Modeling, Simulation & Operational Data Analysis of Compact Globoid Worm Gear Drive for a Slewing Drive Actuator (Slewing Motion Mechanism)

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Declaration

I certify that this research work titled "*Modeling, Simulation & Operational Data Analysis of Compact Globoid Worm Gear Drive for a Slewing Drive Actuator (Slewing Motion Mechanism)"* is my own work. The work has not been presented elsewhere for assessment. The material that has been used from other sources it has been properly acknowledged / referred.

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This thesis has been read by an English expert and is free of typing, syntax, semantic, grammatical and spelling mistakes. Thesis is also according to the format given by the university.

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Abstract

A slewing drive actuator is a power / torque transmission mechanism that gives controlled power / torque output for rotation in desired axis while safely carrying radial & axial loading. These are mainly used for regulating & controlling precise motion in control applications. Applications may include heavy hydraulic machinery, drilling equipment, retraction / extension mechanism, helicopter transmission etc. Slewing actuators function with worm drive technology. The rotation of the worm turns the gear about an axis normal to the axis of the worm decreasing the speed of the gear and increasing its torque. For increased loading requirements including working arrangements with shock loads, smoother operation & higher efficiency, globoid (double-enveloping) worm drive may be used. Globoid worm gear sets have both members throated; the worm is also curved longitudinally to fit the curvature of wheel. The current research study deals with a single axis slew drive lift actuator, whose desired operational data is known & the emphasis is to design the globoid worm gear drive which will satisfactorily perform the desired function. The load rating (load carrying capacity & efficiency) of globoid worm gear pair is calculated by applying mathematical and analytical approach. The calculation of gear tooth geometry & gearing parameters necessary for manufacturing & analysis are determined by following the AGMA gear standards. Parametric modeling of globoid worm gear drive is done in Pro-E Wildfire 4.0 using gear parameters & relations and the motion actuator assembly is modeled in Solid Edge ST3. Computer-Aided Finite Element Model is developed in Ansys Workbench 13.0 and models of bending stress and non-linear contact stress with static deformation mode are analyzed.

Key Words: *Globoid worm, Slew drive, Lift actuator, Bending and Contact stresses*

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CHAPTER 1: INTRODUCTION

The current research study deals with the load calculations and stress analysis of globoid worm gear drive for a single axis slew drive lift actuator. This study is comprised of two parts. First part is devoted to the evaluation of loading capacity of globoid gear drive by calculating all necessary geometrical parameters using gear standards. The second part includes computer-aided modeling and simulation of stresses and deformation produced in the gear drive.

1.1 Background

The globoid worm gear drive is an attractive technology to use in lift actuation purpose. The use of this technology can provide very compact torque & power transmission with good weight saving. The designing of this technology involves careful calculation and selection of the desired parameters to predict actual behavior of stresses.

1.1.1 Slewing Motion: An Introduction

Slewing motion is the rotary motion (rotation) of a mass about an axis. The word slewing is commonly used in precise motion control applications. The slew axis is often combined with other axis (axes) to form a particular motion profile. A slewing drive actuator is a torque or power transmission mechanism which provides controlled power & torque output for rotation in single or multiple axis (axes) while safely carrying radial & axial loads. These are mainly used for regulating & controlling precise movements in control applications. Application areas of slewing drives include Solar Trackers, Hydraulic Machinery, Drilling Equipment, Retraction / Extension Mechanism, Aircraft / Helicopter transmission etc.

Slewing actuators generally function with cylindrical worm gear technology, in which the worm (screw) acts as the driving member for the wheel. The rotation of the screw turns the gear about an axis normal to the axis of the screw. This combination decreases the speed of the gear (driven member) and increases its torque. The gear ratio of worm-gear unit depends upon the ratio of the number of teeth on worm wheel or gear to the number of starts (threads) on the worm. For increased load capacity and smoother operation, slewing actuators may use globoid or double-enveloping worm gear technology.

1.1.2 Worm & Worm gears: Brief Introduction

Worm & Worm gears are commonly used for power transmission between two non-parallel, non-intersecting shafts, generally having 90° shaft angle, consisting of a worm or screw and the mating member known as worm wheel or worm gear. The worm has threads (teeth) wrapped around a cylinder. Worm gear sets are used in application areas where the speed reduction required is in the range 3 to 100:1 and in applications where precise indexing is necessary. The gear ratio of the worm gear set depends upon the worm gear number of teeth and the number of starts of the worm.

The clockwise or anti-clockwise rotation of the worm gear depends upon the rotation of the worm, as well as hand of teeth (left-handed or right-handed). The hand of the helix is same for both worm & worm gear. Worm gear sets are made so that one or both members wrap each other.

1.2 Objective of the Research

The focus of this study will be on a single axis slew drive lift Actuator whose desired operational data is known & the emphasis will be to design the Globoid worm gear drive which will satisfactorily perform the desired function.

The extension / retraction lift mechanism comprises a suspension unit having a transversely extending upper suspension link, a transversely extending lower suspension link; spring and damper unit; and a suspension upright pivotally connected to outboard ends of the links. The retraction mechanism has a retraction linkage, pivotally connected to the medium, and an actuator for moving the suspension between a protracted position and a retracted position. The retraction linkage further comprises an upper retraction link pivotally connected to an outboard end thereof to the upper suspension link and a lower retraction link pivotally connected to an inboard end, and a tie bar pivotally connected between the upper retraction link and the lower retraction link. The actuator is a servo electronic actuator pivotally connected to an outboard end thereof to the lower retraction link and pivotally connected to an inboard end thereof to the suspension. When the retraction linkage is protracted then the retraction linkage holds the suspension unit in a deployed position; and on retraction, in use, the retraction mechanism lifts the suspension unit upward to a retracted position. The retraction mechanism supports the suspension unit spaced apart transversely, the suspension unit supported on the medium only via the retraction mechanism. Tie bar is connected to upper retraction arm at an intermediate location to ensure a large angle of tilt on retraction. The suspension does not have to be compressed on retraction.

A rotation-type lift actuator includes a main housing that covers globoid or double-enveloping worm gear assembly along with supported bearings (axial and radial both). The housing also has a positioning shaft directly coupled to the prime mover (DC motor with necessary speed reduction assembly in this case). A hole at the bottom of the housing is provided for pivoting the actuator with retracting mechanism.

The reason for selecting worm gear set is for elevation (lifting) & locking purpose. In terms of elevation (lifting), it was chosen primarily on the basis that it can be packaged in a very small volume and require a very low input force. For locking purpose, the worm gear set was chosen for its self-locking ability, allowing for infinite locked, elevated positions.

The focus of this research study is on following:

- **a.** Study of globoid worm gear geometry & calculation of gearing parameters necessary for gear manufacturing & analysis.
- **b.** Kinematic & dynamic characteristics of globoid worm gear system
- **c.** (Parametric) Modeling of globoid worm & wheel (gear) without losing its geometry.
- **d.** Development of finite element model and determine models of tooth contact analysis (contact stress simulation) & bending stress analysis.

1.3 Organization (Layout) of the Thesis

This thesis is organized of seven chapters. Chapter 1 gives detailed information about the general introduction, objectives to be achieved and organization of the thesis. Chapter 2 is literature review, which shows brief history of globoid worm gears and previous work which have been done by various researchers. Chapter 3 describes globoid gear geometry and calculation of other geometrical parameters that are necessary to analyze & manufacture gear set. Chapter 4 describes kinematic and dynamic characteristics of double-enveloping gears. Chapter 5 shows the parametric model of globoid gear using 3D design software and a brief introduction of Finite Element analysis. Chapter 6 presents detailed information about the analysis of bending and contact stresses. It also shows the comparison of results using simulations. Chapter 7 provides the conclusion obtained from this work and suggests future work in this area.

CHAPTER 2: LITERATURE REVIEW

There has been a lot of study on gear designing and finite element analysis of gears. Worm gears are good to use when reduction ratio in small space is desired. The work includes finite element analysis of gears by changing tooth profiles to estimate effect of stresses.

2.1 Worm & Worm gears developments & families

A worm's external shape may be a cylinder or it may be hourglass-like. The external shape of the meshing worm wheels may also be a cylinder or one with a (globoidal) throat recess. As a result of this, four options can be chosen to combine the external shapes. However, worm gearing today is normally divided into two types by the worm’s external shape since commonly used worm wheels are almost exclusively globoidal (with the throat recess). The two types are cylindrical (or single-enveloping) worm wheel drives and globoidal (or double-enveloping) worm wheel drives (as shown in Figures below). Cylindrical worm and enveloping wheel create “single enveloping product”. Contoured worm and enveloping wheel create “double enveloping product”.

Single-enveloping worm gear set consists of a cylindrical worm and a throated gear, wrapped around the worm. In action, single-enveloping worm gears have theoretical line contact. As worm gear teeth passes during meshing, the line of contact sweeps across the height of the zone of action and the entire width. An important characteristic of worm gearing is the higher sliding velocity of teeth compared with helical or spur gears. In worm gear drive with low speed ratio, the speed of sliding is higher than the worm pitch line velocity. The torque transferred in the case of single enveloping worm gear drive is dependent on the meshing of one to two teeth of worm wheel. This kind of meshing focuses the work done on small area of contact. The worm is the cylindrical component of drive with the outer dimensions same along the length of the threaded region. The worm wheel teeth are contoured to envelop the threads of mating worm, so the name single enveloping set.

Double-enveloping worm gear drives have both worm & worm gear wrapped around each other and throated. Double enveloping drives involve multiple teeth meshing, even up to one eighth of the circumference of the gear, for torque transmission and multiplication. This is only achievable by contouring the threaded region of worm teeth to the wheel. This contouring gives the second enveloping, thus the name double enveloping worm drive. The added teeth in contact in a globoid gear drive, two to five times more compared with single enveloping drive, increases the region of contact in the mesh by the same amount. Moreover, the conditions of lubrication are also improved by the addition of good direction of contact pattern in the mesh relative to worm direction of rotation. This consequence the load rating of double enveloping gear drives about two to four times the more than the load rating for single enveloping gear drives of same dimensions. The load distribution in addition to better lubrication characteristics in the double enveloping gear mesh make them more efficient and high power product.

Figure 2.1: Cylindrical (Single-Enveloping) Worm & Gear

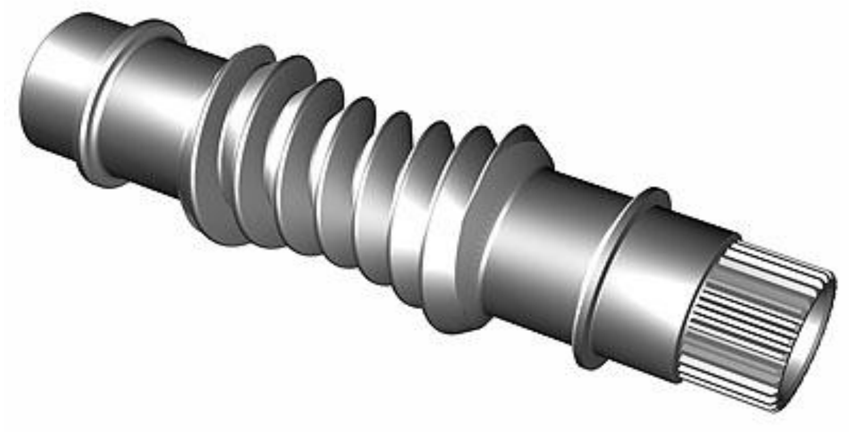

Figure 2.2: Globoid Worm & Globoid (Double-Enveloping) Worm Gear Set

Materials of dissimilar hardness are used for worm & worm gears. The worm-gear sets most commonly consist of a hardened steel worm meshing with a bronze worm wheel. A bronze worm wheel is more ductile, with a lower coefficient of friction. The worm goes through many more contact stress cycles than the worm wheel, so it is advantageous to use the harder, more durable material for the worm. However, the detailed analysis of the application could indicate which material combination will perform satisfactorily.

2.2 History of globoid worm gear development

Globoid or double-enveloping or hourglass gears are one of the varieties of general type worm gears (Lagutin, 1999). Such a gear has a worm in the form of a globoid, i.e., a concave profile that envelopes the worm wheel. In globoid worm gear set, the number of worm wheel teeth engaged with the worm threads is equal to the number of gear wheel pitches located within the arch of the enveloping reference globoid. While the contact ratio of single-enveloping worm gears is, as a rule, less than two, this factor reaches 4-6 and even more for a globoid analogue.

Globoid gears differ from gears with cylindrical worms by a higher load-carrying capacity, especially at large center distances. Besides the multi-pair engagement it occurs due to specified properties of the gearing. First, the instantaneous contact lines are located across the active surfaces of teeth, but not along them, as it takes place in gears with a cylindrical worm. This creates more suitable conditions for the hydrodynamic oil layer formation, and as a consequence it leads to the increased transmission efficiency and prevents the appearance of scuffing- prone zones. Second, the radius of active surfaces reduced curvature in the globoid gear is greater than in the cylindrical worm one, and it can reach significant values, depending on the modification law. It permits to increase the allowable contacting load and thereby to raise the load-carrying capacity of gears or reduce the size of the gearbox, keeping the value of the transmitted power.

The application of globoid gears has been known since the late XIX century. First two patents on methods of worm and gear wheel generation for double-enveloping worm gears were received by Dr. Lorenz, the founder of the well-known Lorenz Co, in 1891 (Litvin, 1998). In the early 30s of the XX century the US Company of the Michigan Tool has greatly succeeded in the development of the technology, tools and equipment to manufacture the Cone gears. These developments promoted the creation of the globoid gear, successfully competing with other manufacturers for more than 70 years (O’Connor, 1994). In Russia the first globoid gears were produced in 1904 at the Baltic Shipyard for winches warships. In the forties globoid gears were widely used in the Soviet Union in metallurgical, mining, materials handling, marine and other engineering industries. V.A. Shishkov, Ya.I. Diker, L.I. Sagin, A.K. Kartsev, V.L. Zhuravlev, P.S. Zak and others contributed to the successful organization of the globoid gear production in Russian engineering (Zak, 1962).

The difficult geometry of globoid worm gears, specific lubrication conditions and the generation of the worm wheel tooth flanks, comprising several zones, motivated many researchers to develop the analytical characteristics of the worm threads and worm wheel teeth engagement. Among them were N.I. Kolchin, B.A. Gessen and F.L. Litvin (see Litvin and Fuentes, 2008). In this paper the authors discuss the recent advances in evaluating the geometry, technology and load- carrying capacity of modified globoid gears.

2.3 Globoid worm gear applications & characteristics

The most important characteristic of a globoid worm-gear set is that it is throated & enveloped to improve the contact between the worm and gear. It indicates simultaneous meshing of multiple teeth, thus sharing the load all the time. The result is obviously increased loading capacity along with smoother operation. Other advantages of double enveloping worm gears include high torque, versatility, flexibility, good lubrication condition, higher load capacity, gear mesh, low transmission error, low (near zero) backlash.

Globoid worm gear drives can be used in the application areas that require higher load carrying capacity (heavy duty machinery), especially at higher centre distances, including working arrangements with shock loads. These drives are also found in aircraft / helicopter transmissions.

2.4 Previous Work

A great deal of research has been made on gear design & analysis, and a good literature is available on gear modeling & simulation. The stress analysis of gears, gear transmission faults, and gear dynamic loading predictions, noise in gear systems, and the design optimization of gear drives are considered as major problems in designing gears. Errichello, Ozguven and Houser analyzed variety of literature on the static & dynamic simulation models for multiple gear drive systems. The first known study related with gear transmission error was made by Harris. He analyzed spur gears behavior at different speeds and explained his result by showing curves of different transmission errors in static mode.

The tooth contact analysis of worm gear based drives was studied by Zhan Dong-an & Wang Shu-ren. They derived basic equations to carry out tooth contact analysis and analyzed mathematically the dynamic model for worm drive system.

DC Sun & Qin Yuan studied the kinematic & dynamic characteristics of worm gear drives. Their work included the selection of a double-enveloping worm gear and its dimensions suitable for use in helicopter transmissions; the 3-D graphics representation of the selected worm gear using the I-DEAS software; the analysis of the kinematics of meshing; the analysis of load sharing among the meshing teeth; and the implementation of the analyses in a computer program.

Manufacturing & load ratings of globoid gears were proposed by Sergey Lagutin. Their paper considers the technique of geometrical design of globoid gears modified by increasing the machine-tool center distance and gear ratio while cutting the worm with respect to the same parameters of the worm gear. The tooth machining by means of fly-blades with 2 or 4 cutters was described. The factors determining the gear main parameters, in particular, the worm pitch diameter and the gear wheel width were recommended. In order to evaluate the load-carrying capacity and efficiency of gears, they recommended choosing or calculating all the input factors. A number of factors, defined earlier by means of logarithmic charts, were described by analytical equations which were more convenient for the CAD. All the proposed methods and algorithms had been verified by the practice of design and manufacture of modified globoid gears for rolling mills, oil pumps, mixers and other equipment at the J S Co "EZTM" (Electrostal Plant of Heavy Machines) and a number of other plants.

Finite element analyses of worm gear pairs were analyzed by Tibor Bercsey. To find the efficiency and to minimize the effects of friction and wear along with the optimization of tribology conditions for heavily loaded worm drives, the contact stresses and pressure along the contact region of meshing teeth zones are required. The purpose of this analysis was to find the contact region and the distribution of varying load among the meshed teeth zones in worm gear teeth pairs.

Worm face gear drives parameters calculation with both cylindrical & conical worms based on application of tilted head cutter was proposed by Faydor L Litvin. His work is based on using grinding tool as gear cutter instead of using hob for cutting teeth. The conjugated worm face generation was also based on the usage of head cutter in tilted position. The contact at the bearing surface of the gear set was limited to a bearing area and oriented in the longitudinal direction. A pre-designed mathematical function of for finding transmission errors in gear drives to reduce undesired vibration & noise from the system was given. The analysis of stresses for the gear set was also made using 3-D computer-aided finite element approach.

Yaping Zhao & Zhao Zhang performed 3-D analysis to study the behavior of meshing teeth in the modified height teeth of globoid worm drive. Their analysis shows that height modification by varying flank improves meshing performance.

CHAPTER 3: GEOMETRY OF GLOBOID GEARS

A worm resembles the thread of a screw which wraps around a root cylinder in a helical fashion. The profile of the thread is either straight or slightly curved, depending upon the type of design used. The teeth of worm in worm-drive system may or may not be of involute form depending upon the designing needs. The worm may have more than one tooth or thread helix. It should be noted that the number of teeth is referred as the number of “start”. The basic dimensional parameters of the worm is pressure angle, module, length of thread, diameter of pitch cylinder, addendum, dedendum, tooth thickness, number of turns of the thread. The worm is coupled with a worm gear.

There are five most common types of worm thread profiles:

- Straight sided axial thickness section
- Straight sided normal space width section
- Involute helicoids
- Milled helicoid by double cone form
- Milled helicoid by circular convex form

3.1 Basic parameters of worm and wheel

To find worm & gear geometry the first step is to define worm & worm gear basic parameters and selecting the thread profile for worm.

For worm geometry, the main parameters of concern are:

- Number of threads on worm
- Axial Module
- Normal Pressure Angle
- Thread Thickness
- Worm pitch diamater
- Rotational speed of the worm (rpm)
- Worm Profile
- Outside diameter of the grinding wheel (for milled helicoid profile)
- Radius of curvature of the grinding wheel (for milled helicoid profile)

For the worm wheel, the main parameters are:

- Number of teeth on worm wheel
- Thickness of gear tooth
- Face width of worm wheel
- Outside addendum coefficient
- Bottom clearance coefficient

A brief description of some important above mentioned terminologies are defined below:

(1) (Normal) Pressure angle (*αn*): Pressure angle is measured in worm axis plane and is equal to half the profile angle of thread. The pressure angle used in worm gear drive is dependent on the lead angle (worm helix angle). The pressure angle must be large enough to avoid undercutting of gear tooth on the side at which the contact ends.

(2) Worm Axial pitch (*px*): The worm axial pitch is the distance measured axially, from a particular point on one tooth to the corresponding point on an adjacent tooth. It is also known as linear pitch.

(3) Number of starts of worm (*N1*): A worm can have a single or more threads or teeth, wound around the cylindrical body of the worm along the length of the worm. The number of threads is also termed as the number of starts. The number of starts should not normally exceed 6.

(4) Angle of Worm helix (*γ*): Worm helix angle is the angle subtended between a line tangent to the pitch helix and a perpendicular plane with reference to the worm axis. This can also be called as the worm lead middle angle.

(5) Addendum (*ba*), Dedendum: If the lead angle is small, the addendum and dedendum should be selected in relation to the axial module. It can be seen that as the lead angle increases, the axial module also increases for the same normal module.

(6) Hand of worm: A worm may be classified as right handed or left handed depending upon the direction of helix of the worm thread in which the thread winds around the root cylinder.

3.2 Standard Design procedure of globoid worm & gear

Worm gear dimensions are usually determined by following the gear standards (e.g. AGMA, GOST, ISO, DIN etc), which were developed based on many years of experience. The American Gear Manufacturing Association (AGMA) has developed the system of national standards, representing a group of correlated regulatory documents governing the calculation and production of modified globoid gears includes the following procedure.

Step 1: Globoid gear pairs, Basic parameters.

The basic parameters are: the center distance, a; the nominal gear ratio, i; the worm pitch diameters, d1; the gear wheel face width, b2. The procedure typically starts with a known speed ratio and a known center distance. For current application, the suggested transmission torque output is 280 N-m (approx), the speed ratio is 50, and the center distance is about 47.5 mm. These parameters are beyond the range covered by the AGMA standards. Hence, while the standards were followed as closely as possible, some modifications had to be made.

Step 2: Globoid gears, Basic worm and basic generating worm.

The standard establishes the pitch profile angle in the axial section in the middle of the worm. It regulates also the worm threads addendum and dedendum as well as the backlash and radial clearance and curvature radii of the fillets. The basic geometrical parameters for generating worm and gear surfaces are calculated.

Step 3: Globoid gears, Tolerances.

The standard regulates the rate of kinematic accuracy, tooth-to-tooth accuracy and gear contact, as well as the accuracy components.

Step 4: Globoid gears, Calculation of geometry.

It establishes the calculation technique for all geometric and tooth-measuring dimensions of globoid gears as well as the machining settings parameters of the worm.

Common terminologies used to describe the globoid worm-gear set are calculated in the table below. Another term that should be introduced is the mid-plane, which refers to the plane passing through the axis of the worm and perpendicular to the axis of worm gear. The pitch circle of the gear is measured in the mid-plane. The pitch circle of the worm is measured at the center of worm. In the globoid worm gear the worm circular pitch and the gear circular pitch are equal. The base circle is used to define the worm profile in the mid-plane. The normal pressure angle is always chosen to be 20 deg. Several of the selected quantities in Table below should be explained. A right hand worm is selected. The number of meshing teeth Np was chosen to be 5 to reduce the load level on the meshing teeth. The choice of $ß$ = 5 deg (apex angle of generating plane) was a compromise between acquiring a better contact condition and maintaining an acceptable thickness of tooth at the top of the thread of worm. It should be considered that for the double-enveloping worm wheel a different choice of dimensions can result in very different surface shape and contact pattern.

In classic Cone globoid gears the active part of the worm threads surface is generated by a rectilinear generatrix (cutting edge), which is located in a mid-plane of a globoid gear wheel and is tangent to some profile circle with the center lying on the gear wheel axis. This generatrix is rotating around the gear wheel center with the machine-tool ratio of angular velocities equal to the gear ratio. In such gears the initial pattern of the active surfaces contacting is a narrow strip located across the gear wheel tooth near its mid-plane. During the running-in period, the wearing out of both gear wheel tooth flanks and worm threads takes place. The so-called enveloping zone is generated at the gear wheel tooth flank, propagating on its considerable area within the natural wearing process.

The worm thread is also being worn out non-uniformly. The maximum wear appears on the thread part, entering the engagement. In the middle of the worm and in the adjacent after it zone the wear reaches a minimum and it is increased again at the thread output of the engagement. Such a running-in process takes about 150 to 200 hours, after that the geometry of teeth and threads active flanks is stabilized and the wear rate is multiply reduced. The variation of the thread geometry in the process of wear is called natural modification

The double-enveloping worm gear has two contact lines on the mating surface during most of the meshing period. In comparison to the single enveloping product that has only one contact line. Hence, the globoid worm-gear set is said to have superior contact properties.

3.3 Calculation of gear tooth geometry & gearing parameters

The nomenclature of double-enveloping worm gear set is shown in the figure below in which the basic layout can be seen.

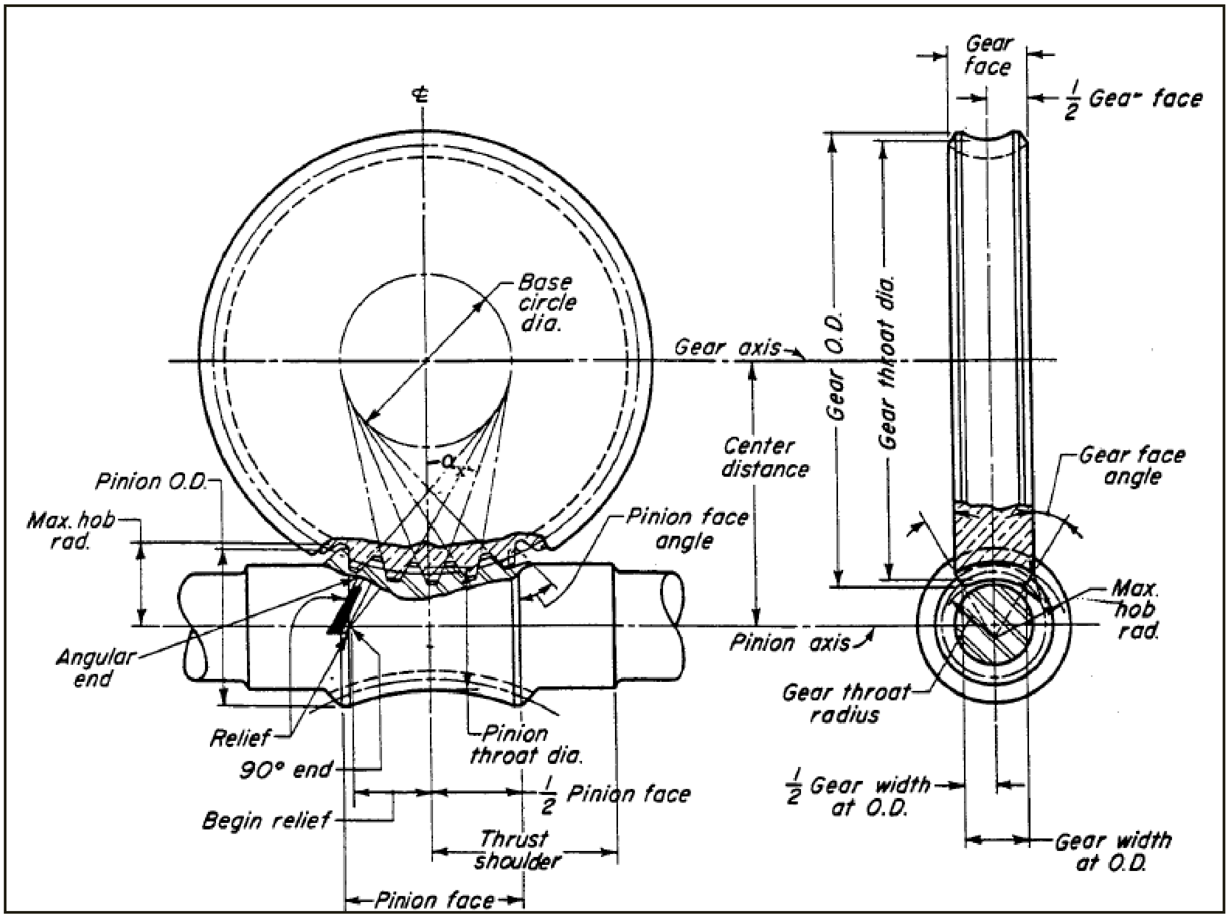

Figure 3.1: Nomenclature of globoid worm & wheel

To calculate gearing parameters and tooth geometry, standards have been established and formulas being developed. Table below lists the calculated dimensions and parameters for the desired globoid gear set, along with the formulas used for the calculation.

Table 3-1: Dimensions/Parameters of globoid worm & wheel

Parameter/Dimension	Symbol	Value	Formula based	Reference/Remarks
Speed Ratio	*i*	50	Known Value	Given
Normal pressure angle	*αn*	20º	Standard Value	Dudley, Practical gear design Handbook
Centre distance	*a*	47.5 mm	Known Value	Given
No of worm starts/threads	*N1*	1	Known Value	Given

Parameter/Dimension	Symbol	Value	Formula based	Reference/Remarks
No of gear teeth	*N2*	50	$N2 = N1 * i$	Dudley, Practical gear design Handbook
Worm pitch diameter*	*d1*	14.65mm (Kd=2)	$d1 = a^{0.875}/\mathrm{Kd}$ $\mathrm{Kd} = 1.7 \sim 2.2$	-do-
Gear pitch diameter	*d2*	80.35mm	$d2 = 2 * a - d1$	-do-
Axial pitch	*px*	5.04 mm	$px = \pi * d2/N2$	-do-
Worm lead angle*	*γ*	6.25 º	$\tan\gamma = px * \frac{N1}{\pi}/d1$	-do-
Normal circular pitch	*pn*	5 mm	$pn = px * \cos\gamma$	-do-
Axial pressure angle	*αx*	20º	$\tan\alpha x = \tan\alpha n \,/\, \cos\gamma)$	-do-
	Δα	0.03	$\Delta\alpha = \frac{px}{2}/d2)$	-do-
Base circle diameter	*db*	27.5 mm	$db = d2 * \sin(\alpha x + \Delta\alpha)$	-do-
Module	*xm*	1.6	$xm = px/\pi$	-do-
No of meshing teeth	*Np*	5	Known Value	Given
Half angle of meshing	*ψa*	16.4º (0.285 rad)	$\psi a = \pi * (Np - 0.45)/N2$	Shen, Meshing of Spatial Mechanisms and SG71 Worm gear
Start angle of meshing	*ψf*	3.614 º	$\psi f = \sin^{-1}(db/d2) - \psi a$	-do-
Thickness of worm thread	*S1*	2.268mm	$S1 = 0.45 * px$	Dudley, Practical gear design Handbook
Thickness of gear tooth	*S2*	2.772mm	$S2 = 0.55 * px$	-do-
Whole depth	*bt*	2.5 mm	$bt = 0.5 * pn$	-do-

Parameter/Dimension	Symbol	Value	Formula based	Reference/Remarks
Working depth	*bk*	2.25 mm	$bk = 0.45 * pn$	-do-
Addendum	*ba*	1.125mm	$ba = 0.225 * pn$	-do-
Clearance	*c*	0.25mm	$c = bt - bk$	-do-
Worm throat diameter*	*da1*	16.9 mm	$da1 = d1 + 2 * ba$	Shen, Meshing of Spatial Mechanisms and SG71 Worm gear
Worm root diameter*	*df1*	11.9 mm	$df1 = da1 - 2 * bt$	Dudley, Practical gear design Handbook
Gear throat diameter*	*da2*	82.6 mm	$da2 = d2 + 2 * ba$	-do-
Gear root diameter*	*df2*	77.6 mm	$df2 = da2 - 2 * bt$	Shen, Meshing of Spatial Mechanisms and SG71 Worm gear
Worm face width	*b1*	22.68mm	$b1 = d2 * sin\psi a$	-do-
Gear face width	*b2*	11.9 mm (1* df1)	$b2 = (0.9 \sim 1.0) * df1$	Shen, Meshing of Spatial Mechanisms and SG71 Worm gear
Apex angle of generating plane	β	5 º	Standard Value	Dudley, Practical gear design Handbook

***At the centre of worm**

3.4 Analysis of globoid gear operational data

To design the Globoid Worm Gear drive it is required to evaluate load carrying capacity and efficiency of the gear unit for the desired mechanism. This data will be analyzed mathematically.

3.4.1 Evaluation of load carrying capacity

The load-carrying capacity of globoid gears is influenced by a combination of different factors; among them are first of all the overall dimensions of the gear unit, materials of the gear wheel rim and worm, characteristics of the lubricant, sliding speed in the meshing, the quality of engagement and manufacturing accuracy in general. It is also important to assign correctly the operating conditions factors.

The center distance ‘a’ is the basic factor here. Zak (1962) carefully investigated the dependence of the globoid gear load-carrying capacity on its center distance. On the basis of considering the lubricating layer state in the engagement and solving the simplified hydrodynamic task he obtained experimentally and analytically, that the allowable torque, “T2” on the globoid gear wheel is related to the center distance a by the power law with an exponent value 2.9. Taking into account other factors affecting load carrying capacity, the calculation formula is as follows:

$$T2 = 55\mathrm{e}^{-5}\ \mathrm{a}^{\mathrm{n}} * \mathrm{Ku} * \mathrm{Kn1} * \mathrm{Km} * \mathrm{Kz} * \mathrm{Kp} * \mathrm{Kt} \tag{3.1}$$

Where,

T2= Allowable output torque (on globoid gear or wheel in N-m)

a= Centre distance of the gear unit (=47.5mm)

n= Exponent Value=4.02

Ku= Co-efficient of gear ratio

u=Gear Ratio=50

Kn1= Co-efficient of worm speed

n1=Worm Speed=31.45 Rpm

Km=Co-efficient of gear wheel rim material

Kz= Co-efficient of modified geometry

Kp =Co-efficient of operating mode of gear unit

Kt = Co-efficient of gears accuracy degree

The loading capacity of globoid gears, given by above relation, has been repeatedly confirmed by their long-term successful operation under different conditions and on a variety of machines and plants. However, the improvement of methods of globoid gears modification, on the one hand, and the new possibilities of CAD, on the other hand, demanded to specify ad correct the techniques of determining certain input coefficients. The choice of coefficients Ku and Kn1, according to Zak method, is carried out by means of curves plotted on graphs with a logarithmic scale, while these curves for the choice of the coefficients have been defined only for a few standard centre distances & gear ratios complicating the calculation of gears not involved into these graphs.

According to the gear ratio i (or u) ≤30, the coefficient Ku is uniquely determined by the formula:

$$\mathrm{Ku} = -0.759 \log \mathrm{u}^2 + 2.79 \log \mathrm{u} - 0.135$$

For the gear ratio, i (or u) >*30*, the coefficient Ku is determined by successive solutions of several simple equations:

$$\mathrm{A1u} = [\log(\log \mathrm{a}) + 0.839] * \log(\mathrm{u}/75) + 1$$

$$\mathrm{A1u} = [\log(\log 47.5) + 0.839] * \log(50/75) + 1$$

$$\mathrm{A1u} = [\log 1.676 + 0.839] * \log(0.666) + 1$$

$$\mathrm{A1u} = 1.065 * (-0.176) + 1$$

$$\mathrm{A1u} = -0.1872 + 1$$

$$\mathrm{A1u} = 0.812$$

$$\mathrm{A2u} = [\log(108/\log \mathrm{a}) * \log \mathrm{a}]/\log(\mathrm{a}/245)$$

$$\mathrm{A2u} = [\log(108/\log 47.5) * \log 47.5]/\log(47.5/245)$$

$$\mathrm{A2u} = -4.257$$

$$\mathrm{Au} = [\mathrm{A1u}/\mathrm{A2u}] - 0.158$$

$$\mathrm{Au} = -0.348$$

$$\mathrm{Ku} = \mathrm{Au} \log(\mathrm{u}) * \log(\mathrm{u}/30) + 1$$

$$\mathrm{Ku} = -0.348 \log(50) * \log(50/30) + 1$$

$$\mathrm{Ku} = 0.868$$

The expression of Kn1 is given by the relation:

$$Kn1 = AN \log n1 * \log (n1 - 3) + 1$$

Similarly, a set of equations provide the coefficients of the worm velocity “Kn1”.

$$A1n = \log [6.75 \log^2 a - 33.92 \log a + 53.61]$$

$$A1n = \log [6.75 \log^2 (47.5) - 33.92 \log (47.5) + 53.61]$$

$$A1n = 1.196$$

$$A2n = [A1n - \log (\log a)] * \log a$$

$$A2n = [1.196 - \log (\log 47.5)] * \log 47.5$$

$$A2n = 1.629$$

$$AN = [-\log (a/95) * (2.97 - 0.796 \log n1)]/A2n - 0.202$$

$$AN = [-\log (47.5/95) * (2.97 - 0.796 \log 31.45)]/(1.629) - 0.202$$

$$AN = 0.125$$

$$Kn1 = 0.125 \log 31.45 * \log (31.45 - 3) + 1$$

$$Kn1 = 1.272$$

The coefficient Km taking into account the gear wheel rim material for phosphor bronze is within the range 1.0 to 0.8, while for a cast iron, it is equal to 0.5.

$$Km = 0.9$$

The coefficient Kz may vary depending on the gear ratio within 1.1to1.2, where the lower value corresponds to the gear ratio ≤10 and the higher one is for >25.

$$Kz = 1.2$$

The proposed technique also expanded the range of values of the service factor, Kp, which is the product of two coefficients Kpv and Krw. The coefficient Kpv of cycle duration is assigned depending on the operating time of gears and ranged from 1 at 100% load to 1.25 at

25%. The coefficient of operating conditions of the gear unit Krw is assigned from 0.65 for heavy duty work with a large (up to 200%) overload to 1.15 for light operating conditions without shocks and short time overloads that don’t exceed 125% of the rated load.

$$\mathrm{Kp} = \mathrm{Kpv} * \mathrm{Krw}$$
$$\mathrm{Kp} = 1 * 0.65$$
$$\mathrm{Kp} = 0.65$$

The coefficient Kt taking into account the degree of accuracy has to be equal to 1 for the 7^{th} degree and equal to 0.8 for the 9^{th} degree.

$$\mathrm{Kt} = 0.9$$

$$T2 = 55\mathrm{e}^{-5}\ \mathrm{a}^{\mathrm{n}} * \mathrm{Ku} * \mathrm{Kn1} * \mathrm{Km} * \mathrm{Kz} * \mathrm{Kp} * \mathrm{Kt}$$
$$T2 = 55.10^{-5}\ 47.5^{\mathrm{n}} * 0.868 * 1.272 * 0.9 * 1.2 * 0.65 * 0.9$$
$$T2 = 55 * 10^{-5} * 5499311 * 0.1658$$
$$T2 = 501.5\ \mathrm{Nm}$$

Gudov et al. (2008) have introduced an additional correction coefficient into the formula:

$$K = 20/\mathrm{L}^{0.25}$$

which allows for the specified service life L in hours.

Graphs of the allowable torque T_2 on the gear wheel shaft in relation to the center distance aw are shown in Figure below for three gear ratios ”u”. The constant input values are:

- Worm input speed (N1): 31.45 RPM
- Service life (L):5,000hrs
- Safety factor (Kp): 0.65
- Alloy steel hardened to 285 HB as worm material

- Phosphor Bronze as gear or wheel material

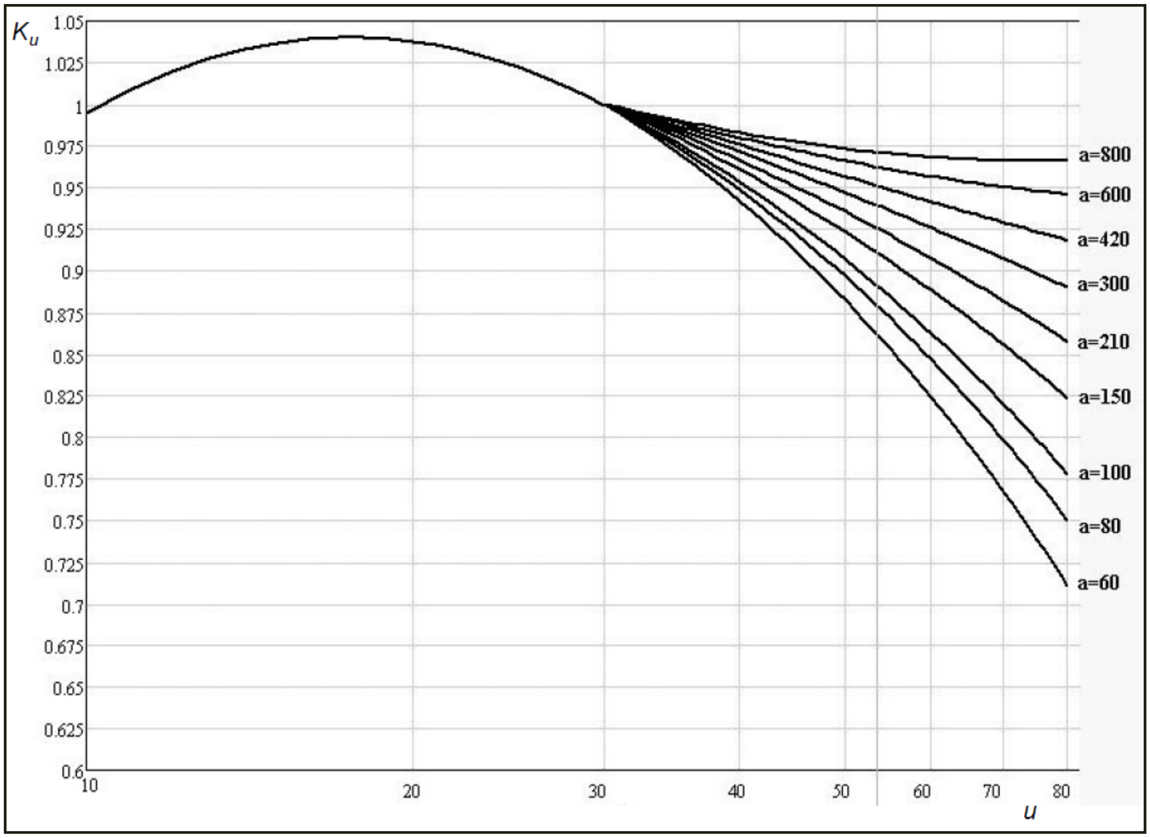

Figure 3.2: Coefficient *K*u the gear ratio influence, with "*a"* in *mm* (about Zak)

The formula determining the loading capacity of cylindrical worm gear drive is similar to the above formula and differs by the values of certain coefficients, e.g., the value of exponent (4.02) in the center distance is different for cylindrical worm gears than for globoid gear drive. Therefore, advantages of globoid gears are especially noticeable at large center distances. This fact is confirmed by comparing data of cylindrical and globoid gears.

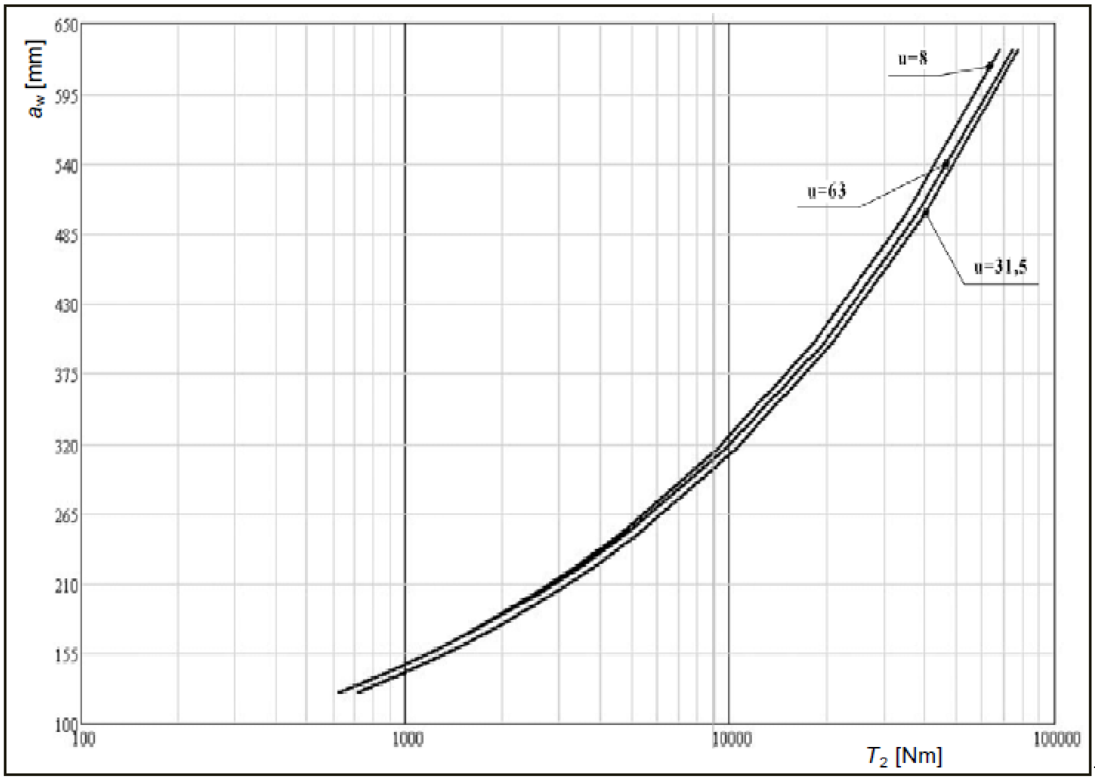

Figure 3.3: Relation between Allowable Torque (T2) & centre Distance (aw)

3.4.2 Efficiency of globoid gear unit

The efficiency of a globoid gear unit is generally determined by considering into account the losses in the engagement. The efficiency is generally expressed as the product:

$$\eta = \eta z * \eta m * \eta t \tag{3.2}$$

where,

η= Efficiency of the globoid gear unit

ηm =Coefficient of losses for oil mixing

ηt =Coefficient of the gear unit under loading

ηz =Coefficient of losses in the engagement

Taking into account the losses in the engagement, the coefficient ηz for globoid gears using the principle of wedge is calculated as:

$$\eta z = \tan \gamma \,/\, \tan(\gamma + \rho)$$

where, ‘γ’ is worm lead angle at the centre of the pitch globoid, ‘ρ’ is a friction angle, which depends on the sliding velocity in the meshing, the antifriction the hardness & roughness of worm threads and the accuracy degree of gears.

Friction Angle is generally defined by

$$\tan\theta = \mu s$$
$$\theta = 19.29°$$
$$\eta z = 22.9\%$$

(μs=0.35 for phosphor bronze-steel combination)

The detailed calculation of the friction angle, as well as the coefficient ηm that allows for losses for oil splashing and mixing, may be found in a number of works including Zak (1962) and will not be considered. We have to repeat only that in globoid gears the contact lines are located across the tooth, perpendicular to the sliding velocity. So the conditions for the lubricating oil layer formation are better, the angle of friction is smaller and the efficiency is higher than in gears with a cylindrical Archimedean or close to it worm.

At last we note that when selecting an electric motor to the drive, comprising a globoid gear transmission, that the product "ηz.ηm” defines its efficiency at full load. If the load on the gearset is less, it is necessary to take into account the dependence of the efficiency on the actual load. This dependence has a shape of an inverted square parabola, the vertex of which corresponds to the rated load and can be taken into the coefficient:

$$\eta t = \mathrm{T}_{2\mathrm{F}} Kp * [(2\mathrm{T}_2 - \mathrm{T}_{2\mathrm{F}} * Kp)/(\mathrm{T}_2)^2] \quad (3.3)$$
$$\eta t = 18.6 * 0.65 * [(2 * 27.92 - 18.6 * 0.65)/\ (27.92)^2]$$
$$\eta t = 0.68 \text{ or } 68\%$$

where,

$\mathrm{T}_{2\mathrm{F}}$ is the actual specified output torque

CHAPTER 4: KINEMATIC & DYNAMIC CHARACTERISTICS OF GLOBOID WORM DRIVE

The kinematic and dynamic analysis of gear mechanisms includes the development of equations describing the position, velocity, and acceleration of all points of interest in the mechanism in relation to a chosen primary variable. For this part of the analysis, several assumptions were made:

1) No friction
2) Rigid bodies
3) No backlash
4) Mass-less members

The kinematic properties of worm gear drives are analyzed with the method of enveloping.

4.1 Method of enveloping

To describe the enveloping method, let us consider two rotating machine components, m and k, which are in meshing. Now attach a coordinate system S_m (x_m,y_m,z_m) to the machine component m; and likewise attach a coordinate system S_k (x_k,y_k,z_k) to the machine component k. The two machine components are rotating with the angular coordinate's ψ_m and ψ_k with respect to some stationary reference frame. Since the two components are in meshing there must be some definite relationship between ψ_m and ψ_k. Hence only one of them is independent, and either can serve as the running parameter. Let ψ denote this running parameter. The surface of machine component m can be expressed as f (x_m,y_m,z_m)=0 in S_m. By coordinate transformation, f=0 becomes a family of surfaces, F (x_k,y_k,z_k,ψ)=0 in S_k. Then the surface of the machine component k is the envelope of F=0, viz.

$$F\ (x_k, y_k, z_k) = 0$$

$$\frac{\partial F}{\partial \varphi} = 0$$

If ψ_k / ψ_m = constant, the surfaces of the two meshing components are said to be conjugate surfaces (Litvin, Theory of Gearing. NASA Reference Publication).The conjugate analysis can be used to model the worm wheel flank generation processes during hobbing process. Many types of worm tooth geometry can be found in application and work is still going on to find new ones. The use of flexible mathematical models capable of analyzing all types of worm gearing is thus very beneficial. The models are represented in the form of matrices. The major advantage of the matrix models is that the geometric parameters and the motional parameters appear in separate matrices. Therefore the equations can be applied to different geometries without further derivations. Furthermore, dealing with matrix models in a computer is very convenient, because the individual matrices are of simple form and easily derived, so that the computer can be left to carry out the multiple matrix operations without the need for complex algebraic analysis. To carry out the analysis, four right hand coordinate systems, S_1, S_2, S_j and S_n, are set up. S_j and S_n are stationary systems. S_j (x_j,y_j,z_j) is located at the worm position. x_j is along the center line of the worm gear and points to the gear, and z_j axis is along the worm axis. S_n (x_n,y_n,z_n) is located at the gear position. x_n is along the center line but opposite to x_j, and z_n is along the gear axis.

S_1 and S_2 are moving systems. S_1(x_1, y_1, z_1) rotates with the worm and its Z_1 axis always coincides with the Z_j axis. The worm rotation angle ψ_1 is the angle between x_1 and x_j. S_2(x_2, y_2, z_2) rotates with the gear and its z_2 axis always coincides with the z_n axis. (In the first enveloping process S_2 turns around the base circle with the grinding wheel.) The rotation angle $\psi_2$is the angle between x_2 and x_n. Thus, S_1 is fixed to the worm and S_2 is fixed to the gear; and the worm and gear rotate about S_j and S_n, respectively. Initially S_1 coincides with S_j, and S_2 coincides with S_n.

The coordinates in S_m can be transformed into S_k by multiplying the S_m co-ordinates matrix with the co-ordinate transformation matrix where ψ is the running parameter of the coordinate transformation.

$$\begin{bmatrix} x_k \\ y_k \\ z_k \\ 1 \end{bmatrix} = A_{\psi}^{(km)} \begin{bmatrix} x_m \\ y_m \\ z_m \\ 1 \end{bmatrix}$$

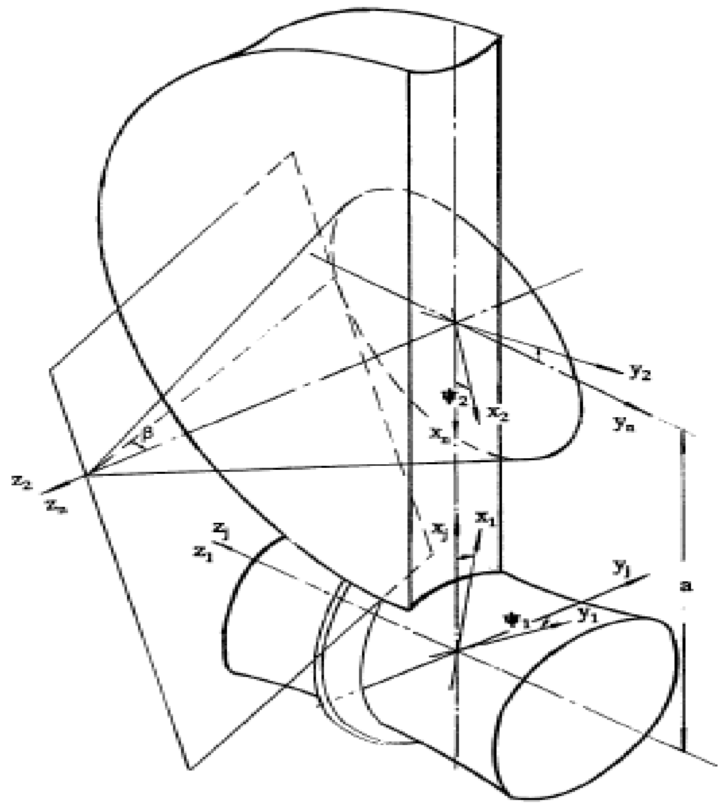

Figure 4.1: Co-ordinate System

The coordinate transformation matrices from **Sj** to **S1**, from **S2** to **Sn**, and from **Sn** to **Sj** are given as:

$$A^{(1j)} = \begin{bmatrix} \cos\psi_1 & \sin\psi_1 & 0 & 0 \\ -\sin\psi_1 & \cos\psi_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A^{(n2)} = \begin{bmatrix} \cos\psi_2 & -\sin\psi_2 & 0 & 0 \\ \sin\psi_2 & \cos\psi_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A^{(jn)} = \begin{bmatrix} -1 & 0 & 0 & a_0 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

To distinguish between the first and the second enveloping processes, ψ is used as the running parameter for the first enveloping process and θ is used for the second enveloping process. The order of operations with respect to different running parameters may be interchanged.

4.1.1 Worm surface

The worm surface is formed by a grinding wheel in the first enveloping process. The surface of the grinding wheel is called the original generating surface. For a disc grinding wheel this original generating surface is a plane. For a cone grinding wheel the original generating surface is a line on the cone. The original generating surface can be expressed in the S_2 system as $f(x_2, y_2, z_2) = 0$. In the first enveloping process S_2 rotates about z_2 with the rotation angle ψ_1, and S_2 rotates about z_1 with the rotation angle ψ_1. As a result of rotation, the original generating surface forms a one-parameter family in S_l. Because ψ_1 / ψ_2 = constant, the running parameter ψ can be either ψ_1 or ψ_2. Then, the envelope of the family of surfaces is the worm surface and represents equation of a straight line. This straight line is on both the generating plane and the enveloping surface. This straight line is the contact line between the worm and the disc grinding wheel at the position ψ_1. It can also be seen that the worm surface is the set of all the straight contact lines.

4.1.2 Gear surface

The same procedure applies to the second enveloping process. In this process the gear hob has the same shape as the worm surface. As the result of rotation, this generating surface forms a one-parameter family of surfaces in S_2. The running parameter for this process can be either θ_1 or θ_2, while ψ becomes a geometric parameter already in existence in the worm surface equation.

From the above description, it can be seen that in the case of double enveloping worm gears, the worm and gear are in line contact, and there are usually two contact lines on each pair of the meshing teeth.

The kinematic models are used in theory of gearing for:

i. Determining generated surfaces as the envelope of a family of generating surfaces.

ii. Determining lines of contact on conjugate surfaces.

iii. Defining the limits for no undercutting of the generated surfaced using generation.

iv. Calculation of the relative normal curvatures and the sliding and rolling velocities at contact points of the mating surfaces.

v. Determining the surfaces of tools used to manufacture gears.

vi. Simulation of mismatched (non-conjugate) gear drives, including prediction of the path of contact, entrainment velocity, and contact gap contour and transmission error.

vii. Prediction of the effects of manufacturing errors on no-load performance.

4.1.3 Equation of Meshing

The equation of meshing is used to determine contact points on the worm surface during the conjugate action. Only those points on the worm surfaces which satisfy this equation will take part in generating the worm wheel surface. The equation of meshing is:

$$\boldsymbol{M} =. \boldsymbol{N}_{\mathbf{1}}^{(\mathbf{1})} . \boldsymbol{B} . \boldsymbol{r}_{\mathbf{1}}^{(\mathbf{1})} = \mathbf{0} \tag{4.1}$$

The normal vector $\mathbf{N_1}$ and the position vector $\mathbf{r_1}$ are functions of the surface parameters, while **B** depends on wheel and worm / hob rotation angle.

4.2 Dynamic model of worm gear drive system

The dynamic model for the worm gear drive design relies on specific conditions of sliding friction contact and backlash between mating gear components. By considering the multi-body contact model approach and partitioning co-ordinate method, an assembly motion of conjugated rigid and inertial bodies is acceptable for gear dynamics. Such design of dynamic system with structural variable properties on the force constraints level can be classified as discontinuous dynamical systems with respect to unpredictable time events caused by system transitions from one dynamic regime of motion to another.

The internal dynamic loads and appropriate varying relationships at the Worm Gear Mesh are represented by compatible set of the Force Transfer Functions. Such force/load parameters are the main object of simulation and qualitative system analysis for diagnostic as well. The corresponding Two-Sided Wedge Mechanism as dynamical translational analogue model of the Worm Gear Drive has been developed which can satisfy all possible regimes of motion. The Design Dynamical Model of the Worm Gear Drive system modeled by a Two-Sided Wedge Mechanism (TSWM) is shown in fig below. The model can be classified as translational and dynamically structural variable multi-body system with respect to the contact type and appropriate force constraints. The worm gear mesh contact is exhibited on a plane model by two slope active lines (a-a) and (b-b) between conjugated wedge-like rigid and inertial bodies via the presence backlash. This model has been developed to simplify visualization of variable internal force distributions according to the sliding friction contact conditions resembling a screw-like worm gear mesh and to separate motion into two different dynamical regimes called as “tractive” and “inverse-tractive” corresponds to relationships between internal force components (reactions) and contact conditions.

The following assumptions are made. The inertial bodies m1 and m2 are realized as assembly motion in generalized rectangular co-ordinate x_1, x_2 with relative motion to each other via one of two slope contact surface-line (a-a) or (b-b) with the presence of negligible clearance. The absolute bodies’ motion along co-ordinate directions are restricted by ideal support guides 3.

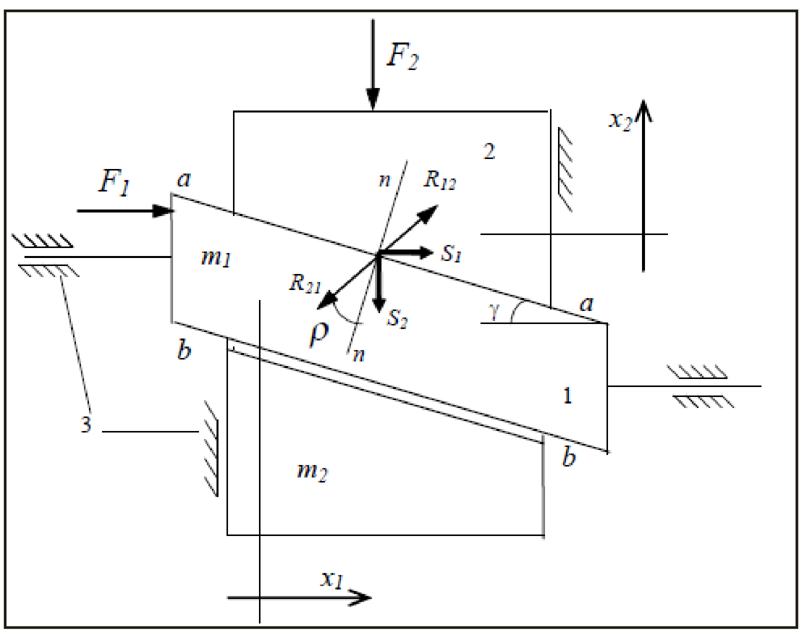

Figure 4.2: Two-Sided Wedge Mechanism

(Translational Model of Worm Drive System)

The masses m1 and m2 can be derived by the following formulas:

$$m1 = I\sum 1 / R1^2,\ m2 = I\sum 2 / R2^2$$

where, $I\sum 1$ is the summarized moment of inertia of the drive elements, connected to the worm shaft, $I\sum 2$ is the summarized moment of inertia of the driven elements, which are connected to the wheel shaft & R_1, R_2 are radii on the pitch cylinders of the worm and worm wheel respectively. The negligible clearance between TSWMs bodies identifies the worm gear mesh backlash and instantaneous transitions during contact-case modifications. The contact conditions at the TSWM are subjected to the pure sliding friction law resembling to the worm gear contact.

4.2.1 External forces

The generalized forces F_1 and F_2 (in above figure), applied to the mass m_1&m_2 respectively, act along the coordinate directions $0x_1$, $0x_2$, and are represented by external force contour components. For the real-life Motor Operated Drive systems, the force F_1 can be expressed according to the motor start characteristic curve as shown in figure below by nonlinear function:

$$F_1 = \frac{M(\omega)}{R_1} = \frac{2F_{\max}}{\dfrac{V_0 - \dot{x}_1}{V_0 - V_{\max}} + \dfrac{V_0 - V_{\max}}{V_0 - \dot{x}_1}}$$

where, $V_o=\omega_o*R_1$ is the synchronous linear velocity; $V_{max}=\omega_{max}*R_1$ is the maximum linear velocity of driving body; x_1 is the current linear velocity of driving body ; $F_{max}=M_{max} / R1$ is the maximum motor force; ω_o, ω_{max},M_{max} are the recommended constant motor parameters & R_1 is the worm pitch radius.The external resistant force F_2^- is applied to the driven body in the opposite direction to the body motion and used as negative value ($F2 <0$), has to be constant over the observed period of time.

4.2.2 Kinematic constraints

In general, the contact at the wedge mechanism and appropriate worm gear mesh undergoes holonomic constraints with constant kinematic ratio. The constraint equation on position level $x_1 = x_1(t)$, $x_2 = x_2(t)$ according to the contact geometry via slope surface between moving bodies can be written as follows:

$$x_2(t) = x_1(t)\, tg\, \gamma$$

$$\text{or, } x_2(t) = x_1(t)\, tg\, \gamma = 0$$

where, $\boldsymbol{\gamma}$ is the constant slope angle, which constitutes a geometrical parameter.

Differentiating above equation gives joint velocities and accelerations of the moving parts as corresponding constraint equations

$$\dot{x}_2(t) = \dot{x}_1(t) tg\gamma$$
$$\ddot{x}_2(t) = \ddot{x}_1(t) tg\gamma$$

A sliding/relative velocity "Vs" acts along the slope contact line between the moving bodies and resemble to the worm gear mesh velocity scheme resulting in the following relation:

$$V_s = \frac{\dot{x}_1(t)}{\cos\gamma} = \frac{\dot{x}_2(t)}{\sin\gamma}$$

which can also be written as:

$$V_s = V_1/\cos\gamma = V_2/\sin\gamma$$

and denote linear velocities in the worm gear mesh.

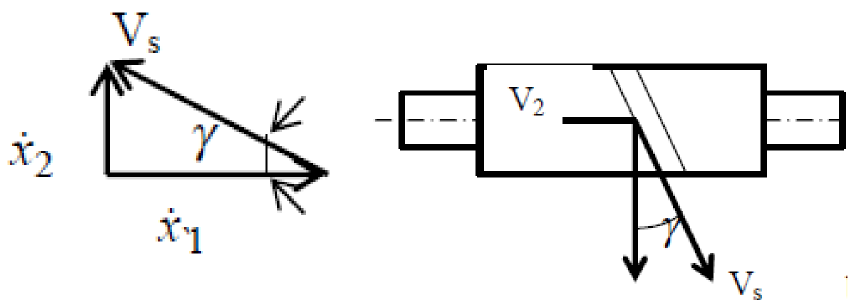

Figure 4.3: Sketch plane of the distribution of the linear velocities at the TSWM and the worm gear mesh

4.2.3 Mathematical Model

The dynamics equations of motion including active internal forces and complement constraint equations are resulted in DAEs form

$$
\begin{aligned}
m_1\ddot{x}_1 &= F_1 - S_1,\\
m_2\ddot{x}_2 &= F_2 - S_2,\\
\ddot{x}_2 &= \ddot{x}_1 tg\gamma,\\
S_1 &= S_2\psi_j, (j = \overline{1,2}).
\end{aligned}
$$

$$\psi_1 = tg(\gamma + \rho), \psi_1 = tg(\gamma - \rho)$$

The mathematical problem formulation results in residual ODEs form with unpredictable discontinuities in the right-hand side within FTFs together with corresponding initial conditions and implementation of non-linear functions constitutes a full mathematical model resulted in:

$$\ddot{x}_1(t) = \frac{F_1 - F_2\psi_j}{m_1 - m_2\psi_j tg\gamma}, (\psi_j = \{\psi_j, j = 1,2\})$$

$$x_1(0) = x_{10}, \dot{x}_1(0) = \dot{x}_{10}$$

where,

$$\psi_1 = -tg(\gamma + \rho(\dot{x}_1)), \psi_2 = -tg(\gamma - \rho(\dot{x}_1)).$$

$$\rho(\dot{x}_1) = 1/\left(a(\dot{x}_1 / \cos\gamma)^b + c\right)$$

$$F_1 = \frac{2F_{\max}}{\dfrac{V_0 - \dot{x}_1}{V_0 - V_{\max}} + \dfrac{V_0 - V_{\max}}{V_0 - \dot{x}_1}}$$

This mathematical model of the TSWM describes an assembly motion in different regimes within the space of independent state coordinates.

CHAPTER 5: 3-D MODELING & FINITE ELEMENT ANALYSIS OF GLOBOID GEAR SET

5.1 Solid modeling based on Pro/E & Solid Edge

Modeling & Simulation of mechanical systems uses computer-aided design softwares for 3-D modeling. 3-D modeling is the geometrical representation of actual object without losing its geometry information. 3-D modeling involves various modeling tools like parametric modeling, feature-based modeling, solid and surface modeling.

Parametric modeling means Mechanical CAD software uses parameters to model the desired geometry. The most significant of these parameters are geometrical dimensions. Geometry modeling is done using standard relations and formulas between various parameters. Parametric modeling is performed when part features are inter-dependent and they are drawn by taking reference of each other. The attributes of the feature can be modified by changing the relationship. The changes will propagate automatically throughout the model. Thus, they develop a relationship among themselves. This relationship is known as the parent-child relation. So, if someone wants to change the placement of the child feature, the alterations can be made in the dimensions of the references and hence the design can be changed as per requirement.

Parametric modeling allows the design engineer to let the characteristic parameters of a product drive the design of that product. During the gear design, the main parameters that would describe the designed gear such as module, pressure angle, pitch circle diameter and tooth thickness could be used as the parameters to define the gear. But, the parameters do not have to be only geometric. They can also be key process information such as case hardening specifications, Quality of grades, metallurgical properties and even load classifications for the gear being designed.

Pro/Engineer uses these parameters, in combination with its features to generate the geometry of the gear and all essential information to create the model. For example, the parametric dimensional values make the shape of the tooth profile and non-geometric parameters information specifies things like the required case hardening depth or nondestructive testing requirements.

An important aspect of feature based modeling in Pro/Engineer is the concept of parent/child relationships. A child feature is one of those references previously created parent feature. For example, the surface of a block might be used as a reference plane to create a slot. A change to the parent feature will also potentially affect the child.

5.1.1 Parametric Modeling of Globoid Worm & Gear

Using Pro/Engineer to design globoid worm & gear, start with using basic parameters and relations necessary for modeling geometry. This article focuses on using mathematical relations for profile modeling. The procedure to model globoid worm is as follows. Open a new part file in Pro-Engineer wildfire 4.0. Click on Tools→Parameters and create the input parameters as mentioned in Table below. These parameters will decide the geometry of the worm.

Table 5-1: Worm Input Parameters

Variable	**Variable type**	**Value (mm)**	**Description**
a	Real number	47.5	Centre Distance
d2	Real number	80.35	Gear Pitch Circle Diameter
pn	Real number	5	Circular Pitch
an	Real number	20°	Pressure Angle
px	Real number	5	Axial Pitch
L	Real number	6**pn* =30	Length of Worm
N	Real number	L / *px* =6	Number of turns of the thread
ba	Real number	1.125	Addendum
S1	Real number	2.268	Thickness of Worm thread
bt	Real number	2.5	Whole Depth

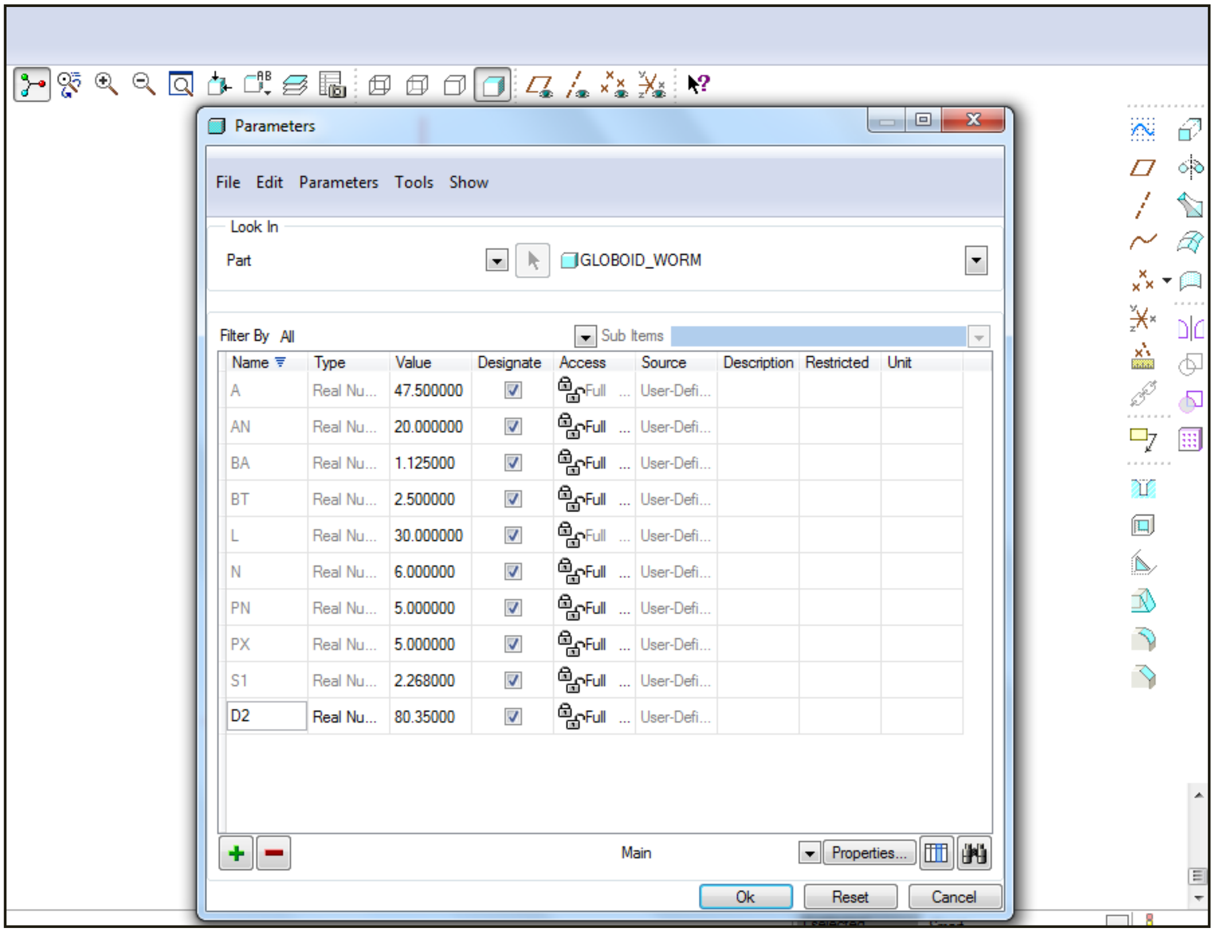

Figure 5.1: Worm input Parameters

Create the basic geometry such as addendum, dedendum and pitch circles in support of the gear tooth. Create the tooth solid feature with helical sweep protrusion. The sweep surface is the helix of the required worm.

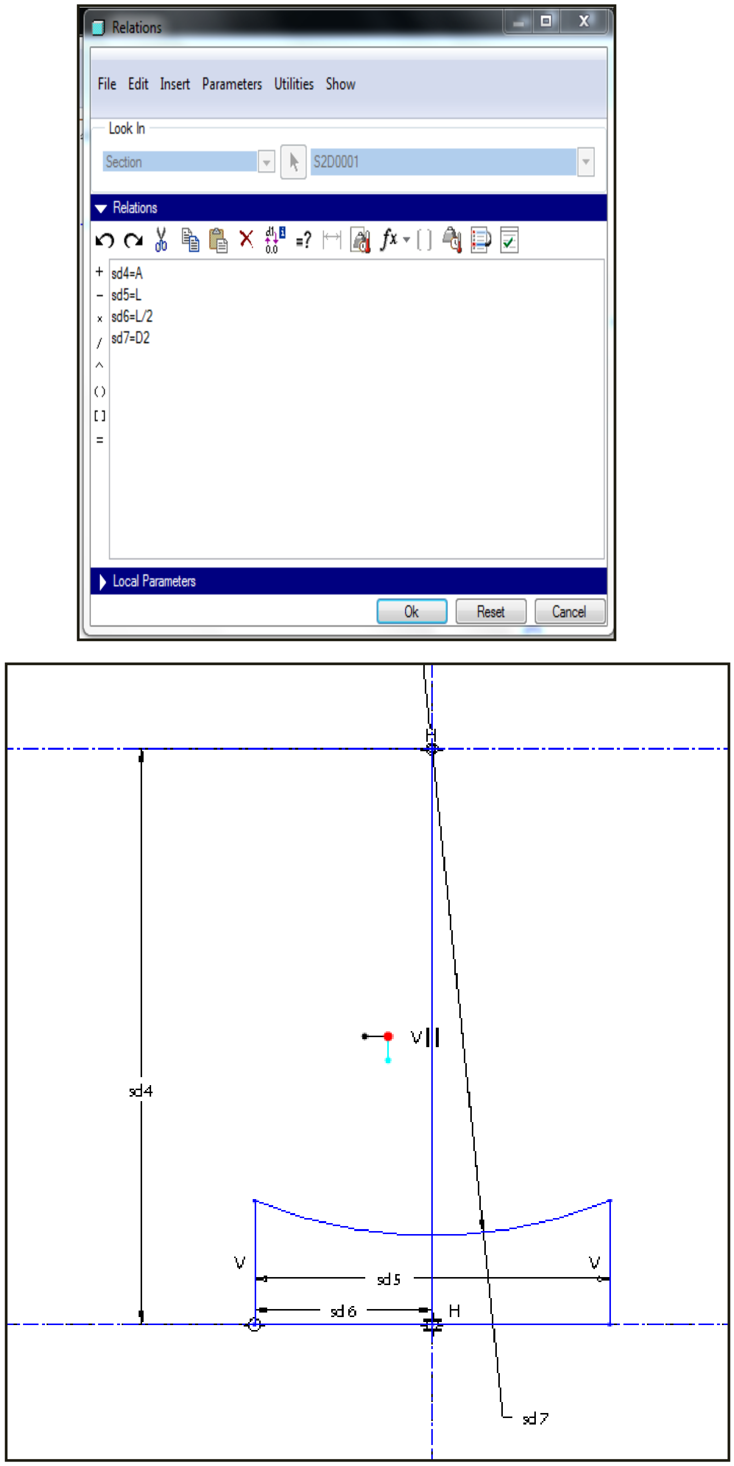

Figure 5.2: Relations for Base Diameter Profile for Globoid Worm

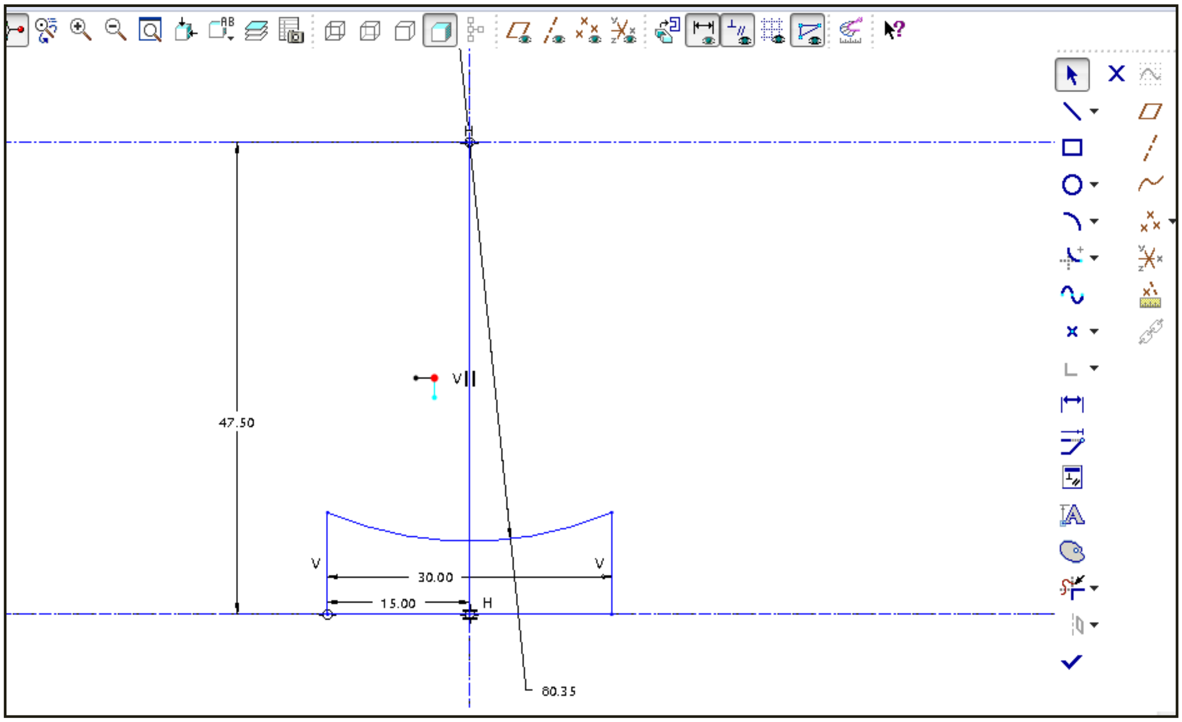

Figure 5.3: Base Diameter Profile for Globoid Worm

Revolve the sketch to get globoidal shape. Use Helical Sweep Profile for Tooth Profile using constant, thru axis and Right-handed attributes. Define sweep profile for the thread, give axial pitch value and define section for worm thread as shown below.

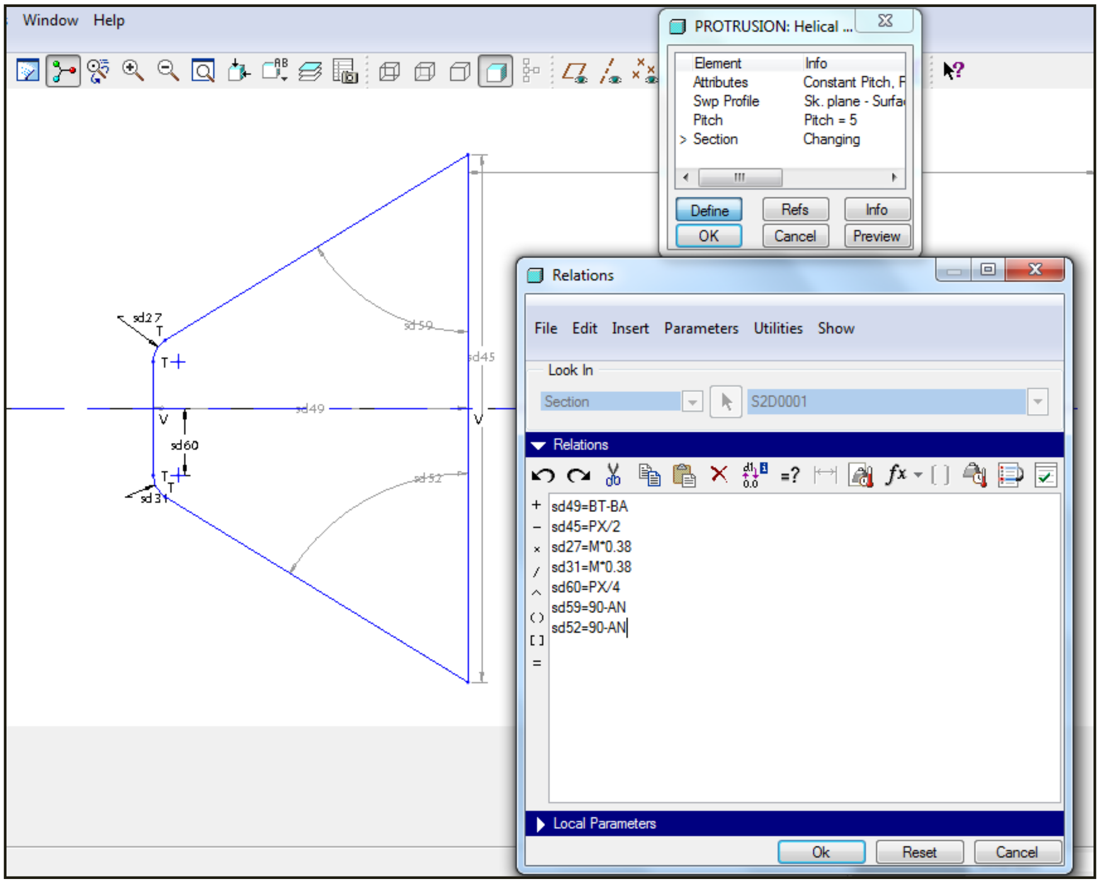

Figure 5.4: Tooth Geometry for Globoid Worm

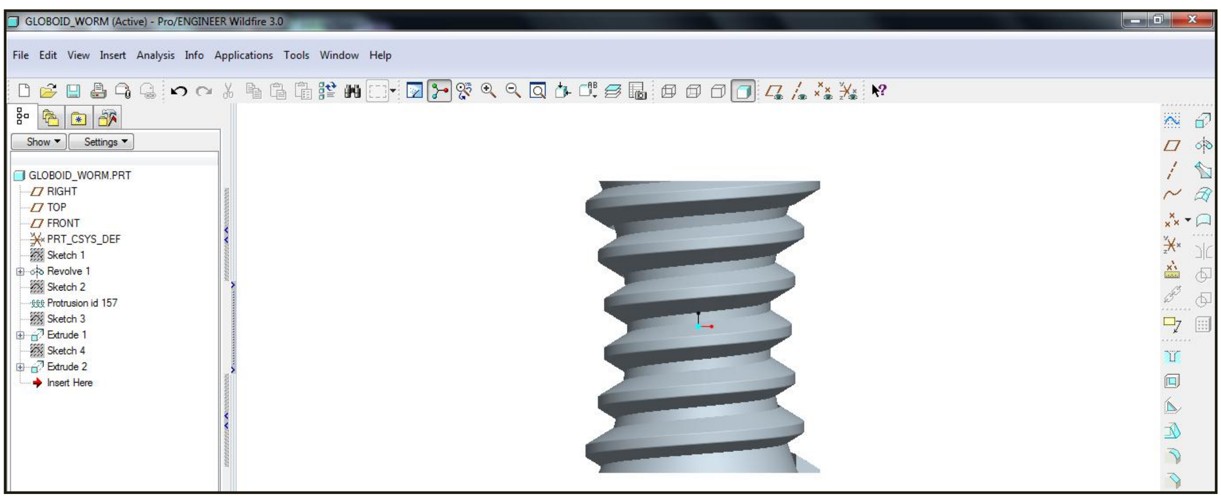

Figure 5.5: 3D Model of Globoid Worm

Worm Gear is also modeled using the parametric procedure. The tooth profile for worm wheel is same as that of globoid worm. Gear helix angle is same as worm lead angle. Use Pattern

Command around the centre line of the gear axis to have multiple teeth. The combined worm and gear set modeled for the said retraction unit are shown below.

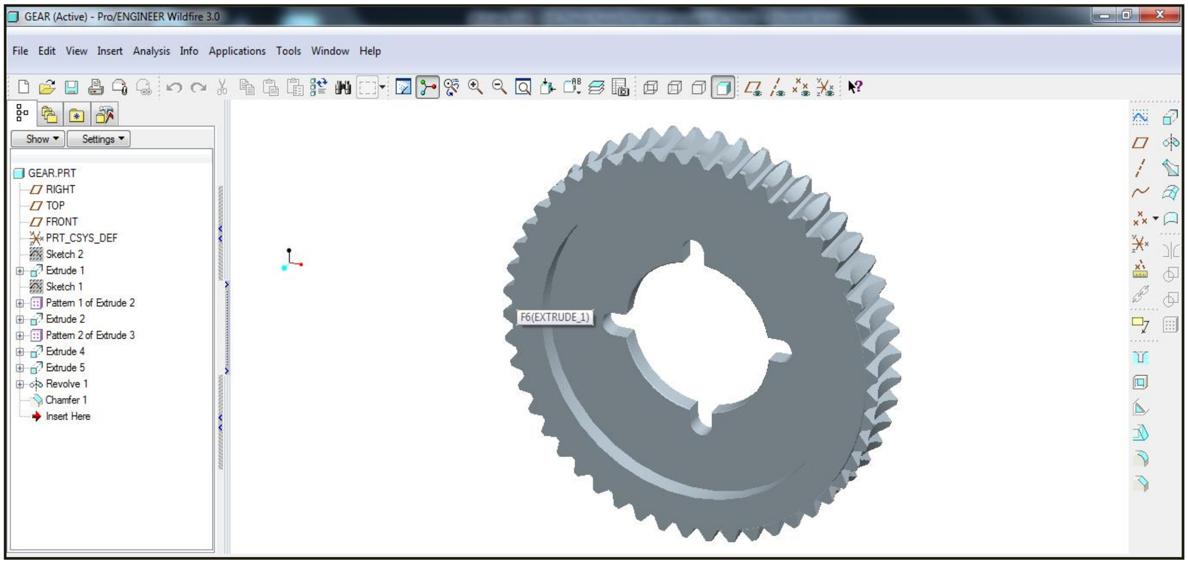

Figure 5.6: 3D Model of Worm Gear

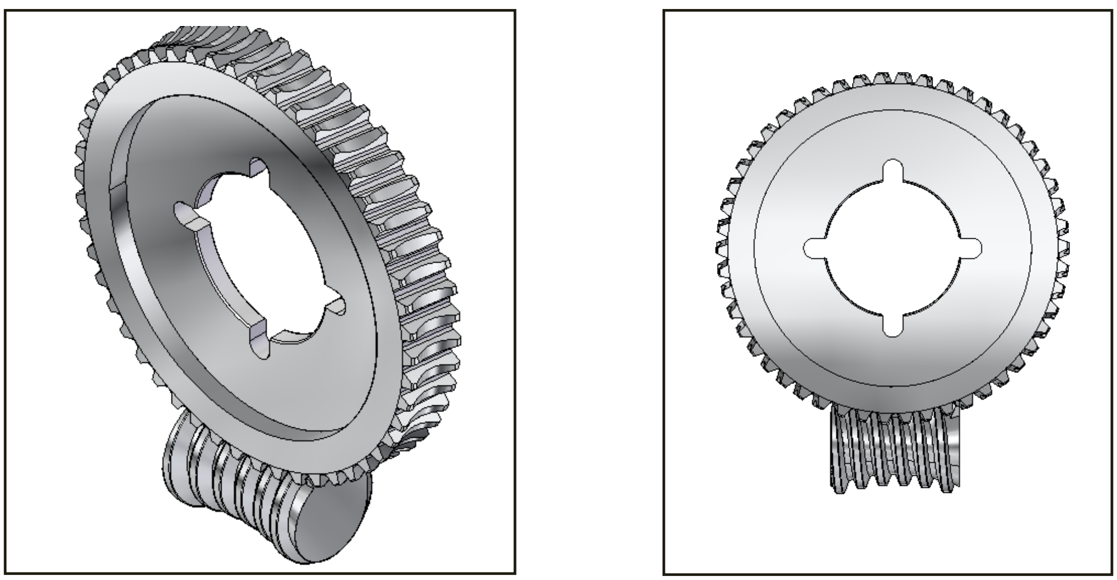

Figure 5.7: Globoid Worm Gear Assembly

The complete assembly of actuator modeled in Solid Edge ST3. The rotation-type slewing actuator includes a main housing that covers double-enveloping worm gear assembly along with supported bearings (axial and radial both). The housing also has a positioning shaft directly coupled to the prime mover (DC motor with necessary speed reduction assembly in this case). A hole at the bottom of the housing is provided for pivoting the actuator with the mechanism. Some modification is made in Globoid worm to mount it with motor via coupler for input drive. Link Connectors are attached with worm wheel to mount actuator with lift mechanism. A feedback device is also coupled via gear drive for actuator zeroing reference.

5.2 Introduction of Finite Element Software: ANSYS

Finite Element Analysis is a computer simulation program used in engineering analysis that uses a numerical technique called the finite element method. This approach provides information regarding the behavior of the mating gears under different constrains like moments, forces or pressure. First step is to import the 3D model into the FEA environment and choose the desired material properties of the parts. Meshing is the process in which the geometry is spatially divided into elements (tetrahedrons). This mesh along with material properties is used to mathematically represent the stiffness and mass distribution of the assembly.

The solver of the FEA application needs required information regarding the type of connection between the mating components, the nature of the contact between worm and worm gear teeth. It is also required to define the supports and loading. In the case of static structural analysis, the solution determines the displacements, stresses, strains, and forces in structures or components caused by external loads that do not induce significant inertia and damping effects. Results show the distribution of stresses along the meshing teeth of the equivalent Von Misses stress as well as other types of stress. Moreover the teeth deformations can also be found out.

The outputs of the solver can be graphs, animations or reports. After the solution is solved, the results provide information regarding the field of interest and the designer have to decide what type of changes are required depending of the case, the material or the shape of the parts for optimization.

ANSYS is the name commonly used for ANSYS mechanical, general-purpose finite element analysis (FEA) computer aided engineering software tools developed by ANSYS Inc. ANSYS Workbench is an analysis tool incorporating pre-processing such as creation of geometry and meshing, solver and post processing modules in a unified graphical user interface. ANSYS Workbench and APDL has same solver so good results are based on good meshing.

These problems that can be solved also include linear structural and contact analysis that is non-linear. Among the various FEM packages, in this work ANSYS is used to perform the analysis. The following steps are used in the solution procedure using ANSYS:

- **i.** The geometry of the gear to be analyzed is imported from solid modeler (e.g., Pro/Engineer) in IGES format this is compatible with the ANSYS.
- **ii.** The element type and materials properties such as Young's modulus and Poisson's ratio are specified.
- **iii.** Meshing the three-dimensional gear model.
- **iv.** The boundary conditions and external loads are applied.
- **v.** The solution is generated based on the previous input parameters.
- **vi.** Finally, the solution is viewed in a variety of displays.

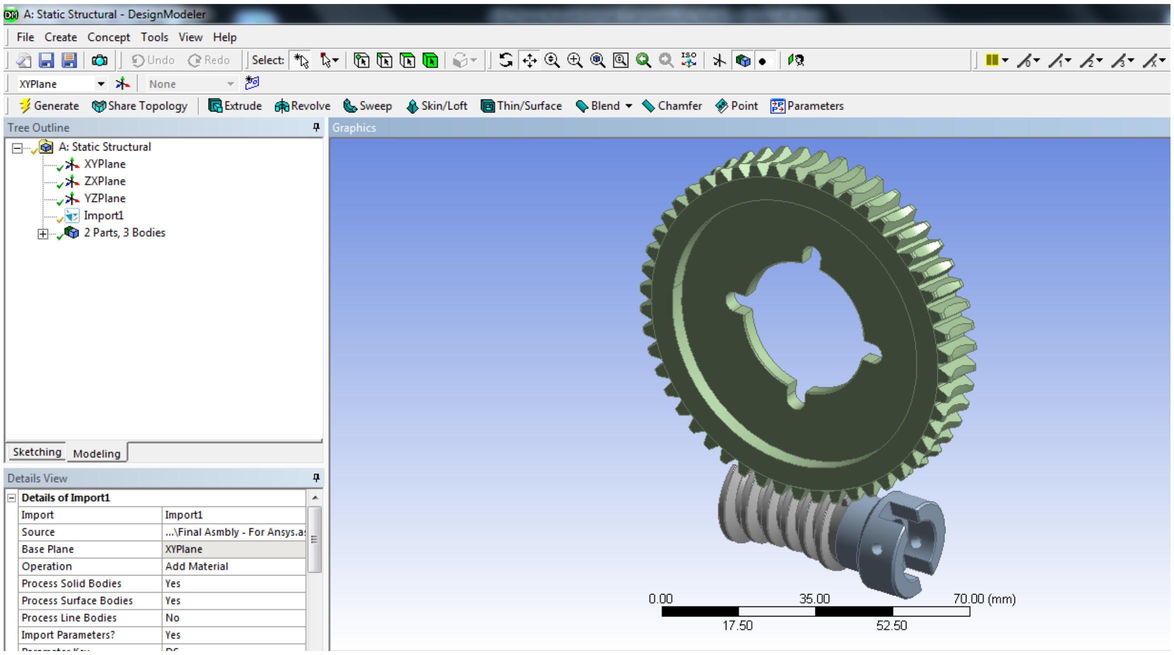

Figure 5.8: Globoid Worm Gear Assembly in ANSYS

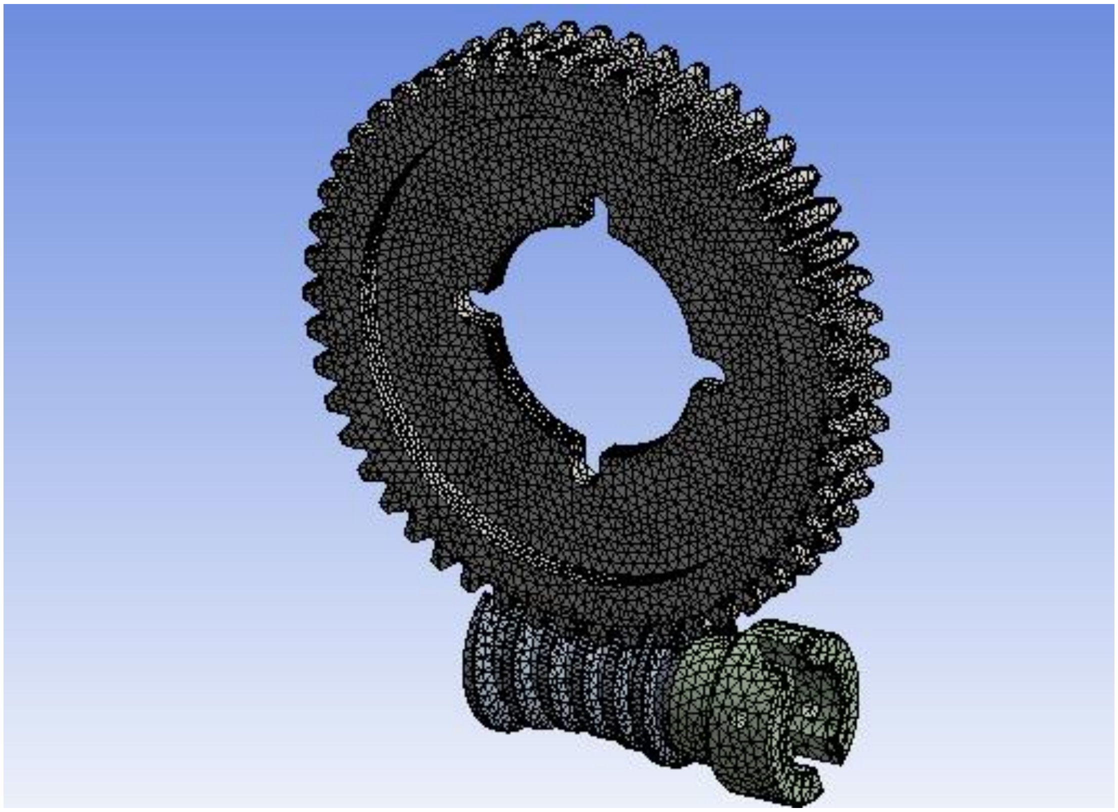

Figure 5.9: Fine Meshing of Globoid Worm Gear Assembly in ANSYS

CHAPTER 6: BENDING STRESS ANALYSIS & CONTACT STRESS SIMULATION

6.1 Bending Stress Analysis

In worm-wheel drive, worm gear teeth are checked against bending. As per AGMA standard, the bending stress in worm-gear drive is found by equations as shown below. [1]

$$\boldsymbol{\sigma_{max} = S_b * \sigma_{lim}} \tag{6.1}$$

$$\boldsymbol{\sigma_{max} = \frac{\frac{F_t}{2}}{\pi} * m_n * b} \tag{6.2}$$

σmax = Max bending stress that may occur

Sb = Safety factor against bending

σlim= Limiting value of load factor

$Ft2$= Max tangential force on worm gear

b= Arc length of tooth width of gear

mn= Profile correction factor associated with lead angle

For the analysis of bending stresses, static structural analysis is performed which includes evaluation of bending stresses at worm gear teeth. The model enables the calculation of all important variables (e.g. deformation, stiffness, stress distribution) as well as the determination of changes in these variables among the meshed teeth. First of all the modeled gear set is imported in Ansys workbench. Automatic fine mesh is generated. The mesh at contact surface is resized with a finer element size. For this analysis, tetrahedron element type is chosen and size of element is taken as “1” mm for all areas except for those sections where the stress concentration is high, it is taken as “0.5” mm. Figure below shows the meshed 3D model. The boundary conditions and external loading is defined.

The boundary conditions are defined by displacement constraints keeping the worm as fixed member. The load is defined in pressure form. Figures below show the equivalent (von-Mises) stress distribution. From the stress distribution, it is shown that the large concentrated stresses are at the root of the worm gear teeth. The maximum von-mises stresses are up to 66 MPa, which is found to be within the desired limit preventing failure of gear tooth due to bending.

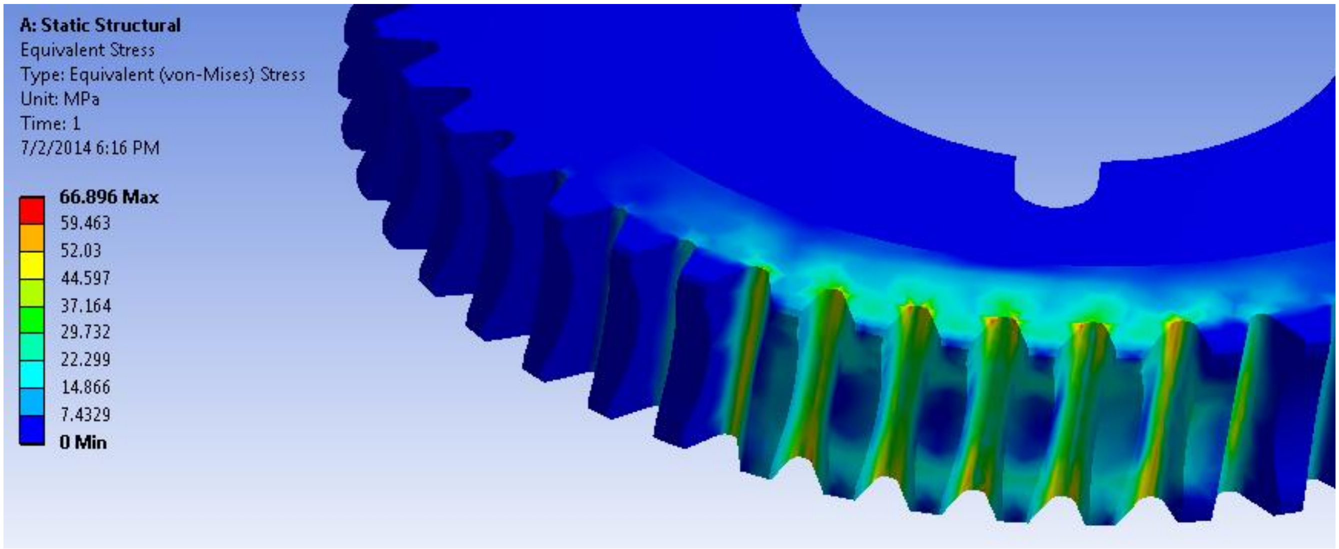

Figure 6.1: Bending Stresses at Worm Gear Teeth

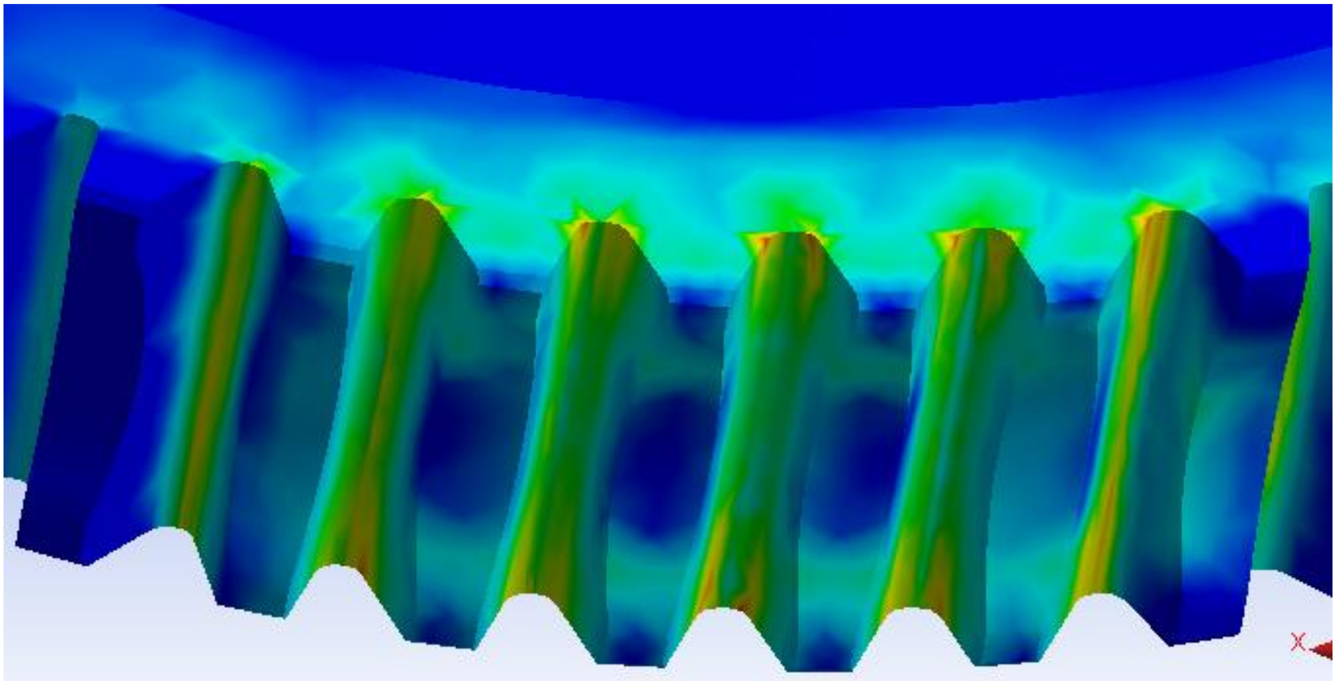

Figure 6.2: Bending Stresses at Worm Gear Teeth (Enlarged View)

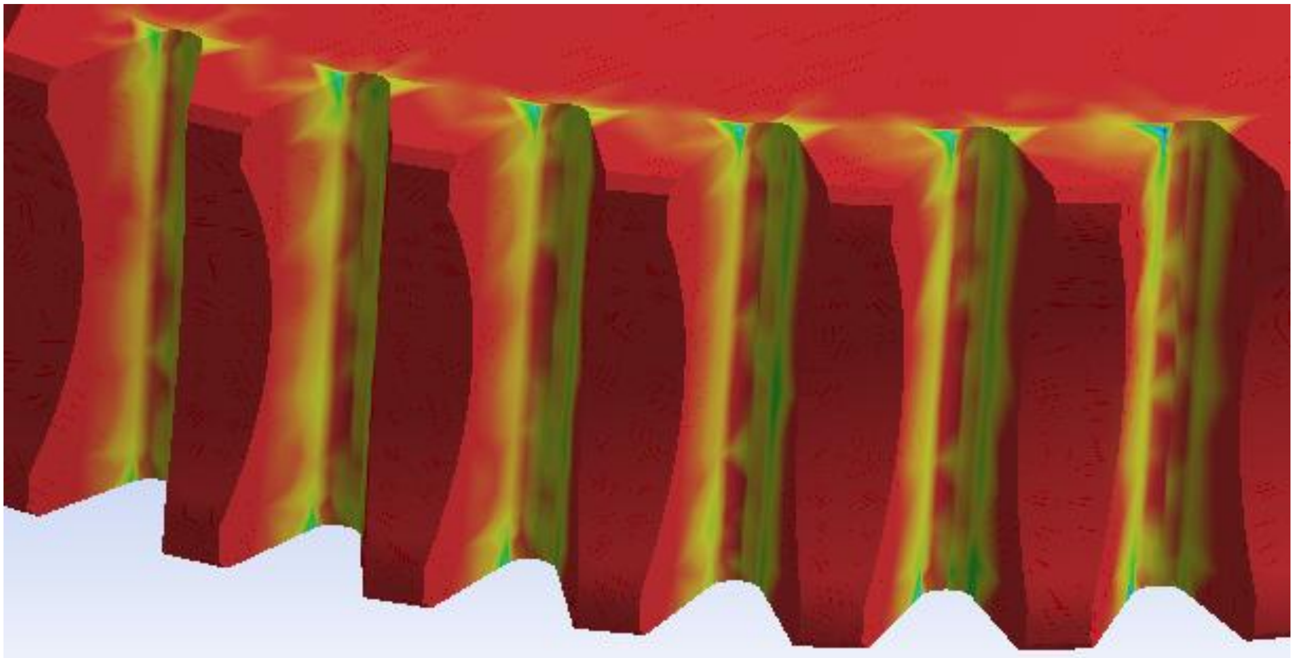

Figure 6.3: Bending Stresses high at Fillet corner

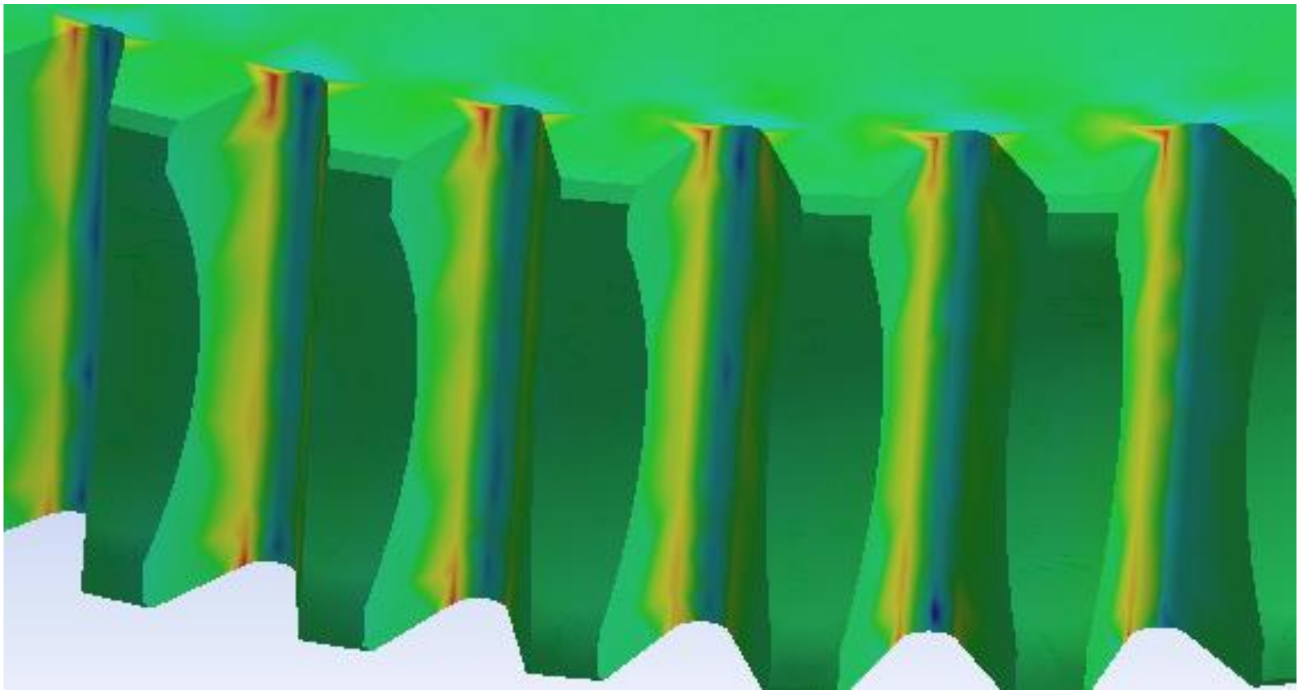

Figure 6.4: Bending Stresses high at Fillet corner (Color Difference)

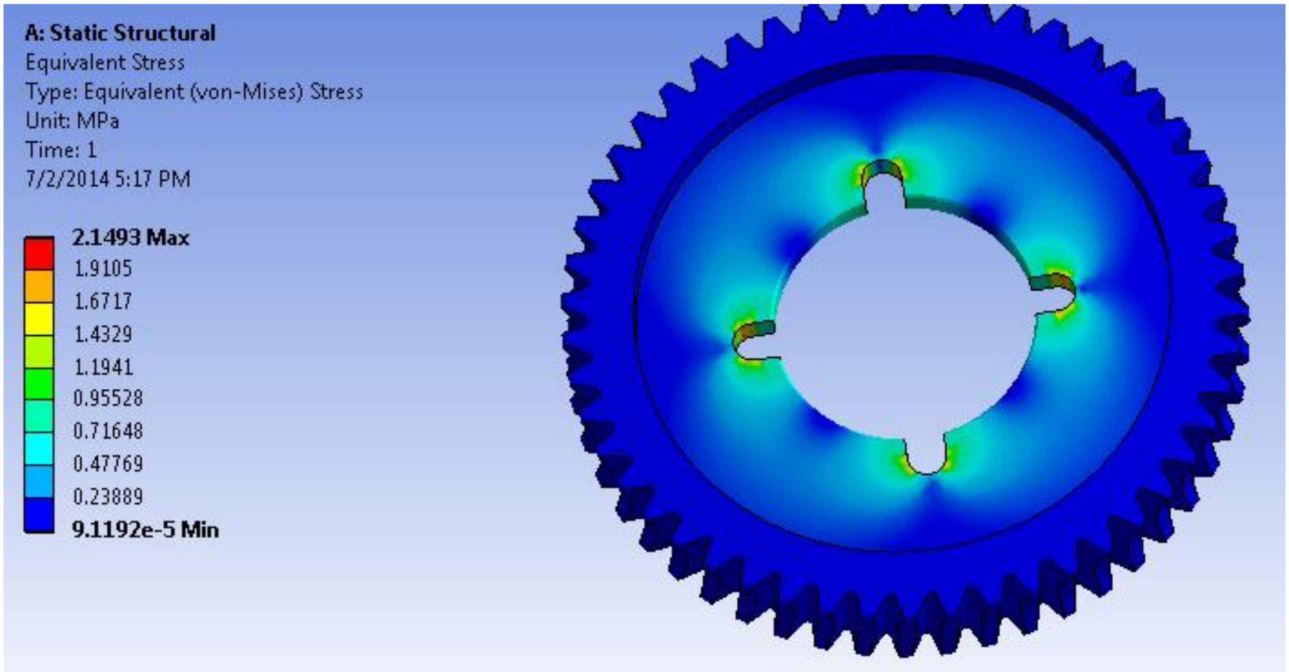

Figure 6.5: Worm Gear weaker edges other than Teeth

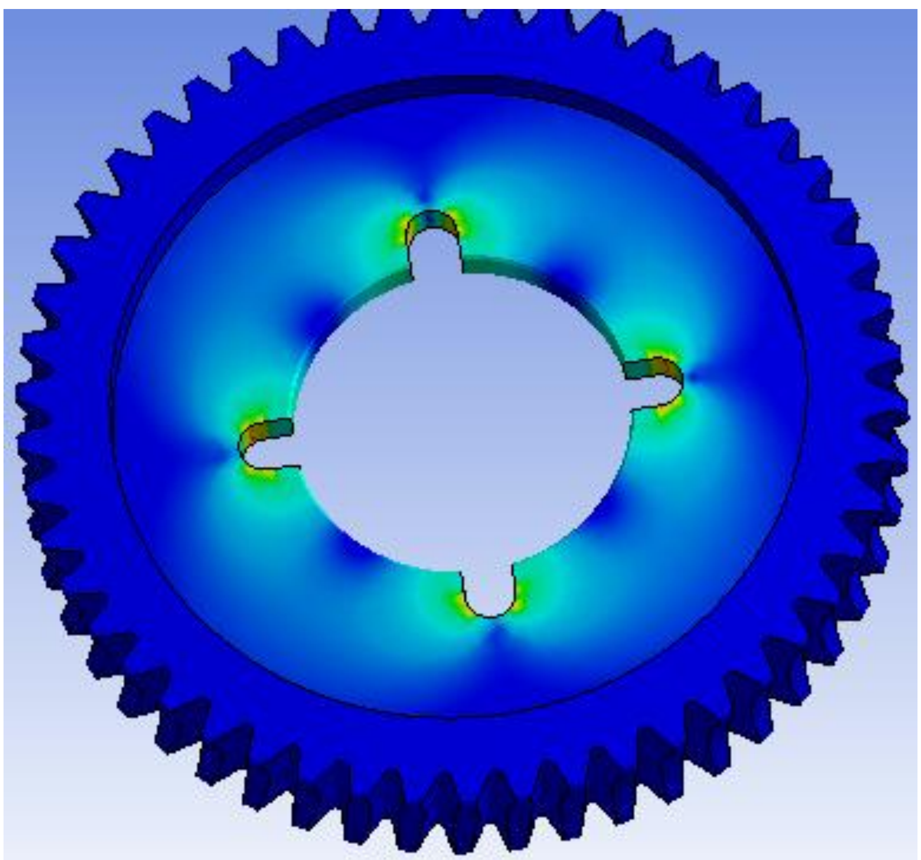

Figure 6.6: Worm Gear weaker edges other than Teeth (Enlarged View)

6.2 Contact Analysis Using FEM

In worm-drive system, it is important to check the system against failure due to the effect of contact stresses. The expression for contact pressure in worm-drive system is given by equation as shown below. [1]

$$\boldsymbol{P = S_c * P_{lim}} \tag{6.3}$$

$$\boldsymbol{P = F_t/2 * dm_1 * dm_2 * K_c} \tag{6.4}$$

P = Actual contact pressure that may occur

Sc = Safety factor

Plim= Limiting value of contact pressure

Ft2= Max tangential force on worm gear

dm1, dm2= Pitch circle diameter of worm & worm gear

Kc= Factor associated with lead angle

For contact stresses, structural contact analysis is performed which includes evaluation of stresses & deformations produced at contact surfaces. The contact load (pressure) is applied along the areas of gear teeth at which load is applied by pinion. These areas are the contact regions of gear and worm teeth. The results are shown in figures below. The stress distribution along contact surfaces shows a visible effect at the teeth root & teeth flank along the contact section (more prominent at the beginning and ending teeth in mesh). The results indicate uniform behavior at the contact region and that the transmitted loads reach their maximum value at the contact vicinity along the contact region. It is also noticeable that the load distribution along different contact regions follows the same trend of the equivalent stress distribution i.e. it peaks at the middle section and tends to decay near the ends of the contact region. The ideal accepted contact pattern is located on the leaving side of middle section and has no edge contact. Edge contacts are undesirable due to possible lubricant diversion / blockage at the entering side, or due to broken contact or improper rolling action at the leaving side. Top and bottom edge contacts are also unacceptable due to improper rolling action. Narrow areas of contact may cause heating and, depending on loading, may be critical due to which welding between gear and worm may occur. For precise motion drives, good rolling action as well as area of contact is desired.

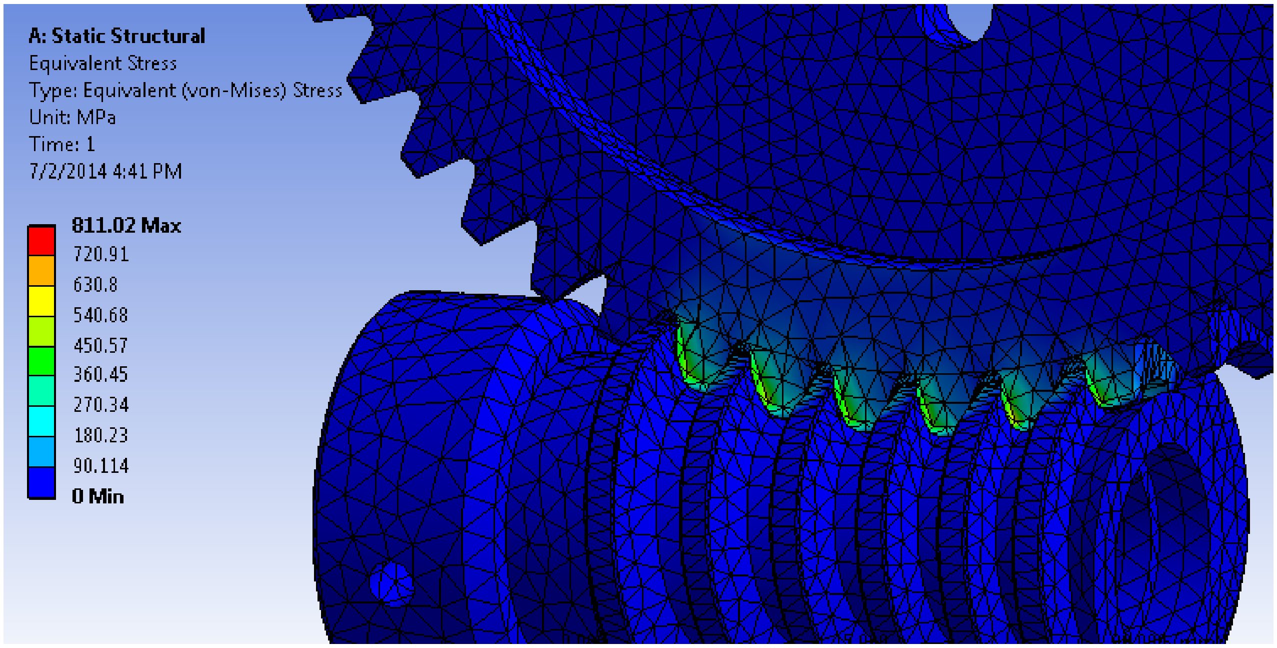

Figure 6.7: Contact Stresses in Worm Gear Set

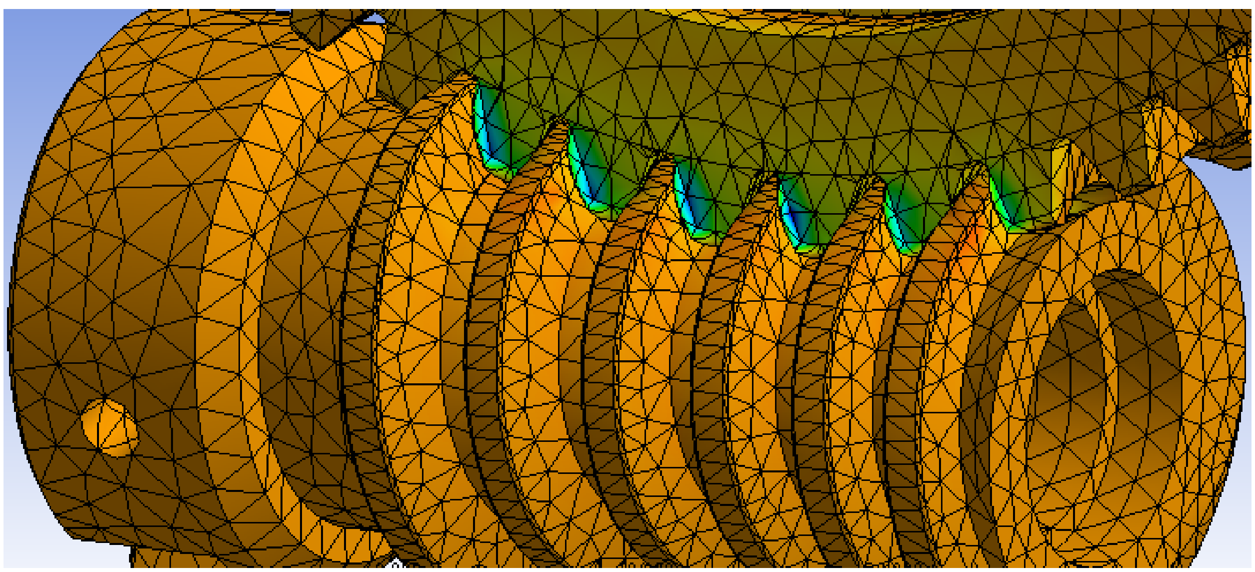

Figure 6.8: Contact Stresses in Worm Gear Set (at Contact areas)

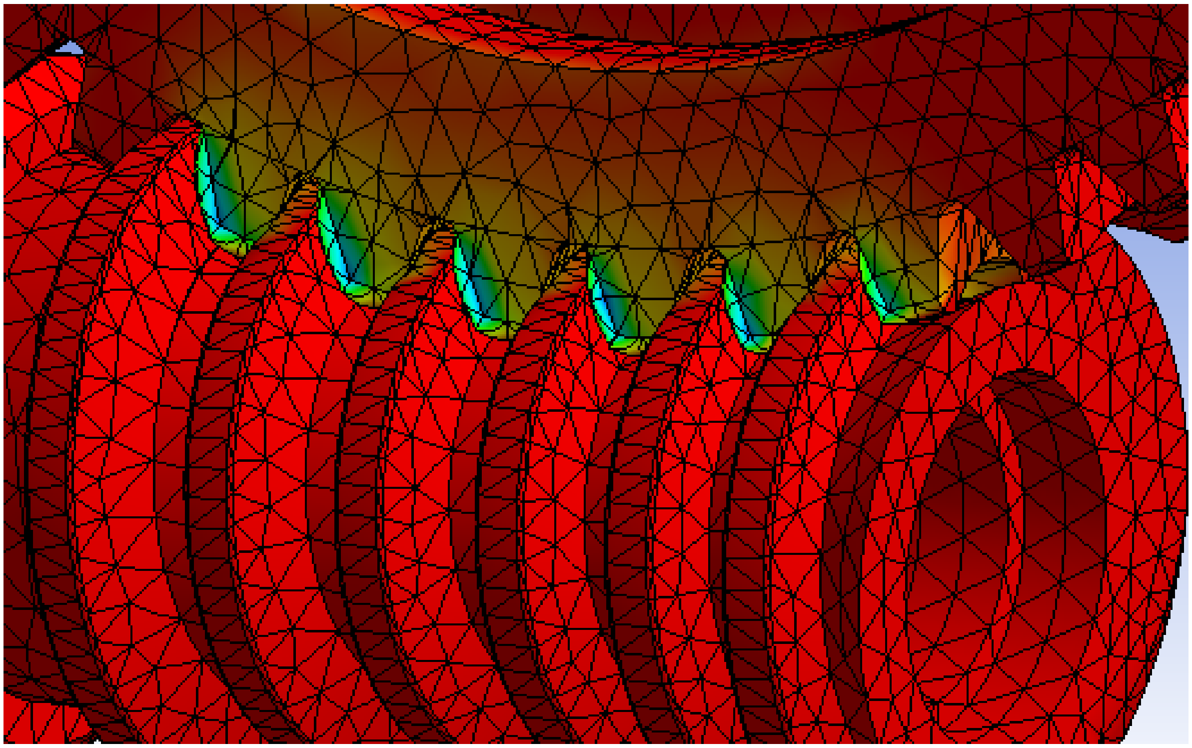

Figure 6.9: Contact Stresses in Worm Gear Set (Color Difference)

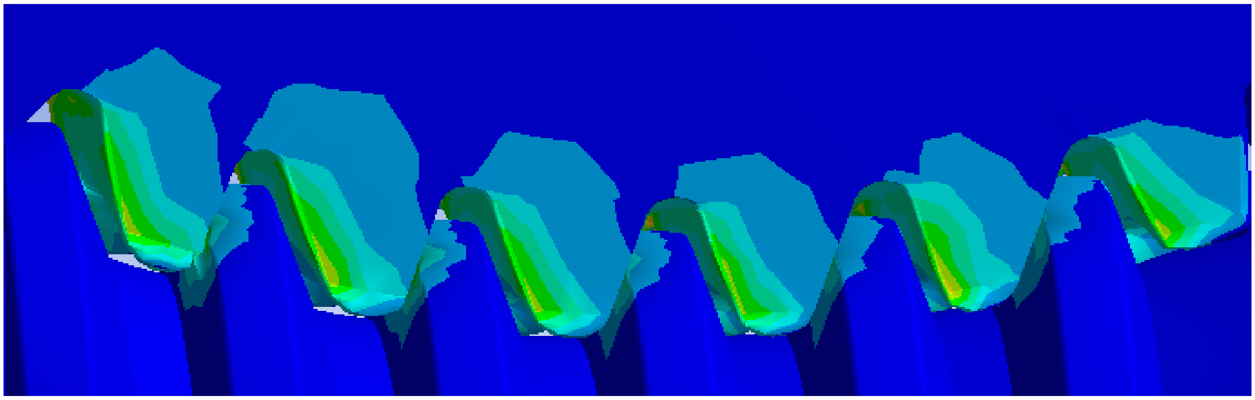

Figure 6.10: Contact Stresses along contact region

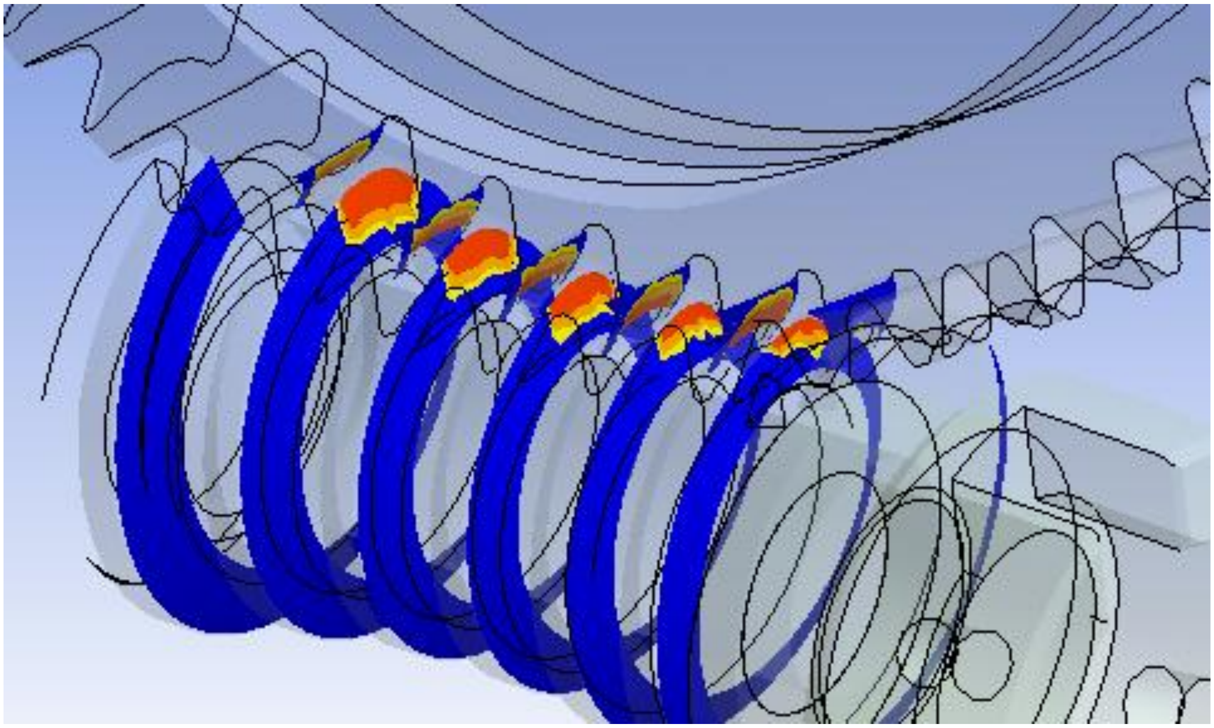

Figure 6.11: Contact areas highlighted in Meshing

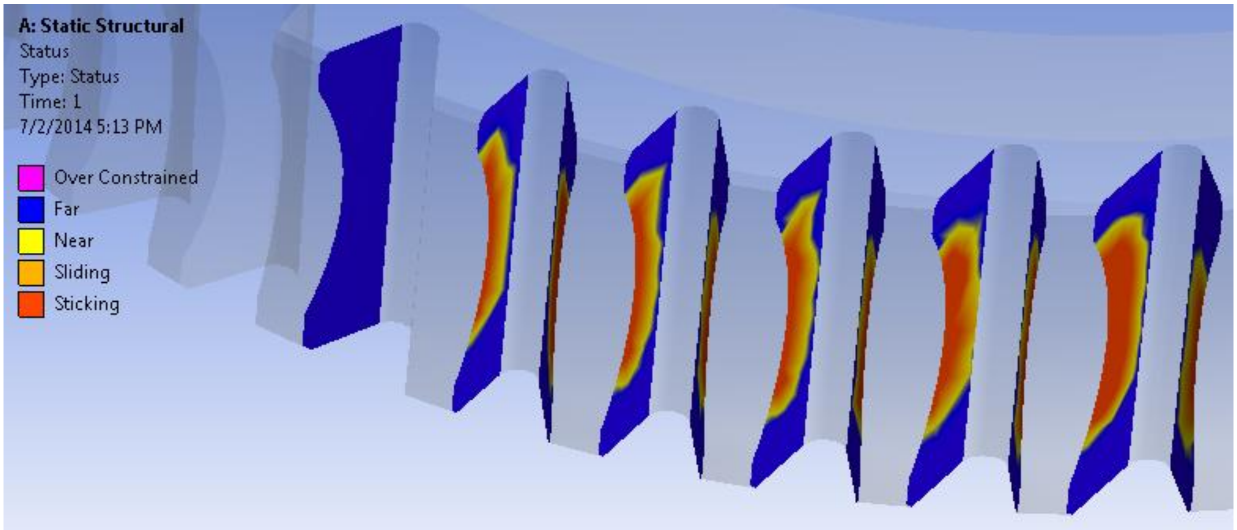

Figure 6.12: Contact Region at different teeth is same in meshing

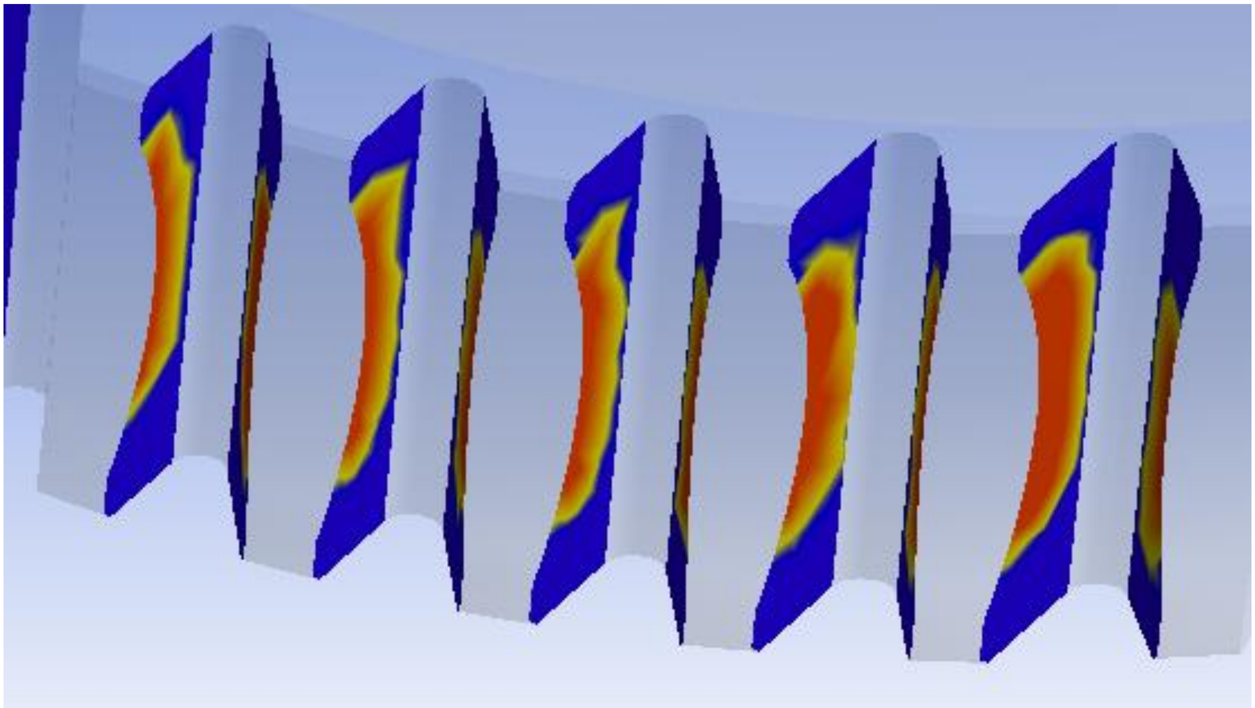

Figure 6.13: Contact Region at different teeth is same in meshing (Enlarged View)

CHAPTER 7: CONCLUSION & FUTURE WORK

7.1 Summary & Conclusion

The focus of this study was to design globoid worm gear drive for a single axis slew drive lift actuator. The calculations of gear tooth geometrical parameters were determined by following the AGMA gear standard. Parametric modeling was done in Pro-E Wildfire 4.0 and motion actuator assembly was modeled in Solid Edge ST3. Computer-Aided FEM was developed in Ansys Workbench 13.0 and models of tooth contact analysis (Contact Stress Simulation) & Bending Stress Analysis are performed. The loading capacity of globoid gear set as determined by mathematical & analytical approach shows that the allowable output torque is 1.8 times greater than the actual torque required for the current application. The desired factor of safety is 1.5, so above torque calculation is valid and applicable. Moreover the overall efficiency of globoid gear set as determined by mathematical approach was found to be 68% which is also within standard range which validated the design. (The overall efficiency doesn't incorporate losses for oil mixing). The bending and contact stresses analyzed by FEM technique also show acceptable stress values, the loading being applied by applying existing relationships of bending moment and contact pressures validating the results of stresses by FEM.

7.2 Recommendation & Future work

- The contact stress can be reanalyzed for a better result by simulating the real contact region between the two mating gears instead of using the equivalent contact region by improving the solution in a high capacity computer.
- Further study can be conducted on the whole actuator with all elements in the system including gear housing, coupler and bearings. Frictional Coefficients can be added in the analysis to see the effects of this variable in terms of variation in contact analysis.
- Simulation of a lubricant film in contact zone.

APPENDIX A

Bending Stresses Theoretical Calculation

$$\boldsymbol{\sigma_{max} = S_b * \sigma_{lim}}$$

$$\boldsymbol{\sigma_{max} = \frac{\frac{F_t}{2}}{\pi} * m_n * b}$$

σ*max* = Max bending stress that may occur

Sb = Safety factor against ben*ding*

σ*lim*= Limiting value of load factor

Ft2= Max tangential force on worm gear

b= Arc length of tooth width of gear

mn= Profile correction factor associated with lead angle

σmax = 280 / 40.175 / 2 / 3.1416 * 1.8 * 1.125 + xm * 40.15 * 44.5 / 57.3

= 1.109 * 2.025 +0.015 *31.156

σmax = 70.48 Mpa

Contact Stresses Theoretical Calculation

$$\boldsymbol{P = S_c * P_{lim}}$$

$$\boldsymbol{P = F_t/2 * dm_1 * dm_2 * K_c}$$

P = Actual contact pressure that may occur

Sc = Safety factor

Plim= Limiting value of contact pressure

Ft2= Max tangential force on worm gear

dm1, *dm2*= Pitch circle diameter of worm & worm gear

Kc= Factor associated with lead angle

P= 280/ 40.175 / 2 * 14.65 * 80.35 * 0.2

P= 820.4 Mpa

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