Annexure A. **Induction Coil Design**

A1. Introduction

An induction furnace comprises a workpiece, an induction coil, load-matching station and power supply. Besides other parameters, optimum heating could be achieved by designing a specific heating coil for a particular application. The heating process involves many factors like electromagnetism, heat transfer and metallurgical phenomena. Physical properties of the workpiece (usually a metallic material) change with temperature and hence variation in induced electromagnetic and heat transfer occurs[215].

A2. Mechanism

The basic mechanism of induction heating is quite simple, where alternating current is produced in a coil (i.e. induction coil) by applying alternating voltage. The alternating current produces magnetic field (of the same frequency) in the vicinity of the coil. The strength of the magnetic field produced depends upon various parameters of the coil like current, geometry and distance. A workpiece, placed in the coil, experiences eddy currents due to the varying magnetic fields around it. The coil currents and induced eddy currents have the same frequencies but opposite in direction. The induced eddy currents generate separate magnetic fields in opposite direction to the magnetic fields of the coil. Therefore, the induction coil produces a net magnetic field of source and induced magnetism. The induced eddy currents in the workpiece generate heat in a thin surface layer, called skin depth, by ohmic or Joule effect[215,216].

Temperature distribution and heating of the workpiece depends upon skin depth generated by a particular induction system having specific magnetic field frequency and physical properties of the material. Induction heating is handy, rapid, localized and economical heating method frequently used in industry[217].

A3. Important Parameters

Electrical resistivity (ρ) and relative magnetic permeability (μ_r) are very important properties of materials affecting all major parameters of an induction system, e.g. skin depth, heat distribution, coil impedance and coil efficiency. In the workpiece, the current distribution becomes non-uniform due to the combined action of various electromagnetic phenomena, which causes asymmetrical temperature profile. The factors causing asymmetrical temperature profile in the workpiece include[215, 218]:

- Skin effect
- Proximity effect
- Ring effect
- End and edge effects

A3.1 Skin Effect

Due to the alternating current, a non-uniformity of current distribution in the cross- section of a conductor occurs, which is called skin effect. Such non-uniform current distribution is associated with alternating currents only. A gradient of maximum to minimum current densities establishes when a conductor is placed in the vicinity or within an induction coil (Figure-A1). Eddy currents have circumferential nature, which concentrates the current flow at the surface of the workpiece with, theoretically, no current flow at its center.

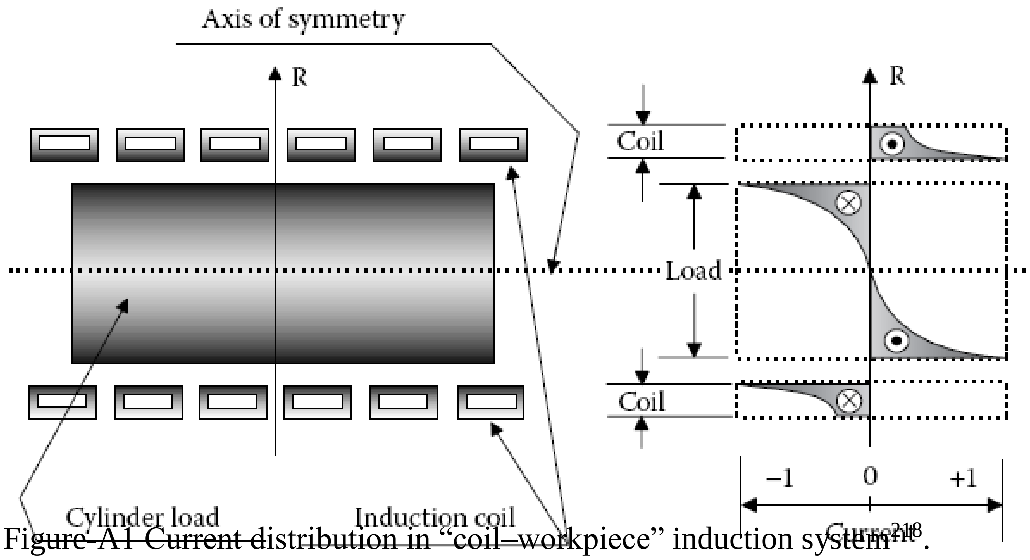


Figure-A1 Current distribution in “coil–workpiece” induction system[218].

The numerical value of the skin depth (in cm) could be calculated from Eq-A1, where resistivity of the workpiece is ρ, relative magnetic permeability is μ and working frequency is f:

$$d = 5000\sqrt{\frac{\rho}{\mu f}} \qquad \text{(A1)}$$

As general criteria, skin depth is the depth at which induced magnetic field strength is reduced to 1/3rd of its surface strength.

A3.2 Proximity Effect

Distortion of the electromagnetic field due to the presence of conductive parts in the close vicinity of the workpiece is called proximity effect. Such conductive parts generate auxiliary magnetic fields distorting the uniformity of the original one.

A3.3 Ring Effect

The effect is predominant in bent or ring shaped current carrying coils. The circular shape of the coil will redistribute the current such that the highest magnetic flux will be at the inner side of the ring, while outer side will be dissipating. The ring effect is even more pronounced in multi-turn coils than single turn ones. Fortunately, ring effect is not always negative. In case of induction heating of a cylindrical workpiece, which is placed inside a coil, ring effect plays a positive role by combined action of skin effect and proximity effect a concentration of current occurs inside of the coil increasing coil efficiency. A decrease in coil efficiency occurs when a coil is place inside a workpiece.

A3.4 End and Edge Effects

End and edge effects are the lateral and transverse heterogeneities of the electromagnetic field at the edges and end area of the workpiece. The effects cause uneven temperature profiles in objects having high aspect ratios e.g. rectangles, cylinders and trapezoids.

To ensure uniform heating, it is essential to have a precise evaluation of electromagnetic field distribution within the workpiece. The above mentioned factors are very important to predict distribution of electromagnetic field, which further renders uniformity in temperature profile in the workpiece.

A4. Coil Design Analyses

“No Toil, No Coil” is a common saying of the induction heating coil designers. Whatever the type of induction heating (with core or coreless) is under consideration, coil is a crucial part, always demanding utmost efforts and time. A major reason for the criticality of coil design is the need of a particular type of coil for a particular application to have optimum results. In the present work, it is required to melt aluminum in a small crucible with maximum stirring action. Therefore, specific design parameters (i.e. geometrical, thermal and electromagnetic) are analyzed to optimize the coil design.

The design is based upon the conditions given in Table-A1. These conditions are set according to the experimental requirements and limitations posed by available induction generator.

Table-A1 Preconditions used for the designing of the induction coil.

Material to melt	Aluminum
Capacity of the furnace	0.1 Kg
Crucible material	Fused alumina
Crucible shape	Cylindrical
Wall thickness of the crucible	2 mm
Gap between coil and the crucible	5mm
Material of coil pipe	ETP Copper
External diameter of coil pipe	5 mm
Internal diameter of coil pipe	4 mm
Frequency of power supply	10 kHz
Current of power supply	100 A

A4.1 Geometrical Analysis

As described in Table-1, aluminum (100 g) is required to melt in a cylindrical alumina crucible. The dimensions of the crucible[157] and volume of the melt are calculated using Eq-A2 and A3, respectively.

$$\frac{H}{D} = (1.6 - 2.0) \tag{A2}$$

$$V = \frac{\pi d^2 H}{4} \tag{A3}$$

where, H and D are height and diameter of molten metal in the crucible, respectively.

The inter diameter ($D_\circ$) of the induction coil[157] is a function of crucible diameter (Dc), crucible wall thickness (Bw) and insulation thickness (Bins) and is determined using Eq-A4:

$$D_\circ = Dc + (Bw + Bins) \tag{A4}$$

Height of the inductor coil ($H_\circ$) depends upon the height of the workpiece or the melt (H), which is usually kept 1.1 to 1.2 times of the H[157]:

$$H_\circ = (1.1 \sim 1.2)H \tag{A5}$$

A4.2 Thermal Analysis

The heat energy required (Qr) to process the aluminum includes energy to melt aluminum (Qa), energy to superheat (Qs), energy to melt slag (Qg), energy for endothermic reaction(Qe) and energy librated by any exothermic reaction(Qx). The system consists pure aluminum therefore, theoretically Qe=Qx. The net required heat energy in this case would be[219]:

$$Qr = Qa + Qs + Qg \tag{A6}$$

and,

$$Qa = mC(T_\circ - T) + E \tag{A7}$$

Where, m is mass of charge in kg, C is specific heat capacity of aluminum and is equal to 1100J/kg.K, $T_\circ$ is melting temperature (933 K for aluminum) and T is room temperature (298 K). E is the heat energy required for any phase transformations, which would be zero for pure aluminum.

Similarly,

$$Qs = mC_{\circ}Ts \tag{A8}$$

where, $C_{\circ}$ is heat capacity of aluminum in molten form (992 J/kg.K), and Ts is the superheating temperature (323 K).

and,

$$Qg = mgCg \tag{A9}$$

where, mg is amount of slag, which is considered 8% of the total melt and Cg is slag heat energy (18KJ/kg).

A4.3 Electromagnetic Analysis

The heat energy produced (Qp) to melt the charge in the form of eddy currents is a function of frequency of the power supply (f), height of the workpiece (H), maximum magnetic flux density (B), diameter of the workpiece (D) and resistivity (ρ) of the work piece (aluminum) is 2.83x10^{-8}Ωm. The function is shown in Eq-A10 [157, 158].

$$Qp = \frac{\pi^3 f^2 H B^2 D^4}{8\rho} \tag{A10}$$

or

$$B = \sqrt{\frac{8\rho Qp}{\pi^3 f^2 D^4 H}} \tag{A11}$$

also

$$Qp = \frac{Qr}{t} \tag{A12}$$

where, t is time in seconds to attain maximum flux.

The current density in the inductor is:

$$Ia = \frac{I}{A} \tag{A13}$$

To produce required amount of magnetic flux density (B), number of turns of the inductor coil could be estimated by Eqs-A14 and A15 [158, 219]:

$$B = \frac{\mu\mu_{\circ} NI}{L} \quad \text{(A14)}$$

or

$$N = \frac{BL}{\mu_{\circ} I} \quad \text{(A15)}$$

where, μ and $\mu_{\circ}$ are relative permeability and permeability of workpiece material and free space, respectively. Relative permeability of non-magnetic materials is equal to 1, and permeability of free space is $4\pi \times 10^{-7}$ Tm^{-1}. L is length of the inductor, which is H in present work. The current passing through the inductor is I in amperes. Results of the theoretical analyses are given in Table-A2.

A4.4 Efficiency of the Designed Induction Heating System

After defining the design parameters it is necessary to evaluate the efficiency of the designed coil prior to its practical implementation.

Julio W et al.[159] proposed a mathematical model (Eq-A16) for the evaluation of heating efficiency of induction coil:

$$\eta_{\circ} = \frac{Rs}{Rs+Rp} \times 100 \quad \text{(A16)}$$

Where, $\eta_{\circ}$ is percentage heat efficiency of the coil, Rp is the primary coil resistance and Rs is load (or secondary) resistance. The losses within the induction coil are associated with the resistance of the coil (Rp) and therefore kept to minimum values. However, Rs defines the extent of induced energy in the workpiece and should be maximized.

A4.4.1 Simulation

To find the values of Rp and Rs, simulation based upon calculated parameter of the induction coil was carried out using finite element methods software for magnetics (FEMM). Geometry of the heating system was defined in FEMM according to Figure-A2, where external diameter of the workpiece (aluminum) was taken as the diameter of the crucible neglecting the actual material of the crucible (alumina), which is inactive for magnetic inductions and copper was taken as coil material.

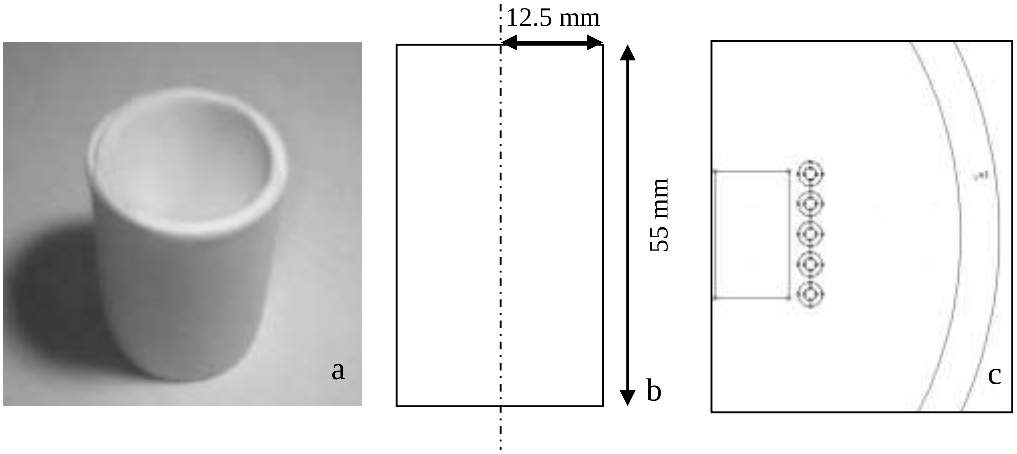


Figure-A2 a) Actual crucible used for melting, b) Transversal cutting of the crucible used for simulation and c) FEMM drawing.

Table-A2 Results of the design analyses

Geometrical analysis	
Height of molten metal (H)	5.5 cm
Diameter of molten metal (D)	3.0 cm
Volume of the molten melt (V)	38.9 cm^3
Inter diameter of the induction coil ($D_\circ$)	45.0 cm
Height of the inductor coil ($H_\circ$)	6.0 cm
Thermal analysis	
Energy to melt aluminum (Qa)	69850 J
Energy to superheat molten aluminum (Qs)	32042 J
Energy to melt slag (Qg)	144 J
Total heat energy required (Qr)	102036 J
Electromagnetic analysis	
Maximum magnetic flux density (B),	0.00409 T
Current density through the coil (Ia)	5.67 $A.mm^{-2}$
Number of turns of the coil (N)	4.47 turns

The simulation was required for axisymmetric situation, therefore only a traversal cutting of the crucible was drawn (Figure-A2c). FEMM first determines the magnetic contributions of 2D slice and then evaluates remaining by full revolution imparting the solid.

All the regions were defined with their respective materials to realize the working problem i.e. workpiece was defined as "Aluminum 1199", conductors of the coil were defined as "Copper" within one circuit "coil" and environment around the workpiece and inductor was defined as "air". The inside area of the copper tube was not used in the simulation hence defined as "No mesh" (Figure-A3a).

As operational and geometrical parameters are already defined in Table-A1 &A2, respectively, a simulation of the heating system was generated for the distribution of magnetic field intensities. Magnetic flux distribution in the workpiece and around the inductor are shown in Figures-A3b and c. The electrical parameters of the heating circuit were evaluated using a postprocessor facility provided by FEMM. The results are shown in Figure-A3d. The parameter "Voltage/Current" directly yields the value of Rp+Rs of Eq-A16. To obtain Rp value, an additional simulation was carried out where no load or workpiece was used (Figure-A4a-d), which was done by eliminating workpiece while keeping all other parameters [159]. Therefore:

$$Rp + Rs = 0.01274\Omega \; or \cong \; 12.7 \; m\Omega$$

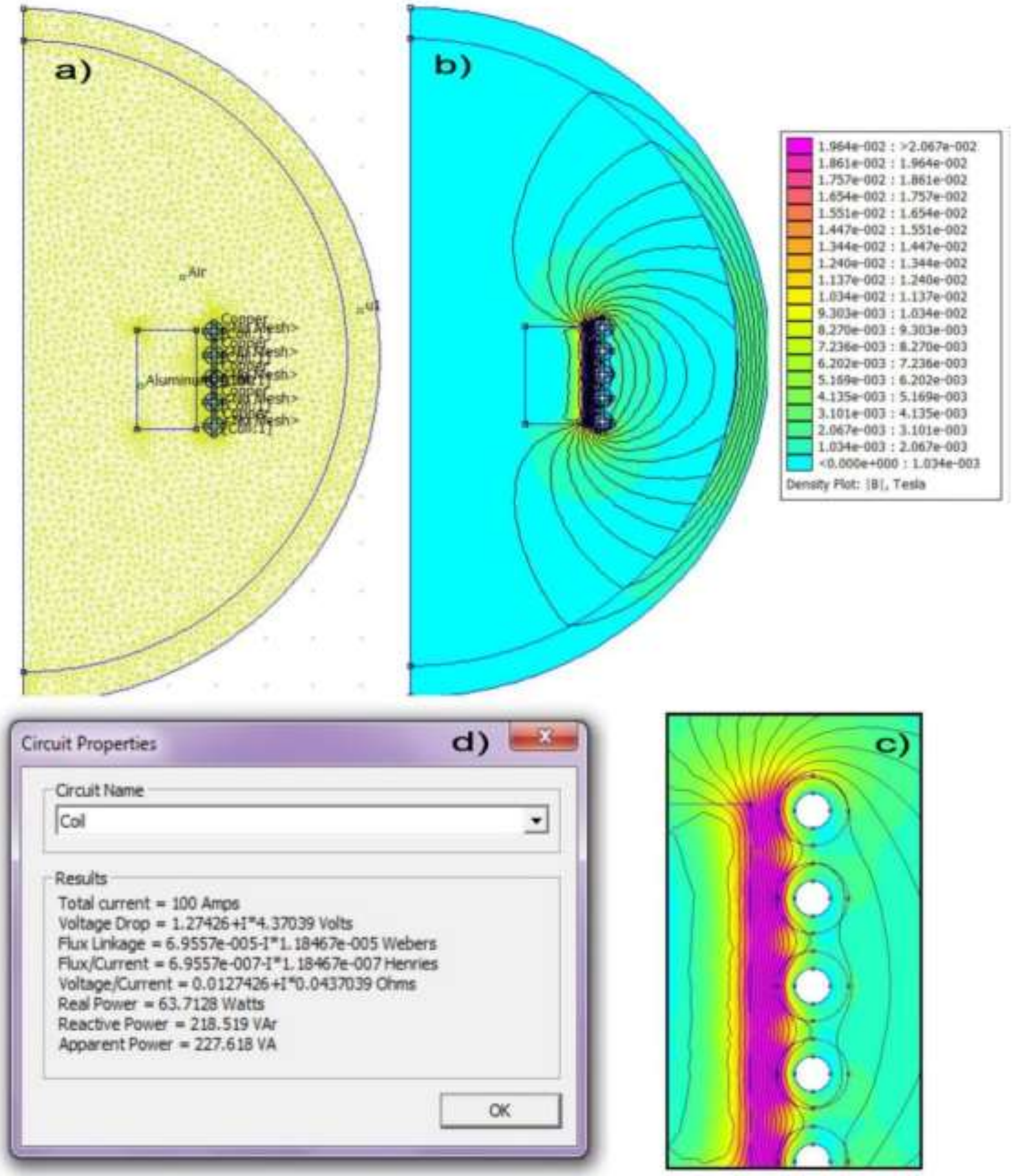
and

$$Rp = 0.004768\Omega or \cong 4.77 m\Omega$$

thus, according to Eq-16:

$$\eta_{\circ} = \frac{Rs}{Rp + Rs} \times 100 = 62.6\%$$

Therefore, a heating efficiency ($\eta_{\circ}$) of >60% is achievable with the designed induction heating system.



Circuit Name: Coil

Results

```
Total current = 100 Amps
Voltage Drop = 1.27426+I*4.37039 Volts
Flux Linkage = 6.9557e-005-I*1.18467e-005 Webers
Flux/Current = 6.9557e-007-I*1.18467e-007 Henries
Voltage/Current = 0.0127426+I*0.0437039 Ohms
Real Power = 63.7128 Watts
Reactive Power = 218.519 VAr
Apparent Power = 227.618 VA
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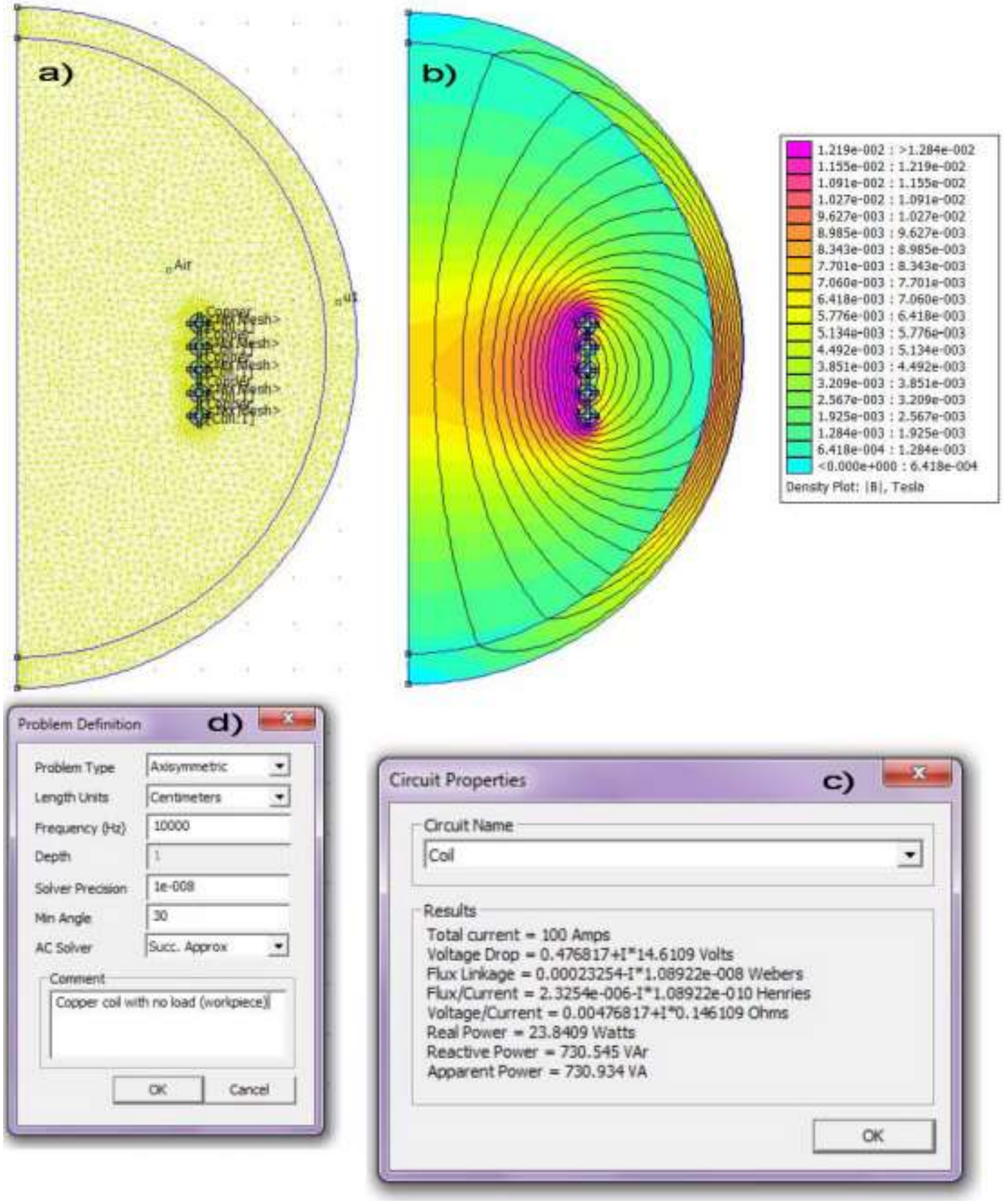
Figure-A3 a) Definition of all the working area and respective materials, b) magnetic flux distribution in the designed system c) a closer view of Figure-3b and d) postprocessor results of the circuit “coil”



Problem Type	Axisymmetric
Length Units	Centimeters
Frequency (Hz)	10000
Depth	1
Solver Precision	1e-008
Min Angle	30
AC Solver	Succ. Approx

Comment
Copper coil with no load (workpiece)

Circuit Name
Coil

Results

```
Total current = 100 Amps
Voltage Drop = 0.476817+I*14.6109 Volts
Flux Linkage = 0.00023254-I*1.08922e-008 Webers
Flux/Current = 2.3254e-006-I*1.08922e-010 Henries
Voltage/Current = 0.00476817+I*0.146109 Ohms
Real Power = 23.8409 Watts
Reactive Power = 730.545 VAr
Apparent Power = 730.934 VA
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Figure-A4 Same as Figure-3 but work piece is eliminated from the work sheet. a) definition of all the working area and respective materials, b) magnetic flux distribution in the designed system c) postprocessor results of the circuit “coil” and d) parameters used to define the problem