

# Optimization of Locators Placement for Minimum Workpiece Positioning Error



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Positioning Error

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MAY, 2017

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I certify that this research work titled “*Optimization of Locators Placement for Minimum Workpiece Positioning Error*” is my own work. The work has not been presented elsewhere for assessment. The material that has been used from other sources it has been properly acknowledged / referred.

Signature of Student

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## **Language Correctness Certificate**

This thesis has been read by an English expert and is free of typing, syntax, semantic, grammatical and spelling mistakes. Thesis is also according to the format given by the university.

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*Dedicated to my exceptional parents especially my mother and adored  
siblings whose tremendous support and cooperation led me to this  
wonderful accomplishment*

## Abstract

Product quality greatly depends upon the fixture quality. Proper fixture design plays a critical role in attaining the required quality of the product. Among other factors, locator's placement is one of the significant factor in fixture design. It plays a vital role in part defect or part loss. In this research, locator's placement of 3-2-1 DOF fixture system is optimized by using genetic algorithm to reduce positional errors. Analytical method is used to calculate the displacement of work-piece placed on the locators.

A model is proposed for the system account for external forces and torques. This model is used to calculate the overall stiffness and displacement of the workpiece. Potential energy of the elastic elements, kinetic energy of inertial elements and work done by external forces are calculated. Langrangian formulation is used to calculate the rigid body displacement of the part placed on the fixture. The positional error can be reduced by placing the locators at optimized locations. The aim of this research is to optimize the placement of six locators using GA. The aim of this algorithm is to find the minimum displacement of workpiece. Initial placement of locators is randomly generated in the algorithm and displacement of workpiece is calculated for every corresponding locator placement. Locator's placement with minimum workpiece displacement is taken as best result.

Case study is also done to check the efficiency and effectiveness of the proposed methodology. Application of proposed model is also discussed as real time problem.

**Key Words:** *Genetic algorithm, Fixturing system, Precision manufacturing, Langrangian formulation*



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## **CHAPTER 1: INTRODUCTION**

Fixtures are work holding devices used to locate, support and hold the work-piece. Fixtures are generally specified by specific operations, for example turning, milling, grinding, shaping etc. A good fixturing system has ability to maintain conformity of product. Most important feature for machining fixtures are assuring precise position of work-piece during machining.

The quality of final product greatly depends on fixtures (Vasundara and Padmanaban, 2013). To reduce the lead time and cost of final product, automation and computerization in fixture design field is required. According to one estimate, about 40% of the part defect loss is due to poor fixture design (Hashemi et al., 2014). Similarly, it was also reported that about 10-20% of the total manufacturing cost is related to fixtures design system in traditional Flexible Manufacturing System (FMS) (Butt et al., 2012). Now-a-days fixtures not only have their application in machining but they are also widely used in assembly operations.

### **1.1. Motivation**

In this modern era, main aim of manufacturing industry is to get best quality at lowest possible cost within minimum possible time range. Quality is related to minimal error which is induced by machining; especially in precision manufacturing system where precision is taken up to micrometers. Locator's position and their placement is one the crucial factor which affects the quality of final product. The need of high quality final is the driving force to design the fixturing system.

In the following section main functions of fixtures are discussed which are “locating”, “supporting” and “holding”.

## 1.2.Functions of Fixture

### 1.2.1. Locating the work-piece

The main purpose of fixtures is to locate the work-piece correctly with correct position and correct orientation without considering any machining force.

There are total 12 possible movements commonly referred to as 6 Degree of Freedom (commonly written as “6 D.O.F”). These 12 movements include 6 axial movements (translation along positive and negative x-axis, y-axis and z-axis) and 6 rotational movements (clock-wise and counter clock-wise rotation along x-axis, y-axis, and z-axis). Fig.1 shows the possible movement of work-piece in machine axis.

Different fixtures configurations are used in literature among, which most common configurations are shown in figure 2. Figure 2 (a) shows fixture configuration for prismatic parts, Fig.2 (b) shows fixture configuration for parts having holes while Figure 2 (c) shows fixture configuration for cylindrical parts. There corresponding clamping configuration are also shown in Figure 3 with dotted arrow having sign (S).

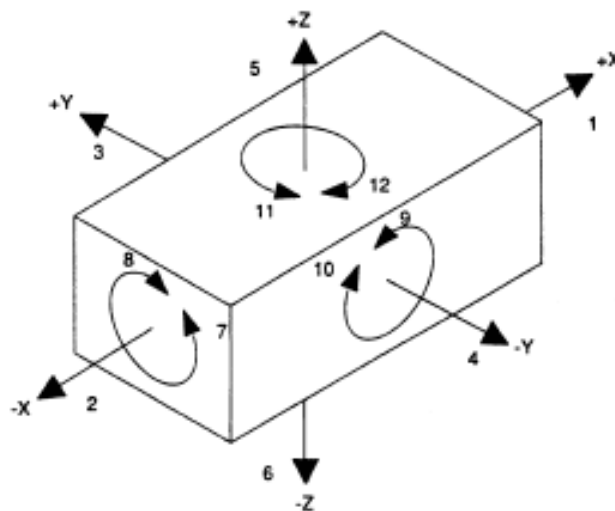


Figure 1 Degree of freedom (12 possible movements of workpiece)

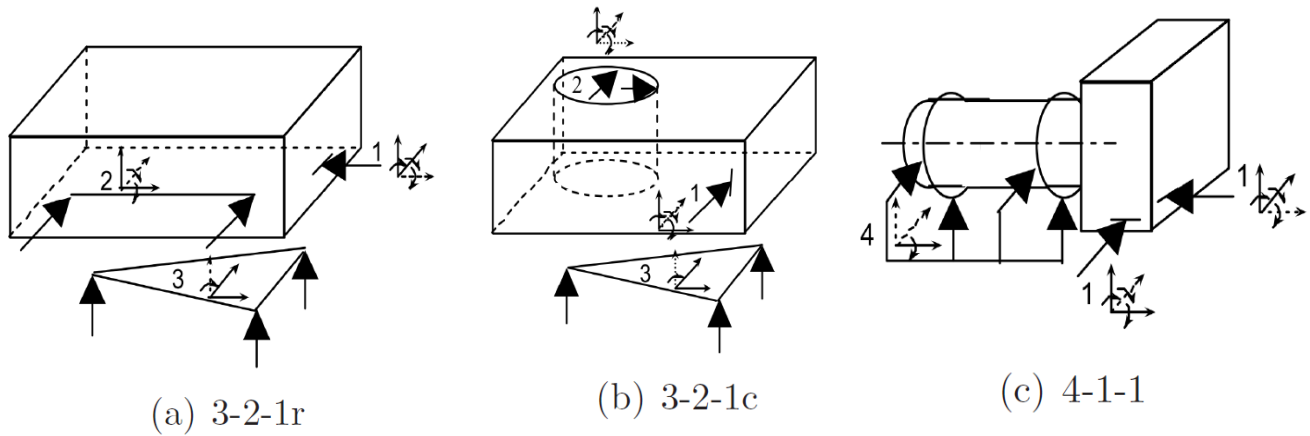


Figure 2 Different Fixture Configurations

### 1.2.2. Supporting the work-piece

The supporting components in fixtures primarily use two techniques: positive stop and friction. A positive stop usually uses the immovable part (pin etc.) which physically hinders the movement of work-piece. Generally, two types of supports are used to stop the displacement of work-piece under machining forces. These supports are: fixed or adjustable support.

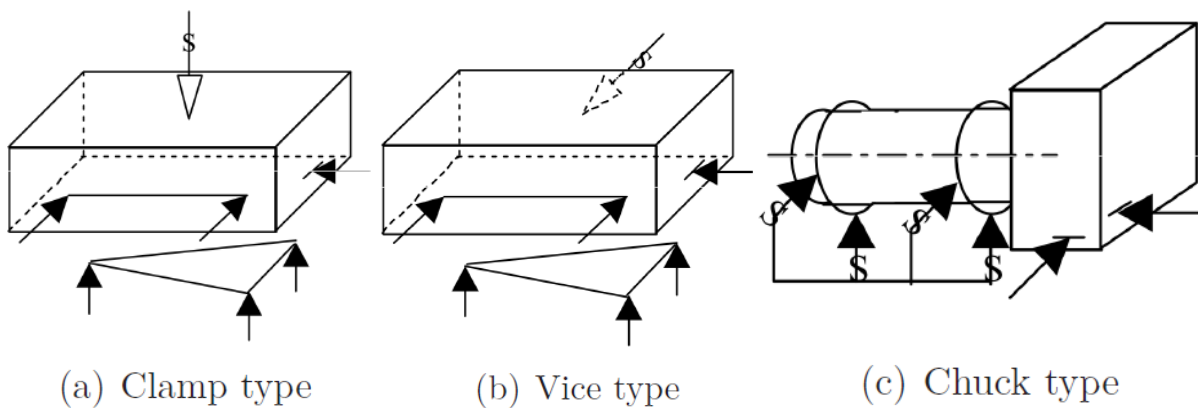


Figure 3 Clamping types (Butt, 2012)



### **1.2.3. Holding the workpiece**

Workpiece holding in fixtures have two specific functions. These are: positioning and clamping. The positioning function includes fixing of workpiece relative to tool and work center must be aligned with machining tool. Clamping not only hold the workpiece during machining operation but also neutralizes the cutting force.

### **1.3. Fixture design**

Basic fixture design must satisfy certain criteria (Nalbandh and Rajyguru, 2013.), which are:

1. Accuracy of locators
2. Total restraint
3. Sufficient rigidity
4. No interference

Fixture design needs a systematic and logical approach. The above criteria indication must be taken into consideration. Any problem is the sign of missing information or due to neglecting some design requirements. Conventionally, fixture design consists of four step problem solving technique: Set-up planning, fixture planning, unit design and verification. Figure 4 describes the functions and details of each step.

There are mainly four components of fixture, which are further classified according to their functionality. An integrated approach is devised for automatic computer aided fixture design comprised of fixture plan, fixture layout and fixture assembly. Fixture database is also provided according to the functionality of fixture's component (Nasr et al., 2011). There are four major components of fixtures which are: baseplate, locator, support and clamp. Figure 5 gives the simple hierarchy of fixtures components.

This research work focuses on fixture planning. Locator's placement has great impact on workpiece positioning. So, optimization of locator's position is one of the crucial factors in minimizing the overall workpiece positioning error. Following section briefs the optimization technique that can be used for this purpose.

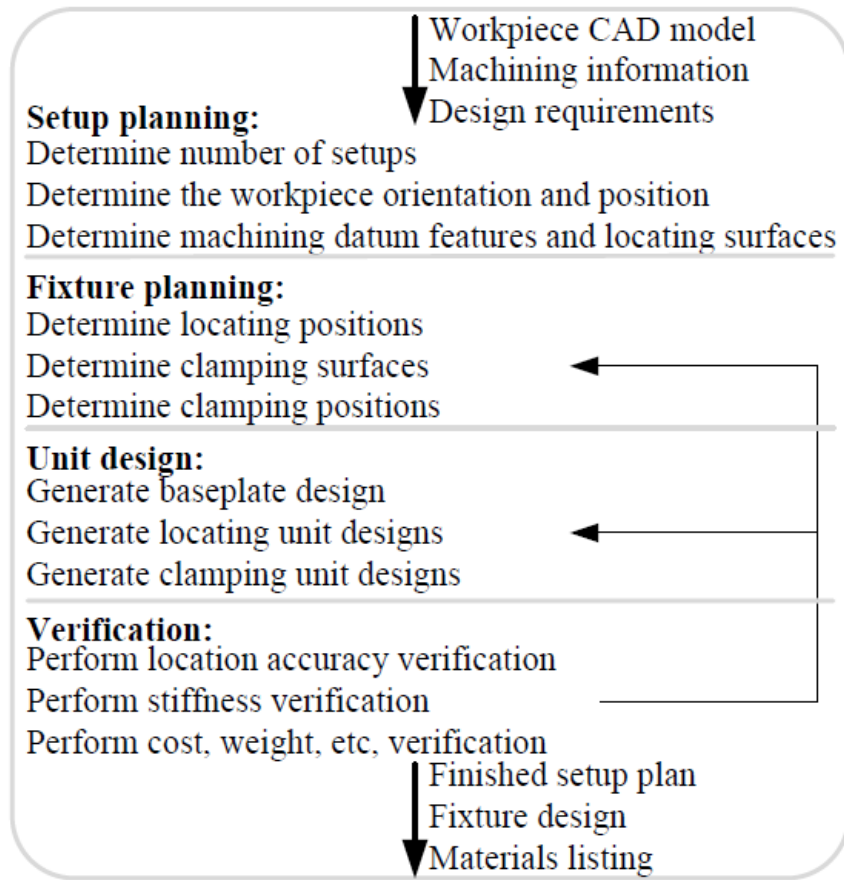


Figure 4 Process of fixture design (Nalbandh and Rajygeru, 2013)

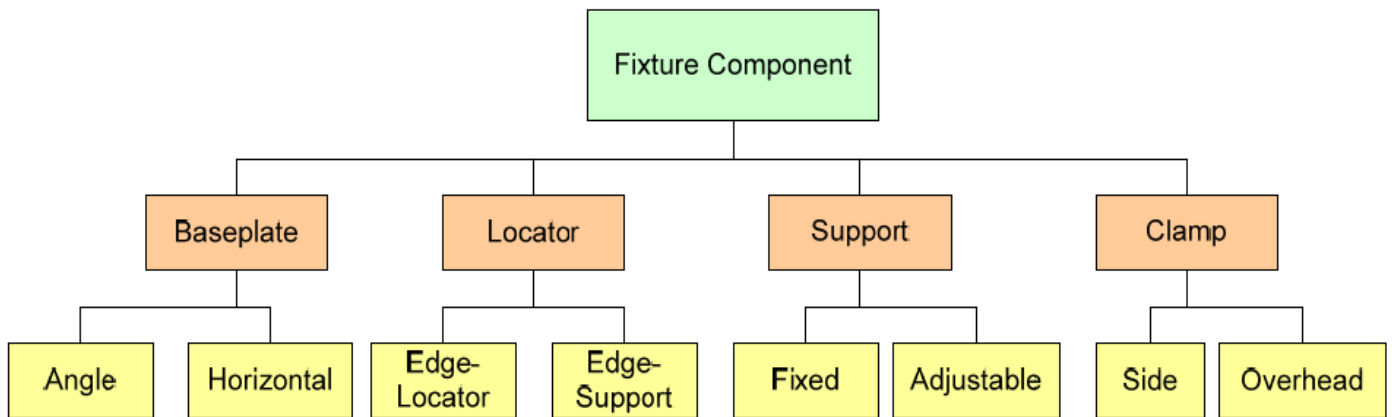


Figure 5 Fixtures components (Nasr et al., 2011)

## 1.4. Optimization Technique

In recent years, fixtures layout and fixture design attained significant attention in modern engineering especially precision manufacturing. Development has been made in different areas of fixture configurations, model related to it, optimization of different type of layouts and fixture design. Following points highlights some of these areas:

1. Automated computer aided fixture design (CAFD).
2. Minimizing the work-piece displacement caused by the clamping and machining forces by using “kinetic model”.
3. Minimizing the error caused by the work-piece and fixture displacement by using “kinematic model”.
4. Optimization of fixture layout to minimize the work-piece elastic deformation by using ANSYS and other numerical techniques.

The possible machining errors which are responsible for work-piece positioning and orientation error and cause misalignment of work-piece given by Butt et al (2012) which are:

1. Inaccuracy due to placement of locators
2. Error due to geometrical defect of work-piece
3. Deformation error due to external forces
4. Machine tool error or kinematic error

Butt et al (2012) discussed the possible causes of these errors. The first error addressed in the fixture configuration. Normally for rectangular part, 3-2-1 fixturing system is mostly used in modern industry. Primary, secondary and tertiary planes are considered for this purpose to locate the workpiece. Three locator form primary datum, two form secondary datum and sixth locator form tertiary datum which is perpendicular to both primary and secondary. The second error is due to work-piece rough surface which cause its dislocation from its mean position. Homogeneous transformation matrix (HTM) and small displacement torsor (STD) were used to calculate the deviation of work-piece from its mean position. The third error is mainly due to cutting forces, machining forces and clamping. Work-piece displaced from its mean position under the clamping and machining forces. “Mechanical model” is used to minimize the work-piece positioning error caused by external forces. The last error of machine tool error is almost

impossible to eliminate, therefore, it must be compensated. The easiest way to compensate error is to change the tool path with the help of NC programming, but this compensation usually requires 4 or 5 axis machine tool.

Techniques like finite element analysis (FEA), analytical modeling and other evolutionary techniques like Ant colony algorithm (ACA), Particle swarm algorithm (PSO), Genetic algorithm (GAs) are used for optimization in literature. In the following sections, ant colony algorithm (ACA), particle swarm optimization (PSO), genetic algorithm (GA) is discussed which can be applicable for optimization of fixturing system.

#### **1.4.1. Ant Colony Optimization**

Natural ants have tendency to search their food by shortest possible path without any visualization aid. They can find new path if they find any hurdle in their way. A natural ant has ability to deposit pheromone through the path on which they are walking. The following ants usually prefer to follow the direction which is rich in pheromone. A pheromone is a substance (usually gland/hormone) ants deposit on ground for food trail.

Common ant behavior towards food is shown in Fig.6. Ants usually follow the shortest possible path (Fig.6-A). By introducing the hurdle which has unequal branches, they cross the hurdle with random fluctuation (Fig.6-B). All ants move at same speed with ability to deposit pheromone at equal rate. So the path with short length will receive the pheromone earlier as compared to the path which is longer in distance. With the passage of time, following ants choose the path with greater concentration of pheromone, thus following the shortest distance as shown in Fig.6-D. This behavior of ants can be used for travelling sales man problem (TSP) to find the shortest possible distance with cities.

This algorithm is first proposed by Marco Dorigo in 1992 in his PhD thesis. His research mainly aims to find the optimal path in the graph based on the ant's behavior. Initially, this algorithm is used to optimize TSP, but with passage of time this concept is diversified for many other numerical problems. Certain assumptions were made when ant's behavior is applied to real world problem. Concept of 'artificial ants' is introduced as agent to move from city to city. At each iteration, artificial ants move from city to city with a constraint not to visit a city more than once and the movement of those ants is biased with the level of pheromone. In real world

problem, pheromone is determined by the solution component. After each iteration, the value of pheromone is updated in order to converge the solution with promising values.

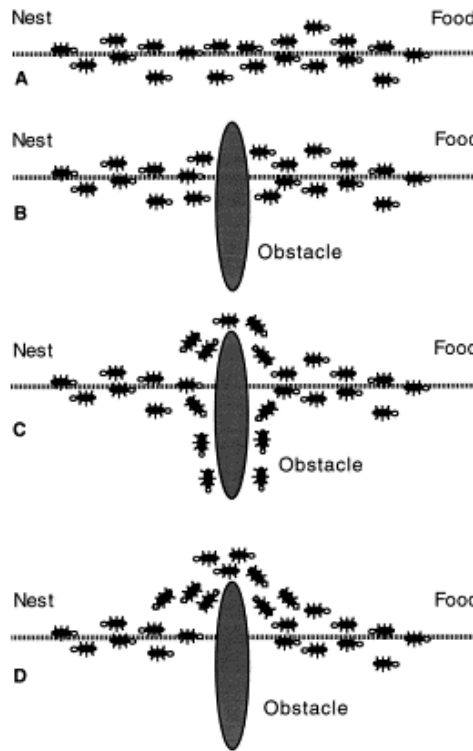


Figure 6 (A) Ants follow normal path (B) Hurdle is introduced (C) Random fluctuation in the path (D) Following ants follow the shortest path (Dorigo and Gambardella, 1997)

Dorigo et al (2006) discussed the application of ACO in modern era. Application of ACO in the field of telecommunication, routing problems is also discussed in the article. Beside this, ACO is also applied to dynamic and stochastic optimization problem and multi objective problems. The article also includes the other ant's behavior inspired algorithms which motivate the researchers to work on it in near future.

### 1.4.2. Genetic Algorithm

Concept of Genetic algorithm is based upon Darwin's theory of selection. It is a heuristic approach which is inspired by process of "natural selection". It works on the principle of "survival to fitness function". In recent years, genetic algorithm (commonly written as GA) is emerged as one the evolutionary technique whose application area is very vast. Now GA has its

application in the field of image processing, numerical function optimization, design and machine learning (David Beasley, 1994).

Genetic algorithm generates the solution to optimization within the search space through natural evolution techniques such as selection, cross over and mutation. As genetic algorithm is a direct analogy of natural behavior, so it is also used to solve real world problems if it is coded suitably. Each chromosome depicts some possible solution of the problem, whose selection is based on its “fitness score” which is evaluated by fitness function. Individuals/chromosomes having good fitness values qualify for reproduction and individuals with low fitness values were dropped out; just like species of individuals competing for food and shelter. Individuals habituated successfully become the part of ecosystem. Such individuals with good hereditary material get chance to produce their off-springs. It is possible that the combination of “fit” individuals might produce “super-fit” off-springs which are more immune to the environment and even have excellent “fitness score”. In this way, best individuals were selected to reproduce, thus, new generation gives the possible values which have good fitness values as compared to their parents. In this way, over many generations, good hereditary material passed on through generations, being mixed and exchanged with other good hereditary material. By giving chance to such good individuals, new promising search space is explored. Figure 7 flow chart shows the generic simple genetic algorithm.

Some common terminologies used in the literature of genetic algorithm are defined below:

- Individual: Any possible solution
- Chromosome: It is a set of genes of an individual.
- Gene: It is a subunit of chromosome, typically represent a trait or combination of trait.
- Fitness: Target function that we are trying to optimize
- Allele: Possible setting of a trait
- Trait: A physical habitat or combination of habitat that may be inherited.
- Genome: Collection of chromosomes (traits) of an individual.
- Genotype: It is the representation of heritable genetic identity. It mostly refers to the typical set of genes carried by an individual.

- Phenotype: It is the physical representation of properties encoded by individual's genotype. Phenotype is simply the description of your physical trait.
- Search space: All possible solutions of the problem.

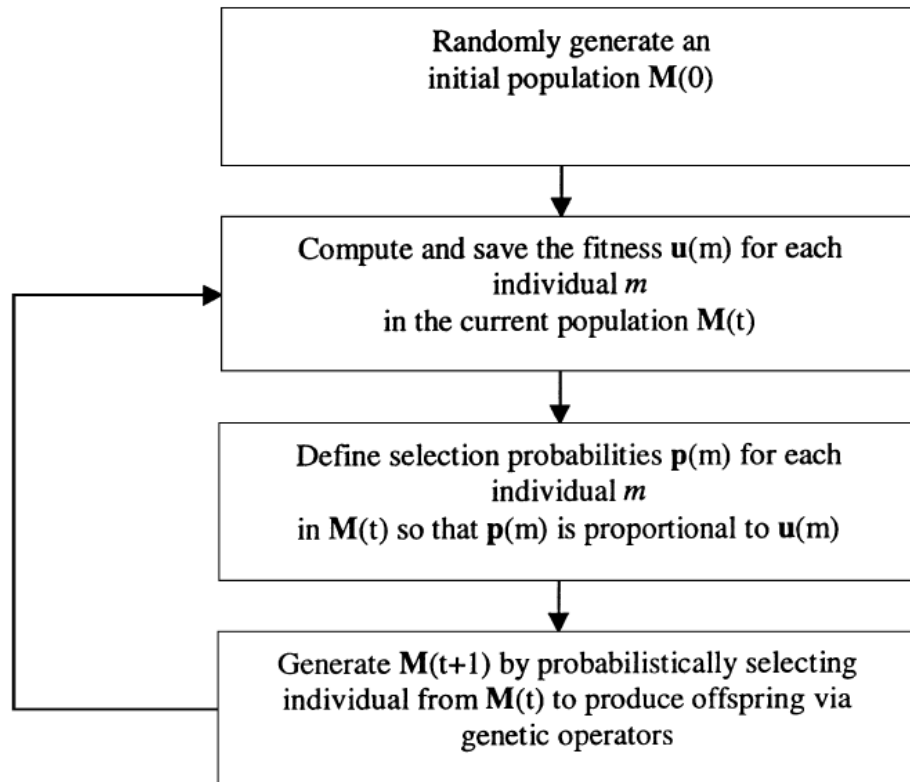


Figure 7 Flow chart for simple genetic algorithm (Subramaniam et al., 1999)

#### A. Simple Genetic Algorithm

Algorithm for Simple GA is given below:

1. *Initial population*: Generates the random population of individuals
2. *Coding*: Individuals were coded authentically for reproduction.
3. *Fitness function*: Each individual is evaluated based on fitness function.
4. *Test*: Fitness score of each individual is checked, if stopping criteria is satisfied, and then end.
5. *New population*: Create new population with the help of genetic operators (reproduction, cross over, mutation).
6. *Reproduction*: Select the parents from the mating pool based upon their fitness function.

7. *Crossover*: By defining crossover probability, parent's chromosome off-springs. If no crossover was performed, off-springs were exactly the same copy of parents.
8. *Mutation*: By defining mutation probability, individual mutate by random swapping of gene.
9. *Replace*: Replace the initial population with new population to proceed
10. *Loop*: Go to step 3

## B. GA Operators

Crossover and mutation are two important operators which are responsible for genetic diversity in the search space. This genetic diversity is the necessity of the process of evolution. It is necessary to understand “coding” and “selection” before “reproduction”. So, GA operators can be explained as:

- Coding
- Selection
- Reproduction.

### i. Coding

The first and foremost important step is to encode the variable into potential parameter. A specific coding method is needed to change the chromosomes into equivalent numerical values. The chromosome must be translated into some parameter, known as “genotype”. This genotype contains the traits that are needed to construct an organism physically, known as “phenotype”. The fitness of any individual depends on its phenotype.

Mostly finite length of binary string of 1's and 0's used as parameter for encoding. It is one of the oldest methods of encoding in GA. Usually chromosome of very small allele can also be encoded through this method. This binary coded value must be decoded in order to check the fitness of that individual. Binary coding works on the principle of  $2^n$ . For example:

0001 is 2 to the zero power, or 1.

0010 is 2 to the 1<sup>st</sup> power, or 2.

1000 is 2 to the 3<sup>rd</sup> power, or 8.

Similarly, one can figure the number 1010 by adding power of 2 i.e.



$$1010 = 2^3 + 0 + 2^1 + 0 = 10$$

$$0101 = 0 + 2^2 + 0 + 2^0 = 5$$

There are other coding methods used instead of binary coding like:

- Array of integers (like real numbers: 43.2, -33.1.....0.0, 8.92)
- Array of letters (like alphabets: A, B, G, N.....)
- Permutation of elements (like E11, E3, E7...)

ii. Define fitness function

A fitness function must be introduced in order to check the “ability” or “fitness score” of each individual. A chromosome has its genome in the coded form which corresponds to its equivalent fitness score. Fitness function is a parameter or set of parameters used to judge the performance of each individual. The higher the fitness function score, the higher will be the chance to get selected for reproduction.

It might be possible to have combinational optimization i.e. to optimize more than one parameter. For example in designing of bridge, there are many variables to optimize like strength to load ratio, completion time, cost, maximum load or may be the combination of these parameters.

iii. Selection

In this stage of genetic algorithm, individuals are selected from the mating pool according to their fitness score. These individuals normally referred as “parents” and individuals after reproduction referred as “off-springs”. Some of the selection methods are discussed below:

- Roulette wheel selection:

It is also known as “fitness proportionate selection”. In this selection criteria, individual get chance to be selected on the basis of its fitness score. The fitter individuals have more chance to get selected as compared to the weaker ones. So, fitter individuals have good probability to survive, but it doesn’t mean that weaker individuals did not survive at all. Weaker ones have chance to get selected but with low probability.

Roulette wheel selection is the analogy of “selection of the fittest” in which individuals with better genotype has more chance to get selected in mating pool as compared to the individuals with weak genotype. Following figure 8 shows an example of roulette wheel selection with an individual of good fitness score (38%) and the weaker one (5%) too.

- Rank selection:

It is explorative technique for selection. In this method individuals are selected based on their rank rather their fitness score. Every individual is assigned by the selection probability with respect to its rank. Conventional roulette wheel selection has a drawback of biased selection based on the absolute fitness value. So, rank selection method is the extension of roulette wheel method in which individuals are sorted relatively rather than absolute fitness.

Rank selection method avoids the premature convergence and stagnation. Following figure 9 shows the difference between roulette wheel selection and rank selection method explained to solve travelling salesman problem in literature (N.M.Razali et al 2011).

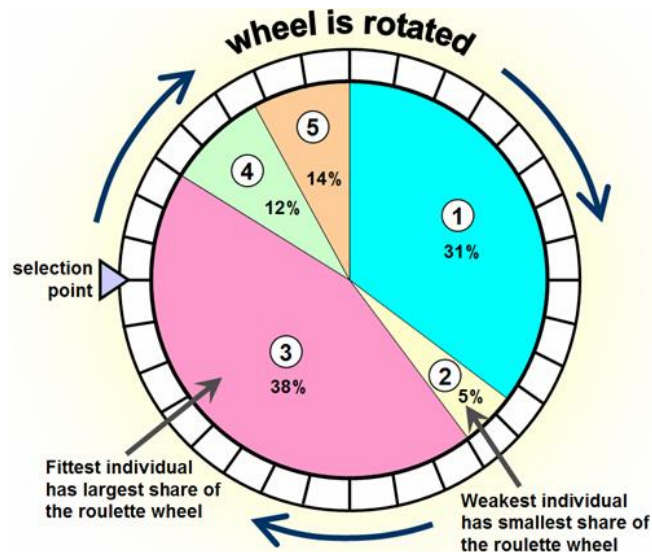


Figure 8 Example of Roulette wheel Selection

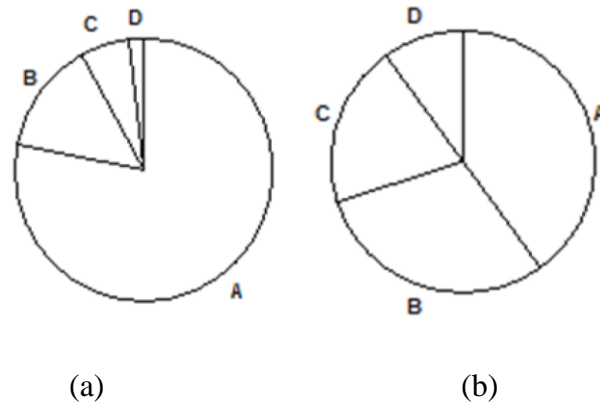


Figure 9 (a) Roulette wheel selection (b) Rank selection method (Noraini and Geraghty, 2011)

- Tournament selection:

Tournament selection is one of the most popular and efficient method in GA to select the parents from the mating pool due to its easy implementation. In tournament selection, ‘n’ parents are selected from the pool by competing against each other. The individual with higher fitness score wins and selected for the reproduction, and the one who loses is simply dropped. Tournament selection preserve diversity in the population by giving each individual a chance to compete. The number of individual competing in each tournament is normally referred as “tournament size”. Mostly two individual compete with one another, so tournament size is 2 (that’s why tournament selection is also referred as “binary selection”).

For example if you want to select 20 individuals from 100, start selecting by competing two individual. Keep the winning individual and again compete that individual with another one. Do it again and again until you get top 20 individuals from the mating pool, just like a cricket tournament in which team keep competing with other teams (like quarter final, semi-final) until it gets through the final match. Following figure 10 helps to understand the idea of tournament selection.

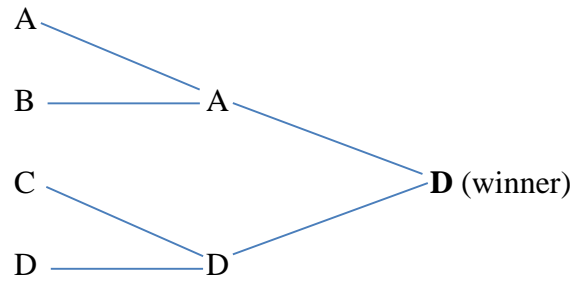


Figure 10 Tournament selection

- Elitism:

Some good potential individuals can be lost with the application of GA operators. So, it is needed to stop losing these potential candidates with good fitness score. Elitist strategy is generally used to keep the best fitted individual in the population and is being copied to the next generation without reproduction (crossover, mutation). Elitism ensures that the convergence of GA will not decrease from one generation to other. This strategy has excellent impact on the performance of GA and its convergence.

iv. Reproduction

In reproduction phase, the most fitted individuals were selected from the mating pool (by using any of the selection criteria) and were selected for reproduction. The selected individuals recombine to form off springs which will form the next generation. The fitted ones has chance to get selected many times, as compared to the weaken ones which possibly had chance to recombine only one's o none. Reproduction phase will depend on two criteria. These are:

- Crossover:

It is recombination of two parents to produce two new off springs. The idea behind crossover is that the new offspring is better than the parents. In this way, new search space may be explored for the problem. Crossover takes two chromosomes, cut them at chosen positions which produce two “heads” and two “tails”. Two new off springs can be made by swapping the corresponding heads and tails. Crossover probability may vary from 0.6 to 1.0. If there is no crossover operation, the off springs are the exact copy of their parents. So, in such case all genes are passed to next generation without any disruption. There were many techniques of crossover like single-point crossover, two-point cross over, random crossover and uniform crossover.

In single point crossover, chromosome is just cut from one position only and swapped the corresponding heads and tail. In two point crossover, chromosomes cut at two points and swapped respectively. In uniform crossover, fixed ratio of mixing is done with probability of exact 0.5. In random crossover, chromosomes are cut randomly and swapped respectively. Figure 11 shows the possible crossover techniques.

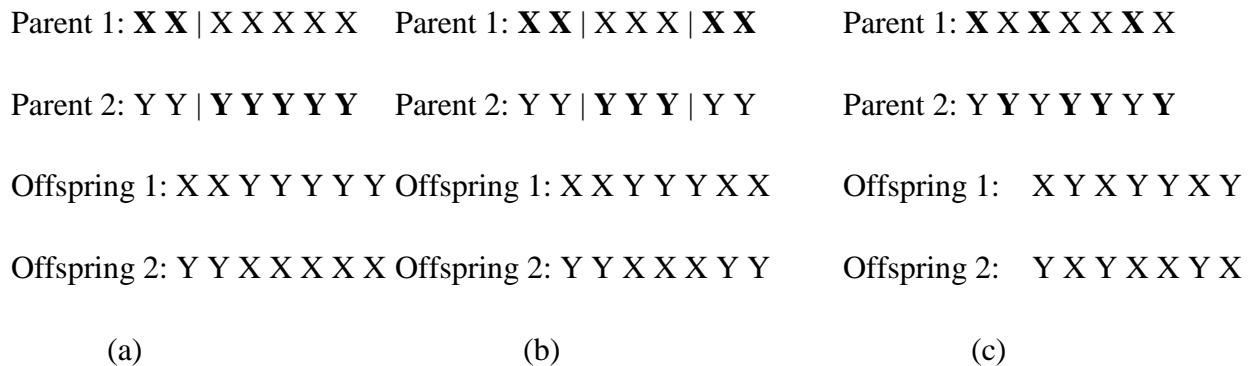


Figure 11 (a) Single-point cross over (b) Two-point crossover (c) Uniform crossover

- Mutation:

Mutation normally applied after crossover. It generally involves the randomly alteration of gene with in an individual. Usually mutation has very small probability ranges from 0.001 to 0.1. It is noted that crossover is more important in exploring the new search space but mutation is important to converge the solution al local optimum point. So, mutation ensures that no point in the search space is neglected from being examined. Figure 12 shows the typical mutation operator.

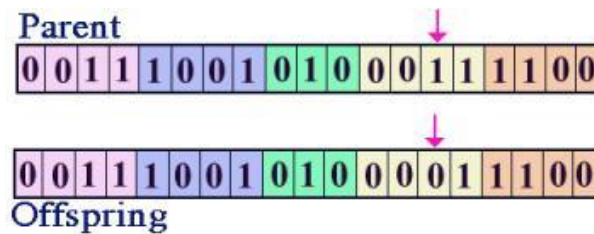


Figure 12 Mutation Operator

### 1.4.3. Particle Swarm Optimization

Particle swarm optimization (PSO) is developed by J. Kennedy and R.C. Eberhart in 1995. Eberhart et al (1995) presented particle swarm optimization and implement its optimization

criteria to global best and local best paradigm. In their paper, they relate the PSO with GAs in many perspectives. In contrast, both techniques initialize the population; both have an objective function on which the candidate are evaluated.

In comparison, PSO has memory about individual as compared to GA. Also PSO tracks the optimal solution of each individual unlike GA where there is problem in interaction with individuals. Application areas including neural networks and robot learning is also discussed Eberhart et al (1995).

PSO is a meta-heuristic approach which is influenced by the behavior of flock of birds in the search of food. A set of potential candidates are chosen and their fitness value is evaluated on the basis of objective function to find the maxima or minima. Blondin, (2009) explained PSO algorithm with the help of simple example.

Figure 13 shows the potential four candidates trying to seeking the global maxima. It is observed that objective function is a ‘block box’ as there is no clue whether any candidate solution is approaching the optimal solution or not. Each candidate has position, its solution according to the objective function and velocity. For tracking each individual’s best candidate solution is saved which is known as ‘individual best position’. Among all the candidates, PSO maintains the best optimal solution known as ‘global best position’.

There are generally three steps of this algorithm which are repeated until stopping criteria is met; these are:

1. Evaluation of candidates in the basis of objective
2. Evaluate the individual and global best position
3. Update the velocity and position of each candidate

First two steps are quite simple to bring the fruitful result; but third step is responsible for optimizing ability of PSO algorithm. Velocity update generally consists of three terms. First one is ‘inertial component’ which is responsible to keep the candidate move in same direction. Higher this component, higher will be the convergence rate. Second term is known as ‘cognitive term’ which compels the candidate to move in search region where it gives high fitness value.

Third term is known as ‘social term’ makes the candidate to move in the region swarm has found so far.

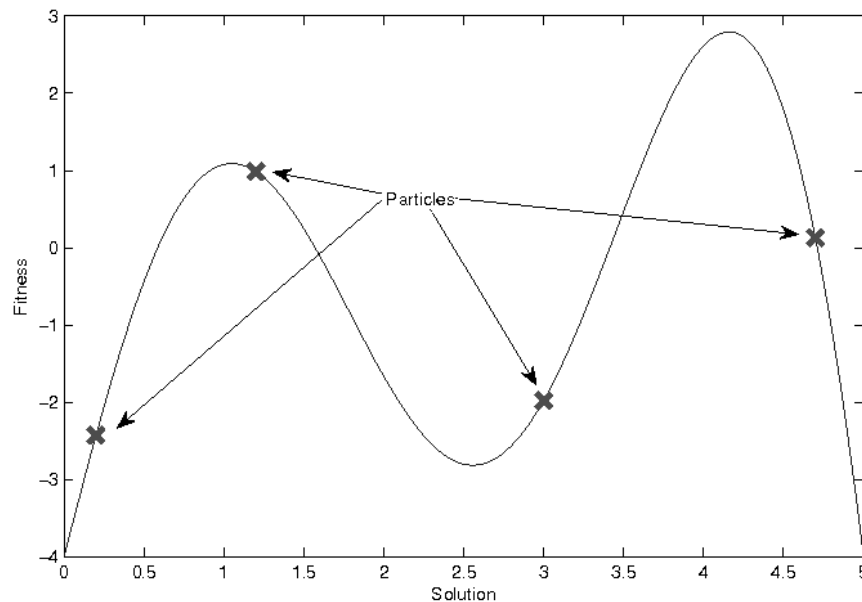


Figure 13 Candidates for maxima problem (Blondin, 2009)

### 1.5.Problem Definition

In current manufacturing system which requires precision, it is need of the time to manufacture quality products which meets customer requirement. To cope with this scenario, it is necessary to introduce efficient and effective fixturing system which is able to produce final product with required quality and precision. One of the defects in final product is induced by bad fixturing system. Error in workpiece position greatly depends upon position of locators. Placement and position of locators must be considered in order to eliminate the workpiece positioning error. But the question is: Is there any direct relationship which shows the relationship between locator’s placement and good quality product in order to minimize the workpiece positioning error?

Optimization of locator’s placement is one of the solutions to minimize the workpiece positioning error. This type of problem cannot be defined by limited research space. Classical optimization techniques need direct relationship between parameters to optimize the problem, but evolutionary techniques do not require any direct relationship between parameters and they are remain operative for the problems with many constraints. So the problem of locator’s placement optimization is solved by applying one of the evolutionary techniques.

### **1.5.1. Problem Description**

The problem statement defined above can be divided into following three parts

1. Define an objective function to build a relationship between locator's placement and workpiece positioning error.
2. Use evolutionary technique to optimize the locator's placement.
3. Build an algo for the problem.

### **1.6.Thesis Construction**

This thesis report comprises of five chapters. Chapter 1 provides the comprehensive introduction of importance and function of fixturing system. Different optimization techniques are also shortly discussed. Beside this motivation and problem statement is also defined which provides the base of this research. Chapter 2 gives a comprehensive detail of the research that has been done in this field so far. Research gap is found between the existing and present work which provides foundation of this work. Chapter 3 gives the proposed methodology to fill the research gap which was found in chapter 2. Chapter 4 and chapter 5 comprises of case study, conclusion, discussion and future work.

### **1.7.Summary**

In this chapter, brief introduction is given on fixtures and their functions. Fixture design and its 4-stage process are also discussed. Beside fixture design, importance of fixture is also reviewed in current scenario of manufacturing industry. Importance of optimization is highlighted in the field of fixture planning. Different optimization techniques (like ACA, PSO, GAs) are discussed in general which can be used for optimization of locator position and its placement. Problem definition is also discussed which highlights the importance of this research work.

In next chapter, literature is reviewed which focuses on different optimization techniques that were used to optimized locator's position and minimize the workpiece positioning error.



## **CHAPTER 2: LITERATURE REVIEW**

In the recent past years, extensive work has been done on the optimization of locator's placement in order to minimize the work-piece positioning error. Single technique or combinations of techniques are used in the literature for the purpose of optimization. Conventional/classical technique like calculus based techniques or linear programming are also used.

GA is emerged as one of the most famous algorithm used to optimize the problem which has not clear parameters and direct relationship for objective function. In this research, GA is chosen to optimize the locator's position and placement. For this purpose, GA is mainly focused in this literature. GA has been used alone or combined with different techniques and other evolutionary methods to optimize the locator's placement and position. GA is used in combination with following fields:

- GA in combination with finite element analysis
- GA in combination with ant colony algorithm
- GA in combination with particle swarm optimization
- GA in combination with neural networks
- Application of GA to find hole positioning error
- GA in combination with analytical method.

Following given the brief literature review on the optimization of locator's placement and positioning which mainly focus the GA.

### **2.1. GA in Combination with FEA & ANSYS**

Krishnakumar and Melkote (2000) presents the fixture layout optimization technique by using genetic algorithm by minimizing the maximum deformation of machined surface caused by clamping and machining forces over the entire tool path. They presented two GA based optimization technique method and then comparing he results over three example problem. First method used to find the optimum fixture layout for entire cutting process by optimizing the

layout for each step and then find the “best” among them. The performance of each optimum layout is evaluated at each instant of the machining process considered in the simulation. The “best” one is considered the optimized layout for whole process. Second method aimed to generate the optimized layout for whole process. Machine loads are applied sequentially on surface and corresponding deformation for each load is computed. GA is then used to minimize the maximum deformation by varying the position of clamps and locators. They conclude that second method is more promising as compared to first one.

Necmettin Kaya (2006) used genetic algorithm approach to minimize the elastic deformation of work-piece in 2D fixture layout by calculating the part deflection in ANSYS. He used the concept of “chromosome library” to reduce the computation time as FE evaluations are minimized from 6000 to 415. He explained his work by applying his approach to two different case studies. The conclusion shows that fixture layout optimization is multi-modal in nature although both case studies did not have any apparent similarities.

## **2.2. GA in Combination with Ant Colony Algorithm**

Padmanaban et al (2009) used ant colony algorithm (ACA) based discrete and continuous optimization techniques to minimize the work-piece deformation caused by the machining forces. Discrete based ACA technique gives solution on nodes only, whereas, ACA based continuous optimization technique gives solution with in the range of distance in which locator is lying. Finite element analysis (FEA) is used to evaluate the dynamic response of work-piece. Same forces are assumed in this article as assumed by Necmettin Kaya (2006). The results shows that ACA based continuous fixture optimization layout produce better result as compared to the ACA based discrete optimization method.

## **2.3. GA in Combination with Particle Swarm Optimization**

Dou et al (2010) presented the particle swarm optimization (PSO) algorithm to minimize the deformation error of work-piece in machining region. An integrated approach of PSO is used with ANSYS parametric design language (APDL) to determine the objective function for given layout. Particle library was also used, which reduced the computation time by 96%. Predefined clamping and forces are used in this article as Necmettin Kaya (2006) did. Effect of PSO was checked in this paper as compared to GAs and ACA.

## **2.4. GA in Combination with Neural Networks**

Subramaniam et al (1999) uses the combination of Genetic algorithm (GA) and Neural network (NN) to develop fixture design system. GA gives a set of optimal solution and provides the alternate optimal solution scheme whereas NN used to get trained on the basis of previous experiments. So, NN gives relationship between the complex parameters. It is concluded that integrated approach of GA and NN yields good results.

## **2.5. Application of GA to find Hole Positioning Error**

Abedini et al (2014) calculated the locator's position error to find the optimal fixture layout in compliance with given tolerance range of work-piece. The position error is then minimized through genetic algorithm. 3-2-1 fixture configuration is used to calculate the 3D work-piece geometry problem. The author tried to relocate the initially misplaced work-piece in the machine reference.

Hole positioning error (HPE) is determined through this illustration, which is an important factor in hole making process; where (HPE) is the difference between theoretical hole position and actual hole position. Ideally, there must be perfect locator's geometry and position, but this is not the case when comes to practical implementation. Usually geometry and position of locators is misaligned which produce the positional and orientation error as misaligned locator refer to the datum error. This datum error is usually responsible for the geometric error in work-piece. Figure 14 shows the position error of hole. The tool path change and position error is usually caused by orientation or height error. More than one datum surfaces are accountable for accurate hole making. Primary datum plane is derived from the co-ordinates of first three locators; secondary datum plane is derived from fourth and fifth locator while tertiary datum plane is derived from the co-ordinates of sixth locator.

Genetic algorithm is used to optimize the locator position in order to minimize the work-piece positioning error. Certain assumptions were made in this article which includes: 1) fixture elements and work-piece are rigid 2) error due to work-piece and tool is not considered 3) each locator has some fixed error. Real variable coding is used to represent the chromosome in this article, and each chromosome is represented by 18 genes. The fitness function for this article is defined as the minimum hole positioning error. Roulette wheel selection method is used for

selection criteria; beside this elitism strategy is also used in this article to keep the best possible chromosomes. Two tests run cases reveals the correct working of genetic algorithm. The application of GA is validated through the case study.

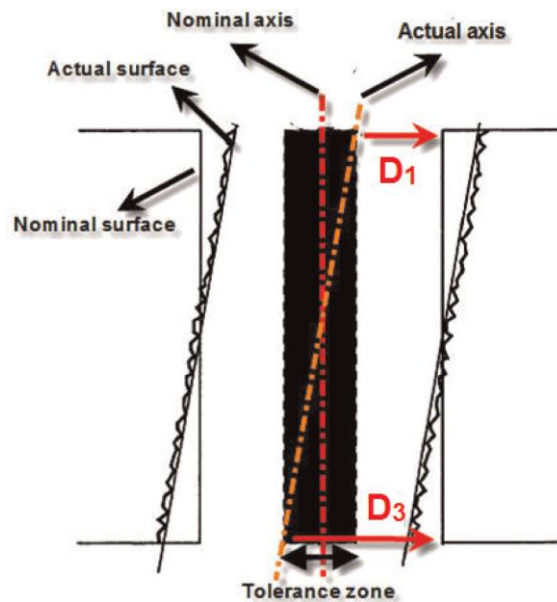


Figure 14 Positional tolerance for cylindrical holes (Abedini et al., 2014)

## 2.6. Linear Programming to find Workpiece Positioning Error

Bo Li and Shreyes N. Melkote (1999) present a model to minimize the localized elastic deformation of work-piece at fixturing points by optimally place the locator and clamps around the work-piece. Linear programming is used to find the optimal placement of locators and clamps. Aim of this paper is to reduce the work-piece locating error by improving fixturing points by considering locators contact point and clamps as elastic element. The whole model consists of four steps: 1) finding initial feasible solution. 2) Find the search line. 3) Set step size. 4) Solve the corresponding equation. The results show that work-piece has minimized location error; also work-piece piece has uniform and lower in magnitude deformation.

## 2.7. GA in combination with Analytical Method

Wu and Chan (1996) present the application of genetic algorithm for fixture configuration optimization problem. A generalize fixture verification system is introduced to validate the each individual by analyzing the different contact types of machining surface and fixturing system. It

covers the vast range of work-piece/fixturing system. Based on the validation system, genetic algorithm is used to find the most statically stable fixturing system.

(Sajid Ullah Butt et al., 2012) presented an analytical model for intricate parts which is capable of performing 6-DOF repositioning of work-piece. Compensations are made through different ways like through change in tool path, moving the cutting tool or moving the part w.r.t machine co-ordinate system. Conventionally, work-piece was mounted directly on the machine without any base plate and compensations were made through the NC part programs. Although this type of compensation is the easiest one but it needs 4 or 5-axis machine tool to perform the required transformation.

This article presents the fixturing system which is capable of holding the intricate work-piece and performing 6-DOF repositioning in the machine co-ordinate without using 4 or 5-axis machine. A high quality base plate is introduced on which work-piece is rigidly mounted. All the compensation is made through this base plate which is moving on the locators. Two models are presented based upon this fixturing system; one is kinematic model which results in relocation of work-piece through axial advancements of six locators with in the machine reference. The initially located is again misplaced due machining and clamping forces. The mechanical model addresses the displacement of work-piece due to these forces considering the locators and clamps to be elastic.

Fixturing system purposed in this article consists of six locators which were placed in 3-2-1 configuration. Kinematic model works if the initial and final positions of workpiece are known. Kinematic model only reorient the base plate to its corrected position in order to make it ready for mechanical actions. Practically, locators can deform under static and dynamic loads. The initially corrected baseplate-workpiece assembly again dislocated due to mechanical forces. If this dislocation cannot be taken into account, it will surely leads to wrong processing.

Abedini et al (2014) use kinematic model to minimize the workpiece positioning error. Workpiece positioning error is minimized through application of genetic algorithm. The main focus is to minimize the hole positioning error of workpiece.

Mechanical model is introduced to deal with the deformation of elastic locator under machining forces. For this purpose, the overall stiffness and mass of the system is calculated. Baseplate and

workpiece assembly is assumed to be rigid and unaffected under load. Once the displacement error is known for baseplate-workpiece assembly, the error is eliminated by re-orientation. Small displacement theory (SDT) is used in mechanical model. Lagrangian formulation is used to calculate the mechanical behavior of fixturing system. Table.1 gives the overview of the literature review.

## **2.8. Research Gap between Existing and Present Work:**

As mention in literature review, research has been done on optimizing the fixturing layout. (Abedini et al., 2014) represent the mathematical model to optimize the workpiece co-ordinates planes, by assuming static load without considering any machining forces. So this article actually optimizes the kinematic model presented by Butt et al (2012). The mechanical model this article is not yet been optimized.

This research aims to optimize the mechanical model for workpiece positioning error through genetic algorithm. The proposed work uses the ‘analytical model’ to calculate the displacement of the workpiece placed on the locators. Potential energy of locators and clamps, kinetic energy of workpiece and work done due to external machining forces are calculated.

Locators and clamps are assumed to be elastic and external machining or clamping force is applied to the workpiece. The proposed system uses the 3-2-1 configuration of locators. Friction between the work-piece and baseplate is assumed to be negligible. The work-piece is assumed to be rigidly mounted and there is no deformation occurs except the points where it contacts the locators.

By using this analytical model, the displacement of the workpiece can be calculated as a function of the placement of the locators. Evolutionary optimization techniques generate new position of locators and the model calculates the workpiece error for each placement. The optimization continues until the algorithm converges to global minima.

Table 1 Existing work on minimizing workpiece positioning error

Author	Optimization Criteria	Analysis	Elastic Elements	Force Applied	Objective
Bo Li et al. (1999)	LP model	Analytical modeling	Locators contact points & clamps	Clamping force	Reduction of workpiece locating error by improving fixturing points
Krishnakumar et al (2000)	GA	FEA	Workpiece	Cutting + clamping force	Minimize the maximum nodal displacement by defining two criteria
N.Kaya (2005)	GA	FEA	Workpiece	Cutting + clamping force	To minimize the maximum nodal values and check the corresponding fitness value for each optimal layout
K.P.Padmanaban et al (2009)	ACA	FEA	Locators and Clamps contact points	Cutting +clamping force	Minimize workpiece elastic deformation
D.Jiangping et al (2011)	PSO	ANSYS + FEA	Locators and clamps	Cutting + clamping force	Minimize the locator's and clamp's displacement for all loaded cases
S.U.Butt et al (2012)	NA	Analytical modeling	Locators and clamps	Machining + clamping forces	Compensation of workpiece displacement by advance six axial locators calculated through mechanical model and kinematic model.
V.Abdeni et al (2014)	GA	Mathematical modeling	Locators	Static force	Minimize the hole positioning error by calculating the locating error.

## 2.9. Mechanical Model Presented by Butt et al. (2012)

Mechanical model includes the calculation of clamping forces, machining forces and energy calculation. Calculations of these forces are given in the following subsections. Mechanical model is formalized by using Lagrangian formulation and small displacement theory. The generic Lagrangian formulation is given as:

$$\frac{\partial}{\partial t} \left( \frac{\partial(T-U)}{\partial \dot{q}_i} \right) - \frac{\partial(T-U)}{\partial q_i} = \frac{\partial W}{\partial q_i} \quad (1)$$

In the above equation, U is the total potential energy contained by all elastic elements, T is the total potential energy contained by all inertial elements and W is the work done by machining and clamping forces.

The main aim is to evaluate the displacement of workpiece under load from the corrected position achieved after the kinematic model. This displacement is the positioning error of workpiece.

### A. Potential Energy of Elastic Elements

For the fixturing system with 3-2-1 configuration, the only elastic elements considered are locators. The total potential energy of the system is the sum of potential energy of all locators. The overall potential energy of locators can be given as:

$$U = \frac{1}{2} \sum \{\Delta X_i\}^T [K]_i \{\Delta X_i\} \quad (2)$$

Where  $[K]_i$  is the stiffness matrix of  $i^{\text{th}}$  locator, where 'i' is the number of locator as given in equation (3).

$$[K]_i = \begin{bmatrix} k_{xx} & 0 & 0 \\ 0 & k_{yy} & 0 \\ 0 & 0 & k_{zz} \end{bmatrix} \quad (3)$$

$\{\Delta X\}_i$  is the displacement vector of  $i^{\text{th}}$  locator. This displacement vector tells about the x, y and z components of displacement of point of contact of  $i^{\text{th}}$  locator with baseplate when the load is applied. This displacement vector is calculated with respect to an imaginary point P which is



assumed to be center of gravity of base plate. Small displacement theory is used so the final displacement was given as:

$$[\Delta X]_i = \begin{Bmatrix} \Delta X_i \\ \Delta Y_i \\ \Delta Z_i \\ 1 \end{Bmatrix} = \begin{bmatrix} 0 & -\Delta\alpha & \Delta\gamma & \Delta X_P \\ \Delta\alpha & 0 & -\Delta\beta & \Delta Y_P \\ -\Delta\gamma & \Delta\beta & 0 & \Delta Z_P \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} X_i - X_P \\ Y_i - Y_P \\ Z_i - Z_P \\ 1 \end{Bmatrix} \quad (4)$$

Above equation gives the displacement of  $i^{\text{th}}$  locator in terms of 6 DOF i.e.  $\{\Delta X_P \ \Delta Y_P \ \Delta Z_P \ \Delta\beta \ \Delta\gamma \ \Delta\alpha\}$ . This equation can be used to find the potential energy of locator as given in equation (2). Hence, we get the potential energy of each locator as the function of six variables (three linear and three rotational). Sum of potential energy of each locator will gives us total potential energy of the system, which is a scalar value.

#### B. Kinetic Energy of Inertial Elements

Workpiece-baseplate assembly is the only inertial element in the fixturing system shown. So, workpiece-baseplate assembly experiences kinetic energy under external forces. Generally, there are two components of kinetic energy; one is linear and other is angular. The general expression of linear kinetic energy is given as:

$$T_v = \frac{1}{2} \{\vec{V}\}^T [M] \{\vec{V}\} \quad (5)$$

Where  $[M]$  is the mass matrix and it is usually a diagonal matrix.  $\{\vec{V}\} = \{\vec{v}_x \ \vec{v}_y \ \vec{v}_z\}$  is the velocity vector. The angular kinetic energy of system ( $T_\Omega$ ) can be written as:

$$T_\Omega = \frac{1}{2} \{\vec{\Omega}\}^T [I] \{\vec{\Omega}\} \quad (6)$$

Where  $[I]$  is the mass matrix and it is also a diagonal matrix.  $\{\vec{\Omega}\} = \{\vec{\omega}_x \ \vec{\omega}_y \ \vec{\omega}_z\}$  is the angular velocity vector. By adding equation (7) and equation (8), we can find the total kinetic energy of the system as a function of their baseplate displacement vector  $\{\Delta X_P \ \Delta Y_P \ \Delta Z_P \ \Delta\beta \ \Delta\gamma \ \Delta\alpha\}^T$ . Similar to potential energy, total kinetic energy of the system can also be calculated as the function of displacement of baseplate. Keeping in view, in our case kinetic energy is negligible but it has greater impact in turning process.

$$T = \frac{1}{2} \{\vec{V}\}^T [M] \{\vec{V}\} + \frac{1}{2} \{\vec{\Omega}\}^T [I] \{\vec{\Omega}\} \quad (7)$$

### C. Work Done by External Forces

Calculation of work done due to external forces and moments (Butt et al, 2012) can be explain by a example. Workpiece-baseplate assembly will experience an external static load i.e.  $F = \{F_x \ F_y \ F_z\}^T$  and an external torque  $T = \{T_x \ T_y \ T_z\}^T$ . Work done by external force and torque is generally given by:

$$W = \{F\} \cdot \{\Delta X_P\} + \{T\} \cdot \{\Delta \Theta\} \quad (8)$$

Where  $\{F\}$  is the force vector,  $\{\Delta X_P\}$  is the displacement of baseplate at point P under force F,  $\{T\}$  is the external torque and  $\{\Theta\}$  is the angular displacement vector due to external torque. Isometric view of baseplate showing the force and troque components is shown in figure 15. If force applied is away from center of gravity, it causes the linear as well as angular displacement of workpiece. The displacement caused by external force can be written in form of homogenous transformation matrix as:

$$\begin{Bmatrix} \Delta X_{Pf} \\ \Delta Y_{Pf} \\ \Delta Z_{Pf} \\ 1 \end{Bmatrix} = \begin{bmatrix} 0 & -\Delta\alpha & \Delta\gamma & \Delta X_P \\ \Delta\alpha & 0 & -\Delta\beta & \Delta Y_P \\ -\Delta\gamma & \Delta\beta & 0 & \Delta Z_P \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} x_f - x_P \\ y_f - y_P \\ z_f - z_P \\ 1 \end{Bmatrix} \quad (9)$$

Where  $\{x_f - x_P \ y_f - y_P \ z_f - z_P\}^T$  is the distance vector between point of action of force and point P.  $\{\Delta X_{Pf} \ \Delta Y_{Pf} \ \Delta Z_{Pf}\}^T$  is the displacement vector,  $\{\Delta\beta \ \Delta\gamma \ \Delta\alpha\}^T$  is the angular displacement.

Equation (7) can be rewritten in vector form with  $i$  external forces and  $j$  external torques as given in equation (9). In this way we get  $\{\Delta X_{Pf} \ \Delta Y_{Pf} \ \Delta Z_{Pf}\}^T$  and as a result W in the form of six variables i.e  $\{\Delta X_P \ \Delta Y_P \ \Delta Z_P \ \Delta\beta \ \Delta\gamma \ \Delta\alpha\}^T$ .

$$W = \sum \begin{Bmatrix} F_x \\ F_y \\ F_z \end{Bmatrix}_i \cdot \begin{Bmatrix} \Delta X_{Pf} \\ \Delta Y_{Pf} \\ \Delta Z_{Pf} \end{Bmatrix}_i + \sum \begin{Bmatrix} T_x \\ T_y \\ T_z \end{Bmatrix}_j \cdot \begin{Bmatrix} \Delta \beta \\ \Delta \gamma \\ \Delta \alpha \end{Bmatrix} \quad (10)$$

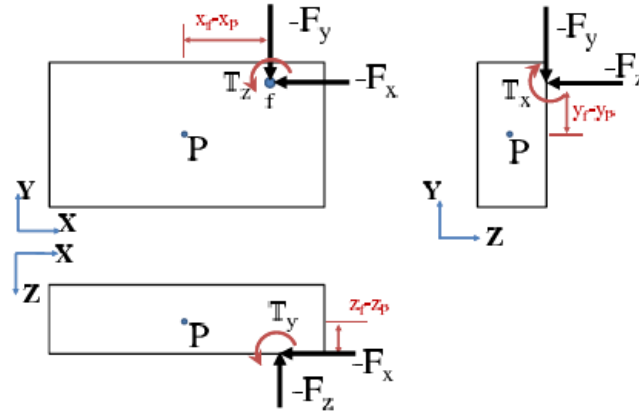


Figure 15 Isometric view of workpiece-baseplate assembly under load (Butt, 2012)

#### D. Clamping Forces

Clamps are used to tightening the workpiece after it is located through locators. As clamps are considered as elastic element, they exert compressive forces on locator. The initial position of workpiece is measured through CMM and deformation of locator under workpiece-baseplate assembly is taken into account. So, only external forces are responsible for further deformation. Usually, clamp is tightened after brought it into the workpiece contact. Two cases are possible for clamps:

- Case I

Clamps can be presented as external static load acting on baseplate at point where it is contacted with baseplate. In this case, total work done is the sum of work done by all external forces (including clamps). The work done of clamping forces can be evaluated as discussed in equation (9).

- Case II

Clamps can be presented as elastic elements. In this case, one end of clamp is in contact with baseplate and other is externally displaced by an unknown force as shown in figure 16. The external displacement is known and it is constant. So, potential energy of clamps is evaluated and is then added to overall potential energy of the system.

In Butt et al (2012) clamps are considered as elastic element. Two clamps ( $C_1$  and  $C_2$ ) are considered in this work as shown in Fig.14. A part of external displacement is shifted the baseplate clamp contact and displacement of  $j^{\text{th}}$  clamp with respect to baseplate  $\{\Delta X_{ij} \ \Delta Y_{ij} \ \Delta Z_{ij}\}^T$  by using HTM is given in equation (10) which is quite similar to equation (4).

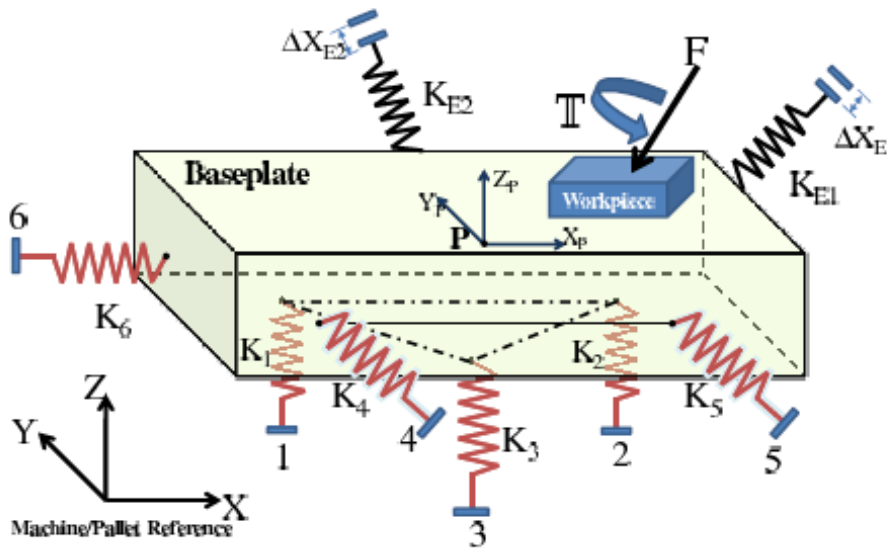


Figure 16 Clamps Modeling according to Pallet Reference (Butt, 2012)

$$\begin{Bmatrix} \Delta X_{ij} \\ \Delta Y_{ij} \\ \Delta Z_{ij} \\ 1 \end{Bmatrix} = \begin{bmatrix} 0 & -\Delta\alpha & \Delta\gamma & \Delta X_P \\ \Delta\alpha & 0 & -\Delta\beta & \Delta Y_P \\ -\Delta\gamma & \Delta\beta & 0 & \Delta Z_P \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} X_{cj} - X_P \\ Y_{cj} - Y_P \\ Z_{cj} - Z_P \\ 1 \end{Bmatrix} \quad (10)$$

Where  $\{x_{cj}-x_P \quad y_{cj}-y_P \quad z_{cj}-z_P\}^T$  is the distance between clamp point and point P. The final displacement of clamps can be written as:

$$\begin{Bmatrix} \Delta X_{Cj} \\ \Delta Y_{Cj} \\ \Delta Z_{Cj} \end{Bmatrix} = \begin{Bmatrix} \Delta X_{Ej} \\ \Delta Y_{Ej} \\ \Delta Z_{Ej} \end{Bmatrix} - \begin{Bmatrix} \Delta X_{ij} \\ \Delta Y_{ij} \\ \Delta Z_{ij} \end{Bmatrix} \quad (11)$$

Where  $\{\Delta X_{Ej} \quad \Delta Y_{Ej} \quad \Delta Z_{Ej}\}^T$  is the external displacement vector of  $j^{\text{th}}$  clamp. The negative sign of  $\{X_{ij}\}$  shows that the clamps always placed in direction opposite to locators. Final potential energy of clamps can be written as:

$$U_C = \frac{1}{2} \sum \{\Delta X_C\}_j^T [K_E]_j \{\Delta X_C\}_j \quad (12)$$

Where  $\{\Delta X_E\}_j = \{\Delta X_{Cj} \quad \Delta Y_{Cj} \quad \Delta Z_{Cj}\}^T$  is the relative displacement of clamps and  $[K_E]_j$  is a stiffness matrix of  $j^{\text{th}}$  clamp and is assumed to be known.

After calculating all the potential energy of elastic elements, kinetic energy of inertial elements, work done by external forces and moments, the Langrangian formulation is used to find the stiffness and mass matrices, linear and angular displacement of workpiece on fixturing system.

## 2.10. Summary

In this chapter, detailed literature review is discussed which encompasses different optimization techniques that has been used so far. The modern work in this field mainly focuses on evolutionary techniques of optimization (Genetic algorithm, neural networks, ant colony optimization. Mechanical model of Butt et al (2012) for fixturing system is discussed in detail. At the end of this chapter, research gap is found which is filled by this research work. Overview of the literature is given in table form also.

In next chapter proposed methodology is given which optimized the mechanical model of (Butt, 2012) by applying one of the evolutionary techniques.

## CHAPTER 3: PROPOSED METHODOLOGY

In this chapter, the proposed methodology, in order to minimize the workpiece positioning error, is detailed. In literature, several optimization techniques were applied to minimize the workpiece displacement as discussed in chapter 2, but no research is done yet to optimize the locator's position for flexible fixturing system as given by Butt (2012). In this present work, a methodology is discussed to optimize the locator's placement using genetic algorithm because it is easily adaptable and we can easily quantify the parameters in GA as compared to other evolutionary techniques. Other techniques require clear relationship between parameters (in form of equations etc.). As discussed earlier in chapter 2, most researchers used GA to optimize the fixturing system. Certain assumptions were made for this work which are taken as constraints for this GA. These assumptions are;

1. Locators are considered as 3-D elastic elements with negligible masses (Butt 2012)
2. 3-D rigid workpiece is taken for this work
3. There is no friction between locator and base plate contact
4. Locator's placement resolution is taken up to  $10\mu\text{m}$
5. Six locators are always in contact with the workpiece

General working and pseudo code of simple genetic algorithm is discussed in chapter 1 (section 1.8). Lagrangian formulation is used to calculate the six displacements of locators (three translational and three rotational) which is resolved in Mathematica®. The flow chart of the genetic algorithm used to find optimized locator's position is given in figure 17. Each step of the flow chart is discussed in the following sections.

Mechanical model presented by (Butt et al., 2012) is used as fitness function in the development of this algorithm. The aim of this research is to minimize the workpiece positioning error. The error can be minimized by optimal placement of locators around workpiece. 3-2-1 fixture configuration around a rectangular workpiece is used for this research. The developed algorithm will be capable of finding optimized placement of six locators which produce minimum possible

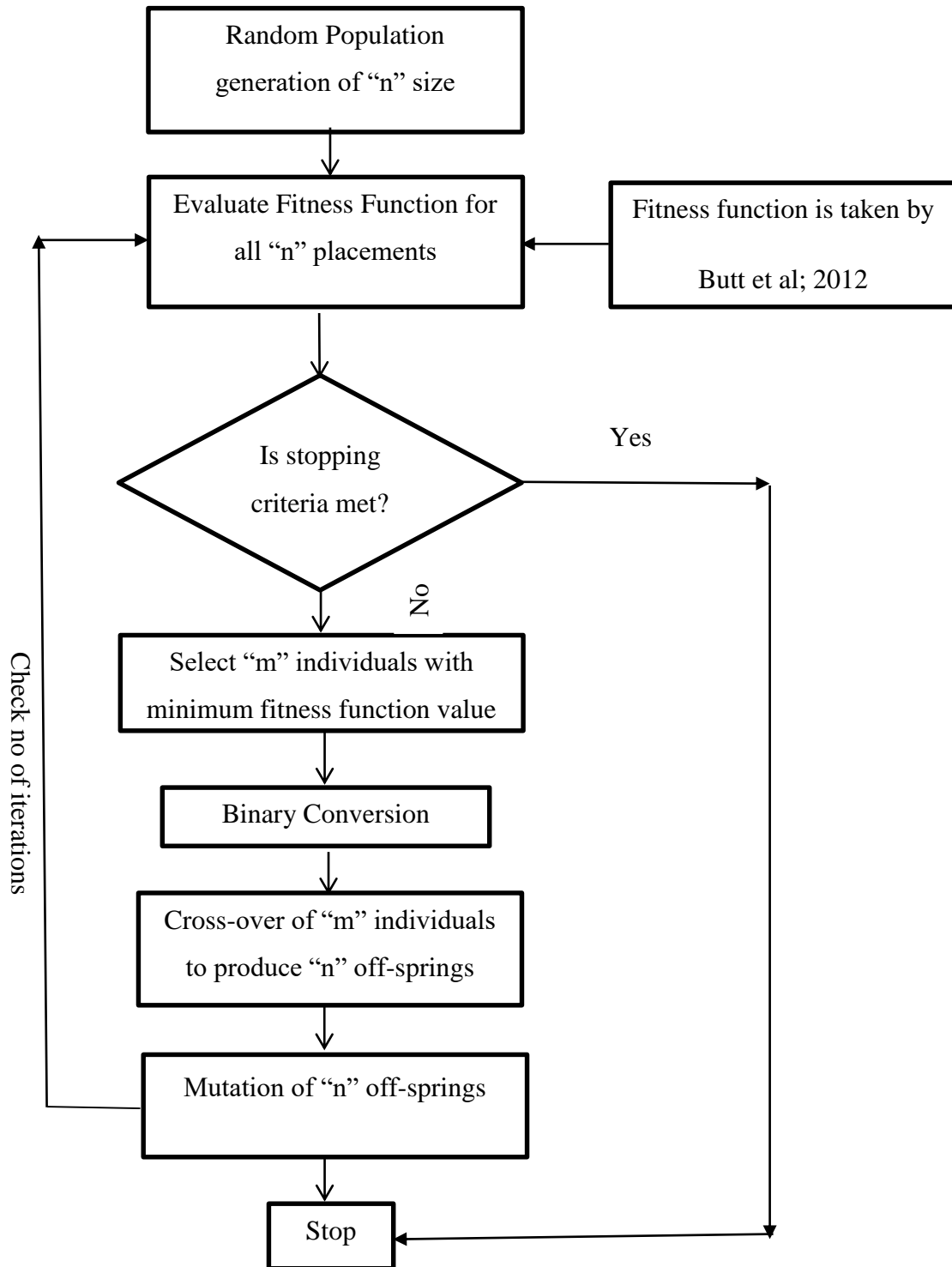


Figure 17 Flow chart of Genetic algorithm for present work

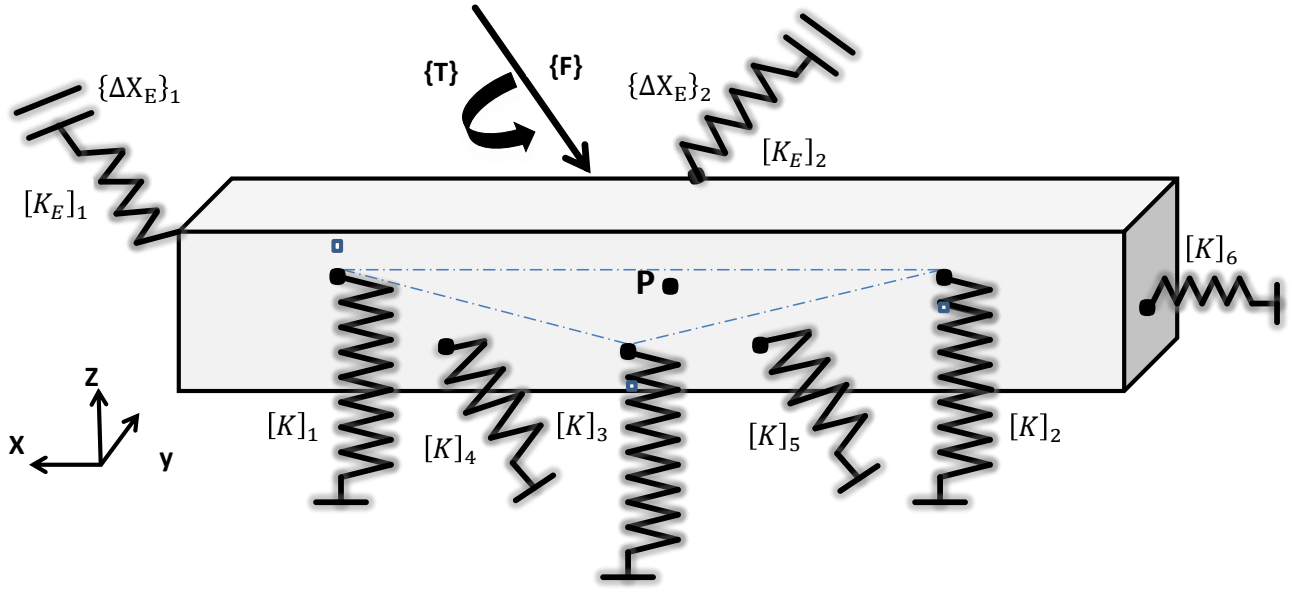


Figure 18 Fixturing System with 3-2-1 Configuration and two Clamps

workpiece displacement for specific loading. Figure 18 shows a fixturing system with simple rectangular part in 3-2-1 elastic locators configuration as presented by Butt et al., (2012).

Fixturing system having 3-2-1 fixture configuration is considered for this study. Two elastic clamps are also considered as three dimensional springs.  $\{F\}$  is taken as machining force on workpiece,  $\{T\}$  is the moment of cutting tool,  $[K_1], [K_2], \dots [K_6]$  are stiffness matrices of respective locators.  $[K_E]_1$  and  $[K_E]_2$  are stiffness matrix of clamps,  $\{\Delta X_E\}_1$  and locators are supposed to move axially for isostatism. Workpiece has no inclination i.e.  $\alpha_B, \beta_B, \gamma_B = 0$ . It might be possible that workpiece has initial inclination due to locator's placement. In that case,  $\alpha_B, \beta_B$  and  $\gamma_B$  can be assigned respective values.

The input for this algorithm is random population generation as locators' placement. Workpiece positioning error is calculated for each locator's placement discussed by Butt et al (2012). We get three linear and three rotational displacements  $(\Delta x_p, \Delta y_p, \Delta z_p, \beta_p, \gamma_p, \alpha_p)$  corresponding to each locator's placement matrix at center of gravity of workpiece. Homogeneous transformation matrix (HTM) is used to calculate the positioning error  $(\Delta x, \Delta y, \Delta z)$  of the point of interest on the workpiece. Algorithm aims to minimize this error norm.

Random population is generated for locator's placement by giving values of x-axis, y-axis and z-axis to each locator. For six locators, a 6x3 matrix is generated. These values of locator's



placement are used as input for Mechanical model discussed by Butt et al (2012). The output of Mechanical model is the positioning error of workpiece. Best configuration (6x3 matrix) is the one which gives minimum norm (workpiece positioning error).

### 3.1. Initial Random Population Generation

Initial values of each locator are taken as random population which is randomly generated by specifying the probable contact area for each locator on workpiece. Figure 19 shows the order of placement of locators (Butt et al; 2012). With the reference of figure 19, placement and position of locators along the axes is determined in this work; this will be discussed in case study in next chapter.

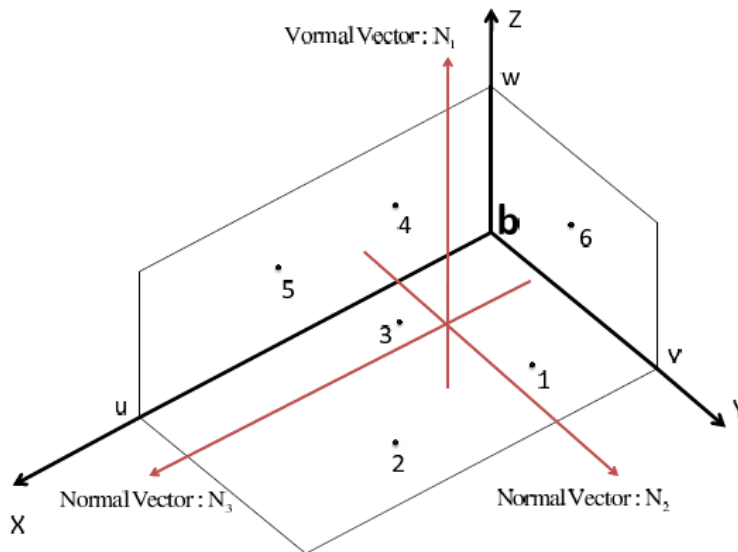


Figure 19 Order for placement of locators (Butt, 2012)

### 3.2. Evaluate the Fitness Function

The objective function for this work ('Mechanical Model' for fixturing system) is taken from Butt (2012). The output of mechanical model through Langrangian formularization is taken as input fitness function for this GA. The Mechanical Model calculates the positioning error by using potential energy of elastic elements, kinetic energy of inertial elements and work done by external forces. We get three translational errors (along x, y, z-axis) and three rotational errors ( $\beta$ ,  $\gamma$ ,  $\alpha$ ) as the component outputs from Langrangian formularization. HTM is used to calculate the positioning error ( $\Delta x$ ,  $\Delta y$ ,  $\Delta z$ ) of the point of interest on the workpiece.

Norm of these three displacement is then taken to calculate the overall error of the system. Figure 20 gives the flow chart of mechanical model to find the six variables (Butt et al; 2012). The fitness function to evaluate the fitness of individuals work is defined as:

$$F_i = \text{Min} \left( \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2} \right) \quad (13)$$

Here it is important to note that we are assuming fixture co-ordinate system (FCS) for the selection of random population. It is important to shift the fixture co-ordinate system (FCS) to machine co-ordinate system (MCS). In the following subsection method is discussed to convert fixture co-ordinate system to machine co-ordinate system.

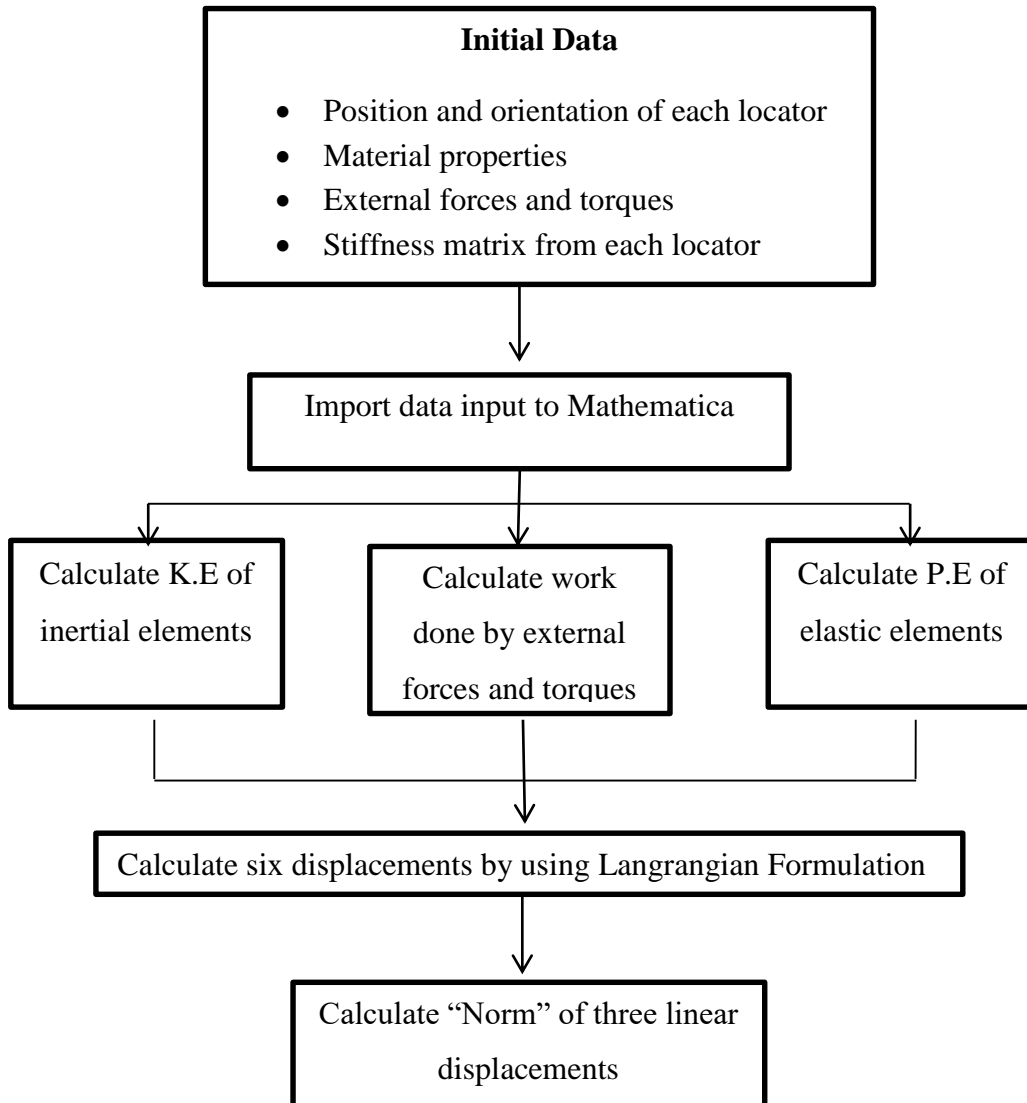


Figure 20 Flow chart for the Calculation of Fitness Function (Butt, 2012)

### 3.2.1. Conversion of Workpiece Co-ordinate to Machine Co-ordinates

If the dimension of rectangular workpiece is given as  $(x, y, z)$ mm. Co-ordinates of point P which is assumed as center of mass of workpiece, is taken as  $(P_x, P_y, P_z)$ mm. It is necessary to convert the workpiece co-ordinate system to machine co-ordinate system; otherwise it might be possible that the locators which are placed according to the fixture co-ordinate system do not touch the surface of workpiece. Compensation must be made in order to convert workpiece co-ordinate system to machine co-ordinate system.

Figure 21 shows the workpiece co-ordinate system (WCS) in accordance with machine co-ordinate system (MCS). Assume the position of locator 1 in example as  $(x_1, y_1, z_1) = (70, 100, 0)$ mm. This calculation is applicable to all six locators. Position of point 'P' (center of gravity) is assumed as known in machine co-ordinate system (MCS) while 'P' in workpiece (WCS) is  $x/2, y/2, z/2$ . Co-ordinates of point P can be calculated by  $(P_x, P_y, P_z) = (x/2 + l_1, y/2 + l_1, z/2 + l_1)$ ; where  $l_1$  is the axial length of locator. Suppose the axial length of locator is 15mm, then the co-ordinates of point P can be calculated as  $(65, 55, 40)$ mm.  $x, y$  and  $z$  are the workpiece dimensions i.e.  $(x, y, z) = (100, 80, 50)$ mm. Position of 'O' in workpiece co-ordinate system is given as:

$$(O_x, O_y, O_z) = (P_x - x/2, P_y - y/2, P_z - z/2) \quad (14)$$

Thus, the co-ordinates of point 'O' is calculated as:

$$(O_x, O_y, O_z) = (15, 15, 15)\text{mm}$$

So, the machine co-ordinates for locator1 can be given by:

$$\text{Position of locator 1} = (O_x + x_1, O_y + y_1, O_z + z_1)$$

i.e. Position of locator 1 =  $(85, 115, 15)$ mm

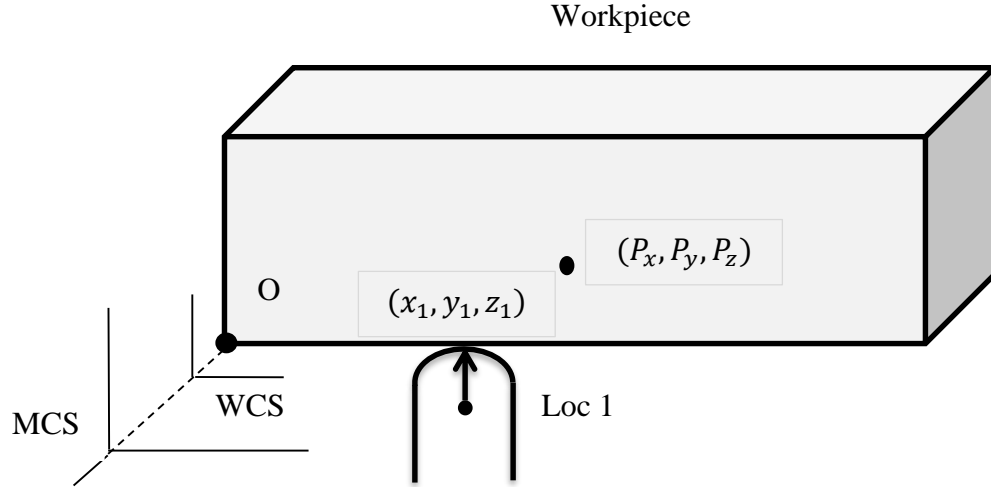


Figure 21 Conversion of Workpiece Co-ordinate System (WCS) to Machine Co-ordinate System (MCS)

So the final relationship between machine co-ordinate system (MCS) and workpiece co-ordinate system (WCS) for the  $i^{\text{th}}$  locator, can be given as;

$$\begin{Bmatrix} X_i \\ Y_i \\ Z_i \end{Bmatrix} = \begin{Bmatrix} P_x \\ P_y \\ P_z \end{Bmatrix} - \begin{Bmatrix} x/2 \\ y/2 \\ z/2 \end{Bmatrix} + \begin{Bmatrix} x_i \\ y_i \\ z_i \end{Bmatrix}$$

Where  $(P_x, P_y, P_z)$  is center of gravity,  $(x, y, z)$  is the workpiece dimension and  $(x_i, y_i, z_i)$  is the position of  $i^{\text{th}}$  locator.

### 3.3. Selection and Binary Conversion

Individuals with minimum norm are selected for reproduction. Size of this selection depends upon the size of population. After selection of best matrices, the next step is to convert each individual element of each matrix into binary form. It is possible that bit length may vary in accordance with values. For example, binary conversion of '5' is '101' (having three number of bits) and that of '10' is '1010' (having four number of bits). So it is important to convert all the individual values into same bit length, otherwise it is impossible to bring these individuals for cross-over and mutation.

In this research resolution is taken up to  $10\mu\text{m}$  i.e.  $10^{-5}\text{m}$ . It is significant to choose a bit length to incorporate the values up to five decimal places. For this purpose, we are assuming  $2^{16}$  which is equal to 65535. So each element of matrix is converted into 16-bit length. So, maximum value of locator placement is  $(x, y, z) = 655.35\text{mm}$ .

### **3.4. Cross-over**

After binary conversion, reproduction is done through cross-over. Z-axis of first three locators, y-axis of 4th and 5th locators, and x-axis of 6th locator are taken as invariant because in placement only literal position are changed and axial position can be calculated as point of contact with workpiece. It is necessary to exclude these individuals from matrices for cross-over. So, cross-over operation is only work with twelve out of eighteen elements by excluding six elements as invariants. These invariants are assigned “zero” value in this algorithm. Cross-over probability can be changed as per need and this probability is only valid for twelve elements of position matrix which are chosen from eighteen elements. The chosen best matrices are brought for cross-over and after swapping two halves of every matrix we again get equal to population size matrices. Figure 22 shows the possible mating with in four matrices as example. Cross-over of four matrices generate population of twelve matrices Random point cross-over is implemented for this algorithm to avoid the premature convergence.

There is no definite point for swapping occurs. Random swapping points are generated by algorithm from where chromosome is cut and swapped the corresponding heads and tails.

### **3.5. Mutation**

Mutation is performed after cross-over. Here again, we exclude the invariants and we only operate with twelve elements left. Mutation helps to explore the new search space and ensures that algorithm does not converge at any local minima. In this work, two mutation operators are considered. One is ‘ $P_M$ ’ which defines the choice of individuals from the twelve potential elements, while other mutation operator is ‘ $P_m$ ’ which defines the number of bits to invert from the chosen element. After mutation, all the values are again converted into real values.

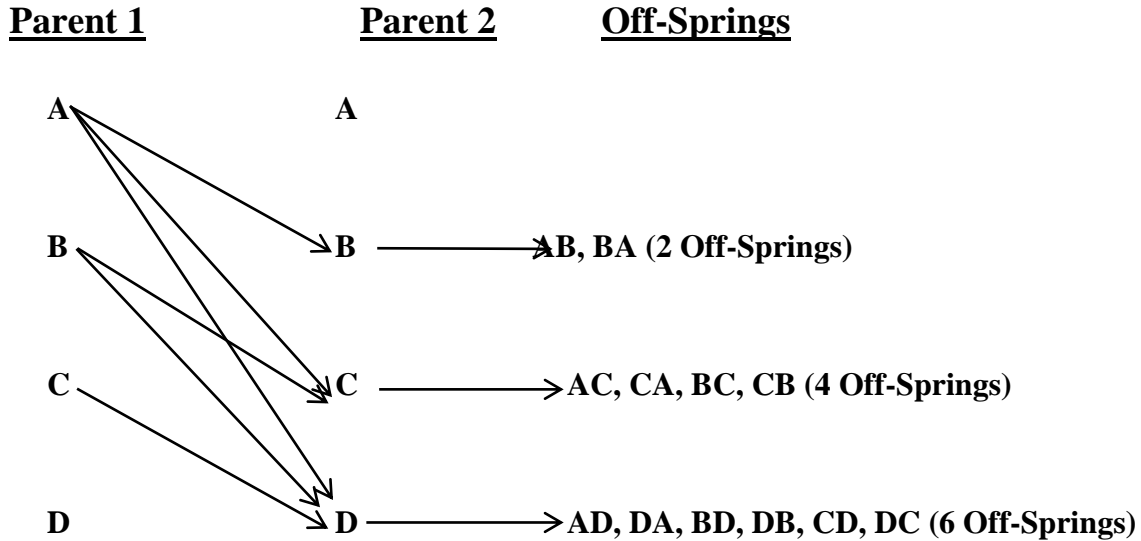


Figure 22 Mating with in Four Parents (Producing Total 12 Off-Springs)

During reproduction (cross-over and mutation), it might be possible that the operated element may fall “*out of range*” after reproduction. It may fall above or below the allotted limits. It is necessary for this algorithm that position of each locator must be in its range. Following section describes the compensation method which is followed to bring the individuals in their range.

### 3.5.1 Applying Limitations

In this section, compensation method is proposed to cope with the problem of ‘*out of range*’. This problem may cause after reproduction by swapping and inversion of bits because it is possible to have drastic change in value after reproduction. In such case, relative weightage is given to new changed value according to given range.

In our case, the range is taken from 0 to 65535, which shows the maximum and minimum values for the range. Three cases are possible for applying limitations: first case is if the new changed value is greater than given range, second case is if the new changed value is less than the given range and third case is if the new changed value is still lies between the given range. For the first two cases compensation is must, but for the third case no compensation is needed.

Compensation formula is developed for first two cases. If we introduce variable i.e.  $m_{ij}$ = initial generated number by algorithm,  $mm_{ij}$ = new changed value after reproduction,  $max_{ij}$ = maximum limit of range and  $min_{ij}$ = minimum limit of range then:

For 1st case (when new changed value is greater than given range), the new value will be,

$$MM_{ij} = m_{ij} + (\max_{ij} - m_{ij}) \times \frac{mm_{ij}}{65535 - m_{ij}} \quad (15)$$

For 2nd case (when new changed value is less than given range), we have,

$$MM_{ij} = m_{ij} - (m_{ij} - \min_{ij}) \times \frac{mm_{ij}}{m_{ij} - 0} \quad (16)$$

A simple example is solved to elaborate the working of the formula below.

### 3.5.2. Example for Limitation Application

Assume the x-range of locator 1 is (0-100) mm and initial random sample for this range is chosen to be 70 mm. Suppose after reproduction (cross-over and mutation) this initial generated number turns to 1900 mm, which is not in the x-range of locator 1, then by using equation 15, we have,

$$MM_{ij} = 70 + (100 - 70) \times \left( \frac{1900}{65535 - 70} \right) = 70.87 \text{ mm}$$

So, the above equation shows only addition of one (mm) after a drastic change of value from 70 to 1900. The following figure 23 shows the relative weightage of values according to given range of actual binary value.

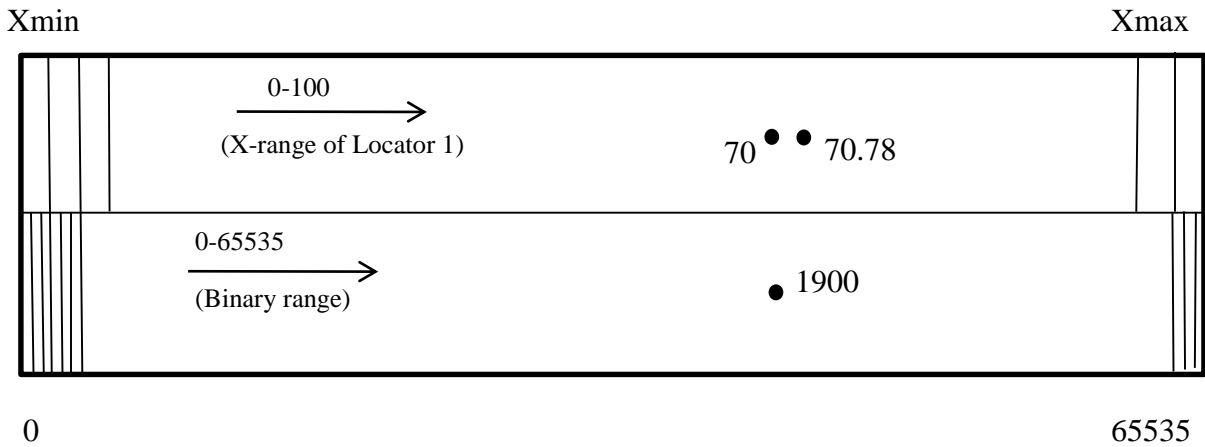


Figure 23 Relative Weightage of Chosen value according to the Binary Range

### **3.6. Summary**

In this chapter, detailed genetic algorithm is discussed which is applied for minimizing the workpiece positioning error by locator placement optimization. Each step of the algorithm is described in detail. Fig.17 shows the detailed flow chart of the algorithm. In next chapter, case study is presented which shows the efficient working of developed algorithm.



## **CHAPTER 4: OPTIMIZATION OF INPUT DATA AND SIMPLE CASE STUDY**

For the effective and efficient working of developed GA, it is necessary to choose right GA parameters according to the nature of problem statement. It might be possible that we cannot get optimized results without choosing right GA parameters. In this chapter input parameters like mutation rate, population size and number of iterations are optimized and applied on simple case study which incorporates all the steps that were elaborated in previous chapter.

### **4.1. Effect of GA Parameter**

In developing algorithm for GA, certain parameters are involved like number of iterations, population size and mutation rate. In the following sections, effect of these parameters, on final result, is discussed for the development of final model.

#### **4.1.1. Effect of Population Size**

Population size depict random sample chosen from search space. It is important to choose appropriate population size for the given problem. If small population size is chosen, it possibly does not find the optimal solution. If very large population size is chosen, it provides high diversity and algorithm may found its optimal solution but at the cost of time. So, large population size is CPU intensive (high computation time) and small population size loses its diversity before finding global optimum solution. It is noticed that, increase in population size beyond a limit does not yield any noticeable convergence (Sarmady, 2007).

For this study population sizes of 30, 90, 132 and 156 have been tested. Table 2 gives the population sizes and their computation time for 'n' iterations. Figure 24 shows the population size and their convergence. It is noticed that minimum workpiece positioning error for all population sizes is approximately 8.79 $\mu\text{m}$ , but small population sizes tend to converge late (with more number of iterations) as compared to large population sizes. Increasing population size

after that, increases time exponentially. Result shows that 156 population size yields the best result.

Table 2 Population Size and their Computation Time

Population Size	Computation Time (seconds)	No of iteration
30	11296.01	350 iteration
90	18178.8	200 iteration
132	8947.52	65 iteration
156	8442.23	50 iteration

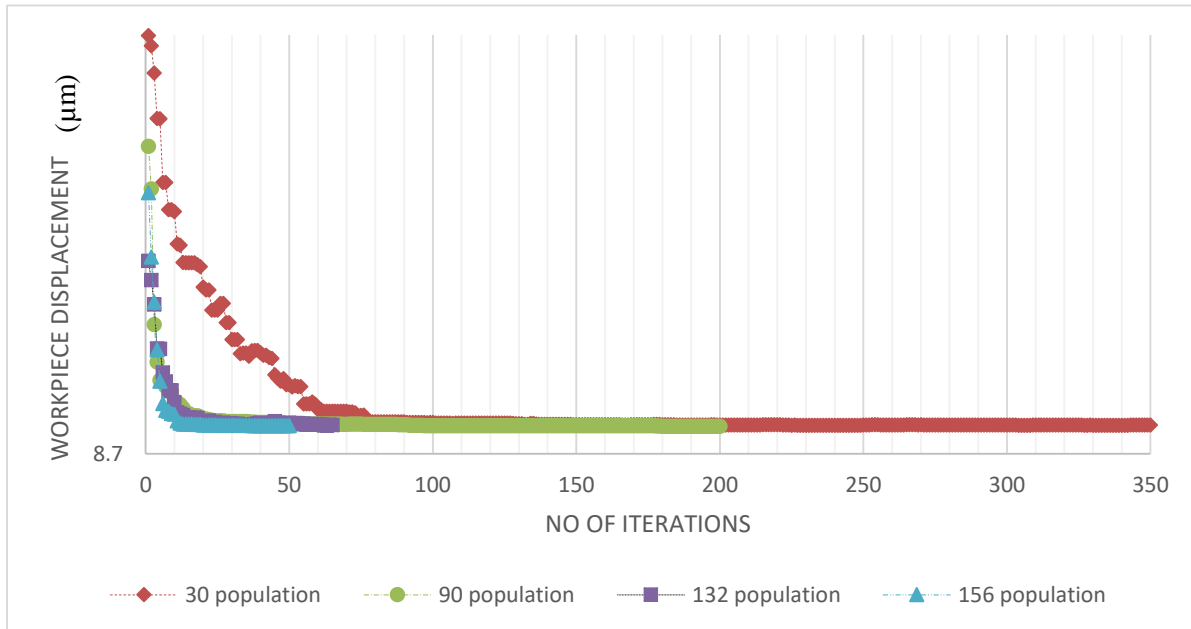


Figure 24 Effect of Population Size

#### 4.1.2. Effect of Mutation Rate

Role of mutation rate is to save the algorithm from pre-mature convergence (local optima). Low mutation rate may result in local optima and high mutation rate may cause distortion of chromosome more than needed without converging at global optima.

For this case study, different mutation rates are tested. We assumed the population size of 156 and fixed 'P<sub>m</sub>' = 0.16; which mean, for a chosen element, any one random bit out of 16 will be

inverted. We tested ‘ $P_M$ ’ (which shows the random selection of elements out of 12 elements) at 1/12, 2/12, 3/12 and 6/12.

Figure.25 shows the effect of different mutation rate. From the figure it is inferred that  $P_M = 0.1667$  yields best results.

#### 4.1.3. Effect of Number of Iterations

Choosing optimum number of iterations is important for the convergence of algorithm but without at the cost of computation time. Algorithm is ran for three times at 40 iterations by considering the parameters which are discussed above i.e. population size = 156,  $P_m = 0.16$ ,  $P_M = 0.1667$ . Figure 26 shows that the algorithm converges all the three times well before 40 iterations. So 40 iterations are chosen as stopping criteria to be on safer side.

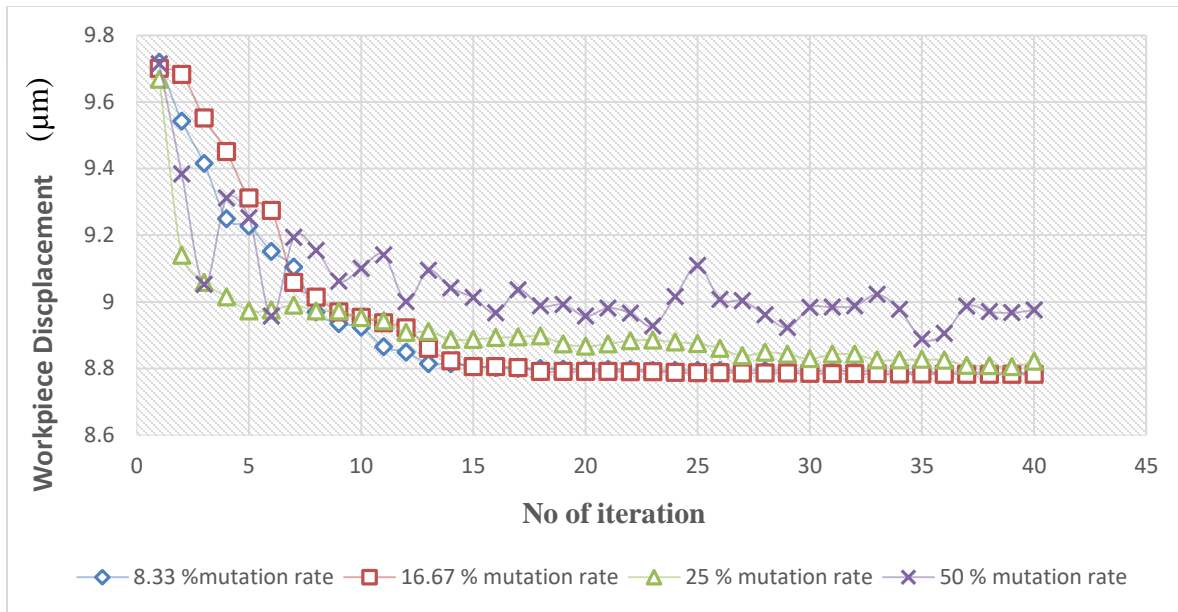


Figure 25 Effect of different mutation rates

## 4.2. Input Data for case study

For this case study, fixturing system with 3-2-1 locator configuration and two clamps is considered. Dimension of rectangular workpiece are given as  $(x, y, z) = (100, 80, 50)$ mm . The position of point P in machine co-ordinate system, which is assumed as center of gravity, is taken as  $(P_x, P_y, P_z) = (75, 65, 50)$ mm.

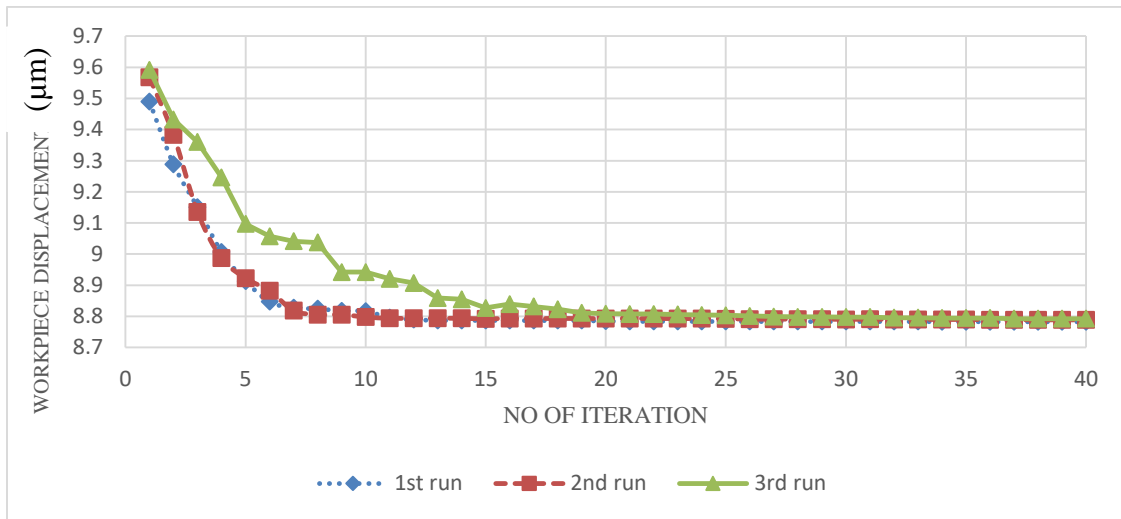


Figure 26 Convergence for 40 iteration number

All locators are placed in order according to the figure 19 with each rectangle represents the probable area of locator's contact. Figure 27 shows the number and range of each locator along with the axes. Table 3 gives the range of each locator in tabular form while table 4 gives an example of random placement of locator's position. The initial data input is taken in meters and will again converted into mm by algorithm.

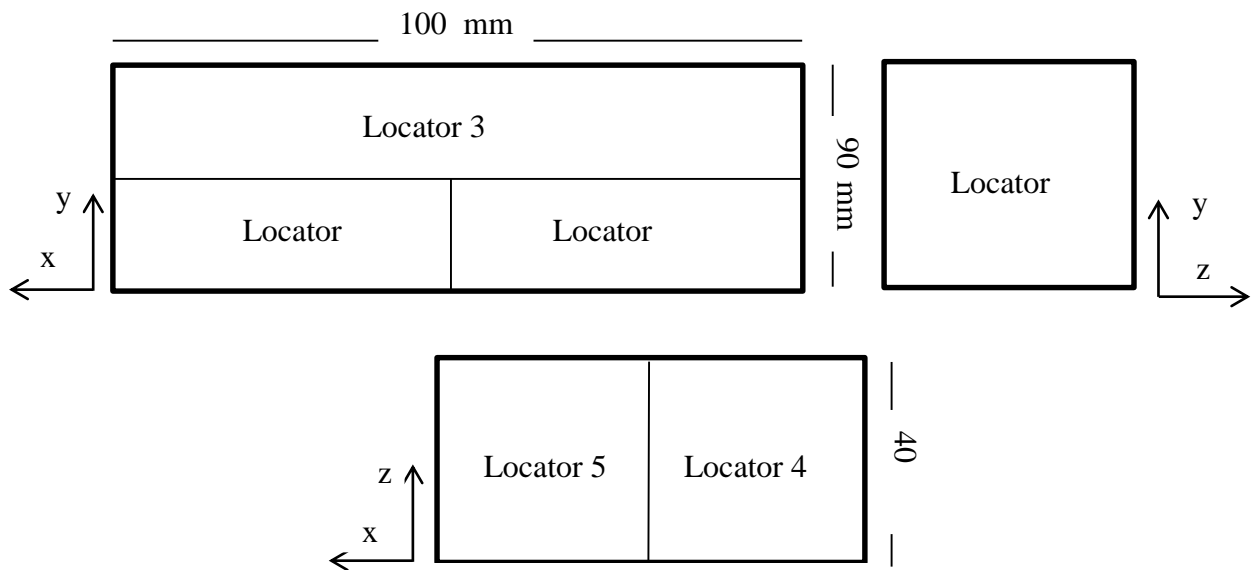


Figure 27 Ranges of Locators defined according to figure 19

Table 3 Range of each Locator in (mm)

Locator No	x-range	y-range	z-range
1	0-50	45-90	-
2	50-100	45-90	-
3	0-100	0-45	-
4	0-50	-	0-40
5	50-100	-	0-40
6	-	0-90	0-40

Table 4 A sample of placement of 6-locators

Locator No	x (mm)	y (mm)	z (mm)
1	40	60	-
2	80	70	-
3	60	40	-
4	35	-	24
5	77	-	32
6	-	60	30

The algorithm generates 156 random population matrices. These sets are represented in matrix form of order 6x3, which shows six locators moving in x, y and z-axis in their region respectively. These matrices are input for fitness function evaluation. Among them, one possible individual from population is shown in table 4; keeping in view the range defined for every locator in table 3. These matrices are input for the Mechanical model which in return gives us error of point of interest. The fitness function to evaluate the fitness of individuals is given by equation.13.

We have to choose 14 best matrices out of 156 set of matrices. Each matrix gives an output as the error (displacement vector) of point of interest; so we get 156 output each comprises of three

error displacements. Norm of these 156 outputs is calculated by using equation.13. As our aim is to minimize the error, we choose matrices corresponding to 14 minimum norms.

In this work, cross-over probability is taken as 0.85. For mutation, ‘P<sub>M</sub>’ is taken as 0.1667 and ‘P<sub>m</sub>’ is taken as 0.16. Following equation shows the binary conversion of example sample population of table 4 into 16-bit length.

$$\begin{bmatrix} 40 & 60 & 0 \\ 80 & 70 & 0 \\ 60 & 40 & 0 \\ 35 & 0 & 24 \\ 77 & 0 & 32 \\ 0 & 60 & 30 \end{bmatrix} = \begin{bmatrix} \{0000000000101000\} & \{0000000000111100\} & \{0000000000000000\} \\ \{0000000001010000\} & \{0000000001000110\} & \{0000000000000000\} \\ \{0000000000111100\} & \{0000000000101000\} & \{0000000000000000\} \\ \{0000000000100011\} & \{0000000000000000\} & \{0000000000011000\} \\ \{0000000001001101\} & \{0000000000000000\} & \{0000000000100000\} \\ \{0000000000000000\} & \{0000000000111100\} & \{0000000000011110\} \end{bmatrix}$$

## 4.2. Output Data

We took 40 iterations as stopping criteria which mean algorithm stops after 40 iterations. Table 5 shows the input parameters for this algorithm. The whole algorithm calculates the six degrees’ displacement of workpiece-baseplate assembly by using Langrangian formulation.

Table 6 shows the optimum position of six locators with minimum error. The convergence of GA for this case study is shown by graph in figure 28. Detailed flow chart for this case study is given in figure 29.

Table 5 GA Input Parameters

No of Iterations	40
Random Population Size	156
Cross-over Probability	0.85
Mutation Probability (P <sub>M</sub> )	0.1667
Mutation Probability (P <sub>m</sub> )	0.16

Table 6 Optimal Position of six locators with minimum error

Locator No	X (mm)	Y (mm)	Z (mm)
1	40.1	115.1	-
2	108.6	61.0	-
3	184.4	1.26	-
4	27.2	-	5.01
5	162.9	-	55.2
6	-	93.9	66.6
Minimum Error (m)		$8.78 \times 10^{-6}$	

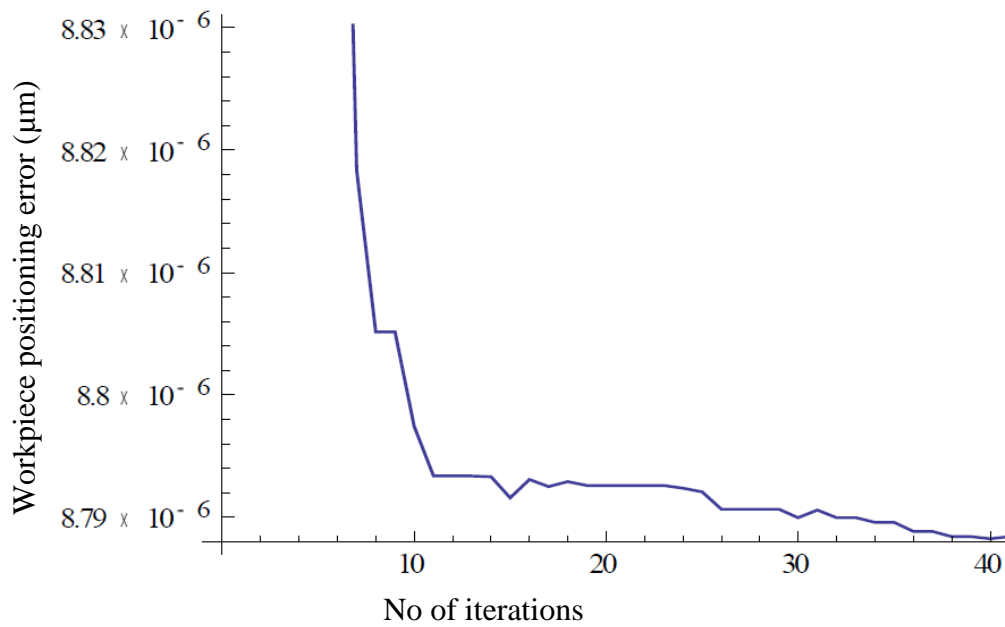


Figure 28 Convergence of GA ('No of generation' at abscissa and 'Positional error' at ordinate)

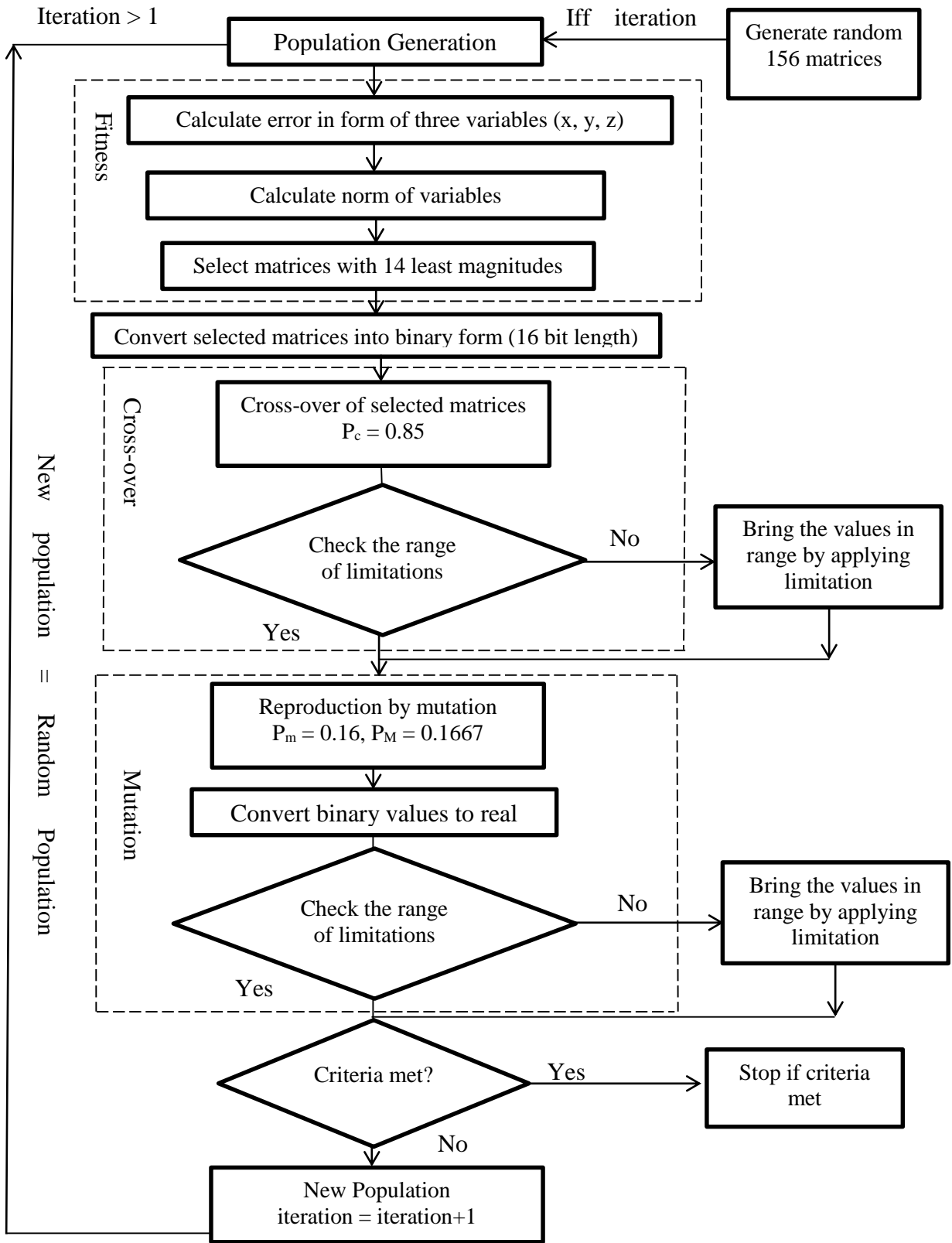


Figure 29 Detail flow chart for case study



### **4.3. Summary**

In this chapter, case study is presented which incorporate all the calculations which were discussed in chapter 3. Effect of GA parameters is discussed briefly. Data input is given with all the GA input parameters and output data shows the results of the developed GA. In next chapter single force results at different points on workpiece will be discussed.

## **CHAPTER 5: OPTIMIZATION OF LOCATOR'S PLACEMENT FOR MACHINING FORCE AT DIFFERENT POINTS**

In this chapter, optimization of locators' placement is performed by considering machining force at different points. In previous chapter, single point force was considered for the case study. This chapter discuss more complex problem, which is analogy of real time problem.

A machining force is considered, which follows a certain path on the workpiece creating a tool path for the machine tool. 3-2-1 fixturing system is used for this problem too. Setup is assumed with dimension of workpiece  $(x, y, z) = (170 \times 90 \times 40)$ mm. position of point P (which is center of gravity) in machine co-ordinate system is taken as  $(P_x, P_y, P_z) = (100, 80, 65)$ mm .

### **5.1. Problem Statement**

For this case study, we are assuming set-up for real time problem, which is much complex as compared to the simple case, explained in the previous chapter. Here we are assuming the machining force which follows a certain path on workpiece, just like machine tool follow path for machining. We can randomly choose different points on machine tool path on workpiece for optimization purpose. If optimized placement for each point is to be used, locators need to move during machining operation which will cause instability.

In this section, a single optimized placement of locators for all forces on different points on tool path is to be calculated. It is assumed that a cutting tool, moving rectangular workspace. Machining is simulated by applying same force at four vertices of rectangular workspace and calculating positioning error of workpiece at the point where machining force/torque is applied.

GA generates random population and for each random population, same force at four individual different points give four different displacements. Sum of all those individual points displacement is then minimized. After each iteration, sum of all individual different points is taken and then subjected to minimization.

Figure.30 shows the flow chart of fitness function of single machining force at different points. Machining force at four vertices of rectangle yields four individual error displacement. Among these four errors at four vertices, we choose the worst case (with maximum workpiece positioning error) to optimized.

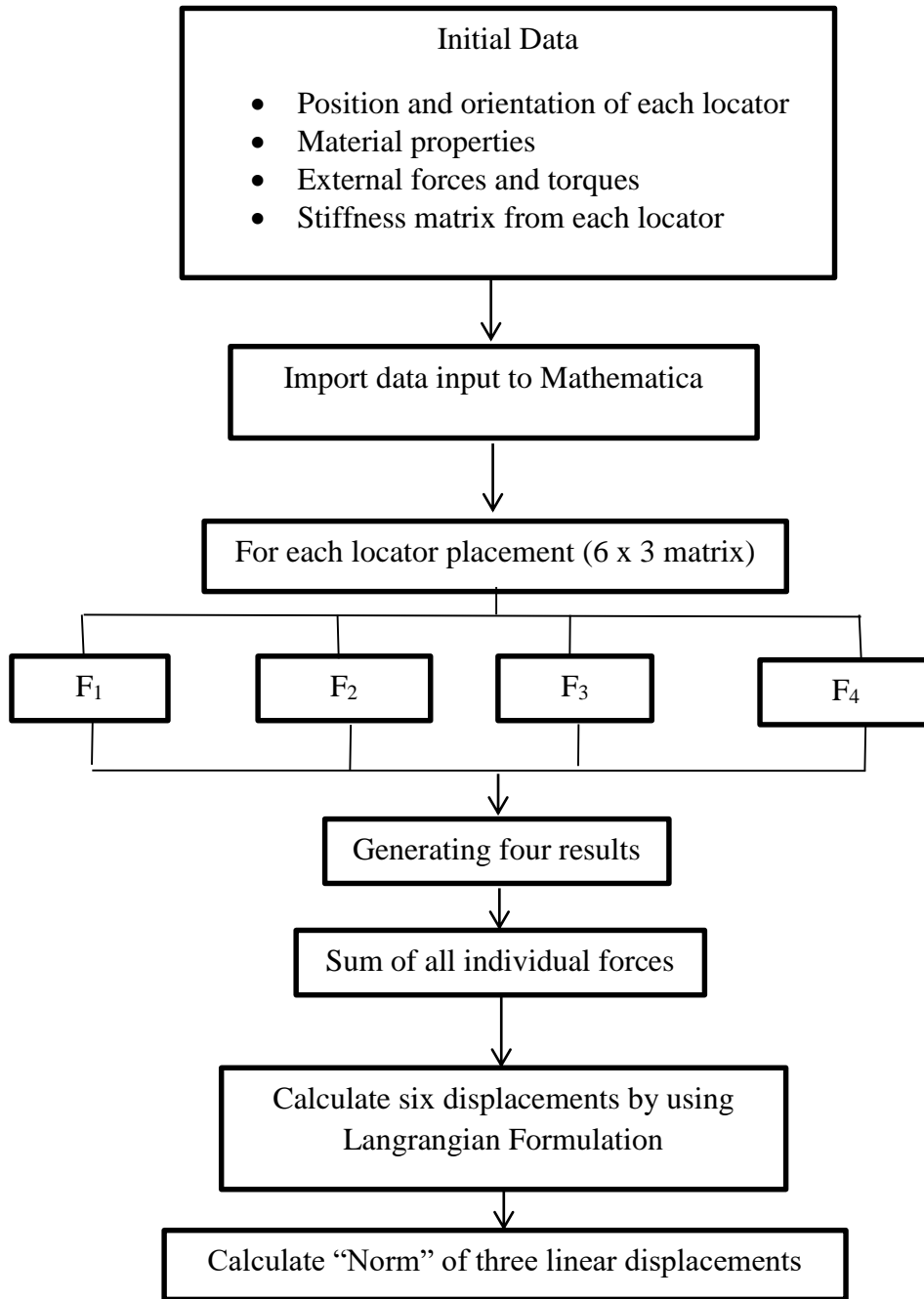


Figure 30 Flow Chart of Fitness Function of Machining Force at four Vertices of Rectangle

## 5.2. Input Data

Figure 31 shows the sketch of the problem statement. Simple rectangular workspace is assumed for this problem with workpiece co-ordinates (75, 35,40), (75, 55, 40), (95,55,40) and (95, 35, 40). We are calculating force at four vertices of rectangle individually as shown in figure 31. Machining tool follow the path F<sub>1</sub> through F<sub>4</sub>. It is assumed that tool start machining at point F<sub>1</sub> and follow the path through F<sub>2</sub>, then F<sub>3</sub>, then F<sub>4</sub> and back to vertice F<sub>1</sub>.

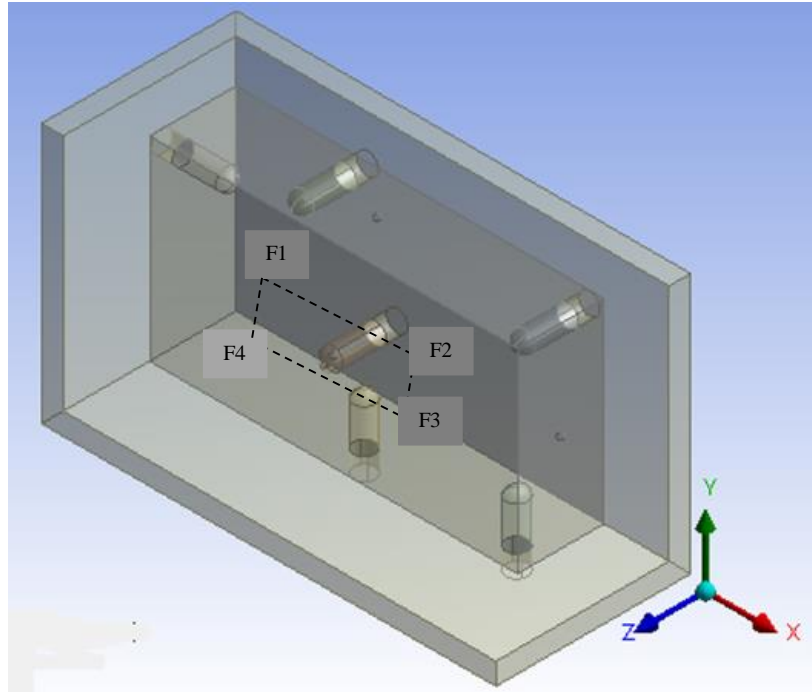


Figure 31 Machining Force follow the Tool Path

Optimal placement is calculated by locator's placement which gives minimum error at the point of applied force. Homogeneous transformation matrix (HTM) is used to calculate the positioning error ( $\Delta x$ ,  $\Delta y$ ,  $\Delta z$ ) of the point of interest on workpiece. Norm is then used to get overall error of the system. Following equation gives the generic positioning error for any point of interest on workpiece.

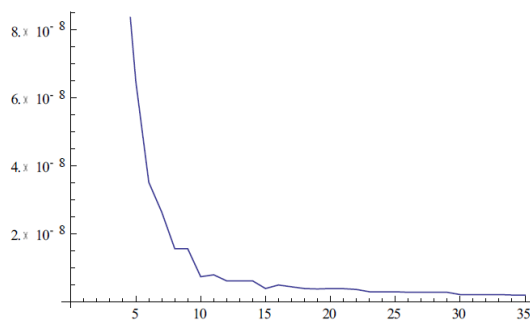
$$\begin{Bmatrix} \Delta x \\ \Delta y \\ \Delta z \\ 1 \end{Bmatrix}_i = [HTM]_i \begin{Bmatrix} X_f - X_P \\ Y_f - Y_P \\ Z_f - Z_P \\ 1 \end{Bmatrix}_i$$

Where  $(x_p \ y_p \ z_p)$  is the point of center of gravity of workpiece for this case and  $(x_{f_i} \ y_{f_i} \ z_{f_i})$  shows the co-ordinates of the point at which  $i$ th machining force is applied. Table 7 gives the locator range for each locator. Same GA parameters were used as in previous chapter. There is an optimized placement of locators for each force on each vertex. First of all, minimum work-piece positioning error is calculated separately on points  $F_1, F_2, F_3$  and  $F_4$ . At all individual four points, GA is converged which gives minimum workpiece positioning error of every point.

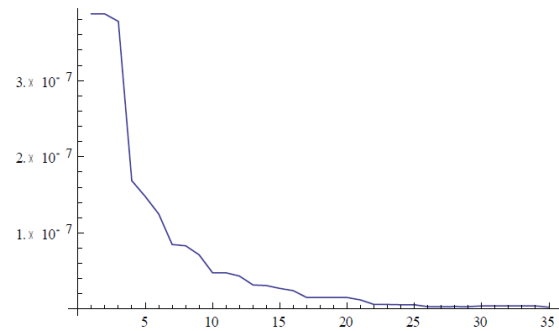
Figure 32 shows the convergence of four individual point forces, Figure 32(a) shows convergence of only  $F_1$ , figure 32(b) shows convergence of only  $F_2$ , figure 32(c) shows convergence of only  $F_3$  and figure 32(d) shows convergence of only  $F_4$ .

Table 7 Locator range for the Problem

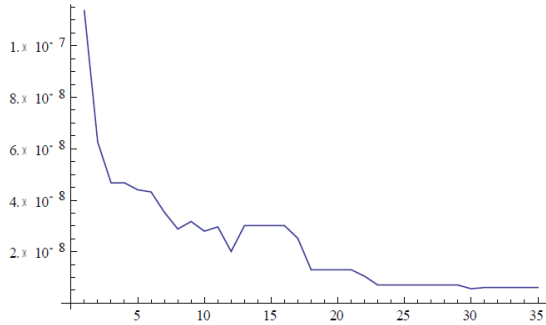
Locator No	x-range	y-range	z-range
1	0-50	45-90	-
2	50-100	45-90	-
3	0-100	0-45	-
4	0-50	-	0-40
5	50-100	-	0-40
6	-	0-90	0-40



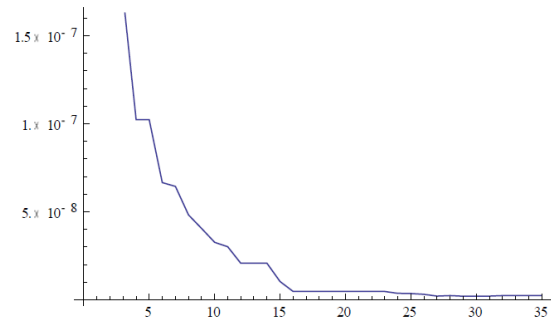
(a)



(b)



(c)



(d)

Figure 32 Converge of GA at four individual Points (a) Convergence of F1 (b)Convergence of F2 (c) Convergence of F3 (d) Convergence of F4

Table 8 gives the best optimal position of locators for machining force at individual point F1. Table 9 gives best position of locators for machining force at individual point F2. Table 10 gives the best optimal position of locators at individual point F3. Table 11 gives best optimal position of locators at individual point F4.

### 5.3. Output Data

After calculating positioning error at individual points, we calculate the minimum workpiece positioning error for whole workpiece by considering the four points at the same time. Now, we tried to find the best placement of locators at which the overall effect of force at these four points will be minimum.

It might be possible that the error may increase for one or two points as compared to the corresponding individual case; but for overall system it provides the locator placement at which positioning error is minimized.

Table 8 Best Optimal Locator's Placement at F<sub>1</sub>

Locator No	X (mm)	Y (mm)	Z (mm)
1	48.73	55.60	-
2	98.59	68.31	-
3	61.65	2965	-
4	33.54	-	23.88
5	96.71	-	19.57
6	-	75.00	20.69
Minimum Error (m)		1.95 x 10 <sup>-9</sup>	

Table 9 Best Optimal Locator's Placement at F<sub>2</sub>

Locator No	X (mm)	Y (mm)	Z (mm)
1	79.65	72.90	-
2	124.31	74.99	-
3	28.01	24.67	-
4	20.10	-	21.56
5	96.38	-	24.70
6	-	72.08	15.87
Minimum Error (m)		2.35 x 10 <sup>-9</sup>	

Table 10 Best Optimal Locator's Placement at F<sub>3</sub>

Locator No	X (mm)	Y (mm)	Z (mm)
1	29.69	52.69	-
2	85.51	56.92	-
3	17.77	34.37	-
4	37.90	-	18.02
5	86.13	-	21.35
6	-	70.75	23.17
Minimum Error (m)		6.04 x 10 <sup>-9</sup>	

Table 11 Best Optimal Locator's Placement at F<sub>4</sub>

Locator No	X (mm)	Y (mm)	Z (mm)
1	66.77	61.23	-
2	144.97	74.10	-
3	23.34	18.69	-
4	24.16	-	22.74
5	89.09	-	24.79
6	-	59.74	17.01
Minimum Error (m)		2.33 x 10 <sup>-9</sup>	

Figure 33 shows the convergence of GA for overall system and Figure 34 shows the minimum positioning error all individual points (F<sub>1</sub>, F<sub>2</sub>, F<sub>3</sub> and F<sub>4</sub>). Table 12 shows the placement of each locator and the corresponding minimum error of whole workpiece.



It is noticeable that the force individual points yield result in ( $10^{-9}$  m) but for overall system it gives result in ( $10^{-7}$  m) i.e. for whole system GA converges at larger value as compared to the force at individual points because for whole system GA accounts for four points for each iteration, whereas for individual point GA only account for force at only single point.

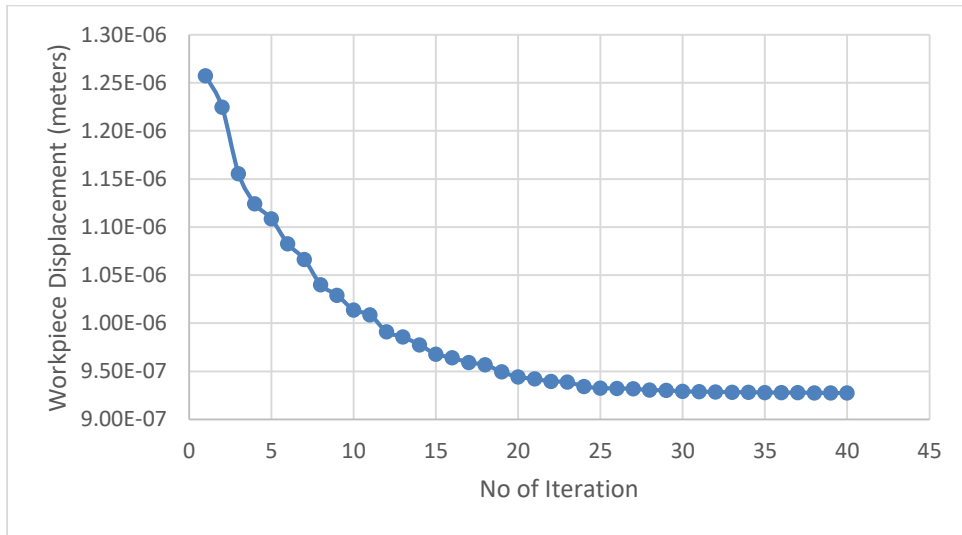


Figure 33 Overall convergence of GA by considering all points at the same time

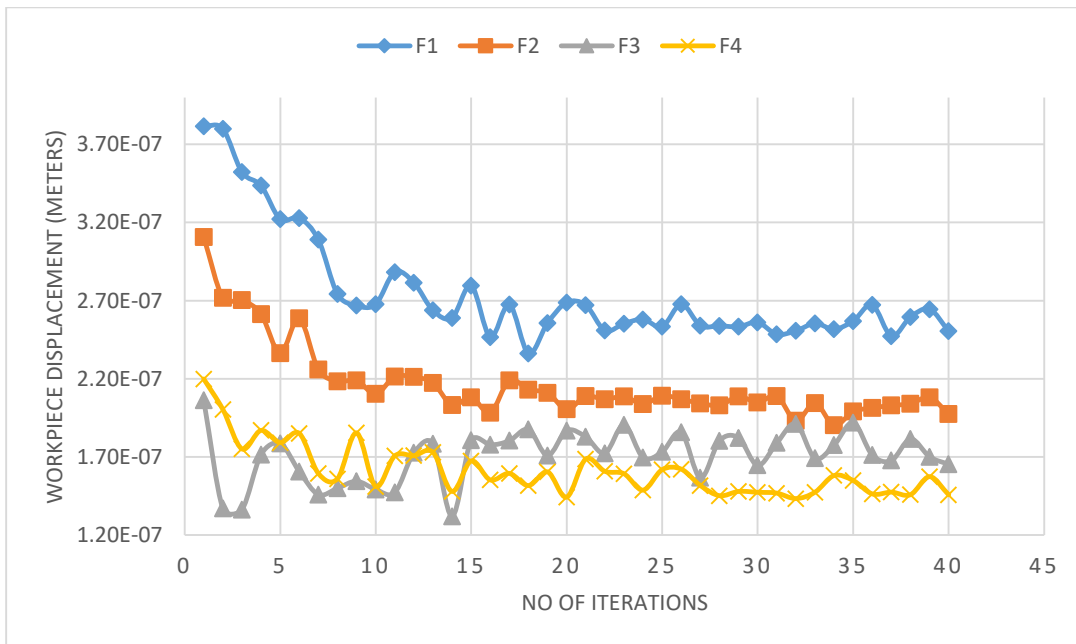


Figure 34 Convergence of four Individual Points

Table 12 Best Locator placement considering all points at same time

Locator No	X (mm)	Y (mm)	Z (mm)
1	17.77	75.0	-
2	154.59	45.01	-
3	31.57	15.00	-
4	15.00	-	24.99
5	91.52	-	24.95
6	-	74.94	19.29
Minimum Error (m)		$9.274 \times 10^{-7}$	

#### 5.4. Summary

In this chapter, validation case is discussed in which minimum workpiece positioning error is calculated by considering single force at different points on which machine tool follow the tool path. Results shows that GA is converged for complex problem too.

## **CHAPTER 6: CONCLUSION & DISCUSSION**

### **6.1. Conclusion and Discussion**

The scope of this research work includes precision manufacturing and quality product. The work consists of developing genetic algorithm to optimize the locator's position and placement in order to minimize the workpiece positioning error. The precise positioning of locators enables us to minimize the workpiece positioning error, thus creating excellent dimensional control in final product which is very much important for good quality. The developed algorithm uses six displacements associated with six locators by using Lagrangian formalization to minimize the error. These six displacements (three translational and three rotational) form the basis of our objective function. Norm of these displacements is indicator of workpiece positioning error. Less norm shows the less workpiece positioning error and vice versa. So we choose the lowest possible norm in 40 iterations. For this work expected precision for locator placement is 10 $\mu$ m.

#### **6.1.1. Application**

The proposed methodology of this research work can be applied to the work of Butt et al., (2012) to find the optimal position of locators for 'hip-bone prosthesis'. This work is significant for the domain which needs very good dimensional control and exceptional good quality (where we considered measurement up to  $\mu$ m). This work is applicable to emerging fields of engineering where position error of workpiece affects the final product. Beside this work is also applicable to aerospace industry, automotive industry and medical field (used for bone replacement i.e. knee replacement, hip-bone prosthesis) where quality of final product is very much important.

#### **6.1.2. Future Work**

There are certain assumptions we made in this research work to simplify it and to get clear vision of obtained output. These assumptions are limitations of this work. We might expand these

assumptions for future work. Following are some possible future work that can be expanded regarding this research work.

- We may include friction and deformation between locators and baseplate contact.
- Locators can be placed at angles with respect to workpiece i.e. they are not restricted with axial movement only.
- Other evolutionary techniques like ant colony optimization, particle swarm optimization can also apply to optimize the locator's position in order to compare the efficiency of both algorithms.

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