the indenter, which manipulates the rigid body translation of the indenter. The plate has been created in 2D axisymmetric deformable with 25.4 mm diameter and 3 mm thickness. The plate is modeled using four nodded bilinear axisymmetric quadrilateral elements. The element type used is “CAX4R”, in which the letter or number indicates the type of element which is of continuum type, axi-symmetric in nature has 4-nodes bilinear and reduced integration with hourglass control respectively. The Reduced integration option available was used to save calculation time.

The material of plate is selected as plexiglas, which has the material mechanical properties as: E=3100 MPa, v=0.37. The two model parts are assembled by creating a zero gap between them to reduce the running time of the job. The contact interactions are defined as service to service contact, which can describe the contact between a deformable surface and a rigid body. The indenter surface was defined as the “master” surface since the indenter is rigid body. The top of the specimen was the “slave” surface. In our work it was assumed that the friction effect is negligible and μ=0 is defined in the models assuming there is no slip between the surfaces in contact.

The projectile is constrained in all directions except in the vertical direction normal to the plate surface (y-axis). Bottom of the plate is constrained axially and YASYMM boundary conditions are applied to the left side of the plate. The boundary conditions of plate and indenter are shown in figure 30.

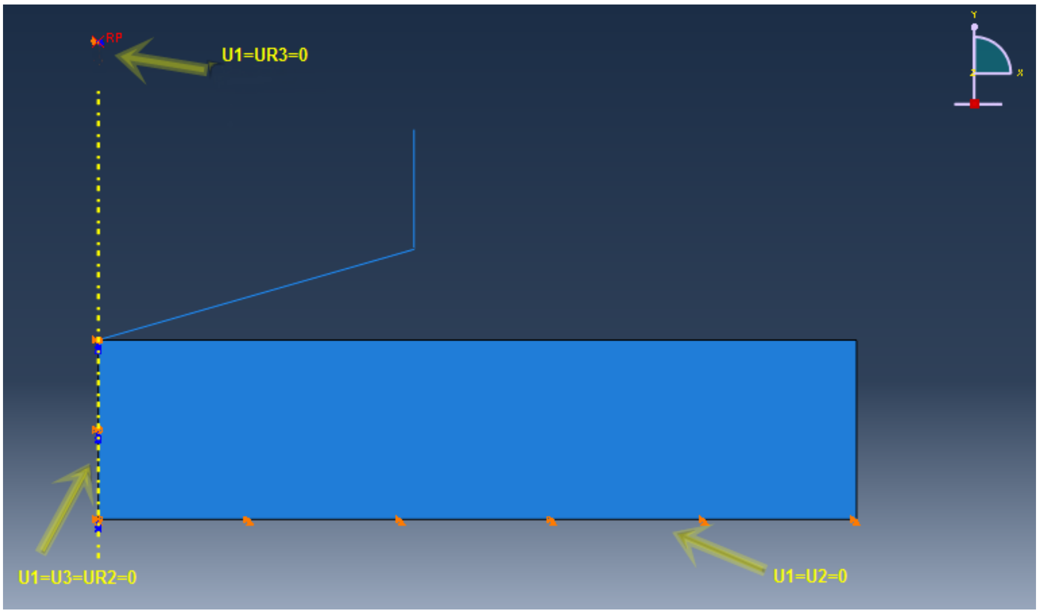


Figure 30: Boundary conditions applied in the model

As shown in figure 31 finer mesh is created around the indenter and the element size is increased making the mesh denser away from the indenter towards the boundary of the model.

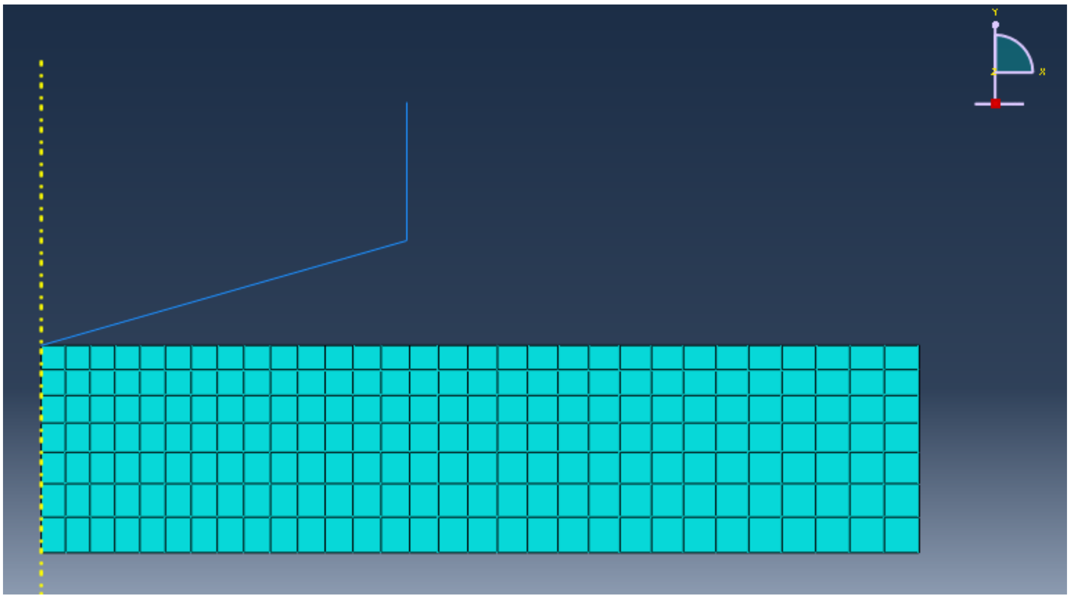
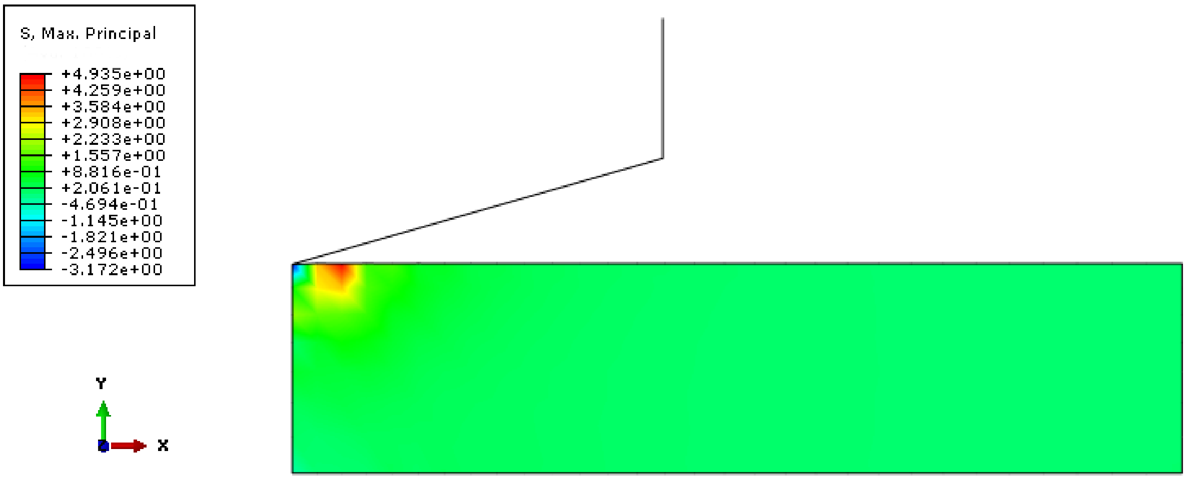


Figure 31: Meshed model of plate

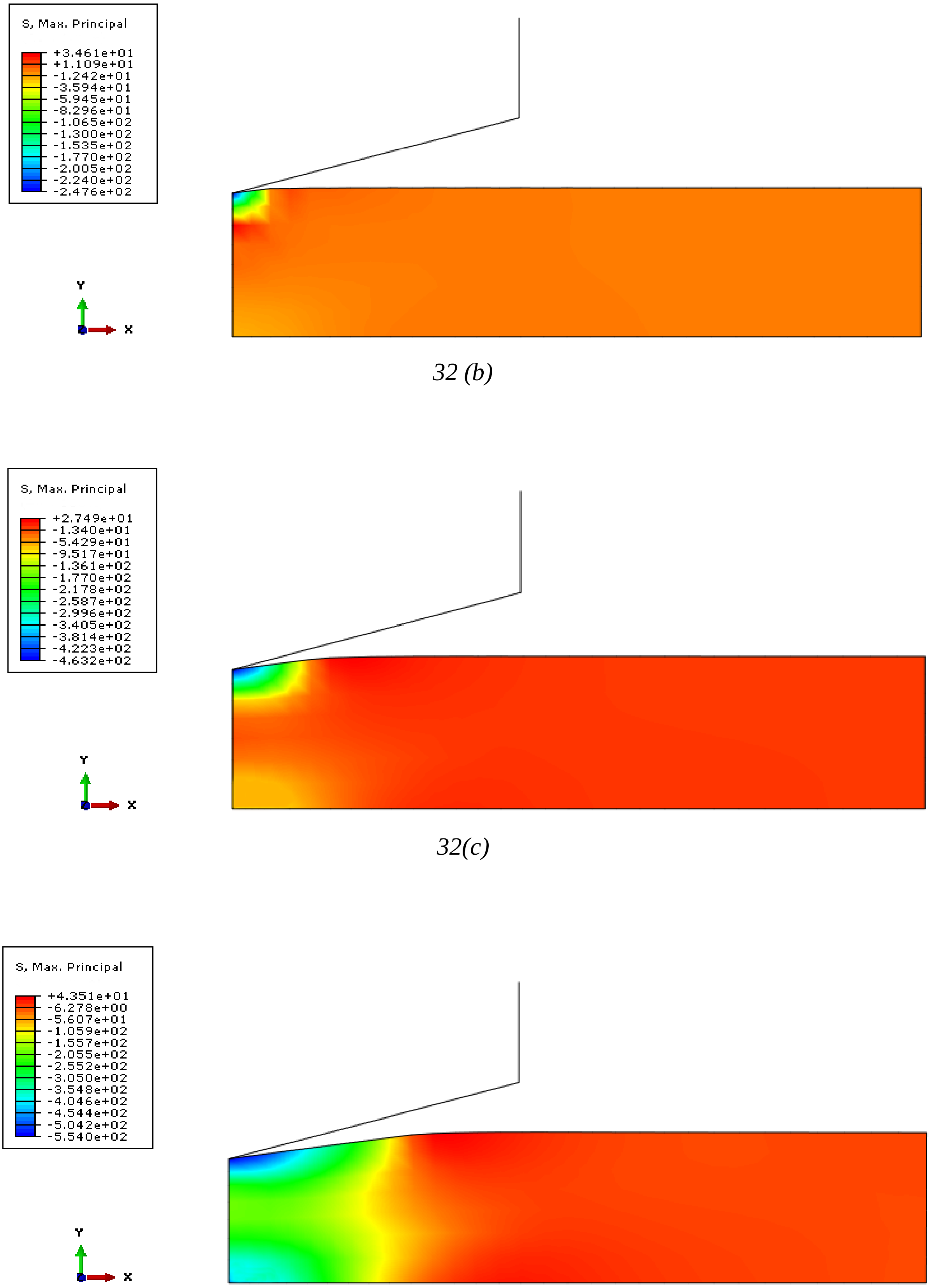
The indentation experiment was simulated by applying a downward displacement of 1.045 mm to the indenter which forces the indenter to press into the material. In Abaqus there are two methods to simulate the process of indentation into a material: load control and displacement control. In the case of the displacement controlled analysis displacement is specified as input, which is equal to the indentation depth. The indentation depth (δ) is calculated by linear proportion to the incremental time. For the applied displacement the reaction load (F) on the indenter is the summation of force over the contact zone along the penetration direction. Hence the F-δ curve is obtained.

6.3 Simulation Results

We can view deformed/undeformed shapes or plots of our job in visualization module. We analysed the results in order to see how the plate responds under the indentation. Figures: 32(a) to 32(d), shows the simulated progression of the penetration of a plexiglas plate, subjected to conical indentation. In addition, the principal stress distributions for the plate at various moments in time for the indentation loading are shown in Figure 32.



32(a)



32 (b)

32(c)

32(d)

Figure 32(a-d): Principal stress distribution on Plexiglas plate

6.3.1 Load displacement curve for elastic model

Load displacement (P-h) curve is the straightforward characterization of indentation results. For each material model, finite element simulation results gave a set of P and h data for each loading step and then these data points are plotted in Excel. The load-displacement curve obtained from simulation by using simple elastic model is shown in figure 33.

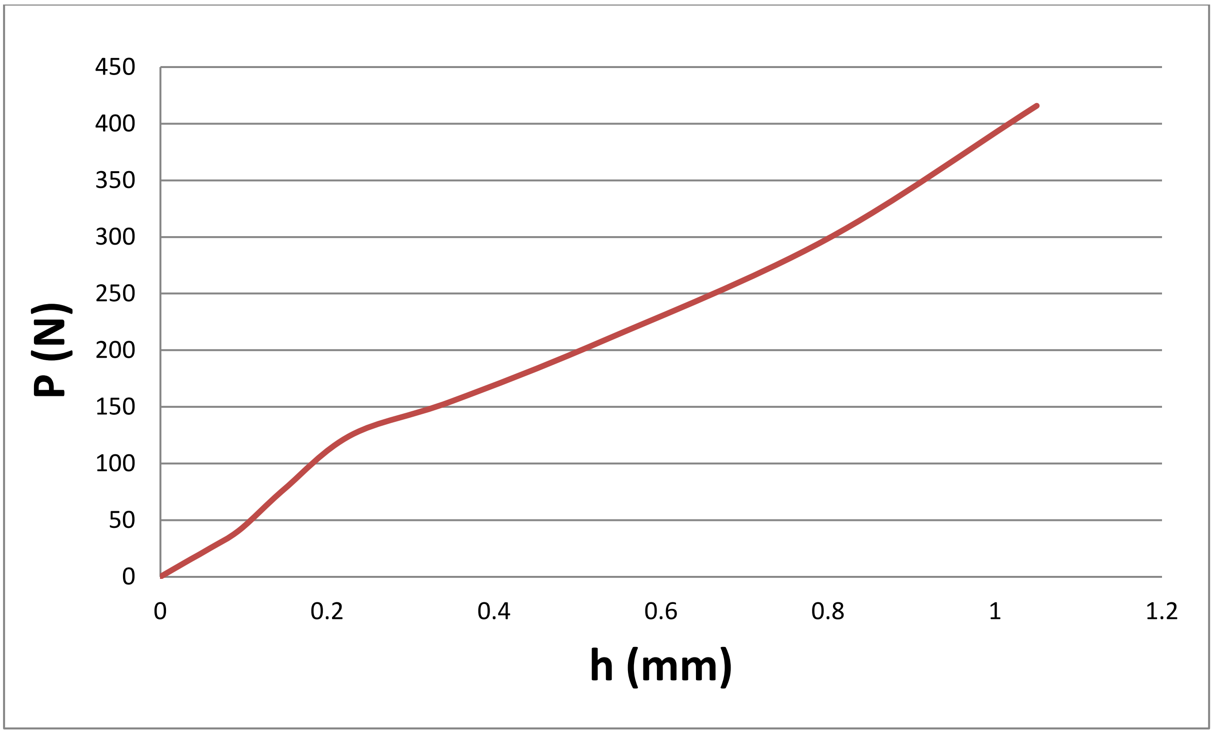


Figure 33: Load displacement curve using elastic model

6.3.2 Comparison between FEM and Experimental load displacement curve

Load displacement curve obtain by using elastic model in current FEM simulation is compared with previously extracted experimental load displacement curve as shown in figure 34.

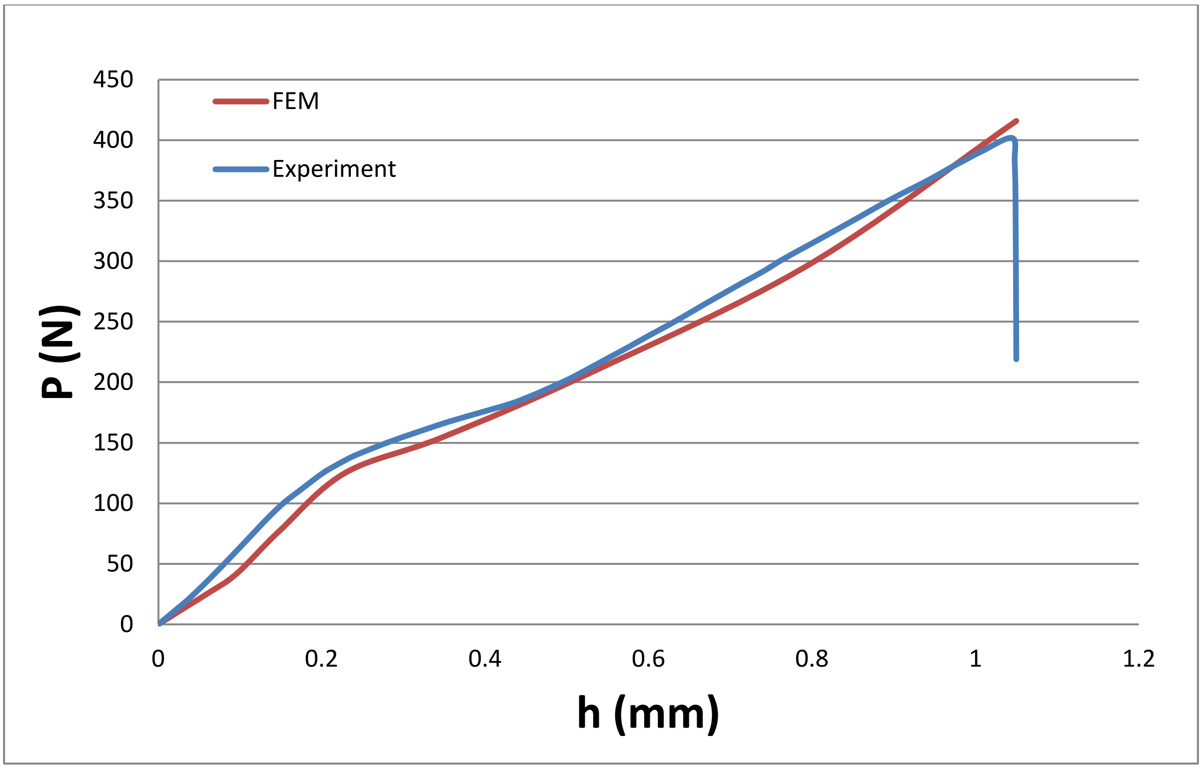


Figure 34: Load displacement curve comparison between FEM and experimental results

The figure 34 indicated the agreement between finite element and experimental results is appreciably good, but we are unable to generate crack in Plexiglas plate by using simple elastic model. Therefore a cracking model needed in order to produce a crack in a material. We used the brittle cracking material model for Plexiglas for prediction of its brittle cracking behavior when subjected to indentation loading.

6.4 Brittle Cracking Model for Plexiglas

The brittle cracking model in Abaqus/Explicit is most accurate in applications where the brittle behavior dominates. It is applicable to all brittle materials whose compressive and pre-cracking tensile can be represented through linear elasticity. This model can be expected to yield more accurate and realistic results than the tensile failure criterion, which is also applicable to brittle materials. In addition brittle cracking model allows more detailed modeling of post-cracking response than the simple tensile failure model. It does not track individual macro-cracks, but captures the effect of cracking through constitutive calculations. The basic Plexiglas properties and brittle cracking parameters are extracted from the stress strain data obtained in previous chapter data and final values are fixed after an admissible result is obtained. The Plexiglas properties are given in Table 6.

Table 6

Plexiglas mechanical and brittle cracking properties

Plexiglas Properties	Values	
Density	1.2x10-6 (kg/mm^3)	
Elasticity modulus	3100 (MPa)	
Poisson ratio	0.37	
Brittle Cracking	Direct stress after cracking	Direct cracking strain
	36.8	0
	34.1	0.000333
	21.1	0.000667
Direct cracking failure strain	1x10-6	
Brittle shear	Shear retention factor	Crack opening strain
	1	0
	0.5	0.001
	0.25	0.002
	0.125	0.003

6.4.1 Simulation Results

Figures 35a and 35b shows the contours of principle stress distributions for the Plexiglas plate when subjected to conical indentation. It can be seen that the principle stresses are maximum near the indenter tip during the loading phase. Figure 8a illustrate the maximum principle stress distribution at peak loading force just before the crack start to initiate, whereas figure 8b illustrate the principle stress distribution after cracking occurs in a plate. Observation of these stresses concedes that the principle stresses decrease significantly at peak indentation load indicating the crack initiation.

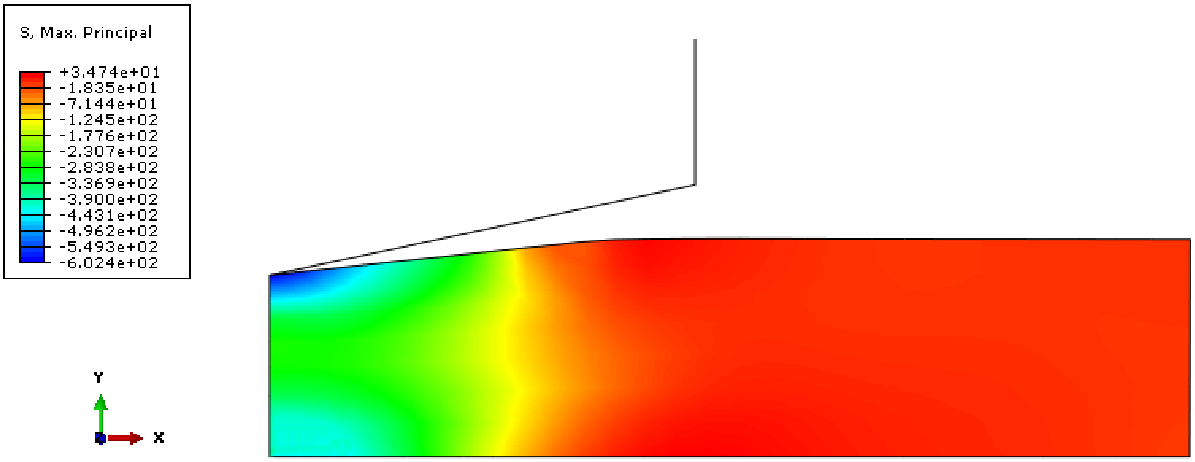


Figure 35(a): Maximum principle stress distribution before cracking

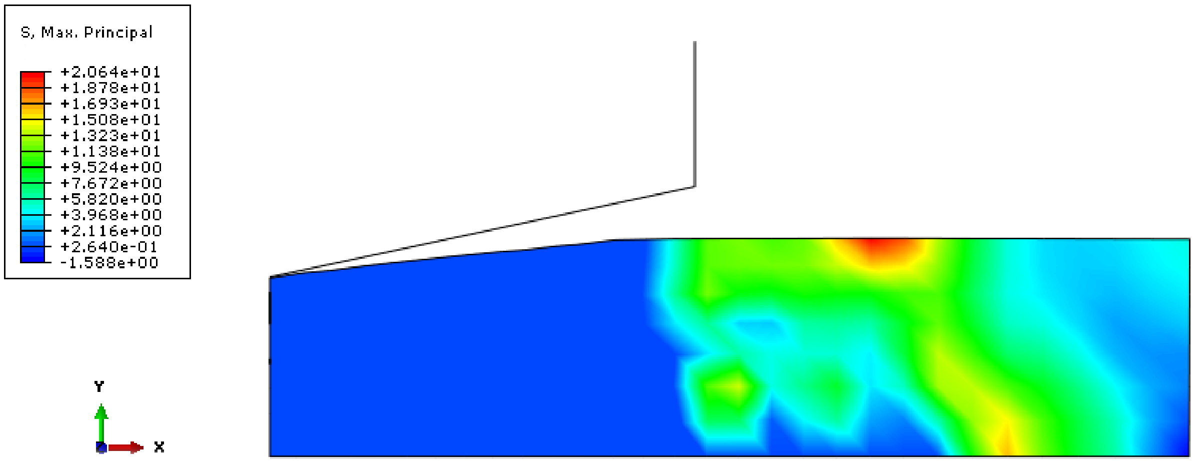


Figure 35(b): Maximum principle stress distribution after cracking

Figure 36 shows a symbol plot which allows us to visualize the magnitude and direction of principle stresses. The length of the arrow specifies the magnitude of the stresses and the direction of the arrow indicates its direction at each node. The symbol plot in figure clearly reveals that maximum principle tensile stresses develop normal to Plexiglas surface during the indentation loading signifying the radial crack development.

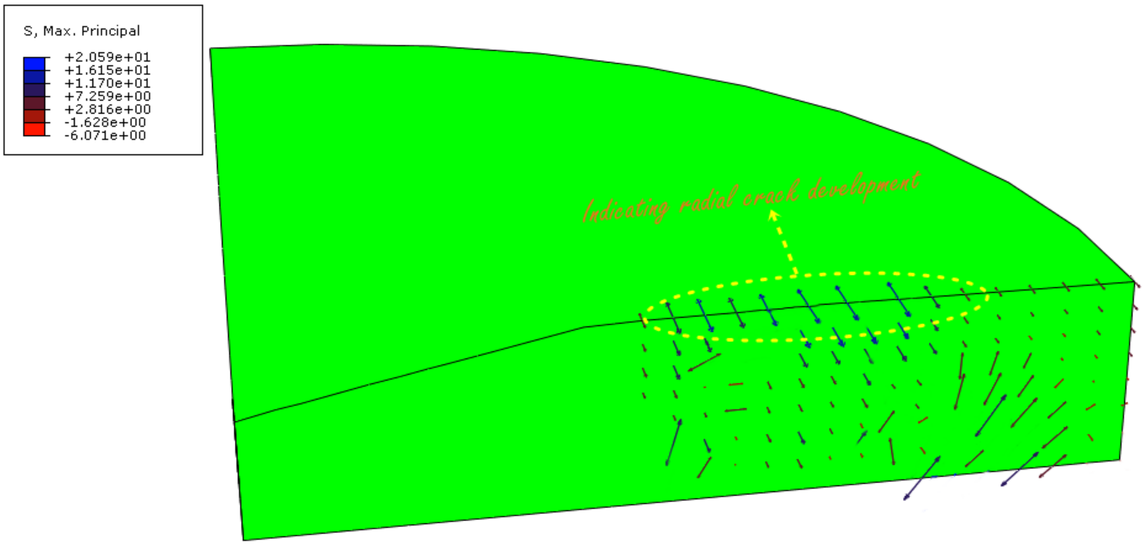


Figure 36: symbol plot of principle stresses

A plot of energy-displacement curve is shown in figure 37, illustrate the overall energy absorbed by a Plexiglas sample for the above case. The amount of energy in the system rises up to a point "A", just before the fracture initiates and after that the system loses its energy. The total energy in a system just before fracture is equal to 11268 J. It is approximately the same amount energy we estimated in the previous chapter i.e. from the stress-strain curve.

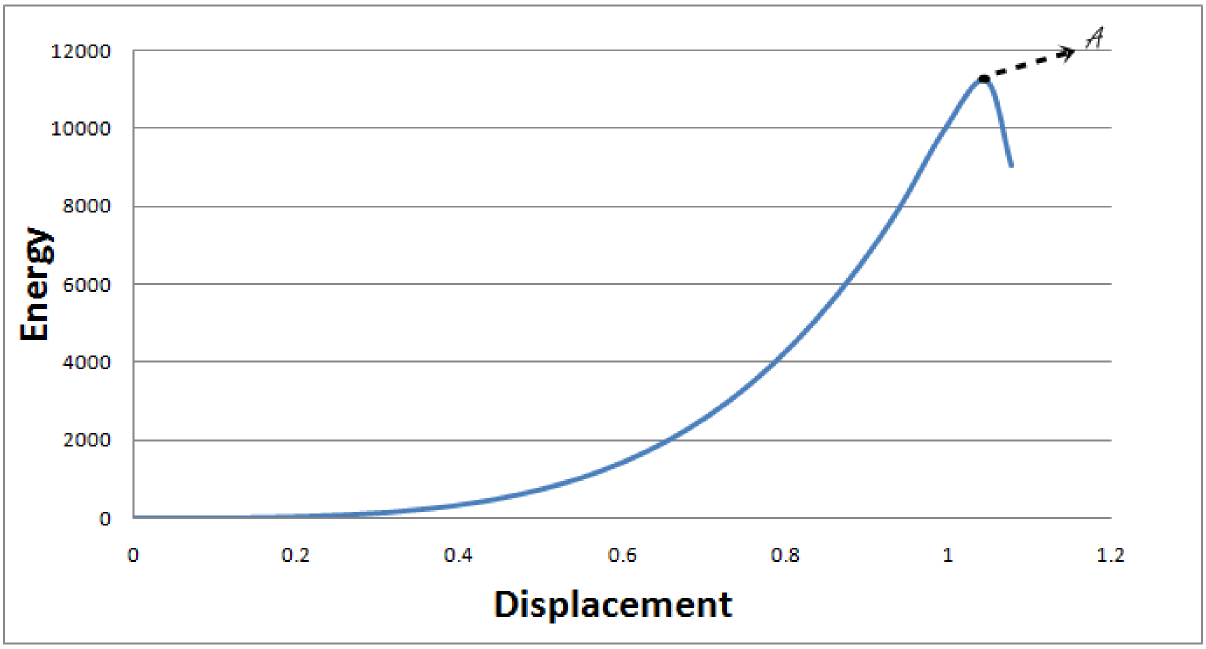


Figure 37: Energy-displacement curve

6.4.2 Load displacement curve for brittle cracking model

The load-displacement curve obtained from simulation by using brittle cracking material model is shown in figure 38.

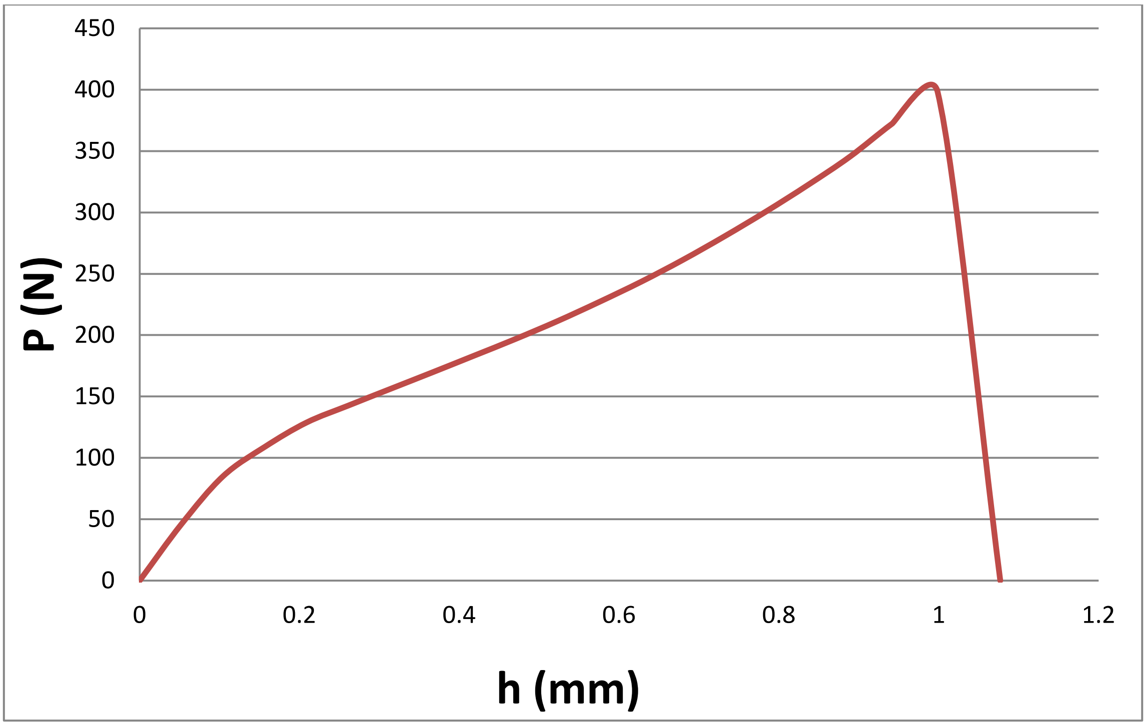


Figure 38: Load displacement curve using brittle cracking model

6.4.3 Comparison between FEM and Experimental load displacement curve

Load displacement curve obtained by using brittle cracking model in current FEM simulation is compared with previously extracted experimental load displacement curve is shown in figure 39.

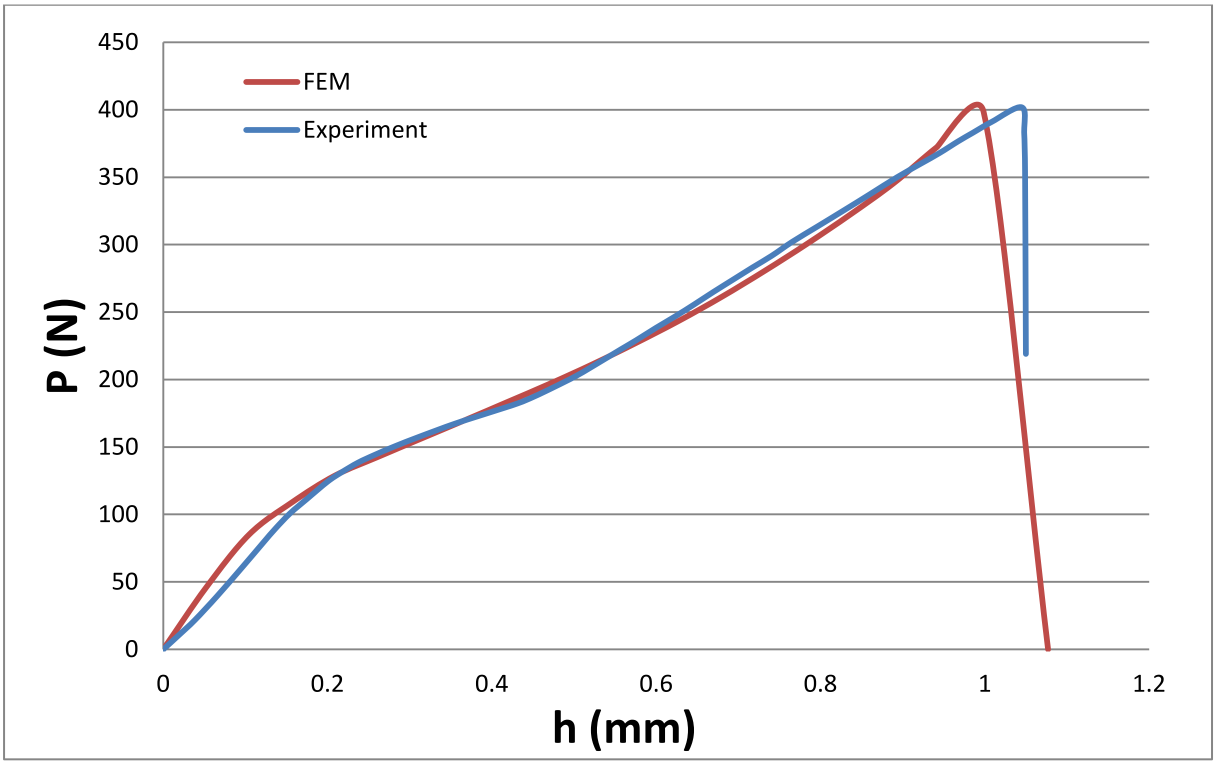


Figure 39: Load displacement curve comparison between FEM and experimental results

The figure 39 indicates the model predictions of P-h relationship agree well with the experimental results. Hence by using brittle cracking material model we are able to predict the crack initiation in Plexiglas plate during the indentation process. In the case of conical indentations, the numerical results obtained match well with experimental data and provide evidence that during the loading stage radial cracks are initiated.

CHAPTER 7

CONCLUSION AND FUTURE WORK

Modeling and simulation of materials have become essential with the intention of decreasing the cost of experimental investigations. Simulation is used to capture the material response under different conditions and is extensively used as a time saving and cost effective alternative. The material models are used in these simulations which also need material constants for each type of material used. Through various experiments models and material data are developed. For decades, emphasis has been placed on developing the constitutive models for metals. These well established models are used to predict plastic behavior of metals. Nonmetals are difficult to model and characterize because damage and fracture mechanisms are not easily definable. Thus unlike numerous available material models for metals, well established models for nonmetals are not very common. Models available today for non metals are either completely new or in modified form of the existing material models that are used for metals. Brittle cracking of non metal is currently a hot research topic in the literature.

In this thesis, the brittle cracking behavior of poly methyl methacrylate (PMMA) material, subjected to sharp tip indentation loading has been investigated. Modified Vickers testing apparatus is used to acquire the experimental data in the form of load displacement curve. The mechanical properties like hardness, yield stress and fracture toughness have been determined by analyzing the geometrical dimensions of the indent and from the analysis of a load-displacement curve. In addition, stress strain graph is generated from the force displacement data. The energy absorbed by Plexiglas sample before fracture can be determined by calculating the area under this stress-strain graph. After getting the stress strain graph, a mathematical model is also created for this graph by doing curve fitting.

Furthermore, a numerical simulation based study is made to investigate the brittle cracking in Plexiglas plate when subjected to indentation loading. By using simple elastic model the agreement between finite element and experimental results is appreciably good, but we are unable to generate crack in Plexiglas plate. The brittle cracking model in ABAQUS/Explicit is used to produce cracks in Plexiglas. For this purpose the experimental data is used to extract parameters of brittle cracking model available in the Abaqus materials library. The crack prediction in Plexiglas plate by simulation of the experimental data has been achieved successfully and a good correlation is observed between experiments and simulations results. The amount of total energy absorbed by a Plexiglas plate before fracture obtained from the finite element analysis and from the experimental results shows a good agreement.

The subject thesis is an attempt towards the accurate prediction of crack in Plexiglas plate when subjected to sharp tip indentation loading. So with the help of brittle cracking model we were able to initiate crack in Plexiglas plate. However, in our analysis we mainly focused on the prediction of crack's initiation but not on the crack evolution during the indentation process. Therefore, there is a lot of room for further research in the concept at hand.

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