Independent Suspension System, Modeling and Analysis of Multi-axle Vehicle

Author

SANA FATIMA

NUST201464486MCEME35114F

Supervisor

DR. RAJA AMER AZIM

DEPARTMENT OF MECHANICAL ENGINEERING

COLLEGE OF ELECTRICAL & MECHANICAL ENGINEERING

NATIONAL UNIVERSITY OF SCIENCES AND TECHNOLOGY

ISLAMABAD

AUGUST, 2018

Independent Suspension System, Modeling and Analysis of Multi-axle Vehicle

Author

SANA FATIMA

NUST201464486MCEME35114F

A thesis submitted in partial fulfillment of the requirements for the degree of

MS Mechanical Engineering

Thesis Supervisor:

DR. RAJA AMER AZIM

Thesis Supervisor's Signature: ____

DEPARTMENT OF MECHANICAL ENGINEERING

COLLEGE OF ELECTRICAL & MECHANICAL ENGINEERING

NATIONAL UNIVERSITY OF SCIENCES AND TECHNOLOGY, ISLAMABAD

AUGUST, 2018

Declaration

I certify that this research work titled "Independent *Suspension System, Modeling and Analysis of Multi-axle Vehicle"* is my own work. The work has not been presented elsewhere for assessment. The material that has been used from other sources it has been properly acknowledged / referred.

Signature of Student

Sana Fatima

NUST201464486MCEME35114F

Language Correctness Certificate

This thesis has been read by an English expert and is free of typing, syntax, semantic, grammatical and spelling mistakes. Thesis is also according to the format given by the university.

Signature of Student

Sana Fatima

NUST201464486MCEME35114F

Signature of Supervisor

Dr. Raja Amer Azim

Copyright Statement

Acknowledgements

First and foremost, I thank my ALLAH (SWT) the Almighty for helping and guiding me to finish this research.

I would like to thank my supervisor Dr. Raja Amer Azim for his endless support, valuable suggestions, and motivation that he has afforded me all along the course of this research. Special thanks for his good support and positive comments and continuous supervision during each stage of this thesis.

I am thankful to Dr. Imran Akhter, Dr. Wasim Akram and Dr. Hasan Aftab Saeed for their support, professional guidance and insightful help.

Deep appreciation to my family for their support and continues help especially to my mother and brothers for their endless love, assistance, unfailing support and continuous encouragement throughout my years of study. My special thanks and love to my supportive husband.

Finally, I must express my very profound gratitude to all my friends who guided me throughout this work.

To the soul of my dear father,

To my lovely mother, to my brothers and husband, for their encouragement and continues support.

Abstract

Suspension system of the vehicle separates the vehicle body from road's irregularities. If suspension system is not present, road shocks and vibration affect the passenger and payload. Independent, dependent and semi-independent suspension systems are used in vehicles depending upon the type of vehicle. Independent suspension system performs better off road and 3D road profile and in heavy vehicles it is preferred than other types of suspension systems. Independent suspension provides better roll over resistance and contact patch novel forces. Independent suspension system controls three types of vehicle motions including pitch, roll and heave. Mathematical models for two axle vehicle with four independent suspension systems are available in literature as the classical case. Mathematical models of multi-axle vehicle are required to develop their design and for simulation purposes. Design parameters set according to the system requirements affect the performance of vehicle's suspension. A complete mathematical model is required to evaluate the suspension system behavior. System becomes complicated with axles more than two requiring independent suspensions more than four. This thesis presents the 11-DOF full-car model of a four axle vehicle with eight independent suspensions. Overall system is based on eight passive type suspensions operating independently to handle the vehicle vibrations. Generic mathematical model in index notation is also developed to design a system with n number of axles. Model is implemented in MATLAB Simulink to generate the behavior of heave, pitch and roll on different road inputs and results are compared with a commercial software simulation results.

Key Words: *Independent suspension system, Multi-axle vehicle, Heavy vehicles*

Table of Contents

List of Figures

List of Tables

Nomenclature

C	Unsprung damping coefficient
K	Unsprung stiffness coefficients
$\boldsymbol{K_t}$	Tire stiffness coefficients
m	Unsprung Mass
M	Sprung Mass
Li	Distance from CG to ith Axle
Ixx	Mass moment of inertia about X-Axis
Iyy	Mass moment of inertia about Y-Axis
Izz	Mass moment of inertia about Z-Axis
$\boldsymbol{\varphi}$	Roll Angle
$\boldsymbol{\theta}$	Pitch Angle
w	The lateral offset of the wheel from the center of gravity
DOF	Degree of freedom

CHAPTER 1: INTRODUCTION

1.1 Overview

From the time of the wheel's invention, vehicles are the common and main source of transportation. With the development of new technologies, vehicle performance parameters are improved including engine performance, fuel consumption, suspension, stability, and safety. These parameters vary with the type of vehicle. Parameters for light vehicles are different from heavy vehicles. A suspension system is very important in vehicles performance.

Role of a suspension system is to isolate the automobile body from road irregularities. If suspension system is not present in a vehicle the vibrations caused by road irregularities transferred to the passenger and load. Parts of the suspension system are tires, tire stiffness, springs, dampers, and linkages. These linkages are the connection between wheels and vehicle and they allow the relative motion between them. Suspension system performs a key part in a automobile's comfort, handling, stability, and road grip. A suspension system is required for the vehicle because irregularities on the roads are always present. These irregularities present on the road apply force on the vehicle to move up and down perpendicular to the road surface. The scale of the force is determined by the dimension of irregularities. Automobile's body is characterized on the bases of spring's location i.e. between the wheels and frame, sprung mass and unsprung mass.

1.1.1 Sprung mass

Mass of the vehicle which includes the body, frame, internal components, passengers, and cargo is called sprung mass. It doesn't include the mass of suspension system elements but supported above the suspension system.

1.1.2 Unsprung mass

The mass of the suspension system including tires, dampers, springs, wheel axles, wheel bearings, wheel hubs, and linkages is called unsprung mass. If the vehicle's brakes are within the wheel, their mass is also considered part of the unsprung mass.

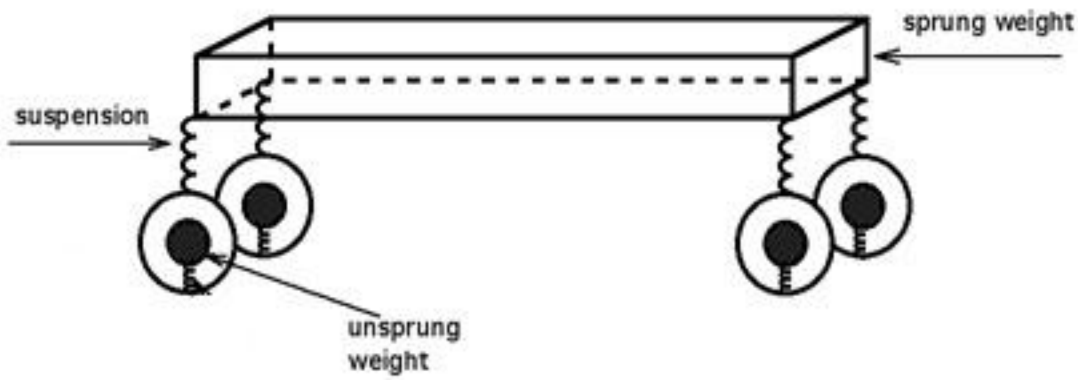


Figure 1.1: Sprung and unsprung weights

A vertical force is applied to each wheel when wheels of the vehicle pass over the bump on the road. This vertical force will transfer to the frame if the suspension system is not present in the vehicle and wheels will miss connection with the road. Then under the downward force of gravity, the wheels can smash back into the road exterior. A suspension system is needed to grip the energy of the perpendicularly accelerated wheels to allow the comfort and stability of the passenger and vehicle body.

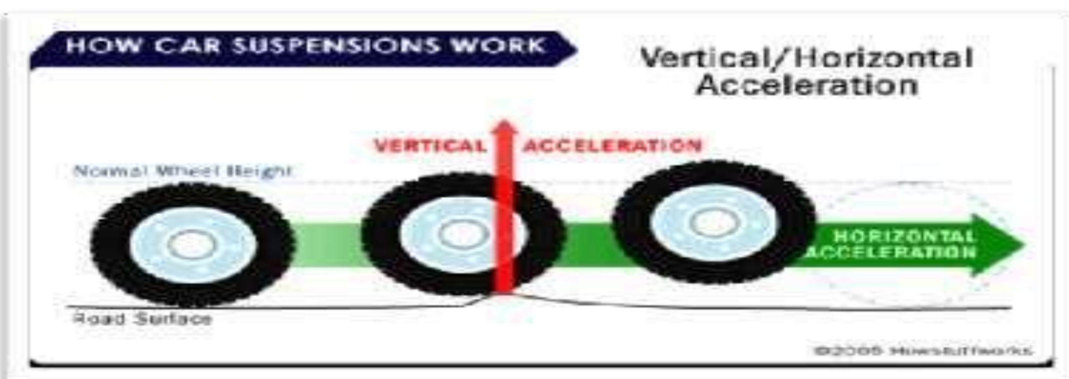


Figure 1.2: How car suspensions work

1.2 Types of the suspension system

Suspension system can be classified into three basic types depending upon the movement of wheels on the same axle. They are independent, dependent and semi-independent suspension systems.

1.2.1 Independent Suspension System

Independent suspension system is the type of suspension in which left and the right wheels of the same axle move vertically independently when the vehicle moves on an uneven surface. There is no mechanical connection present between two wheels of the same axle in this type of suspension so force acting on one wheel does not affect the other wheel. This type of suspension has less unsprung mass due to which ride quality and vehicle handling is better. Independent suspension system has the advantage of less space, easy steerability, handling and low weight. Double wishbones and MacPherson Strut are the examples of an independent suspension system.

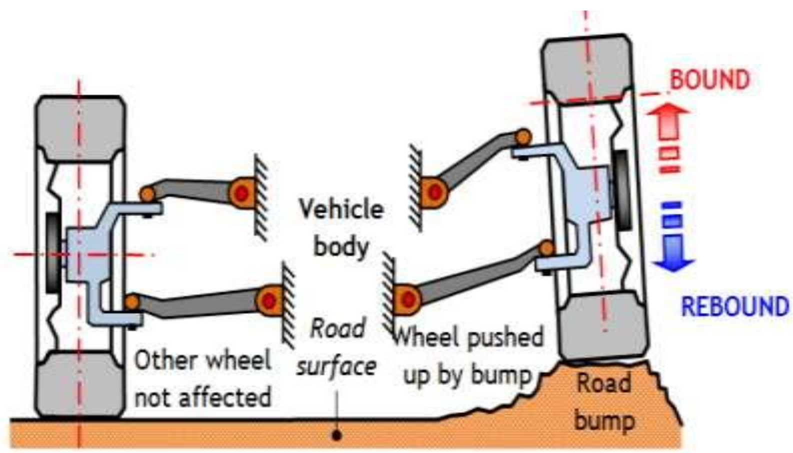


Figure 1.3: Independent Suspension System

1.2.2 Dependent Suspension System

The type of suspension system in which inflexible connection is present between the two wheels of the same axle is called a dependent suspension system. There is an inflexible mechanical connection present between two wheels of the same axle in this type of suspension so force acting on one wheel affects the other wheel. Dependent suspension system can endure shocks with a great capacity than independent suspension but it is less stable. Solid axle is the example of a dependent suspension system.

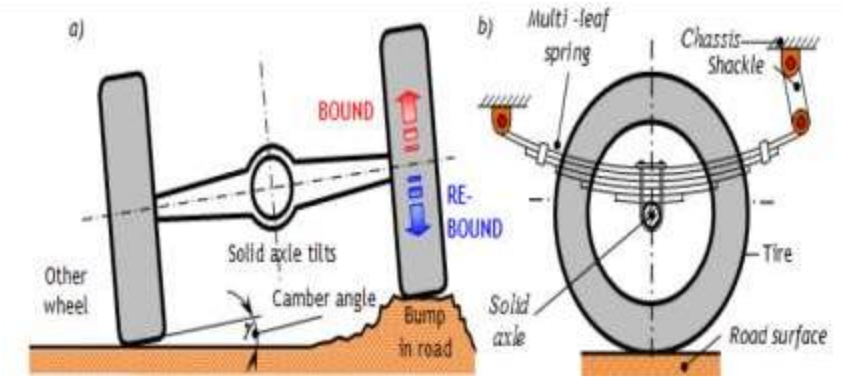


Figure 1.4: Dependent Suspension System

1.2.3 Semi-independent System

The suspension system in which, the wheel move relative to one another as in independent suspension but the position of one wheel has some effect on the other wheel is called semi-independent suspension system. It has both the features of dependent as well as independent suspension systems. This type of suspension system is achieved by twisting suspension parts. Twist beam is the example of a semi-independent suspension system.

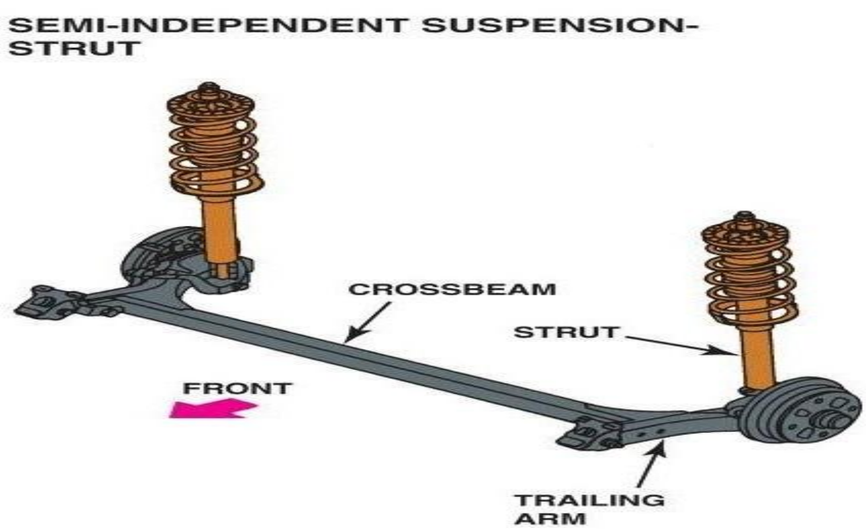


Figure 1.5: Semi-independent Suspension System

1.3 Aims and Objectives

Suspension system mainly focuses on the stability and ride comfort of vehicles. Vehicles carrying heavy payloads usually have more than two axles. Strong driving force, stability, and ride comfort are necessary parameters for these vehicles. Mostly heavy vehicles use an independent suspension system. In off-road vehicles, independent suspension system provides more comfort and stability than dependent suspension. In independent suspension, if the wheel on one side hits the road irregularities it won't upset the wheel on another side at the same axle.

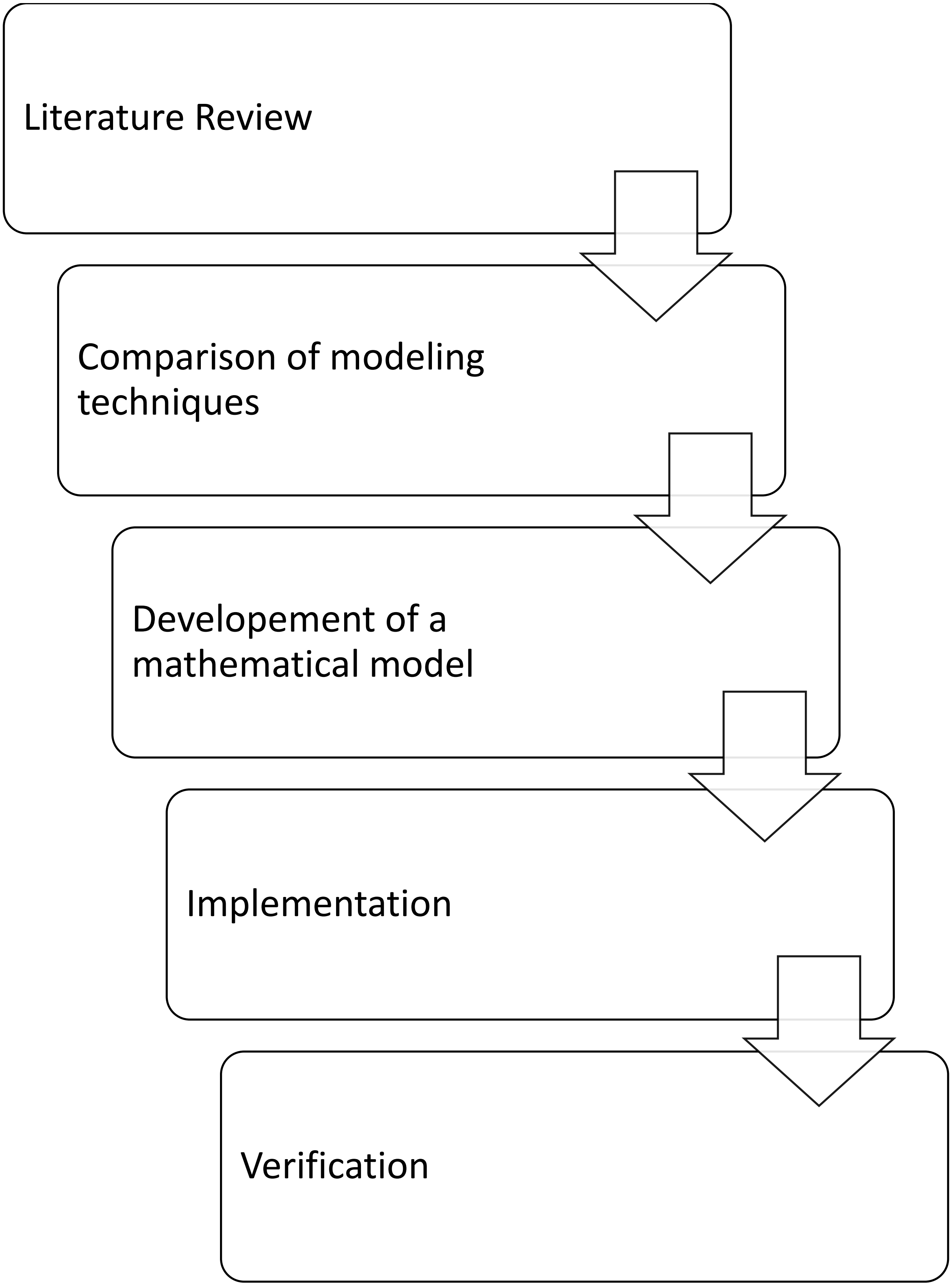
This thesis focuses on mathematical modeling of independent suspension systems. The mathematical model for 4 axle vehicle with 8 independent suspension systems is derived. This model is implemented in MATLAB Simulink and results are compared with TruckSIM simulation.

The main objective of this study is to propose a mathematical model for independent suspension system in index notation which can be used for multi-axle vehicles and results are verified using commercial software. The considerable objectives to help this aim are

- Complete analysis to develop a broad understanding of the work which has already been done for independent suspension.
- Literature review to develop an understanding of the modeling.
- Making a mathematical model and setting appropriate assumptions for the model.
- Implementing the mathematical model in MATLAB Simulink.
- Making a model for simulation in TruckSIM.
- Comparing the results of both software; hence, validating the model.

1.4 Methodology

Extensive literature study is done to fully understand the suspension system models. Different modeling techniques are studied. On the bases of literature review, a complete mathematical model is developed. This model is implemented on MATLAB Simulink and results are verified using commercial software.


Literature Review
Comparison of modeling techniques
Developement of a mathematical model
Implementation
Verification

CHAPTER 2: LITERATURE REVIEW

In this section, relevant work regarding modeling and simulation of independent suspension has been reviewed.

2.1 Stanislav PEHAN et al.[1]

The main purpose of Stanislav and Sašo's work is to design the suspension mechanism that fulfills requirements about stability, safety, and maneuverability. They developed a very brief concept of suspension. This paper shows a design of the suspension mechanism calculated to use in a terrain vehicle with four wheels steer and drive. The main purpose is focused to design an autonomous suspension system that offers improved contact of the tire with the road surface and less horizontal displacement of the tire.

2.2 L.Davis et al.[2]

The pitch and lift control scheme is suggested by Davis and Scully. The natural frequencies of the Roll and Pitch were determined by the vehicle suspension dynamics and the moment of inertia whereas their damping was determined by the speed-dependent dynamics. The programmed Stability Augmentation System (SAS) gave roll and pitch suitable damping and natural frequencies to increase ride quality to considerably decrease the effects of wayside or other obstacles on vehicle motion. Modeling equations were derived and converted into a state space. They concluded that it is impossible to decrease body motion at the wheel frequency by using control inputs only between the sprung and unsprung masses, as the system has pole suppression at the wheel frequency resulting from an uncontrollable mode.

2.3 Jain et al.[3]

The work of Jain and Kumar was the result of a comprehensive study of various available independent suspension systems to develop the Suspension System for Society of Automotive Engineers (SAE) vehicles. They provided various aspects of suspension kinematics and dynamics as well as methods of analyzing the suspension system to obtain optimal suspension geometry.

2.4 Nam P Suh et al.[4]

Deo and Suh have proposed a new design of a customizable vehicle suspension system with independent control of stiffness, damping and ride height, capable of delivering the desired performance based on user preferences, road conditions and maneuvers. They built a suspension prototype to demonstrate the concept. Axiomatic Design Theory was used for the development of the concept, design and manufacture of the prototype, as well as for the design and implementation of the control system of the suspension system. The mechanical design of the proposed system has been decoupled from the functional requirements (FR) of stiffness and ride height. In addition, the vehicle load (noise factor) affected the driving height. A customizable suspension feedback control system has been designed and implemented to decouple the system and make it robust to the noise factor.

2.5 Sujithkumar et al.[5]

Sujithkumar and his associates focus on modeling and predicting the response of the vehicle suspension under the effect of road disturbances. A mathematical model has been developed and the control algorithm has been verified in order to reduce undesirable movements of the suspended mass, such as lifting, pitching and rolling. The semi-active suspension provides a better ride comfort. Force fraction consumption required for the active suspension. Their work demonstrated the comparative performance of passive suspension, as well as two semi-active suspension control logic, skyhook on-off and continuous skyhook, through simulations. The driving comfort was evaluated for the effective acceleration of the center of gravity in the vertical direction (Z), which is the main contribution to the measurement of the VOR. Standardized tire strength (NTF), which directly represents stability, has also been studied. The variations in the NTF were also less with the continuous control of skyhooks. Therefore, the semi-active control logic finds a promising application in the construction of multiple-axle multipurpose armored vehicles. The reduction in effective acceleration at the center of gravity indicates a reduction in hunter fatigue and military payloads. In addition, the reduction in acceleration had a promising chance of improving the shooting accuracy of the platform.

2.6 Kamesh Jagtap et al.[6]

This article reviews the suspension system of an all-terrain vehicle (ATV). ATVs have to face different soil conditions like mud, ice, rocks, bumpy roads etc. Therefore, they must have all-terrain properties. Therefore, it must be robust enough to sustain wave motion occurring in such frequencies. When designing a suspension system of an ATV, different terrain conditions must be considered. All parameters contributing to the construction of the suspension system for an automobile are discussed in this paper. For an ATV, the values of such commercial vehicle properties vary. Taking into account all parameters and assuming some data, the design of the suspension system for an ATV was initiated. This paper gives an idea of how an independent suspension system is involved in the development of an ATV.

2.7 Hamed, Maomar [7]

The mathematical model of the 4-axle vehicle with 7 degrees of freedom (DOF) was developed. The vehicle parameters and their properties were analyzed under the condition of various suspension errors. The main objective of this study was to consider the shaking response of the suspension system and advance an approach to checking the condition of individual vehicles based on cost-effective vibration measurement.

The main focus was on the investigation of online state checking for vehicle suspension systems. The core task was to extract modal vehicle constraints from academic models and from shaking signals measured by a supervised suspension system.

2.8 Yucheng Liu.[8]

This article briefly introduces recent innovations and some representative patents (since 2000) in this area and explains the features and benefits of these inventions. Future trends in research in the design of the suspension system are also discussed. This article provides insight into current and future developments of the suspension system to improve controllability, stability and ride comfort.

2.9 V.R. Patil et al.[9]

This article introduces various techniques that can be used to improve the vehicle's stability and road-holding. The highway application requires more speed and stability for the vehicle. It is not possible to achieve with constant spring stiffness or damping. In this work, various techniques are developed to overcome this problem.

2.10 Ayman A. Aly et al.[10]

An active suspension system has been proposed to improve driving comfort. A four-part system with 2 degrees of freedom (DOF) is designed and built on the basis of the four-wheel independent suspension concept to simulate the actions of an active vehicle suspension system. The purpose of a suspension system is to support the body of the vehicle and increase driving comfort. The purpose of the work described in this article is to illustrate the use of intelligent technology in controlling a continuously damping automotive suspension system. driving comfort

Improved by reducing the bodywork acceleration caused by bodywork in smooth road bumps and the real roughness of the road. This article also describes the model and control used in the study and discusses the results of the vehicle response obtained from a series of road entry simulations.

2.11 Saurav Talukdar et al.[11]

The journey is considered one of the criteria for evaluating the performance of a vehicle. However, the assessment of the driving quality of a car varies from person to person. There are many mathematical models used to study vehicle dynamics. The development of electronics, sensors and actuators has allowed the use of control systems to improve the driving quality of a vehicle. This work deals with some of the most commonly used driving patterns. 1/4 model car, model half-body of freedom (DoF) and half-body model of 4 DoF. A complete analysis of these models from a designer's point of view, eg. As a variation of the stiffness of the suspension, the damping coefficient, etc., is presented. The reaction of these models to a random excitation is studied. The above models are based on the rigid body hypothesis. They do not contain a tire model that takes into account the rigidity and damping properties of the tire. The flexible ring tire model will be integrated with the 4DF Half Car model and the influence of tire flexibility on vehicle travel will be analyzed. A flexible 2 DoF model modeling the chassis as an Euler beam on springs is also analyzed and compared to the rigid 2 DoF model. The semi-active suspensions allow to vary the characteristics of the suspension to improve the quality of the pipe. These types of systems are popular with high-end cars. A simulation of MR dampers (semi-active suspension) with the Bouc-Wen model has been integrated into the quad model with PID regulator.

2.12 J B Ashtekar et al.[12]

The suspension system is made to have driving comfort. With the right suspension, vibrations are not transferred from the tire to the passenger. Depending on the profile of the roadway, vibrations are transmitted to the chassis (unsprung mass) on the seat and body via the strut passenger.

In this case, the most important factor is the vertical acceleration generated in the suspended mass. A reduction in the vertical acceleration is therefore reflected in a better ride comfort. Therefore, a suspension analysis for vertical acceleration must be performed. In this article, the effect of suspension parameters, i. H. Suspended mass, unsprung mass, damping value, stiffness of the suspension spring and stiffness of the tire are accentuated on the vertical acceleration. For analytical purposes, the single car quarter model is considered. The equation of motion is obtained in the differential form. These motion equations form the basis of the Simulink model in MATLAB. This simulation will help simulate the parameters of the suspension. Thus, vertical accelerations are obtained under different working conditions and the maximum values of the accelerations are obtained. The same suggestions are then given to the suspension test stand. These values are then checked on the chassis test bench.

2.13 Puneet Gandhia et al.[13]

Since the suspension system shows an important part in refining relaxation and steadiness, various methods are observed and compared in a 4 DOF semi-trailer model in this work. Adaptive Neuro-Fuzzy Interface System provides optimal results in terms of amplitude and settling time.

2.14 Y. Samim Unlusory et al.[14]

To improve the speed limit profile, the goal achievement method is used. Selected points on a vehicle are examined for vertical and longitudinal acceleration. Various bump profiles are used in this study to reduce their impact over a wide range of speeds.

2.15 Yahaya Md. Sam2 et al.[15]

The author uses an LQR controller to study active suspension of a complete vehicle. A linear dynamic model is observed, including displacement, acceleration, suspension motion, wheel displacement and displacement, pitch and yaw angles. With this controller, an improvement in comfort and handling is achieved.

2.16 D. Geoff Rideout.[16]

In order to observe the dynamic reactions of large vehicles, it is necessary to develop a model and simulate it with road conditions. Flexible frame effects should be considered or not, depending on assumptions. The author presented a bond-graph model considering longitudinal dynamics, transversal frame vibrations, and bounce motion. The effects of various aspects are taken into account, including the speed, pitch, and acceleration of the payload.

CHAPTER 3: MATHEMATICAL MODELING

3.1 Comparison of Modeling Techniques

There are two techniques used to model a dynamic system, the Lagrangian Method, and the Newtonian Method. The Lagrangian approach is normally used where forces and torques are not easy to show on a free body diagram. However, in this case, a free body diagram can be constructed. Forces and torques can be shown on a diagram. So for this case, the Newtonian method is used to model the mathematical equations of the dynamic system. Newton's second law of motion is useful to derive the equations and automobile models are developed.

The relationship between a body and forces acting on that body and effect of those forces is described by Newton's second law of motion. The submission of forces affecting the frame is equal to the mass of that frame multiplied by the acceleration of the frame, which is described in the equation 3.1

$$\sum F = ma \tag{3.1}$$

3.2 Development of Mathematical Model

Longer vehicles with heavy loads have multiple axles. From the structural point of view, additional axle help reduce the span of the vehicle and spread the load over a wider area. This change the vehicle dynamics and things get further complicated when we are using independent suspensions. Independent suspension system does not have any solid axle connecting the wheels. Instead, wheels are directly connected to the chassis. Developing a math model involves few assumptions. Different models are used to study vehicles suspension. The scope of study defines the use of vehicle model.

3.2.1 Single Wheel Model

Single wheel assembly with its spring and damper in 2 axle vehicle is also called single wheel model. Single wheel model is developed to know the basics of vehicle's suspension systems modeling. A single wheel model represents one corner of a vehicle. It describes only vertical motion.

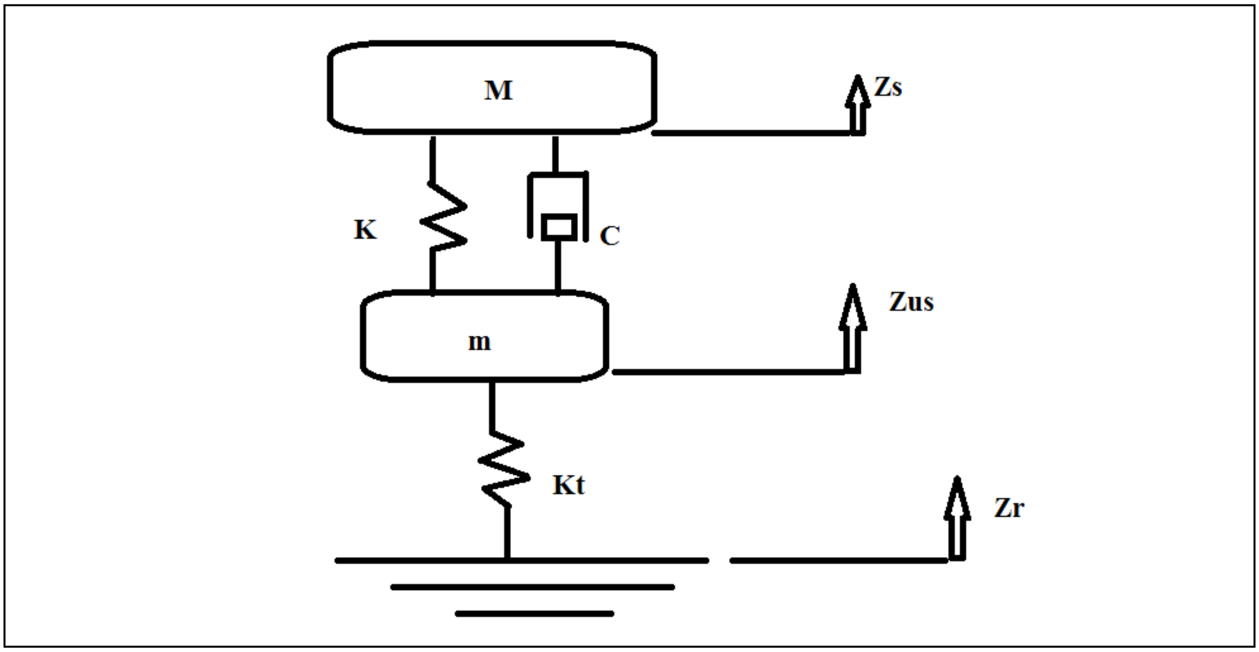


Figure 3.1: Single Wheel Model

Equations for single wheel model are developed using Newton's second law of motion.

Unsprung mass equation

$$m\ddot{z}_u = -K(z_u - z_s) - C(\dot{z}_u - \dot{z}_s) + K_t(z_r - z_u) \tag{3.2}$$

Sprung mass equation

$$M\ddot{z} = K(z_u - z_s) + C(\dot{z}_u - \dot{z}_s) \tag{3.3}$$

3.2.2 Single Track Model

If we cut through the central axis of the vehicle and consider wheels of one side it is called single track model. Single track model for 2 axle vehicle is below.

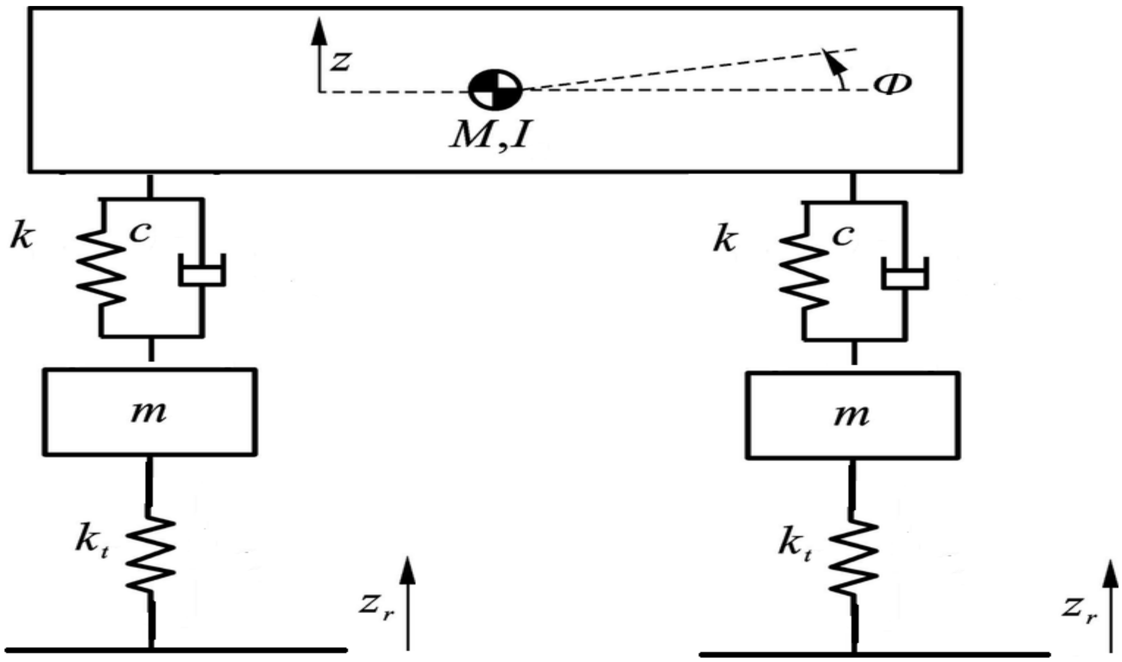


Figure 3.2: Single Track Model

Equations for single track model are developed by applying Newton's second law of motion

Unsprung mass equations

$$m\ddot{z}_{1u} = -K(z_{1u} - z_{1s}) - C(\dot{z}_{1u} - \dot{z}_{1s}) + K_t(z_{1r} - z_{1u}) \tag{3.4}$$

$$m\ddot{z}_{2u} = -K(z_{2u} - z_{2s}) - C(\dot{z}_{2u} - \dot{z}_{2s}) + K_t(z_{2r} - z_{2u}) \tag{3.5}$$

Sprung mass equations

$$M\ddot{z}_s = K(z_{1u} - z_{1s}) + K(z_{2u} - z_{2s}) + C(\dot{z}_{1u} - \dot{z}_{1s}) + +C(\dot{z}_{2u} - \dot{z}_{2s}) \tag{3.6}$$

Pitch equation for half car model is

$$I\ddot{\theta} = l[K(z_{1u} - z_{1s}) + K(z_{2u} - z_{2s}) + C(\dot{z}_{1u} - \dot{z}_{1s}) + +C(\dot{z}_{2u} - \dot{z}_{2s})] \tag{3.7}$$

3.2.3 Full Car Model

All wheels attached to the chassis are considered in full car model. This model captures the full extent of motion induced by automobile suspension. The elementary aim of suspension is to provide the drive ease. It is achieved by reducing sprung mass acceleration without compromising the lateral dynamics. Full car model can capture the heave, roll and pitching movement of the vehicle.

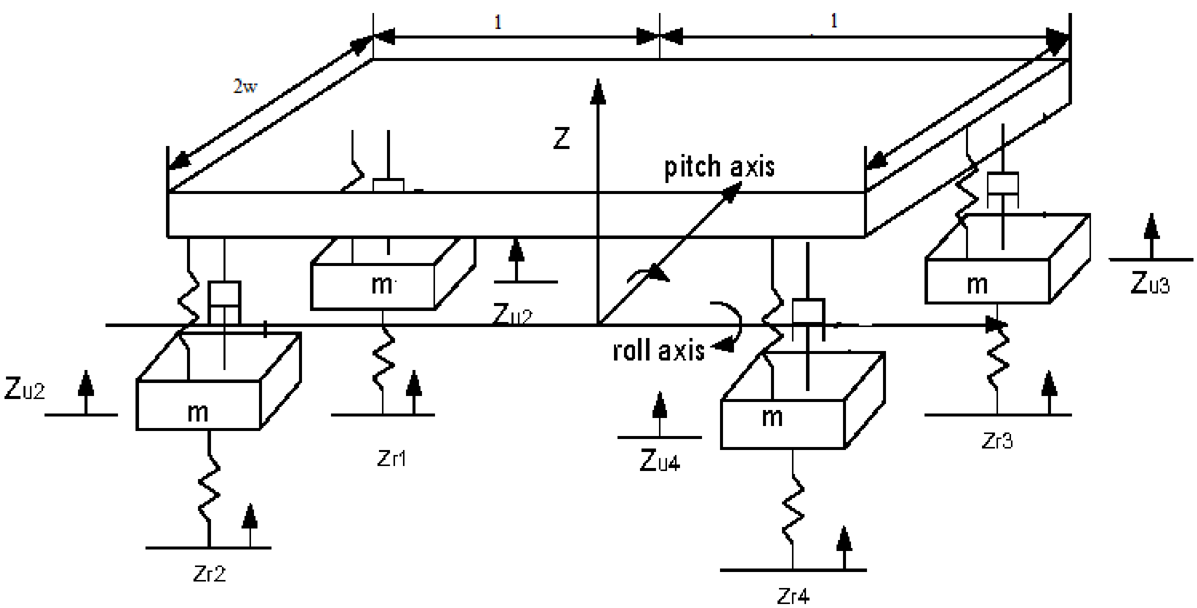


Figure 3.3: Full car model

The mathematical model for 2 axle vehicle can describe all three types of motion i.e., heave, pitch, and roll.

Unsprung mass equations for 2 axle vehicle are

$$m\ddot{z}_{11u} = -K(z_{11u} - z_{11s}) - C(\dot{z}_{11u} - \dot{z}_{11s}) + K_t(z_{11r} - z_{11u}) \tag{3.8}$$

$$m\ddot{z}_{21u} = -K(z_{21u} - z_{21s}) - C(\dot{z}_{21u} - \dot{z}_{21s}) + K_t(z_{21r} - z_{21u}) \tag{3.9}$$

$$m\ddot{z}_{12u} = -K(z_{12u} - z_{12s}) - C(\dot{z}_{12u} - \dot{z}_{12s}) + K_t(z_{12r} - z_{12u}) \tag{3.10}$$

$$m\ddot{z}_{22u} = -K(z_{22u} - z_{22s}) - C(\dot{z}_{22u} - \dot{z}_{22s}) + K_t(z_{22r} - z_{22u}) \tag{3.11}$$

Sprung mass equations

$$\begin{aligned} M\ddot{z}_s = {} & K(z_{11u} - z_{11s}) + K(z_{12u} - z_{12s}) + K(z_{21u} - z_{21s}) + K(z_{22u} - z_{22s}) \\ & + C(\dot{z}_{11u} - \dot{z}_{11s}) + C(\dot{z}_{12u} - \dot{z}_{12s}) + C(\dot{z}_{21u} - \dot{z}_{21s}) + C(\dot{z}_{22u} - \dot{z}_{22s}) \end{aligned} \tag{3.12}$$

Pitch equation for half car model is

$$\begin{aligned} I_{yy}\ddot{\theta} = l[& K(z_{11u} - z_{11s}) + K(z_{12u} - z_{12s}) + K(z_{21u} - z_{21s}) + K(z_{22u} - z_{22s}) \\ & + +C(\dot{z}_{11u} - \dot{z}_{11s}) + C(\dot{z}_{12u} - \dot{z}_{12s}) + C(\dot{z}_{21u} - \dot{z}_{21s}) \\ & + C(\dot{z}_{22u} - \dot{z}_{22s})] \end{aligned} \tag{3.13}$$

Rolling equation for half car model is

$$\begin{aligned} I_{xx}\ddot{\varphi} = w[& K(z_{11u} - z_{11s}) + K(z_{12u} - z_{12s}) + K(z_{21u} - z_{21s}) + K(z_{22u} - z_{22s}) \\ & + C(\dot{z}_{11u} - \dot{z}_{11s}) + C(\dot{z}_{12u} - \dot{z}_{12s}) + C(\dot{z}_{21u} - \dot{z}_{21s}) + C(\dot{z}_{22u} - \dot{z}_{22s})] \end{aligned} \tag{3.14}$$

3.3 State space model for 2 axle vehicle

State vector contains the displacements of suspension attachment nodes, displacement of chassis in z direction, roll and pitch angles and their respective time rates. U (t) is input vector, which contains road disturbance which are in the form of displacement.

$$\dot{X}(t) = AX(t) + BU(t) \tag{3.15}$$

Where $X(t)$ is the state vector which consists of vertical displacements at each wheel, heave, pitch, roll and their respective time rates.

$$\dot{X}^T = [X_1 \quad X_2] \tag{3.16}$$

$$X_1 = [z_{11u} \quad z_{21u} \quad z_{12u} \quad z_{22u} \quad z \quad \theta \quad \varphi] \tag{3.17}$$

$$X_2 = [\dot{z}_{11u} \quad \dot{z}_{21u} \quad \dot{z}_{12u} \quad \dot{z}_{22u} \quad \dot{z} \quad \dot{\theta} \quad \dot{\varphi}] \tag{3.18}$$

$$U^T = [z_{11r} \quad z_{21r} \quad z_{12r} \quad z_{22r}] \tag{3.19}$$

In equation 19, $U(t)$ is input or control matrix.Elements of control vector are the road input at each wheel.

Order of the matrices is given in the equation 20

$$\dot{X}_{14\times1} = A_{14\times14}X_{14\times1} + B_{14\times4}U_{4\times1} \tag{3.20}$$

Block matrix $A(t)$ in equation 21 is the state matrix

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \tag{3.21}$$

Matrix A (t) has the 4 sub-matrices A_{11}, A_{12}, A_{21}, and A_{22}.

Matrix A_{11} is a 7×7 Null matrix

$$A_{11} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{3.22}$$

Matrix A_{12} is a 7×7 Identity matrix

$$A_{12} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{3.23}$$

Matrix A_{21} is a 7×7 Stiffness matrix. It represents suspension stiffness elements in the equation.

$$A_{21} = \begin{bmatrix} \frac{-k-kt}{m} & 0 & 0 & 0 & \frac{k}{m} & \frac{-lk}{m} & \frac{wk}{m} \\ 0 & \frac{-k-kt}{m} & 0 & 0 & \frac{k}{m} & \frac{-lk}{m} & \frac{wk}{m} \\ 0 & 0 & \frac{-k-kt}{m} & 0 & \frac{k}{m} & \frac{-lk}{m} & \frac{wk}{m} \\ 0 & 0 & 0 & \frac{-k-kt}{m} & \frac{k}{m} & \frac{-lk}{m} & \frac{wk}{m} \\ \frac{k}{m} & \frac{k}{m} & \frac{k}{m} & \frac{k}{m} & \frac{-4k}{m} & 0 & 0 \\ \frac{lk}{Iyy} & \frac{lk}{Iyy} & \frac{lk}{Iyy} & \frac{lk}{Iyy} & 0 & \frac{-4lk}{Iyy} & 0 \\ \frac{wk}{Ixx} & \frac{wk}{Ixx} & \frac{wk}{Ixx} & \frac{wk}{Ixx} & 0 & 0 & \frac{-wk}{Ixx} \end{bmatrix} \tag{3.24}$$

Matrix A_{22} is a 7×7 damping matrix and represents the damping elements in the equation.

$$A_{22} = \begin{bmatrix} \frac{-c}{m} & 0 & 0 & 0 & \frac{c}{m} & \frac{-lc}{m} & \frac{wc}{m} \\ 0 & \frac{-c}{m} & 0 & 0 & \frac{c}{m} & \frac{-lc}{m} & \frac{wc}{m} \\ 0 & 0 & \frac{-c}{m} & 0 & \frac{c}{m} & \frac{-lc}{m} & \frac{wc}{m} \\ 0 & 0 & 0 & \frac{-c}{m} & \frac{c}{m} & \frac{-lc}{m} & \frac{wc}{m} \\ \frac{c}{m} & \frac{c}{m} & \frac{c}{m} & \frac{c}{m} & \frac{-4c}{m} & 0 & 0 \\ \frac{lc}{Iyy} & \frac{lc}{Iyy} & \frac{lc}{Iyy} & \frac{lc}{Iyy} & 0 & \frac{-4lc}{Iyy} & 0 \\ \frac{wc}{Ixx} & \frac{wc}{Ixx} & \frac{wc}{Ixx} & \frac{wc}{Ixx} & 0 & 0 & \frac{-wc}{Ixx} \end{bmatrix} \tag{3.25}$$

B matrix represents the tire stiffness and it has 2 sub-matrices B_{11} and B_{12}.

$$B^T = [B_{11} \quad B_{12}] \tag{3.26}$$

B_{11} is 7×4 null matrix.

$$B_{11} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{3.27}$$

B_{12} is a 7×4 stiffness matrix and represents the tire stiffness elements.

$$B_{12} = \begin{bmatrix} \frac{kt}{m} & 0 & 0 & 0 \\ 0 & \frac{kt}{m} & 0 & 0 \\ 0 & 0 & \frac{kt}{m} & 0 \\ 0 & 0 & 0 & \frac{kt}{m} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (3.28)$$

3.4 Generic Mathematical Model for the multi-axle vehicle

Generic mathematical model for vehicles with n-number of axles is developed. Index notation used to define the whole mathematical model is

$$\boldsymbol{z_{ijk}} \boldsymbol{= displacement\ between\ masses\ on\ each\ axle.}$$

Where

Table 3-1: Index Representation

Index	**Representation**
i	Number of axles starting from the front
j	j = side of the vehicle

	j=1→ Right side of the vehicle j=2→ Left side of the vehicle
k	k=input type k=u→unsprung k=s→sprung k=r→road

Following are the assumptions on which this mathematical model is based on.

- Vehicle chassis has the equal wheelbase
- Center of gravity lies in the middle of the vehicle
- All axles are equidistant
- Each suspension strut is modeled with a spring and a damping element having coefficients K and C respectively
- Tires are modeled with a spring of stiffness coefficient K_t
- The roll angle is $\emptyset$ with its positive direction according to the right-hand rule
- Pitch angle is denoted by θ
- Variable Z defines the heave displacement
- The input to the system is displacement in z-direction applied at the free end of the tire by road disturbances
- Each wheel experiences the roll, pitch and heave forces when there is an input to the system
- Unsprung masses, stiffness, and damping coefficients are same for all axles

The generic model for n number of axles is obtained by applying Newton's second law of motion.

Unsprung Mass Equations is

$$m\ddot{z}_{iju} = -K\left(z_{iju} - z_{ijs}\right) - C\left(\dot{z}_{iju} - \dot{z}_{ijs}\right) - K_t\ (z_{ijr} - z_{iju}) \tag{3.29}$$

Sprung Mass Equation is

$$M\ddot{z} = \sum_{i=1}^{n} K(z_{i1u} - z_{i1s}) + C(\dot{z}_{i1u} - \dot{z}_{i1s}) + \sum_{i=1}^{n} K(z_{i2u} - z_{i2s}) + C(\dot{z}_{i2u} - \dot{z}_{i2s}) \tag{3.30}$$

Pitch Equation is

$$I_{yy}\ddot{\theta} = l\left[\sum_{i=1}^{n} K(z_{i1u} - z_{i1s}) + C(\dot{z}_{i1u} - \dot{z}_{i1s}) + \sum_{i=1}^{n} K(z_{i2u} - z_{i2s}) + C(\dot{z}_{i2u} - \dot{z}_{i2s})\right] \tag{3.31}$$

Roll Equation is

$$I_{xx}\ddot{\varphi} = w\left[\sum_{i=1}^{n} K(z_{i1u} - z_{i1s}) + C(\dot{z}_{i1u} - \dot{z}_{i1s}) + \sum_{i=1}^{n} K(z_{i2u} - z_{i2s}) + C(\dot{z}_{i2u} - \dot{z}_{i2s})\right] \tag{3.32}$$

3.5 Mathematical Formulation for 4 axle vehicle

Full car model for 4 axle vehicles is developed. Suspension system of a vehicle is defined by unsprung mass, sprung mass, rolling and pitching equations. Each wheel experience a force called the unsprung force. Application of Newton’s second law of motion on each wheel yields 8 second order differential equations.

$$m\ddot{z}_{11u} = -K(z_{11u} - z_{11s}) - C(\dot{z}_{11u} - \dot{z}_{11s}) + K_t(z_{11r} - z_{11u}) \tag{3.33}$$

$$m\ddot{z}_{12u} = -K(z_{12u} - z_{12s}) - C(\dot{z}_{12u} - \dot{z}_{12s}) + K_t(z_{12r} - z_{12u}) \tag{3.34}$$

$$m\ddot{z}_{21u} = -K(z_{21u} - z_{21s}) - C(\dot{z}_{21u} - \dot{z}_{21s}) + K_t(z_{21r} - z_{21u}) \tag{3.35}$$

$$m\ddot{z}_{22u} = -K(z_{22u} - z_{22s}) - C(\dot{z}_{22u} - \dot{z}_{22s}) + K_t(z_{22r} - z_{22u}) \tag{3.36}$$

$$m\ddot{z}_{31u} = -K(z_{31u} - z_{31s}) - C(\dot{z}_{31u} - \dot{z}_{31s}) + K_t(z_{31r} - z_{31u}) \tag{3.37}$$

$$m\ddot{z}_{32u} = -K(z_{32u} - z_{32s}) - C(\dot{z}_{32u} - \dot{z}_{32s}) + K_t(z_{32r} - z_{32u}) \tag{3.38}$$

$$m\ddot{z}_{41u} = -K(z_{41u} - z_{41s}) - C(\dot{z}_{41u} - \dot{z}_{41s}) + K_t(z_{41r} - z_{41u}) \tag{3.39}$$

$$m\ddot{z}_{42u} = -K(z_{42u} - z_{42s}) - C(\dot{z}_{42u} - \dot{z}_{42s}) + K_t(z_{42r} - z_{42u}) \tag{3.40}$$

Effects of all these wheel forces can be represented by sprung mass equation, by applying Newton’s second law of motion on sprung mass yields

$$\begin{aligned} M\ddot{z}_s = {} & K(z_{11u} - z_{11s}) + K(z_{12u} - z_{12s}) + K(z_{21u} - z_{21s}) + K(z_{22u} - z_{22s}) \\ & + K(z_{31u} - z_{31s}) + K(z_{32u} - z_{32s}) + K(z_{41u} - z_{41s}) \\ & + K(z_{42u} - z_{42s}) + C(\dot{z}_{11u} - \dot{z}_{11s}) + C(\dot{z}_{12u} - \dot{z}_{12s}) \\ & + C(\dot{z}_{21u} - \dot{z}_{21s}) + C(\dot{z}_{22u} - \dot{z}_{22s}) + C(\dot{z}_{31u} - \dot{z}_{31s}) \\ & + C(\dot{z}_{32u} - \dot{z}_{32s}) + C(\dot{z}_{41u} - \dot{z}_{41s}) + C(\dot{z}_{42u} - \dot{z}_{42s}) \end{aligned} \tag{3.41}$$

Second type of motion induced by suspension is roll. Rolling motion is the rotation of vehicle chassis about longitudinal axis. Applying second law of motion in polar form and summing moments gives

$$I_{xx}\ddot{\varphi} = w[K(z_{11u} - z_{11s}) + K(z_{12u} - z_{12s}) + K(z_{21u} - z_{21s}) + K(z_{22u} - z_{22s}) \\ + K(z_{31u} - z_{31s}) + K(z_{32u} - z_{32s}) + K(z_{41u} - z_{41s}) + K(z_{42u} - z_{42s}) \\ + C(\dot{z}_{11u} - \dot{z}_{11s}) + C(\dot{z}_{12u} - \dot{z}_{12s}) + C(\dot{z}_{21u} - \dot{z}_{21s}) + C(\dot{z}_{22u} - \dot{z}_{22s}) \\ + C(\dot{z}_{31u} - \dot{z}_{31s}) + C(\dot{z}_{32u} - \dot{z}_{32s}) + C(\dot{z}_{41u} - \dot{z}_{41s}) + C(\dot{z}_{42u} - \dot{z}_{42s})] \quad (3.42)$$

Pitching is the 3rd type of the motion induced by suspension. It is rotation of vehicle chassis about lateral axis. Applying second law of motion in polar form and summing moments gives

$$I_{yy}\ddot{\theta} = l[K(z_{11u} - z_{11s}) + K(z_{12u} - z_{12s}) + K(z_{21u} - z_{21s}) + K(z_{22u} - z_{22s}) \\ + K(z_{31u} - z_{31s}) + K(z_{32u} - z_{32s}) + K(z_{41u} - z_{41s}) + K(z_{42u} - z_{42s}) \\ + +C(\dot{z}_{11u} - \dot{z}_{11s}) + C(\dot{z}_{12u} - \dot{z}_{12s}) + C(\dot{z}_{21u} - \dot{z}_{21s}) + C(\dot{z}_{22u} - \dot{z}_{22s}) \\ + C(\dot{z}_{31u} - \dot{z}_{31s}) + C(\dot{z}_{32u} - \dot{z}_{32s}) + C(\dot{z}_{41u} - \dot{z}_{41s}) + C(\dot{z}_{42u} - \dot{z}_{42s})] \quad (3.43)$$

Translation of the vehicle body displacements can be expressed as

$$z_{12s} = z_s - l\theta - w\varphi \quad (3.44)$$

$$z_{22s} = z_s - l\theta - w\varphi \quad (3.45)$$

$$z_{32s} = z_s - l\theta - w\varphi \quad (3.46)$$

$$z_{42s} = z_s + l\theta - w\varphi \quad (3.47)$$

$$z_{11s} = z_s - l\theta + w\varphi \quad (3.48)$$

$$z_{21s} = z_s - l\theta + w\varphi \tag{3.49}$$

$$z_{31s} = z_s - l\theta + w\varphi \tag{3.50}$$

$$z_{41s} = z_s + l\theta + w\varphi \tag{3.51}$$

Translation of the vehicle body velocities can be expressed as

$$\dot{z}_{12s} = \dot{z}_s - l\dot{\theta} - w\dot{\varphi} \tag{3.52}$$

$$\dot{z}_{22s} = \dot{z}_s - l\dot{\theta} - w\dot{\varphi} \tag{3.53}$$

$$\dot{z}_{32s} = \dot{z}_s - l\dot{\theta} - w\dot{\varphi} \tag{3.54}$$

$$\dot{z}_{42s} = \dot{z}_s + l\dot{\theta} - w\dot{\varphi} \tag{3.55}$$

$$\dot{z}_{11s} = \dot{z}_s - l\dot{\theta} + w\dot{\varphi} \tag{3.56}$$

$$\dot{z}_{21s} = \dot{z}_s - l\dot{\theta} + w\dot{\varphi} \tag{3.57}$$

$$\dot{z}_{31s} = \dot{z}_s - l\dot{\theta} + w\dot{\varphi} \tag{3.58}$$

$$\dot{z}_{41s} = \dot{z}_s + l\dot{\theta} + w\dot{\varphi} \tag{3.59}$$

Putting values of translation in unsprung mass equations

$$m\ddot{z}_{11u} = -K(z_{11u} - z_s + l\theta - w\varphi) - C\left(\dot{z}_{11u} - \dot{z}_s + l\dot{\theta} - w\dot{\varphi}\right) + K_t(z_{11r} - z_{11u}) \tag{3.60}$$

$$m\ddot{z}_{12u} = -K(z_{12u} - z_s + l\theta + w\varphi) - C(\dot{z}_{12u} - \dot{z}_s + l\dot{\theta} + w\dot{\varphi}) + K_t(z_{12r} - z_{12u}) \tag{3.61}$$

$$m\ddot{z}_{21u} = -K(z_{21u} - z_s + l\theta - w\varphi) - C(\dot{z}_{21u} - \dot{z}_s + l\dot{\theta} - w\dot{\varphi}) + K_t(z_{21r} - z_{21u}) \tag{3.62}$$

$$m\ddot{z}_{22u} = -K(z_{22u} - z_s + l\theta + w\varphi) - C(\dot{z}_{22u} - \dot{z}_s + l\dot{\theta} + w\dot{\varphi}) + K_t(z_{22r} - z_{22u}) \tag{3.63}$$

$$m\ddot{z}_{31u} = -K(z_{31u} - z_s + l\theta - w\varphi) - C(\dot{z}_{31u} - \dot{z}_s + l\dot{\theta} - w\dot{\varphi}) + K_t(z_{31r} - z_{31u}) \tag{3.64}$$

$$m\ddot{z}_{32u} = -K(z_{32u} - z_s + l\theta + w\varphi) - C(\dot{z}_{32u} - \dot{z}_s + l\dot{\theta} + w\dot{\varphi}) + K_t(z_{32r} - z_{32u}) \tag{3.65}$$

$$m\ddot{z}_{41u} = -K(z_{41u} - z_s - l\theta - w\varphi) - C(\dot{z}_{41u} - \dot{z}_s - l\dot{\theta} + w\dot{\varphi}) + K_t(z_{41r} - z_{41u}) \tag{3.66}$$

$$m\ddot{z}_{42u} = -K(z_{42u} - z_s - l\theta + w\varphi) - C(\dot{z}_{42u} - \dot{z}_s - l\dot{\theta} - w\dot{\varphi}) + K_t(z_{42r} - z_{42u}) \tag{3.67}$$

Putting values of translation in sprung mass equation

$$\begin{aligned} M\ddot{z}_s = {} & K(z_{11u} - z_s + l\theta - w\varphi) + K(z_{12u} - z_s + l\theta + w\varphi) \\ & + K(z_{21u} - z_s + l\theta - w\varphi) + K(z_{22u} - z_s + l\theta + w\varphi) \\ & + K(z_{31u} - z_s + l\theta - w\varphi) + K(z_{32u} - z_s + l\theta + w\varphi) \\ & + K(z_{41u} - z_s - l\theta - w\varphi) + K(z_{42u} - z_s - l\theta + w\varphi) \\ & + C(\dot{z}_{11u} - \dot{z}_s + l\dot{\theta} - w\dot{\varphi}) + C(\dot{z}_{12u} - \dot{z}_s + l\dot{\theta} + w\dot{\varphi}) \\ & + C(\dot{z}_{21u} - \dot{z}_s + l\dot{\theta} - w\dot{\varphi}) + C(\dot{z}_{22u} - \dot{z}_s + l\dot{\theta} + w\dot{\varphi}) \\ & + C(\dot{z}_{31u} - \dot{z}_s + l\dot{\theta} - w\dot{\varphi}) + C(\dot{z}_{32u} - \dot{z}_s + l\dot{\theta} + w\dot{\varphi}) \\ & + C(\dot{z}_{41u} - \dot{z}_s - l\dot{\theta} - w\dot{\varphi}) + C(\dot{z}_{42u} - \dot{z}_s - l\dot{\theta} + w\dot{\varphi}) \end{aligned} \tag{3.68}$$

Equation of rolling motion of sprung mass after transformation:

$$
\begin{aligned}
I_{xx}\ddot{\varphi} = w[&K(z_{11u} - z_s + l\theta - w\varphi) + K(z_{12u} - z_s + l\theta + w\varphi) \\
&+ K(z_{21u} - z_s + l\theta - w\varphi) + K(z_{22u} - z_s + l\theta + w\varphi) \\
&+ K(z_{31u} - z_s + l\theta - w\varphi) + K(z_{32u} - z_s + l\theta + w\varphi) \\
&+ K(z_{41u} - z_s - l\theta - w\varphi) + K(z_{42u} - z_s - l\theta + w\varphi) \\
&+ C\left(\dot{z}_{11u} - \dot{z}_s + l\dot{\theta} - w\dot{\varphi}\right) + C\left(\dot{z}_{12u} - \dot{z}_s + l\dot{\theta} + w\dot{\varphi}\right) \\
&+ C\left(\dot{z}_{21u} - \dot{z}_s + l\dot{\theta} - w\dot{\varphi}\right) + C\left(\dot{z}_{22u} - \dot{z}_s + l\dot{\theta} + w\dot{\varphi}\right) \\
&+ C\left(\dot{z}_{31u} - \dot{z}_s + l\dot{\theta} - w\dot{\varphi}\right) + C\left(\dot{z}_{32u} - \dot{z}_s + l\dot{\theta} + w\dot{\varphi}\right) \\
&+ C\left(\dot{z}_{41u} - \dot{z}_s - l\dot{\theta} - w\dot{\varphi}\right) + C\left(\dot{z}_{42u} - \dot{z}_s - l\dot{\theta} + w\dot{\varphi}\right)]
\end{aligned}
\tag{3.69}
$$

Equation of pitching motion of sprung mass after transformation:

$$
\begin{aligned}
I_{yy}\ddot{\theta} = l[&K(z_{11u} - z_s + l\theta - w\varphi) + K(z_{12u} - z_s + l\theta + w\varphi) \\
&+ K(z_{21u} - z_s + l\theta - w\varphi) + K(z_{22u} - z_s + l\theta + w\varphi) \\
&+ K(z_{31u} - z_s + l\theta - w\varphi) + K(z_{32u} - z_s + l\theta + w\varphi) \\
&+ K(z_{41u} - z_s - l\theta - w\varphi) + K(z_{42u} - z_s - l\theta + w\varphi) \\
&+ C\left(\dot{z}_{11u} - \dot{z}_s + l\dot{\theta} - w\dot{\varphi}\right) + C\left(\dot{z}_{12u} - \dot{z}_s + l\dot{\theta} + w\dot{\varphi}\right) \\
&+ C\left(\dot{z}_{21u} - \dot{z}_s + l\dot{\theta} - w\dot{\varphi}\right) + C\left(\dot{z}_{22u} - \dot{z}_s + l\dot{\theta} + w\dot{\varphi}\right) \\
&+ C\left(\dot{z}_{31u} - \dot{z}_s + l\dot{\theta} - w\dot{\varphi}\right) + C\left(\dot{z}_{32u} - \dot{z}_s + l\dot{\theta} + w\dot{\varphi}\right) \\
&+ C\left(\dot{z}_{41u} - \dot{z}_s - l\dot{\theta} - w\dot{\varphi}\right) + C\left(\dot{z}_{42u} - \dot{z}_s - l\dot{\theta} + w\dot{\varphi}\right)]
\end{aligned}
\tag{3.70}
$$

CHAPTER 4: IMPLEMENTATION AND VERIFICATION

The mathematical model is solved in MATLAB Simulink and results are verified in TruckSIM. Vehicle template is created in TruckSIM as per data given in Table 4-1 and results are generated.

Table 4-1: Vehicle Parameters

Parameters	**Values**	**Units**
Vehicle Length	4	m
Vehicle Height	1.93	m
Vehicle Width	2.65	m
Spring Stiffness Coefficient **K**	700	kN/m
Damping Coefficient **C**	30	kN.s/m
Vehicle's Sprung Mass **M**	7000	kg
Vehicle's Unsprung Mass **m**	250	kg
Roll Inertia **Ixx**	3628	kgm^2
Pitch Inertia **Iyy**	55666	kgm^2
Yaw Inertia **Izz**	11999.5	kgm^2
The lateral offset of the wheel from the center of gravity **w**	1.95	m

4.1 Road Profile

The vehicle is moving in a straight line. A small sharp bump is introduced to study the vehicle response while passing on the bump and factors used for the bump in TruckSIM are registered in table 4-2.

Table 4-2: Bump Factors

Parameters	**Values**	**Units**
Height of the Bump	7	cm
Width of the Bump	7	m
Length of the Bump	40	cm

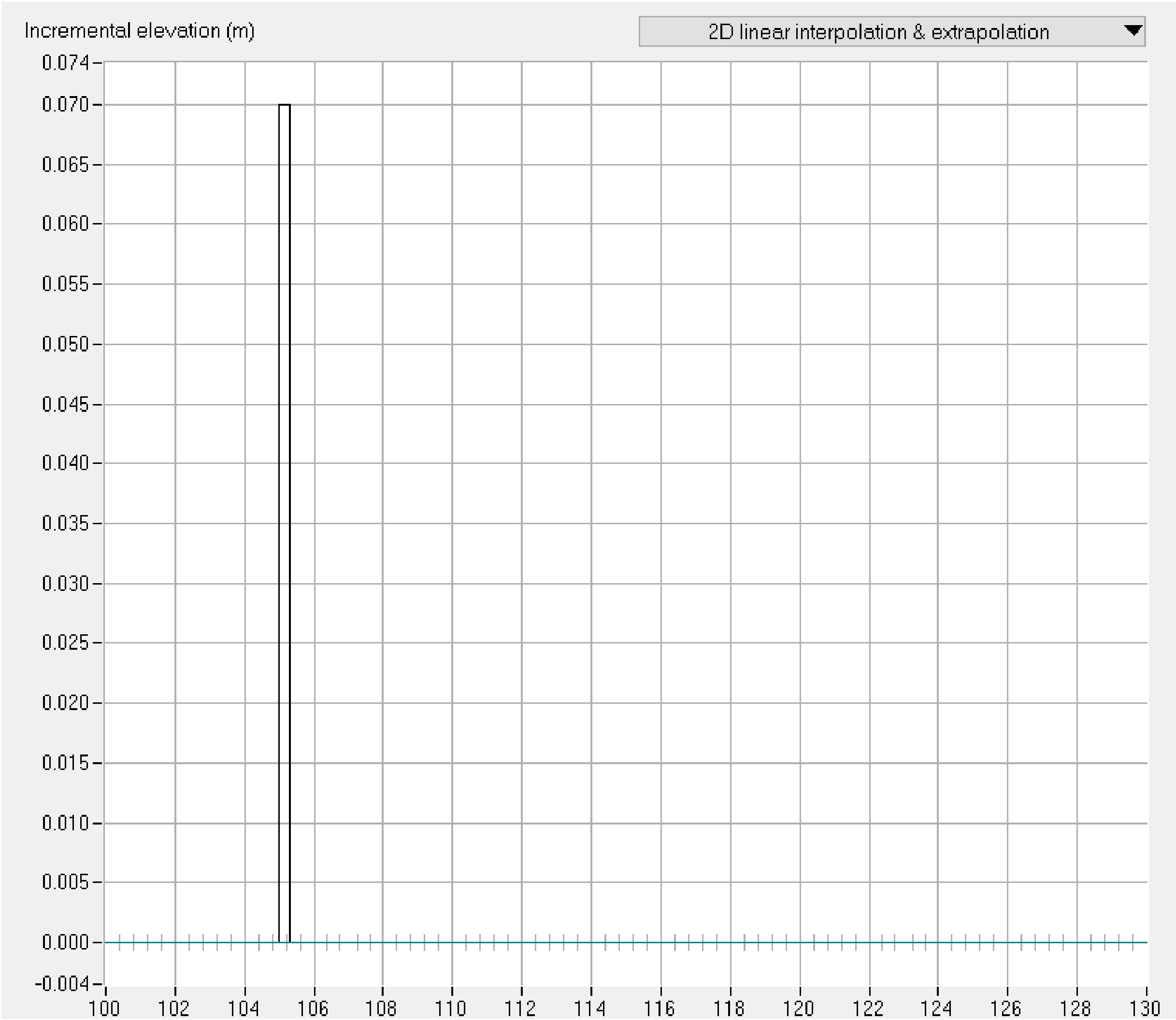


Figure 4.1: Road Profile

Flat road with the total length of the 130 meters is used in TruckSIM. Bump is introduced approximately at 105 meters. It is a small sharp bump with the elevation of 0.07 meters.

4.2 Implementation in MATLAB Simulink

State space model is solved using the default MATLAB toolbox, SIMULINK. The mathematical model is solved with the same input functions used for simulation in commercial software and under the same driving conditions. Simulation results are generated in MATLAB and then compared with TruckSIM results. In MATLAB SIMULINK step input is given to the system which replicates the bump input in the TruckSIM.

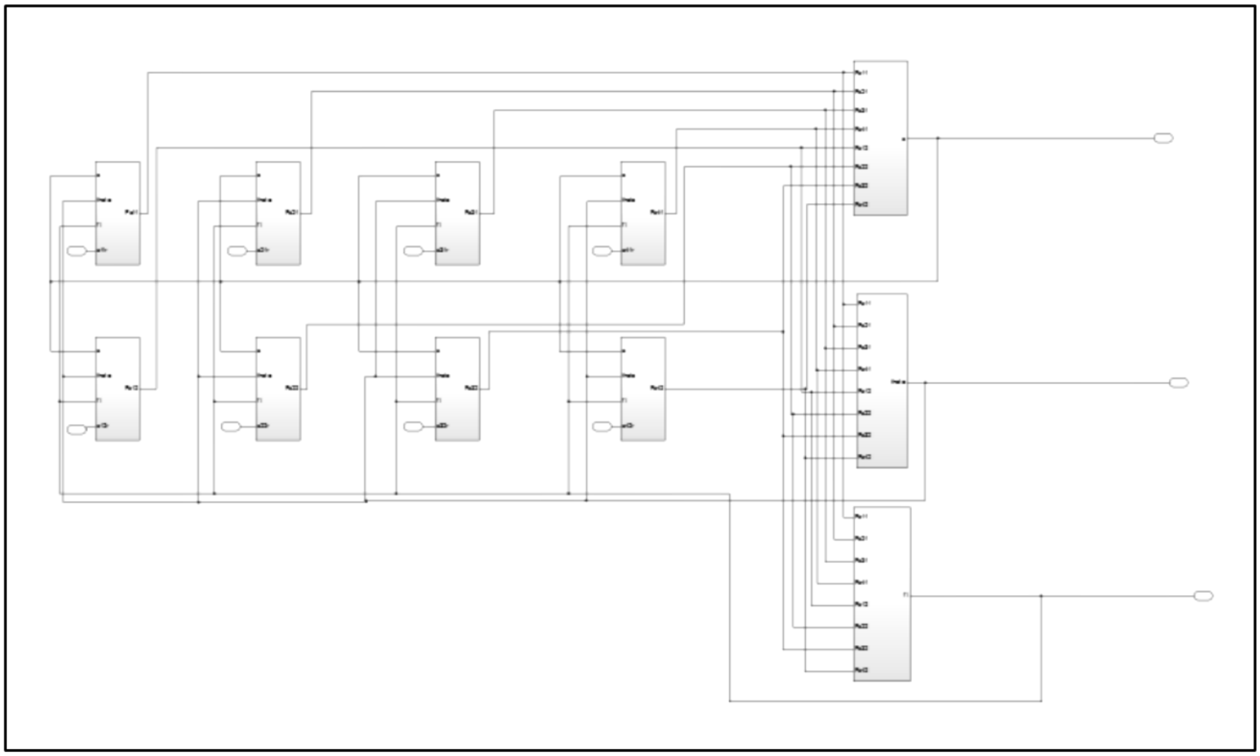


Figure 4.2: Mathematical model of 4 axle vehicle in Simulink

Figure 10 represents the mathematical model of 4 axle vehicle in MATLAB Simulink. Small 8 blocks are representing the 8 unsprung mass equations. 3 large blocks are representing the sprung mass, pitch and roll equations.

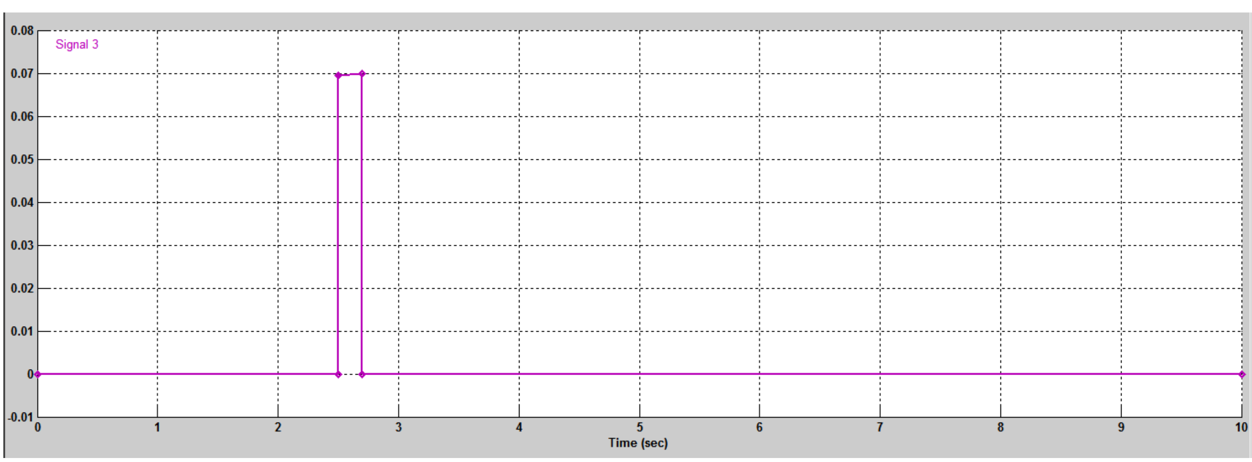


Figure 4.3: Input Function in MATLAB Simulink

In figure 4.3 a step input is given, representing the road input to the system. The step is showing the bump in the road. Bump is of 0.07m height.

4.2.1 Heave

The center of gravity (CG) displacement to the ground on a road with large amplitude disturbances is shown in Figure 4.4. It can be shown from the figure that when the vehicle passes on the bump then the stable state is disturbed. Firstly vehicle displaces with large amplitude then it settles due to dampers present in the suspension system. Dampers absorb all the disturbances caused by the road. This graph shows how wheels on each axle behave when the vehicle moves on the bump. It can be shown from the figure that the difference between all amplitudes is equal which satisfy the equidistant axle assumption.

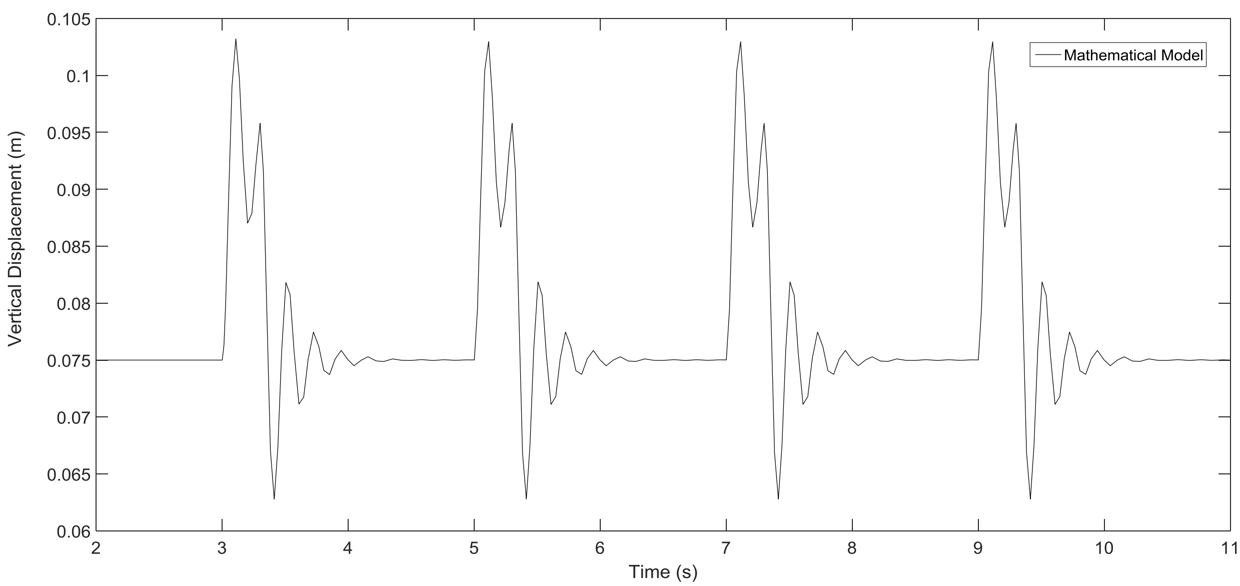


Figure 4.4: Heave Graph in SIMULINK

The vehicle moves with the speed of 0.7 km/h on the road for about 10 seconds. The vertical axis represents the vertical displacement in meters and the horizontal axis represents the time in seconds. Figure 10 shows that when the effect of disturbance caused by the road lasts between 3 to 5 seconds for axle 1, 5 to 7 seconds for axle 2, 7 to 9 seconds for axle 3 and 9 to 11 seconds for axle 4. Vehicles heave motion is between 0.063 and 0.103 meters.

4.2.2 Pitch

Motion about the lateral axis is called the pitch and it is a measure of how far a vehicle is titled up or down [21]. When the vehicle moves on the bump the weight of the vehicle is transferred forward onto the front wheels and body of the vehicle will lean forward. Figure 4.5 shows the behavior of the vehicle's pitch while passing on the bump.

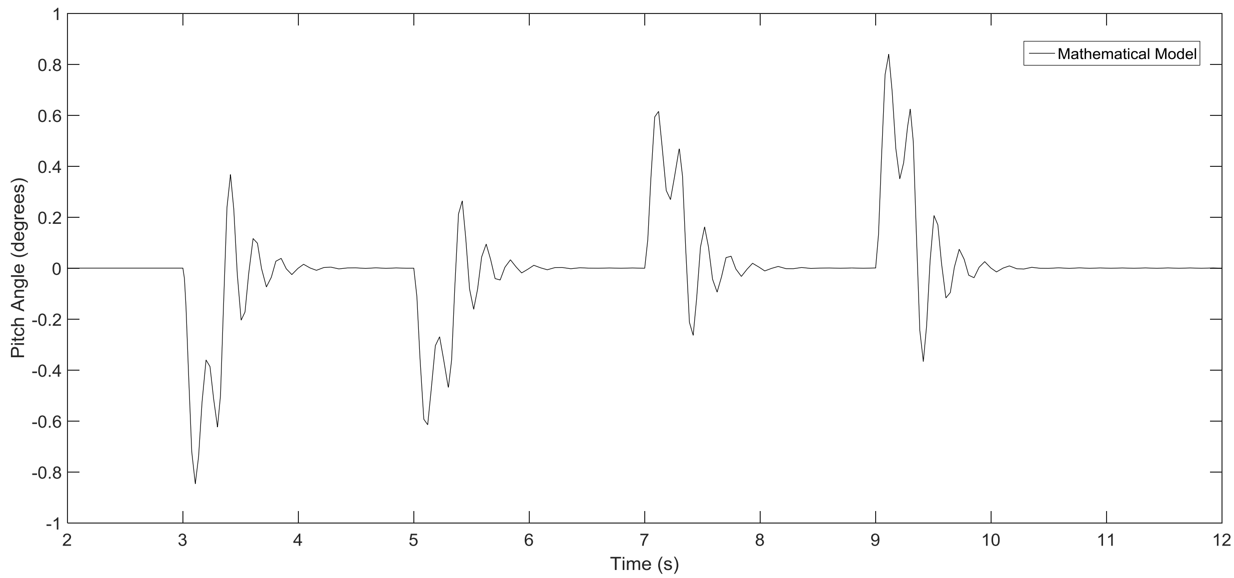


Figure 4.5: Pitch Graph in SIMULINK

The perpendicular axis denotes the pitch angle in degrees and the parallel axis denotes the interval in seconds. At time 3 seconds the 1st axle moves on the bump and pitching motion is induced and vehicle leans forward. Vehicle leans back at 7 seconds transferring the weight on the rear axles. Vehicles pitching motion is between -0.8 and +0.8 degrees.

4.2.3 Roll

Roll is the movement of the vehicle's body that defines how well a vehicle can distribute its weight while turning. Roll is shifting of the weight of the vehicle to the right or left [22]. As in this case, the vehicle is moving on a straight road with zero turning so there is no roll in the system. Figure 4.6 shows a straight line representing no roll in the system. The perpendicular axis denotes the roll angle in degrees and the parallel axis denotes the interval in seconds

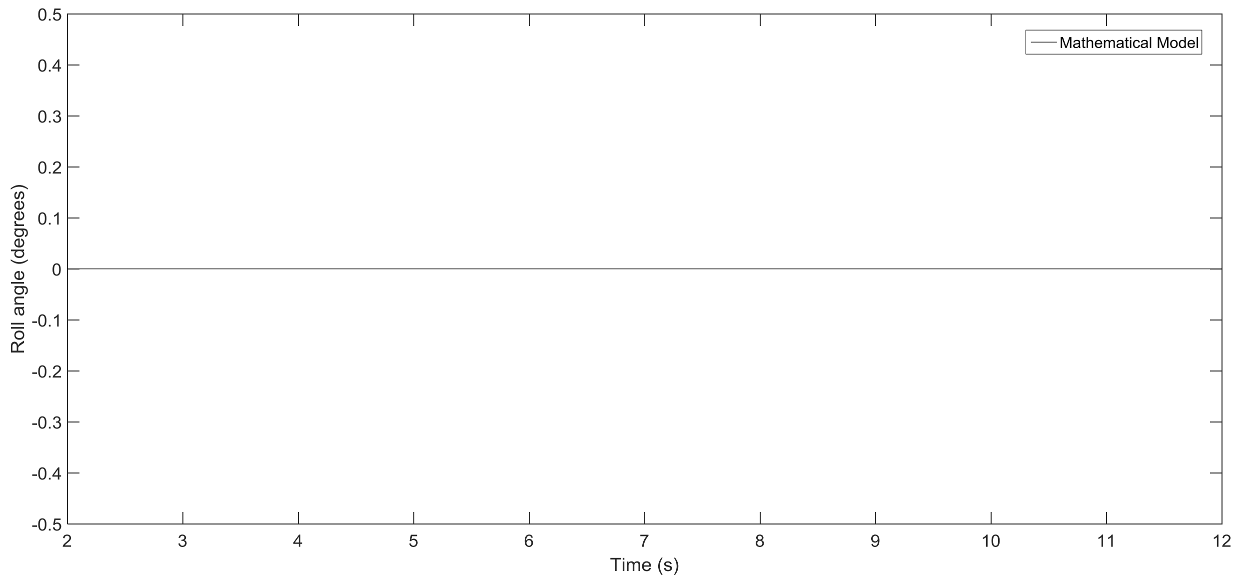


Figure 4.6: Roll Graph in SIMULINK

4.3 Implementation in TruckSIM:

The simulation model of 4 axle vehicle is created in TruckSIM to analyze the behavior of the vehicle on road input. Factors used in this model are registered in Table 2. Graphs for heave, pitch and roll are generated. Road parameters are given in Table 3. Straight road with a small sharp bump is introduced.

4.3.1 Heave

The center of gravity (CG) displacement to the ground on a road with large amplitude disturbances is shown in Figure 4.7. It can be shown from the figure that when the vehicle passes on the bump then the stable state is disturbed. Firstly vehicle displaces with large amplitude then it settles due to dampers present in the suspension system. Dampers absorb all the disturbances caused by the road. This graph shows how wheels on each axle behave when the vehicle moves on the bump. It can be shown from the figure that the difference between all amplitudes is equal which satisfy the equidistant axle assumption.

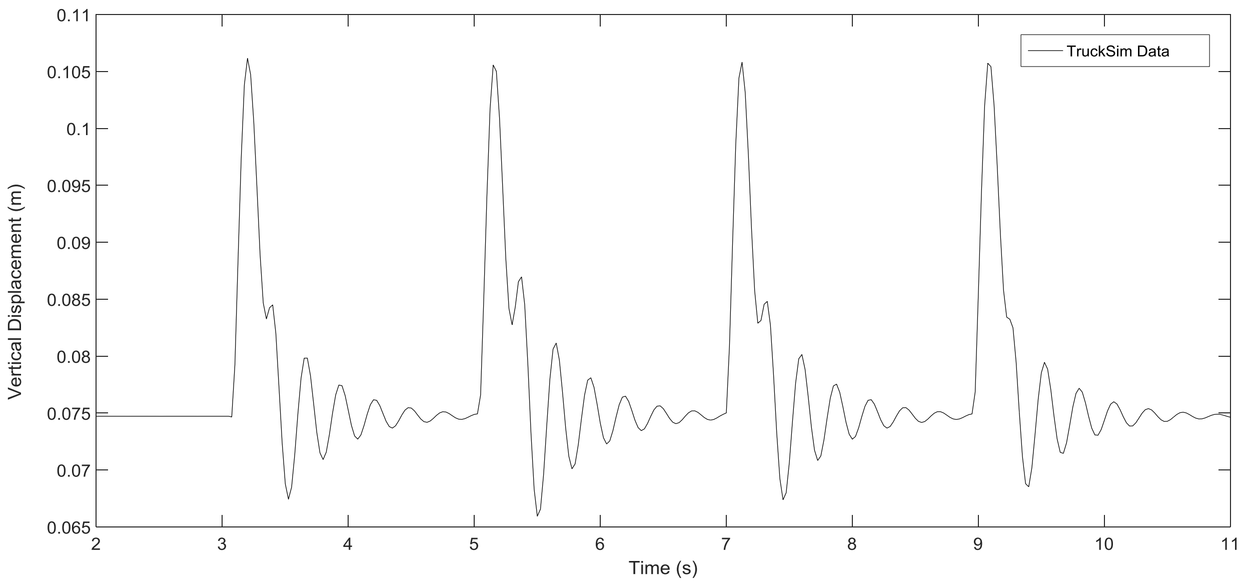


Figure 4.7: Heave Graph in TruckSIM

The vehicle moves with the speed of 0.7 m/s on the road for about 10 seconds. The vertical axis represents the vertical displacement in meters and the horizontal axis represents the time in seconds. Figure 13 shows that when the effect of disturbance caused by the road lasts between 3 to 5 seconds for axle 1, 5 to 7 seconds for axle 2, 7 to 9 seconds for axle 3 and 9 to 11 seconds for axle 4. Vehicles heave motion is between 0.067 and 0.107 meters.

4.3.2 Pitch

Motion about the lateral axis is called the pitch and it is a measure of how far a vehicle is titled up or down [21]. When the vehicle moves on the bump the weight of the vehicle is transferred forward onto the front wheels and body of the vehicle will lean forward. Figure 4.8 shows the behavior of the vehicle's pitch while passing on the bump.

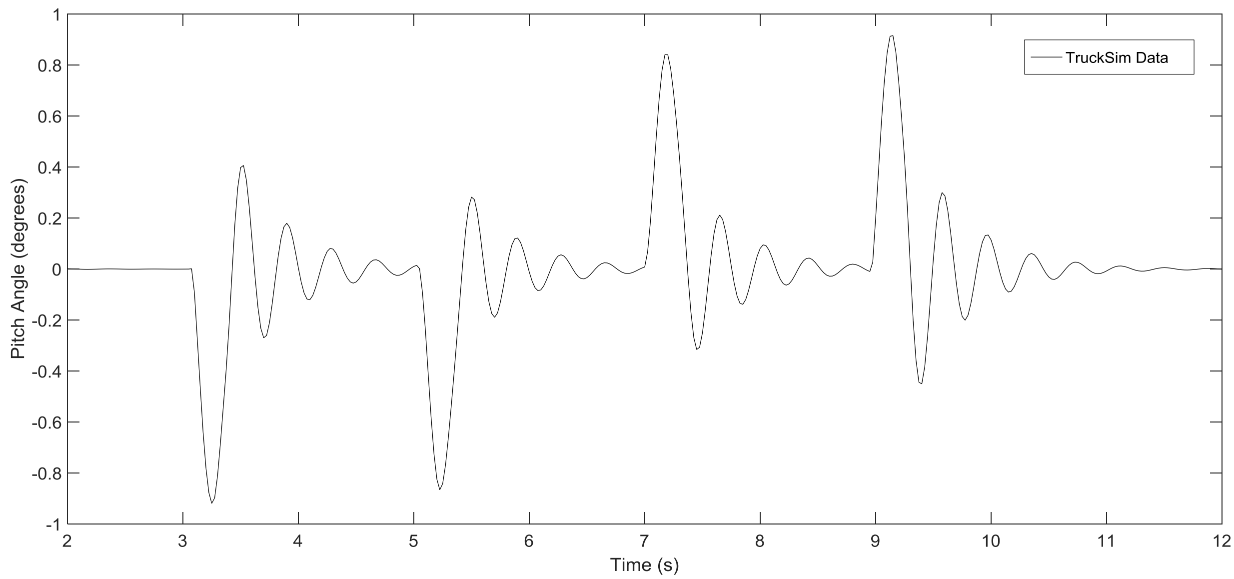


Figure 4.8: Pitch Graph in TruckSIM

The perpendicular axis denotes the pitch angle in degrees and the parallel axis denotes the interval in seconds. At time 3 seconds the 1st axle moves on the bump and pitching motion is induced and vehicle leans forward. Vehicle leans back at 7 seconds transferring the weight on the rear axles. Vehicles pitching motion is between -0.9 and +0.9 degrees.

4.3.3 Roll

Roll is the movement of the vehicle's body that defines how well a vehicle can distribute its weight while turning. Roll is shifting of the weight of the vehicle to the right or left [22]. As in this case, the vehicle is moving on a straight road with zero turning so there is no roll in the structure. Figure 4.9 displays small disturbances in the structure. The perpendicular axis denotes the roll angle in degrees and the parallel axis denotes the interval in seconds.

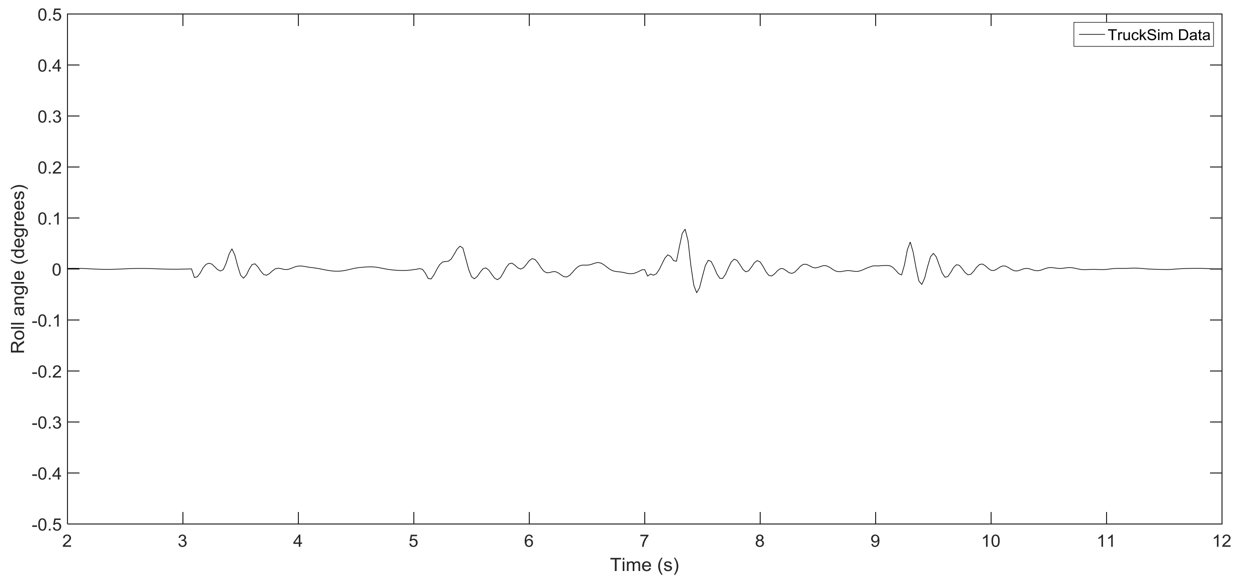


Figure 4.9: Roll in TruckSIM

4.4 Verification

Results from MATLAB Simulink and TruckSIM are compared in MATLAB to analyze the behavior of heave, pitch, and roll in both software.

4.4.1 Heave

The center of gravity (CG) displacement to the ground on a road is called heave or bounce. Figure 4.10 shows the amplitude differences of heave in TruckSIM and MATLAB Simulink. The dotted line on the graph represents the TruckSIM data and the full line represents the MATLAB Simulink data. It can be seen from the graphs that the behavior of heave is same in both software and there is a difference of approximately 4mm. This difference is due to few parameters which are considered in TruckSIM but not in Simulink. The graph shows that each axle in both software experiences the road disturbance at the almost same time.

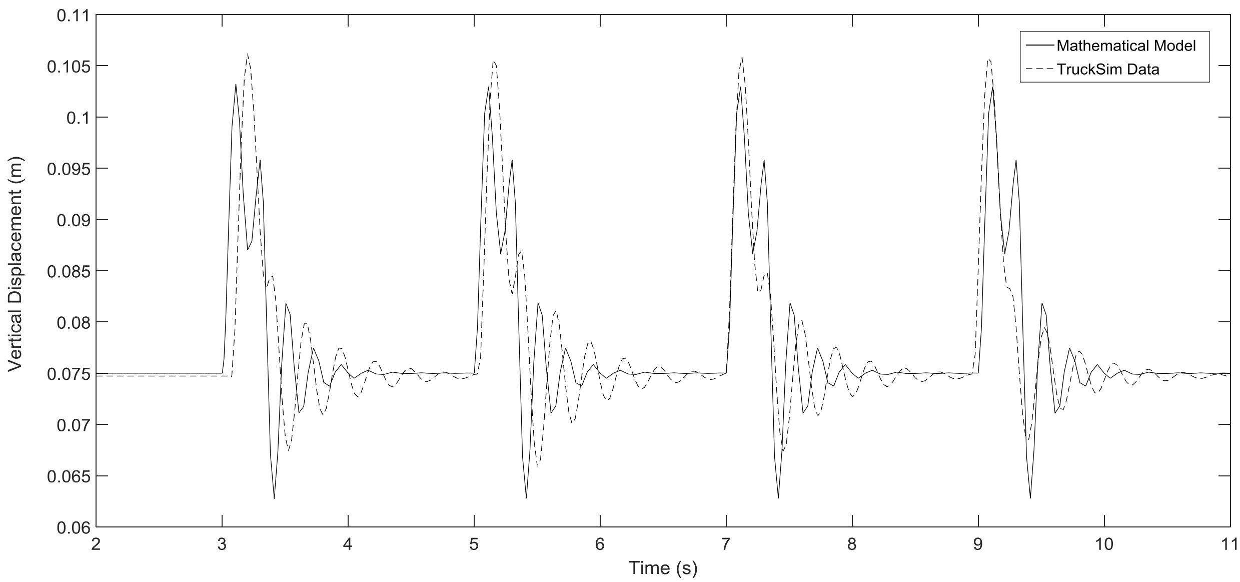


Figure 4.10: Comparison Graph of Heave

4.4.2 Pitch

Motion about the lateral axis is called the pitch and it is a measure of how far a vehicle is titled up or down [21]. When the vehicle moves on the bump the weight of the vehicle is transferred forward onto the front wheels and body of the vehicle will lean forward. Figure 4.11 shows the behavior of vehicle's pitch in TruckSIM and MATLAB Simulink while passing on the bump. The perpendicular axis denotes the pitch angle in degrees and the parallel axis represents the interval in seconds. The dotted line on the graph represents the TruckSIM data and the full line represents the MATLAB Simulink data. It can be shown from the figure that there is a small difference of 0.1 degrees in both results. The trend of the pitch is the same in both software.

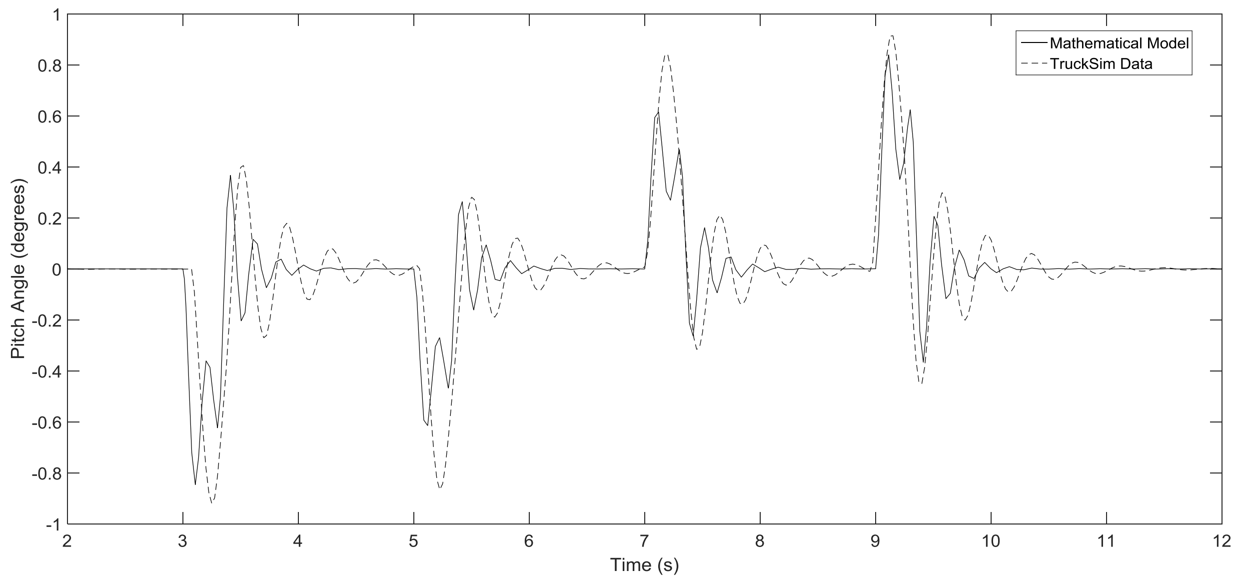


Figure 4.11: Comparison Graph of Pitch

4.4.3 Roll

Roll is the movement of a vehicle's body that defines how well a vehicle can distribute its weight while turning. Roll is shifting of the weight of the vehicle to the right or left [22]. As in this case, the vehicle is moving on a straight road with zero turning so there is no roll in the system. Figure 4.12 shows the comparison of roll angle in TruckSIM and MATLAB Simulink. The perpendicular axis denotes the roll angle in degrees and the parallel axis denotes the interval in seconds. The dotted line represents the TruckSIM data and the full line represents the MATLAB Simulink data.

TruckSIM considers few parameters which are not considered in the mathematical model. Due to these parameters, there is a very small roll angle variation. Largest amplitude of roll angle is less than 0.1 degrees.

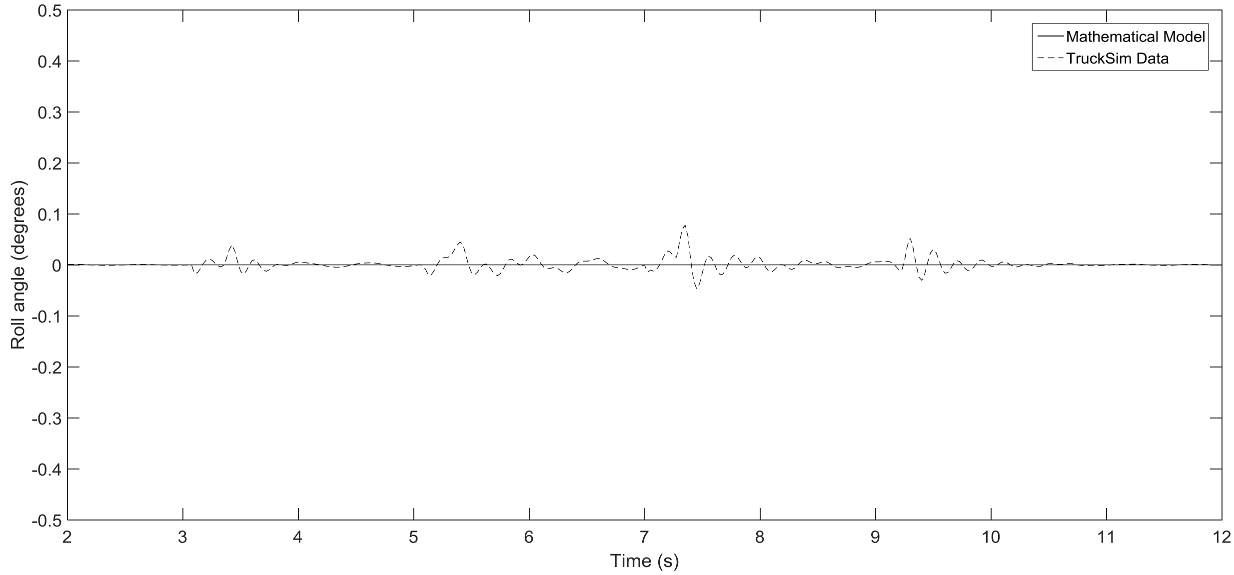


Figure 4.12: Comparison Graph of Roll

CHAPTER 5: CONCLUSION AND RECOMMENDATIONS

5.1 Conclusion

Analysis of independent suspension system of multi-axle heavy vehicles is the main focus of this research. Different suspension system analyses were fully understood to characterize the system dynamics and behavior based on simulations between a vehicle and a road condition. Full vehicle mathematical model is developed to know the automobile dynamics, following the classification of the modal factors of the automobile. Validated model for independent suspension system is proposed in this thesis for n number of axles. The mathematical model for 2 axle vehicle is available in literature but such an extensive study for 6 axle vehicle with the validated model in Simulink is not available. These types of models are needed for designing a vehicle and simulators. The mathematical model has been developed for multi-axle vehicle suspension system and simulation results are verified. The parametric study was done to generate the simulation results. By changing the different parameters behavior of pitch, heave and roll changes is analyzed. The reaction of the automobile has been evaluated by using an 11-DOF modal. Complete automobile model is currently ready to be used for more innovative analysis of the suspension of the vehicle.

5.2 Recommendations

- The mathematical model for lateral dynamics of a multi-axle vehicle
- Comparison of mathematical model with experimental data of a fully instrumented vehicle
- Non-linear model
- Model can be used for controller design, variable suspension and variable tire models.

REFERENCES

[1] Lajqi, S., Pehan, S., Lajqi, N., Gjelaj, A., Psenicnik, J., & Emin, S. (2013). Design of independent suspension mechanism for a terrain vehicle with four wheels drive and four wheels steering. *Annals of the Faculty of Engineering Hunedoara*, *11*(1), 101.

[2] Campos, J., Davis, L., Lewis, F. L., Ikenaga, S., Scully, S., & Evans, M. (1999, June). Active suspension control of ground vehicle heave and pitch motions. In *Proceedings of the 7th IEEE Mediterranean Control Conference on Control and Automation* (pp. 222-233).

[3] D N Siddhartha Jain, Ratnesh Kumar: Kinematic and dynamic analysis of independent suspension system. Manipal Institute of Technology India.

[4] Deo, H. V., & Suh, N. P. (2004, June). Axiomatic design of customizable automotive suspension. In *Anais do ICAD2004-the 3rd International Conference on Axiomatic Design*.

[5] Trikande, M. W., Jagirdar, V. V., & Sujithkumar, M. (2014). Evaluation of semi-active suspension control strategies for 8x8 armoured vehicle using stochastic road profile inputs. *IFAC Proceedings Volumes*, *47*(1), 941-948.

[6] Jagtap, K., Rathod, Y., Shedge, A., Gramopadhye, M., & Diware, V. (2016). SUSPENSION SYSTEM FOR AN ALL-TERRAIN VEHICLE: A REVIEW. *International Journal of Engineering Research and General Science*, *4*(3).

[7] Hamed, M. (2016). *Characterisation of the Dynamics of an Automotive Suspension System for On-line Condition Monitoring* (Doctoral dissertation, University of Huddersfield).

[8] Liu, Y. (2008). Recent innovations in vehicle suspension systems. *Recent Patents on Mechanical Engineering*, *1*(3), 206-210.

[9] Hokale, S., & Patil, V. R. The Adoptive Control on Ground Clearance of Vehicle with Variable Speed: A.

[10] Aly, A. A., & Salem, F. A. (2013). Vehicle suspension systems control: a review. *International journal of control, automation and systems*, *2*(2), 46-54.

[11] Talukdar, S., Mazumdar, A., Mullasseril, M., Kalita, K., & Ujjwal, A. (2012). *Mathematical modeling in vehicle ride dynamics* (No. 2012-01-0056). SAE Technical Paper.

[12] Ashtekar, J. B., & Thakur, A. G. (2014). Simulink model of suspension system and it's validation on suspension test rig. *International journal of mechanical engineering and robotics research*, 2278-0149.

[13] Gandhi, P., Adarsh, S., & Ramachandran, K. I. (2017). Performance Analysis of Half Car Suspension Model with 4 DOF using PID, LQR, FUZZY and ANFIS Controllers. *Procedia Computer Science*, *115*, 2-13.

[14] Başlamişli, S. C., & Ünlüsoy, Y. S. (2009). Optimization of speed control hump profiles. *Journal of Transportation Engineering*, *135*(5), 260-269.

[15] Darus, R., & Sam, Y. M. (2009, March). Modeling and control active suspension system for a full car model. In *Signal Processing & Its Applications, 2009. CSPA 2009. 5th International Colloquium on* (pp. 13-18). IEEE.

[16] Rideout, D. G. (2012). Simulating Coupled Longitudinal, Pitch and Bounce Dynamics of Trucks with Flexible Frames. *Modern Mechanical Engineering*, *2*(04), 176.

[17] Sprung and unsprung mass image Retrieved from https://en.wikipedia.org/wiki/Unsprung_mass

[18] Semi-independent strut image Retrieved from https://slideplayer.com/slide/5793945/

[19] Pitch Retrieved from http://www.machinedesign.com/engineering-essentials/what-s-difference-between-pitch-roll-and-yaw

[20] Roll Retrieved from https://carsexplained.wordpress.com/2017/02/21/fundamentals-of-car-science-pitch-and-roll/

Completion Certificate

It is to certify that the thesis titled "Independent Suspension System, Modeling and Analysis of Multi-axle Vehicle" submitted by registration no. NUST201464486MCEME35114F, NS Sana Fatima of MS-80 Mechanical Engineering is complete in all respects as per the requirements of Main Office, NUST (Exam branch).

Supervisor: ______________
Dr. Raja Amer Azim

Date: ____ August, 2018