Liquid Level Control of Coupled Tanks System using Linear Model Predictive Control

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THESIS

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Abstract

Process Control is used extensively in industry and the requirements of process industry is increasing gradually, as optimal production rate with assurance of quality is required. Model Predictive Control (MPC) is an optimal control scheme which works on the principle of receding horizon. This work investigates the utilization of Discrete-time Linear Model Predictive Control in controlling nonlinear Coupled Tanks System. Initially, a basic Model Predictive Control scheme based on Generalized Predictive Control is applied. Secondly, a modified form of the basic approach is employed in which the error between the actual output of the system and the calculated output is considered in predicting the future outputs. In these two schemes, approximation of the control signal requires a large control horizon resulting in high computational burden. In the interest of reducing the number of parameters defining the control signal and good performance MPC scheme which utilizes Laguerre functions to capture the control trajectory is used. The operational constraints of the coupled tanks system are also incorporated in the Laguerre polynomials based predictive control scheme. Simulation results have been included which demonstrate the performance of controllers when used to control SISO and MIMO nonlinear Coupled Tanks System.

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SYMBOLS

a	Laguerre Scaling Factor
A1	Area of Cross-Section of Tank 1
A2	Area of Cross-Section of Tank 2
a0	Discharge Coefficient of Channel 1 and Channel 2
a1	Discharge Coefficient of Channel 3
s1	Channel 1 Section Area
s2	Channel 2 Section Area
s3	Channel 3 Section Area
q1	Input to Tank 1
q2	Input to Tank 2
y1	Tank 1 Output
y2	Tank 2 Output
Qi1	Pump Flow Rate in to Tank 1
Qi2	Pump Flow Rate in to Tank 2
Q01	Flow Rate out of Tank 1
Q02	Flow Rate out of Tank 2
Q03	Flow Rate of Liquid between Tanks
H1	Height of Liquid in Tank 1
H2	Height of Liquid in Tank 2
Np	Prediction Horizon
Nc	Control Horizon

N1, N2	Minimum and Maximum Prediction Horizon
$\Delta u(k)$	Change of Process Input
$u(k)$	Manipulated Input
Δ	Difference Operator
J	Cost Function
$\xi(t)$	White Noise Disturbance
Kob	Optimum Observer Gain
$y(k)$	Process Output
Xm	State Variable Vector
X(k+t/ki)	Future State Variables of MPC
Y(k+ t/ki)	Predicted Outputs of MPC
rw	Tuning Parameter for Closed Loop Performance of Basic MPC
δ	Delta Function
Γ	Z-transform of Discrete-time Laguerre Function
N	Number of Laguerre Terms
L(k)	Discrete-time Laguerre Function
d(k+i)	Error in Prediction of System
Rs	Vector Containing Set point Information
η	Laguerre Parameter Vector
g	Gravitational Constant
A	System Matrix
B	Input Matrix
C	Output Matrix

ABBREVIATIONS

CARIMA	Controlled Auto-Regressive Integrated Moving Average
CTS	Coupled Tanks System
DMC	Dynamic Matrix Control
GPC	Generalized Predictive Control
IMC	Internal Model Control
LMPC	Linear Model Predictive Control
LQG	Linear Quadratic Gaussian
MIMO	Multi-Input/Multi Output
MPC	Model Predictive Control
PID	Proportional-Integral-Derivative
P-PID	Predictive Proportional-Integral-Derivative
QP	Quadratic Programming
RHC	Receding Horizon Control
SISO	Single-Input/Single-Output
TITO	Two-Input/Two-Output
RMSE	Root-Mean-Square Error

Chapter 1

INTRODUCTION

1.1 Motivation

Process control is used extensively in industry as it enables automation and a complex process can be operated easily from a central control system. For more than fifty years Proportional-Integral-Derivative (PID) control has dominated the field of process control as the industrial standard. The reason for the popularity of PID controller is its simplicity, flexibility, ability to deliver adequate performance for most control problems, low computational cost and having a design algorithm that has not been altered in all these years. However, PID does have some demerits. PID controllers have difficulty handling processes with nonlinear dynamics, processes with delay and processes with signal noise. Tuning a PID controller is not a simple task and requires training and experience. Thus PID control leaves margin for improvement in production, production quality and operating costs. Majority of the alternate control schemes require computers capable of high clock speed, which was a big issue in the past as computers back then were very expensive. However, the computing technology has grown extensively over the years and computers have become cheaper and more powerful. Thus, it is now possible to implement modern control schemes which require high computational power. The requirements of process industry increase by the day as optimal production rate with assurance of quality is demanded. Model Predictive Control is considered as one of the most significant developments in control engineering. Model Predictive Control (MPC) is an optimal model based control scheme which utilizes the receding horizon control. In MPC optimal control law is computed by solving an optimization problem and a plant model is used to predict the future output response of a plant.

Liquid level and flow control is a basic problem in process industries. The coupled tanks system is one of the most commonly used coupled multivariable system. The coupled tanks design used in this research is CE105 apparatus, it is a popular design that mimics a real industrial process and used and used by researchers to investigate basic and advanced control engineering principles. It can be configured as a SISO or as a MIMO system via manipulation of the control valves. Level control in such design is difficult because the levels of both tanks interact. Normally, multivariable systems are decoupled and separate controllers are used to control the states but even after decoupling constraint handling remains an issue. The receding horizon approach and constraint handling capability of Model Predictive Control make it an appropriate choice for liquid level control of the coupled tanks system.

1.2 Objective of the Thesis

The primary objective of this research is to control the liquid level of nonlinear coupled tanks system (CTS) using Linear Model Predictive Control (LMPC) in MATLAB, Simulink environment. Following are the sub-objectives taken together from the main objective:

- In depth study of predictive control approaches
- Mathematical modeling of coupled tanks system CE105
- Developing algorithm of MPC schemes in MATLAB script
- Application of MPC schemes to nonlinear coupled tanks system
- Comparative study of MPC approaches based on simulated results in MATLAB/Simulink environment
- Inclusion of hard constraints in the MPC scheme and simulations

1.3 Thesis Organization

The thesis has been organized into six chapters.

Chapter 1: Gives the motivation behind Model Predictive Control for the Coupled Tanks System, objectives of the thesis and the organization of the thesis.

Chapter 2: This chapter includes the major relevant research work studied in depth to help us in this thesis work.

Chapter 3: In this chapter we discuss the nonlinear coupled tanks system. In this chapter, the mathematical model of the plant is developed using first principle method with detailed description of model parameters. The system is linearized and the linearized perturbation models for both SISO and TITO systems are derived.

Chapter 4: This chapter provides all the details related to discrete linear model predictive control methodology. In this research we started with a single MPC scheme but based on our finding we ended up using three linear MPC approaches. This chapter gives detailed information on all the three algorithms. The chapter also covers the constraint handling capabilities of MPC.

Chapter 5: This chapter shows the performance of controller when applied firstly to the SISO and then the MIMO nonlinear coupled tanks system using the MPC approaches. We will discuss the performance of controller for the liquid level control and the effect on performance when control parameters are varied. All the simulated results are shown and discussed.

Chapter 6: In this chapter the findings of this research are discussed and future enhancements are suggested so that further improvements can be made to the results obtained.

Chapter

2

LITERATURE REVIEW

2.1 Introduction

This chapter will cover research findings related to coupled tanks system, predictive control approaches and their comparison, model predictive control and control of coupled tanks system using linear model predictive control.

2.2 Major Research Studies

2.2.1 Advances in PID Control, Predictive Control of Non-Minimum Phase Systems [1]

This research is mainly focused on predictive control strategies used to cope with problems such as non-minimum systems showing time delay or undershoots. In this work the classical and the modern predictive control approaches were compared.

Researchers that addressed the problem concerning non-minimum phase systems having time delay or undershoot characteristics from a predictive control point of view mainly used on of two strategies: a classical non-optimal approach or a modern optimization based predictive approach [2]. The classical predictive controllers that are most widely considered include the Smith predictor structure and the internal model control (IMC) structure [3-5]. Modern predictive controllers consider generalized predictive control (GPC) [2].

The smith predictor and internal model control are two of the most popular classical predictive control approaches used to cope with the undesirable time delay characteristics. Padé approximation is used to estimate the time delay by rational transfer function.

The Smith Predictor Control is a feedback control scheme used to eliminate the output delay, Plant. Let the plant representation be, $G_p(s) = G(s)e^{-Ts}$ .Where T>0 is the time delay. The internal loop that simulates the dynamics of the plant be defined as, $G_m(s) = \hat{G}(s)e^{-\hat{T}s}$. Where $\hat{T}$ is the approximation of time delay. Which results in feedback $Y_f(s) = \hat{G}(s)U(s)$. Thus the resulting feedback obtained only depends on model of the plant. The structure of Smith Predictor controllers are shown below.

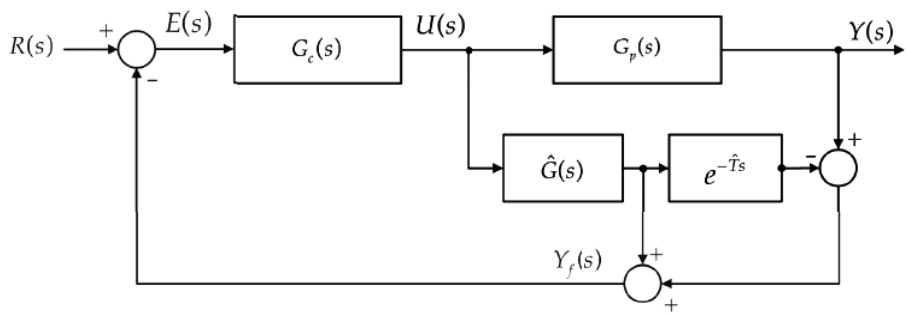

Fig. 2.1 The smith predictor structure

The Internal Model Control can be viewed as an extension of the Smith Predictor [6]. The transfer function of controller is given as,

$$C(s) = \frac{G_{IMC}}{1 - G_m(s)G_{IMC}}$$

And the system transfer function,

$$T(s) = \frac{G_P(s)C(s)}{1 - G_p(s)C(s)}$$

The controller C(s) drives the output to track the reference input. Thus resulting in output Y(s)=R(s).

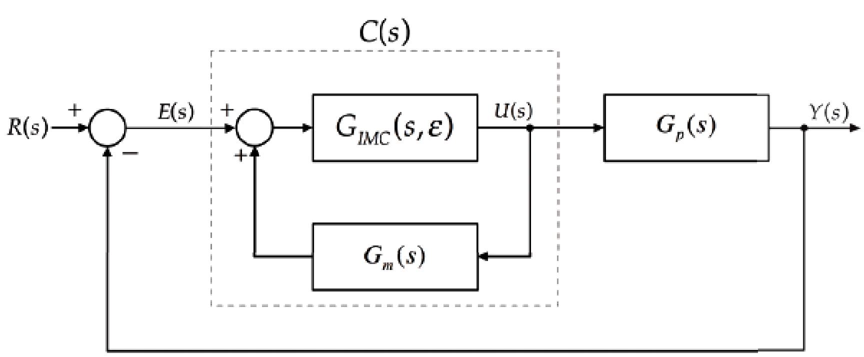

Fig. 2.2 The internal model control structure

The modern predictive approach used in this research formulates the control problem in terms of generalized predictive control algorithm [7, 8]. This method has found wide-spread application in process industries and research in the field is very active [9].

The conclusion of this research was that, the classical approaches effectively deal with non-minimum-phase systems but require approximations of time delays such as Padé approximation to be included in the control algorithm. The modern approach is applicable to non-minimum phase systems, open loop unstable plants having variable dead time. The modern approach outperforms the classical approaches with the novelty that time delays do not require approximations to be included in the algorithm.

2.2.2 Generalized Predictive Control [10]

A novel method generalized predictive control or GPC is developed as a general purpose controller capable of controlling majority of the processes.

- Non-minimum phase systems
- Open-loop unstable plant
- Plant with variable or unknown dead time
- Plant having unknown order

This method uses the Controlled Auto-Regressive and Moving-Average (CARIMA) plant model. Appropriate quadratic function of the future errors is minimized. Simulation studies demonstrate the superior performance of GPC.

2.2.3 Discrete Time Model Predictive Control Design Using Laguerre Functions [11]

In the model predictive control methodology, the general approach for expanding the projected control trajectory is by using the difference of control signal $\Delta u(k)$. In the previous approaches, in case of complicated process dynamics and high demands on closed loop performance, approximation of the control sequence required large number of terms.

In this research an appropriate approximation of the projected control signal related to Laguerre networks is introduced. It is assumed that within a moving horizon window, the difference of control signal behaves like the impulse response of a stable system and like the impulse response of a stable system decays after some time the difference of control system also exponentially decays after some initial time period.

The quadratic optimization is solved in terms of Laguerre parameters and the simulation results show smoother control input and that the number of terms used in the optimization procedure can be reduced greatly.

2.2.4 A Case Study of Predictive Control from the ALSTOM Gasifier Problem [12]

This work studies the control formulation for ALSTOM gasifier. The novelty of this work is that a linear state space model is used as controller internal model to control a nonlinear multivariable system. MPC approach is used with some changes to the standard algorithm [8].

MPC is known for its capability to handle multivariable interaction and process constraints in the most natural way. A linear state space model identified at 0% load condition is used as the internal model.

MATLAB simulations show that performance deterioration occurs at 50% and 100% load conditions but the controller is able to achieve the control specifications under the process constraints.

2.2.5 Fuzzy Sliding Mode Controller Applied to the Coupled Tanks System [13]

This paper is aimed develop a control strategy for level control of CE105 SISO Coupled Tanks. The reference levels used in the experiments to verify the efficiency of the controller is a pulse train with varied amplitude.

The experimental results presented in this paper indicate that the approach used has considerable advantages compared to classical sliding mode control.

2.2.6 Multivariable Predictive PID Control of Quadruple Tank [14]

This paper is concerned with the design of MIMO predictive PID controller for control of a MIMO control problem, linear quadruple tank system. In the paper the MPC approach similar to [8] has been used to obtain the P-PID structure. The linear system model is developed using First Principle method.

Simulations show a comparison between PID and P-PID control of linear quadruple tank system. The results conclude that linear P-PID is more effective than the conventional PID controller for control of quad tank system.

2.3 Summary

In this chapter a brief overview of the research work in the field of predictive control, Different predictive control approaches that have been adopted by researchers and their findings, model predictive control and control of coupled tanks system using discrete linear model predictive control were discussed. The research works and their results along with the advantages were explained.

Chapter

3

THE COUPLED TANKS SYSTEM

3.1 Introduction

In this chapter the nonlinear plant (coupled tanks system) will be discussed. The linearized state space model of the plant and the model parameters will determined. In this chapter, the mathematical modeling of the plant will be performed using first principle method. The system linearization and the linearized perturbation models for both SISO and TITO systems will be derived.

3.2 Overview of the Coupled Tanks System

In the process industries the control of liquid level in tanks and regulated flow between tanks is a basic problem. The coupled tanks system consists of two vertical tanks interconnected by a flow channel as shown in Fig. 3.1. Each tank has an independent pump for inflow of liquid in to the tank. The sectional area of the outlets present at the base of each tank and the channel connecting the two tanks can be varied with rotary valves. Models like coupled tanks system are essential in various industries such as, petrochemical, paper making and water treatment industries. The unique thing about coupled tanks system is that, since the tanks are coupled together the levels interact and must be controlled. The coupled tanks system can be configured as a Single Input Single Output (SISO) or as a Multi Input Multi Output system.

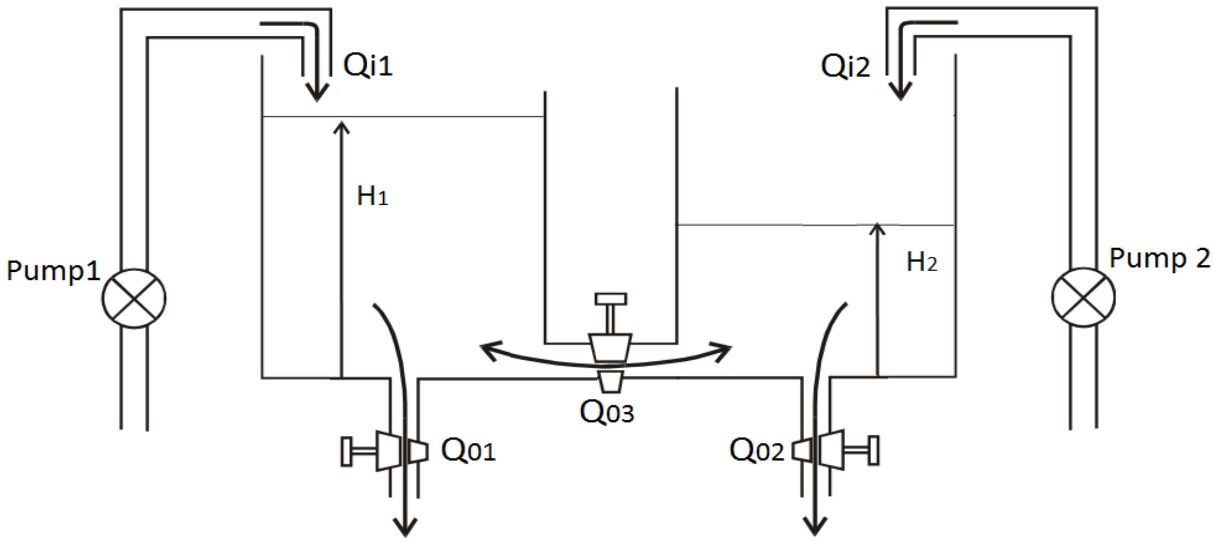

Fig. 3.1 The Coupled Tanks System

3.3 Determination of Model Parameters

The system model used in model predictive control can be determined by various methods [15]. The formulation of control law for the model predictive control scheme used in this thesis is based on the state-space model of the system and thus the state-space model of the coupled tanks system has to be determined with minimal model-plant mismatch. There are two ways of determining the linearized state-space model of the coupled tanks system:

- By performing a series of tests on the plant, signals such as step, sine, pseudo-random are applied to the system and the output is saved. Then a system identification scheme is employed to obtain the linear plant model.

- By using the First Principles method. In this method the plant dynamics can be described by using nonlinear equations. For the derivation of the nonlinear equations the complete knowledge of the process in the system such as thermodynamic, chemical processes and the physical specification is required. The nonlinear system is then linearized by using an appropriate linearization method.

For this research the First Principles method is used to obtain the nonlinear process model.

3.3.1 Mathematical Modeling of MIMO Coupled Tanks System Using First Principle Approach

The first principle method uses an application of conservation of mass known as mass balance.

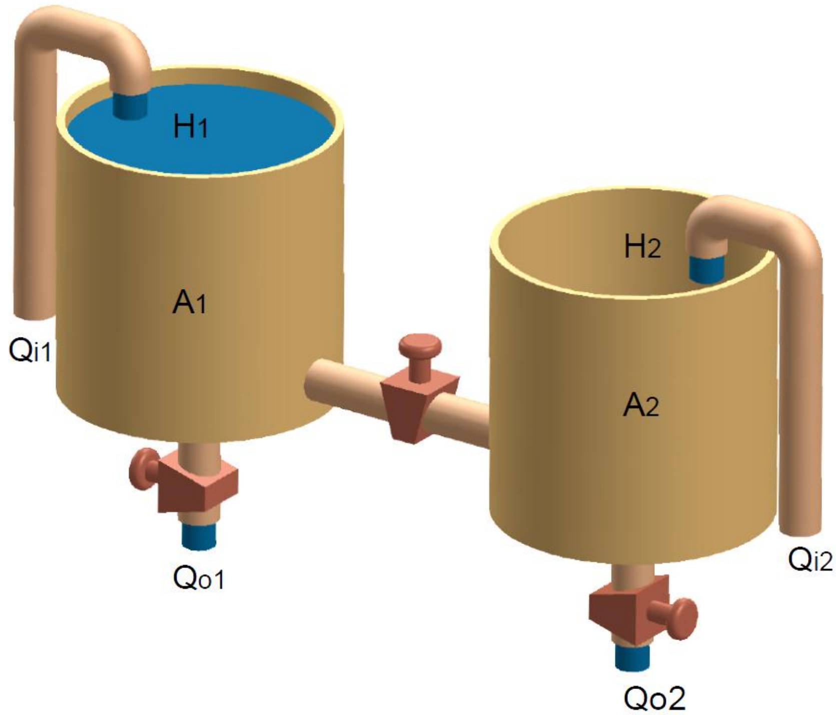

Fig. 3.2 Dynamics of the Coupled Tanks System

Since area of the tank is constant, the time rate of change of liquid level in the tank can be expressed by accounting for liquid entering and leaving the tank. The coupled tanks dynamics can be seen in Fig. 3.2.

Considering mass balance, the dynamic equation of each tank is developed.

Tank 1 flow balance equation,

$$A_1 \frac{dH_1}{dt} = Q_{i1} - Q_{01} - Q_{03} \tag{3.1}$$

Tank 2 flow balance equation,

$$A_2 \frac{dH_2}{dt} = Q_{i2} - Q_{02} - Q_{03} \tag{3.2}$$

Where,

H1= height of liquid in tank 1
H2= height of liquid in tank 2
A1= cross-sectional area of tank 1
A2= cross-sectional area of tank 2
Qi1= pump flow rate in to tank 1
Qi2= pump flow rate in to tank 2
Q01= flow rate of liquid out of tank 1
Q02= flow rate of liquid out of tank 2
Q03= flow rate of liquid between tanks

By Bernoulli's equation for a non-viscous, incompressible fluid in steady flow, flow rate of liquid through the orifices can be calculated,

$$Q_{01} = s_1 a_0 \sqrt{2g}\sqrt{H_1} = \alpha_1 \sqrt{H_1} \tag{3.3}$$

$$Q_{02} = s_2 a_0 \sqrt{2g}\sqrt{H_2} = \alpha_2 \sqrt{H_2} \tag{3.4}$$

$$Q_{03} = s_3 a_1 \sqrt{2g}\sqrt{H_1 - H_2} = \alpha_3 \sqrt{H_1 - H_2} \tag{3.5}$$

Where,
a0= discharge coefficient of channel 1 and channel 2
a1= discharge coefficient of channel 3
s1= channel 1 sectional area
s2= channel 2 sectional area
s3= channel 3 sectional area
α1, α2 and α3= the respective proportionality constants which depend on the coefficients of discharge, the cross-sectional area of each tank and gravitational constant.

Using values from Eq. (3.3), (3.4) and (3.5) in Eq. (3.1) and (3.2) the non-linear equations that describe the dynamics of the Multi-Input and Multi-Output system are derived:

$$A_1 \frac{dH_1}{dt} = Qi_1 - \alpha_1 \sqrt{H_1} - \alpha_3 \sqrt{H_1 - H_2} \tag{3.6}$$

$$A_2 \frac{dH_2}{dt} = Qi_2 - \alpha_2 \sqrt{H_2} + \alpha_3 \sqrt{H_1 - H_2} \tag{3.7}$$

3.3.1.1 Linearized Perturbation MIMO Model

Considering a small variation of q_1 and $q_{2,}$ in both the control inputs respectively. Let h1 and h2 be the resulting change in heights of the two tanks due to this variation. Considering this variation Eq. (3.6) and (3.7) can be written as,

For Tank 1

$$A_1 \frac{d(H_1 + h_1)}{dt} = (Q_{i1} + q_1) - \alpha_1 \sqrt{H_1 + h_1} - \alpha_3 \sqrt{H_1 - H_2 + h_1 - h_2} \tag{3.8}$$

For Tank 2

$$A_2 \frac{d(H_2 + h_2)}{dt} = (Q_{i2} + q_2) - \alpha_2 \sqrt{H_2 + h_2} + \alpha_3 \sqrt{H_1 - H_2 + h_1 - h_2} \tag{3.9}$$

Subtracting Eq. (3.6) and (3.7) from Eq. (3.8) and (3.9):

$$A_1 \frac{dh_1}{dt} = q_1 - \alpha_1 (\sqrt{H_1 + h_1} - \sqrt{H_1}) - \alpha_3 (\sqrt{H_1 - H_2 + h_1 - h_2} - \sqrt{H_1 - H_2}) \tag{3.10}$$

$$A_2 \frac{dh_2}{dt} = q_2 - \alpha_2 (\sqrt{H_2 + h_2} - \sqrt{H_2}) + \alpha_3 (\sqrt{H_1 - H_2 + h_1 - h_2} - \sqrt{H_1 - H_2}) \tag{3.11}$$

For small perturbations, in accordance with the Binomial expansion.

$$(1+x)^n \approx 1+nx \tag{3.12}$$

Thus,

$$\sqrt{(H_1+h_1)} = \sqrt{H_1}(1+\frac{h_1}{H_1})^{0.5} \tag{3.13}$$

$$\approx \sqrt{H_1}(1+\frac{h_1}{2H_1})$$

Therefore,

$$(\sqrt{(H_1+h_1)} - \sqrt{H_1}) \approx (\frac{1}{2}\frac{h_1}{\sqrt{H_1}}) \tag{3.14}$$

Similarly,

$$(\sqrt{(H_2+h_2)} - \sqrt{H_2}) \approx (\frac{1}{2}\frac{h_2}{\sqrt{H_2}}) \tag{3.15}$$

And,

$$(\sqrt{(H_2-H_1+h_1-h_2} - \sqrt{H_1-H_2}) \approx (\frac{1}{2}\frac{h_1-h_2}{\sqrt{H_1-H_2}}) \tag{3.16}$$

Simplifying Eq. (3.10) and (3.11) to obtain the linearized MIMO coupled tanks system,

$$A_1\frac{dh_1}{dt} = q_1 - \frac{\alpha_1}{2\sqrt{H_1}}h_1 - \frac{\alpha_3}{2\sqrt{H_1-H_2}}(h_1-h_2) \tag{3.17}$$

$$A_2\frac{dh_2}{dt} = q_2 - \frac{\alpha_2}{2\sqrt{H_2}}h_2 + \frac{\alpha_3}{2\sqrt{H_1-H_2}}(h_1-h_2) \tag{3.18}$$

3.3.2 Mathematical Modeling of SISO Coupled Tanks System Using First Principle Approach

The coupled tanks can be configured as a single input single output system as shown in figure below,

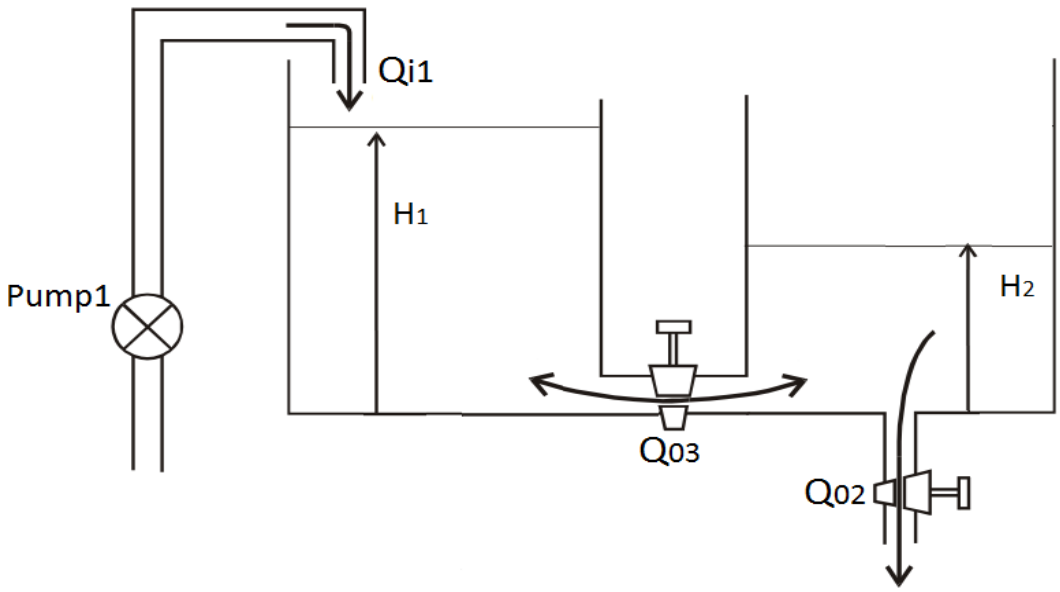

Fig. 3.3 Dynamics of the SISO Coupled Tanks System

In this configuration input from pump 2 is stopped and outlet valve of tank 1 is also closed thus, Q_{i2} and Q_{01} are zero.

The nonlinear equations that describe the dynamics of the SISO system can be found simply by considering the MIMO system equation i.e Eq. (3.6) and (3.7)

Tank 1 flow balance equation,

$$A_1 \frac{dh_1}{dt} = Q_{i1} - \alpha_3 \sqrt{H_1 - H_2} \tag{3.19}$$

Tank 2 flow balance equation,

$$A_2 \frac{dh_2}{dt} = \alpha_2 \sqrt{H_2} + \alpha_3 \sqrt{H_1 - H_2} \tag{3.20}$$

3.3.2.1 Linearized Perturbation SISO Model

The linearized perturbation model for the SISO configuration can be found in the same way as was found for the MIMO system.

The linearized SISO coupled tanks system,

$$A_1\frac{dh_1}{dt}=q_1-\frac{1}{2}\frac{\alpha_3 h_1}{\sqrt{H_1-H_2}}+\frac{1}{2}\frac{\alpha_3 h_2}{\sqrt{H_1-H_2}} \tag{3.21}$$

$$A_2\frac{dh_2}{dt}=\frac{1}{2}\frac{\alpha_3 h_1}{\sqrt{H_1-H_2}}-(\frac{1}{2}\frac{\alpha_2}{\sqrt{H_2}}+\frac{1}{2}\frac{\alpha_3}{\sqrt{H_1-H_2}})h_2 \tag{3.22}$$

3.4 State Space Representation of the System

The controller used in this thesis uses state space representation to formulate the optimal control solution. Hence, the state space model is formulated using the derived nonlinear equations of the system.

Eq. (3.17) and (3.18) can be written in matrix form as,

$$\begin{pmatrix}\frac{dh_1}{dt}\\ \frac{dh_2}{dt}\end{pmatrix}=\begin{pmatrix}\frac{-1}{A}\left(\frac{\alpha_1}{2\sqrt{H_1}}+\frac{\alpha_3}{2\sqrt{H_1-H_2}}\right) & \frac{1}{A}\left(\frac{\alpha_3}{2\sqrt{H_1-H_2}}\right)\\ \frac{1}{A}\left(\frac{\alpha_3}{2\sqrt{H_1-H_2}}\right) & \frac{-1}{A}\left(\frac{\alpha_2}{2\sqrt{H_1}}+\frac{\alpha_3}{2\sqrt{H_1-H_2}}\right)\end{pmatrix}\begin{pmatrix}h_1\\ h_2\end{pmatrix}+\begin{pmatrix}\frac{1}{A} & 0\\ 0 & \frac{1}{A}\end{pmatrix}\begin{pmatrix}q_1\\ q_2\end{pmatrix} \tag{3.23}$$

$$\begin{pmatrix}y_1\\ y_2\end{pmatrix}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}h_1\\ h_2\end{pmatrix} \tag{3.24}$$

Where,

q1= input to tank 1

q2= input to tank 2

y1= tank 1 output i.e the resulting perturbation level due to variation in input q1 and q2

y2= tank 2 output i.e the resulting perturbation level due to variation in input q1 and q2

Now,

q1 = u1, q2= u2

h1= x1, h2=x2

The continuous time state space equations for the MIMO system i.e a two input two output system can be described as,

$$\begin{pmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{pmatrix} = \begin{pmatrix} \frac{-1}{A}\left(\frac{\alpha_1}{2\sqrt{H_1}} + \frac{\alpha_3}{2\sqrt{H_1 - H_2}}\right) & \frac{1}{A}\left(\frac{\alpha_3}{2\sqrt{H_1 - H_2}}\right) \\ \frac{1}{A}\left(\frac{\alpha_3}{2\sqrt{H_1 - H_2}}\right) & \frac{-1}{A}\left(\frac{\alpha_2}{2\sqrt{H_2}} + \frac{\alpha_3}{2\sqrt{H_1 - H_2}}\right) \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} \frac{1}{A} & 0 \\ 0 & \frac{1}{A} \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \tag{3.25}$$

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \tag{3.26}$$

Assuming,

$$Ap = \begin{pmatrix} \frac{-1}{A}\left(\frac{\alpha_1}{2\sqrt{H_1}} + \frac{\alpha_3}{2\sqrt{H_1 - H_2}}\right) & \frac{1}{A}\left(\frac{\alpha_3}{2\sqrt{H_1 - H_2}}\right) \\ \frac{1}{A}\left(\frac{\alpha_3}{2\sqrt{H_1 - H_2}}\right) & \frac{-1}{A}\left(\frac{\alpha_2}{2\sqrt{H_2}} + \frac{\alpha_3}{2\sqrt{H_1 - H_2}}\right) \end{pmatrix}$$

$$Bp = \begin{pmatrix} \frac{1}{A} & 0 \\ 0 & \frac{1}{A} \end{pmatrix}$$

$$Cp = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Thus, in standard state space representation,

$$\begin{pmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{pmatrix} = Ap\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + Bp\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \tag{3.27}$$

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = Cp\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \tag{3.28}$$

Similarly, from Eq. (3.21) and (3.22) the continuous time state space equations for the SISO system can be described as,

$$\begin{pmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{pmatrix} = \begin{pmatrix} \frac{-1}{A}\left(\frac{\alpha_3}{2\sqrt{H_1 - H_2}}\right) & \frac{1}{A}\left(\frac{\alpha_3}{2\sqrt{H_1 - H_2}}\right) \\ \frac{1}{A}\left(\frac{\alpha_3}{2\sqrt{H_1 - H_2}}\right) & \frac{-1}{A}\left(\frac{\alpha_2}{2\sqrt{H_2}} + \frac{\alpha_3}{2\sqrt{H_1 - H_2}}\right) \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} \frac{1}{A} \\ 0 \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \tag{3.29}$$

$$y_1 = \begin{pmatrix} 0 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \tag{3.30}$$

Assuming,

$$Ap = \begin{pmatrix} \dfrac{-1}{A}\left(\dfrac{\alpha_3}{2\sqrt{H_1 - H_2}}\right) & \dfrac{1}{A}\left(\dfrac{\alpha_3}{2\sqrt{H_1 - H_2}}\right) \\ \dfrac{1}{A}\left(\dfrac{\alpha_3}{2\sqrt{H_1 - H_2}}\right) & \dfrac{-1}{A}\left(\dfrac{\alpha_2}{2\sqrt{H_2}} + \dfrac{\alpha_3}{2\sqrt{H_1 - H_2}}\right) \end{pmatrix}$$

$$Bp = \begin{pmatrix} \dfrac{1}{A} \\ 0 \end{pmatrix}$$

$$Cp = \begin{pmatrix} 0 & 1 \end{pmatrix}$$

Thus, in standard state space representation,

$$\begin{pmatrix} \dfrac{dx_1}{dt} \\ \dfrac{dx_2}{dt} \end{pmatrix} = Ap\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + Bp(u_1) \tag{3.31}$$

$$y_1 = Cp\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \tag{3.32}$$

3.5 Summary

In this chapter the nonlinear plant (coupled tanks system) was discussed. The scheme to be used to control this system required the linearized state space model of the plant. In this chapter, the mathematical model of the plant was developed using first principle method with detailed description of model parameters. The system was linearized and the linearized perturbation models for both SISO and TITO systems were derived. The continuous time state space formulation was also presented.

Chapter 4

LINEAR MODEL PREDICTIVE CONTROL

4.1 Introduction

This chapter provides basics and in-depth study related to discrete linear model predictive control methodology. In this research to begin with a single MPC scheme is employed but based on the findings three linear MPC approaches are used. This chapter gives detailed information on all the three algorithms. The most distinct ability of MPC to handle constraints is also covered along with the formation of quadratic optimization problem.

4.2 Model Predictive Control

Model Predictive Control refers to a specific procedure in controller design from which many kinds of algorithms can be developed for different systems, linear or nonlinear, discrete or continuous. The main difference in the various control methods of MPC is mainly the way the control problem is developed. The use of a plant model in controller design is the main difference between Model Predictive Control and the classical PID control. Thus, an appropriate model of the plant is required for controller design. One drawback of MPC is that while the control law is easily implemented its derivation is very complex, but the controlling capabilities of MPC make it worth the effort. A wide range of predictive control algorithms have been developed over the years. For example, Model Predictive Heuristic Control (MPHC) scheme proposed by Richalet et al. in 1978 [16] which used a linear impulse response model, Dynamic Matrix Control (DMC) proposed by Cutler and Ramaker in 1979 [17], Generalized Predictive Control (GPC) proposed by Clarke et al. in 1987 [10]. The major differences between all these approaches are the type of plant models used and the cost function to be optimized.

MPC is capable of controlling multivariable systems and operational constraints by solving an optimization problem in which the constraints are defined.

4.2.1 Model Predictive Control Strategy

All MPC approaches share the same fundamental framework, and can be summarized as in Fig. 4.1

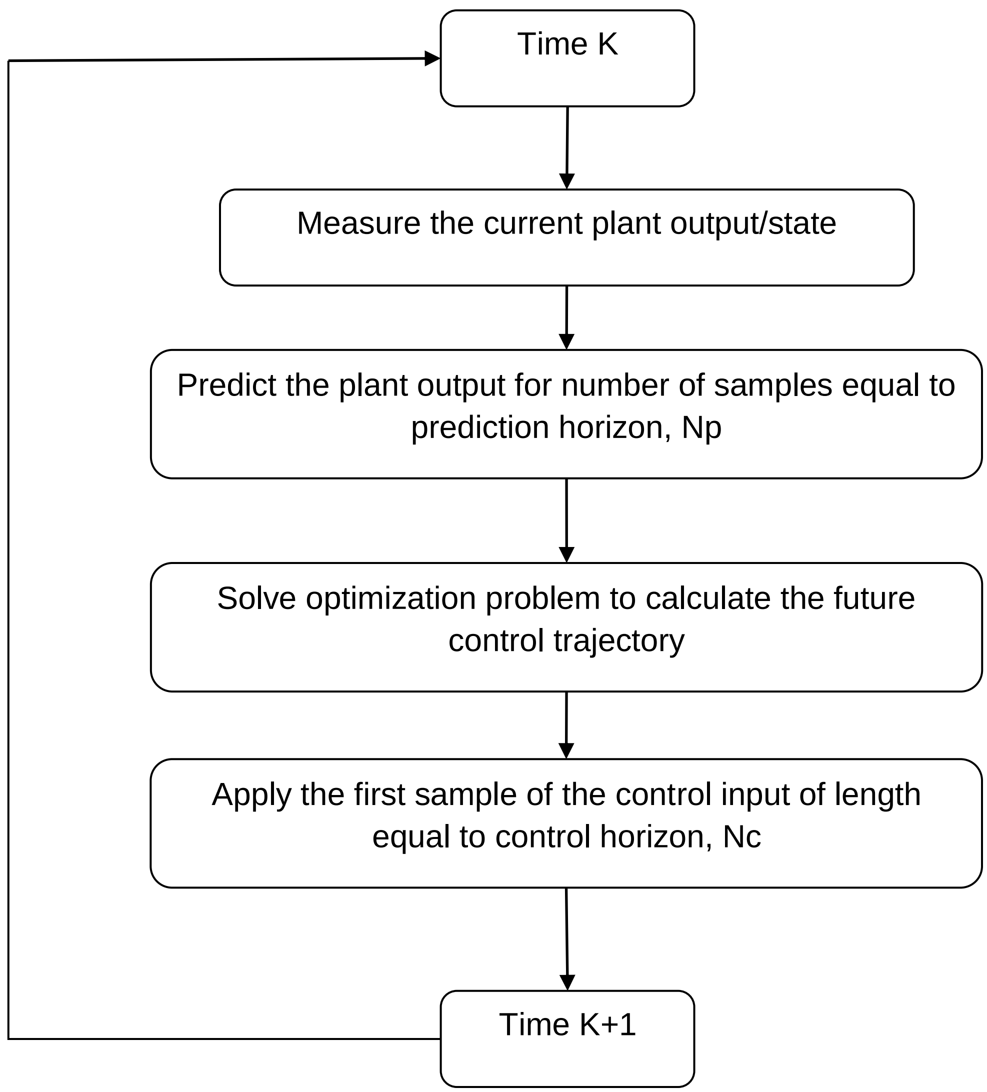

Fig. 4.1 Predictive Control Algorithm

I. A suitable process model is used to predict the plant output. The output is predicted for one optimization window. Where, the length of the optimization window is equal to prediction horizon denoted by Np.

II. The future optimal control trajectory denoted by,

$$\Delta u(k), \Delta u(k+1), \ldots, \Delta u(k+N_c-1)$$

The optimal control trajectory is calculated by minimizing desired parameters, such as tracking error and constraints formulated into the cost function. Where, length of control trajectory is equal to the control horizon, denoted by Nc.

III. Only the first sample of the predicted control trajectory is applied and remaining sequence is rejected.

IV. The Receding Horizon scheme is then applied.

4.3 The Receding Horizon Control

In conventional controls one has to deal with finite horizons and infinite horizons. Feedback control systems such as power generation systems, building automation systems, chemical plants etc. run for a long time periods. In such systems finite horizon solutions are not feasible and infinite horizon solutions are necessary. Thus, the solution has to be extended. But the infinite sequence does not effectively minimize the defined optimal cost function and may result in an unstable process behavior.

The basic principle of receding horizon is to solve an optimization problem for the present state of the system for a finite time horizon of sampling instants equal to N [18]. After the current control action is obtained by solving the optimization problem and the first sample of the control sequence is applied is applied to the plant. The system state is measured for the next sample and the procedure is repeated, which defines the receding horizon control law. This is illustrated below,

I. Firstly, the current output of the process is measured and the current state is estimated.

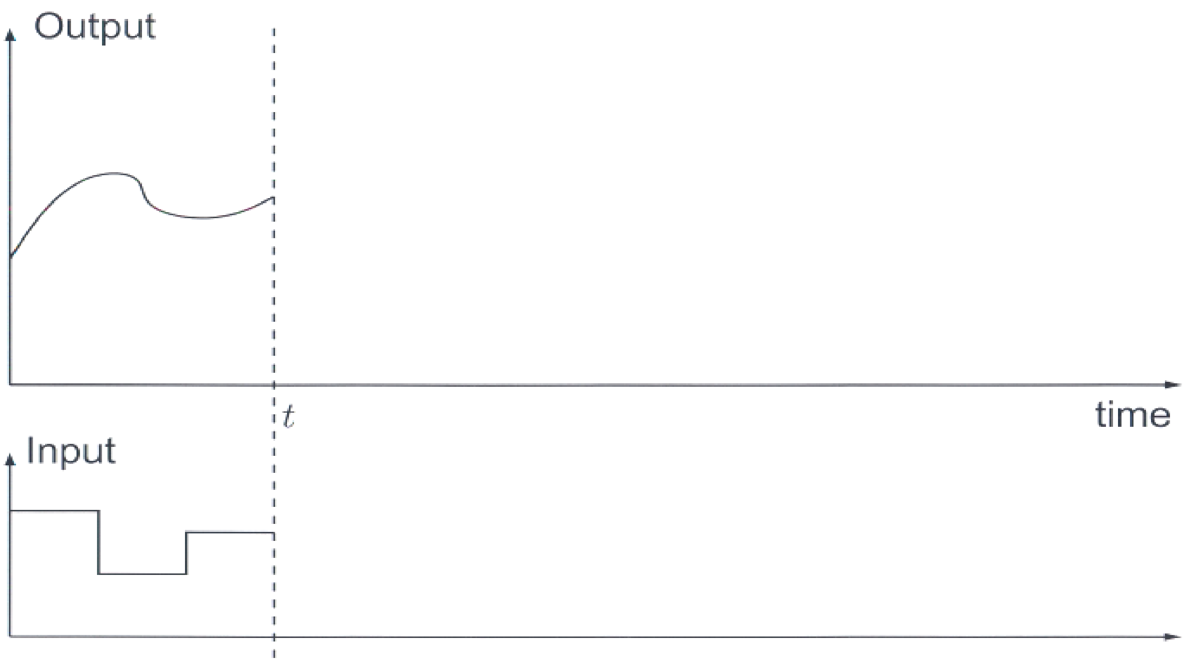

Fig. 4.2 current system output and state

II. Solve an optimization problem to obtain the future control trajectory over the finite horizon i.e the control sequence which is the best solution to obtain the desired output of that specific process.

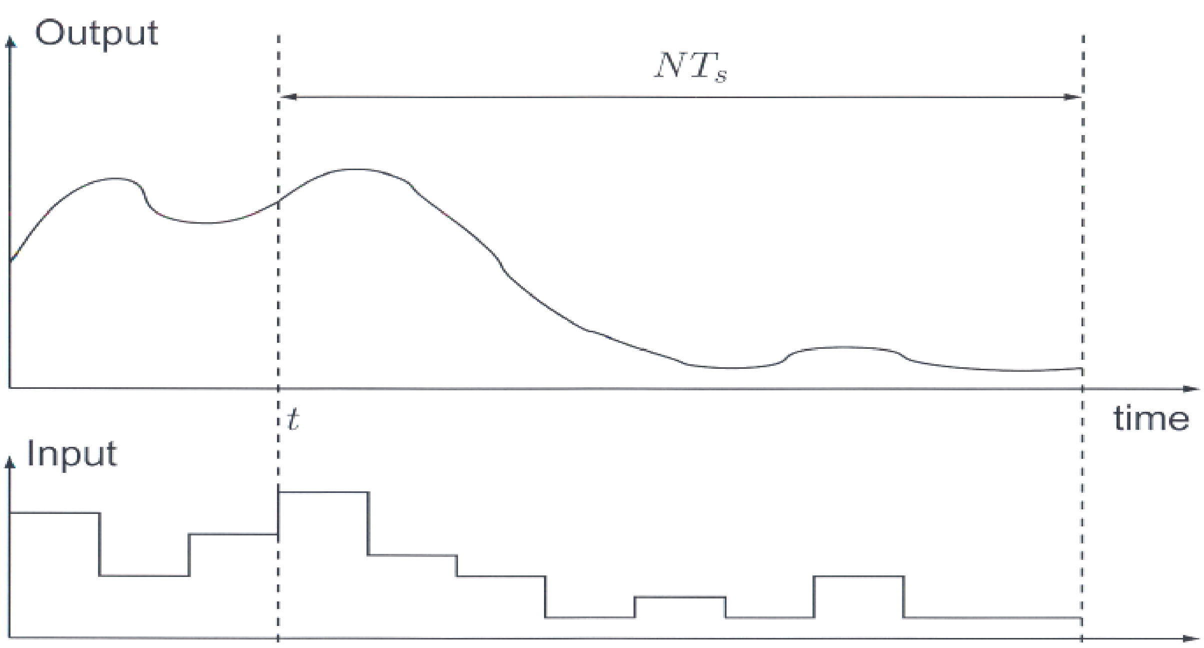

Fig. 4.3 optimal input sequence over a finite horizon

III. The first sample of the optimal control sequence is applied to the plant and the system state is measured from the next sample and the optimization window is moved to this sample.

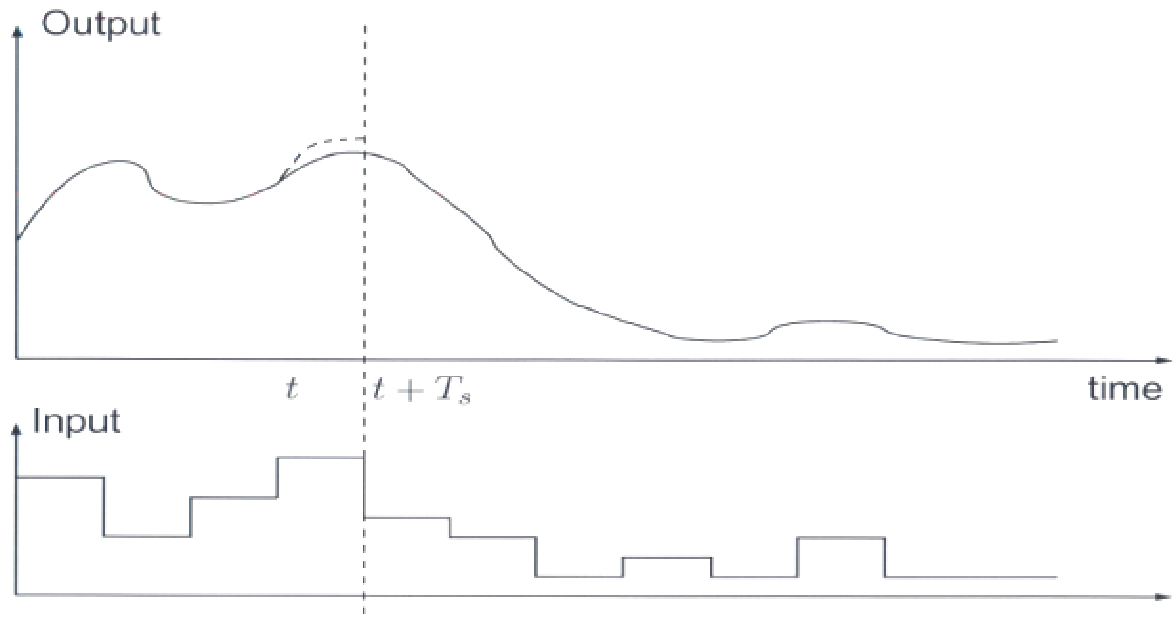

Fig. 4.4 only first element of optimal control is applied

IV. A new optimal input sequence is computed and implemented as was done previously and the procedure is repeated for future sampling instants.

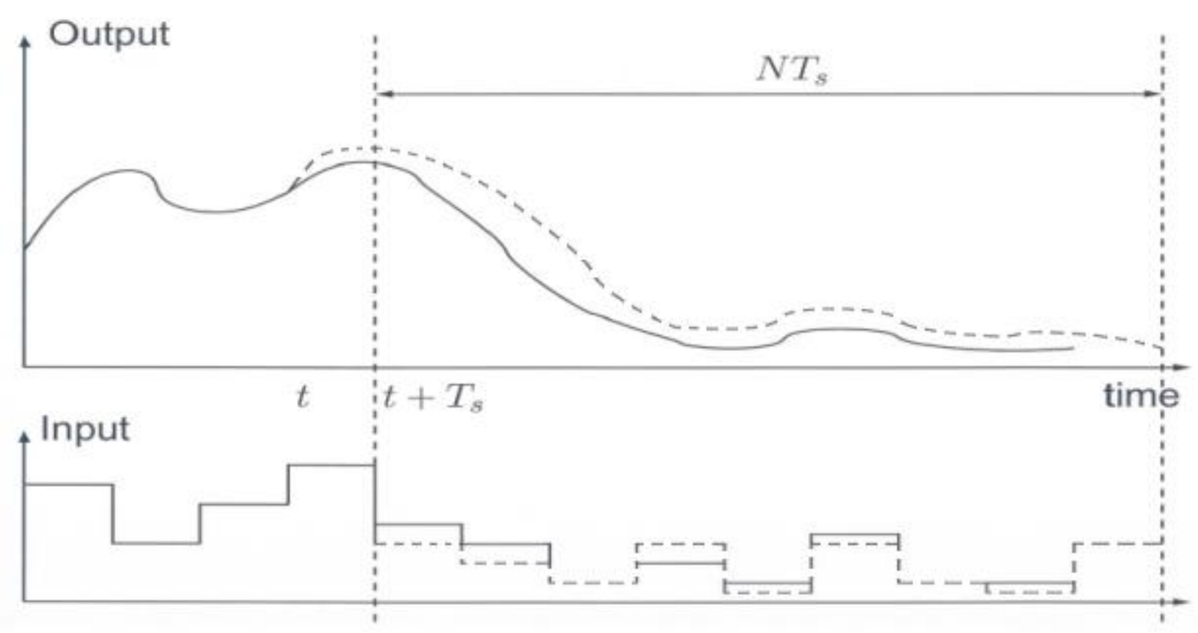

Fig. 4.5 optimal input sequence calculated for future instants

4.4 Set Point Improvement

Process control systems in practice are subject to constraints. Such as Actuators having slew rate limitation, System safety limits such as temperature and pressure limitations etc. The operating points are generally set to satisfy economic targets. The control system operating point is thus usually selected near the limits. Hence constraints violation can result. The figure below shows constraint violations occurring when a linear control strategy other than MPC is adopted.

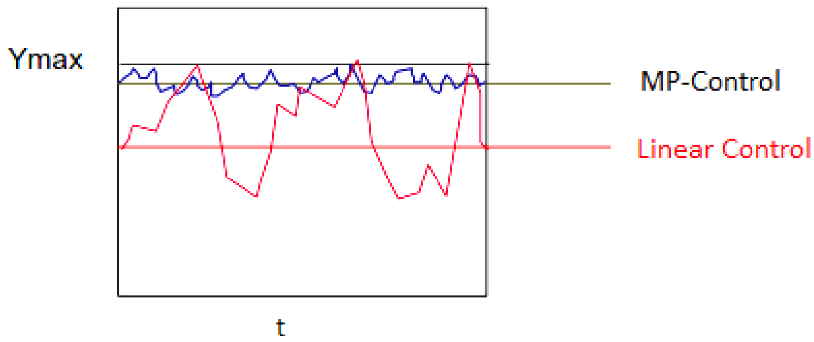

Fig. 4.6 constraint violation comparison

Model predictive control approach allows operating point to be selected close to the constraints. It is not usually possible to operate close to the constraints because most controllers are unable to deal with disturbances like MPC can. This improves the performance. The illustration in Fig. 4.7 shows variances of certain controlled output and the improvement achieved when the predictive control approach is adopted [15]..

a) The high variance and the Gaussian shape results when a badly tuned linear controller is used. In this case if a set-point is selected close to the constraint it is a high probability that the constraint will be violated and thus a set-point with a relatively lower value has to be adjusted.

b) The low variance and the Gaussian distribution results when an optimal linear controller is used. In such a case as evident from Fig. 4.6 the set-point can be adjusted relatively closer to the system constraint.

c) The controller is aware of the system constraint and thus one can select an operating point very close to the defined constraint.

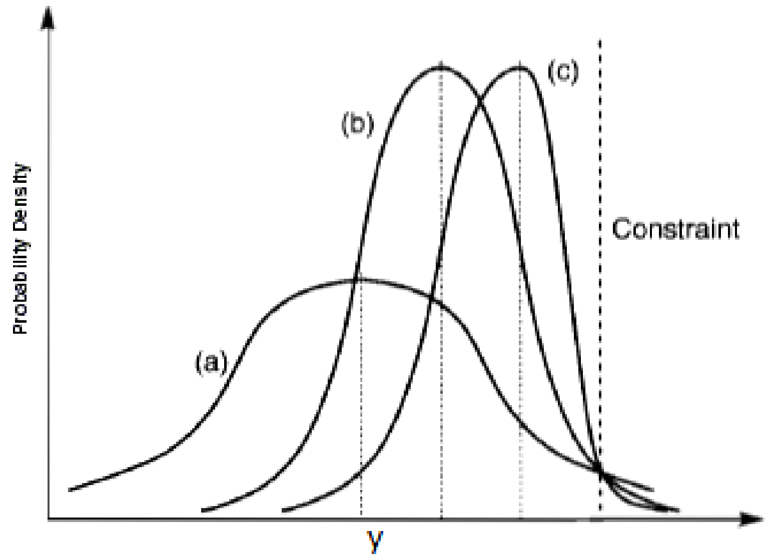

Fig. 4.7 set point improvement by using MPC

4.5 Model Predictive Control Algorithm

The basic methodology of discrete-time model predictive control is finding the optimal control trajectory i.e. ΔU, which is the incremental variation of the control signal. Based on this fundamental approach a restricted model approach was proposed by Tan et al. [7] [8]. Tan formulated the control problem in terms of Generalized Predictive Control which was proposed by D.W.Clarke [10]. This is one of the most widely used model predictive control approach. In this thesis in order to control the nonlinear SISO coupled tanks system this discrete-time approach is first employed.

4.5.1 MPC Based On Generalized Predictive Control

This algorithm was introduced by Tan et al [7]. This algorithm itself has many variants, but all the versions share the same methodology. This basic MPC approach uses GPC to formulate the control problem.

4.5.1.1 Generalized Predictive Control

Generalized Predictive Control (GPC) was introduced by D.W.Clarke in 1987 [10], at that time much progress was already made in adaptive and self-tuning controls but there was a lack of a general purpose control algorithm that could be applied to a the majority of processes. This general purpose control scheme was to be able to control following unconstrained linear plants:

- non-minimum phase systems
- open-loop unstable systems
- systems having variable or unknown dead-time
- systems with unknown order

The GPC approach by Clarke was applicable to all these processes. The idea of GPC is to calculate the future control signals in such a way that it minimizes a cost function over the defined prediction horizon. GPC used Controlled Auto Regressive Moving Average (CARIMA) model to represent the process dynamics.

$$A(q^{-1})y(t) = B(q^{-1})u(t-1) + C(q^{-1})\frac{\xi(t)}{\Delta} \qquad (4.1)$$

Where,

A and B are polynomials in the backward shift operator q^{-1}

$$x(t) = C(q^{-1})\frac{\xi(t)}{\Delta}$$

Δ is the difference operator 1- q^{-1}

y(k) is the system output

u(k) is the manipulated input

$\zeta(t)$ is the white noise disturbance

The GPC Cost function is defined as,

$$J(N1, N2) = E\{\sum_{J=N_1}^{N_2}[y(t+j) - w(t+j)]^2 + \sum_{j=1}^{N_1}\lambda(j)[\Delta u(t+j-1)]^2\} \qquad (4.2)$$

Where,

y(t+j) is the output prediction

w(t+j) is the set point

$\lambda(j)$ is the control weighting sequence

N1 and N2 are the minimum and maximum prediction horizon

Minimizing the defined GPC cost function the optimal control law can be found.

The difference between the various methods of MPC is mainly the way the control problem is formulated. Generalized predictive control is to date is the most popular methods of MPC.

4.5.1.2 Basic MPC Algorithm

Considering a single input single output system:

$$X_m(k+1) = A_m X_m(k) + B_m u(k) \tag{4.3}$$

$$y(k) = C_m X_m(k) \tag{4.4}$$

where, u(k) is the manipulated variable, y(k) is the process output and Xm is the state variable vector having a dimension of 2.

The system has u(k) as input. To design a predictive controller this needs to be altered, thus model is augmented with integrator and the result is obtained in state space form. Firstly, applying difference operation on both sides of Eq. (4.3):

$$X_m(k+1) - X_m(k) = A_m(X_m(k) - X_m(k-1)) + B_m(u(k) - u(k-1)) \tag{4.5}$$

The difference of state and output can be defined as,

$$\Delta X_m(k+1) = X_m(k+1) - X_m(k)$$
$$\Delta X_m(k) = X_m(k) - X_m(k-1)$$
$$\Delta u(k) = u(k) - u(k-1)$$

To obtain integral effect the difference of state variable must be connected to the system output. For this an augmented state vector is defined:

$$X(k) = \left(\Delta X_m(k)^T \quad y(k)\right)^T \tag{4.6}$$

Where, script T indicates matrix transpose. Now the state equation can be written as,

$$\Delta X_m(k+1) = A_m \Delta X_m(k) + B_m \Delta u(k) \tag{4.7}$$

Form this state the output can be calculated as,

$$y(k+1) = C_m A_m \Delta X_m(k) + y(k) + C_m B_m \Delta u(k) \tag{4.8}$$

$$y(k+1) - y(k) = C_m(X_m(k+1) - X_m(k)) = C_m \Delta X_m(k+1) \tag{4.9}$$

Eq. (4.8) and (4.9) in standard state space formulation can be written as,

$$\begin{pmatrix} \Delta X_m(k+1) \\ y(k+1) \end{pmatrix} = \begin{pmatrix} A_m & 0_m^T \\ C_m A_m & 1 \end{pmatrix} \begin{pmatrix} \Delta X_m(k) \\ y(k) \end{pmatrix} + \begin{pmatrix} B_m \\ C_m B_m \end{pmatrix} \Delta u(k) \tag{4.10}$$

$$y(k) = \begin{pmatrix} 0_m & 1 \end{pmatrix} \begin{pmatrix} \Delta X_m(k) \\ y(k) \end{pmatrix} \tag{4.11}$$

Or simply,

$$X(k+1) = AX(k) + B\Delta u(k) \tag{4.12}$$

$$y(k) = CX(k) \tag{4.13}$$

Where, 0_m= [0 0…0] is a 1 x n vector, n being the order of the system. $\Delta X_m(k+1) = X_m(k+1) - X_m(k)$ and augmented state vector $X(k)=[\ \Delta X_m(k)^T\ y(k)\ ]^T$

The first step in this procedure was to augment the system with an integrator this augmented model will be used in future. The future states of the system are calculated by using Eq.(412). The future states are predicted for one optimization windows. Assuming current sampling instant $k_{i,}$ with ki >0. The future control trajectory is considered as,

$$\Delta u(k), \Delta u(k+1), \Delta u(k+2), \ldots, \Delta u(k+N_C-1) \tag{4.14}$$

The future state variables are considered as,

$$X(k+1), X(k+2), \ldots, X(k+m \mid k), \ldots, X(k+N_P \mid k) \tag{4.15}$$

Using Eq.(412) these states can be found,

$$
\begin{aligned}
X(k_i+1 \mid k_i) &= AX(k_i) + B\Delta u(k_i) \\
X(k_i+2 \mid k_i) &= A^2 X(k_i) + AB\Delta u(k_i) + B\Delta u(k_i+1) \\
&\vdots \\
&\vdots \\
X(k_i+N_P \mid k_i) &= A^{N_P} X(k_i) + A^{N_P-1} B\Delta u(k_i) + A^{N_P-N_C} B\Delta u(k_i+N_C-1)
\end{aligned}
$$

Where, N_c is called the control horizon. It dictates the number of parameters used to capture the future control trajectory and N_p is the prediction horizon. It is also the length of the optimization window.

The predicted output can be calculated from the predicted states that have been calculated,

$$
\begin{aligned}
y(k+1 \mid k) &= C_m A_m X(k) + C_m B_m \Delta u(k) \\
y(k+2 \mid k) &= C_m A_m^2 X(k) + C_m A_m B_m \Delta u(k) + C_m B_m \Delta u(k+1) \\
&\vdots \\
&\vdots \\
y(k+N_P \mid k) &= C_m A_m^{N_P} X(k) + C_m A_m^{N_P-1} B_m \Delta u(k) + C_m A_m^{N_P-2} B_m \Delta u(k+1) + \cdots \\
&\cdots + C_m A_m^{N_P-N_C} B_m \Delta u(k+N_C-1)
\end{aligned}
$$

From the outputs predicted it can be seen that these predictions are formulated in terms of current state information and the future control increments.

The predicted output can be written in matrix form as,

$$Y = FX(k_i) + \phi \Delta U \tag{4.16}$$

Where,

$$Y = \begin{pmatrix} y(k_i+1 \mid k_i) & y(k_i+2 \mid k_i) & \ldots & y(k_i+N_P \mid k_i) \end{pmatrix}^T$$

$$\Delta U = \begin{pmatrix} \Delta u(k_i) & \Delta u(k_i+1) & \ldots & \Delta u(k_i+N_C-1) \end{pmatrix}^T$$

And,

$$\phi = \begin{pmatrix} CB & 0 & 0 & \cdots & 0 \\ CAB & CB & 0 & \cdots & 0 \\ CA^2B & CAB & CB & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ CA^{N_P-1}B & CA^{N_P-2}B & CA^{N_P-3}B & \cdots & CA^{N_P-N_C}B \end{pmatrix}$$

$$F = \begin{pmatrix} CA \\ CA^2 \\ CA^3 \\ \vdots \\ CA^{N_P} \end{pmatrix}$$

After the prediction is found the optimization problem must be defined and optimal solution is found by finding the solution to this optimization problem. The main aim of the controller is to minimize the difference between the predicted output and the control objective i.e the required set-point.

Let the data vector containing the set point information be R_s^T=[1 1 .. 1]r(k_i). The quadratic cost function for this algorithm:

$$J = (R_S - Y)^T (R_S - Y) + \Delta U^T R_d \Delta U \quad (4.17)$$

To minimize the errors b/w the predicted output and set point signal

For the case of large weighting factor rw , the cost function is interpreted as the condition where it is considered how large ΔU might become

Where, $Rd = r_w \times I_{Nc \times Nc}$ with rw ≥ 0 is used as a tuning parameter for desired closed-loop performance.

In order to find optimal ΔU that minimizes J, Eq. 4.16 that is the predicted output is used.

$$J = (R_S - Fx(k_i))^T (R_S - Fx(k_i)) - 2\Delta U^T \phi^T (R_S - Fx(k_i)) + \Delta U^T (\phi^T \phi + R_d)\Delta U \qquad (4.18)$$

The condition to find minimum cost is thus,

$$\frac{\partial J}{\partial \Delta U} = 0$$

Eq. (4.18) can now be simplified to obtain the optimal control,

$$\Delta U = (\phi^T \phi + R_d)^{-1} \phi^T (R_S - Fx(k_i)) \qquad (4.19)$$

This gives us the required controls,

$$\Delta U = \begin{pmatrix} \Delta u(k_i) & \Delta u(k_i+1) & \ldots & \Delta u(k_i + N_C - 1) \end{pmatrix}^T$$

After, the optimal control is calculated now the Receding Horizon Principle is applied and only the first sample of the control sequence is applied to the system and the rest is discarded. The process is than repeated on the next sample where the window is receded.

4.5.2 Model Predictive Control Using Laguerre Functions

Laguerre functions when used with DMPC have many benefits such as , improvements in feasibility [19], independent tuning parameters for multivariable systems and smoother response. The usage of Laguerre functions to improvement in performance of predictive control is an area that needs further investigation [20].

In the previous basic MPC approach the control trajectory was defined as,

$$\Delta U = \begin{pmatrix} \Delta u(k_i) & \Delta u(k_i+1) & \ldots & \Delta u(k_i + N_C - 1) \end{pmatrix}^T$$

Where, Nc is the control horizon i.e the dimension of the control sequence.

At a sample instant ki any element in the control trajectory can be defined by using the discrete Dirac delta function i.e δ function.

$$\Delta u(k_i + i) = \begin{pmatrix} \delta(i) & \delta(i-1) & \ldots & \delta(i - N_C + 1) \end{pmatrix} \Delta U \tag{4.20}$$

Where, δ(i)=1 for i=0 and δ(i)=0 for i≠0

From this fact it can be concluded that Δu(k+i) can be represented by a discrete polynomial functions. In this case discrete time Laguerre functions will be used to approximate the control trajectory.

Laguerre functions are orthonormal functions and can be used to approximate the increments of control signals. First the single-input case for MPC algorithm using Laguerre functions is described and is then extended to multi-input case. The z-transforms of discrete-time Laguerre functions are written as:

$$\Gamma_1(z) = \frac{\sqrt{1-a^2}}{1-az^{-1}}$$

$$\Gamma_2(z) = \frac{\sqrt{1-a^2}}{1-az^{-1}} \left(\frac{z^{-1}-a}{1-az^{-1}} \right)$$

$$\vdots$$

$$\vdots$$

$$\Gamma_N(z) = \frac{\sqrt{1-a^2}}{1-az^{-1}} \left(\frac{z^{-1}-a}{1-az^{-1}} \right)^{N-1} \tag{4.21}$$

where, 0≤ a<1 is the scaling factor and N=1,2,… is the number of Laguerre terms.

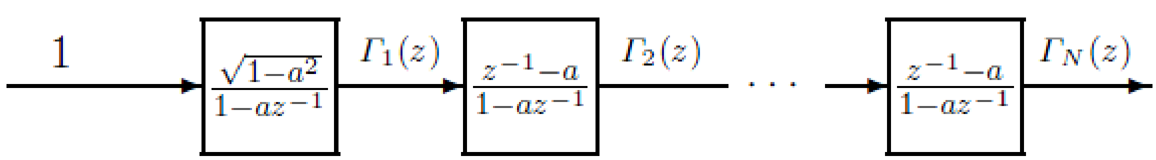

Fig. 4.8 Discrete Laguerre Network

Discrete-time Laguerre functions can be found by using the following relation:

$$\Gamma_N(z) = \Gamma_{N-1}(z)\left(\frac{z^{-1}-a}{1-az^{-1}}\right) \tag{4.22}$$

Defining the inverse z-transforms as,

$l_1(k)$,$l_2(k)$,…..,$l_N(k)$

In vector form these can be expressed as,

$$L(k) = \left[l_1(k) \quad l_2(k) \quad \dots \quad l_N(k)\right] \tag{4.23}$$

Thus the inverse z-transforms in vector form is L(k). Using the realization Eq. (4.22), the discrete-time Lagueree functions can be written in the form of difference equation with initial condition L(0):

$$L(k+1) = A_l L(k) \tag{4.24}$$

Where,

$$L(0)^T = \sqrt{\beta}(1 \quad -a \quad -a^2 \quad \dots \quad (-1)^{N-1}a^{N-1})$$

$$\beta = (1-a^2)$$

$$A_l = \begin{pmatrix} a & 0 & 0 & \cdots & 0 \\ \beta & a & 0 & \cdots & 0 \\ -a\beta & \beta & a & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ (-1)^{N-2}a^{N-2}\beta & (-1)^{N-3}a^{N-3}\beta & (-1)^{N-4}a^{N-4}\beta & \cdots & a \end{pmatrix}$$

Thus at time k, Laguerre functions $l_1(k)$, $l_2(k)$, ….., $l_N(k)$ are used to capture the control trajectory Δu(ki), Δu(ki+1),……, Δu(ki+k) with a set of Laguerre coefficients.

$$\Delta u(ki+m) = \sum_{i-1}^{N} C_i(k_i) l_i(m) \tag{4.25}$$

Where, m is the future sampling instant and N is the number of terms used in the expansion. Thus in this approach Nc is no longer used and now number of parameters N and factor a are used to describe the properties of the trajectory. For a single input single output system the trajectory can be found as,

$$\Delta u(ki+m)=\sum_{i-1}^{N}C_i(k_i)l_i(m)=L(m)^T\eta \tag{4.26}$$

Where,

m=0,1,2…N_p

$\eta=\begin{bmatrix} c_1 & c_2 & \cdots & c_N \end{bmatrix}$

The value of $x(k_i+m)$ for single-input is found as:

$$x(k_i+m\,|\,k_i)=A^m x(k_i)+\sum_{i-0}^{m-1}A^{m-i-1}BL(i)^T\eta \tag{4.27}$$

If the system is extended from single-input to multi-input input. The control variable will be formulated as:

$$\Delta u(k)=\begin{bmatrix}\Delta u_1(k) & \Delta u_2(k) & \cdots & \Delta u_P(k)\end{bmatrix}^T \tag{4.28}$$

Where, p is the number of inputs and $\Delta u_i(k)=L_i(k)\eta_i$

Now the prediction of future state at time m:

$$x(k_i+m\,|\,k_i)=A^m x(k_i)+\phi(m)^T\eta \tag{4.29}$$

Where,

$$\phi(m)=\sum_{j=0}^{m-1}A^{m-j-1}\begin{bmatrix}B_1L_1(j)^T & B_2L_2(j)^T & \dots & B_mL_m(j)^T\end{bmatrix} \tag{4.30}$$

The control law can be realized as,

$$\Delta u(k)=\begin{pmatrix} L_1(k)^T & 0 & \cdots & 0 \\ 0 & L_2(k)^T & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \cdots & L_P(k)^T \end{pmatrix}\eta \tag{4.31}$$

With, $\eta^T = \begin{pmatrix} \eta_1^T & \eta_2^T & \cdots & \eta_m^T \end{pmatrix}$

The cost function in terms of Laguerre parameter η is given by,

$$J = \eta^T \Omega \eta + 2\eta^T \Psi x(k_i) + \sum_{m=1}^{N_P} x(k_i)^T (A^T)^m Q A^m x(k_i) \tag{4.32}$$

Where,

$$\Omega = \sum_{m=1}^{N_P} \phi(m) Q \phi(m)^T + R_L$$

$$\Psi = \sum_{m=1}^{N_P} \phi(m) Q A^m$$

After optimal value of η is found the receding horizon control law can be realized,

$$\Delta u(k) = \begin{pmatrix} L_1(0)^T & 0 & \cdots & 0 \\ 0 & L_2(0)^T & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \cdots & L_P(0)^T \end{pmatrix} \eta$$

4.5.3 A Modified Basic MPC Scheme

A simple MPC approach based on the algorithm by [8] was used by [12]. The procedure followed in this algorithm is the same as the Basic MPC approach. In the research [12, 21] minor modifications were made in the algorithm the biggest change was the consideration of the error in the predicted by using the plant model and the actual response of the plant. This small change resulted major improvements in the overall performance of the system. This algorithm also extends the applicability to MIMO high order systems.

Around the operating point the dynamics of the system can be approximated by the following discrete-time state space equations with sampling time k:

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y &= Cx(k) + Du(k) \end{aligned} \tag{4.33}$$

At a sampling instant, with the measured output of the system $y_m(k)$ and the systems current state x(k) the future output can be predicted within the prediction horizon by using the future input. The future input is to be determined within the control horizon.

Defining the error in prediction of the system,

$$d(k+i) = d_k = y_m(k) - Cx(k) \tag{4.34}$$

Assuming a prediction horizon of P the future output can be predicted as,

$$Y = \phi U + \Psi x(k) + Ld_k \tag{4.35}$$

Where,

$$Y = \begin{bmatrix} y^T(k+1) & y^T(k+2) & \cdots & y^T(k+P) \end{bmatrix}^T$$

$$U = \begin{bmatrix} u^T(k) & u^T(k+1) & \cdots & u^T(k+M-1) \end{bmatrix}^T$$

$$\phi = \begin{pmatrix} CB & 0 & \cdots & 0 \\ CAB & CB & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ CA^{P-1}B & CA^{P-2}B & \cdots & \sum_{i=M}^{P} CA^{P-i}B \end{pmatrix}$$ with i =M to P

$$\Psi = \begin{pmatrix} CA \\ \vdots \\ CA^P \end{pmatrix}$$

$$L = \begin{pmatrix} I \\ \vdots \\ I \end{pmatrix}$$

The input and output reference vectors are defined as:

$$Yr = \begin{bmatrix} y_r^T(k+1) & \cdots & y_r^T(k+P) \end{bmatrix}^T$$

$$Ur = \begin{bmatrix} u_r^T(k) & \cdots & u_r^T(k+M-1) \end{bmatrix}^T$$

In order to minimize the operating cost the following optimization problem has to be solved,

$$J = 0.5(Y - Y_r)^T Q(Y - Y_r) + 0.5(U - U_r)^T R(U - U_r) \quad (4.36)$$

The optimal solution to the optimization problem,

$$U = -K_x x(k) + K_y (y_r - d_k) \quad (4.37)$$

Where,

$$K_x = S^{-1} X_1$$
$$K_y = S^{-1} X_2$$

4.6 Constrained Control Using Laguerre Functions

Practical systems exhibit operational constraints and one major advantage of Model Predictive Control is its ability to handle operational constraints for multivariable systems [9, 22] . The idea is to incorporate constraints in the cost function. Thus, solving the constrained optimization problem requires quadratic programming [23]. In this thesis work, hard constraints are considered on the control variable and the control variable incremental variation. The difficulty lies in formulation of the considered constraints in the form of inequality linear equations.

When the control signal trajectory is expressed using Laguerre functions the locations of the operational constraints can be chosen freely. This flexibility can potentially reduce the number of constraints and thus the computational load. One more benefit of Laguerre functions is that Laguerre functions have an exponential decay factor and thus the control variable incremental variation converges to zero after the transient period. Due to this reason the number of constraints is reduced because they must be applied only in the transient period.

Constrained control with hard constraints requires an optimization problem to be solved using quadratic programming. In order to solve the optimization problem the system state at the current instant and the lower and upper limits of the operational constraint must be known. The optimal solution is found by minimizing the cost function J while making certain that the defined constraints are not violated.

In this system usually the following constraints are considered:

1. Constraints on Input Amplitude

$$u_{\min} \le u(k_i + k) \le u_{\max}$$

2. Constraints on Input Slew Rate

$$\Delta U_{\min} \le \Delta U(k_i + k) \le \Delta U_{\max}$$

for i=0…… Nc-1

Firstly, considering constraints on system input

Since,

$$
\begin{aligned}
&\Delta u(k) = u(k) - u(k-1) \\
&u(k) = \Delta u(k) + u(k-1) \\
&u(k+1) = \Delta u(k+1) + \Delta u(k) + u(k-1) \\
&\qquad\vdots \\
&\qquad\vdots
\end{aligned}
$$

Hence,

$$
\begin{pmatrix} u(k) \\ u(k+1) \\ u(k+2) \\ \vdots \\ \vdots \\ u(k+N_C-1) \end{pmatrix} = \begin{pmatrix} I \\ I \\ I \\ \vdots \\ \vdots \\ I \end{pmatrix} u(k-1) + \begin{pmatrix} I & 0 & 0 & 0 & \cdots & 0 \\ I & I & 0 & 0 & \cdots & 0 \\ I & I & I & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \cdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ I & I & I & I & \cdots & I \end{pmatrix} \begin{pmatrix} \Delta u(k) \\ \Delta u(k+1) \\ \Delta u(k+2) \\ \vdots \\ \vdots \\ \Delta u(k+N_C-1) \end{pmatrix}
$$

Or,

$$U = S_1 u(k-1) + S_2 u'$$

Constraint definition can be given as,

$$U_{\min} \le [S_1 u(k-1) + S_2 u'] \le U_{\max}$$

Where,

$$
\begin{aligned}
U_{\min} &= [U_{\min} \quad U_{\min} \quad \cdots \quad U_{\min}]^T \\
U_{\max} &= [U_{\max} \quad U_{\max} \quad \cdots \quad U_{\max}]^T
\end{aligned}
$$

This can be written separately as inequalities,

$$S_2 u' \le u_{\max} - S_1 u(k-1) \tag{4.38}$$

$$-S_2 u' \le -u_{\min} + S_1 u(k-1) \tag{4.39}$$

Now, considering constraints on control variable incremental variation

$$\Delta U_{\min} \le \Delta U(k_i + k) \le \Delta U_{\max}$$

Can be expressed as two inequalities,

$$-\Delta U \le -\Delta U_{\min} \tag{4.40}$$

$$\Delta U \le \Delta U_{\max} \tag{4.41}$$

Combining the inequalities of all constraints in matrix form,

$$\begin{pmatrix} -S2 \\ S2 \\ -I \\ I \end{pmatrix} u' \le \begin{pmatrix} -u_{\min} + S_1 u(k-1) \\ u_{\max} - S_1 u(k-1) \\ -\Delta U_{\min} \\ \Delta U_{\max} \end{pmatrix} \tag{4.42}$$

or, Mu’$\le \gamma$

Since the constraints are linear inequalities and the cost function is quadratic optimization requires standard quadratic programming problem to be solved. The Lagurre model based cost function is defined in terms of Laguerre network parameter vector η and thus the constraints should also be formulated in terms of η.

$$\mathrm{M}\,\eta \le \gamma \tag{4.43}$$

4.6.1 Quadratic Programming Solutions

To formulate MPC control law the quadratic problem has to be solved. In quadratic problem both the control objective and the constraints derived above are combined as a single optimization problem. The solution to this problem can be found through various methods. The quadratic optimization is a very difficult field in its self, much research has been done for finding optimal solutions to these problems [9, 15, 23, 24]. Most common methods used to find the minimized solution are Active Set Method and Interior Reflective Method [23].

Quadratic programming is the mathematical problem of finding a vector x that minimizes a quadratic function such as:

$$\min_x \{\frac{1}{2} x^T M x + f^T x\}$$

Subject to the linear constraints:

$Ax \le w$	(inequality constraint)
$Ax = w$	(equality constraint)
$qw \le x \le vw$	(bound constraint)

In this research the Active-Set algorithm is employed with inequality constraints. MATLAB is used to implement this algorithm. In Active Set Method a set of constraints considered as the working sequence set are defined at each step of the algorithm. An equality constraint problem is solved at every step of the algorithm. The algorithm was implemented by using the MATLAB Optimization Toolbox [25].

4.7 Summary

In this chapter the structure of MPC and the detailed methodology used in this research were discussed. The chapter includes derivations of the MPC approaches that have been employed. The operational constraints were incorporated in the cost function and the MPC control law was found using the MATLAB optimization toolbox to solve the quadratic programming problem.

Chapter 5

RESULTS AND ANALYSIS

5.1 Introduction

In the previous chapter different MPC approaches were developed and the inclusion of hard constraints in the algorithm was discussed. In this chapter the performance of controller when applied firstly to the SISO and then the MIMO nonlinear coupled tanks system is discussed. The performance of controller for the liquid level control and the effect on performance when control parameters are varied is evaluated. Based on the best possible performance achieved using each control methodology, the merits and demerits of these control algorithms will be explained. A comparison between the outcome of the MPC approaches applied and PID controller is also presented.

The analysis for the nonlinear SISO system will be done firstly, for all approaches and then the results and analysis for MIMO CTS will be presented. The results will also demonstrate the constraint handling capabilities of model predictive control in which the optimal control was found by using MATLAB optimization toolbox to solve the quadratic problem formulated in the last chapter.

The parameters of the actual hardware model CE105 coupled tanks system [26, 27] were used in the simulations [13]. The nonlinear systems will be linearized at an operating point, but the simulation will be performed for a different reference level i.e a pulse train whose amplitude varies between 0 and a certain desired tank level. This is done to analyze, whether the linear model used by the controller to control the nonlinear system would be able to achieve set points other than the point at which the model was linearized. All the simulations were carried out in MATLAB and Simulink.

5.2 Implementation of MPC on SISO Coupled Tanks System

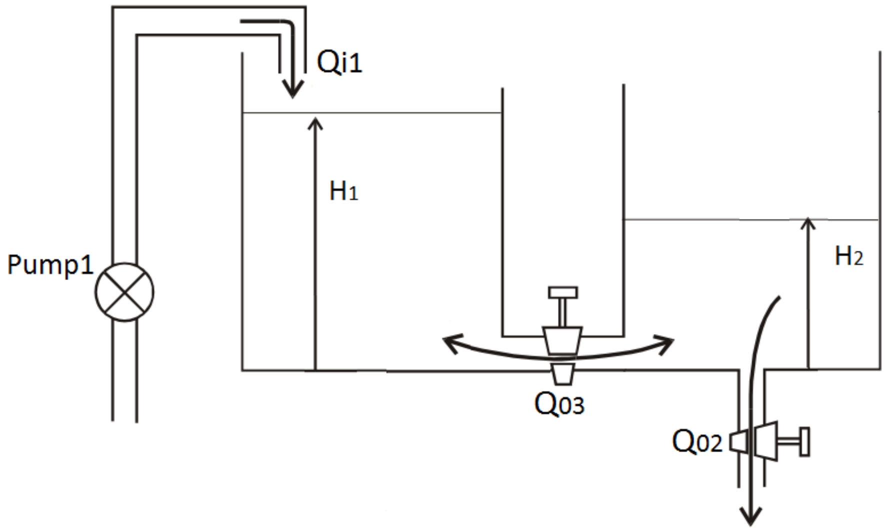

Fig. 5.1 layout of SISO coupled tanks system

The state space model of the SISO continuous time model as derived above is given by,

$$\begin{pmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{pmatrix} = \begin{pmatrix} \frac{-1}{A}\left(\frac{\alpha_3}{2\sqrt{H_1-H_2}}\right) & \frac{1}{A}\left(\frac{\alpha_3}{2\sqrt{H_1-H_2}}\right) \\ \frac{1}{A}\left(\frac{\alpha_3}{2\sqrt{H_1-H_2}}\right) & \frac{-1}{A}\left(\frac{\alpha_2}{2\sqrt{H_2}}+\frac{\alpha_3}{2\sqrt{H_1-H_2}}\right) \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} \frac{1}{A} \\ 0 \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

$$y_1 = \begin{pmatrix} 0 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

The parameters of the coupled tanks system as given by [13, 26], are presented in the table below,

TABLE I. PARAMETERS OF THE COUPLED TANK SYSTEM

Symbol	Quantity	Value
A_1	tank 1 section area	9350.10^{-6} m^2
A_2	tank 2 section area	9350.10^{-6} m^2

Symbol	Quantity	Value
s_1,s_2,s_3	channel sectional area	$78.50.10^{-6}$ m^2
a_0	discharge coefficient of channel 1	≈1
a_0	discharge coefficient of channel 2	≈1
a_1	discharge coefficient of channel 3	≈0.5
h_{max}	maximum liquid level	0.25 m
g	gravitational constant	9.8 m/s^2

In our calculations the pump and the sensor gains were ignored, which add to the input and output gain respectively.

The operating points of the system H1 and H2 depends on the tank capacity and can be any value between 0 and 0.25m. The only limitation in choosing these values is a large difference between them as the input pump has limited input capability and the states of the system are coupled. The operating points chosen for the SISO system were H_1=0.09 and H_2=0.08.

Now, all the parameters required to define the system completely have been identified hence the state dynamics can be defined.

State space model matrices for SISO CTS

$$A = \begin{pmatrix} -0.0929 & 0.0929 \\ 0.0929 & -0.1586 \end{pmatrix}$$

$$B = \begin{pmatrix} 106.9518 \\ 0 \end{pmatrix} \tag{5.1}$$

$$C = \begin{pmatrix} 0 & 1 \end{pmatrix}$$

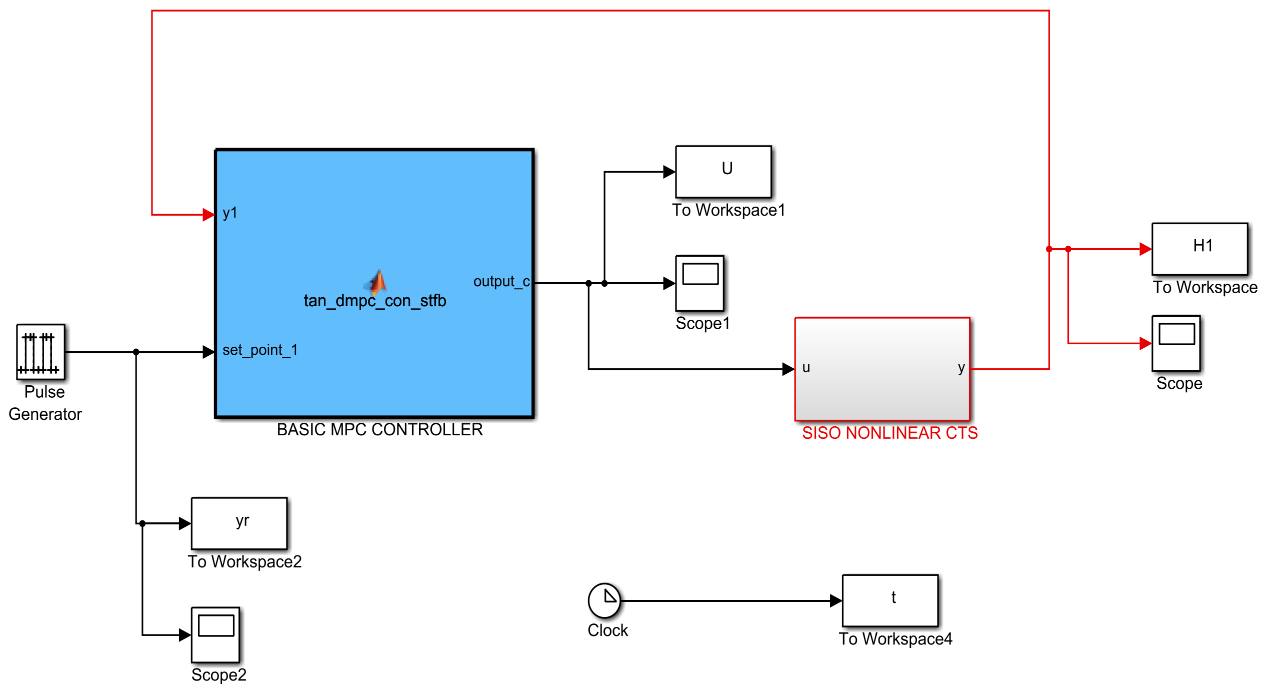

Fig. 5.2 Simulink model of MPC implementation to nonlinear SISO CTS

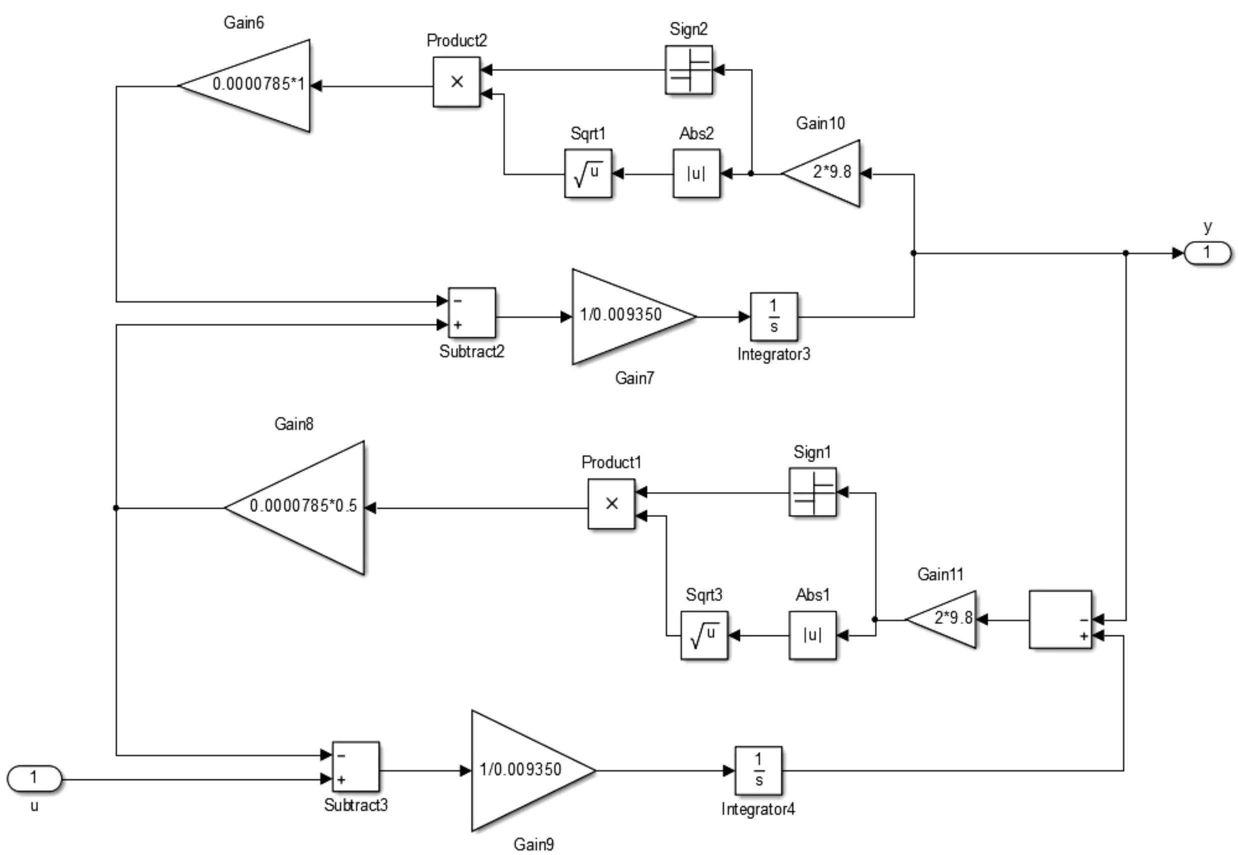

Fig. 5.3 Simulink schematic of SISO coupled tanks system

MATALB/Simulink environment was used to test the performance of Linear MPC when used to control the nonlinear coupled system. Fig. 5.1 shows the Simulink schematics of application of discrete model predictive controller to the SISO CTS. The MPC algorithm was first written in MATLAB script and then converted and implemented in Simulink using embedded MATLAB functions. The schematic of the SISO coupled tanks system as shown in Fig. 5.2 is made in the Simulink environment.

The reference level for H_2 used to evaluate the response of MPC controllers applied to CTS is a pulse train whose amplitude varies between 0 and 0.12m.

5.2.1 Application of Basic MPC Approach to Nonlinear SISO CTS

The basic MPC approach based on GPC is a restricted model approach. The controller is implemented on SISO CTS and a pulse train is used as reference as shown if Fig. 5.3 below the system does not attain the desired set-point. There are lots of undershoots and overshoots in the plant response which are undesirable. In this result the control horizon was set to 2 and prediction horizon to 40. The optimal parameter values are attained through trial and error.

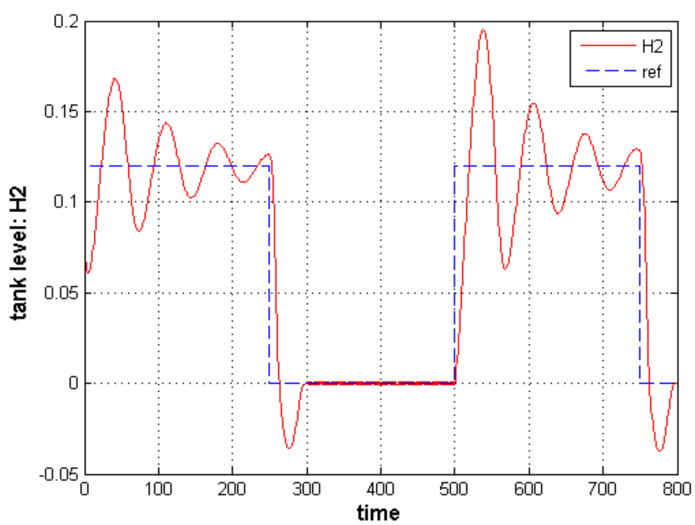

Fig. 5.4 Basic MPC approach with control horizon of 2

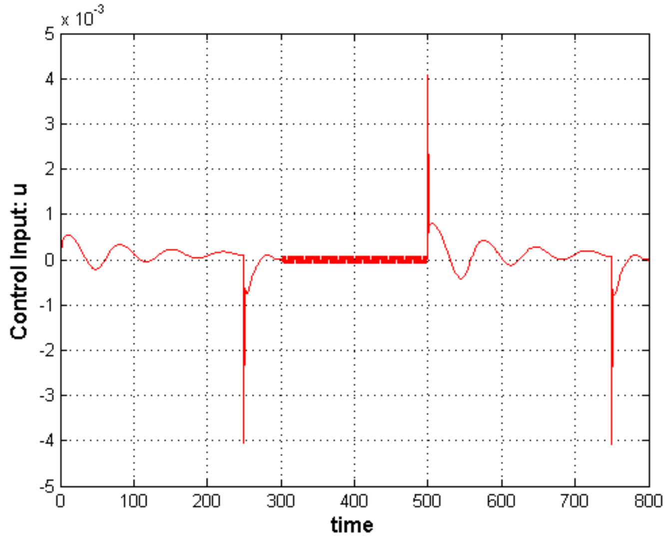

Fig. 5.5 Control Input with Basic MPC and a control horizon of 2

The control horizon is represented by Nc, it dictates the number of parameters used to capture the future control trajectory. Np represents the prediction horizon. The future states of the system are predicted for number of samples equal to the prediction horizon provided the current states of the system are known. Hence it can be concluded that the higher the prediction and control horizon are the better the performance of the controller will be. The prediction horizon is always greater than the control horizon. To begin with the prediction horizon is already kept at a high enough value, so we will try increasing the value of control horizon and see if the performance is improved. Firstly, we will increase the value of control horizon to 10.

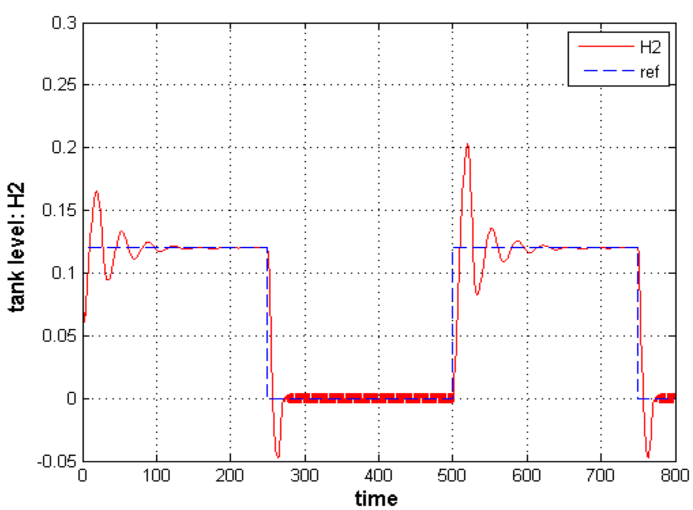

Fig. 5.6 Basic MPC approach with control horizon of 10

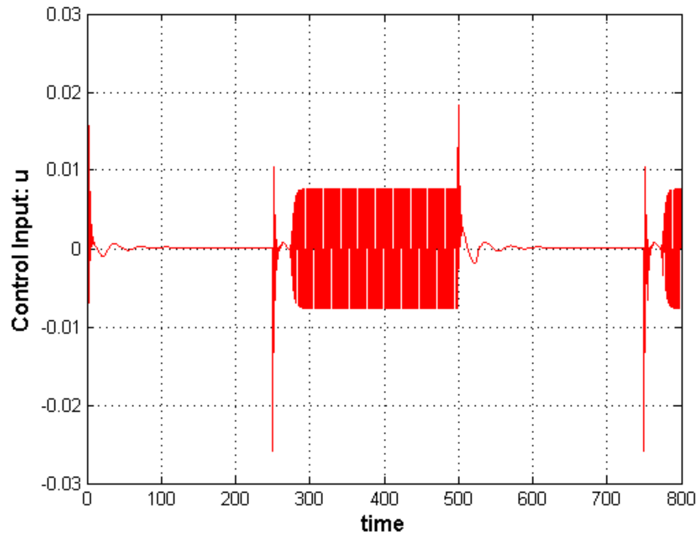

Fig. 5.7 Control Input with Basic MPC and a control horizon of 10

It can be seen from the results obtained from Fig. 5.5 and Fig 5.6 that after the control horizon parameter is tuned to 10 the controller now attains the required set-point but the performance is not good. Hence, increasing the control horizon further.

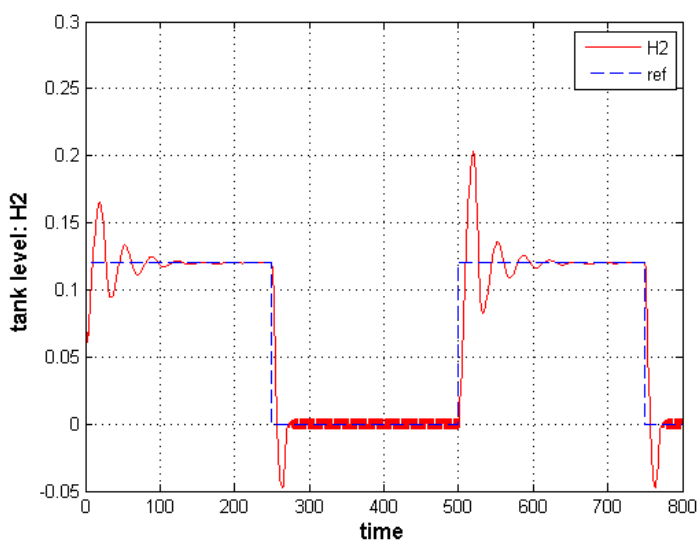

Fig. 5.8 Basic MPC approach with control horizon of 30

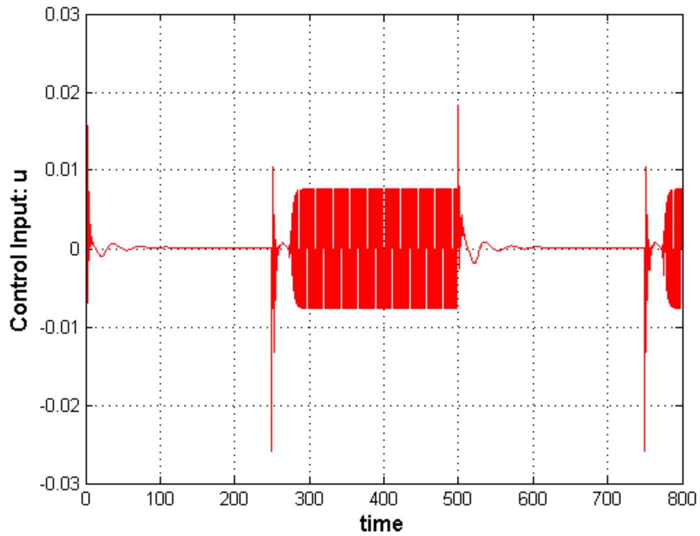

Fig. 5.9 Control Input with Basic MPC and a control horizon of 30

Irrespective of the value of the control horizon the overshoots and undershoots are not eliminated. The results show that in order to improve performance a long control horizon is required and not much improvement is seen after the control horizon exceeds the value 10. The Root-Mean-Square Error (RMSE) can be used as a measure of the differences between the actual system output and the desired set point. The RMSE values for the varied control horizon are shown in table below.

TABLE II. RMSE WITH VARIED CONTROL HORIZON

Control Horizon	**RMSE**
2	0.02852
10	0.01934
30	0.01944

5.2.2 Application of MPC Based On Laguerre Functions to Nonlinear SISO CTS

MPC based on Laguerre functions has tuning parameters *a* i,e scaling factor of the Laguerre network and *N* i.e the number of Laguerre functions used to generate the control trajectory.

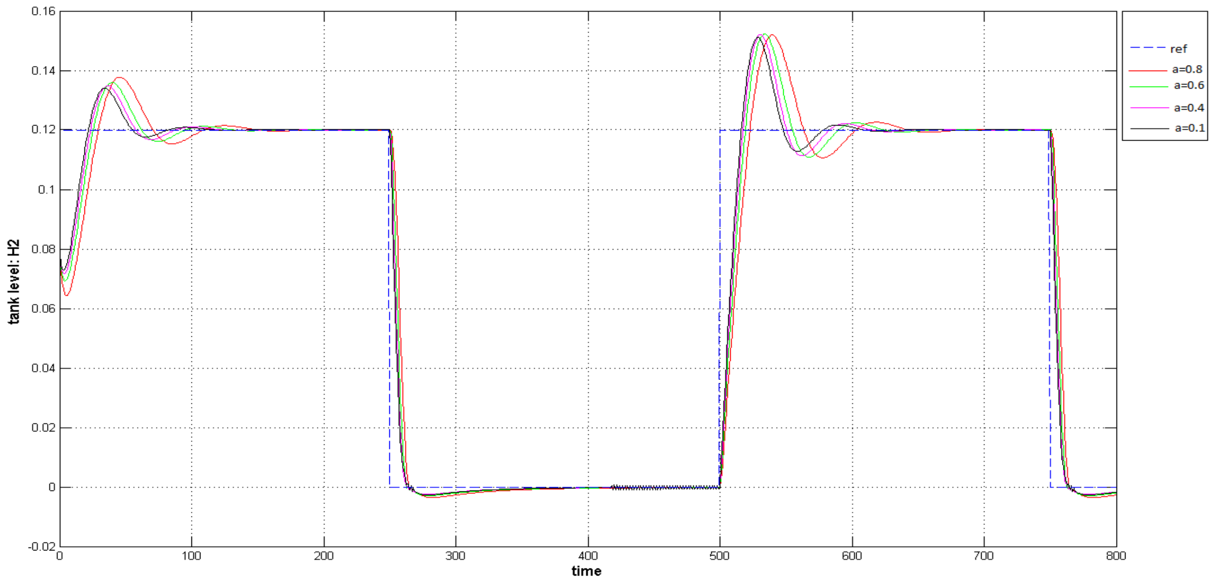

Fig. 5.10 comparison of response with constant N and varied a

These parameters are required to be selected by the user freely. For tuning purpose initially lower value of N is used, the Laguerre terms. The scaling factor is always in the range 0<a<1. Laguerre functions are actually exponential functions with a decay factor a. Hence by using the Laguerre functions the projected control signal is forced to decay exponentially. Firstly the scaling factor by which the best performance is achieved will be determined by keeping the Laguerre terms constant and then do the same to tune N by using the scaling factor selected earlier.

Initially stating with N=2 and different values of a. Fig 5.9 shows that the response speed increases as the scaling factor is reduced and the overshoots and undershoots are reduced as scaling factor is reduced. Fig 5.10 shows that best performance is achieved at the scaling factor of 0.1. To have a quantified verification the Root-Mean-Square Error (RMSE) values for each case is shown in the table below.

TABLE III. RMSE WITH VARIED SCALING FACTOR

No. of Laguerre Terms (N)	Scaling Factor (a)	RMSE
N=2	0.8	0.02096
	0.6	0.01844
	0.4	0.01692
	0.1	0.0158

Now, by keeping the scaling factor at 0.1 the number of Laguerre terms i.e. N will be varied.

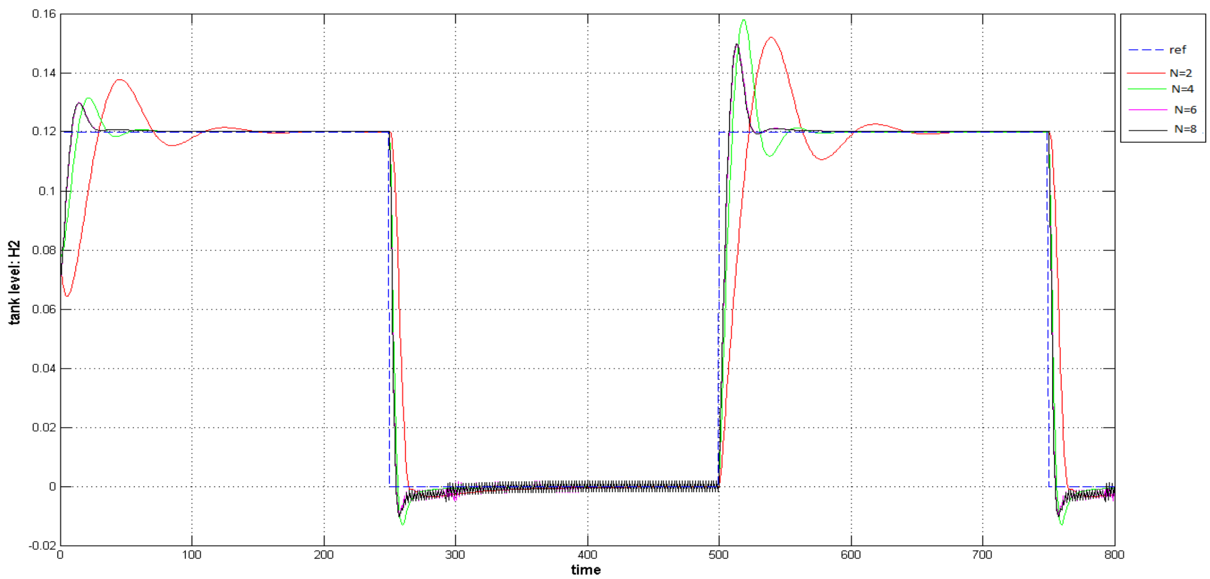

Fig. 5.11 comparison of response with tuned a and varied N

Fig. 5.11 shows that when the number of Laguerre terms is increased from 2 to 4 a significant improvement in performance is seen. But, when the value of N is increased above 6 the system performance remains unaltered. Hence it can be concluded that as N increases the control becomes more aggressive. The RMSE for each case is shown in table below.

TABLE IV. RMSE WITH VARIED NUMBER OF LAGUERRE TERMS

Scaling Factor (a)	No. of Laguerre Terms (N)	RMSE
a=0.1	2	0.0158
	4	0.01307
	6	0.01128
	8	0.01127

The result also shows that as Laguerre terms are increased while improvement in performance is seen but noise is introduced at the lower value of the pulse reference.

Hence with the tuned parameters N = 6 and a=0.1 the system response and the control sequence are shown below.

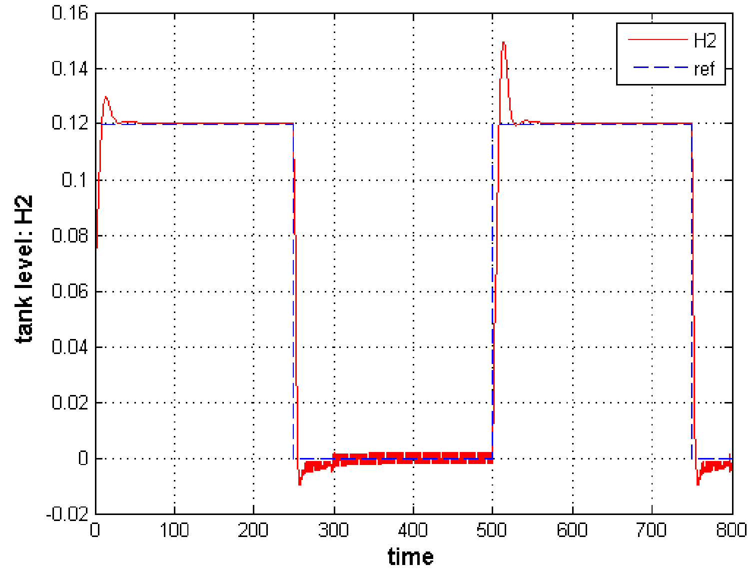

Fig. 5.12 output response with MPC based on Laguerre functions

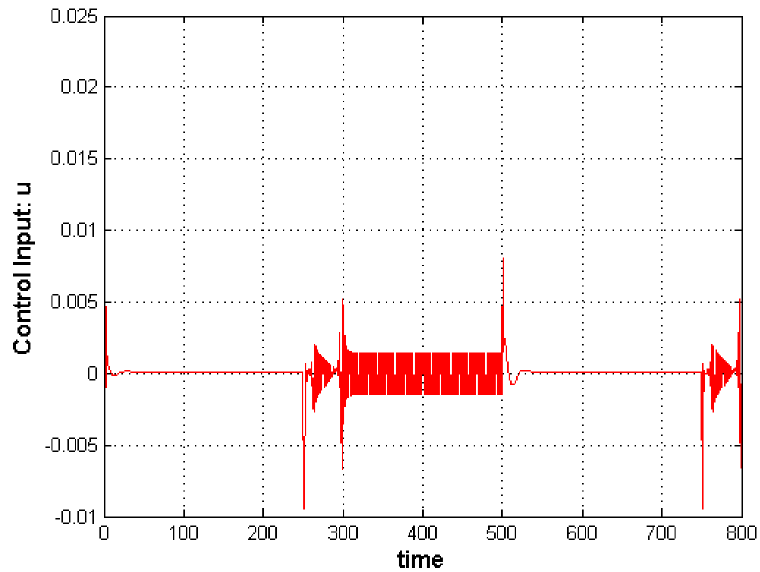

Fig. 5.13 Control input with MPC based on Laguerre polynomials

Simulation results shown in Fig. 5.11 and Fig 5.12 indicate improvement in performance when MPC with Laguerre functions is employed for control of SISO CTS as compared to the performance of basic MPC Fig. 5.7. The size of undershoot and overshoot are reduced considerably. In basic MPC approaches approximation of control signal required a large number of parameters defined as the control horizon resulting in high computational load. Laguerre functions behave like exponential functions with a decay factor a. Thus the incremental control signal decays exponentially. Therefore, when Laguerre polynomials are used less parameters are required to define the control sequence. As shown in the results only 6 parameters were required to capture the control trajectory hence reducing the on-line computational cost significantly.

The issue concerning the reduction of noise in the steady state without degradation of performance could not be coped with by tuning controller parameters.

5.2.3 Application of Modified Basic MPC Scheme to Nonlinear SISO CTS

A more recent variation of the basic MPC approach [12] is employed. By using a prediction horizon of 40 we will observe the controller performance at different control horizons.

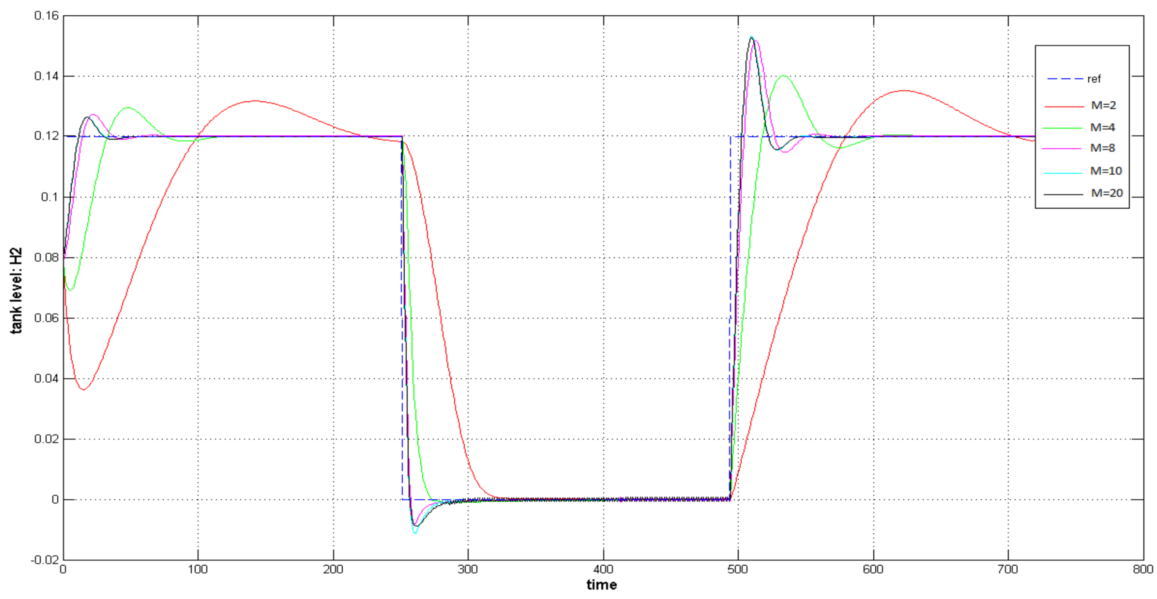

Fig. 5.14 comparison of response with varied control horizon M

The RMSE with varied Control Horizon is shown below,

TABLE V. RMSE WITH VARIED CONTROL HORIZON

Control Horizon	RMSE
2	0.03665
4	0.01862
8	0.01269
10	0.01151
20	0.01141

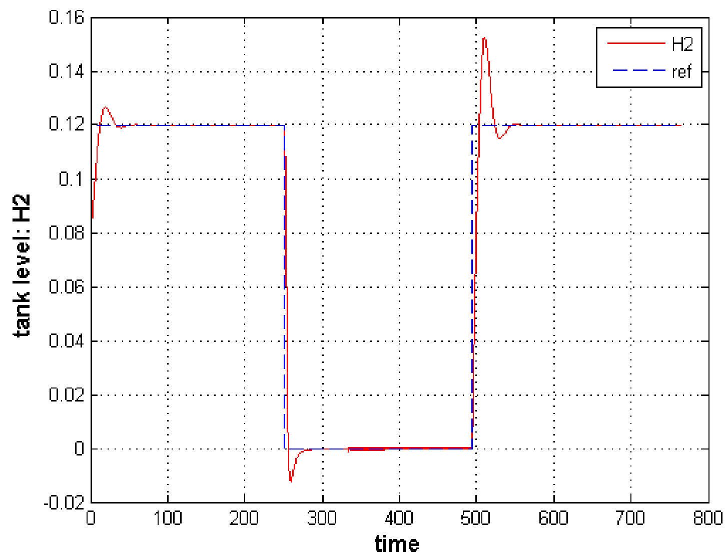

Fig. 5.15 output response with modified basic MPC

It can be seen by the responses shown above that the performance achieved at a control horizon of 20 is nearly the same as was obtained by when MPC with Laguerre functions was used as shown in Fig. 5.12. The noise in the lower level of the pulse train is reduced in comparison to the Laguerre approach. But here a large control horizon was used to achieve this result.

5.3 Comparison between MPC and PID Control of SISO CTS

The proportional-integral-derivative controller (PID controller) is a feedback control mechanism widely used in the process industries. The PID controller compares the measured process variable and the desired reference point and uses the difference between them i.e error to compute the control input to the plant. The PID controller minimizes the error by adjusting the plant control input. The PID structure is shown in Fig. 5.16

The main advantage of MPC over PID controllers is the ability to handle constraints, non-minimum phase processes, changes in system parameters (robust control) and its straightforward applicability to large multivariable processes [28] .

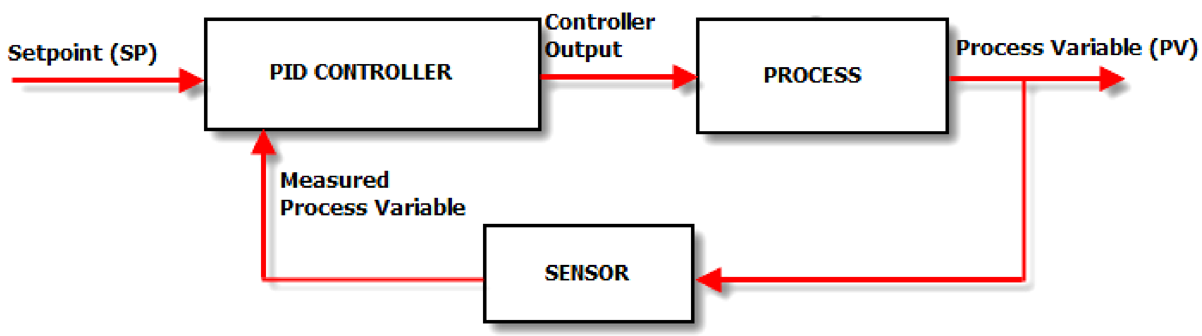

Fig. 5.16 PID control structure

The PID controller is applied to the coupled tanks system for comparison of overall control achieved by the PID and the MPC control. The Fig. 5.16 shows that the desired setpoint of 0.12 is nearly attained at the edge of the high pulse of the pulse train, this can be seen more clearly in Fig. 5.18.

Compared to the basic MPC approach the PID control has less overshoots and undershoots but as shown in Fig. 5.19 the basic MPC attains the setpoint more quickly.

The results in Fig. 5.12 and Fig. 5.15 show that the MPC based on Laguerre functions and the modified basic MPC scheme both perform much better than the PID controller.

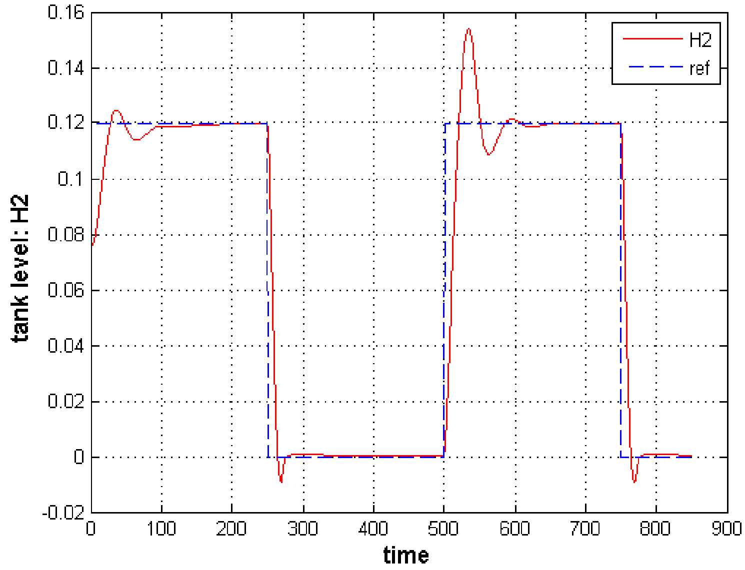

Fig. 5.17 output response with PID control

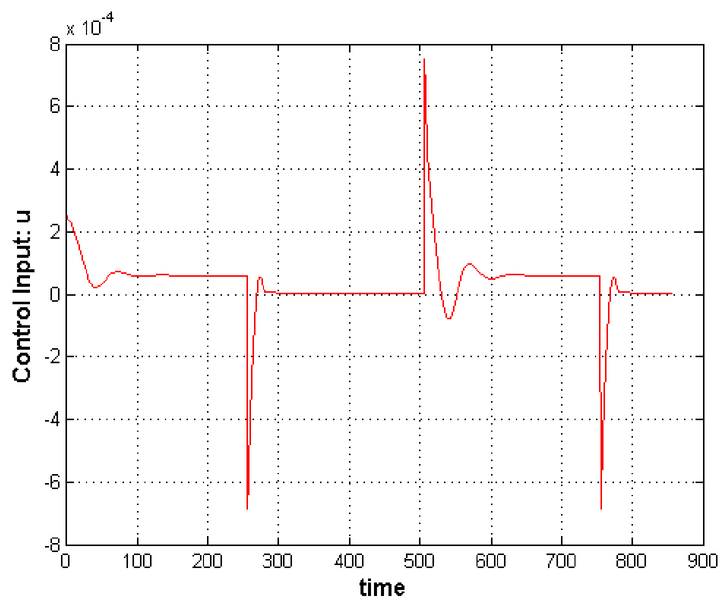

Fig. 5.18 Control input with PID control

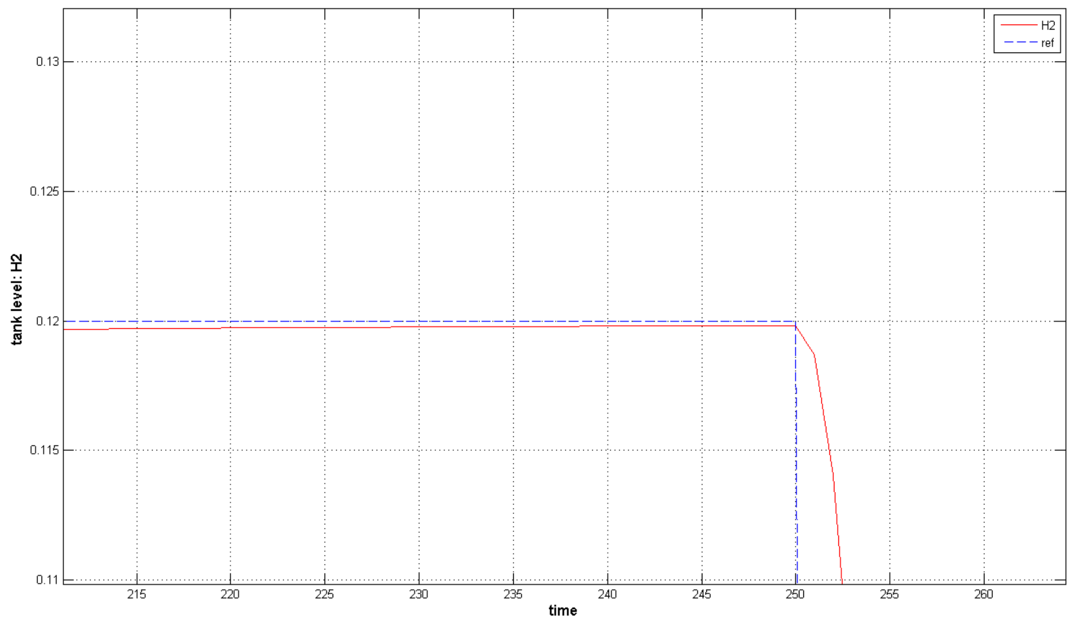

Fig. 5.19 output with PID control zoomed in

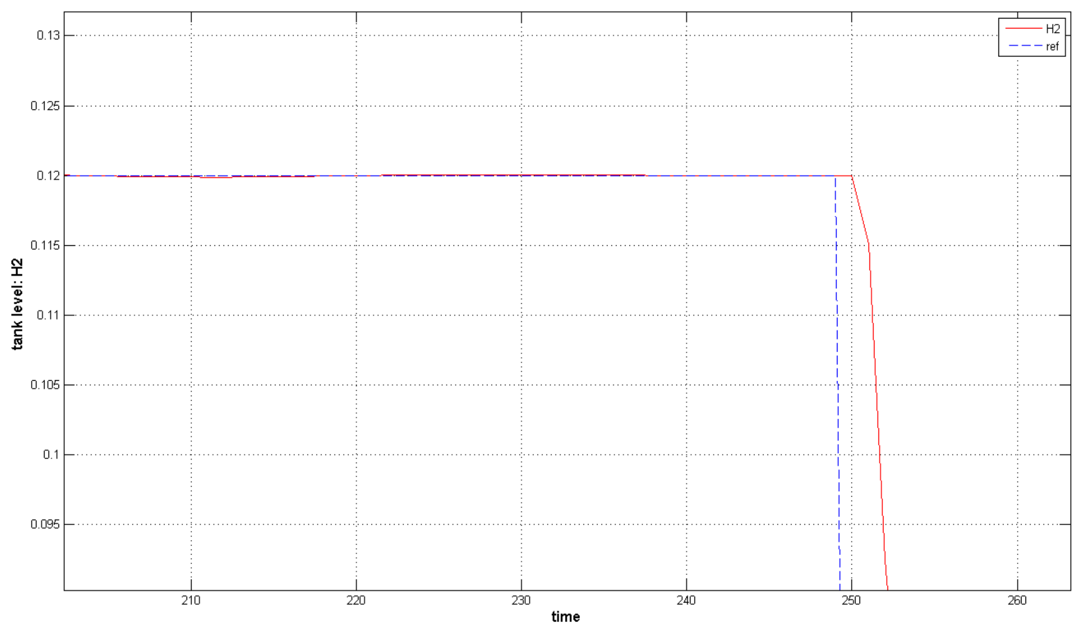

Fig. 5.20 output response with Basic MPC control zoomed in

The Root-Mean-Square Error (RMSE) and Mean Absolute Error (MAE) can be used as a measure of the differences between the actual system output and the desired set point depicting the accuracy of the control scheme. The Root-Mean-Square Error and Mean Absolute Error Comparison of the three MPC approaches and the PID control are shown in the table and column chart below.

TABLE VI. RMSE AND MAE RESULTS FOR COMPARISON OF SISO CONTROL

METHOD	RMSE	MAE
PID Control	0.01934	0.00773
Basic MPC	0.01937	0.008792
Modified Basic MPC	0.01141	0.003428
MPC Based on Laguerre Functions	0.01128	0.00296

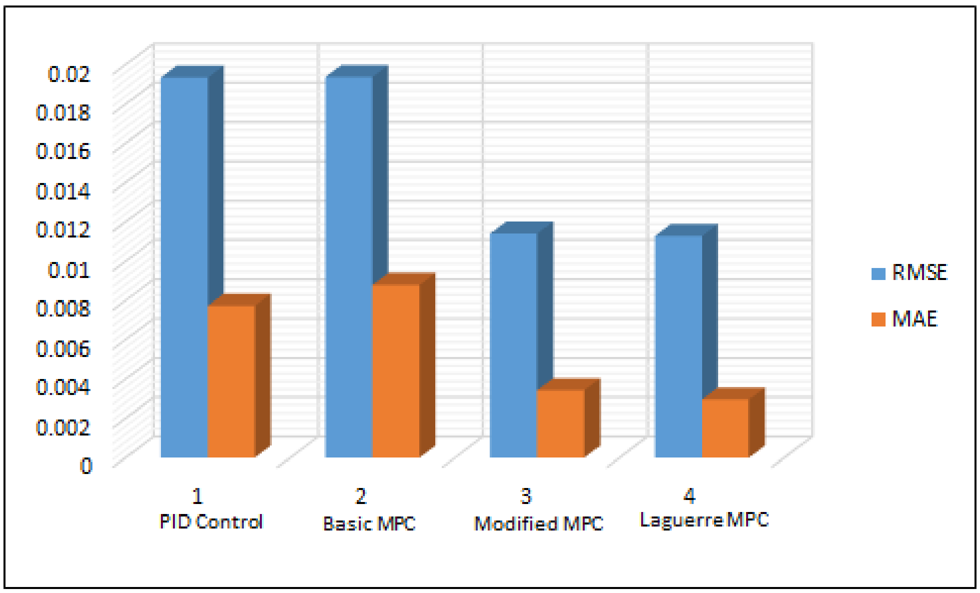

Fig. 5.21 RMSE and MAE results for SISO control

5.4 Implementation of MPC on MIMO Coupled Tanks System

The state space model of the SISO continuous time model as derived above is given by,

$$\begin{pmatrix} \frac{dh_1}{dt} \\ \frac{dh_2}{dt} \end{pmatrix} = \begin{pmatrix} \frac{-1}{A}\left(\frac{\alpha_1}{2\sqrt{H_1}} + \frac{\alpha_3}{2\sqrt{H_1 - H_2}}\right) & \frac{1}{A}\left(\frac{\alpha_3}{2\sqrt{H_1 - H_2}}\right) \\ \frac{1}{A}\left(\frac{\alpha_3}{2\sqrt{H_1 - H_2}}\right) & \frac{-1}{A}\left(\frac{\alpha_2}{2\sqrt{H_1}} + \frac{\alpha_3}{2\sqrt{H_1 - H_2}}\right) \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} + \begin{pmatrix} \frac{1}{A} & 0 \\ 0 & \frac{1}{A} \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \end{pmatrix}$$

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}$$

To describe the parameters of the state space model we will use the values already used for the SISO system shown in TABLE 1. The state dynamics can be defined.

<u>State space model matrices for MIMO CTS</u>

$$A = \begin{pmatrix} -0.3096 & 0.09288 \\ 0.09288 & -0.1586 \end{pmatrix}, B = \begin{pmatrix} -0.0619 & 0 \\ 0 & -0.0619 \end{pmatrix},$$

$$C = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, D = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

The simulations for the MIMO controllers were again carried out in MATLAB/Simulink. The Simulink model for the nonlinear MIMO system is shown below,

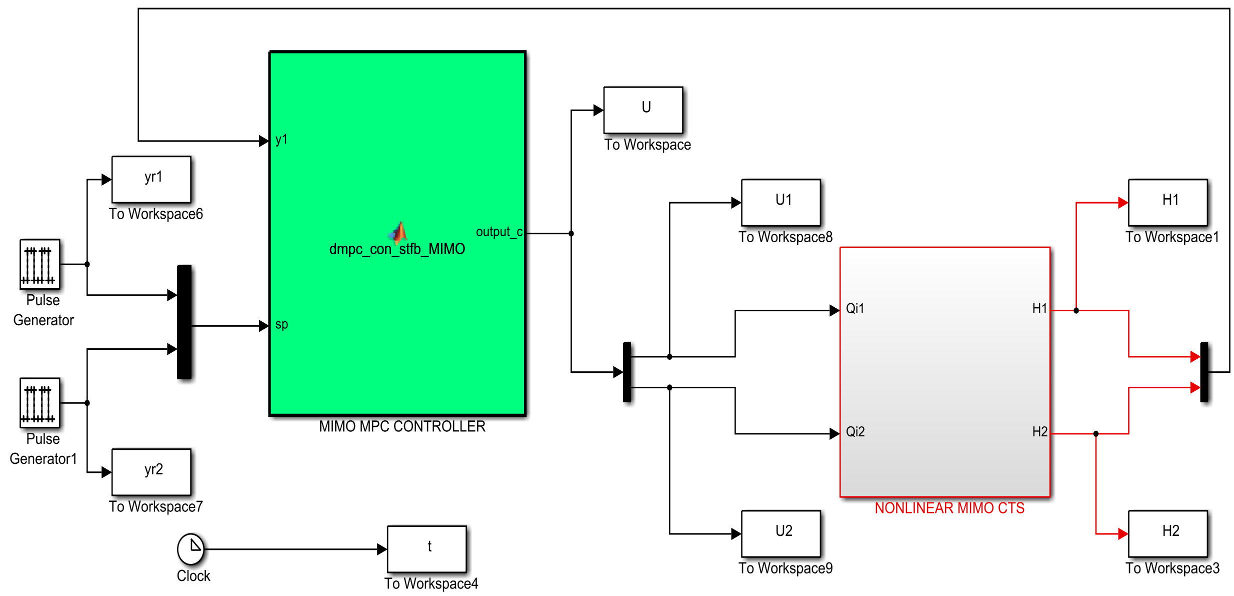

Fig. 5.22 Simulink model of MPC implementation to nonlinear MIMO CTS

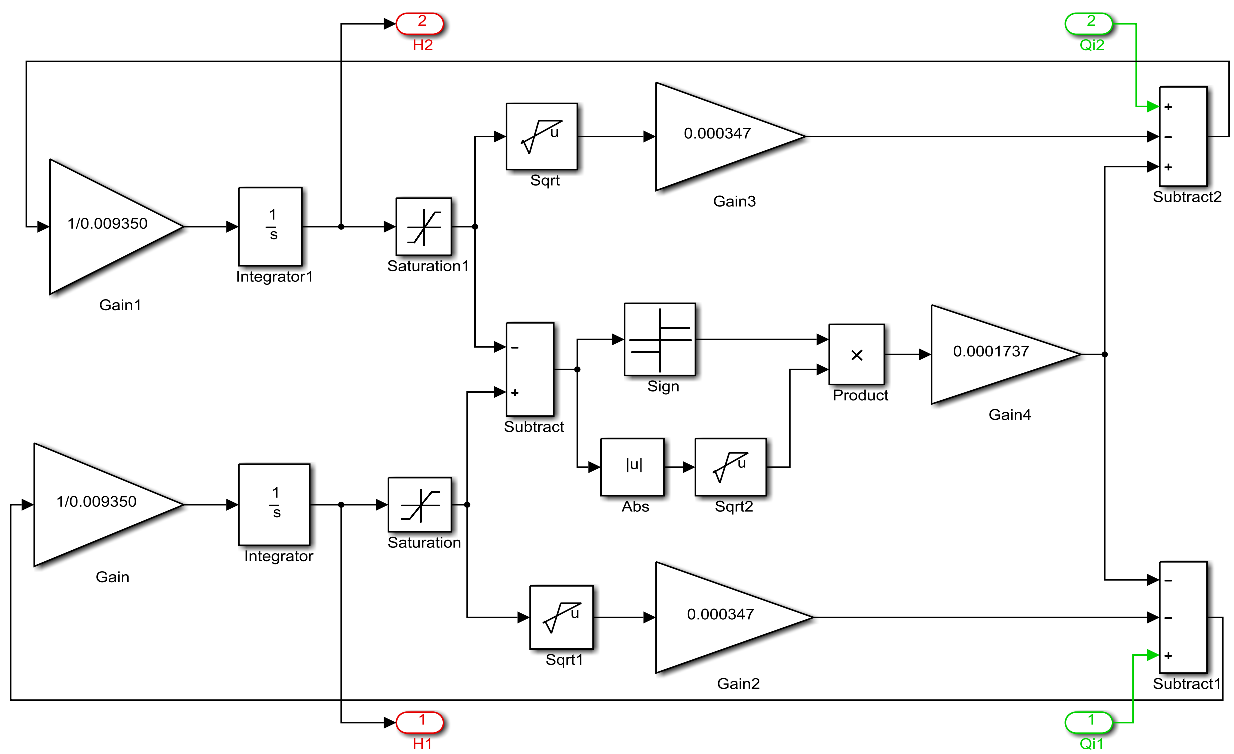

Fig. 5.23 Simulink schematic of MIMO coupled tanks system

The reference levels used for H_1 and H_2 are pulse train with amplitude variations of 0 to 0.12m and 0 to 0.15m respectively. Application of MPC approaches i.e Laguerre Network approach and the Modified basic MPC approach to the nonlinear MIMO CTS is presented next.

5.4.1 Application of MPC Based On Laguerre Functions to Nonlinear MIMO CTS

The Laguerre parameters were tuned using the same methodology which was adopted for tuning the SISO controller. The parameters values are a=0.4 and N=4.

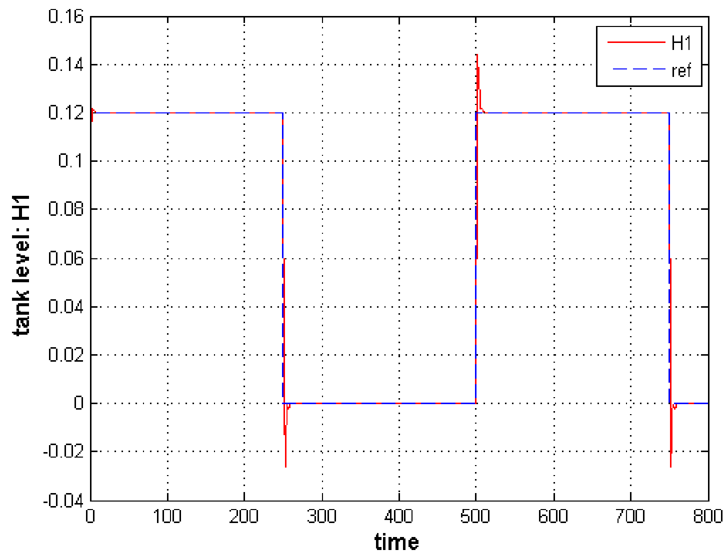

Fig. 5.24 tank 1 level response using MPC based on Laguerre functions

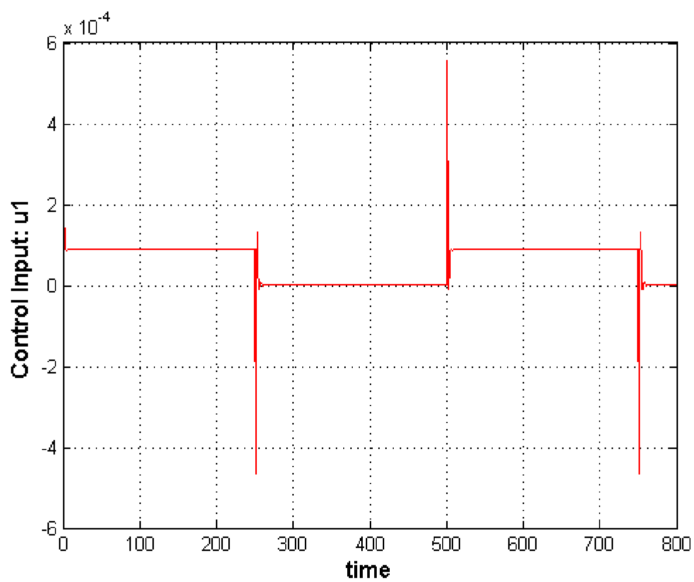

Fig. 5.25 tank 1 level control using MPC based on Laguerre functions

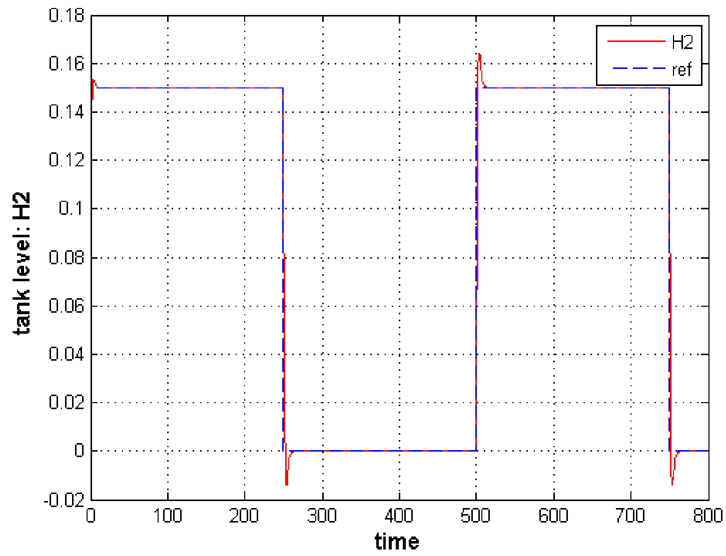

Fig. 5.26 tank 2 level response using MPC based on Laguerre functions

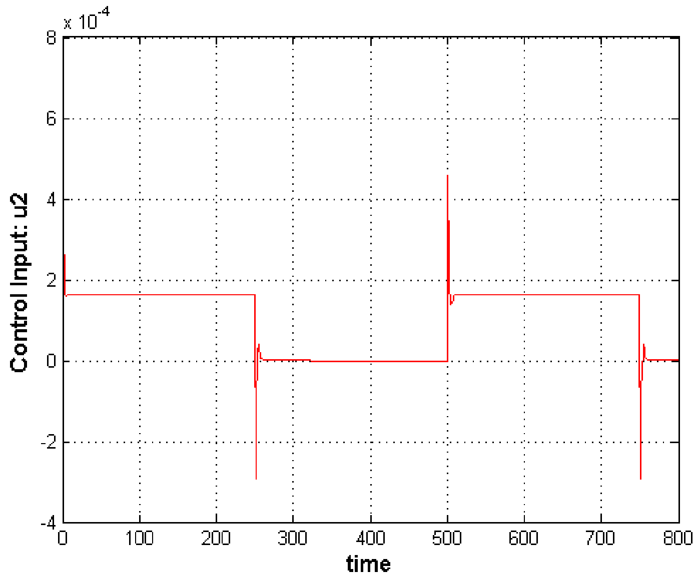

Fig. 5.27 tank 2 level control using MPC based on Laguerre functions

The performance of MPC with Laguerre functions with MIMO plant is adequate. Tight control is observed throughout the simulation.

5.4.2 Application of Modified Basic MPC Scheme to Nonlinear MIMO CTS

In this MPC approach the predicted output considers the error between the output of the actual nonlinear plant and the output calculated from the linear plant model. This is very helpful especially at different operating points and there always exists some difference in the responses of the linearized system and the actual nonlinear system. The tuned parameters for this simulation are prediction horizon Np=40 and control horizon M=10.

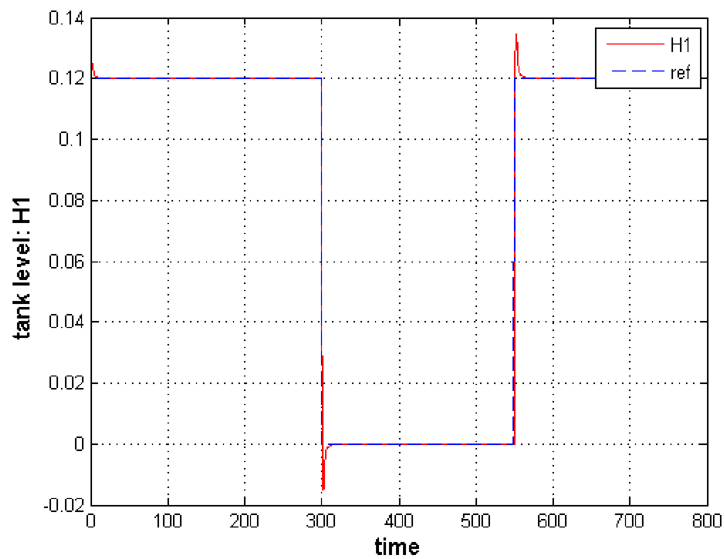

Fig. 5.28 tank 1 level response using modified basic MPC

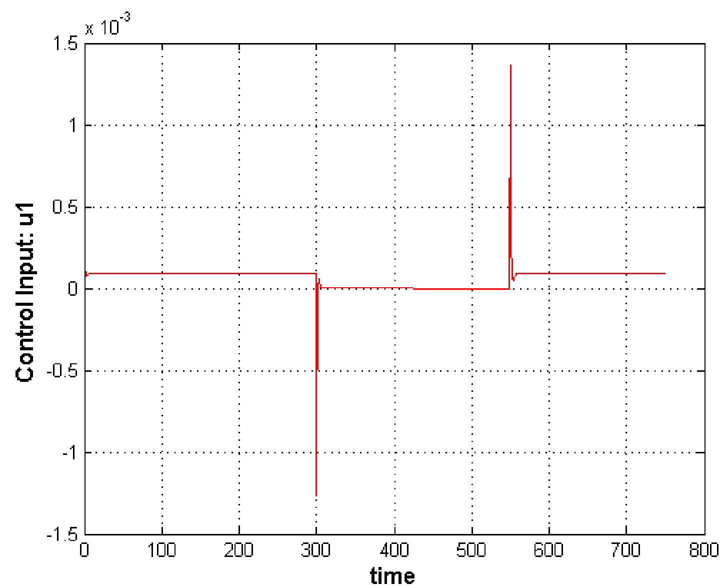

Fig. 5.29 tank 1 level control using modified basic MPC

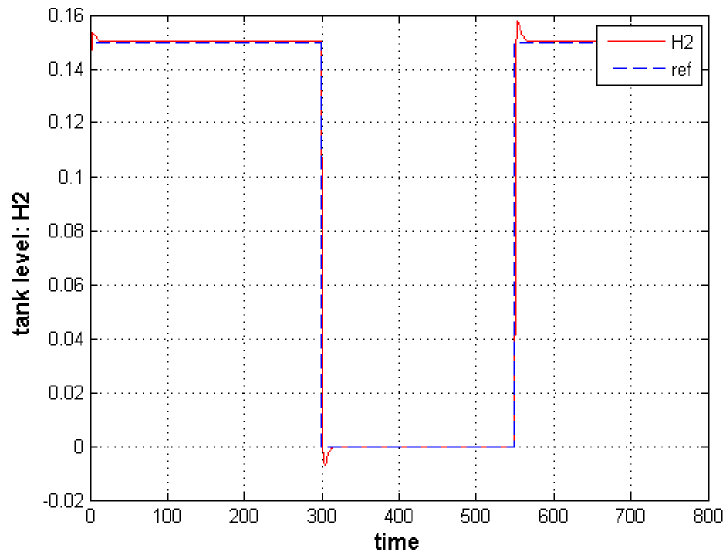

Fig. 5.30 tank 2 level response using modified basic MPC

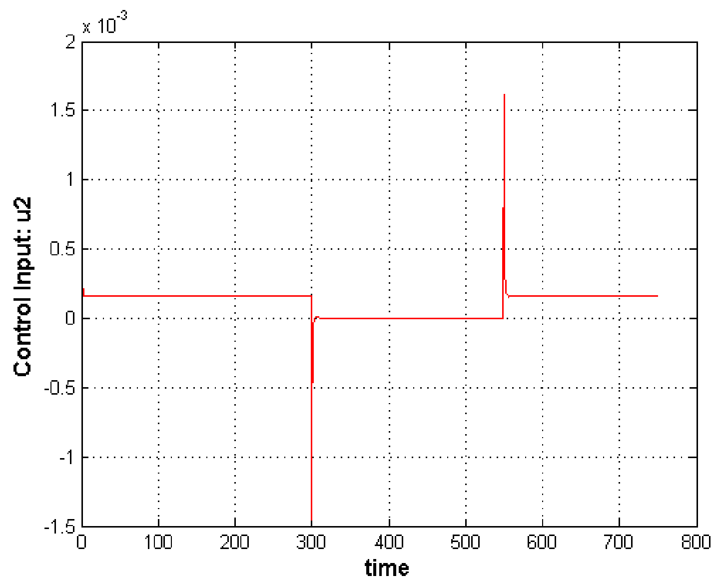

Fig. 5.31 tank 2 level control using modified basic MPC

From these results we can observe that there is not much difference in performance of both the approaches used. The control of both these controllers is adequate for the MIMO CTS. The advantage of Laguerre based control is the less computational load, this can be very beneficial in real time implementation. The Root-Mean-Square Error and Mean Absolute Error Comparison of the two MIMO MPC approaches is presented below.

TABLE VII. RMSE AND MAE RESULTS COMPARISON FOR MIMO SYSTEM

METHOD	RESPONSE	RMSE	MAE
Modified Basic MPC	Tank 1 Level	0.005422	0.0006154
	Tank 2 Level	0.00731	0.0008391
MPC Based on Laguerre Functions	Tank 1 Level	0.006375	0.0002836
	Tank 2 Level	0.00768	0.0007152

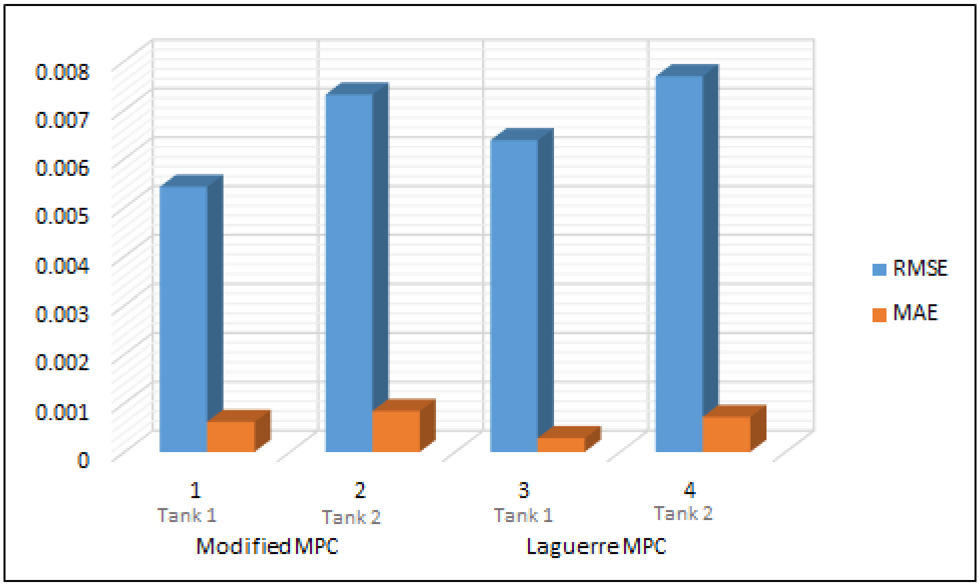

Fig. 5.32 RMSE and MAE column chart for MIMO CTS system

5.5 Constrained Control of Nonlinear Coupled Tanks System

One of the most unique features of MPC is the inclusion of constraint information in the constraint and the ability of the controller to handle hard constraints. Implementation of hard constraints requires a quadratic optimization problem to be worked out. The optimization law combines the operation limits and the control objective in to a single problem. We will consider a set of constraints to study the controller constraint handling capability.

5.5.1 Constrained Control of Nonlinear SISO CTS

The first set of constraints considered for the SISO system:

1. Constraints on Input Amplitude

$$-0.004 \leq u(ki+k) \leq 0.004$$

2. Constraints on Input Slew Rate

$$-0.004 \leq \Delta U(ki+k) \leq 0.004$$

5.5.1.1 Constrained Control of SISO CTS using MPC based on Laguerre Functions

Considering the set of constraints defined above and enforcing constraints using quadratic programming.

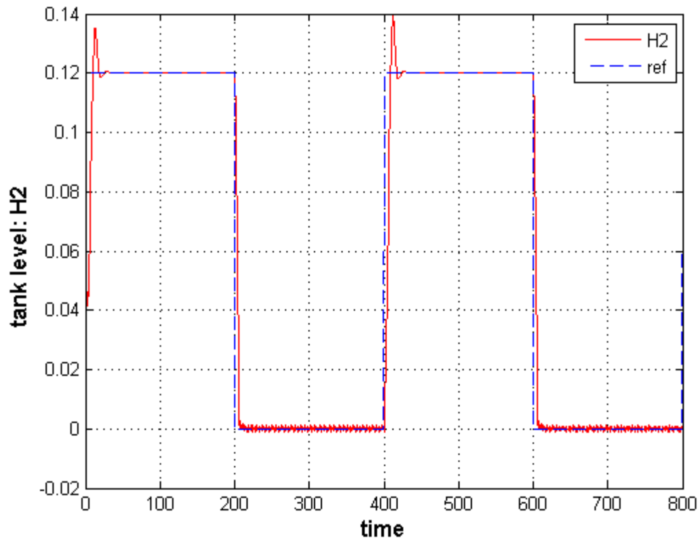

Fig. 5.33 system output with input and slew rate constraints

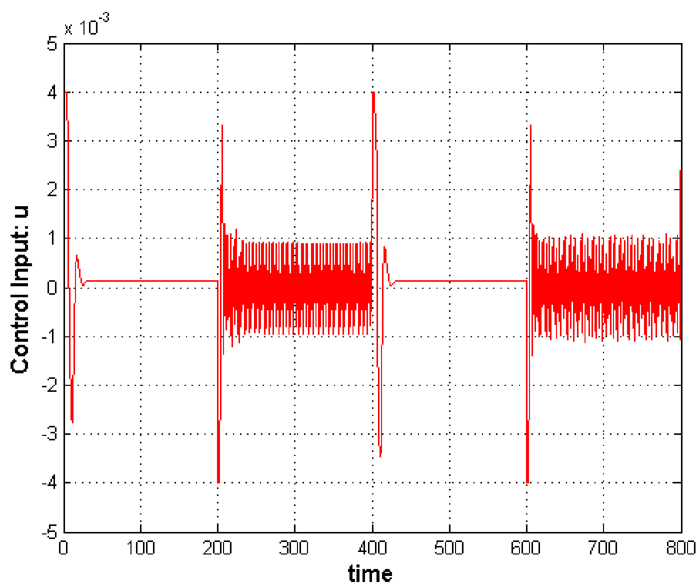

Fig. 5.34 control input with input and slew rate constraints

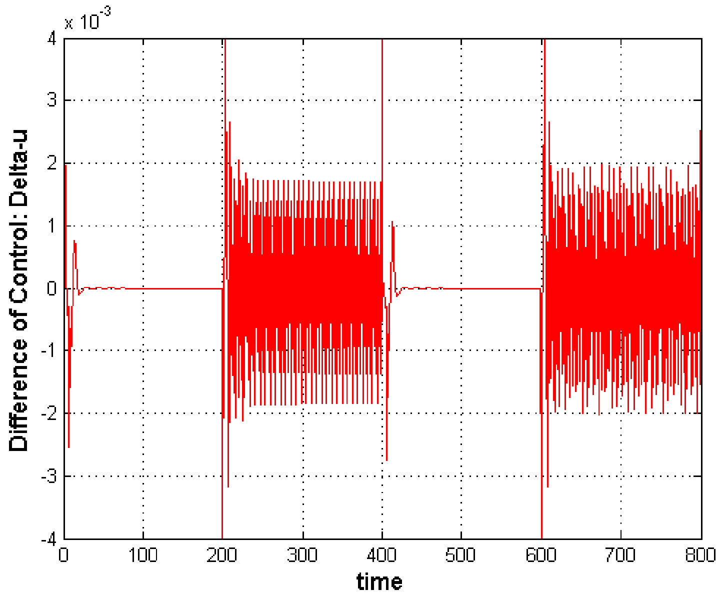

Fig. 5.35 slew rate with input and slew rate constraints

5.5.1.2 Constrained Control of SISO CTS using Modified MPC scheme

The results obtained when the modified MPC was applied to SISO tank system are presented below.

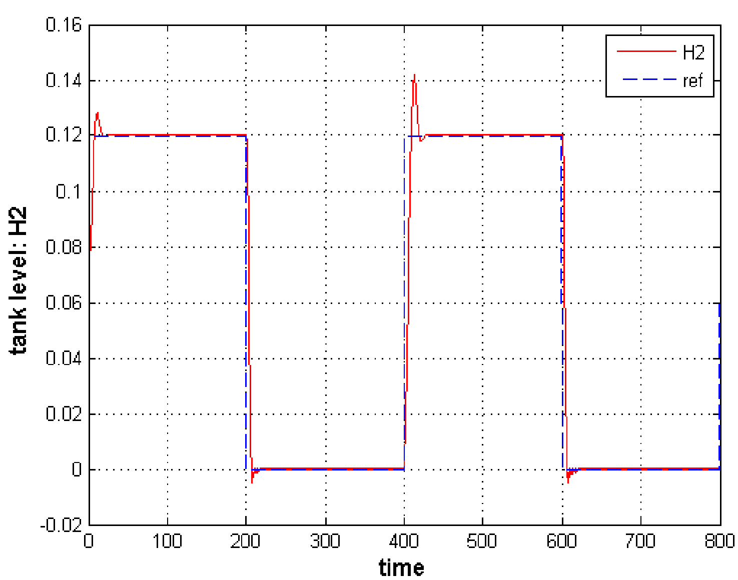

Fig. 5.36 system output with input and slew rate constraints

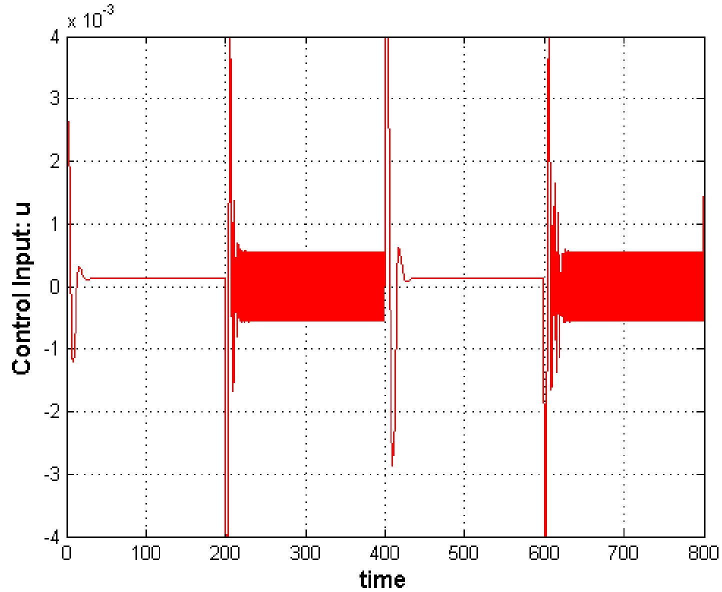

Fig. 5.37 control input with input and slew rate constraints

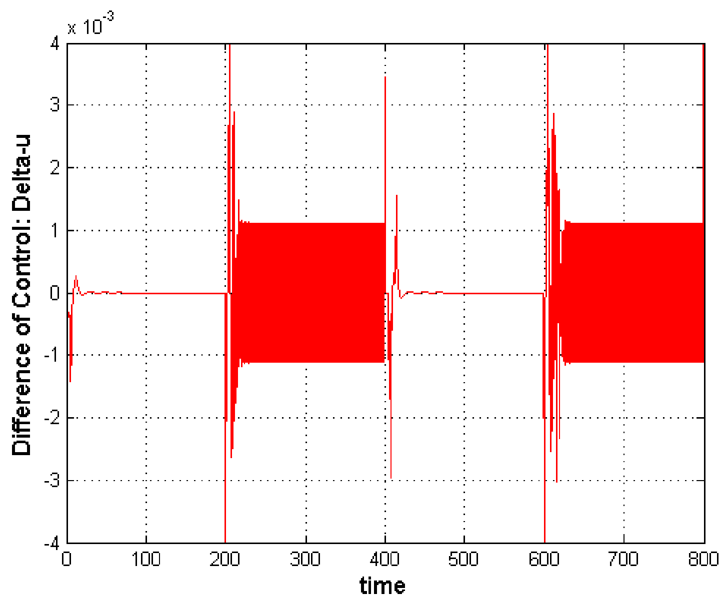

Fig. 5.38 slew rate with input and slew rate constraints

The RMSE and MAE Comparison of the two SISO MPC approaches with enforced constraints is presented below. Both controllers efficiently handle the applied hard constraints. Soft constraints can also be defined in MPC algorithm.

TABLE VIII. RMSE AND MAE RESULTS FOR COMPARISON OF CONSTRAINED SISO CONTROL

METHOD	RMSE	MAE
Modified Basic MPC	0.01454	0.003235
MPC Based on Laguerre Functions	0.01381	0.00312

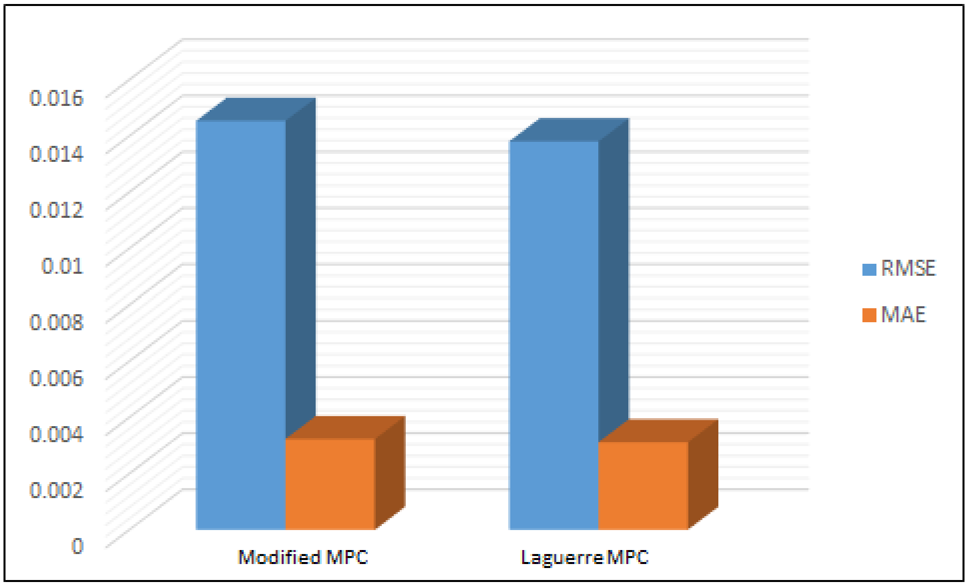

Fig. 5.39 RMSE and MAE results column chart for constrained SISO control

5.5.2 Constrained Control of Nonlinear MIMO CTS

One of the major attraction of Model Predictive Control is the ability to handle multivariable systems with constraints. MPC provides the flexibility of defining constraints for each manipulated variable and output individually.

The constraints are now implemented on the Two-Tank system with MIMO configuration using MPC based on Laguerre function and the modified MPC scheme. The objective is to achieve the desired MPC control law without violating the constraints defined. A similar set of constraints are considered for both inputs and slew rates.

The set of constraints considered for the MIMO system:

1. Constraints on Input Amplitude :- $0.001 \le u1(ki+k) \le 0.001$

   $0.001 \le u2(ki+k) \le 0.001$

2. Constraints on Input Slew Rate: $-0.001 \le \Delta U1(ki+k) \le 0.001$

   $-0.001 \le \Delta U2(ki+k) \le 0.001$

5.5.2.1 Constrained Control of Nonlinear MIMO CTS using MPC based on Laguerre Functions

Considering a set of constraints to study the controller constraint handling capability.

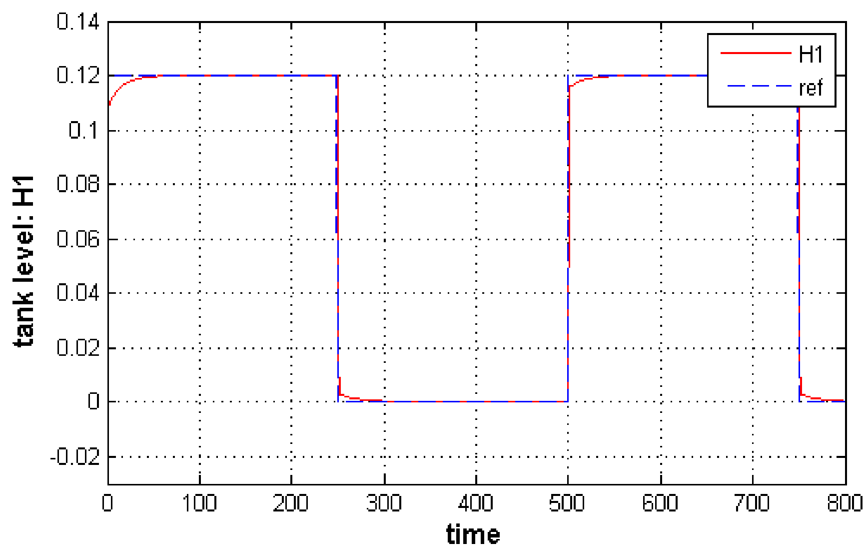

Fig. 5.40 Tank 1 level output with input and slew rate constraints

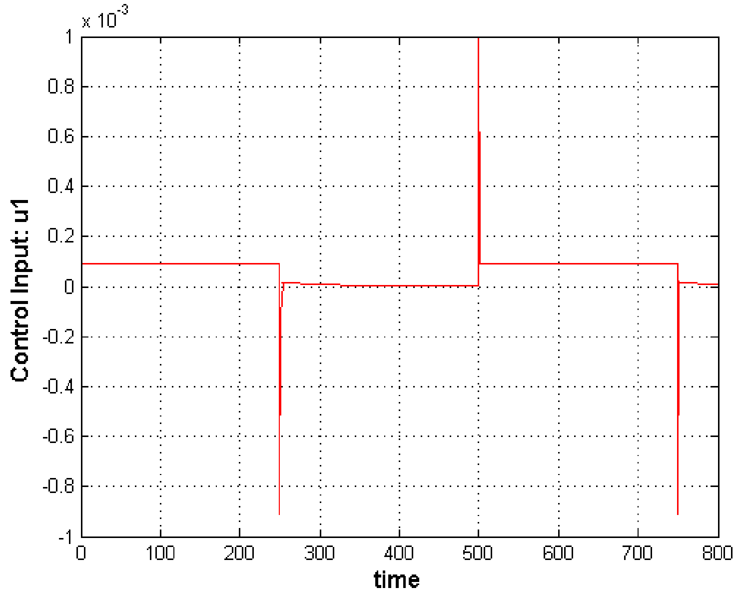

Fig. 5.41 Control input to tank 1 with input and slew rate constraints

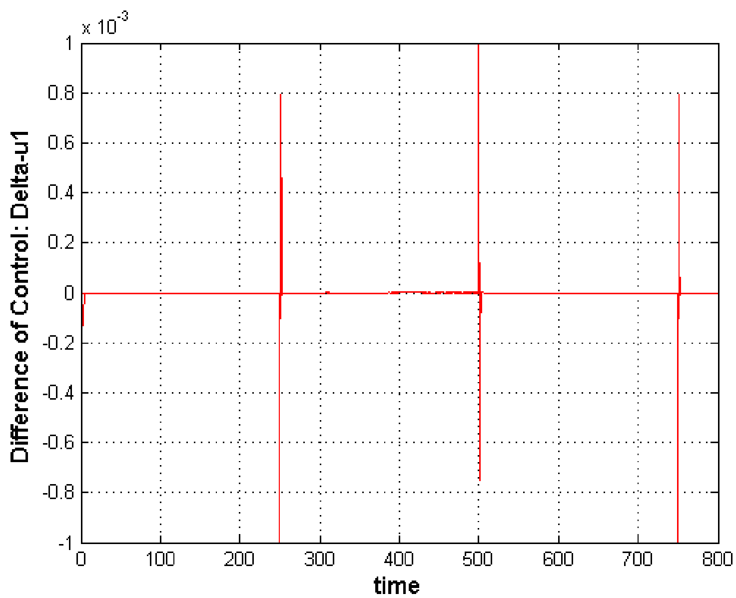

Fig. 5.42 Slew rate for tank 1 with input and slew rate constraints

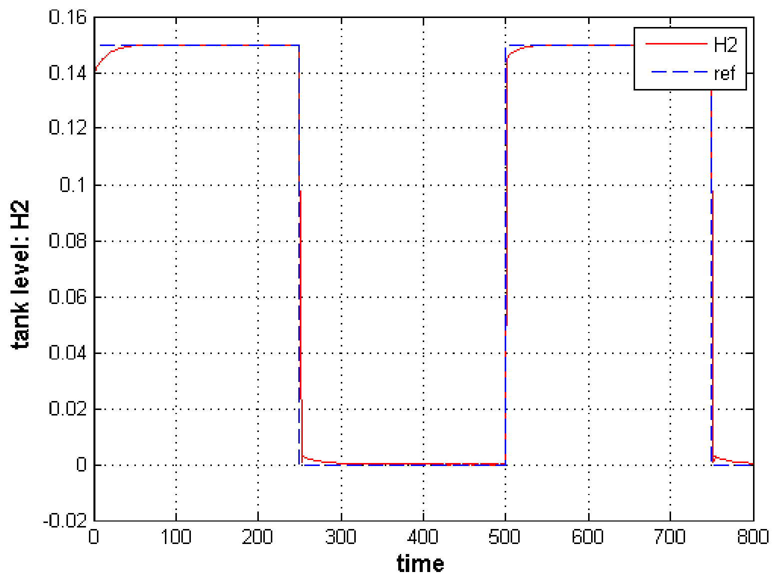

Fig. 5.43 Tank 2 level output with input and slew rate constraints

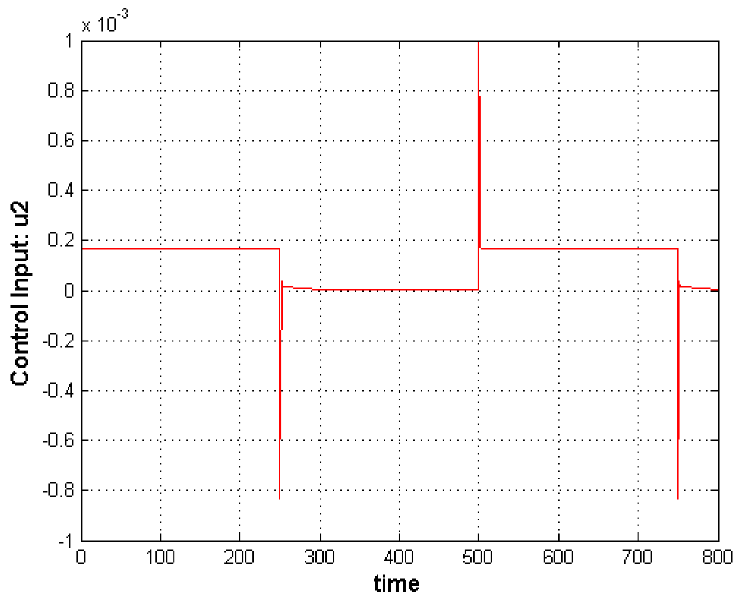

Fig. 5.44 Control input to tank 2 with input and slew rate constraints

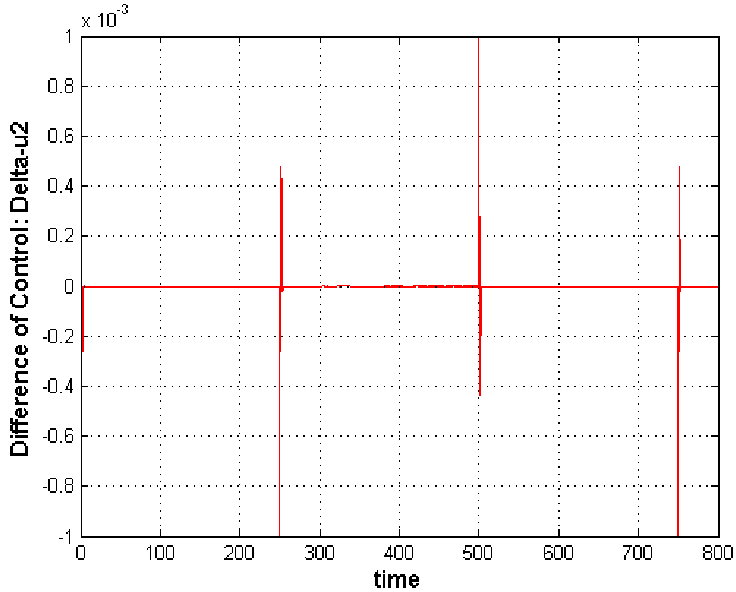

Fig. 5.45 Slew rate for tank 2 with input and slew rate constraints

5.5.2.2 Constrained Control of Nonlinear MIMO CTS using Modified MPC Scheme

Considering the set of constraints defined for system input amplitude and input slew rate to study the controller constraint handling capability.

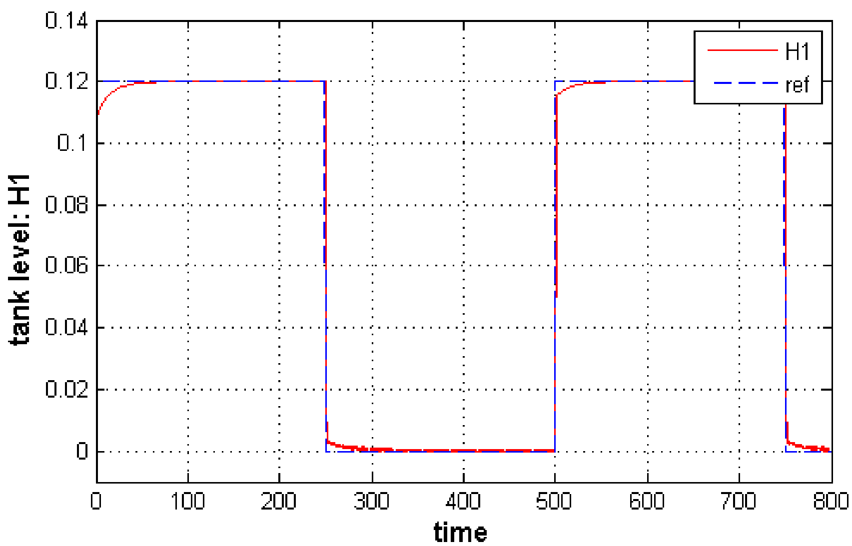

Fig. 5.46 Tank 1 level output with input and slew rate constraints

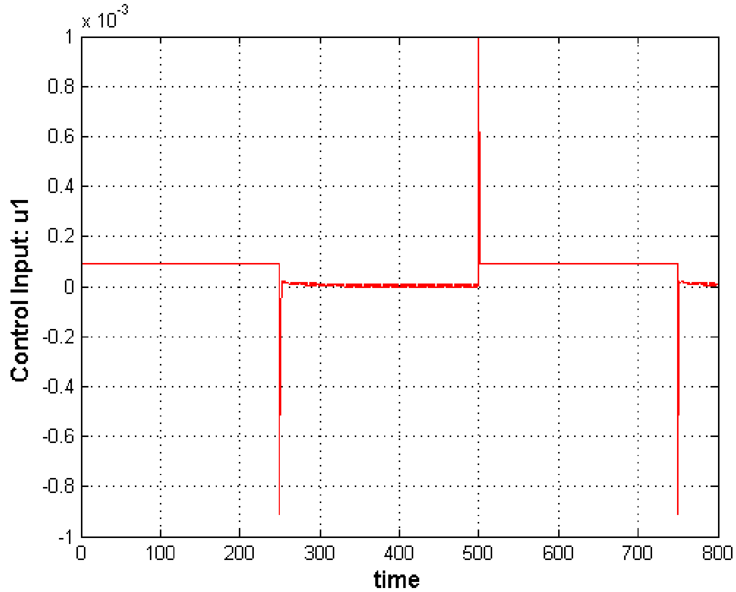

Fig. 5.47 Control input to tank 1 with input and slew rate constraints

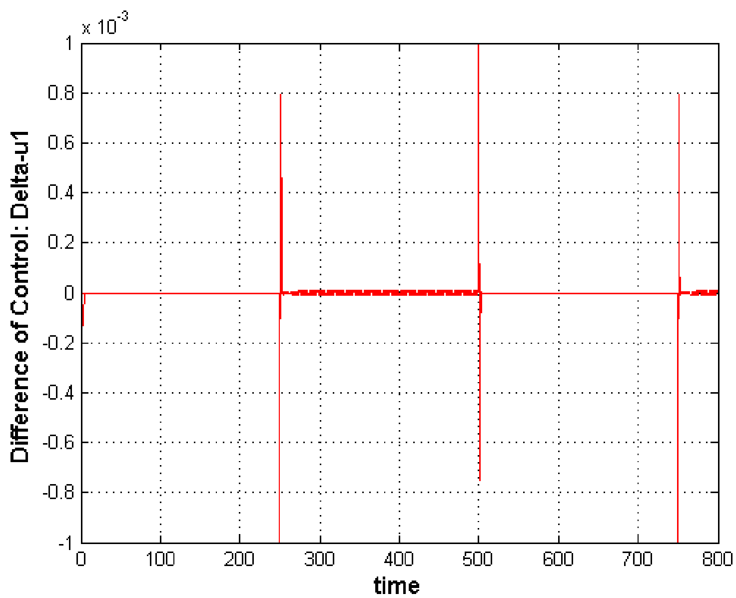

Fig. 5.48 Slew rate for tank 1 with input and slew rate constraints

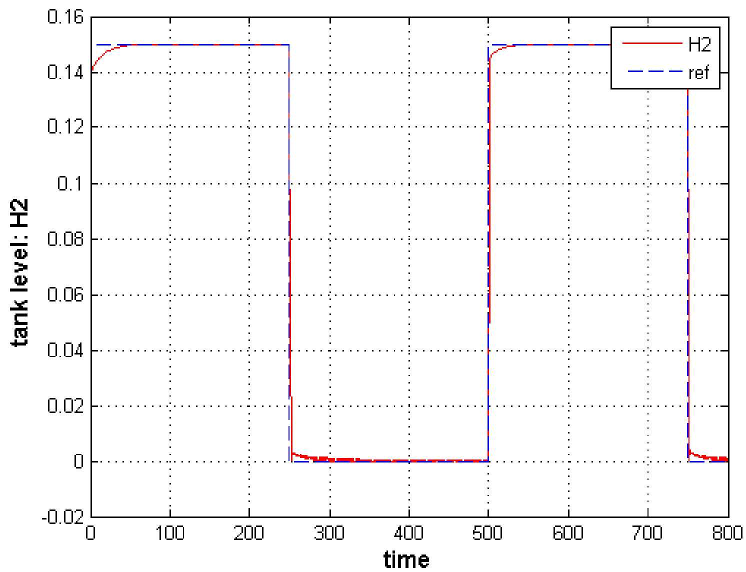

Fig. 5.49 Tank 2 level output with input and slew rate constraints

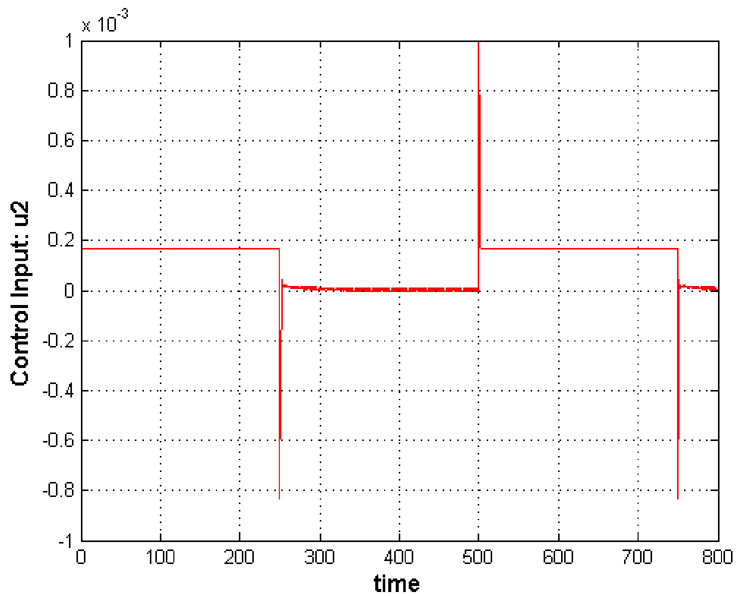

Fig. 5.50 Control input to tank 2 with input and slew rate constraints

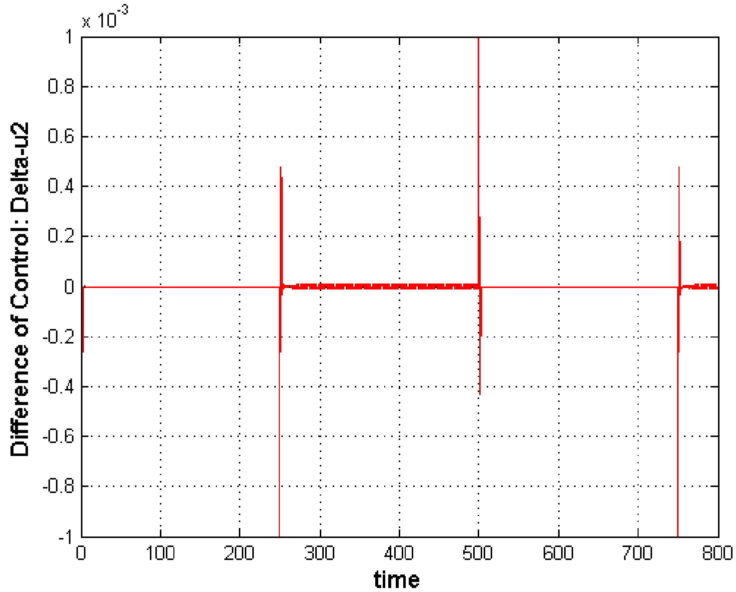

Fig. 5.51 Slew rate for tank 2 with input and slew rate constraints

The RMSE and MAE Comparison of MIMO MPC approaches with enforced constraints is presented below.

TABLE IX. RMSE AND MAE RESULTS COMPARISON FOR MIMO SYSTEM

METHOD	RESPONSE	RMSE	MAE
Modified Basic MPC	Tank 1 Level	0.006529	0.0012
	Tank 2 Level	0.007979	0.00134
MPC Based on Laguerre Functions	Tank 1 Level	0.006056	0.000968
	Tank 2 Level	0.007738	0.001139

While both the control methodologies achieve the control law without violating the defined constraints. It is apparent from the results shown and the error quantities calculated that the constrained control of MIMO system is achieved more accurately with MPC based on Laguerre functions.

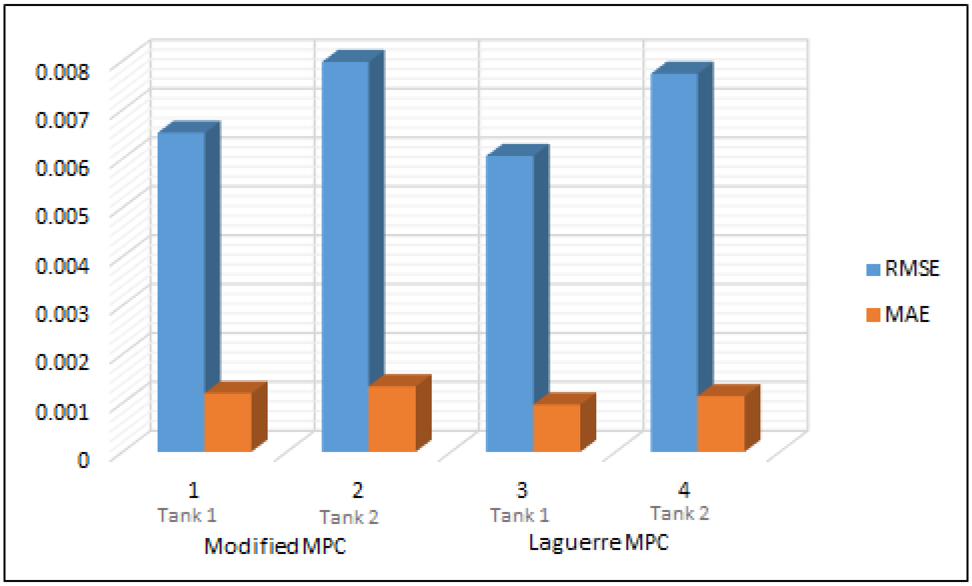

Fig. 5.52 RMSE and MAE results column chart for constrained MIMO system

5.6 Summary

In this chapter the performance of the MPC controllers employed were presented. The control schemes were firstly applied to SISO nonlinear system and then to MIMO nonlinear system. The performance of controller for the liquid level control and the effect on performance when control parameters are varied were evaluated. The results presented show that the performance achieved by the modified Basic MPC and Laguerre functions based MPC is nearly the same but in Laguerre based MPC scheme the number of terms used in the optimization problem are reduced. Thus, the computational load is reduced significantly when MPC utilizing Laguerre polynomials is used. A comparison between the outcome of the MPC approaches applied and PID controller was also presented. Hard constraints are considered on the control variable and the control variable incremental variation for both SISO and MIMO configurations of CTS. From the results obtained and the error calculations of the controller outputs it was seen that the Laguerre functions based MPC achieved the constrained control with more precision.

Chapter 6

CONCLUSION AND FUTURE RECOMMENDATIONS

6.1 Conclusion

In this thesis discrete-time linear model predictive control was used to control nonlinear SISO and MIMO coupled tanks system. From the simulation results, it is clear that the level control of nonlinear coupled tanks is successful by using linear MPC. This was the prime objective of this thesis work. Three MPC approaches were used in the analysis, a basic MPC scheme based on GPC, a modified form of the basic MPC approach and MPC based on Laguerre functions. In the basic MPC approach and modified basic MPC scheme, approximation of control signal required a large number of parameters i.e. a high control horizon resulting in increased computational load. Laguerre functions behave like exponential functions with a decay factor a. Thus the incremental control signal decays exponentially. Therefore, when Laguerre polynomials were used less parameters were required to define the control sequence. The MPC based on Laguerre functions proved to be the most suitable procedure for set point tracking and ability to handle hard operational constraints while being able to achieve the online optimal solution with a comparatively lower computational cost. It can be concluded that linear model predictive control methodology is quite capable of controlling highly nonlinear systems.

The nonlinear system considered in this thesis was the coupled tanks system, it is a design that mimics the industrial process, the actual parameters and dimensions of the hardware were used. Using the first principle mass balance approach the nonlinear models for the CTS were derived in both SISO and MIMO configurations. The nonlinear systems were then linearized at an operating point. The simulation results shown were performed for a different set point, this was done to analyze, whether the linear model used by the controller to control the nonlinear system would be able to achieve set points other than the point at which the model was linearized. The simulations showed that different set points were achieved with good performance. MATLAB code was written for all the three MPC approaches and then converted to embedded MATLAB functions so that they could be applied to the nonlinear Simulink models. The tuning of MPC parameters was done by using the heuristic trial and error strategy. Ability to handle hard constraints is the most unique features of MPC. The tuning of MPC parameters was done by using the heuristic trial and error strategy. In MPC methodology hard constraints can be included in the control algorithm. Quadratic optimization Active-Set was algorithm was implemented by using the MATLAB Optimization Toolbox [25] utilizing the inherit capability of MPC to cope with hard constraints. The MPC controller ability to achieve the desired set point for the nonlinear system without violating constraints on system input as well as control input slew rate was demonstrated using MPC using Laguerre functions and modified MPC scheme. The Root-Mean-Square Error and Mean Absolute Error calculated for both the SISO and MIMO constrained control problem showed that the Laguerre based MPC scheme achieved the objective with more accuracy.

The research works [29-41]were very also helpful for completion this thesis work.

6.2 Future Work Recommendation

The following recommendations are suggested for further improvements to this research work:

- In this research First Principle methodology was used and the mass balance equations were linearized at a particular operating point. Although MPC was able to achieve set points other than the linearization because of its robust properties. The linear state space model itself only represents the nonlinear model only in a small region. A better approach would be a data driven model which could be obtained by using the system identification approach. By using a data driven model a more generalized linear model can be obtained. This may also improve the controller performance at different operating points.

- If the states of the system are not directly measurable i.e. the output matrix in the state space model is not a diagonal or an identity matrix then an observer design must be included in the control scheme. Since the linearized model is available, a linear observer may be designed. But we were unable to observe the states of our highly nonlinear coupled tanks system. Thus in case one is not unable to measure the states directly design of high gain observer and extended high gain observer for SISO and MIMO system must be carried out.

- Further validation of simulated result by application to the actual hardware i.e CE105 CTS apparatus.

- Before the application of proposed control scheme the pump dynamics must be added to control algorithm along with large disturbance rejection mechanisms.

- This thesis does not cover the robustness properties of the proposed control mechanisms in terms of modelling errors. This is the major improvement that can be used to improve the effectiveness of the control scheme further.

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