Trajectory Planning and Control for Lane change of Autonomous Vehicle

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ISLAMABAD

May, 2019

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A thesis submitted in partial fulfillment of the requirements for the degree of

MS Electrical Engineering

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DECLARATION

I certify that this research work titled "*Trajectory Planning and Control for Lane Change of Autonomous Vehicle"* is my own work. The work has not been presented elsewhere for assessment. The material that has been used from other sources it has been properly acknowledged / referred.

Signature of Student

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LANGUAGE CORRECTNESS CERTIFICATE

This thesis has been read by an English expert and is free of typing, syntax, semantic, grammatical and spelling mistakes. Thesis is also according to the format given by the university.

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ACKNOWLEDGEMENTS

I would like to express my profound gratitude to ALLAH Almighty, for His countless blessings and divine direction throughout my life.

A special thanks to my supervisors Dr. Aamer Iqbal Bhatti and Dr. Muwahida Liaquat for their tremendous supervision, and a very reliable encouragement throughout the course of this thesis. I wouldn't have been here today without their help.

I would like to thank Dr. Fahad Mumtaz Malik and Dr. Usman Ali for their meaningful assistance and guidance.

I would also like to thank family members who believed in me and assisted me to continue hard work in the most tough times, especially to my parents for their continuous support, prayers and motivation throughout my educational career. I would like to appreciate Mr. Mansoor Mehmood, Mr. Sohaib Mehmood and Mr. Shehram Mehmood for guiding me towards the better future.

Finally, I am grateful to all the individuals mainly Ehtsham Rasool, Muhammad Bilal Sheikh, Hammad Abid, Sarmad Saeed, Syed Hassan Shah and Aamer aziz for their valuable assistance to my study.

I dedicate this thesis to my parents, loving family and especially to Bisma Talat, Abdullah Mansoor, Sabeen Qureshi, Ali Mansoor, Eshaal Qureshi and Muhammad Ahmed

ABSTRACT

In recent years' autonomous vehicle is becoming active research area. One of the most important features of autonomous vehicle is lane-change without driver assistance and its important part of level 2 autonomy. Wrong maneuver can lead to serious accidents on highways. Lane-change of autonomous vehicle is challenging control problem and various techniques like model predictive control, cooperative lane change, geographical information system has been used in past to get lane-change maneuver. Lane-change problem can be divided into trajectory planning and trajectory control. The current work addresses lane changes of autonomous vehicle by generating trajectory with quintic polynomial and controlling it with the help of controller (PID). Two cases are discussed for lane change; considering previous vehicle and without previous vehicle. Collision avoidance technique is used in case of previous vehicle to avoid collision. Results obtained shows the feasible trajectory for smooth lane change in both cases. Collision avoidance technique shows how collision can be avoided in case of previous vehicle.

Keywords: Lane-change, Autonomous vehicle, Trajectory planning, Trajectory control, collision avoidance

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ACRONYMS

LKS	Lane Keeping system
NHTSA	National Highway Safety Authority of the United States
LIDAR	Light Detection And Ranging Sensors
LCE	Lane change Execution
GPS	Global Positioning System
ACC	Adaptive cruise control
CACC	Cooperative adaptive cruise control
GIS	Geographical information system
CLCE	Cooperative Lane change Execution
FLCE	Forced Lane change Execution

LIST OF SYMBOLS

Symbol	Description
v_{des_i}	Desired Velocity (initial)
v_{des_f}	Desired Velocity (final)
d	Final position in longitudinal direction
ω	Final position in lateral direction
T	Total time for Lane-change
R_{Host}	Radius of host vehicle
$R_{\mathrm{Pr}e}$	Radius of previous vehicle
L_{Host}	Length of host vehicle
L_{pre}	Length of previous vehicle
x_{Host}	Longitudinal Position of host vehicle
$x_{\mathrm{Pr}e}$	Longitudinal Position of previous vehicle
y_{Host}	Lateral Position of host vehicle
$y_{\mathrm{Pr}e}$	Lateral Position of previous vehicle
$dist$	Distance between vehicles

CHAPTER 1: INTRODUCTION

An autonomous vehicle recognized as a self-driving or also known as unmanned vehicle. An autonomous vehicle is a vehicle which uses different types of sensor to travel between one point to other without human intervention or input.

1.1 Historic Background

After the invention of an autonomous vehicle concept researchers were motivated to explore this technology. So, in 1925 FRANCIS HOUDINA tested his first autonomous vehicle in the streets, there was no human interference or input in the vehicle, which was controlled by the radio. With the passage of time the automobile industries improved in their technology and in year 1969 John McCarthy upgraded the autonomous vehicle and introduced a new technology which used camera to get information around the surroundings [1]. In 1995 after many years of research on autonomous vehicle a minivan covered a distance of 2797 miles without any human input or control which was designed by the POMERLEAU and TODD JOCHEM [2]. In 2003 one of the most well-known company Toyota introduced an automatic parking assistance system in their Toyota Prius hybrid model, and after the invention and perfectly working of that technology Lexus also introduced the same system for their Lexus LS sedan model. [3], Whereas an American company Ford introduced an Active Park Assistance system in their vehicles in the year 2009 [4], and BMW which is also a well-known automobile company introduced the same technology one year late in their vehicles. [5].

Google started developing its self-driving vehicle project in 2009. A few years later, Google has announced about their own autonomous vehicle which have been cooperatively driven around 300,000 miles under the computer control without any driver and without any accident [6] [7].

After 2013, several major automobile companies which includes Ford, BMW, General Motors, Mercedes Benz, and many other companies started to work on their personal unmanned vehicle technology. And also, these companies which includes Nissan, tesla, BMW and Audi are planning to release their unmanned vehicle in near 2020[8].

1.2 Automation Levels

The two main bodies defining the classification of Automation are National Highway Safety Authority of the United States (NHTSA) and Society of Automotive Engineers SAE. According to SAE following are the Six levels of automation [9].

1.2.1 LEVEL 0

Level 0 is also named as the no automation

System ability

There is not a single automatic feature in this level which can control the vehicles automatically. Level zero vehicle only contain some indication sensors like heating sensor fuel indicator etc.

Driver involvement

In this level driver needs to be alert all the time because all the responsibilities like look after around the vehicle, accelerating, deaccelerating, braking and the throttle are under controlled by the driver only.

1.2.2 LEVEL 1

Level 1 is also named as the driver assistance

System ability

In level one, system can control some features of the vehicles like acceleration and speed of the vehicle. Level one vehicles are only capable to control the distance and speed in longitudinal direction.

Driver involvement

In this level, drivers need is very important although it give some relief to driver from the throttle but still need to be alert and should keep their hands on the steering all the time. Driver can over take the control at any time to avoid the accident.

1.2.3 LEVEL 2

Level 2 is also known as the Partial Automation

System ability

It’s an advance version of the level one in which some functions are controlled automatically like throttle, braking, and steering of the vehicle. In level 2 automation, system perform many control actions like cruise control, automatic braking system and keeping the vehicle in particular lane. This level works only on the long distance traveling and on motorways where vehicle don’t care about the nearby people and traffic signals.

Driver involvement

In this level, system is able to perform the main task automatically but it is driver's responsibility to take care about the surroundings. Driver may take break from driving task on highways but still need to be alert about the surrounding to take control in case of emergency.

1.2.4 LEVEL 3

Level 3 is also named as the Conditional Automation

System ability

It’s an advance version of the Partial Automation the main upgradation in this level is, it can observe the obstacles and the traffic signals and can be driven in urban areas as well unlike the partial automation.

Driver involvement

The main advantage of Level 3 from other levels is that, starting from this level, the car itself monitors the environment using different sensors. Driver can trust on this level fully but still need to be alert for emergency case because it cannot find the path in the worst rainy weather condition.

1.2.5 LEVEL 4

Level 4 is also named as the High Automation

System ability

It's an advance version of conditional automation, the main upgradation in this level is its equipped with advance sensors which can predict and look after the surroundings without the human input. It's almost the complete autonomous vehicle.

Driver involvement

In the Level 4 vehicle can sense the environment without the driver but there is still need a driver for the emergency situations. In the rainy weather there is a possibility of the failure of system. For the activation of the automatic mode vehicle indicates weather the situation of the surrounding is safe or not then driver can activate the system.

1.2.6 LEVEL 5

Level 5 is also known as the Complete Automation

System ability

All driving tasks performed by the control system. The vehicle senses the situation and environment and act accordingly without human interference. Pedals and steering are removed in most of the cases.

Driver involvement

It's an advance and final version of the level 4 automation. This is a complete autonomous vehicle which comes without the steering wheel, throttle and brake pedals. So, there is no requirement of driver to control the vehicle, only one task is need to be performed by the driver or passenger is to set the destination. After that vehicle find its route by itself and perform all the driving tasks independently.

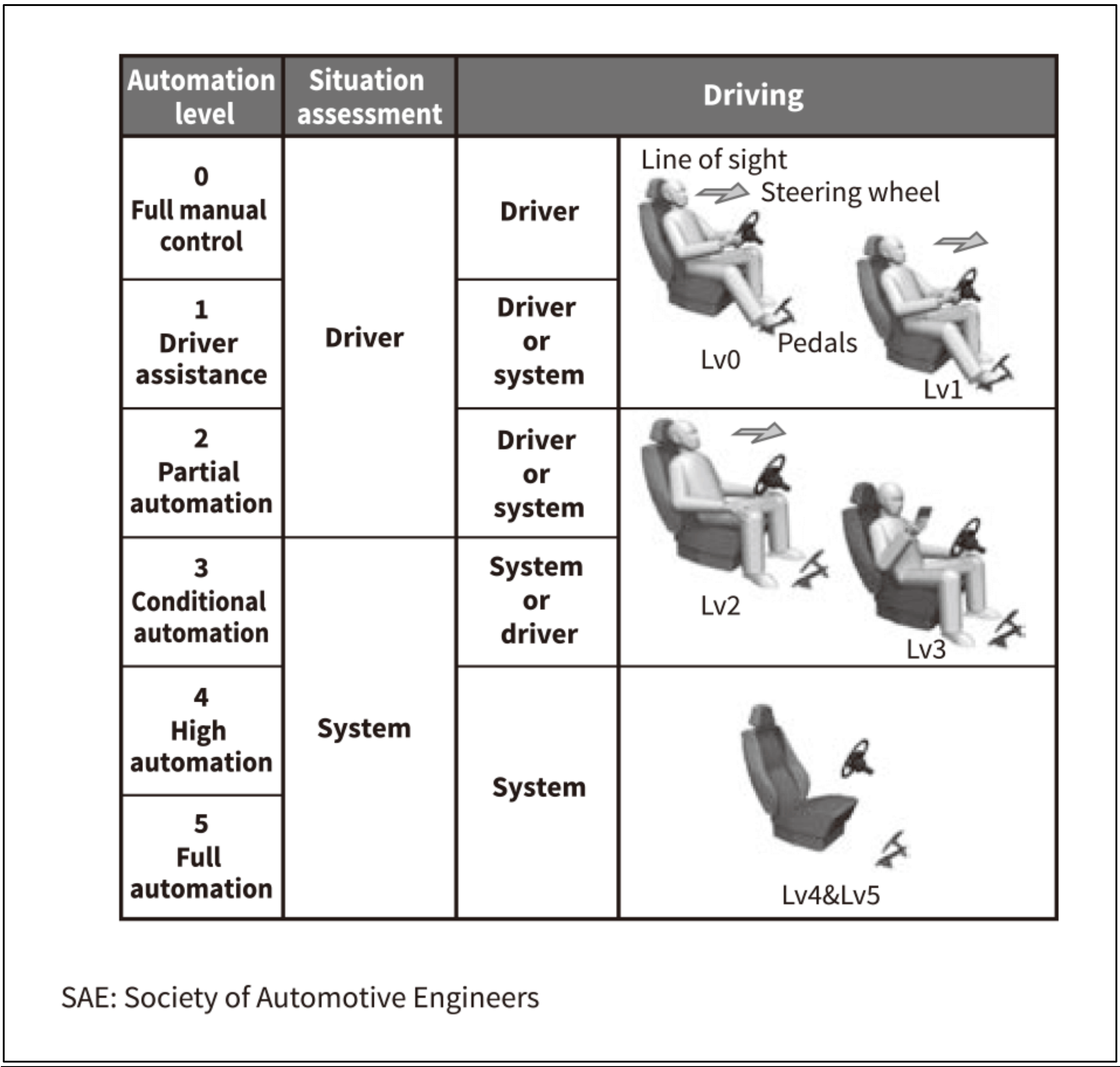

Figure 1.1 SAE Automation Levels

1.3 Sensors/Equipment's used in Autonomous Vehicles

- <u>Light Detection and Ranging Sensors (Lidar):</u>

LiDAR can detect the position of objects and people around the vehicle simultaneously and evaluate the path and speed at which they are moving. Using that particular information, a computer system generates the detailed map to ensure the safest path for autonomous vehicle [10].

- Radar sensors:

Radar sensors are used to observe different short range and long-range obstacles. They detect the speed of other vehicles [10].

- Cameras:

Cameras in autonomous vehicles are mainly used for lane detection and gaining information relating road.

- GPS:

Global Positioning System helps to determine the location of the vehicle by getting inputs from satellites.

- Ultrasound Sensor:

Ultrasound sensors are used in the obstacle detection near the vehicle, they are helpful during parking.

1.4 Impact of Autonomous vehicles on society

1. Safety and convenience

Data of highways traffic accidents indicates that most of the accidents occur due to human fault as per the data of highway traffic it is approximately equal to 90%, and that can be minimized by using autonomous vehicles [11-13]. An autonomous vehicle can detect possible danger faster and better than human.

2. Traffic Management:

Traffic jams are irritating but its common in many urban cities. Autonomous vehicles effectively communicate with each other and they can help in smooth flow of traffic and reducing traffic jams.

3. Influence on Economy:

Autonomous vehicles are more efficient than humans, and they are also fuel efficient. Due to a smaller number of accidents insurance cost will be reduced too and the vehicles automatic parts will reduce construction costs.

4. Better Parking Facility:
Autonomous vehicles will make it possible to have feasible parking in cities. With autonomous features in vehicle and different sensor used it will be very convenient to find parking slot in different scenarios.

1.5 Motivation

Unmanned vehicles are the future of automotive industry. With several automatic modules already present in market, it is expected that very soon autonomous vehicles will become a reality. Autonomous lane-change is one of the best examples of an independent modules of the autonomous vehicle system. These autonomous features are very useful and help to perform driving tasks efficiently and accurately.

1.6 Objective

The main objectives are

- Trajectory planning and control of autonomous vehicle while considering previous vehicle.
- To apply collision avoidance technique in order to avoid collision with previous vehicle.

1.7 Thesis outline

Chapter 2: In chapter 2 literature review is described in detail covering Adaptive Cruise Control (ACC), Cooperative Adaptive Cruise Control (CACC), systems related lane change, different approaches related to lane change and gap analysis.

Chapter 3: In this chapter trajectory generation and control is defined but general case is considered in where no vehicle is considered in adjacent lane.

Chapter 4: In this chapter trajectory generation and control is described with considering previous vehicle in adjacent lane. Collision avoidance technique is also derived.

Chapter 5: The whole thesis is concluded in this chapter and ideas for future work are also discussed.

CHAPTER 2: LITERATURE REVIEW

2.1 Adaptive Cruise Control (ACC)

Driving safety can be increased by the help of Adaptive cruise control (ACC), by applying this automated system the distance between two vehicles can be reduced. Huge amount of work has been done on longitudinal control of autonomous vehicles [14].

2.2 Cooperative Adaptive Cruise Control (CACC)

Cooperative adaptive cruise control (CACC) is an incorporation to adaptive cruise control (ACC). CACC ensure that leader and follower maintain safe distance and travel at specific speed. CACC also ensures that variation in acceleration and deceleration remain in comfort zone [15].

2.3 Lane change

In recent years, problem of lane changes on highways got more attention as its one of the most common behavior of automobiles, with recent research on self-driving vehicles, lane-change has become a vital field of experimentation and very important control problem. The aim of changing the lane for an autonomous vehicle is to move to neighboring lane automatically under particular limitations.

2.4 Systems related Lane change

Lane exits are the major cause of deadly accidents while driving on highways. Almost 1.5 million accidents annually are caused by distracted drivers as per reports by the National Highway Transportation Safety Administration (NHTSA) where a large proportion of which are caused due to unexpected lane change [16]. Automotive industries are working on this problem and lateral systems of three types have been developed in the past that focus on lane change incidents. Three types are lane departure warning systems (LDWS), lane keeping systems (LKS) and yaw stability control systems. [17]

2.4.1 Lane departure warning system (LDWS)

Lane departure warning system (LDWS) is modeled to minimize traffic accidents. It is a mechanism which observes car position in respect of lane and give alert when vehicle change its lane. The Auto Vue LDW system designed by Tiers, Inc is one of the examples of a commercial LDW system under development. The Auto Vue device consists of cameras, computer and software. Device is small in size and attaches to dashboard or windshield.

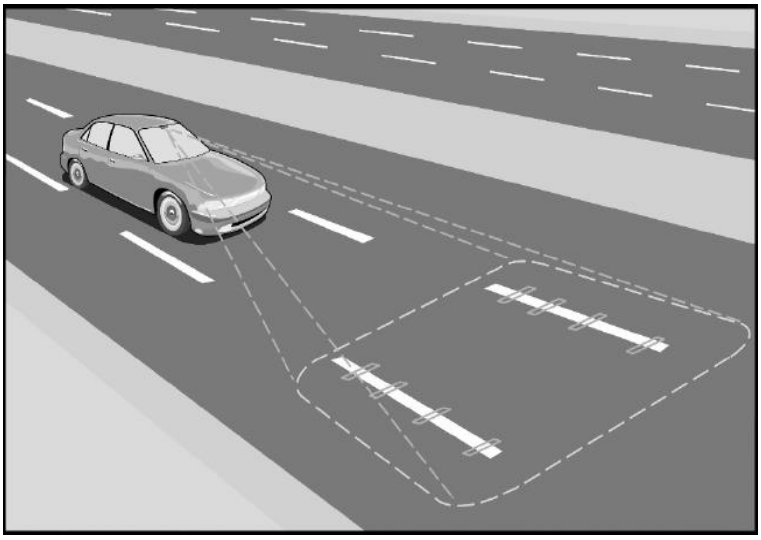

Figure 2.1 LDWS

It is designed to identify between the road and lane markings. The camera traces lane markings and sends the captured data to the computer. This information with the vehicle's speed is combined by the computer. Using image recognition techniques, the computer can detect when an automobile starts moving towards an unplanned lane change. In such case the computer gets alert and alarms the driver of the decision. Auto Vue is operating efficiently throughout the day, and in almost every weather condition when the lane marks can be seen easily. Trucks made by Mercedes and Freightliner use lane departure warning systems designed by Tiers [17].

2.4.2 Lane keeping system (LKS)

Lane-keeping system (LKS) manages the steering wheel automatically so that the vehicle is kept in its lane. Many universities have established research groups and verified aforementioned system. Research groups at California PATH established this system formed by the help of cylindrical magnets fixed at regular intervals in the middle of the lane. Lateral position was measured by the magnetic field generated by the permanent magnets [18].

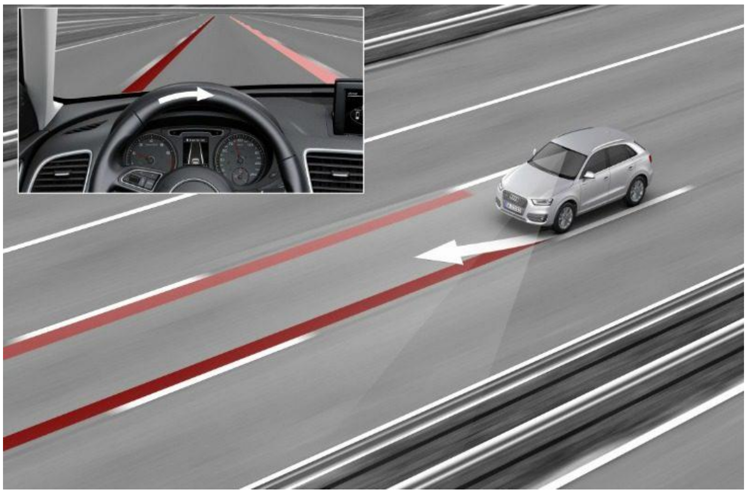

Figure 2.2 LKS

Researchers at Berkeley and at Carnegie Mellon [19] have established a system related measuring lateral position by using camera sensors and established vision based lateral control system. Research groups at the Minnesota University have established lane departure warning and lane keeping systems based on the use of GPS for lateral position measurements [20]. Many automotive producers like Nissan are developing such systems. A recently introduced system called LKS in Japan on Nissan’s Cima model, offers automatic steering control in corresponding with the vehicle driver[21].

2.4.3 Yaw stability Control Systems

Vehicles are prevented from drifting and spinning through a system known as vehicle stability control system that has been established and recently introduced by several automotive industries. These systems are also frequently stated as yaw control systems.

Figure 2.3 Stability Control

The purpose of this control system is to re-establish the velocity of the vehicle to the motion that is projected by the driver. A lot of work has been done by many industries on these control systems during the past ten years. They have run simulations and experimental prototype vehicles to explore this system and detect any loopholes. Some of these systems have also been introduced on production vehicles. Examples contain the BMW DSC3 and the Mercedes ESP [17].

2.5 Lane change strategies

Lane change problem is approached by different ways in recent years. In [22] lane change and path planning is accomplished by using Geographical information system (GIS) data which includes information related road lanes, traffic signs, speed limits, traffic signs and crosswalks. By using GIS data autonomous vehicle determines whether to change lane or not. Lane change execution (LCE) in [23] is further described as Cooperative lane change execution (CLCE) and Forced lane change execution (FLCE). CLCE can be described as if other vehicles driver in courtesy give space to complete Lane change where as in FLCE driver have to take care of other vehicle because of prevention behavior. In [24] specific inter vehicle gap and time instance is considered by estimating the longitudinal trajectory. In [25] traffic situation is assessed by system based on external environment and it give signal to driver for lane change, image processing technique is used for detection of lanes and vehicles. In [26] Dublin curve is used to generate the path but it was unrealistic and the reason was discontinuous curvature. In

[27] B-spline curves are used for path generation but the process involved complex calculations.

2.6 Gap Analysis

In area of autonomous vehicle significant research is done on Adaptive cruise control in which specific distance is maintained between the host and previous vehicle, but in case of ACC only longitudinal direction is considered in most of the research like in [9(paper)]. In case of lane-change numerous researchers applied model predictive control for planning of trajectory. Although good results were provided in many scenarios. But for planning of trajectory MPC approach is complex with respect to polynomial approach. This work presents the trajectory planning of autonomous vehicle with polynomial approach in which previous vehicle is also considered. The overall approach is simple, having low complexity and also ensures the collision prevention for convenient lane change.

CHAPTER 3: VEHICLE TRAJECTORY GENERATION AND CONTROL

The overall lane change process involves two main steps. First, trajectory planning where desired trajectory is planned by putting the values of position, velocity and acceleration at initial and final points. Second, trajectory tracking or trajectory control in which desired trajectory is controlled by the help of controller.

3.1 Kinematic model of vehicle

Kinematic model of car is shown in Figure 1, there is configuration of $\left(x \quad y \quad \theta\right)$,where x, y represents the midpoint of rear end, θ represents the angle between car heading and x-axis,

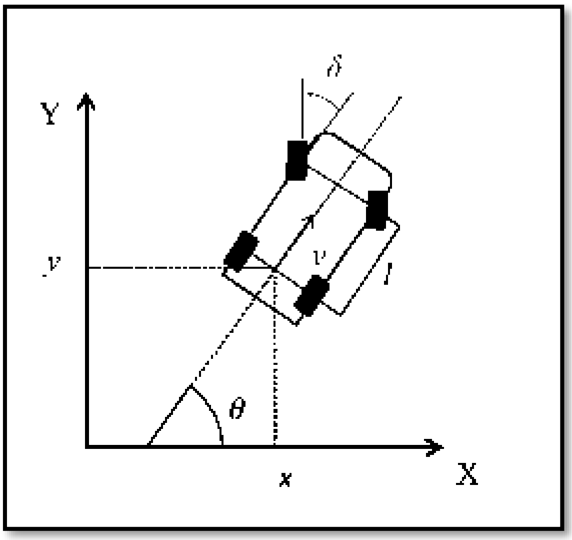

Figure 3.1 Model of Car (Coordinates)

L is the total distance between front and rear end. $u=(v,w)$ is the controlling vector, where v is the linear velocity and w is the angular velocity [29].

$$\dot{x}=v\cos\theta \tag{3.1}$$

$$\dot{y}=v\sin\theta \quad , \quad u=\begin{bmatrix} v \\ w \end{bmatrix} \tag{3.2}$$

$$\dot{\theta}=w \tag{3.3}$$

3.2 Reference trajectory design for lane change (without previous vehicle)

First, we are considering the case in which previous vehicle is not considered. Two 5th order polynomials are use, one for longitudinal direction and one for lateral direction. The polynomial equations are derived by having six constraints three initial values for position, velocity and acceleration and three final values for position, velocity and acceleration. For simplification, two assumptions are made. First is that initial accelerations in both directions and final accelerations in both directions are assumed to be zero. Second is that the velocity of host vehicle is kept same for the whole time of lane change.

3.2.1 Trajectory calculation parameters

To design lane-change trajectory total six constraints are defined, three initial values of position, velocity and acceleration in longitudinal and lateral direction and similarly three final values of position, velocity and acceleration in longitudinal and lateral direction. To meet all six constraints fifth order polynomial function of time is formulated for both lateral and longitudinal direction. Longitudinal direction is denoted as x and y denotes the lateral direction. Polynomials (3.4), (3.5) shows the positions in terms of time.

$$x(\mathrm{t}) = a_5 t^5 + a_4 t^4 + a_3 t^3 + a_2 t^2 + a_1 t + a_0 \tag{3.4}$$

$$\mathrm{y}(\mathrm{t}) = b_5 t^5 + b_4 t^4 + b_3 t^3 + b_2 t^2 + b_1 t + b_0 \tag{3.5}$$

Taking first derivative of equations (3.4), (3.5) to get the velocity equations in term of time

$$\dot{x}(\mathrm{t}) = 5a_5 t^4 + 4a_4 t^3 + 3a_3 t^2 + 2a_2 t + a_1 \tag{3.6}$$

$$\dot{\mathrm{y}}(\mathrm{t}) = 5b_5 t^4 + 4b_4 t^3 + 3b_3 t^2 + 2b_2 + b_1 \tag{3.7}$$

Taking second derivative of equations (3.6), (3.7) to get the acceleration equations in term of time

$$\ddot{x}(\mathrm{t}) = 20a_5 t^3 + 12a_4 t^2 + 6a_3 t^1 + 2a_2 \tag{3.8}$$

$$\ddot{\mathrm{y}}(\mathrm{t}) = 20b_5 t^3 + 12b_4 t^2 + 6b_3 t^1 + 2b_2 \tag{3.9}$$

Writing in matrix form for easier calculations (longitudinal direction)

$$\begin{bmatrix} x_0 \\ \dot{x}_0 \\ \ddot{x}_0 \\ x_f \\ \dot{x}_f \\ \ddot{x}_f \end{bmatrix} = \begin{bmatrix} t_0^{\ 5} & t_0^{\ 4} & t_0^{\ 3} & t_0^{\ 2} & t_0^{\ 1} & 1 \\ 5t_0^{\ 4} & 4t_0^{\ 3} & 3t_0^{\ 2} & 2t_0^{\ 1} & 1 & 0 \\ 20t_0^{\ 3} & 12t_0^{\ 2} & 6t_0^{\ 1} & 2 & 0 & 0 \\ t_f^{\ 5} & t_f^{\ 4} & t_f^{\ 3} & t_f^{\ 2} & t_f^{\ 1} & 1 \\ 5t_f^{\ 4} & 4t_f^{\ 3} & 3t_f^{\ 2} & 2t_f^{\ 1} & 1 & 0 \\ 20t_f^{\ 5} & 12t_f^{\ 5} & 6t_f^{\ 5} & 2 & 0 & 0 \end{bmatrix} \begin{bmatrix} a_5 \\ a_4 \\ a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix} \tag{3.10}$$

Writing in matrix form for easier calculations (lateral direction)

$$\begin{bmatrix} y_0 \\ \dot{y}_0 \\ \ddot{y}_0 \\ y_f \\ \dot{y}_f \\ \ddot{y}_f \end{bmatrix} = \begin{bmatrix} t_0^{\ 5} & t_0^{\ 4} & t_0^{\ 3} & t_0^{\ 2} & t_0^{\ 1} & 1 \\ 5t_0^{\ 4} & 4t_0^{\ 3} & 3t_0^{\ 2} & 2t_0^{\ 1} & 1 & 0 \\ 20t_0^{\ 3} & 12t_0^{\ 2} & 6t_0^{\ 1} & 2 & 0 & 0 \\ t_f^{\ 5} & t_f^{\ 4} & t_f^{\ 3} & t_f^{\ 2} & t_f^{\ 1} & 1 \\ 5t_f^{\ 4} & 4t_f^{\ 3} & 3t_f^{\ 2} & 2t_f^{\ 1} & 1 & 0 \\ 20t_f^{\ 5} & 12t_f^{\ 5} & 6t_f^{\ 5} & 2 & 0 & 0 \end{bmatrix} \begin{bmatrix} b_5 \\ b_4 \\ b_3 \\ b_2 \\ b_1 \\ b_0 \end{bmatrix} \tag{3.11}$$

Now six unknown coefficients need to be calculated for both lateral and longitudinal direction. First, we will find six unknown coefficients for longitudinal direction. Boundary conditions for longitudinal direction are given below in equations (3.12) and (3.13)

$$X_0 = \begin{pmatrix} x_0 \\ \dot{x}_0 \\ \ddot{x}_0 \end{pmatrix} = \begin{pmatrix} 0 \\ v_{des_i} \\ 0 \end{pmatrix} \tag{3.12}$$

$$X_f = \begin{pmatrix} x_f \\ \dot{x}_f \\ \ddot{x}_f \end{pmatrix} = \begin{pmatrix} d \\ v_{des_f} \\ 0 \end{pmatrix} \tag{3.13}$$

Where v_{des_i} and v_{des_f} are the initial and final desired velocity which is kept same as per our assumption and d is the total traveled distance in longitudinal direction.

Now putting initial and final time as t=0 and t_f =T, where T is the total time taken for lane change. Equation (3.11) will become

$$\begin{bmatrix} x_0 \\ \dot{x}_0 \\ \ddot{x}_0 \\ x_f \\ \dot{x}_f \\ \ddot{x}_f \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ T^5 & T^4 & T^3 & T^2 & T^1 & 1 \\ 5T^4 & 4T^3 & 3T^2 & 2T^1 & 1 & 0 \\ 20T^5 & 12T^5 & 6T^5 & 2 & 0 & 0 \end{bmatrix} \begin{bmatrix} a_5 \\ a_4 \\ a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix} \quad (3.14)$$

Now from above matrix form we can easily calculate values of first three coefficients and they are given below

$$a_0 = 0 \quad (3.15)$$

$$a_1 = V_{des} \quad (3.16)$$

$$a_2 = 0 \quad (3.17)$$

Now to calculate rest of three coefficients we will do some more calculation

$$x_f = a_5T^5 + a_4T^4 + a_3T^3 + a_2T^2 + a_1T + a_0$$

$$x_f = a_5T^5 + a_4T^4 + a_3T^3 + V_{des}T \quad (3.18)$$

$$\dot{x}_f = 5a_5T^4 + 4a_4T^3 + 3a_3T^2 + 2a_2T + a_1$$

$$\dot{x}_f = 5a_5T^4 + 4a_4T^3 + 3a_3T^2 + V_{des} \quad (3.19)$$

$$\ddot{x}_f = 20a_5T^3 + 12a_4T^2 + 6a_3T + 2a_2$$

$$\ddot{x}_f = 20a_5T^3 + 12a_4T^2 + 6a_3T \quad (3.20)$$

From equation (3.18), we get

$$d = a_5T^5 + a_4T^4 + a_3T^3 + V_{des}T \quad (3.21)$$

$$d - V_{des}T = a_5T^5 + a_4T^4 + a_3T^3 \quad (3.22)$$

From equation (3.19), we get

$$V_{des_i} = 5a_5T^4 + 4a_4T^3 + 3a_3T^2 + V_{des_i} \tag{3.23}$$

$$0 = 5a_5T^4 + 4a_4T^3 + 3a_3T^2 \tag{3.24}$$

From equation (3.20), we get

$$0 = 20a_5T^3 + 12a_4T^2 + 6a_3T^1 \tag{3.25}$$

Now we have written the equations (3.22), (3.24) and (3.25) in form of three unknown coefficients, so for easier calculation written in matrix form given in equation (3.26)

$$\begin{bmatrix} T^3 & T^4 & T^5 \\ 3T^2 & 4T^3 & 5T^4 \\ 6T^1 & 12T^2 & 20T^3 \end{bmatrix} \begin{bmatrix} a_3 \\ a_4 \\ a_5 \end{bmatrix} = \begin{bmatrix} d - V_{des}T \\ 0 \\ 0 \end{bmatrix} \tag{3.26}$$

$$\begin{bmatrix} a_3 \\ a_4 \\ a_5 \end{bmatrix} = \begin{bmatrix} T^3 & T^4 & T^5 \\ 3T^2 & 4T^3 & 5T^4 \\ 6T^1 & 12T^2 & 20T^3 \end{bmatrix}^{-1} \begin{bmatrix} d - V_{des}T \\ 0 \\ 0 \end{bmatrix} \tag{3.27}$$

Finally, the six coefficients for longitudinal direction are calculated and shown in equation (3.28) below

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix} = \begin{bmatrix} 0 \\ V_{des} \\ 0 \\ \frac{10}{T^3}(\mathrm{d} - \mathrm{V}_{des}\mathrm{T}) \\ -\frac{15}{T^4}(\mathrm{d} - \mathrm{V}_{des}\mathrm{T}) \\ \frac{6}{T^5}(\mathrm{d} - \mathrm{V}_{des}\mathrm{T}) \end{bmatrix} \tag{3.28}$$

Now we will find six unknown coefficients for lateral direction by applying the same procedure as applied for longitudinal direction. Boundary conditions for lateral direction are given below in equations (3.29) and (3.30)

$$Y_0 = \begin{pmatrix} y_0 \\ \dot{y}_0 \\ \ddot{y}_0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{3.29}$$

$$Y_f = \begin{pmatrix} y_f \\ \dot{y}_f \\ \ddot{y}_f \end{pmatrix} = \begin{pmatrix} \omega \\ 0 \\ 0 \end{pmatrix} \tag{3.30}$$

By applying the same procedure as applied for longitudinal direction the six coefficients for lateral direction are calculated and shown in equation (3.31) below

$$\begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ (10/\mathrm{T}^3)w \\ -(15/\mathrm{T}^4)w \\ (6/\mathrm{T}^5)w \end{pmatrix} \tag{3.31}$$

Trajectories in both longitudinal direction and lateral direction are given in equation (3.32) and equation (3.33) respectively.

$$x(\mathrm{t}) = \frac{6}{T^5}(\mathrm{d}-\mathrm{V}_{des}\mathrm{T})\,\mathrm{t}^5 - \frac{15}{T^4}(\mathrm{d}-\mathrm{V}_{des}\mathrm{T})\,\mathrm{t}^4 + \frac{10}{T^3}(\mathrm{d}-\mathrm{V}_{des}\mathrm{T})\,\mathrm{t}^3 + V_{des}t \tag{3.32}$$

$$\mathrm{y}(\mathrm{t}) = \frac{6}{T^5}(\omega)\,\mathrm{t}^5 - \frac{15}{T^4}(\omega)\,\mathrm{t}^4 + \frac{10}{T^3}(\omega)\,\mathrm{t}^3 \tag{3.33}$$

By putting values of desired velocity, final position in longitudinal and lateral direction and total time for lane change, reference trajectory will be attained.

Symbol	Description
v_{des_i}	Desired Velocity (initial)
v_{des_f}	Desired Velocity (final)
d	Final position in longitudinal direction
ω	Final position in lateral direction
T	Total time for Lane-change
R_{Host}	Radius of host vehicle
R_{Pre}	Radius of previous vehicle
L_{Host}	Length of host vehicle
L_{pre}	Length of previous vehicle
x_{Host}	Longitudinal Position of host vehicle
x_{Pre}	Longitudinal Position of previous vehicle
y_{Host}	Lateral Position of host vehicle
y_{Pre}	Lateral Position of previous vehicle
$dist$	Distance between vehicles

Table 3.1 Common symbols used in algorithm

3.3 Example/scenario

Let’s take the time T=6, the total time taken by the vehicle to complete the lane change. Initial and final values are given in equation (3.34) and (3.35)

$$\begin{pmatrix} x_0 \\ \dot{x}_0 \\ \ddot{x}_0 \\ x_f \\ \dot{x}_f \\ \ddot{x}_f \end{pmatrix} = \begin{pmatrix} 0 \\ 20 \\ 0 \\ 100 \\ 20 \\ 0 \end{pmatrix} \tag{3.34}$$

$$\begin{pmatrix} y_0 \\ \dot{y}_0 \\ \ddot{y}_0 \\ y_f \\ \dot{y}_f \\ \ddot{y}_f \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 4 \\ 0 \\ 0 \end{pmatrix} \tag{3.35}$$

Having above initial and final conditions the reference trajectory is generated by the help of equations (3.32) and (3.33).and shown in figure (3.2)

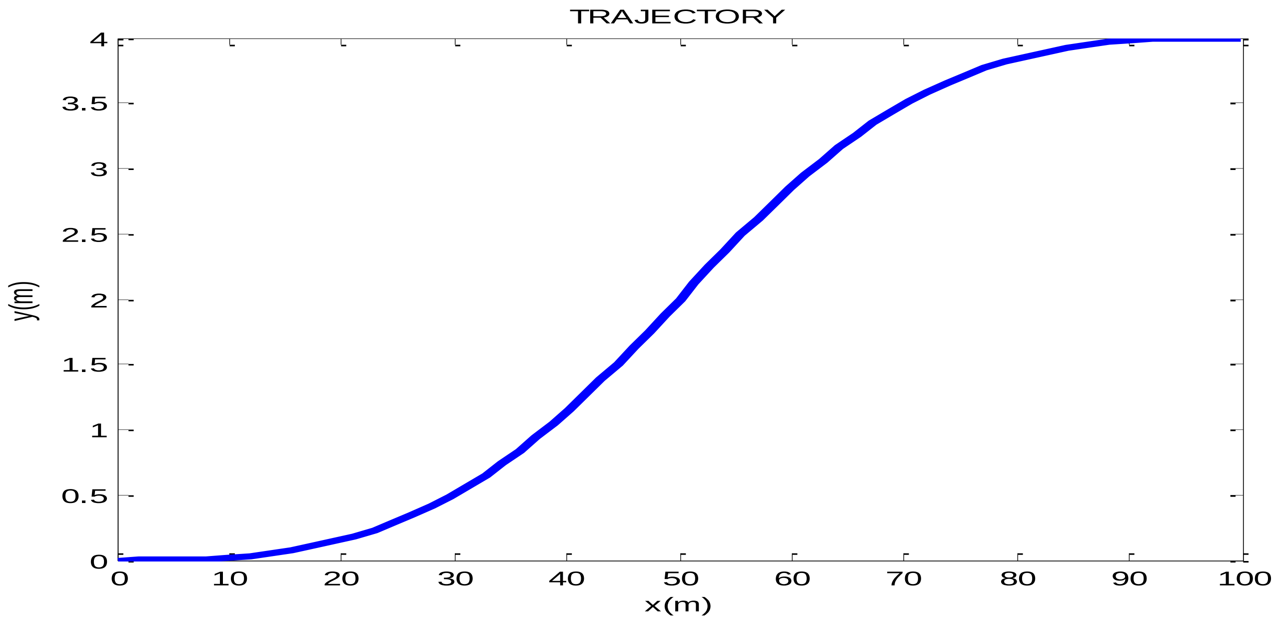

Figure 3.2 Reference Trajectory

Graphs for position, velocity and acceleration for both longitudinal and lateral direction are shown in figures below

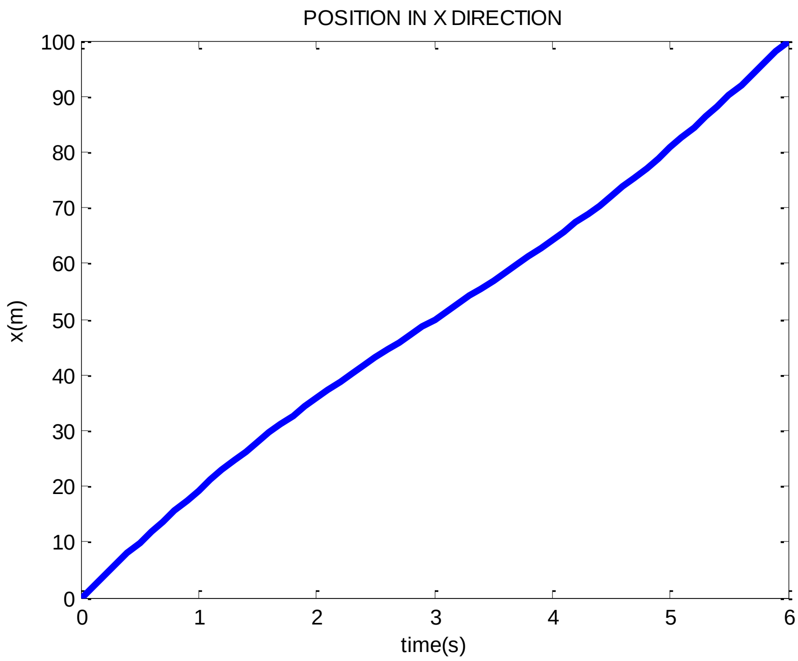

Figure 3.3 Position in longitudinal direction

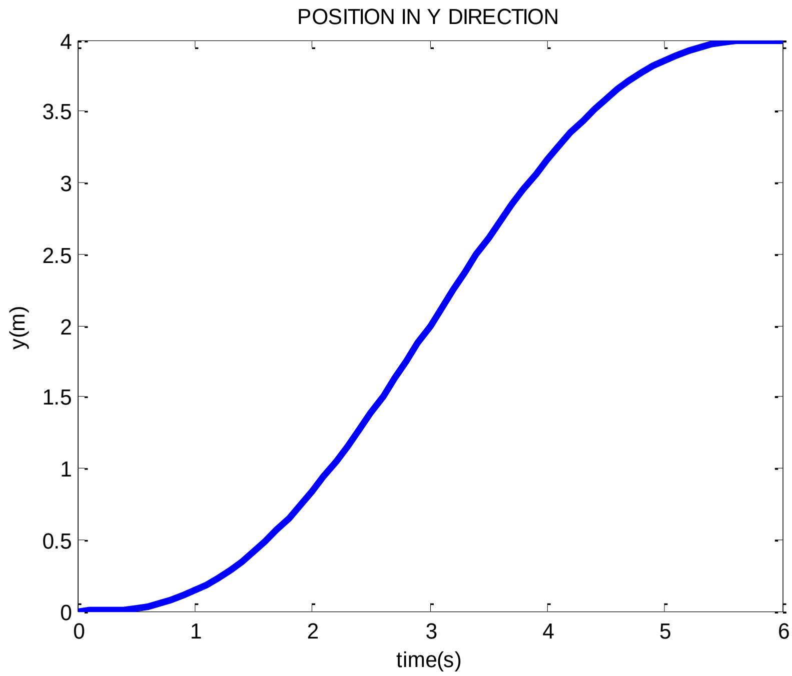

Figure 3.4 Position in lateral direction

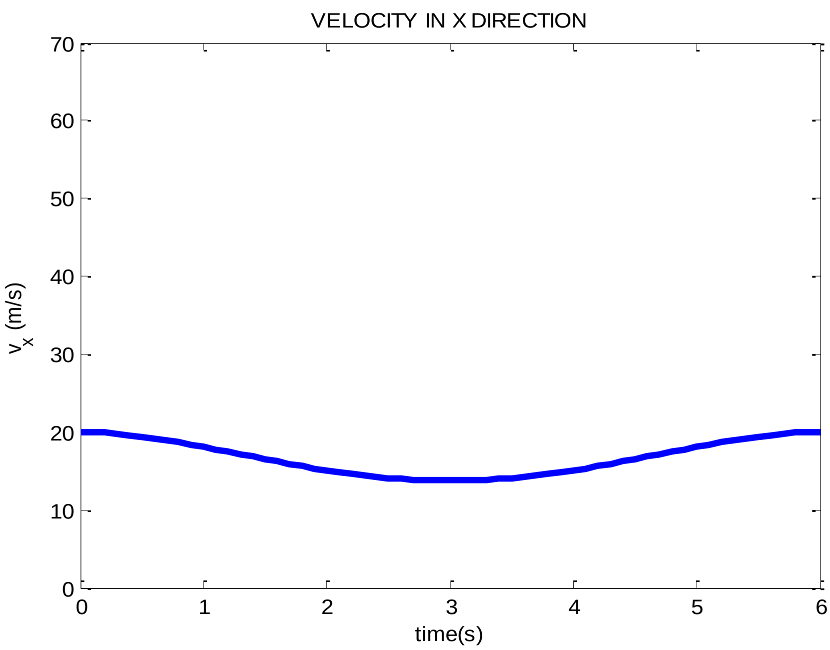

Figure 3.5 Velocity in longitudinal direction

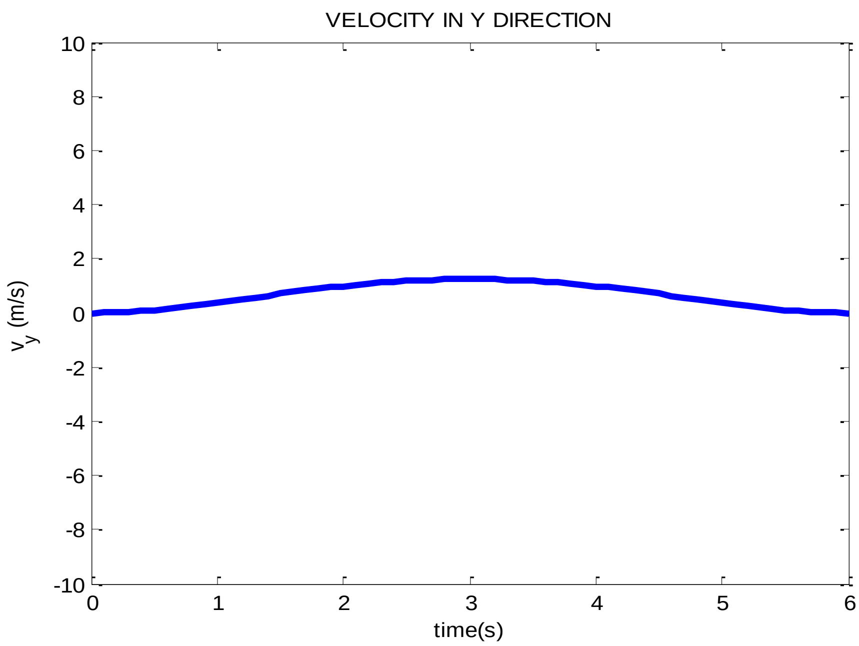

Figure 3.6 Velocity in lateral direction

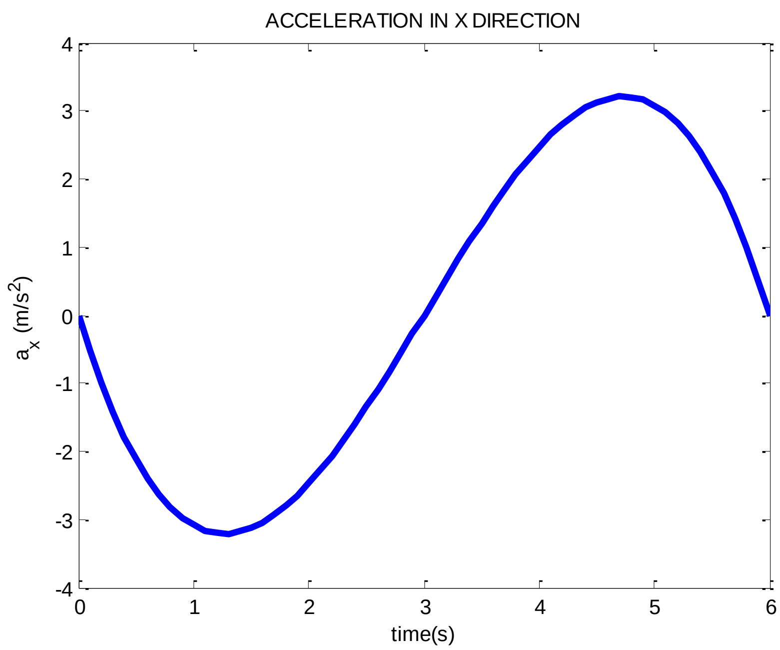

Figure 3.7 Acceleration in longitudinal direction

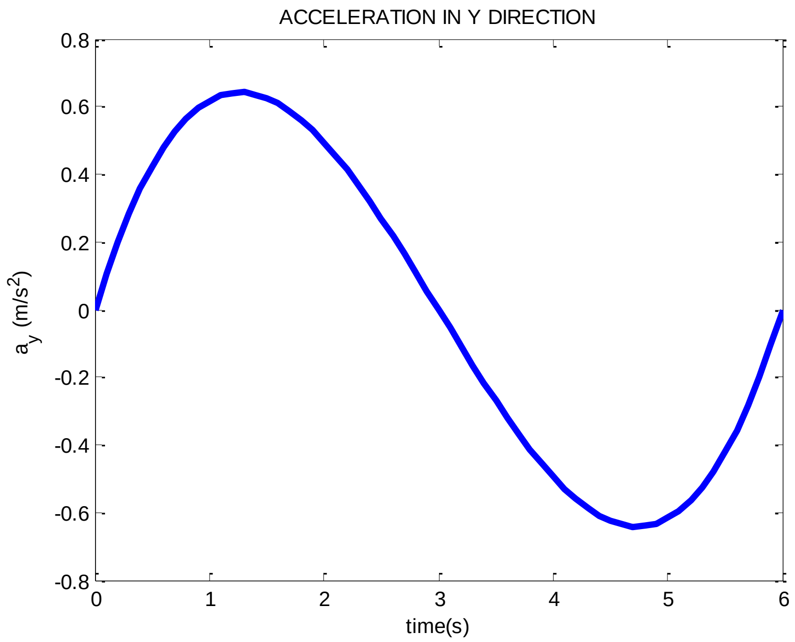

Figure 3.8 Acceleration in lateral direction

3.4 Trajectory control

After getting the reference trajectory as shown in Figure 2, PID controller is applied to track the reference values. Longitudinal (x_r) and lateral (y_r) coordinates are used as reference to PID controller.

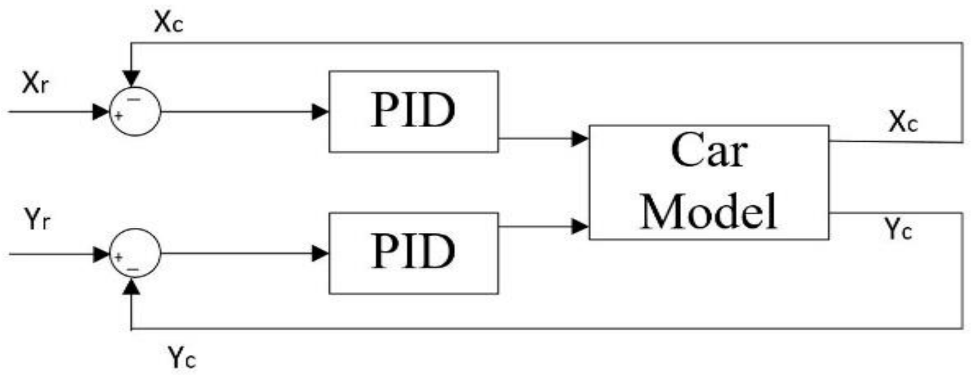

Figure 3.9 General Block Diagram

PID controller is applied on car model and output taken is controlled Longitudinal (x_c) and lateral (y_c) coordinates. Simulation is performed on SIMULINK / MATLAB and controlled trajectory is obtained as shown in figure (3.10)

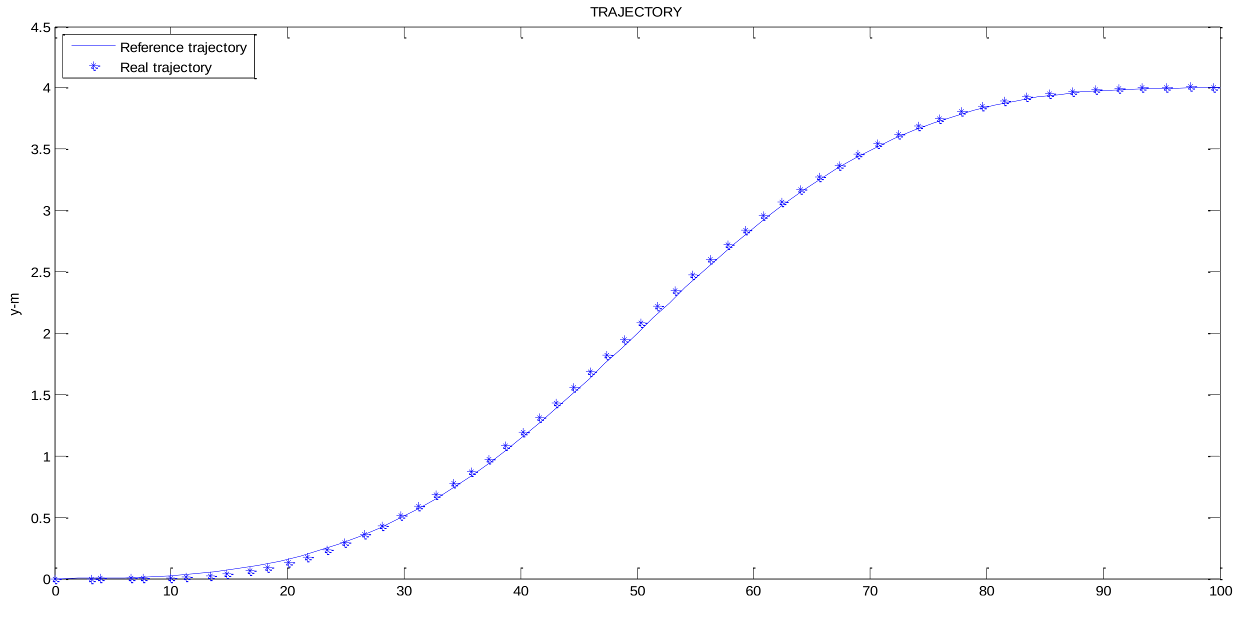

Figure 3.10 Controlled Trajectory

Figure (3.11) shows the error in longitudinal and lateral direction

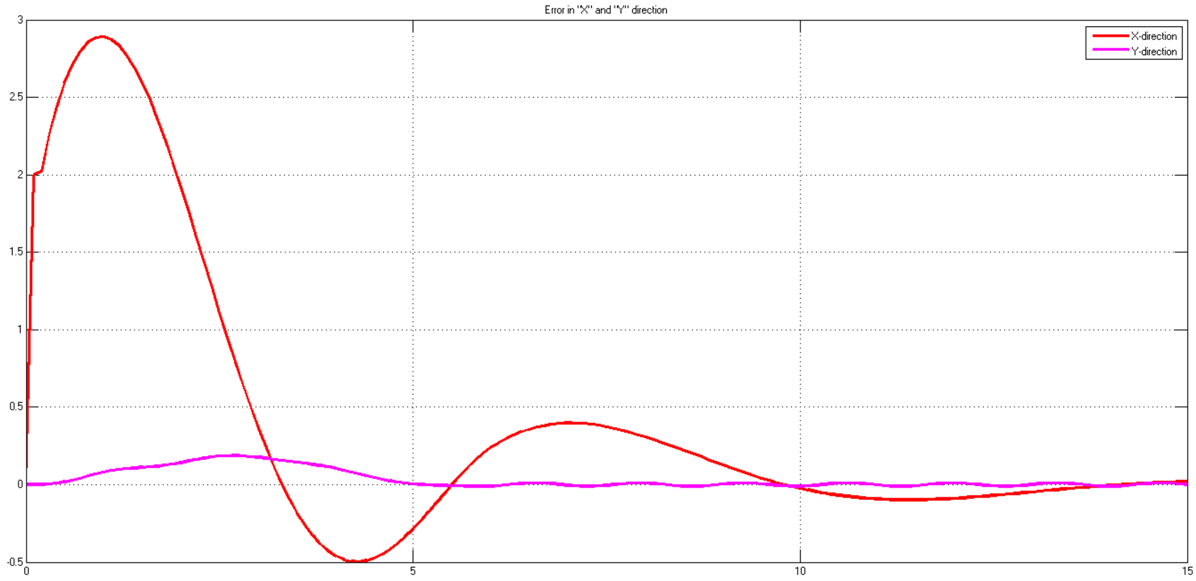

Figure 3.11 Error in both directions

CHAPTER 4: TRAJECTORY GENERATION AND CONTROL OF VEHICLE WITH ONE OBSTACLE

The trajectory generated in chapter 3 was designed considering that vehicle is moving on road without any vehicle around. Now we are considering the case in which previous vehicle is considered, there is major chance of collision if same algorithm as described in chapter 3 is applied in case where previous vehicle is present in adjacent lane. So collision avoidance technique need to be developed to avoid any collision between the vehicles.

4.1 Collision avoidance

For collision avoidance the vehicle is modeled as one circle for simplicity. The circle encloses the whole vehicle with radius R. By looking at figure (4.1) it can be observed that there is some extra space around the vehicle and that's good in case of little uncertainty caused sensors. Although accurate method to model the vehicle is also discussed in this chapter in which vehicle is modelled as series of circle.

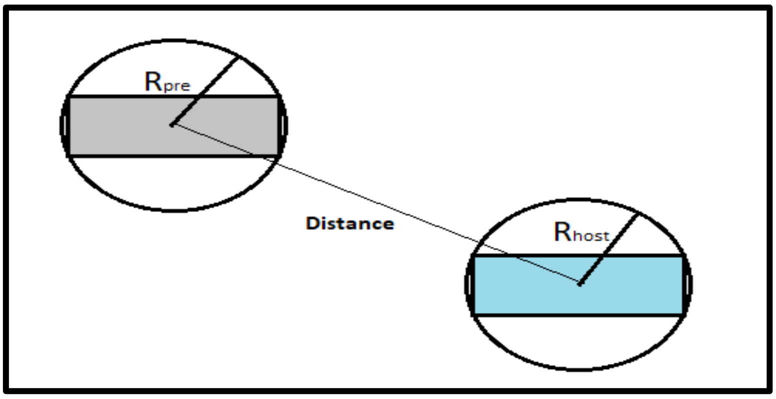

Figure 4.1 Distance Between Vehicles

Figure (4.1) shows both host vehicle and previous vehicle is represented by one circle with radius and R_{Host} respectively R_{Pre}. Both the radii are half the length of vehicle. Now the distance between the host vehicle and previous vehicle can be described as

$$dist = \sqrt{(x_{Host} - x_{Pre})^2 + (y_{Host} - y_{Pre})^2} \tag{4.1}$$

$$dist > (R_{Host} + R_{Pre}) \tag{4.2}$$

To avoid collision between two vehicles the above equation (4.2) must true for whole lane change process. The circle around the vehicle cover all the parts of vehicle and as long as the distance from mid-point of host vehicle to mid-point of previous vehicle is greater than the sum of their respective radii there will be no collision.

4.2 Trajectory generation with previous vehicle

The trajectory generated in this section will consider previous vehicle in adjacent lane, for that purpose extra coefficient is added to longitudinal direction by making it 6th order polynomial.

$$x(\mathrm{t}) = a_6 t^6 + a_5 t^5 + a_4 t^4 + a_3 t^3 + a_2 t^2 + a_1 t + a_0 \qquad (4.3)$$

Taking first derivative of equation (4.3) to get the velocity equation in term of time

$$\dot{x}(\mathrm{t}) = 6a_6 t^5 + 5a_5 t^4 + 4a_4 t^3 + 3a_3 t^2 + 2a_2 t + a_1 \qquad (4.4)$$

Taking second derivative of equation (4.4) to get the acceleration equations in term of time

$$\ddot{x}(\mathrm{t}) = 30a_6 t^4 + 20a_5 t^3 + 12a_4 t^2 + 6a_3 t + 2a_2 \qquad (4.5)$$

Writing in matrix form for easier calculations (longitudinal direction)

$$\begin{bmatrix} x_0 \\ \dot{x}_0 \\ \ddot{x}_0 \\ x_f \\ \dot{x}_f \\ \ddot{x}_f \end{bmatrix} = \begin{bmatrix} t_0{}^6 & t_0{}^5 & t_0{}^4 & t_0{}^3 & t_0{}^2 & t_0{}^1 & 1 \\ 6t_0{}^5 & 5t_0{}^4 & 4t_0{}^3 & 3t_0{}^2 & 2t_0{}^1 & 1 & 0 \\ 30t_0{}^4 & 20t_0{}^3 & 12t_0{}^2 & 6t_0{}^1 & 2 & 0 & 0 \\ t_f{}^6 & t_f{}^5 & t_f{}^4 & t_f{}^3 & t_f{}^2 & t_f & 1 \\ 6t_f{}^5 & 5t_f{}^4 & 4t_f{}^3 & 3t_f{}^2 & 2t_f & 1 & 0 \\ 30t_f{}^4 & 20t_f{}^3 & 12t_f{}^2 & 6t_f & 2 & 0 & 0 \end{bmatrix} \begin{bmatrix} a_6 \\ a_5 \\ a_4 \\ a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix} \qquad (4.6)$$

Now we have seven unknown coefficients need to be calculated for longitudinal direction. First we will find six unknown coefficients for longitudinal direction and separate the new coefficient "a_6". Boundary conditions for longitudinal direction are given below in equations (4.7) and (4.8)

$$X_0 = \begin{pmatrix} x_0 \\ \dot{x}_0 \\ \ddot{x}_0 \end{pmatrix} = \begin{pmatrix} 0 \\ v_{des_i} \\ 0 \end{pmatrix} \tag{4.7}$$

$$X_f = \begin{pmatrix} x_f \\ \dot{x}_f \\ \ddot{x}_f \end{pmatrix} = \begin{pmatrix} d \\ v_{des_f} \\ 0 \end{pmatrix} \tag{4.8}$$

Where v_{des_i} and v_{des_f} are the initial and final desired velocity and d is the total traveled distance in longitudinal direction.

Now putting initial and final time as t_0 =0 and t_f =T, where T is the total time taken for lane change. Equation (4.6) will become

$$\begin{bmatrix} x_0 \\ \dot{x}_0 \\ \ddot{x}_0 \\ x_f \\ \dot{x}_f \\ \ddot{x}_f \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & 0 \\ T^6 & T^5 & T^4 & T^3 & T^2 & T^1 & 1 \\ 6T^5 & 5T^4 & 4T^3 & 3T^2 & 2T^1 & 1 & 0 \\ 30T^4 & 20T^3 & 12T^2 & 6T & 2 & 0 & 0 \end{bmatrix} \begin{bmatrix} a_6 \\ a_5 \\ a_4 \\ a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix} \tag{4.9}$$

From above equation (4.9) three coefficients can be calculated shown below

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 0 \\ V_{des_i} \\ 0 \end{bmatrix} \tag{4.10}$$

Now to find remaining coefficients we will do the calculations as shown below

$$x_f = a_6T^6 + a_5T^5 + a_4T^4 + a_3T^3 + a_2T^2 + a_1T + a_0 \tag{4.11}$$

$$d - V_{des}T - a_6T^6 = a_5T^5 + a_4T^4 + a_3T^3 \tag{4.12}$$

$$\dot{x}_f = 6a_6T^5 + 5a_5T^4 + 4a_4T^3 + 3a_3T^2 + 2a_2T + a_1 \tag{4.13}$$

$$v_{des_f} - v_{des_i} - 6a_6T^5 = 5a_5T^4 + 4a_4T^3 + 3a_3T^2 \tag{4.14}$$

$$\ddot{x}_f = 30a_6T^4 + 20a_5T^3 + 12a_4T^2 + 6a_3T + 2a_2 \tag{4.15}$$

$$-30a_6T^4 = 20a_5T^3 + 12a_4T^2 + 6a_3T \tag{4.16}$$

Now the above equations are written in form of three unknown coefficients, and the extra coefficient a_6 is also separated because it's very important coefficient as collision avoidance mainly depends on value of a_6.

Equation (4.17) shows the values of all longitudinal coefficients

$$\begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{pmatrix} = \begin{pmatrix} 0 \\ v_{des_i} \\ 0 \\ -1/\mathrm{T}^3(6v_{des_i}T - 10\,\mathrm{d} + 4v_{des_f}T + a_6T^6) \\ 1/\mathrm{T}^4(8v_{des_i}T - 15\,\mathrm{d} + 7v_{des_f}T + 3a_6T^6) \\ -3/\mathrm{T}^5(v_{des_i}T - 2\,\mathrm{d} + v_{des_f}T + a_6T^6) \end{pmatrix} \tag{4.17}$$

The extra coefficient a6 should be free and independent as its design coefficient, so we need to isolate the a6 coefficient to calculate the feasible value and to avoid collision.

Equations (4.18), (4,19) and (4.20) shows the values of coefficients in which a_6 is included

$$a_3 = -1/\mathrm{T}^3(6v_{des_i}T - 10\,\mathrm{d} + 4v_{des_f}T) - a_6T^3 \tag{4.18}$$

$$a_4 = 1/\mathrm{T}^4(8v_{des_i}T - 15\,\mathrm{d} + 7v_{des_f}T) + 3a_6T^2 \tag{4.19}$$

$$a_5 = 3/\mathrm{T}^5(v_{des_i}T - 2\,\mathrm{d} + v_{des_f}T) - 3a_6T^1) \tag{4.20}$$

Equation (4.21) shows the isolated a_6 from all other coefficients

$$a_i = \alpha_i + \beta_i a_6 \tag{4.21}$$

$$\begin{pmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \end{pmatrix} = \begin{pmatrix} 0 \\ v_{des_i} \\ 0 \\ 2/\mathrm{T}^3(-3v_{des_i}T + 5\,\mathrm{d} - 2v_{des_f}T) \\ -1/\mathrm{T}^4(-8v_{des_i}T + 15\,\mathrm{d} - 7v_{des_f}T) \\ 3/\mathrm{T}^5(-v_{des_i}T + 2\,\mathrm{d} - v_{des_f}T) \end{pmatrix} \tag{4.22}$$

$$\begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \\ \beta_5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ -\mathrm{T}^3 \\ 3\mathrm{T}^2 \\ -3\mathrm{T} \end{pmatrix} \tag{4.23}$$

$$x(\mathrm{t}) = a_6t^6 + a_5t^5 + a_4t^4 + a_3t^3 + a_2t^2 + a_1t + a_0 \tag{4.24}$$

$$x(\mathrm{t}) = \mathrm{C}_1 + \mathrm{C}_2\mathrm{a}_6 \tag{4.25}$$

Now the design coefficient a6 is free to design.

$$\begin{aligned} C_1 = {} & 3/\mathrm{T}^5(-v_{des_i}T + 2\,\mathrm{d} - v_{des_f}T)\,\mathrm{t}^5 - 1/\mathrm{T}^4(-8v_{des_i}T + 15\,\mathrm{d} - 7v_{des_f}T)\,\mathrm{t}^4 \\ & + 2/\mathrm{T}^3(-3v_{des_i}T + 5\,\mathrm{d} - 2v_{des_f}T)\,\mathrm{t}^3 + v_{des_i}t \end{aligned} \tag{4.26}$$

$$C_2 = t^6 - 3Tt^5 + 3T^2t^4 - T^3t^3 \tag{4.27}$$

4.3 Vehicle representation

For accurate trajectory generation for lane change vehicle is now represented as series of circles instead of one circle covering the whole vehicle. Radius of circle in this case is equal to the width of the vehicle.

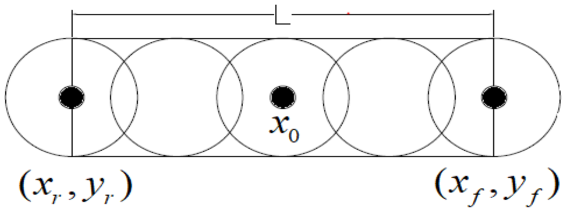

Figure 4.2 Vehicle Representations

Rectangle is used to represent the vehicle, where L is the length of vehicle, x_0 is the center point of vehicle, (x_r, y_r) shows the rear end of vehicle and (x_f, y_f) shows the front end of the vehicle

$$P = P_r + \mu(P_f - P_r) \tag{4.28}$$

In equation (4.28) P_r is the vector of rear circle center and P_f is the vector of front circle center. Now writing the above equation in Cartesian form

$$x = x_r + u(x_f - x_r) \tag{4.30}$$

$$y = y_r - u(y_f - y_r) \tag{4.31}$$

After writing in Cartesian form, u is the parameter connecting the front and rear circle. For example, if value of u is equal to 1 then

$$x = x_f \tag{4.32}$$

$$y = y_f \tag{4.33}$$

if value of u is equal to 0 then

$$x = x_r \tag{4.34}$$

$$y = y_r \tag{4.35}$$

Now we will apply our collision avoidance technique, in order to have collision free trajectory equation (4.37) must hold

$$dist = \sqrt{(x_{Host} - x_{\Pr e})^2 + (\mathrm{y}_{Host} - y_{\Pr e})^2} \tag{4.36}$$

$$dist > (\mathrm{R}_{Host} + R_{\Pr e}) \tag{4.37}$$

$$(x_{Host} - x_{\mathrm{pr}e})^2 + (\mathrm{y}_{Host} - y_{\mathrm{pr}e})^2 > (\mathrm{R}_{Host} - R_{\mathrm{pr}e}) \tag{4.38}$$

To build relationship between collision avoidance and longitudinal trajectory Putting equations (4.30), (4.31) in equation (4.38)

$$\begin{aligned}&[x_{Host(\mathrm{r})} + u_{Host}(x_{Host(\mathrm{f})} - x_{\mathrm{Host(r)}}) - x_{\mathrm{Pr}e(\mathrm{r})} - u_{\Pr e}(x_{\mathrm{Pre(f)}} - x_{\mathrm{Pr}e(\mathrm{r})})]^2 + \\ &[\mathrm{y}_{Host(\mathrm{r})} - \mathrm{y}_{\mathrm{Pre(r)}} - u_{\Pr e}(\mathrm{y}_{\mathrm{Pre(f)}} - \mathrm{y}_{\mathrm{Pre(r)}})]^2 > (\mathrm{R}_{Host} + R_{\Pr e})^2\end{aligned} \tag{4.39}$$

Now in order to simplify the equation assuming that lateral yaw angle is very small, also we are taking the same values for $\mathrm{R}_{Host}, R_{\mathrm{pr}e}$ because the length of both vehicles is kept same. So equation (4.39) becomes

$$\left[x_{Host_0} - \frac{L_{Host}}{2} + \mu_{Host} L_{Host} - x_{pre} - \mu_{pre}(x_{pre_f} - x_{pre_r})\right]^2 + \left[\mathrm{y}_{Host_r} - y_{\mathrm{pre}_r} - \mu_{pre}(\mathrm{y}_{pre_f} - y_{pre_r})\right]^2 > (\mathrm{R}_{Host} - R_{\mathrm{pr}e})^2 \tag{4.40}$$

Putting the longitudinal trajectory equation (4.25) in equation (4.40) we get

$$\left[C_1 + C_2 a_6 - \frac{L_{Host}}{2} + \mu_{Host} L_{Host} - x_{pre} - \mu_{pre}(x_{pre_f} - x_{pre_r})\right]^2 + \left[\mathrm{y}_{Host_r} - y_{pre_r}\right]^2 - (\mathrm{R}_{Host} - R_{\mathrm{pr}e})^2 > 0 \tag{4.41}$$

Writing the above equation in simpler form

$$(\lambda_1 + C_2 a_6)^2 + \lambda_2 > 0 \tag{4.42}$$

Where,

$$\lambda_1 = C_1 + L_{Host}(\mathrm{u}_{Host} - 0.5) - x_{\mathrm{Pr}e(\mathrm{r})} - u_{\Pr e} L_{\Pr e} \tag{4.43}$$

$$\lambda_2 = [\mathrm{y}_{Host(\mathrm{r})} - \mathrm{y}_{\mathrm{Pre(r)}}]^2 - (\mathrm{R}_{Host} + R_{\Pr e})^2 \tag{4.44}$$

Rewriting the equation in terms of a_6

$$C_2^{\,2}a_6^{\,2}+2\lambda_1C_2a_6+\lambda_1^{\,2}+\lambda_2>0 \tag{4.45}$$

The roots of equation (4.45) are

$$a_6<\frac{-2\lambda_1C_2-\sqrt{(2\lambda_1C_2)^2-4C_2^{\,2}(\lambda_1^{\,2}+\lambda_2)}}{2C_2^{\,2}} \tag{4.46}$$

$$a_6>\frac{-2\lambda_1C_2+\sqrt{(2\lambda_1C_2)^2-4C_2^{\,2}(\lambda_1^{\,2}+\lambda_2)}}{2C_2^{\,2}} \tag{4.47}$$

Where,

$$C_1=a_5t^5+a_4t^4+a_3t^3+a_2t^2+a_1t+a_0 \tag{4.48}$$

$$C_2=t^6-3Tt^5+3T^2t^4-T^3t^3 \tag{4.49}$$

As long as the equation (4.45) holds the trajectory will be safe and no collision will be observed between the host car and previous car. Equations (4.46), (4.47) shows the two roots of quadratic equation if the value of a6 is chosen outside these roots the will be no collision.

4.4 Simulation of trajectory generation in presence of previous vehicle

In order to validate the collision avoidance technique, we are considering an example in which boundary conditions are as follows, equation (4.50) shows boundary conditions for host vehicle in longitudinal direction

$$\begin{pmatrix} x_{0(\text{host})} \\ \dot{x}_{0(\text{host})} \\ \ddot{x}_{0(\text{host})} \\ x_{f(\text{host})} \\ \dot{x}_{f(\text{host})} \\ \ddot{x}_{f(\text{host})} \end{pmatrix} = \begin{pmatrix} 0 \\ 25 \\ 0 \\ 100 \\ 28 \\ 0 \end{pmatrix} \tag{4.50}$$

Equation (4.51) shows boundary conditions for host vehicle in lateral direction

$$\begin{pmatrix} y_{0(\text{host})} \\ \dot{y}_{0(\text{host})} \\ \ddot{y}_{0(\text{host})} \\ y_{f(\text{host})} \\ \dot{y}_{f(\text{host})} \\ \ddot{y}_{f(\text{host})} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 3 \\ 0 \\ 0 \end{pmatrix} \tag{4.51}$$

Equation (4.52) shows boundary conditions for previous vehicle in longitudinal direction

$$\begin{pmatrix} x_{0(\text{pre})} \\ \dot{x}_{0(\text{pre})} \\ \ddot{x}_{0(\text{pre})} \\ x_{f(\text{pre})} \\ \dot{x}_{f(\text{pre})} \\ \ddot{x}_{f(\text{pre})} \end{pmatrix} = \begin{pmatrix} -17 \\ 28 \\ 0 \\ 100 \\ 28 \\ 0 \end{pmatrix} \tag{4.52}$$

Equation (4.53) shows boundary conditions for previous vehicle in lateral direction

$$\begin{pmatrix} y_{0(\text{pre})} \\ \dot{y}_{0(\text{pre})} \\ \ddot{y}_{0(\text{pre})} \\ y_{f(\text{pre})} \\ \dot{y}_{f(\text{pre})} \\ \ddot{y}_{f(\text{pre})} \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ 0 \\ 3 \\ 0 \\ 0 \end{pmatrix} \tag{4.53}$$

Now by the help of the above boundary condition the first step is to define the range of a_6 to have collision free trajectory, the range of a_6 is found by equations (4.46), (4.47) and shown in figure (4.3)

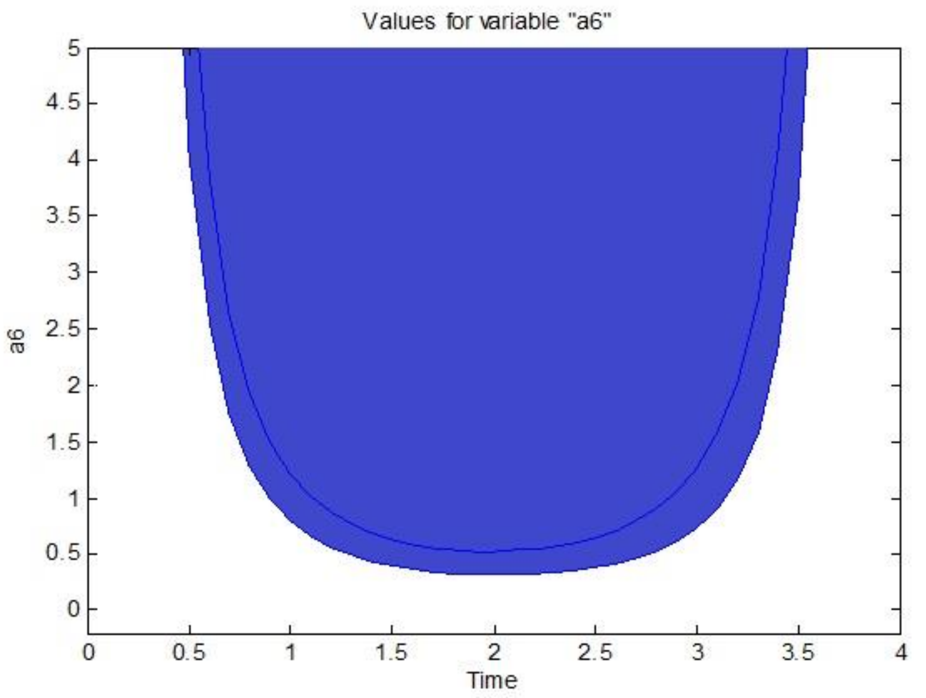

Figure 4.3 Range For “a6”

Now value of a6 should be outside the roots as shown in Figure 6. For clarification two values of a6 are taken, one from inside and one from outside the bounds as shown in Figure 6.

Selecting (a6=0.5) from inside the bounds we see there is clear collision as shown in Figure 7.

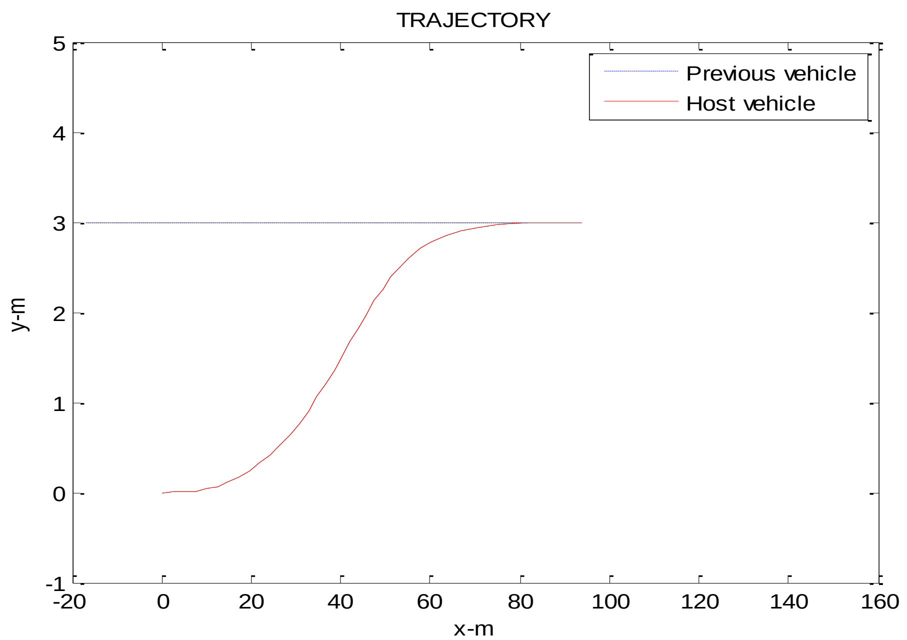

Figure 4.4 Generated Trajectory with Previous Vehicle (a6=0.5)

As we can observe that there is definite collision between previous vehicle and host vehicle, the relative distance between both vehicles is shown in Figure (4.5)

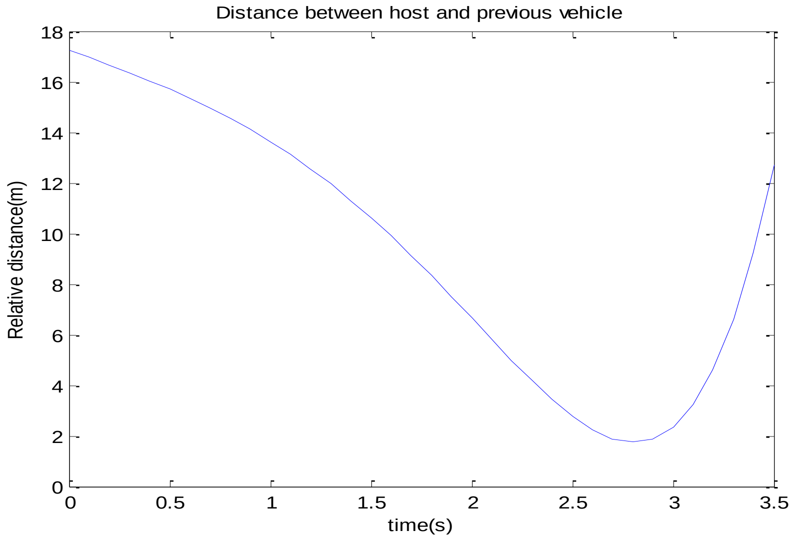

Figure 4.5 Relative distance Between Host and Previous Vehicle

The above graph indicates that the relative distance became very low, so there will be collision. So, for the safer and collision free trajectory value of a6 should be selected outside the roots. Taking the value of $a_6 = -0.05$ gives the following result shown in Figure (4.6) and relative distance between host and previous vehicle is shown in Figure (4.7).

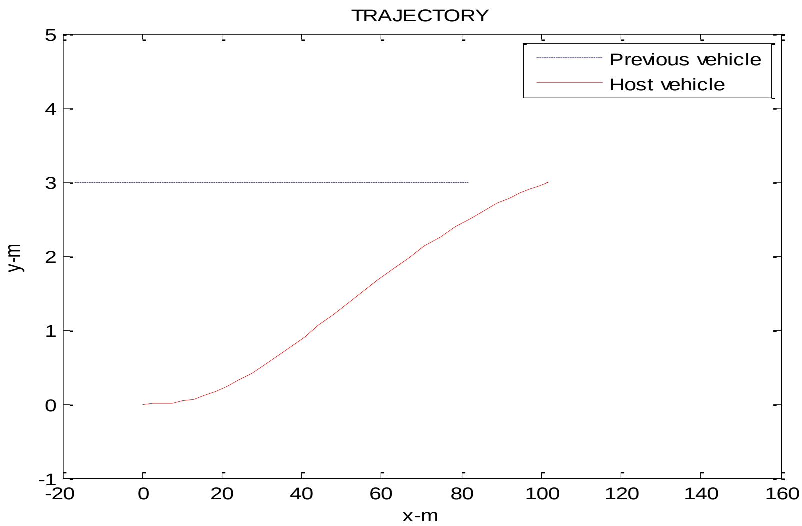

Figure 4.6 Generated Trajectory with Previous Vehicle (a6= 0.05)

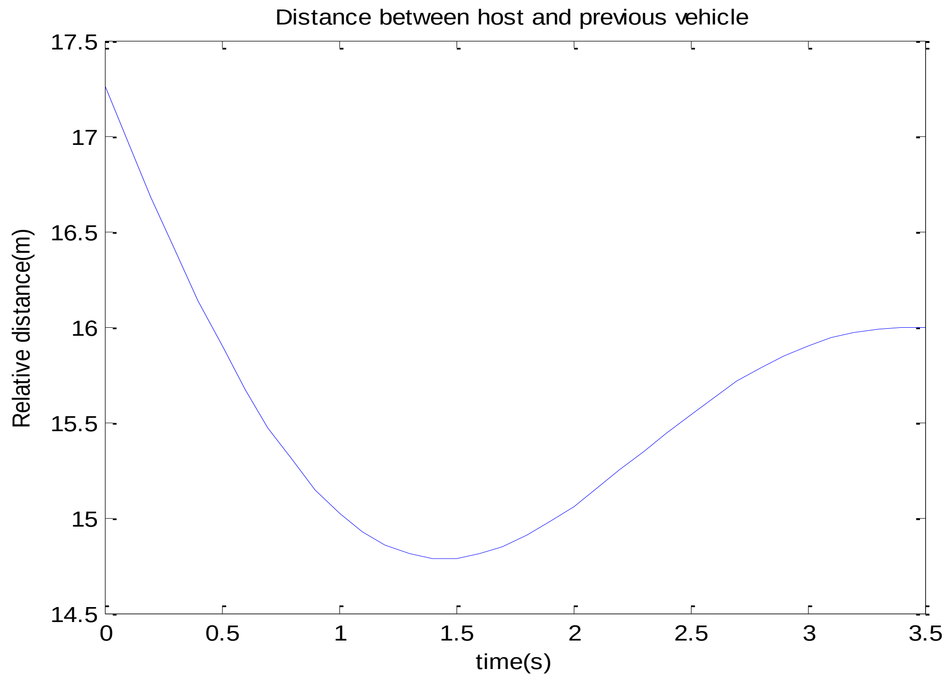

Figure 4.7 Relative distance Between Host and Previous Vehicle

After getting the reference trajectory, PID controller is applied exactly as it was applied in previous case, and controlled trajectory is achieved in case of including previous vehicle.

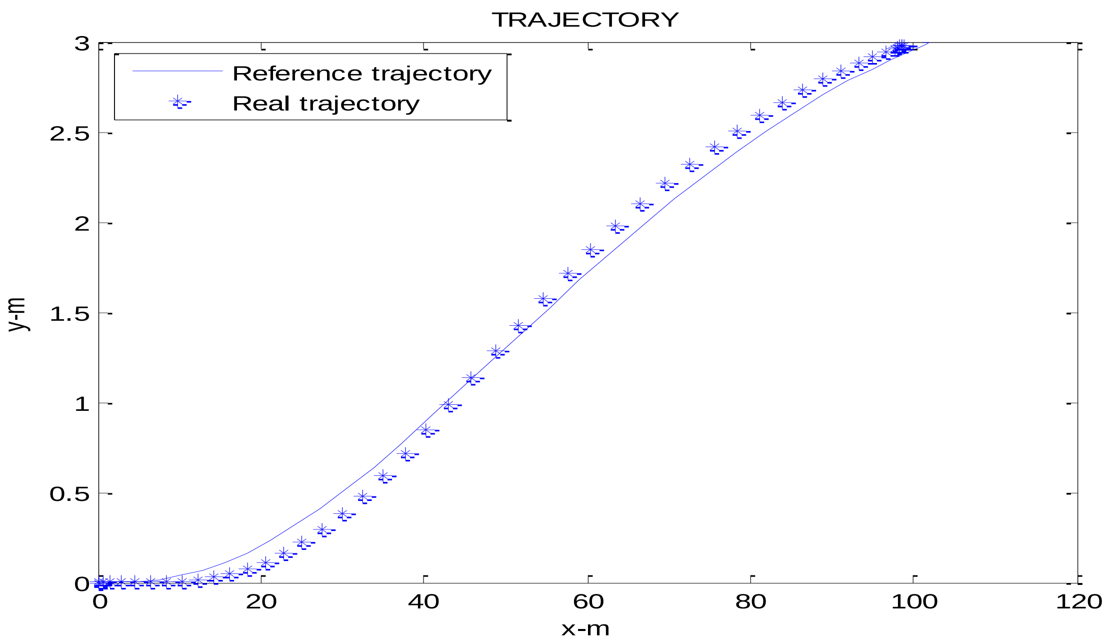

Figure 4.8 Controlled trajectory

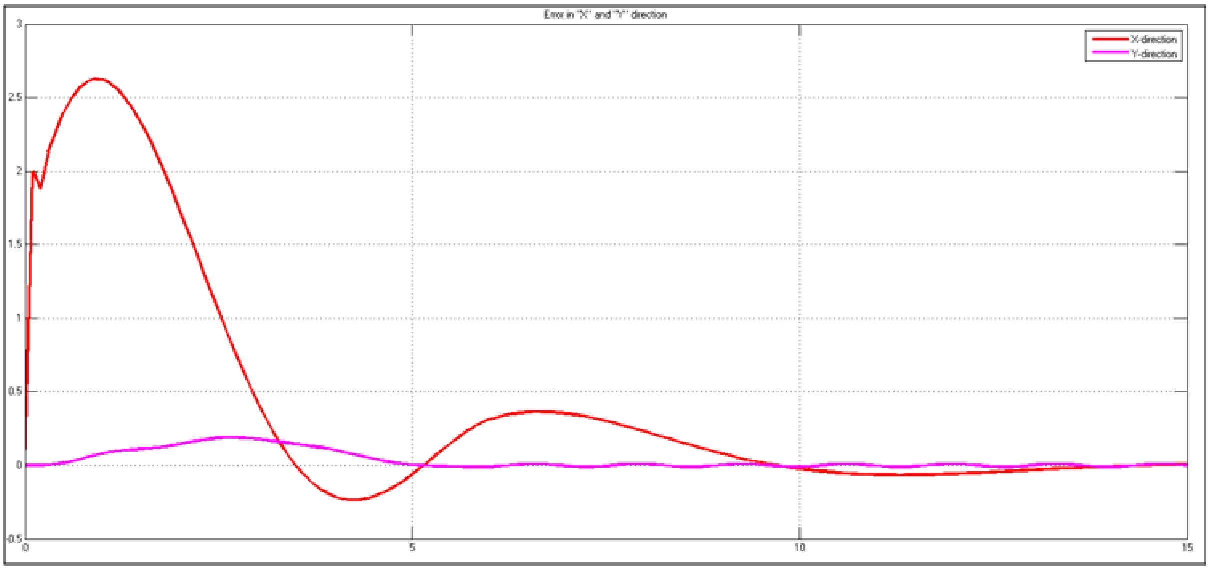

Figure 4.9 Errors in both directions

4.5 Simulation results by adding steering angle model

Kinematic model of car is shown in Figure 3.1, there is configuration of $\begin{pmatrix} x & y & \theta & \delta \end{pmatrix}$, where x, y represents the midpoint of rear end, θ represents the angle between car heading and x-axis and δ is the steering angle

$$\begin{aligned} \dot{x} &= v\cos\theta \\ \dot{y} &= v\sin\theta \\ \dot{\theta} &= v/l(\tan(\delta)) \\ \dot{\delta} &= w \end{aligned} \qquad u = \begin{bmatrix} v \\ w \end{bmatrix} \qquad (4.54)$$

L is the total distance between front and rear end. $u = (v, w)$ is the controlling vector, where v is the linear velocity and w is the angular velocity [29].

By updating the vehicle model by adding steering angle, reference trajectory is generated by the same way as generated before and similarly collision avoidance is applied. The final controlled trajectory with error graphs and is shown in figures.

Feasible trajectory is generated in this case as well, Simulink blocks diagrams are shown in figures

4.5.1 Results with PI Controller

Controlled trajectory is shown in figure below

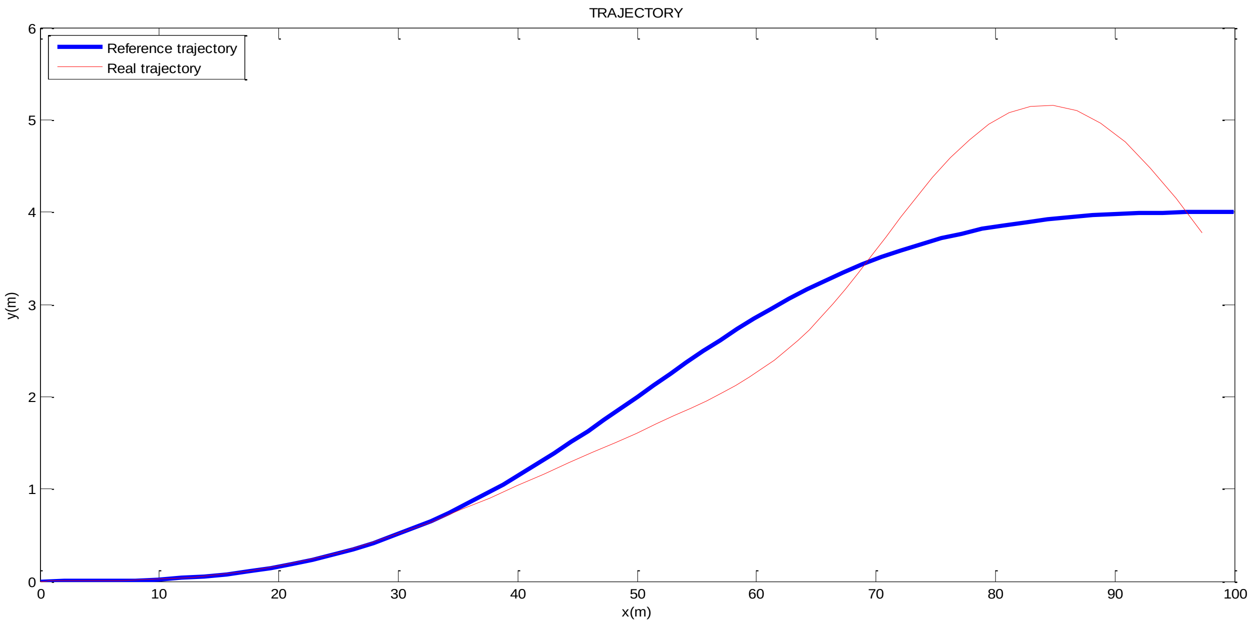

Figure 4.10 Controlled trajectory (with PI Controller)

Figure (4.11),(4.12) shows the Position in both directions

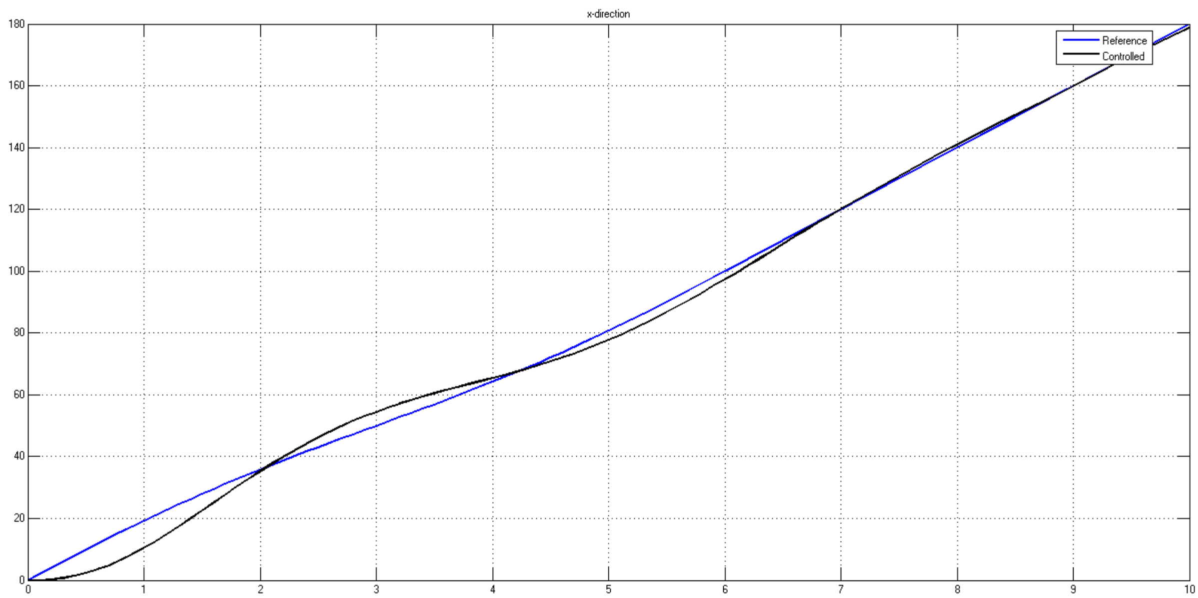

Figure 4.11 Position in X direction

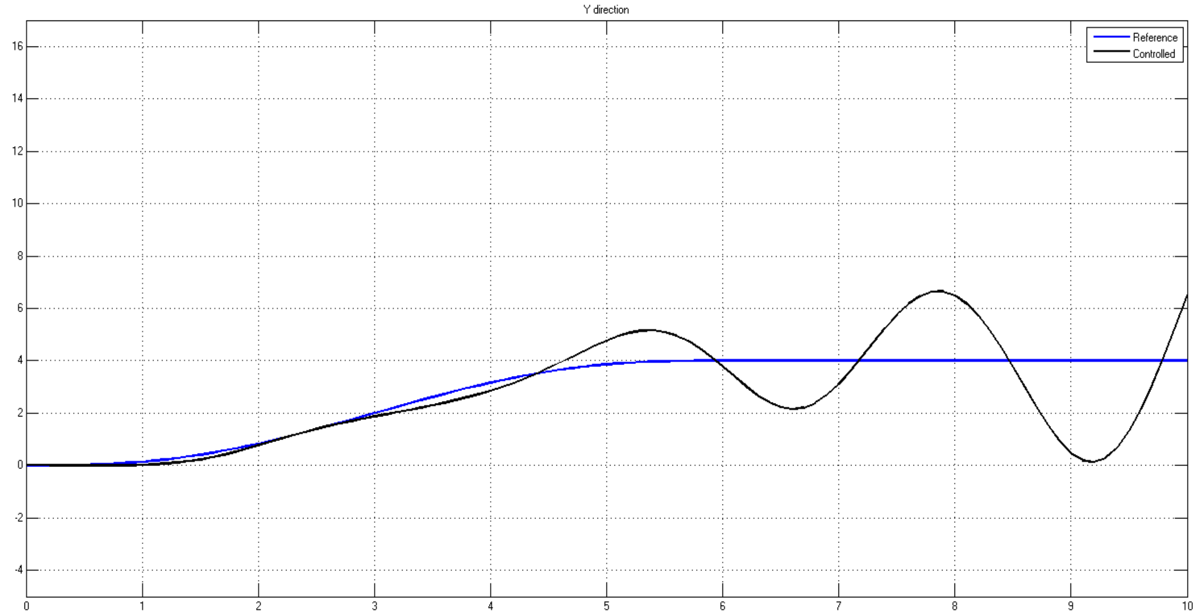

Figure 4.12 Position in Y direction

Figure (4.13) shows the error in longitudinal and lateral direction

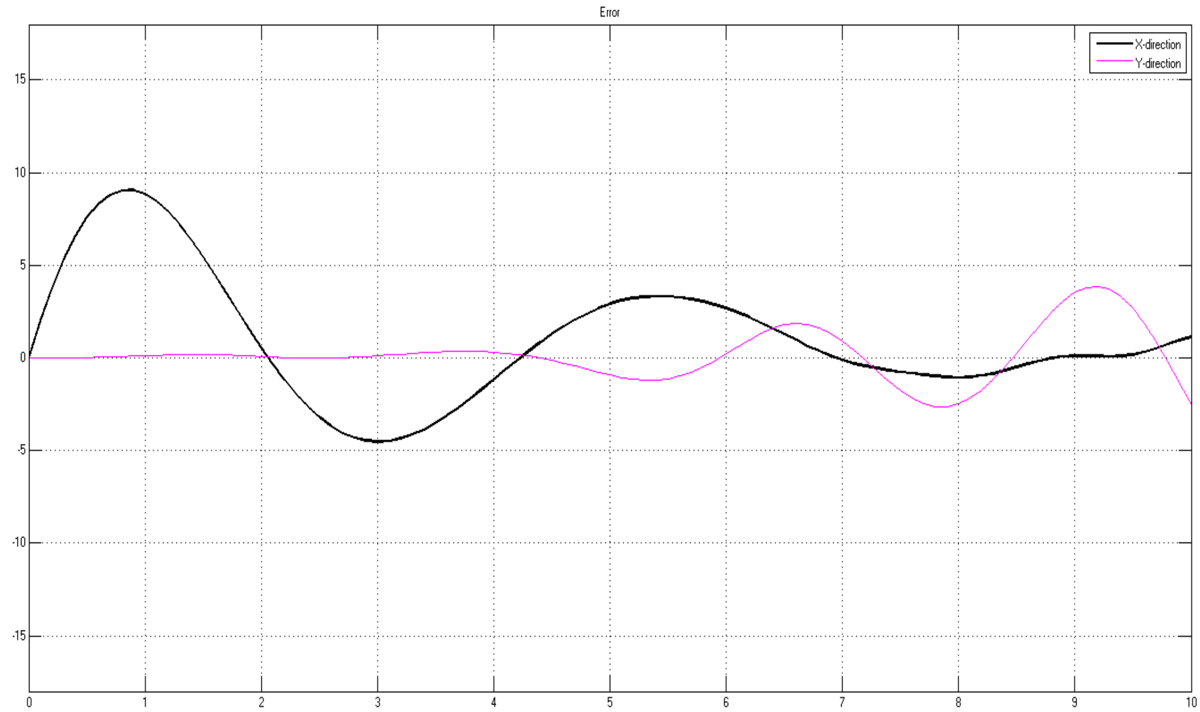

Figure 4.13 Error in X and Y direction

4.5.2 Results with PID Controller

Controlled trajectory is shown in below figure

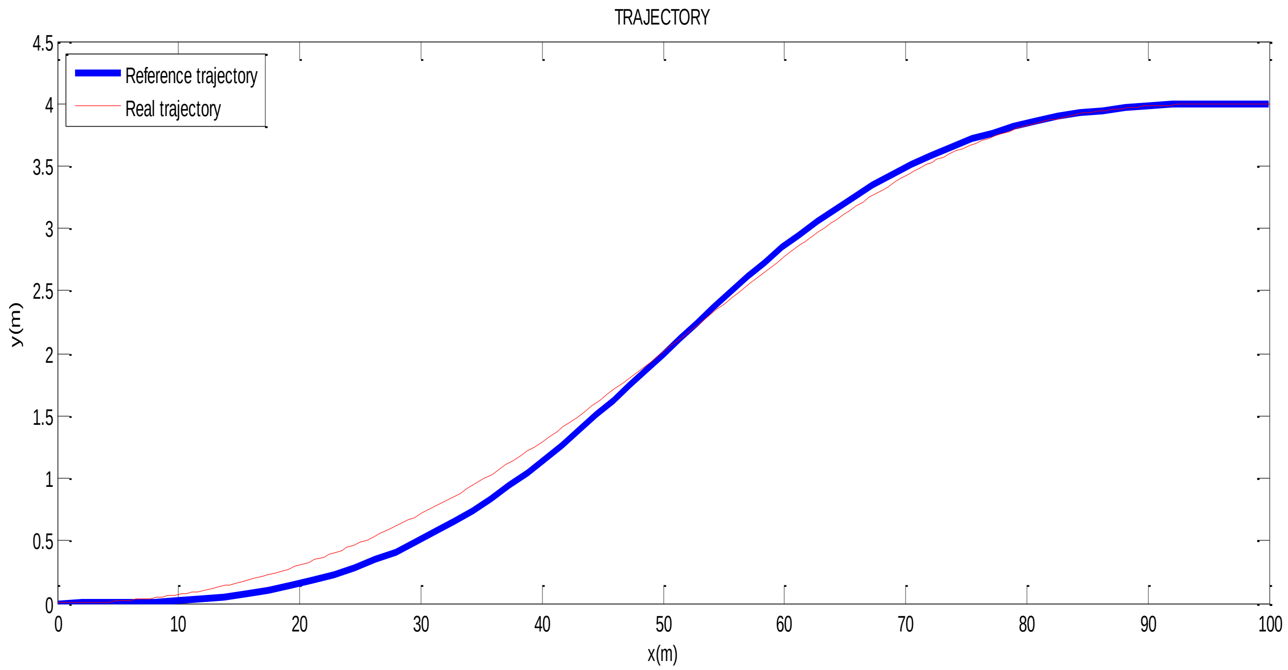

Figure 4.14 Controlled Trajectory

Figure (4.15) shows the error in longitudinal direction

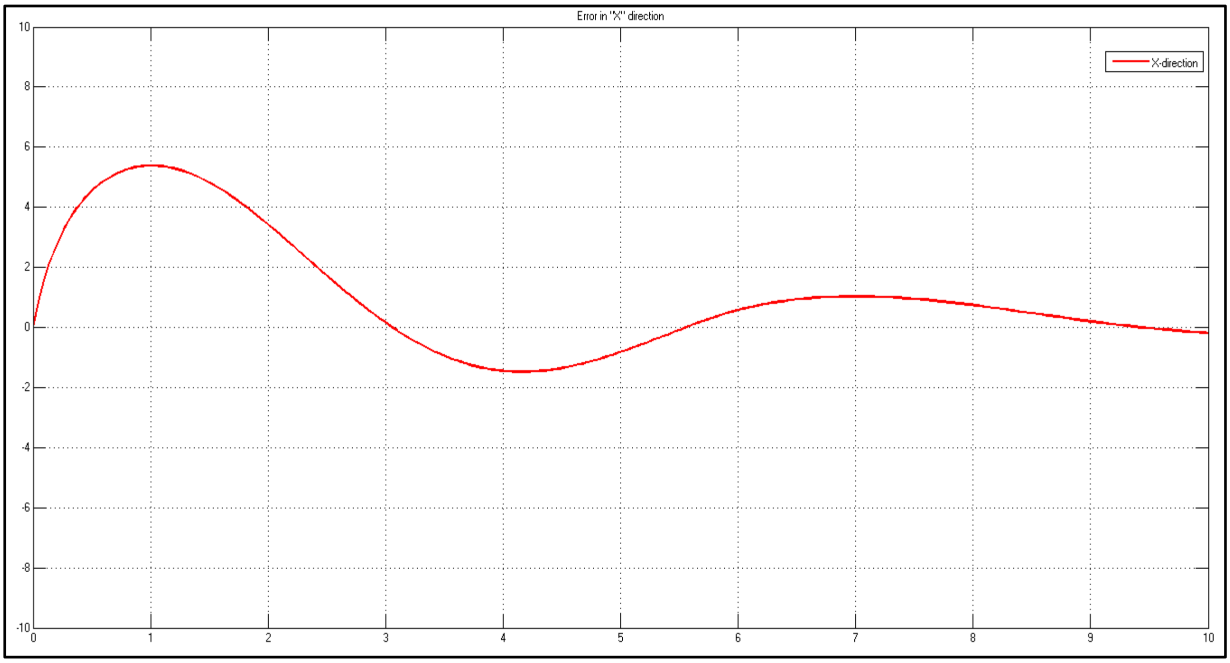

Figure 4.15 Error in x direction

Figure (4.16) shows the error in lateral direction

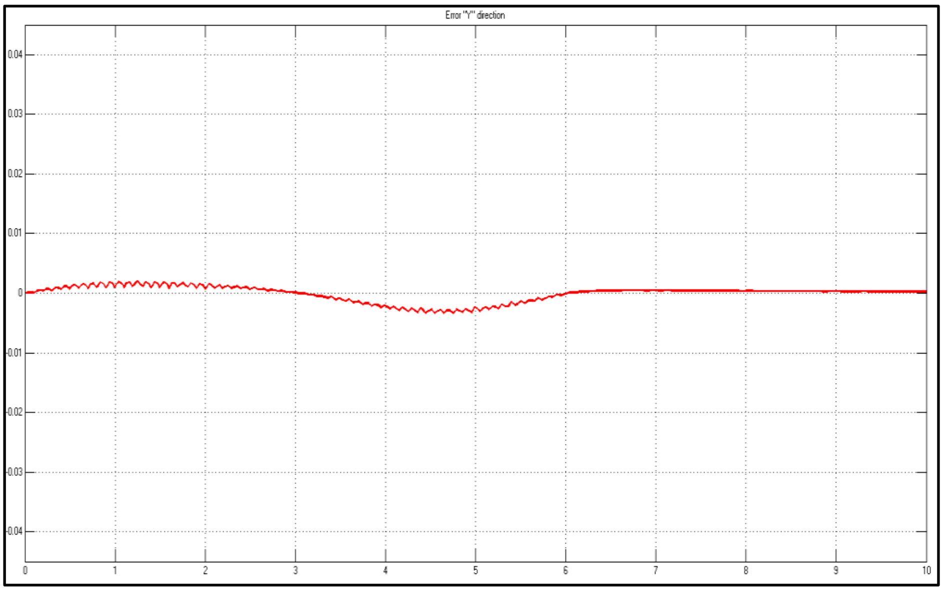

Figure 4.16 Error in y direction

PID controller shows better results as compare to PI controller. Error in both lateral and longitudinal directions are higher in PI controller that's why the reference trajectory wasn't tracked with accuracy where as by using PID controller the reference trajectory was tracked with minimum error.

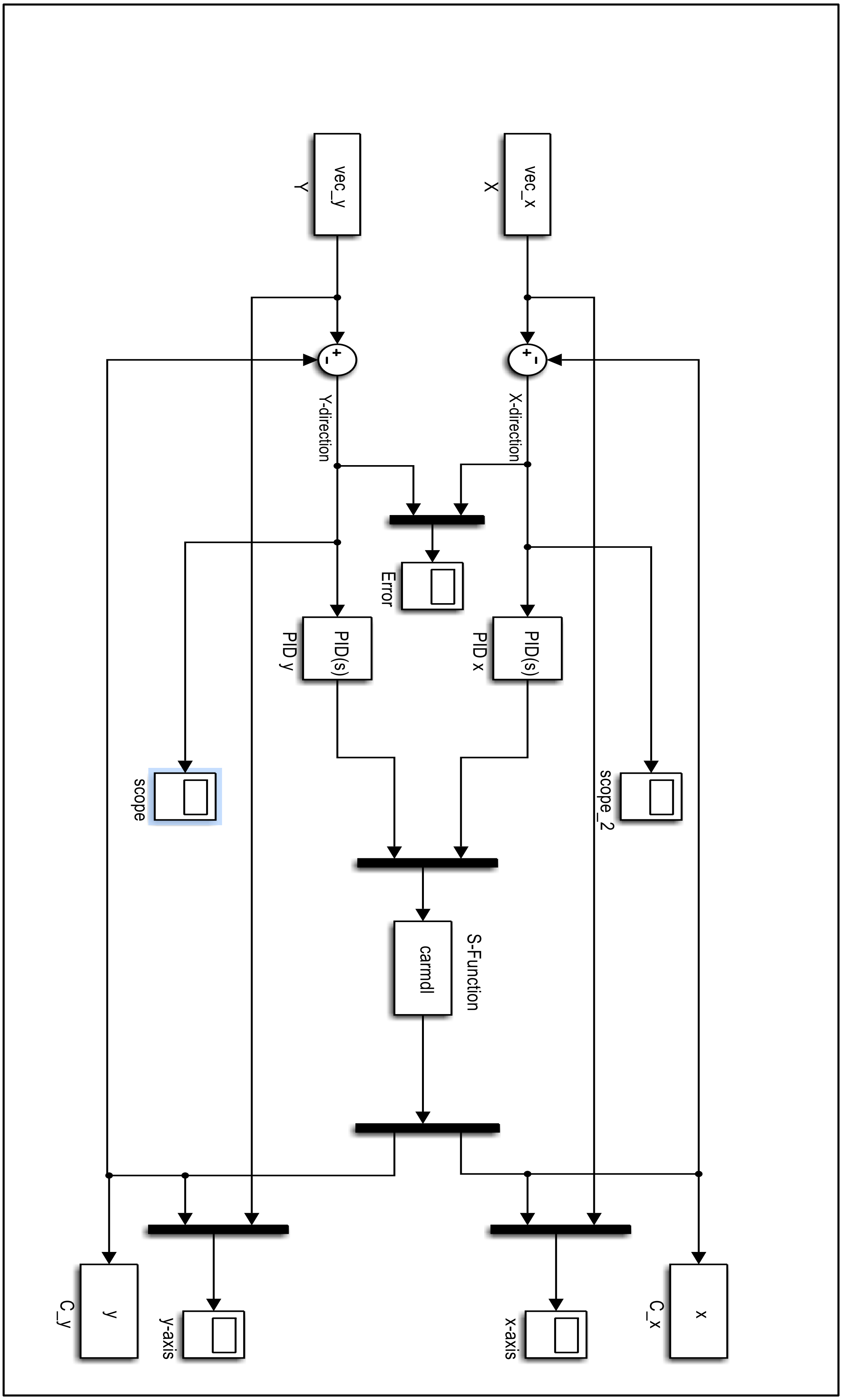

Figure 4.17 Simulink model (for 1st case)

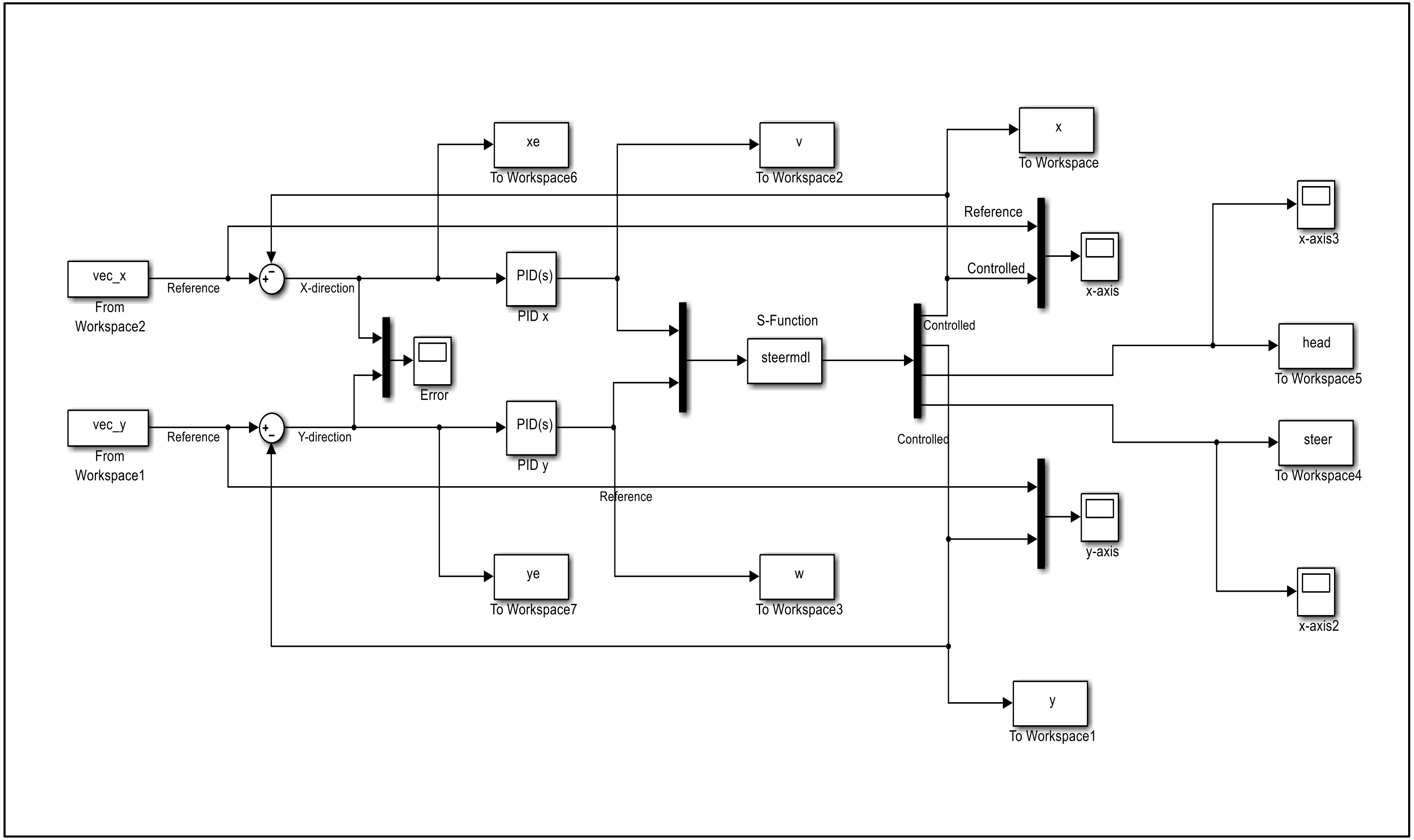

Figure 4.18 Simulink model (for 2nd case)

CHAPTER 5: CONCLUSION AND FUTURE WORK

5.1 CONCLUSION

The current work addresses the lane change of autonomous vehicle by using polynomial approach. Kinematic model of car is used. Two cases are discussed, in first case general scenario is considered where no vehicle is considered in adjacent lane where as in the second case previous vehicle is considered in adjacent lane. Feasible and smooth trajectory is generated by polynomial equation given the boundary conditions of position, velocity and acceleration. PID controller is used to track the generated trajectory. Collision avoidance technique is applied to avoid collision in case of previous vehicle. Simulation results shows the planned trajectory and controlled trajectory in both cases, with and without previous vehicle. In case of previous vehicle, collision prevention technique ensured the smooth and feasible trajectory for lane-change.

5.2 FUTURE WORK

- Combing the current work of lane change with adaptive cruise control will give complete level 2 automation.
- In future dynamic model with engine states can be considered instead of kinematic model.
- Obstacles other than cars can be considered for example pedestrians, crossing areas, speed breakers etc.
- To consider the real time scenarios where number of vehicles in current and adjacent lanes are more than one.

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COMPLETION CERTIFICATE

"It is certified that the contents of thesis document titled "*Trajectory Planning and Control for Lane Change of Autonomous Vehicle"* submitted by Mr. Adeel Mehmood Registration No. NUST2016-MS-16 (EE) 00000172436 have been found satisfactory for the requirement of degree".

Thesis Advisor: _______________________

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