Design of Universal Joints for Heavy Loads

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ISLAMABAD

AUGUST, 2018

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A thesis submitted in partial fulfillment of the requirements for the degree of

MS Mechanical Engineering

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Declaration

I certify that this research work titled "Design of Universal Joint for Heavy Loads" is my own work. The work has not been presented elsewhere for assessment. The material that has been used from other sources it has been properly acknowledged / referred.

Signature of Student
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Language Correctness Certificate

This thesis has been read by an English expert and is free of typing, syntax, semantic, grammatical and spelling mistakes. Thesis is also according to the format given by the university.

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Signature of Supervisor

Copyright Statement

Acknowledgements

I would like to acknowledge and thank all the important people who have helped me not only during research but throughout MS degree

Firstly, I would like to show my gratitude towards my supervisor Dr. Hasan Aftab Saeed for his guidance, insight and help during the research phase.

I would also like to thank Dr. Wasim Akram as without his support completion of the project would not be possible.

Finally, I would like to thank my parents as without their support and encouragement, this would not have been possible.

Dedicated to my parents, my teachers and my wife whose tremendous support and cooperation led me to this wonderful accomplishment

Abstract

Rotation requirement of joint is in two directions. It will replace three jacks of a leveling tripod with two jacks to reduce complexity. A joint with 2 DOF capability will be designed and developed for experimentation. The purpose of this project was to reduce complexity of the initial design and to study, design and develop a joint that fulfills the minimum DOF requirement for heavy loads.

After the initial benchmarking and evaluation of possible joints, universal joint came out to be most appropriate for the tripod. After the Initial Design, kinematics analysis was performed on universal joint in which transformation matrices were calculated from base to the end effector. Kinematic Envelop was calculated keeping in mind the extreme movement of the tripod in order to control the movement of tripod. The angles calculated were used in transformation matrices to calculate joint linear velocities, accelerations and angular velocities and accelerations.

Using the inertial load of the joint and applied force of 20T on universal joint, we calculated the forces and moments on all the extreme positions of tripod by applying these forces on kinematics equations that we calculated during kinematics analysis.

Stress analysis was also performed on the universal joint during design stage which led us to our final design.

Scaled down (1:5) universal joint was also designed and developed which was tested for load of 4T. The purpose was to compare the simulated results with experimental results and deduce our results of full scale model.

Table of Contents

List of Figures

List of Tables

Symbols

L1	Length of Lower Shaft of Universal Joint
L2	Length of Upper Shaft of Universal Joint
m1	Mass of Lower shaft of Universal Joint
m2	Mass of Upper Shaft of Universal Joint
θ_i	Universal Joint Angles
${}^{i}_{i+1}T$	Homogenous Transformation Matrix from I to i+1
${}^{i}_{i}w$	Angular Velocity on node i
${}^{i}_{i}v$	Linear Velocity on node i
${}^{i}_{i}f$	Force acting on node i
$\dot{\theta}_\iota$	Joint Rate at node i
$\ddot{\theta}_\iota$	Joint accelerations at node i
Ci	cos θ_i
Si	sin θ_i
x_i	x-axis at node i
y_i	y-axis at node i
z_i	z-axis at node i
I	Node no 0,1,2,3

CHAPTER 1: INTRODUCTION

A universal joint, also commonly called Cardan joint, Hooke’s joint or Universal Coupling is a joint between two rigid rods that allow the rods to rotate about in any direction. A couple of cylindrical pins located perpendicular to each other are connected by a cross yokes.

Robert Hooke is generally considered as an inventor of universal joint. But in reality the structure of (semicircular yokes with hollow cylinders connected by perpendicular pins) was being used way before Robert Hooke’s time. However, initially it was considered that it gives same or close to same output as input shaft.

Robert Hooke did extensive research and deduced that at inclined angles the output shaft gave variable output as compared to input shaft. Hooke worked on more complex design to overcome the problem and came up with “double universal joint” which can annul the variable output problem in comparison to input shaft. Hooke’s this research caused his name to be attached with invention of universal joint.

In this thesis, since both the shafts are fixed and there is no rotational shaft movement and universal joint only requirement is to compensate movement of actuators, we will be using “Single Universal Joint”.

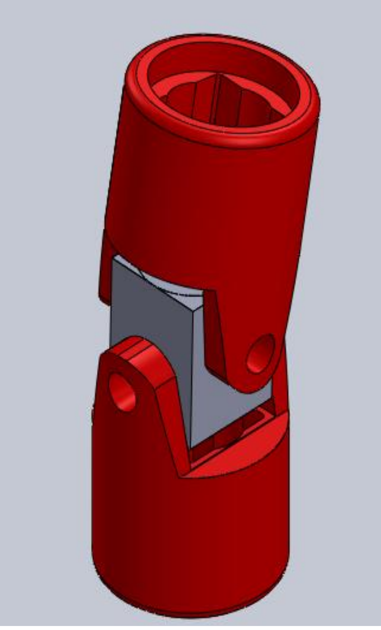


Figure 1.1 Universal Joint Structure

1.1 Universal Joint Design / Structure:

A universal joint uses basic mechanical principals, there are four point of attachments (usually with needle bearing), two per shaft. Each of these shaft ends called yoke arm or yoke end, attaches to a cross piece, providing a flexible joint which can take large loads.

The installation of universal joint is very simple but it requires a high level of fabrication skills and a well-equipped engineering shop.

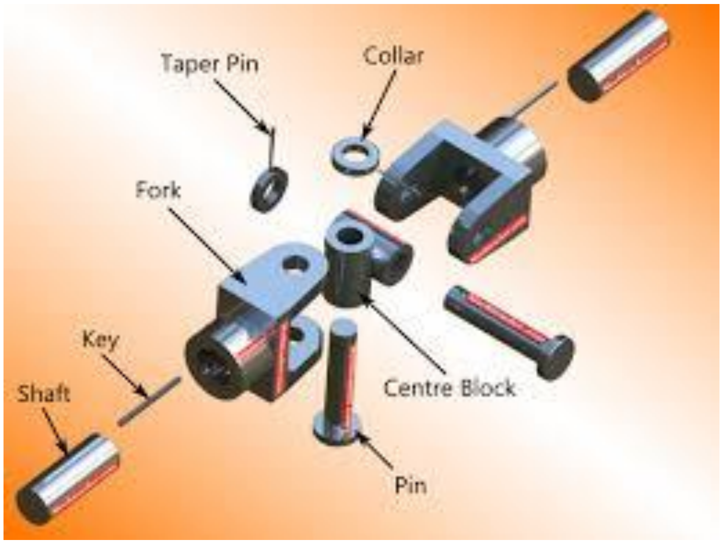


Figure 1.2 Universal Joint Assembly

1.2 Design Approach:

Rotation requirement of joint is in two directions. It will replace three jacks of a leveling tripod with two jacks to reduce complexity.

This thesis requires knowledge of design and analysis, kinematics, kinetics and flexible body dynamic analysis. A joint with 2 DOF capability will be designed and developed for experimentation. All the required analysis and experimentation will be conducted as per laid down criterion. The kinematic analysis, kinetic Analysis, stress analysis, motion envelop analysis will be performed at design stages. The joint will be manufactured through forging and tested for 20 T load on an experimental setup.

Figure 1.3 Sample tripod mechanism supported by joints and movement controlled with actuators

1.3 Evaluation Matrix:

Before selecting universal joint as replacement of installed joint, we will consider other possible joints that could be used to replace spherical joint along with advantages and disadvantages of using those joints using the “Evaluation Matrix”. The possible Joints are:

- Revolute Joint
- Knuckle Joint
- Screw Joint
- Spherical Joint
- Universal Joint

As it can be seen from the evaluation table below, it can deduced that universal joint is the best choice for replacement of the currently installed spherical joint due to the following reasons:

- They fulfill minimum requirement of DOF.
- They have comparatively moderate cost and complexity.
- This joint will be manufactured through forging and tested for 20 T load on an experimental setup.

Criteria	**Revolute joint**	**Knuckle joint**	**Screw Joint**	**Spherical joint**	**Universal Joint**
Rotation along x-axis	+	+	-	+	+
Rotation along y-axis	-	-	-	+	+
Complexity	S	S	S	-	S
Withstand high loads	-	-	S	+	+
Cost	S	S	S	-	S
$\sum +$	1	1	0	3	3
$\sum -$	2	2	2	2	0
$\sum S$	2	2	3	0	2

Table 1.1 Evaluation Matrix

1.4 Initial Proposal:

Initially, the tripod is supported by 3 leveling jacks with 3 spherical joints to compensate the movement of the leveling jack. In order to reduce complexity of the system we propose to replace the middle spherical joint with universal joint and reduce 3 jack mechanisms to 2 jack mechanism. The universal joint will be able to compensate the movements of leveling jacks by rotating in 2 directions, thereby giving 2 DOF.

When jack 1 and 2 are at same height but elevated from joint at point 3 as shown in Figure 1.1. The downward movement of both jacks has to be compensated by joint at 3, thus giving 1 DOF.

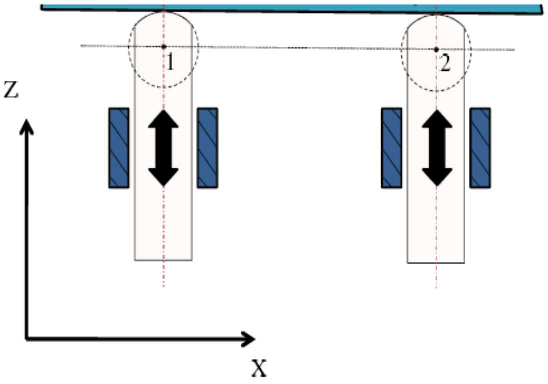


Figure 1.4

When jack 1 is elevated while 2 and 3 are at same height as shown in Figure 1.2. The down ward movement of jack 1 will be compensated by joint 3. (2 DOF)

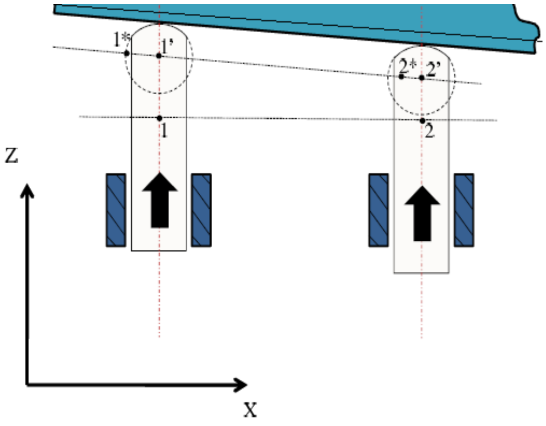
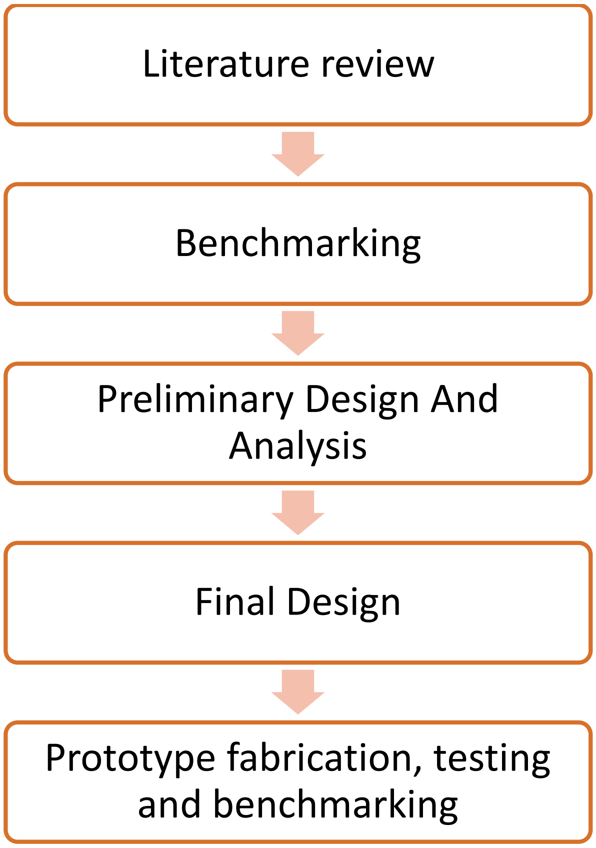


Figure 1.5

1.5 Methodology:



CHAPTER 2: LITERATURE REVIEW

In this section, relevant work regarding kinematic, kinetic and stress analysis of universal joint has been thoroughly discussed.

2.1 Dr. Robert L Williams II [1]:

The main purpose of this paper is to remove singularity problem in industrial robots. In this paper, Omni wrist industrial commercial robot is studied in detail. Forward position, velocities and inverse position and velocities of double cardan universal joint have been studied in detail. In order to link the base frame with the end effector, three forward and three inverse maps are presented for both position and velocities. These derived equations are enough to control the end effector.

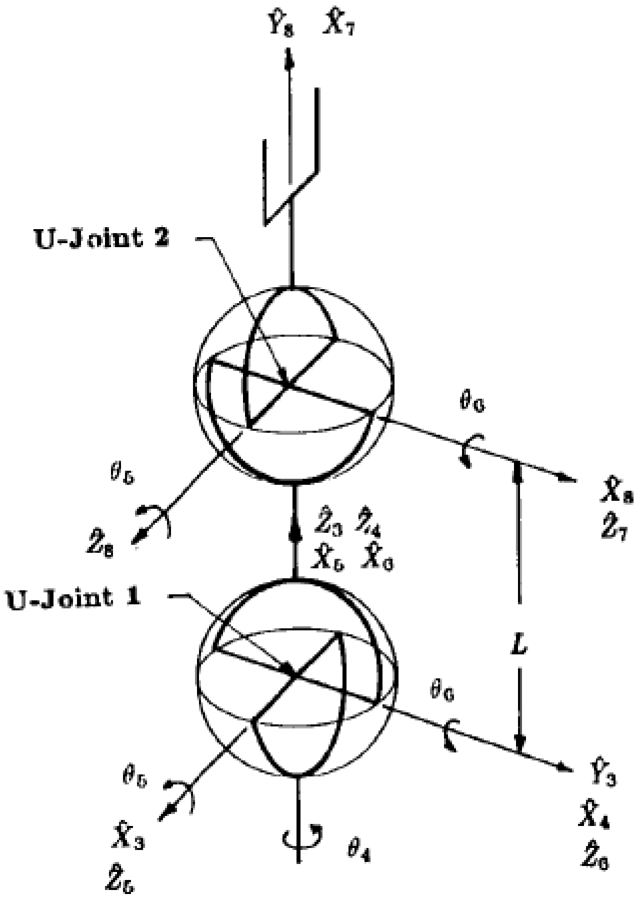


Figure 2.1 Double Universal Joint

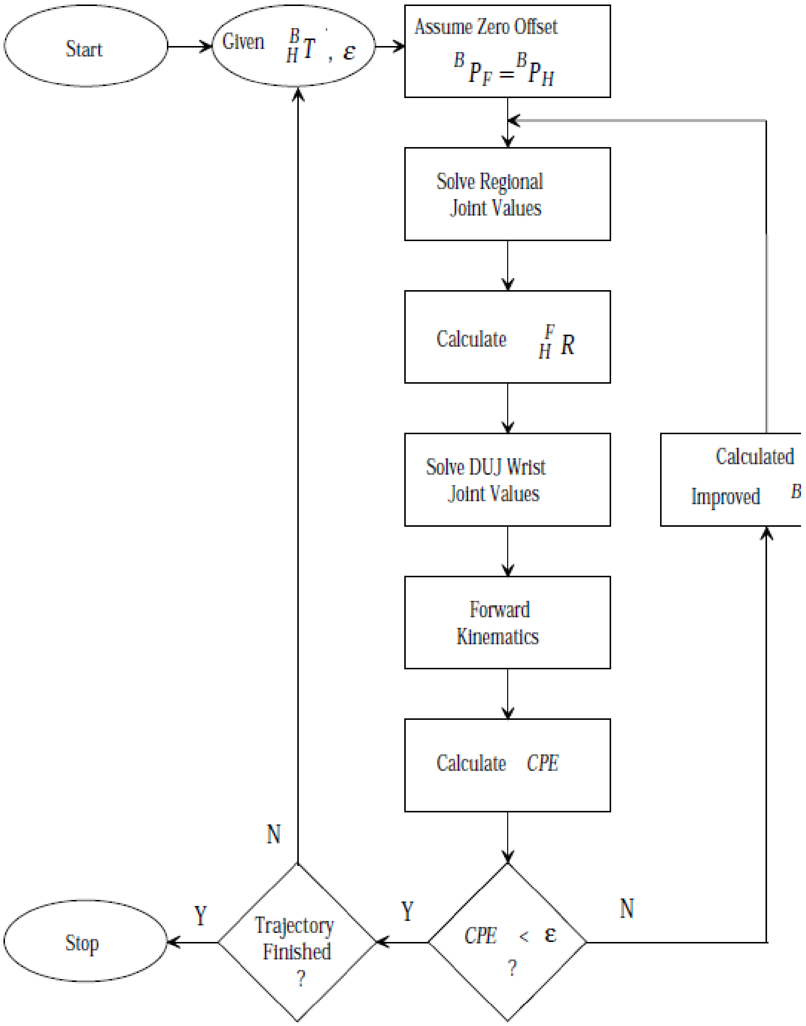
2.2 Dr. Robert L Williams II:

As discussed in Dr. Robert L William paper on study of Double joint universal joint [1], double universal joint robot will completely eliminate singularity problem which effects the performance of commercial robot wrists. With the removal of singularity problem, an offset problem arises which effects position and orientation of inverse kinematics problem. This paper focuses on solving the inverse position kinematics problem of manipulators with the double universal joint. Two solution methods were used in this research paper on double universal joint for inverse kinematics problem.

1. **Closed Form Solution**

$$\begin{matrix} x & & Px & & L(c4s6+s4s5c6) \\ y & = & Py & - & L(s4s6-c4s5c6) \\ z & & Pz & & Lc5s6 \end{matrix}$$

2. **Iterative Solution Method**



2.3 E. Pennestr`ı, L. Vita, P.P. Valentini

This research paper is divided in two parts. The first part, dual numbers is introduced which is for the formulation of Kinematics and Static analysis of universal joint. In this paper, universal joint with manufacturing error is treated as RCCC linkage.

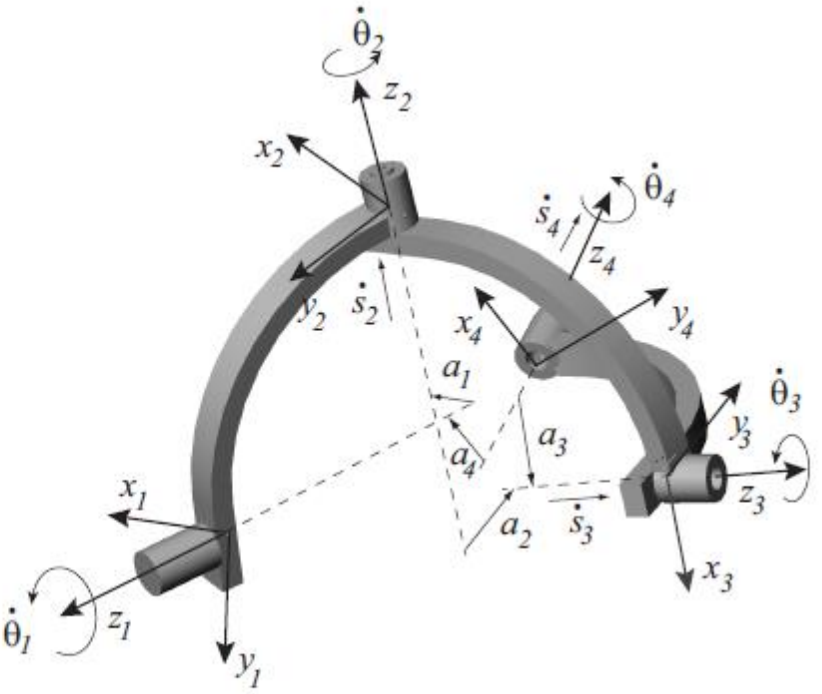


Figure 2.2 Kinematic equivalent of RCCC linkage

The second part deals with dynamic analysis of universal joint along with the mechanical efficiency of the universal joint. Numerical results have also been calculated and experimentally validated

2.5 Farzad Vesali, Mohammad Ali Rezvani and Mohammad Kashfi

In this research paper, the causes of failures in universal joint has been thoroughly discussed along with bearing surfaces failure causes. Analysis have been performed on universal joint behavior due to loading and different fatigue theories are also applied.

2.6 R.N Jazar

In this paper, Forward Acceleration Kinematics, Inverse Acceleration Kinematics and Rigid Link Recursive Acceleration of a rigid body with respect to a global frame has been discussed thoroughly.

2.7 Tatjana Lazović, Aleksandar Marinković, Svetislav Marković

In this paper, mathematical modeling of Universal Joint, needle bearing dimension, mechanical behavior and load distribution, to fully increase efficiency and load carrying capacity. Due to inappropriate lubrication or leaked seals, the universal joint service life is limited by corrosion and wear and tear. The universal joint repairs means additional machining of worn out needles. The change in needle dimension means dynamics and load carrying capacity of needle bearing is changed.

CHAPTER 3: INITIAL DESIGN

3.1 Different Views of Initial Design:

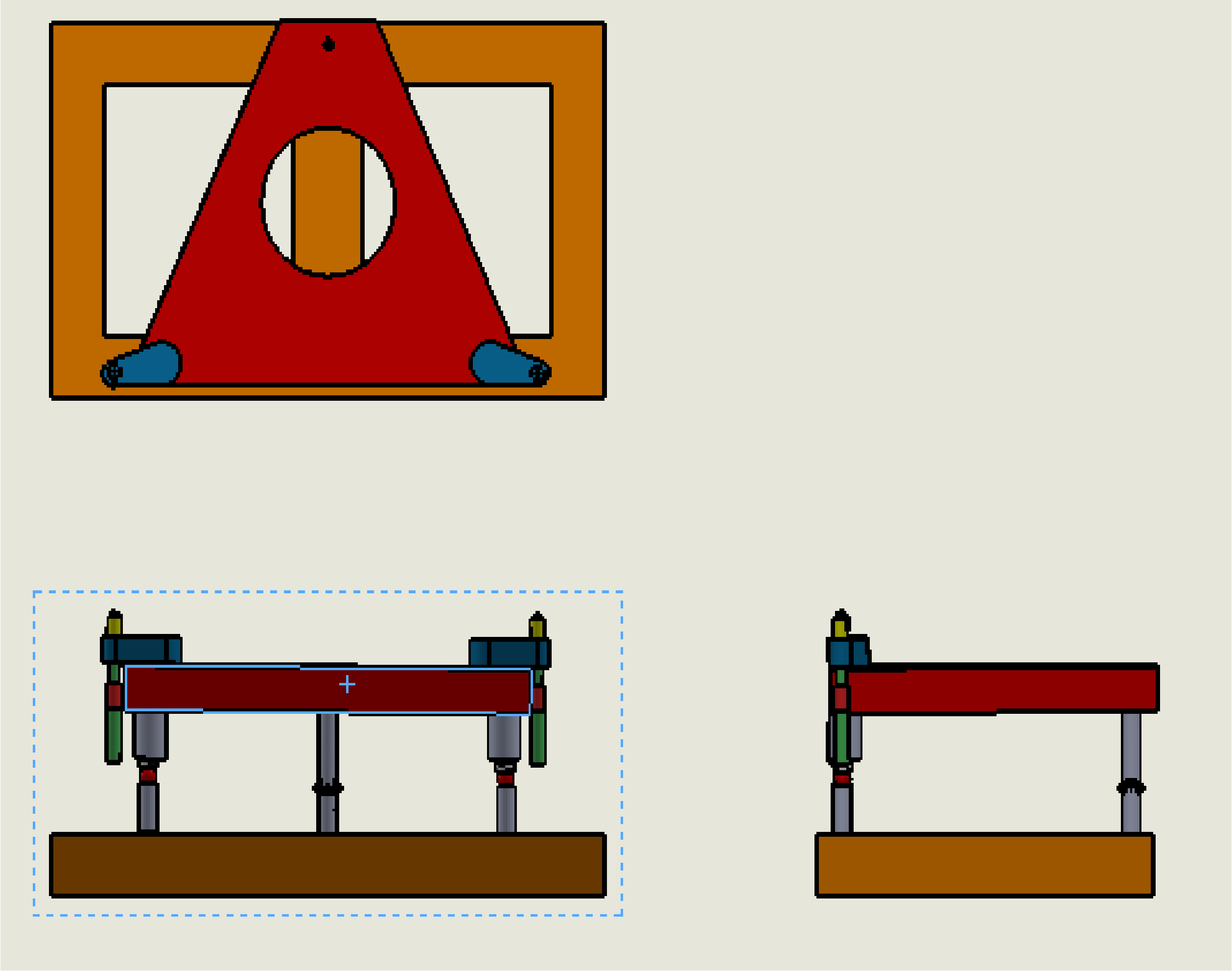


Figure 3.1 Top, Front and Side View of Initial Design

3.2 Different Components of Initial Design:

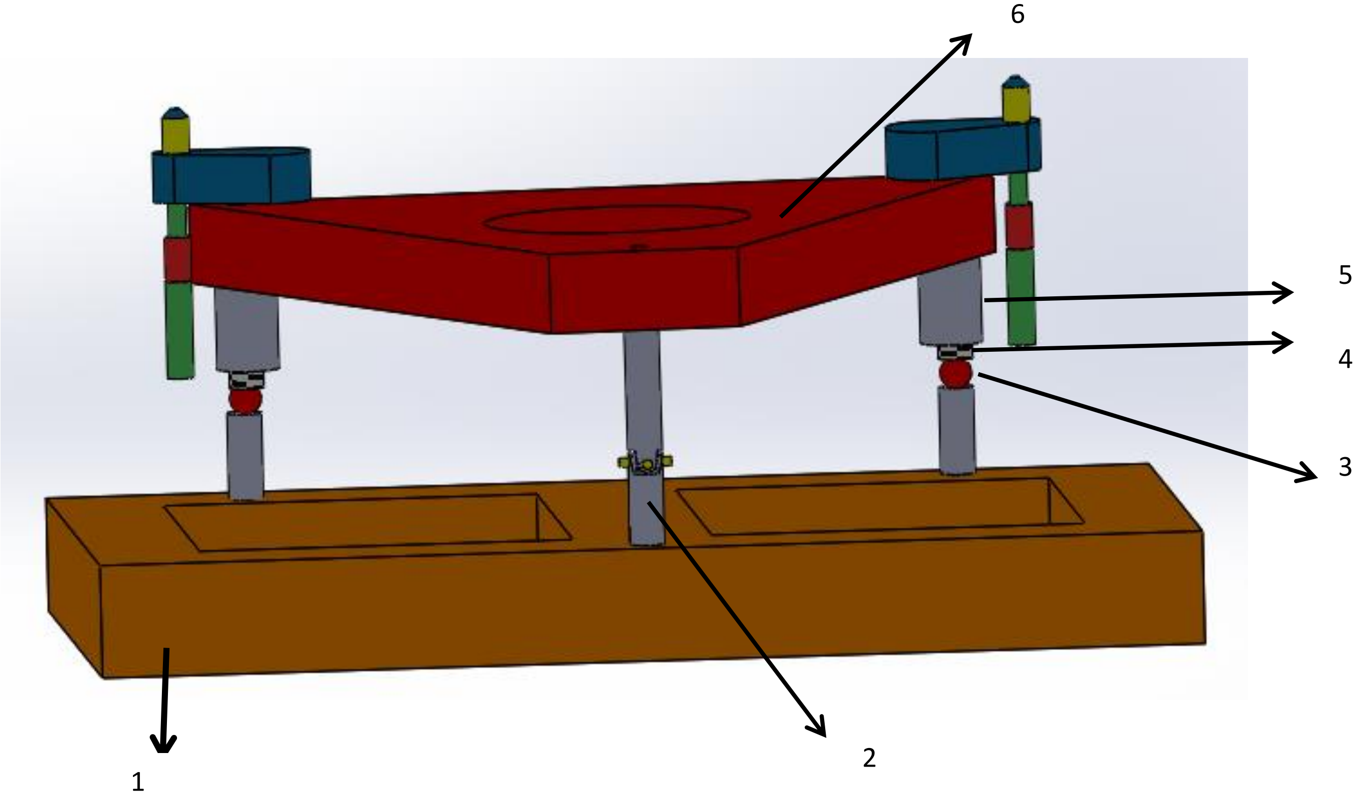


Figure 3.2 Different Components of Initial Design

The Initial design includes:

1. Base plate
2. Universal joint (accommodates actuator movement by rotating about 2 directions.)
3. Spherical Joints
4. Linear actuator opened system (40mm). Extendable up to 200mm
5. Linear actuator
6. Top plate

CHAPTER 4: KINEMATICS

Kinematics is the study of motion that considers only the study of motion without taking into account of the reason causing it. In kinematic Analysis of Universal Joint we will study positions, link velocities and link accelerations considering its geometry and other variables.

The relation between these motions and the inertial forces and applied forces on universal joint will be discussed in next chapter.

Firstly, we will consider the position and orientation of universal joint links. As universal joint geometry is complex, we will attach link frames i-e x, y, z frames to each nodes as shown in figure below.

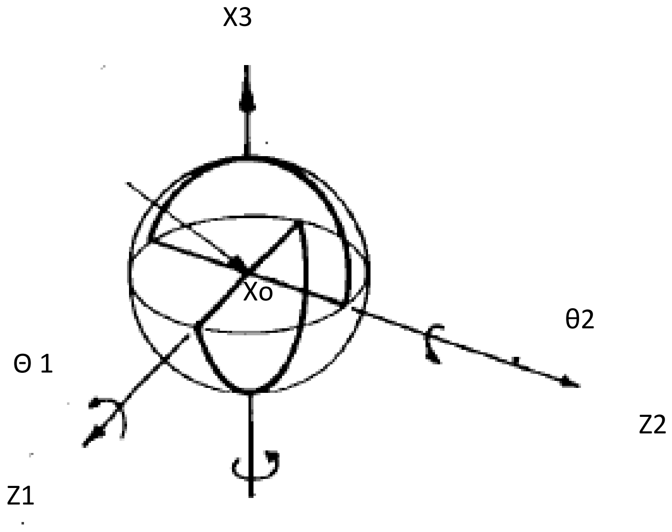


Figure 4.1 Universal Joint Schematic

Universal Joint rotates about 2 axis thereby giving two degree of freedom. The link frames attached to the universal joint starts from node 0 given to the base frame, then node 1 at which it rotates about Z axis and then node 2 at which it rotates about Z axis there by giving 2 Degree of freedom.

In order to control the movement of Universal Joint, we calculate the angles at which the universal joint rotates as the tripod moves. Then at those angles, we will calculate the change in forces in coming chapters. Since it is unnecessary to calculate all the angles we will consider only extreme points at which the tripod will move.

4.1 Kinematic Envelop:

In order to fully capture kinematic envelop of universal joint in this structure. We have to consider extreme points of actuators and analyze kinematic envelop of universal joint on those points. Initially both linear actuators systems are 40 mm open as shown in the figure below.

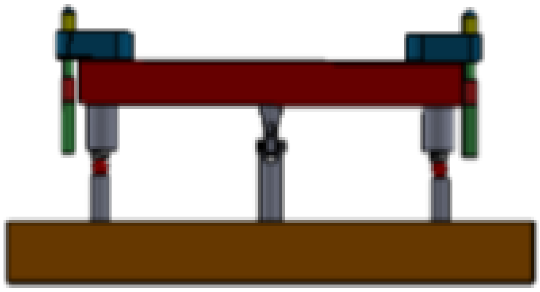


Figure 4.2 Initial positions of the tripod

In the first scenario, we will consider both linear actuators to be open to their full extent i-e 200mm. As it can be seen from the figure, the universal joint will rotate 6.32±.50 degrees about x-axis in order to accommodate the movement of linear actuators

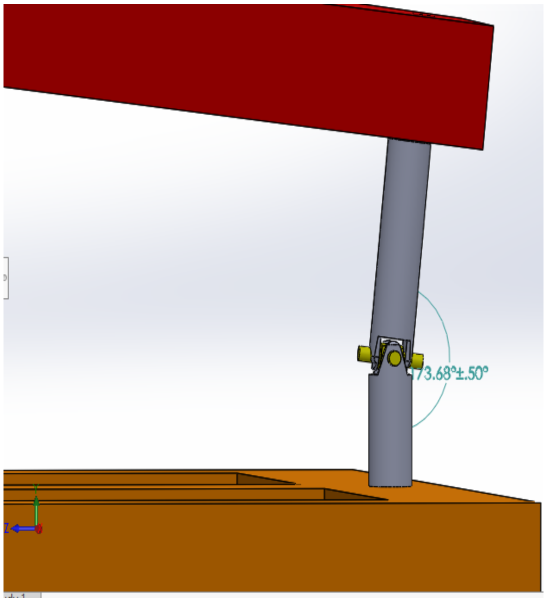
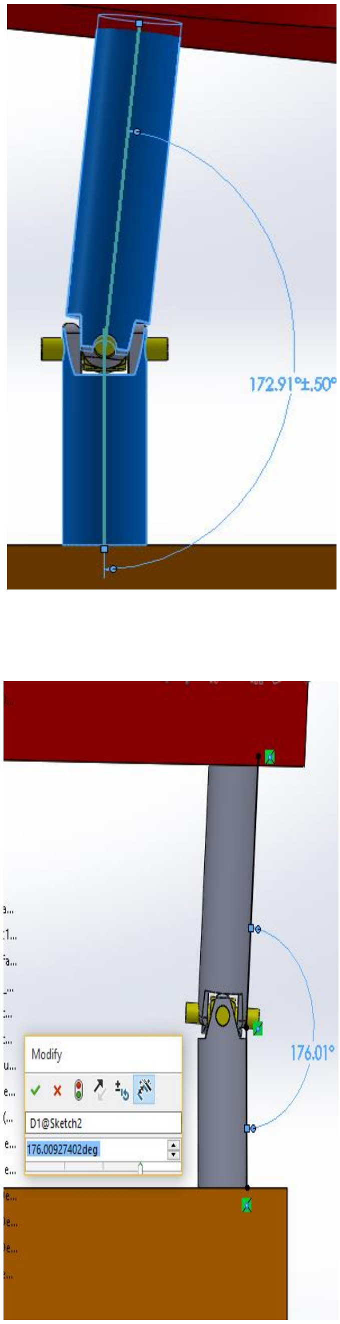


Figure 4.3 U-Joint rotating about x-axis

In the second scenario, one linear actuator system is fully open i-e 200mm while the other linear actuator is at its initial position i-e 40mm. As it can be seen from the figure, the universal joint will rotate about along x-axis 7.09±.50 degrees and 3.99±.50 degrees about z-axis to accommodate the movement of linear actuators



D1@Sketch2

176.00927402deg

Figure 4.4 U joint rotating about x and y axis

In kinematic analysis we will study the motion of the subject without allowing for the force that origins it. Within kinematic analysis, we will consider the position, velocity and accelerations including the higher order derivatives. To cope with the geometry of the universal joint, frames will be attached to different part of the mechanism and these frames will be linked to one another.

4.2 Position and Orientation of Universal joint:

Forward Kinematics finds the orientation and position of universal joint and how the universal joint position changes as it rotates around y and x axis

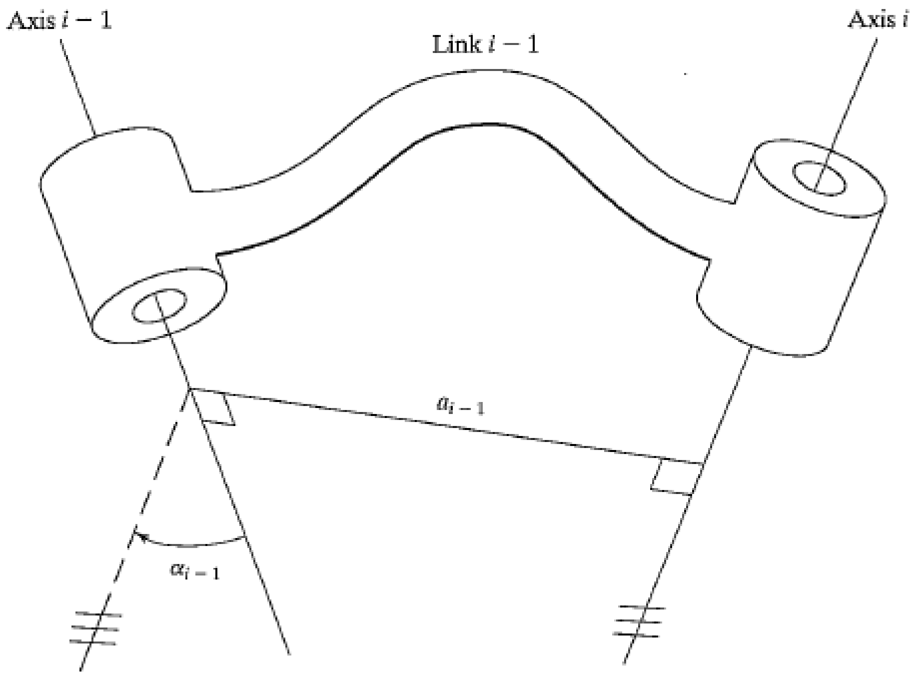


Figure 4.5 Kinematic Function for a Link

In the universal joint schematic diagram, the links are numbered starting from the fixed base that is marked link 0 as can be seen from Figure 5.1. After that comes link 1, which is the first moving link which rotates about to give 1 DOF. The second moving link that is Link 2, which is the second pin giving 2 DOF. The third link is the top of the universal joint which is fixed with the top of tripod plate and load is directly applied over it.

For the design of universal joint, many attributes will be considered including strength of material, its type, weight, shape, inertia and location. But during the formulation of the kinematic equations. Two consecutive links will be considered as rigid links. In order to differentiate between two consecutive nodes, we will consider these variables:

- Link length measured along two consecutive frames written as a_{i-1}
- Link twist which measured the relative position of two frames is α_{i-1}
- Third parameter is the link offset written as d_i
- Fourth Parameter is the link twist written as θ_i

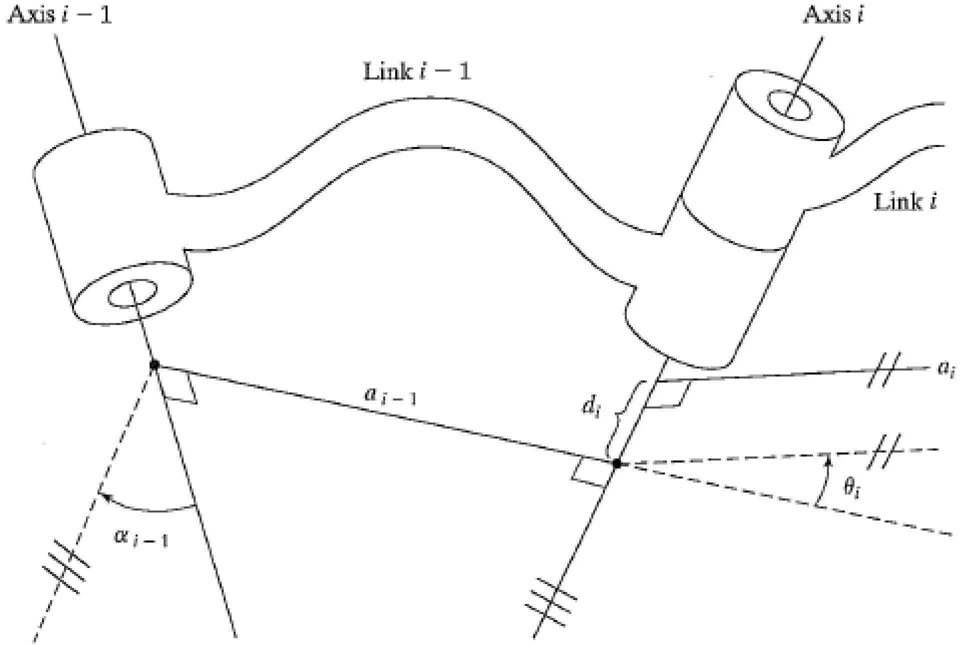


Figure 4.6 showing link offset d and joint twist θ

General Homogenous transformation matrix:

$$ {}^{i-1}_{i}T = \begin{matrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i . c_{ai-1} & c\theta_i . c_{ai-1} & -s_{ai-1} & -s_{ai-1} . di \\ s\theta_i . s_{ai-1} & c\theta_i . c_{ai-1} & c_{ai-1} & c_{ai-1.di} \\ 0 & 0 & 0 & 1 \end{matrix} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 4.1 $$

4.3 Denavit – Hartenberg Notation:

Any kinematic link, is characterized by four unique variables for each link. Two for the link itself and two for the adjacent link.

For revolute joints, Θi wii be the joint variable, and the other remaining three quantities would be fixed link parameters.

The mechanism by means of these quantities is a convention usually called Denavit Hartenberg notation.

As shown in figure 5.1, the link frames are interconnected with other link frames to convention, the following definitions of the link parameters are valid

- a i = the distance from Zi to Zi+1 measured along Xi
- α i = the angle from Zi to Zi+1 measured along Xi
- d i = the distance from Xi-1 to Xi measured along Zi
- Θ i = the angle from Xi-1 to Xi measured about Zi

Summarizing step by step purpose of attaching frame i with links:

- Identifying the link frames and numbering then 0 to n.
- Identifying common point of intersection.
- Identifying links that rotate as Z-axis in direction where it rotates about.
- Identifying perpendicular of Z-axis as X-axis
- Identify Y-axis to complete the coordinate system.
- Apply link parameters to each node and complete the D-H table.

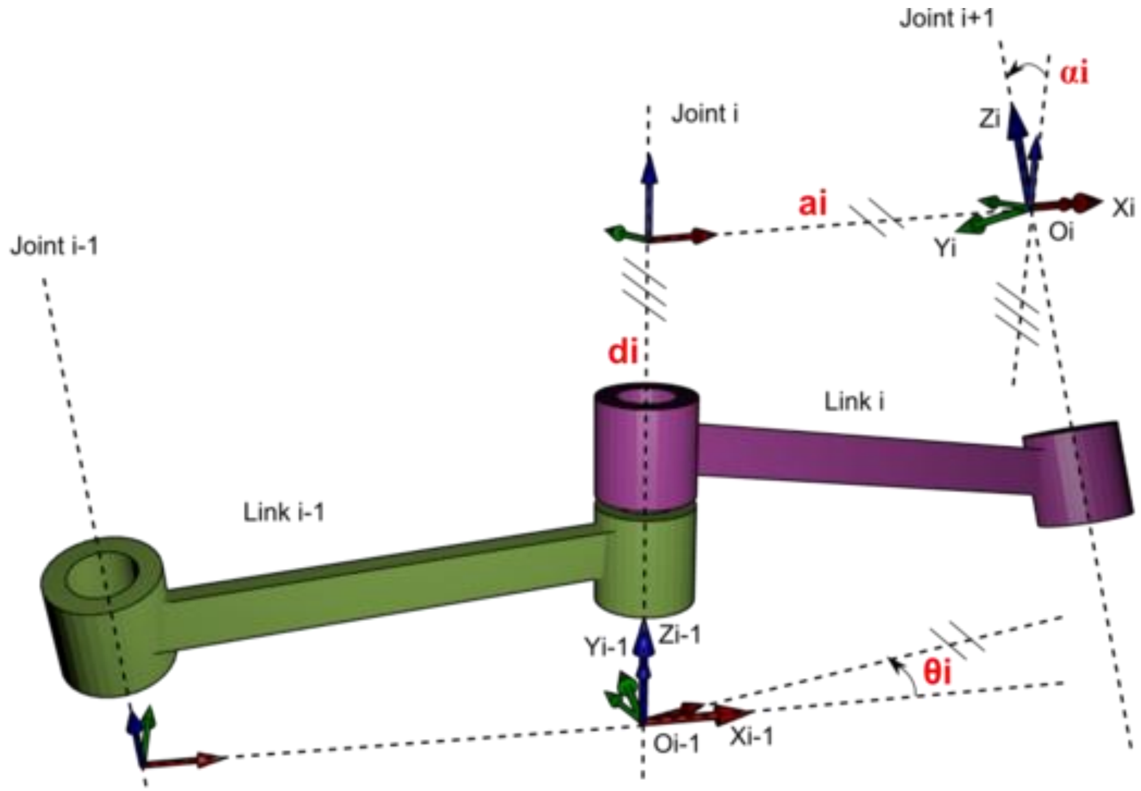
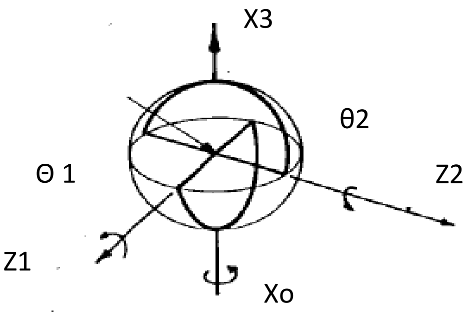


Figure 4.7 Link Frames are attached to the frame {i} is rigidly attached to link i

4.4 D-H table:

Apply the procedure to the nodes of universal joint as shown in figure below, we deduce the following D H table:



i	α i-1	a i-1	Di	Θi
1	0	L1	0	Θ1 +90
2	+90	0	0	Θ2
3	-90	L2	0	0

Table 4.1 D H table of Universal Joint

General Homogenous transformation matrix:

$$ {}^{i-1}_{i}T = \begin{matrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i.c_{ai-1} & c\theta_i.c_{ai-1} & -s_{ai-1} & -s_{ai-1}.di \\ s\theta_i.s_{ai-1} & c\theta_i.c_{ai-1} & c_{ai-1} & c_{ai-1.di} \\ 0 & 0 & 0 & 1 \end{matrix} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 4.2 $$

Putting Values of DH table in Transformation matrices, we get:

i	${}^{i-1}_{i}\boldsymbol{T}$
1	${}^{0}_{1}T = \begin{matrix} c1 & -s1 & 0 & l1 \\ s1 & c1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}$
2	${}^{1}_{2}T = \begin{matrix} c2 & -s2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s2 & c2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}$
3	${}^{2}_{3}T = \begin{matrix} 1 & 0 & 0 & l2 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}$

Table 4.2 Transformation Matrix

$$ {}^{0}_{3}T = {}^{0}_{1}T * {}^{1}_{2}T * {}^{2}_{3}T \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots . .4.3 $$

$$ {}^{0}_{3}T = \begin{matrix} c2.c1 & s1 & s2 & c2l2.c1 + l1 \\ c2s1 & -c1 & s2.s1 & c2l2 \\ s3 & 0 & -c3 & s3.l2 \\ 0 & 0 & 0 & 1 \end{matrix} $$

Now using the angles found in kinematic envelop, we use the transformation matrix to find the position and orientation of universal joint.

4.5 Forward Velocity:

Now we consider the problem of calculating the linear and angular velocities of the links of the universal joint. The manipulator is chain of bodies, each one capable of motion relative to its neighbors.

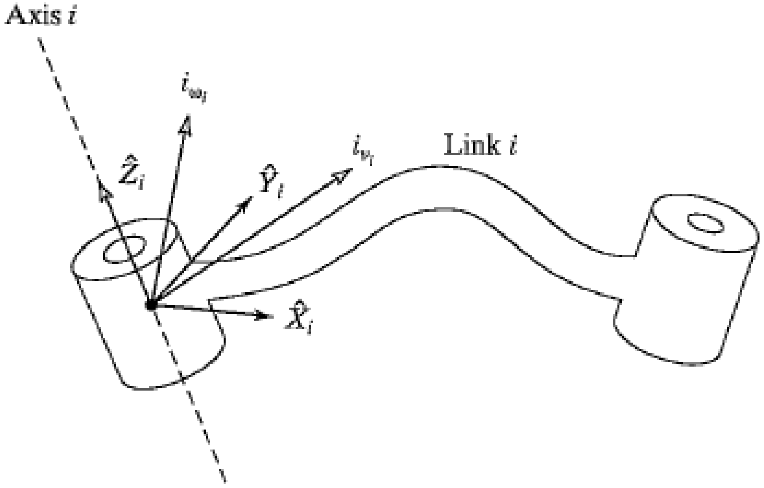


Figure 4.8 the velocity of link I is given by vector vi and wi, written in frame i.

As shown in Figure 5.1, we will start from the base link and move iteratively to successive link till the end effector. The velocity i will be added to the velocity of i+1 and so on.

As shown in Figure 6.1, we can assume each link as a rigid body with linear and angular velocity vectors describing its motion. Further, we will link its velocities with respect to link frame itself rather than the base frame.

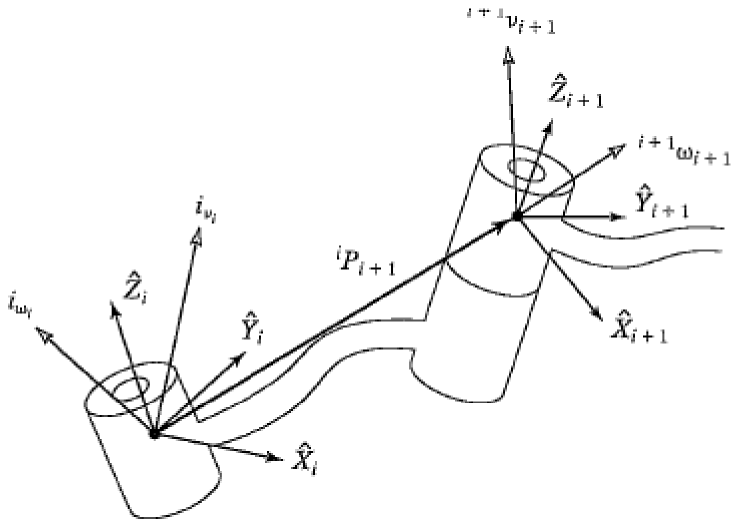


Figure 4.9 Velocities vector neighboring links

The forward velocity solves for Cartesian rates given joint rates using velocity recursion equations

$${}^{i+1}_{i+1}w = {}^{i+1}_{i}R.{}^{i}_{i}w + \dot{\theta}_{i+1}.{}^{i+1}_{i+1}\check{Z} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 4.3$$

$$\left\{{}^{i+1}_{i+1}v\right\} = \left[\left[{}^{i+1}_{i}R\right]\right].(\left\{{}^{i}_{i}v\right\} + \left\{{}^{i}_{i}w\right\} * \{{}_{i+1}{}^{i}P\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 4.4$$

Using the transformation matrices, we will calculate the forward velocity solution.

For $\boldsymbol{i = 0}$

$${}^{1}_{1}w = \begin{matrix} 0 \\ 0 \\ \dot{\theta}_1 \end{matrix} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 4.5$$

$${}^{1}_{1}v = \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 4.6$$

For $i = 1$

$${}^{2}_{2}w = \begin{matrix} S_2.\dot{\theta}_1 \\ C_2.\dot{\theta}_1 \\ \dot{\theta}_2 \end{matrix} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 4.7$$

$${}^{2}_{2}v = \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 4.8$$

For $i = 2$

$${}^{1}_{1}w = \begin{matrix} S_2.\dot{\theta}_1 \\ \dot{\theta}_2 \\ -C_2\dot{\theta}_1 - \theta_3 \end{matrix} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 4.9$$

$${}^{1}_{1}v = \begin{matrix} 0 \\ C_2.l_2\dot{\theta}_1 \\ -\dot{\theta}_2 l_2 \end{matrix} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 4.10$$

$${}^{0}_{3}v = {}^{0}_{3}R * {}^{3}_{3}v \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 4.11$$

$${}^{0}_{3}v = \left\{ \begin{matrix} s_1.c_2.\dot{\theta}_1.l_2 \\ c_1 c_2 \theta_1 l_2 - s_1.s_2.\dot{\theta}_2.l_2\} \\ c_2.\dot{\theta}_2.l_2 \end{matrix} \right. \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 4.12$$

4.6 Static Forces:

As shown in figure 5.1, all the rigid links of universal joints have their own masses causing them to have inertial forces. These inertial forces will propagate from one link to another, this along with the applied loads will cause static forces and torques on each link.

Defining symbols we get:

$f_i = forces\ exerted\ on\ link\ i\ by\ previous\ link$

$n_i = torque\ exerted\ on\ link\ i\ by\ previous\ link$

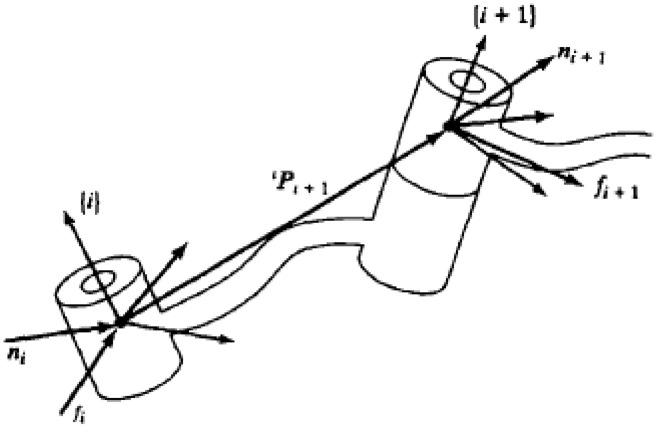


Figure 4.10 Static forces on a single link

From last link and working to the first link (base), we formulate the force and moment relationship as shown in equations below:

$${}^i_if = {}^i_{i+1}R.\,{}^{i+1}_{i+1}f \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 4.13$$

$${}^i_in = {}^i_{i+1}R.\,{}^{i+1}_{i+1}n + {}^i_{i+1}P * {}^i_if \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 4.14$$

CHAPTER 5: DYNAMICS

So far we have discussed motion of study of motion of links including velocities, accelerations and static forces without considering the forces that causes it. In this chapter, we will discuss the calculated equation of motion in previous chapters and apply forces on it that are causing the motion. Also we will apply the extreme positions, that we deduced in kinematic envelops and calculate reactive forces on those angles.

Dynamics of any rigid link is the study of motion of rigid bodies under the action of forces.

5.1 The iterative Newton Euler Dynamics:

We will consider each link of universal joint as a rigid body. By applying center of mass and inertia tensor of each link, mass distribution is completely characterized. Newton's Equation along its rotational analog, Euler's Equation explains how forces, inertias and accelerations relate.

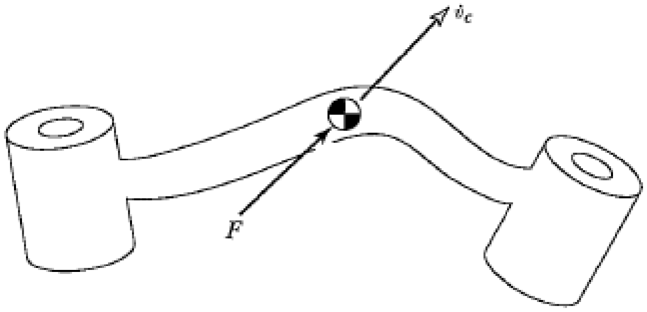


Figure 5.1 A Force F acting at the center of mass of a body causes the body to accelerate

5.1.1 Newton's Equation:

The Force F acting at the center of mass of rigid body and causing acceleration is given by the Newton Equation as shown in Figure 7.1

$F = m\dot{v}_c$……………………………………………………………………………......5.1

5.1.2 Euler's Equation:

As shown in Figure 6.2, moment N acting on rigid link rotating the body with angular velocity ω and angular acceleration $\dot{\omega}$

$N = I^c\dot{w} + w * {}^c_w I$ ……………………………………………………………………………………….5.2

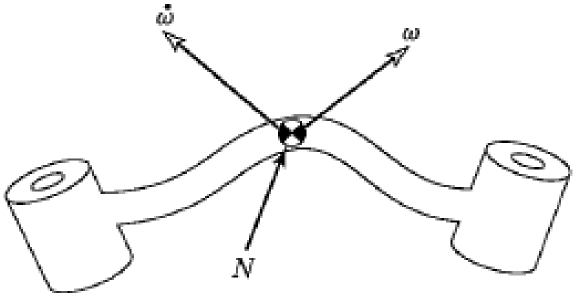


Figure 5.2 A moment N is acting on a body and the body is rotating with the velocity.

5.2 Inertia Tensor:

The moment of inertia determines the torque that is required to produce angular acceleration about a rotational axis. It shows how mass determines force needed to produce desired acceleration. It is also known as rotational inertia.

Considering a rigid body having 3 degree of freedom, there are infinite number of rotational axes. In order to characterize the link's mass distribution, we introduce Inertia Tensor.

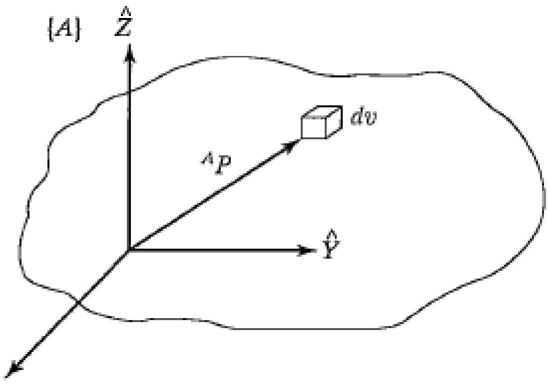


Figure 5.3 showing mass distribution of an object

As universal joint as shown in figure 5.1, consist of two yokes (hollow cylinders) and two pins (cylinders), the following Inertia Tensor would be used

$$I^c = \begin{array}{ccc} 0.083 * m(3(r_1^2 + r_2^2) + h^2 & 0 & 0 \\ 0 & 0.083 * m(3(r_1^2 + r_2^2) + h^2 & 0 \\ 0 & 0 & 0.083 * m(3(r_1^2 + r_2^2) \end{array}$$

In the matrix above the values of I_{xx} ,I_{yy}, I_{zz} are mass moments of inertia. In this matrix, mass moments are principal moments of inertia and reference frames aligned are principal axes.

5.3 The iterative Newton Euler Dynamics Algorithm

The complete algorithm for computing joint torques from the motion of the joints is composed of two parts. First, link velocities and accelerations are iteratively calculated from node 1 to node 3 and the Newton-Euler is applied to each link. Second, forces and moments are calculated extensively from link 3 to link. The equations are summarized below:

Outward iteration i: 0-3

In order to compute inertial forces acting on link 1; it is necessary to compute the rotational velocity and linear and rotational accelerations of the center of mass of each link of universal joint at any given instinct. These computations will be done in an iterative way, starting with link 1 and moving successively, link by link outward to last node.

This equation calculated rotational velocity link by link in outward iteration

$$\{{}^{i+1}_{i+1}w\} = [{}^{i+1}_{i}R].{}^{i}_{i}w + \theta\imath + 1.\{{}^{\imath+1}_{\imath+1}Z\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots .5.3$$

Using the equation for angular velocity we calculate angular accelerations in outward iterations.

$${}^{i+1}_{i+1}\dot{w} = \left[{}^{i+1}_{i}R\right] * {}^{i}_{i}\dot{w} + \left[{}^{i+1}_{i}R\right] * {}^{i}_{i}w + \dot{\theta}_{i+1} * {}^{i+1}_{i+1}\hat{Z} + \ddot{\theta}_{\imath+1} * {}^{i+1}_{i+1}\hat{Z} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots .5.4$$

Here we will find the linear accelerations of each link

$${}^{i+1}_{i+1}\dot{v} = \left[{}^{i+1}_{i}R\right] * ({}^{i}_{i}\dot{w} * {}_{i+1}{}^{i}P + {}^{i}_{i}w * \left({}^{i}_{i}w * {}_{i+1}{}^{i}P)\right) + {}^{i}_{i}\dot{v} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots .5.5$$

We apply accelerations to center of mass of each link

$${}^{i+1}_{i+1}\dot{v}c = ({}^{i+1}_{i+1}\dot{w} * {}^{i+1}_{i+1}Pc + {}^{i+1}_{i+1}w * \left({}^{i+1}_{i+1}w * {}^{i+1}_{i+1}Pc\right) + {}^{i+1}_{i+1}\dot{v} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots .5.6$$

$${}^{i+1}_{i+1}F = m_{i+1} * {}^{i+1}_{i+1}\dot{v}c \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots .5.7$$

$${}^{i+1}_{i+1}N = {}^{Ci+1}_{\quad i+1}I.{}^{i+1}_{i+1}\dot{w} + {}^{i+1}_{i+1}w * {}^{Ci+1}_{\quad i+1}I.{}^{i+1}_{i+1}w \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 5.8$$

Inward iteration i: 3-1

In order wo calculate torque and net forces applied on each link we apply inward iteration.

${}^{i}_{i}f = {}^{i}_{i+1}R . {}^{i+1}_{i+1}f + {}^{i}_{i}F$ … . . .5.9

$\tau i = {}^{i}_{i}nT \; {}^{i}_{i}Z$ ….5.10

Now Applying these equations to find the forces at extreme angles that we calculated in kinematic envelop in previous chapter.

For $\boldsymbol{\theta_1 = 0, \theta_2 = 0}$

- **For $\boldsymbol{i = 1}$**

$${}^{1}_{1}f = \begin{matrix} 196311.3N \\ -16.65N \\ -2.5N \end{matrix}$$

- **For $\boldsymbol{i = 2}$**

$${}^{2}_{2}f = \begin{matrix} 196218.7N \\ -2.5N \\ 16.5N \end{matrix}$$

- **For $\boldsymbol{i = 3}$**

$${}^{3}_{3}f = \begin{matrix} 196200N \\ 6.6N \\ 2.5N \end{matrix}$$

$${}^{3}_{3}n = \begin{matrix} 1.147 * 10^{-5}Nm \\ -0.40Nm \\ +0.40Nm \end{matrix}$$

For $\boldsymbol{\theta_1 = 6.32, \theta_2 = 0}$

- **For $\boldsymbol{i = 1}$**

$${}^{1}_{1}f = \begin{matrix} 195092.62N \\ 21471.5N \\ 1.71N \end{matrix}$$

- **For $\boldsymbol{i = 2}$**

$$ {}^{2}_{2}f = \begin{matrix} 195092.2N \\ 21574N \\ 1.71N \end{matrix} $$

- **For $\boldsymbol{i = 3}$**

$$ {}^{3}_{3}f = \begin{matrix} 195000N \\ 21574N \\ 1.71N \end{matrix} $$

$$ {}^{3}_{3}n = \begin{matrix} +368.914Nm \\ -333.504Nm \\ +3451.84Nm \end{matrix} $$

For $\boldsymbol{\theta_1 = 7.09, \theta_2 = 3.90}$

- **For $\boldsymbol{i = 1}$**

$$ {}^{1}_{1}f = \begin{matrix} 194992.2N \\ 24213.79N \\ -132230.5N \end{matrix} $$

- **For $\boldsymbol{i = 2}$**

$${}^{2}_{2}f = \begin{array}{c} 194992.2N \\ 24213.79N \\ -13230.5N \end{array}$$

- **For $\boldsymbol{i = 3}$**

$${}^{3}_{3}f = \begin{array}{c} 194200N \\ 24220N \\ -13242N \end{array}$$

$${}^{3}_{3}n = \begin{array}{c} -557.17Nm \\ +5434.066Nm \\ +1753.6Nm \end{array}$$

CHAPTER 6: STRESS ANALYSIS

The stress at any point on the universal joint depends on the nature of load acting on it. The stresses may be present as follows:

6.1 Bending Stress:

$$\sigma_b = \frac{32M}{\pi d_0^3 * (1-k^4)} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots 6.1$$

M: Bending moment at the point of interest

d_0: Outer diameter of the shaft

k: Ratio of inner to outer diameters of the shaft (k = 0 for a solid shaft because inner diameter is zero)

6.2 Axial Stress:

$$\sigma_a = \frac{4\alpha F}{\pi d_0^2 * (1-k^2)} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots 6.2$$

Where column action factor=$\alpha = \frac{1}{1-0.0044\frac{L}{K}}$ if L/K <115

$\alpha = \frac{\sigma yc}{\pi^2 nE}$ if L/K >115

Where,

n = 1.0 for hinged end

n = 2.25 for fixed end

n = 1.6 for ends partly restrained, as in bearing

K = least radius of gyration, L= shaft length

σ yc = yield stress in compression

Combining Bending and axial stress, we get:

$$\sigma x = \frac{32M}{\pi d_0^3 * (1-k^4)} \pm \frac{\sigma yc}{\pi^2 nE} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots 6.3$$

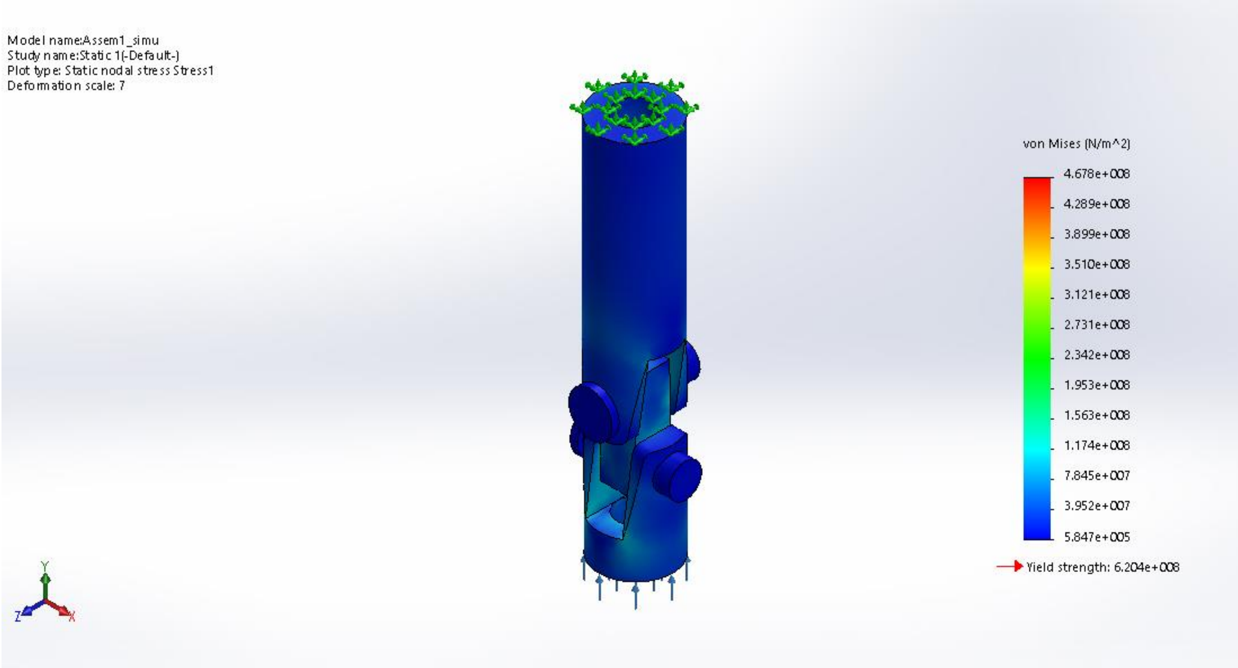
6.3 Stress Analysis Simulations:

Stress Analysis Simulation was performed on Solidworks Simulation tool. The following material characteristics were considered:

Material	Alloy Steel
Mass Density	7700 ${Kg}/{m^3}$
Yield Strength	620.4 MPa

6.3.1 Simulation Result for combined Universal Joint Assembly:

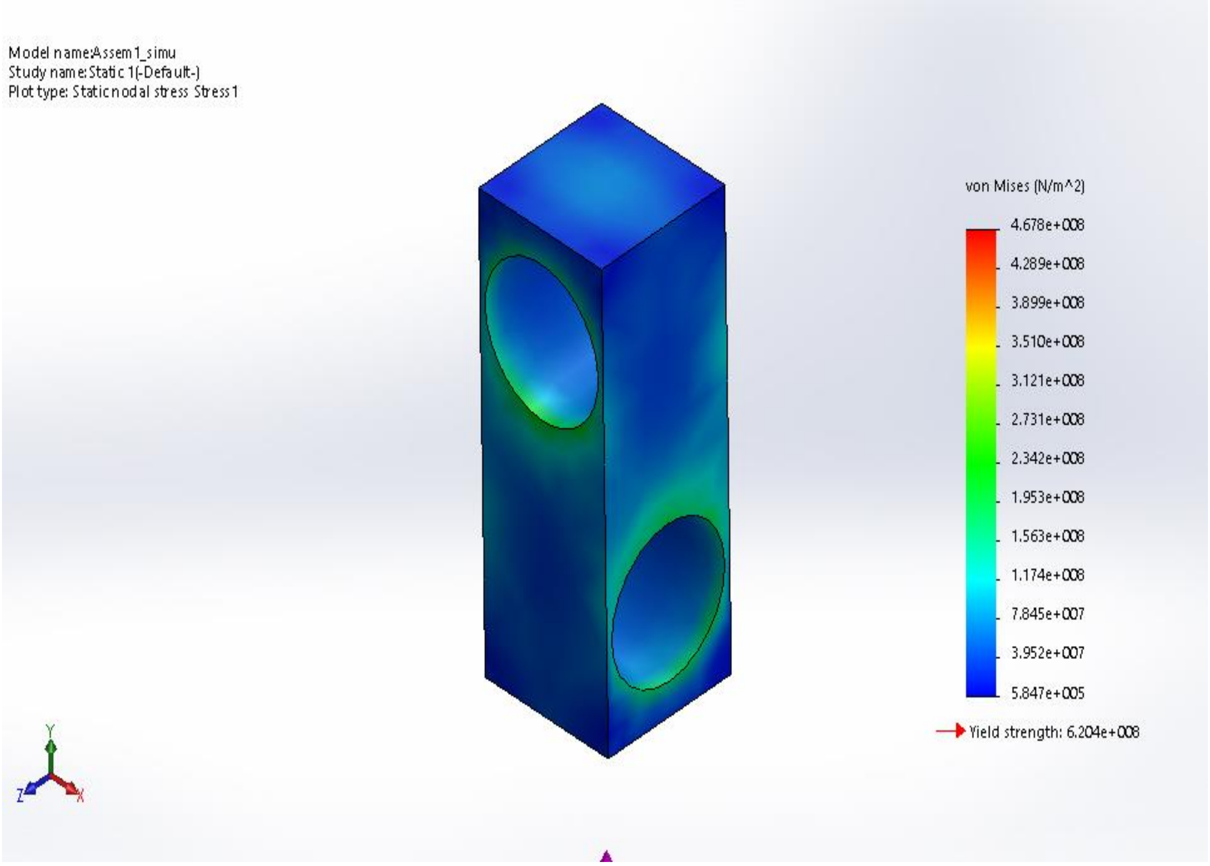
Name	Type	Min	Max
Stress1	VON: von Mises Stress	584711 N/m^2 Node: 2665	4.67785e+008 N/m^2 Node: 2827



Assem1_simu-Static 1-Stress-Stress1

Figure 6.1 Simulation Result for combined Assembly

6.3.2 Simulation Result for Isolated Center Block:

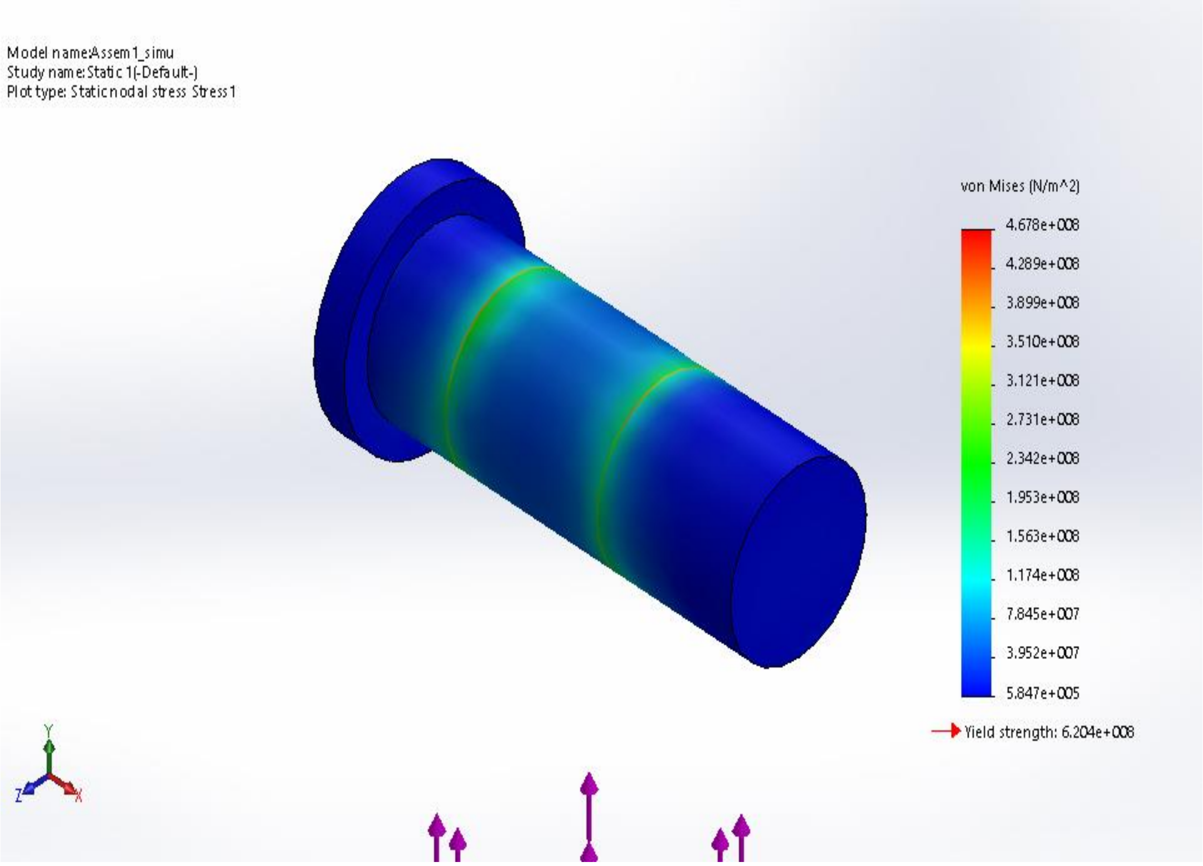
<table>
<tr><th>Name</th><th>Type</th><th>Min</th><th>Max</th></tr>
<tr><td>Stress1</td><td>VON: von Mises Stress</td><td>584711 N/m^2
Node: 2665</td><td>4.67785e+008 N/m^2
Node: 2827</td></tr>
<tr><td colspan="4">

Assem1_simu-Static 1-Stress-Stress1</td></tr>
</table>

Figure 6.2 Simulated Result of Central Block

6.3.3 Simulation Result for Isolated Supporting Pins:

Name	Type	Min	Max
Stress1	VON: von Mises Stress	584711 N/m^2 Node: 2665	4.67785e+008 N/m^2 Node: 2827



Assem1_simu-Static 1-Stress-Stress1

Figure 6.3 Simulated Result of Supporting Pin

CHAPTER 7: FINAL DESIGN

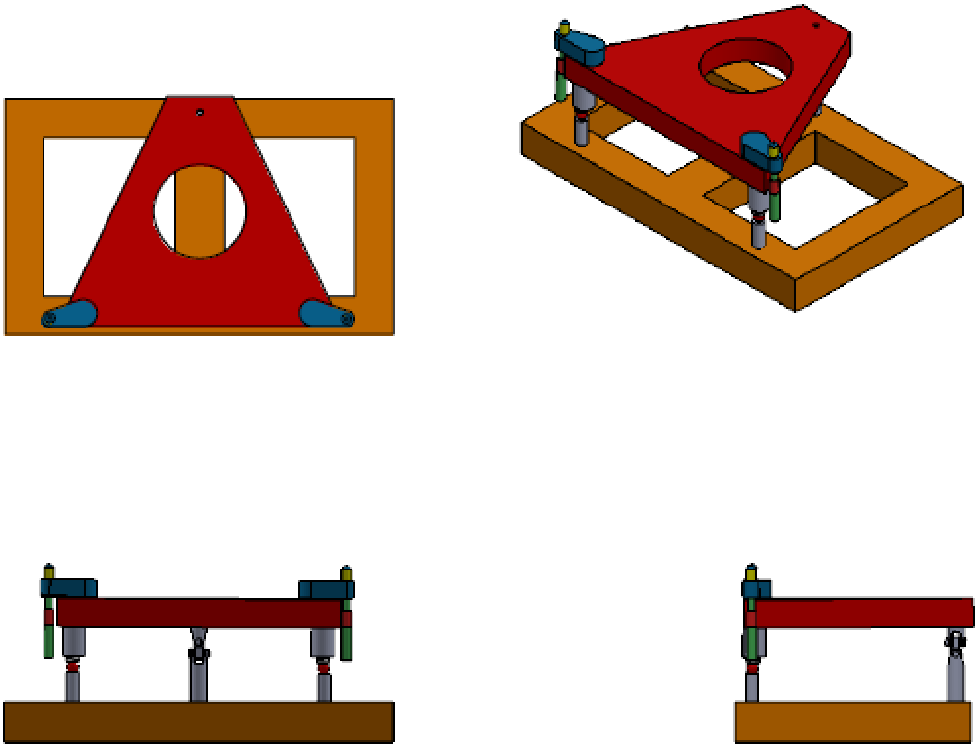


Figure 7.1 Final Design of Tripod Assembly

7.2 Detailed Dimensioning of Universal Joint Components:

7.2.1 Dimensions of Universal Joint (Lower shaft and Yoke):

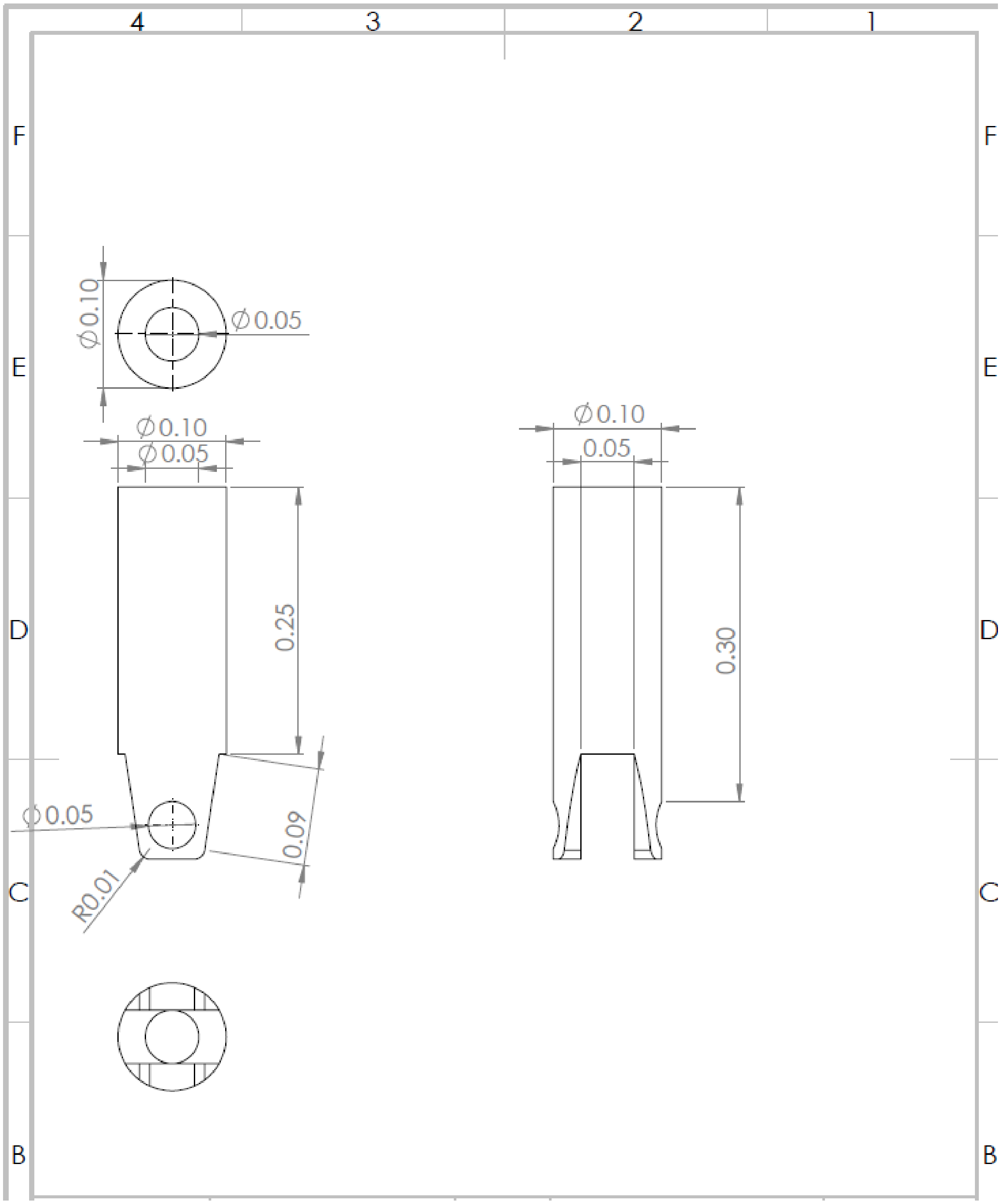


Figure 7.2 Dimension of Universal Joint (Lower Yoke)

7.2.2 Dimension of Universal Joint (Upper Shaft and Yoke):

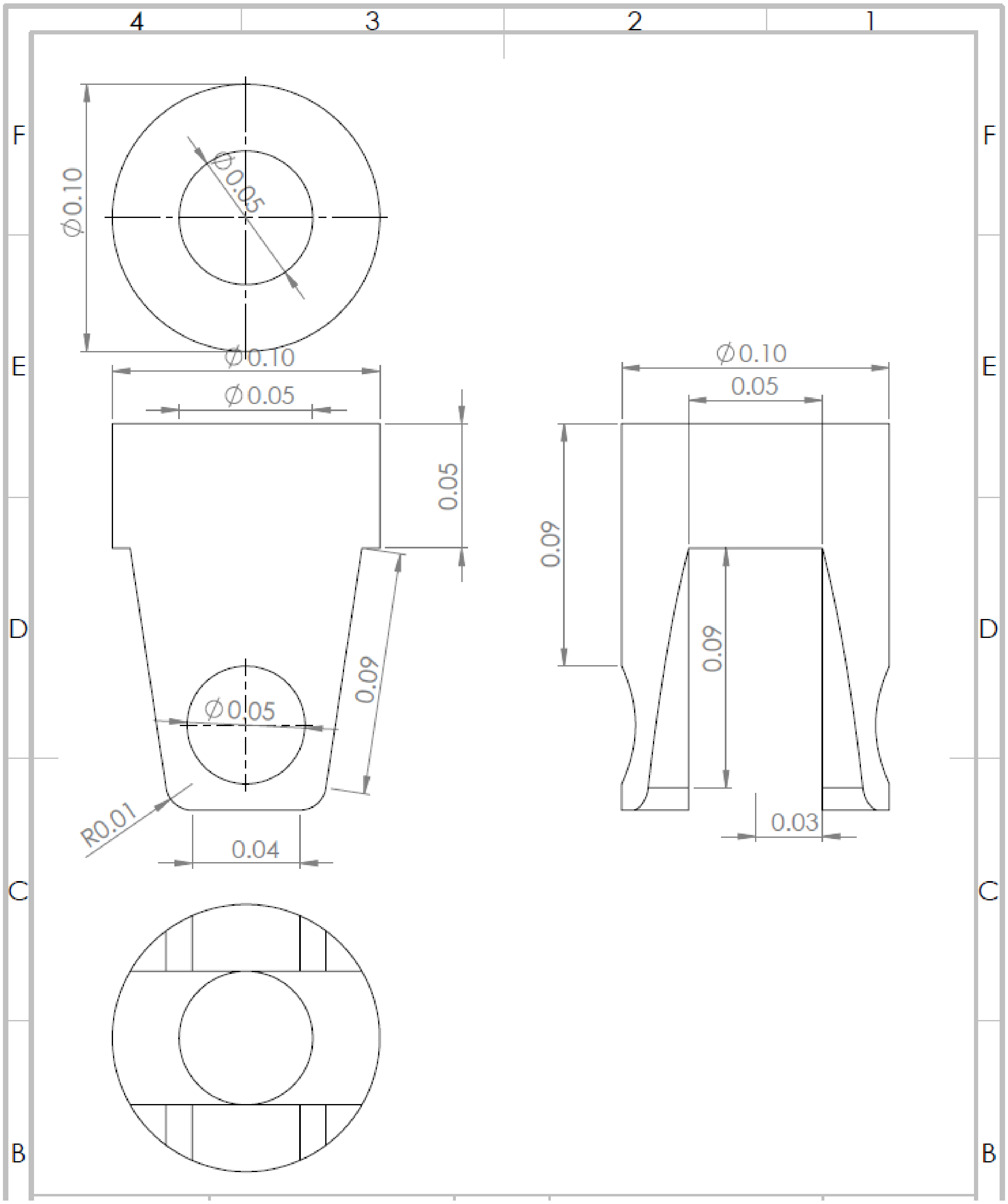


Figure 7.3 Dimension of Universal Joint (Upper Yoke)

7.2.3 Dimension of Universal Joint (Central Block):

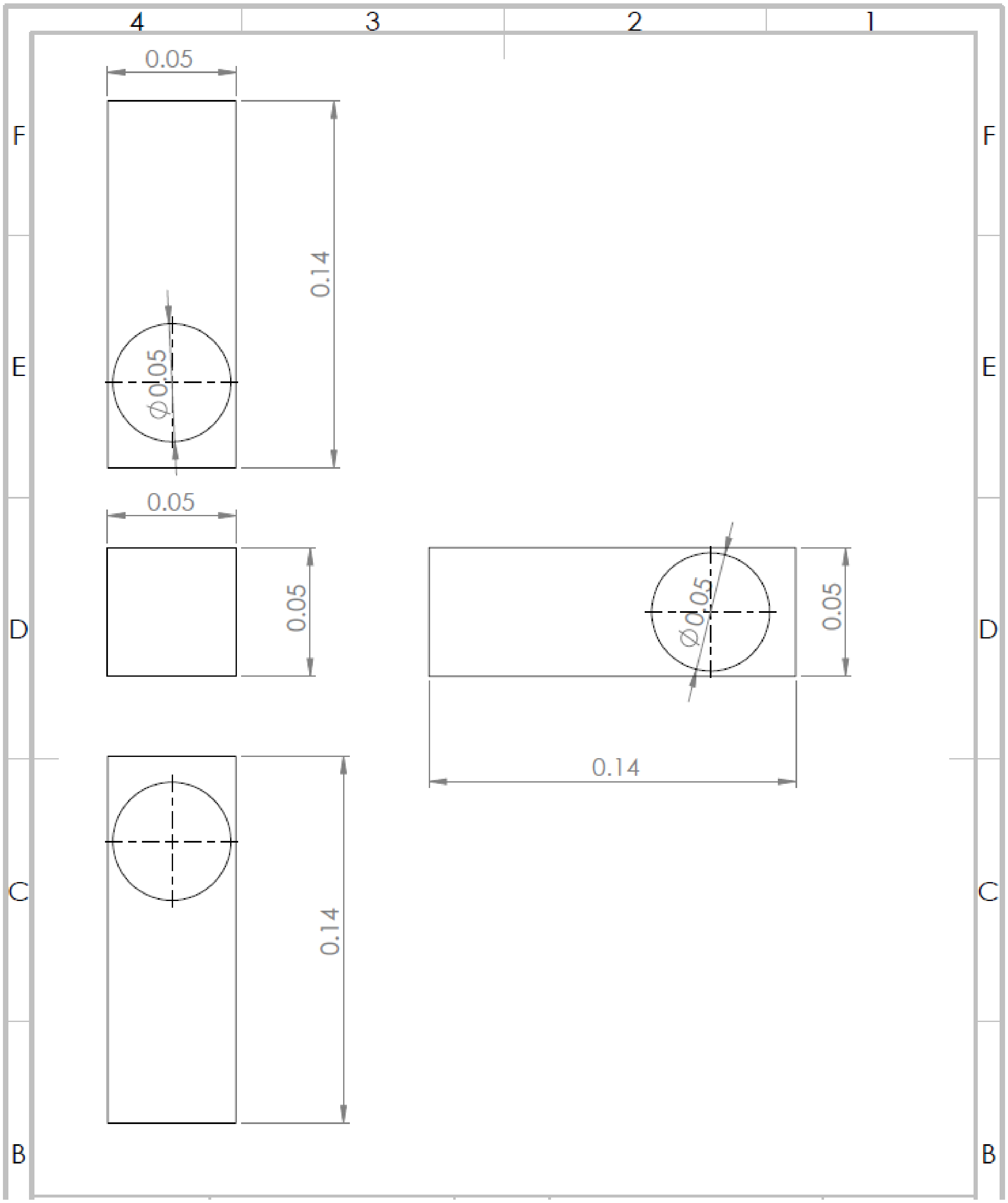


Figure 7.4 Dimension of Universal Joint (Central Block)

7.2.4. Dimension of Universal Joint (Supporting Pin):

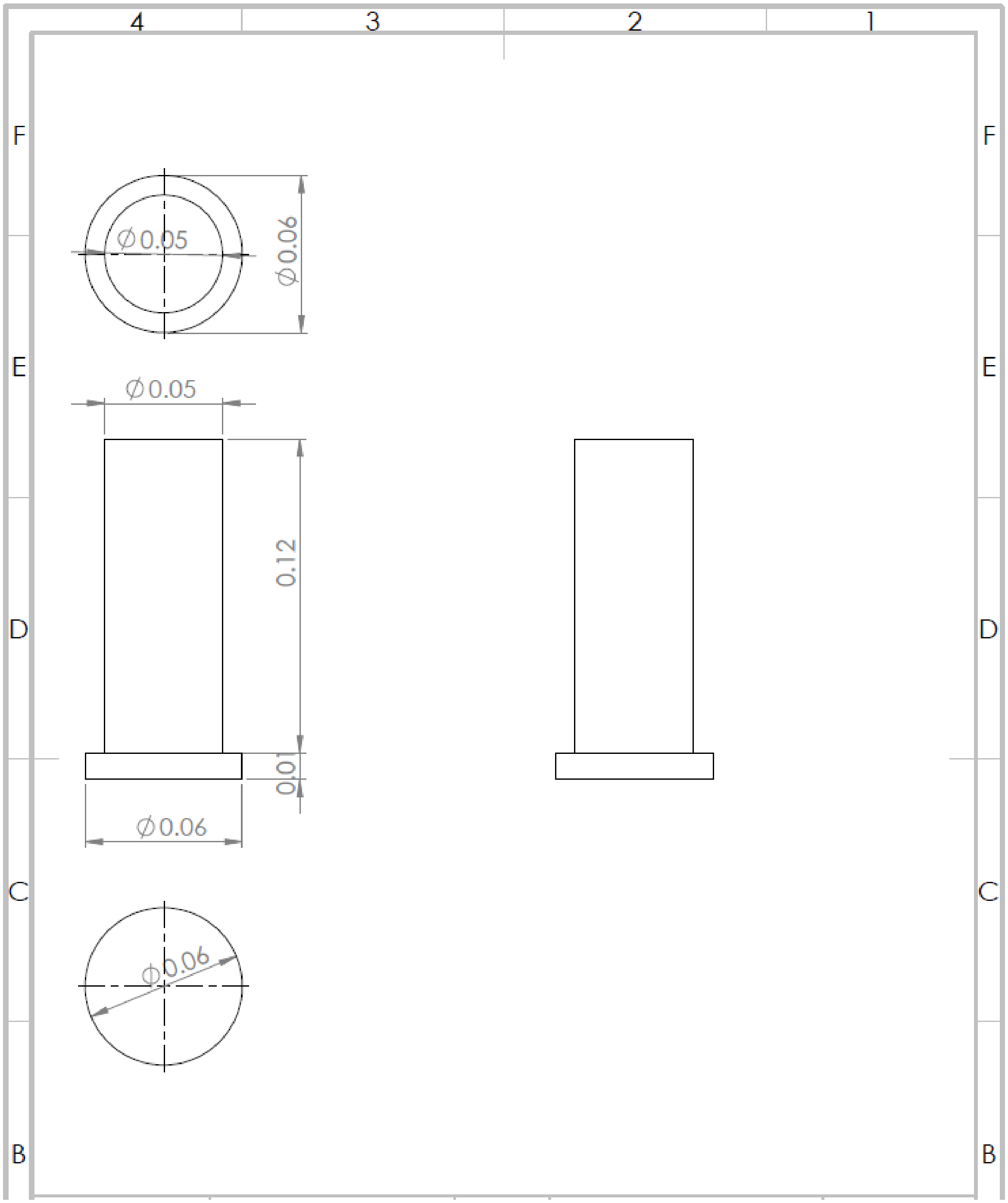


Figure 7.5 Dimension of Universal Joint (Supporting Pin)

CHAPTER 8: SCALED DOWN DEIGN FOR EXPERIMENTAL TESTING

The Design prototype is scaled down to 1:5 for fabrication and experimental testing under load of 4 T.

8.1 Final Scaled Down Design:

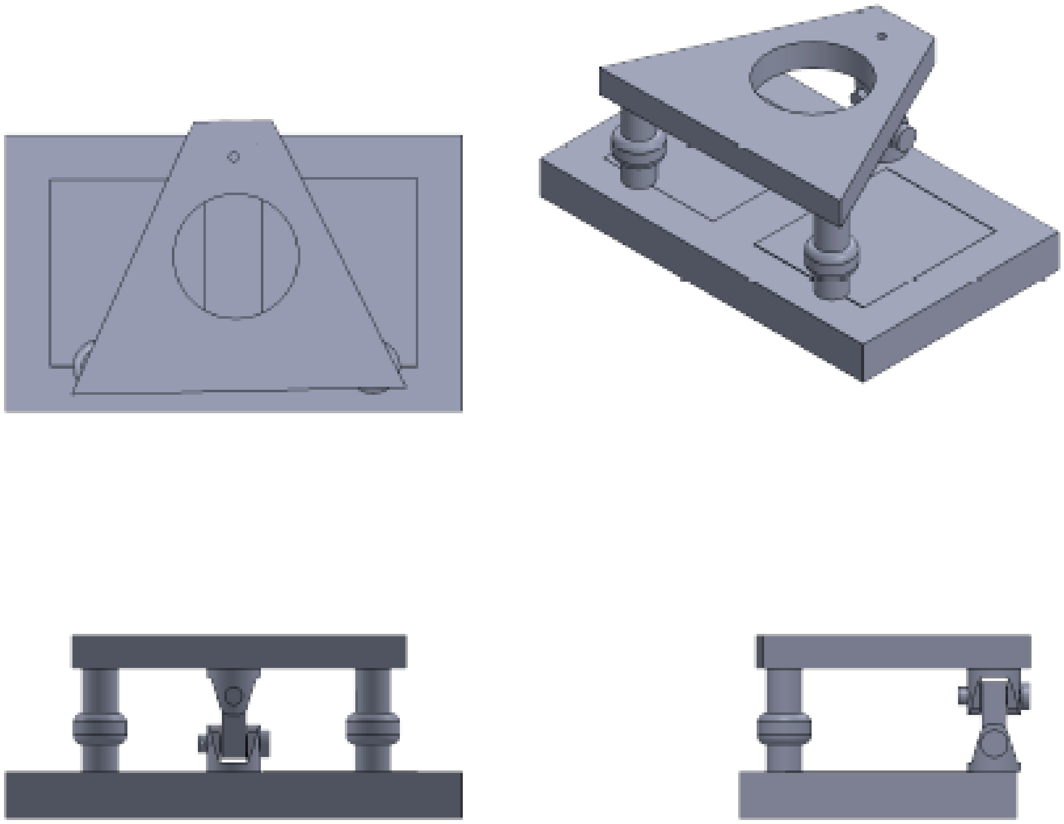


Figure 8.1Final Design (Scaled Down)

8.2 Simulation Results of Scaled Down Universal Joint:

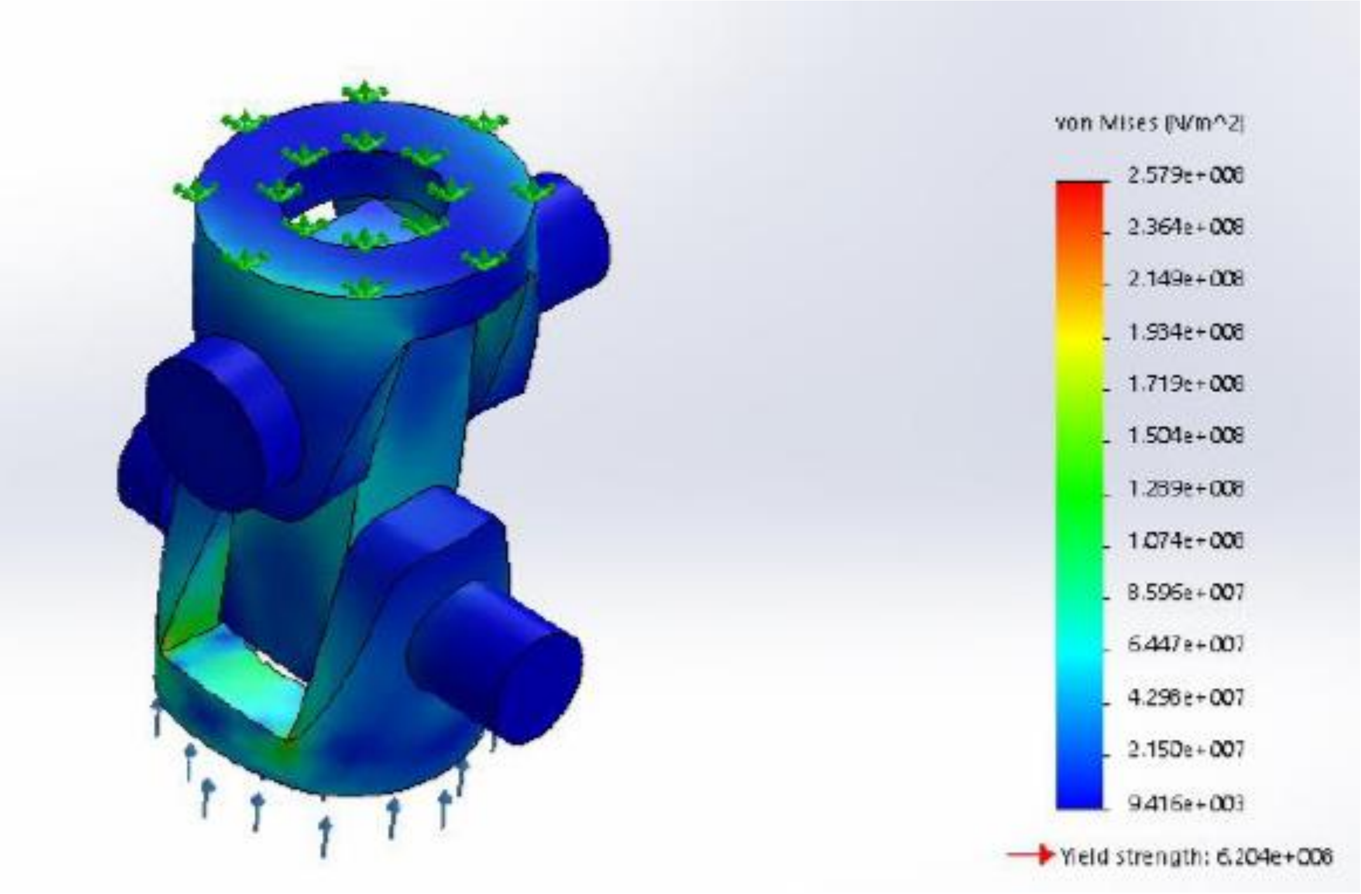


Figure 8.2 Simulation Result for combined Assembly (Scaled Down)

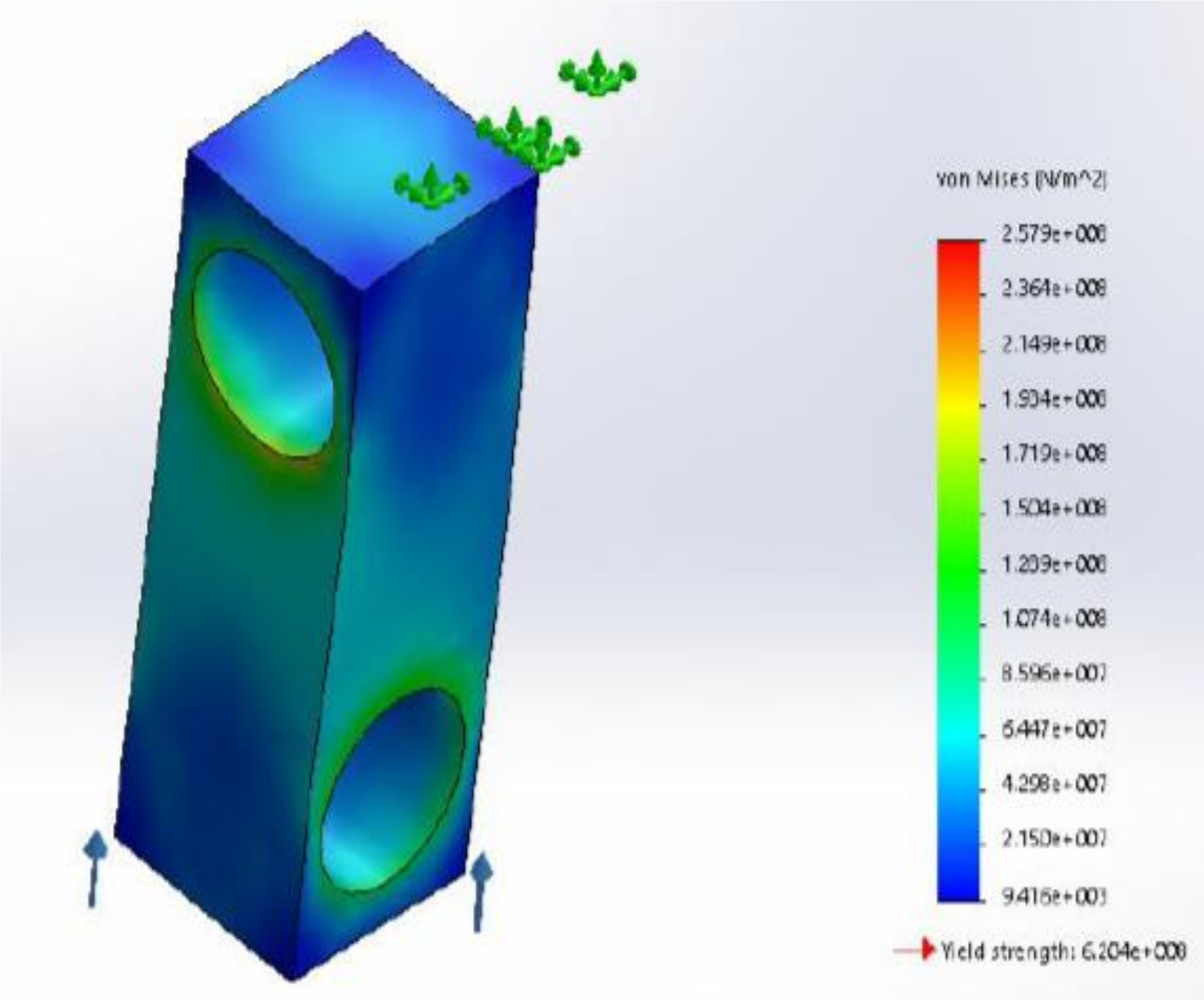


Figure 8.3 Simulation Result for central body (Scaled Down)

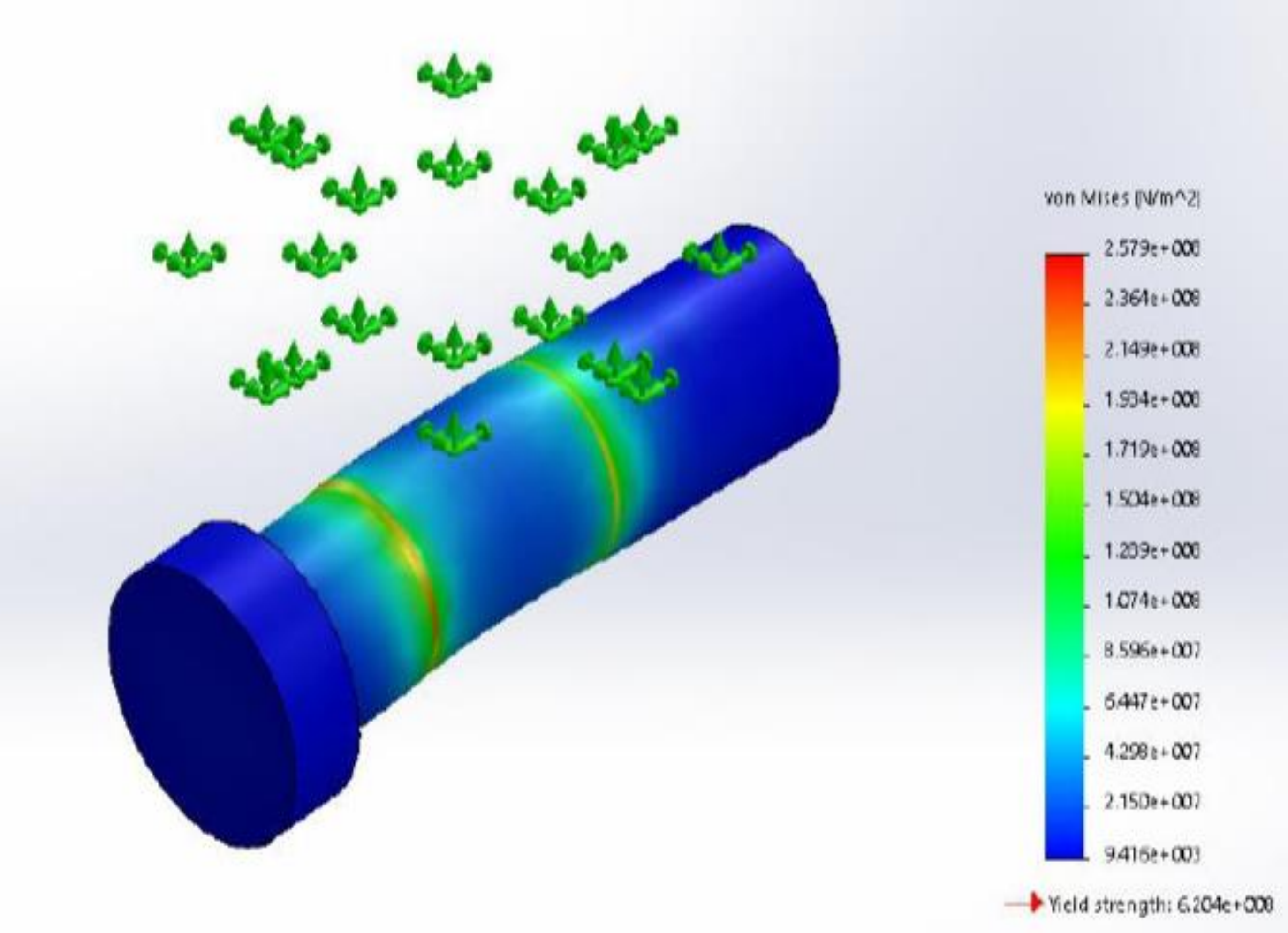


Figure 8.4 Simulation Result for connecting pins (Scaled Down)

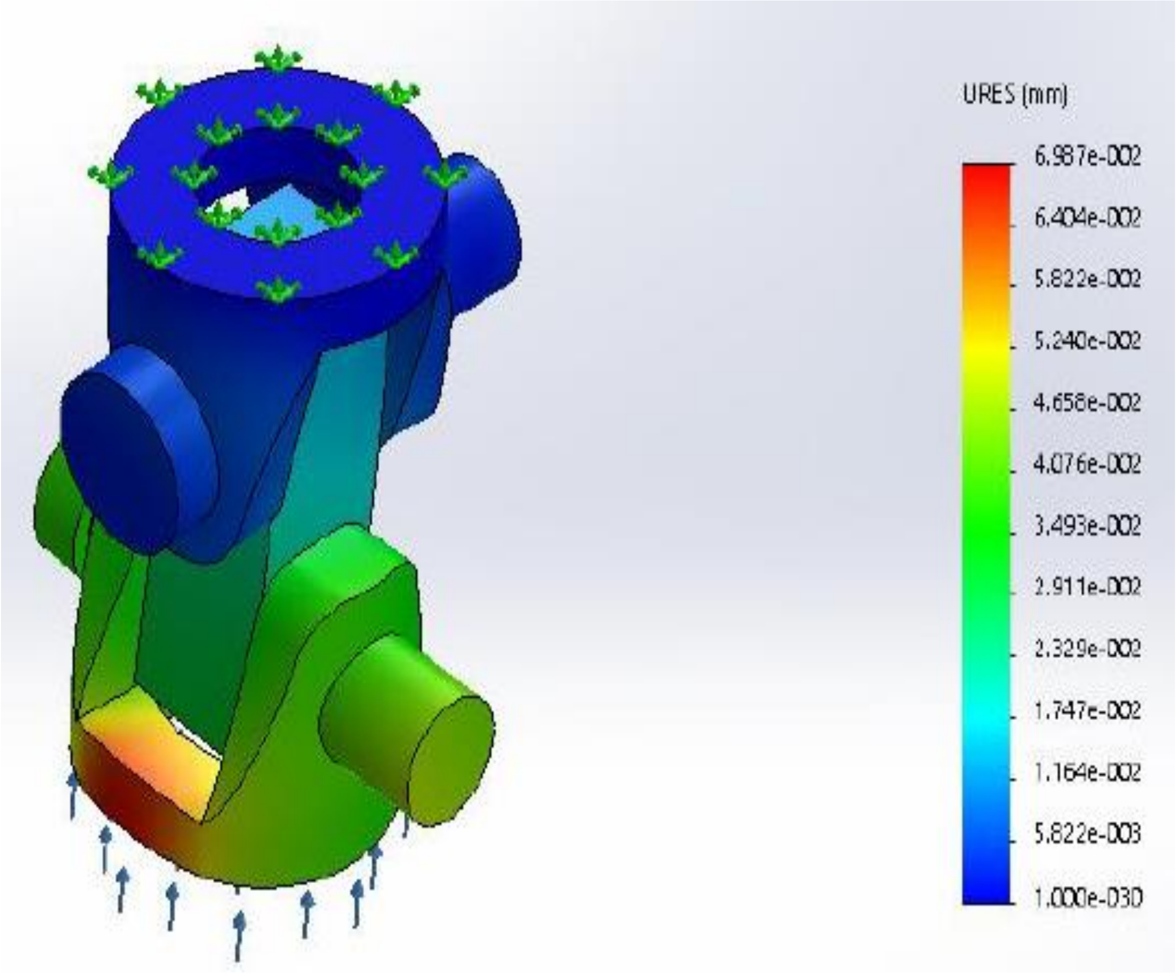


Figure 8.4 Simulation Result for Displacement (Scaled Down)

8.3 Detailed Dimensioning of Universal Joint (Scaled Down):

8.3.1 Detailed Dimensioning of Upper and Lower Yoke:

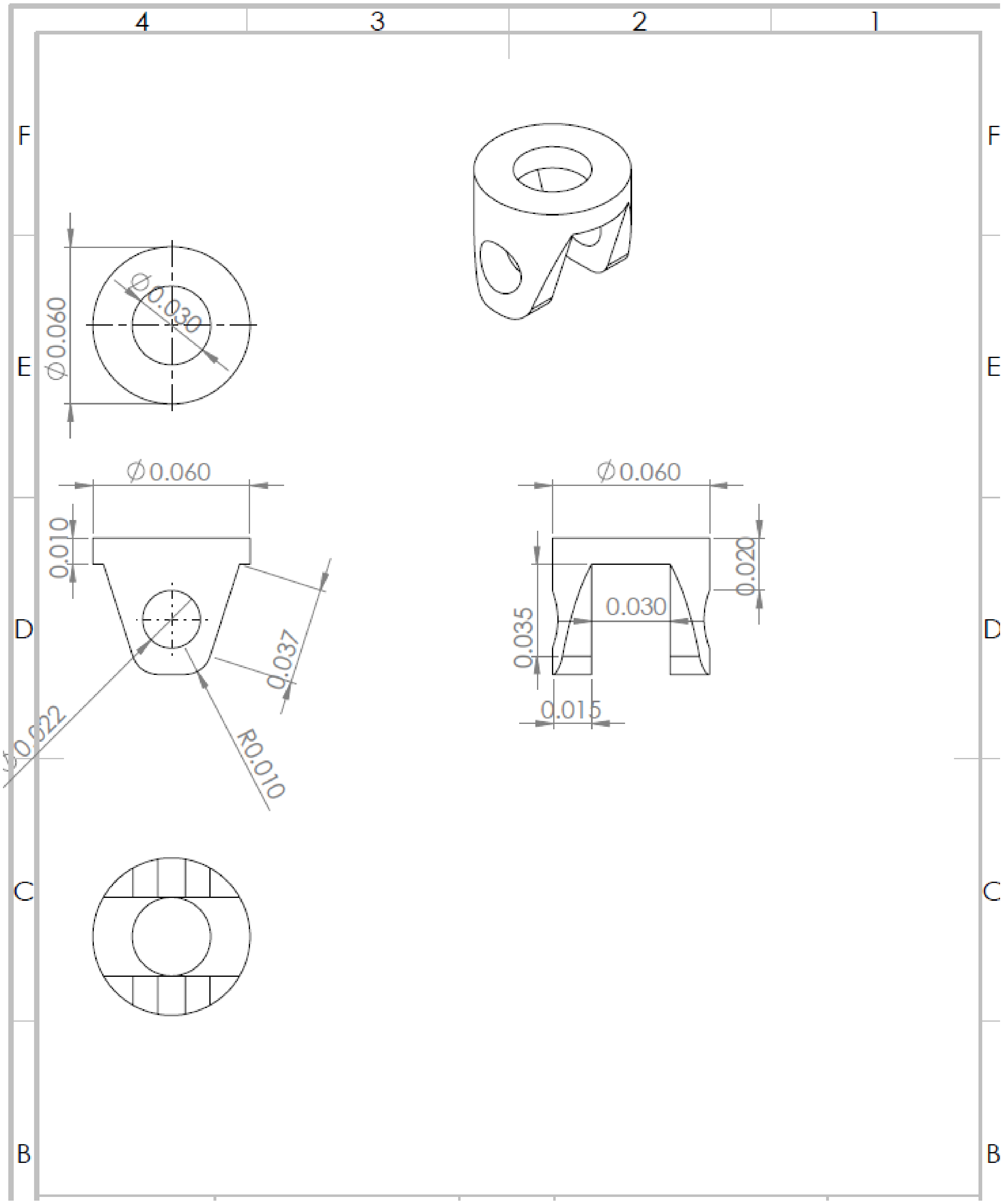


Figure 8.5 Dimensions of upper and lower yoke

8.3.2 Detailed Dimensioning of Central Block:

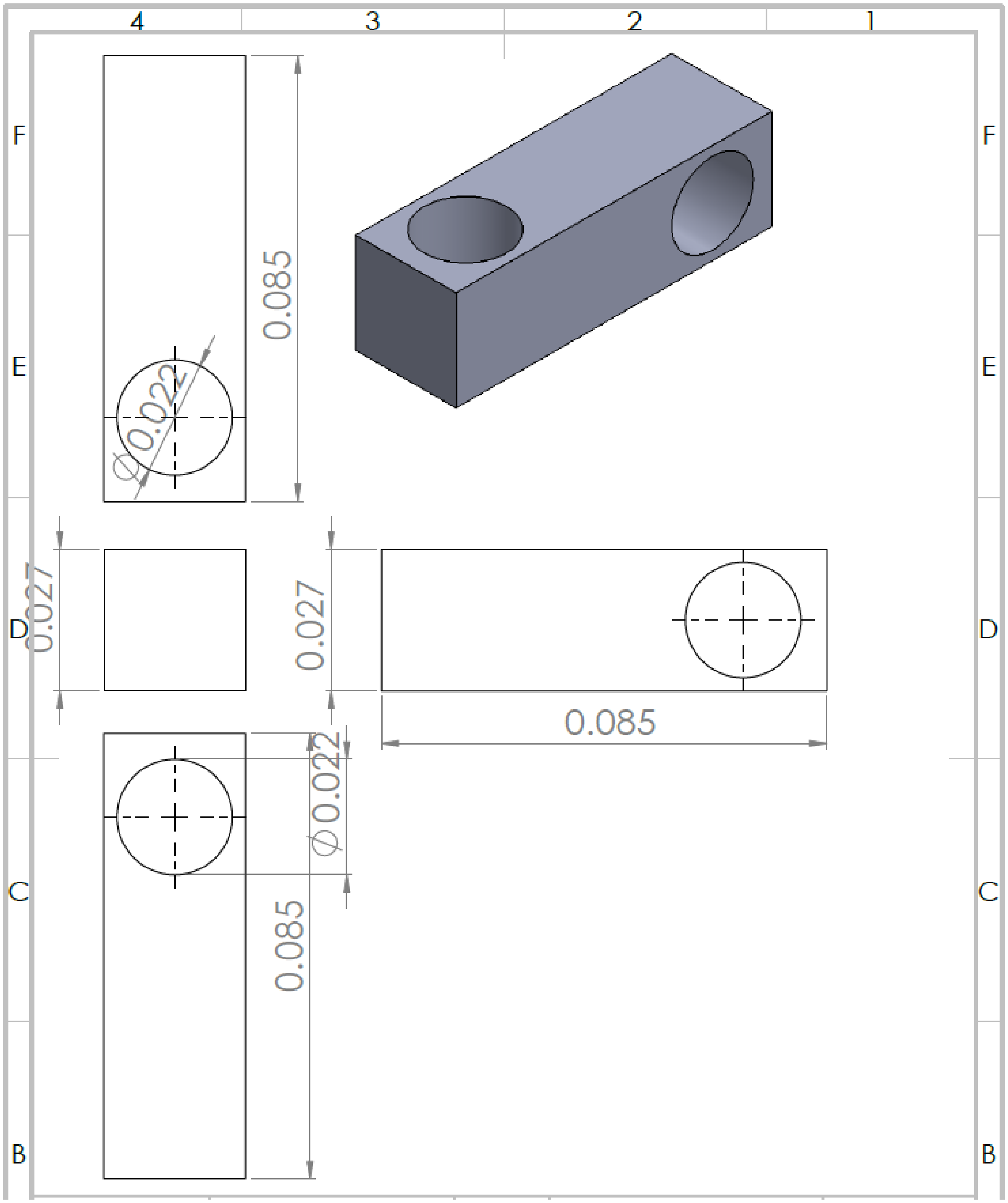


Figure 8.6 Dimension of Central Block

8.3.3 Detailed Dimension of Supporting Pins:

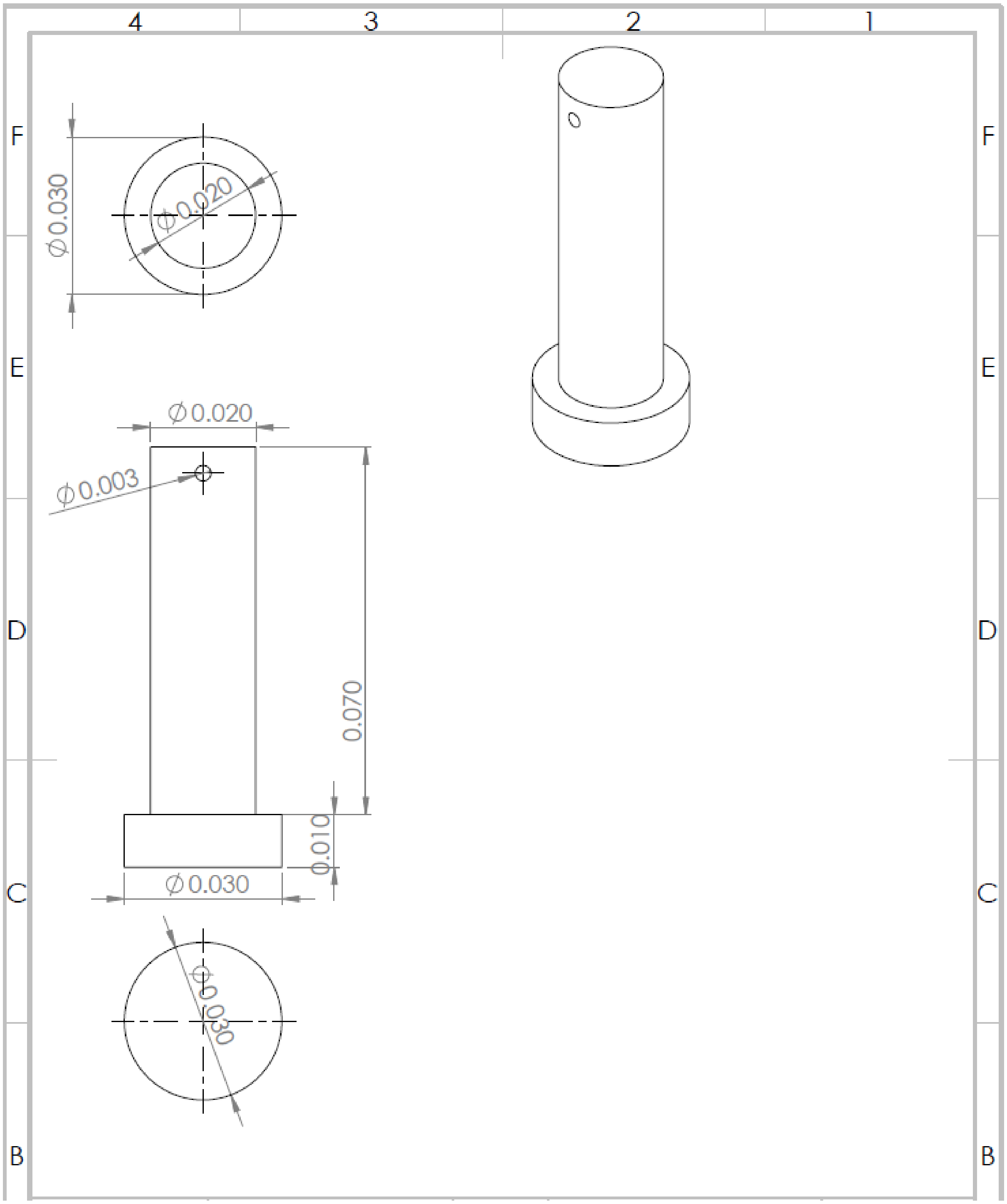


Figure 8.7 Dimension of Supporting Pins

8.4 Experimental Results:

No	Material Yield Strength	Applied Load (N)	Experimental Result (MPa)	FEA Result (MPa)
1	250 MPa	5000	32.9	34.39
2		10000	42.8	68.78
3		15000	55.75	101.2
4		20000	72.4	107.6
5		25000	94.2	157.6
6		30000	122.48	170.2
7		35000	159.23	200.6
8		40000	207.0	230.0

Table 8.1 Experimental Results

CHAPTER 9: CONCLUSION

By replacing spherical joint (central) with a universal joint of the tripod, the leveling mechanism becomes simpler as with the universal joint; three jack mechanism becomes two jack there-by reducing the complexity of the system. As the number of the jacks reduces, controlling the tripod mechanism becomes comparatively easier. Also spherical joint was allowing an extra DOF which was rotation about z-axis, so introducing universal joint (2 DOF), will restrict tripod unnecessary rotation.

Universal Joint’s kinematic envelop was calculated against all the possible positions of the elevating jacks. Kinematic and Dynamic Analysis were also performed on the Universal Joint in which velocity, accelerations of all the nodes of Universal Joint were calculated against applied forces and movements of elevating jacks. Finally, Stress Analysis was performed against load of 20T, which led us to our final design.

We also scaled down our design prototype to 1:5 for fabrication and testing purpose against load of 4T. The purpose of scaled down design was to fabricate a prototype so that the design experimental results can be compared to simulated results.

CHAPTER 10: RECOMMENDATIONS

1. Full Scale Design Prototype should be fabricated and experimented so that its results can be compared with the simulated results.
2. The remaining joints should also be analyzed so that to further optimize the design.
3. The sensors, actuators of the tripod leveling mechanism should be studied thoroughly to further optimize the system.

CHAPTER 11: REFERENCES

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[6] Farzad Vesali, Mohammad Ali Rezvani* and Mohammad Kashfi: Dynamics of universal joints, its failures and some propositions for practically improving its performance and life expectancy.

CERTIFICATE OF COMPLETENESS

It is hereby certified that the dissertation submitted by NS Muhammad Raza Ahmed Khan, Reg No. **NUST201464481MCEME35114F**, Titled: ***Design of Universal Joints for Heavy Loads*** has been checked/reviewed and its contents are complete in all respects.

Supervisor's Name: **Dr. Hasan Aftab Saeed**

Signature: ____________________

Date: ____________________