**NATIONAL UNIVERSITY OF SCIENCES & TECHNOLOGY**

**COLLEGE OF ELECTRICAL AND MECHANICAL ENGINEERING,**

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# Integration and 3D Modeling of Machining Tolerances in Co-Evolution Paradigm for Reconfigurable Manufacturing Systems - Small Displacement Torsors

By

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**Submitted to the Faculty of Department of Mechanical Engineering, National University of Sciences and Technology College of Electrical and Mechanical Engineering, Rawalpindi Pakistan, in partial fulfillment of the requirement for the degree of**
**Master of Science in Mechanical Engineering**

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# 1. INTRODUCTION

Reconfigurable manufacturing system is one of the latest additions in the field of manufacturing systems. The basis of the design of RMS is the part family that they have to manufacture. The concept of RMS is introduced because of the need to reduce the cost and time. It is because of the product design and volume changes in this modern world are very rapid. It has the capability of responding to market changes and deliver product at minimum cost, more variety and virtuous quality [1].

The norms for the development of process plans and structural configurations are now altered for reconfigurable systems therefore the conventional approaches are no longer applicable. Conventional approaches first develop the process plans and then focus on the kinematic configurations or vice versa. These approaches minimize the initial cost of the production without taking into consideration the quality of the product. Basing on the concept of co-evolution of manufacturing the process plans and kinematic configurations of the RMS will be generated. In this coordinated evolution the product, processes and production systems are taken in consideration simultaneously and have equal importance [2].

In RMS the capabilities of the manufacturing systems and machines change according to each configuration. Therefore a flexible process planning approach should be used. Out of the various approaches the generation of machine configurations given by “El Maraghy” has been used in this work. In this approach the process plans changes as the features of a product are changed and accordingly the kinematic configurations. In this approach the cutting tool charts, sequence tables and precedence matrix are the inputs and in the output we get the process plans [3]. The output obtained from the above activity is exemplified in the arrangement of a hierarchical tree structure [4].

The process plans obtained consists of a large solution space. This solution space is optimized basing on the quality in terms of geometric deviations. For this purpose the tolerance analysis of the generated process plans will be performed. If the tolerance value of the process plans comes out greater than the desired value then the process plan is not capable. The geometric deviations of the manufactured part should be within the defined tolerance limits. Functionality of a design is measured in the form tolerance analysis of individual parts [5].

Various approaches deal with the generation of design solutions for such systems. In this work two and three dimensional analysis of the generated design solutions using algorithmic approach for reconfigurable manufacturing systems will be carried out. In this analysis quality is considered as the key performance indicator. The tolerance analysis that is performed will lead us to the quality of the product. Small Displacement Torsors (SDT) is used for the representation of tolerances.

## 1.1. Motivation

Various approaches have been used for modeling of machining tolerances. In case of tolerance analysis of RMS not much work is done. Tolerance analysis of the generated solutions of reconfigurable manufacturing system has been done in 1D only.

Reconfigurable manufacturing systems are the most under consideration manufacturing system that have high potential for research and development. Therefore two or three dimensional analysis using SDT will be performed. The topic has potential to compete internationally at both levels i.e. research and industrial application.

## 1.2. Objective

The main objective of this work is to develop a methodology for the tolerance analysis of generated solutions of reconfigurable manufacturing system. Using this methodology 2D and 3D tolerance analysis is performed. The result obtained gives the variation in the part manufactured from the nominal one. The end value of the tolerance is written in the form of torsor. The same methodology is applied on the machine configuration for the validation of the results obtained from the tolerance analysis. Comparison of this proposed methodology (Algorithmic Approach) is done with another existing methodology in the literature i.e. Model of Indeterminate. The algorithmic approach is then integrated to the co evolution paradigm.

The objective of integrating the tolerance analysis in the optimized generated process plans is to evaluate them on the basis of quality. The generated solution quality will be measured in terms of geometric deviations. Basing on this analysis a platform for co evolution will be developed. The value of tolerances obtained will enable us to segregate the number of solutions that are obtained. The solution space can be optimized on the basis of this tolerance analysis. The main factors that will be considered are the design requirements as described by the designer, the cost that will be enable us to manufacture it and the time that is required to complete the job. These factors will take us to the desired solution basing on the geometric variations calculated. It will eventually define the quality of the part family that is to be manufactured from reconfigurable manufacturing system.

## 2. LITERATURE SURVEY

Tolerance is defined as the permissible amount of variation that a manufactured part can vary. If the accuracy of the part is increased then the cost of that item also increases. There are basically three types of dimensional tolerances: Form tolerances, Orientation tolerances and Position tolerances. Form tolerances consist of straightness, circularity, flatness and cylindricity. In orientation tolerances we have perpendicularity, parallelism, angularity etc. Position tolerances consist of position, symmetry and concentricity [6-8].

Tolerance is a vital part of design and manufacturing. Tolerance has its importance throughout the product life cycle. During design functionality is the major concern. Ideally every designer wants to have tolerance approaching zero [9]. However constraint is put by manufacturing. Deciding factor of tolerances are the constraints of functionality, manufacturing and assembly [10]. The role of tolerances in product life cycle is shown in figure 2-1:

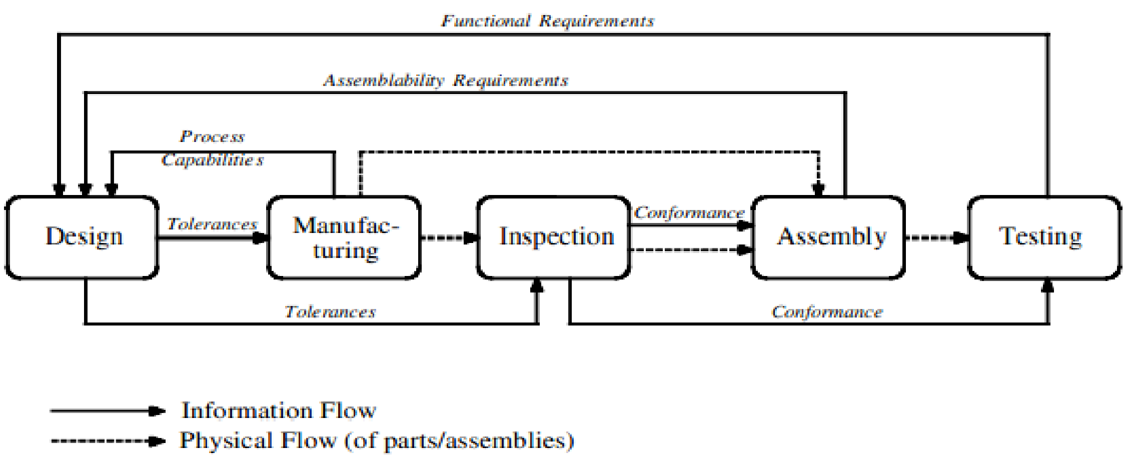


Figure 2-1: Importance of Tolerances during development of product

Existing research in the field of tolerancing can be classified into seven distinct categories [11]:

1. Tolerancing Schemes
2. Tolerance Modeling and Representation
3. Tolerance Specification
4. Tolerance Analysis
5. Tolerance Synthesis
6. Tolerance Transfer

7. Tolerance Evaluation

Tolerancing schemes are of two types: parametric and geometric tolerances. Parametric tolerances consist of the conventional plus and minus tolerances while in geometric tolerances we have locations, profiles, orientations etc. [12]

Tolerance modeling and representation is an efficient way of defining the tolerances mathematically or electronically. It consists of different solid modeling techniques in order to represent the tolerances. Tolerance is incorporated as an intrinsic feature into the product definition. A lot of techniques are used to model and represent tolerances [11].

Tolerance specification is concerned with TYPES and VALUES of tolerances. Designer specifies tolerances on the basis of knowledge and practical experience. It is preferably carried out in conformance with the given standards of ISO and ANSI [11].

Tolerance analysis has a lot of work in terms of number of research publications in the field of mechanical tolerancing. Verification of functionality of a design on the basis of the variability of the individual parts is done using this method. The way the analysis is carried out may be of 1D, 2D or 3D [11].

Tolerance synthesis is carried out in a direction opposite to tolerance analysis i.e. "From the tolerance of the function of interest to the individual tolerances". The optimal tolerance values are achieved while the tolerance types are assumed to be fixed [11].

Connection between manufacturing and design are met using Tolerance transfer keeping in view [13]:

- Machines precision
- Design Allowance
- Machining errors, setup errors and tool wears inclusion

Assessment of manufactured part data from coordinate measuring machine is done using Tolerance evaluation. Inconsistent inspection systems and differences in interpretation of geometric dimensioning and tolerancing are main concerns in this field [11].

Among the seven distinct categories of tolerances my area of focus will be tolerance analysis and synthesis. In order to represent the tolerances the technique that I have used is small displacement torsors (SDT). The concept of small displacement torsor (SDT) is first mentioned by Bourdet to solve the general problem of fitting of a geometrical surface model to a set of points (a cloud of points) in three-dimensional metrology [14].

A set of 3D tolerance propagation model is on the concept of SDT and is firstly presented by Bourdet in 1996 [15]. Rigid body displacement consisting of three rotations and translations is mathematically represented using SDT. Assuming the displacements are small, the linearization is applied and form of a torsor T at point A comes out to be as:

$$T = \begin{bmatrix} \alpha & u \\ \beta & v \\ \Upsilon & w \end{bmatrix}_A$$

"Villeneuve" used the idea of SDT to model the process. In this work he has done the 3D geometrical deviation model of work-piece, set-ups and machining operations [16]. The geometrical deviation of surface $P_i$ of the manufactured part by machining operation $M_k$ in setup $S_j$ is described by the small displacement torsor $T_{P, Pi,}$:

$$T_{P,P_i} = T_{R,P_{i\,(Sj)}} - T_{R,P\,(Sj)}$$

where $T_{R,\ Pi\ (Sj)}$ represents the deviation of surface $P_i$ in machining operation $M_k$. It depends upon the variation of machining operation.

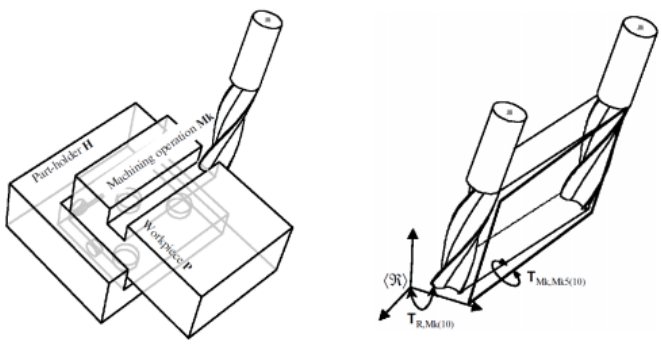


Figure 2-2: SDT of a machining setup [16]

"Vignat" established a model of manufactured parts for simulating and storing the manufacturing defects in 3D based on SDT and the model of "Villeneuve". Deviations generated during a virtual manufacturing process are collected in this model. The two phenomenon which generate the defects are: positioning and machining. The deviations due to these phenomena are accumulated over the successive setups. The positioning deviation is the deviation between nominal part and nominal machine whereas the machining deviation is the deviation between manufactured surface I relation to the nominal machine. Using hierarchically organized elementary connections positioning of part on part holder is realized. The SDT parameters are the deviations of the manufactured surface in relation to nominal position in MMP [17-18].

There are other methods to model the geometric tolerances. One of the approaches is Screw Model proposed by Bourdet in 1976. The vector field of displacement vector and rotation vectors in this model are known as screw [19-20]. Spiewak propose the kinematic and geometric simulation of a machining phase of milling operation [21]. But this study was limited to a single phase and succession of phases cannot be integrated. Vectorial tolerancing approach is presented by "Wirt". CAPP, NC machining, inspection and other activities are also represented in form of vectors in this method [22-24]. Uncertainty tensor was proposed by Clément. This method permits to analyze the dispersion due to part setup but does not take into account the errors due to machining [25]. The kinematic modeling of manufacturing errors proposed by Bénéat allows to simulate the manufacturing of a part. The modeling is based on the representation of machining errors by jacobian matrices [26].

Tichadou proposed a chart representation of the manufacturing process. These charts model the successive setup and for each setup the positioning surface and their hierarchy and the machined surfaces. It makes it possible to highlight the influential paths. They propose then two analysis methods. The first one uses a small displacement torsor model. The second one is based on the use of CAD software in which they model a manufacturing process with defects. They then virtually measure the realized part and check its conformity [27].

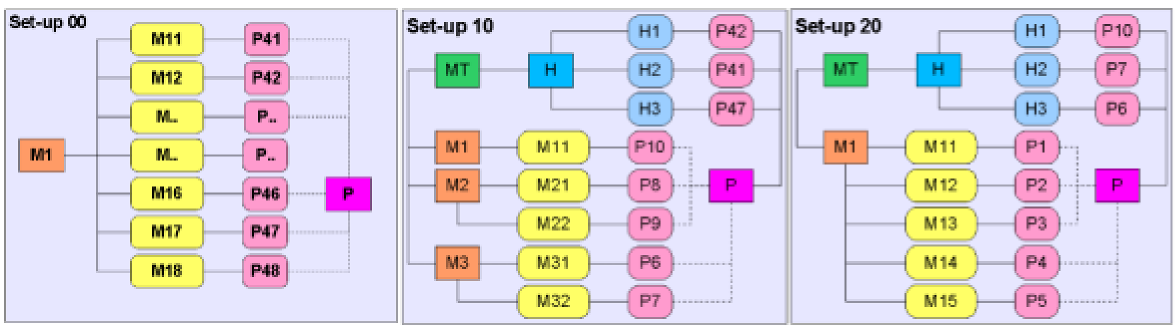


**Figure 2-3: Charts of three phases [27]**

The abbreviations in figure 2-3 charts represent:

- Hh is part holder surface
- Mm is machining operation
- Mmj is machining operation surface

Concept of generation of structural configurations is given by Baqai. It is based on the inputs of functional specifications and process plans. Structural configurations are knowledge helps in generation of flexible manufacturing systems process plans [28]. Baqai developed concept of parallel structures and post for reconfigurable manufacturing system in this approach. In comparison to the Tichadou work, the phases have been replaced by posts in case of RMS. Tichadou approach works for series structures.

A methodology is given by "Baqai" in which an algorithm is developed. This algorithm provides a method for generation of process plans for a manufacturing system. Also it gives the architectural configurations of the manufacturing system [29]. Using this concept the quality analysis of a reconfigurable manufacturing system has been done. In this study only one dimensional tolerance analysis is done on part cover intermediate shaft (CAI). Solutions for single post and multiple posts are generated in this work [30].

Bourdet presented a methodology for the 3D symbolic representation of chain of dimensions basing on the small displacement torsor. It is basically the study of geometric behavior of successive actual states of the part during the manufacturing process. It requires relations to be written between the functional conditions, the geometric defects of the parts and the gap in the links. These relations can be obtained from the model of indeterminate which is a generalization of the ΔL method [31-32].

# 3. Small Displacement Torsors

Small displacement torsor (SDT) is the small displacement of surface or solid between two positions which are close to each other. There is an associated surface which is generated from the real manufactured surface. Associated surface is extracted from the real surface by scattering 'n' points on the real surface. The variation of this associated surface is measured from the nominal surface which is the ideal one. These variations are represented in the form of a small displacement torsor. A SDT is a represented in the form of a 3 by 2 matrix. This matrix consists of two vectors [33]:

- A rotation vector 'R' called RESULTANT
- A displacement vector 'D(O)' called MOMENT expressed at O

Considering a three dimensional reference frame O (x, y, z). A SDT is thus noted as:

T=[R D (O)]

R and D (O) can be expressed as:

$$\mathbf{R} = \alpha.\mathbf{x} + \beta.\mathbf{y} + \Upsilon.\mathbf{z}$$

$$\mathbf{D\ (O)} = u.\mathbf{x} + v.\mathbf{y} + w.\mathbf{z}$$

Where α, β, ϒ are the rotations about x, y and z axis and u, v, w are the translations from O. Therefore the torsor can be expressed as:

$$T = \begin{bmatrix} \alpha & u \\ \beta & v \\ \Upsilon & w \end{bmatrix}_O$$

## 3.1. Types of Torsors:

There are four types of torsors which are used to model the deviations in a machining process plan in the form of small displacement torsors. Mathematically there are represented in the form of a 3 by 2 matrix.

- Error Torsor
- Deviation Torsor
- Play or Connection Torsor
- Global Torsor

### 3.1.1. Error Torsor:

The displacement between a theoretical nominal surface and the position of the real surface is represented in the form of error torsor. It depends on the topology of the surface. Error torsor represents the dimensional errors of machined surfaces. The error torsor between two planes is shown in figure 3-1 :

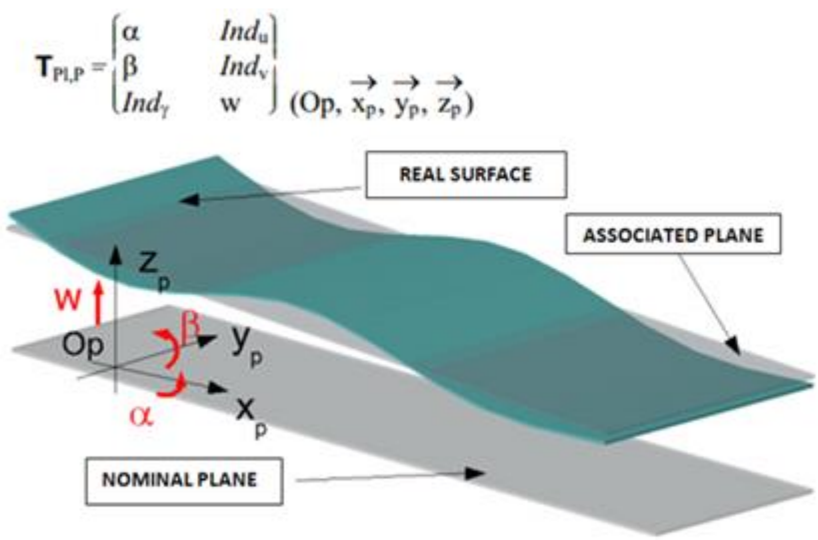


Figure 3-1: Error Torsor [34]

The variations between the associated plane and the nominal plane are measured. These variations are rotation along x-axis, rotation along y-axis and displacement along the z-axis. Variations are indicated in red color in figure 3-1.

### 3.1.2. Deviation Torsor:

Deviation torsor is the deviation of difference in position between two surfaces of the same part. It is basically a combination of two or more error torsors. In figure 3-2 the torsor between surfaces of the work piece ‘P’ is measured i.e. ‘Pi’ and ‘Pj’. Part ‘P’ shown is the nominal one. The variation of surfaces ‘Pi’ and ‘Pj’ are measured from the respective surfaces of part ‘P’ in form of error torsor. The combination of these error torsors gives the deviation torsor between ‘Pi’ and ‘Pj’.

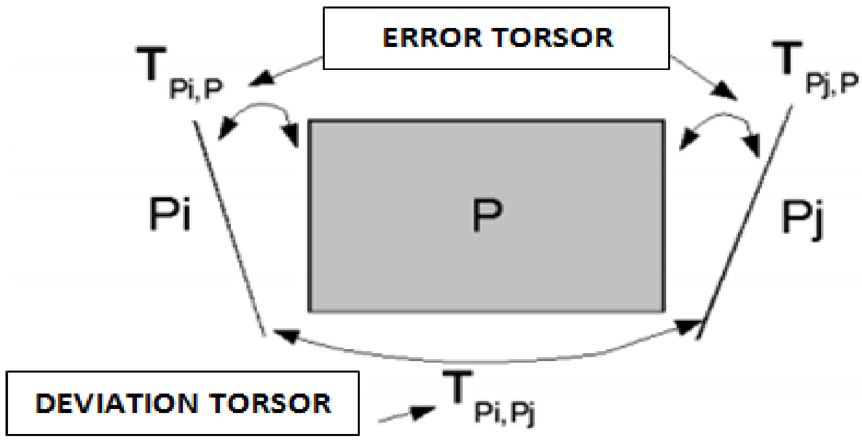


Figure 3-2: Deviation Torsor [34]

The deviation torsor $T_{Pi,\,Pj}$ will be expressed as:

$$T_{Pi,Pj} = T_{Pi,P} - T_{Pj,P}$$

### 3.1.3. Play or Connection Torsor:

Play or connection torsor is the positioning error between two surfaces of two different parts. It represents contact error between part holder and part surfaces and also between machining operations and machined surfaces.


Figure 3-3: Play Torsor [34]

The variations between the plane 1 and plane 2 are represented in the form of SDT.

### 3.1.4. Global Torsor:

It characterizes the defects of position of a solid in comparison with its nominal position. Global torsor is used to give deviations in machine tool and tool positions.

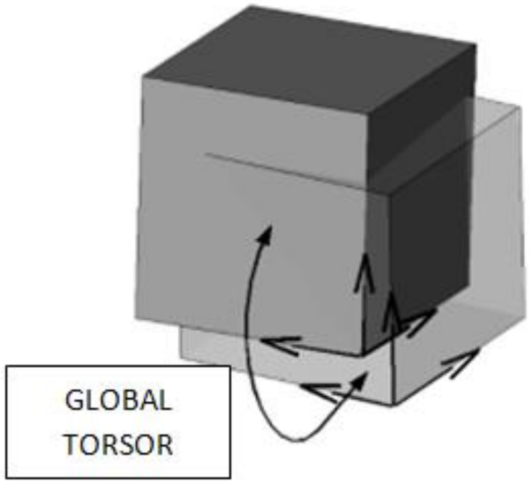


Figure 3-4: Global Torsor [34]

All types of torsors that exist between two solids i.e. Solid A and Solid B are shown in the figure 3-5. Solid A and Solid B in the darker shade are the nominal parts while the transparent ones are the manufactured Solids A and B. Four types of torsors in figure 3-5 help in identifying the geometric variations that has incurred during the manufacturing of the respective parts.

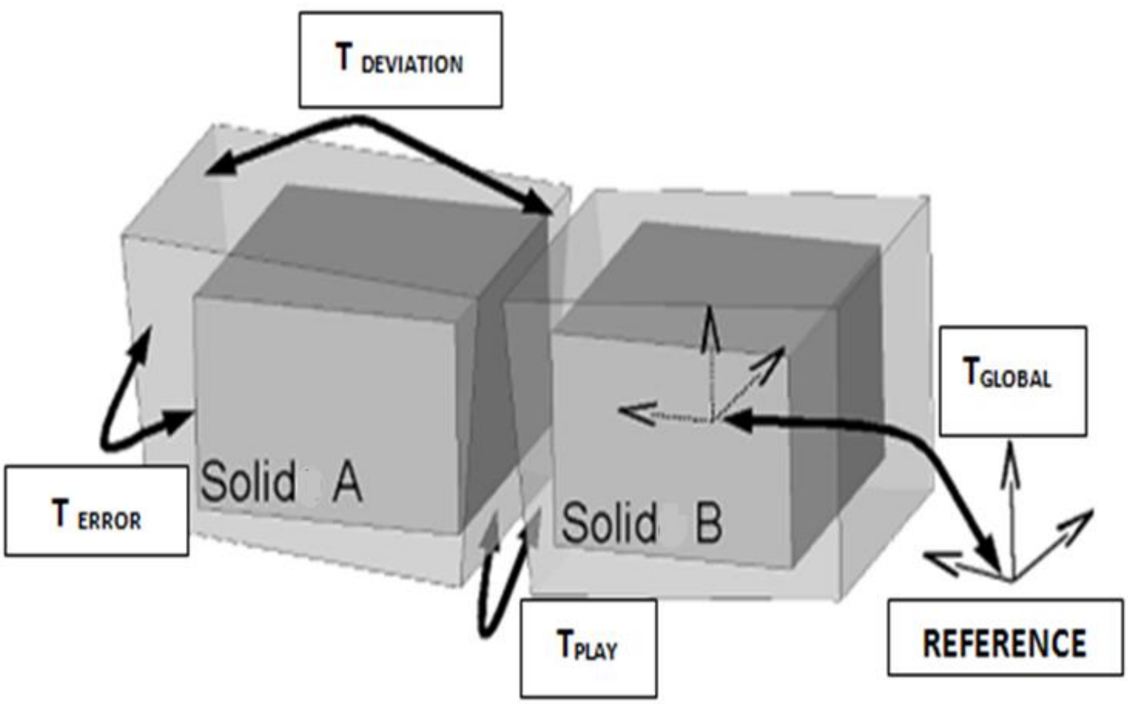


Figure 3-5: Torsors [34]

## 3.2. Torsor Setup in a Machining Phase

Torsors between the elements in a machining phase are as shown in figure 3-6.

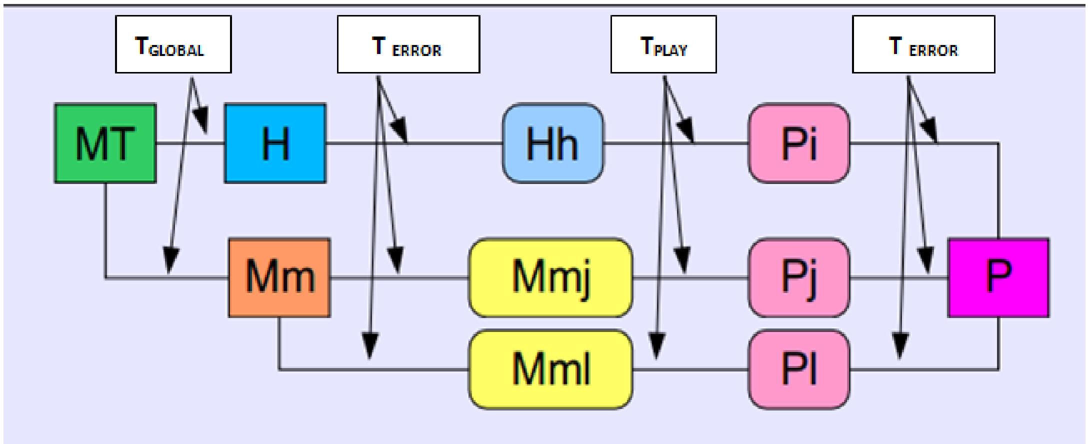


Figure 3-6: Torsor setup in a machining phase [27]

In the above figure the characters represents:

- P is the nominal Part
- H is work-piece
- MT represents machine tool
- Mm is the machining operation
- Pi represents work-piece surface
- Hh is surface of work-piece
- Mmj is machining surface

Similarly the torsors summary in a tabular form in a machining phase that exists between the above mentioned characters can be represented in a similar fashion in figure 3-7:

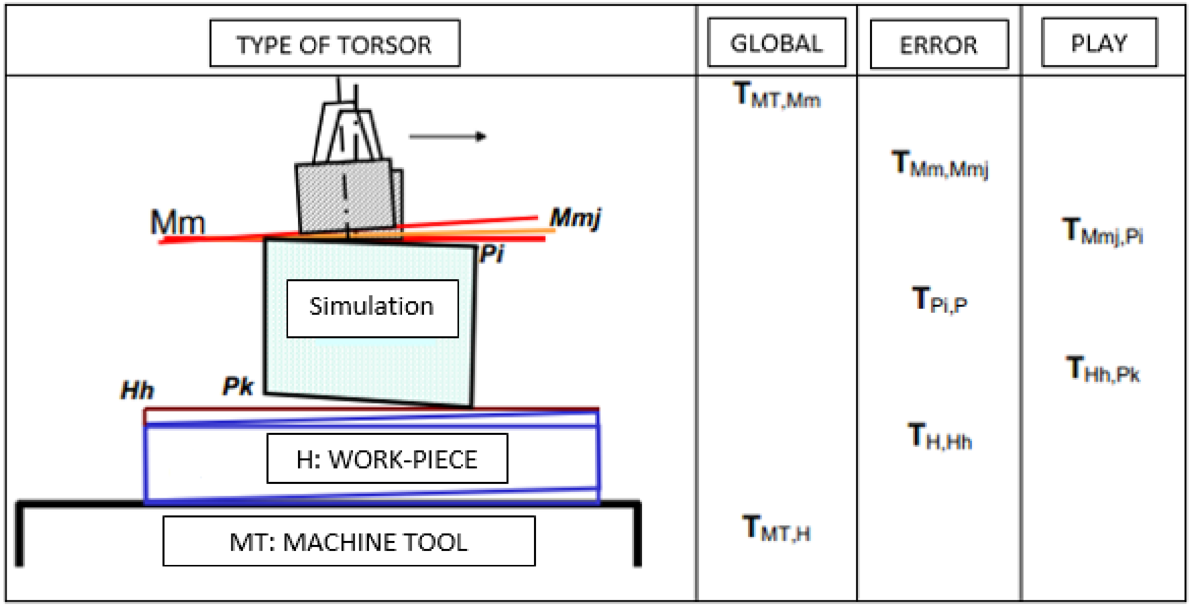


Figure 3-7: Torsors in a Machining Phase [34]

## 3.3. Torsor Chain

Addition of set of different torsors present in the component loop are used to simulate the geometrical behavior [34]. The summation of these SDTs shown in the form a chain linking the respective surfaces will express the condition.  An example in the form of a torsor chain is given in figure 3-8.

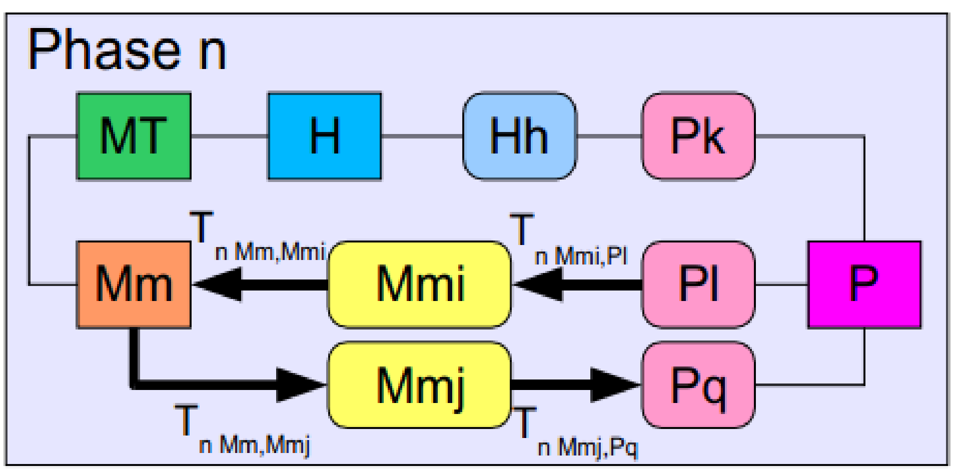


Figure 3-8: Torsor chain of a graph representing phase n [27]

The closed loop of the torsor chain for the geometric deviation between two surfaces, Pl and Pq, machined in the same phase n with the same tool is written as:

$$T\left(P_l, P_q\right) = -T_n\left(M_{mi}, P_l\right) - T_n\left(M_m, M_{mi}\right) + T_n\left(M_m, M_{mj}\right) + T_n\left(M_{mj}, P_q\right)$$

For a multi-phase process plan these chains can pass from one phase graph to another. These conditions can be between two machining surfaces, one machining surface and one positioning surface or between two surfaces realized in different phases. The representation of the geometric condition between two surfaces with the help of a torsor chain becomes our point of interest. This representation will help in carrying out a one, two or three dimensional analysis of the generated process plans and structures [33-34].

If structure i appears 2 times in the chain then the second occurrence is not taken into account in the accumulation of defects. If spindle i appears 2 times in the string chain the second occurrence is not taken into account in the accumulation of defects. If the element “position” appears only once in the chain then this case is not taken into account when the total defects [33].

# 4. Methodology

An iterative approach is used for the tolerance analysis of the generated solutions of reconfigurable manufacturing systems. 2 dimensional and 3 dimensional tolerance analysis can be performed using this proposed methodology. Tolerances are represented in the form of small displacement torsors. For each geometric variation that incurs during the manufacturing process torsors are written for that variation in this method.

## 4.1. Proposed Methodology:

An iterative approach is proposed for the tolerance analysis of generated solutions of RMS. This approach gives us the absolute value of tolerances which reflexes the capability of the manufacturing system. It starts with the initialization of one of the generated process plans and goes on till all process plans are analyzed. Torsor chains are generated and the torsor equations are obtained. In these equations each element represents a torsor whose value is determined in the next step. After that the data values are plugged into the torsors and the deviations are obtained. Basing on the value of tolerance we can decide whether the process plan is feasible or not as per the design requirements given by the designer. The approach in the form of a flow chart is shown in the figure 4-1:

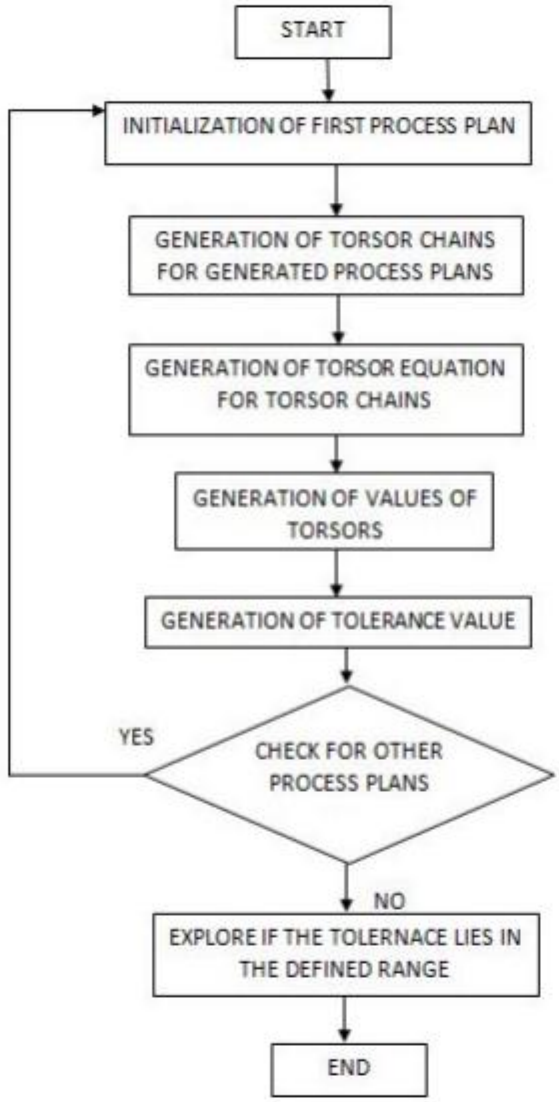


**Figure 4-1: Algorithm [35-36]**

## 4.2. Application

The part CAI (cover indeterminate shaft) is selected as a part on which the proposed method is applied and the results are generated. The machining features along with their respective machining operations are indicated in figure 4-2. It has five axial and one plane feature.

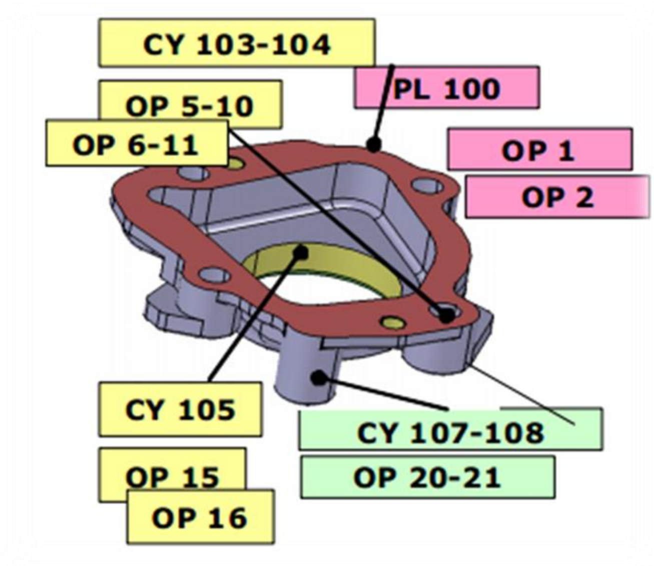


Figure 4-2: Part CAI [1]

The single post generated solutions of the mentioned part are used for the tolerance analysis. Single post generated solution and the liaisons between the interacting surfaces are indicated in figure 4-3 and figure 4-4 respectively.

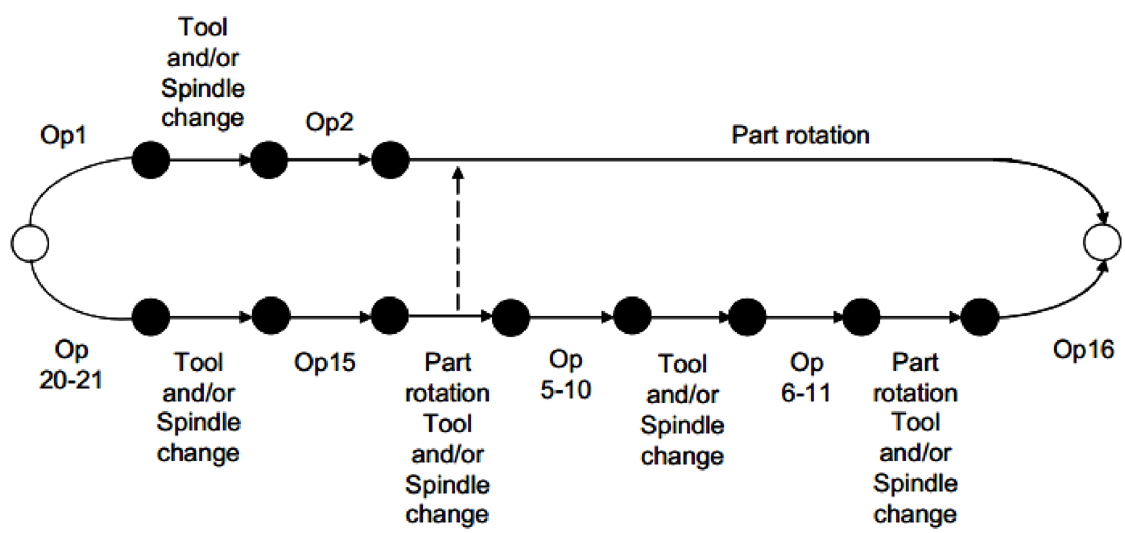


Figure 4-3: Single post generated solution [1]

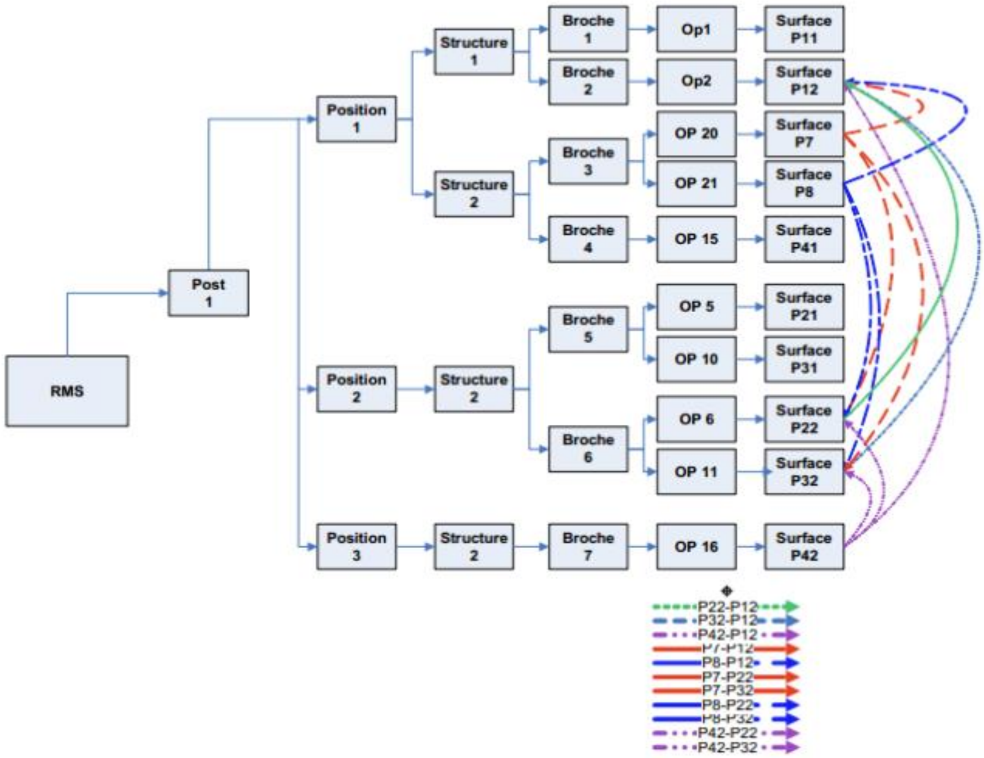


**Figure 4-4: Liaisons between the interacting surfaces [1]**

The graphical representation helps in identifying the each torsor that exist between each element of the manufacturing system. The torsor equation written to evaluate the interaction between two surfaces will be the summation of all the torsors that exist between the elements that comes in that respective torsor chain. There are 11 interacting surfaces as shown in figure 4-4. Each interaction is in the form of a two or three dimensional torsor chain. These values can be obtained after evaluating the tolerance analysis using the above proposed methodology. Following things are assumed in the processing of the graphs shown above:

- Second occurrence of any same element in a torsor chain is not taken into account while writing the torsor equation
- Single occurrence of position element in a torsor chain is not taken into account while writing the torsor equation
- Absolute Values of tolerances are used in this work

### 4.2.1. Two Dimensional Analysis

Using the above mentioned heuristics of the graphs first of all 2D analysis of the part CAI is performed using the algorithmic method. The interaction between all the surfaces is kept under consideration and is represented in the form of torsor.

- First of all we write the torsor chain equations between the interacting surfaces

$$T_{P22-P11} = \Delta T_{Tooling\ 12} + \Delta T_{Broche\ 2} + \Delta T_{Structure\ 1} + \Delta T_{Position\ 1} + \Delta T_{Position\ 2} + \Delta T_{Structure\ 2} + \Delta T_{Broche\ 6} + \Delta T_{Tooling\ 22} \quad (1)$$

$$T_{P32-P12} = \Delta T_{Tooling\ 12} + \Delta T_{Broche\ 2} + \Delta T_{Structure\ 1} + \Delta T_{Position\ 1} + \Delta T_{Position\ 2} + \Delta T_{Structure\ 2} + \Delta T_{Broche\ 6} + \Delta T_{Tooling\ 32} \quad (2)$$

$$T_{P42-P12} = \Delta T_{Tooling\ 12} + \Delta T_{Broche\ 2} + \Delta T_{Structure\ 1} + \Delta T_{Position\ 1} + \Delta T_{Position\ 3} + \Delta T_{Structure\ 2} + \Delta T_{Broche\ 7} + \Delta T_{Tooling\ 42} \quad (3)$$

$$T_{P7-P12} = \Delta T_{Tooling\ 12} + \Delta T_{Broche\ 2} + \Delta T_{Structure\ 1} + \Delta T_{Structure\ 2} + \Delta T_{Broche\ 3} + \Delta T_{Tooling\ 7} \quad (4)$$

$$T_{P8-P12} = \Delta T_{Tooling\ 12} + \Delta T_{Broche\ 2} + \Delta T_{Structure\ 1} + \Delta T_{Structure\ 2} + \Delta T_{Broche\ 3} + \Delta T_{Tooling\ 8} \quad (5)$$

$$T_{P7-P22} = \Delta T_{Tooling\ 7} + \Delta T_{Broche\ 3} + \Delta T_{Structure\ 2} + \Delta T_{Position\ 1} + \Delta T_{Position\ 2} + \Delta T_{Broche\ 6} + \Delta T_{Tooling\ 22} \quad (6)$$

$$T_{P7-P32} = \Delta T_{Tooling\ 7} + \Delta T_{Broche\ 3} + \Delta T_{Structure\ 2} + \Delta T_{Position\ 1} + \Delta T_{Position\ 2} + \Delta T_{Broche\ 6} + \Delta T_{Tooling\ 32} \quad (7)$$

$$T_{P8-P22} = \Delta T_{Tooling\ 8} + \Delta T_{Broche\ 3} + \Delta T_{Structure\ 2} + \Delta T_{Position\ 1} + \Delta T_{Position\ 2} + \Delta T_{Broche\ 6} + \Delta T_{Tooling\ 22} \quad (8)$$

$$T_{P8-P32} = \Delta T_{Tooling\ 8} + \Delta T_{Broche\ 3} + \Delta T_{Structure\ 2} + \Delta T_{Position\ 1} + \Delta T_{Position\ 2} + \Delta T_{Broche\ 6} + \Delta T_{Tooling\ 32} \quad (9)$$

$$T_{P42-P22} = \Delta T_{Tooling\ 42} + \Delta T_{Broche\ 7} + \Delta T_{Structure\ 2} + \Delta T_{Position\ 3} + \Delta T_{Position\ 2} + \Delta T_{Broche\ 6} + \Delta T_{Tooling\ 22} \quad (10)$$

$$T_{P42-P32} = \Delta T_{Tooling\ 42} + \Delta T_{Broche\ 7} + \Delta T_{Structure\ 2} + \Delta T_{Position\ 3} + \Delta T_{Position\ 2} + \Delta T_{Broche\ 6} + \Delta T_{Tooling\ 32} \quad (11)$$

- In next step each value of torsor is written. The translation component along z-axis and the rotation components along x-axis and y-axis are invariant in two dimensional tolerance analysis. It is represented by "*I*"

$$T_{P22-P12} = \begin{bmatrix} I & \Delta x_{T12} \\ I & \Delta y_{T12} \\ \Delta\gamma_{T12} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{B2} \\ I & \Delta y_{B2} \\ \Delta\gamma_{B2} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{S1} \\ I & \Delta y_{S1} \\ \Delta\gamma_{S1} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{P1} \\ I & \Delta y_{P1} \\ \Delta\gamma_{P1} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{P2} \\ I & \Delta y_{P2} \\ \Delta\gamma_{P2} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{S2} \\ I & \Delta y_{S2} \\ \Delta\gamma_{S2} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{B6} \\ I & \Delta y_{B6} \\ \Delta\gamma_{B6} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{T22} \\ I & \Delta y_{T22} \\ \Delta\gamma_{T22} & I \end{bmatrix}$$

$$T_{P32-P12} = \begin{bmatrix} I & \Delta x_{T12} \\ I & \Delta y_{T12} \\ \Delta\gamma_{T12} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{B2} \\ I & \Delta y_{B2} \\ \Delta\gamma_{B2} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{S1} \\ I & \Delta y_{S1} \\ \Delta\gamma_{S1} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{P1} \\ I & \Delta y_{P1} \\ \Delta\gamma_{P1} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{P2} \\ I & \Delta y_{P2} \\ \Delta\gamma_{P2} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{S2} \\ I & \Delta y_{S2} \\ \Delta\gamma_{S2} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{B6} \\ I & \Delta y_{B6} \\ \Delta\gamma_{B6} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{T32} \\ I & \Delta y_{T32} \\ \Delta\gamma_{T32} & I \end{bmatrix}$$

$$T_{P42-P12} = \begin{bmatrix} I & \Delta x_{T12} \\ I & \Delta y_{T12} \\ \Delta\gamma_{T12} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{B2} \\ I & \Delta y_{B2} \\ \Delta\gamma_{B2} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{S1} \\ I & \Delta y_{S1} \\ \Delta\gamma_{S1} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{P1} \\ I & \Delta y_{P1} \\ \Delta\gamma_{P1} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{P3} \\ I & \Delta y_{P3} \\ \Delta\gamma_{P3} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{S2} \\ I & \Delta y_{S2} \\ \Delta\gamma_{S2} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{B7} \\ I & \Delta y_{B7} \\ \Delta\gamma_{B7} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{T42} \\ I & \Delta y_{T42} \\ \Delta\gamma_{T42} & I \end{bmatrix}$$

$$T_{P7-P12} = \begin{bmatrix} I & \Delta x_{T12} \\ I & \Delta y_{T12} \\ \Delta\gamma_{T12} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{B2} \\ I & \Delta y_{B2} \\ \Delta\gamma_{B2} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{S1} \\ I & \Delta y_{S1} \\ \Delta\gamma_{S1} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{S2} \\ I & \Delta y_{S2} \\ \Delta\gamma_{S2} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{B3} \\ I & \Delta y_{B3} \\ \Delta\gamma_{B3} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{T7} \\ I & \Delta y_{T7} \\ \Delta\gamma_{T7} & I \end{bmatrix}$$

$$T_{P8-P12} = \begin{bmatrix} I & \Delta x_{T12} \\ I & \Delta y_{T12} \\ \Delta\gamma_{T12} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{B2} \\ I & \Delta y_{B2} \\ \Delta\gamma_{B2} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{S1} \\ I & \Delta y_{S1} \\ \Delta\gamma_{S1} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{S2} \\ I & \Delta y_{S2} \\ \Delta\gamma_{S2} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{B3} \\ I & \Delta y_{B3} \\ \Delta\gamma_{B3} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{T8} \\ I & \Delta y_{T8} \\ \Delta\gamma_{T8} & I \end{bmatrix}$$

$$T_{P7-P22} = \begin{bmatrix} I & \Delta x_{T7} \\ I & \Delta y_{T7} \\ \Delta\gamma_{T7} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{B3} \\ I & \Delta y_{B3} \\ \Delta\gamma_{B3} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{S2} \\ I & \Delta y_{S2} \\ \Delta\gamma_{S2} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{P1} \\ I & \Delta y_{P1} \\ \Delta\gamma_{P1} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{P2} \\ I & \Delta y_{P2} \\ \Delta\gamma_{P2} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{B6} \\ I & \Delta y_{B6} \\ \Delta\gamma_{B6} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{T22} \\ I & \Delta y_{T22} \\ \Delta\gamma_{T22} & I \end{bmatrix}$$

$$T_{P7-P32} = \begin{bmatrix} I & \Delta x_{T7} \\ I & \Delta y_{T7} \\ \Delta\gamma_{T7} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{B3} \\ I & \Delta y_{B3} \\ \Delta\gamma_{B3} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{S2} \\ I & \Delta y_{S2} \\ \Delta\gamma_{S2} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{P1} \\ I & \Delta y_{P1} \\ \Delta\gamma_{P1} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{P2} \\ I & \Delta y_{P2} \\ \Delta\gamma_{P2} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{B6} \\ I & \Delta y_{B6} \\ \Delta\gamma_{B6} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{T32} \\ I & \Delta y_{T32} \\ \Delta\gamma_{T32} & I \end{bmatrix}$$

$$T_{P8-P22} = \begin{bmatrix} I & \Delta x_{T8} \\ I & \Delta y_{T8} \\ \Delta\gamma_{T8} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{B3} \\ I & \Delta y_{B3} \\ \Delta\gamma_{B3} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{S2} \\ I & \Delta y_{S2} \\ \Delta\gamma_{S2} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{P1} \\ I & \Delta y_{P1} \\ \Delta\gamma_{P1} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{P2} \\ I & \Delta y_{P2} \\ \Delta\gamma_{P2} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{B6} \\ I & \Delta y_{B6} \\ \Delta\gamma_{B6} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{T22} \\ I & \Delta y_{T22} \\ \Delta\gamma_{T22} & I \end{bmatrix}$$

$$T_{P8-P32} = \begin{bmatrix} I & \Delta x_{T8} \\ I & \Delta y_{T8} \\ \Delta\gamma_{T8} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{B3} \\ I & \Delta y_{B3} \\ \Delta\gamma_{B3} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{S2} \\ I & \Delta y_{S2} \\ \Delta\gamma_{S2} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{P1} \\ I & \Delta y_{P1} \\ \Delta\gamma_{P1} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{P2} \\ I & \Delta y_{P2} \\ \Delta\gamma_{P2} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{B6} \\ I & \Delta y_{B6} \\ \Delta\gamma_{B6} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{T32} \\ I & \Delta y_{T32} \\ \Delta\gamma_{T32} & I \end{bmatrix}$$

$$T_{P42-P22} = \begin{bmatrix} I & \Delta x_{T42} \\ I & \Delta y_{T42} \\ \Delta\gamma_{T42} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{B7} \\ I & \Delta y_{B7} \\ \Delta\gamma_{B7} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{S2} \\ I & \Delta y_{S2} \\ \Delta\gamma_{S2} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{P3} \\ I & \Delta y_{P3} \\ \Delta\gamma_{P3} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{P2} \\ I & \Delta y_{P2} \\ \Delta\gamma_{P2} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{B6} \\ I & \Delta y_{B6} \\ \Delta\gamma_{B6} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{T22} \\ I & \Delta y_{T22} \\ \Delta\gamma_{T22} & I \end{bmatrix}$$

$$T_{P42-P32} = \begin{bmatrix} I & \Delta x_{T42} \\ I & \Delta y_{T42} \\ \Delta\gamma_{T42} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{B7} \\ I & \Delta y_{B7} \\ \Delta\gamma_{B7} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{S2} \\ I & \Delta y_{S2} \\ \Delta\gamma_{S2} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{P3} \\ I & \Delta y_{P3} \\ \Delta\gamma_{P3} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{P2} \\ I & \Delta y_{P2} \\ \Delta\gamma_{P2} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{B6} \\ I & \Delta y_{B6} \\ \Delta\gamma_{B6} & I \end{bmatrix} + \begin{bmatrix} I & \Delta x_{T32} \\ I & \Delta y_{T32} \\ \Delta\gamma_{T32} & I \end{bmatrix}$$

- Then the tolerance values are obtained using the data values. The variations in tooling, structure, position, broche (Spindle) along the respective axis are substituted in the torsors respectively. Hence the value of tolerance is calculated.

The data values basing on the experimental data and the input from the experienced machine operators are as shown in table 4-1. The associated error/ deviation in different activities related to tool, spindle, structure, position and part change activities in part manufacturing process are catered through these values.

Table 4-1: Data Values

| Values are in mm/ deg for translations/ rotation | | | | | | |
|---|---|---|---|---|---|---|
| **Elements** | **Δx** | **Δy** | **Δz** | **Δα** | **Δβ** | **Δγ** |
| **Tool** | .002 | .002 | .002 | .08 | .08 | .08 |
| **Spindle** | .004 | .004 | .004 | .08 | .08 | .08 |
| **Structure** | .003 | .003 | .003 | .08 | .08 | .08 |
| **Position** | .004 | .004 | .004 | .08 | .08 | .08 |
| **Post** | .006 | .006 | .006 | .08 | .08 | .08 |

Using the above values and incorporating the Varignon relationship the result comes out to be [34, 37]:

$$\mathrm{T}_{\mathrm{P22-P11}} = \begin{bmatrix} I & 0.026 \\ I & 0.026 \\ 0.64 & I \end{bmatrix}$$

$$\mathrm{T}_{\mathrm{P32-P12}} = \begin{bmatrix} I & 0.026 \\ I & 0.026 \\ 0.64 & I \end{bmatrix}$$

$$\mathrm{T}_{\mathrm{P42-P12}} = \begin{bmatrix} I & 0.026 \\ I & 0.026 \\ 0.64 & I \end{bmatrix}$$

$$\mathrm{T}_{\mathrm{P7-P12}} = \begin{bmatrix} I & 0.018 \\ I & 0.018 \\ 0.48 & I \end{bmatrix}$$

$$\mathrm{T}_{\mathrm{P8-P12}} = \begin{bmatrix} I & 0.018 \\ I & 0.018 \\ 0.48 & I \end{bmatrix}$$

$$\mathrm{T}_{\mathrm{P7-P22}} = \begin{bmatrix} I & 0.023 \\ I & 0.023 \\ 0.56 & I \end{bmatrix}$$

$$\mathrm{T}_{\mathrm{P7-P32}} = \begin{bmatrix} I & 0.023 \\ I & 0.023 \\ 0.56 & I \end{bmatrix}$$

$$T_{P8-P22} = \begin{bmatrix} I & 0.023 \\ I & 0.023 \\ 0.56 & I \end{bmatrix}$$

$$T_{P8-P32} = \begin{bmatrix} I & 0.023 \\ I & 0.023 \\ 0.56 & I \end{bmatrix}$$

$$T_{P42-P22} = \begin{bmatrix} I & 0.023 \\ I & 0.023 \\ 0.56 & I \end{bmatrix}$$

$$T_{P42-P32} = \begin{bmatrix} I & 0.023 \\ I & 0.023 \\ 0.56 & I \end{bmatrix}$$

*I represent* the invariant values that have no effect on the end result. The end result gives the absolute geometric variation between the associated surfaces of the manufactured part with the nominal surfaces of the part to be manufactured.

### 4.2.2. Three Dimensional Analysis

Similarly the 3D tolerance analysis is performed between the interacting surfaces on Part CAI. Same steps are followed as that of the 2D tolerance analysis. In 3D there will no invariant value in 3 by 2 matrix of torsor. Initially the same torsor equations are obtained as that of eq 1-11. Then the value of each torsor is evaluated i.e.

$$T_{P22-P12} = \begin{bmatrix} \Delta\alpha_{T12} & \Delta x_{T12} \\ \Delta\beta_{T12} & \Delta y_{T12} \\ \Delta\gamma_{T12} & \Delta z_{T12} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{B2} & \Delta x_{B2} \\ \Delta\beta_{B2} & \Delta y_{B2} \\ \Delta\gamma_{B2} & \Delta z_{B2} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{S1} & \Delta x_{S1} \\ \Delta\beta_{S1} & \Delta y_{S1} \\ \Delta\gamma_{S1} & \Delta z_{S1} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{P1} & \Delta x_{P1} \\ \Delta\beta_{P1} & \Delta y_{P1} \\ \Delta\gamma_{P1} & \Delta z_{P1} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{P2} & \Delta x_{P2} \\ \Delta\beta_{P2} & \Delta y_{P2} \\ \Delta\gamma_{P2} & \Delta z_{P2} \end{bmatrix}$$
$$+ \begin{bmatrix} \Delta\alpha_{S2} & \Delta x_{S2} \\ \Delta\beta_{S2} & \Delta y_{S2} \\ \Delta\gamma_{S2} & \Delta z_{S2} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{B6} & \Delta x_{B6} \\ \Delta\beta_{B6} & \Delta y_{B6} \\ \Delta\gamma_{B6} & \Delta z_{B6} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{T22} & \Delta x_{T22} \\ \Delta\beta_{T22} & \Delta y_{T22} \\ \Delta\gamma_{T22} & \Delta z_{T22} \end{bmatrix}$$

$$T_{P32-P12} = \begin{bmatrix} \Delta\alpha_{T12} & \Delta x_{T12} \\ \Delta\beta_{T12} & \Delta y_{T12} \\ \Delta\gamma_{T12} & \Delta z_{T12} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{B2} & \Delta x_{B2} \\ \Delta\beta_{B2} & \Delta y_{B2} \\ \Delta\gamma_{B2} & \Delta z_{B2} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{S1} & \Delta x_{S1} \\ \Delta\beta_{S1} & \Delta y_{S1} \\ \Delta\gamma_{S1} & \Delta z_{S1} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{P1} & \Delta x_{P1} \\ \Delta\beta_{P1} & \Delta y_{P1} \\ \Delta\gamma_{P1} & \Delta z_{P1} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{P2} & \Delta x_{P2} \\ \Delta\beta_{P2} & \Delta y_{P2} \\ \Delta\gamma_{P2} & \Delta z_{P2} \end{bmatrix}$$
$$+ \begin{bmatrix} \Delta\alpha_{S2} & \Delta x_{S2} \\ \Delta\beta_{S2} & \Delta y_{S2} \\ \Delta\gamma_{S2} & \Delta z_{S2} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{B6} & \Delta x_{B6} \\ \Delta\beta_{B6} & \Delta y_{B6} \\ \Delta\gamma_{B6} & \Delta z_{B6} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{T32} & \Delta x_{T32} \\ \Delta\beta_{T32} & \Delta y_{T32} \\ \Delta\gamma_{T32} & \Delta z_{T32} \end{bmatrix}$$

$$T_{P42-P12} = \begin{bmatrix} \Delta\alpha_{T12} & \Delta x_{T12} \\ \Delta\beta_{T12} & \Delta y_{T12} \\ \Delta\gamma_{T12} & \Delta z_{T12} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{B2} & \Delta x_{B2} \\ \Delta\beta_{B2} & \Delta y_{B2} \\ \Delta\gamma_{B2} & \Delta z_{B2} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{S1} & \Delta x_{S1} \\ \Delta\beta_{S1} & \Delta y_{S1} \\ \Delta\gamma_{S1} & \Delta z_{S1} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{P1} & \Delta x_{P1} \\ \Delta\beta_{P1} & \Delta y_{P1} \\ \Delta\gamma_{P1} & \Delta z_{P1} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{P3} & \Delta x_{P3} \\ \Delta\beta_{P3} & \Delta y_{P3} \\ \Delta\gamma_{P3} & \Delta z_{P3} \end{bmatrix}$$
$$+ \begin{bmatrix} \Delta\alpha_{S2} & \Delta x_{S2} \\ \Delta\beta_{S2} & \Delta y_{S2} \\ \Delta\gamma_{S2} & \Delta z_{S2} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{B7} & \Delta x_{B7} \\ \Delta\beta_{B7} & \Delta y_{B7} \\ \Delta\gamma_{B7} & \Delta z_{B7} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{T42} & \Delta x_{T42} \\ \Delta\beta_{T42} & \Delta y_{T42} \\ \Delta\gamma_{T42} & \Delta z_{T42} \end{bmatrix}$$

$$T_{P7-P12} = \begin{bmatrix} \Delta\alpha_{T12} & \Delta x_{T12} \\ \Delta\beta_{T12} & \Delta y_{T12} \\ \Delta\gamma_{T12} & \Delta z_{T12} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{B2} & \Delta x_{B2} \\ \Delta\beta_{B2} & \Delta y_{B2} \\ \Delta\gamma_{B2} & \Delta z_{B2} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{S1} & \Delta x_{S1} \\ \Delta\beta_{S1} & \Delta y_{S1} \\ \Delta\gamma_{S1} & \Delta z_{S1} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{S2} & \Delta x_{S2} \\ \Delta\beta_{S2} & \Delta y_{S2} \\ \Delta\gamma_{S2} & \Delta z_{S2} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{B3} & \Delta x_{B3} \\ \Delta\beta_{B3} & \Delta y_{B3} \\ \Delta\gamma_{B3} & \Delta z_{B3} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{T7} & \Delta x_{T7} \\ \Delta\beta_{T7} & \Delta y_{T7} \\ \Delta\gamma_{T7} & \Delta z_{T7} \end{bmatrix}$$

$$T_{P8-P12} = \begin{bmatrix} \Delta\alpha_{T12} & \Delta x_{T12} \\ \Delta\beta_{T12} & \Delta y_{T12} \\ \Delta\gamma_{T12} & \Delta z_{T12} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{B2} & \Delta x_{B2} \\ \Delta\beta_{B2} & \Delta y_{B2} \\ \Delta\gamma_{B2} & \Delta z_{B2} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{S1} & \Delta x_{S1} \\ \Delta\beta_{S1} & \Delta y_{S1} \\ \Delta\gamma_{S1} & \Delta z_{S1} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{S2} & \Delta x_{S2} \\ \Delta\beta_{S2} & \Delta y_{S2} \\ \Delta\gamma_{S2} & \Delta z_{S2} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{B3} & \Delta x_{B3} \\ \Delta\beta_{B3} & \Delta y_{B3} \\ \Delta\gamma_{B3} & \Delta z_{B3} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{T8} & \Delta x_{T8} \\ \Delta\beta_{T8} & \Delta y_{T8} \\ \Delta\gamma_{T8} & \Delta z_{T8} \end{bmatrix}$$

$$T_{P7-P22} = \begin{bmatrix} \Delta\alpha_{T7} & \Delta x_{T7} \\ \Delta\beta_{T7} & \Delta y_{T7} \\ \Delta\gamma_{T7} & \Delta z_{T7} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{B3} & \Delta x_{B3} \\ \Delta\beta_{B3} & \Delta y_{B3} \\ \Delta\gamma_{B3} & \Delta z_{B3} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{S2} & \Delta x_{S2} \\ \Delta\beta_{S2} & \Delta y_{S2} \\ \Delta\gamma_{S2} & \Delta z_{S2} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{P1} & \Delta x_{P1} \\ \Delta\beta_{P1} & \Delta y_{P1} \\ \Delta\gamma_{P1} & \Delta z_{P1} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{P2} & \Delta x_{P2} \\ \Delta\beta_{P2} & \Delta y_{P2} \\ \Delta\gamma_{P2} & \Delta z_{P2} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{B6} & \Delta x_{B6} \\ \Delta\beta_{B6} & \Delta y_{B6} \\ \Delta\gamma_{B6} & \Delta z_{B6} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{T22} & \Delta x_{T22} \\ \Delta\beta_{T22} & \Delta y_{T22} \\ \Delta\gamma_{T22} & \Delta z_{T22} \end{bmatrix}$$

$$T_{P7-P32} = \begin{bmatrix} \Delta\alpha_{T7} & \Delta x_{T7} \\ \Delta\beta_{T7} & \Delta y_{T7} \\ \Delta\gamma_{T7} & \Delta z_{T7} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{B3} & \Delta x_{B3} \\ \Delta\beta_{B3} & \Delta y_{B3} \\ \Delta\gamma_{B3} & \Delta z_{B3} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{S2} & \Delta x_{S2} \\ \Delta\beta_{S2} & \Delta y_{S2} \\ \Delta\gamma_{S2} & \Delta z_{S2} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{P1} & \Delta x_{P1} \\ \Delta\beta_{P1} & \Delta y_{P1} \\ \Delta\gamma_{P1} & \Delta z_{P1} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{P2} & \Delta x_{P2} \\ \Delta\beta_{P2} & \Delta y_{P2} \\ \Delta\gamma_{P2} & \Delta z_{P2} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{B6} & \Delta x_{B6} \\ \Delta\beta_{B6} & \Delta y_{B6} \\ \Delta\gamma_{B6} & \Delta z_{B6} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{T32} & \Delta x_{T32} \\ \Delta\beta_{T32} & \Delta y_{T32} \\ \Delta\gamma_{T32} & \Delta z_{T32} \end{bmatrix}$$

$$T_{P8-P22} = \begin{bmatrix} \Delta\alpha_{T8} & \Delta x_{T8} \\ \Delta\beta_{T8} & \Delta y_{T8} \\ \Delta\gamma_{T8} & \Delta z_{T8} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{B3} & \Delta x_{B3} \\ \Delta\beta_{B3} & \Delta y_{B3} \\ \Delta\gamma_{B3} & \Delta z_{B3} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{S2} & \Delta x_{S2} \\ \Delta\beta_{S2} & \Delta y_{S2} \\ \Delta\gamma_{S2} & \Delta z_{S2} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{P1} & \Delta x_{P1} \\ \Delta\beta_{P1} & \Delta y_{P1} \\ \Delta\gamma_{P1} & \Delta z_{P1} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{P2} & \Delta x_{P2} \\ \Delta\beta_{P2} & \Delta y_{P2} \\ \Delta\gamma_{P2} & \Delta z_{P2} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{B6} & \Delta x_{B6} \\ \Delta\beta_{B6} & \Delta y_{B6} \\ \Delta\gamma_{B6} & \Delta z_{B6} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{T22} & \Delta x_{T22} \\ \Delta\beta_{T22} & \Delta y_{T22} \\ \Delta\gamma_{T22} & \Delta z_{T22} \end{bmatrix}$$

$$T_{P8-P32} = \begin{bmatrix} \Delta\alpha_{T8} & \Delta x_{T8} \\ \Delta\beta_{T8} & \Delta y_{T8} \\ \Delta\gamma_{T8} & \Delta z_{T8} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{B3} & \Delta x_{B3} \\ \Delta\beta_{B3} & \Delta y_{B3} \\ \Delta\gamma_{B3} & \Delta z_{B3} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{S2} & \Delta x_{S2} \\ \Delta\beta_{S2} & \Delta y_{S2} \\ \Delta\gamma_{S2} & \Delta z_{S2} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{P1} & \Delta x_{P1} \\ \Delta\beta_{P1} & \Delta y_{P1} \\ \Delta\gamma_{P1} & \Delta z_{P1} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{P2} & \Delta x_{P2} \\ \Delta\beta_{P2} & \Delta y_{P2} \\ \Delta\gamma_{P2} & \Delta z_{P2} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{B6} & \Delta x_{B6} \\ \Delta\beta_{B6} & \Delta y_{B6} \\ \Delta\gamma_{B6} & \Delta z_{B6} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{T32} & \Delta x_{T32} \\ \Delta\beta_{T32} & \Delta y_{T32} \\ \Delta\gamma_{T32} & \Delta z_{T32} \end{bmatrix}$$

$$T_{P42-P22} = \begin{bmatrix} \Delta\alpha_{T42} & \Delta x_{T42} \\ \Delta\beta_{T42} & \Delta y_{T42} \\ \Delta\gamma_{T42} & \Delta z_{T42} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{B7} & \Delta x_{B7} \\ \Delta\beta_{B7} & \Delta y_{B7} \\ \Delta\gamma_{B7} & \Delta z_{B7} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{S2} & \Delta x_{S2} \\ \Delta\beta_{S2} & \Delta y_{S2} \\ \Delta\gamma_{S2} & \Delta z_{S2} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{P3} & \Delta x_{P3} \\ \Delta\beta_{P3} & \Delta y_{P3} \\ \Delta\gamma_{P3} & \Delta z_{P3} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{P2} & \Delta x_{P2} \\ \Delta\beta_{P2} & \Delta y_{P2} \\ \Delta\gamma_{P2} & \Delta z_{P2} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{B6} & \Delta x_{B6} \\ \Delta\beta_{B6} & \Delta y_{B6} \\ \Delta\gamma_{B6} & \Delta z_{B6} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{T22} & \Delta x_{T22} \\ \Delta\beta_{T22} & \Delta y_{T22} \\ \Delta\gamma_{T22} & \Delta z_{T22} \end{bmatrix}$$

$$T_{P42-P32} = \begin{bmatrix} \Delta\alpha_{T42} & \Delta x_{T42} \\ \Delta\beta_{T42} & \Delta y_{T42} \\ \Delta\gamma_{T42} & \Delta z_{T42} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{B7} & \Delta x_{B7} \\ \Delta\beta_{B7} & \Delta y_{B7} \\ \Delta\gamma_{B7} & \Delta z_{B7} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{S2} & \Delta x_{S2} \\ \Delta\beta_{S2} & \Delta y_{S2} \\ \Delta\gamma_{S2} & \Delta z_{S2} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{P3} & \Delta x_{P3} \\ \Delta\beta_{P3} & \Delta y_{P3} \\ \Delta\gamma_{P3} & \Delta z_{P3} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{P2} & \Delta x_{P2} \\ \Delta\beta_{P2} & \Delta y_{P2} \\ \Delta\gamma_{P2} & \Delta z_{P2} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{B6} & \Delta x_{B6} \\ \Delta\beta_{B6} & \Delta y_{B6} \\ \Delta\gamma_{B6} & \Delta z_{B6} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{T32} & \Delta x_{T32} \\ \Delta\beta_{T32} & \Delta y_{T32} \\ \Delta\gamma_{T32} & \Delta z_{T32} \end{bmatrix}$$

In the end the tolerance values are obtained using the same data values as given in the table above. The result comes out to be:

$$T_{P22-P11} = \begin{bmatrix} 0.64 & 0.026 \\ 0.64 & 0.026 \\ 0.64 & 0.026 \end{bmatrix}$$

$$T_{P32-P12} = \begin{bmatrix} 0.64 & 0.026 \\ 0.64 & 0.026 \\ 0.64 & 0.026 \end{bmatrix}$$

$$T_{P42-P12} = \begin{bmatrix} 0.64 & 0.026 \\ 0.64 & 0.026 \\ 0.64 & 0.026 \end{bmatrix}$$

$$T_{P7-P12} = \begin{bmatrix} 0.48 & 0.018 \\ 0.48 & 0.018 \\ 0.48 & 0.018 \end{bmatrix}$$

$$T_{P8-P12} = \begin{bmatrix} 0.48 & 0.018 \\ 0.48 & 0.018 \\ 0.48 & 0.018 \end{bmatrix}$$

$$T_{P7-P22} = \begin{bmatrix} 0.56 & 0.023 \\ 0.56 & 0.023 \\ 0.56 & 0.023 \end{bmatrix}$$

$$T_{P7-P32} = \begin{bmatrix} 0.56 & 0.023 \\ 0.56 & 0.023 \\ 0.56 & 0.023 \end{bmatrix}$$

$$T_{P8-P22} = \begin{bmatrix} 0.56 & 0.023 \\ 0.56 & 0.023 \\ 0.56 & 0.023 \end{bmatrix}$$

$$T_{P8-P32} = \begin{bmatrix} 0.56 & 0.023 \\ 0.56 & 0.023 \\ 0.56 & 0.023 \end{bmatrix}$$

$$T_{P42-P22} = \begin{bmatrix} 0.56 & 0.023 \\ 0.56 & 0.023 \\ 0.56 & 0.023 \end{bmatrix}$$

$$T_{P42-P32} = \begin{bmatrix} 0.56 & 0.023 \\ 0.56 & 0.023 \\ 0.56 & 0.023 \end{bmatrix}$$

The interaction between P22-P12, P32-12 and P42-12 has the maximum value because of different tooling operation, spindle changes, positioning and structure changes are involved in it. The torsor obtained at the end incorporates all the errors because of the tooling operation, spindle, positioning and post changes involved.

Resultant torsor gives an idea about the probable variation in the manufacturing of the part. These variations should be included in the drawings. In case of RMS design evaluation the solutions are selected and then ranked. Each generated solution is checked with respect to its corresponding torsor chain or set of torsor chains and then ranking is done basing on the results obtained [38].

## 4.3. Multi Post Tolerance Analysis

Tolerance analysis of the multi post generated solution of part CAI is performed. There is no tool change and spindle change involved in the multi post kinematic configuration. Accordingly the deviation associated to them are considered zero which is not the case in single post generated solution. The corresponding structure configuration for this process plan is shown in figure 4-5:

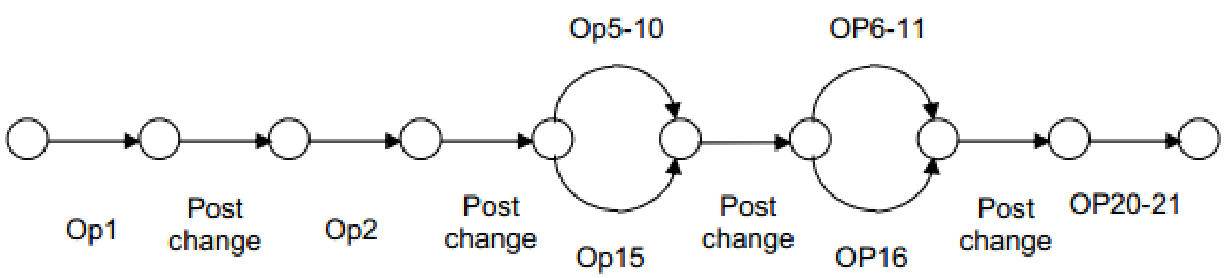


**Figure 4-5: Five Post Solution of Part CAI [28]**

The structure configuration shown above has five posts and two parallel structures. The post change activities involved in this case are four. The distances between the posts are considered in the tolerance analysis. Varignon relationship incorporates the errors introduced due to the post change involved. The graphical representation of the above configuration can be represented as shown in figure 4-6:

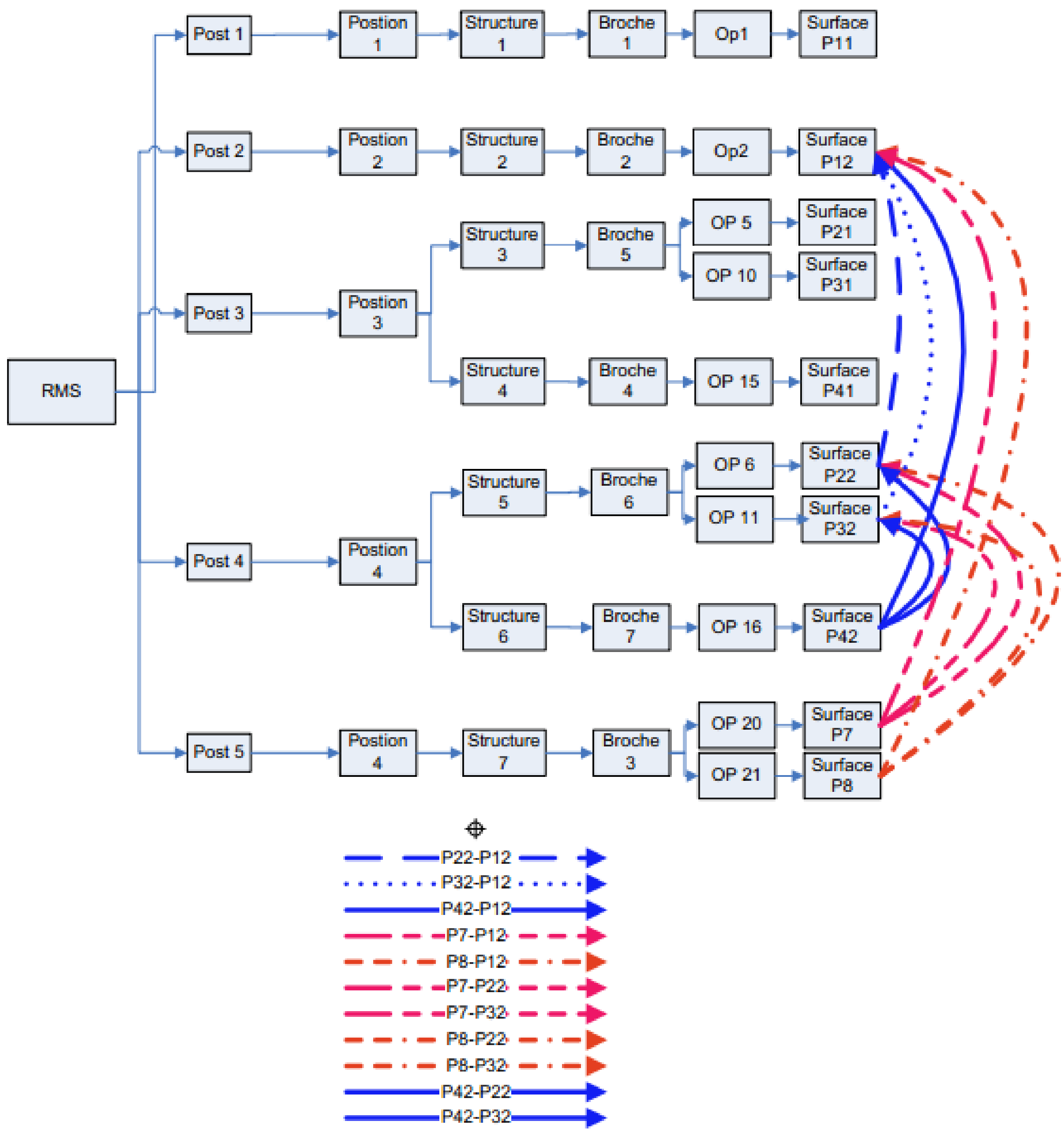


Figure 4-6: Graphical Representation of Five Post Solution of Part CAI

The possible interactions between the surfaces are also shown in the graphical representation of five post solution of part CAI. These possible interactions are obtained from the functional drawings of the part. Each liaison is in the form of chain between the interacting elements.

Using the defined algorithm the tolerance analysis is performed and the results obtained are shown in tabular form in table 4-2.

**Table 4-2: Multi Post Tolerance Analysis Results**

| SURFACE | TORSOR EQUATION | TOLERNACE VALUE |
|---|---|---|
| P22-P12 | $\Delta T_{\text{Tooling 12}} + \Delta T_{\text{Broche 2}} + \Delta T_{\text{Structure 2}} + \Delta T_{\text{Post 2}} + \Delta T_{\text{Post 4}} + \Delta T_{\text{Structure 5}} + \Delta T_{\text{Broche 5}} + \Delta T_{\text{Tooling 22}}$ | $\begin{bmatrix} 0.64 & 0.15 \\ 0.64 & 0.45 \\ 0.64 & 0.30 \end{bmatrix}$ |
| P32-P12 | $\Delta T_{\text{Tooling 12}} + \Delta T_{\text{Broche 2}} + \Delta T_{\text{Structure 2}} + \Delta T_{\text{Post 2}} + \Delta T_{\text{Post 4}} + \Delta T_{\text{Structure 5}} + \Delta T_{\text{Broche 5}} + \Delta T_{\text{Tooling 32}}$ | $\begin{bmatrix} 0.64 & 0.15 \\ 0.64 & 0.45 \\ 0.64 & 0.30 \end{bmatrix}$ |
| P42-P12 | $\Delta T_{\text{Tooling 12}} + \Delta T_{\text{Broche 2}} + \Delta T_{\text{Structure 2}} + \Delta T_{\text{Post 2}} + \Delta T_{\text{Post 4}} + \Delta T_{\text{Structure 6}} + \Delta T_{\text{Broche 6}} + \Delta T_{\text{Tooling 42}}$ | $\begin{bmatrix} 0.64 & 0.15 \\ 0.64 & 0.45 \\ 0.64 & 0.30 \end{bmatrix}$ |
| P7-P12 | $\Delta T_{\text{Tooling 12}} + \Delta T_{\text{Broche 2}} + \Delta T_{\text{Structure 2}} + \Delta T_{\text{Post 2}} + \Delta T_{\text{Post 5}} + \Delta T_{\text{Structure 7}} + \Delta T_{\text{Broche 7}} + \Delta T_{\text{Tooling 7}}$ | $\begin{bmatrix} 0.64 & 0.21 \\ 0.64 & 0.66 \\ 0.64 & 0.45 \end{bmatrix}$ |
| P8-P12 | $\Delta T_{\text{Tooling 12}} + \Delta T_{\text{Broche 2}} + \Delta T_{\text{Structure 2}} + \Delta T_{\text{Post 2}} + \Delta T_{\text{Post 5}} + \Delta T_{\text{Structure 7}} + \Delta T_{\text{Broche 7}} + \Delta T_{\text{Tooling 8}}$ | $\begin{bmatrix} 0.64 & 0.21 \\ 0.64 & 0.66 \\ 0.64 & 0.45 \end{bmatrix}$ |
| P7-P22 | $\Delta T_{\text{Tooling 7}} + \Delta T_{\text{Broche 7}} + \Delta T_{\text{Structure 7}} + \Delta T_{\text{Post 5}} + \Delta T_{\text{Post 4}} + \Delta T_{\text{Structure 5}} + \Delta T_{\text{Broche 5}} + \Delta T_{\text{Tooling 22}}$ | $\begin{bmatrix} 0.64 & 0.06 \\ 0.64 & 0.21 \\ 0.64 & 0.15 \end{bmatrix}$ |
| P7-P32 | $\Delta T_{\text{Tooling 7}} + \Delta T_{\text{Broche 7}} + \Delta T_{\text{Structure 7}} + \Delta T_{\text{Post 5}} + \Delta T_{\text{Post 4}} + \Delta T_{\text{Structure 5}} + \Delta T_{\text{Broche 5}} + \Delta T_{\text{Tooling 32}}$ | $\begin{bmatrix} 0.64 & 0.06 \\ 0.64 & 0.21 \\ 0.64 & 0.15 \end{bmatrix}$ |
| P8-P22 | $\Delta T_{\text{Tooling 8}} + \Delta T_{\text{Broche 7}} + \Delta T_{\text{Structure 7}} + \Delta T_{\text{Post 5}} + \Delta T_{\text{Post 4}} + \Delta T_{\text{Structure 5}} + \Delta T_{\text{Broche 5}} + \Delta T_{\text{Tooling 22}}$ | $\begin{bmatrix} 0.64 & 0.06 \\ 0.64 & 0.21 \\ 0.64 & 0.15 \end{bmatrix}$ |
| P8-P32 | $\Delta T_{\text{Tooling 8}} + \Delta T_{\text{Broche 7}} + \Delta T_{\text{Structure 7}} + \Delta T_{\text{Post 5}} + \Delta T_{\text{Post 4}} + \Delta T_{\text{Structure 5}} + \Delta T_{\text{Broche 5}} + \Delta T_{\text{Tooling 32}}$ | $\begin{bmatrix} 0.64 & 0.06 \\ 0.64 & 0.21 \\ 0.64 & 0.15 \end{bmatrix}$ |
| P42-P22 | $\Delta T_{\text{Tooling 42}} + \Delta T_{\text{Broche 6}} + \Delta T_{\text{Structure 6}} + \Delta T_{\text{Structure 5}} + \Delta T_{\text{Broche 5}} + \Delta T_{\text{Tooling 22}}$ | $\begin{bmatrix} 0.48 & 0.018 \\ 0.48 & 0.018 \\ 0.48 & 0.018 \end{bmatrix}$ |
| P42-P32 | $\Delta T_{\text{Tooling 42}} + \Delta T_{\text{Broche 6}} + \Delta T_{\text{Structure 6}} + \Delta T_{\text{Structure 5}} + \Delta T_{\text{Broche 5}} + \Delta T_{\text{Tooling 32}}$ | $\begin{bmatrix} 0.48 & 0.018 \\ 0.48 & 0.018 \\ 0.48 & 0.018 \end{bmatrix}$ |

## 4.4. Validation of Results of Tolerance Analysis

The validation of results of tolerance analysis is done by performing the tolerance analysis of the machine configuration using the same algorithmic approach. The end value of the elements of the resultant torsor is compared. Machining configuration of the setup which is used to manufacture the same part CAI is considered for validation. The generic machining configuration of a reconfigurable manufacturing system is shown in figure 4-7:

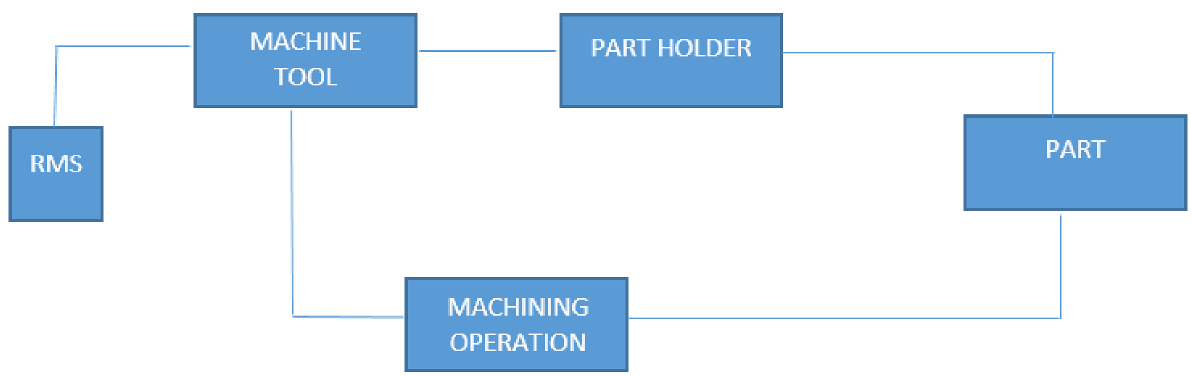


**Figure 4-7: Graphical Representation of Machine Configuration**

The machine tool consists of the number of tools that are used for the manufacturing of a particular part family. Part holder has the surfaces that are in contact with the machine tool and the part. Similarly the machining operation contains the number of operations involved in the manufacturing of the part CAI. The more elaborated form of the machining configuration is shown in figure 4-8:

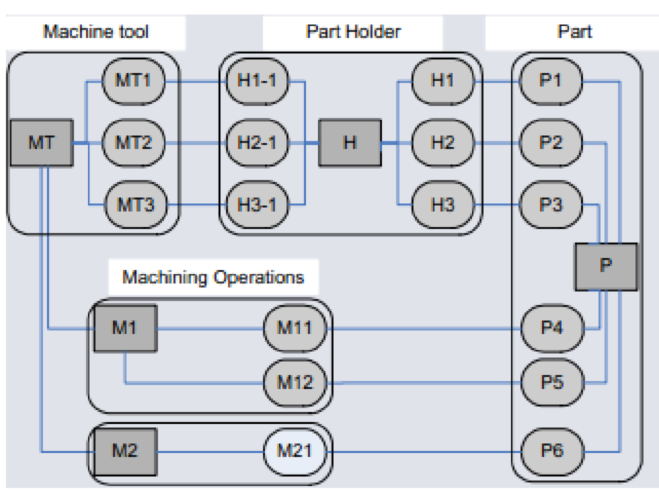


**Figure 4-8: Machine Configuration**

The interaction between the part surfaces P3 and P4 is taken into consideration. The torsor chain equation of the machining configuration is:

$$T = \Delta T_{\text{(part - part holder)}} + \Delta T_{\text{(part holder - machine tool)}} + \Delta T_{\text{(machine tool - machining operation)}} + \Delta T_{\text{(machining operation – part)}}$$

Substituting the values of the torsors:

$$T = \Delta T_{P3\text{-}H3} + \Delta T_{H3\text{-}H} + \Delta T_{H\text{-}H31} + \Delta T_{H31\text{-}MT3} + \Delta T_{MT3\text{-}MT} + \Delta T_{MT\text{-}M1} + \Delta T_{M1\text{-}M11} + \Delta T_{M11\text{-}P4}$$

Plugging in the data values in the above equation the result comes out to be:

$$T = \begin{bmatrix} 0.56 & 0.021 \\ 0.56 & 0.021 \\ 0.56 & 0.021 \end{bmatrix}$$

The end value of the torsor gives the geometric variations that will incur due to the machine configuration. These tolerance values of the torsors are now compared with the result obtained as a result of tolerance analysis done on generated process plans of part cover intermediate shaft. The maximum tolerance value obtained as a result of the tolerance analysis done on all possible interacting surfaces is compared with the result obtained from the machining configuration tolerance analysis.

The difference of 0.08 degrees in the rotation vector elements and 0.005 mm in elements of the translation vector elements of the final torsor exists. Considering the magnitude of difference values which are approximately equivalent the results obtained are validated.

# 5. COMPARISON OF EXISTING METHODOLGY WITH THE PROPOSED METHOLOGY

In this chapter the three dimensional tolerance analysis is done with an existing approach. Model of Indeterminate is selected by which the 3D tolerance analysis is done. The results of this approach are then compared with the algorithmic approach as presented in the chapter 4. Both approaches are applied on the same part Cover Intermediate Shaft (CAI).

## 5.1. Model of Indeterminate

Model of indeterminate is a method is used to write the three dimensional chains of deviations in order to have the correct functioning of a rigid structure. It incorporates the deviations that are caused due to orientation changes and position changes. This method also incorporates the intrinsic parameters that are involved in causing the geometric variations. This method works on the principle of conversion of functional conditions of the mechanism to the functional dimensions. These functional conditions correspond to the each stage of manufacturing of the part family.

In this approach in order to represent the each geometric error small displacement torsors are used. The study of geometric behavior of successive actual states of the part during the manufacturing process requires relations to be written between the functional conditions, the geometric defects of the parts and the gap in the links. These relations can be obtained from the model of indeterminate which is a generalization of the ΔL method. This method is based on the following steps:

- Step 1: Deviation torsors for the calculated surfaces
- Step 2: Gap torsors for pairs of relative positioning calculated surfaces
- Step 3: Geometric loop closing equation of the calculated surfaces
- Step 4: Compatibility relations
- Step 5: Resolve and obtain system of equations

From the above system of equations the following results can be obtained:

- Indeterminate values as function of differences in gap and deviation torsors
- Degrees of freedom of the system or mechanism from the indeterminate variables which are not determined by resolving the system of equations
- Chains of deviations in 3d from the compatibility relations between the gap and geometric defects of the surfaces if the system of equation is over-constrained [31] [32]

## 5.2. Application

The model of indeterminate is applied on the single post generated solutions of the same part CAI.

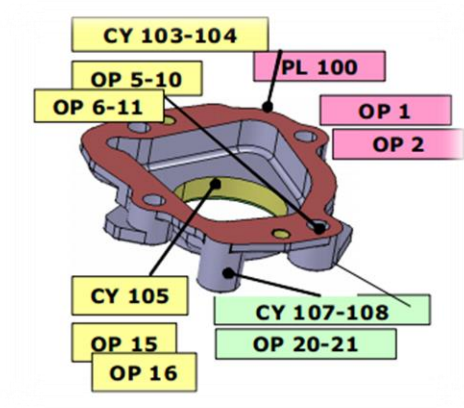


Figure 5-1: Cover Intermediate Shaft

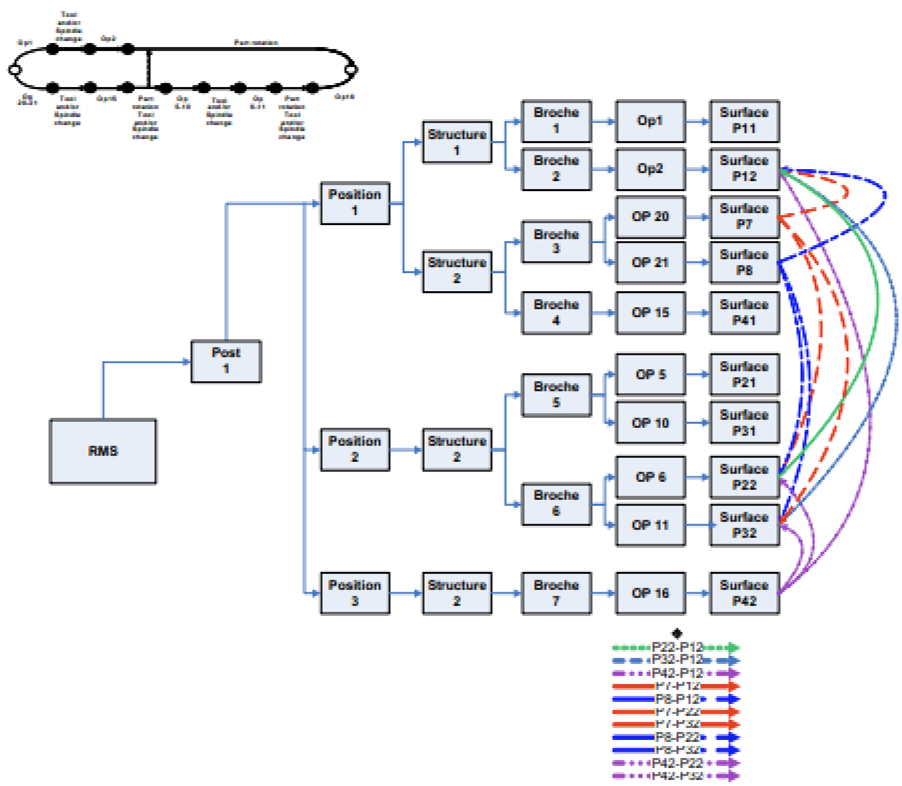


Figure 5-2: Single Post Generated Solution

The figure 5-2 shows all possible liaisons between the interacting surfaces. The interaction between the surfaces P32-P12 is kept under consideration. In this method the steps as mentioned above are followed. This method gives a system of equations that leads to part tolerances. If the system of equation is indeterminate then few assumptions and conditions are applied to obtain the end result. In case of unconstrained system each relation will lead a chain to a loop which is closed by a functional condition. In over-constrained system specific relations will express condition of compatibility between gaps and defects.

STEP 1: Deviation torsors of the respective calculated surfaces $P_{32}$ & $P_{12}$ are:

$$E_{P32} = \begin{bmatrix} a_{32} & u_{32} \\ b_{32} & v_{32} \\ c_{32} & w_{32} \end{bmatrix} \tag{1}$$

$$E_{P12} = \begin{bmatrix} a_{12} & u_{12} \\ b_{12} & v_{12} \\ c_{12} & w_{12} \end{bmatrix} \tag{2}$$

STEP 2: Gap torsors between the interacting surfaces is:

$$T_{(P32/P12)} = \begin{bmatrix} J\,(r_x, P_{32}, P_{12}) & J\,(t_x, P_{32}, P_{12}) \\ J\,(r_y, P_{32}, P_{12}) & J\,(t_y, P_{32}, P_{12}) \\ J\,(r_z, P_{32}, P_{12}) & J\,(t_z, P_{32}, P_{12}) \end{bmatrix} \tag{3}$$

where J represent the components of the gap torsor.

STEP 3: Loop equations

$$T\,(P_{32}, P_{12}) = E\,(P_{32}/P_i) + D\,(P_i/R) - D\,(P_j/R) - E\,(P_{12}/P_j) \tag{4}$$

$$T\,(P_{22}, P_{11}) = E\,(P_{22}/P_i) + D\,(P_i/R) - D\,(P_j/R) - E\,(P_{11}/P_j) \tag{5}$$

$$T\,(PL_{100}, PL_{101}) = E\,(PL_{100}/P_i) + D\,(P_i/R) - D\,(P_j/R) - E\,(PL_{101}/P_j) \tag{6}$$

where D (P/R) represents the part torsor.

STEP 4: Compatibility relations

$$0 = -J\,(r_x, P_{32}, P_{12}) + J\,(r_x, P_{22}, P_{11}) + a_{32} - a_{12} + a_{22} - a_{11} \tag{7}$$

$$0 = -J\,(r_x, PL_{100}, PL_{101}) + J\,(r_x, P_{22}, P_{11}) + a_{100} - a_{101} + a_{22} - a_{11} \tag{8}$$

$$0 = -J\,(r_y, P_{32}, P_{12}) + J\,(r_y, P_{22}, P_{11}) + b_{32} - b_{12} + b_{22} - b_{11} \tag{9}$$

$$0 = -J\,(r_y, PL_{100}, PL_{101}) + J\,(r_y, P_{22}, P_{11}) + b_{100} - b_{101} + b_{22} - b_{11} \tag{10}$$

$$0 = -J\,(r_z, P_{32}, P_{12}) + J\,(r_z, P_{22}, P_{11}) + c_{32} - c_{12} + c_{22} - c_{11} \tag{11}$$

$$0 = -J\,(r_z, PL_{100}, PL_{101}) + J\,(r_z, P_{22}, P_{11}) + c_{100} - c_{101} + c_{22} - c_{11} \tag{12}$$

$$0 = -J\,(t_x, P_{32}, P_{12}) + J\,(t_x, P_{22}, P_{11}) + u_{32} - u_{12} + u_{22} - u_{11} \tag{13}$$

$$0 = -J\left(t_y, P_{32}, P_{12}\right) + J\left(t_y, P_{22}, P_{11}\right) + v_{32} - v_{12} + v_{22} - v_{11} \tag{14}$$

$$0 = -J\,(t_z, P_{32}, P_{12}) + J\,(t_z, P_{22}, P_{11}) + w_{32} - w_{12} + w_{22} - w_{11} \tag{15}$$

From the above system of equations (eq. no 07 to 15) the desired result are obtained by plugging in the data values from table 1 and chain of deviations can be evaluated. Also by applying the angular conditions and relative positioning conditions, system of equations (eq. no 07 to 15) can be solved. Solving the above system of equation the result comes out to be:

$$J\,(r_x, P_{32}, P_{12}) = J\,(r_x, PL_{100}, PL_{101}) \tag{16}$$

$$J\left(r_y, P_{32}, P_{12}\right) = J\left(r_y, PL_{100}, PL_{101}\right) \tag{17}$$

$$J\,(r_z, P_{32}, P_{12}) = J\,(r_z, PL_{100}, PL_{101}) \tag{18}$$

$$J\,(t_x, P_{32}, P_{12}) = J\,(r_x, P_{32}, P_{12}) - 0.32 \tag{19}$$

$$J\left(t_y, P_{32}, P_{12}\right) = J\left(r_y, P_{32}, P_{12}\right) - 0.32 \tag{20}$$

$$J\,(t_z, P_{32}, P_{12}) = J\,(r_z, P_{32}, P_{12}) - 0.32 \tag{21}$$

The above equations give the elements of T $_{(P32/P12)}$. Equation no. 16 to 18 gives the angular components of the torsor. They are dependent on the components of gap torsor of planes 100 and 101. Similarly equations 19 to 21 which are derived from equation 13 to 15 by plugging in the values from table 1 gives the translation components of the torsor.

The above equations give the elements of T $_{(P32/P12)}$

$$T_{P32-P12} = \begin{bmatrix} 0.5 & 0.18 \\ 0.5 & 0.18 \\ 0.5 & 0.18 \end{bmatrix}$$

## 5.3. Comparison with the Algorithmic Method

Considering the same interaction between the surfaces P32-P12 when algorithmic method is applied on it the result comes out to be:

$T_{P32-P12}$ =
$\Delta T_{Tooling\ 12} + \Delta T_{Broche\ 2} + \Delta T_{Structure\ 1} + \Delta T_{Position\ 1} + \Delta T_{Position\ 2} + \Delta T_{Structure\ 2} +$
$\Delta T_{Broche\ 6} + \Delta T_{Tooling\ 32}$

$$T_{P32-P12} = \begin{bmatrix} \Delta\alpha_{T12} & \Delta x_{T12} \\ \Delta\beta_{T12} & \Delta y_{T12} \\ \Delta\gamma_{T12} & \Delta z_{T12} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{B2} & \Delta x_{B2} \\ \Delta\beta_{B2} & \Delta y_{B2} \\ \Delta\gamma_{B2} & \Delta z_{B2} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{S1} & \Delta x_{S1} \\ \Delta\beta_{S1} & \Delta y_{S1} \\ \Delta\gamma_{S1} & \Delta z_{S1} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{P1} & \Delta x_{P1} \\ \Delta\beta_{P1} & \Delta y_{P1} \\ \Delta\gamma_{P1} & \Delta z_{P1} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{P2} & \Delta x_{P2} \\ \Delta\beta_{P2} & \Delta y_{P2} \\ \Delta\gamma_{P2} & \Delta z_{P2} \end{bmatrix}$$
$$+ \begin{bmatrix} \Delta\alpha_{S2} & \Delta x_{S2} \\ \Delta\beta_{S2} & \Delta y_{S2} \\ \Delta\gamma_{S2} & \Delta z_{S2} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{B6} & \Delta x_{B6} \\ \Delta\beta_{B6} & \Delta y_{B6} \\ \Delta\gamma_{B6} & \Delta z_{B6} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{T32} & \Delta x_{T32} \\ \Delta\beta_{T32} & \Delta y_{T32} \\ \Delta\gamma_{T32} & \Delta z_{T32} \end{bmatrix}$$

Using the same data values which are used in the previous chapter. The tolerance comes out to be:

$$T_{P32-P12} = \begin{bmatrix} 0.64 & 0.026 \\ 0.64 & 0.026 \\ 0.64 & 0.026 \end{bmatrix}$$

## 5.3. Evaluation of the Results obtained by both Methods:

The results obtained from the tolerance analysis of part CAI are approximately same in magnitude either obtained by the model of indeterminate or algorithmic approach. The tolerance value obtained from the model of indeterminate are less as compared to the algorithmic approach in case of the angular components of the torsor while they are greater in case of the translational components of the torsor. There is a difference of 0.14 degrees in the values of the angular components and 0.154 mm difference in translational components of the torsor.

The model of indeterminate requires more computation in comparison to the algorithmic approach. Algorithmic approach is an iterative process which leads to tolerances just like a closed loop system. In this approach the indeterminate values are taken as zero. In the second approach the number of equations to deal with is greater in number. There are loop equations, compatibility equations and a resolving technique which gives the part tolerances. In this method tolerance evaluation between the interacting surfaces is dependent on components of other torsors. On the other hand in the first approach the calculation of torsor for each interacting surface is independent. The benefit of model of indeterminate is that we can determine the indeterminate values in the torsor. There effect can be incorporated in the design process. In the second approach there might be a fact we come across an indeterminate system. Then different conditions and assumptions are applied to solve that system.

# 6. INTEGRATION OF TOLERANCE ANALYSIS WITH CO-EVOLUTION PARADIGM FOR RMS

In this chapter the algorithm method described in previous chapter is integrated to the Co-evolution paradigm for reconfigurable manufacturing systems. Tolerance analysis is performed on the part CAI using the data values of three different machines of different manufacturers. The solutions are segregated on the basis of the tolerances obtained. Basing on these values of the tolerances along with the process plans the machining processes can be selected and the desired quality of the end product can be maintained.

## 6.1. Tolerance Analysis

The three dimensional tolerance analysis of the generated solutions of the process plans of Part CAI is performed using three different machining setups. These data values are obtained from the industry. These values are based on pure experimental results and the input from the machine operators of the respective machines. The associated error/ deviation in different activities related to tool, spindle, structure, position and part change activities in part manufacturing process are catered through these values.

The analysis is performed on the single post generated solution as used in the previous chapter of the part cover intermediate shaft. The solution in the graphical form is shown in figure 6-1:

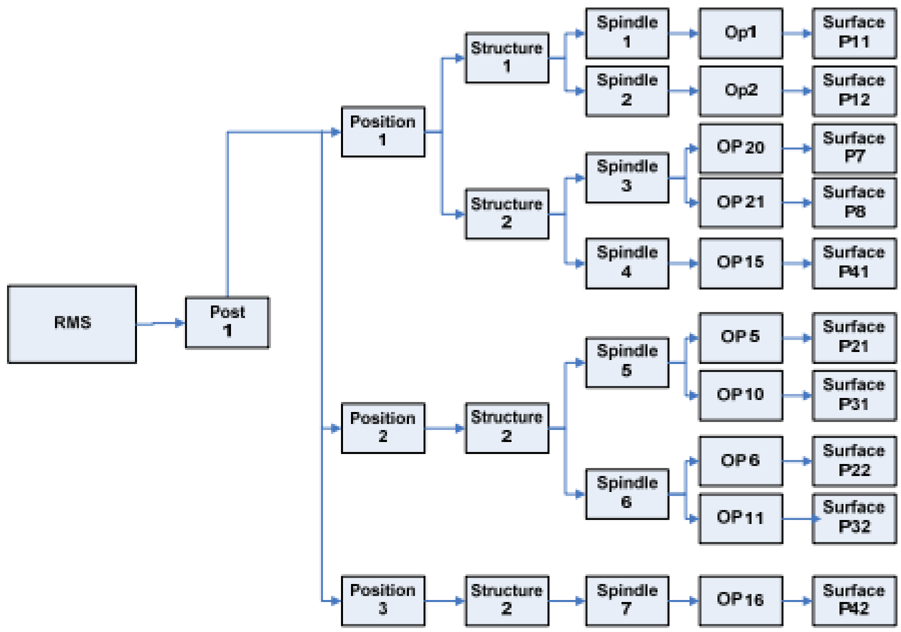


Figure 6-1: Graphical Solution of Single Post [1]

### 6.1.1. Machine "A" Tolerance Analysis

The three dimensional tolerance analysis is performed using the data values of all the machines. The variation only lies at the step of the algorithmic approach where the tolerance is evaluated. This is basically due to the change in their data values.

Table 6-1: Data Values of machine A

| Values are in mm/ deg for translations/ rotation | | | | | | |
|---|---|---|---|---|---|---|
| **Elements** | **Δx** | **Δy** | **Δz** | **Δα** | **Δβ** | **Δγ** |
| **Tool** | .003 | .003 | .003 | .05 | .05 | .05 |
| **Spindle** | .005 | .005 | .005 | .05 | .05 | .05 |
| **Structure** | .004 | .004 | .004 | .05 | .05 | .05 |
| **Position** | .002 | .002 | .002 | .05 | .05 | .05 |
| **Post** | .008 | .008 | .008 | .05 | .05 | .05 |

The tolerance evaluation is represented in the tabular form in table 6-2:

**Table 6-2: 3D Tolerance Analysis Results of machine A**

| Surface | Torsor Equations | Tolerance Value |
|---|---|---|
| P22-P11 | $\Delta T_{\text{Tooling 12}} + \Delta T_{\text{Broche 2}} + \Delta T_{\text{Structure 1}} + \Delta T_{\text{Position 1}} + \Delta T_{\text{Position 2}} + \Delta T_{\text{Structure 2}} + \Delta T_{\text{Broche 6}} + \Delta T_{\text{Tooling 22}}$ | $\begin{bmatrix} 0.4 & 0.028 \\ 0.4 & 0.028 \\ 0.4 & 0.028 \end{bmatrix}$ |
| P32-P12 | $\Delta T_{\text{Tooling 12}} + \Delta T_{\text{Broche 2}} + \Delta T_{\text{Structure 1}} + \Delta T_{\text{Position 1}} + \Delta T_{\text{Position 2}} + \Delta T_{\text{Structure 2}} + \Delta T_{\text{Broche 6}} + \Delta T_{\text{Tooling 32}}$ | $\begin{bmatrix} 0.4 & 0.028 \\ 0.4 & 0.028 \\ 0.4 & 0.028 \end{bmatrix}$ |
| P42-P12 | $\Delta T_{\text{Tooling 12}} + \Delta T_{\text{Broche 2}} + \Delta T_{\text{Structure 1}} + \Delta T_{\text{Position 1}} + \Delta T_{\text{Position 3}} + \Delta T_{\text{Structure 2}} + \Delta T_{\text{Broche 7}} + \Delta T_{\text{Tooling 42}}$ | $\begin{bmatrix} 0.4 & 0.028 \\ 0.4 & 0.028 \\ 0.4 & 0.028 \end{bmatrix}$ |
| P7-P12 | $\Delta T_{\text{Tooling 12}} + \Delta T_{\text{Broche 2}} + \Delta T_{\text{Structure 1}} + \Delta T_{\text{Structure 2}} + \Delta T_{\text{Broche 3}} + \Delta T_{\text{Tooling 7}}$ | $\begin{bmatrix} 0.3 & 0.024 \\ 0.3 & 0.024 \\ 0.3 & 0.024 \end{bmatrix}$ |
| P8-P12 | $\Delta T_{\text{Tooling 12}} + \Delta T_{\text{Broche 2}} + \Delta T_{\text{Structure 1}} + \Delta T_{\text{Structure 2}} + \Delta T_{\text{Broche 3}} + \Delta T_{\text{Tooling 8}}$ | $\begin{bmatrix} 0.3 & 0.024 \\ 0.3 & 0.024 \\ 0.3 & 0.024 \end{bmatrix}$ |
| P7-P22 | $\Delta T_{\text{Tooling 7}} + \Delta T_{\text{Broche 3}} + \Delta T_{\text{Structure 2}} + \Delta T_{\text{Position 1}} + \Delta T_{\text{Position 2}} + \Delta T_{\text{Broche 6}} + \Delta T_{\text{Tooling 22}}$ | $\begin{bmatrix} 0.35 & 0.024 \\ 0.35 & 0.024 \\ 0.35 & 0.024 \end{bmatrix}$ |
| P7-P32 | $\Delta T_{\text{Tooling 7}} + \Delta T_{\text{Broche 3}} + \Delta T_{\text{Structure 2}} + \Delta T_{\text{Position 1}} + \Delta T_{\text{Position 2}} + \Delta T_{\text{Broche 6}} + \Delta T_{\text{Tooling 32}}$ | $\begin{bmatrix} 0.35 & 0.024 \\ 0.35 & 0.024 \\ 0.35 & 0.024 \end{bmatrix}$ |
| P8-P22 | $\Delta T_{\text{Tooling 8}} + \Delta T_{\text{Broche 3}} + \Delta T_{\text{Structure 2}} + \Delta T_{\text{Position 1}} + \Delta T_{\text{Position 2}} + \Delta T_{\text{Broche 6}} + \Delta T_{\text{Tooling 22}}$ | $\begin{bmatrix} 0.35 & 0.024 \\ 0.35 & 0.024 \\ 0.35 & 0.024 \end{bmatrix}$ |
| P8-P32 | $\Delta T_{\text{Tooling 8}} + \Delta T_{\text{Broche 3}} + \Delta T_{\text{Structure 2}} + \Delta T_{\text{Position 1}} + \Delta T_{\text{Position 2}} + \Delta T_{\text{Broche 6}} + \Delta T_{\text{Tooling 32}}$ | $\begin{bmatrix} 0.35 & 0.024 \\ 0.35 & 0.024 \\ 0.35 & 0.024 \end{bmatrix}$ |
| P42-P22 | $\Delta T_{\text{Tooling 42}} + \Delta T_{\text{Broche 7}} + \Delta T_{\text{Structure 2}} + \Delta T_{\text{Position 3}} + \Delta T_{\text{Position 2}} + \Delta T_{\text{Broche 6}} + \Delta T_{\text{Tooling 22}}$ | $\begin{bmatrix} 0.35 & 0.024 \\ 0.35 & 0.024 \\ 0.35 & 0.024 \end{bmatrix}$ |
| P42-P32 | $\Delta T_{\text{Tooling 42}} + \Delta T_{\text{Broche 7}} + \Delta T_{\text{Structure 2}} + \Delta T_{\text{Position 3}} + \Delta T_{\text{Position 2}} + \Delta T_{\text{Broche 6}} + \Delta T_{\text{Tooling 32}}$ | $\begin{bmatrix} 0.35 & 0.024 \\ 0.35 & 0.024 \\ 0.35 & 0.024 \end{bmatrix}$ |

### 6.1.2. Machine "B" Tolerance Analysis

Using the same steps as above of Machine A the 3D tolerance analysis is performed. The data values of Machine B are as per table 6-3:

**Table 6-3: Data Values of machine B**

| Values are in mm/ deg for translations/ rotation | | | | | | |
|---|---|---|---|---|---|---|
| **Elements** | **Δx** | **Δy** | **Δz** | **Δα** | **Δβ** | **Δγ** |
| **Tool** | .006 | .006 | .006 | .07 | .07 | .07 |
| **Spindle** | .008 | .008 | .008 | .07 | .07 | .07 |
| **Structure** | .003 | .003 | .003 | .07 | .07 | .07 |
| **Position** | .002 | .002 | .002 | .07 | .07 | .07 |
| **Post** | .007 | .007 | .007 | .07 | .07 | .07 |

The tolerance evaluation is represented in the tabular form in table 6-4:

**Table 6-4: 3D Tolerance Analysis Results of machine B**

| **Surface** | **Torsor Equations** | **Tolerance Value** |
|---|---|---|
| P22-P11 | $\Delta T_{\text{Tooling 12}} + \Delta T_{\text{Broche 2}} + \Delta T_{\text{Structure 1}} + \Delta T_{\text{Position 1}} + \Delta T_{\text{Position 2}} + \Delta T_{\text{Structure 2}} + \Delta T_{\text{Broche 6}} + \Delta T_{\text{Tooling 22}}$ | $\begin{bmatrix} 0.56 & 0.038 \\ 0.56 & 0.038 \\ 0.56 & 0.038 \end{bmatrix}$ |
| P32-P12 | $\Delta T_{\text{Tooling 12}} + \Delta T_{\text{Broche 2}} + \Delta T_{\text{Structure 1}} + \Delta T_{\text{Position 1}} + \Delta T_{\text{Position 2}} + \Delta T_{\text{Structure 2}} + \Delta T_{\text{Broche 6}} + \Delta T_{\text{Tooling 32}}$ | $\begin{bmatrix} 0.56 & 0.038 \\ 0.56 & 0.038 \\ 0.56 & 0.038 \end{bmatrix}$ |
| P42-P12 | $\Delta T_{\text{Tooling 12}} + \Delta T_{\text{Broche 2}} + \Delta T_{\text{Structure 1}} + \Delta T_{\text{Position 1}} + \Delta T_{\text{Position 3}} + \Delta T_{\text{Structure 2}} + \Delta T_{\text{Broche 7}} + \Delta T_{\text{Tooling 42}}$ | $\begin{bmatrix} 0.56 & 0.038 \\ 0.56 & 0.038 \\ 0.56 & 0.038 \end{bmatrix}$ |
| P7-P12 | $\Delta T_{\text{Tooling 12}} + \Delta T_{\text{Broche 2}} + \Delta T_{\text{Structure 1}} + \Delta T_{\text{Structure 2}} + \Delta T_{\text{Broche 3}} + \Delta T_{\text{Tooling 7}}$ | $\begin{bmatrix} 0.42 & 0.034 \\ 0.42 & 0.034 \\ 0.42 & 0.034 \end{bmatrix}$ |
| P8-P12 | $\Delta T_{\text{Tooling 12}} + \Delta T_{\text{Broche 2}} + \Delta T_{\text{Structure 1}} + \Delta T_{\text{Structure 2}} + \Delta T_{\text{Broche 3}} + \Delta T_{\text{Tooling 8}}$ | $\begin{bmatrix} 0.42 & 0.034 \\ 0.42 & 0.034 \\ 0.42 & 0.034 \end{bmatrix}$ |

| | | |
|---|---|---|
| P7-P22 | $\Delta T_{\text{Tooling } 7} + \Delta T_{\text{Broche } 3} + \Delta T_{\text{Structure } 2} + \Delta T_{\text{Position } 1} + \Delta T_{\text{Position } 2} + \Delta T_{\text{Broche } 6} + \Delta T_{\text{Tooling } 22}$ | $\begin{bmatrix} 0.49 & 0.035 \\ 0.49 & 0.035 \\ 0.49 & 0.035 \end{bmatrix}$ |
| P7-P32 | $\Delta T_{\text{Tooling } 7} + \Delta T_{\text{Broche } 3} + \Delta T_{\text{Structure } 2} + \Delta T_{\text{Position } 1} + \Delta T_{\text{Position } 2} + \Delta T_{\text{Broche } 6} + \Delta T_{\text{Tooling } 32}$ | $\begin{bmatrix} 0.49 & 0.035 \\ 0.49 & 0.035 \\ 0.49 & 0.035 \end{bmatrix}$ |
| P8-P22 | $\Delta T_{\text{Tooling } 8} + \Delta T_{\text{Broche } 3} + \Delta T_{\text{Structure } 2} + \Delta T_{\text{Position } 1} + \Delta T_{\text{Position } 2} + \Delta T_{\text{Broche } 6} + \Delta T_{\text{Tooling } 22}$ | $\begin{bmatrix} 0.49 & 0.035 \\ 0.49 & 0.035 \\ 0.49 & 0.035 \end{bmatrix}$ |
| P8-P32 | $\Delta T_{\text{Tooling } 8} + \Delta T_{\text{Broche } 3} + \Delta T_{\text{Structure } 2} + \Delta T_{\text{Position } 1} + \Delta T_{\text{Position } 2} + \Delta T_{\text{Broche } 6} + \Delta T_{\text{Tooling } 32}$ | $\begin{bmatrix} 0.49 & 0.035 \\ 0.49 & 0.035 \\ 0.49 & 0.035 \end{bmatrix}$ |
| P42-P22 | $\Delta T_{\text{Tooling } 42} + \Delta T_{\text{Broche } 7} + \Delta T_{\text{Structure } 2} + \Delta T_{\text{Position } 3} + \Delta T_{\text{Position } 2} + \Delta T_{\text{Broche } 6} + \Delta T_{\text{Tooling } 22}$ | $\begin{bmatrix} 0.49 & 0.035 \\ 0.49 & 0.035 \\ 0.49 & 0.035 \end{bmatrix}$ |
| P42-P32 | $\Delta T_{\text{Tooling } 42} + \Delta T_{\text{Broche } 7} + \Delta T_{\text{Structure } 2} + \Delta T_{\text{Position } 3} + \Delta T_{\text{Position } 2} + \Delta T_{\text{Broche } 6} + \Delta T_{\text{Tooling } 32}$ | $\begin{bmatrix} 0.49 & 0.035 \\ 0.49 & 0.035 \\ 0.49 & 0.035 \end{bmatrix}$ |

### 6.1.3. Machine "C" Tolerance Analysis

Using the same steps as above of Machine A & B the 3D tolerance analysis is performed. The data values of Machine C are shown in table 6-5:

**Table 6-5: Data Values of machine C**

| Values are in mm/ deg for translations/ rotation | | | | | | |
|---|---|---|---|---|---|---|
| **Elements** | **Δx** | **Δy** | **Δz** | **Δα** | **Δβ** | **Δγ** |
| **Tool** | .004 | .004 | .004 | .09 | .09 | .09 |
| **Spindle** | .006 | .006 | .006 | .09 | .09 | .09 |
| **Structure** | .005 | .005 | .005 | .09 | .09 | .09 |
| **Position** | .003 | .003 | .003 | .09 | .09 | .09 |
| **Post** | .009 | .009 | .009 | .09 | .09 | .09 |

The tolerance evaluation is represented in the tabular form in table 6-6:

**Table 6-6: 3D Tolerance Analysis Results of machine C**

| Surface | Torsor Equations | Tolerance Value |
|---|---|---|
| P22-P11 | $\Delta T_{\text{Tooling 12}} + \Delta T_{\text{Broche 2}} + \Delta T_{\text{Structure 1}} + \Delta T_{\text{Position 1}} + \Delta T_{\text{Position 2}} + \Delta T_{\text{Structure 2}} + \Delta T_{\text{Broche 6}} + \Delta T_{\text{Tooling 22}}$ | $\begin{bmatrix} 0.72 & 0.036 \\ 0.72 & 0.036 \\ 0.72 & 0.036 \end{bmatrix}$ |
| P32-P12 | $\Delta T_{\text{Tooling 12}} + \Delta T_{\text{Broche 2}} + \Delta T_{\text{Structure 1}} + \Delta T_{\text{Position 1}} + \Delta T_{\text{Position 2}} + \Delta T_{\text{Structure 2}} + \Delta T_{\text{Broche 6}} + \Delta T_{\text{Tooling 32}}$ | $\begin{bmatrix} 0.72 & 0.036 \\ 0.72 & 0.036 \\ 0.72 & 0.036 \end{bmatrix}$ |
| P42-P12 | $\Delta T_{\text{Tooling 12}} + \Delta T_{\text{Broche 2}} + \Delta T_{\text{Structure 1}} + \Delta T_{\text{Position 1}} + \Delta T_{\text{Position 3}} + \Delta T_{\text{Structure 2}} + \Delta T_{\text{Broche 7}} + \Delta T_{\text{Tooling 42}}$ | $\begin{bmatrix} 0.72 & 0.036 \\ 0.72 & 0.036 \\ 0.72 & 0.036 \end{bmatrix}$ |
| P7-P12 | $\Delta T_{\text{Tooling 12}} + \Delta T_{\text{Broche 2}} + \Delta T_{\text{Structure 1}} + \Delta T_{\text{Structure 2}} + \Delta T_{\text{Broche 3}} + \Delta T_{\text{Tooling 7}}$ | $\begin{bmatrix} 0.54 & 0.03 \\ 0.54 & 0.03 \\ 0.54 & 0.03 \end{bmatrix}$ |
| P8-P12 | $\Delta T_{\text{Tooling 12}} + \Delta T_{\text{Broche 2}} + \Delta T_{\text{Structure 1}} + \Delta T_{\text{Structure 2}} + \Delta T_{\text{Broche 3}} + \Delta T_{\text{Tooling 8}}$ | $\begin{bmatrix} 0.54 & 0.03 \\ 0.54 & 0.03 \\ 0.54 & 0.03 \end{bmatrix}$ |
| P7-P22 | $\Delta T_{\text{Tooling 7}} + \Delta T_{\text{Broche 3}} + \Delta T_{\text{Structure 2}} + \Delta T_{\text{Position 1}} + \Delta T_{\text{Position 2}} + \Delta T_{\text{Broche 6}} + \Delta T_{\text{Tooling 22}}$ | $\begin{bmatrix} 0.63 & 0.031 \\ 0.63 & 0.031 \\ 0.63 & 0.031 \end{bmatrix}$ |
| P7-P32 | $\Delta T_{\text{Tooling 7}} + \Delta T_{\text{Broche 3}} + \Delta T_{\text{Structure 2}} + \Delta T_{\text{Position 1}} + \Delta T_{\text{Position 2}} + \Delta T_{\text{Broche 6}} + \Delta T_{\text{Tooling 32}}$ | $\begin{bmatrix} 0.63 & 0.031 \\ 0.63 & 0.031 \\ 0.63 & 0.031 \end{bmatrix}$ |
| P8-P22 | $\Delta T_{\text{Tooling 8}} + \Delta T_{\text{Broche 3}} + \Delta T_{\text{Structure 2}} + \Delta T_{\text{Position 1}} + \Delta T_{\text{Position 2}} + \Delta T_{\text{Broche 6}} + \Delta T_{\text{Tooling 22}}$ | $\begin{bmatrix} 0.63 & 0.031 \\ 0.63 & 0.031 \\ 0.63 & 0.031 \end{bmatrix}$ |
| P8-P32 | $\Delta T_{\text{Tooling 8}} + \Delta T_{\text{Broche 3}} + \Delta T_{\text{Structure 2}} + \Delta T_{\text{Position 1}} + \Delta T_{\text{Position 2}} + \Delta T_{\text{Broche 6}} + \Delta T_{\text{Tooling 32}}$ | $\begin{bmatrix} 0.63 & 0.031 \\ 0.63 & 0.031 \\ 0.63 & 0.031 \end{bmatrix}$ |
| P42-P22 | $\Delta T_{\text{Tooling 42}} + \Delta T_{\text{Broche 7}} + \Delta T_{\text{Structure 2}} + \Delta T_{\text{Position 3}} + \Delta T_{\text{Position 2}} + \Delta T_{\text{Broche 6}} + \Delta T_{\text{Tooling 22}}$ | $\begin{bmatrix} 0.63 & 0.031 \\ 0.63 & 0.031 \\ 0.63 & 0.031 \end{bmatrix}$ |
| P42-P32 | $\Delta T_{\text{Tooling 42}} + \Delta T_{\text{Broche 7}} + \Delta T_{\text{Structure 2}} + \Delta T_{\text{Position 3}} + \Delta T_{\text{Position 2}} + \Delta T_{\text{Broche 6}} + \Delta T_{\text{Tooling 32}}$ | $\begin{bmatrix} 0.63 & 0.031 \\ 0.63 & 0.031 \\ 0.63 & 0.031 \end{bmatrix}$ |

## 6.2. Results and Discussion

Three dimensional tolerance analyses are performed on the part CAI. The evaluation of tolerance values is done between all the interacting surfaces and associated value of torsor is obtained. The maximum value between the two interacting surfaces out of all the surfaces defines the tolerance limit for that particular process plan and structure. This value should be less than the value defined by the designer.

The maximum value obtained in our case is:

$$T_{max} = T_{P22\text{-}P12} / T_{P32\text{-}P12} / T_{P42\text{-}P12}$$

- For machine A the value of torsor is

$$\begin{bmatrix} 0.4 & 0.028 \\ 0.4 & 0.028 \\ 0.4 & 0.028 \end{bmatrix}$$

- For machine B the value of torsor is

$$\begin{bmatrix} 0.56 & 0.038 \\ 0.56 & 0.038 \\ 0.56 & 0.038 \end{bmatrix}$$

- For machine C the value of torsor is

$$\begin{bmatrix} 0.72 & 0.036 \\ 0.72 & 0.036 \\ 0.72 & 0.036 \end{bmatrix}$$

The interaction between P22-P12, P32-12 and P42-12 has the maximum value because of different tooling operation, spindle changes, positioning and structure changes are involved in it. The torsor obtained at the end incorporates all the errors because of the tooling operation, spindle, positioning and post changes involved [35].

Resultant torsor gives an idea about the probable variation in the manufacturing of the part. These variations should be included in the drawings. In case of RMS design evaluation the solutions are selected and then ranked. Each generated solution is checked with respect to its corresponding torsor chain or set of torsor chains and then ranking is done basing on the results obtained [38].

From the end result obtained of different machines the solutions can be ranked basing on the desired requirements. The value of the resultant torsor of Machine 'A' has the smallest value as compared to the machine 'B' and 'C'. Machine 'B' angular variation is less as compared with the machine 'C' while the translation geometric variations of machine 'C' are less as that of 'B'.

The tolerances defined on the drawing that satisfies the design requirement are matched with the geometric variations represented in the form of torsor obtained as a result of tolerance analysis. Also we know that as the tolerance value of a manufacturing system is reduced the cost of that system is increased. Therefore the evaluation will be done without compromising the quality of the end result. In figure 6-2 manufacturing phases of Part CAI are shown.

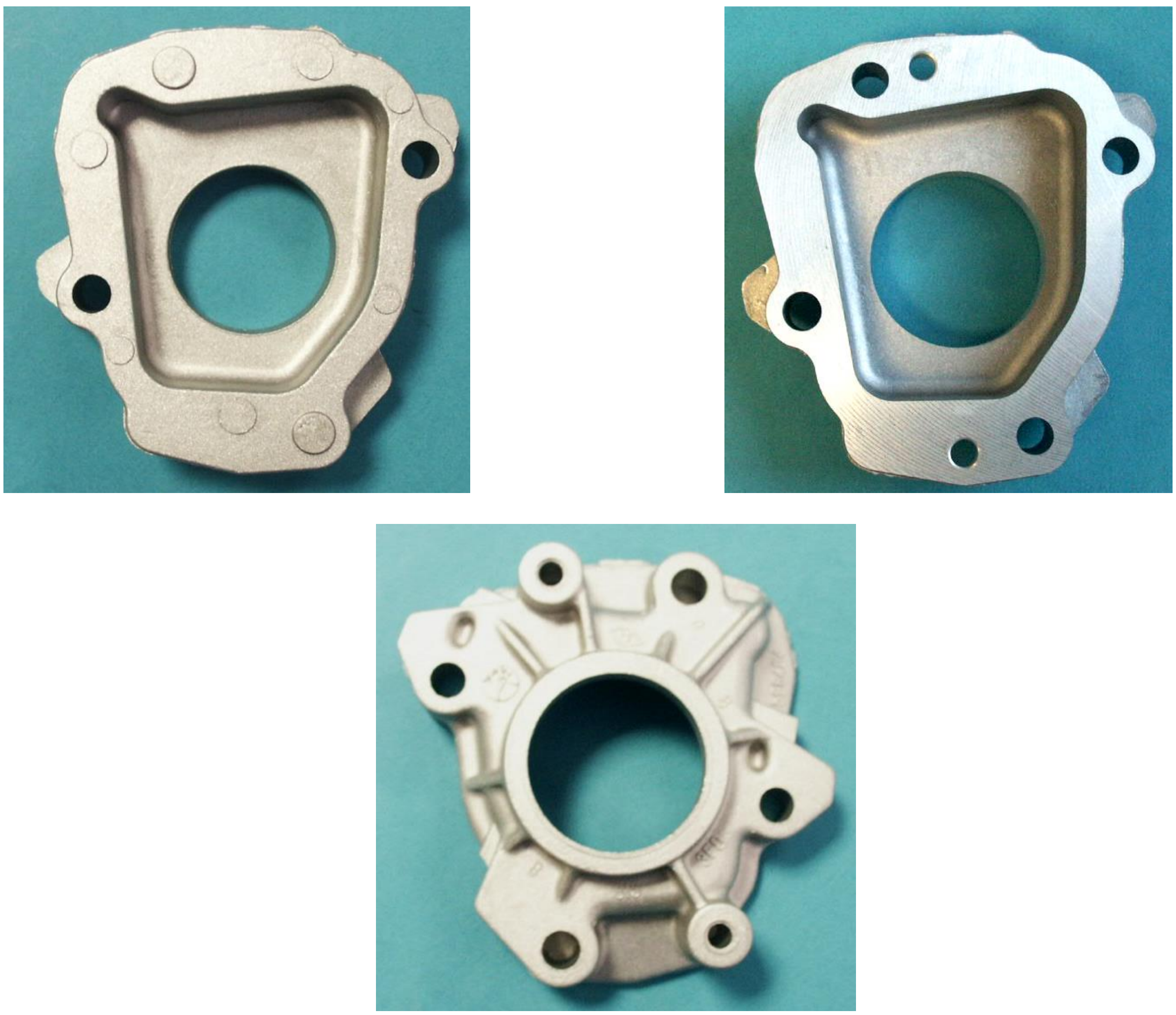

**Figure 6-2: Part CAI [1]**

Different part families can be segregated by doing the tolerance analysis. From the final value of the torsor obtained the ranking of solutions is done. If the value is greater than the design requirements then that solution can be neglected or ranked accordingly.

## Conclusion

The geometric variations obtained as a result of this tolerance analysis are accommodated during the design of a manufacturing system. Integration of this algorithmic approach in Co-Evolution paradigm, the solutions of the RMS are ranked basing on the value of tolerances obtained. Process plans generated solution space is reduced using this approach. Machines for Manufacturing of a part are selected using this approach. Selection of solutions generated for manufacturing system is done using this approach.

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