ATTITUDE CONTROL OF VTOL-UAVS

Submitted by:

Maryam Heidarian

Supervised by:

Cdr. Dr. Attaullah Memon

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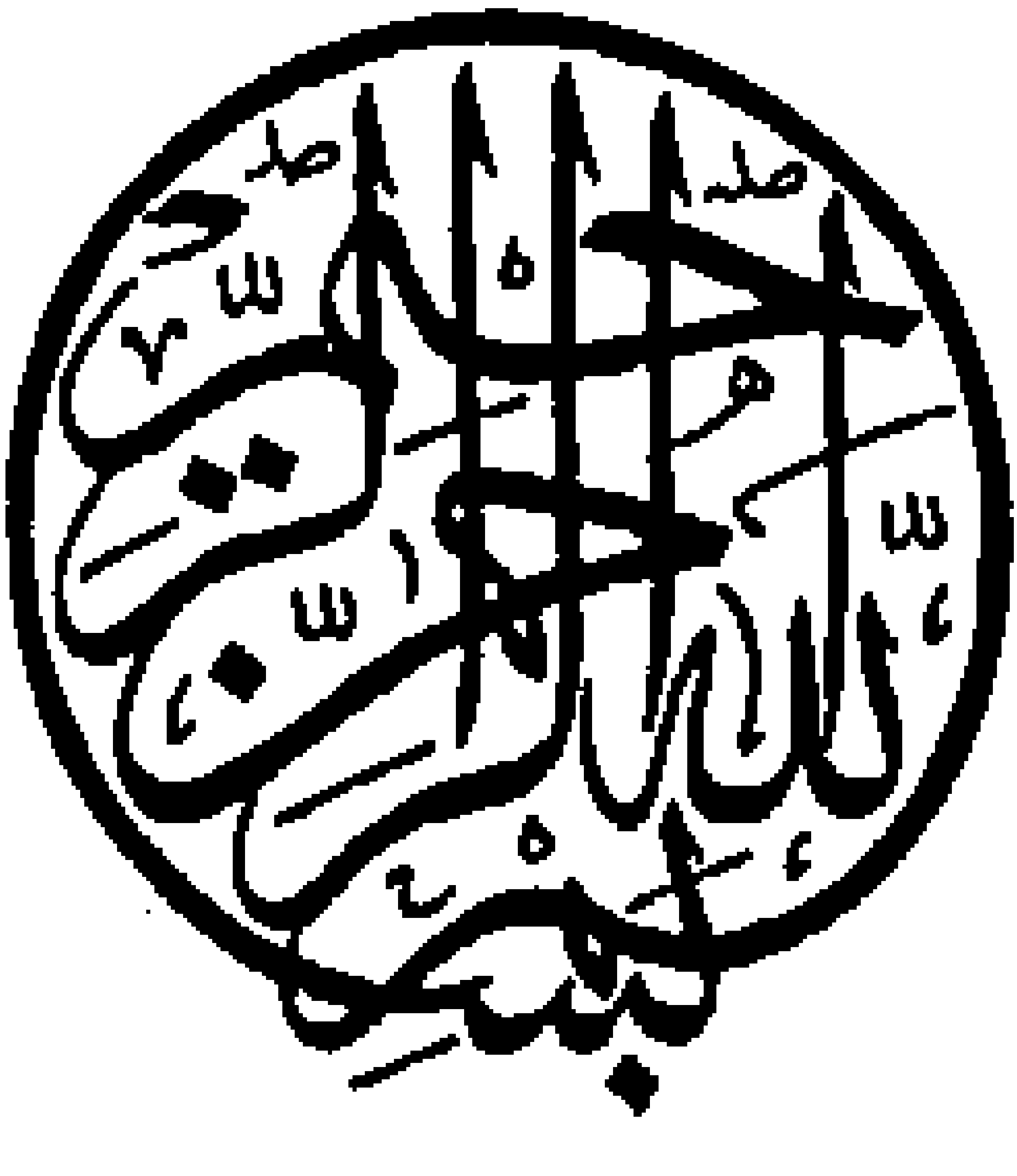
بسم الله الرحمن الرحيم

National University of Sciences Technology, Pakistan

Pakistan Navy Engineering College, Karachi

Title of Thesis:

Attitude Control of VTOL-UAVs

submitted by:

Maryam Heidarian (MSEE Control)

supervised by:

Cdr. Dr. Attaullah Memon PN

Assistant Professor

Guidance and Examination Committee:

Capt. Dr. M Junaid Khan PN

Assistant Professor

Cdr. Dr. M Farhan PN

Assistant Professor

Dr. M Bilal Kadri

Assistant Professor

In the memory of my grandmother

Khojasteh Mahdipour

1936-2013

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ABSTRACT

VTOL-UAVs are Unmanned Aerial Vehicles which have the ability of Vertical Take Off and Landing and the capability of fast target acquisition. Due to this, their applications have shown a growing interest in performing certain tasks which require high maneuverability and robustness with respect to unknown external disturbances. Use of VTOL-UAVs has been envisaged in a variety of applications in environmental protection, intervention in hostile sites, natural risk management, remote inspections, rescue missions, agriculture and film production.

The particular concern of the researchers' society in these miniature aerial vehicles has been related to two main comparable VTOL-UAVs, helicopters and quad copters (quad rotors). We consider a new control scheme to obtain asymptotic attitude stability of a modeled quad rotor as a representative of VTOL aerial vehicles. The modeled quad rotor is a symmetric VTOL-UAV with four rigid mono-directional propellers, which has been modeled based on quaternion representation with taking Coriolis and gyroscopic torques into account. The quaternion representation is one of the most successful methods which can describe the rotation of the aerial vehicle about the fixed axis of reference. In order to successfully control a VTOL-UAV, a composite control scheme comprising of two different controllers is required, namely: the attitude controller and the position controller. Design of these two controllers constitutes challenging tasks and the same have been addressed separately in the literature.

The most simple and practical attitude stabilization approach is based on measurement of angular velocity, in order to find quad rotor's angular rotations. Therefore rigorous and optimized design of robust attitude controller is essentially required in order to deal with dynamical inaccuracies. Our algorithm to stabilize attitude control of quad rotor is twofold: firstly, designing a control torque which can stabilize the

attitude of the quad rotor dynamically and secondly, determining the rotor torque to obtain the designed control torque, while considering that the real dynamical input to the quad rotor is angular speed of rotors.

The main contribution of this thesis is to propose a novel attitude stabilization control scheme which can improve and simplify the robust attitude controller. In the proposed approach, two nearly equivalent PD^2 control laws (model independent as well as model dependent) have been used to obtain exponential stability of attitude angles and asymptotic stability of attitude angular velocity of the modeled quad rotor. The proposed control design has been tested using simulations and yields good performance under prescribed uncertainties and disturbances.

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KEY TO SYMBOLS AND ABBREVIATIONS

VTOL	Vertical Take Off and Landing
UAV	Unmanned Aerial Vehicle
PVTOL	Planar Vertical Take Off and Landing
GAS	Globally Asymptotically Stable
GES	Globally Exponentially Stable
PD	Proportional Derivative
PD^2	Proportional Derivative Derivative
ISS	Input-to-State Stable

Chapter 1

INTRODUCTION

Flight control of VTOL-UAVs is an area that poses interesting problems for control researchers. The classical control strategy for these vehicles assumes a linear model obtained for a particular operating point. Applying modern nonlinear control theory can improve the performance of the controller and enable the tracking of aggressive trajectories.

1.1 VTOL

The main idea of vertical flight aircrafts is going back to early Chinese tops, a toy first has been used around 400 BC. The earliest version of these tops consisted of feathers which were connected to a stick. The stick has been whirled so fast between two hands to generate lift and then released into free flight. 2000 years later, Mikhail Lomonsov, developed a small coaxial rotor modeled based on the Chinese top but powered by a wound-up spring device. This vehicle could fly freely and climb to a good altitude.

Launoy and Bienvenu, In 1783, used a coaxial version of the Chinese top in a model consisting of a counter rotating set of feathers powered by a string wound around the

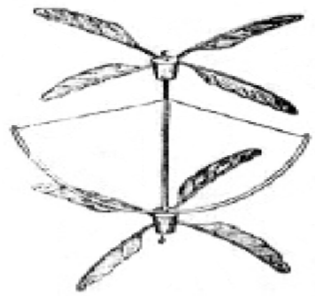

Figure 1.1: Sketch of Launoy and Bienvenu's model

rotor shaft and tensioned by a crossbow. When the tension was released, the blades spun and the aircraft ascended into the air (see Figure 1.1).

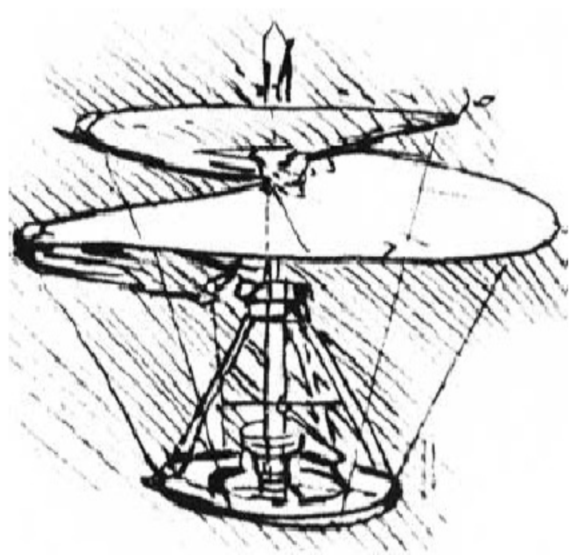

Figure 1.2: Sketch of Da Vinci's screw

Leonardo Da Vinci also has designed a sophisticated human-carrying helicopter-like vehicle, "aerial-screw" or "air gyroscope " aircraft, which is dated in 1483 but was first published nearly three centuries later. His proposed model included a helical surface formed out of iron wire, with linen surfaces made "airtight with starch ". Da Vinci describes that the machine should be "rotated with speed that said screw bores through the air and climbs high ". (See Figure 1.2)

Da Vinci clearly did not build his machine. If he had, he would have discovered the omission of any means to counteract the torque generated by the rotation of screw.

1.2 UNMANNED AERIAL VEHICLES

It is more than two decades since the first VTOL-UAVs have been fully demonstrated in practical researches. VTOL-UAVs are Unmanned Aerial Vehicles which have the ability of Vertical Take Off and Landing and the capability of fast target acquisition. Due to this, their applications have shown a growing interest in performing certain tasks which require high maneuverability and robustness with respect to unknown external disturbances.

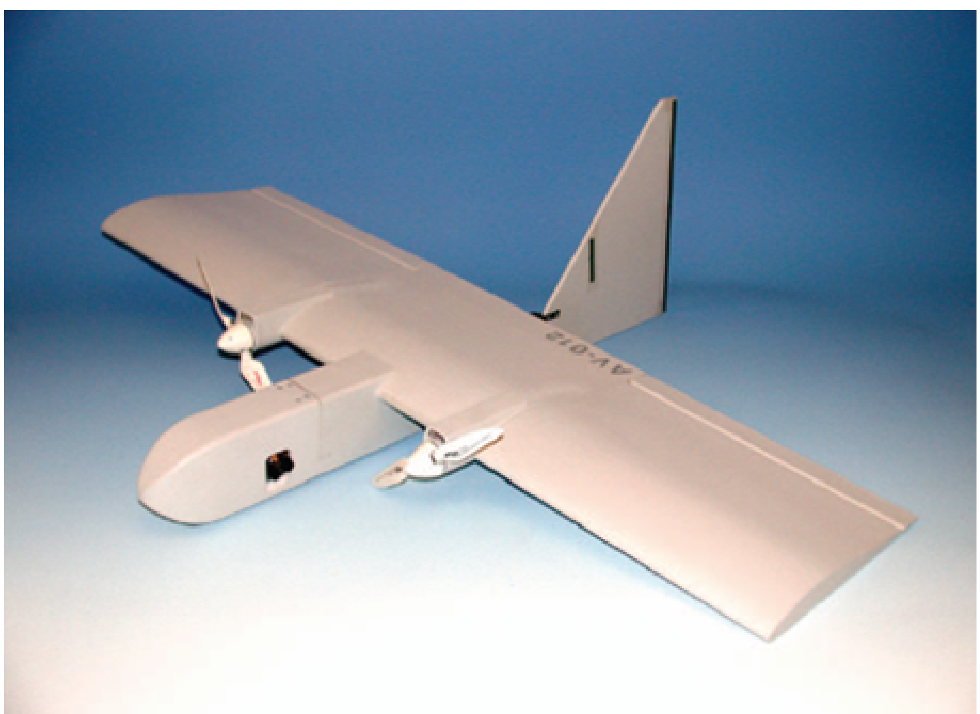

Figure 1.3: Dragon Eye UAV [1]

Basically UAV (Unmanned Aerial Vehicle) is a remotely operated device in which the position in space is controlled through a remote-control system while the horizontal speed will be measured via on board cameras. Attitude of these vehicles will be controlled through on board automatic controllers which provide required orientations for safe operators' desired maneuvers. Figure 1.3 shows a sample UAV.

1.3 ADVANTAGES OF VTOL-UAVs

Since VTOL-UAVs can take off and land vertically, they have certain advantages in missions where there is a space limitation or a fast reaction is required. These properties mostly are required for surveillance and inspection tasks. VTOL-UAVs also can be used both as individual vehicles and in multiple vehicle teams [2].

Recently, the particular concern of the researchers' society in these miniature aerial vehicles has been related to two main comparable VTOL-UAVs:

- helicopters

  A classical helicopter consists of a main rotor and a tail rotor.

- quad copters (quad rotors)

  A quad rotor is a helicopter which has four propellers in cross configuration. Figure 1.4 shows general view of a quad rotor.

There are several advantages to quad rotors over comparably-scaled helicopters:

First, quad rotors do not require mechanical linkages to vary the rotor blade pitch

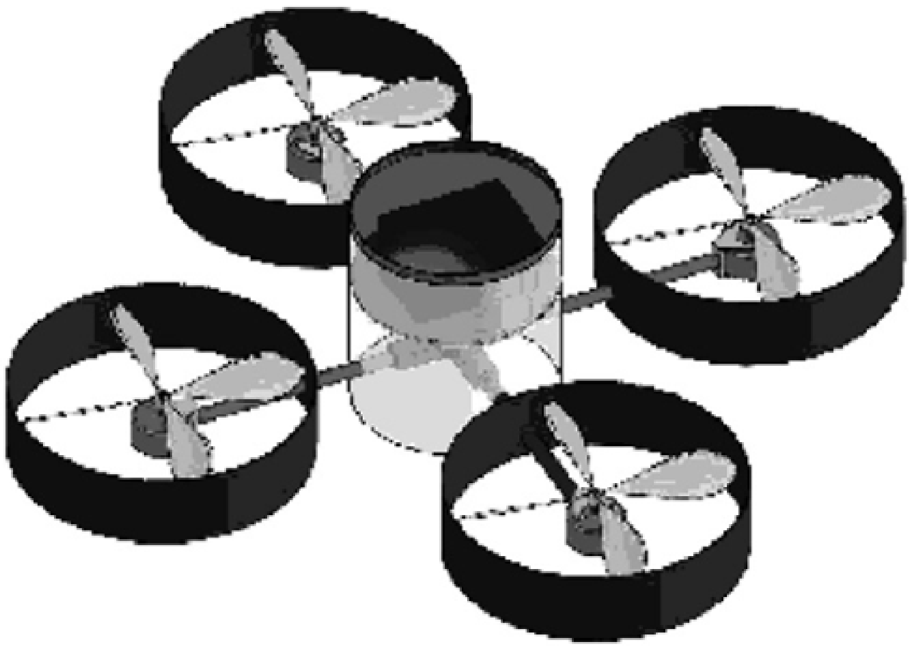

Figure 1.4: General view of a quad rotor

angle as they spin. This simplifies the design and maintenance of the vehicle.

Second, existence of these four, shows that every individual of them has to be very smaller than one main rotor on helicopter with same dynamical size. This property guarantees less kinetic energy storage during flight. This reduces the damage caused by the rotors in case of hitting anything and also shows that a quad rotor is mechanically less complicated in design and maintenance, than a classical helicopter. For small-scale UAVs, this makes the vehicles safer for close interaction. Some small-scale quad rotors have frames that enclose the rotors, permitting flights through more challenging environments, with lower risk of damaging the vehicle or its surroundings.

1.4 QUAD ROTOR

Quad rotor concept is not new. In 1907, about four years after the Wright brothers' first successful powered flights in fixed-wing airplanes at Kitty Hawk in the United States, Paul Cornu constructed a vertical flight vehicle which was reported to have carried a human off the ground for the first time. It is reported this vehicle had a few, short time flights with thirty centimeters altitude, which has never been satisfactorily verified. The 22 hp gasoline motor used in this vehicle was hardly powerful enough to have sustained effective hovering flight out of ground.

In 1907, the Breguet Brothers built first helicopter, Gyroplane No.1. It was a quad rotor powered by a 40 hp motor. It is reported that, this vehicle have carried a pilot off the ground briefly. This vehicle also like the Cornu aircraft, never flew completely freely because of its lacked stability and proper means of control.

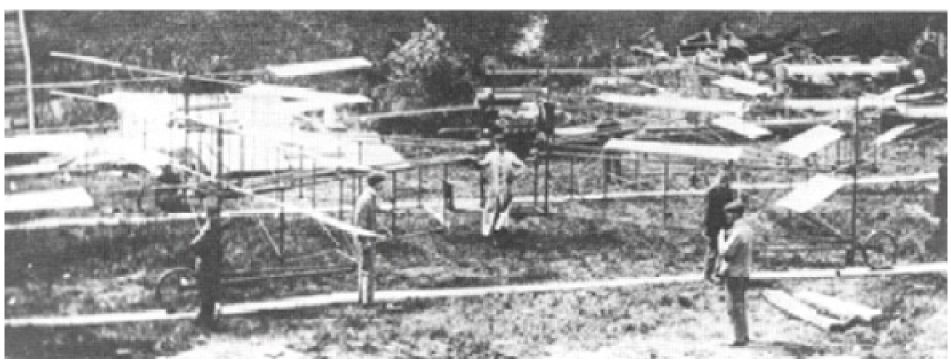

Figure 1.5: L. and J. Breguet tested the first four-rotor helicopter called Gyroplane No.1 in 1907

The Peugeot Engineer, Etienne Oemichen, designed a quad rotor and first flew his 800kg Oemichen No.2 in 1922. As well as the four main rotors, it featured five additional rotors for lateral stability, two to control forward movement and one at the nose for steering. On 4 may 1924, it became the first rotorcraft to complete the

1 km closed circuit flight with an average speed of 2.2 m/s. Oemichen's had four subsequent models, all featuring a single main rotor with two tail rotors, none were successful.

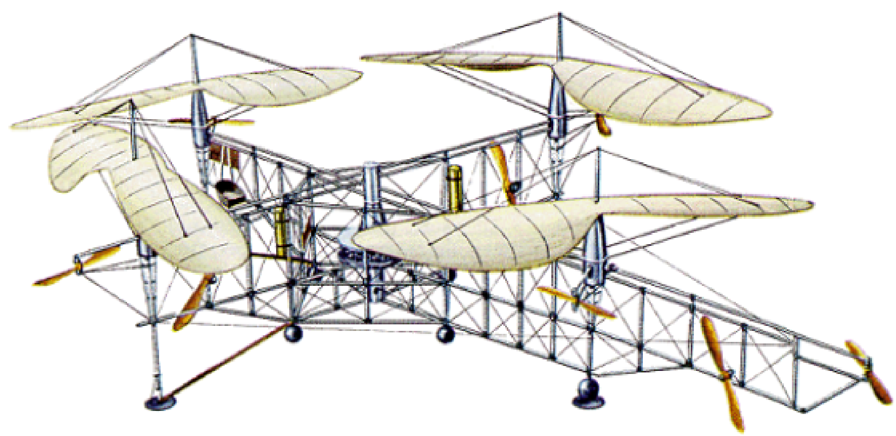

Figure 1.6: Oemichen No.2, the first aircraft to successfully complete an enclosed 1 km circuit

Even though the concept of the quad rotor helicopter is simpler than the traditional cyclic and collective pitch helicopter; it was not further developed until the introduction of a control system to control the helicopter.

The new generation quad rotors mostly have become unmanned. Due to the availability of high speed brush less motors, inertial measurement units based on MEMS technology and high power to weight ratio ($> 150W/Kg$), Li-polymer batteries, unmanned quad rotors can now be designed and fabricated [3] but their control is still a challenge.

1.5 APPLICATIONS

VTOL-UAVs has been envisaged in a variety of applications in:

> environmental protection, intervention in hostile sites, natural risk management, remote inspections, rescue missions, agriculture, film production and many more.

1.6 FUNDAMENTAL ISSUES

VTOL-UAVs are dynamically unstable, even with improved control laws an experienced operator is always required to remotely control these vehicles on space. In order to successfully control a VTOL-UAV, a composite control scheme comprising of two different controllers is required, namely:

- the attitude controller
- the position controller

Designs of these two controllers constitute challenging tasks and mostly have been concerned separately in the literature. The quad rotor control problem is same as control problem of VTOL-UAVs.

1.7 REVIEW OF PREVIOUS WORKS

In this section we will discuss the literature review of work done in the related field. In fact many books and research papers were consulted but here only few examples of research papers have been discussed briefly.

In 2002, a dynamical model of quad rotor based on quaternion representation has been derived from Newton-Euler equations [4].
Hamel et al., in [5], identify dynamics of the vehicle beyond the basic nonlinear equations of motion, with gyroscopic torque and Coriolis terms.
Based on the proposed model in [5], Tayebi and Mc Gilvray [6] have represented a model independent PD controller with asymptotic stability and a model dependent PD^2 controller with exponential stability. Good performance of controllers in both simulation and real time has been proven.
A method to obtain attitude control stabilization of a quad rotor through using backstepping technique and adding saturation functions has been analyzed in [7].

Precise measurement of the angular velocity and the initial orientation of the planer is required for attitude stabilization of these vehicles. Due to various mechanical uncertainties (related to gyroscope), there may be some errors in these measurements. Using inertial measurements units (IMU's) information to estimate these required values is one of the possible ways to reduce the errors which has been used in [8].
In the more recent work, [9] has considered control designs that do not necessarily require exact knowledge of the angular velocity of the quad rotor UAV.

Still the most simple and practical attitude stabilization approach is based on measurement of angular velocity, in order to find quad rotor's angular rotations, especially in case of small movements. Therefore rigorous and optimized design of robust attitude controller is essentially required in order to deal with dynamical inaccuracies.

The main contribution of this paper is to propose a novel attitude stabilization control scheme which can improve and simplify the robust attitude controller. Despite that only attitude control of a quad rotor is not adequate to make a real aerial maneuver, study of this problem may improve design's difficulties.

1.8 CHAPTERS ORGANIZATION

The rest of this report is organized as follows:

Chapter 2

Chapter 2 states the background materials and problem formulation which presents the necessary mathematical foundation for our design strategy.

Chapter 3

The control design and its main results have been presented in this chapter in detail.

Chapter 4

Chapter 4 shows the simulation results of the proposed control laws.

Chapter 5

And finally this chapter concludes the thesis work and contains the future recommendations.

Chapter 2

ATTITUDE CONTROL of QUAD ROTORS

The quad rotor control problem is similar to that of controlling a planar vertical take off and landing (PVTOL) aircraft, which evolves in a vertical plane [10]-[12]. In view of quad rotor's configuration, the quad rotor shares some similarities with the PVTOL aircraft. Indeed, if the roll, pitch and yaw angles are set to zero, the quad rotor reduces to a PVTOL and can be viewed as two PVTOLs connected with orthogonal axes [13].

2.1 INTRODUCTION TO QUAD ROTORS

A quad rotor simply consists of four lift generating propellers mounted on motors. These motors have been located at the lateral sides of a cross shaped frame with an angle of 90 degrees between the arms. Center of gravity (a point which is the average location of the mass of the aircraft) is placed at the intersection of the line joining rotors 1 and 3 and the line joining rotors 2 and 4 which is middle of the connecting links. Quad rotor is assumed to be symmetric as can be appreciated in Figure 2.1.

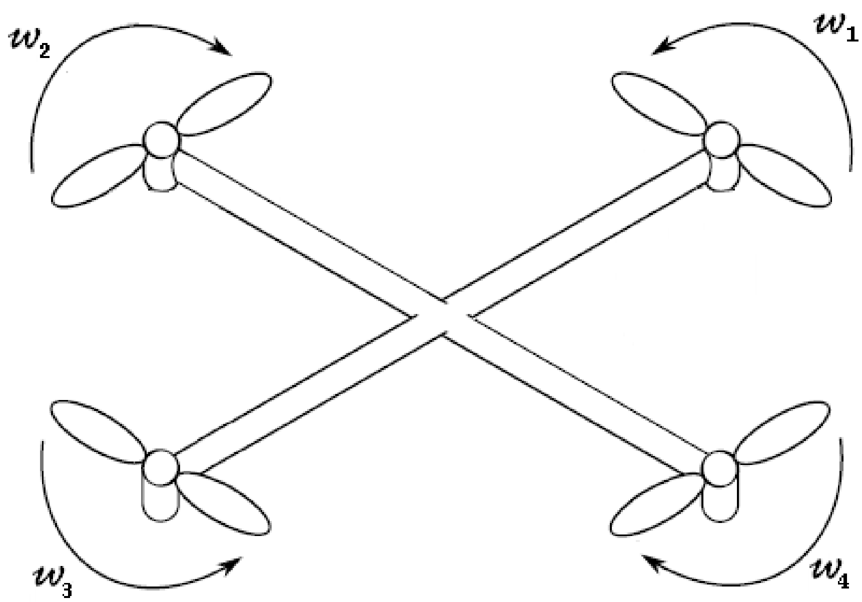

Figure 2.1: A symmetric quad rotor

Control of these vehicles has been succeeded by varying the angular speed of rotors, $w_i,\ i \in 1, 2, 3, 4$.

In flight dynamics, there are three very important parameters which are relative to the orientation of the air vehicle. These three have been illustrated in Figure 2.2 and are as below:

- Roll movement

  Roll motion is an up and down movement of the lateral sides of the quad rotor. It changes the direction that lateral sides of quad rotor is facing, to upper or down of its previous location. In the other words, roll is when the quad rotor rotates about the front/back (X) axis.

- Pitch movement

  Pitch motion is an up or down movement of the front of the quad rotor. It changes the direction that quad rotor is facing, to upper or down of its direction

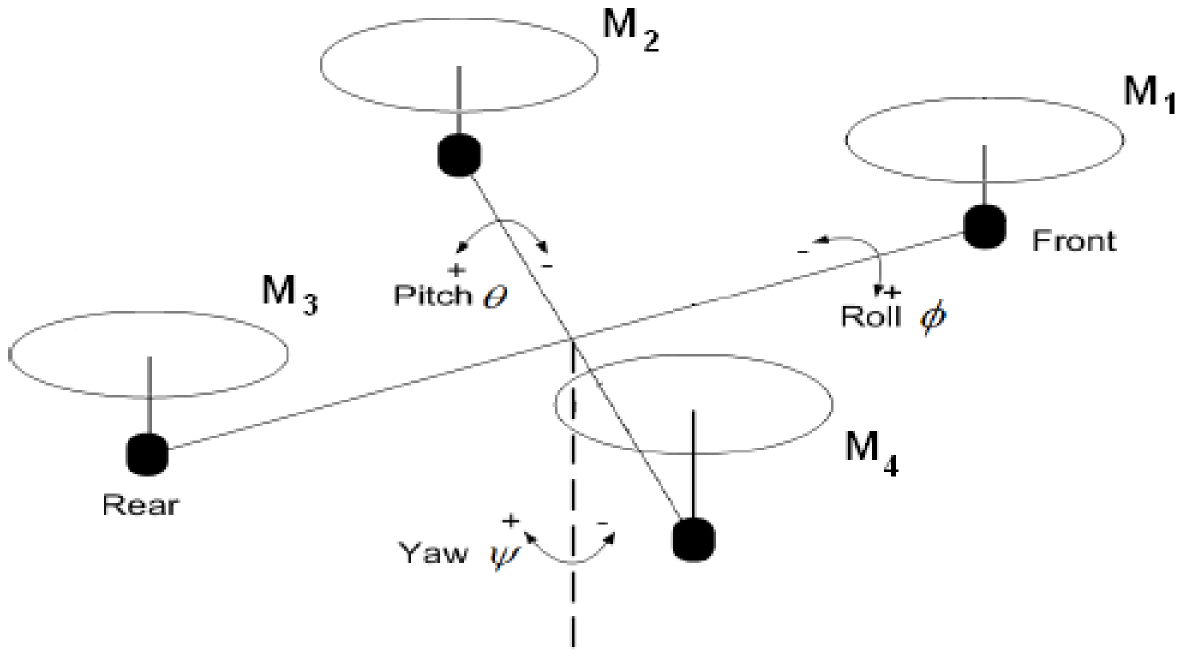

Figure 2.2: Aircrafts' movements

of motion. In the other words, pitch is when the quad rotor rotates about the side-to-side (Y) axis.

- Yaw movement

  Yaw motion is a movement of the front of the quad rotor from side to side. It changes the direction that quad rotor is facing, to the left or right of its direction of motion. In the other words, yaw is when the quad rotor rotates about the up-down (Z) axis.

As we can see in Figure 2.3:

A pitch movement is possible with increasing (reducing) the speed of the rear motor while reducing (increasing) the speed of the front motor. This means, the pitch torque is a function of the difference $w_1 - w_3$.

Similarly, roll movement is obtained by using the lateral motors. Thus, the roll torque will be a function of $w_2 - w_4$.

The yaw motion is also obtained by increasing (reducing) the speed of the front and rear motors together while reducing (increasing) the speed of the lateral motors together. This means, the yaw torque is a function of $w_1 + w_3 - w_2 - w_4$.

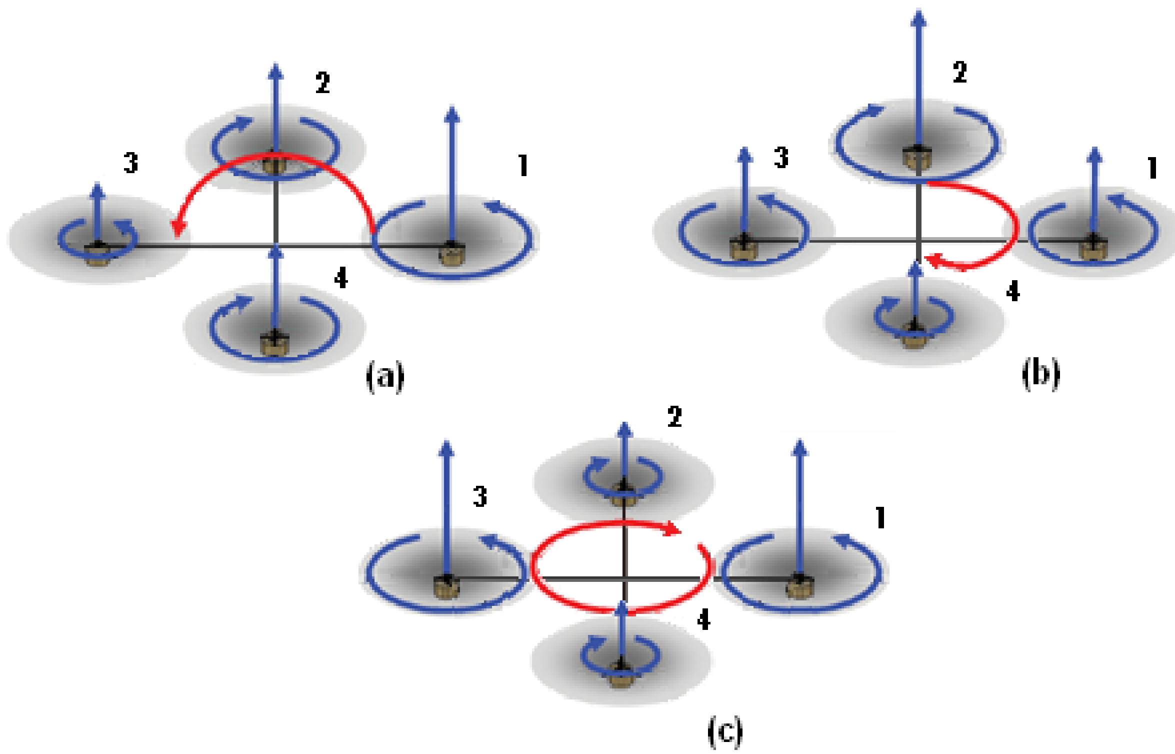

Figure 2.3: a) Pitch b) Roll c) Yaw movements

2.2 SYSTEM DESCRIPTION

Pitch, roll and yaw movements can be accomplished while keeping the total thrust, T constant. Also the vertical movement is generated by increasing the total thrust. The main thrust, with considering Figure 2.3, can be expressed by

$$T = \sum_{i=1}^{4} |f_i| \tag{2.1}$$

$$f_i = b\ w_i^2 \tag{2.2}$$

Where f_i is the vertically upward lifting force produced by related motor. As seen in Figure 2.1, the rotation direction of two of the rotors are clockwise while the other two are counterclockwise, this is in order to balance the movements and prevent the yaw drift caused by the reactive torques. We have shown the reactive torque of i_{th} rotor by:

$$Q_i = l\ w_i^2 \tag{2.3}$$

Since, each motor turns in a fixed direction, the produced force f_i is always positive. Thus:

$$T = b \sum_{i=1}^{4}\ w_i^2 \tag{2.4}$$

Where $b > 0$ and $l > 0$ are constants and are dependent on the density of air, size,

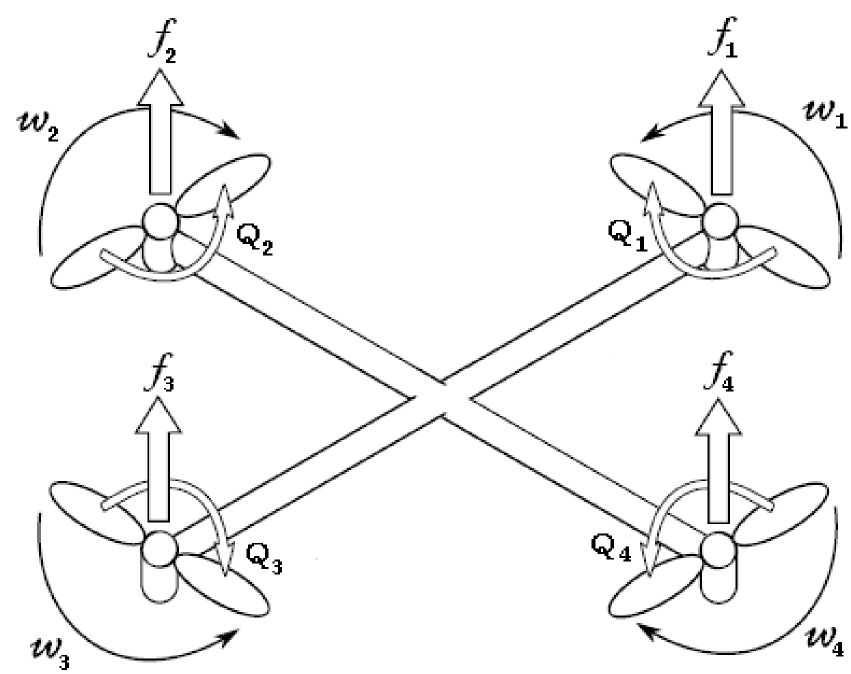

Figure 2.4: Illustration of forces

shape and pitch angle of the propellers and other factors [14].

The generalized torques i.e roll torque τ_ϕ, pitch torque τ_θ, yaw torque τ_ψ (Figure 2.2) have to be as below

$$\tau_\phi = db(\ w_2^2 -\ w_4^2) \tag{2.5}$$

$$\tau_\theta = db(\ w_1^2 -\ w_3^2) \tag{2.6}$$

$$\tau_\psi = l(\ w_1^2 +\ w_3^2 -\ w_2^2 -\ w_4^2) \tag{2.7}$$

Where d is the length of arms between the motors and the center of gravity.

2.2.1 REFERENCE AXIS

A good knowledge around the dynamical model is required to improve the performance of the aircraft. To discuss the quad rotor dynamics, it is necessary to set up a system of reference axes or coordinate system.

The vector additions' laws (essentially, commutativity) have not been followed by finite rotation vectors of rigid bodies, due to this unique dynamical problem, we cannot find the attitude of the aircraft from integrating the angular velocities. Thus, modeling of the quad rotor dynamically requires defining two reference frames:

- Initial frame

  Initial frame I is defined by the set of unit vectors $\{x_I, y_I, z_I\}$. This frame is an earth fixed coordinate frame with origin at the defined home location, as can be seen in Figure 2.4, the unit vector x_I is directed North, y_I is directed East, and z_I is directed upward the center of the earth.

- Body fixed-frame

  Body fixed-frame B with orthogonal axes is defined by the set of unit vectors

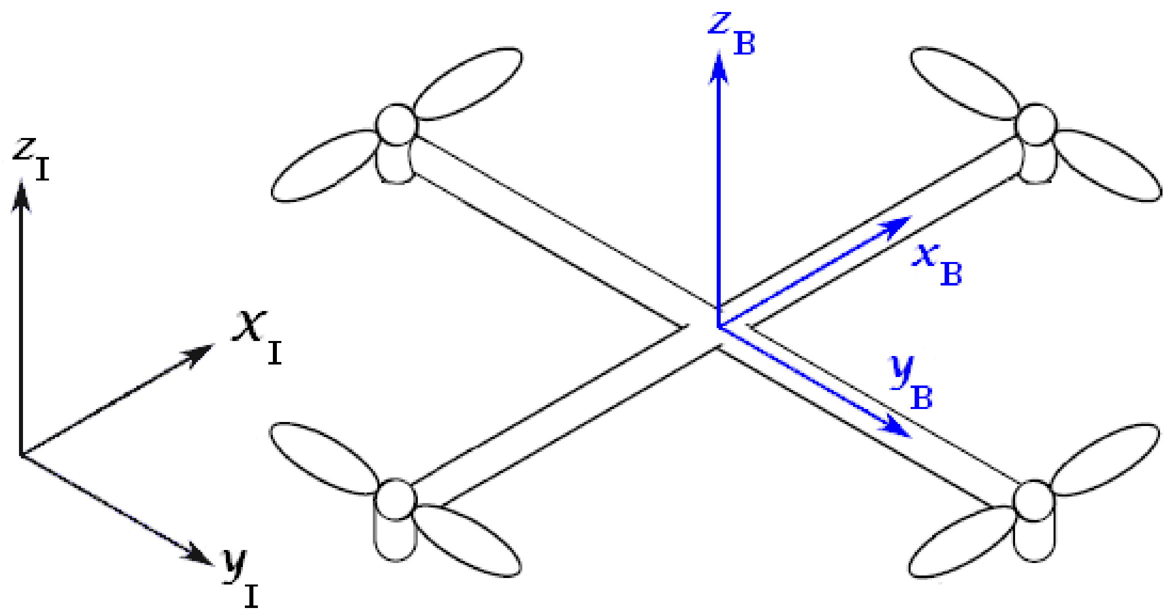

Figure 2.5: Reference frames

$\{x_B, y_B, z_B\}$, as can be seen in Figure 2.4. The origin of the vehicle frame is at the center of mass of the quad rotor. However the frame of B is aligned with the inertial frame of I. In the other words, the unit vector x_B point North, y_B point East and z_B points upward the center of the earth.

2.2.2 QUATERNION REPRESENTATION

In order to define the orientation of the aircraft between these two reference frames, one can use an Euler angle description [15], in which a 3×3 direction matrix will represent the rotation of the aircraft with respect to the body fixed-frame. This rotation matrix (R), is given by:

$$R = \begin{pmatrix} c_\theta c_\psi & c_\psi s_\theta s_\phi - s_\psi c_\phi & c_\psi s_\theta c_\phi + s_\psi s_\phi \\ c_\theta s_\psi & s_\psi s_\theta s_\phi + c_\psi c_\phi & s_\psi s_\theta c_\phi - c_\psi s_\phi \\ -s_\theta & s_\phi c_\theta & c_\phi c_\theta \end{pmatrix}$$

where $c_x \triangleq \cos x, s_x \triangleq \sin x, x\epsilon\{\phi, \theta, \psi\}$, with ϕ, θ and ψ denoting, respectively, the roll, the pitch and the yaw.

The Euler description has an inherent geometric singularity problem at $c_\theta = 0$. In order to overcome this problem, one can use quaternion representation [16]-[21], which defines the rotation of the aircraft with four parameters. The quaternion description is benefiting the Euler's theorem which states that any rotation of an aircraft can be described by a single rotation about affixed axis [15]. This globally nonsingular representation of the orientation, is given by vector $(q, q_0)^T$, with

$$q = \hat{k}\sin(\frac{\lambda}{2})$$

$$q_0 = \cos\frac{\lambda}{2}$$

where γ is the equivalent rotation angle about the axis described by the unit vector $\hat{k} = (\hat{k_1}, \hat{k_2}, \hat{k_3})^T$, subject to the constraint

$$q^T q + q_0^2 = 1$$

The rotation matrix R is related to the quaternion through the Rodrigues formula [15], [22] :

$$\begin{aligned} R &= I + 2q_0 s(q) + 2s(q)^2 \\ &= I + \sin\gamma s(\hat{k}) + (1 - \cos\gamma)s(\hat{k})^2 \end{aligned}$$

in which $s(x)$ is a skew-symmetric matrix, defined by:

$$s(x) = \begin{pmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{pmatrix}$$

An algorithm for the quaternion extraction is presented in [19]. In fact, q and q_0 are obtained from R, [23], as follows:

$$s(q) = \frac{1}{2\sqrt{1 + trR}}(R - R^T)$$

in which trR denotes the trace of the matrix R. The result is shown in the next section.

2.3 DYNAMICAL MODEL

The dynamical model of the quad rotor has been obtained via a Newton-Euler approach in [6]. The basic assumption is that

Quad rotor and its propellers are rigid.

External aerodynamic effects (air friction, wind pressure and ...) can be neglected.

A simplified model with consideration of coriolis and gyroscopic torques is given as below:

$$\dot{q} = \frac{1}{2}\begin{pmatrix} -(\bar{q})^T \\ s(\bar{q}) + q_0 I_{3\times3} \end{pmatrix}\Omega \tag{2.8}$$

$$I_f\dot{\Omega} = -s(\Omega)I_f\Omega - G_a + \tau_a \tag{2.9}$$

$$I_r\dot{w}_i = \tau_i - Q_i \qquad i\epsilon\{1, 2, 3, 4\} \tag{2.10}$$

q represents the quaternion equations which can be evaluated by:

$$q = \begin{pmatrix} q_0 \\ \bar{q} \end{pmatrix} = \begin{pmatrix} \cos\frac{\phi}{2}\cos\frac{\theta}{2}\cos\frac{\psi}{2} + \sin\frac{\phi}{2}\sin\frac{\theta}{2}\sin\frac{\psi}{2} \\ \sin\frac{\phi}{2}\cos\frac{\theta}{2}\cos\frac{\psi}{2} - \cos\frac{\phi}{2}\sin\frac{\theta}{2}\sin\frac{\psi}{2} \\ \cos\frac{\phi}{2}\sin\frac{\theta}{2}\cos\frac{\psi}{2} + \sin\frac{\phi}{2}\cos\frac{\theta}{2}\sin\frac{\psi}{2} \\ \cos\frac{\phi}{2}\cos\frac{\theta}{2}\sin\frac{\psi}{2} - \sin\frac{\phi}{2}\sin\frac{\theta}{2}\cos\frac{\psi}{2} \end{pmatrix} \tag{2.11}$$

The parameters ϕ, θ and ψ respectively represent the roll, pitch and yaw angular displacements about their related axes, and has been defined by the following relations [24]:

$$\dot{\phi} = \Omega_1 + (\Omega_2 \sin\phi + \Omega_3 \cos\phi)\tan\theta \tag{2.12}$$

$$\dot{\theta} = \Omega_2 \cos\phi - \Omega_3 \sin\phi \tag{2.13}$$

$$\dot{\psi} = (\Omega_2 \sin\phi + \Omega_3 \cos\phi)\sec\theta \tag{2.14}$$

$\Omega = (\Omega_1, \Omega_2, \Omega_3)^T$ describes the angular velocity of the quad rotor expressed in the body-fixed frame B.

Also $I_f \epsilon \mathbb{R}^{3\times 3}$ is a symmetric positive-definite constant inertia matrix of the airframe with respect to the previous frame.

G_a denotes the gyroscopic torques which is due to the combination of the rotation of the airframe and the four rotors, and can be shown as:

$$G_a = I_r s(\Omega) z_I (w_1 + w_3 - w_2 - w_4)$$

Where $z_I = (0,0,1)^T$ describes the unit vector in the frame I.

$\tau_a = (\tau_\phi, \tau_\theta, \tau_\varphi)^T$ shows the control torque.

I_r is a constant which represent moment of inertia of the rotor.

And τ_i , $i\epsilon\{1,2,3,4\}$ represents the rotors torque.

Note the term $-s(x)I_f\Omega$ in (2.9), is due to Coriolis torque.

2.4 CHAPTER SUMMARY

This chapter briefly discussed background material which is required to understand the concepts of attitude control of quad rotors as a representative of PVTOLs. It described forces and torques of quad rotors and their relationships. Then we modified a model for the dynamics of a symmetric quad rotor with four rigid mono-directional propellers, based on quaternion representation with taking Coriolis and gyroscopic torques into account.

Chapter 3

CONTROL STRATEGIES

In this chapter, we aim to design a control algorithm to stabilize the attitude of quad rotor with consideration of our proposed dynamical model in previous chapter.
The proposed algorithm to stabilize the attitude of quad rotor consists of two parts.

- First, we design the control torque (τ_a) that stabilizes the attitude of quad rotor dynamically.
- Then, we design a rotor torque (τ_i) to obtain the designed control torque (τ_a), while considering that the real dynamical input to the quad rotor is angular speed of rotors.

3.1 DESIGNING THE CONTROL TORQUE τ_a

The dynamical model of the quad rotor, as described in the last chapter, possesses a cascade structure, in which

τ_a controls Ω

and

Ω controls q

i.e. $(\tau_a \to \Omega \to q)$. This means:

$$\dot{\Omega} = g(\Omega, \tau_a)$$

$$\dot{q} = f(q, \Omega)$$

From (2.8), we have

$$\begin{aligned} \dot{q} &= f(q, \Omega) \\ &= F(\Omega) q \end{aligned} \tag{3.1}$$

$$F(\Omega) = \frac{1}{2} \begin{pmatrix} 0 & -\Omega_1 & -\Omega_2 & -\Omega_3 \\ \Omega_1 & 0 & -\Omega_3 & \Omega_2 \\ \Omega_2 & \Omega_3 & 0 & -\Omega_1 \\ \Omega_3 & -\Omega_2 & \Omega_1 & 0 \end{pmatrix} \tag{3.2}$$

Similarly from (2.9), we have

$$\begin{aligned} \dot{\Omega} &= g(\Omega, \tau_a) \\ &= I_f^{-1}(-s(\Omega) I_f \Omega - G_a + \tau_a) \end{aligned} \tag{3.3}$$

With this in mind, our goal is to find a suitable control law $\tau_a = H(q, \Omega)$. We achieve this objective through the following two steps:

- By finding desired angular velocity $\Omega_d = h(q)$ such that when Ω_d is given as input to (2.9), the solution to the nonlinear equation $\dot{q} = f(q, h(q))$ is asymptotically stable.

- By ensuring that the angular velocity Ω asymptotically tracks the desired angular velocity Ω_d, i.e. $\lim_{t \to \infty} (sup|\Omega - \Omega_d|) = 0$.

3.1.1 FINDING DESIRED ANGULAR VELOCITY

The desired angular velocity Ω_d, has to be chosen in such a way that the solution of the nonlinear differential equation $\dot{q} = f(q, \Omega_d)$ converges to its equilibrium point. The equilibrium point, with assuming $0 \leq q_0 \leq 1$ is $q_e = (1, 0, 0, 0)^T$.

The quaternion regulation error can be described by

$$\begin{aligned} \widetilde{q} &= q - q_e \\ &= (q_0 - 1, q_1, q_2, q_3)^T \end{aligned}$$

Comparing $\tilde{q}$ with $\dot{q}$ will give us:

$$\begin{aligned} \dot{\tilde{q}} &= \frac{1}{2} \begin{pmatrix} -(\bar{q})^T \\ s(\bar{q}) + (q_0 - 1)I \end{pmatrix} \Omega \\ &= \frac{1}{2} \begin{pmatrix} -q_1 & -q_2 & -q_3 \\ q_0 - 1 & -q_3 & q_2 \\ q_3 & q_0 - 1 & -q_1 \\ -q_2 & q_1 & q_0 - 1 \end{pmatrix} \Omega \\ &= B(q)\Omega \end{aligned} \tag{3.4}$$

Theorem 1. *Let α be a positive constant and Q is any positive definite symmetric matrix. If the desired angular velocity is given by*

$$\Omega_d = \alpha I B(q)^T Q q \tag{3.5}$$

Then, under the stated assumptions and conditions, the overall quaternion system will be globally exponentially stable (GES). Furthermore, the desired quaternion regulation *settling time can be obtained by choosing a suitable value of* α*.*

◊

Proof. For simplicity, we will consider $Q = 2I$ in Equation (3.5) so that

$$\Omega_d = -\alpha\bar{q} \tag{3.6}$$

Consider the stable unforced system

$$\begin{aligned} \dot{\tilde{q}} &= N(\tilde{q}, \Omega) \\ &= N(\tilde{q}, 0) = 0 \end{aligned}$$

Substituting the value of Ω_d from (3.6) we have

$$\dot{\tilde{q}} = -B(q)\alpha\bar{q}$$

Defining

$$\begin{aligned} V &= \frac{1}{2}(\tilde{q})^T\tilde{q} \\ &= 1 - q_0 \end{aligned}$$

With substitutions from (2.8) and (3.6), we have

$$\begin{aligned} \dot{V} &= -\dot{q_0} \\ &= -\frac{1}{2}\bar{q}^T\alpha_i(\bar{q}) \end{aligned}$$

Using the property of quaternion representation [17] that $\bar{q}^T\bar{q} + {q_0}^2 = 1$, and since $0 \leq q_0 \leq 1$, we get:

$$
\begin{aligned}
\dot{V} &= -\frac{1}{2}\alpha(1+q_0)V \\
&< 0
\end{aligned}
$$

which shows that for the desired input the system is input-to-state stable (ISS). This means $\lim_{t\to\infty} \tilde{q} = 0$ and from definition of $\tilde{q}$ we can conclude $\lim_{t\to\infty} q = q_e$.

According to the treatment given in [25], it can be shown that the system is globally asymptotically stable (GAS). For exponential stability with substituting (3.6) into (2.8) we have

$$\dot{q_0} = \frac{1}{2}\alpha(1-q_0^2)$$

$$\dot{\bar{q}} = -\frac{1}{2}\alpha q_0 \bar{q}$$

The time response of $q(t)$, by solving these differential equations can be found as

$$q_0(t) = 1 - 2c_1 \frac{e^{-\alpha t}}{1+c_1 e^{-\alpha t}} \tag{3.7}$$

$$\bar{q}(t) = \frac{1+c_1}{1+c_1 e^{-\alpha t}} e^{-0.5\alpha t} q(0) \tag{3.8}$$

where c_1 can be defined as $c_1 = \frac{1-q_0(0)}{1+q_0(0)}$.

From (3.7) and (3.8) we can conclude that the quaternion system (2.8) is globally exponentially stable (GES) [25].

Also from (3.7) and (3.8), it can be seen that the parameter α is related to the settling time of the quaternion regulation and according to definition of the regulation settling time t_q in [26], this relationship is given as:

$$t_q = \frac{4.6}{0.5\alpha} = \frac{9.2}{\alpha} \tag{3.9}$$

□

Definition 1. *According to [26], the settling time of a regulated system is called the regulation settling time, which is defined as*

> *The time required for the time response curve to reach and stay within a range around zero of size specified by absolute percentage of the initial value (usually* 2% *or* 5%*).*

In the other words, for the regulation settling time, the 4-factor in step response should be replaced by 4.6-factor in the regulation response, As we did it in(3.9).

3.1.2 DESIRED ANGULAR VELOCITY TRACKING

In the next step, we design τ_a such that it makes the angular velocity Ω, asymptotically follow the desired angular velocity (3.6).

The angular velocity tracking error can be described as $\tilde{\Omega} = \Omega - \Omega_d$.
Assume that

$$\dot{\tilde{\Omega}} = -\lambda f(\tilde{\Omega}) \tag{3.10}$$

in which λ is a positive constant and $f(\tilde{\Omega})$ is any function of $\tilde{\Omega}$ which satisfies

$$\tilde{\Omega} f(\tilde{\Omega}) > 0 \qquad \tilde{\Omega} \neq 0 \tag{3.11}$$

$$f(\tilde{\Omega}) = 0 \qquad \tilde{\Omega} = 0 \tag{3.12}$$

Defining the Lyapunov function candidate as

$$V = \tfrac{1}{2}(\tilde{\Omega})^T \tilde{\Omega}$$

The time derivative of V while considering (3.10) is

$$\begin{aligned}\dot{V} &= -\lambda\tilde{\Omega}f(\tilde{\Omega}) \\ &< 0\end{aligned}$$

which shows that $\lim_{t\to\infty}\tilde{\Omega} = 0$ and subsequently from (3.10), we have $\lim_{t\to\infty}\Omega = \Omega_d$.

A model-independent control law, τ_a can now be designed as

$$\tau_a = s(\Omega)I_f\Omega + G_a + I_f\dot{\Omega} \tag{3.13}$$

Using the definition of $\tilde{\Omega}$, we have that $\dot{\Omega} = \dot{\tilde{\Omega}} + \dot{\Omega}_d$, which with respect to (3.10) gives

$$\tau_a = s(\Omega)I_f\Omega + G_a - \lambda I_f f(\tilde{\Omega}) + I_f\dot{\Omega}_d \tag{3.14}$$

From (3.6), we have $\dot{\Omega}_d = J(q)\dot{q}$, where $J(q)$ is Jacobian matrix of Ω_d as given by

$$J(q) = \begin{pmatrix} 0 & -\alpha_1 & 0 & 0 \\ 0 & 0 & -\alpha_2 & 0 \\ 0 & 0 & 0 & -\alpha_3 \end{pmatrix}$$

Finally from (2.8) we get

$$\dot{\Omega}_d = J(q)F(\Omega)q$$

which yields

$$\tau_a = s(\Omega)I_f\Omega + G_a - \lambda I_f f(\tilde{\Omega}) + I_f J(q)F(\Omega)q \tag{3.15}$$

Remark 1. *As an example, one of the functions that can satisfy Equations (3.11) and (3.12) is*

$$f(\tilde{\Omega}) = sat(\tilde{\Omega}) = \begin{cases} \tilde{\Omega} & |\tilde{\Omega}| < a \\ sgn(\tilde{\Omega}) & |\tilde{\Omega}| \geq a \end{cases}$$

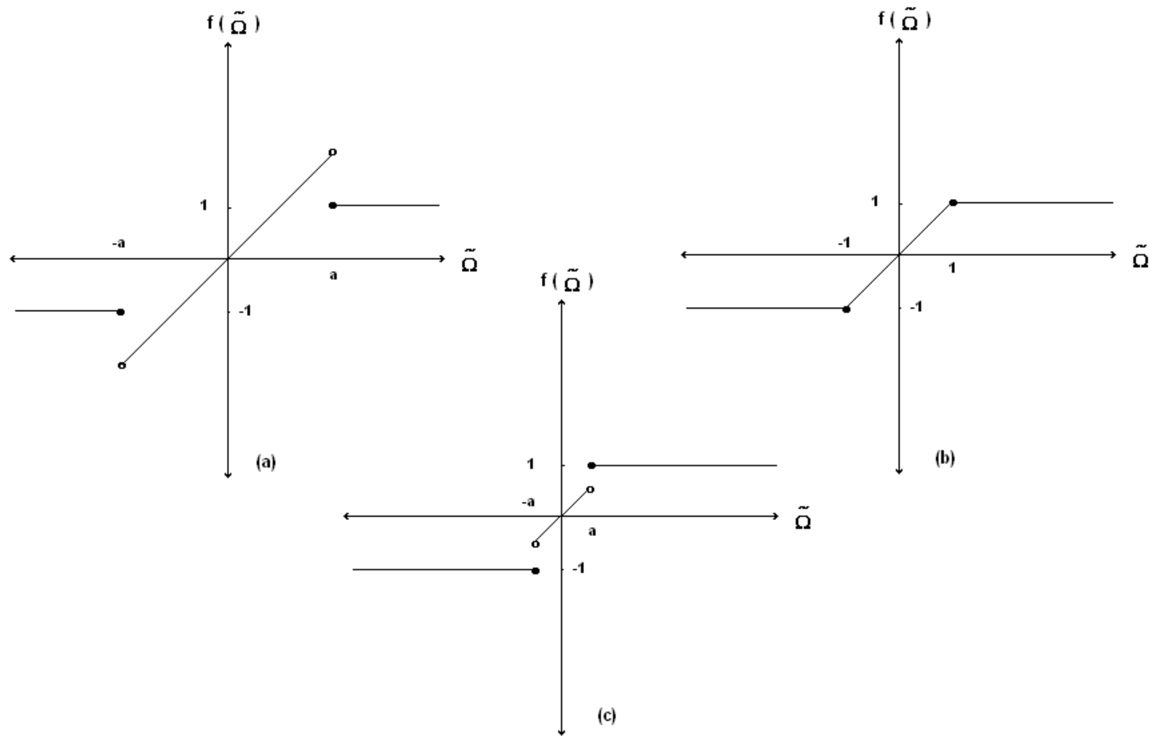

Figure 3.1: $f(\tilde{\Omega}) = sat(\tilde{\Omega})$ a) $a > 1$ b) $a = 1$ c) $a < 1$

Where the positive constant a is the width of the boundary layer of the saturation function (Figure 3.1).

Remark 2. *The control law (3.15) will ensure the asymptotic stability of the quad rotor if and only if the regulation of Ω to its equilibrium point (zero) is faster than regulation of q to q_e which means that*

$$t_\Omega < t_q \tag{3.16}$$

Here, the angular velocity tracking error settling time t_Ω approximately is:

$$t_\Omega = \frac{a + \alpha}{\lambda \alpha} \tag{3.17}$$

Notice that the boundary layer width a, has to be sufficiently small such that it is *in the angular velocity settling range. Also the control law (3.15) with respect to the airframe inertia uncertainties ΔI_f is robustly stable if the angular velocity tracking parameter λ is*

$$\lambda > \lambda_0 = a(a + \frac{1}{2}\|J\|_\infty)\frac{\delta}{\sigma_{min}(I_{f0})}$$

where I_{f0} is nominal value of airframe's inertia matrix and $\Delta I_f \in \{y| \|y\|_\infty \leq \delta\}$.

It is important to point out here that one can simply design a model-dependant control law as

$$\begin{aligned} \tau_a &= \dot{\Omega} \\ &= -\lambda f(\tilde{\Omega}) + J(q)F(\Omega)q \end{aligned} \tag{3.18}$$

Further, with both the control laws of (3.15) and (3.18), the regulation problem of attitude angles to their desired values (i.e. at zero as in hovering case) results in a globally exponentially stable (GES) system, and the regulation settling time is a function of α.

3.2 DESIGNING THE ROTOR TORQUE τ_i

Having achieved the task of designing control torque (model dependent or model independent), we now proceed to designing τ_i such that the angular speed of the rotors (w_i's), follow the desired angular speeds generated by our designed control torque $\tau_a = (\tau_\phi, \tau_\theta, \tau_\psi)^T$.

From Equations (2.4) through (2.7), we find the desired angular speeds as

$$\begin{pmatrix} w_{d_1}^2 \\ w_{d_2}^2 \\ w_{d_3}^2 \\ w_{d_4}^2 \end{pmatrix} = \begin{pmatrix} 0 & bd & 0 & -bd \\ bd & 0 & -bd & 0 \\ l & -l & l & -l \\ b & b & b & b \end{pmatrix}^{-1} \begin{pmatrix} \tau_\phi \\ \tau_\theta \\ \tau_\psi \\ T \end{pmatrix}$$

$$= A^{-1} \begin{pmatrix} \tau_\phi \\ \tau_\theta \\ \tau_\psi \\ T \end{pmatrix} \tag{3.19}$$

in which the parameters b, d and $l > 0$ are assumed to be positive, in order to ascertain that the matrix A remains nonsingular. Also, notice that the total thrust T with respect to the required airframe torques has to be sufficiently large.

For making the angular speeds of rotors asymptotically approach their related w_{d_i}'s, we can simply define τ_i with respect to (2.10), as

$$\tau_i = Q_i + I_r \dot{w_{d_i}} - h_i f(\tilde{w}_i) \tag{3.20}$$

where $\tilde{w}_i = w_i - w_{d_i}$ and h_i is a positive constant which can be found by trial and error. For having a smooth and nice result it is better that, with increasing t_q we decease our chosen value for h_i. With replacing (3.20) in (2.10), we will get

$$I_r \dot{\tilde{w}}_{d_i} = -h_i f(\tilde{w}_i)$$

$$\dot{\tilde{w}}_{d_i} = -\frac{h_i}{I_r} f(\tilde{w}_i) \tag{3.21}$$

One of the functions that can make (3.21) exponentially stable is

$$f(\tilde{w}_i) = sat(\tilde{w}_i) = \begin{cases} \tilde{w}_i & |\tilde{w}_i| < a \\ sgn(\tilde{w}_i) & |\tilde{w}_i| \geq a \end{cases} \tag{3.22}$$

Exponential stability of (3.21), i.e. $\lim_{t\to\infty} \tilde{w}_i = 0$, or equivalently $\lim_{t\to\infty} w_i = w_{d_i}$, ensures that w_i's asymptotically track w_{d_i}'s.

Notice that in Equation (3.20), one of the possible means of finding $\dot{w_{d_i}}$ can be by using the dirty derivative filter [6]:

$$\dot{w_{d_i}} = \tfrac{s}{1+T_f s} w_{d_i}$$

3.3 CHAPTER SUMMARY

In this chapter we proposed an algorithm for attitude stabilization of a quad rotor as a representative of VTOL-UAVs. We designed two nearly equivalent control laws (model independent as well as model dependent) to obtain exponential stability of attitude angles and asymptotic stability of attitude angular velocity of the quad rotor UAV. Also in the proposed algorithm we can regulate the attitude parameters (attitude angles and attitude angular velocity) to their desired values, as fast as required.

Chapter 4

SIMULATION RESULTS

Analysis and design of control systems has become very easy due to the development of modern simulation tools like MATLAB. Such tools give very deep insight into analysis of systems.

This chapter describes the simulations and results of our proposed strategy in MATLAB environment.

4.1 SYSTEM IDENTIFICATION

As an example for testifying the performance, we consider a quad rotor with dynamical parameters as in table 4.1.

From the described procedure we know the required parameters for the control laws, can be found according to our desired regulation settling time (attitude angles settling time).

Table 4.1: Considered dynamical values

Dynamical Parameters	Considered Values	Units
d	0.225	m
I_r	3.4×10^{-5}	$kg.m^2$
I_{f_ϕ}	4.9×10^{-3}	$kg.m^2$
I_{f_θ}	4.9×10^{-3}	$kg.m^2$
I_{f_ψ}	8.8×10^{-3}	$kg.m^2$
b	2.9×10^{-5}	-
l	1.1×10^{-6}	-
T	1.5	N
T_f	0.008 (Cutoff frequency of 20 Hz)	S

Note that, there is a compromise between choosing a small value of t_q and having large peak value for the angular velocity Ω, since with replacing the control law (3.15) or (3.18) in (2.9) we will find that peak value of the angular velocity Ω would be a function of

α in major

λ and a in minor

In another words, if we decrease t_q, we have increased α and decreased λ which will increase the peak value. Also choosing a as small as possible will cause to smaller peak values.

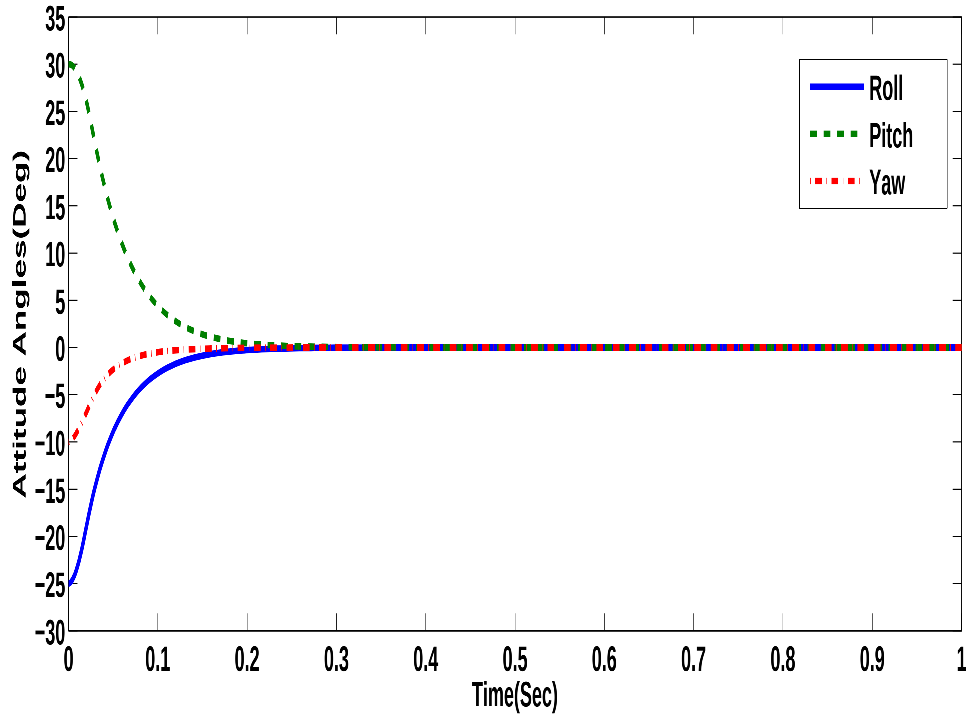

Figure 4.1: Attitude angles, Model dependent controller, simulation 1

In this report we will consider a as:

$$a = 0.02\alpha \tag{4.1}$$

4.2 SIMULATION 1

Consider our desired t_q as $0.2s$, from (3.9) and (4.1) respectively we will have

$$\alpha = \frac{9.2}{0.2} = 46$$

$$a = 0.02 \times 46 = 0.92$$

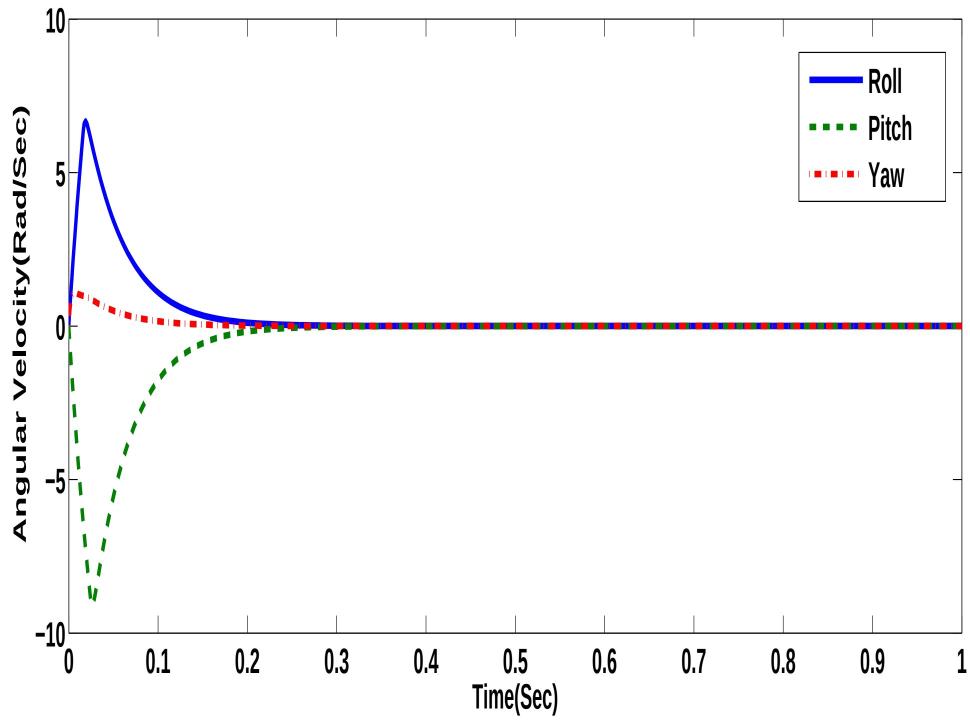

Figure 4.2: Angular velocities, Model dependent controller, simulation 1

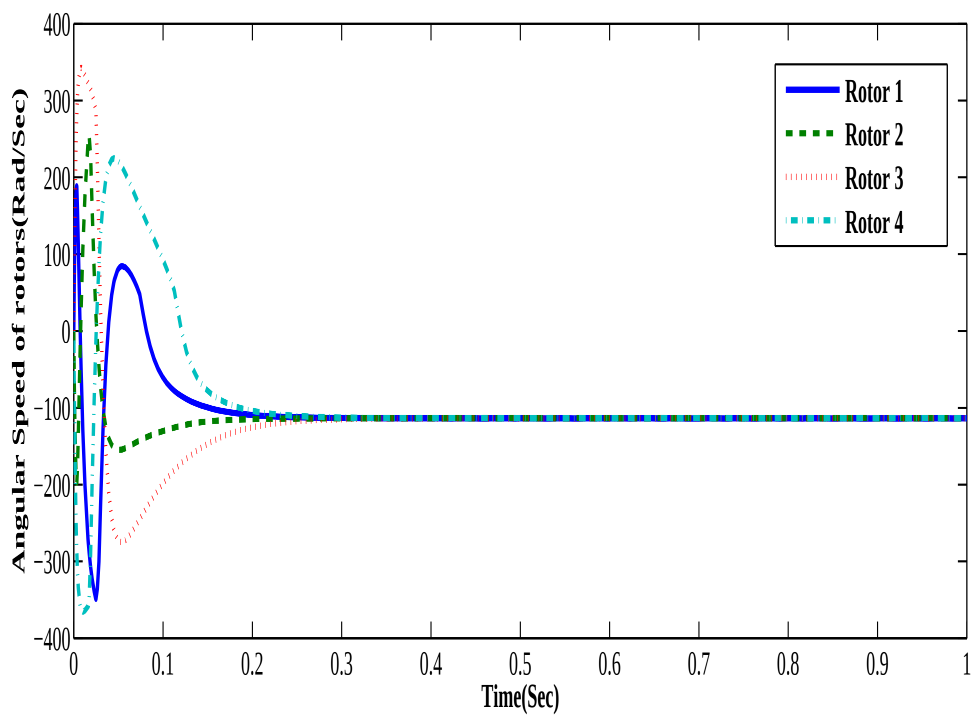

Figure 4.3: Angular Speed of Rotors, Model dependent controller, simulation 1

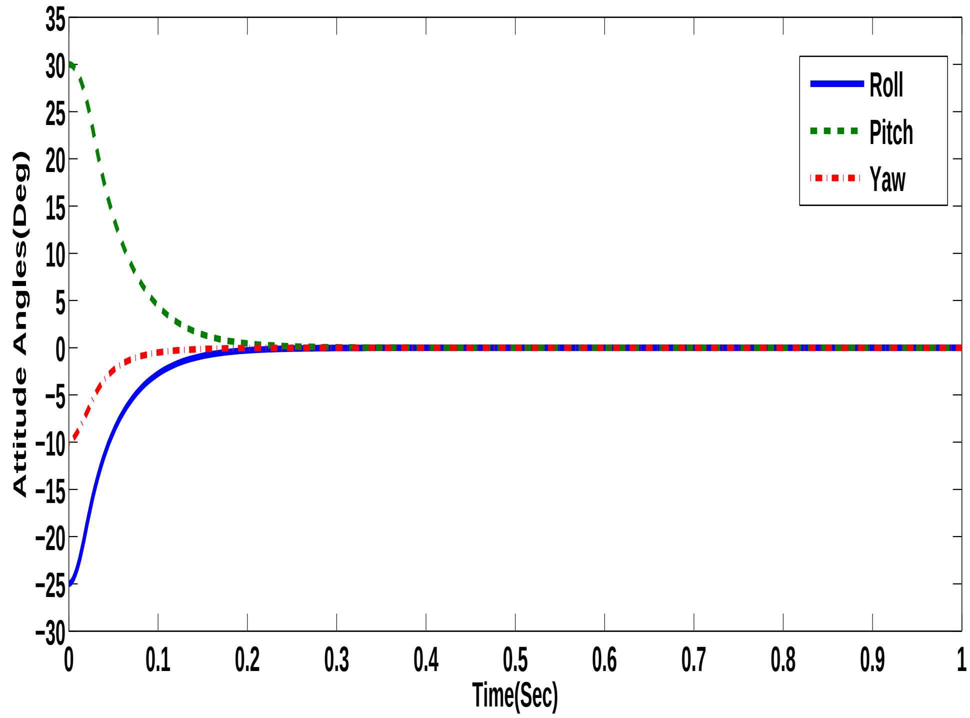

Figure 4.4: Attitude angles, Model independent controller, simulation 1

One can choose $t_\Omega = 0.1s$ which will satisfy (3.16).

Then from (3.17):

$$\lambda = \frac{0.92 + 46}{0.92 \times 0.1} = 510$$

With these parameters, we will simulate the system with model dependent control law (3.15) in Figures 4.1 and 4.2 and 4.3, and model independent control law (3.18) in Figures 4.4 and 4.5 and 4.6.

In this Simulation the desired final situation of quad rotor is hovering. Considered parameters' values have been shown in Table 4.2.

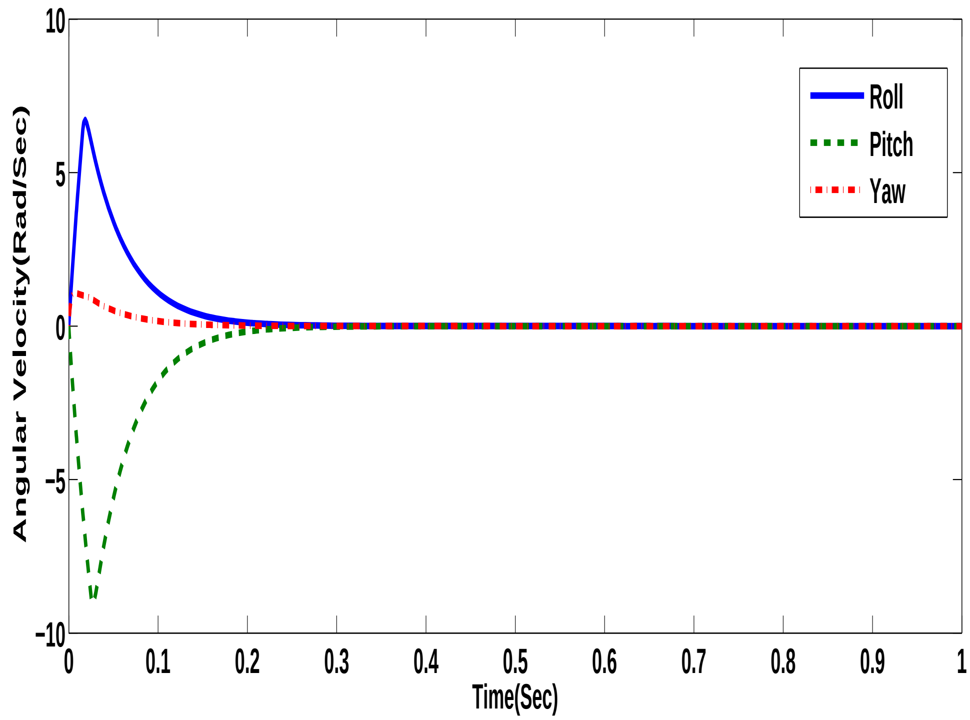

Figure 4.5: Angular velocities, Model independent controller, simulation 1

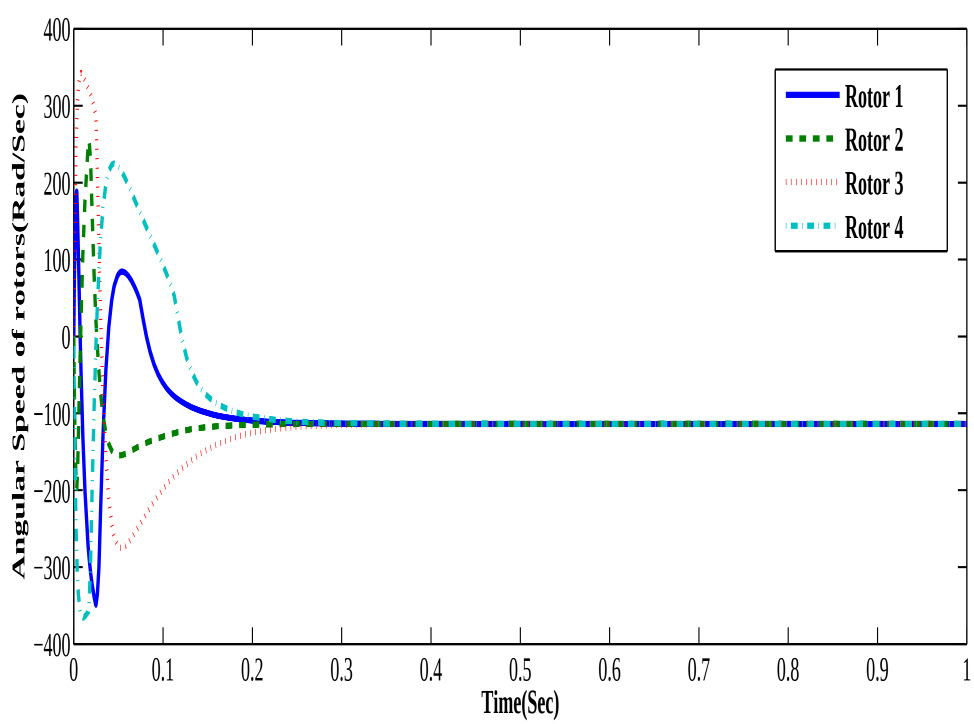

Figure 4.6: Angular Speed of Rotors, Model independent controller, simulation 1

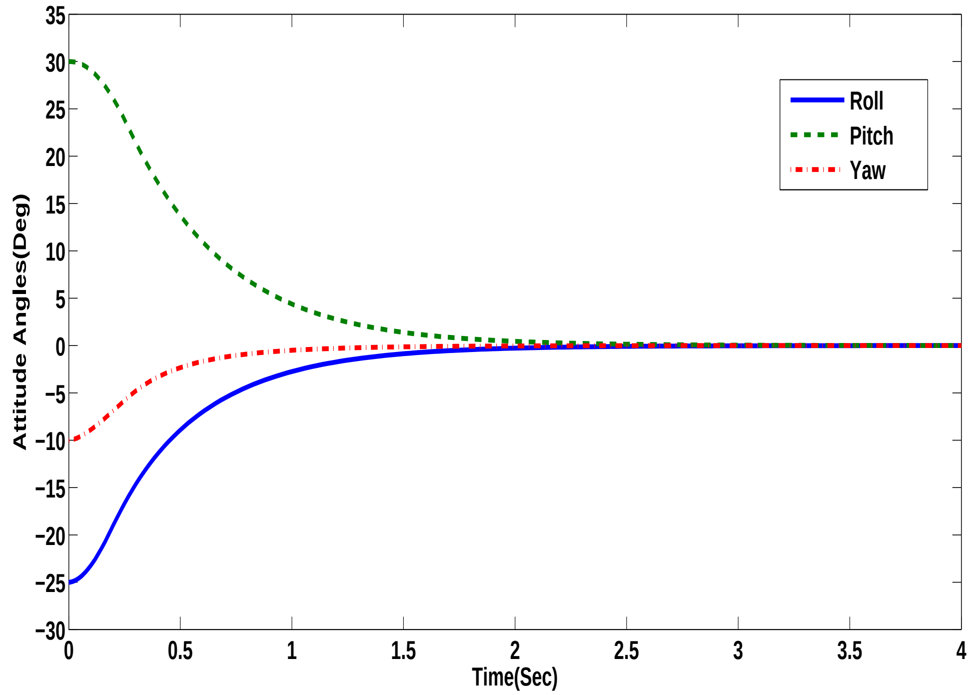

Figure 4.7: Attitude Angles, Model dependent controller, simulation 2

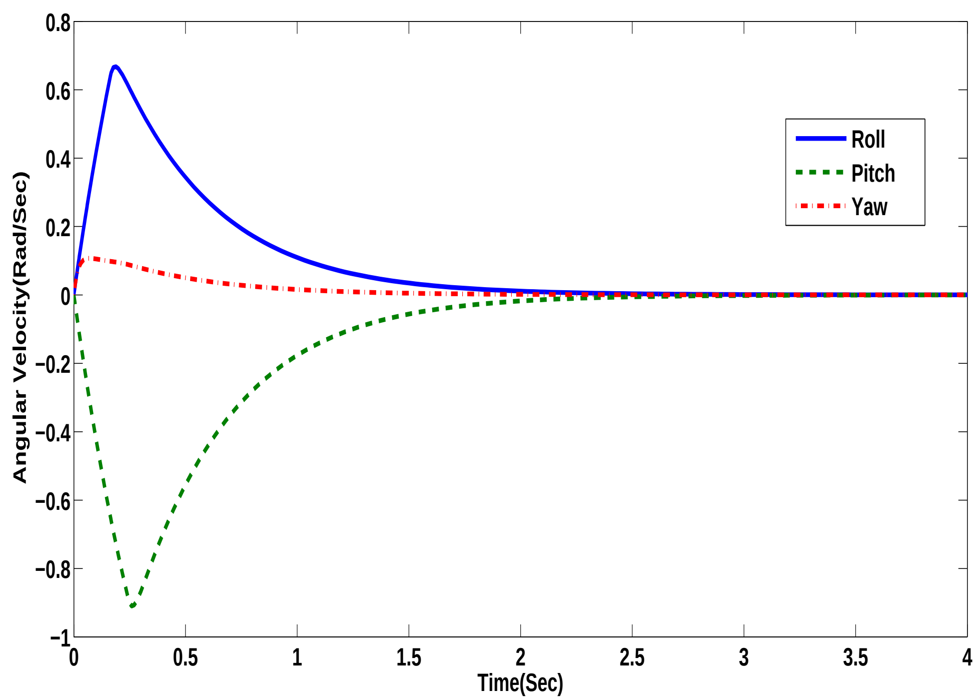

Figure 4.8: Angular velocities, Model dependent controller, simulation 2

Table 4.2: Considered Values in Simulation 1

Parameters	Considered Values	Units
t_q	0.2	S
t_Ω	0.1	S
α	46	-
a	0.92	-
λ	510	-
ϕ Initial Roll angle	-25	Deg
θ Initial Pitch angle	30	Deg
ψ Initial Yaw angle	-10	Deg
h_i $i \,\epsilon \{1, 2, 3, 4\}$	0.002	-

4.3 SIMULATION 2

For second Simulation, let's consider $t_q = 2s$. Then

$$\alpha = \frac{9.2}{2} = 4.6$$

$$a = 0.02 \times 4.6 = 0.092$$

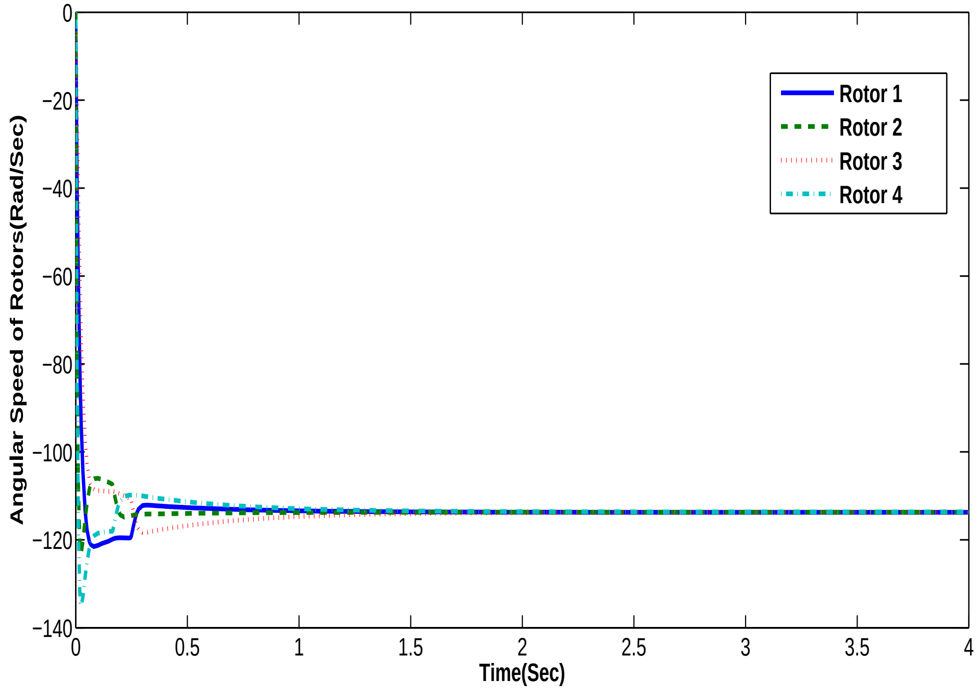

Figure 4.9: Angular Speed of Rotors, Model dependent controller, simulation 2

One can chose $t_\Omega = 1$, then

$$\lambda = \frac{0.092 + 4.6}{0.092 \times 1} = 51$$

assume rest of parameters be same as simulation 1.

Result of simulation of the system with control law (3.15) has been shown in Figures 4.5 and 4.6, and result for control law (3.18) in Figure 4.7 and 4.8. Considered parameters' values have been shown in Table 4.3.

As we can see in these two simulations, the desired final situation of quad rotor is hovering. In two next simulations we'll survey the system when the quad rotor will be regulated to a desired final situation which is not hovering.

Table 4.3: Considered Values in Simulation 2

Parameters	Considered Values	Units
t_q	2	S
t_Ω	1	S
α	4.6	-
a	0.092	-
λ	51	-
ϕ Initial Roll angle	-25	Deg
θ Initial Pitch angle	30	Deg
ψ Initial Yaw angle	-10	Deg
h_i $i \, \epsilon \, \{1, 2, 3, 4\}$	0.002	-

4.4 SIMULATION 3

Here, the system and the parameters are same as simulation 1 (Table 4.2). The only difference is we will assume that the quad rotor will be regulated to:

$$\phi_r = 10°, \theta_r = -15°, \psi_r = 5°$$

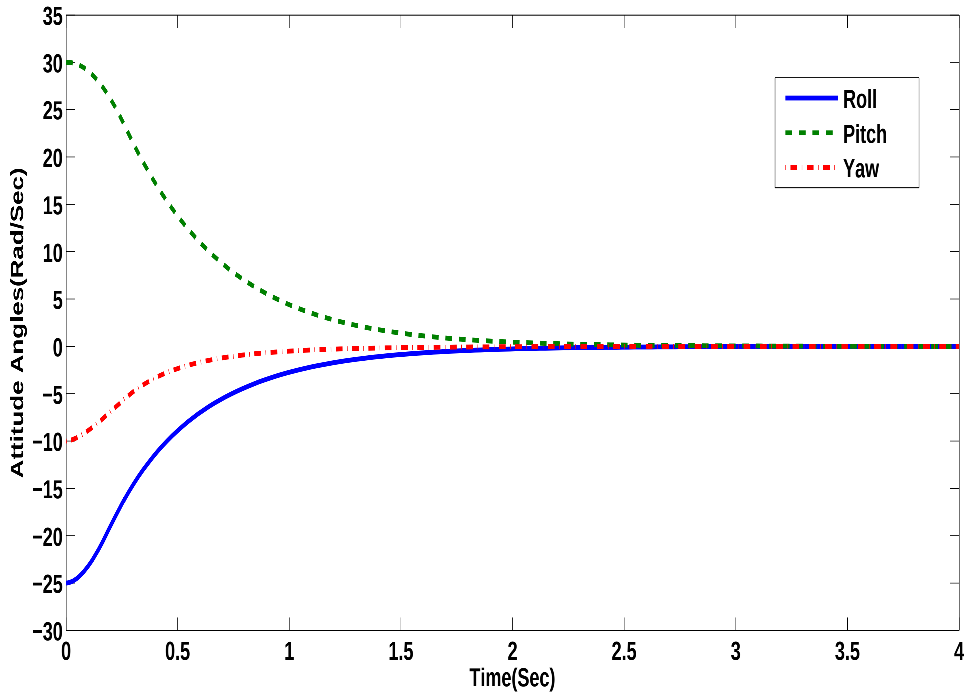

Figure 4.10: Attitude Angles, Model independent controller, simulation 2

Performance of the system in this case with model dependent control law (3.15) has been shown in Figure 4.9 and 4.10, and performance of the system with model independent control law (3.18) has been shown in Figures 4.11 and 4.12.

4.5 SIMULATION 4

Finally, Figures 4.13 and 4.14 will show the performance of the system with model dependent controller (3.15), while simulation parameters are as Table 4.3, and quad rotor will be regulated to:

$$\phi_r = 10°, \theta_r = -15°, \psi_r = 5°$$

Also, Figures 4.15 and 4.16 respectively show trajectories of attitude angles and angular velocities of model independent controller (3.18), for same parameters as above.

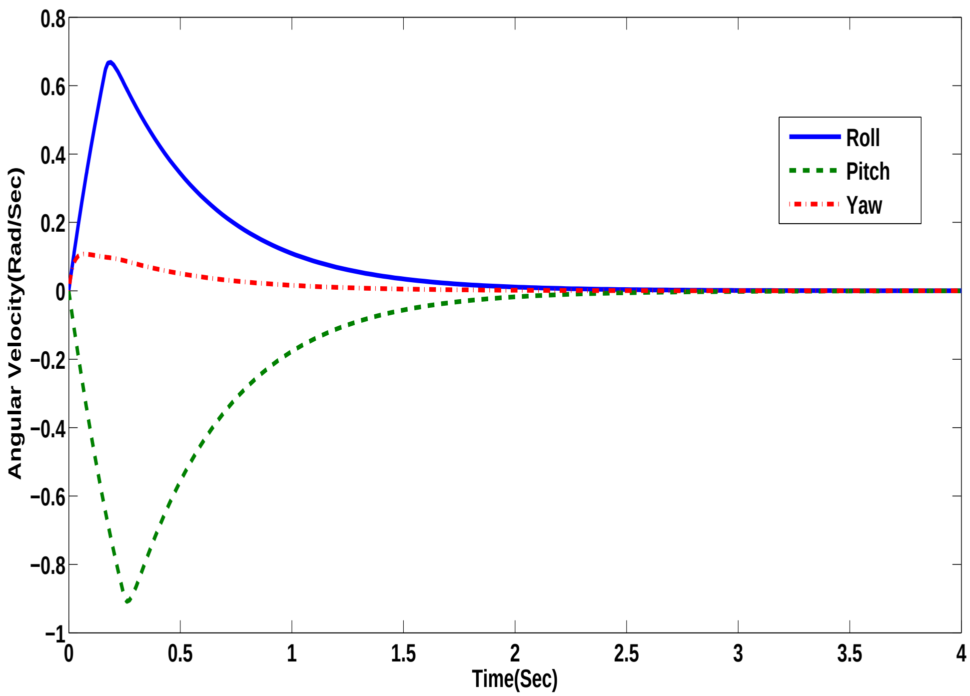

Figure 4.11: Angular velocities, Model independent controller, simulation 2

4.6 CHAPTER SUMMARY

MATLAB has been adopted in analysis and design of control system for its simplicity and comprehensiveness by researchers.

This chapter has depicted the performance of the proposed control laws (model dependent and model independent), which have been simulated over MATLAB software.

Here, first we have calculated different control law parameters (i.e. a, λ, α) for our desired settling times and then applied that parameters into the proposed control laws (model dependent and model independent), in order to find numerical control

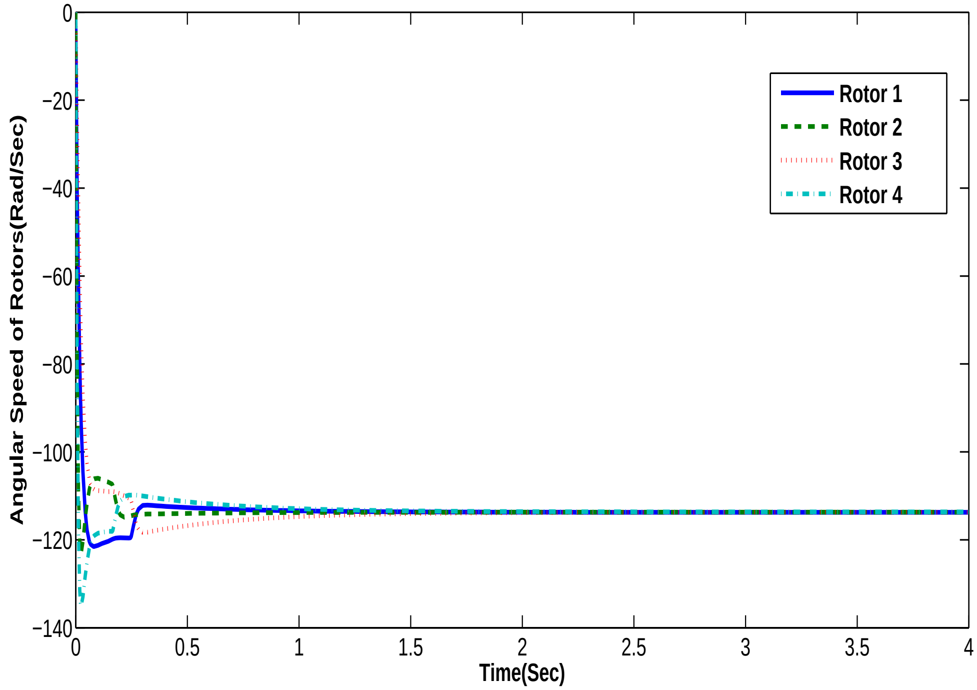

Figure 4.12: Angular Speed of Rotors, Model independent controller, simulation 2

laws. We have then testified them with simulating over MATLAB software.

As seen, figures are ascertaining that, under the described procedure, attitude parameters of quad rotor (i.e. attitude angles and attitude angular velocities) can be regulated to their required final values within the desired settling time.

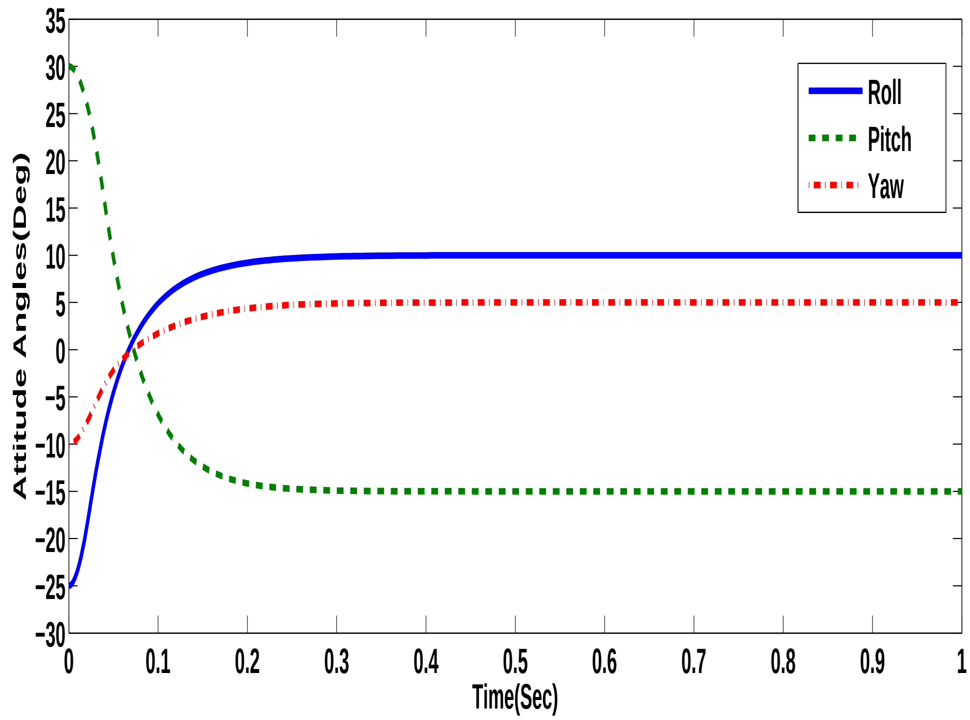

Figure 4.13: Attitude Angles, Model dependent controller, simulation 3

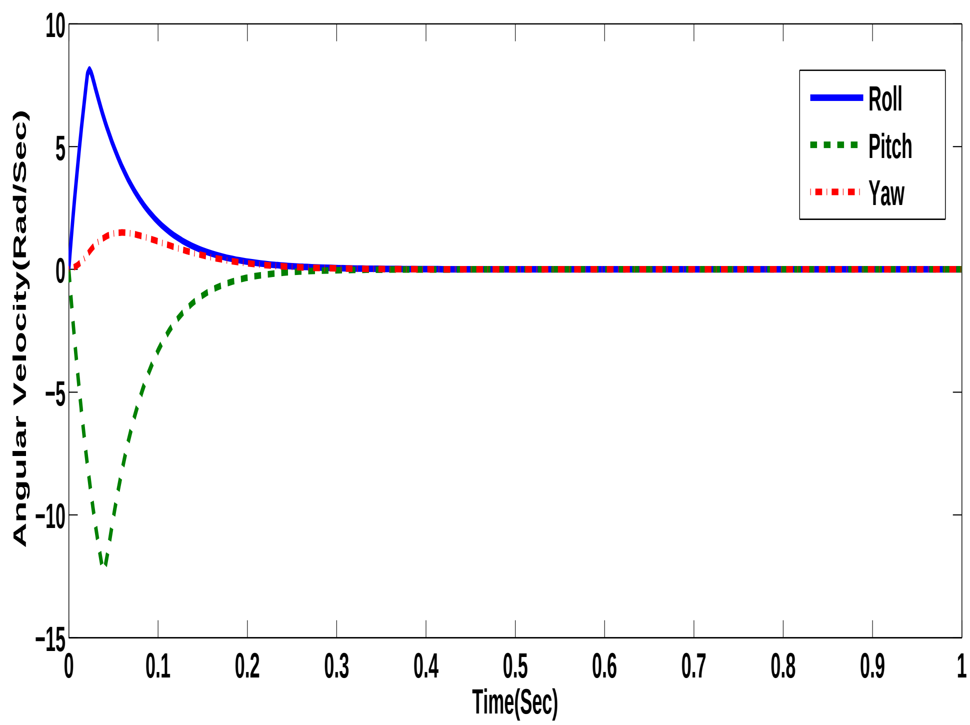

Figure 4.14: Angular velocities, Model dependent controller, simulation 3

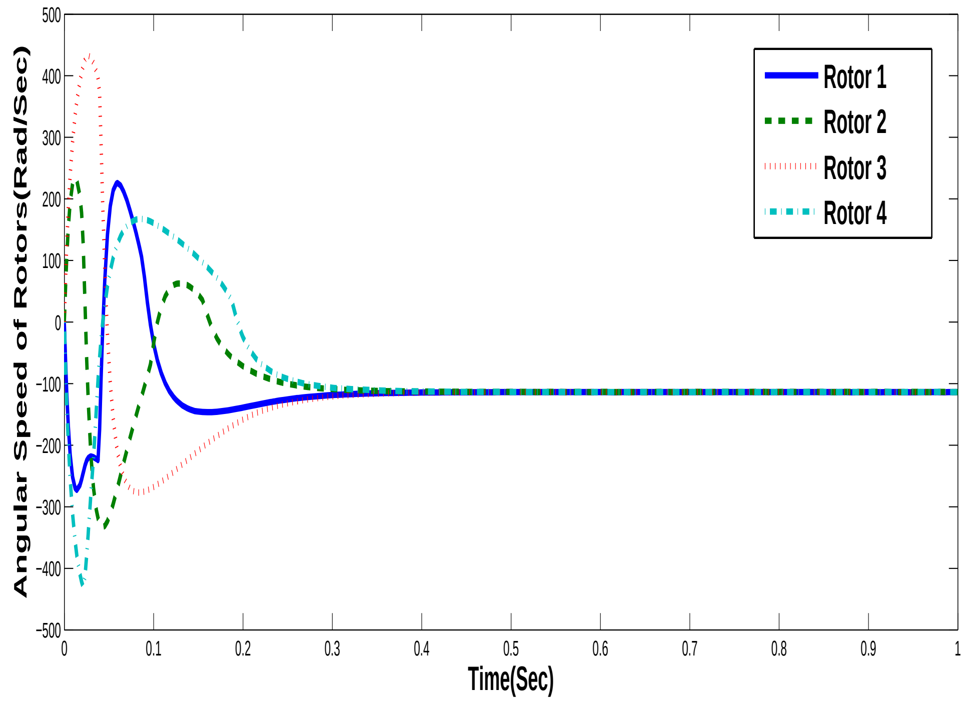

Figure 4.15: Angular Speed of Rotors, Model dependent controller, simulation 3

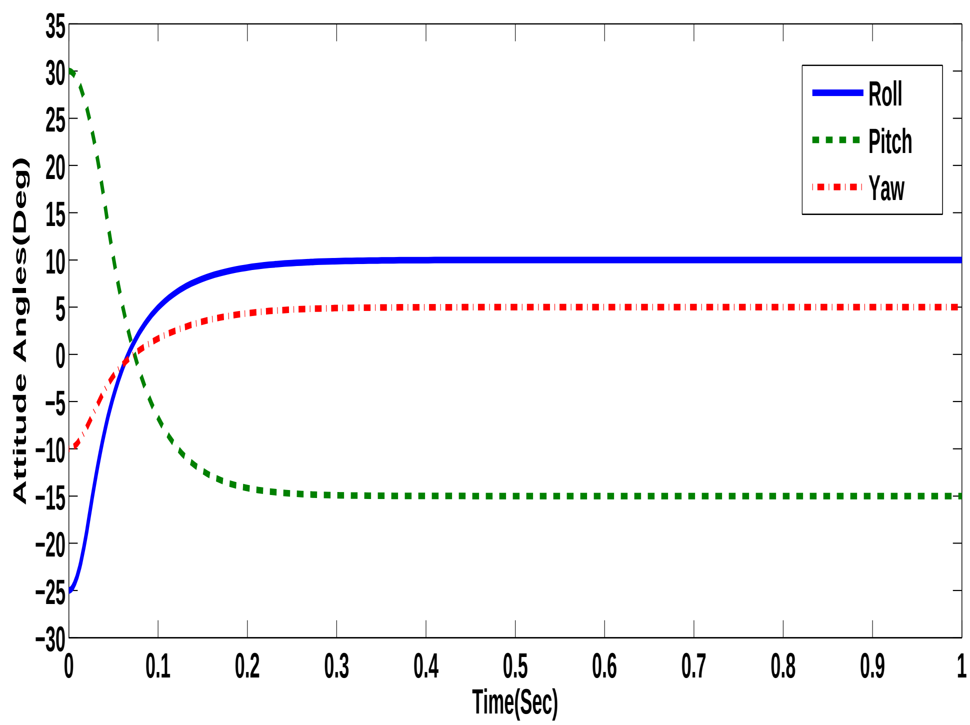

Figure 4.16: Attitude Angles, Model independent controller, simulation 3

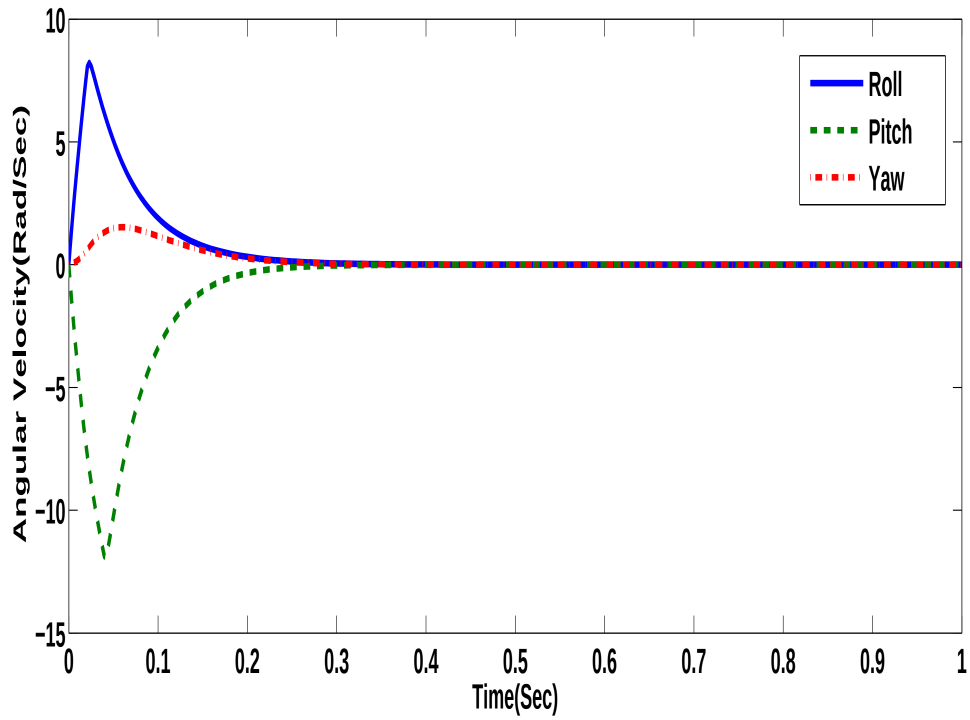

Figure 4.17: Angular velocities, Model independent controller, simulation 3

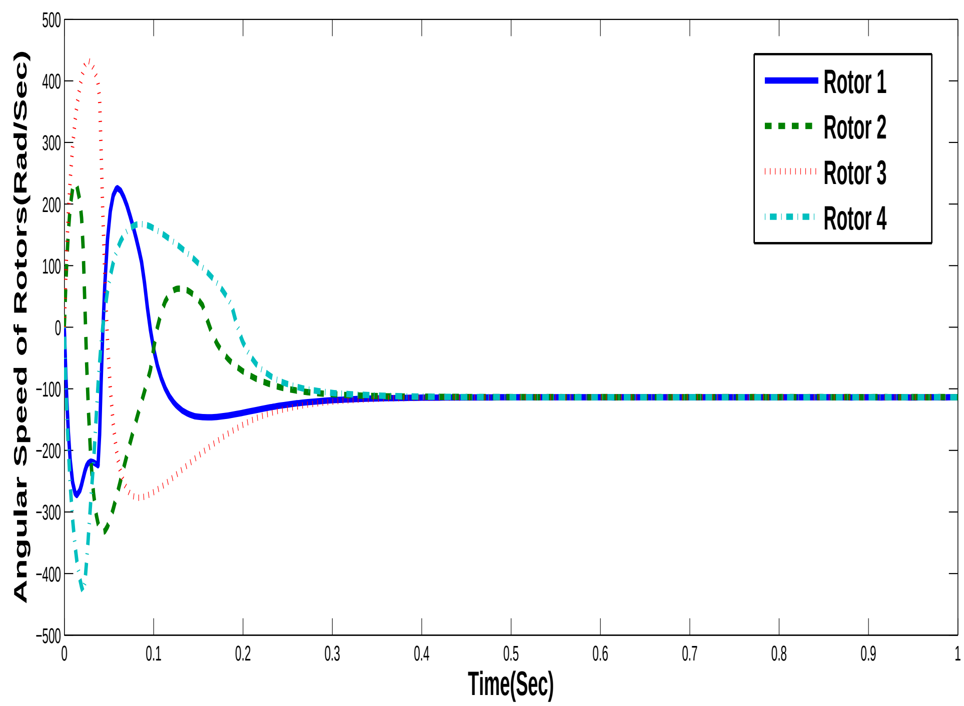

Figure 4.18: Angular Speed of Rotors, Model independent controller, simulation 3

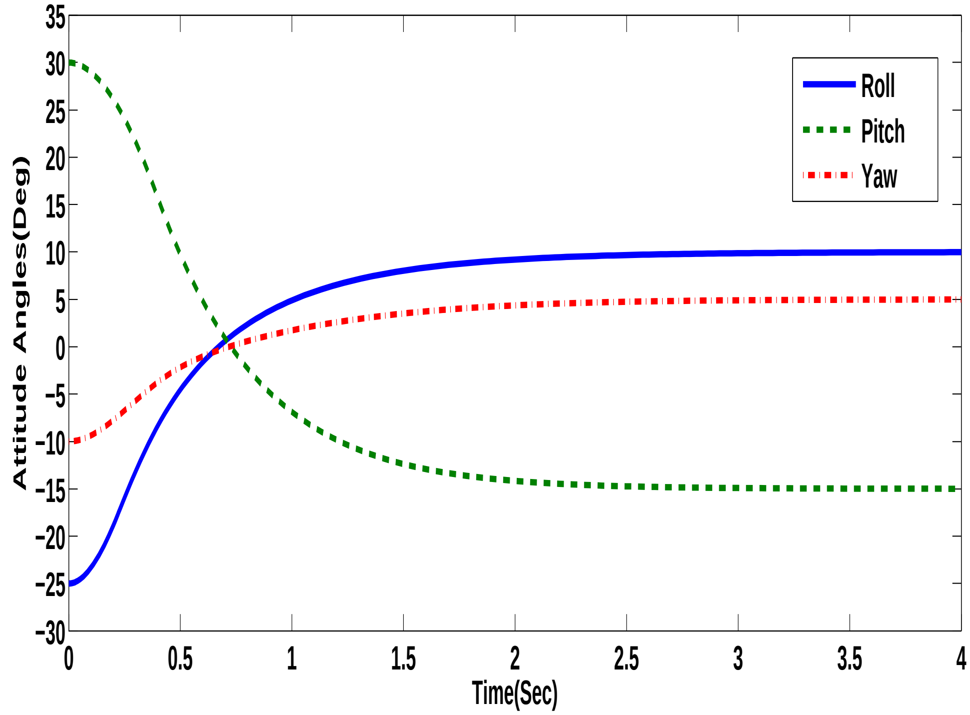

Figure 4.19: Attitude Angles, Model dependent controller, simulation 4

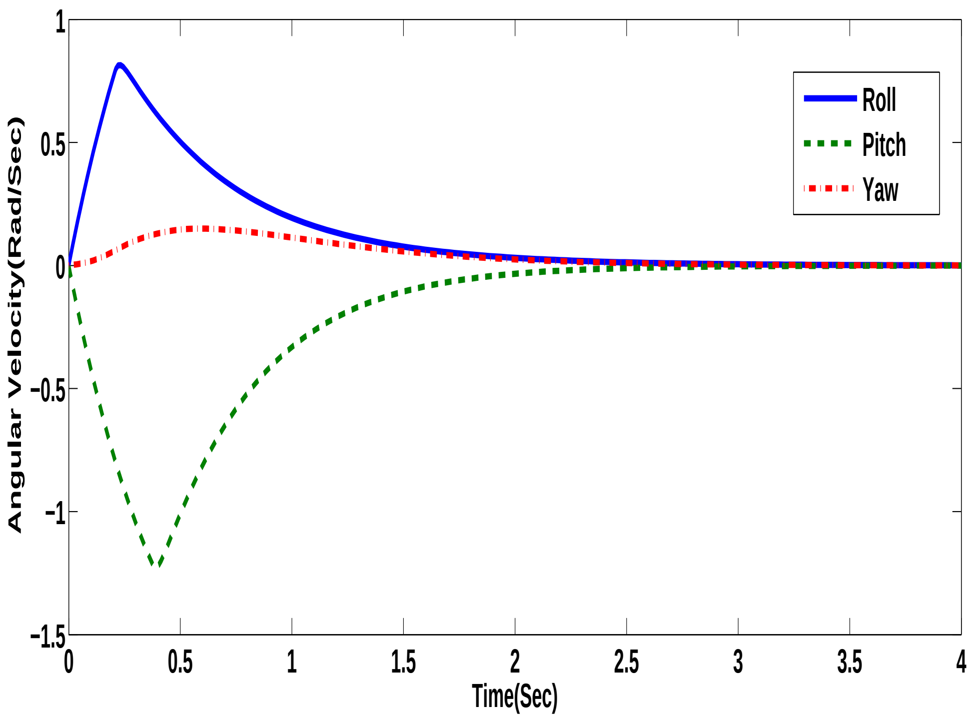

Figure 4.20: Angular velocities, Model dependent controller, simulation 4

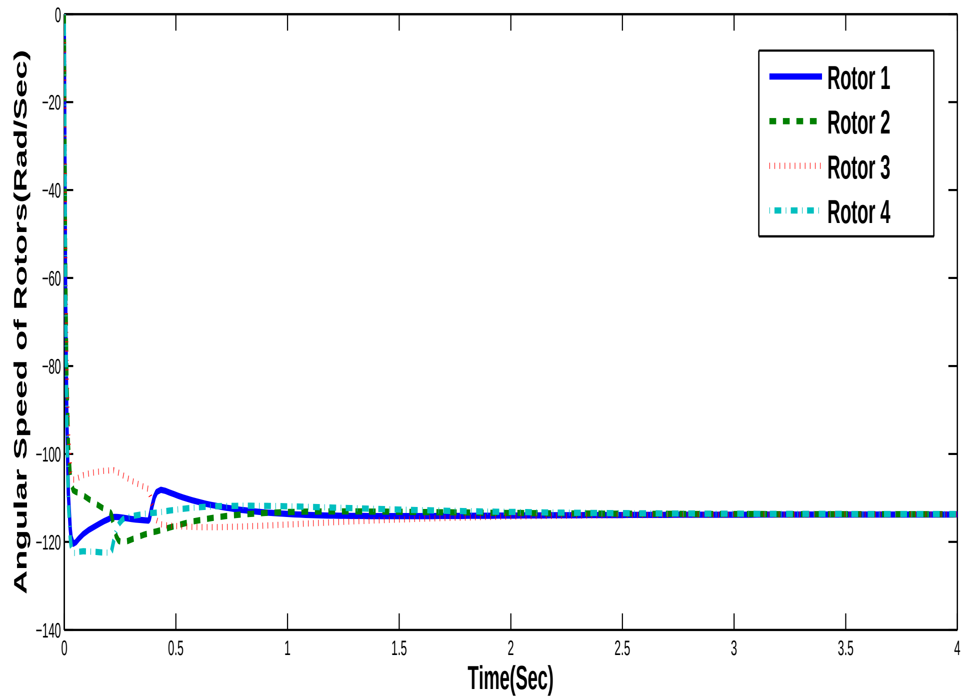

Figure 4.21: Angular Speed of Rotors, Model dependent controller, simulation 4

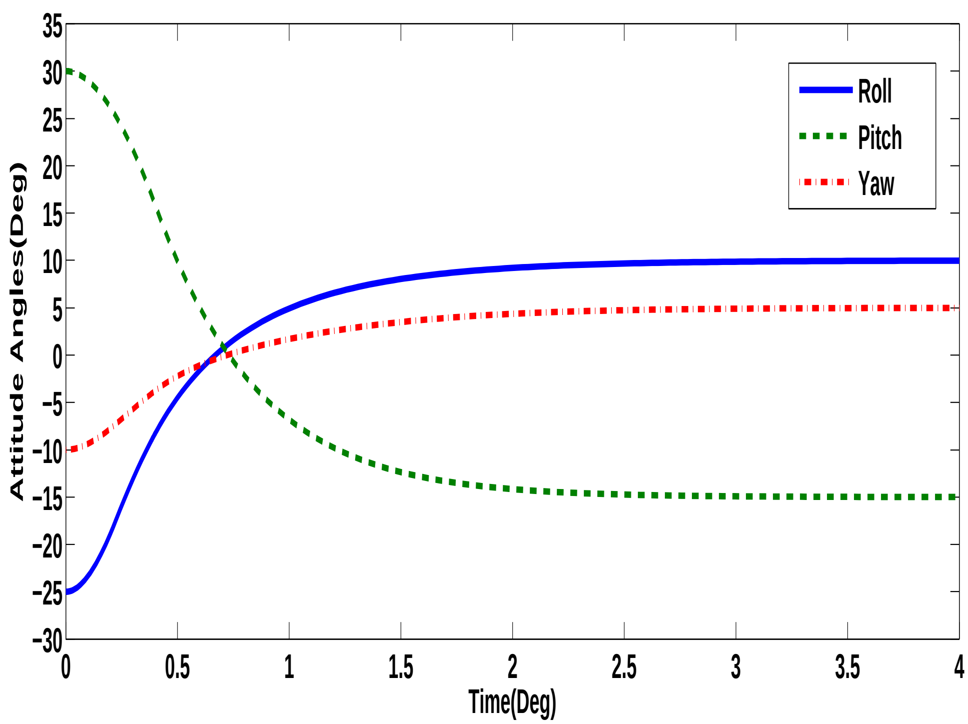

Figure 4.22: Attitude Angles, Model independent controller, simulation 4

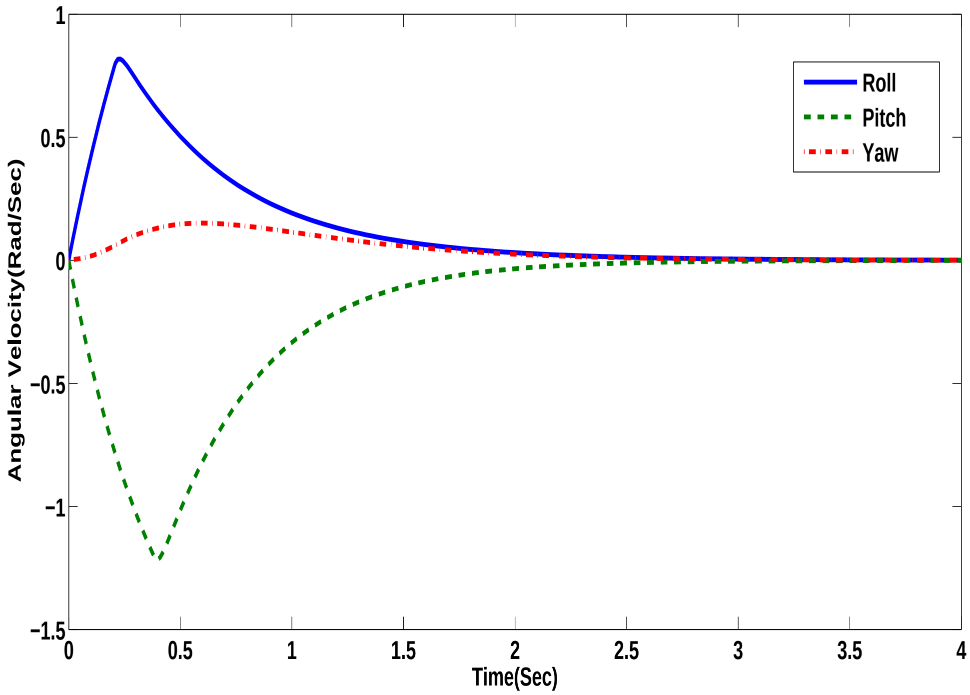

Figure 4.23: Angular velocities, Model independent controller, simulation 4

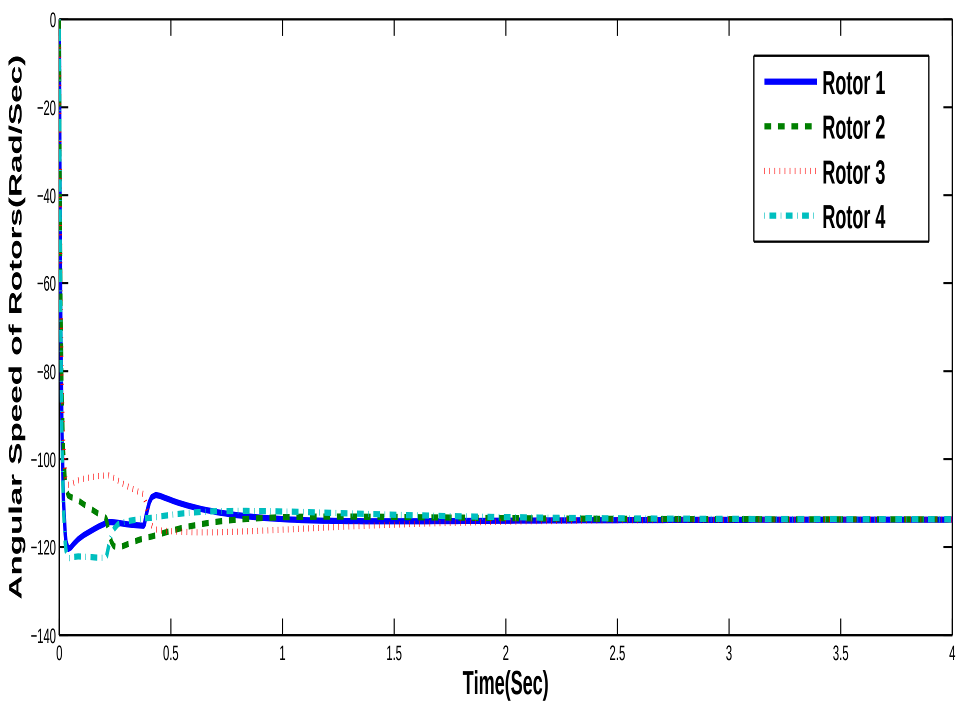

Figure 4.24: Angular Speed of Rotors, Model independent controller, simulation 4

Chapter 5

CONCLUSION

In this thesis, we presented a novel control approach to obtain asymptotic attitude stability of a quad rotor as a representative of VTOL-UAVs.

5.1 WORK'S REVIEW

First, we considered a symmetric quad rotor with four rigid mono-directional propellers as shown in Figure 5.1.

Then we modeled the described quad rotor in equations from (2.8) to (2.10), which is based on quaternion representation with taking Coriolis and gyroscopic torques into account.

finally, based upon this dynamical model, we defined two nearly equivalent model-dependent control law as:

$$\tau_a = s(\Omega)I_f\Omega + G_a - \lambda I_f f(\tilde{\Omega}) + I_f J(q)F(\Omega)q \quad (5.1)$$

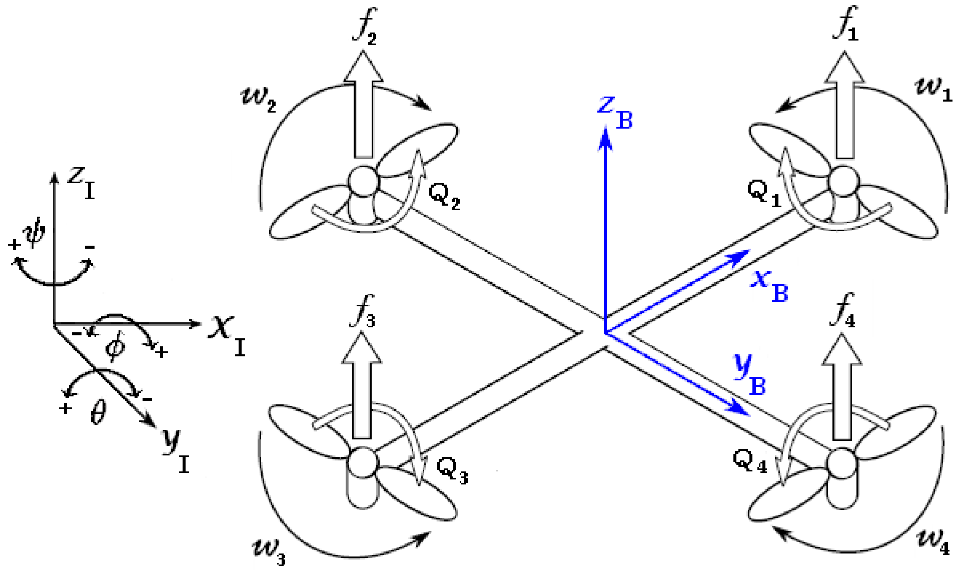

Figure 5.1: The final description

and model-independent control law as:

$$\tau_a = -\lambda f(\tilde{\Omega}) + J(q)F(\Omega)q \tag{5.2}$$

The performance of these two control laws have been tested over simulation.

5.2 CONCLUSION

In the proposed robust control laws for attitude control of the quad rotor, the regulation of attitude angles to their desired values is shown to be globally exponentially stable, and the desire settling time values can be adjusted by the operator. In the proposed methodology, the tracking of desire angular velocity is shown to be globally asymptotically stable. The performance of the proposed control design has been ascertained using Simulation which yields good performance under prescribed uncertainties and disturbances.

5.3 FUTURE WORK

More recently, a growing interest for stabilization control of quad rotors as a representative of VTOL-UAvs has been shown among the research community, which is because of their enormous applications in different fields.

The main contribution of this thesis is to propose a novel attitude stabilization control scheme which can improve and simplify the robust attitude controller. Despite that only attitude control of a quad rotor is not adequate to make a real aerial maneuver, we study this problem to improve the performance by overcoming various design difficulties.

The positive results obtained in this development towards attitude control of quad rotors, reinforce our conviction that, in spite of the natural high instability of these systems, a reliable control is still possible.

There are a number of improvements that can be made on this controller for the future. These improvements can be as:

- Implementing the attitude controller practically
- Enhancing the control with position controller
- Developing a fully autonomous vehicle
- Implementing a fast quad rotor controller
- Implementing quad rotors in multiple vehicle teams
- ...

5.4 PUBLICATION

M. Heidarian, A. Y. Memon, “Attitude Control of VTOL-UAVs ,” In proc. of the 8th International United Kingdom Automatic Control Council Conference 2012, Cardiff, UK, Sep 2012, pp. 363-368 [28].

Appendix A

DEFINITIONS

A.1 Lipschitz

A function satisfying

$$\|f(t,x) - f(t,y)\| \leq L \parallel x - y \parallel$$

is said to be Lipschitz in x, and the positive constant L is called a Lipschitz constant.

A.2 Class K

A continuous function $\alpha : [0, a) \to [0, \infty)$ is said to belong to class K if it is strictly increasing and $\alpha(0) = 0$.

A.3 Class K_∞

A continuous function $\alpha : [0, a) \to [0, \infty)$ is said to belong to class K_∞ if $a = \infty$ and $\alpha(r) \to \infty$ as $r \to \infty$.

A.4 ISS

Consider the system

$$\dot{x} = f(x, u(t))$$

where

- f is locally Lipschitz in (x, u)
- $f(0, 0) = 0$
- $u(t)$ is piecewise continuous and bounded

The system $\dot{x} = f(x, u(t))$ is said to be input-to-state stable (ISS), if there exist $V(x)$ such that

$$\alpha_1(\|x\|) \leq V(t, x) \leq \alpha_2(\|x\|)$$

$$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f(t, x, u) \leq -W_3(x) \qquad \forall \|x\| \geq \rho(\|u\|) > 0$$

$\forall (x, u)$, where α_1, α_2 are class K_∞ functions, ρ is a class K function, and $W_3(x)$ is a continuous positive definite function. Then the system is input-to-state stable with $\gamma = \alpha_1^{-1} \circ \alpha_2 \circ \rho$.

A.5 GAS

Consider the autonomous (i.e., unforced, no explicit time dependence) system:

$$\dot{x} = f(x)$$

in which, the origin is assumed to be an equilibrium point, that is, $f(0) = 0$.

If a Lyapunov function $V(x)$ can be found such that:

1.$V(x) > 0$ for $x \neq 0$

V(x)=0 forx=0

2.$\dot{V}(x) < 0$ negative definite for $x \neq 0$

3.$V(x) \rightarrow \infty$ as $\|x\| \rightarrow \infty$

then the origin is globally asymptotically stable (GAS).

A.6 GES

Consider the autonomous system

$$\dot{x} = f(x)$$

which is GAS. The equilibrium point is globally exponentially stable (GES), if there exist positive constant c, k, and λ such that

$$\|x(t)\| \leq k\|x(t_0)\|e^{-\lambda(t-t_0)} \qquad \forall \|x(t_0)\| < c$$

For more information about these definitions refer to [25] and[27].

Appendix B

NOMENCLATURE

$w_1 \dots w_4$	angular speed of rotors
ϕ, θ, ψ	quad rotor's angular displacement (roll, pitch, yaw)
T	total thrust generated by rotors
$f_1 \dots f_4$	upward lifting forces of rotors
$Q_1 \dots Q_4$	reactive torques of rotors
$\tau_\phi, \tau_\theta, \tau_\psi$	roll, pitch, yaw torque
d	length of arms of quad rotor
x_I, y_I, z_I	axes of inertia frame
x_B, y_B, z_B	axes of body-fixed frame
R	rotation matrix

$q_0 \ldots q_3$	quaternion description of quad rotor's rotation
I	identity matrix
$\Omega = (\Omega_1 \ldots \Omega_3)^T$	angular velocity of quad rotor
I_f	inertia matrix of quad rotor
$\tau_a = (\tau_\phi, \tau_\theta, \tau_\psi)^T$	control torque
$G_a = (G_\phi, G_\theta, G_\psi)^T$	gyroscopic torques
I_r	moment of inertia of rotor
$\tau_i = (\tau_1 \ldots \tau_4)^T$	rotor torques
$\Omega_d = (\Omega_{d_1} \ldots \Omega_{d_3})^T$	desired angular velocity
t_q	regulation settling time
α	regulation settling time parameter
t_Ω	angular velocity tracking settling time
λ	angular velocity tracking parameter
a	boundary layer width for t_Ω
$w_{d_i} = (w_{d_1} \ldots w_{d_4})^T$	desired angular speed

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