

# **MULTI-OBJECTIVE CONTROLLER SYNTHESIS FOR MIMO SYSTEM**

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**THESIS**

**Submitted to:**

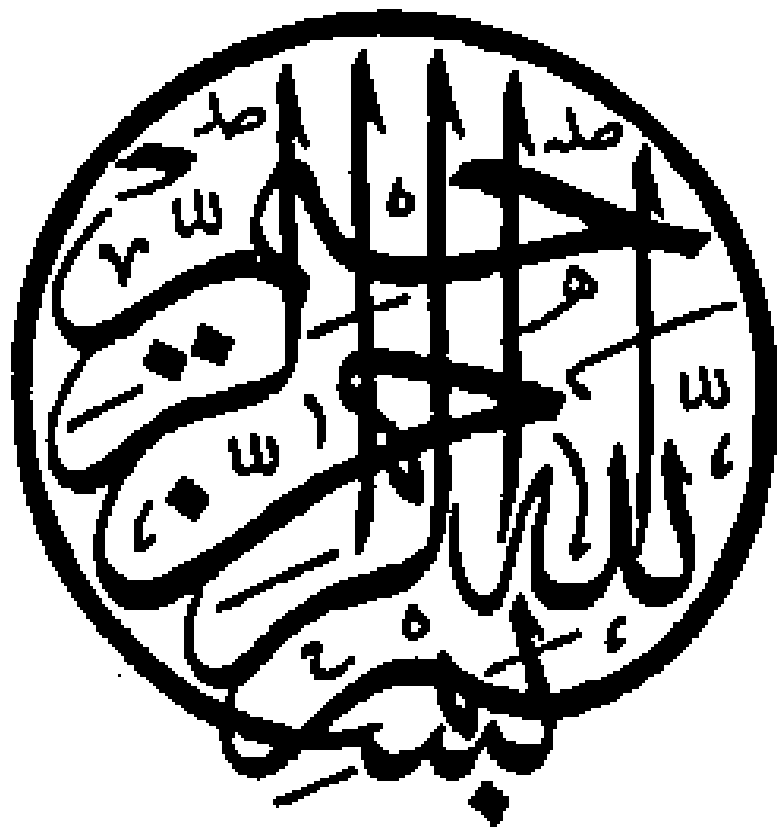
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## ABSTRACT

The aim of this work is to study and understanding of multi-objective optimization problem. In this work the optimization problem include  $H_2$  and  $H_\infty$  of some closed loop transfer function with Pole constraints. Multi-objective  $H_2/H_\infty$  state feedback control with Pole constraints is an approach for synthesizing multi-objective controller for MIMO system. In this technique design task such as disturbance rejection, robust stabilization of systems with uncertainty can be expressed by  $H_\infty/H_2$  performance and by placing the closed loop poles in some region of left-Half plane one can achieve the desired transient response. This thesis covers synthesizing multi-objective controller for pitch attitude hold autopilot of an aircraft. MATLAB software is used for simulation and verification of performance.

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