DYNAMIC RESPONSE OF BOLTED JOINTS UNDER HARMONIC EXCITATIONS

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In the name of Allah,
the most Beneficent and the
most Merciful

DECLARATION

I hereby declare that I have developed this thesis entirely on the basis of my personal efforts under the sincere guidance of my supervisor Dr.Hasan Aftab Saeed. All the sources used in this thesis have been cited and the contents of this thesis have not been plagiarized. No portion of the work presented in this thesis has been submitted in support of any application for any other degree of qualification to this or any other university or institute of learning.

Kanza Shoaib

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Dedicated to my parents, siblings and husband

ABSTRACT

Three dimensional finite element modeling of two flexible plates connected by a shear bolted lap joint is done in order to determine the dynamic characteristics of the model. Energy dissipation occurs due to friction and micro slip between the contacting surfaces of the plates when they are subjected to vibrations. Hysteresis curves are drawn to calculate this dissipated energy under different harmonic loadings.

This work involves use of ANSYS to analyze the three dimensional model. A FE analysis will be performed on the contacting surfaces to observe the variation in different dynamic parameters of the joint. And by using Proper Orthogonal Decomposition (POD) concept, the POM modes will be determined for the system to show how they can be helpful in the reduction of the model.

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CHAPTER 1
INTRODUCTION

1.1. Introduction

In the designing of structure, the study of dynamic characteristics of structures plays an important role. There are certain interesting aspects of the dynamics of nonlinear systems that are different from those of their linear counterparts. Two of the most commonly encountered nonlinearities in the dynamics of structures are the cubic nonlinearities in displacements in the context of the duffing oscillator and the dynamic characteristics of shear lap joints.

Dynamics of nonlinear systems is different from linear systems in certain aspects such as the presence of internal resonance, the loss of validity of the superposition principle and the single frequency excitation of higher harmonics. This present study is to examine the dynamic characteristics of joints under harmonic excitations. A generic model for the dynamics of joints is developed while keeping in mind the fact that joints play an important role in the dissipation of energy in vibrating built-up structures, so this simplified model can be used in replacement of any complex joint within the system. Therefore, this work is done to create a generic 3D model for joints for a wide variety of excitation conditions.

1.1.1. **Categories of Joint Dynamics**

Metherell and Diller (1968) analyzed shear lap joint without including the inertial effects of the comprising plates which leads to the study of the dynamic characteristics of lap joints. Two main categories of the joint dynamic studies are; the application of constitutive methods to investigate joint behavior and the phenomenological treatment of joints.

The phenomenological approach; the first category is about the macro-behavior of the joint dynamics and is studied as observed computationally or experimentally. Reproducing the hysteresis behavior of joints with only a few degrees of freedom (DOFs) is emphasized. As changing the amplitude or frequency of the excitation can significantly change the shape of the hysteresis curve so, these methods are limited in their scope. This results in necessary retuning of the parameters to fit the new hysteresis curve.

In the second approach, the joint is either treated analytically or is modeled by the finite element (FE) method. Normally the number of elements required in the FE model is quite large and nonlinear component of force arising from friction is associated with each of the DOFs. Though only one DOF is enough for the characterization of joint dynamics, the inertia of the system is usually ignored in the pseudo-static analytical approaches developed till now. This leads to ambiguous results at higher frequencies of excitation. This approach is only valid for the case of single frequency harmonic excitations.

1.1.2. Friction between Contacting Surfaces

The dissipation of energy through friction between contacting rough surfaces forms the basis of the phenomenon of joint dynamics. A complete analysis of the problem has not yet performed, although various attempts have been made to analyze the rough contact. The analysis becomes more difficult when, under the action of normal clamping force, one rough surface is moved against the other. So, up till now the problem was analyzed by making certain simplifying assumptions. These include the bristle concept of surface asperities, the statistical treatment of the contact based on the contact behavior of a single isolated asperity, or approximating the contact between two deformable rough surfaces with one between a rigid rough and a deformable flat surface. This study is now extended by making a 3D contact between two deformable surfaces to get a more accurate analysis done.

To have a clear understanding of dynamic parameters, it is natural to carry out first a detailed FE analysis of the contact problem since these parameters play an important role in the dynamics of joints.

1.1.3. Proper Orthogonal Decomposition

The energy dissipation in joints subjected to micro-slip is a highly nonlinear phenomenon. Nonlinear forces act at each DOF as this is a main problem in developing a simplified model of the joint dynamics. It is important here to mention that the length of a joint that encounters micro-slip is a complex function of the excitation amplitude and frequency. Active length is the term normally used for the length of the joint that experiences micro-slip, while grey length is the term used for the length that does not experience micro-slip. This means that if the same mesh is used the number of DOFs varies according to the amplitude and frequency of the excitation. It should also be pointed out that structures found in practice usually contain a number of joints. These facts suggest that before the model is integrated with the model of the remaining structure the reduction of nonlinear DOFs in a joint model is highly desirable.

Proper orthogonal decomposition, also known as the singular value decomposition, principal mode synthesis, Karhunen-Loève decomposition, is usually applied in the context of model reduction of both linear and nonlinear systems. This approach has the advantage that system dynamics can be reproduced with a small number of orthogonal functions known as the proper orthogonal modes (POMs). There is a proper value (singular value) attached to every POM that is an indicator of its importance in the dynamics of the system.

The POD can be applied if the system response is deterministic in the sense that the system response remains constant for the same set of inputs (Azeez and Vakakis (2001)). Therefore, this method can be a strong candidate which is always excited from the point at which they are integrated with the rest of the system for the decomposition of the dynamics of joints.

1.2. Motivation

After overcoming the computational time problem, much work on nonlinear phenomenon has started being encountered during design phase of structures. Dissipation of vibration energy through joints within built up structures is one of them. Energy is dissipated mainly by friction and the micro-slip experienced by the relative motion of contacting points. Friction is a highly nonlinear phenomenon and its simulation is very difficult and time consuming. So a simplified and generic model of joint is highly required which can be incorporated instead of every complex joint within structures.

1.3. Objective

The main objective of this thesis is to utilize a general purpose finite element code, e.g. ANSYS 14.1, for modeling a shear lap bolted joint between two flexible plates using 3D elements unlike that used by Khattak (2006). This model will serve as a basis for incorporated joint model developed by Khattak (2006) in an FE model. Energy dissipation behavior of the joint in terms of hysteresis curve will be determined from the time history of the nodal displacements. POD technique will be used to find the modes for this model which will be able to span the space of nodal displacements.

`1.4. Methodology

The sequence or methodology followed for the completion of work is summarized in the form of block diagram which is shown below-

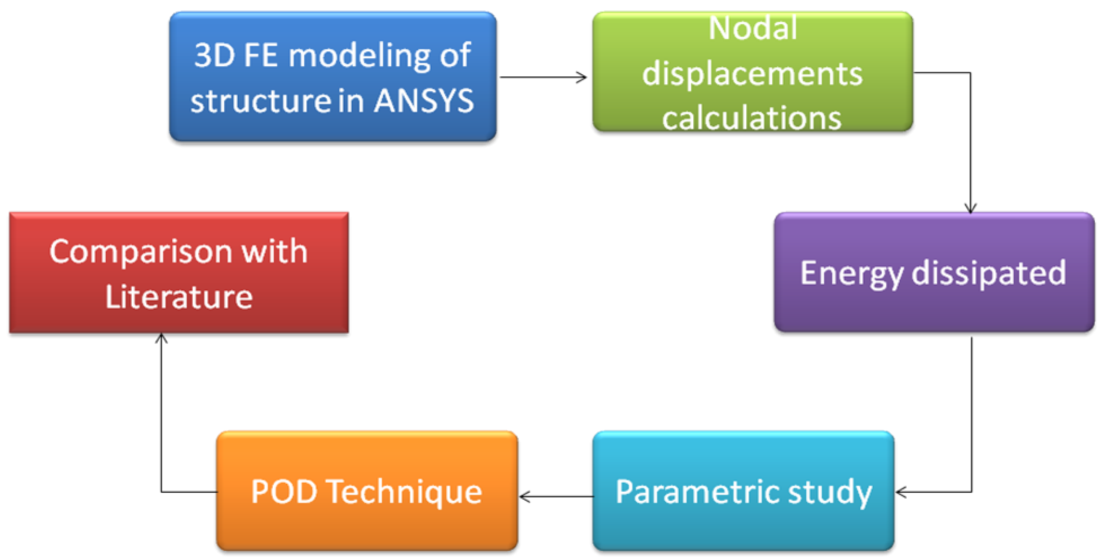

Figure 1. Methodology block diagram

CHAPTER 2

BACKGROUND AND LITERATURE REVIEW

2.1. Introduction

As the computational capabilities are getting better day by day, so many nonlinear phenomena have started being modeled during the design of structures and dynamics of joints connecting the structures is one of them; of which lap joints are more under study. Mathematically joint dynamics is expressed as-

$$\mathbf{K\,x + C\,(dx/dt) + M\,(d^2x/dt^2) = F - f_N} \qquad \textbf{........................(2.1)}$$

where K represents the stiffness matrix, C damping matrix and M mass matrix. x is the nodal displacement vector, F is excitation force and f_N represents the frictional forces.
Here C is directly proportional to K and is given as-

$$\mathbf{C} = \varepsilon * \mathbf{K} \qquad \textbf{........................(2.2)}$$

where $\varepsilon = 10^{-5}$ to get a better convergence and avoid high resonances which are both major interests of the mechanical systems.

This friction f_N involved plays a vital role in dissipating energy from the system. Within fabricated structures, joints account for almost 90% of total dissipated energy.

2.2. FEM Joint Modeling

Many researchers have done modeling and analysis of the joint interface to study it's dynamic characteristics till now, where most of them have used the phenomenological approach for this job i.e. frictional force to be calculated by the relative displacement. Research work of some of them which was related to this study is presented here-

Ingrid A. Rashquinha and Daniel P. Hess (1997) developed a dynamic model of a fastened assembly. He modeled dynamics of the individual components of the structural assemblies with lumped parameter models. A nonlinear lumped parameter fastener model is then used to couple the structural component models. The result is a set of nonlinear differential equations that model the dynamics of the assembly. To illustrate this approach, he modeled and analyzed an assembly consisting of a cantilevered beam with a component fastened to the beam with a threaded fastener. This work was initiated on the general premise that dynamic models of fastened assemblies could be used to optimize fastener placement and orientation. This is an area of ongoing research, the goals of which are to develop general design guidelines and criteria that minimize maintenance and failure due to fastener loosening.

Ibrahim and Pettit (2003) have discussed the linear and non linear problems that effects the dynamics of the bolted joints which includes different preload experiences, energy dissipation between joints, variation with respect to many joint parameters and fatigue and failure modes. In the same time period, Song has simulated dynamic response of bolted joints using finite element method. He has developed a two dimensional adjusted Iwan beam element which comprises of springs and frictional sliders which behaves non linearly due to stick slip phenomena. This element has 6 joint parameters which are to be determined. At a specific location on the beam, acceleration responses are calculated while varying joint parameters and further these parameters are validated by comparing the simulated results to the experimental results of the system.

C.T.McCarthy and M.A.McCarthy (2004) has done the finite element analysis of three dimensional bolt hole in a single lap composite bolted joint. They summarized that the clearance in a bolt hole occurs by increasing rotations of bolt and decreasing its contact area and stiffness and so it majorly affects the stiffness and onset load of the joint and it should be kept under solid consideration while designing.

M. J. Oldfield, H. Ouyang and J. E. Mottershead (2005) worked on a single joint rig. They have taken experimental results from a setup made by joining two plates with a bolted joint, excited at resonant frequencies. Micro-slip is noticed at the interface and thus time dependent displacement is recorded. Hysteresis curves are drawn between torque and angular displacement for many preload and excitation conditions. In the hysteretic curves, a deviation is clearly visible because of the micro-slip making the presence of super harmonics in frequency more visible. They recorded all time dependent data and used analytical approach to express it.

Hamid Ahmadian and Hassan Jalali in same year (2005)proposed a nonlinear model for bolted lap joints and interfaces, capable of representing the dominant physics involved in the joint such as micro/macro-slip. He modeled joint using a nonlinear spring to represent the softening phenomenon of the joint interface due to slip. An approximate solution for the dynamical behavior of assembled structure is obtained using the method of multiple scales.

The solution provides frequency response function of the beam at any desired location due to a point excitation at a certain location. The obtained frequency response function is compared with the corresponding experimental counterparts to identify the parameters of the bolted joint interface. In the identification procedure joint interface parameters are fine tuned so that the differences between calculated and measured frequency responses are minimized.

Hamid Ahmadian and Hassan Jalali (2006) further presented a generic element formulation to illustrate non-linear characteristics of a single bolted lap joint. This generic element formulation was comprised of a general stiffness and damping matrix representing the nonlinearities within the joint interface. An assumption is made that the mass matrix is all known, whereas stiffness and damping matrices were obtained by the comparisons of estimated values and measured responses. Generic element parameters were also observed by optimized experimental and analytical results comparison. But a similar behavior was seen between all observed and predicted results which proved that the given generic model can be used to model the joints accurately. This concept could be extended further to other kind of joints as well to get simple and compact representation of structures for easy determination of their dynamic behavior.

Segalman (2006) introduced different methods to incorporate damping within dynamics of mechanical structures and read different parameters significance from these experiments. He also advised methods to get equivalent linear system of representation for the structures to have response at specific loadings. Although some of the techniques were already there but a major problem with them was that they only validate the linear model of same magnitude and type as that of the calibration. Also physical presence of structure is highly required for its calibration. Right now these newly advised techniques for the modeling of joints through dynamics codes have still much to be done and need time for proper implementation.

O. Damisaa and V.O.S. Olunloyob (2007) discussed that layered structures that are under dynamic loading with varying frequency and non-uniform pressure at the interface effects the dissipated energy from the system and also the logarithmic damping decrement involved in the slip damping phenomenon. He calculated the system's response, the slip at the interface, energy dissipation and the optimized pressure applied on the interface surface for a composite cantilever beam under different excitation types. He concluded that dissipated energy under dynamic loading is lower than that of the static loading. Also frequency ratio highly effects the value of transverse displacement and the slip at the interface whereas the combined effect of frequency ratio and pressure gradient again normalizes the displacement and slip values.

Jason D. Miller and D. Dane Quinn (2008) discussed that in the analysis of structural systems, modeling mechanical joints in an accurate and computationally efficient manner is of great importance composed of a large number of connected components. He himself decomposed an interface model into a series-series Iwan model together with an elastic chain, subjected to interfacial shear loads. For the simulation of frictional damping a reduced-order formulation of the resulting model significantly reduced the computational requirements . Results were presented as the interface subjected to harmonic loading of varying amplitude. The models presented were able to qualitatively reproduce experimentally observed dissipation scaling. Finally, the interface models were embedded within a larger structural system to illustrate their effectiveness in capturing the structural damping induced by mechanical joints.

Khattak (2009) discussed the dynamic characteristic of joints for different types of geometries under different excitation conditions. He figured out that in order to get a reduced model for joint, the main hurdles are the nonlinearities involved because of the occurrence of micro-slip during the phenomenon. He applied POD technique to get to a reduced model for joints without violating any laws, where this technique helps in reducing the linear system of equations and the nonlinear part is fully determined first and then reduced before the phase of integration. By using this technique, calculation time was majorly reduced. This reduced order model was made for a joint that is in isolation from the system and is externally excited, so that a general reduced model can be obtained and further it can be accommodated for different geometries and excitation conditions. It can also be used in accordance with the structures to read their dynamics. He verified his reduced model with the full model and result's compatibility was seen.

Hadjila Bournine, DavidJ.Wagg and SimonA.Neild (2010) analyzed the frictional damping within a column comprised of two bolted beams. They declared that the dynamic friction can enhance the damping effects of this structure significantly. A complete analysis was done to see the frictional effects and the effects by changing the tension within the bolts, on the structure dynamics. When the tension within the bolt was low, the column properties were similar to that of a single beam with an exponential decay, whereas when the tension within the bolt was high, the column properties were equivalent to a beam of twice thickness. These results were verified by getting the variation in the natural frequency of beam column model under both loading conditions. If the dynamic friction is there and tension within the bolts will be adjusted properly, there will be chances of getting friction damping value ten times higher than defined viscous damping.

Z.Y. Qin, S.Z. Yan and F.L. Chu (2010) analyzed the dynamics of a clamp band joint model along with the flange of interface ring which is subjected to axial excitation. They studied the sliding contact and the friction produced between the parts of the system and calculated the system parameters by having nonlinear finite element analysis. For this analysis, a scaled model of the connector strip clamp was made and static experiments verified the common model of the clamping band. They studied the forced response of this dynamic model under axial excitation and nonlinearity effect due to the clamp band joint. The results calculated on the joint model are quite in line with the experimental data; therefore the said model was verified. Along this, the proposed model showed that the clamp band joint reduced the stiffness of the system and generates nonlinearity in the system. During payload response, jump phenomenon was also visualized due to the stiffness change in the system. The parametric study showed that by increasing the wedge angle, the resonance frequency decreases and the amplitude of resonance increases. Additionally, it was concluded that changes in preload had no significant effect on the response of system as the excitation remain within the allowable design load.

Yu Luan and Zhen-Qun Guan (2011) investigated the static behavior of the bolted flange connections. For this purpose a simple nonlinear model was developed in which the mechanical properties of the joint were modeled by bi-linear spring. The results revealed that the bi-linear springs of different compressive and tensile modules show accurate axial stiffness; also they provide additional flexibility to the system as compared to the linear beam model. To visualize the dynamic characteristics of simply bolted flange, a mass - spring system with two degree of freedom was developed. There confirmed the occurrence of coupling of longitudinal vibration and lateral vibration which is due to the coupling element in the stiffness matrix. The impact behavior of this mass spring system was studied which showed that that transverse impact can excite the coupling longitudinal vibrations and the longitudinal impact only excites longitudinal vibrations.. In addition to this it was also established that the longitudinal frequency doubles the transverse frequency under transverse impact . Finally, a reduced nonlinear dynamic model was proposed to accommodate the results obtained from the tests in longitudinal and transverse directions but a linear beam model cannot generate exact longitudinal response under lateral impact.

I. Ullah and M. Yasin (2011) presented the dynamic response of shear lap bolted joints subjected to multi-harmonic loading where the response was obtained for all detailed as well as reduced order models. They studied micro-slip and energy dissipated at the joint interface while considering the Coulomb Friction Model. The detailed nonlinear dynamic model and the reduced model obtained by applying the Proper Orthogonal Decomposition technique to the FEM model, both are tested at harmonic and multi-harmonic vibrations at frequencies lower and higher than that of resonance and the results i.e. slip and energy dissipated were compared for both cases. And they proved that reduced order model extracted by using POD to the full solution are compact and computationally less intensive and all the similar results could be obtained without any defined loss of accuracy.

J. Abad, J.M. Franco, R. Celorrio and L. Lezáun (2011) studied the joint behavior by analyzing a pre stressed bolted lap joint under relative displacement. They presented a 3D FEM model with the help of design techniques of experiments in order to adjust the contact parameters. They validated the theoretical results obtained by elasto-plastic analysis with their experimental ones. They analyzed the preload effect and displacement variation along with the nonlinear joint behavior and calculated the equivalent stiffness and energy dissipated from the hysteresis curves drawn. To adjust the contact parameters of 3D FEM model effecting the hysteretic behavior, a tensile test was applied according to the design techniques of experiments, which helped a lot in reducing the computational cost. The numerical results after tuning these parameters were in good co-ordination with the experimental ones where maximum error of 5% in stiffness and 10% in dissipated energy was recorded. Thus they illustrated the nonlinear joint behavior under the influence of displacement and preload value over stiffness and dissipated energy of hysteresis.

Morteza Iranzad and Hamid Ahmadian (2012) modeled a thin layer interface of a bolted joint within an assembled structure with elasto plastic material behavior. A constitutive relation was introduced by them to represent the joint behavior in three separate phases i.e. sticktion, micro-slip and macro-slip. Thin layer parameters were identified under constant force amplitudes applied to the nonlinear dynamic model of joint and by lowering the differences between response predictions and experimental results. Finally model was verified under comparison of model predictions to the observed experimental results for different amplitude force levels and a good agreement was seen between the results. Thus the strategy they proposed for modeling nonlinear effects in joints was simple, accurate, with low computational power and applicable to joints FEM model.

2.3. POD technique

The proper orthogonal decomposition (POD) is a statistical method that represents data in a compact form. This method can be used for two purposes i.e. to get a reduced order model by projecting data of more degree of freedom to a space of less degree of freedom and extracts the unique and relevant data for this purpose. It extracts the spatial structures or modes from time dependent data to estimate the system response. These mode shapes are then used to reduce the models through Galerkin reconstruction process.

The output data of a system is discretized in time and space. For n number of observations, p –dimensional vectors are collected giving an (mxn) matrix i.e.

$$\mathbf{X} = [\mathbf{x}_1 \cdot\cdot\cdot \mathbf{x}_n] = [\mathbf{x}_{11} \cdot\cdot\cdot \mathbf{x}_{1n}$$

$$\cdot\cdot\cdot \;\; \cdot\cdot\cdot \;\; \cdot\cdot\cdot$$

$$\mathbf{x}_{m1} \cdot\cdot\cdot \mathbf{x}_{mn}] \qquad \text{......................(2.3)}$$

Single value decomposition of this matrix gives[1]

$$\mathbf{X = U\,S\,V^T} \qquad \text{........................(2.4)}$$

Where U is a (mxm) orthonormal matrix comprised of left singular vectors, S is a (mxn) semi positive definite matrix and V is an (nxn) orthonormal matrix with right singular vectors. Indeed

$$\mathbf{X\,X^T = U\,S^2\,U^T} \qquad \text{........................(2.5)}$$

$$\mathbf{X^T\,X = V\,S^2\,V^T} \qquad \text{........................(2.6)}$$

Major advantage of using SVD to compute POD is that instead of going into the detail of eigen vectors, an additional information is obtained from the vectors of V matrix. Each vector of V consists of the time modulation of proper orthogonal modes which are normalized by the singular values. This information about modes is very important in the dynamics of structures and plays a vital role to update the model of nonlinear systems.

This POD method is well applied to many fields like structural mechanics, fluid dynamics, thermal processes and signal processing. Thus many researchers have used it in their respective fields and have evolved many new things using this technique. Some of the highlighted work related to this study is given below.

M. F. A. Azeezs and A. F. Vakakis (2000) applied K-L method on the structures under vibrations, of which they mainly focused on vibro-impacting beams and overhung rotors. They first applied this method on an impacted beam model to extract the modes and to visualize the energy transfer patterns. These K-L modes are then used to generate lower level models with the help of Galerkin approach. Then they applied K-L method to an overhung rotor that also experienced vibro-impacts to study the nonlinear effects on the dynamics of structure, visualize the energy transfer patterns at both low and high modes, confirm the reduced models obtained for dynamics and to prove that this method is a useful tool to study real time changes within the system due to vibrations. They also did an experimental study on overhung rotor under vibro impacts at the end.

Tapan K. Sengupta and S. Dey (2004) presented the relationship between space and time dependence for dynamical fluid models with bypass transition. Their work is useful for reduced order modeling of models with fluid-structure interaction. They used analytical and experimental results for by-pass transition presented by Sengupta and used POD technique to study the unsteady viscous flow of fluid. The output of the dynamical system was obtained in the form of DNS results. These results showed that even for strong excitations, only a small number of modes are required to read the spatial structures of the fluid motion. Where ever local flow information was required, linearity property of POD was highly recommendable.

Currently many more people are working on this technique to get through dynamic characterization and order reduction of systems. This method can be extended to many applications within structural dynamics like damage detection, modal analysis, sensor validating process, active control and much more.

CHAPTER 3

MODELING OF SYSTEM

3.1. Introduction

As chapter 2 has discussed all the research carried out in the finite element modeling of bolted joints and dependence of different parameters on system response. Khattak (2006) developed a generic 1D FEM model of a bolted shear lap joint between a rigid and a flexible plate which was parameter free still didn't violate any physics laws. He studied the dynamics of an isolated joint while keeping some assumptions like joint will not fail under loading; no material nonlinearity is going to be considered etc. Some simplifications were done to the model, they were:

(1) Macro-slip doesn't exist in the joint
(2) Any kind of force or displacement is imposed only at the joint's free end
(3) Pressure applied due to the presence of joints is uniformly distributed on the joint's top surface
(4) Elastic compliance doesn't exist within contact areas
(5) Force of Coulomb friction is considered

POD technique was used to get a reduced order model which can reproduce all the results similar to the detailed one. He proved that the proposed simplified model can be used instead of a complex joint model within a structure and thus can be very useful computationally without any loss of accuracy. Here, first his work is verified by modeling a three dimensional joint with one rigid and one flexible plate in ANSYS under same loading and boundary conditions, and then extended with both flexible plates experiencing friction between them.

3.2. Physical system description

Physical model of the system consists of one rigid and one flexible steel plates connected through a shear lap bolted joint. The upper plate acts as a cantilever beam i.e. it's one face is constrained in all DOFs and it is bound to move along the lower one. An imposed horizontal displacement on the lower plate is applied to its free end as an effect of excitation.

Instead of modeling a true bolt between the plates, an equivalent uniform normal pressure is applied on the top surface of the overlapping area of plates. Thus a little simplification is made to get a quick convergent solution.

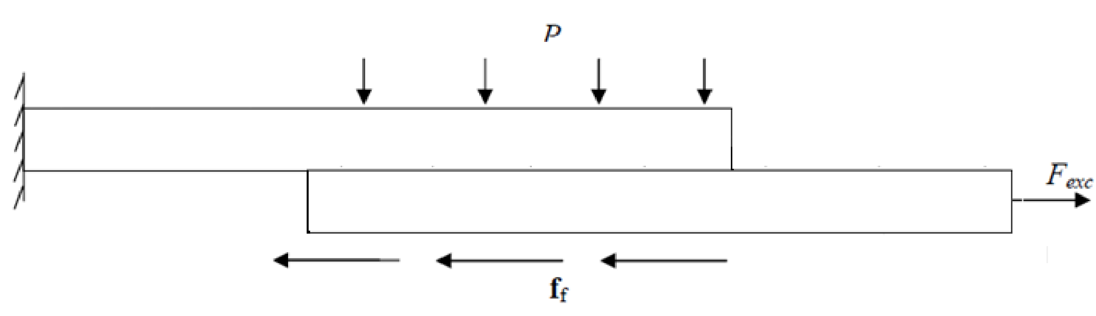

Figure 2. Free body diagram of test model

The joint is going to be studied in isolation from the system to get a clear understanding of the nonlinear dynamics of the joint.

3.3. Finite Element Modeling

The above described geometry is modeled in ANSYS 14.1 to carry out a complete 3D FEM analysis and study the respective parameter changes.

3.3.1 Model Geometry

The overlapping area of plates is modeled in ANSYS, where dimensions of overlapping area of both plates is 500x50 mm. Thickness of both plates is 10 mm. Both plates are made up of steel with material properties i.e. Yield strength $S_y=6\times10^8$Pa, Ultimate strength $S_u=8\times10^8$ pa and Poisson ratio= 0.33. However, the elastic modulus of both the plates is different i.e. rigid plate is modeled with elastic modulus $E=2\times10^{14}$Pa and flexible with $E=2\times10^{11}$Pa.

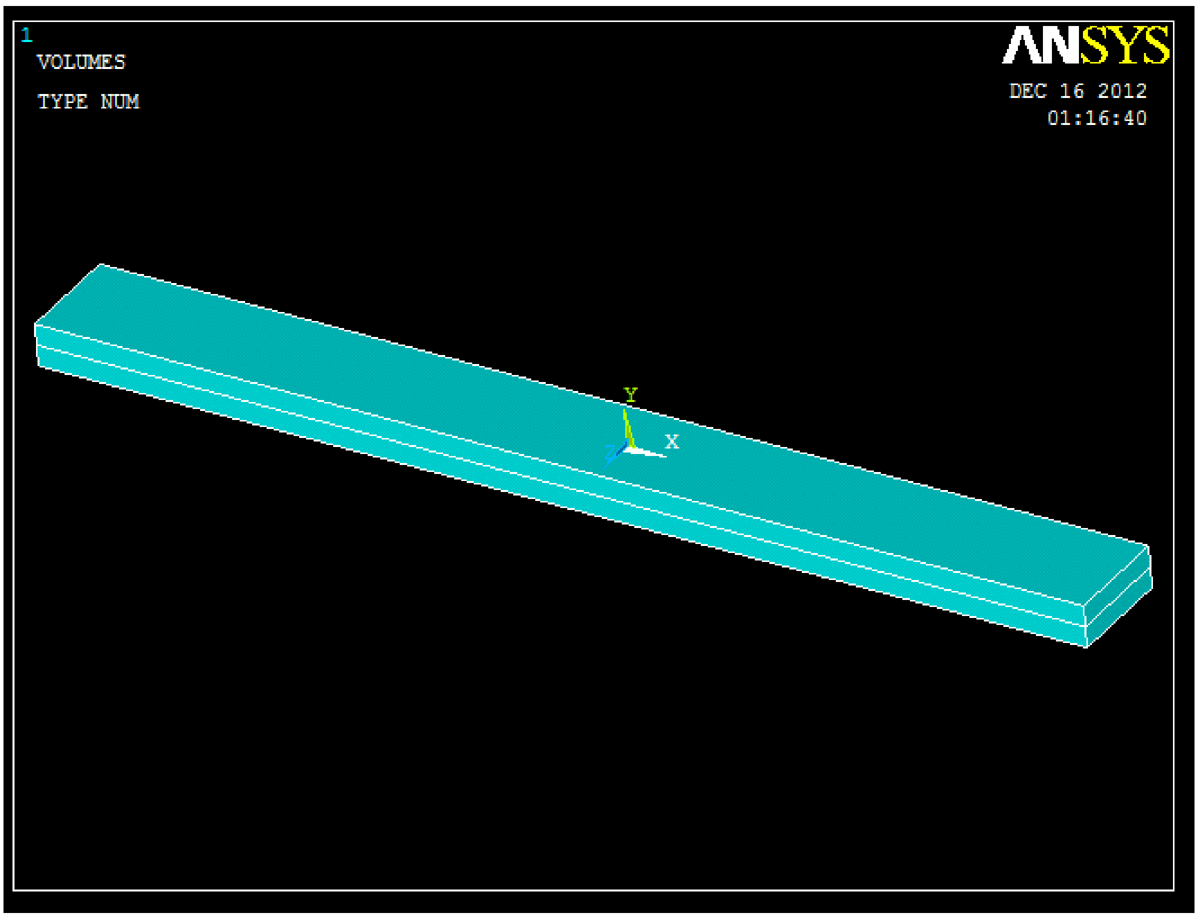

Figure 3. Modeling of volumes in ANSYS

3.3.2 Finite Element Mesh

As it is a contact problem, so mesh of the volumes should be selected with care such that behavior of contact can be captured while having a reasonable mesh size at the same time. 3 or 4 sided mapped meshing is done with 200 elements along length, 15 along width and 3 along thickness of each beam. The model is meshed with solid 8 node 185 brick elements. Finally the model resulted in 9,000 elements.

This number of elements are decided after having a complete mesh sensitivity study as the required displacement value starts converging at free end under 1000Hz frequency with 200 elements along length. If there will be no non-linear forces, less number of elements can be an option.

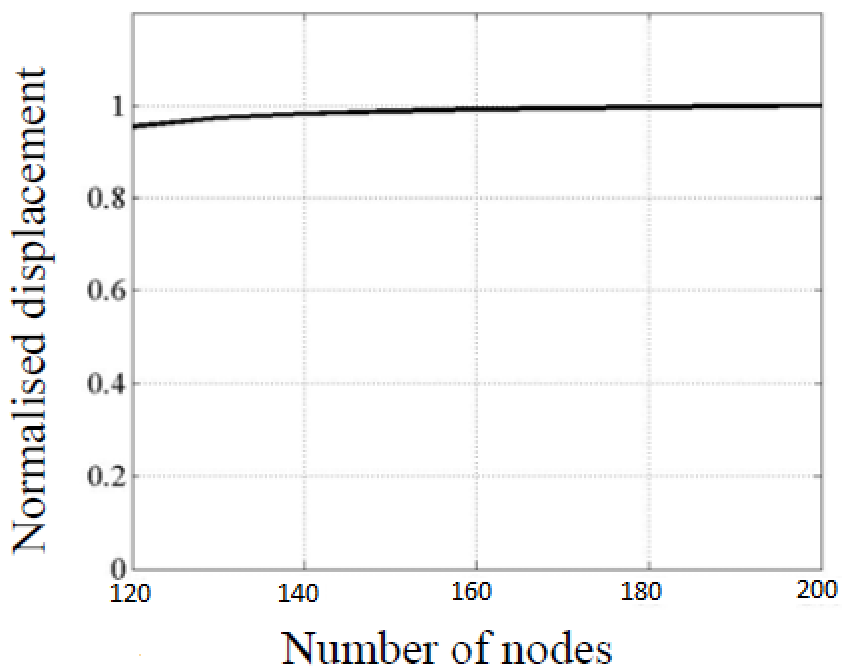

Figure 4. Mesh size selection along length

Mesh size of both the plates is selected to be same, in order to get the nodes of upper plates coinciding with that of lower one and if coupling will be required, it will be easy to get it done. Because of the displacement of these active region nodes, energy dissipation from the system occurs. After meshing, the model looks like-

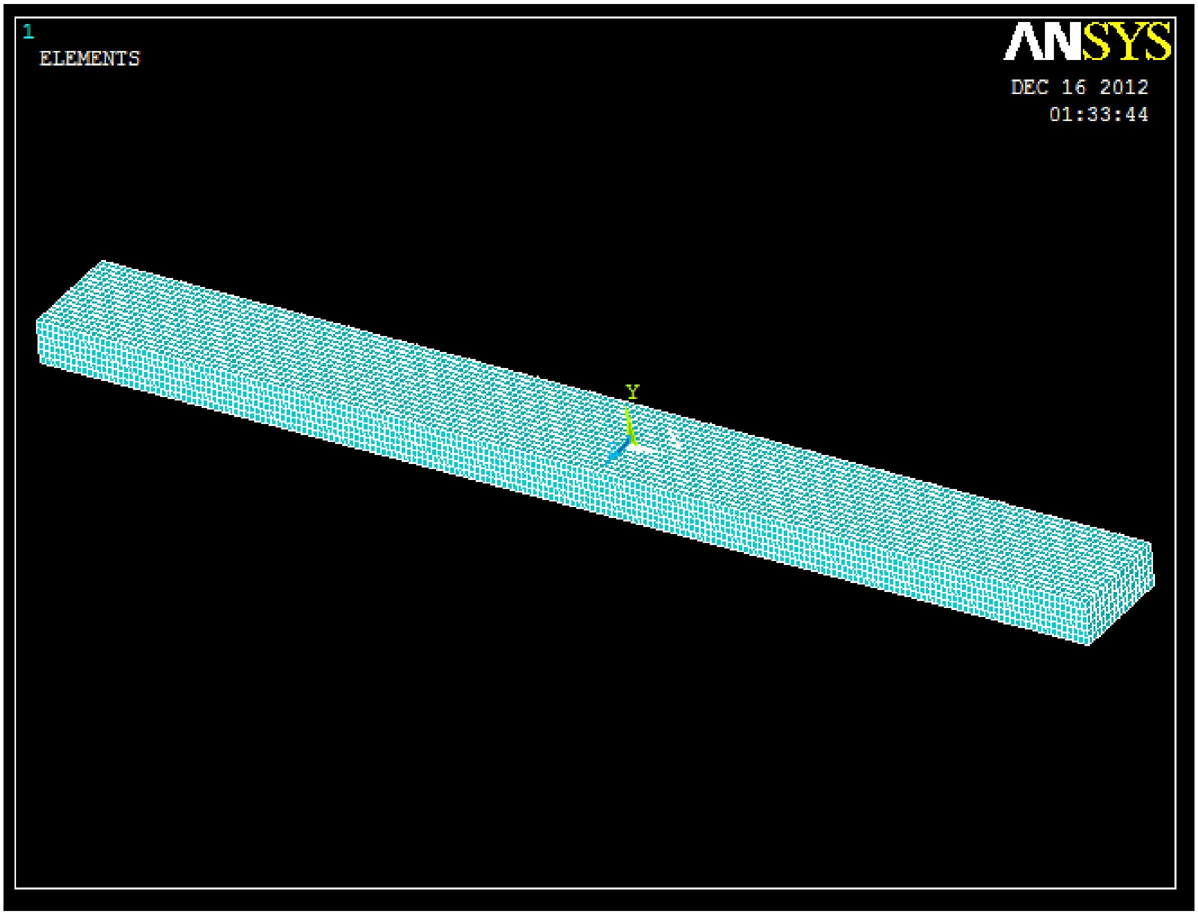

Figure 5. Meshed volumes in ANSYS

3.3.3 Establishment of Contact

Before the application of load, to establish a contact between the plates is necessary, because without the presence of initial contact, the solution will never converge and both the plates will not be bound to move on one another, or not fulfill the joint requirement. Thus, a standard contact is generated by the contact manager between the lower contacting surface of upper plate and upper surface of lower plate. It is a surface to surface contact modeled between two plates with element types CONTA174 and TARGE170 and friction coefficient of 0.7 and normal penalty stiffness 1.0.

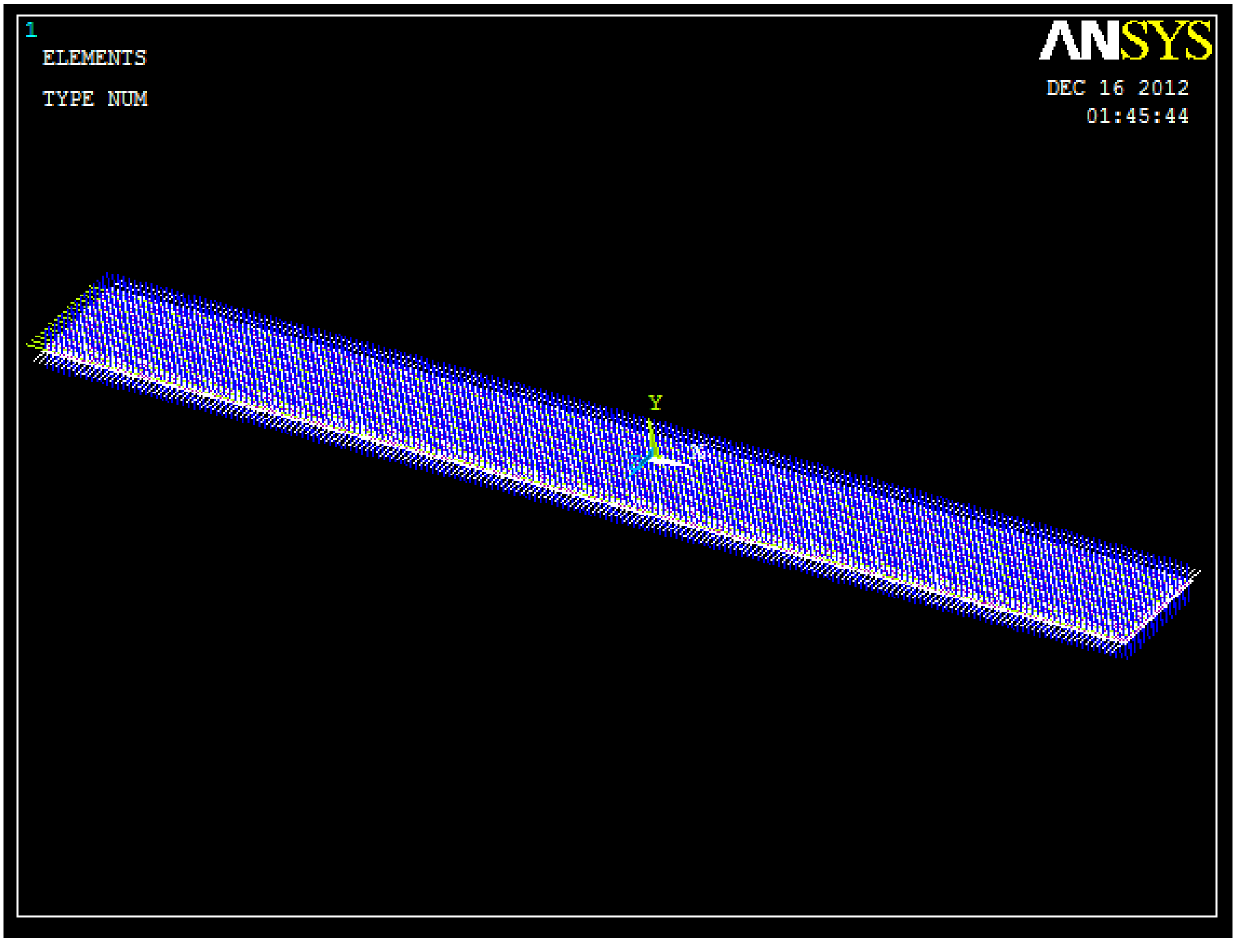

Figure 6. Contact area between two plates

3.3.4 Coupling

A few nodes are coupled from the center of the interface of the beams throughout the thickness. This coupling will impose a zero displacement at the center of the joint, modeling the bolt characteristic of relative zero displacement with respect to the plate.

As the length of the plates is too long, so there is a possibility that solution will diverge even after the establishment of standard contact. This coupling will also avoid slippage of plates under applied loading, and will not let the contact open even in case of high amplitude of excitation.

Since the stiffness matrix of the model is singular, the coupling in fact makes it non singular. This coupling will not disturb the physics of the system as well.

3.3.5 Load steps

As the excitation loads are supposed to be applied gradually to the system, so solution to it is carried out in divided time steps. Here boundary conditions and loadings are applied in two steps and time interval of each step is decided according to the excitation frequency.

If excitation frequency is 1000 Hz, the time for each load step taken will be 0.001 sec, if 100 Hz, it will be 0.01 sec and so on.

3.3.5.1 Load Step 1

In the first load step, all the boundary conditions are applied and the equivalent pressure of bolts is attained. As the loading applied will be ramped so before the application of external excitation in the second step, it is necessary to build up the required pressure due to bolts on the system.

Boundary Conditions

As upper plate is a cantilever beam, so it's right face is constrained in all degree of freedom. It is not desired to let the plates move in y direction or to let the plate bend in any case, so the lower plate is constrained in y direction.

Selection of Clamping Pressure

Instead of modeling complete bolt design, case is simplified and a uniform pressure is applied on that respective area. As M10 bolts are used, so the equivalent pressure is to be calculated to apply on the overlapping top surface area. The general layout for placing M10 bolts is given as-

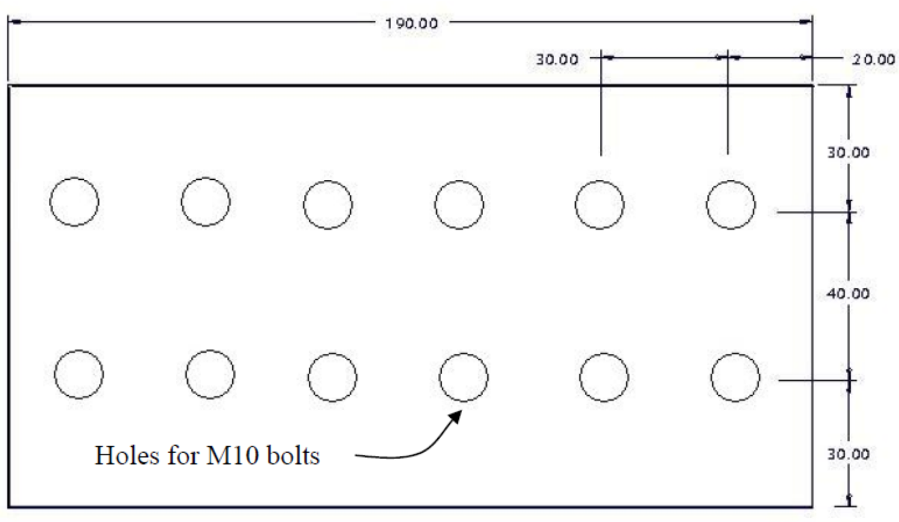

Figure 7. General layout of M10 bolts

To clamp two plates of surface area of 500x50 mm, required bolts are calculated according to the given layout. The clamping force of a bolt depends on the fraction of total cross sectional area of the bolts to the total area of the joint and the strength of the bolt. For the current case, fraction of areas comes out to be 5% and proof load of a M10 bolt is 1000MPa. Taking 70% of the proof load, the total clamping pressure of M10 bolts comes out to be 35MPa. Initially it was applied to the model and then a major distortion in the elements was seen which lead to convergence difficulties and finally the pressure of 8 MPa is decided. In the first load step, this pressure is applied as a ramped transition.

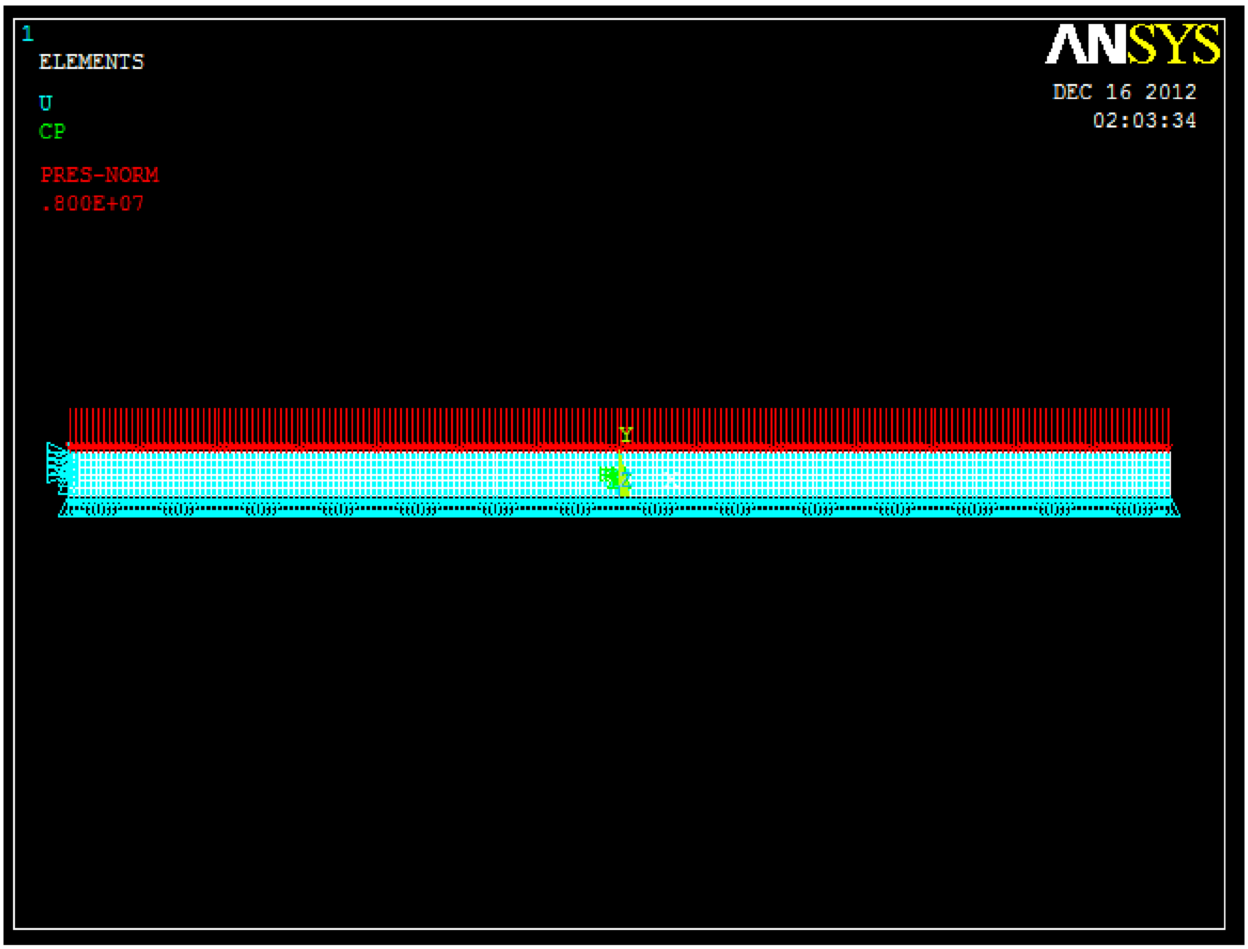

Figure 8. Boundary conditions in ANSYS

This pressure is considered uniform as for a single bolt contact pressure is similar to the shape of a cone. However when several bolts are used along the length of the joint, these cones overlap to form a uniform distribution of pressure.

3.3.5.2 Load step 2

Imposed Displacement

Now a simplified bolted lap joint model representation is ready. In order to analyze the response of this system under external vibrations, it should be simulated with an imposed force or displacement at the free end. Let the system be subjected to a shearing force of 90% of the frictional limit, for this the respective calculations are-

As pressure on overlapping area is 8 MPa, so equivalent force on that area will be

$$F = P \times A \qquad \text{.......................(3.1)}$$

$$F = (8 \times 10^6) \times (250 \times 10^{-3} \times 50 \times 10^{-3})$$

$$F = 100 \times 10^3 N = 100\ kN$$

Frictional force is defined as

$$F_f = \mu \times N \qquad \text{.......................(3.2)}$$

$$F_f = 0.7 \times 100 \times 10^3$$

$$F_f = 70 \times 10^3 \mathrm{N} = 70\ \mathrm{kN}$$

Applied force is 90% of this frictional force to get a slip in 90% of the joint length

$$F = 90\%\ F_f \qquad \text{.......................(3.3)}$$

$$F_f = \frac{90}{100} \times 70 \times 10^3$$

$$F_f = 63 \times 10^3 \mathrm{N} = 63\ \mathrm{kN}$$

Shear stress due to this 90% of frictional force is

$$\sigma = \frac{F_f}{A_c} \qquad \text{.......................(3.4)}$$

$$\sigma = \frac{63 \times 10^3}{(50 \times 10^{-3}) \times (10 \times 10^{-3})}$$

$$\sigma = 126 \times 10^6 = 126\ \mathrm{MPa}$$

Stiffness of a beam is given as

$$k = \frac{A \times \mathrm{E}}{L} \qquad \text{......................(3.5)}$$

$$k = \frac{(50 \times 10^{-3}) \times (10 \times 10^{-3}) \times 2 \times 10^{11}}{(250 \times 10^{-3})}$$

$$k = 400 \times 10^{6}\ \mathrm{N/m}$$

By the applied force, the system should displace

As, $$F = k \times x \qquad \text{......................(3.6)}$$

$$x = \frac{F}{k}$$

$$x = \frac{63 \times 10^{3}}{400 \times 10^{6}}$$

$$x = 0.1575 \times 10^{-3}\mathrm{m} = 15.75 \times 10^{-5}\mathrm{m}$$

Thus, an imposed displacement is applied on the right face of lower beam i.e. in second load step the external vibration is simulated with a sinusoidal displacement of 7.87×10^{-5} m amplitude in x-direction. In other words, it is displaced 7.87×10^{-5} m to the +ive x-direction, then back to mean position, then 7.87×10^{-5} m to the –ive x-direction, back to mean position and the cycle goes on.

Excitation Frequency

The first extensional resonance of a plate with dimension 500×50×10 mm is calculated as-

$$\omega_n = \frac{(2n+1)}{2l}\pi c \qquad \text{......................(3.7)}$$

At n=0,

$$\omega_n = \frac{\pi c}{2l}$$

And

$$f = \frac{\omega_n}{2\pi} = \frac{\pi c}{2l \times 2\pi} = \frac{c}{4l} \qquad \textbf{........................(3.8)}$$

Where

$$c = \left(\frac{E}{\rho}\right)^{1/2} = \left(\frac{2 \times (10)^{11}}{7800}\right)^{1/2} = 5063.7$$

And

$$l = 250 \times (10)^{-3} m$$

So

$$f = \frac{5063.7}{4 \times 250 \times (10)^{-3}} = 5064\ Hz$$

When system is excited at this frequency, active length changes and is more than 250 mm. Thus excitation frequency is selected well below the system resonance i.e. 1000 Hz

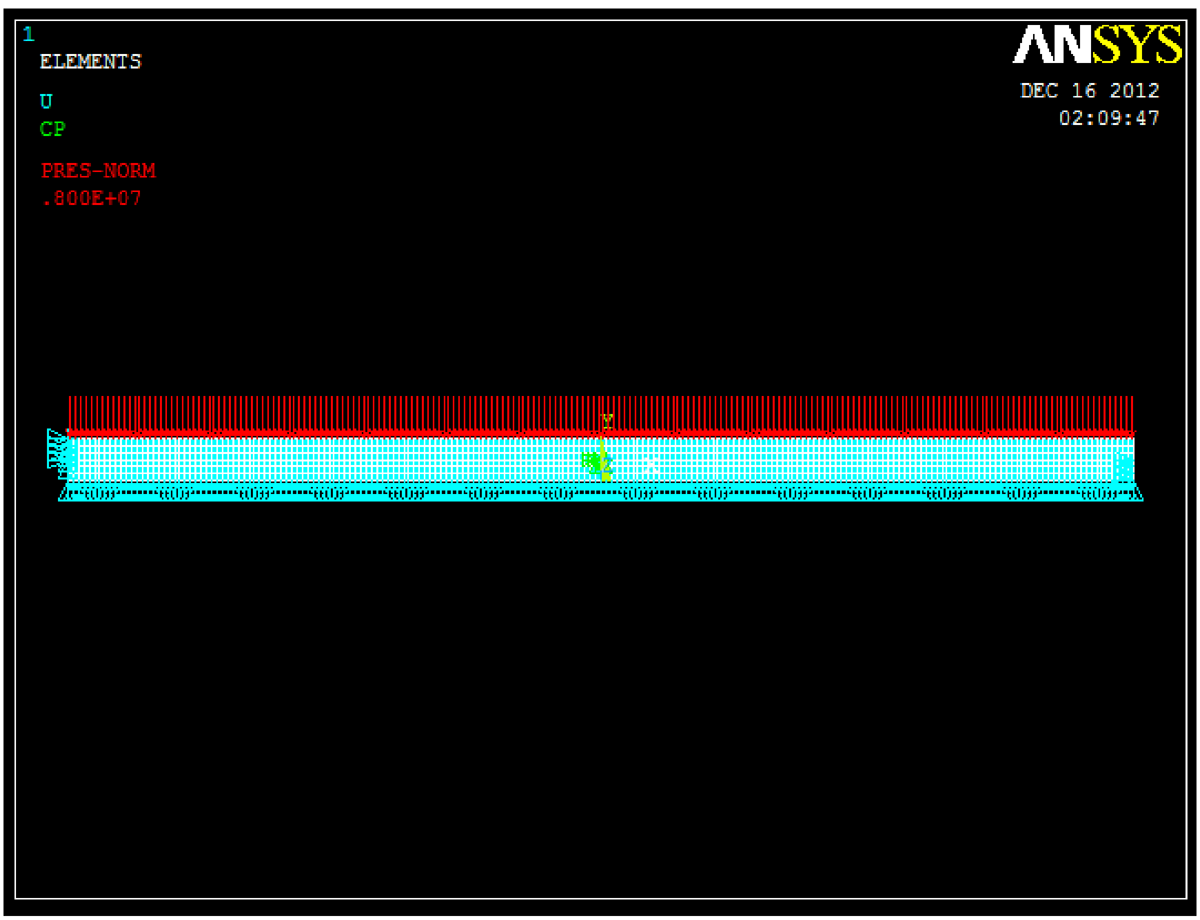

Figure 9. Complete loading in ANSYS

3.3.6 Solution

After the application of the external vibration at specified frequency whether by the applied force or imposed displacement, a transient response of the system is calculated. At each time step, force is recorded against value of the imposed displacement or displacement is recorded at applied force. This imposed displacement and force is then plotted and hysteresis curve is obtained. The area under this hysteresis curve will represent the energy loss due to the friction and micro slip between the plates or the displacement of the active nodes present in the interface region of bolted joint.

3.3.7 Modeling of both flexible plates

This simplified 3D generated model is extended to both flexible plates, by changing the material properties i.e. both with elastic modulus $E = 2\times10^{11}$Pa. Apply the boundary and loading conditions in the same way as discussed above to simulate the nonlinear dynamics of the joints while doing all the calculations accordingly.

CHAPTER 4

RESULTS AND DISCUSSION

The model that is to be analyzed is a lap joint holding two steel beams together under a pressure equivalent to the number of bolts. It is a shear joint and shear joints are generally used under longitudinal loading rather than the bending one. Dynamics of this joint is studied while considering both beams flexible in 3D unlike that of Khattak (2006) who analyzed model in 1D by pressing a deformable beam against a rigid one. However, for a better understanding of joint dynamics, the joint is preferably investigated in isolation from the structure.
Furthermore it is assumed that under given loading joint will not fail or there will be a patch of length always that will not experience relative displacement. Such patch of length is known as the 'grey length' of the joint while the one which undergoes micro-slip is known as 'active length'. Two types of nodes i.e. sticking and slipping nodes are there in the active length whereas only sticking nodes exists in the grey length as they won't allow any kind of relative displacement. Thus, no macro-slip but micro-slip in a small portion of length will be considered during this analysis.

4.1. 3D FEM model

The above system is simulated by 3D FEM model. The mid nodes along the length are given zero imposed displacement and midpoint is taken as the reference point. The upper beam is under pressure equivalent to that of joints. This uniformly distributed pressure is applied normal to the beam. Upper beam is modeled as a simply supported beam while the lower flexible beam is allowed to move against the upper beam in the presence of frictional forces when it is subjected to an imposed harmonic displacement in longitudinal direction at its free end. Resultant model is three dimensional and is modeled with FEM technique. Brick elements are used to model the structure and around 200 elements are taken along its length.

Verification Model

Beams of size 500×50×10 mm are modeled in 3D to verify with idealized system of Khattak with which he verified his model with Csaba (1998), using 200 brick elements along length. Upper plate is made rigid while lower flexible with friction coefficient taken between them is 0.7; pressure applied due to bolts is equivalent to 8MPa or 100kN force on the upper plate. Imposed displacement of 4.375×10^{-5} m at 1000Hz frequency is applied at the free end in positive x direction and then with same amplitude in negative x direction, which can move 50% of the joint's length. On the given boundary and loading conditions, the displacement profile of the system is plotted as-

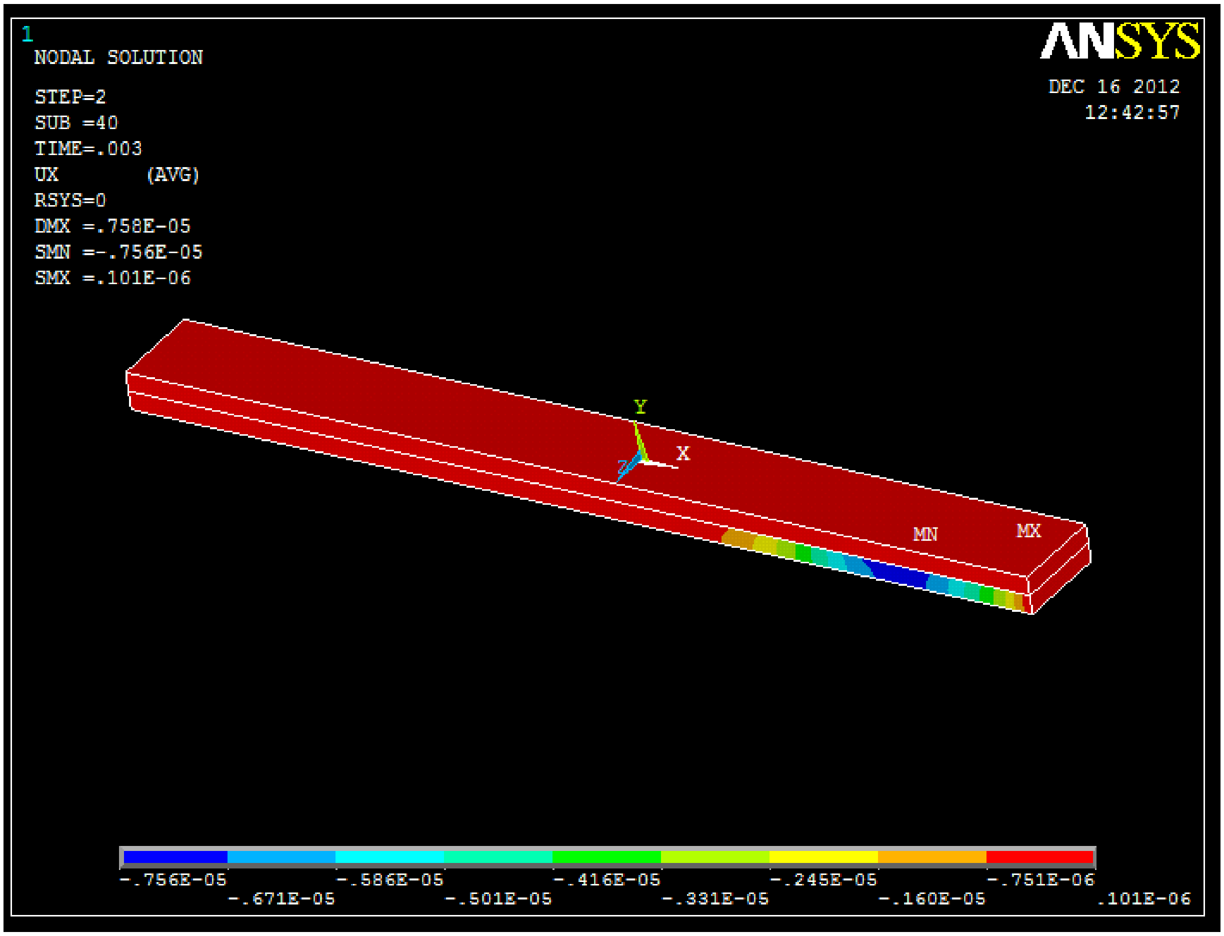

Figure 10. Displacement profile of Idealized Model

Recorded displacement and force for this case are given as-

Figure 11. Displacement graph

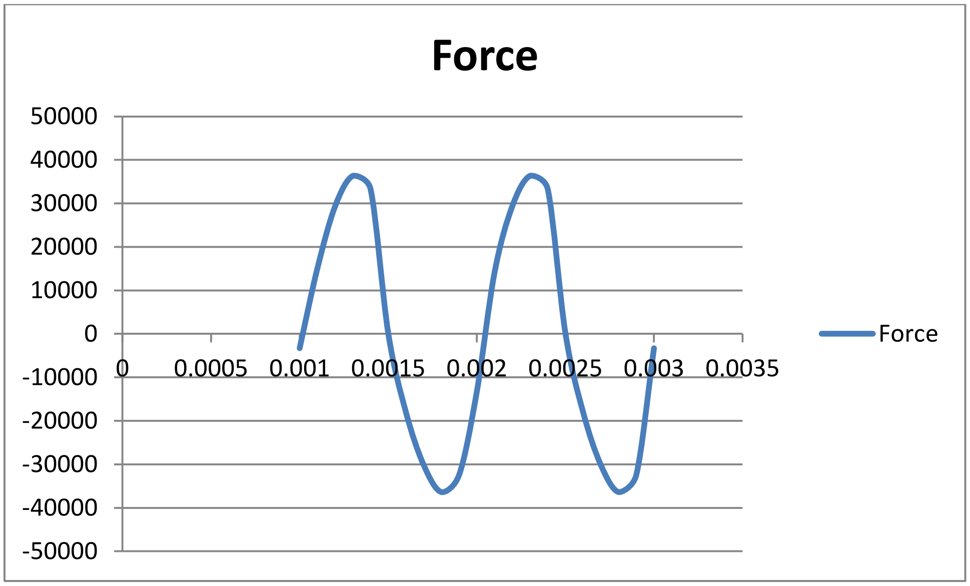

Figure 12. Force Graph

The hysteresis plot between imposed displacement and force is drawn and dissipated energy from the joint is calculated which comes out to be 2.0692J, whereas with same system drawn in 1D by Khattak: dissipated energy was calculated to be 2.0431. An error of 1.27% exists which is due to the meshing difference i.e. if the model is more finely meshed, this error can be reduced but computational time will increase, so an acceptable compromise is done between accuracy and computation time –

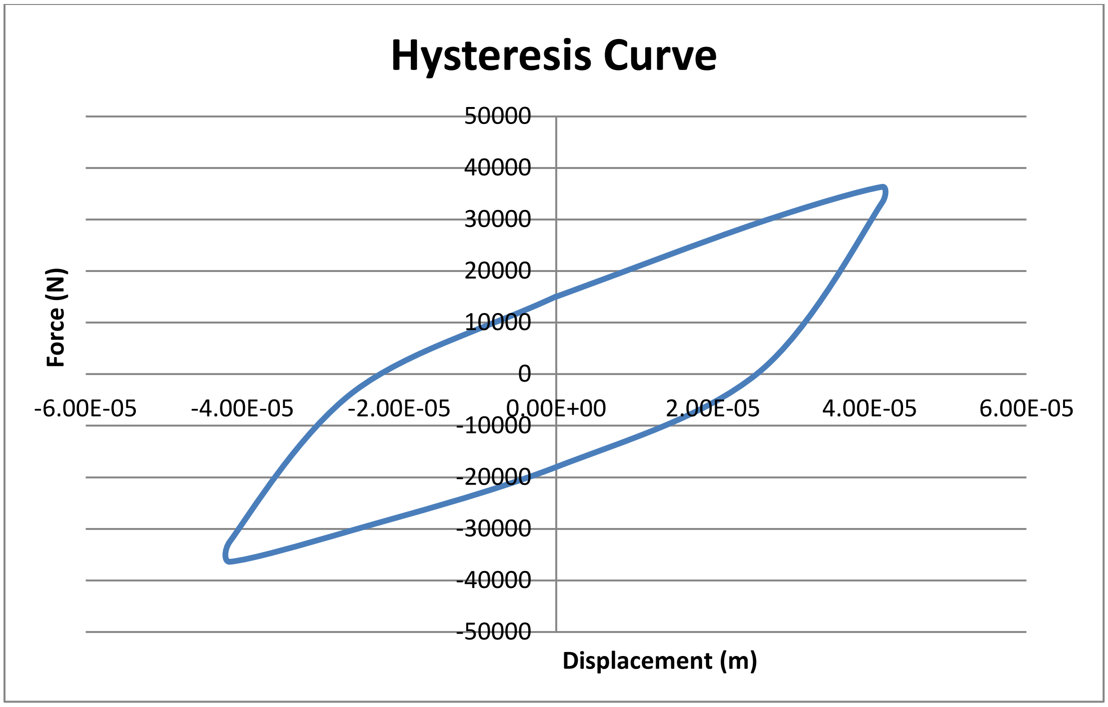

Figure 13. Three Dimensional Verification Model

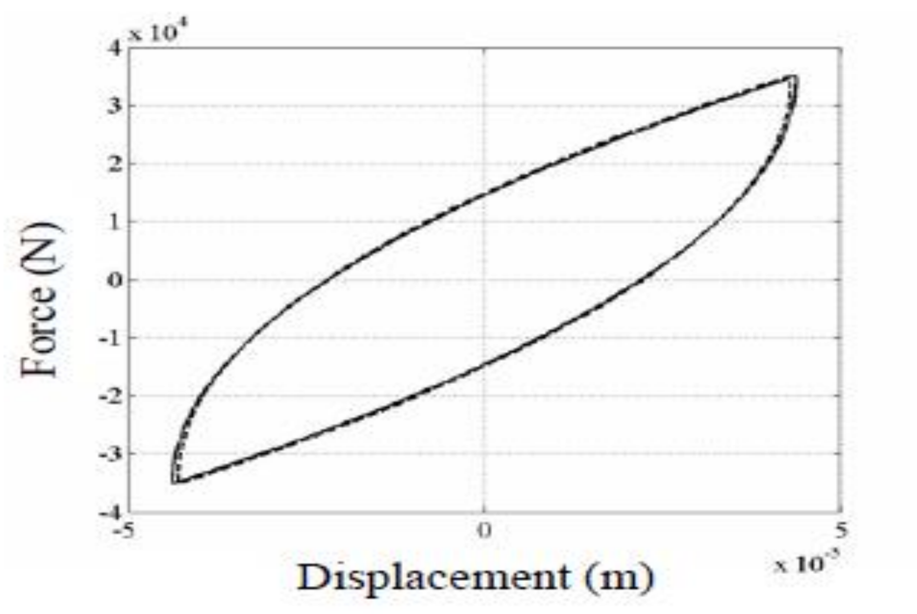

Figure 14. One Dimensional Verification Model

Reference Model

Khattak modeled his reference system with same geometry, initial and boundary conditions as that of the idealized system, but displaced with 90% of the frictional force i.e. with amplitude of 7.87×10^{-5} m at 1000Hz frequency applied at the free end of the lower plate, to cause a slip in 90% of the joint length. Same reference system is made in 3D, and the displacement profile obtained is like-

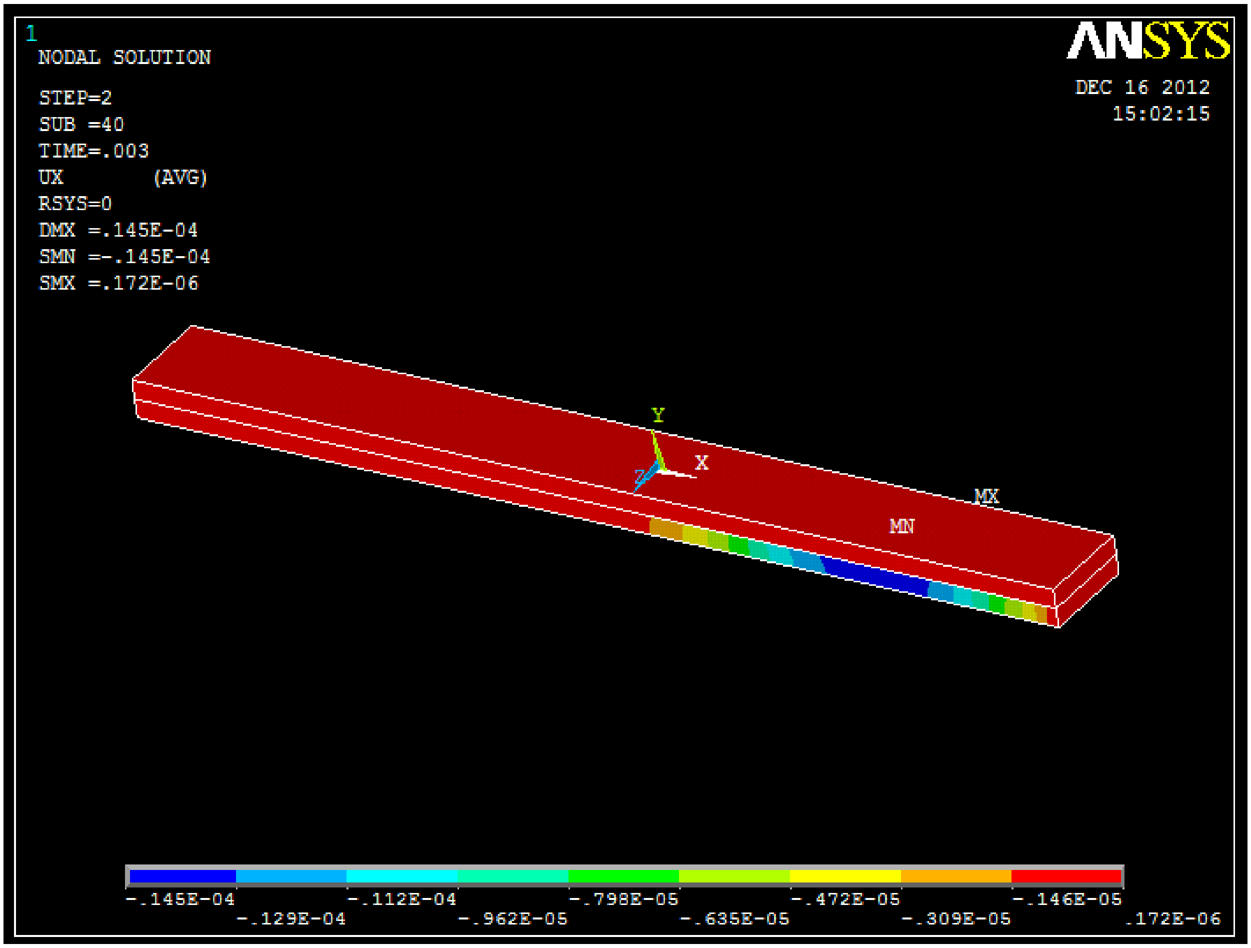

Figure 15. Displacement profile of Reference Model

Recorded displacement and force for this case are given as-

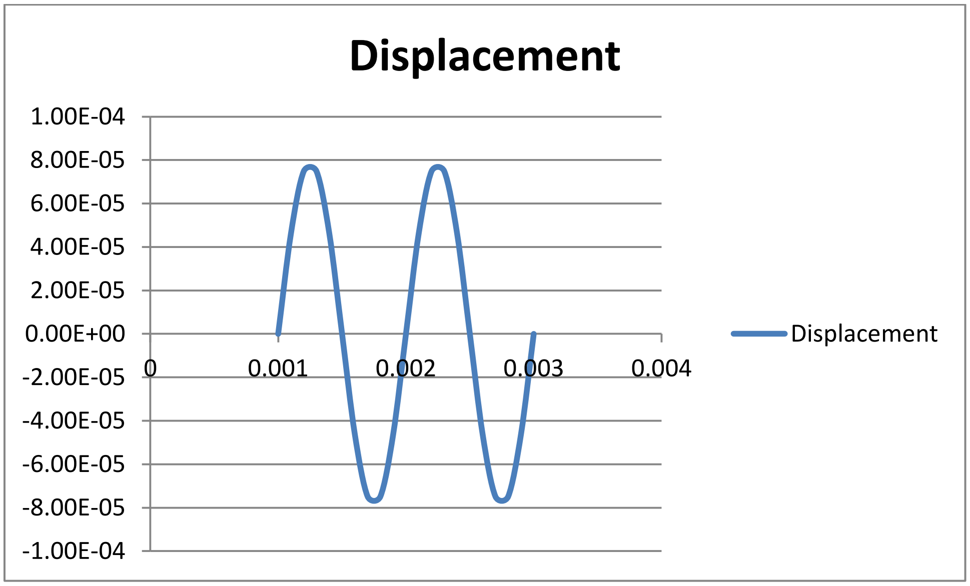

Figure 16. Displacement graph

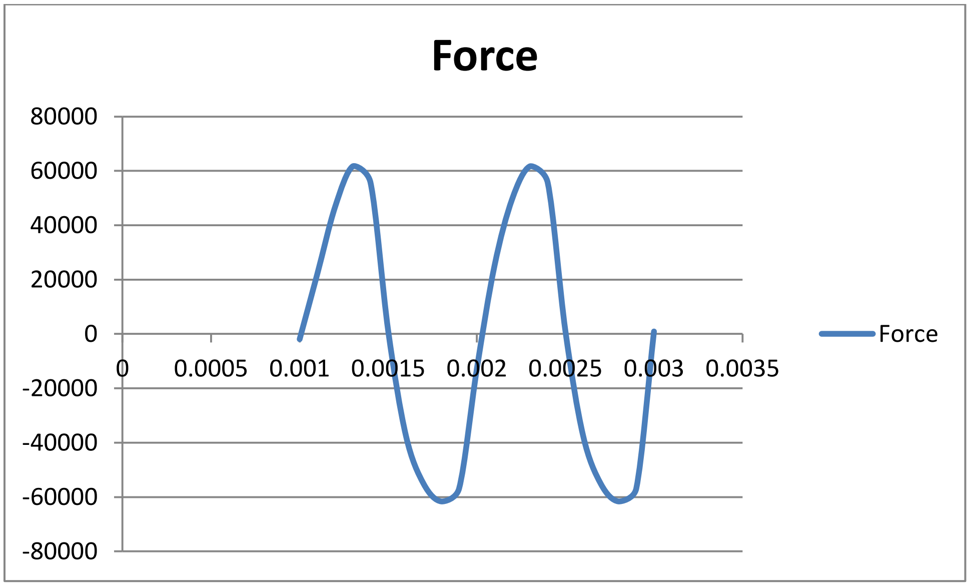

Figure 17. Force graph

The hysteresis plot between imposed displacement and force is drawn and dissipated energy from the joint is calculated which comes out to be 5.6265J,

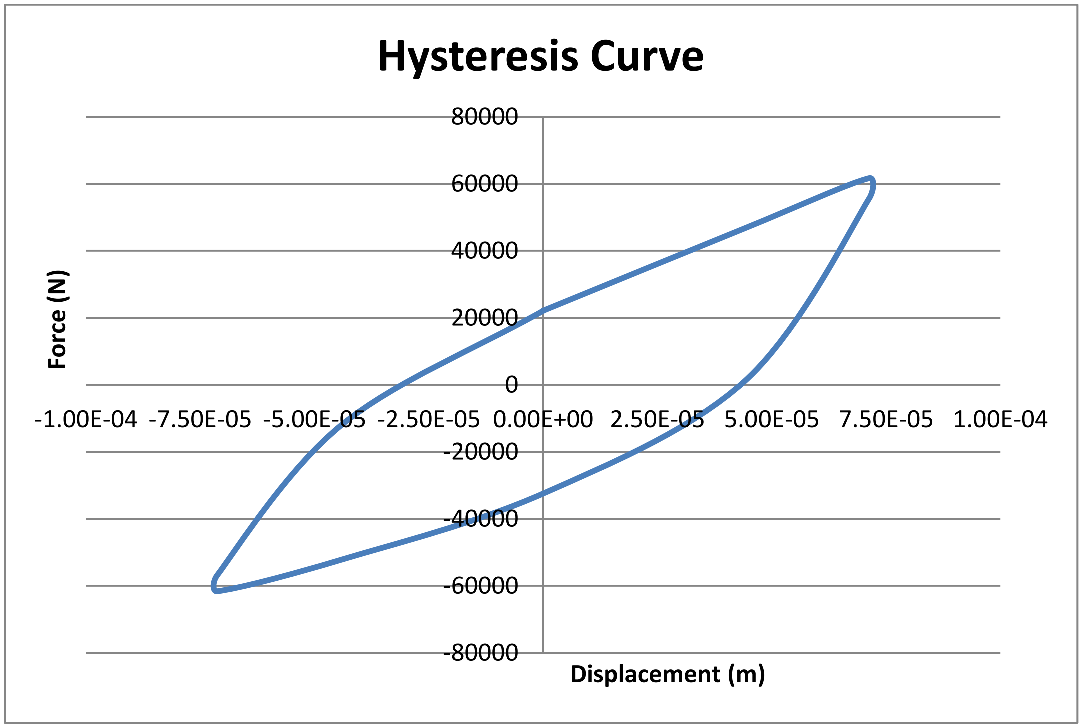

Figure 18. 3D reference model

Both Flexible Plates Model

Now, the above reference system is modeled with both flexible plates, i.e. two flexible steel plates of 500×50×10 mm dimensions are drawn, with same element type and mechanical properties, meshed with 200 elements along length,15 along width and 3 along thickness of each plate. Standard flexible contact is generated between them. Upper plate is constrained in all DOFs from the right end and is subjected to a uniform pressure of 8MPa while lower plate is displaced to 90% of the friction limit i.e. 7.87×10^{-5}m in x direction. The nodal contour displacement plot of this system is shown below-

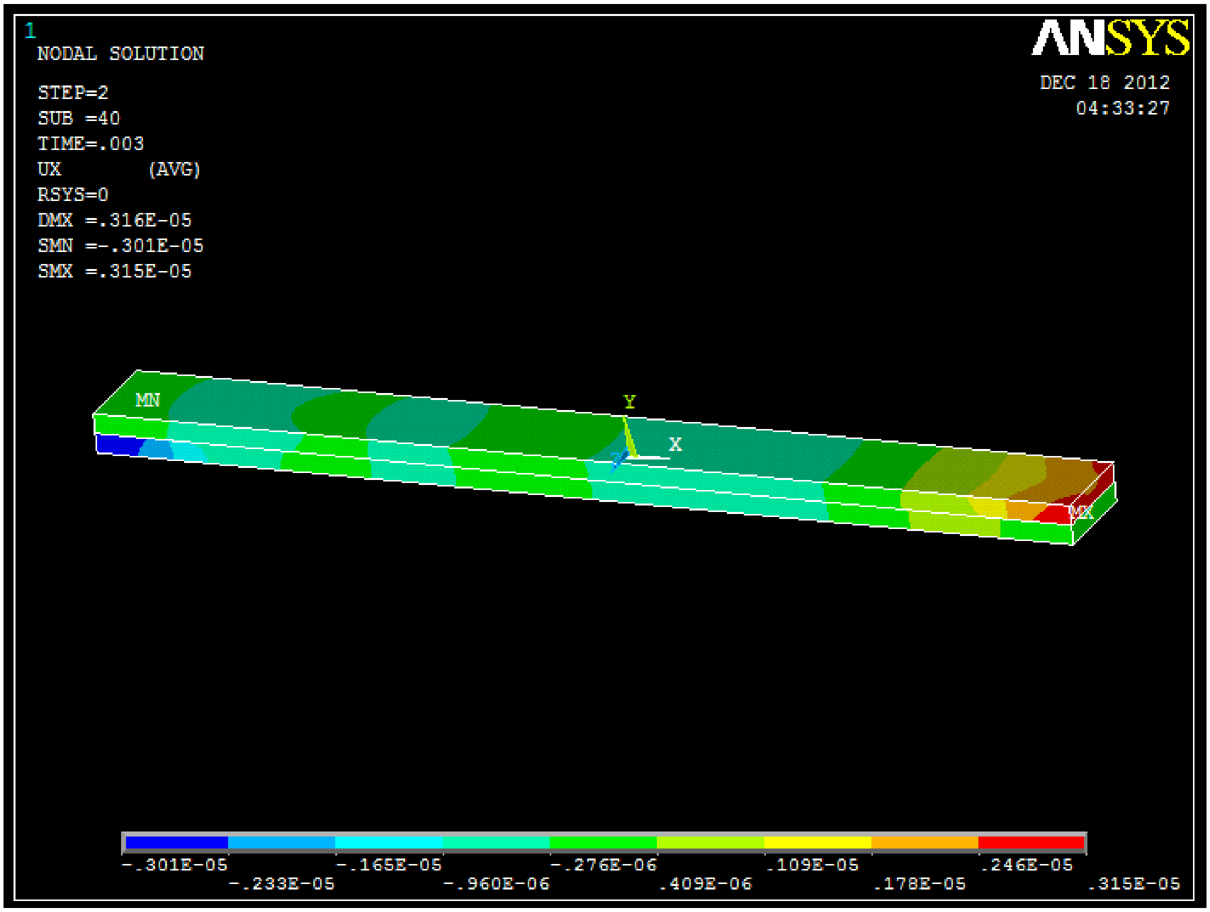

Figure 19. Displacement profile of Flexible Reference Model

Recorded displacement and force for this case are given as-

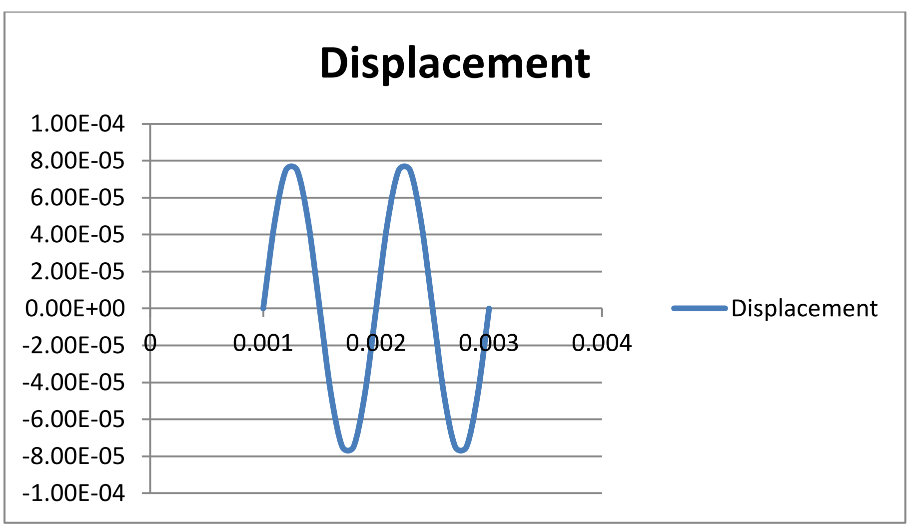

Figure 20. Displacement graph

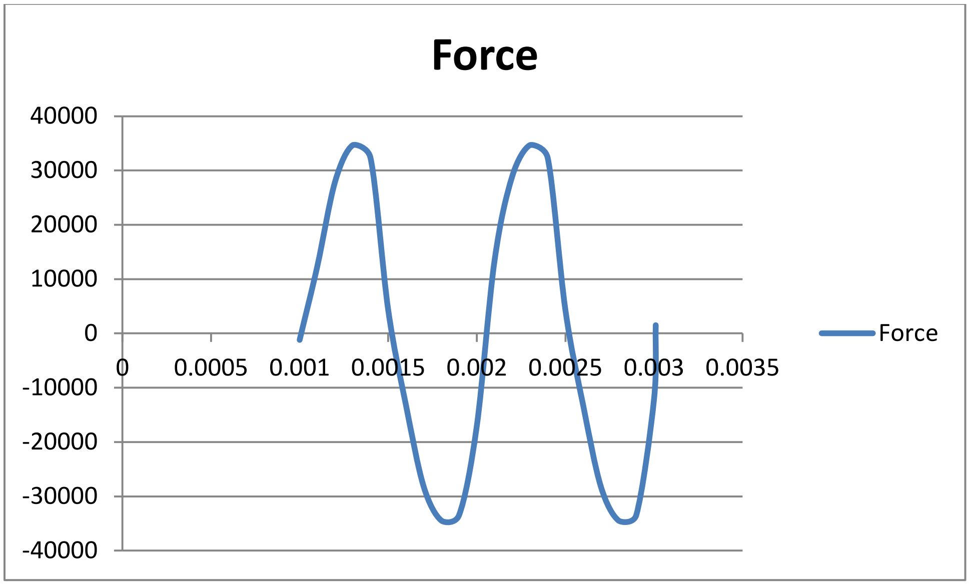

Figure 21. Force graph

The hysteresis plot between imposed displacement and force is drawn and dissipated energy from the joint is calculated which comes out to be 2.762J, which is almost half than that of the same model designed with one rigid and one flexible plate. This is because the relative displacement will be more in rigid flexible case as compared to both flexible case so on application of same force to both systems energy dissipated will be more for rigid flexible case than both flexible one.

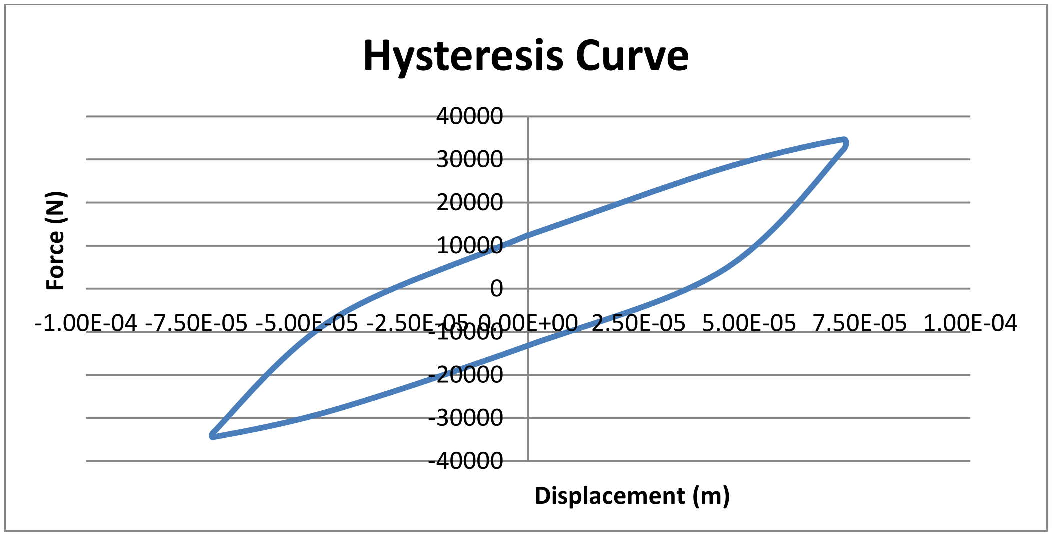

Figure 22. Hysteresis Curve for 3D reference system

Now, to get this reference model displaced by 90% of the active length, an imposed displacement of 31.5×10^{-5}m is applied at the lower beam in second load step and the nodal contour displacement plot obtained is shown below-

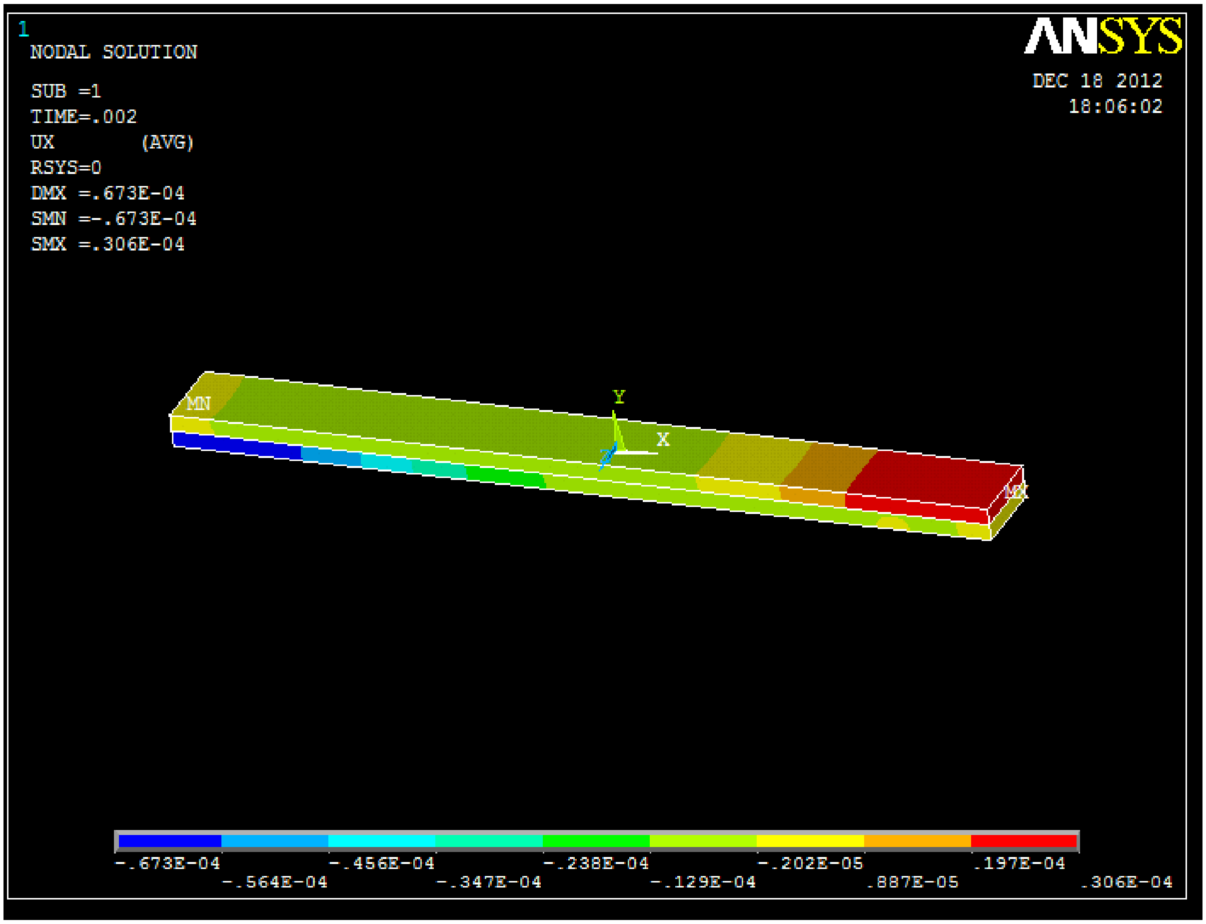

Figure 23. Displacement profile of 3D Flexible Reference Model

Recorded displacement and force for this case are given as-

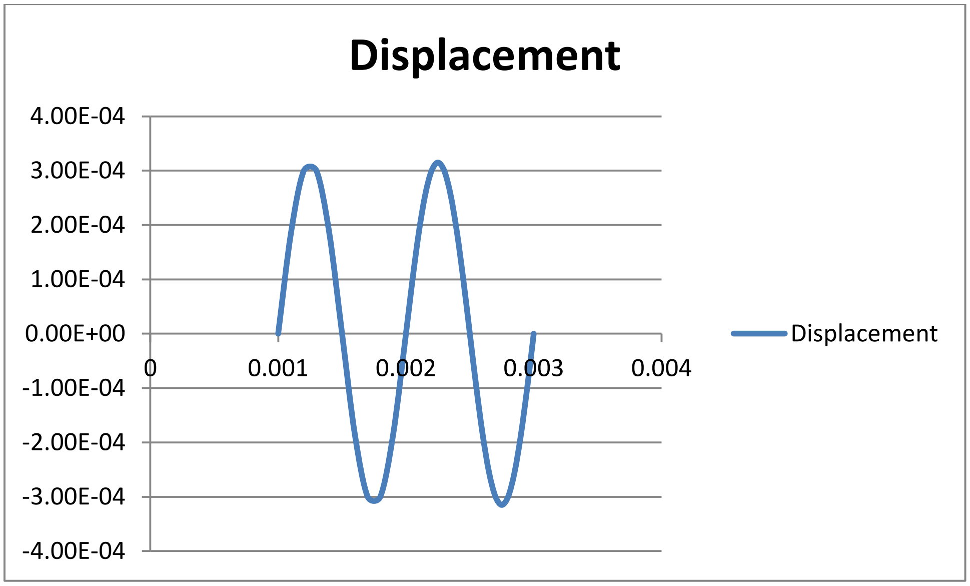

Figure17.1. Displacement graph

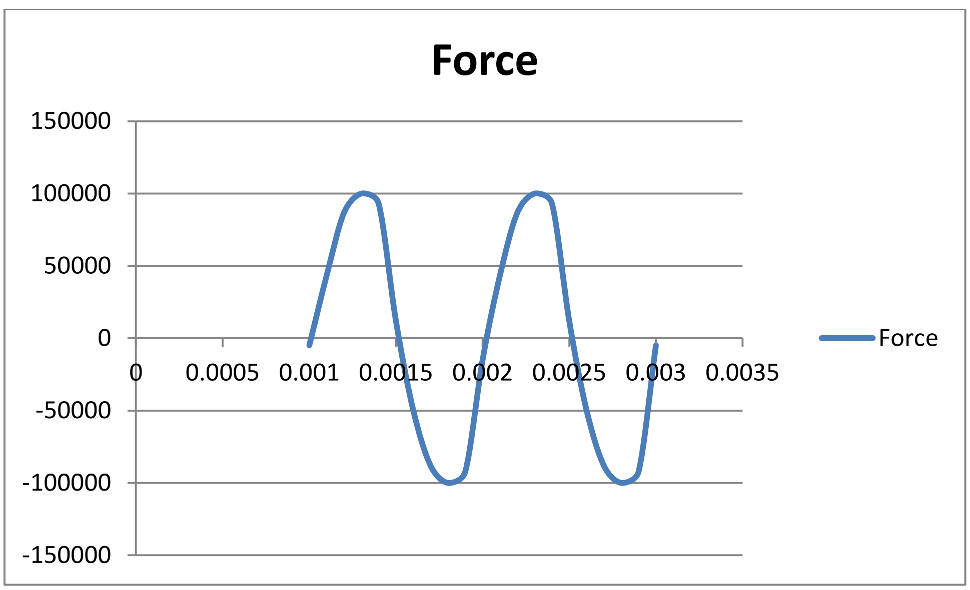

Figure 24. Force graph

The hysteresis plot between imposed displacement and force is drawn and dissipated energy from the joint is calculated which comes out to be 33.56J.

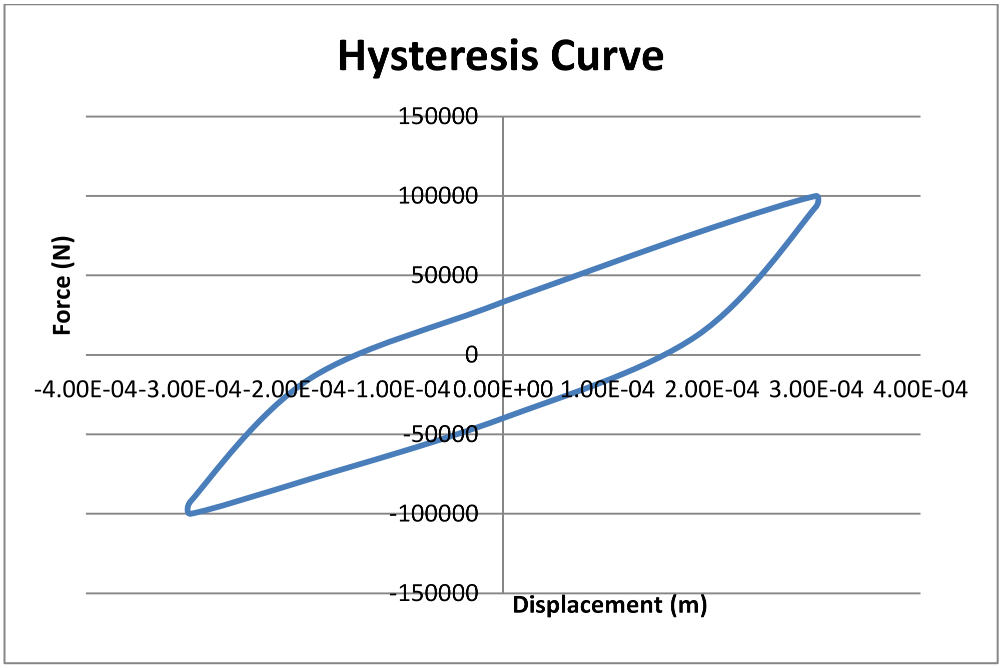

Figure 25. Hysteresis Curve for 3D flexible reference system

It is known that contact analysis is a nonlinear problem and joint dynamics again includes high nonlinearities, which makes it a highly nonlinear system. This nonlinearity tends to increase with the increase in the frequency of the external harmonic vibrations. This is proved by plotting systems response at different excitation frequencies i.e. reference case geometry in 3D with both flexible plates is analyzed under 1, 100 and 1000 Hz frequencies and their dissipated energies are 1.4131, 1.6366 and 2.762J respectively and are drawn below.

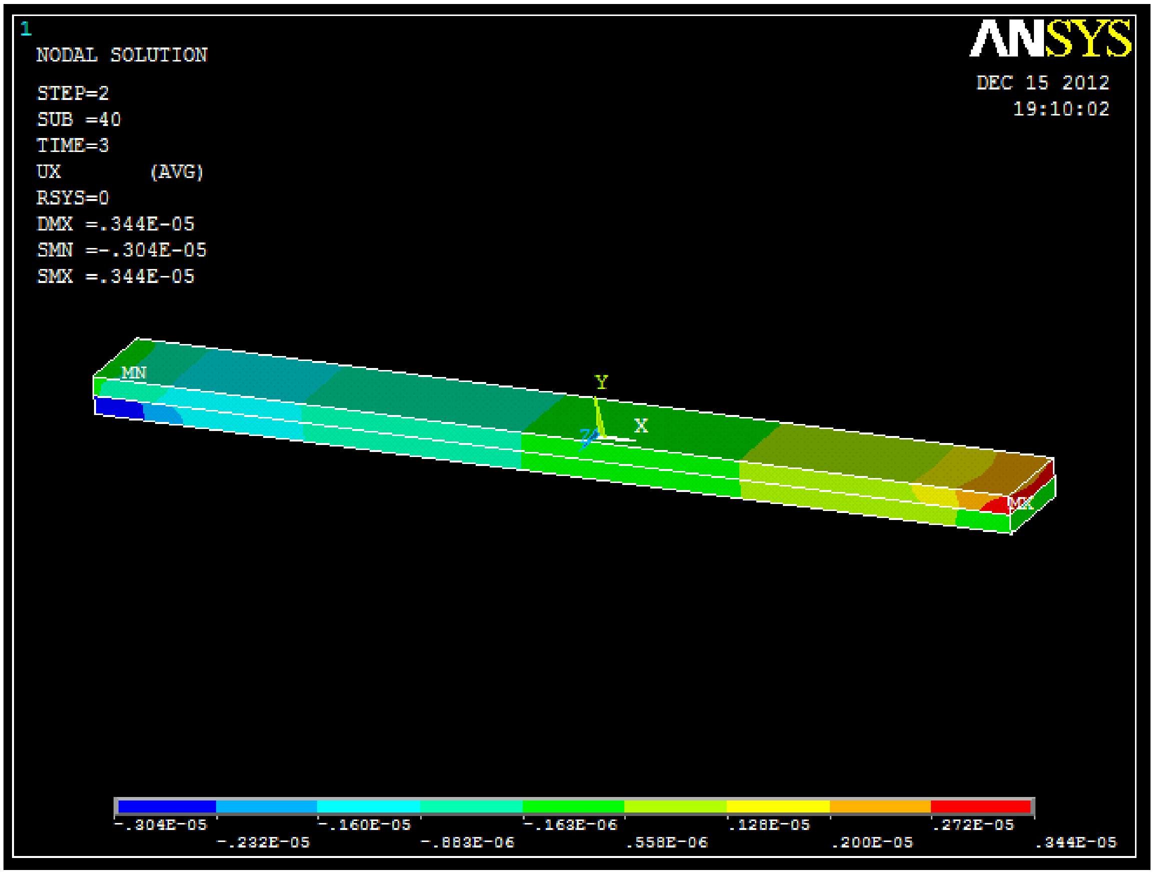

Figure 26. Under displacement of 7.87×10^{-5}m with 1Hz frequency

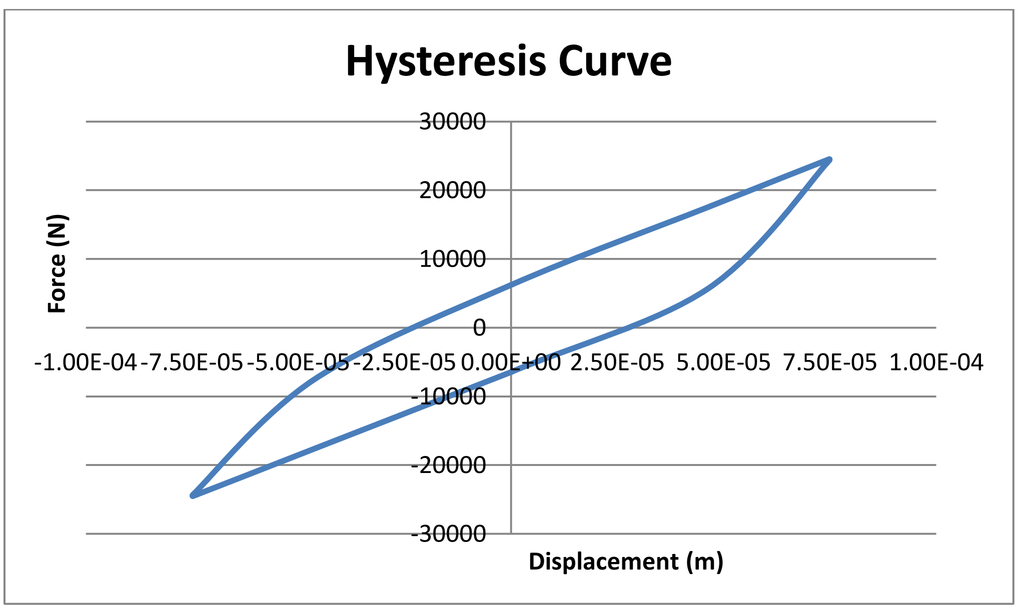

Figure 27. Hysteresis Plot at 1Hz frequency

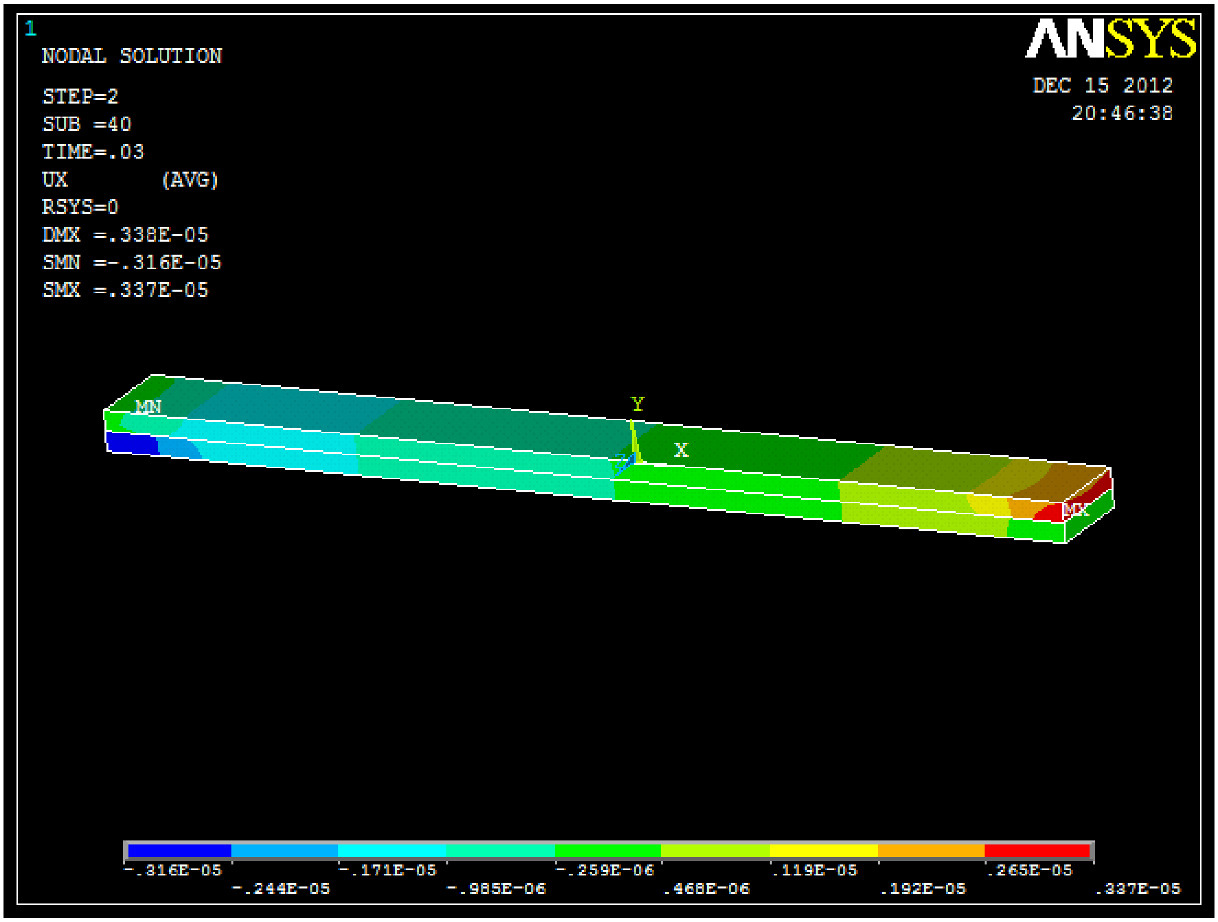

Figure 28. Under displacement of 7.87×10^{-5}m with 100Hz frequency

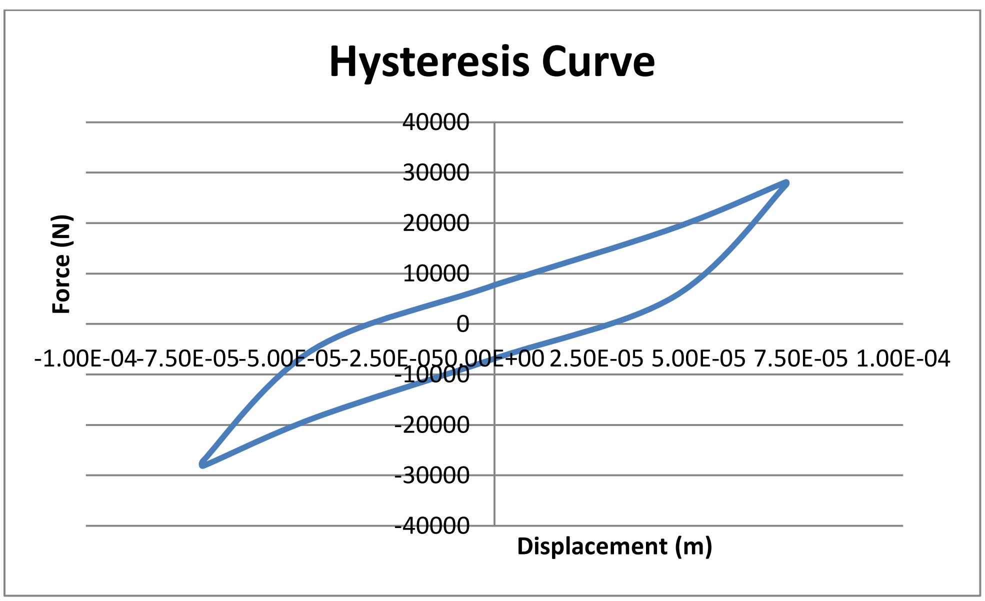

Figure 29. Hysteresis Plot at 100 Hz frequency

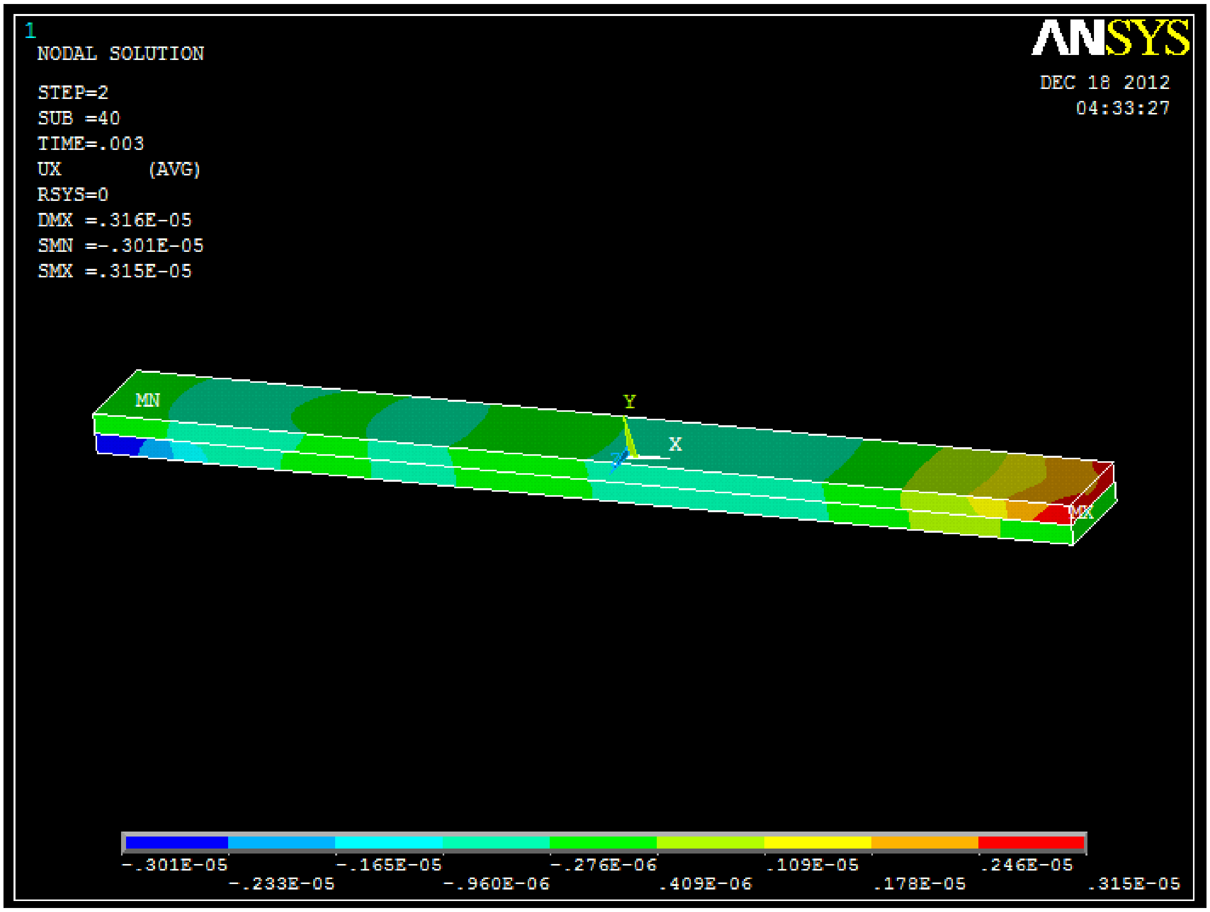

Figure 30. Under displacement of 7.87×10^{-5}m with 1000Hz frequency

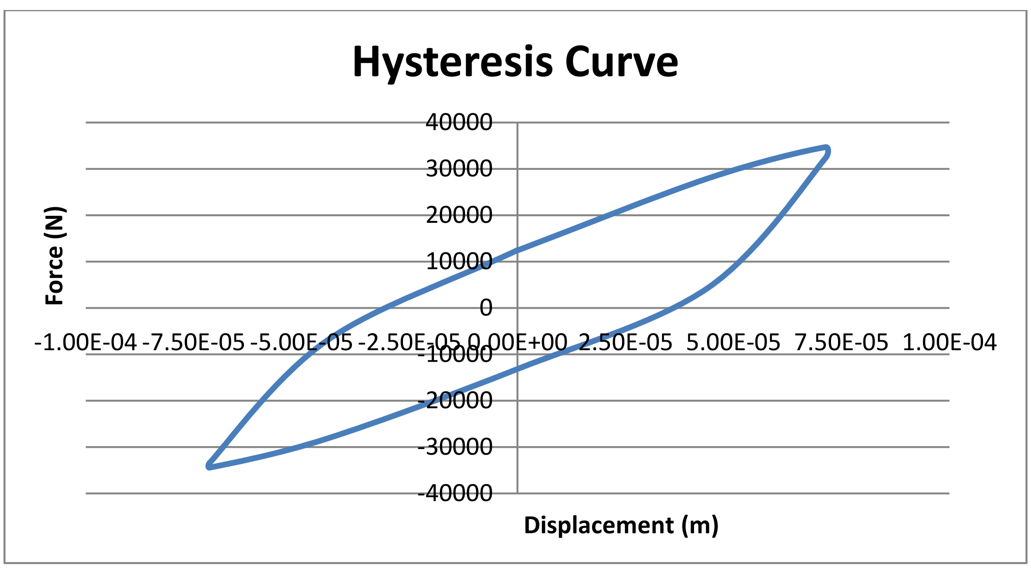

Figure 31. Hysteresis Plot at 1000Hz frequency

Overall, to see the behavior of system response with frequency variation, a graph is plotted which is drawn below. It is clearly visible that as at low frequencies there is an increase in the dissipated energy but not that significant and as frequency increases to resonance, a rapid increase in response is seen in that region. also the resonant frequency is decreasing with the increasing amplitude similar to the case of softening systems.

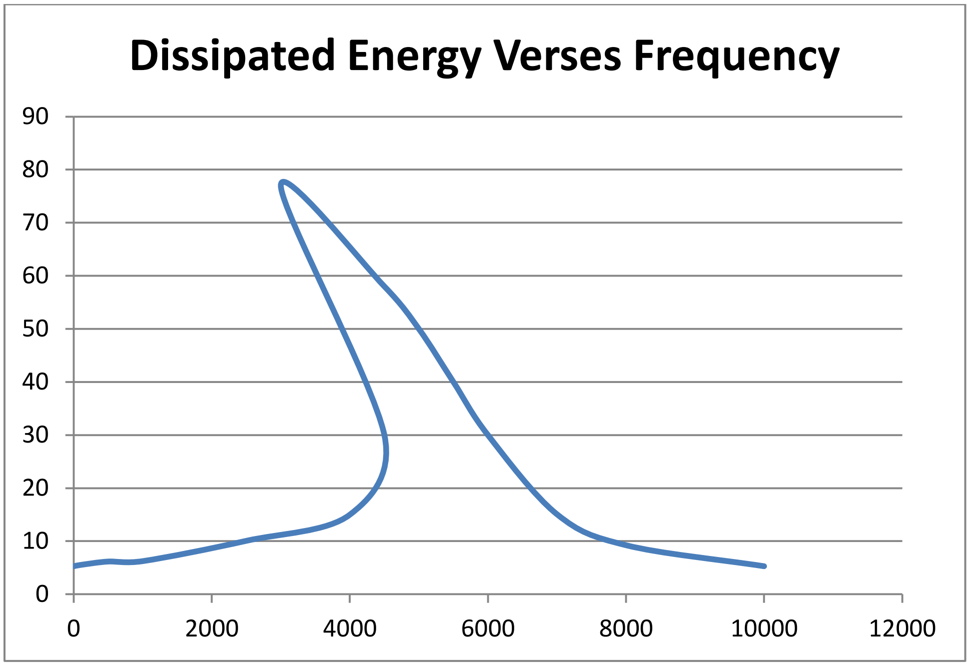

Figure 32. System response with frequency variation

The nonlinearity in the response also tends to increase with the increase in the amplitude of the external harmonic vibrations. This is proved by plotting systems response at different excitation amplitudes i.e. reference case geometry is analyzed under 7.87×10^{-5}m, 19.5×10^{-5}m and 31.5×10^{-5}m displacement amplitude at 1000 Hz frequency and their dissipated energies are 2.762, 12.06 and 33.56J respectively and are drawn below.

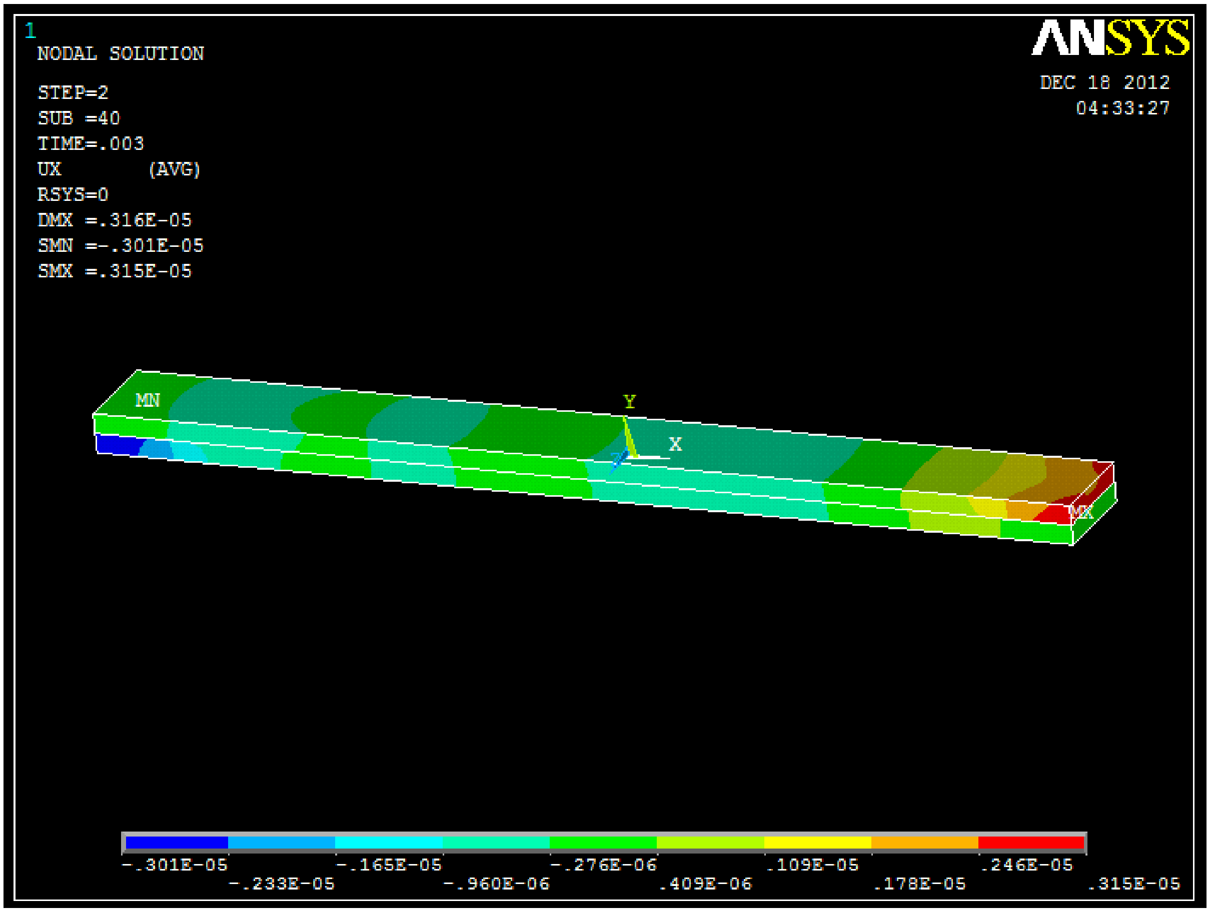

Figure 33. Under displacement of 7.87×10^{-5}m with 1000Hz frequency

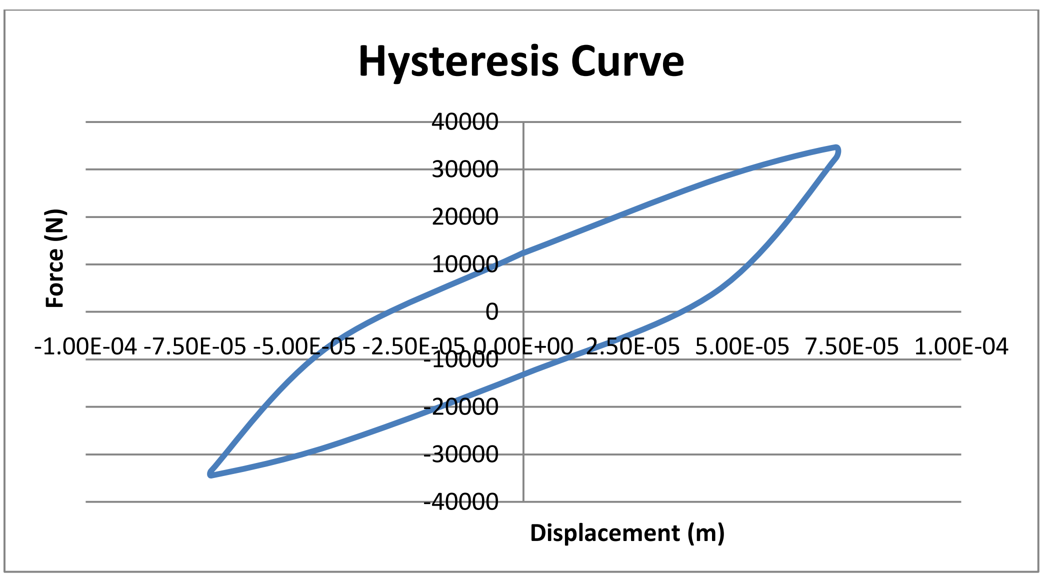

Figure 34. Hysteresis Plot at 7.87×10-5m amplitude

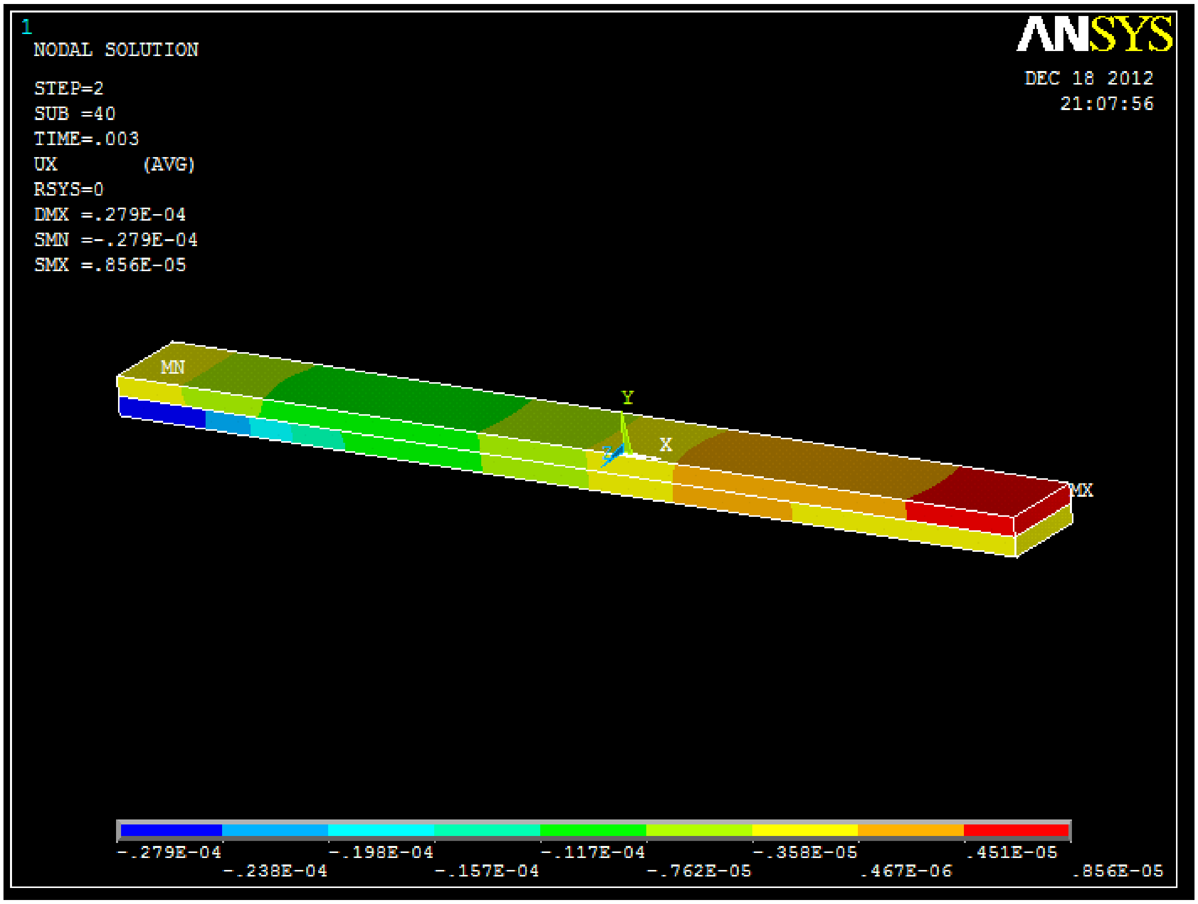

Figure 35. Under displacement of 19.5×10^{-5}m with 1000Hz frequency

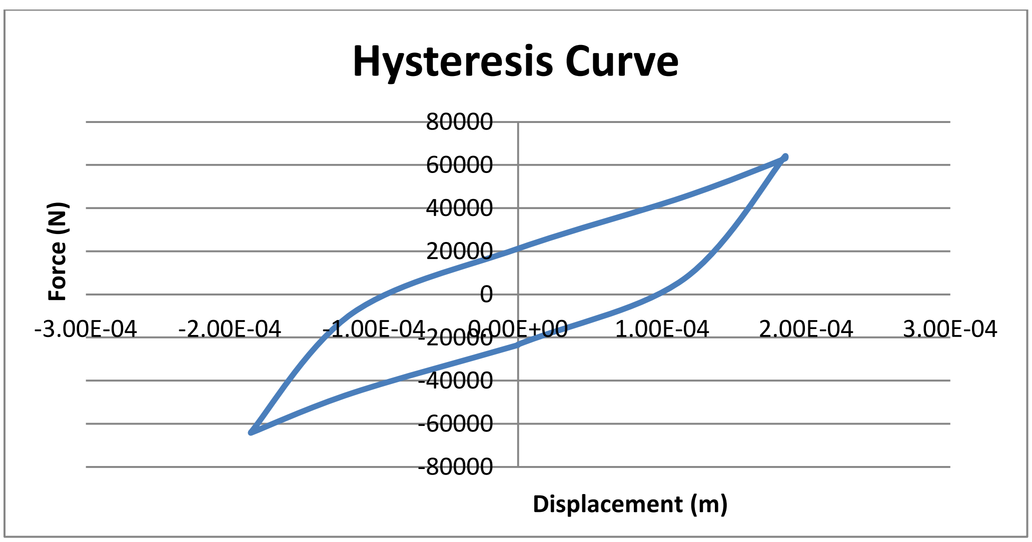

Figure 36. Hysteresis Plot at 19.5×10-5m amplitude

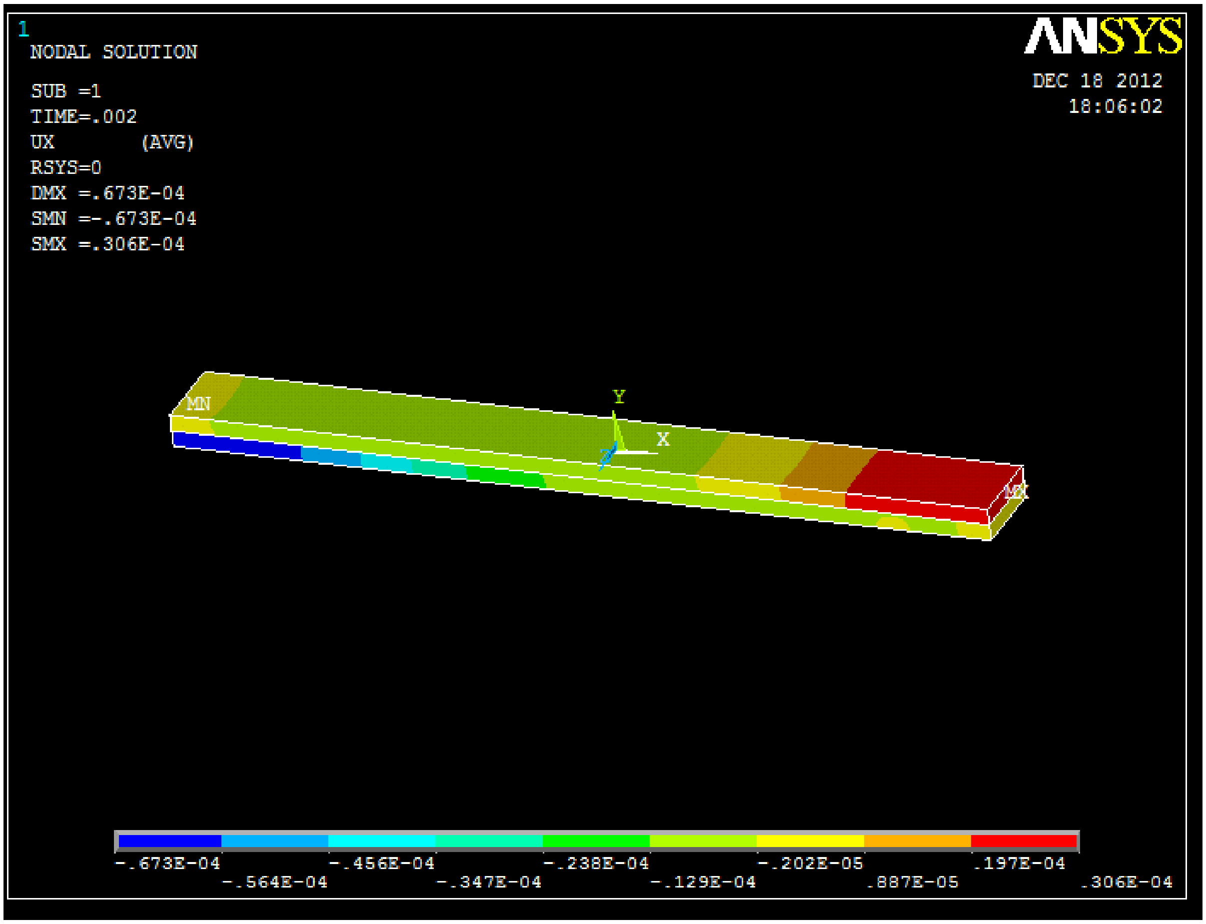

Figure 37. Under displacement of 31.5×10^{-5}m with 1000Hz frequency

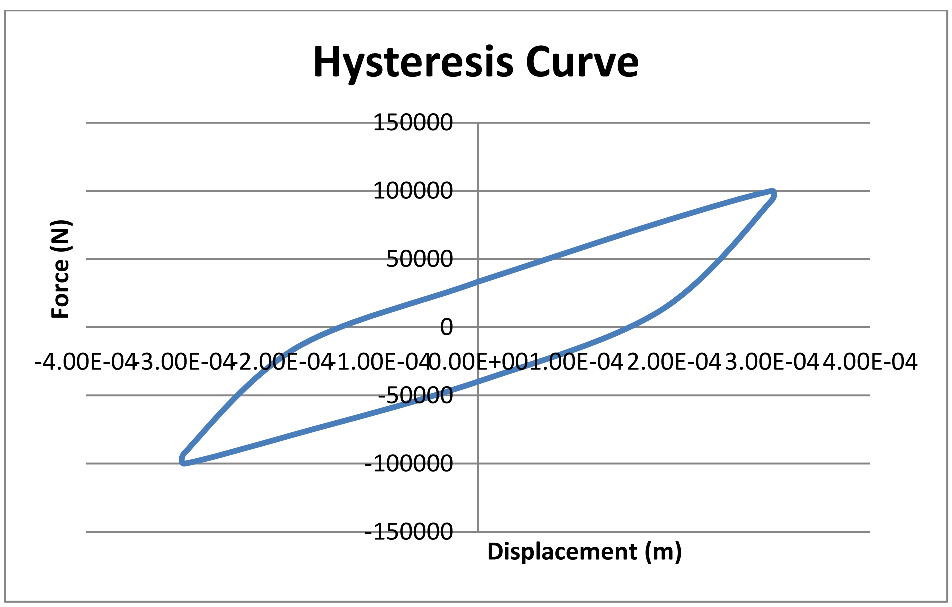

Figure 38. Hysteresis Plot at 31.5×10-5m amplitude

4.2. Proper Orthogonal Decomposition

Proper orthogonal decomposition POD is a technique which is used to decompose the nodal displacement matrix of the system obtained from its FEM analysis where $d(t)_{p*n}$ matrix denotes p as no of time steps and n as the number of nodes.
Mathematically SVD is expressed as-

$$\mathbf{d(t)_{p*n} = U_{p*p}\, \Sigma_{p*n}\, V_{n*n}} \qquad \text{........................(4.1)}$$

Where column vectors of V are the proper orthogonal modes POMs which describes the coherent spatial structures. For grey length region where relative displacement is zero, all zero values are seen in column vectors of V. The matrix **Σ** is a diagonal matrix whose diagonal values represents the proper values PVs of POMs respectively in descending order. These PVs decide the weightage of the POMs accordingly and they can be normalized by dividing each PV with equivalent PV and multiplying by 100. After scaling, it is easy to span the space with least number of POMs. These POMs will surely have amplitude of zero at active-grey interface. Also thy pretty much look like the normal mode shapes obtained for any built in rod.

POD technique provides a basis that can be able to extract to the highlighted patterns of given data and is applied to joint dynamics keeping in mind that applied forces or displacements are generally imposed on the free end of the joint, which confirms the presence of reduced basis that can span complete set of displacements at each node for different excitation conditions. This technique is when applied to nonlinear problems, proper orthogonal modes (POMs) obtained are used only to minimize the linear set of equations. A few among these POMs can then be used to regenerate the response of system very well. But there will be an increase in the computational time as stable time is achieved in a little long run.

Again beam of size 500×50×10 mm is modeled in 3D with one rigid and one flexible beam, which is the reference case using 200 brick elements along length. Friction coefficient is taken as 0.7, pressure applied due to bolts is equivalent to 8MPa or 100kN force on those nodes. Imposed displacement of 7.87×10^{-5} m at 1000Hz frequency is applied at the free end which can move 90% of the joint length containing major amount of nodes which will help in extracting smooth and accurate number of POMs. Here PVs obtained for reference case are normalized which are given in the table below and POMs are generated for the nodes at the right side of the reference point.

POD of Displacement		
Serial No	Proper Values	Cumulative Sum
1	83.247	83.247
2	13.55	96.79
3	1.516	98.30
4	0.69	98.99
5	0.51	99.50

Figure 39. PVs for right side nodes

First five POMs for right side nodes of the current case are drawn below.

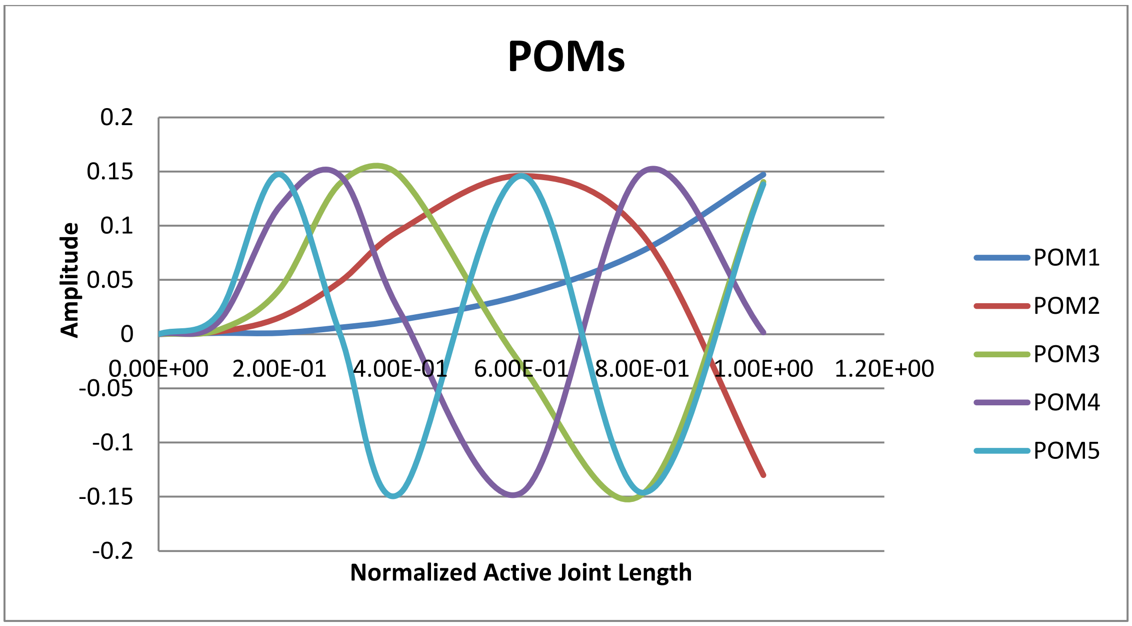

Figure 40. POMs for right side nodes

These modes are compared with that of Khattak's model, which are like-

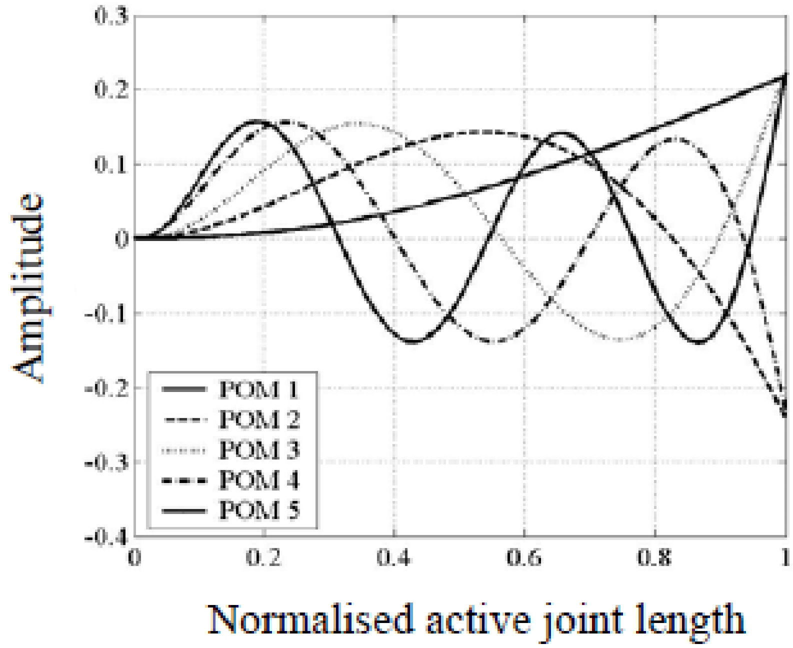

Figure 41. POMs of Khattak's reference mode

The mode shapes of the 3D model and that of the Khattak's model are quite similar in shape and are also of same magnitude.

By using proper orthogonal decomposition technique, the displacement matrix [u] of time histories of nodal displacements obtained from FE analysis is decomposed to obtain proper orthogonal modes matrix, out of which first few modes according to proper orthogonal values whose contribution to total sum of orthogonal values is more than, say, 99% are extracted to reproduce the system. Displacement matrix is now reduced thus lowering the DOFs of the system.

Mathematically, by using POD technique [u] matrix gets decomposed as-

$$[u] = [U]\,[\Sigma]\,[V^T]$$

Where the columns of V matrix are POMs out of which first few are selected on the base of PVs. Let new selected modes are given as-

$$V_1 = \{v_1 v_2 v_3 \ldots\ldots v_j\} \qquad \text{where } j<n$$

With $$[u] = V_1[u']$$

Where u' is the displacement of the reduced model.

In $$Ku + C\dot{u} + M\ddot{u} = f_{ln} - f_f$$

Where f_{ln} are the time dependent excitation forces and f_f are the frictional forces. Pre multiplying this equation with V_1^T results in the reduced order system equation i.e.

$$K'u' + C'\dot{u}' + M'\ddot{u}' = f_{ln}' - f_f'$$

Where $$K' = V_1^T\,K\,V_1$$

$$C' = V_1^T\,C\,V_1$$

$$M' = V_1^T\,M\,V_1$$

$$f_{ln}' = V_1^T f_{ln}$$

$$f_f' = V_1^T f_f$$

All the above equations are straight forward except the one of frictional forces which is responsible for nonlinear behavior of the system. This system will result in first order system

$$\dot{y} = A\,y + b$$

Where $y = \{y_1 y_2\}^T$ by using state space formulation with vectors $y_1 = u'$ and $y_2 = \dot{u}'$

And $$A = \begin{bmatrix} 0_{k\times k} & I_{k\times k} \\ -M'^{-1}K' & -M'^{-1}C' \end{bmatrix}$$

$$B = \begin{bmatrix} 0_{k\times k} \\ X(f_{ln} - f_f) \end{bmatrix}$$

Where $$X = M'^{-1}V_1^T$$

For the reduced system, the velocity vector $\dot{u}'$is expanded for each time step to the full velocity vector using $[\dot{u}]$ = V_1 $[\dot{u}']$ in order to determine nonlinear frictional force vector.This expansion of reduced solutionto the full one for obtaining the full nonlinear force vector and then the compression of nonlinear force vector to the reduced one gives rise to additional computational cost at each time step. Inspite of this drawback, the method results in computational savings for the number of modes used in the analysis are much smaller when compared with the size of the system.

As the analysis in my thesis is done while considering joint in isolation from the structure but when in real the complete jointed structure needs to be analyzed, it can be divided into two sub-structures or regions namely linear and nonlinear region. The linear substructure will be the region without frictional dissipation while nonlinear structure will be the region with frictional dissipation. As system of equations for dynamics of structures is-

$$Ku + C\dot{u} + M\ddot{u} = f_{ln} - f_f$$

The vectors in above equation will also be partitioned according to the linear and nonlinear regions as $u = \{u^1\ u^2\}^T$, $f_{ln} = \{f_{ln}^1\ f_{ln}^2\}^T$ and $f_f = \{f_f^1\ f_f^2\}^T$where 1 and 2 denotes the nonlinear and linear regions respectively. All the entries of $f_f^2$for linear region will be zero and only the last entry of f_{ln}^2will be nonzero corresponding to free end excitation.

Total DOFs of the system n will be equal to n_1+n_2i.e. number of DOFs of linear and nonlinear regions. Similar is the case with stiffness, damping and mass matrices.

$$K = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}$$

$$C = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$$

$$M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$$

And the transformation matrix for the above system will be

$$V_{2(n\times(j+n_2))} = \begin{bmatrix} V_{1(n_1\times j)} & 0_{n_1\times n_2} \\ 0_{n_2\times j} & I_{n_2\times n_2} \end{bmatrix}$$

Where V_1 is the matrix whose columns are POMs. This transformation matrix shows that only the nonlinear region is decomposed while keeping the linear part unchanged.

POMs for both flexible plates system are calculated, they are same on both sides of zero slip and this is because the geometry, loading and boundary conditions are all same on both sides of the middle of the joint.

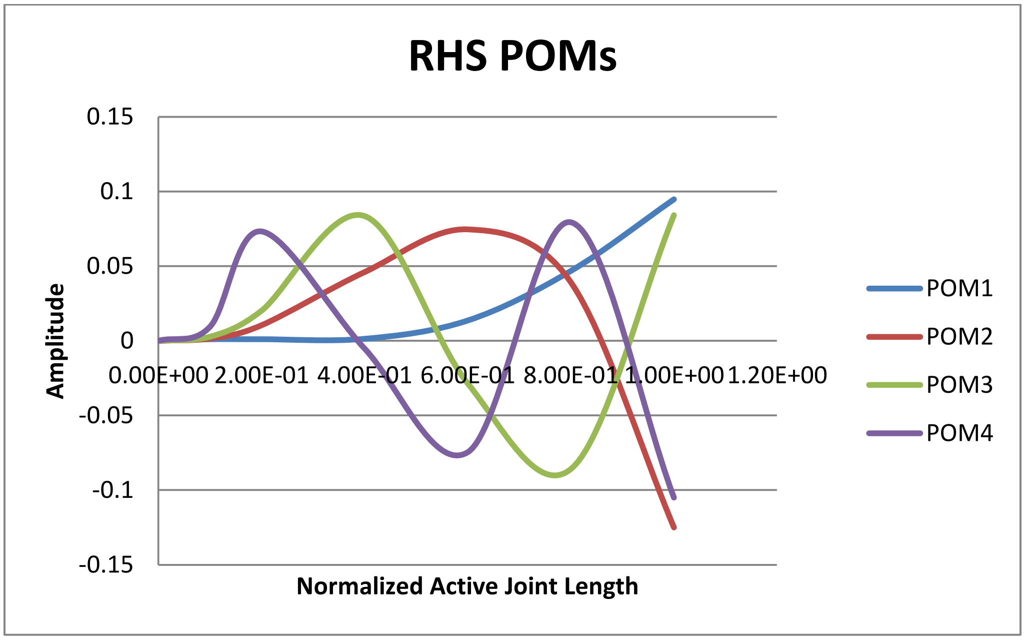

Figure 42. Right Hand Side POMs

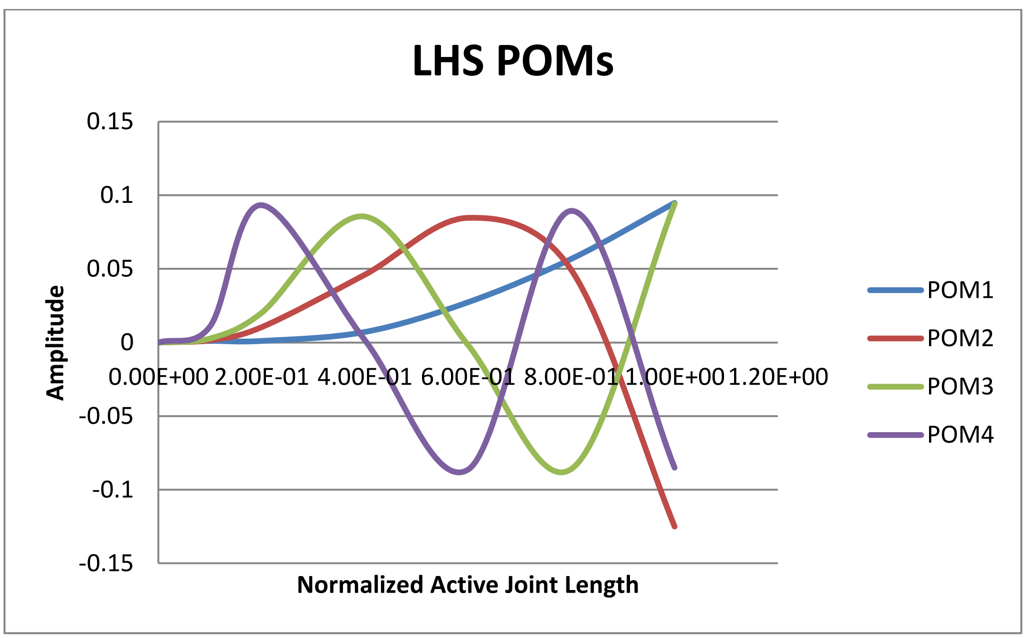

Figure 43. Left Hand Side POMs

The stick slip behavior is clearly seen in the right nodes of the coupled ones, of which four equally distanced nodes behavior is shown below-

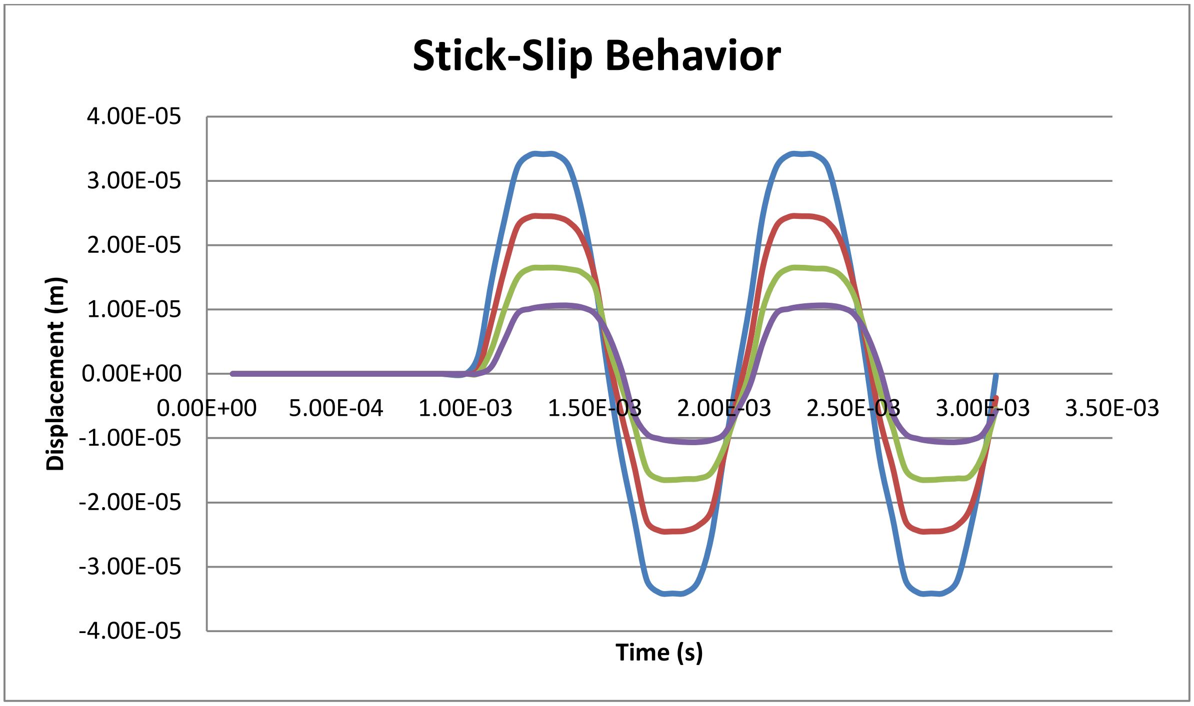

Figure 44. Stick slip behavior of active nodes

This figure shows that as the nodes will be far away from the free end, the more they will show the stick slip behavior or not all the nodes move at a time under an excitation but the effective active length.

An important feature in order to reduce the joint model is the selection of active length of the joint as POMs used are supposed to be scaled according to this active length. At low frequencies, active length depends directly on the excitation amplitude whereas at frequencies close to that of resonance, this active length is a high order function of excitation frequency and amplitude. Thus the system's response obtained is quite reasonable at low frequencies if active length is also known.

CHAPTER 5

CONCLUSION AND FUTURE WORK

5.1. Conclusions

Investigation of dynamic characteristics of shear lap bolted joints is done is this thesis. A three dimensional generic model of a joint is generated and proper orthogonal decomposition technique is used to evolve the joint dynamics. The concluding remarks after doing this study are summarized below-

5.1.1 FE Modeling Of Joint

A simplified three dimensional FEM model is generated in ANSYS and analyzed under different loading conditions and by varying joint parameters. The results obtained from the analysis are concluded below-

- This new 3D generic model of joint with both flexible plates can be incorporated in any bolt assembled structure to get a simplified model with less computational cost. It is a replacement of the Khattak's 1D model as it is more accurate and still supports all his results.
- Dissipation of energy from a joint is a nonlinear phenomenon and it is calculated through hysteresis curves drawn between the applied excitation force and the relative displacement of the plates. Under same conditions, dissipated energy in 3D model is twice that of Khattak's 1D model.
- With the increase in the frequency of the excitation, energy of dissipation increases but not significantly. But as frequency continues to increase till resonance, the response of the system shoots up in that vicinity and immediately drops after that i.e. a mirrored response can be drawn at the resonant point.
- Dissipated energy at the interface is directly proportional to the excitation amplitude i.e. it increases and decreases linearly with the amplitude of vibration.

5.1.2 Proper Orthogonal Decomposition

The proper orthogonal decomposition technique is used to decompose model of the joint subjected to harmonic loading, which gives a reduced order model by minimizing the linear system of equations. The conclusions made by the application of this technique are-

- The shear lap model of joint dynamics is developed. This POD method is applicable to joint model at varying external excitations and parameters.

- From the time dependent data obtained from the analysis, the complete space can be spanned with the help of proper orthogonal modes extracted from this technique.
- Proper Orthogonal Modes obtained from presented 3D model are quite similar in pattern to that of Khattak's 1D model with a change in amplitude which is because of different loading condition but still that difference is justified.
- The method helps to minimize the linear system of equations which cut down computational cost by lowering degree of freedom of the system.

5.2. Limitations

The limitations of my developed model are-

- The model is applicable to shear lap friction joints at relatively low frequencies compared to the resonances (natural frequencies) of the model and for sinusoidal excitations.
- The joint model can be embedded in an FEA model containing several lap shear joints.

5.3. Future work

As the joint model presented in this thesis is a better alternative to the models presented yet. It is capable to simulate joint dynamics more efficiently and accurately but still it can be extended in certain areas which are pointed below.

- As Khattak modeled the contact surfaces at asperity level but no 3D asperity modeling is done here. Thus this 3D system can be further modeled with asperity details but it will require huge amount of computational resources as the degrees of system will get so much increased.
- Calculate the response of this new joint model under different types of loadings e.g. impulse, step and sinusoid and by varying parameters like friction coefficient, clamping pressure etc.
- Determine the active length for bolted joint model which can be further used to scale the proper orthogonal modes obtained from POD technique. This task will be difficult to handle near resonant frequencies.

- As Khattak has reduced his model using nonlinear functions obtained from the proper orthogonal decomposition of the time dependent data and used nonlinear forces to investigate the response of joint for different excitation conditions, same can be applied to this both flexible plates model in extension to POD results.

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