#### **Dynamic Online Fuzzy Modelling of Nonlinear Systems**

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#### THESIS

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# **Executive Summary**

Modelling of complex non-linear systems using the process data is a challenging task. A lot of nonlinear techniques are used in systems modeling, but fuzzy logic based techniques proved to be very effective and efficient especially when the dynamics of the plant or system are complex and totally unknown. This work presents the fuzzy modelling of a cooling coil system using the input-output process data. The modelling is carried out in three steps. First, offline identification, in which, all the measured input-output data is processed at once to model the system. Second, the online identification, in which, the measured input-output data is fed sample by sample. Third, online dynamic modeling, in which, the estimated output is fed back to the model along with the measured input-output data.

Fuzzy modelling with self-learning evolving structure is also very efficient modelling technique and is of prime focus nowadays. This work also contains the modelling of cooling coil system using Evolving Fuzzy Models (EFMs).

Simulation results have been presented, respectively, which demonstrate the efficiency of evolving fuzzy models, and online dynamic modelling

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# List of Symbols and Abbreviations

## SYMBOLS

μ	Membership function
у	Estimated system output
x	System input
a	Consequent parameters
$\wedge$	And operator
Υ	Measured system output
ζ	Consequent parameters vector
$\zeta_i$	$i^{th}$ rule-center in evolving fuzzy models
$\Lambda_i$	$i^{th}$ rule potential
ρ	Spatial distance
$ heta_i$	$i^{th}$ rule-consequent in evolving fuzzy models
σ	Training data quality
К	Forgetting factor

#### ABBREVIATIONS

FLC	Fuzzy Logic Controller
TS	Takagi-Sugeno
EFM	Evolving Fuzzy Model
RLS	Recursive Least-Square
GA	Genetic Algorithm
MIMO	Multiple Input Multiple Output
SISO	Single Input Single Output
MbPC	Model-based Predictive Control
MAC	Model based Adaptive Control
eMAC	Evolving Model based Adaptive Control
eR	Evolving fuzzy Rule-base
eTS	Evolving Takagi-Sugeno
FTC	Fault Tolerant Control
MPC	Model Predictive Control
VAV	Variable Air Volume
ADC	Analog to Digital Converter
DAC	Digital to Analog Converter
MISO	Multiple Input Single Output
FRM	Fuzzy Relational Model
RMSE	Root-Mean-Square Error



### **INTRODUCTION**

#### **1.1 Motivation**

Models of real systems are of fundamental importance in virtually all disciplines. Models are useful for system analysis, i.e., for gaining a better understanding of the dynamics of the system. Models are used to predict or simulate a system's behavior. In engineering, models are required for the design of new processes and for the analysis of existing processes. Advanced techniques for the design of controllers, optimization, supervision, prediction, analysis, simulation, fault detection and diagnosis components are also based on models of processes or the known dynamics of the system.

Since the quality of the model typically determines an upper bound on the quality of the final solution, modeling is often the bottleneck in the designing and development of the whole system or a process. As a consequence, a strong demand for advanced modeling and identification schemes arises [24, Sec1.1, Pg. 18].

There are a number of modelling techniques for complex, nonlinear systems like classical methods, fuzzy modelling, evolving structures based modelling like artificial neural networks (ANNs), evolving fuzzy models and etc. Fuzzy rule-based systems proved to be a convenient tool for modeling complex systems. This is due to their capacity to capture their typical imprecision, which makes classical methods inefficient. At present, Fuzzy Logic Controllers (FLC) is considered one of the most important applications of fuzzy rule-based systems.

However, the learning process of proper rules for a given problem is still an important research area. Therefore, different solutions for this problem have been developed, many of them based on evolving fuzzy models [25].

An evolving fuzzy model (EFM) based on a Takagi – Sugeno (TS) fuzzy model is a very good choice for modeling complex nonlinear systems, when the time required to collect a complete set of training data is too long for the model to be identified. The proposed learning scheme is computationally efficient, undemanding and is viable for use in model-based self-learning controllers. The ability of the EFM is to evolve the rule-base, efficiently, to model the behavior of the system in new regions of the operating space. The EFM approach generates an accurate model with relatively few rules in a computationally undemanding and efficient manner, even if the data are noisy and incomplete [11].

System identification or modelling has always been very challenging to me since I started studying control systems courses during graduation, therefore, it is the most significant factor of motivation in choosing the work area.

#### **1.2 Objective of the Project**

The purpose of this work is to model the cooling coil system using static, dynamic and evolving fuzzy modeling in MATLAB/Simulink. The work is carried out in the following parts:

- Offline static fuzzy model identification;
- Online static fuzzy model identification;
- Online dynamic fuzzy model identification;
- Evolving fuzzy model identification;

#### **1.3 Project Organization**

The project is organized into six chapters described as follows;

**Chapter 1**: Describes the motivation for the modeling of cooling coil system, objectives of the project and its organization.

**Chapter 2:** Includes the major relevant and state-of-the-art research work studied in-depth to carry out this project.

**Chapter 3:** Discusses the nonlinear cooling coil system. In this chapter, the offline static model of the system was developed using TS fuzzy rule-based modeling technique with recursive least

square (RLS) scheme for parameter estimation. The results for the developed model were also presented in this chapter.

**Chapter 4:** In this chapter, the online static and online dynamic model of the cooling coil system were developed using fuzzy rule-based modeling technique with RLS scheme for parameter estimation. The results for the developed models, i.e. online static and online dynamic model were also discussed in this chapter.

**Chapter 5:** In this chapter, the evolving fuzzy models were discussed and the evolving fuzzy model of the cooling coil was developed with simulation results.

**Chapter 6:** In this chapter the findings of this research are discussed and future recommendations are suggested so that further improvements can be made to the results obtained.



## LITERATURE REVIEW

#### **2.1 Introduction**

This chapter will cover literature review for the subject work focusing on Takagi-Sugeno based fuzzy modeling techniques and fuzzy relational modeling based evolving fuzzy modeling technique.

#### 2.2 Major Research Studies

#### 2.2.1 Fuzzy Identification of Systems and Its Application to Modeling and Control [1]

This work presents a mathematical fuzzy model of a system. Generally, the modelling by inputoutput data is categorized by two approaches; one is a mathematical modelling of a system using first principle method and the other is the system identification. A mathematical model requires simplicity and generality for control applications. The fuzzy identification is quite simple in comparison with the other approaches. In fuzzy modelling, the input space is partitioned and linear input-output relation is tried to form. The output of fuzzy model is obtained by combining the inferences that were applied to an input.

This work presents the identification of a system using its input-output training data set. The model identification is divided into two parts; the structure identification and the parameter identification. The structures are usually kept fixed in this work, therefore, omitting the structure identification, while focusing on parameter identification.

Finally, the two examples from industrial processes are taken in this work. One is a water cleaning process in which fuzzy controller is developed by fuzzifying the operator's control actions. The other example is a converter in the steel-making process which is fuzzily modeled and controlled.

#### 2.2.2 Recursive Least-Squares Method with Membership Functions [2]

Traditional linear regression is widely used for parameter estimation. But sometimes linear regression seems to be not sufficient to correlate a process accurately. In this work, membership functions and recursive least-squares algorithm are combined to overcome the shortcomings of traditional linear regression by dividing the model into multiple linear sub-models. The consequent parameters of the sub-models can be tuned recursively to achieve good performance indicators, excellent fit and approximation. This combination can be used for nonlinear process to achieve good performance which was not possible with the traditional linear regression.

#### 2.2.3 Multivariable GA-Based Identification of TS Fuzzy Models: MIMO Distillation Column Model Case Study [6]

Fuzzy modelling is an efficient tool for the approximation and estimation of nonlinear dynamical systems using measured input-output data. This approach is popular because it utilizes heuristic knowledge to develop quantitative model of complex nonlinear systems.

Takagi-Sugeno (TS) models have got a lot of attention amongst other fuzzy logic based modelling techniques in nonlinear system identification. These models provide promising results because of using mathematical functions as rule consequents.

TS model is developed in two steps – firstly, the fuzzy sets (membership functions) in the rule antecedent are determined, mostly, manually with expert knowledge or by some data-driven technique. The evaluation, of antecedent fuzzy sets seems to be bottleneck in the identification process, which is a nonlinear optimization problem. Then in the second step, the mathematical consequent parameters are approximated and estimated.

The accuracy, of the developed model based on TS method, depends upon the fuzzy sets and consequent mathematical functions. The accuracy of the model can be improved by tuning the fuzzy set and the consequent functions to minimize the mean squared error between the model output and the measured/actual output.

In this work, a multi-input and multi-output (MIMO) dynamical system is considered to be identified by using nonlinear identification approach by combining Genetic Algorithms (GA) and Takagi-Sugeno fuzzy system. In this approach, the GA is used for tuning the antecedent fuzzy sets and recursive least squares (RLS) for consequent linear sub-model parts of the defined fuzzy rules.

#### 2.2.4 Identification of Fuzzy Models for A Glass Furnace Process [26]

Linear identification is a subject of intense research, even if it is well-established and saturated. There are three types of models are used i.e. white-box models, gray-box models and black-box models depending upon the knowledge of the nonlinearities present in the measured signals. In white-box models, the nonlinearities are well-defined, in gray-box models, the nonlinearities are partially-defined and in black-box models, there is no physical insight of the nonlinearities.

When there's no clear picture of which nonlinear combination of measured signals to incorporate in the regressors, then the neural networks and fuzzy models are of high priority to the researchers and engineers.

In this work, the authors have developed a nonlinear model of the glass furnace processing plant with gas as an input and throat temperature as a output by using Takagi-Sugeno fuzzy system and recursive least square (RLS) technique for consequent parameter tuning.

# 2.2.5 Fuzzy Clustering for the Identification of Takagi-Sugeno Fuzzy Models of MIMO Dynamical Systems [4]

Fuzzy systems are widely used to model complex nonlinear dynamical systems, especially, single input single output (SISO) systems. Modelling nonlinear multi-input multi-output (MIMO) processes is a challenging task. This work presents the Takagi-Sugeno based fuzzy modelling to model MIMO dynamical systems with fuzzy clustering approach. This new technique is used to model a high-purity distillation column as an example and results are compared by linear and other fuzzy clustering based techniques.

#### 2.2.6 On-line identification of computationally undemanding evolving fuzzy models [11]

This work presents the modelling nonlinear dynamic systems using Evolving Fuzzy Modelling (EFM) approach in which rule-base is evolved by an incremental learning method. EFMs proved to be very efficient and good technique for modelling complex nonlinear systems when time to collect set of training data is too short. The learning technique is computationally efficient and undemanding, useful for model based self-learning controllers. The authors presented three examples of the EFM – first, is of modelling a simple nonlinear system, the second is of modelling cooling-coil in air conditioning system and the third one is of application of EFM in model-based predictive control (MbPC). The results substantiate the efficiency and ability of EFMs to evolve rule-base to extrapolate the behavior of the system in new operating regions. In

all the example cases, the EFMs proved to generate accurate models with relatively few rules with less computation even if the measured data set is noisy and incomplete.

#### 2.2.7 An Evolving Fuzzy Model for Embedded Applications [12]

This work presents an evolving fuzzy model (EFM) technique to model the nonlinear dynamic systems with incremental self-learning method to construct rule-base. The EFM, based on TS type fuzzy model with fixed consequents, is very good approach for modeling complex nonlinear dynamic systems. EFMs proved to be very efficient and good technique for modelling complex nonlinear systems when time to collect the set of training data is very short. The learning technique is computationally efficient and is useful for model-based self-learning controllers. The EFMs proved to evolve the rule-base for extrapolating the behavior of the system in new operating regions and to generate accurate models with relatively few rules with less computation even if the measured data set is noisy and incomplete.

#### 2.2.8 Evolving Fuzzy Model-based Adaptive Control [13]

This work presents the adaptive controller based on evolving fuzzy model which author calls it evolving fuzzy model-based adaptive controller (eMAC) which is useful for nonlinear uncertain systems. There are two fuzzy models used in the prediction of the behavior of the system – one model is evolved using Takagi-Sugeno (TS) based fuzzy model which is learnt online from the measured data and the other is TS fuzzy model identified offline from the general linear model of the system. The technique is applied to nonlinear dynamic system which has significant time delay and simulation results substantiate the improvement in the performance of control system incorporating eMAC. The proposed technique is tested on cooling coil of air-handling unit.

#### 2.2.9 Identification of Evolving Fuzzy Rule-Based Models [14]

An evolving fuzzy rule-based (eR) modelling approach is presented in this work. In this approach, the non-iterative update of rule-base structure and parameters is carried out by unsupervised learning. The rule-base evolves by more adding informative rules judging from the potential of the data. The rule-base is updated by new more informative data instead of retraining the whole structure. The adapting and evolving nature of these models, along with transparent and compact form of fuzzy rules, makes them a very good choice for identification and control of complex nonlinear dynamic processes in comparison to neural networks. The author has demonstrated the results by applying the proposed technique on an example system of airconditioning system, which illustrates the capability, viability and efficiency of the approach.

This approach can be used in other benchmark problems of adaptive nonlinear control, robotics, performance analysis, fault detection, prediction and behavior modelling.

#### 2.2.10 An Approach to Online Identification of Takagi-Sugeno Fuzzy Models [15]

This work presents the modelling technique in which the Takagi-Sugeno (TS) model structures and parameters are updated recursively by combining supervised and unsupervised learning. The rule-base and consequent parameters of the TS model evolve continuously by adding new rules when new data with higher potential is available. In this approach, the developed model of the system is called evolving Takagi-Sugeno (ETS) model. The adapting and evolving nature of these models, along with transparent and compact form of fuzzy rules, makes them a very good candidate for identification and control of complex nonlinear dynamic processes in comparison to neural networks. The author has demonstrated the results by applying the proposed technique on an example system of air-conditioning system, which illustrates the capability, viability and efficiency of the approach. This approach can be used in other benchmark problems of adaptive nonlinear control, robotics, performance analysis, fault detection, prediction and behavior modelling.

#### 2.2.11 On-line Identification of MIMO Evolving Takagi-Sugeno Fuzzy Models [16]

Online identification based evolving Takagi-Sugeno (ETS) fuzzy model has been recently introduced and proved to be efficient modelling technique for designing flexible system models having even less a priori information. Mostly, the research in this area is limited to single input single output (SISO) systems. In this work, the approach is extended to the multiple-input multiple-output (MIMO) systems. In this case, the online unsupervised learning of fuzzy rule-base antecedents by recursive, non-iterative clustering and supervised estimation of the linear sub-models parameters by Kalman-filtering are also extended. In this approach, the radius of circle of influence of each fuzzy rule is a vector instead of a scalar as was in the ETS approach for SISO systems so that different operating regions of data space can be covered by each input. Like in original ETS, in MIMO system modelling, the fuzzy rule-base and the parameters of the fuzzy model are evolved by adding new rules or by modifying the existing rules. The author has demonstrated some simulation results using well-known benchmark. The author proposes the further applications of MIMO ETS to predictive modeling of the speech spectrum magnitude and classification of multi-channel source modulation.

#### 2.2.12 Fault Tolerant Control Using Evolving Fuzzy Modeling [17]

In this work the evolving fuzzy modelling (EFM) based fault-tolerant control (FTC) is presented. Usually, fault-tolerant control is carried out in two steps – first, fault detection and second, fault accommodation. In fault accommodation, evolving TS fuzzy models are used while in fault detection; fuzzy model-based approaches are used. Information from fault detection model is fed into the fault accommodation model using model predictive control (MPC) scheme. For the process liable to faults, the evolving fuzzy modelling approach improves the control performance by continuously evaluating the control performance and online clustering. The proposed approach has been applied to an example system of distillation column process by incorporating two simulated faults i.e. load process fault and the change in the heating temperature. The evolving fuzzy modelling based fault tolerant control accommodated the simulated faults.

# 2.2.13 Online Identification of Evolved Takagi Sugeno Fuzzy Model for CO<sub>2</sub> Sequestration Process [18]

Greenhouse gas emissions from large local industries are causing the problems increasing nowadays with increasing number of industries. Carbon capture and storage proved to be very efficient in reducing these emissions. Among different methods, Carbon dioxide (CO<sub>2</sub>) sequestration has received a lot of attention because of its long term storage and low cost. This sequestration requires over-monitoring of critical parameters which influence the CO<sub>2</sub> sequestration performance due to over-pressurization and cap simulator (ECLIPSE-100) environment and evolving Takagi-Sugeno (ETS) based online fuzzy modelling. This scheme constructs an ETS based fuzzy rule-base structure recursively using recursive least squares (RLS) estimation to track reservoir dynamic changes in CO<sub>2</sub> sequestration.

Chapter 3

System Overview and Modeling Techniques

#### **3.1 Introduction**

In this chapter, the complex nonlinear plant i.e. cooling coil of air-conditioning system is described and discussed. Different techniques for modelling the cooling coil of air-conditioning system using its input-output data are proposed and static offline fuzzy modelling is discussed in detail showing the formulation of the technique, the governing equations, and the results obtained.

#### 3.2 Introduction and description of the Cooling Coil of an air-conditioning System

The schematic diagram for laboratory air-conditioning system is shown in Figure 1. In this airconditioning system the air from the environment enters into the system and is heated by the electric heater. The air is heated in order to simulate the seasonal variations and daytime temperature. The pump circulates the air into whole duct. Then after the pump, the air goes into the cooling coil subsystem to reduce the temperature of the air depending upon the position of the control valve. The air is heated again before going into VAV (Variable Air Volume) box. The purpose of the VAV box is to vary the airflow rate in order to keep the temperature of the coming air constant when in cooling mode.

The electric heater after the VAV box is used to warm the air to simulate the heat generated by the occupants of the room. The air is then mixed with the incoming outside air to save the energy while the rest of the air leaves the system as exhaust. The heating coil remains turned off throughout the experiment.

#### **3.2.1 Experimental Setup**

There are sensors in the test rig are for instrumentation of temperature and flow rate at different places on the test rig. These sensors are to monitor the output air temperature, input ambient temperature, cold water temperature, air mass flow rate and signal from the control valve. The control computer uses Analog-to-Digital converters (ADC) and Digital-to-Analog (DAC) to communicate with the sensors and actuators. The control computer, based on the knowledge from the sensors, commands the control valve of the cooling coil to control the mass flow rate throughout the coil. The value of the control signal ranges between 0 and 1, but it is converted to  $0 \le Vv \le 10$  Volts to feed to the actuator.



Figure 1: Schematic Diagram of air-conditioning unit

The sampling time of the control computer is 10 seconds. The control signal is pulse is with a period of 2 hours and value between 0.1 and 1.0. A control signal of 0.1 closes the control valve and signal of 1.0 opens it completely. The air mass flow rate is kept fixed at 0.29 kg/sec.

#### 3.2.2 Response of the cooling coil

The response of the cooling coil system is shown in Figure 2. The figure shows the four parameters which were monitored and logged. The four parameters are the control signal, ambient temperature, cold water temperature and the output air temperature. All these parameters were logged on May 20, 2008 for approximately 5 hours. As each sample is taken at every 10 seconds, therefore, there are 1764 samples of the data. This data will be used for training purposes in modelling the system.

As the data shows, the output air temperature varies between  $6^{0}$ C and  $15^{0}$ C, the cold water temperature varies between 50C and 80C, the ambient temperature varies between  $15^{0}$ C and  $16^{0}$ C and as previously described control signal varies between 0.1 and 1.0.



Figure 2: Response of the cooling coil system

As it can be seen from the data that when the control valve is fully open, the output air temperature falls steeply but takes long time to achieve the steady state value and when the valve is fully closed, the temperature rises quickly.

The cold water temperature also changes when changes are applied to the valve. When the valve is opened, the cold water temperature rises and when the valve is closed, the cold water temperature falls below by few degrees. This change is due to the heat exchange between the cold water and the warm inlet air. When the valve is opened, the exchange of heat between the cold water and the inlet air takes place and the temperature of the cold water rises. When the valve is closed, the temperature of cold water falls because there's no heat exchange takes place. The cold water returns in the coils with the same temperature [10].

The upward and downward trends of the output air temperature shows that the time constant of the system is different at upward transition and downward transition, making the system highly nonlinear in dynamics.

#### **3.3 Modelling the cooling coil system**

As the data is available for the plant i.e. cooling coil system and will be used to model the system. The data shows that the system is multiple input single output (MISO) system. The inputs of the system are command signal to control valve for cold water inlet, the ambient temperature and the cold water temperature. The output of the system is temperature of the outlet air.

In this work, the cooling coil system is modeled using four techniques. These techniques are defined as follows;

#### 3.3.1 Static offline fuzzy modeling

In this fuzzy modeling technique, the model of the system is developed offline i.e. the modeling system (could be any computer, or embedded system) is disconnected from the system to be modeled, cooling coil system in this case. All the input-output is processed and used in the modeling technique at once. This type of modeling technique is more useful when forecasting of statistical systems is to be done or for the systems which have enough time between collecting the data and modeling the system.

The advantage of this technique is that all the nonlinearities, noises, and uncertainties are processed and modeled and makes the model a very good fit for whole span of the each of the inputs.

This technique is computationally demanding and requires a lot of memory to process the data, and modeling is slower one but the model output copies the actual output within first few samples.

The term static tells that the there's no delay in the system or no model output feedback is provided to the model. There are only three inputs used in the modeling technique. All the fuzzy basics of the modeling will be covered in detail in this chapter.

#### 3.3.2 Static online fuzzy modeling

In this fuzzy modeling technique, the model of the system is developed online i.e. the modeling system (computer or embedded system) is connected to the actual system to be modeled. Each input and output of the actual system is also fed to the modeling computer or system. Therefore, modeling is carried out during the run. The modeling computer processes the single sample of the training data at each instant and identification is carried out with the passage of time. Initially, the error between the actual output and the model output is greater but as the more data is available with time, the error decreases and the actual and the model output get closer to each other.

As defined in the earlier subsection, this modeling technique is also static, i.e. no feedback to or delays in the model. There are only three inputs to the model.

This technique is computationally undemanding as it processes only one sample at each instant but the identification is done during the run and initial errors are larger.

All the fuzzy basics of this modeling technique will be covered in the next chapter.

#### 3.3.3 Dynamic online fuzzy modeling

This modeling technique is similar to the previous one i.e. static online fuzzy modeling with a difference that there's some delay in the model. The modeling is carried out during the run but with one extra input i.e. model output with some delay. This technique has the edge to have very less error in the actual and the model output in comparison to the static one. All the basics of the technique are similar to the static one but with a difference of one extra input to the system.

The fuzzy basics and the governing equations with obtained results will be presented in the next chapter.

#### 3.3.4 Evolving fuzzy modeling

In this technique, the modeling is carried out online i.e. the inputs and output of the actual system are also fed to the model. The processing is done on each sample of the training data. The rulebase of the model is evolved or developed during the run and region of influence. If the number of rules is equal to the user defined number of rules, then the rules are deleted or modified on the basis of the potential of the data. Once the rule-base is formed, the consequent parameters are estimated using recursive-least-square (RLS) method or any other suitable estimation technique.

This technique is clustering based and highly undemanding computation wise. Once, the system is modeled with this technique, can handle or extrapolate the system behavior with minimum divergence.

This area of research is of prime focus, nowadays, due to its less computation and highly accurate results.

#### 3.4 Static offline fuzzy modeling

As previously mentioned, the static offline fuzzy modeling technique is used when the modeling system or computer is disconnected from the actual system to be modeled. Once the input-output data is collected and logged by putting sensors at the inputs and outputs, and then this data is used in modeling the system. The fuzzy basics, the governing equations, the estimation techniques and the results are discussed in the following lines;

The model identification roots back to the work of Tomohiro Takagi, Michio Sugeno and their co-workers. The TS type systems are usually based on the rule-base with fuzzy antecedents and functional consequent parameters in contradiction to that of Mamdani systems or models with fuzzy consequents. The TS models take fuzzified inputs and give crisp output which lessens the computational cost, effort and time with easiness to implement while the Mamdani models give crisp value at the output after defuzzifying the output. As the dynamic model should be efficient and quick enough to trace the outputs of the actual plant, therefore, TS type models are used in dynamic modeling.

Usually, the TS models have the rule-base defined as:

 $R_i$ : IF  $x_1$  is  $\mu_1^i$  and  $x_2$  is  $\mu_2^i$  and  $x_3$  is  $\mu_3^i$  and... and  $x_n$  is  $\mu_n^i$  THEN  $y_i = a_0^i + a_1^i x_1 + a_2^i x_2 + a_3^i x_3 + ... + a_n^i x_n$ 

Where ' $R_i$ ' is the *i*<sup>th</sup> fuzzy rule and 'x' is the input vector  $[x_1, x_2,...,x_n]^T$ ; ' $\mu_n^{i}$ ' is the antecedent fuzzy sets or the premise variables for the inputs; ' $y_i$ ' is the output of the *i*<sup>th</sup> linear subsystem; ' $a_j^{i}$ ' are the consequent parameters of the each subsystem, while j = 0, 1, 2, ..., n.

The model identification or fuzzy modelling is carried out in the following steps;

#### 3.4.1 Selection of fuzzy sets

In this step, the fuzzy sets, their type and the number of fuzzy sets are selected, depending upon the system too be modeled, for the universe of discourse of each input. The mostly used membership functions are triangular, Gaussian and binomial. Each input variable is normalized to make the model generalized and to obtain the better results, hence, the universe of discourse for each input variable would be 0~1.

#### 3.4.2 Rule-base

In this step, the rules-base or inference mechanism for the Takagi-Sugeno (TS) system are defined as [1];

 $R_1: IF x_1 is \mu_1^{-1} and x_2 is \mu_2^{-1} and \dots and x_n is \mu_n^{-1} THEN y = a_0^{-1} + a_1^{-1} x_1 + a_2^{-1} x_2 + a_3^{-1} x_3 + \dots + a_n^{-1} x_n$ 

$$R_{N}: IF x_{1} is \mu_{1}^{N} and x_{2} is \mu_{2}^{N} and \dots and x_{n} is \mu_{n}^{N} THEN y = a_{0}^{N} + a_{1}^{N} x_{1} + a_{2}^{N} x_{2} + a_{3}^{N} x_{3} + \dots + a_{n}^{N} x_{n}$$

And the fuzzified output 'y' for the input  $x = [x_1, x_2, ..., x_n]^T$  is obtained as;

$$y = \frac{\sum_{i=1}^{N} \left( [\mu_{1}^{i} \wedge \mu_{2}^{i} \wedge ... \wedge \mu_{n}^{i}] \times [a_{0}^{i} + a_{1}^{i}x_{1} + a_{2}^{i}x_{2} + ... + a_{n}^{i}x_{n}] \right)}{\sum_{i=1}^{N} [\mu_{1}^{i} \wedge \mu_{2}^{i} \wedge \mu_{3}^{i} \wedge ... \wedge \mu_{n}^{i}]}$$
(3.1)

Let,

$$\gamma_i = \frac{[\mu_1^i \wedge \mu_2^i \wedge \mu_3^i \wedge \dots \wedge \mu_n^i]}{\sum_{i=1}^N [\mu_1^i \wedge \mu_2^i \wedge \mu_3^i \wedge \dots \wedge \mu_n^i]}$$

Then,

$$y = \sum_{i=1}^{N} \gamma_i \times [a_0^i + a_1^i x_1 + a_2^i x_2 + \dots + a_n^i x_n]$$

If there are 'q' data samples for each input-output set, 'N' is the total number of rules and 'p' is the order of the TS model, ' $\chi[q \times N(p+1)]$ ' is the structural matrix, ' $\Upsilon[q \times 1]$ ' is the measured output vector and ' $\zeta[n(p+1) \times 1]$ ' is the vector of consequent parameters, which are to be estimated, then,

$$\begin{pmatrix} \gamma_1^1 & \dots & \gamma_N^1 & x_1^1 \gamma_1^1 & \dots & x_1^1 \gamma_N^1 & x_n^1 \gamma_1^1 & \dots & x_n^1 \gamma_N^1 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \dots & \vdots & \ddots & \vdots \\ \gamma_1^q & \dots & \gamma_N^q & x_1^q \gamma_1^q & \dots & x_1^q \gamma_N^q & x_n^q \gamma_1^q & \dots & x_n^q \gamma_N^q \end{pmatrix}$$

While,

$$\Upsilon = [y_1, y_2, \dots, y_q]^T$$

And,

$$\zeta = [a_0^1, a_0^2, a_0^3, \dots, a_0^N, a_1^1, a_1^2, a_1^3, \dots, a_1^N, \dots, a_n^1, a_n^2, a_n^3, \dots, a_n^N]$$

#### 3.4.3 Consequent parameters estimation

The consequent parameter vector ' $\zeta$ ' is the vector which is estimated to get the final model output and is estimated, offline or online, depending upon the estimation scheme based on the modeling application. In this case, the recursive least square (RLS) technique is used to estimate the consequent parameters. RLS works very well for both the offline and online system modelling, therefore, is a strong candidate in this case. The RLS technique is also called the stable-state Kalman filter [1].

The governing equation for the RLS technique is given by;

$$\zeta_{i+1} = \zeta_i + \delta_{i+1} \times \chi_{i+1}^T \times (\Upsilon_{i+1} - \chi_{i+1} \times \zeta_i)$$

$$\delta_{i+1} = \delta_i - \frac{\delta_i \times \chi_{i+1}^T \times \chi_{i+1} \times \delta_i}{1 + \chi_{i+1} \times \delta_i \times \chi_{i+1}^T}$$
(3.2)

While,

$$\delta_i = \alpha \times I_{N \times (p+1)}$$

And,

 $\zeta_i = O_{N \times (p+1)}$ 

' $\alpha$ ' is a large initializing number, *I* is the identity square matrix and  $O_m$  is a zeros matrix with dimensions  $[N \times (p+1)]$ .

#### 3.4.4 Static offline fuzzy modeling of Cooling Coil System

In modeling the cooling coil system, which is our main focus, the equations, mentioned in the above subsections, will be tailored accordingly.

In fuzzification of the inputs, the three inputs are normalized and fuzzified using triangular membership functions. There are two membership functions are chosen for each input.

In rule-base or inference mechanism, the Takagi-Sugeno (TS) zero-order system is chosen for modeling the cooling coil system. The zero-order system has the rule base of the form;

While 'n' is the total number of inputs, i.e. three in this case and 'N' is the total number of rules which will be eight  $(2 \times 2 \times 2)$  in this case, and hence, eight consequent parameters. The regression vector for the static model would be;

$$x(k) = [x_1(k), x_2(k), x_3(k)]$$

Therefore, there are eight parameters to be estimated by the model using recursive least square (RLS) technique using the equations (3.2)

#### 3.4.5 Results of Static offline fuzzy model of Cooling Coil System

By using the setup defined in the previous subsections, the modeled and the actual outputs are shown in Figure 3.



Figure 3: Actual output and the static offline fuzzy model output of the system

The absolute error between actual output and the estimated output of the model is shown in Figure 4. The error plot shows that there's a very small error between the actual output of the system and the estimated output of the model. The error at the transitions is somehow greater but in the steady state the error is very small. The number of membership functions can be increased or Guassian membership functions can be used to achieve more improvement.

The Figure 5 shows the root mean squared error (RMSE) between the actual output and the estimated output of the model.

Figure 6 shows the consequent parameters' convergence. The plot shows that almost all the parameters are converged. Some converged in few samples while some took more samples to get converged.



Figure 4: Error between actual and estimated output



Figure 5: Root Mean Squared Error for Fuzzy Offline Model of the System



Figure 6: Convergence of the consequent parameters

#### **3.5 Conclusion**

In this chapter, the introduction of the example system, i.e. cooling coil is discussed in detail with its dynamics. Then, static offline fuzzy modeling technique, with its mathematical foundations, is applied on the system. The results are also discussed in the last which shows the model is copying the output of the system, and giving the sense of the dynamics of the system.



# Static and Dynamic Online Modeling Approach

#### **4.1 Introduction**

In this chapter the two modeling technique has been described. One is static online modeling and other is dynamic online modeling. Both the techniques are Takagi-Sugeno based fuzzy modeling techniques. The dynamic online modeling has improved results as compared to static online modeling.

#### 4.2 Online fuzzy modeling

As described earlier, the online modeling is a modeling in which the measured input-output data set is fed into the modeling computer during the system running. The modelling computer gets the data sample one by one and processes it online to model the system. This online modeling is usually used in model based predictive control, adaptive control, fault diagnosis, health monitoring and forecasting.

The term static shows that there's no feedback or delay in the model. Therefore, the modeling computer takes only the inputs of the actual system. The pictorial representation of the static modeling is shown in Figure 7.

The term dynamic shows that there's feedback or delays in the model. Therefore the modeling computer takes model output (i.e. estimated output) with single or multiple delays along with the inputs of actual system. The pictorial presentation of the static modeling is shown in Figure 8.









The Takagi-Sugeno (TS) based static online fuzzy modeling has the same basics as in the static offline fuzzy modeling, except the consequent parameter estimation. The foundation of the modeling technique is given below;

#### 4.2.1 Selection of fuzzy sets

In this step, the fuzzy sets, their type and the number of fuzzy sets are selected, depending upon the system too be modeled, for the universe of discourse of each input. The mostly used membership functions are triangular, Gaussian and binomial. Each input variable is normalized to make the model generalized and to obtain the better results, hence, the universe of discourse for each input variable would be 0~1.

#### 4.2.2 Rule-base

In this step, the rules-base or the inference mechanism for the Takagi-Sugeno (TS) system is defined as [1];

 $R_1: IF x_1 is \mu_1^{\ l} and x_2 is \mu_2^{\ l} and \dots and x_n is \mu_n^{\ l} THEN y = a_0^{\ l} + a_1^{\ l} x_1 + a_2^{\ l} x_2 + a_3^{\ l} x_3 + \dots + a_n^{\ l} x_n$ 

•				•						•	
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 $R_N$ : IF  $x_1$  is  $\mu_1^N$  and  $x_2$  is  $\mu_2^N$  and ... and  $x_n$  is  $\mu_n^N$  THEN  $y = a_0^N + a_1^N x_1 + a_2^N x_2 + a_3^N x_3 + ... + a_n^N x_n$ 

And the fuzzified output 'y' for the input  $x = [x_1, x_2, ..., x_n]^T$  is obtained as;

$$y = \frac{\sum_{i=1}^{N} \left( [\mu_{1}^{i} \land \mu_{2}^{i} \land \dots \land \mu_{n}^{i}] \times [a_{0}^{i} + a_{1}^{i}x_{1} + a_{2}^{i}x_{2} + \dots + a_{n}^{i}x_{n}] \right)}{\sum_{i=1}^{N} [\mu_{1}^{i} \land \mu_{2}^{i} \land \mu_{3}^{i} \land \dots \land \mu_{n}^{i}]}$$
(4.1)

Let,

$$\gamma_i = \frac{\left[\mu_1^i \wedge \mu_2^i \wedge \mu_3^i \wedge \dots \wedge \mu_n^i\right]}{\sum_{i=1}^{N} \left[\mu_1^i \wedge \mu_2^i \wedge \mu_3^i \wedge \dots \wedge \mu_n^i\right]}$$

Then,

$$y = \sum_{i=1}^{N} \gamma_i \times [a_0^i + a_1^i x_1 + a_2^i x_2 + \dots + a_n^i x_n]$$

If there are 'q' data points for each input-output set, 'N' is the total number of rules and 'p' is the order of the TS model, ' $\chi[q \times N(p+1)]$ ' is the structural matrix, ' $\Upsilon[q \times 1]$ ' is the measured output vector and ' $\zeta[n(p+1) \times 1]$ ' is the vector of consequent parameters, which are to be estimated, then,

$$\begin{pmatrix} \gamma_1^1 & \dots & \gamma_N^1 & x_1^1 \gamma_1^1 & \dots & x_1^1 \gamma_N^1 & x_n^1 \gamma_1^1 & \dots & x_n^1 \gamma_N^1 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \dots & \vdots & \ddots & \vdots \\ \gamma_1^q & \dots & \gamma_N^q & x_1^q \gamma_1^q & \dots & x_1^q \gamma_N^q & x_n^q \gamma_1^q & \dots & x_n^q \gamma_N^q \end{pmatrix}$$

While,

$$\Upsilon = [y_1, y_2, \dots, y_q]^T$$

And,

$$\zeta = [a_0^1, a_0^2, a_0^3, \dots, a_0^N, a_1^1, a_1^2, a_1^3, \dots, a_1^N, \dots, a_n^1, a_n^2, a_n^3, \dots, a_n^N]$$

#### 4.2.3 Consequent parameters estimation

The consequent parameter vector ' $\zeta$ ' is the vector which is estimated to get the final model output and is estimated, offline or online, depending upon the estimation scheme based on the modeling application. In this case, the recursive least square (RLS) technique is used to estimate the consequent parameters. RLS works very well for both the offline and online system modelling, therefore, is a strong candidate in this case. The RLS technique is also called the stable-state Kalman filter [1].

The online consequent parameters, using RLS technique, are identified by [10];

$$\zeta_{i} = \zeta_{i-1} + \frac{\delta_{i-1} \times \chi_{i}^{T} \times (\Upsilon_{i} - \chi_{i} \times \zeta_{i-1})}{1 + \chi_{i} \times \delta_{i-1} \times \chi_{i}^{T}}$$

$$\delta_{i} = \delta_{i-1} - \frac{\delta_{i-1} \times \chi_{i}^{T} \times \chi_{i} \times \delta_{i-1}}{1 + \chi_{i} \times \delta_{i-1} \times \chi_{i}^{T}}$$

$$(4.2)$$

while,

$$\delta_i = \alpha \times I_{N \times (p+1)}$$

And,

$$\zeta_i = O_{N \times (p+1)}$$

'a' is a large initializing number, *I* is the identity square matrix and  $O_m$  is a zeros matrix with dimensions  $[N \times (p+1)]$ .

#### 4.3 Static online fuzzy modeling of Cooling Coil System

In modeling the cooling coil system, which is our main focus, the equations, mentioned in the above subsections, will be tailored accordingly.

In fuzzification of the inputs, the three inputs are normalized and fuzzified using triangular membership functions. There are two membership functions are chosen for each input.

In rule-base or inference mechanism, the Takagi-Sugeno (TS) zero-order system is chosen for modeling the cooling coil system. The zero-order system has the rule base of the form;

While 'n' is the total number of inputs, i.e. three in this case and 'N' is the total number of rules which will be eight  $(2 \times 2 \times 2)$  in this case, and hence, eight consequent parameters. The regression vector for the static model would be;

$$x(k) = [x_1(k), x_2(k), x_3(k)]$$

Therefore, there are eight parameters to be estimated by the model using recursive least square (RLS) technique using the equations (4.2).

#### 4.3.1 Results of Static online fuzzy model of Cooling Coil System

By using the setup defined in the previous sections, the modeled and the actual output are shown in Figure 9.



Actual and Estimated Plant Output

Figure 9: Output of actual and statically online modeled system

The absolute error between actual output and the estimated output of the model is shown in Figure 10. The error plot shows that there's a very small error between the actual output of the system and the estimated output of the model. The error at the transitions is somehow greater but in the steady state the error is very small. The number of membership functions can be increased or Guassian membership functions can be used to achieve more improvement.

Figure 11 shows the root mean squared error (RMSE) for the fuzzy online model of the system.

Figure 12 shows the consequent parameters' convergence. The plot shows that almost all the parameters are converged. Some converged in few samples while some took more samples to get converged.



Figure 10: Error between actual and estimated output



Figure 11: Root mean squared error (RMSE)



Figure 12: Convergence of the consequent parameters

#### 4.4 Dynamic online fuzzy modeling of Cooling Coil System

In modeling the cooling coil system, dynamically, the equations will be the same as in previous section, with one extra input to the modeling computer i.e. feedback from the model output.

In fuzzification of the inputs, all the inputs are normalized and fuzzified using triangular membership functions. There are two membership functions chosen for each input.

In rule-base or inference mechanism, the Takagi-Sugeno (TS) zero-order system is chosen for modeling the cooling coil system. The zero-order system has the rule base of the form;

While 'n' is the total number of inputs, i.e. three in this case and 'N' is the total number of rules which will be sixteen  $(2 \times 2 \times 2 \times 2)$  in this case, and hence, sixteen consequent parameters. The regression vector for the static model would be;

$$x(k) = [y(k-1), x_1(k), x_2(k), x_3(k)]$$

Therefore, there are eight parameters to be estimated by the model using recursive least square (RLS) technique using the equations (4.2)

#### 4.4.1 Results of dynamic online fuzzy model of Cooling Coil System

By using the setup defined in the previous sections, the estimated and the actual outputs of the model are shown in Figure 13.

The absolute error between actual output and the estimated output of the model is shown in Figure 14. The error plot shows that there's a very small error between the actual output of the system and the estimated output of the model. The error at the transitions is somehow greater but in the steady state the error is very small. The number of membership functions can be increased or Guassian membership functions can be used to achieve more improvement.



Figure 13: Actual and dynamically modeled output of the system



Figure 14: Error between actual output and dynamically modeled output

Figure 15 shows the root mean squared error for the dynamic online fuzzy model of the cooling coil system. This error is decreasing gradually but at the transitions there's spike in the error but the envelop shows that the spikes are decreasing gradually.

Figure 16 shows the consequent parameters' convergence. The plot shows that almost all the parameters are converged. Some converged in few samples while some took more samples to get converged.



Figure 15: Root mean squared error for the dynamic online fuzzy model of the system

#### 4.5 Conclusion

In this chapter two techniques have been discussed i.e. static online fuzzy modeling and dynamic online fuzzy modeling of cooling coil system. The difference is of the feedback of the output with delays back to the model in the later technique. Therefore, the number of input for the earlier technique is three and for later, it is four. Hence, having two membership functions for each input, the consequent parameters to be estimated are increased in the later technique with zeroth order Takagi-Sugeno (TS) model.



Figure 16: Convergence of parameters of dynamically modeled system

By comparing few factors i.e. RMSE of the prediction error, parameter convergence time and number of consequent parameters for the two techniques, one can understand the accuracy and efficiency of the techniques. The comparison of RMSE of both the techniques shows that the later technique is more accurate and efficient than the earlier one. The envelop or trend of the RMSE of the static online model shows that the RMSE is increasing gradually with increasing number of samples while the dynamic online is not, or is approximately constant.

The convergence rate of the static online modeling is fast as compared to the dynamic one but some parameters are not converged in the end, while in the dynamic, the convergence rate is slower but all the parameters are finally converged and hence giving more accurate results as compared to the static one, therefore, will be more likely to validate the model.

The number of consequent parameters, as discussed above, is more in the dynamic model than in the static model, therefore, somehow more computation in the dynamic model.

The above lines conclude that the dynamic model is more accurate, and validated than the staticmodelwithcomputationaltradeoff.

# Chapter 5

# **Evolving Fuzzy Model**

#### **5.1 Introduction**

In this chapter, the online evolving fuzzy modeling technique is presented to model the cooling coil system. Firstly, the evolving fuzzy modeling is introduced and its governing theory and equations are explained, then finally, its application to cooling coil system.

As its name implies, the model is fuzzy based but is evolving. The rule base is evolved at each sample when new information is available. The rule-base structure and parameters are updated non-iteratively by incremental unsupervised learning [14]. The rule-base evolves either by adding new rule or replacing the existing rule. If the user-defined maximum number of rules are achieved, the new rule replaces the rule with lowest potential otherwise the rule is added to the rule-base depending upon the new information or data is sufficiently outside the region of influence of the nearest rule in the rule-base. The scheme is computationally efficient, undemanding and fast [11]. The adaptive nature of this scheme in coordination with fuzzy rules makes this a favorable technique for modeling complex, nonlinear systems in comparison to neural network based techniques.

#### 5.2 Evolving Fuzzy Models

A number of online fuzzy modeling techniques have been presented in various forms and at various places. The modeling roots back to the Mamdani rules based fuzzy models [19]. Nowadays, Takagi-Sugeno (TS) fuzzy models and fuzzy relational models (FRMs) are widely used and explored in different modeling applications for model-based self-tuning controllers [20].

Different techniques have been evaluated for Mamdani based or Takagi-Sugeno based fuzzy modeling from the process data. The bottleneck of these techniques is computational demand and complexity. The computational demands and processing the FRM based models is very much less than that of T-S based models as FRMs do not require any online optimization and estimation of rule-bases or –confidences and require very less prior knowledge if uniformly spaced sets are used. However, FRMs are less accurate than Mamdani or TS based models and require a large set of input-output process data, hence large amount of memory, to achieve accurate models [21].

Recursive least-squares algorithms are used to estimate the rule consequents for online identified TS models [22, 15]. In this work, a computationally efficient method for achieving equivalent FRM results using TS model with fixed consequents is presented [11]. Online clustering technique, is used to find the location of the TS model's rule bases in the input space, which is very much similar to that of proposed by Angelov and his co-workers [14] with exception that the rule is created based on the measured distance between new data and the existing rule, rule deletion is based on the potential of the existing rule and the rule consequents are estimated using simple fuzzy identification techniques [11].

#### **5.2 Mathematical foundation for EFMs**

Evolving fuzzy models can be obtained by carrying out following procedure;

#### 5.2.1 Defining fuzzy rule-base

The fuzzy rule-base or rule-confidence ' $\chi$ ' is the zeroth order Takagi-Sugeno model as described in the previous chapters.

$$\Re_i : If(x_1 = X_{i1}) and ...and(x_j = X_{ij}) and ...and(x_n = X_{in}) Then(y = b_i)$$
(5.1)

Where,

 $\mathfrak{R}_i$ : is the *i*<sup>th</sup> rule i.e.  $i \in [1, 2, 3, ..., R]$ ,

And '*R*' is the number of user-defined rules.

' $x_{j}$ ' is the  $j^{th}$  input i.e. j = 1, 2, ..., n,

and 'n' is the total number of inputs,

'X<sub>ij</sub>' is the  $i^{th}$  fuzzy set for the  $j^{th}$  input,

And the center-value of  $X_{ij}$  is defined by  $x_{ij}$  such as

$$\mu_{X_{ij}}(x_{ij}) = 1$$

Where  $\mu_{X_{ij}}(x_{ij})$  is the membership function of the fuzzy set  $X_{ij}$ ,

And 'y' is the model output,

 $b_i$  is the consequent parameter of the  $i^{th}$  rule.

The center of each fuzzy set  $x_{ij}$  and the consequent parameters ' $b_i$ ' are used define the rule center ' $z_i$ ' for the *i*<sup>th</sup> rule. Thus,

$$\Xi_i = [x_{i1}^* \dots x_{ij}^* \dots x_{in}^* b_i]^T$$

The rule centers  $\zeta_i$  for each rule are stored in a rule-base matrix  $\Phi$  i.e.

$$\Phi = [\zeta_1 ... \zeta_i ... \zeta_R]^T$$

The training data vector at the  $k^{th}$  sampling instant  $\xi(k)$  contains the current input vector x(k) appended by measured system output y(k). Therefore,

$$\xi(k) = [x(k); y(k)]$$

where,

 $x(k) = [x_1(k)...x_i(k)...x_n(k)]^T$ 

#### 5.2.2 Online evolution of rule-base

The core function of the EFM is to choose the current data as the new rule or to replace an existing rule with the current data.an online clustering scheme does this based on the measure of the distance between current data and the nearest of the first rule. Rule potential is measured to choose which rule should be deleted if user-defined total number of rules is achieved.

#### 5.2.2.1 Rule-base initialization

The rule-base initializes the rule-center of first rule by first data. The potential of the first rule is set to zero because it could be the outlier and hence might not be representing the behavior of the system. Therefore,

 $\Xi_i = \xi(1)$  and  $\Lambda_1(1) = 0$ 

Where,

 $\Lambda$  is the rule potential.

#### 5.2.2.2 Rule creation

The spatial distance between the current data  $\xi(k)$  and the *i*<sup>th</sup> rule-center  $\zeta_i$  in the existing rulebase is given by the Euclidean distance where,

$$\rho_i(k) = \left\| \xi(k) - \Xi_i \right\| = \sqrt{\sum_{j=1}^{n+1} \left| \xi_j(k) - \zeta_{ij} \right|^2}$$
(5.2)

If the minimum distance  $\rho_{\psi}(k)$  ( $\psi$  is the index of the nearest/closest rule) is such that the current data are sufficiently far from the region of influence of the closest rule, then the new data are added s a new rule, otherwise ignored [11]. A threshold  $T_{Th}$ , fraction of r, is used to control the overlapping of the new rule to an existing rule. Thus the current data  $\xi(k)$  are appended as a new rules if  $\rho_{\psi}(k) > (T_{Th} \cdot r)$ . As described in the previous sub-subsection, the potential of the new rule is initialized as zero with the assumption that the current data might be an outlier.

#### 5.2.2.3 Rule deletion

Each rule has a corresponding rule-potential  $\Lambda_i(k)$ , based on the Cauchy form of spatial proximity [14], which shows how much a rule is representative of the observed data. The potential of the  $i^{th}$  rule  $\Lambda_i(k)$  is given by;

$$\Lambda_{i}(k) = 1 - \frac{1}{1 + \frac{1}{k} \cdot \sum_{l=1}^{k} \left( \frac{1}{\sum_{j=1}^{n+1} \left[ \xi_{j}(l) - \zeta_{ij} \right]^{2}} \right)}$$
(5.3)

It can be noted that the defined rule potential is more sensitive to data close to the center of the rule than the data far from the rule. The potential can be updated recursively as;

$$\Lambda_{i}(k) = \frac{(k-1) \cdot \Lambda_{i}(k-1) + \frac{1 - \Lambda_{i}(k-1)}{\sum_{j=1}^{n+1} \left(\xi_{j}(k) - \zeta_{ij}\right)}}{k - \Lambda_{i}(k-1) + \frac{1 - \Lambda_{i}(k-1)}{\sum_{j=1}^{n+1} \left(\xi_{j}(k) - \zeta_{ij}\right)}}$$
(5.4)

If the total number of user-defined rules 'R' are achieved, the existing rule with minimum potential is deleted and replaced with current data as a new rule.

As the rule creation technique is based on distance in the input-output space, there are a lot of chances that an outlier could be added as a new rule but this outlier would have a very low potential and continue to have a lowest potential and ultimately would be deleted when total number of rules reached the user-defined maximum rules [11].

#### 5.2.3 Online estimation of the rule consequents

A computationally efficient and undemanding estimation technique, based on the fuzzy identification scheme proposed by Ridley [23], is used for rule consequents of FRMs.

The rule consequent of the newly created rule is initialized, to reduce the noisy effect and possibility of the new rule to be an outlier, as follows;

$$\theta_{i+1} = \frac{\left(\sum_{i=1}^{R} [\mu_i(x(k) \cdot \theta_i(k))]\right) + (\mu_i(x(k) \cdot y(k)))}{\left(\sum_{i=1}^{R} [\mu_i(x(k))\right) + \mu_i(x(k))}$$
(5.5)

The current input x(k) fires the next created rule i+1,

 $\mu_i(x(k)) = 1$ 

Then,

$$\theta_{i+1} = \frac{\left(\sum_{i=1}^{R} [\mu_i(x(k) \cdot \theta_i(k))]\right) + y(k)}{\left(\sum_{i=1}^{R} [\mu_i(x(k))\right) + 1}$$
(5.6)

And

 $\sigma_{i+1} = 1$ 

Where,

' $\sigma$ ' depicts the quality of the new data and ' $\mu_i$ ' is the multidimensional membership function which shows the antecedent of the new  $i^{th}$  rule.

The consequents of the current rule bases are updated to accommodate the new data. The consequent parameter  $\theta_i(k)$  of the *i*<sup>th</sup> rule is updated as;

$$\theta_{i}(k) = \frac{\sum_{j=1}^{k} \left( \kappa^{(k-j)} \cdot \mu_{i}(x(k) \cdot y(j)) \right)}{\sum_{j=1}^{k} \left( \kappa^{(k-j)} \cdot \mu_{i}(x(j)) \right)}$$
(5.7)

Where, ' $\mu_i(x(k))$ ' is the degree to which the current data fires the  $i^{th}$  rule and  $\kappa(0 \le \kappa \le 1)$  is the forgetting factor. The value of ' $\kappa$ ', near to zero, discards mostly the previous data and value near to one accounts mostly the previous data.

The estimation technique, in its recursive form, can be written as;

$$\theta_i(k) = \frac{\kappa \cdot \theta_i(k-1) \cdot \sigma_i(k-1) + y(k) \cdot \mu_i(x(k))}{\sigma_i(k)}$$
(5.8)

Where  $\sigma_i(k)$ , is the frequency and quality of the training data which is used to train the *i*<sup>th</sup> rule, is given by;

$$\sigma_i(k) = \kappa \cdot \sigma_i(k-1) + \mu_i(x(k))$$
(5.9)

#### 5.3 Modeling the Cooling Coil System using EFMs

As mentioned earlier, the modeling based on Takagi-Sugeno (TS) fuzzy models and fuzzy relational models (FRMs) proved to be very efficient and computationally undemanding. Therefore, the combination is applied to our cooling coil system using its measured input-output data. The developed EFM model of the cooling coil system has outstanding results with minimum error compared to the previous techniques mentioned in the previous chapters.

The training data is first normalized, as in the previous techniques, to feed to the EFM. The purpose of normalizing is to make the modeling computer generic to respond to other sets of training data of the same system or the other system, and hence require no major changes in the modeling computer. The model output is also de-normalized to get the actual results of the model based on the system's inputs.

#### 5.3.1 Rule-base initialization

As described earlier, the fuzzy rule-base, for modeling the cooling coil system, is initialized by putting first data as the first rule-center. The first rule-potential is initialized as zero.

 $\Xi_i = \xi(1)$  and  $\Lambda_1(1) = 0$ 

#### 5.3.2 Rule creation

The new rule is created based on the spatial distance between current data and the existing rulecenter. If the data are far from the region of influence of the closest rule, the data is added as a new rule, otherwise the existing rule-base remains unchanged. Therefore, as previously mentioned, the new rule is added if  $\rho_{\psi}(k) > (T_{Th} \cdot r)$ , while  $\rho_{\psi}(k)$  is the minimum spatial distance.

In our case i.e. modeling of cooling coil system, the maximum number of rules is defined to 100, therefore, R=100, and  $T_{Th} = 2$  with r = 0.05

#### 5.3.3 Rule deletion

When user-defined maximum number of rules is achieved in the current rule-base, then the rulebase, with minimum potential, is replaced with the current data. The potential of each rule is calculated recursively at each sample using the equations mentioned in the previous section.

The purpose of deletion is to get the rule-base providing closest results of the system. The model is evolved or continuously improved at each sample using the current data. The current data may be an outlier, which could contaminate the rule-base. The outlier, having minimum potential, will ultimately be deleted and replaced with new data.

#### 5.3.4 Estimation of rule consequents

Once the rule-base is obtained, the rule consequents are estimated then. In our model, the number of parameters to be estimated is equal to the number of rules in the rule-base, therefore, the number of rule consequents would be 100. The consequents are also estimated, using the equation described in previous section, at each sample. The forgetting factor ' $\kappa$ ' is set to 0.75

#### **5.4 Results**

In this section, the results obtained are shown. The results show that the applied combinational technique has very accurate results with very much less error than the other technique applied previously.

Figure 17 shows that EFM output is precisely copying the system output. Figure 18 is showing the absolute error between the actual and modeled output. The error is very small. This very small error shows the efficiency of the technique used to model the system.



Figure 17: Actual and EFM output of the cooling coil system



Figure 18: Absolute error between actual and EFM output

Figure 19 shows that the RMS error is getting smaller and smaller as the new data is coming into the modeling computer.



Figure 19: RMS error between actual and EFM output

#### **5.5 Conclusion**

In this chapter, the mathematical foundation of EFMs, its application to the example system i.e. cooling coil system, and its results are discussed. The EFM modeling proved to be the most accurate and efficient technique amongst all the modeling techniques applied to the cooling coil system. This technique is also the computationally efficient technique and online also. The RMSE is also smallest of that of all other techniques and the envelop shows the decreasing error.

The models obtained in the techniques or schemes applied in the previous chapters are less accurate and efficient than that of obtained by EFMs. The RMSE is lowest of the all other schemes and with decreasing trend. This would be very good candidate to validate the model.

# Chapter 6

# **Conclusion And Future Recommendations**

#### 6.1 Conclusion

In this work, the cooling coil system was modeled using different techniques which included the static offline modeling, static online modeling, dynamic online modeling, and evolving fuzzy modeling. The simulation results are also presented to support the techniques. The results show that the modeled system is closely copying the actual system. As you scroll down, the techniques proves to be more accurate i.e. the EFM model has highly accurate results, the dynamic online modeling has accurate results but less than the EFM model and static online modeling has also the accurate results but less than the dynamic online modeling and so on. The comparison can be carried on the behalf of some parameters like RMSE of the prediction error, number of rules defined and etc.

The static offline fuzzy modeling is used where the initial guess of the system is to be required like in some model based or inverse model based modeling techniques. The static offline modeling is also used in forecasting the statistical systems or meteorological systems.

The static online fuzzy modeling and dynamic online fuzzy modeling techniques are used in model based predictive control, internal model controllers, or model based adaptive control according to the application and results required. In some applications, the static modeling would be enough while in other applications; dynamic modeling would appropriate to get accurate results.

The EFMs proved to be highly accurate and précised in modeling complex nonlinear systems with minimum error. The combo technique i.e. FRMs (using TS model) and RLS (for estimation of consequents) proved to be highly efficient and computationally undemanding in achieving the results. The evolution based on clustering, makes the system more quick and accurate in finding the right rule-base.

For comparison, the results obtained using each technique i.e. static offline fuzzy modeling, static online fuzzy modeling, dynamic online fuzzy modeling and evolving fuzzy modeling are shown in Figure 20, Figure 21, Figure 22, and Figure 23 respectively;



Figure 20: Actual output and the static offline fuzzy model output of the system



Figure 21: Output of actual and statically online modeled system



Figure 22: Actual and dynamically modeled output of the system



Figure 23: Actual and EFM output of the cooling coil system

#### 6.2 Future Work Recommendation

The recommendations and future work are suggested as follows;

- In static offline fuzzy modeling, static online fuzzy modeling, and dynamic online fuzzy modeling, the Takagi-Sugeno (TS) type-zero system was used. Therefore, there were only constant parameters were estimated. For future work, the higher type of the system could be used to make the results more accurate.
- In dynamic online fuzzy modeling, there is only one sample delay in the model; the number of delays can be increased to achieve more accurate results.
- In EFM modeling, a number of delays can be inserted in the model, i.e. in inputs and the output feedback to make the system more accurate.
- After detailed analyses, the irrelevant or not significant inputs could be removed from the model to make the system quicker and computationally undemanding.
- The developed EFM model can be extended to model based adaptive control or model based predictive control for controlling the temperature of the room or the environment in more efficient way.

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