Affirmation of Enhanced Product Design using GD&T and Concept of Torsors in Tolerance Analysis

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A thesis submitted in partial fulfillment of the requirements for the degree of MS Mechanical Engineering

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Declaration

I certify that this research work entitled *"Affirmation of Enhanced Product Design using GD&T and Concept of Torsors in Tolerance Analysis"* is my own work. The work has not been presented elsewhere for assessment. The material that has been used from other sources it has been properly acknowledged / referred.

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Language Correctness Certificate

This thesis has been read by an English expert and is free of typing, syntax, semantic, grammatical and spelling mistakes. Thesis is also according to the format given by the university.

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Copyright Statement

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Dedicated to

my beloved Parents & my dearest friend Memona Sadiq

Abstract

Cost and quality of a product are prime drivers in manufacturing. It is important to optimize these variables for maximum benefit. It is to be noted that higher quality means a product which has lesser errors and thus more close to ideal product. The quality of product is associated with precision of manufacturing system but such machines are expensive. Therefore, a method of machine selection is devised based on tolerance range. Dimensional and geometrical tolerances are applied for manufacturing a quality product. In this work, a two part assembly is taken as a case study example. Tolerances are applied at three different conditions i.e. Least material condition (LMC), Maximum material condition (MMC) and Regardless of feature size (RFS). Then tolerance stack-up analysis is performed for all three conditions separately. The resulted minimum and maximum values for allowable variation range is compared which gives us an optimized method to design the product for manufacturing.It is followed by the torsor linkage model for case study parts in which effect of Geometric tolerances on part precision and selection of machine to manufacture the part is analyzed. Standard tolerance classes have been used to define the parts under study. Part geometry is defined in terms of form, orientation and position which are sufficient to quantify the possible variations which might cause errors. Angle components of torsors for parallelism and perpendicularity constraints are controlled and evaluated at several standard deviations. The results are in compliance to proposed idea that a less precise machine can produce high quality product within a specific tolerance range.

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List of Acronyms

ANOVA Analysis of Variance

ASME The American Society of Mechanical Engineers

CAD Computer Aided Drawing

CAM Computer Aided Manufacturing

CMM Coordinate Measuring Machine

DLM Direct Linearization Method

DOF Degrees of Freedom

EMS Estimated Mean Shift

FEA Finite Element Analysis

GD&T Geometric Dimensioning and Tolerancing

GMC Geometric Manufacturing Condition

GUI Graphical User Interface

HTM Homogeneous Transformation Matrix

ISO International Organization for Standardization

LMC Least Material Condition

MM Millimeter

MAX Maximum

MCS Monte Carlo Simulation

MIN Minimum

MMC Maximum Material Condition

PDF Probability Density Function

RFS Regardless of Feature Size

RSS Root Sum Square

SDT Small Displacement Torsor

T-map Tolerance Map

TTRS Technologically and Topologically Related Surfaces

WC Worst Case

2D Two Dimensional

3D Three Dimensional

Chapter 1

Introduction

Geometric Dimensioning and Tolerancing is a quite recent field in terms of research. During the whole life cycle of a product, proper GD&T application means optimization, referring to cost effectiveness and enhanced quality. With the current growth rate and evolution of high tech merchandises, the need of GD&T is more than ever. Whenever a product is designed, the design engineer assigns certain nominal dimensions to the separate components and their assembly. Upon manufacturing, the geometric variations in these components might be inevitable but these variations can greatly be reduced to an acceptable limit. Tolerance analysis is one of the methods to ensure high grade of product by giving controlled freedom to the technician working on job floor operating with machines. Throughout the years engineers and manufacturers have been looking out for methods to make their manufacturing processes more efficient and manufacturing techniques more advanced. Geometric Dimensioning and Tolerancing (GD&T) is essential to interpret the engineering drawings. With the help of product tolerancing, cost of production can be decreased by reducing the gap between engineering design for performance and manufacturing. Therefore, most of the automotive industries, consumer packaging and aviation businesses seek to gain in competitive benefit by using tolerance analysis methods. In this work, the approach of using small displacement torsors to model the variations in a specific part is explained. Torsors represent the parameters which might cause errors between Real and Theoretical Surface.

1.1 Aim and Goal

Now-a-days when any country's economic growth and development depends on its advancement of industrial expertise, the use of correct and better manufacturing methods is far more increasing than ever. When a part is manufactured, its dimensional accuracy and precision are closely monitored for optimized performance. Manufacturing requires a design process and drawing is first step in it. To start with, one has to have the understanding of engineering drawing and the ASME standard that is followed by practitioners all over the world.

1.1.1 Engineering Drawing

An engineering drawing can be defined as a technical document which helps to outline the design specifications of the part to be manufactured. This graphical language is universal and helps to communicate and share ideas between professional people living in any part of the world. It bridges the gap between the design or engineer and the person working on the job floor [Thomas E, 1953]. Thus engineering drawing is the most crucial phase of manufacturing. It is necessary to get the drawings interpreted properly without any errors. All engineering drawings are made using ASME standard for GD&T.

1.1.2 ASME Standard for GD&T

ASME (Y14.5-2009) standard for GD&T is a document containing set of rules, symbols, conventions and definitions used for defining a product's engineering drawing. With the help of this document the manufacturing design procedure is validated. Therefore GD&T is an exact language for manufacturing and industrial engineers who work with drawings of engineering parts. Moreover, it is internationally recognized and interpreted. It helps to define the drawing in such a way that it reduces any doubts and eliminates the probability of any controversy or assumptions during manufacturing design and inspection processes [Y14.5-2009, 2009].

The domain of tolerancing plays a lead role to manufacture a quality product within cost limitation. The effects of tolerance on an entity can be controlled by scrutinizing types of tolerances and its application.This study is related to the manufacturing technology and innovation needs of national industry. The capability of designing, manufacturing and then verifying the product,specially the high precision parts will make the country self sufficient in manufacturing and production enterprise. These high precision parts are required for making intricate performing parts of automobiles, the defense technology like missiles, tanks and other war fare equipment.As the reliance of all industries like automotive, aviation,consumer packaging, electronics, heavy machinery and medical devices on tolerance analysis increases, many of them seek to gain a competitive advantage.

1.2 Proposition

There are many advantages of Geometric dimensioning and tolerancing in product and manufacturing systems design, since by improving the life cycle time of a product. Moreover the improvement of quality has a positive effect on the production. The profitability also increases by decreasing the rejected parts ratio. So, one can say that profitability is directly affected by precision of a manufacturing machine. The more precise the machine, less will be the errors in product and it will remain within the specified tolerance limits. But a high precision machine will cost more than its counter part with lower precision. Here a decision is to be made about the selection of machine based on its precision, flexibility and cost, for manufacturing a certain product with some tolerance linked to it. Tolerances can be geometric and dimensional. The work in this thesis proposes that torsors linked to a workpiece can be manipulated to an extent so that a rather less precise machine may produce a higher quality part thus saving cost of manufacturing. The geometric tolerances defined for a product can be indirectly controlled by keeping torsor components within limits. There are boundary conditions defined in GD&T which can effect tolerance range of product. This affect of boundary conditions on dimensional tolerances is studied for a simple assembly case of parts.

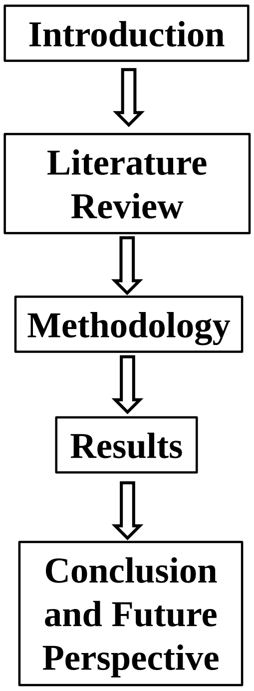

Figure 1.1. Disposition of thesis report

1.3 Thesis Disposition

This thesis report consists of five main sections. The first chapter is of introduction which includes the aim and goal along with proposition. The second chapter will give the reader detailed background and state of the art in the subject under discussion. It includes definition, types and effects of tolerance. Then a few tolerance analysis techniques like Stack-up analysis, Worst case analysis, statistical analysis are given. The difference between ideal and non-ideal features is explained followed state of art in torsors. Chapter 3 is about methodology of proposed work. It describes the tolerance constraints in terms of torsors, boundary conditions and the case-study of assembly of two parts. The results of the case study are shown in chapter 4. Results conform with the proposed solution. The conclusion, future prospects and possible research directions are provided in chapter 5 which is the last chapter.

1.4 Summary

This chapter communicated problem statement, aim and goal, proposition and thesis organization. The next section presents literature review to make proposition and solution of this study comprehensible. All related definitions and terms have been explained thoroughly.

Chapter 2

Literature Review

In this chapter, a foundation is made for the reader to grasp the required knowledge of the research on GD&T based on literature review. In the beginning, the basics of Geometric Dimensioning and Tolerancing is explained with its importance in engineering drawing. Then, tolerance is discussed in accordance to its effects, types and control methods. Different tolerance analysis techniques are presented. Since final product consists of separate parts and their assembly. Error is generated at every step of manufacturing process, therefore, for the product to remain within tolerance limits, the tolerancing must be controlled at both separate parts and assembly level.

2.1 Geometric Tolerance

Tolerance can be defined as a variation of physical dimension with respect to the ideal dimension that is required. Actually whenever a product is designed for manufacturing, a certain allowance is given to keep variations within the limit. No two products produced are ever exact or perfect in terms of their dimensions. So the given tolerance helps to keep dimensional variation in control. Tolerancing is a very wide domain in which manufacturing specifications can only be matched with customer needs if tolerance specification is performed correctly. Product design, assembly process, machine capability etc are all included in the network of tolerancing. In the figure 2.1 'Tolerance specification' acts as a bridge linking customer needs and production equipment. A balance has to be maintained between process and product in terms of its design, analysis and verification. If functionality of a part is complex, the product design will automatically be complex. So tolerance specification will be less flexible. Manufacturing cost will rise with tight limits of tolerance specification. Thus making its assembly and inspection process not only complicated but expensive. The customer needs of complex functionality product, impacts the choice of production equipment.

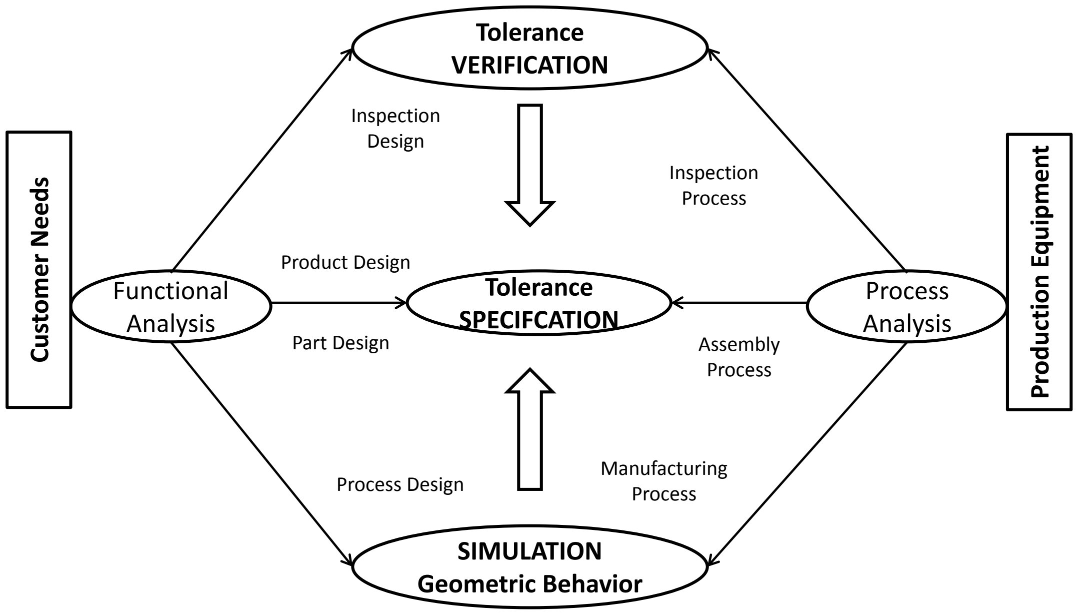

Figure 2.1. Tolerancing and its domains [François Villeneuve, 2010]

2.1.1 Effects of Tolerance on a Product at design and manufacturing level

Part tolerances are fundamental at both engineering design and manufacturing stage. The requirements of performance of a product suggests the level of tolerance values at design stage and for the manufacturing stage. Tolerance values are chosen keeping cost, assemblability and type of process to be used for manufacturing [Chase K W, 1997] Fig 2.2. Tolerance value influences performance of part. If a product is designed for high performance and quality finish, then there will be tight limits of tolerance selected and the cost of manufacturing of a product will dramatically increase. Because high performance products would require intricate and difficult manufacturing process. Also the assembly process becomes more strenuous because the tolerance limits are tight.

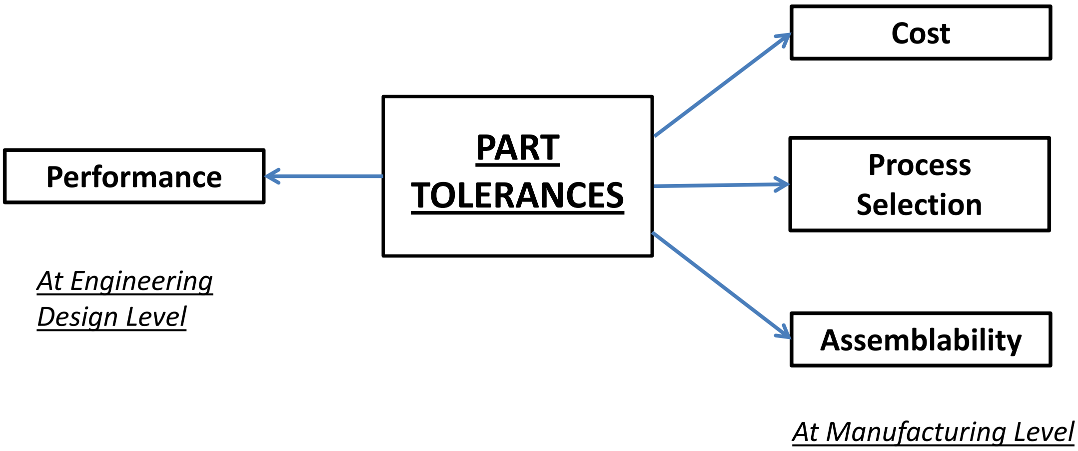

Figure 2.2. Effects of Tolerance at Design and Manufacturing level [Dantan J Y, 2012]

2.1.2 Types of Geometric Tolerance

There are 6 classes of geometric tolerances used in Geometric Dimensioning and tolerancing (GD&T ASME Y14.5). These classes explain the deviation limits for size, form, profile, orientation, location and run-out. In the orientation, we consider parallelism, perpendicularity and angularity. For location of a feature, we consider the position and concentricity(table 2.1) [Y14.5-2009, 2009]. These classes of tolerance are differentiated on basis of features of an entity which are discussed in section **??**. There are certain characteristics for specific types of tolerance and they are characterized by their respective symbols on the engineering drawing of a product. Dimensional tolerance is linear but combining geometric tolerance classes with dimensional tolerance regulate the shape of product within tighter bounds.

2.2 Tolerance Analysis Techniques

Tolerance analysis is a key feature of inspection process which includes tolerance verification. The next few topics are related to different methods in literature used for tolerance analysis. Each method has its own pros and cons according to ease to solution.

Table 2.1. Classes of Tolerance [Y14.5-2009, 2009]

Application	Types of Tolerance	Characteristic	Symbol
INDIVIDUAL FEATURES	Form	Straightness	−
		Flatness	
		Circularity	○
		Cylindericity	
RELATED FEATURES	Profile	Profile of a line	⌒
		Profile of a surface	
RELATED FEATURES	Orientation	Angularity	∠
		Perpendicularity	⊥
		Parallelism	∥
	Location	Position	⊕
		Concentricity	◎
		Symmetry	≡
	Runout	Circular Runout	↗
		Total runout	

2.2.1 Tolerance Stack-up Analysis

Tolerance analysis includes the procedures to find out the accumulated variations in the parts which may be separate or assembled together to form an assembly of mechanical systems. Tolerance analysis can be performed by stack-up analysis. It helps the engineers to evaluate and analyze the effect of geometric dimensioning and tolerancing of each product in assembly. In any mechanical stack-up, the total effect of accumulative variations and other dimensions is calculated. Tolerance stack-up analysis assist the engineers by:

- Providing the link of dimensions of features of a part
- Giving a way to calculate product/parts assembly tolerances as a whole
- Serving as a comparison tool for several proposals
- Helping to make engineering drawings, whole and comprehensive

2.2.2 Statistical Tolerance Analysis

Statistical tolerance analysis utilizes principles of statistics for assignment of tolerance. Statistical approach is flexible for tolerance allocation purpose. It gives designer to relax the tolerance range, although the product remains within quality conformance [Srinivasan, 1999]. In statistical tolerance analysis, extreme values are not important. Instead, a distribution of tolerance variation is studied. Root Sum Squared (RSS) is one type of statistical tolerance analysis method. A product is composed of more than one individual part component. [Parkinson, 1991] In RSS method, square root of the squares of individual component tolerance values is summed up to predict the tolerance of product assembly as shown in equation 2.1.

$$Tolerance_{assembly} = \sum \sqrt{Tolerance^2_{components}} \tag{2.1}$$

Mostly, researchers have taken statistical approach towards tolerance analysis because of its applicability. Some have done experimental study using this approach. Barakallah et al have experimentally validated a statistical model of tolerance analysis. Three phases of manufacturing milling process plan are used for case study. The statistical model finds deviation torsor of machining set up. Total variance of machined and nominal surfaces of 50 parts is calculated to evaluate the process capability of experimental set-up. It is found that the difference in simulated values and experimental values of geometric tolerance variance is very small [Barkallah M, 2012]. Machine tool vibrations of a turning operation is reduced by doing statistical tolerance analysis resulting in a regression model. Dimensional accuracy of machined parts is improved by finding the optimum cutting conditions and identifying prime parameters affecting accuracy of machine tool [Rahman M A, 2014]. A study is done in which Form deviations are integrated with orientation deviations in statistical tolerance analysis of parts. It suggested that the functional requirements change with respect to type of deviations considered during analysis [M Chahbouni, 2014].

Statistical tolerance analysis is frequently applied in automotive, robotics and aeronautical manufacturing industry. In these fields, Tolerance analysis followed by tolerance adjustment is very important. CAD/CAM softwares along with process capability indices assist in tolerance analysis of any product. One of the initial computer aided tolerance control systems can be found in work of Ahluwalia et al using CAD/CAM in tolerance stacking [Ahluwalia R S, 1984]. However most of the CAD/CAM softwares have limitations while dealing with dimensional and geometric tolerances. The research group lead by Wilma Polini have devised a method to do tolerance analysis by a seven step model based on a tolerance measuring e-tool. They did tolerance analysis on the surfaces of an aeronautical assembly parts which are made of composite materials i.e carbon fiber [Polini, 2011b]. Guzman et al inferred that statistical tolerance analysis based on ANOVA and process capability indexes improves the stability of manufacturing process. But results of such analysis is dependent on soundness of previous tolerance values data of a company [Luis García Guzmán, 2003]. Yang et al demonstrated prominence of tolerance analysis at design stages of a planar mechanism with five revolute joints. They solved a Robotic mechanism using kinematics and statistical tolerance analysis. The result provided maximum and minimum values of Link deviations plus values of joint clearance deviation [J X Yang, 2011].

For 3D manufacturing modeling simulation, a comparison between two approaches of small displacement torsor (SDT) and CAM integration shows that both perspectives give same result. However, the selection of approach depends on different advantages. These advantages are as per criteria of tolerance analysis, tolerance synthesis, accuracy, automation, virtual metrology and process plan management [Stephane Tichadou, 2005].

There is another statistical method called "Monte-Carlo" which is non-linear model utilized for the tolerance analysis. In work of Huiwen yan et al Monte Carlo statistical tolerance analysis is performed. The validity of this Statistical model is checked by finding the Upper and Lower limits of tolerance deviation in a nominal dimension of a top column assembly [Huiwen Yan, 2015]. Stuppy et al also applied Monte-Carlo statistical model simulation on a crank-shaft mechanism for tolerance analysis of a moving system. Elastic deformations and joint clearance are included in it. The results of this study are visualized in MATLAB with a Graphical User Interface (GUI) [J Stuppy, 2010].

For review of other tolerance analysis methods, a detailed study on tolerance analysis methods which are Direct Linearization Method (DLM), unified jacobian torsor, Tmaps and matrix model is presented by [Hua Chen, 2014].

2.2.3 Worst Case Analysis

Worst Case Analysis (WCA)is one of the techniques used for tolerance analysis. In WCA stack-up tolerance analysis extreme limits of dimensions and tolerances are taken into account instead of statistical distribution. Although WCA is computationally inexpensive but the manufacturing cost increases greatly due to the tight limits. However, there is an advantage that maximum manufactured parts will be assembled. In Worst Case, all the individual tolerance values of components are added to give the total tolerance of whole assembly, shown in equation 2.2

$$Tolerance_{assembly} = \sum Tolerance_{components} \tag{2.2}$$

Sahani et al presented an automated graphical method to calculate the tolerances of parts using Worst Case (WC) and Root Sum Square method (RSS) which is statistical approach. This algorithm can be utilized for tolerance analysis. But there is one limitation that it can not deal with more than one type of tolerance simultaneously [Sahani A K, 2013]. Manufacturing cost factor is also important. The cost of manufacturing is reduced by optimizing the orientation tolerance i.e angularity, by finding out minimum and maximum clearance for parts assembly. Later on the tolerances can be reassigned as per functionality [A K Sahani, 2014]. Similarly Mansuy et al have improved the algorithm for calculation of coefficients related to functional tolerances in worst case tolerance analysis. The results of this new algorithm is tested by optimizing the cost function for manufacturing of parts under consideration [Mathieu Mansuy, 2011]. Besides Worst case (WC) and Root Sum Square (RSS) method, there are other statistical models like Modified RSS (Root sum square), Estimated Mean Shift (EMS) and Spott's Model which can also be used for tolerance analysis. These models are more efficient than worst case model. In the work done by Dinesh Shringi et al,coherence of these unconventional Tolerance analysis models is verified by application of cost function analysis for a bearing shaft produced using eight manufacturing operations [Dinesh Shringi, 2013].

2.2.4 Process Stability

Process stability is defined by the variability of processes. In manufacturing, the stable process is the one with least variation from mean of specified target variable. Figure 2.4 show an unstable process. In unstable process,there is a large variation from target, thus making it out of control process. Whereas in figure 2.3, a stable process can be seen. Here,the variation from target is almost zero. So it is a well-controlled process. Nonetheless, sole stability does not produce a well working process. Process capability along with Process stability form a good working process. If a system is incapable of meeting up to the specified requirements, then its stability is of no use.

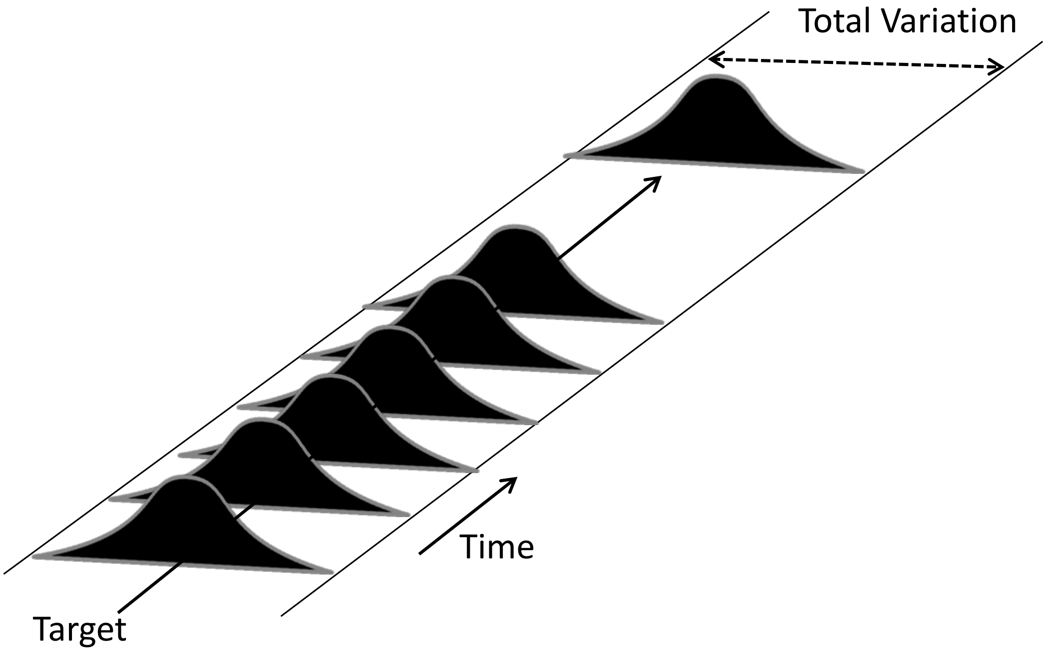

Figure 2.3. A Stable Process [Westin, 2010]

2.2.5 Process Capability

Process capability in manufacturing is defined as ability of any manufacturing process to achieve the required performance. It is a quantitative measure to show whether a process will attain the defined quality specifications. In modern manufacturing, Process capability is computed to evaluate Process control on a manufacturing system. [B. Ramirez, 2006] There are limits of a control variable in process capability analysis. A process should lie

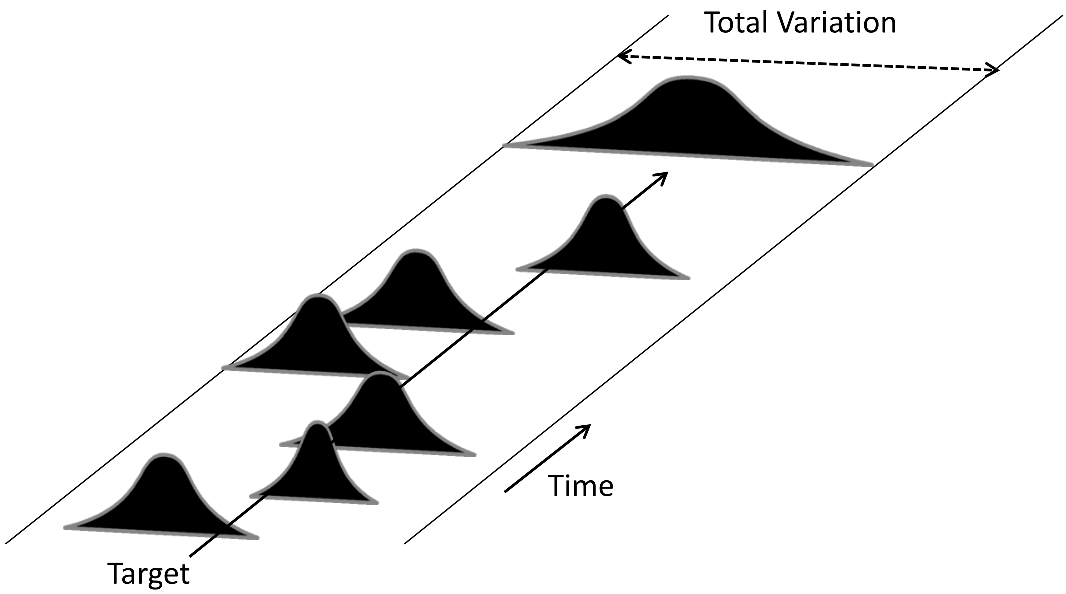

Figure 2.4. An Unstable Process [Westin, 2010]

within the upper and lower limits of process control. If the process is within limits, it is capable of meeting specification. If it is going out of limits, then process will not meet the specifications. It is shown in figure 2.5.

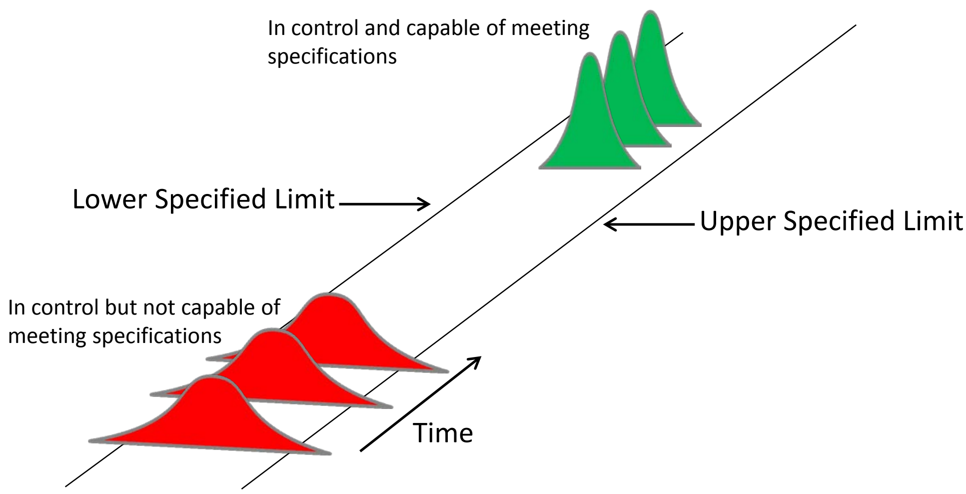

Figure 2.5. Process Capability [Westin, 2010]

2.3 Features and their Application in Manufacturing

The tolerances are bound with the shape,size and appearance of a product. Any product design is explained by its features. As the number and types of features in a single part increases, the complexity is intensified. The drawing of work piece shows dimensions of features which are perfect. But the features of manufactured work-piece are never 100 percent conformed to ideal drawing dimensions. The disparity between the two is error. So ideal and non-ideal features have to be understood. Geometric features are characteristics of any part by which its shape and functionality might be defined. It can be a point, a line, a certain volume, surface or axis etc. [Bernard, 2004].

2.3.1 Link of Geometric Features with Nominal Model and Skin Model of a Part

The 'Skin Model' is model of a part which has defects in it. It shows the interface of part with its environment. Whereas a 'Nominal Model' is perfect in shape without any defects. Now the Tolerance is dimensional condition on geometric features of a part. It establishes quantitative bounds on dimensions of features. Type of manufacturing operations give respective shape and features to a part. Therefore, geometric features of a skin model can be defined by operations. Refer figure 2.6 [Ballu A., 2001].

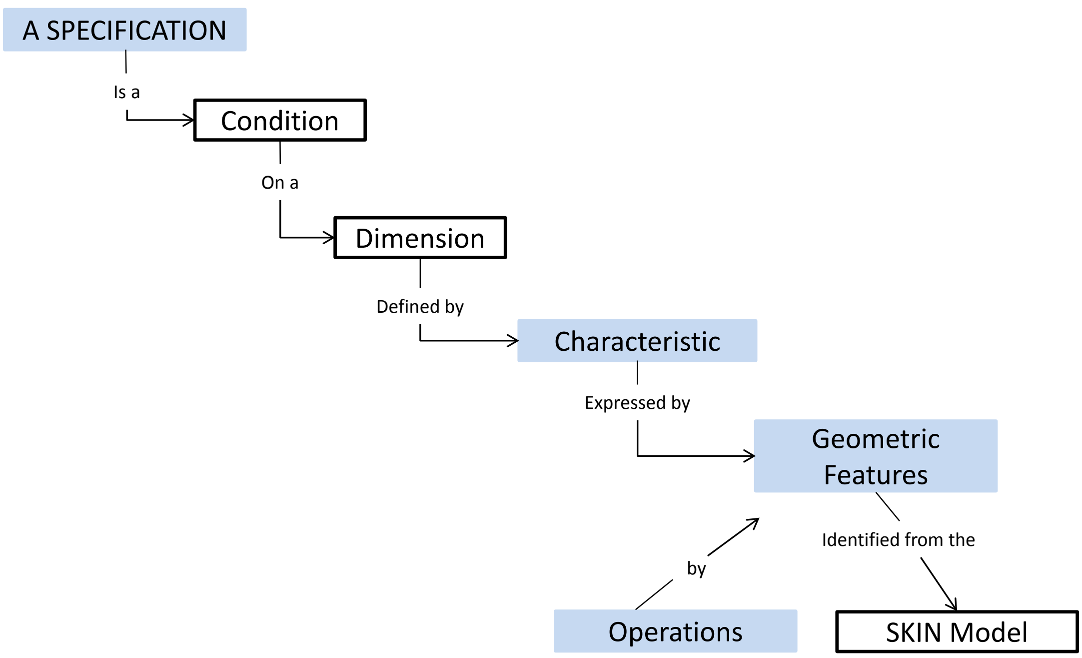

Figure 2.6. Link of Geometric features & Skin Model [François Villeneuve, 2010]

2.3.2 Ideal Feature and non-ideal Features

Ideal features are the features which are flawless and perfect. It can be the plane surface of any nominal model or any tolerance zone. Ideal features are the perfect features without any variations from the set nominal values. Ideal features are expressed in a 'Nominal Model' of a part. Non-ideal features are actual real features of a part after it is manufactured. Of-course the non-ideal features are real features or skin model which vary in dimensions from the ideal or nominal features. The non-ideal features are expressed in a 'Skin Model' [François Villeneuve, 2010]. It is for us to find out the variations between the ideal and nominal features and methods to keep these variations minimum.

2.3.3 Error between Ideal and Non-ideal Features

A part which is defined by one or more features can have errors in its skin model, due to deviation from dimensions assigned in its nominal model. These errors can be calculated and then minimized by measuring maximum distance, minimum distance, square root of squares of distance or by other functions. An example in figure 2.8 shows maximum distance between a line segment and a straight line. These distances can be positive or negative and are called 'Signed Distances'. Refer Figure2.7.

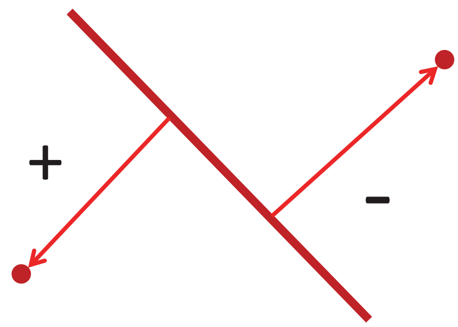

Figure 2.7. Signed Distance of a point from a straight line in a plane [François Villeneuve, 2010]

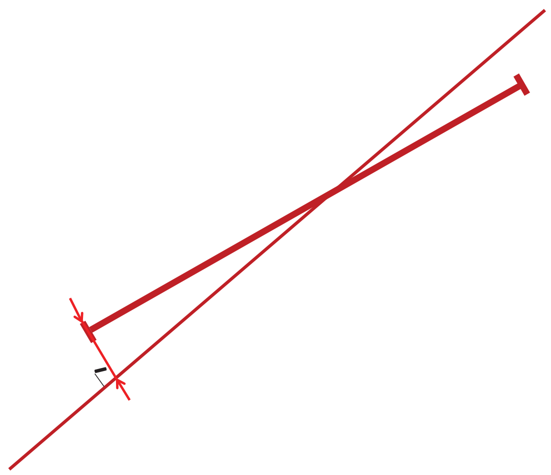

Figure 2.8. Maximum difference between an ideal and real feature of line [François Villeneuve, 2010]

2.4 Boundary Conditions

The applicable geometric tolerance of any feature can be controlled by its size, where boundary conditions i.e Least Material Condition (LMC), Maximum Material Condition (MMC) and Regardless of Feature Size (RFS) are applied [Y14.5-2009, 2009]. Limits of tolerance depending upon the type of features is varied by applying boundary conditions.

2.4.1 Least Material Condition(LMC)

Least Material Condition also known as LMC, is a boundary condition in which least material exists in a certain tolerance range provided by the designer.
For simplicity:
If it is a hole or internal feature: LMC = Largest hole size (least material in part)
If it is a pin or external feature: LMC = Smallest size of the pin

2.4.2 Maximum Material Condition (MMC)

Maximum Material Condition also known as MMC, is a boundary condition in which maximum material exists in the tolerance value range assigned by the designer.
When you have a feature that GD&T is called on:
If it is a hole or internal feature: MMC = smallest hole size

If it is a pin or external feature: MMC = largest size of the pin

2.4.3 Regardless of Feature Size (RFS)

Regardless of Feature Size is abbreviated by RFS, which means the tolerance value of a geometric characteristic does not change because of feature size. It will remain the same. RFS condition has minimum total tolerance range.

2.5 Torsors

Bourdet et al were the pioneers of introducing the concept of torsor in tolerance analysis. With a simple example of two mating parts forming a prismatic link, they also showed applicability of gap torsor, deviation torsor and variation torsor in detail [Bourdet P, 1995]. Torsors have an advantage over previous models. They can easily handle tri-dimensional tolerance variations as compared to computational limitations of one dimensional tolerance analysis models [Bourdet P, 1996]. Torsors is a mathematical expression that shows the relative variation between real geometry and associated geometry of an element. Its advantage is that variations in angles and displacements can easily be viewed. The torsor is defined by equation 2.3 and its visual interpretation is shown in figure 2.9

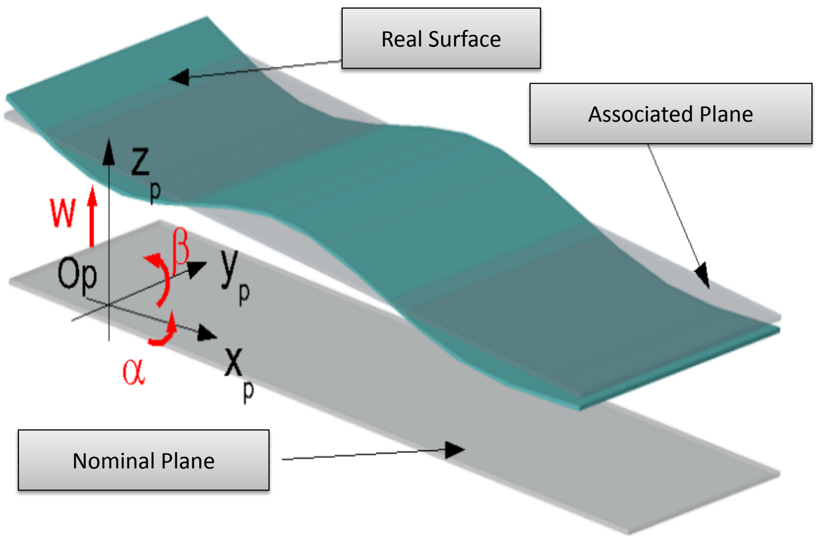

Figure 2.9. Torsor [François Villeneuve, 2010]

$$T = \begin{bmatrix} u & \alpha \\ v & \beta \\ w & \gamma \end{bmatrix} \tag{2.3}$$

Where,

T= torsor

u,v,w= translations in x,y,z axis respectively

α,β,γ= rotations in x,y,z axis respectively

In literature, Small Displacement Torsor (SDT) model has been applied on machining tools, fixture assembly set-up, workpiece machining surfaces and machining set-ups for tolerance analysis purpose. Calibration of machine tools is done using SDT. Small displacement torsors (SDT) which is easier to manipulate for purpose of tolerance analysis because its provides linear system of equations [Frayssinet H, 2004]. Six parameters of small displacement torsor can be reduced for cone, cylinder or any other symmetrically shaped part with the help of axi-symmetric deviation and clearance domains, thus making tolerance analysis and tolerance synthesis effortless [Max Giordano, 2007]. Tolerance analysis of workpiece fixture assembly using SDT is performed by J N Asante. His fixture assembly study includes work piece, locators, clamping and machining errors. Small displacement torsor parameters are used to design the analytical model of workpiece. The comparison shows a very minor difference between the results of analytical and experimental data thus verifying its effectiveness [Asante, 2009]. Machining process reliability is evaluated by modeling machining surfaces variations in work of Laifa et al. They formulated a model by combining small displacement torsor and functional constraints. The inequality constraints are generated in the end which shows tolerance limits set by designer. This analysis model is yet to be validated by experiment [Laifa M, 2014]. Refer to figure 2.10 to see the proper framework for modeling of 3D functional tolerances using SDT done by Laifa et al. They modeled geometrical tolerances of orientation and position using SDT.

There is a study which shows that SDT improves the reliability of tolerance analysis results. Wang et al conducted a tolerance analysis simulation on thin walled C-section composite beam before and after pre-loading. The outcome of study showed better re-

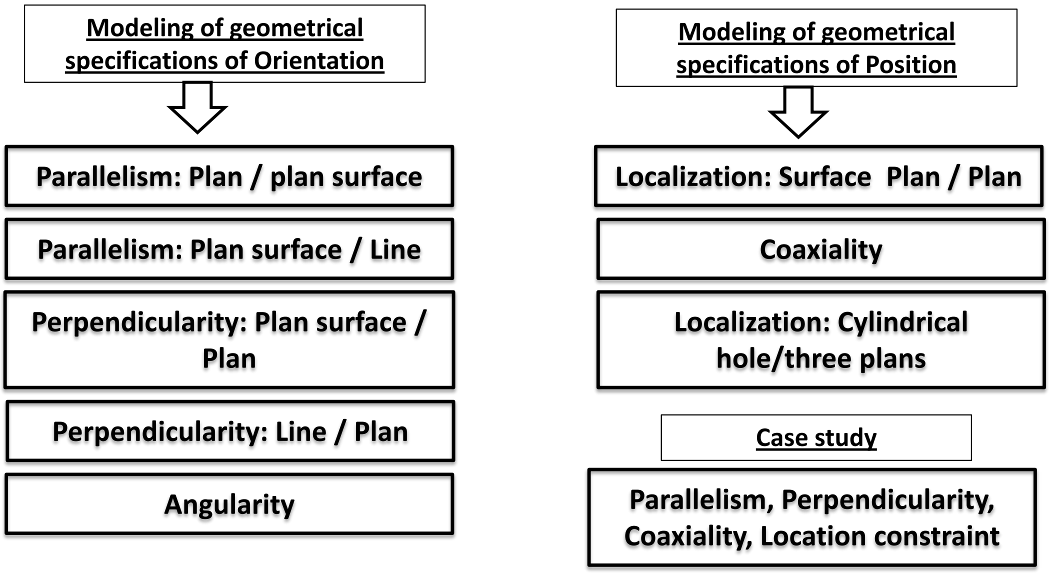

Figure 2.10. Modeling of 3D functional Tolerances by SDT [Laifa M, 2014]

sults with less variation in tolerance when SDT is used. [Hua Wang, 2016] A model that combines Jacobian with Torsors is known as Jacobian Torsor theory. In their work, Zuo et al implemented Jacobian Torsor theory for a 3-2-1 fixture layout of experimental machining set-up. The limits of tolerance are defined by the functional requirement of work piece. Error sources and its propagation is studied in terms of machining feature deviations. Then comparison of results for nominal,actual and predicted features is done using MATLAB [Zuo X, 2013].

2.5.1 Types of Torsors

There are different types of torsors according to the features on which they are defined. First is error torsor, which is simply the difference between associated and nominal surface. A Defect torsor represents the positioning error between two surfaces of same work piece. In figure 2.11 error torsor and defect torsor are interpreted on a work-piece P. The real surfaces of workpiece are Pi and Pj. So the Error torsors are $T_{Pi,P}$ and $T_{Pj,P}$, a difference between nominal and real features of same work piece. Defect torsor is the difference between two defect torsors that is $T_{Pi,Pj}$ Deviation torsor is similar to defect torsor and represents the deviation of difference in position between two surfaces of the same work piece. Gap Torsor exists between the gaps of two joints. It defines the positioning error

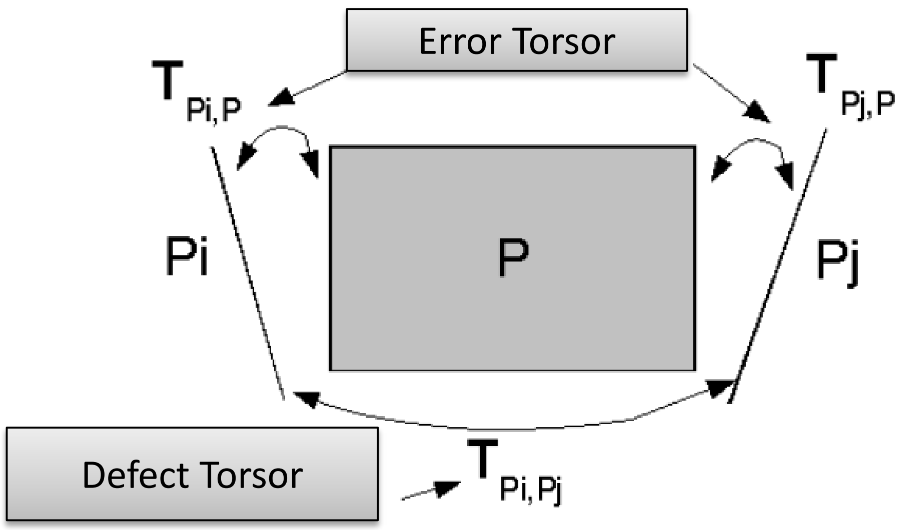

Figure 2.11. Torsors defined in a work piece [François Villeneuve, 2010]

between two surfaces of two different solids. Figure 2.12 has two parts in it, $PartA$ and $PartB$. The actual surface of part A is A_{si} and of part B is B_{sj}. Gap torsor between part A and B is $T_{A_{si}/B_{sj}}$.

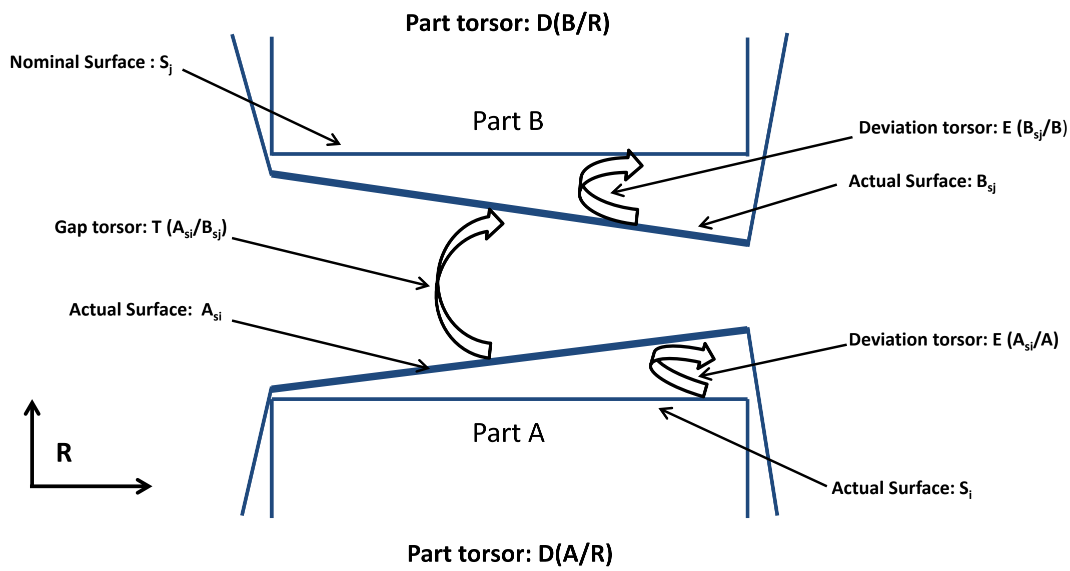

Figure 2.12. Interaction of different torsors of a joint [François Villeneuve, 2010]

2.5.2 Relative and Absolute Deviation Torsor

Deviation torsors of surfaces is categorized into two types that are Relative deviation torsor and Absolute deviation torsor. Relative torsor is a torsor defined by between two different surfaces of a part relative to each other. Here the real surface is compared with the theoretical relative position. Given the equation:

$$E_{AB} = E_B - E_A \tag{2.4}$$

where, E_B, E_A are two torsors

Absolute torsor compares the deviations of the surfaces with respect to Nominal location. For example E_A and E_B. The torsor E_{AB} is the relative torsor. Error torsor is absolute deviation torsor,whereas defect torsor or gap torsor are relative deviation torsors according to definition.

2.5.3 Classes of invariance

Table 2.2. Classes of invariance [François Villeneuve, 2010]

Invariance Classes	Degree of Invariance	Examples
Complex	None	Ellipsoid
Prismatic	A translation along one direction	Two parallel Cylinders
Revolute	A rotation around a straight line	Circle cone Torus
Helical	A rotation around a straight line combined with translation parallel to this line	Helical surface
Cylindrical	A rotation around a straight line and a translation parallel to this line	Cylinder
Planar	A rotation around a straight line and two translations perpendicular to this line	Plane
Spherical	Three rotations around a point	Sphere

Torsor has six components, $u, v, w, \alpha, \beta, \gamma$. It is always easier to work with less variables. Also the computation cost decreases. So, some of these components in a torsor may be selected as zero. There are seven classes of invariance with different degrees of invariance. Invariance means that there will be no effect in overall geometry by displacement in a certain direction. All ideal features can be classified (refer to table 2.2) according to their degree of invariance. [Clement A., 1991] For example, three rotations around a point are three degrees of invariance for a sphere. This means,a torsor describing sphere will have only 3 effective components i.e u, v, w. The rotations α, β, γ are redundant.

2.6 Scope of the Thesis

A table 2.3 is given below which comprehensively shows the work done by different researchers based on investigation technique and class of tolerance. Most of the researchers have used tolerance analysis methods which can be deterministic or statistical. It is easier for the reader to quickly refer to this table for an overview of state of the art. This thesis covers tolerance analysis using statistical and deterministic approach. Both geometric and dimensional tolerances are taken into account while seeing its combined effect on product in terms of manufacturing flexibility and precision. The methodology uses GD&T, torsors and boundary conditions at the same time.

Key: (for table 2.3)

TS= Tolerance Synthesis; TA= Tolerance Analysis

ST= Stochastic; DT=Deterministic

DIM= Dimensional Tolerance; GEO= Geometric Tolerance

Table 2.3. Comparison of existing work done by different researchers

Researchers	**Type of investigation**	**Technique used**	**Class of tolerance**	**Application**
Luis García Guzmán et al. (2003)	TA	ST	DIM	Tolerance adjustment to predict variations for design of stamping parts and welded assemblies production process [Luis García Guzmán, 2003].
Frayssinet H. Et al. (2004)	TA	DET	DIM	Calibration of machine tools is explored using SDT [Frayssinet H, 2004].

J N Asante (2009)	TA	DET	DIM	Tolerance analysis on work-piece fixture assembly using SDT is done [Asante, 2009].
M. Kamali Nejad et al. (2009)	TA	DET	GEO	Geometrical deviations are calculated by developing an algorithm for a model of manufactured part in a multistage machining process and optimization of that algorithm is done using GA and SQP [M Kamali Nejad, 2009].
J. Stuppy et al. (2010)	TA	ST	DIM	Monte-carlo method is used for analysis of a moving mechanism of crankshaft [J Stuppy, 2010].
J. X. Yang et al. (2011)	TA	ST	DIM	5R planar mechanism is solved for tolerance analysis using link dimensions and joint clearance [J X Yang, 2011].
Mathieu Mansuy et al. (2011)	TA; TS	DET	GEO	A new algorithm for worst case analysis of stack-up assemblies is applied [Mathieu Mansuy, 2011].
Wilma Polini (2011)	TA	ST	GEO	Different tolerance analysis techniques for a simple case study are compared [Polini, 2011a].
M. Barkallah et al. (2012)	TA	ST	GEO	Experimental study is done using numerical method to analyze milling process plan errors [Barkallah M, 2012].
Xiao et al. (2013)	TA	DET	DIM	Jacobian-torsor theory is utilized for depiction of error propagation in machining process [Zuo X, 2013].
P. Beaucaire et al. (2013)	TA	ST	DIM	Co-axial connector case study is taken as over constrained mechanism with gaps and tolerance analysis is done using Monte-Carlo and probability methods [P Beaucairea, 2013].
Yann Ledoux et al. (2013)	TA	ST	DIM	A probabilistic approach of tolerance analysis which combines dimensional and architectural specifications is applied on a turbine model [Yann Ledoux, 2013].

Elena Luminiţa Olteanu et al.(2013)	TA	DET	DIM	Determined position and kinematics (Force & Moment) of a cutting tool in drilling process [Elena Luminiţa Olteanu, 2013].
A K Sahani et al. (2014)	TA	DET	DIM	Tolerance Stack-up analysis and cost allocation is done on a dovetail assembly considering geometric constraint of angularity [A K Sahani, 2014].
M. Laifa (2014)	TA	DET	GEO	3D tolerancing in machining process is done by combining functional requirements with SDT [Laifa M, 2014].
M. Chahbouni et al. (2014)	TA	ST	GEO	Effect of form deviations on statistical tolerance analysis of parts is studied [M Chahbouni, 2014].
M. A. Rehman et al. (2014)	TA	ST	DIM	Machine tool vibrations in a turning process using SDT is investigated [Rahman M A, 2014].
Hai Li et al.(2014)	TA	DET	GEO	Deviation propagation thoery is used to do tolerance analysis at a mechanical assembly application of turbo generator stator core lamination at design phase.
Huiwen Yan et al. (2015)	TA	ST	DIM	Monte-carlo technique is applied on a top column assembly model for tolerance analysis [Huiwen Yan, 2015].
Hua Wang et al. (2016)	TA	ST	GEO	Small Displacement torsor model is applied on tolerance analysis simulation of C-section composite beam. [Hua Wang, 2016].
Proposed in this thesis	TA	DET + ST	GEO + DIM	The combined effect of geometric and dimensional tolerances is seen on a case study parts using GD&T, torsors and boundary conditions. Their effect is measured in terms of flexibility and precision of manufacturing systems.

2.7 Overview

In this chapter, literature review was presented to form the background base knowledge for comprehension of this thesis. The meaning and definitions of tolerance, features,boundary conditions was presented. Also, the work done by researchers in tolerance analysis, torsors along with type of technique used in their study is briefed. In latter subsection, scope of this study is given. The next chapters follow up methodology implemented on a case study, closing with results, conclusion and future perspective.

Chapter 3

Case Study (Part I): Application of Boundary Conditions on Dimensional Tolerance

This chapter inscribes application of boundary conditions on dimensional tolerance for a case study consisting of two parts. The case study is taken to visualize the tolerance range for gap produced by assembling.

3.1 Application of Boundary Conditions on Dimensional Tolerance- A Case Study

Tolerance is defined as the allowable variation in the dimensions of any feature, surface, angles or axis. It specifies the safety limits of a product within which a machine might deviate from its nominal or ideal dimension. Thus less tolerance would mean more precision at the expense of higher cost. Tolerance gives the range for deviation between non-ideal (real) and ideal (theoretical) surfaces of features. Only dimensional tolerance is considered for this case study. At design stage, parts are assigned different tolerance values on the engineering drawing. Overall effect of tolerancing is vital to calculate when one or more parts are assembled to form a mechanism. The functional requirements of any assembly

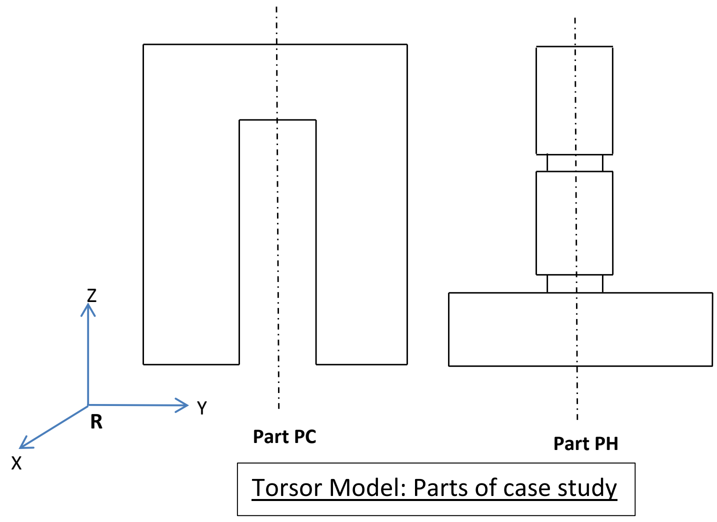

Figure 3.1. Case Study Parts PC and PH

is assessed by tolerance stack-up analysis. It shows the gaps and the interaction of parts in the assembly, thus determining the quality of manufactured parts. Stack up analysis can be done graphically and variations can be calculated using worst case analysis.

A two part assembly (see figure 3.1) consisting of part cylinder (PC) and part hole (PH) is taken for experimentation. The detailed dimensions of the part PC and Part PH are shown in figure 3.3 and 3.2 respectively. Part PH has external features and part PC has internal features.

In this work, we have used an assembly of two simple parts for the purpose of evaluating tolerance analysis at worst case, when different Material Boundary Conditions are applied. The 2-dimensional viewed parts cylinder (PC) and part hole (PH) are assembled in the hole together as shown in figure 3.4 Part PC has a square base with a cylindrical feature of varying diameter. In this part, four references are selected. Reference A is picked on the bottom surface of square base. Reference B is the axis of cylinder. References C and D are upper surfaces of cylinder and square base respectively. Other part PH has a hole in center. Both dimensional and geometric tolerances are applied on these parts. The goal is to analyze the consequences of Boundary conditions on gaps, when these two parts are assembled. These two parts are simplified versions of a complex manufactured assembly for easier evaluation of tolerance analysis. The designer specifies tolerance range for the parts. One part with different boundary conditions (Least Material condition: LMC,Maximum Material Condition: MMC, Regardless of Feature Size:

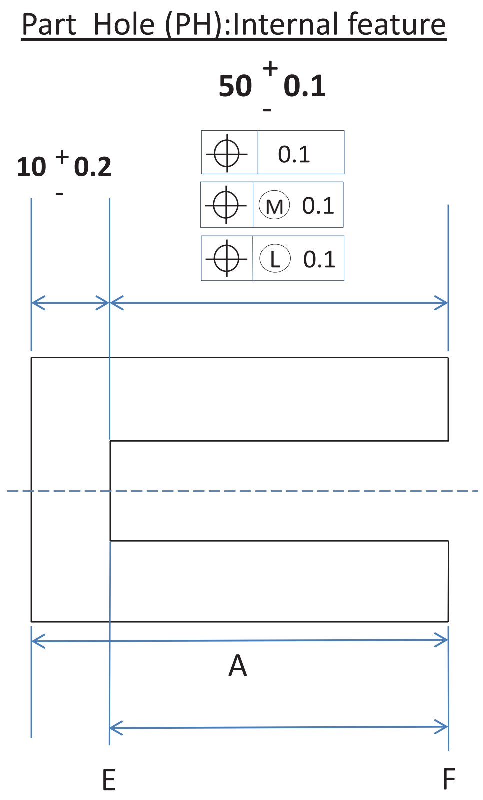

Figure 3.2. Stack up analysis of Part PH

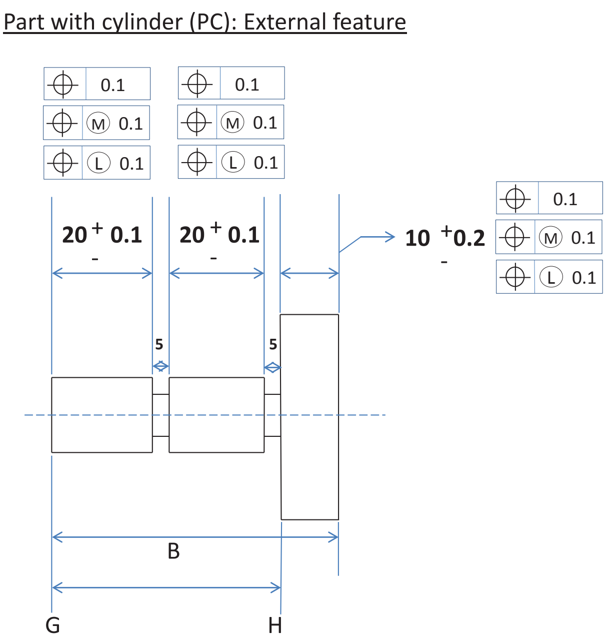

Figure 3.3. Stack up analysis of Part PC

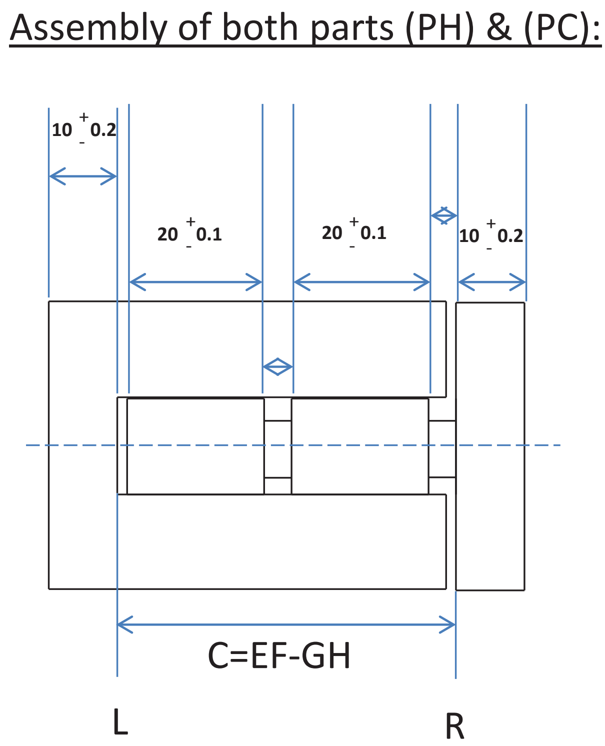

Figure 3.4. Stack up analysis of Parts assembly

RFS) provides fluctuating tolerance limits.

To evaluate the effects of LMC, MMC and RFS boundary conditions on the overall assembly of a product, manual computation of maximum and minimum values are obtained by tolerance stack-up analysis. The detailed stack-up GD&T drawing of both parts and their assembly is provided separately in figures 3.2, 3.3 and 3.4. Control Feature Frame (i) represents first case of RFS while (ii) represents case the second case of MMC and (iii) represents the tolerance LMC,which is the third case. Here on parts PH and PC location tolerance of 0.1 is applied on dimensional tolerances. Our aim is to analyze the maximum and minimum value of A, B and gap 'C' at variable Boundary Conditions. Fist we calculate values of A, B, C1 and C2 at RFS condition and later we will assume boundary conditions to be MMC & LMC. Gap C1 exists when $EF > GH$ while in case $GH > EF$, gap C2 is visible (figure 3.4).

3.2 Tolerance Stack up Results

Now that the case study parts and their GD&T conditions have been explained, tolerance stack up analysis is enacted. Stack-up is applied on all three dimensions A, B and C gap. The result is in form of maximum and minimum values of these dimensions.

3.2.1 Application of RFS condition

We begin with application of RFS condition on parts PH and PC followed by stack-up analysis of its assembly. Stack-up tolerance analysis of part PH gives maximum and minimum value of 'A' dimension (fig. 3.2). Part PC provides the values of 'B' dimension (fig. 3.3) and we obtain total deviation of tolerance limits for assembly as 'C' in terms of gaps (fig. 3.4). When RFS is applied, geometrical tolerance is added as such in the dimensional tolerance. Hence we get range of 'B' as following:

$$B - [(10 \pm 0.2) + (\pm 0.1)] – 5 - [(20 \pm 0.1) + (\pm 0.1)] - 5 - [(20 \pm 0.1) + (\pm 0.1)] = 0$$

$$B = 60 \pm 0.7$$

$$B_{MAX} = 60.7 \quad (3.1)$$

$$B_{MIN} = 59.3 \quad (3.2)$$

The stack-up analysis of part PH is performed and limits are obtained by applying RFS on dimensional tolerance of 50. The maximum and minimum values for 'A' are:

$$A - [(50 \pm 0.1) + (\pm 0.1)] - (10 \pm 0.2)] = 0$$

$$A = 60 \pm 0.4$$

$$A_{MAX} = 60.4 \quad (3.3)$$

$$A_{MIN} = 59.6 \quad (3.4)$$

When parts are assembled, gaps are likely to appear. So it is important to recognize the location of gap and the amount of error produced by that gap. The value of 'C' is the gap in assembly of part PC and PH. The magnitude & location of the assembly gap (C1 & C2) can be found out by the values of EF & GH. Maximum values of C1 & C2 at RFS condition for the assembly (fig. 3.4) are observed:

$$C_{1MAX} = EF_{MAX} - GH_{MIN} = [(50+0.1)+(0.1)]-[(20-0.1)+(-0.1)]+5+[(20-0.1)+(-0.1)+5]$$

$$EF_{MAX} - GH_{MIN} = 50.2\text{–}49.6$$

$$C_{1MAX} = 0.6 \tag{3.5}$$

$$C_{2MAX} = EF_{MIN} - GH_{MAX} = [(50-0.1)+(-0.1)]-[(20+0.1)+(+0.1)]+5+[(20+0.1)+(+0.1)+5]$$

$$EF_{MIN} - GH_{MAX} = 49.8\text{–}50.4$$

$$C_{2MAX} = -0.2 \tag{3.6}$$

3.2.2 Application of MMC condition

The tolerance analysis for part PC is implemented at Maximum Material Condition. In MMC, geometrical tolerances are chosen carefully by calculating the virtual and resultant conditions for given feature. The limits for 'B' in case of MMC are given as:

$$B - [(10 + 0.2) + (0.1)]\text{–}5 - [(20 + 0.1) + (0.1)]\text{–}5 - [(20 + 0.1) + (0.1)] = 0$$

$$B_{MAX} = 60 + 0.7$$

$$B_{MAX} = 60.7 \tag{3.7}$$

$$B - [(9.8) - (0.5)]\text{–}5 - [(19.9) - (0.3)]\text{–}5 - [(19.9) - (0.3)] = 0$$

$$B_{MIN} = 59.6 - 1.1$$

$$B_{MIN} = 58.5 \tag{3.8}$$

Similarly the maximum and minimum values of 'A' for the part PH can be calculated by:

$$A - [(50 + 0.1) + (0.1)] - (10 + 0.2)] = 0$$

$$A_{MAX} = 60 + 0.4$$

$$A_{MAX} = 60.4 \tag{3.9}$$

$$A - [(50 - 0.1) - (0.3)] - (10 - 0.2)] = 0$$

$$A_{MIN} = 59.7 - 0.3$$

$$A_{MIN} = 59.4 \tag{3.10}$$

We determine the maximum value of gap 'C1 and C2' for assembly of part PH and PC at Maximum Material Condition:

$$C_{1MAX} = EF_{MAX} - GH_{MIN} = [(50+0.1)+(0.1)] - [(19.9)-(0.3)]+5-[(19.9)-(0.3)+5]$$

$$EF_{MAX} - GH_{MIN} = 50.2–49.2$$

$$C_{2MAX} = EF_{MIN} - GH_{MAX} = [(50-0.1)-(0.3)]-[(20+0.1)+(0.1)]+5+[(20+0.1)+(0.1)+5]$$

$$EF_{MIN} - GH_{MAX} = 49.6–50.4$$

$$C_{1MAX} = 1.0 \tag{3.11}$$

$$C_{2MAX} = -0.8 \tag{3.12}$$

3.2.3 Application of LMC condition

Now limit values of 'B' for the part PC are calculated for Least Material Condition being applied on dimensional tolerance of 10, 20 and 20 units respectively (see fig. 3.3). The resultant and virtual condition for LMC provides geometrical tolerance values used in tolerance stack-up analysis:

$$B - [(10.2) + (0.5)]–5 - [(20.1) + (0.3)] - 5 - [(20.1) + (0.3)] = 0$$

$$BMAX = 60.4 + 1.1$$

$$BMAX = 61.5 \tag{3.13}$$

$$B - [(9.8) - (0.1)]–5 - [(19.9) - (0.1)]–5 - [(19.9) - (0.1)] = 0$$

$$BMIN = 59.6 - 0.3$$

$$BMIN = 59.3 \tag{3.14}$$

By applying LMC on the part PH which is an internal feature, we get maximum and minimum values of ‘A’ by tolerance stack-up:

$$A - [(50 + 0.1) + (0.3)] - (10 + 0.2) = 0$$

$$AMAX = 60.6 \tag{3.15}$$

$$A - [(50 - 0.1) - (0.1)] - (10 - 0.2) = 0$$

$$AMIN = 59.6 \tag{3.16}$$

By finding out maximum and minimum values for assembly with the help of tolerance stack-up analysis at LMC, the gap magnitude of ‘C1 & C2’ is given:

$$C_{1MAX} = EF_{MAX} - GH_{MIN} = [(50+0.1)+(0.3)] - [(19.9)-(0.1)]+5+[(19.9)-(0.1)+5]$$

$$EF_{MAX} - GH_{MIN} = 50.4–49.6$$

$$C_{1MAX} = 0.8 \tag{3.17}$$

$$C_{2MAX} = EF_{MIN} - GH_{MAX} = [(50-0.1)-(0.1)] - [(20.1)+(0.3)]+5+[(20.1)+(0.3)+5]$$

$$EF_{MIN} - GH_{MAX} = 49.8–50.8$$

$$C_{2MAX} = -1.0 \tag{3.18}$$

Table 3.1. Results of Maximum and Minimum values of Tolerance Stack-up Analysis

	RFS	MMC	LMC
A_{MAX}	60.4	60.4	60.6
A_{MIN}	59.6	59.4	59.6
B_{MAX}	60.7	60.7	61.5
B_{MIN}	59.3	58.5	59.3
C_{1MAX}	0.6	0.1	0.8
C_{2MAX}	-0.2	-0.8	-1.0

Table 3.2. Range (R), Mean (μ) and Standard Deviation (σ) of features/dimensions measured according to RFS, MMC &LMC

Feature	Boundary Condition								
	RFS			MMC			LMC		
	R	μ	σ	R	μ	σ	R	μ	σ
A	0.8	60	0.133	0.1	59.9	0.167	1.0	60.2	0.167
B	1.4	60	0.233	2.2	59.6	0.367	1.7	60.4	0.367
C	0.8	0.2	0.133	1.8	0.1	0.3	1.8	-0.1	0.3

3.2.4 Allowable Tolerance Variation Range

The results are shown in table 3.1. By analyzing this table, it can be noted that when RFS is used for separate parts, the range between maximum and minimum limits of ‘A’ is 0.8 units whereas for same parts the range increases by 0.2 units at MMC and LMC boundary conditions. This range for ‘B’ between maximum and minimum values of tolerance stack-up analysis increases by 0.8 units when MMC or LMC is applied. Moreover, last two columns of table show the gap in assembly represented by ‘C’. At RFS, gap of 0.6 units in assembly appears on left side represented by C1 (shown in fig.3.4). The value of C2 shows the maximum gap that can appear on right side (fig. 3.4) when parts are assembled. Similarly at MMC and LMC gap value on both sides of assembly increases.

Thus, we can conclude that intelligent use of GD&T will result in intelligent variation range. Assuming that the manufacturing process follows a normal distribution, Mean and Standard Deviation of A, B and C can be calculated as shown in table 2 and the plots are shown in fig. 3.5, fig.3.6 and fig.3.7.

The plot shows accuracy and precision required to manufacture the same feature if worker follows MMC, LMC and RFS. Thinner curve shows more precision required for the manufacturing process while the Mean shows accuracy/bias of the manufacturing process.

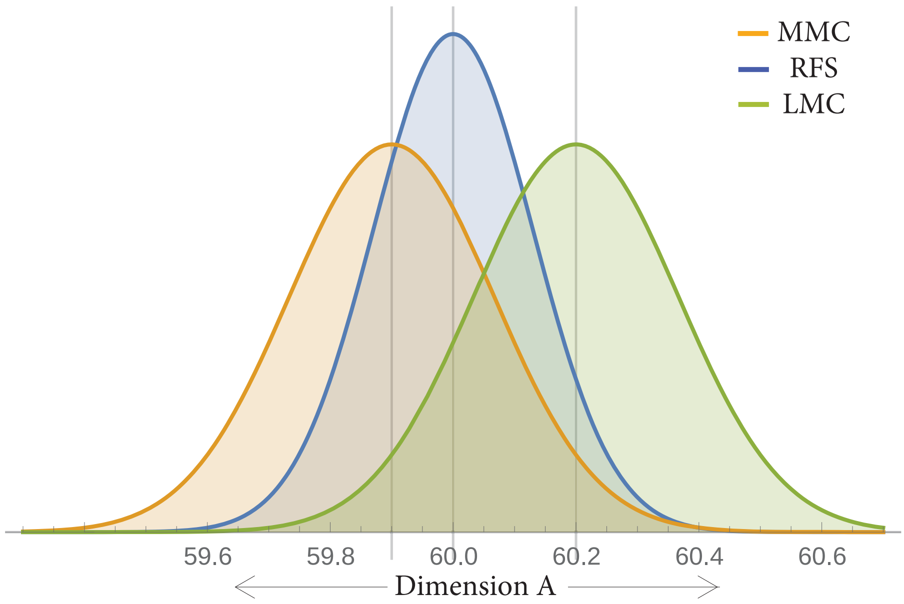

Figure 3.5. Normal Distribution of dimension 'A'

Less bias means the product is close to required dimension. Table 3.1 and fig. 3.5 to 3.7 refers that geometric conditions greatly affect the total tolerance of a part. Therefore, boundary conditions must be applied according to the requirement and capability of a machine. This same stack-up analysis can be expanded for more complex assemblies. The selection of boundary conditions may vary depending upon functional requirements of the product.

3.3 Synthesis

This chapter extensively illustrated the proposed methodology. A case study was explained with all of its parameters. Then, effect of boundary conditions on dimensional tolerance range is seen for the case study. The results of boundary condition normal distribution graphs are found to be useful when choosing a certain manufacturing machine with required precision. The upcoming chapter is second part of same case study workpieces. However, in next section geometric constraints on tolerance are applied instead of dimensional tolerance.

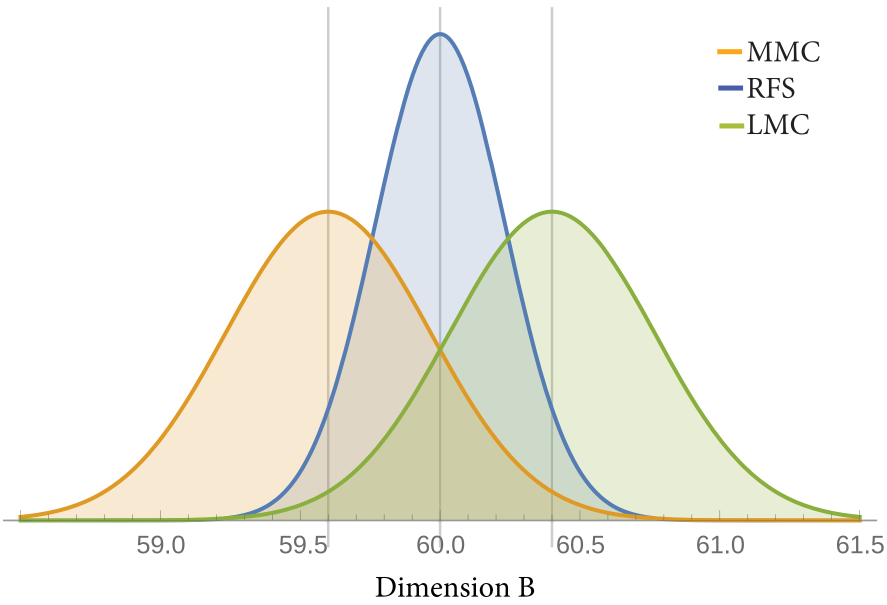

Figure 3.6. Normal Distribution of dimension 'B'

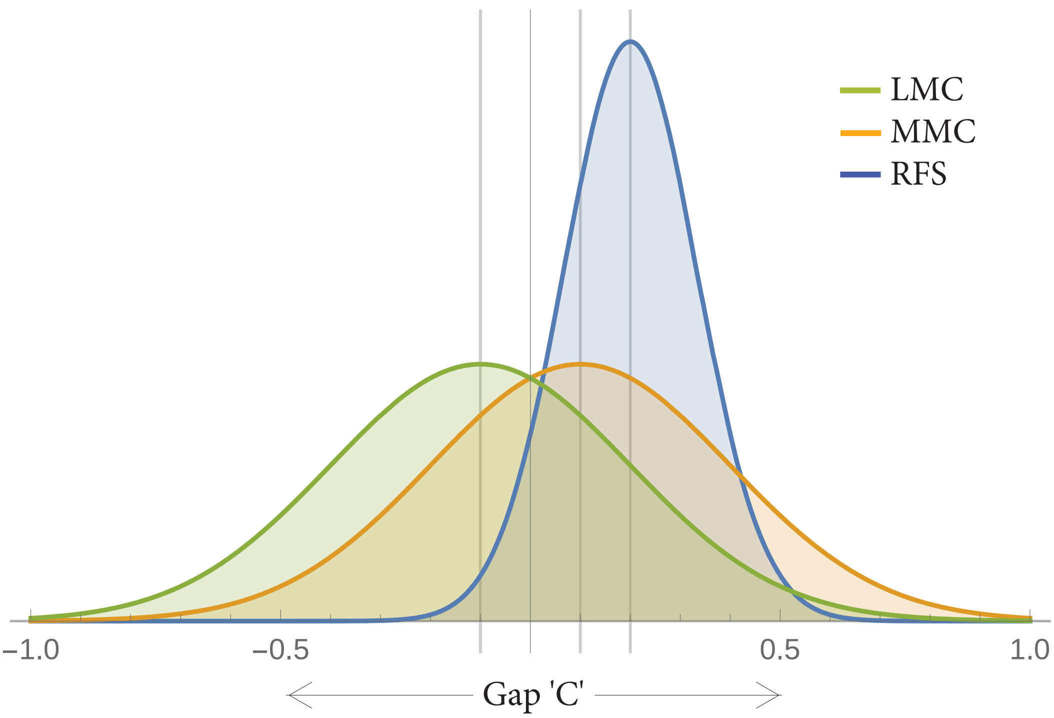

Figure 3.7. Normal Distribution of gap 'C'

Chapter 4

Case Study (Part II): Application of Geometric constraints using Torsors

Torsors and geometric constraints are implemented for the same case study parts to see the effectiveness of proposed model. This second constituent of case study is the application of geometric constraints using small displacement torsors on parts. Assembly of parts with least geometric error is functional requirement. Figure 4.1 has two parts being assembled using a gripper. The parts are same i.e part PC and part PH. First of all, the torsors linkage graph is presented to view effects of geometric errors throughout the assembly. Then equations linking tolerance with torsor parameters is applied with orientation and location constraints.

4.1 Geometric Characteristics and Assembly Model

Geometric tolerance includes orientation tolerances and location tolerance as seen in table2.1. Geometric tolerancing widens tolerance range in terms of inner and outer boundaries of a feature. When simple tolerance of 0.2 is applied on the diameter of 10, total tolerance range is from 9.8-10.2. By incorporating location tolerance on the same feature, boundaries range increases with inner boundary of 9.5 and outer boundary value of 10.5.

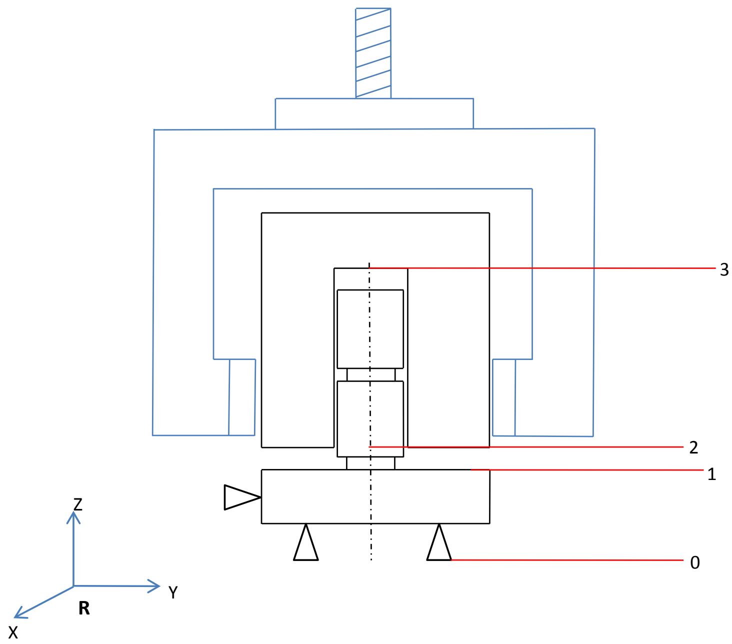

Figure 4.2. Geometric Characteristics of two parts Assembly

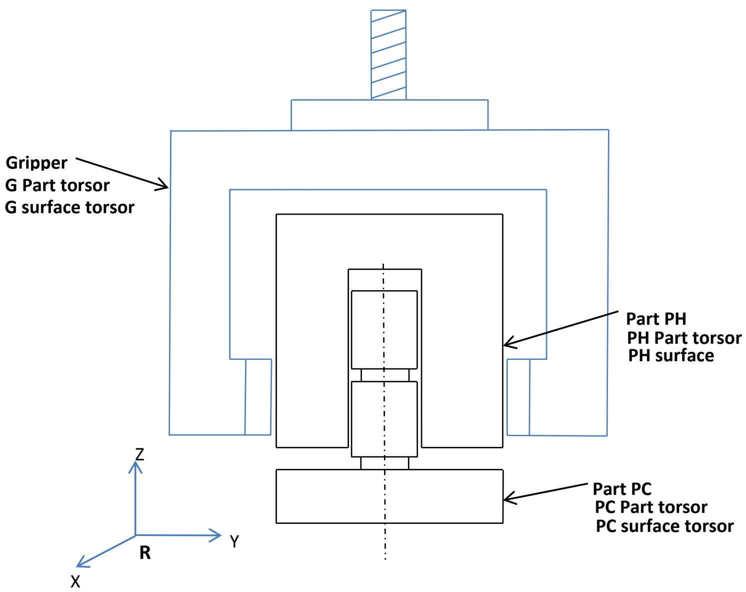

Figure 4.1. Assembly of parts

Figure 4.2 shows the geometric characteristics of assembly. Geometric characteristics are defined on the features of assembly parts. These features are numbered as 0, 1, 2 & 3. Feature '0' and feature '1' is plane. Feature '2' is an axis. Feature '3' is also a plane. From '0-1' parallelism constraint is applied. '1-2' is a perpendicularity constraint and from '2-3' only displacement of a torsor is considered for simplification.

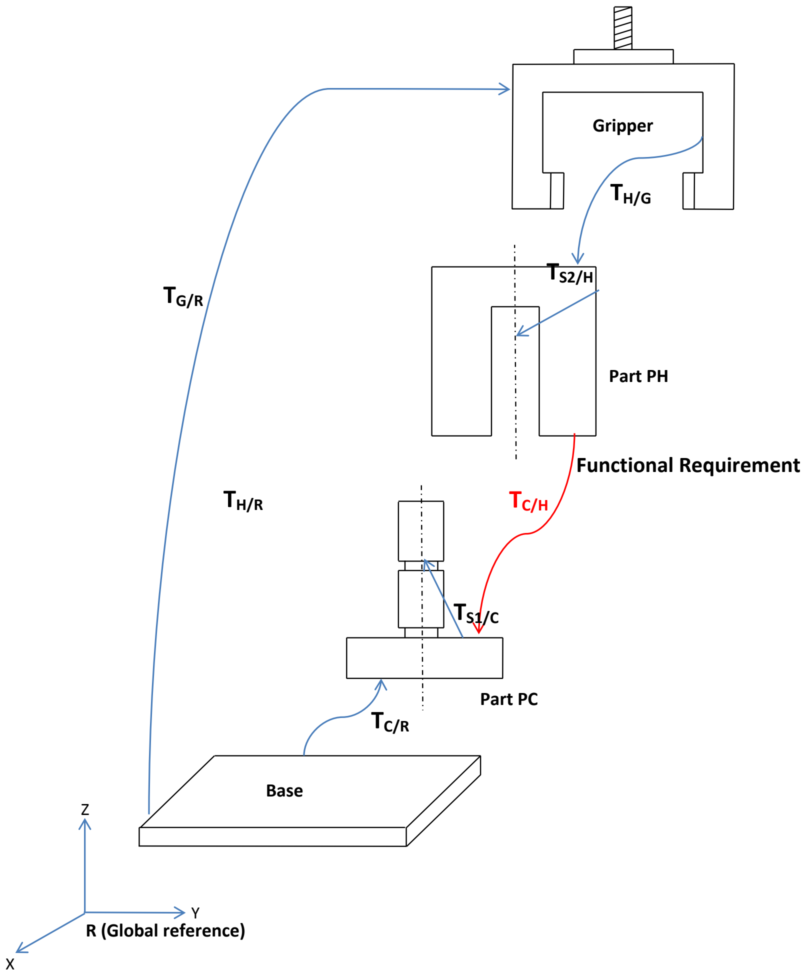

Figure 4.3. Torsor Model Linkage Graph

A torsor model linkage graph is shown in fig.4.3. Part PC is placed on the base platform and the gripper holds part PH to assemble on top of part PC. Reference frame has been named 'R' and gripper is 'G'. Part PC and part PH are represented by 'C' and 'H' respectively. The torsor linkage chain is given by:

$$T_{G/R} + T_{H/G} + T_{C/H} + T_{R/C} = 0 \tag{4.1}$$

Coming up next in this chapter are equations showing parallelism constraint (0-1) and perpendicularity constraint (1-2) in figure 4.2. Initially torsors of respective surfaces are calculated then the difference between two torsors is found. The difference gives the defect torsor. In the final equation,variables x, y and z represent the dimensions of parts, where x=y=50mm and z=60mm.

4.1.1 Parallelism Constraint

Parallelism tolerance is the range or limit subjected on a feature, on basis of parallelity. It comes under one of geometric constraints. In figure 4.4, the visualization of parallelism is given. There are three features which are datum, theoretical and real. Real features vary from the theoretical ones. However, this variation is kept within limits by applying parallelism constraint.

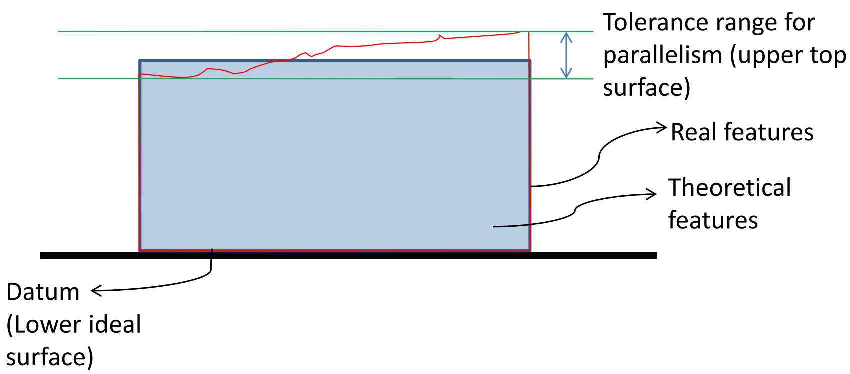

Figure 4.4. Parallelism constraint

Let us consider the following surfaces to define paralellism constraint: E_1 is datum surface, E_2 is Real surface and R is Reference (work-piece). Displacement of a point 'A' of E_1 with respect to E_2 is given by 'D_A' in following equation:

$$D_A = D_O + AO \wedge \Omega \tag{4.2}$$

Where,
'AO' is Vector representing distance of point A from origin 'O' in 3D space
'D_O' is Displacement vector representing distance from origin 'O' in 3D space
Ω is rotation matrix
$\wedge$ represents cross product
The datum feature is a plane and toleranced feature is also a plane. To find the torsor between two plane feature in terms of parallelism,we calculate the defect torsor of datum feature with reference and defect torsor of real feature with reference of work-piece (fig . 4.5):

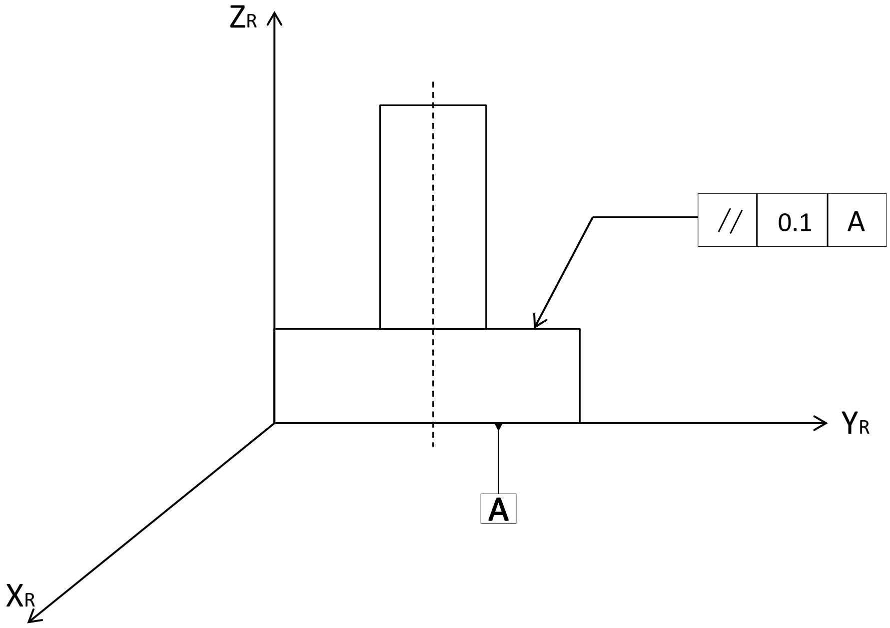

Figure 4.5. Parallelism constraint applied on part

$$T_{E1,R} = \begin{bmatrix} \alpha_{E1,R} & 0 \\ \beta_{E1,R} & 0 \\ 0 & \mathcal{W}_{E1,R} \end{bmatrix}$$

$$T_{E2,R} = \begin{bmatrix} \alpha_{E2,R} & 0 \\ \beta_{E2,R} & 0 \\ 0 & \mathcal{W}_{E2,R} \end{bmatrix}$$

The gap torsor of datum feature with real feature becomes:

$$T_{E1,E2} = T_{E1,R} - T_{E2,R} = \begin{bmatrix} \alpha_{E1,R} & 0 \\ \beta_{E1,R} & 0 \\ 0 & \mathcal{W}_{E1,R} \end{bmatrix} - \begin{bmatrix} \alpha_{E2,R} & 0 \\ \beta_{E2,R} & 0 \\ 0 & \mathcal{W}_{E2,R} \end{bmatrix}$$

$$T_{E1,E2} = \begin{bmatrix} \alpha_{E1,R} - \alpha_{E2,R} & 0 \\ \beta_{E1,R} - \beta_{E2,R} & 0 \\ 0 & \mathcal{W}_{E1,R} - \mathcal{W}_{E2,R} \end{bmatrix} \tag{4.3}$$

For finding the displacement of any point A on datum feature E1 w.r.t real feature E2, putting eq.4.3 in eq.4.2:

$$D_A = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} x \\ y \\ z \end{bmatrix} \wedge \begin{bmatrix} \alpha_{E1,R} - \alpha_{E2,R} \\ \beta_{E1,R} - \beta_{E2,R} \\ 0 \end{bmatrix} \tag{4.4}$$

Only rotation is taken into account because orientation depends only on rotation of angles.

$$D_A = \begin{bmatrix} -z(\beta_{E1,R} - \beta_{E2,R}) \\ -z(\alpha_{E1,R} - \alpha_{E2,R}) \\ x(\beta_{E1,R} - \beta_{E2,R}) - y(\alpha_{E1,R} - \alpha_{E2,R}) \end{bmatrix} \tag{4.5}$$

For small displacements consideration, rotation matrix (K) is used [Butt, 2012]:

$$K = \begin{bmatrix} 1 & -\alpha & \gamma \\ \alpha & 1 & -\beta \\ -\gamma & \beta & 1 \end{bmatrix} \tag{4.6}$$

$$K = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{4.7}$$

To consider the displacement of every single point of datum feature w.r.t real feature, dot product of D_A is taken with the normal of datum plane:

$$\bar{D}.\bar{n}_D = \begin{bmatrix} -z(\beta_{E1,R} - \beta_{E2,R}) \\ -z(\alpha_{E1,R} - \alpha_{E2,R}) \\ x(\beta_{E1,R} - \beta_{E2,R}) - y(\alpha_{E1,R} - \alpha_{E2,R}) \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\bar{D}.\bar{n}_D = 0 + 0 + x(\beta_{E1,R} - \beta_{E2,R}) - y(\alpha_{E1,R} - \alpha_{E2,})$$

$$\bar{D}.\bar{n}_D = x(\beta_{E1,R} - \beta_{E2,R}) - y(\alpha_{E1,R} - \alpha_{E2,R}) \tag{4.8}$$

$\bar{D}.\bar{n_D}$ is in fact the tolerance of parallelism constraint. From equation 4.8, it is deduced that parallelism of any feature is controlled by angle α along y axis and angle β along x

axis.

4.1.2 Perpendicularity Constraint

Orientation of a workpiece also involves perpendicularity constraint besides parallelism. Visualization of perpendicularity constraint applied on the axis of a part is seen in figure 4.6. The red is real part and variation of its axis from ideal part axis is regulated by perpendicularity constraint.

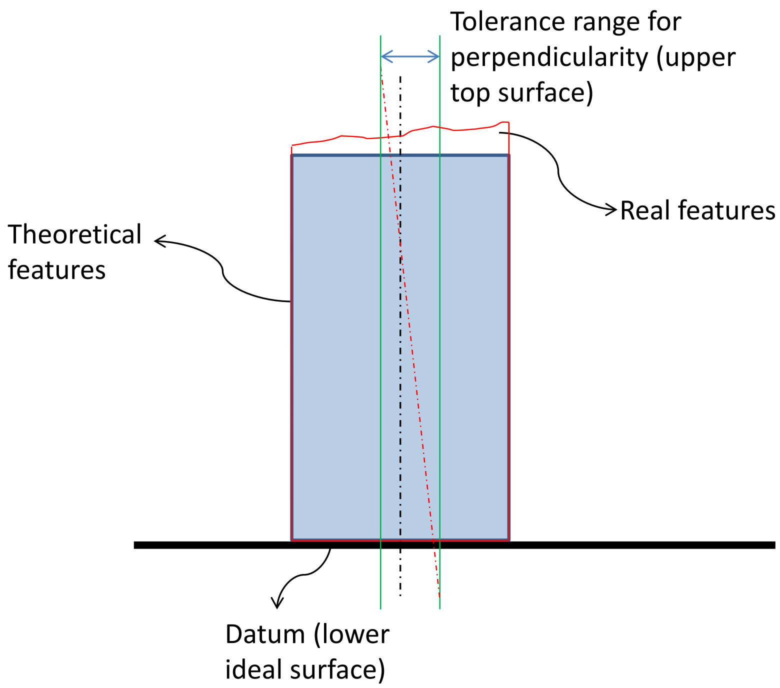

Figure 4.6. Perpendicularity constraint

Previously in parallelism constraint, both datum and real features were planes. But in this perpendicularity constraint Datum feature is a plane and toleranced feature is an axis. So we find the torsor between a plane and axis in terms of perpendicularity. It is to be noted that this perpendicularity constraint forms a tolerance zone in shape of a circle. The axis feature of part must remain within the boundaries of circle diameter (which is tolerance range)to be acceptable. First we calculate the defect torsor of datum feature with reference and defect torsor of real feature with reference of work-piece for perpendicularity constraint:

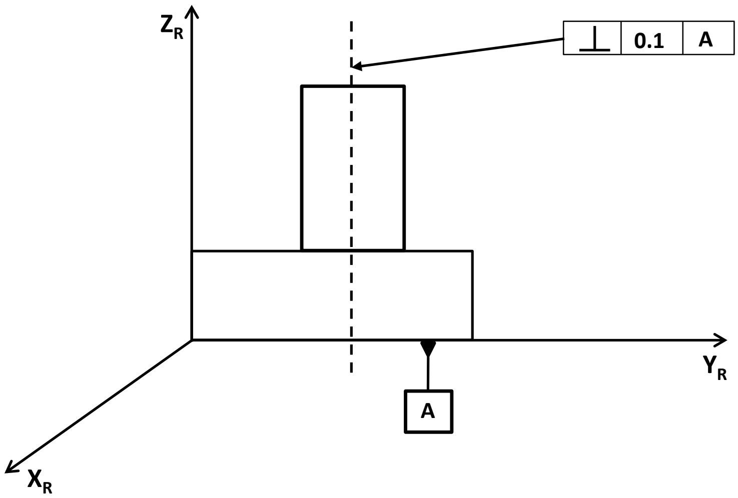

Figure 4.7. Perpendicularity constraint applied on part

$$T_{E1,R} = \begin{bmatrix} \phi_{E1,R} & 0 \\ \psi_{E1,R} & 0 \\ 0 & \mathcal{W}_{E1,R} \end{bmatrix}$$

$$T_{E2,R} = \begin{bmatrix} \phi_{E2,R} & \mathcal{U}_{E2,R} \\ \psi_{E2,R} & \mathcal{V}_{E2,R} \\ 0 & 0 \end{bmatrix}$$

The gap torsor of datum feature with real feature becomes:

$$T_{E1,E2} = T_{E1,R} - T_{E2,R} = \begin{bmatrix} \phi_{E1,R} & 0 \\ \psi_{E1,R} & 0 \\ 0 & \mathcal{W}_{E1,R} \end{bmatrix} - \begin{bmatrix} \phi_{E2,R} & \mathcal{U}_{E2,R} \\ \psi_{E2,R} & \mathcal{V}_{E2,R} \\ 0 & 0 \end{bmatrix}$$

$$T_{E1,E2} = \begin{bmatrix} \phi_{E1,R} - \phi_{E2,R} & 0 \\ \psi_{E1,R} - \psi_{E2,R} & 0 \\ 0 & 0 \end{bmatrix} \tag{4.9}$$

For finding the displacement of any point A on datum feature E1 w.r.t real feature E2, putting eq.4.9 in eq.4.2:

$$D_A = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ z \end{bmatrix} \wedge \begin{bmatrix} \phi_{E1,R} - \phi_{E2,R} & 0 \\ \psi_{E1,R} - \psi_{E2,R} & 0 \\ 0 & 0 \end{bmatrix} \tag{4.10}$$

Again,only rotation is taken into account because perpendicularity effects orientation of a part which depends only on rotation of angles.

$$D_A = \begin{bmatrix} -z(\psi_{E1,R} - \psi_{E2,R}) \\ z(\phi_{E1,R} - \phi_{E2,R}) \\ 0 \end{bmatrix} \tag{4.11}$$

The tolerance zone generated is shown in figure 4.8 where 0.1 mm geometric tolerance

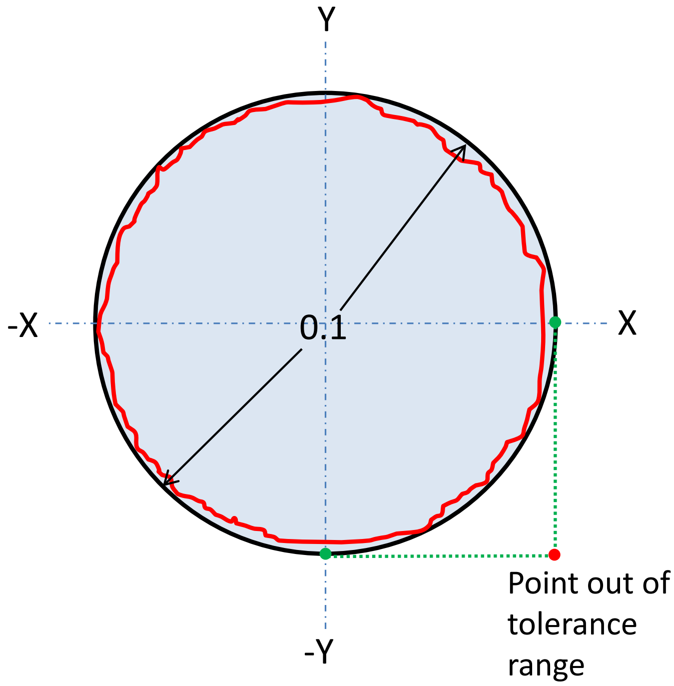

Figure 4.8. Tolerance range for Perpendicularity constraint

range is defined by distinctive circle diameter in black color. The real feature in red color is within the tolerance zone. If distance on x-axis and y-axis are dealt separately then a risk of point going out of tolerance limits arises. As in figure 4.8 two points in green color are taken at extremes of x and y axis. Despite the fact that both point are within tolerance range individually but the overall toleranced feature is out of limits. Since perpendicularity constraint generates a tolerance zone in shape of a circle, so we use the equation of circle to see effects of angles on tolerance:

$$\|\bar{D}\| = \sqrt{(-z(\psi_{E1,R} - \psi_{E2,R}))^2 + (z(\phi_{E1,R} - \phi_{E2,R}))^2} = tolerance/2$$

$$(-z(\psi_{E1,R} - \psi_{E2,R}))^2 + (z(\phi_{E1,R} - \phi_{E2,R}))^2 = (tolerance/2)^2 \tag{4.12}$$

The equation 4.12 infers that perpendicularity tolerance of part is controlled by angle ϕ and angle ψ along z axis dimension only.

4.1.3 Location Constraint

Finally the defect torsor of datum feature with reference and defect torsor of real feature with reference of workpiece is determined for the location constraint:

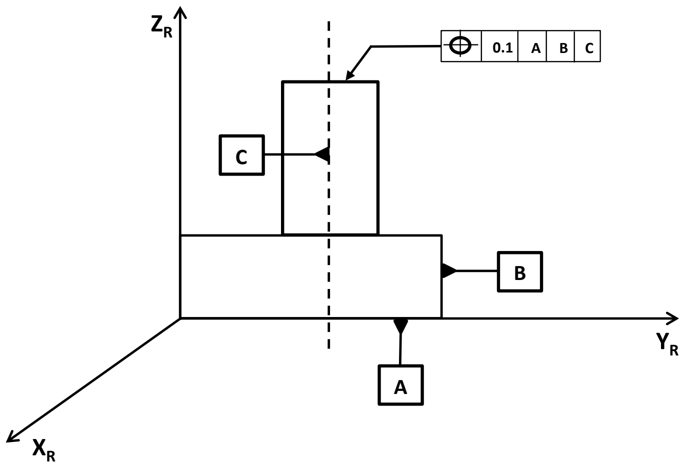

Figure 4.9. Location constraint

$$T_{E1,R} = \begin{bmatrix} \alpha_{E1,R} & 0 \\ \beta_{E1,R} & 0 \\ 0 & \mathcal{W}_{E1,R} \end{bmatrix}$$

$$T_{E2,R} = \begin{bmatrix} \alpha_{E2,R} & 0 \\ \beta_{E2,R} & 0 \\ 0 & \mathcal{W}_{E2,R} \end{bmatrix}$$

The gap torsor of datum feature with real feature becomes:

$$T_{E1,E2} = T_{E1,R} - T_{E2,R} = \begin{bmatrix} \alpha_{E1,R} & 0 \\ \beta_{E1,R} & 0 \\ 0 & \mathcal{W}_{E1,R} \end{bmatrix} - \begin{bmatrix} \alpha_{E2,R} & 0 \\ \beta_{E2,R} & 0 \\ 0 & \mathcal{W}_{E2,R} \end{bmatrix}$$

$$T_{E1,E2} = \begin{bmatrix} \alpha_{E1,R} - \alpha_{E2,R} & 0 \\ \beta_{E1,R} - \beta_{E2,R} & 0 \\ 0 & \mathcal{W}_{E1,R} - \mathcal{W}_{E2,R} \end{bmatrix} \tag{4.13}$$

For finding the displacement of any point A on datum feature E1 w.r.t real feature E2, putting eq.4.13 in eq.4.2:

$$D_A = \begin{bmatrix} 0 \\ 0 \\ \mathcal{W}_{E1,R} - \mathcal{W}_{E2,R} \end{bmatrix} + \begin{bmatrix} x \\ y \\ z \end{bmatrix} \wedge \begin{bmatrix} \alpha_{E1,R} - \alpha_{E2,R} \\ \beta_{E1,R} - \beta_{E2,R} \\ 0 \end{bmatrix} \tag{4.14}$$

Both displacement and rotation is taken into account for the location constraint.

$$D_A = \begin{bmatrix} 0 \\ 0 \\ \mathcal{W}_{E1,R} - \mathcal{W}_{E2,R} \end{bmatrix} + \begin{bmatrix} -z(\beta_{E1,R} - \beta_{E2,R}) \\ -z(\alpha_{E1,R} - \alpha_{E2,R}) \\ x(\beta_{E1,R} - \beta_{E2,R}) - y(\alpha_{E1,R} - \alpha_{E2,R}) \end{bmatrix} \tag{4.15}$$

$$D_A = \begin{bmatrix} 0 \\ 0 \\ \mathcal{W}_{E1,R} - \mathcal{W}_{E2,R} + x(\beta_{E1,R} - \beta_{E2,R}) - y(\alpha_{E1,R} - \alpha_{E2,R}) \end{bmatrix} \tag{4.16}$$

For small displacements, rotation matrix (K) is used from equation 4.6. To consider the displacement of every single point of datum feature w.r.t real feature, dot product of D_A is taken with the normal of datum plane (see fig.4.9):

$$\bar{D}.\bar{n}_D = \begin{bmatrix} 0 \\ 0 \\ \mathcal{W}_{E1,R} - \mathcal{W}_{E2,R} + x(\beta_{E1,R} - \beta_{E2,R}) - y(\alpha_{E1,R} - \alpha_{E2,R}) \end{bmatrix} \cdot \begin{bmatrix} \gamma \\ -\beta \\ 1 \end{bmatrix}$$

$$\bar{D}.\bar{n}_D = 0 + 0 + \mathcal{W}_{E1,R} - \mathcal{W}_{E2,R} + x(\beta_{E1,R} - \beta_{E2,R}) - y(\alpha_{E1,R} - \alpha_{E2,R})$$

$$\bar{D}.\bar{n}_D = \mathcal{W}_{E1,R} - \mathcal{W}_{E2,R} + x(\beta_{E1,R} - \beta_{E2,R}) - y(\alpha_{E1,R} - \alpha_{E2,R}) \tag{4.17}$$

The equation 4.17 enumerates that geometric tolerance of location constraint is effected by angle β is x axis, angleα in y axis and w displacement parameter of torsor.

4.2 Flowcharts of proposed approach

The strategy to use the proposed methodology and get application based results is provided in flowcharts 4.11 and 4.10. Flowchart 4.10 exhibits the path to determine acceptability of parts established on application of boundary conditions. In the beginning total maximum and minimum linear dimensional tolerances are calculated for boundary conditions. These maximum and minimum values of tolerances are used to generate probability density plots. At this level, acceptable precision range is compared to Probability density graphs. Last step is to decide whether parts are within specified requirements or rejected.

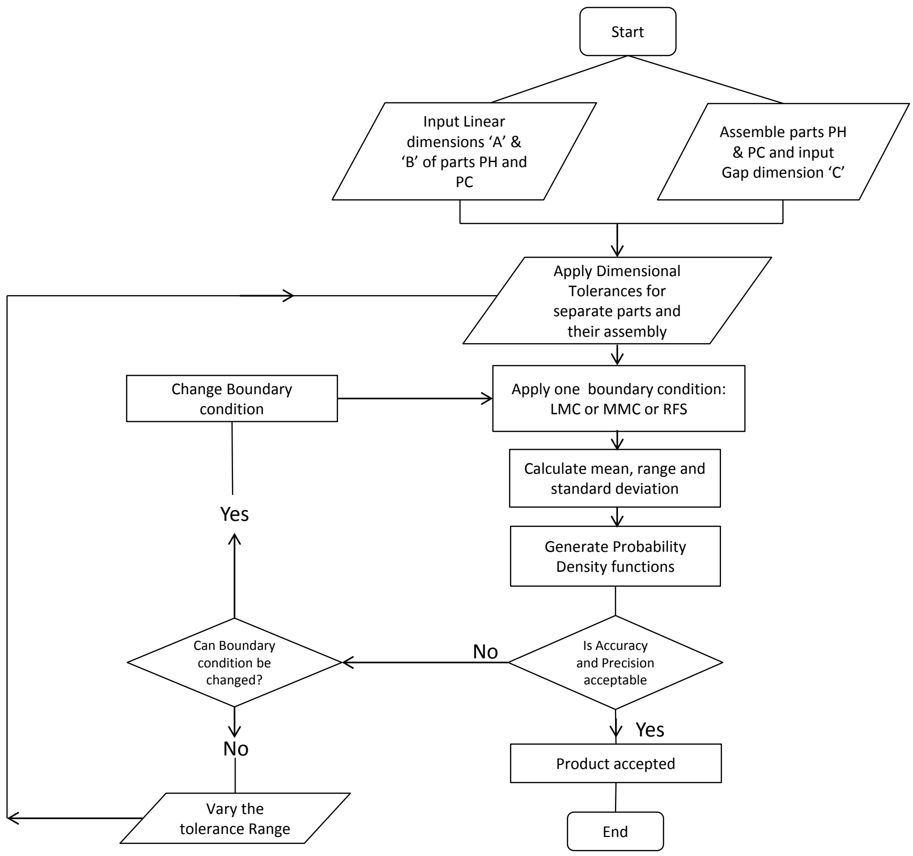

Figure 4.10. Deterministic approach flow chart

The second portion of case study where torsors are used to determine geometrical tolerance range for product is suggested in flowchart 4.11. The program begins with inserting x,y and z linear dimensions of work-piece in constraint equations of parallelism and perpendicularity. Angles for constraints are generated for varying standard deviations. It is followed by normal distribution graphs for resulted geometrical tolerances. In the end tolerance limits are checked. If they are within limits specified by designer then product is satisfactory. If the tolerance limits are not within specified range then, either a higher precision machine should be used or the geometric tolerance should be relaxed a little bit.

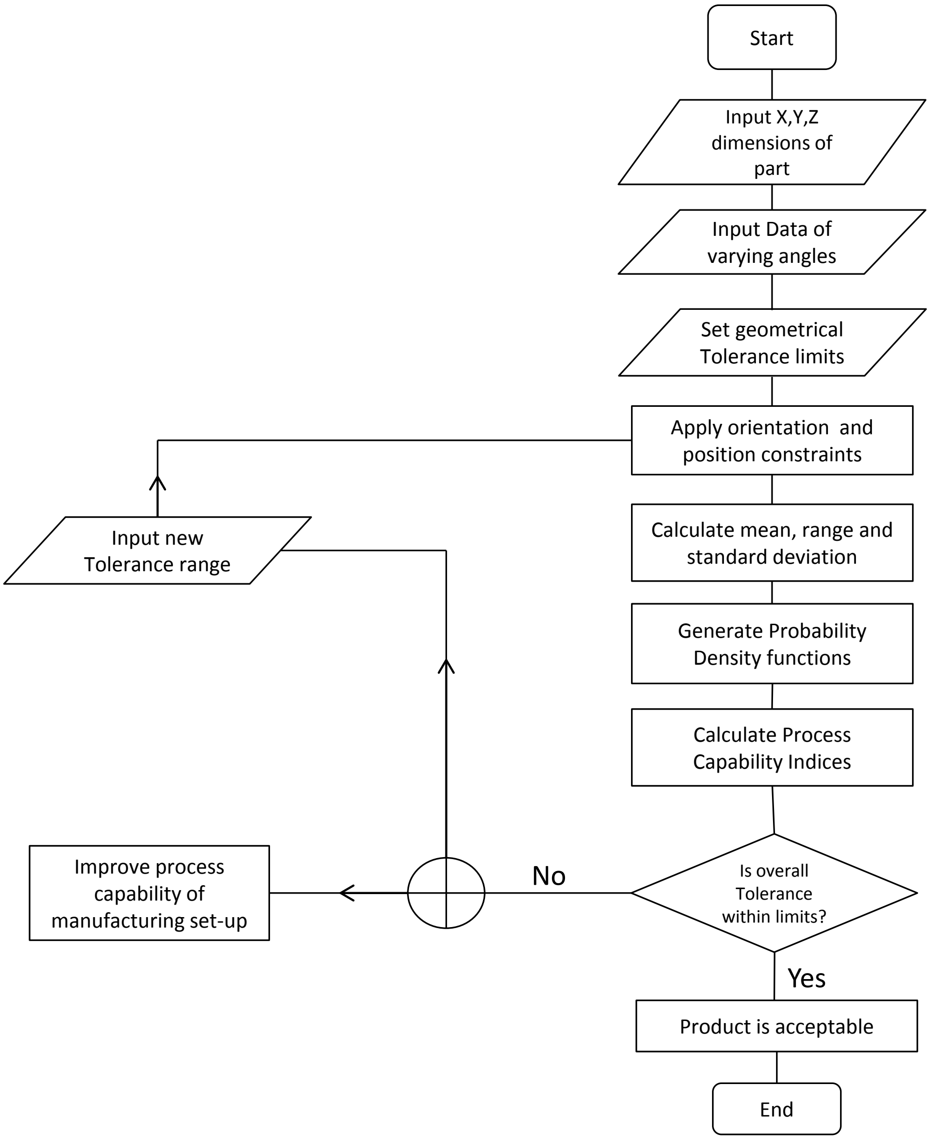

Figure 4.11. Flow chart for Torsors and geometrical tolerances

4.3 Flexibility and Precision of Manufacturing Systems

Statistical analysis of tolerance is most important for manufacturing field because parts are manufactured in hundreds to thousands in number. In statistics, Normal distribution is widely used. It is also known as a Bell curve or Gaussian curve. There are seven features of normal distributions due to which tolerance results are analyzed using this distribution.

1. Normal distributions are symmetric around their mean.

2. The mean, median, and mode of a normal distribution are equal.

3. The area under the normal curve is equal to 1.0.

4. Normal distributions are denser in the center and less dense in the tails.

5. Normal distributions are defined by two parameters, the mean (μ) and the standard deviation (σ).

6. 68% of the area of a normal distribution is within one standard deviation of the mean.

7. Approximately 95.45% of the area of a normal distribution is within two standard deviations of the mean and 6 sigma covers 99.99994% of area.

One sigma for manufactured parts means that 68.27% parts are within tolerance range and six sigma would increase accepted parts up to 99.99994%. This standard deviation can only be increased if parts are manufactured on a high precision machine. So the cost of process will increase with the standard deviation. A precise manufacturing system has less flexibility in terms of tolerance values. The results for this study comply with proposed idea that a less precise machine is able to produce a high quality part thus saving cost. These results are obtained by running a program on MATLAB. This MATLAB program gave the varying range of geometric tolerances for different angle values followed by tolerance ranges for parts production.

4.4 Input Data

Input data for the MATLAB program are the angle components of torsor in constraint equations. The two equations of parallelism and perpendicularity constraints are as follows:

$$Tol_{parallelism} = x(\beta_{E1,R} - \beta_{E2,R}) - y(\alpha_{E1,R} - \alpha_{E2,R}) \tag{4.18}$$

$$Tol_{perpendicularity} = 2 * \sqrt{(-z(\psi_{E1,R} - \psi_{E2,R}))^2 + (z(\phi_{E1,R} - \phi_{E2,R}))^2} \tag{4.19}$$

At first values of α, β, ϕ and ψ angles for both constraints are generated separately for parts. The number of parts is 1000. These values are initiated at three different standard deviations i.e 1σ, 2σ and 3σ. Then they are put in above equations 4.18 and 4.19 to obtain tolerance values range for parallelism and perpendicularity constraints.

4.5 Output Data

The output is in the form of normal distribution curves. These curves are generated for different values of standard deviation of input data. A normal distribution curve for 6σ can be seen in figure 4.12. Upper and lower specification limits are defined on the curve and within this 6σ limits, 99.99994% parts are defect free. Only 0.3ppm defects per million are present in 6σ curve. The percentage of defect free parts for 1σ is 68.27%, for 2σ it is 95.45%, for 3σ its 99.73%. The comprehensive values of angles and tolerances for the applied geometric constraints is given in Table 4.1. Units for angles α, β (symbols of angles for parallelism) and ϕ, ψ (symbols of angles for perpendicularity) are in radians (rad) and the tolerance (Tol) is in millimeters (mm). From the table, it is evident that as standard deviation varies from 1σ to 3σ, tolerance range is contracted.

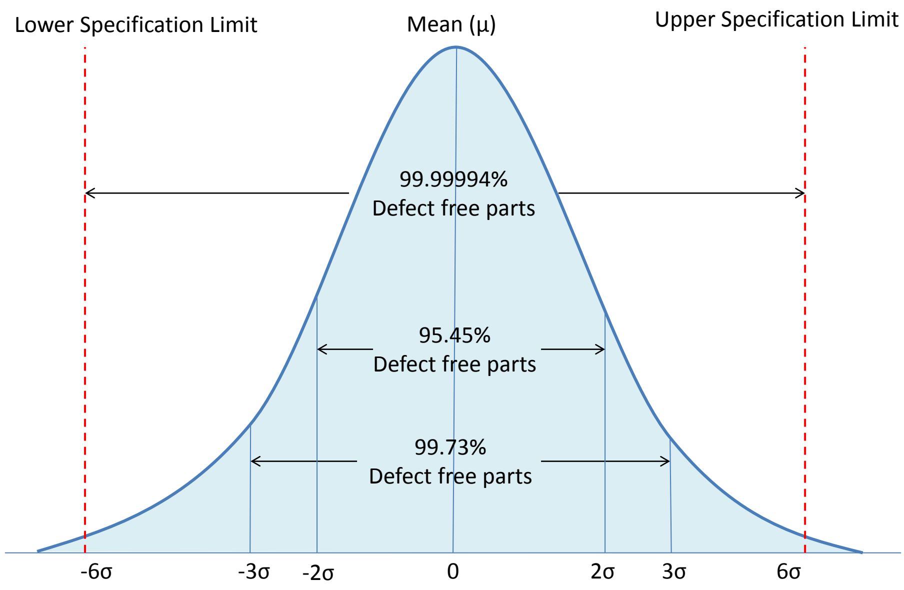

Figure 4.12. Normal Distribution Curve for 6σ

Table 4.1. Results

Constraint Type	Angles (rad); Tol(mm)	Standard Deviation		
		1σ	2σ	3σ
Parallelism	α_{Max}	0.0037	0.0016	0.0012
	β_{Max}	0.008	0.0017	0.0012
	α_{Min}	-0.0032	-0.0016	-9.7791exp^{-4}
	β_{Min}	-0.0031	-0.0015	-0.0011
Parallelism	Tol_{Max}	0.2902	0.1303	0.0911
	Tol_{Min}	-0.2716	-0.1252	-0.0836
Perpendicularity	ϕ_{Max}	0.0030	0.0018	0.0011
	ψ_{Max}	0.0040	0.0017	9.6109exp^{-4}
	ϕ_{Min}	-0.0034	-0.0015	-9.4968exp^{-4}
	ψ_{Min}	-0.0036	-0.0016	-0.0011
Perpendicularity	Tol_{Max}	0.1108	0.0490	0.0286
	Tol_{Min}	8.8483exp^{-4}	4.652exp^{-4}	4.3443exp^{-4}

Figure 4.13, 4.14 and 4.15 shows normal distribution of parallelism tolerance for 1σ,2σ and 3σ respectively. Further on Figure 4.16, 4.17 and 4.18 shows normal distribution of perpendicularity tolerance for 1σ, 2σ, and 3σ respectively. The specified or design

parallelism tolerance in case study was 0.1mm. In graph 4.13, only 682.7 parts out of 1000 are defect free. From table 4.1 maximum value of tolerance comes out to be 0.2902mm and minimum value is -0.2716mm. Graph 4.14 shows that 950 parts are within parallelism geometric tolerance limit with maximum value 0.1303mm and minimum value of -0.1252mm.

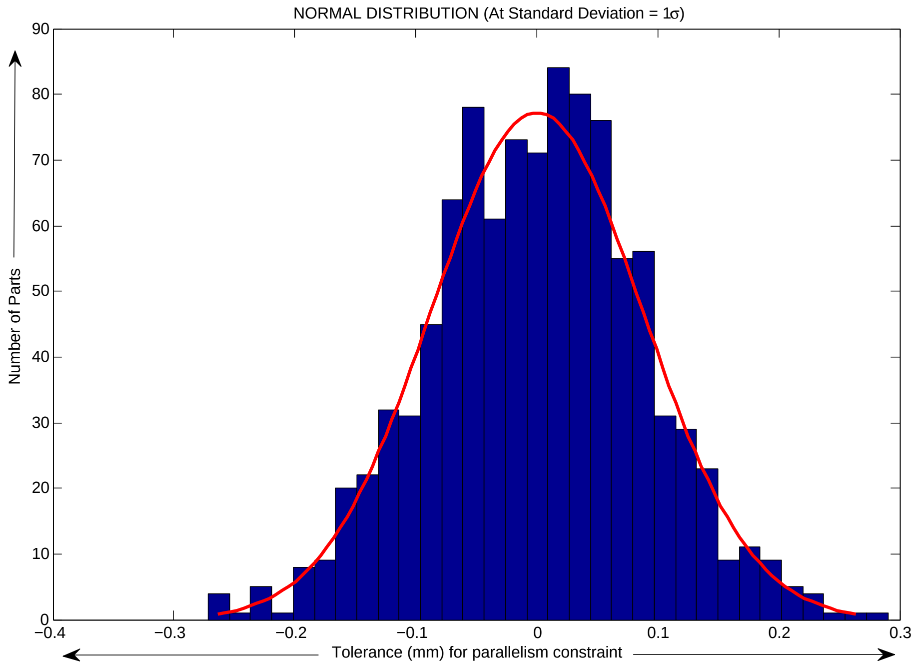

Figure 4.13. Normal Distribution of Parallelism Tolerance when $\sigma = 1$

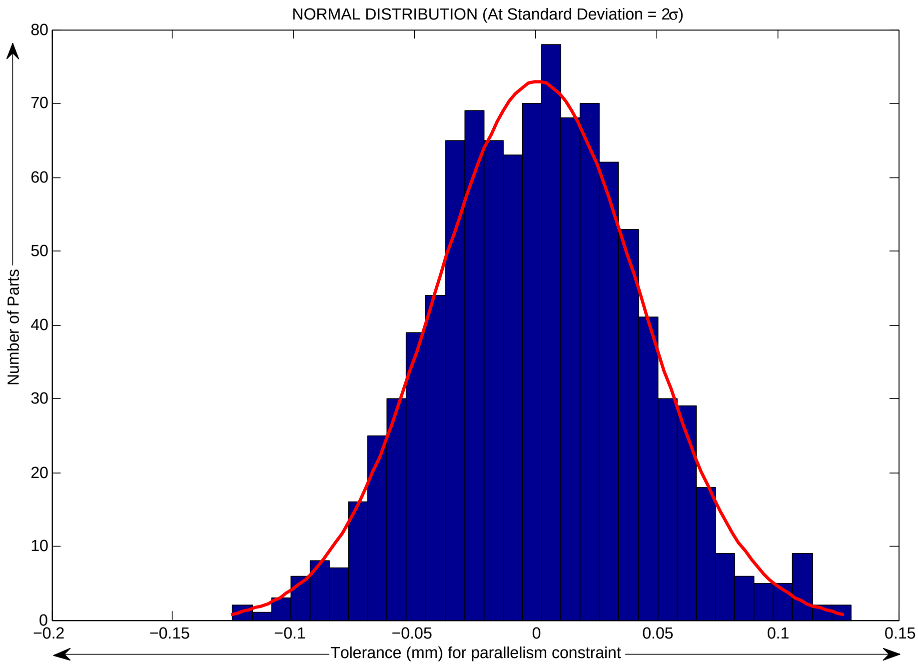

Figure 4.14. Normal Distribution of Parallelism Tolerance when $\sigma = 2$

Normal distribution in graph 4.15 is for tolerance range at 3σ standard deviation. Range of geometric tolerance is in between -0.0836mm to 0.0911mm. It is most precise tolerance range.

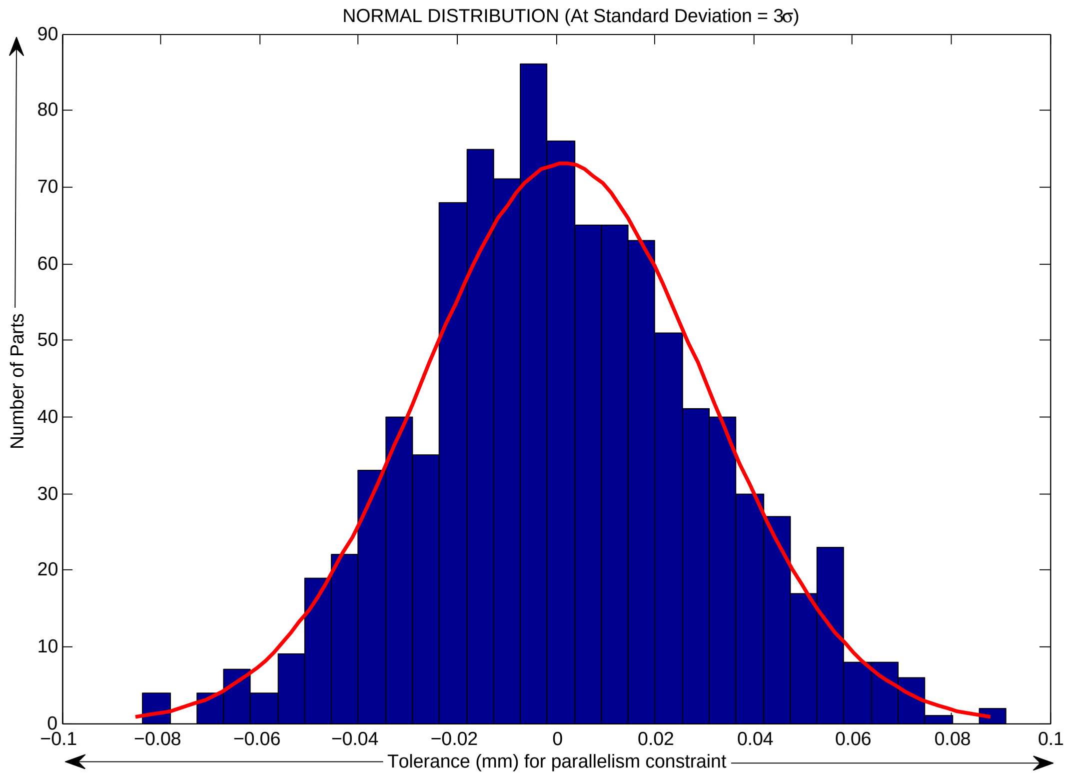

Figure 4.15. Normal Distribution of Parallelism Tolerance when $\sigma = 3$

Figure 4.16 is normal distribution of perpendicularity constraint tolerance values for 1000 parts at 1σ. The maximum value of tolerance is 0.1108mm and minimum value is 0.0008483mm. This tolerance range changes for 2σ, from 0.0004652mm to 0.0490mm, refer to graph 4.17. For standard deviation of 3σ minimum tolerance value is 0.00043443mm and maximum is 0.0286mm. In figure 4.18, only 2.7 parts out of 1000 may deviate from perpendicularity constraint. The perpendicularity tolerance of 0.1mm specification is observed even by just improving precision from 1σ to 2σ. It is seen that the tolerance range values for perpendicularity constraint in figure 4.16, 4.17 and 4.18 are positive values only. This is due to the constraint equation of perpendicularity 4.19 in which x and y axis values are considered simultaneously.

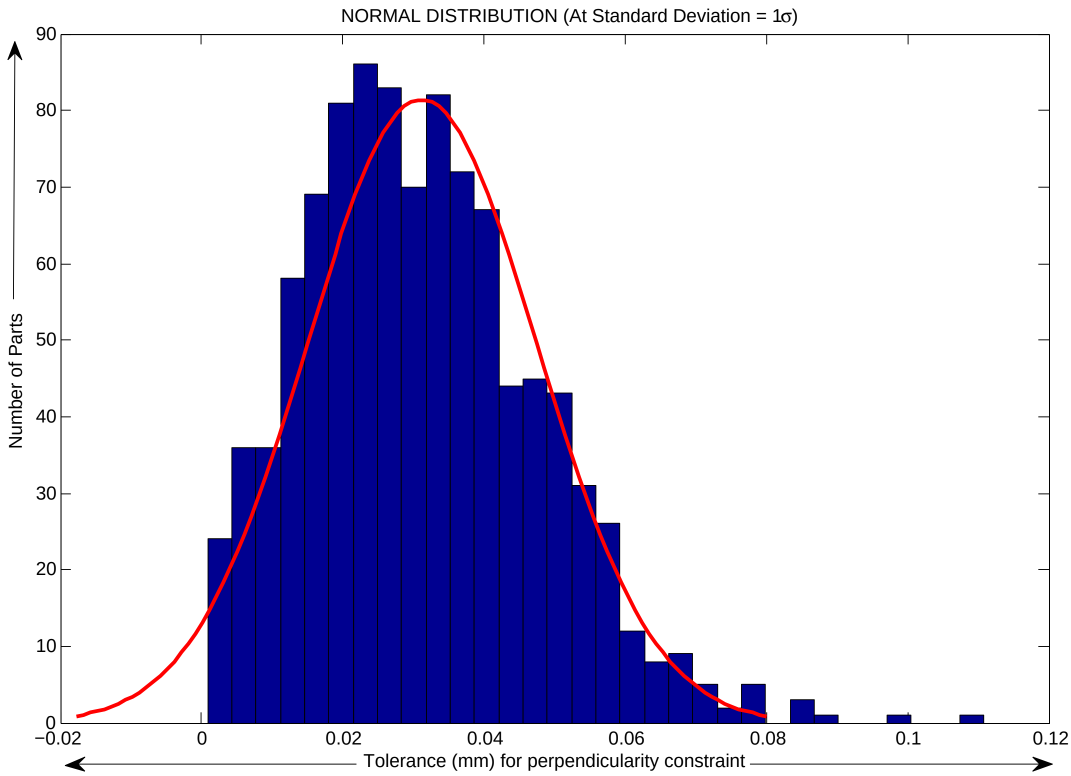

Figure 4.16. Normal Distribution of Perpendicularity Tolerance when $\sigma = 1$

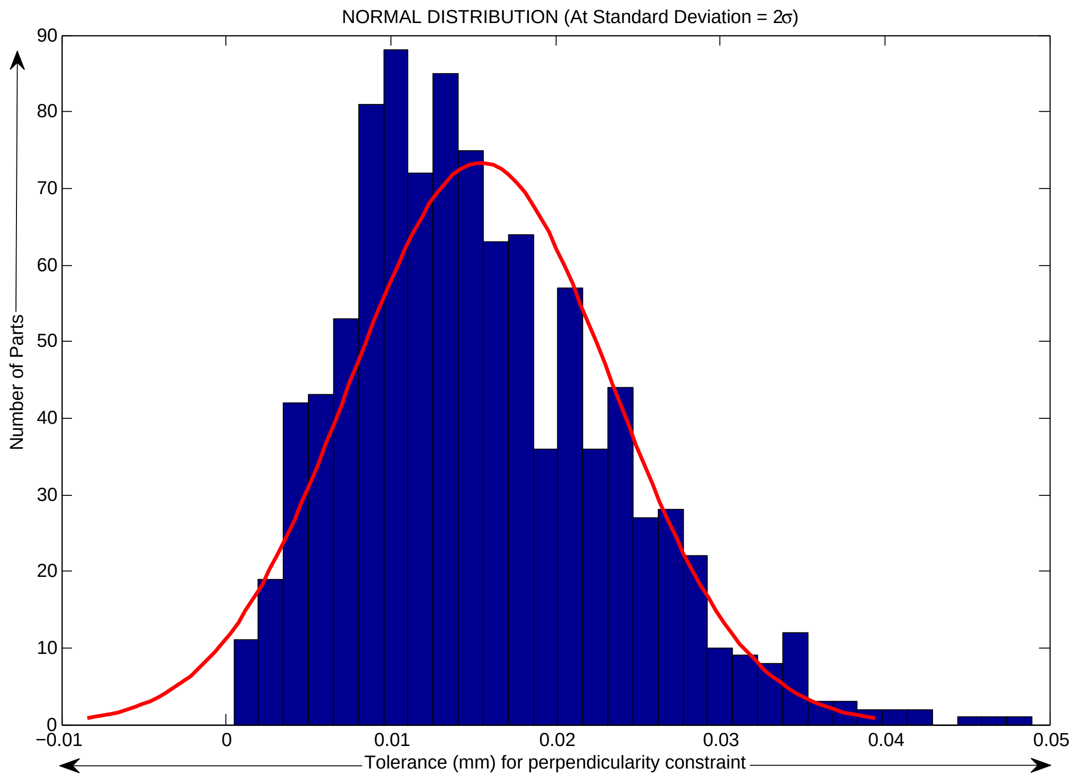

Figure 4.17. Normal Distribution of Perpendicularity Tolerance when $\sigma = 2$

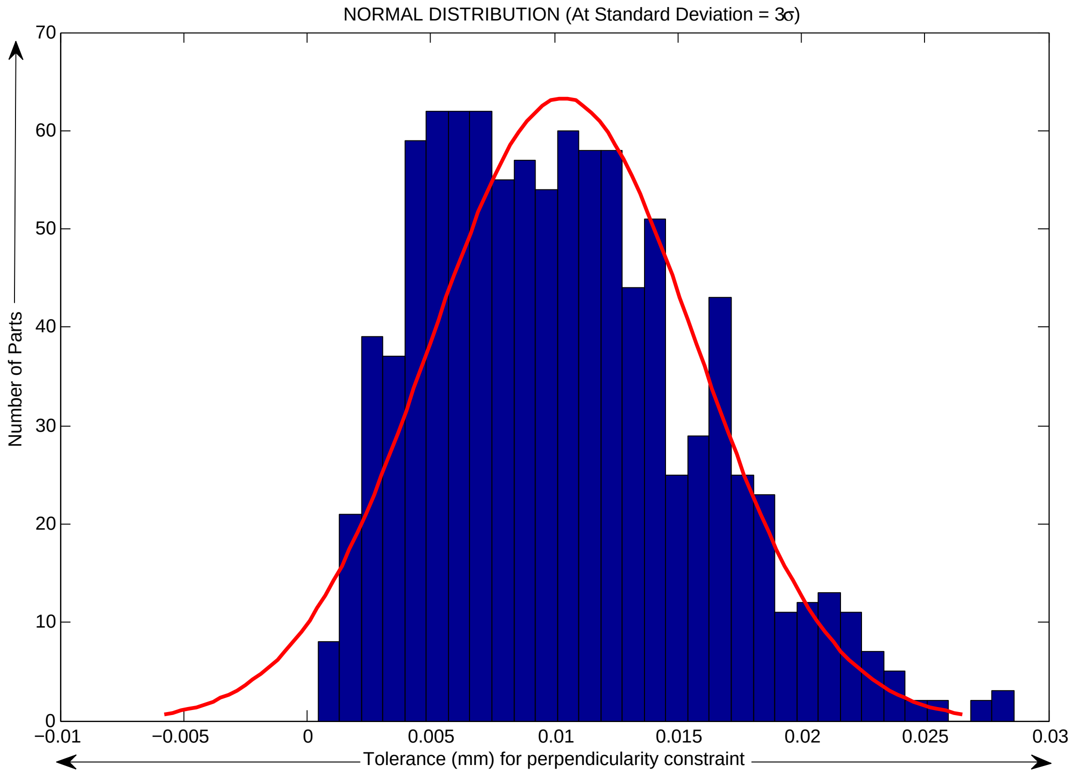

Figure 4.18. Normal Distribution of Perpendicularity Tolerance when $\sigma = 3$

If dot product of normal vector is implemented in one of the axis directions either x-axis or y-axis with equation 4.11,then the tolerance in those specified axis direction can be obtained. The normal distribution of tolerance values of perpendicularity constraint in x-axis direction are shown in figure 4.19,4.20 and 4.21 for standard deviation of 1σ, 2σ and 3σ respectively. Similarly, the graphs 4.22,4.23 and 4.24 show normal distribution of tolerance values of perpendicularity constraint in y-axis direction for standard deviation of 1σ, 2σ and 3σ respectively. The precision of machines required to produce parts with tolerance constraint range at 3σ is higher than that to manufacture parts at 1σ. However, the flexibility (in terms of tolerance limits) of machines at 1σ is definitely higher than the other. Consider a case where a manufacturer has 3 machines available on job floor with 1σ,2σ and 3σ precision. He gets the order to manufacture a part with 0.1(mm) geometric tolerance constraints of parallelism and perpendicularity. The cost of manufacturing by using 3σ precision machine is double than the 2σ precision machine. But it is observed that even 2σ machine can produce product within design parameters. Then feasible option is to work with 2σ machine yet getting favorable results.

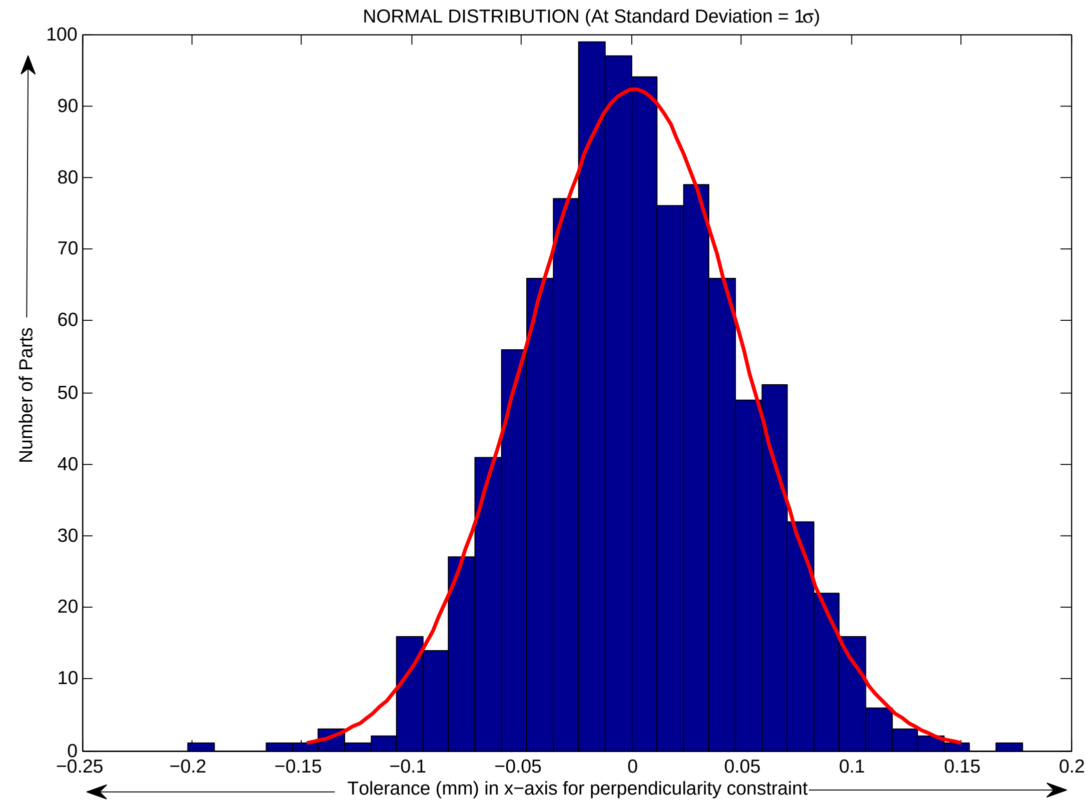

Figure 4.19. Normal Distribution of Perpendicularity Tolerance (on x-axis) when $\sigma = 1$

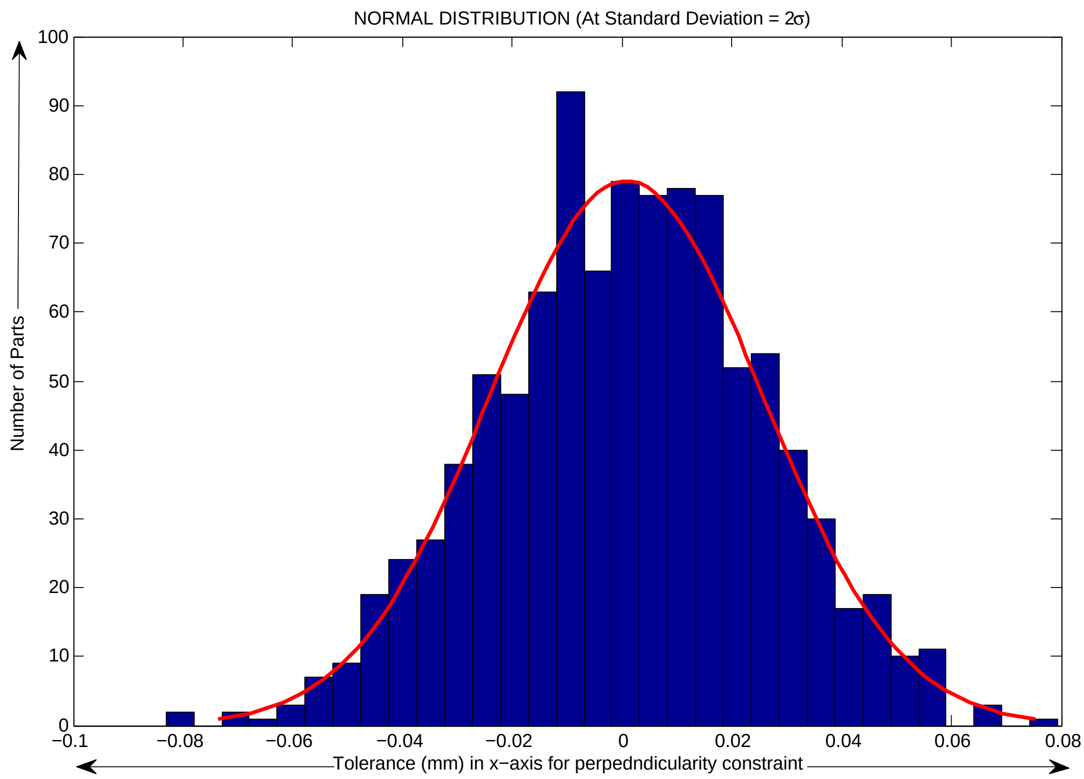

Figure 4.20. Normal Distribution of Perpendicularity Tolerance (on x-axis) when $\sigma = 2$

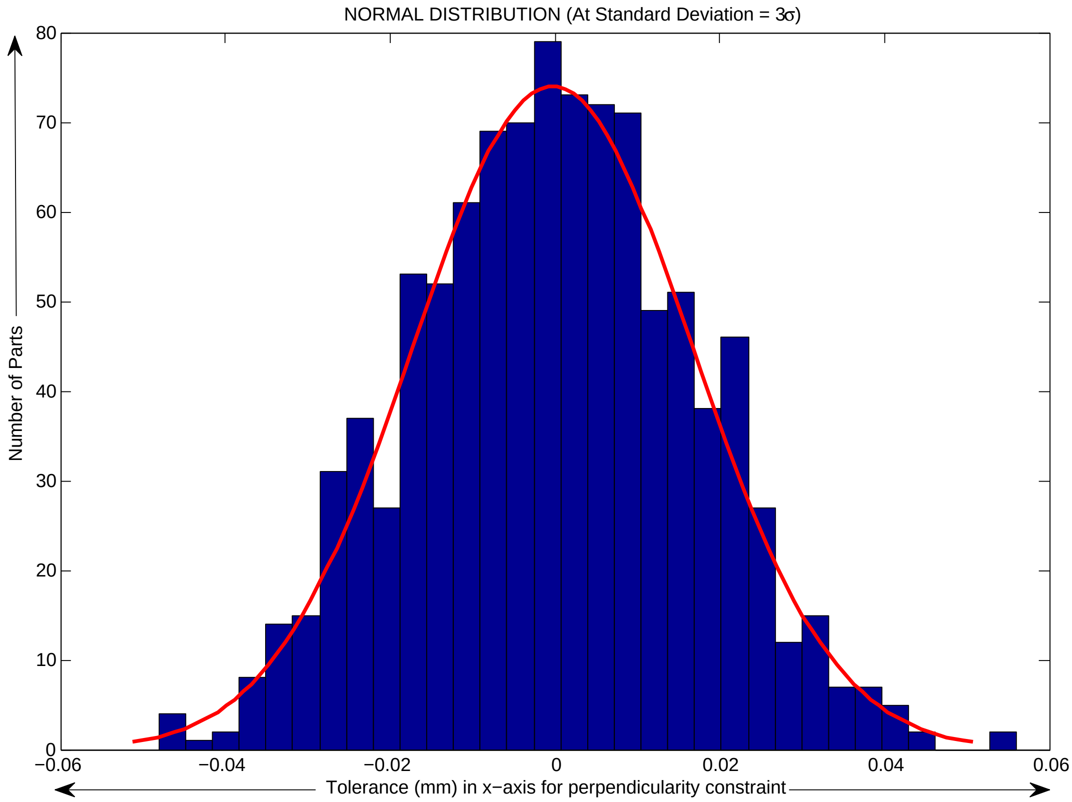

Figure 4.21. Normal Distribution of Perpendicularity Tolerance (on x-axis) when $\sigma = 3$

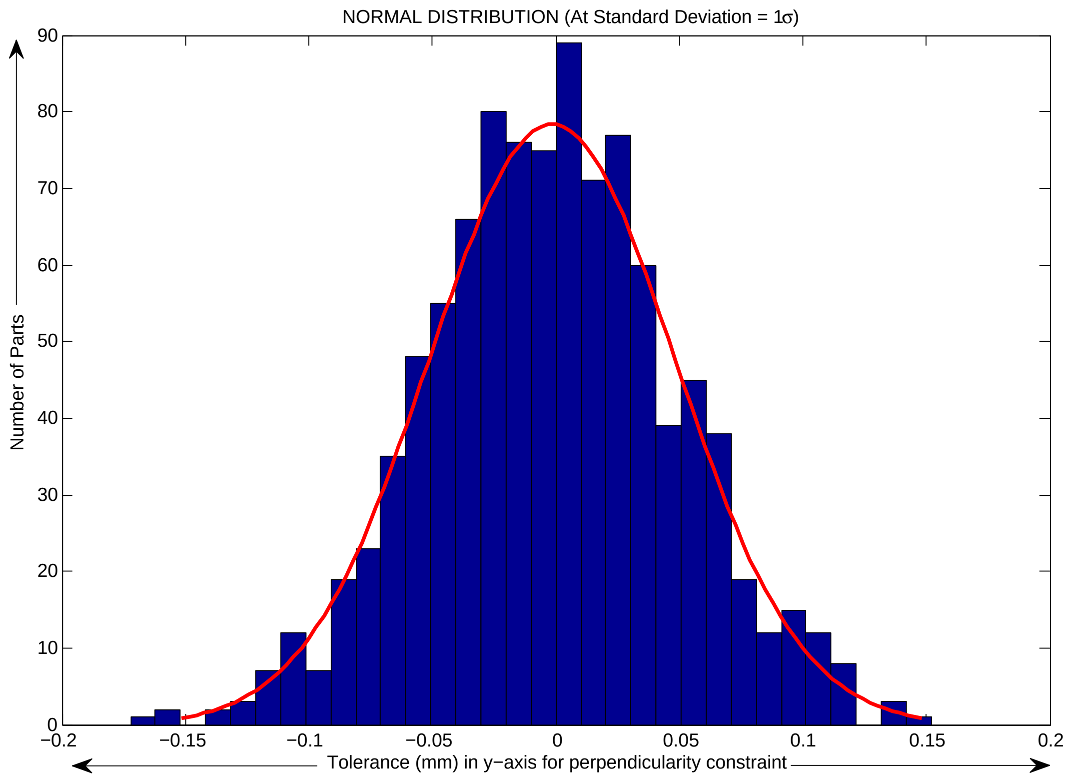

Figure 4.22. Normal Distribution of Perpendicularity Tolerance (on y-axis) when $\sigma = 1$

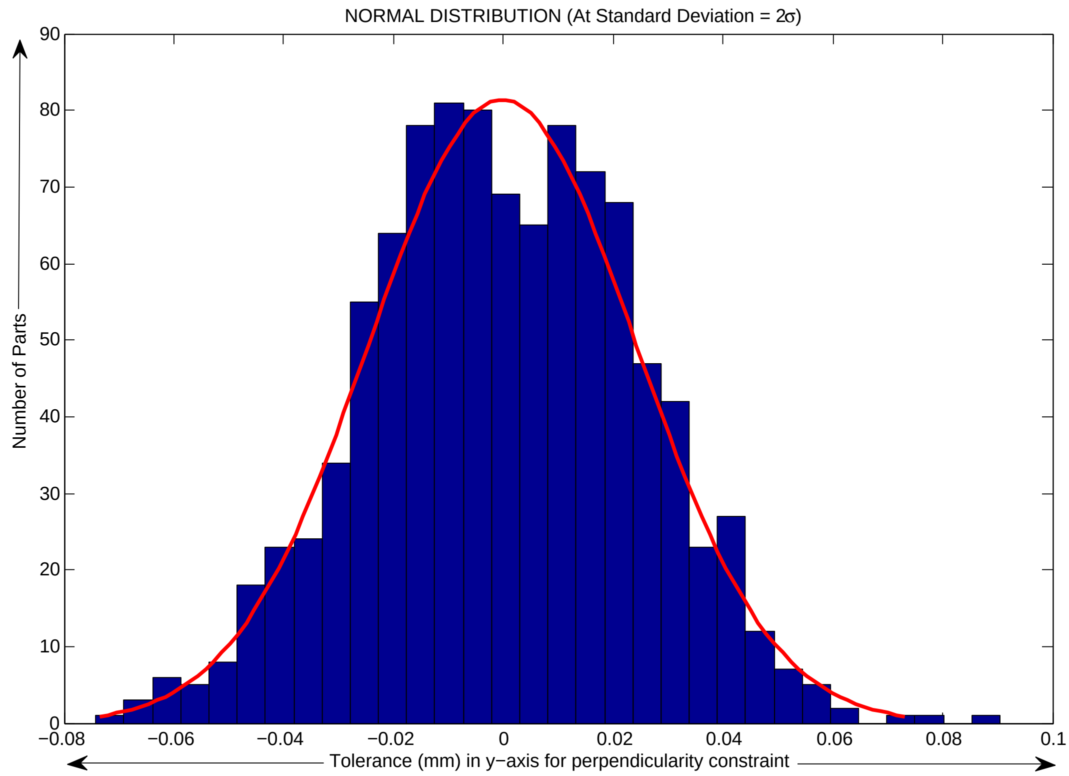

Figure 4.23. Normal Distribution of Perpendicularity Tolerance (on y-axis) when $\sigma = 2$

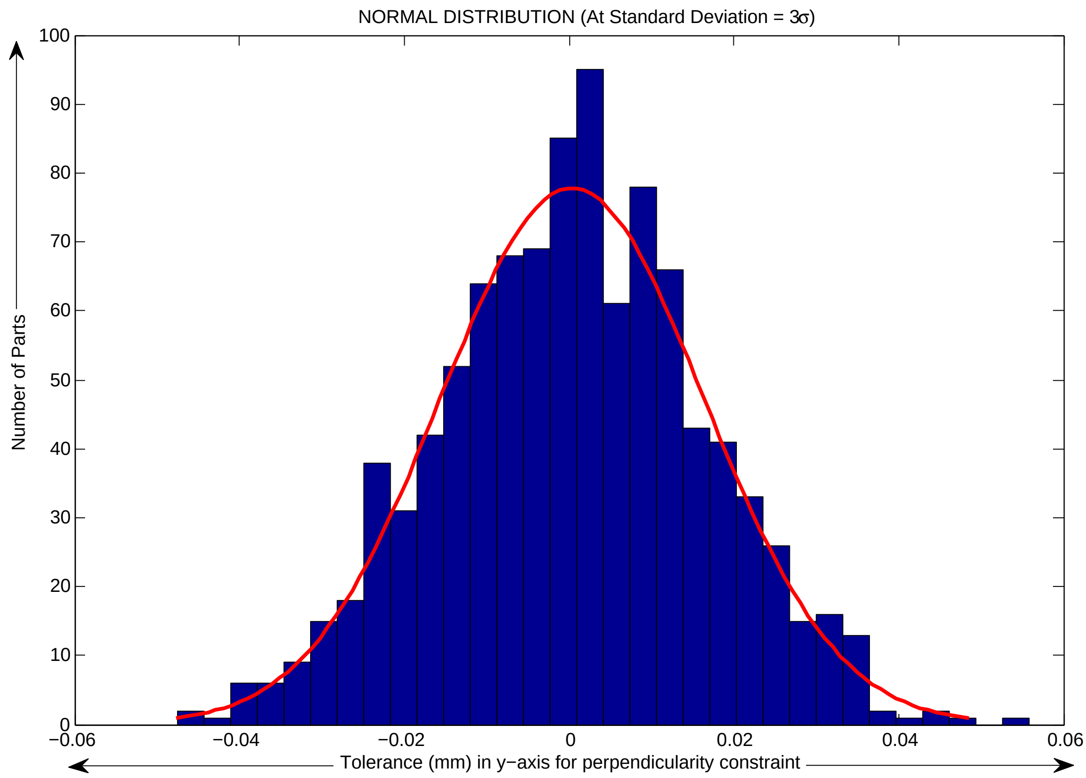

Figure 4.24. Normal Distribution of Perpendicularity Tolerance (on y-axis) when $\sigma = 3$

Figure 4.25 shows a graphical comparison of Parallelism Tolerance and Standard Deviation. The lines in green and blue shows maximum and minimum tolerance data points and red line shows the mean or average tolerance values. The limit of 0.1(mm) tolerance is shown by dotted lines. All tolerance values converge towards the mean value of tolerance as standard deviation increases. From 1σ to 2σ, convergence is very rapid. It infers that as the precision of machine increases, the tolerance limits get tighter. However, a manufacturer might use a machine with 2.2 σ precision instead of 3σ to save cost of manufacturing.

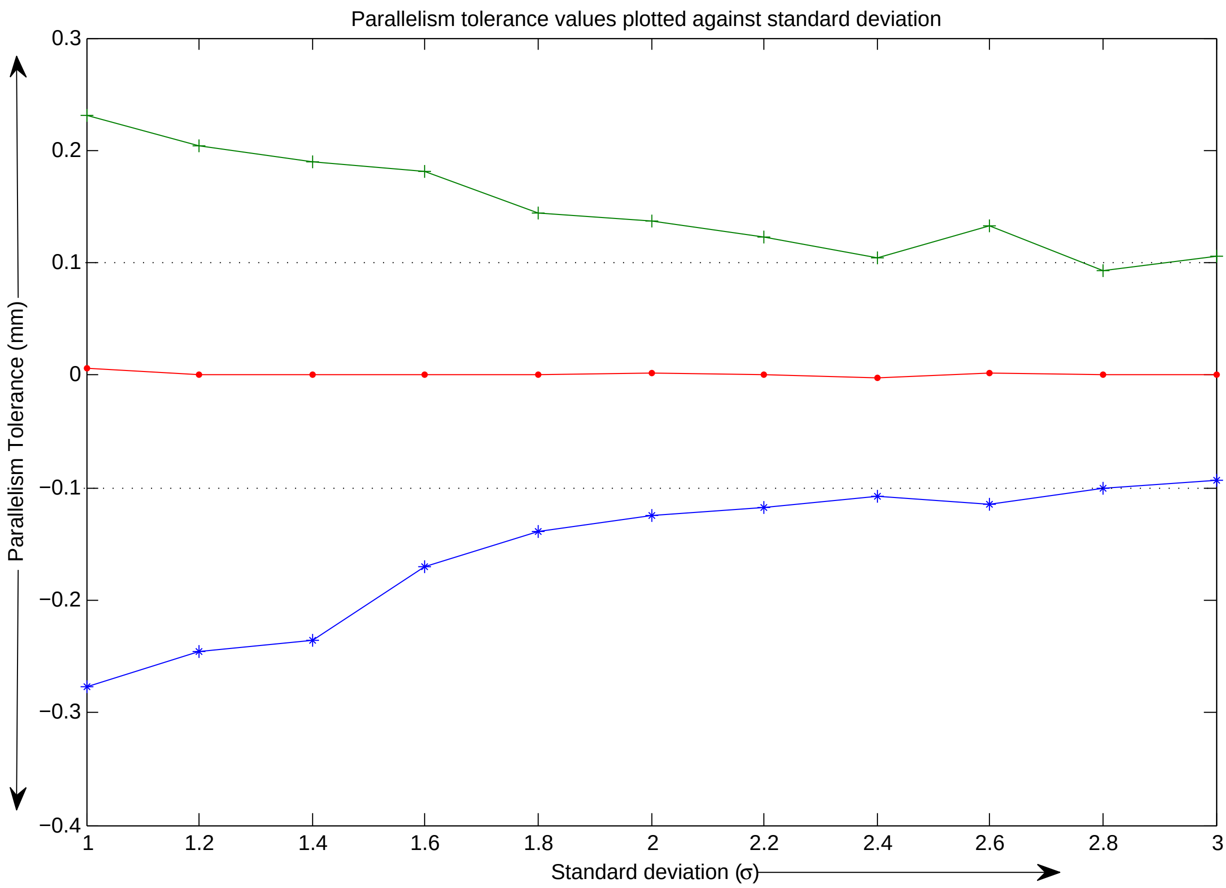

Figure 4.25. Comparison of Parallelism tolerance (mm) values with standard deviation (σ)

4.6 Synopsis

In this section of case study, geometric constraints parallelism, perpendicularity and location equations in terms of tolerance are found. Flowcharts provide application of proposed approach. The results in form of graphs are given in the end which present outcomes and their interpretation. The outcomes of whole study and its interpretation for useful application in manufacturing field was also explained. It includes the input and output variables chosen for the simulation purpose in MATLAB program for simulation. The next chapter is the concluding chapter of this thesis. It discusses conclusion of study as a whole, its limitations and future perspectives.

Chapter 5

Conclusion and Perspective

The topic of this thesis falls under the subject of geometric tolerances and precision manufacturing. GD&T helps to upgrade the quality of product. But this higher quality raises the manufacturing cost. This study proposes a method to control tolerances of product by keeping within cost limits. The literature review shows the gap for research. Both dimensional and geometric tolerances are taken into account. Boundary conditions and torsors are combined with these tolerances to see acceptable tolerance range varying as per availability of different precision manufacturing machines.

There are certain limitations of this work. The type of tolerances selected in case study are only orientation tolerances. It might be possible that other classes of tolerances are taken into account for further research. At methodology stage, errors between the gripper and baseplate is assumed zero for simplification. Other techniques may be used for the tolerance analysis of constraints. In future work, the program algorithm can be enhanced with increase in applied types of tolerances. Experimental set up may be designed and tested for the validation of proposed methodology.

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Certificate of Completeness

It is hereby certified that the dissertation submitted by *NS Hana Amin Khan* Reg. No. *NUST201362436MCEME35113F*, Titled:*Affirmation of Enhanced Product Design using GD&T and Concept of Torsors in Tolerance Analysis* has been checked/reviewed and its contents are complete in all respects.

Signature of Supervisor
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