DESIGN OF A NONLINEAR OUTPUT FEEDBACK CONTROLLER FOR A HEAVY WEIGHT TORPEDO

Submitted by:

Malik M Usman Adeel

Supervised by:

Cdr. Dr. Attaullah Y.Memon PN

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Submitted by:

Malik M Usman Adeel(MSEE Control) (Reg No NUST201260495MPNEC45012F)

Supervised by:

Cdr. Dr. Attaullah Y. Memon PN

Assistant Professor

Guidance and Examination Committee:

Capt Dr Junaid Khan TI(M) PN

Assistant Professor

Cdr. Dr. Syed Sajjad Haider Zaidi PN

Assistant Professor

Cdr Dr Hammad Raza PN

Assistant Professor

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ABSTRACT

This thesis report presents a nonlinear robust output feedback control for a class of autonomous underwater vehicles. In specific, we consider the nonlinear mathematical model of a heavy-weight torpedo and design a sliding mode control to achieve robust stabilization in the presence of parametric uncertainties and model perturbations. The closed-loop analysis using Lyapunov methods is provided. The state feedback sliding mode control is extended to an output feedback design by using a fourth order observer for diving plane. It is shown that the proposed output feedback controller recovers the performance of the state feedback arbitrarily fast in the presence of parametric uncertainties and model perturbations. Then the nonlinear model of the system is transformed in to normal form and nonlinear control is proposed for path tracking in diving plane. Simulation results are provided which show the performance of the proposed control design.

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Chapter 1

INTRODUCTION

1.1 Introduction

With the advancement in automatic control techniques and technology, Autonomous Underwater Vehicles (AUVs) are becoming more popular due mainly to their usability for a variety of scientific, commercial as well as military applications. In particular, AUVs are usually deployed to explore unknown or hostile environments far too hazardous for humans or manned vehicles. Whenever an AUV cruises underwater, it is difficult to measure precisely the intrinsic hydrodynamic coefficients and the added mass-inertia terms acting on an AUV. Due to these complex and nondeterministic dynamics of an AUV, its control and navigation becomes a difficult and challenging task.

1.2 Literature Review

For the past two decades, researchers have presented various methods to achieve stabilization and tracking of these vehicles. In particular, [3] and [4] have presented higher order sliding mode control for autonomous underwater vehicles. In [5], multivariable sliding mode control for unmanned underwater vehicles has been presented. Thor I. Fossen, in his contributions [6] and [7], has provided detailed mathematical description and control approaches for various marine vehicles including submerged vehicles (AUVs), surface ships and other high speed crafts. From the control design viewpoint, usually we do not have access to all the states of the system since the measurement of the complete state vector is not feasible or economically viable as it involves various sensors and thus can result in an increased cost of the system. Moreover, measurement causes noise and can induce time delays associated with the sensor dynamics. To overcome these problems, usually an observer is used which estimates the required states for the control purpose. In [9], an observer for a 6 degrees-of-freedom AUV is proposed, which estimates the velocity of the vehicle.

1.3 Scope of the Thesis

In this thesis, we focus on design of an observer based control for a heavy-weight torpedo. A robust state feedback control is synthesized first using the sliding mode control technique, and the closed-loop stability analysis is provided. The state feedback design is then extended to output feedback using a 4th order linear observer which estimates the complete state vector, thus eliminating the requirement of any sensors for measurement of the state variables except for the output variable which is desired to be tracked. Stability of output feedback control is proved with Lyapunov method in error state space. Then the system nonlinear model is transformed in to normal form and nonlinear control based on state feedback and sliding mode control is proposed. In the end, proposed control techniques are validated through numerical simulation results.

1.4 Thesis Organization

The rest of the thesis is organized as follows. In Chapter II, the mathematical modelling of torpedo is presented and the problem formulation is given. Chapter III presents the robust control design using sliding mode control technique, the closed-loop stability analysis, the state feedback design is extended to output feedback using a linear observer and normal form transformation alongwith nonlinear control. Simulation results are provided in Chapter IV. Finally, Chapter V discusses the conclusions and presents the avenues for future work.

Chapter 2

MATHEMATICAL MODELLING

2.1 Equations of Motion

Traditionally submerged vehicles are modelled in terms of position and orientation frame work using 6 degrees of freedom i.e. surge, sway and heave in translational axis and roll, pitch and yaw in rotational axis as presented in [6]. The non-linear equations of motion is

$$\mathbf{M}\dot{\mathbf{v}} + \mathbf{C}(\mathbf{v})\mathbf{v} + \mathbf{D}(\mathbf{v})\mathbf{v} + \mathbf{G}(\eta) = \tau (2.1)$$

where

$\mathbf{M}$: Inertia and added inertia matrix

$\mathbf{C}$: Coriolis matrix

$\mathbf{D}$: Hydrodynamic damping terms matrix

$\mathbf{G}$: Gravity and buoyancy forces vector

$\mathbf{v}$: Vector of linear and angular velocities in the body fixed frame of torpedo given as

$$\mathbf{v} = \begin{bmatrix} \mu & v & w & p & q & r \end{bmatrix}^T \tag{2.2}$$

in which μ denotes surge, v denotes sway, w denotes heave, p denotes roll, q denotes pitch and r denotes yaw.

η : Vector of position and attitude in inertial frame of torpedo given as

$$\eta = \begin{bmatrix} x & y & z & \phi & \theta & \psi \end{bmatrix}^T \tag{2.3}$$

in which x, y, z are the positions and ϕ, θ, ψ are the orientations corresponding to surge, sway, heave and roll, pitch, yaw respectively.

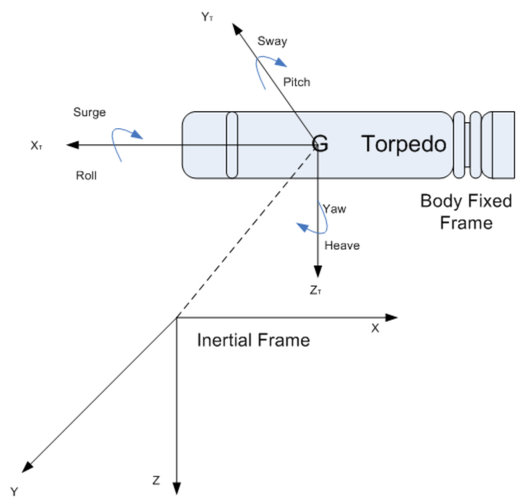

Figure 2.1: Inertial Frame and Body Fixed Frame

τ : Control input vector acting in the body fixed frame of torpedo given as

$$\tau = \begin{bmatrix} f(\delta_\eta) & f(\delta_s) & f(n) \end{bmatrix}^T = \begin{bmatrix} X & Y & Z & K & M & N \end{bmatrix}^T \tag{2.4}$$

in which δ_η denotes the diving plane angle, δ_s denotes the rudder angle and n denotes the number of propeller revolutions and defined by vector of control inputs and external forces i.e. X, Y, Z, K, M and N.

2.2 Mass and Inertia

We now proceed with the analysis of individual matrices. The matrix of inertia is given by

$$\mathbf{M} = M_{RB} + M_A \tag{2.5}$$

Where M_{RB} show the inertia terms of the rigid body and be termed as the actual mass of the body. M_A show the added inertia terms.Usually it is considered that the added mass is the amount of water connected to the outer surface of water which results in a new system having considerably larger mass than the actual rigid body mass of the underwater vechicle. This is a misconception and the added mass-inertia or virtual mass is the pressure-induced forces and moments due to a forced harmonic motion of the body which are proportional to the acceleration of the body [6]. The mass-inertia matrix for the rigid body is given as

$$M_{RB} = \begin{bmatrix} m & 0 & 0 & 0 & mz_G & -my_G \\ 0 & m & 0 & -mz_G & 0 & mx_G \\ 0 & 0 & m & my_G & -mx_G & 0 \\ 0 & -mz_G & my_G & I_x & -I_{xy} & -I_{xz} \\ mz_G & 0 & -mx_G & -I_{yx} & I_y & -I_{yz} \\ -my_G & mx_G & 0 & -I_{zx} & -I_{zy} & I_z \end{bmatrix} \tag{2.6}$$

and the added mass inertia matrix is given as

$$M_A = -\begin{bmatrix} \frac{\partial X}{\partial \dot{\mu}} & \frac{\partial X}{\partial \dot{v}} & \frac{\partial X}{\partial \dot{w}} & \frac{\partial X}{\partial \dot{p}} & \frac{\partial X}{\partial \dot{q}} & \frac{\partial X}{\partial \dot{r}} \\ \frac{\partial Y}{\partial \dot{\mu}} & \frac{\partial Y}{\partial \dot{v}} & \frac{\partial Y}{\partial \dot{w}} & \frac{\partial Y}{\partial \dot{p}} & \frac{\partial Y}{\partial \dot{q}} & \frac{\partial Y}{\partial \dot{r}} \\ \frac{\partial Z}{\partial \dot{\mu}} & \frac{\partial Z}{\partial \dot{v}} & \frac{\partial Z}{\partial \dot{w}} & \frac{\partial Z}{\partial \dot{p}} & \frac{\partial Z}{\partial \dot{q}} & \frac{\partial Z}{\partial \dot{r}} \\ \frac{\partial K}{\partial \dot{\mu}} & \frac{\partial K}{\partial \dot{v}} & \frac{\partial K}{\partial \dot{w}} & \frac{\partial K}{\partial \dot{p}} & \frac{\partial K}{\partial \dot{q}} & \frac{\partial K}{\partial \dot{r}} \\ \frac{\partial M}{\partial \dot{\mu}} & \frac{\partial M}{\partial \dot{v}} & \frac{\partial M}{\partial \dot{w}} & \frac{\partial M}{\partial \dot{p}} & \frac{\partial M}{\partial \dot{q}} & \frac{\partial M}{\partial \dot{r}} \\ \frac{\partial N}{\partial \dot{\mu}} & \frac{\partial N}{\partial \dot{v}} & \frac{\partial N}{\partial \dot{w}} & \frac{\partial N}{\partial \dot{p}} & \frac{\partial N}{\partial \dot{q}} & \frac{\partial N}{\partial \dot{r}} \end{bmatrix}$$

According to the notation used in SNAME (Society of Naval Architects and Marine Engineers,1950) [13],

$$X_{\dot{\mu}} = \frac{\partial X}{\partial \dot{\mu}} ......N_{\dot{r}} = \frac{\partial N}{\partial \dot{r}}$$

So we can write M_A as

$$M_A = -\begin{bmatrix} X_{\dot{\mu}} & X_{\dot{v}} & X_{\dot{w}} & X_{\dot{p}} & X_{\dot{q}} & X_{\dot{r}} \\ Y_{\dot{\mu}} & Y_{\dot{v}} & Y_{\dot{w}} & Y_{\dot{p}} & Y_{\dot{q}} & Y_{\dot{r}} \\ Z_{\dot{\mu}} & Z_{\dot{v}} & Z_{\dot{w}} & Z_{\dot{p}} & Z_{\dot{q}} & Z_{\dot{r}} \\ K_{\dot{\mu}} & K_{\dot{v}} & K_{\dot{w}} & K_{\dot{p}} & K_{\dot{q}} & K_{\dot{r}} \\ M_{\dot{\mu}} & M_{\dot{v}} & M_{\dot{w}} & M_{\dot{p}} & M_{\dot{q}} & M_{\dot{r}} \\ N_{\dot{\mu}} & N_{\dot{v}} & N_{\dot{w}} & N_{\dot{p}} & N_{\dot{q}} & N_{\dot{r}} \end{bmatrix} \tag{2.7}$$

Adding the mass-inertia matrix of rigid body (2.6) and the added mass-inertia matrix

(2.7) yields

$$\mathbf{M} = \begin{bmatrix} m - X_{\dot{\mu}} & -X_{\dot{v}} & -X_{\dot{w}} & -X_{\dot{p}} & mz_G - X_{\dot{q}} & -my_G - X_{\dot{q}} \\ -Y_{\dot{\mu}} & m - Y_{\dot{v}} & -Y_{\dot{w}} & -mz_G - Y_{\dot{p}} & -Y_{\dot{q}} & mx_G - Y_{\dot{r}} \\ -Z_{\dot{\mu}} & -Z_{\dot{v}} & m - Z_{\dot{w}} & my_G - Z_{\dot{p}} & -mx_G - Z_{\dot{q}} & -Z_{\dot{r}} \\ -K_{\dot{\mu}} & -mz_G - K_{\dot{v}} & my_G - K_{\dot{w}} & I_x - K_{\dot{p}} & -I_{xy} - K_{\dot{p}} & -I_{xz} - K_{\dot{r}} \\ mz_G - M_{\dot{\mu}} & -M_{\dot{v}} & -mx_G - M_{\dot{w}} & -I_{yx} - M_{\dot{p}} & I_y - M_{\dot{p}} & -I_{yz} - M_{\dot{r}} \\ -my_G - N_{\dot{\mu}} & mx_G - N_{\dot{v}} & -N_{\dot{w}} & -I_{zx} - N_{\dot{p}} & -I_{zy} - N_{\dot{p}} & I_z - N_{\dot{r}} \end{bmatrix} \tag{2.8}$$

2.3 Coriolis and Centripetal Force

The matrix of coriolis and centripetal terms is given as

$$\mathbf{C} = C_{RB} + C_A \tag{2.9}$$

Where C_{RB} show coriolis and centripetal terms of the rigid body and C_A show the added terms and given by;

$$C_{RB} = \left[\begin{matrix} 0 & 0 & 0 & m(y_G q + z_G r) \\ 0 & 0 & 0 & -m(y_G p + w) \\ 0 & 0 & 0 & -m(z_G p - v) \\ -m(y_G q + z_G r) & m(y_G p + w) & m(z_G p - v) & 0 \\ m(x_G q - w) & -m(z_G r + x_G p) & m(z_G q + \mu) & I_{yx} q + I_{xx} p - I_z r \\ m(x_G r + v) & m(y_G r - \mu) & -m(x_G p + y_G q) & -I_{yz} r - I_{xy} p + I_y q \end{matrix} \right.$$

$$
\begin{matrix}
-m(x_G q - w) & -m(x_G r + v) \\
m(z_G r + x_G p) & -m(y_G r - \mu) \\
-m(z_G q + \mu) & m(x_G p + y_G q) \\
-I_{yx} q - I_{xx} p + I_z r & I_{yz} r + I_{xy} p - I_y q \\
0 & -I_{xx} r - I_{xy} q + I_x p \\
I_{xx} r + I_{xy} q - I_x p & 0
\end{matrix}
\Bigg] \tag{2.10}
$$

and

$$
C_A = \begin{bmatrix}
0 & 0 & 0 & 0 & -\gamma_3 & \gamma_2 \\
0 & 0 & 0 & \gamma_3 & 0 & -\gamma_1 \\
0 & 0 & 0 & -\gamma_2 & \gamma_1 & 0 \\
0 & -\gamma_3 & \gamma_2 & 0 & -\gamma_6 & \gamma_5 \\
\gamma_3 & 0 & -\gamma_1 & \gamma_6 & 0 & -\gamma_4 \\
-\gamma_2 & \gamma_1 & 0 & -\gamma_5 & \gamma_4 & 0
\end{bmatrix} \tag{2.11}
$$

where;

$$\gamma_1 = X_{\dot{\mu}}\mu + X_{\dot{v}}v + X_{\dot{w}}w + X_{\dot{p}}p + X_{\dot{q}}q + X_{\dot{r}}r$$

$$\gamma_2 = X_{\dot{v}}\mu + Y_{\dot{v}}v + Y_{\dot{w}}w + Y_{\dot{p}}p + Y_{\dot{q}}q + Y_{\dot{r}}r$$

$$\gamma_3 = X_{\dot{w}}\mu + Y_{\dot{w}}v + Z_{\dot{w}}w + Z_{\dot{p}}p + Z_{\dot{q}}q + Z_{\dot{r}}r$$

$$\gamma_4 = X_{\dot{p}}\mu + Y_{\dot{p}}v + Z_{\dot{p}}w + K_{\dot{p}}p + K_{\dot{q}}q + M_{\dot{r}}r$$

$$\gamma_5 = X_{\dot{q}}\mu + Y_{\dot{q}}v + Z_{\dot{q}}w + K_{\dot{q}}p + M_{\dot{q}}q + M_{\dot{r}}r$$

$$\gamma_6 = X_{\dot{r}}\mu + Y_{\dot{r}}v + Z_{\dot{r}}w + K_{\dot{r}}p + M_{\dot{r}}q + N_{\dot{r}}r$$

Adding both the matrices (2.10) and (2.11) yields

$$\mathbf{C} = \begin{bmatrix} 0 & 0 & 0 & m(y_G q + z_G r) & -m(x_G q - w) - \gamma_3 & -m(x_G r + v) + \gamma_2 \\ 0 & 0 & 0 & -m(y_G p + w) + \gamma_3 & m(z_G r + x_G p) & -m(y_G r - \mu) - \gamma_1 \\ 0 & 0 & 0 & -m(z_G p - v) - \gamma_2 & -m(z_G q + \mu) + \gamma_1 & m(x_G p + y_G q) \\ -m(y_G q + z_G r) & m(y_G p + w) - \gamma_3 & m(z_G p - v) + \gamma_2 & 0 & -I_{yx} q - I_{xx} p + I_z r - \gamma_6 & I_{yz} r + I_{xy} p - I_y q + \gamma_5 \\ m(x_G q - w) + \gamma_3 & -m(z_G r + x_G p) & m(z_G q + \mu) - \gamma_1 & I_{yx} q + I_{xx} p - I_z r + \gamma_6 & 0 & -I_{xx} r - I_{xy} q + I_x p - \gamma_4 \\ m(x_G r + v) - \gamma_2 & m(y_G r - \mu) + \gamma_1 & -m(x_G p + y_G q) & -I_{yz} r - I_{xy} p + I_y q - \gamma_5 & I_{xx} r + I_{xy} q - I_x p + \gamma_4 & 0 \end{bmatrix} \tag{2.12}$$

2.4 Hydrodynamic Damping

The hydrodynamic damping terms severely affects the movement of vehicle at higher speeds and its effect can nott be neglected. It mainly consists of drag and lift forces

$$\mathbf{D} = D_{Drag} + D_{Lift} \tag{2.13}$$

At lower speeds the effect of lift force is negligible as compared to the drag force, so it can be neglected. The drag force is consists of two parts, the linear drag force and the quadratic or nonlinear drag force

$$D_{Drag} = D_{Linear} + D_{Quadratic} \tag{2.14}$$

[8] has presented the matrices for D_{Linear} and $D_{Quadratic}$ which are as

$$D_{Linear} = \begin{bmatrix} X_\mu & 0 & 0 & 0 & 0 & 0 \\ 0 & Y_v & 0 & 0 & 0 & 0 \\ 0 & 0 & Z_w & 0 & 0 & 0 \\ 0 & 0 & 0 & K_p & 0 & 0 \\ 0 & 0 & 0 & 0 & M_q & 0 \\ 0 & 0 & 0 & 0 & 0 & N_r \end{bmatrix} \tag{2.15}$$

$$D_{Quadratic} = -\begin{bmatrix} X_{\mu|\mu|}|\mu| & 0 & 0 & 0 & 0 & 0 \\ 0 & Y_{v|v|}|v| & 0 & 0 & 0 & 0 \\ 0 & 0 & Z_{w|w|}|w| & 0 & 0 & 0 \\ 0 & 0 & 0 & K_{p|p|}|p| & 0 & 0 \\ 0 & 0 & 0 & 0 & M_{q|q|}|q| & 0 \\ 0 & 0 & 0 & 0 & 0 & N_{r|r|}|r| \end{bmatrix} \tag{2.16}$$

now we can write (2.13) as

$$\mathbf{D} = \begin{bmatrix} X_\mu - X_{\mu|\mu|}|\mu| & 0 & 0 & 0 & 0 & 0 \\ 0 & Y_v - Y_{v|v|}|v| & 0 & 0 & 0 & 0 \\ 0 & 0 & Z_w - Z_{w|w|}|w| & 0 & 0 & 0 \\ 0 & 0 & 0 & K_p - K_{p|p|}|p| & 0 & 0 \\ 0 & 0 & 0 & 0 & M_q - M_{q|q|}|q| & 0 \\ 0 & 0 & 0 & 0 & 0 & N_r - N_{r|r|}|r| \end{bmatrix} \tag{2.17}$$

2.5 Gravitational Moments and Restoring Forces

In the terminology of hydrodynamics, restoring force consists of gravitational and buoyant forces. Let m be the mass of the submerged vehicle, ∇ be the fluid volume displaced by the vehicle, g be the gravitational acceleration taken positive for downward movement and ρ be the density of the fluid. Since the weight is defined as $W = mg$ and the buoyancy as $B = \rho g \nabla$. Then according to [13] SNAME [1950], the vector of gravitational moments and restoring forces is given as

$$g(\eta) = \begin{bmatrix} (W-B)\sin\theta \\ -(W-B)\cos\theta\sin\phi \\ -(W-B)\cos\theta\cos\phi \\ -(y_G W - y_B B)\cos\theta\cos\phi + (z_G W - z_B B)\cos\theta\sin\phi \\ (z_G W - z_B B)\sin\theta + (x_G W - x_B B)\cos\theta\cos\phi \\ -(x_G W - x_B B)\cos\theta\sin\phi - (y_G W - y_B B)\sin\theta \end{bmatrix} \tag{2.18}$$

For any vehicle to be neutrally buoyant, its weight and buoyancy must be equal. In this case the vehicle will not move up or down as in the case of positive or negative buoyancy respectively. If the buoyancy and the weight are equal, also the geometric center lying at the gravitational center of the vehicle, then $g(\eta)$ can be neglected for obvious reasons.

2.6 Symmetry of Geometrical Shape

While cruising at higher speeds the AUVs have very high degree of coupling between the added mass, coriolis terms and hydrodynamic damping terms. In this scope of work, it is considered that the torpedo will maneuver at relatively slower speed with maximum speed of 10 knots and it has gravitational and geometric symmetry in all three planes i.e. x, y and z , thus, the center of gravity coincides with the center of geometry of the vehicle. This assumption suggests that the off-diagonal terms in the matrix of added mass M_A, added coriolis terms C_A and matrix of damping terms **D** and terms $x_G, y_G, z_G, x_B, y_B \& z_B$ can be neglected.

The simplified mass inertia matrix (2.6) will become

$$M_{RB} = \begin{bmatrix} m & 0 & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 \\ 0 & 0 & 0 & I_x & -I_{xy} & -I_{xz} \\ 0 & 0 & 0 & -I_{yx} & I_y & -I_{yz} \\ 0 & 0 & 0 & -I_{zx} & -I_{zy} & I_z \end{bmatrix} \tag{2.19}$$

The simplified matrix of added mass (2.7) will thus become

$$M_A = - \begin{bmatrix} X_{\dot{\mu}} & 0 & 0 & 0 & 0 & 0 \\ 0 & Y_{\dot{v}} & 0 & 0 & 0 & 0 \\ 0 & 0 & Z_{\dot{w}} & 0 & 0 & 0 \\ 0 & 0 & 0 & K_{\dot{p}} & 0 & 0 \\ 0 & 0 & 0 & 0 & M_{\dot{q}} & 0 \\ 0 & 0 & 0 & 0 & 0 & N_{\dot{r}} \end{bmatrix} \tag{2.20}$$

or

$$M_A = -diag\{X_{\dot{\mu}}, Y_{\dot{v}}, Z_{\dot{w}}, K_{\dot{p}}, M_{\dot{q}}, N_{\dot{r}}\}$$

Using (2.19) and (2.20) in (2.5), (2.8) will thus become

$$
\mathbf{M} = \begin{bmatrix} m - X_{\dot{\mu}} & 0 & 0 & 0 & 0 & 0 \\ 0 & m - Y_{\dot{v}} & 0 & 0 & 0 & 0 \\ 0 & 0 & m - Z_{\dot{w}} & 0 & 0 & 0 \\ 0 & 0 & 0 & I_x - K_{\dot{p}} & -I_{xy} & -I_{xz} \\ 0 & 0 & 0 & -I_{yx} & I_y - M_{\dot{p}} & -I_{yz} \\ 0 & 0 & 0 & -I_{zx} & -I_{zy} & I_z - N_{\dot{r}} \end{bmatrix} \tag{2.21}
$$

The simplified coriolis and centripetal force terms matrix (2.10) will become

$$
C_{RB} = \begin{bmatrix} 0 & 0 & 0 & 0 & mw & -mv \\ 0 & 0 & 0 & -mw & 0 & m\mu \\ 0 & 0 & 0 & mv & -m\mu & 0 \\ 0 & mw & -mv & 0 & -I_{yx}q - I_{xx}p + I_z r & I_{yz}r + I_{xy}p - I_y q \\ -mw & 0 & m\mu & I_{yx}q + I_{xx}p - I_z r & 0 & -I_{xx}r - I_{xy}q + I_x p \\ mv & -m\mu & 0 & -I_{yz}r - I_{xy}p + I_y q & I_{xx}r + I_{xy}q - I_x p & 0 \end{bmatrix} \tag{2.22}
$$

The simplified matrix of added coriolis and centripetal force terms (2.11) will thus become

$$
C_A = \begin{bmatrix} 0 & 0 & 0 & 0 & -Z_{\dot{w}}w & Y_{\dot{v}}v \\ 0 & 0 & 0 & Z_{\dot{w}}w & 0 & -X_{\dot{\mu}}\mu \\ 0 & 0 & 0 & -Y_{\dot{v}}v & X_{\dot{\mu}}\mu & 0 \\ 0 & -Z_{\dot{w}}w & Y_{\dot{v}}v & 0 & -N_{\dot{r}}r & M_{\dot{q}}q \\ Z_{\dot{w}}w & 0 & -X_{\dot{\mu}}\mu & N_{\dot{r}}r & 0 & -K_{\dot{p}}p \\ -Y_{\dot{v}}v & X_{\dot{\mu}}\mu & 0 & -M_{\dot{q}}q & K_{\dot{p}}p & 0 \end{bmatrix} \tag{2.23}
$$

Using (2.22) and (2.23) in(2.9), (2.12) will thus become

$$\mathbf{C} = \begin{bmatrix} 0 & 0 & 0 & 0 & mw - Z_{\dot{w}}w & -mv + Y_{\dot{v}}v \\ 0 & 0 & 0 & -mw + Z_{\dot{w}}w & 0 & m\mu - X_{\dot{\mu}}\mu \\ 0 & 0 & 0 & mv - Y_{\dot{v}}v & -m\mu + X_{\dot{\mu}}\mu & 0 \\ 0 & mw - Z_{\dot{w}}w & -mv + Y_{\dot{v}}v & 0 & -I_{yx}q - I_{xx}p + I_z r - N_{\dot{r}}r & I_{yz}r + I_{xy}p - I_y q + M_{\dot{q}}q \\ -mw + Z_{\dot{w}}w & 0 & m\mu - X_{\dot{\mu}}\mu & I_{yx}q + I_{xx}p - I_z r + N_{\dot{r}}r & 0 & -I_{xx}r - I_{xy}q + I_x p - K_{\dot{p}}p \\ mv - Y_{\dot{v}}v & -m\mu + X_{\dot{\mu}}\mu & 0 & -I_{yz}r - I_{xy}p + I_y q - M_{\dot{q}}q & I_{xx}r + I_{xy}q - I_x p K_{\dot{p}}p & 0 \end{bmatrix} \tag{2.24}$$

And the matrix for the hydrodynamic damping terms matrix (2.17) while considering only the linear drag force, will thus become

$$\mathbf{D} = \begin{bmatrix} X_\mu & 0 & 0 & 0 & 0 & 0 \\ 0 & Y_v & 0 & 0 & 0 & 0 \\ 0 & 0 & Z_w & 0 & 0 & 0 \\ 0 & 0 & 0 & K_p & 0 & 0 \\ 0 & 0 & 0 & 0 & M_q & 0 \\ 0 & 0 & 0 & 0 & 0 & N_r \end{bmatrix} \tag{2.25}$$

Similarly, for any vehicle to be neutrally buoyant, its weight and buoyancy must be equal. If the buoyancy and the weight are equal and the geometric center lying at the gravitational center of the vehicle, then $g(\eta)$ can be neglected for obvious reasons.

2.7 Derivation of Kinematic Dynamic Model

Thor I. Fossen [6] has presented a generalized form for representing the mathematical model of AUVs which is as under

$$\dot{\mathbf{X}} = f(x) + g(x)u$$

$$\dot{\mathbf{X}} = \begin{bmatrix} -\mathbf{M}^{-1}\left[\mathbf{C}+\mathbf{D}\right] & -\mathbf{M}^{-1}G \\ J & 0 \end{bmatrix} \mathbf{X} + \begin{bmatrix} \mathbf{M}^{-1} \\ 0 \end{bmatrix} u \tag{2.26}$$

Where $\mathbf{M}$ is the mass inertia matrix, $\mathbf{C}$ is the coriolis and centripetal terms matrix, $\mathbf{D}$ is the hydrodynamic damping terms matrix, J is the kinematic transformation matrix, G is the differential of vector of gravitational moments and restoring forces $g(\eta)$ with respect to the vector of position and altitude η and $u = \tau$ i.e. the control input vector. The description of J is given below

$$J = \begin{bmatrix} J_1 & 0 \\ 0 & J_2 \end{bmatrix} \tag{2.27}$$

in which

$$J_1(\phi,\theta,\psi) = \begin{bmatrix} \cos\psi\cos\theta & -\sin\psi\cos\phi + \cos\psi\sin\theta\sin\phi & \sin\psi\sin\phi + \cos\psi\cos\phi\sin\theta \\ \sin\psi\cos\theta & \cos\psi\cos\phi + \sin\phi\sin\theta\sin\psi & -\cos\psi\sin\phi + \sin\theta\sin\psi\cos\phi \\ -\sin\theta & \cos\theta\sin\phi & \cos\theta\cos\phi \end{bmatrix} \tag{2.28}$$

and

$$J_2(\phi,\theta,\psi) = \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi/\cos\theta & \cos\phi/\cos\theta \end{bmatrix} \tag{2.29}$$

2.7.1 Dynamic Model

Using (2.21),(2.24),(2.25) and (2.27) in (2.26) will results in the 6 degrees of freedom nonlinear equations of motion for surge, sway, heave, roll, pitch and yaw respectively

$$m[\dot{\mu} - vr + wq] = X \tag{2.30}$$

$$m[\dot{v} + \mu r - wp] = Y \tag{2.31}$$

$$m[\dot{w} - \mu q + vp] = Z \tag{2.32}$$

$$I_x\dot{p} + (I_z - I_y)qr + I_{xy}(pr - \dot{q}) - I_{yz}(q^2 - r^2) - I_{xz}(pq + \dot{r}) = K \tag{2.33}$$

$$I_y\dot{q} + (I_z - I_x)pr - I_{xy}(qr + \dot{p}) + I_{yz}(pq - \dot{r}) + I_{xz}(p^2 - r^2) = M \tag{2.34}$$

$$I_z\dot{r} + (I_y - I_x)pq - I_{xy}(p^2 - q^2) - I_{yz}(pr + \dot{q}) + I_{xz}(qr - \dot{p}) = N \tag{2.35}$$

2.7.2 Kinematic Model

The motion of autonomous underwater vehicle in body fixed frame is defined relative to inertial frame, however acceleration on point of earth surface is negligible in comparison with inertial frame. Therefore, in this case, earth fixed frame is considered as inertial frame. Moreover, v i.e. vector of linear and angular velocities is measured with respect to body fixed frame and η i.e. the vector of positions and orientations is measured with respect to inertial frame. Thus, in order to have same reference frame and simplify the measurements, we transform η i.e. the vector of positions and orientations from earth fixed frame to body fixed by the following transformation

$$\dot{\eta} = J(\eta)v \tag{2.36}$$

where J is kinematic transformation matrix and is given by (2.27). Subsituting values of J, η (2.3) and v (2.2) in (2.36), we get

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} \cos\psi\cos\theta & -\sin\psi\cos\phi + \cos\psi\sin\theta\sin\phi & \sin\psi\sin\phi + \cos\psi\cos\phi\sin\theta & 0 & 0 & 0 \\ \sin\psi\cos\theta & \cos\psi\cos\phi + \sin\phi\sin\theta\sin\psi & -\cos\psi\sin\phi + \sin\theta\sin\psi\cos\phi & 0 & 0 & 0 \\ -\sin\theta & \cos\theta\sin\phi & \cos\theta\cos\phi & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & 0 & 0 & 0 & \cos\phi & -\sin\phi \\ 0 & 0 & 0 & 0 & \sin\phi/\cos\theta & \cos\phi/\cos\theta \end{bmatrix} \begin{bmatrix} \mu \\ v \\ w \\ p \\ q \\ r \end{bmatrix} \tag{2.37}$$

The kinematics equations of AUV will thus become

$$\begin{aligned} \dot{x} &= \mu\cos\psi\cos\theta - v(\sin\psi\cos\phi + \cos\psi\sin\theta\sin\phi) \\ &\quad + w(\sin\psi\sin\phi + \cos\psi\cos\phi\sin\theta) \qquad (2.38) \\ \dot{y} &= \mu\sin\psi\cos\theta + v(\cos\psi\cos\phi + \sin\phi\sin\theta\sin\psi) \\ &\quad - w(\cos\psi\sin\phi + \sin\theta\sin\psi\cos\phi) \qquad (2.39) \end{aligned}$$

$$\dot{z} = -\mu \sin\theta + v\cos\theta\sin\phi + w\cos\theta\cos\phi \tag{2.40}$$

$$\dot{\phi} = p + q\sin\phi\tan\theta + r\cos\phi\tan\theta \tag{2.41}$$

$$\dot{\theta} = q\cos\phi - r\sin\phi \tag{2.42}$$

$$\dot{\psi} = q\frac{\sin\phi}{\cos\theta} + r\frac{\cos\phi}{\cos\theta} \tag{2.43}$$

Since the shape of torpedo is symmetric in the x-y and x-z planes, we can decouple its motion in the steering and diving planes. Thus, torpedo's motion can be studied separately in the steering and diving planes.

2.8 System Model in Diving Plane

Here we are considering motion of torpedo in diving plane only so the terms which are related to motion in steering plane are not considered and forward speed i.e. surge μ is considered to be constant i.e. $\mu = \mu_o$. Specifically, we can say

$$p = \phi = r = \psi = v = y = 0 \tag{2.44}$$

The dynamic model and the kinematics model defined by the set of equations (2.30) - (2.35) and (2.38) - (2.43) is thus simplified and will result in the following equations

$$m[\dot{\mu}_o + wq] = X \tag{2.45}$$

$$m[\dot{w} - \mu_o q] = Z \tag{2.46}$$

$$I_y\dot{q} = M \tag{2.47}$$

$$\dot{x} = \mu_o\cos\theta + w\sin\theta \tag{2.48}$$

$$\dot{z} = -\mu_o\sin\theta + w\cos\theta \tag{2.49}$$

$$\dot{\theta} = q \tag{2.50}$$

According to [3], the effect of (2.45) and (2.48) on motion of torpedo in diving plane is negligble and thus these equations can be decoupled from the system. Moreover, in [3], vector of control inputs and external forces Z and M are given as;

$$\begin{aligned} Z &= Z_{\dot{w}}\dot{w} - X_{\dot{\mu_o}}\mu_o q + Z_w w + Z_q q + Z_{\delta\eta}\delta_\eta && (2.51) \\ M &= M_{\dot{q}}\dot{q} + M_w w + M_q q + M_{\delta\eta}\delta_\eta && (2.52) \end{aligned}$$

The notion of [13] is used in above expressions. Thus, the set of equations (2.45) - (2.50) becomes

$$\begin{aligned} m[\dot{w} - \mu_o q] &= Z_{\dot{w}}\dot{w} - X_{\dot{\mu_o}}\mu_o q + Z_w w + Z_q q + Z_{\delta\eta}\delta_\eta \\ I_y\dot{q} &= M_{\dot{q}}\dot{q} + M_w w + M_q q + M_{\delta\eta}\delta_\eta \\ \dot{z} &= -\mu_o \sin\theta + w\cos\theta \\ \dot{\theta} &= q \end{aligned}$$

Rearranging and solving above for $\dot{w}, \dot{q}, \dot{\theta}$ and $\dot{z}$, we get the desired nonlinear model of the torpedo in the diving plane

$$\begin{aligned} \dot{w} &= \frac{Z_w}{m - Z_{\dot{w}}}w + \frac{m\mu_o - X_{\dot{\mu}}\mu_o + Z_q}{m - Z_{\dot{w}}}q \\ &\quad + \frac{Z_{\delta\eta}}{m - Z_{\dot{w}}}\delta_\eta \\ \dot{q} &= \frac{M_w}{I_y - M_{\dot{q}}}w + \frac{M_q}{I_y - M_{\dot{q}}}q && (2.53) \\ &\quad + \frac{M_{\delta\eta}}{I_y - M_{\dot{q}}}\delta_\eta \\ \dot{\theta} &= q \\ \dot{z} &= w\cos\theta - \mu_o \sin\theta \end{aligned}$$

In order to simplify the calculations while keeping the main features of torpedo, we can linearize the system for design of control law instead of using nonlinear model. The linearizing the model at $\theta \approx 0$, the state space form of torpedo's model in the diving plane is given as;

$$\dot{\mathcal{X}} = A\mathcal{X} + Bu \tag{2.54}$$

The state vector $\mathcal{X}$, and the matrices A and B are described by

$$\mathcal{X} = [w \quad q \quad \theta \quad z]^T$$

$$A = \begin{pmatrix} \frac{Z_w}{m-Z_{\dot{w}}} & \frac{m\mu_O - X_{\dot{\mu}}\mu_O + Z_q}{m-Z_{\dot{w}}} & 0 & 0 \\ \frac{M_w}{I_y - M_{\dot{q}}} & \frac{M_q}{I_y - M_{\dot{q}}} & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & -\mu_o & 0 \end{pmatrix}$$

$$B = \begin{pmatrix} \frac{Z_{\delta_\eta}}{m-Z_{\dot{w}}} \\ \frac{M_{\delta_\eta}}{I_y - M_{\dot{q}}} \\ 0 \\ 0 \end{pmatrix}$$

and $u = \delta_\eta$ is the diving plane angle. In the above derivation, surge μ of the torpedo is considered to be constant i.e. $\mu = \mu_o$. For the purpose of designing control u for the system in the next section, it has been verified that the pair (A, B) is controllable.

Chapter 3

CONTROL DESIGN AND ANALYSIS

3.1 State Feedback Sliding Mode Control

In this section, we design the Sliding Mode Control (SMC) for stabilization of the system and provide closed loop stability analysis. The proposed control design approach is based on the methodology presented in [3] and proceeds as follows. The proposed control is composed of two parts, the nominal control or stabilizing control $\hat{u}$, and non-linear control or tracking control $\bar{u}$.

To proceed with the control design, we consider the torpedo's linear model;

$$\dot{\mathcal{X}} = A\mathcal{X} + Bu \tag{3.1}$$

The control objective is that the torpedo states $\mathcal{X}$ tracks the desired reference $\mathcal{X}_d$.

Towards that end, we define the error state vector as

$$\tilde{\mathcal{X}} = \mathcal{X} - \mathcal{X}_d = \begin{bmatrix} x_1 - x_{1d} \\ \vdots \\ x_n - x_{nd} \end{bmatrix} \tag{3.2}$$

where $\mathcal{X}_d$ is the desired state vector.

The sliding surface is defined as;

$$\sigma = S^T \mathcal{X} \tag{3.3}$$

In the error coordinates, the sliding surface can be rewritten as;

$$\sigma = S^T \tilde{\mathcal{X}} \tag{3.4}$$

or

$$\sigma = \begin{bmatrix} s_1 & \cdots & s_n \end{bmatrix} \begin{bmatrix} x_1 - x_{1d} \\ \vdots \\ x_n - x_{nd} \end{bmatrix} \tag{3.5}$$

Our first objective will be to choose S, such that $\lim_{t\to\infty} \dot{\sigma} \to 0$ i.e. $\lim_{t\to\infty} \sigma \to 0$, which will ensure that $\lim_{t\to\infty} \tilde{\mathcal{X}} = \lim_{t\to\infty}(\mathcal{X} - \mathcal{X}_d) \to 0$. Towards that end, consider the Lyapunov function

$$V(\sigma) = \frac{1}{2}\sigma^2 \tag{3.6}$$

In order to determine the surface coefficients S which achieves the aforementioned objective, we need to determine conditions under which $\dot{V}(\sigma)$ can be rendered negative definite, e.g.

$$\dot{V}(\sigma) = \sigma\dot{\sigma} \leq -\eta^2 |\sigma|^2 \tag{3.7}$$

for some $\eta > 0$, where η is a design parameter. The condition (3.7) can be further written as

$$\dot{\sigma} \leq -\eta^2 sgn(\sigma) \tag{3.8}$$

which will ensure the convergence of system trajectories to the sliding surface in finite time. Towards that end, we proceed by differentiating the sliding surface defined in (3.5) along the trajectories of the system, we get the following expression

$$\dot{\sigma} = S^T\dot{\tilde{\mathcal{X}}} = S^T(A\mathcal{X} + Bu - \dot{\mathcal{X}}_d) \leq -\eta^2 sgn(\sigma) \tag{3.9}$$

or

$$\begin{aligned} S^T A\mathcal{X} + S^T Bu - S^T\dot{\mathcal{X}}_d &\leq -\eta^2 sgn(\sigma) \\ S^T Bu &\leq -S^T A\mathcal{X} - S^T\dot{\mathcal{X}}_d - \eta^2 sgn(\sigma) \\ u &\leq \frac{-S^T A\mathcal{X} - S^T\dot{\mathcal{X}}_d - \eta^2 sgn(\sigma)}{S^T B} \end{aligned}$$

or

$$u \leq -(S^T B)^{-1}S^T A\mathcal{X} - (S^T B)^{-1}S^T\dot{\mathcal{X}}_d - (S^T B)^{-1}\eta^2 sgn(\sigma) \tag{3.10}$$

We can split the above equation in two parts, i.e. the nominal control and the non-linear control.

$$u \leq \hat{u} + \bar{u} \tag{3.11}$$

where the nominal control is given by

$$\hat{u} = -(S^T B)^{-1} S^T A \mathcal{X} - (S^T B)^{-1} S^T \dot{\mathcal{X}}_d \tag{3.12}$$

and the switching controller is given by

$$\bar{u} = -(S^T B)^{-1} \eta^2 sgn(\sigma) \tag{3.13}$$

When $\mathcal{X}_d$ is constant, the equation (3.12) can be simplified as

$$\hat{u} = -(S^T B)^{-1} S^T A \mathcal{X} = -K \mathcal{X} \tag{3.14}$$

In order to ensure that the trajectories of the closed loop system under nominal control (3.14) converge to the sliding surface i.e. $\lim_{t\to\infty} \sigma \to 0$; the feedback gain matrix K is chosen such as to place the eigenvalues of the closed loop system at $\begin{bmatrix} 0 & \lambda_2 & \lambda_3 & \cdots & \lambda_n \end{bmatrix}$. This yields the closed loop system which can be represented by the following equations

$$\begin{aligned} \dot{\mathcal{X}} &= A\mathcal{X} + Bu \\ \dot{\mathcal{X}} &= (A - BK)\mathcal{X} \end{aligned}$$

since $\sigma = S^T \tilde{\mathcal{X}} = 0$ and $\dot{\sigma} = S^T \dot{\tilde{\mathcal{X}}} = 0$

Then, $\dot{\sigma} = S^T \dot{\mathcal{X}} = S^T (A - BK)\mathcal{X} = 0$, which can be further written as

$$S^T (A - BK) = 0 \Rightarrow (A - BK)^T S = 0 \tag{3.15}$$

It can be seen that S i.e. the coefficients of sliding surface, is the eigenvector of $(A - BK)^T$ associated to the null eigenvalue. Using equations (3.11), (3.13) and (3.14), the system (2.54) can now be written in the error coordinates (3.2) as

$$\dot{\tilde{\mathcal{X}}} = A\mathcal{X} + B[-(S^T B)^{-1} S^T A\mathcal{X} - (S^T B)^{-1} \eta^2 sgn(\sigma)] \tag{3.16}$$

It is well known that the discontinuous control element i.e. $sgn(\sigma)$ can cause chattering. In order to remove the effect of chattering, we use the traditional substitute i.e. $sat(\frac{\sigma}{\epsilon})$ in which ϵ is a small design parameter [10]. The controller (3.11) is therefore modified as

$$u = \hat{u} + \bar{u} = -(S^T B)^{-1} S^T A\mathcal{X} - (S^T B)^{-1} \eta^2 sat(\frac{\sigma}{\epsilon}) \tag{3.17}$$

which yields the closed loop system (3.16) as described by the following equation

$$\dot{\tilde{\mathcal{X}}} = A\mathcal{X} + B[-(S^T B)^{-1} S^T A\mathcal{X} - (S^T B)^{-1} \eta^2 sat(\frac{\sigma}{\epsilon})] \tag{3.18}$$

It can be shown that with the sliding mode control (3.17), the trajectories of the closed loop system (3.18) will reach the σ in finite time, and stay inside the boundary layer $\sigma \leq \epsilon$ thereafter. The foregoing conclusions can be summarized in the following theorem.

Theorem. *Consider the closed loop system comprising of (2.54) and (3.17). Then, under the given assumptions there exist the matrix K, $\eta > 0$ and $\epsilon > 0$, such that the trajectories of the closed loop system (3.18) are bounded and* $\lim_{t\to\infty} \tilde{\mathcal{X}} \to 0$.

3.2 Output Feedback Control

In this section, we extend the state feedback control design to an output feedback by incorporating a 4^{th} order linear observer for diving plane. To proceed with the observer design, we consider the system

$$\begin{aligned} \dot{\mathcal{X}} &= A\mathcal{X} + Bu \\ Y &= C\mathcal{X} \end{aligned}$$

where $C = [\begin{array}{cccc} 0 & 0 & 0 & 1 \end{array}]$. It has been verified that the pair (A, C) is observable. Our goal is to design an observer based control law i.e.

$$u = -K(\hat{\mathcal{X}}) - (S^T B)^{-1}\eta^2 sat(\frac{\hat{\sigma}}{\epsilon}) \tag{3.19}$$

in which $\hat{\mathcal{X}}$ is the output of the observer. The output of the observer $\hat{\mathcal{X}}$ in diving plane is given by;

$$\begin{aligned} \dot{\hat{x}}_1 &= \frac{Z_w}{m - Z_{\dot{w}}}\hat{x}_1 + \frac{m\mu_o - X_{\dot{\mu}}\mu_o + Z_q}{m - Z_{\dot{w}}}\hat{x}_2 \\ &\quad + \frac{Z_{\delta\eta}}{m - Z_{\dot{w}}}u + L_1(x_4 - \hat{x}_4) \\ \dot{\hat{x}}_2 &= \frac{M_w}{I_y - M_{\dot{q}}}\hat{x}_1 + \frac{M_q}{I_y - M_{\dot{q}}}\hat{x}_2 \\ &\quad + \frac{M_{\delta\eta}}{I_y - M_{\dot{q}}}u + L_2(x_4 - \hat{x}_4) \\ \dot{\hat{x}}_3 &= \hat{x}_2 + L_3(x_4 - \hat{x}_4) \\ \dot{\hat{x}}_4 &= \hat{x}_1 - \mu_o\hat{x}_3 + L_4(x_4 - \hat{x}_4) \end{aligned} \tag{3.20}$$

We now define the estimation error as $e_i = x_i - \hat{x}_i$, where $i = 1, 2, 3, 4$. With this, the closed loop system can now be described in estimation error coordinates as:

$$\begin{aligned} \dot{e}_1 &= \frac{Z_w}{m - Z_{\dot{w}}}e_1 + \frac{m\mu_o - X_{\dot{\mu}}\mu_o + Z_q}{m - Z_{\dot{w}}}e_2 - L_1 e_4 \\ \dot{e}_2 &= \frac{M_w}{I_y - M_{\dot{q}}}e_1 + \frac{M_q}{I_y - M_{\dot{q}}}e_2 - L_2 e_4 \\ \dot{e}_3 &= e_2 - L_3 e_4 \end{aligned} \tag{3.21}$$

$$\dot{e}_4 = e_1 - \mu_o e_3 - L_4 e_4$$

in which it is assumed that the system's parameters are known. Consider the Lyapunov function

$$V(e) = \sum \frac{1}{2} e_i^2 \tag{3.22}$$

where $i = 1, 2, 3, 4$. Then,

$$\dot{V}(e) = e_1 \dot{e_1} + e_2 \dot{e_2} + e_3 \dot{e_3} + e_4 \dot{e_4} \tag{3.23}$$

or

$$\begin{aligned} \dot{V}(e) \quad = \quad & e_1\left(\frac{Z_w}{m - Z_{\dot{w}}} e_1 + \frac{m\mu_o - X_{\dot{\mu}}\mu_o + Z_q}{m - Z_{\dot{w}}} e_2 - L_1 e_4\right) \\ & + e_2\left(\frac{M_w}{I_y - M_{\dot{q}}} e_1 + \frac{M_q}{I_y - M_{\dot{q}}} e_2 - L_2 e_4\right) \\ & + e_3(e_2 - L_3 e_4) + e_4(e_1 - \mu_o e_3 - L_4 e_4) \end{aligned} \tag{3.24}$$

Putting the values in the above equation and rearranging, we get

$$\begin{aligned} \dot{V}(e) \quad \leq \quad & -\left(-\frac{Z_w}{m - Z_{\dot{w}}} + \frac{m\mu_o - X_{\dot{\mu}}\mu_o + Z_q}{2(m - Z_{\dot{w})}} - \frac{L_1}{2}\right. \\ & + \left.\frac{M_w}{2(I_y - M_{\dot{q})}} + \frac{1}{2}\right) |e_1|^2 \\ & - \left(-\frac{M_q}{I_y - M_{\dot{q}}} + \frac{m\mu_o - X_{\dot{\mu}}\mu_o + Z_q}{2(m - Z_{\dot{w})}} - \frac{L_2}{2}\right. \\ & + \left.\frac{M_w}{2(I_y - M_{\dot{q})}} + \frac{1}{2}\right) |e_2|^2 \\ & - \left(\frac{1}{2} - \frac{L_3}{2} - \frac{\mu_o}{2}\right) |e_3|^2 \\ & - \left(L_4 - \frac{1}{2} - \frac{L_2}{2} - \frac{L_3}{2} - \frac{1}{2} - \frac{\mu_o}{2}\right) |e_4|^2 \end{aligned} \tag{3.25}$$

or

$$\dot{V}(e) \leq -\alpha_1 |e_1|^2 - \alpha_2 |e_2|^2 - \alpha_3 |e_3|^2 - \alpha_4 |e_4|^2 \tag{3.26}$$

where α_1, α_2, α_3 and α_4 are positive numbers obtained by selecting suitable observer gains L_1, L_2, L_3 and L_4 to achieve $\dot{V}(e) < 0$. Thus, the convergence is ensured. By a suitable choice of the observer gains L_1, L_2, L_3 and L_4, it can be shown that the performance of the observer-based control approaches to that of the state feedback control. The observed states are used in the control law which stabilizes the closed-loop system.

3.3 Transformation to Normal Form and Sliding Mode Control

3.3.1 Transformation to Normal Form

In this section, we transform the nonlinear system in to normal form using diffeomorphism and design a nonlinear a full state feedback sliding mode control in order to achieve desired result with minimal control effort and more insight to the closed loop system. To proceed further we consider the system is given by;

$$\begin{aligned} \dot{\mathcal{X}} &= F(x) + G(x)u \\ Y &= h(x) \end{aligned} \tag{3.27}$$

where F, G and h are sufficiently smooth in a domain $D \subset R^n$. Our goal is to design a nonlinear state feedback control law of the form;

$$u = \alpha(x) + \beta(x)v \tag{3.28}$$

such that there exits a diffeomorphism $T : D \to R^n$ so that $D_z = T(D)$ contains the origin and the change of variable $z = T(x)$ tranforms the system (3.27) in to the form

$$\dot{z} = Az + B\gamma(x)[u - \alpha(x)] \tag{3.29}$$

with (A, B) contrallable and $\gamma(x)$ nonsigular for all $x \in D$. Thus, the nonlinear model of the torpedo can be written in terms of (3.27) as under

$$\begin{aligned}
\dot{x}_1 &= \frac{Z_w}{m - Z_{\dot{w}}} x_1 + \frac{m\mu_o - X_{\dot{\mu}}\mu_o + Z_q}{m - Z_{\dot{w}}} x_2 \\
&\quad + \frac{Z_{\delta\eta}}{m - Z_{\dot{w}}} u \\
\dot{x}_2 &= \frac{M_w}{I_y - M_{\dot{q}}} x_1 + \frac{M_q}{I_y - M_{\dot{q}}} x_2 \\
&\quad + \frac{M_{\delta\eta}}{I_y - M_{\dot{q}}} u \\
\dot{x}_3 &= x_2 \\
\dot{x}_4 &= x_1 \cos x_3 - \mu_o \sin x_3 \\
y &= x_4
\end{aligned} \tag{3.30}$$

For ease of calculation (3.30) can be written in the form

$$\begin{aligned}
\dot{x}_1 &= a_{11}x_1 + a_{12}x_2 + b_{11}u \\
\dot{x}_2 &= a_{21}x_1 + a_{22}x_2 + b_{22}u \\
\dot{x}_3 &= x_2 \\
\dot{x}_4 &= x_1 \cos x_3 - a_{41} \sin x_3 \\
y &= x_4
\end{aligned} \tag{3.31}$$

where a_{11}, a_{12}, b_{11}, a_{21}, a_{22}, b_{22} and a_{41} are coresponding coefficients. To proceed further with the transformation, first we have to find out the relative degree of the system. The derivatives of output are given by

$$\begin{aligned}\dot{y} &= \dot{x}_4 = x_1 \cos x_3 - a_{41} \sin x_3 \\ \ddot{y} &= \dot{x}_1 - x_1 \sin x_3 \dot{x}_3 - a_{41} \cos x_3 \dot{x}_3 = (.) + \cos x_3 b_{11} u\end{aligned}$$

where (.) contains terms which are function of x. Thus, the system has relative degree 2 in R^4. Therefore, the diffeomorphism will be of the form

$$T(x) = \begin{bmatrix} \phi_1(x) \\ \phi_2(x) \\ --- \\ h(x) \\ L_f h(x) \end{bmatrix} \tag{3.32}$$

where $\phi_1(x)$ and $\phi_2(x)$ are chosen such that $\phi(0) = 0$ and $\frac{\partial \phi_i}{\partial x} G(x) = 0$, for $i = 1, 2$. $h(x)$ is the output and $L_f h(x)$ is the Lie Derivative of h with respect to f and is given by

$$L_f h(x) = \frac{\partial h}{\partial x} F(x) = x_1 \cos x_3 - a_{41} \sin x_3 \tag{3.33}$$

With $G(x) = \begin{bmatrix} b_{11} & b_{22} & 0 & 0 \end{bmatrix}^T$ and satisfying the conditions $\phi(0) = 0$ and $\frac{\partial \phi_i}{\partial x} G(x) = 0$, for $i = 1, 2$, we choose

$$\begin{aligned}\phi_1(x) &= x_3 \\ \phi_2(x) &= \frac{x_1}{b_{11}} - \frac{x_2}{b_{22}}\end{aligned}$$

Thus the diffeomorphism $T(x)$ on some domain containing the origin is given by

$$T(x) = \begin{bmatrix} x_3 \\ \frac{x_1}{b_{11}} - \frac{x_2}{b_{22}} \\ x_4 \\ x_1 \cos x_3 - a_{41} \sin x_3 \end{bmatrix} \tag{3.34}$$

Using above mentioned diffeomorphism , we select following varaibles to proceed with the change of variable in order to get the desired normal form.

$$\begin{aligned} \eta_1 &= x_3 \\ \eta_2 &= \frac{x_1}{b_{11}} - \frac{x_2}{b_{22}} \\ \xi_1 &= x_4 \\ \xi_2 &= x_1 \cos x_3 - a_{41} \sin x_3 \end{aligned} \tag{3.35}$$

Now from (3.35), we can write x_1, x_2, x_3, and x_4 in terms of η_1, η_2, ξ_1 and ξ_2.

$$\begin{aligned} x_1 &= \xi_2 \sec \eta_1 + a_{41} \tan \eta_1 \\ x_2 &= \frac{b_{22}}{b_{11}} \xi_2 \sec \eta_1 + \frac{a_{41} b_{22}}{b_{11}} \tan \eta_1 - b_{22} \eta_2 \\ x_3 &= \eta_1 \\ x_4 &= \xi_1 \end{aligned} \tag{3.36}$$

Substituting values of x_1, x_2, x_3, and x_4 from (3.36) in to (3.31), we get our desired normal form of the system

$$\begin{aligned} \dot{\eta}_1 &= \frac{b_{22}}{b_{11}} \xi_2 \sec \eta_1 + \frac{a_{41} b_{22}}{b_{11}} \tan \eta_1 - b_{22} \eta_2 \\ \dot{\eta}_2 &= \left(\frac{a_{11}}{b_{11}} - \frac{a_{22}}{b_{11}} + \frac{a_{12} b_{22}}{(b_{11})^2} - \frac{a_{21}}{b_{22}}\right)[\xi_2 \sec \eta_1 + a_{41} \tan \eta_1] - \left(\frac{a_{12} b_{22}}{b_{11}} - a_{22}\right)\eta_2 \end{aligned}$$

$$
\begin{aligned}
\dot{\xi}_1 &= \xi_2 \\
\dot{\xi}_2 &= [a_{11} + (a_{12} - a_{41})\frac{b_{22}}{b_{11}}][\xi_2 + a_{41}\sin\eta_1] - (a_{12} - a_{41})b_{22}\cos\eta_1\eta_2 \\
&- \frac{b_{22}}{b_{11}}{\xi_2}^2\tan\eta_1\sec\eta_1 - 2a_{41}\frac{b_{22}}{b_{11}}\xi_2\tan^2\eta_1 + b_{22}\xi_2\tan\eta_1\eta_2 \\
&- a_{41}(\frac{a_{41}b_{22}}{b_{11}})\tan^2\eta_1\sin\eta_1 + a_{41}b_{22}\tan\eta_1\sin\eta_1\eta_2 + b_{11}\cos\eta_1 u \\
y &= \xi_1
\end{aligned} \tag{3.37}
$$

Above equation can also be written as

$$
\begin{aligned}
\dot{\eta}_1 &= d_1\xi_2\sec\eta_1 + d_2\tan\eta_1 - d_3\eta_2 \\
\dot{\eta}_2 &= d_4\xi_2\sec\eta_1 + d_5\tan\eta_1 - d_6\eta_2 \\
\dot{\xi}_1 &= \xi_2 \\
\dot{\xi}_2 &= d_7\xi_2 + d_8\sin\eta_1 - d_9\cos\eta_1\eta_2 - d_{10}{\xi_2}^2\tan\eta_1\sec\eta_1 \\
&- d_{11}\xi_2\tan^2\eta_1 + d_{12}\xi_2\tan\eta_1\eta_2 - d_{13}\tan^2\eta_1\sin\eta_1 \\
&+ d_{14}\tan\eta_1\sin\eta_1\eta_2 + d_{15}\cos\eta_1 u \\
y &= \xi_1
\end{aligned} \tag{3.38}
$$

where $d_1, d_2, ..., d_{15}$ are the corresponding coefficients.

3.3.2 Analysis and Control Design

The normal form (3.38) divides the system into two parts, i.e. an internal part $\eta_1 \& \eta_2$ and an external part $\xi_1 \& \xi_2$. It is pertinent to highlight that in (3.30), there is control u appearing in two states and in (3.38), it appears in only one state that is in external part only. Thus, internal part becomes unobservable by the control. However, it has been verified that the zero dynamics of (3.38) i.e $\dot{\eta} = f_0(\eta, 0)$ are bounded input bounded output stable in the domain of interest and the system is minimum phase. Therefore, the objective is to design a control for only external part of the system such that the output $y = \xi_1$ i.e. the depth of torpedo tracks the desired reference ξ_d. The proposed control is composed of two parts i.e. stablizing or state feedback control $\hat{u}$ and tracking or sliding mode control $\bar{u}$.

$$\begin{aligned}
\hat{u} &= -\frac{1}{d_{15}\cos\eta_1}(d_7\xi_2 + d_8\sin\eta_1 - d_9\cos\eta_1\eta_2 - d_{10}{\xi_2}^2\tan\eta_1\sec\eta_1 - d_{11}\xi_2\tan^2\eta_1 \\
&\quad + d_{12}\xi_2\tan\eta_1\eta_2 - d_{13}\tan^2\eta_1\sin\eta_1 + d_{14}\tan\eta_1\sin\eta_1\eta_2) \qquad (3.39)
\end{aligned}$$

$$\bar{u} = -\frac{1}{d_{15}\cos\eta_1}(Ksat\frac{s}{\mu}) = -\frac{1}{d_{15}\cos\eta_1}(Ksat\frac{k_1(\xi_1 - \xi_d) + \xi_2}{\mu}) \qquad (3.40)$$

where $K\&k_1$ are gains, s is the sliding surface and μ is design parameter. By combining (3.39) and (3.40), the overall control for the normal form of the system becomes;

$$\begin{aligned}
u &= \hat{u} + \bar{u} = -\frac{1}{d_{15}\cos\eta_1}(d_7\xi_2 + d_8\sin\eta_1 - d_9\cos\eta_1\eta_2 \\
&\quad - d_{10}{\xi_2}^2\tan\eta_1\sec\eta_1 - d_{11}\xi_2\tan^2\eta_1 + d_{12}\xi_2\tan\eta_1\eta_2 \qquad (3.41) \\
&\quad - d_{13}\tan^2\eta_1\sin\eta_1 + d_{14}\tan\eta_1\sin\eta_1\eta_2 + Ksat\frac{k_1(\xi_1 - \xi_d) + \xi_2}{\mu})
\end{aligned}$$

Here, it is important to note that the term η_1 that is diving angle of the torpedo ($\eta_1 = x_3 = \theta$) appears in the denominator of the control. This implies that if $\eta_1 \rightarrow 90 \Rightarrow \cos\eta_1 \rightarrow 0 \Rightarrow u \rightarrow \infty$. It highlights the fact that the if a torpedo is made to dive at right angle then it is impossible to control it and track the desired path. Thus, the normal form of the system gives insight to the mathematical proof of aforementioned fact.

Chapter 4

SIMULATION RESULTS

This chapter proceeds with the performance analysis of the proposed control design by the help of numerical simulations. The goal is to track a changing reference depth z_d The controller parameters chosen for this purpose are surge $\mu = \mu_o = 10m/s$, the controller gain $\eta = 20$, the boundary layer parameter ϵ to reduce the effect of chattering is 0.3. The feedback gain matrix K and the observer gains are chosen such the Lyapunov criteria for stability i.e. $V(e) \leq 0$ for all $t \geq 0$.

4.1 Constant Depth Tracking - Actual Parameters

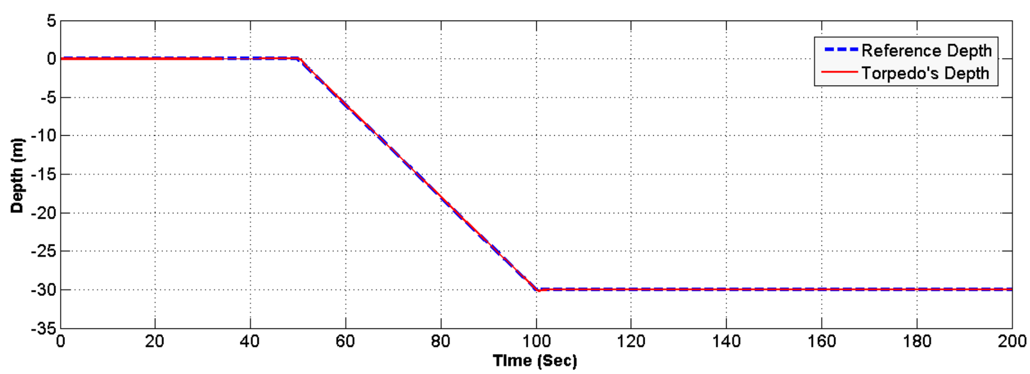

Figure 4.1: Tracking for Constant Depth - Actual Parameters

In figure (4.1), torpedo is commanded to track a constant depth. The state feed-back control (3.17) derived from linearized model (2.54) of torpedo is applied to nonlinear system (2.53) with nominal parameters and the torpedo successfully stabilizes itself at the desired depth. It is because of the robustness of the slidng mode control used in overall statefeed back control that the linear control is applicable to nonlinear system.

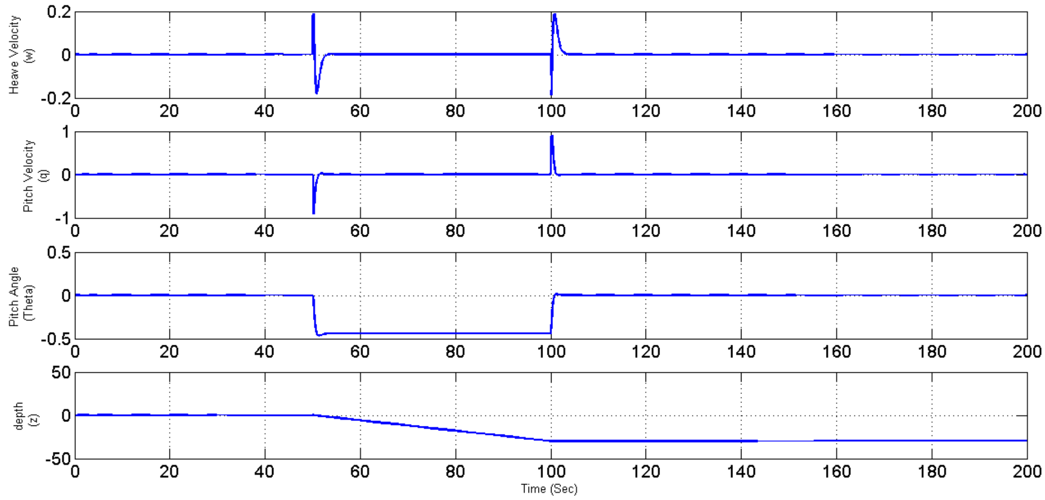

Figure 4.2: Response of States for Tracking Constant Depth - Actual Parameters

Figure (4.2) shows the response of all states of the system (2.53) i.e. heave velocity w, pitch velocity q, pitch angle θ and depth z. It is noteworthy that when there is change in the reference depth i.e. desired output, there is a sharp change in all states. This is because of the fact that when system is commanded to obtain a certain output, all states drive themselves under the effect of control and stablizes when the desired results are achieved.

Figure (4.3) shows the comparison between the control effort of nominal control(3.12), nonlinear control (3.13) and the overall control (3.17). It can be seen that there is a peak in control effort when chnage in depth is commanded and control effort reduces to zero when desired output is achieved.

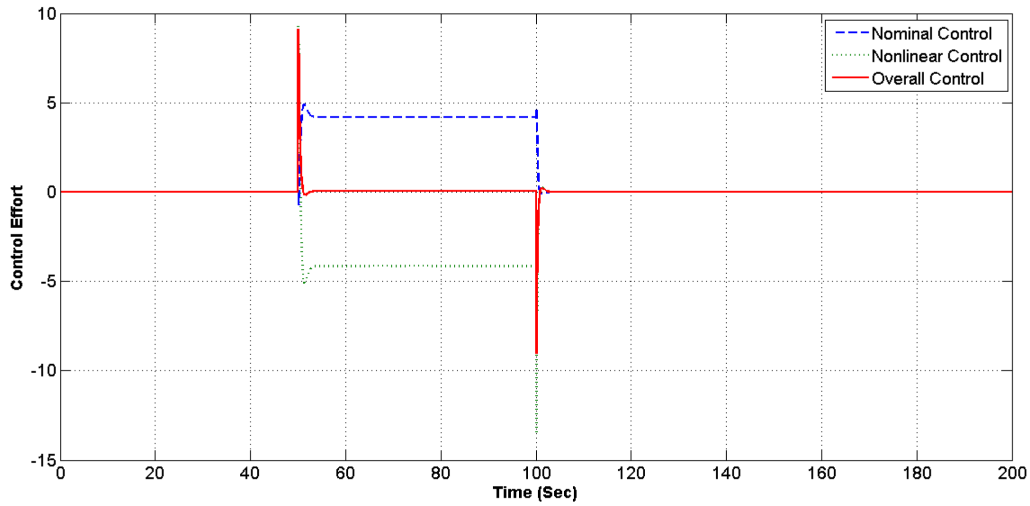

Figure 4.3: Control Effort for Tracking Constant Depth - Actual Parameters

4.2 Varying Depth Tracking - Actual Parameters

In figure (4.4), torpedo is commanded to track a varying depth. The state feedback control (3.17) is applied to nonlinear system (2.53) with nominal parameters and the torpedo successfully tracks the desired varying depth. However, in practical implimentation, the performance of the control may degrade because of the noise/ disturbance induced in the system due to the measurement of the states via sensors.

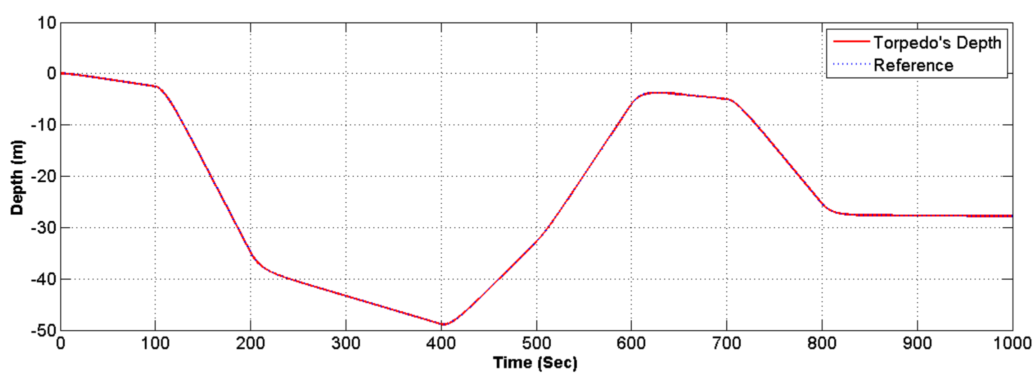

Figure 4.4: Tracking for Varying Depth - Actual Parameters

Figure (4.5) shows the comparison between the control effort of nominal control(3.12),

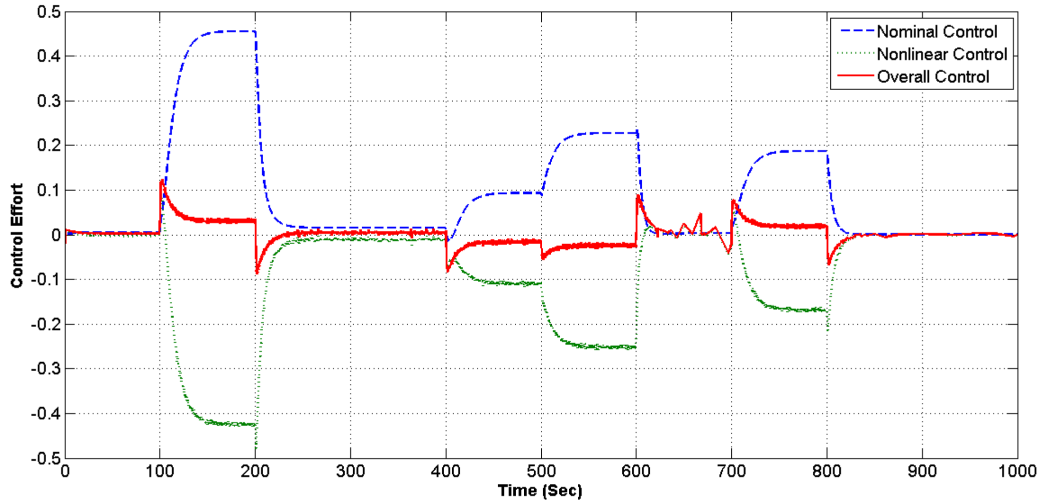

Figure 4.5: Control Effort for Varying Depth - Actual Parameters

nonlinear control (3.13) and the overall control (3.17) for tracking a varying depth with nominal parameters of the system.

4.3 Varying Depth Tracking - Parameters Perturbation

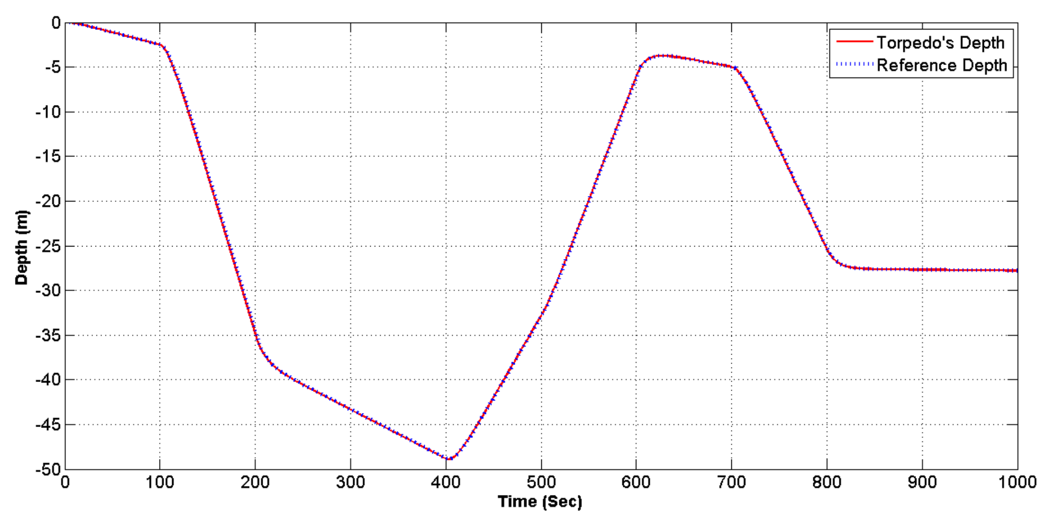

Figure 4.6: Tracking for Varying Depth - Perturbed Parameters

Figure (4.6) shows the performance of state feedback sliding mode control (3.17) when the hydrodynamic coefficients and inertia terms in (2.54) are perturbed to 20% of the actual values. Torpedo trajectory is successfully tracking the varying reference depth. This shows the robustness of proposed sliding mode control law.

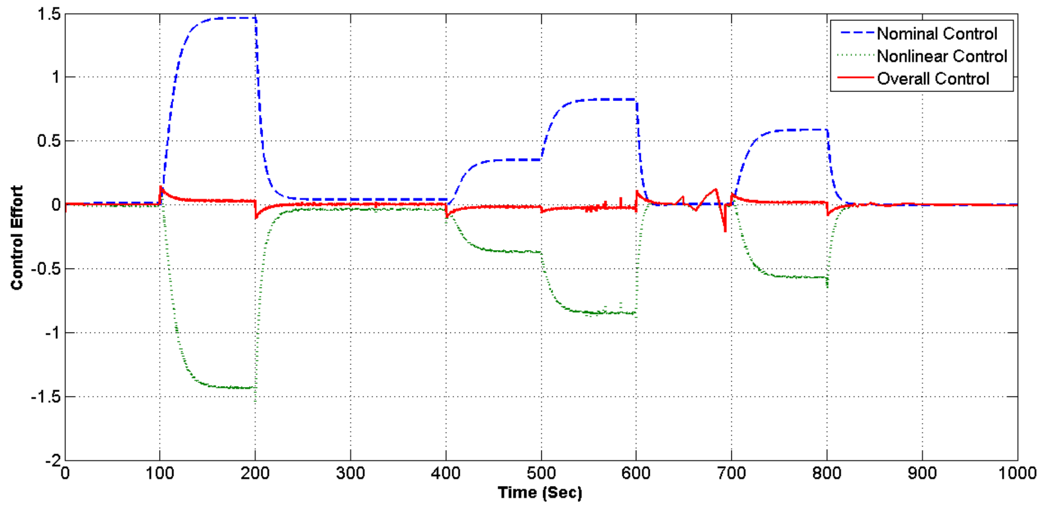

Figure 4.7: Control Effort for Varying Depth - Perturbed Parameters

Figure (4.7) shows the comparison between the control effort of nominal control(3.12), nonlinear control (3.13) and the overall control (3.17) for tracking a varying depth with parameters of the system have been perturbed to 20% of the nominal values. It can be seen that the control effort has increased significantly when parameters are perturbed in comparison of control effort for nominal parameters (Figure 4.5).

4.4 Varying Depth Tracking - Output Feedback Observer Based Control

Figures (4.8) to (4.10) show the performance of the output feedback observer based control design (3.19). In figure (4.8), the control (3.19) derived from linear observer (3.20) has been applied to the nonlinear system (2.53). Torpedo is commanded to

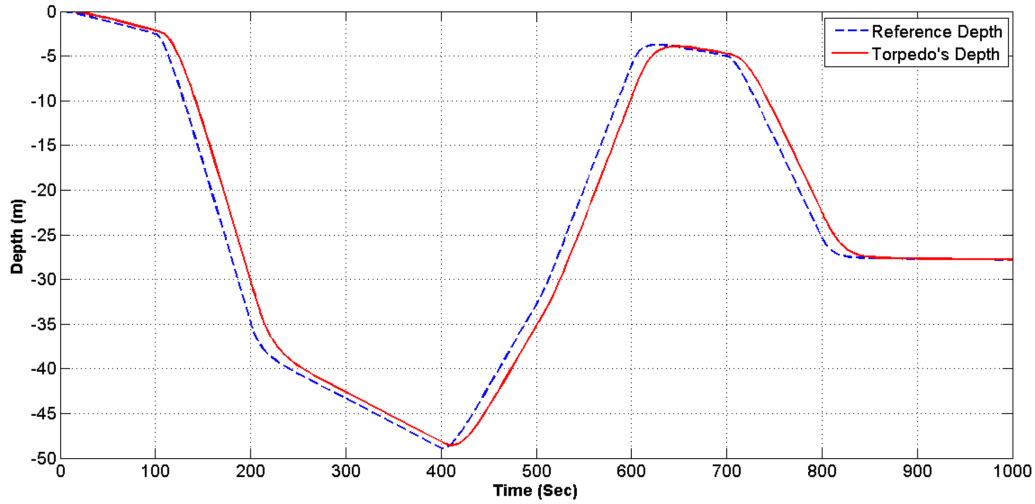

Figure 4.8: Tracking for Varying Depth - Output Feedback Observer Based Control

track a time varying depth and it successfully tracks the desired depth with minimal of error.

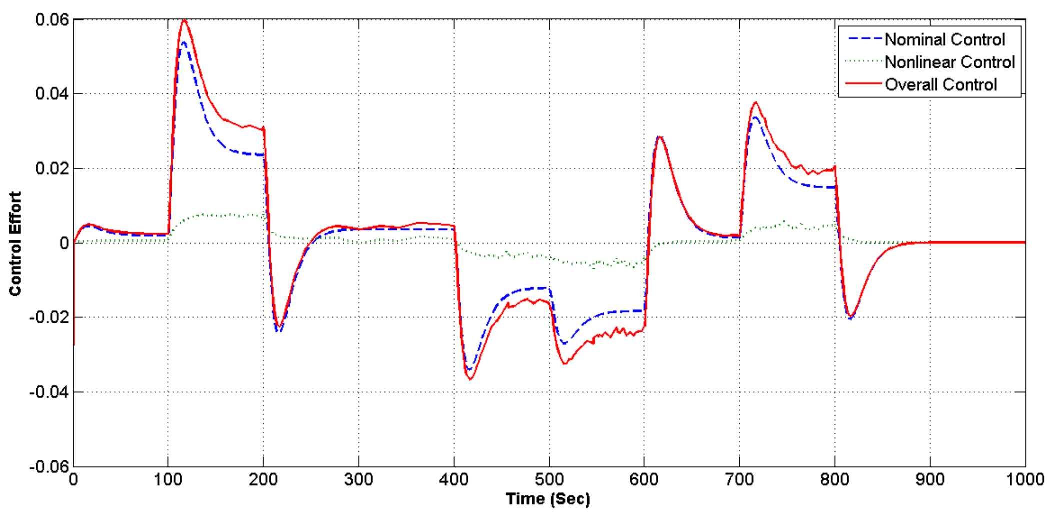

Figure 4.9: Control Effort for Varying Depth - Output Feedback Observer Based Control

Figure (4.9) shows the control effort of output feedback observer based control (3.19). A significant reduction in control effort can be noticed with the use of output feedback control in comparison with the use of state feedback control (Figure 4.5).

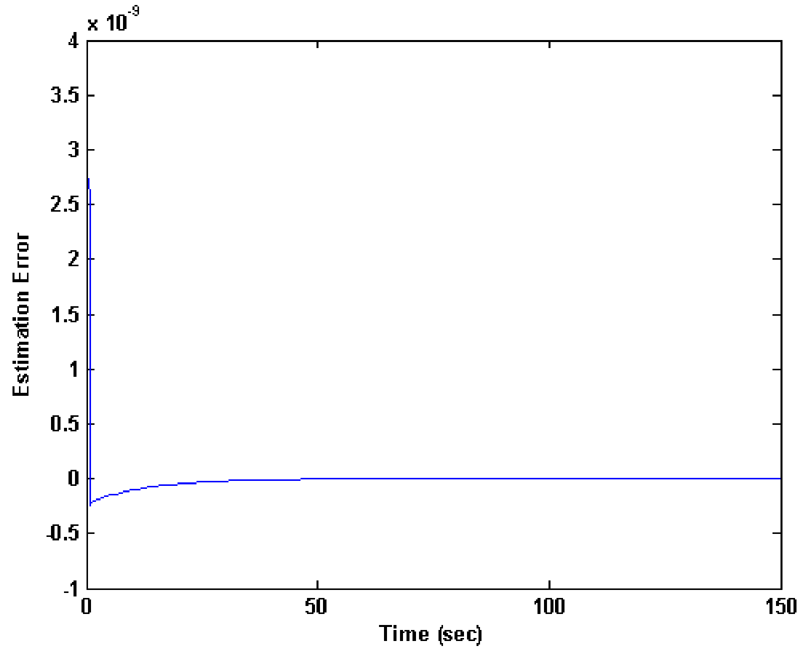

Figure 4.10: Estimation Error - Depth $(z - \hat{z})$

In Figure (4.10), it is shown that for a constant depth reference, the observer state converge to the actual state arbitrarily fast.

4.5 Varying Depth Tracking - Normal Form Actual Parameters

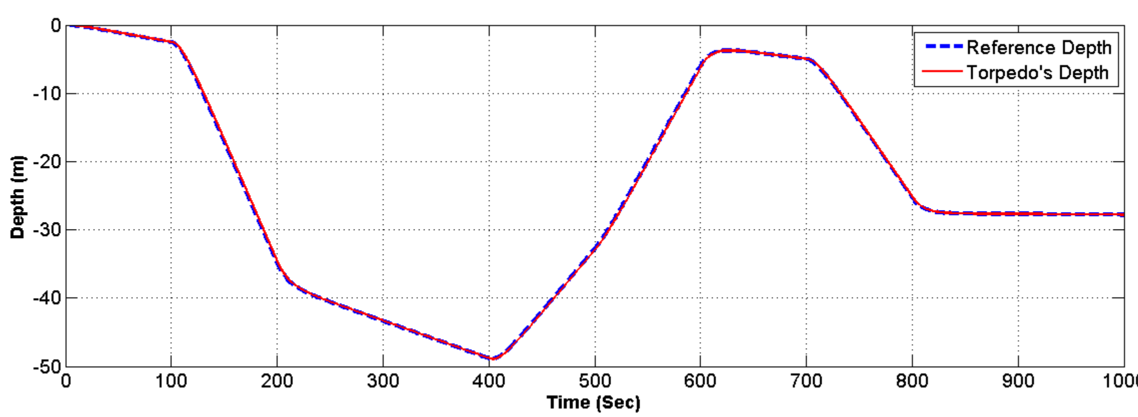

Figure 4.11: Tracking for Varying Depth - Normal Form Actual Parameters

Figure (4.11) shows the performance of the system when the nonlinear model of the system (3.30) is tranformed to normal form (3.38). Torpedo is commanded to track a varying depth reference. The state feedback sliding mode control (3.41) is applied to normal form of nonlinear system (3.38) with nominal parameters and the torpedo successfully tracks the desired varying depth.

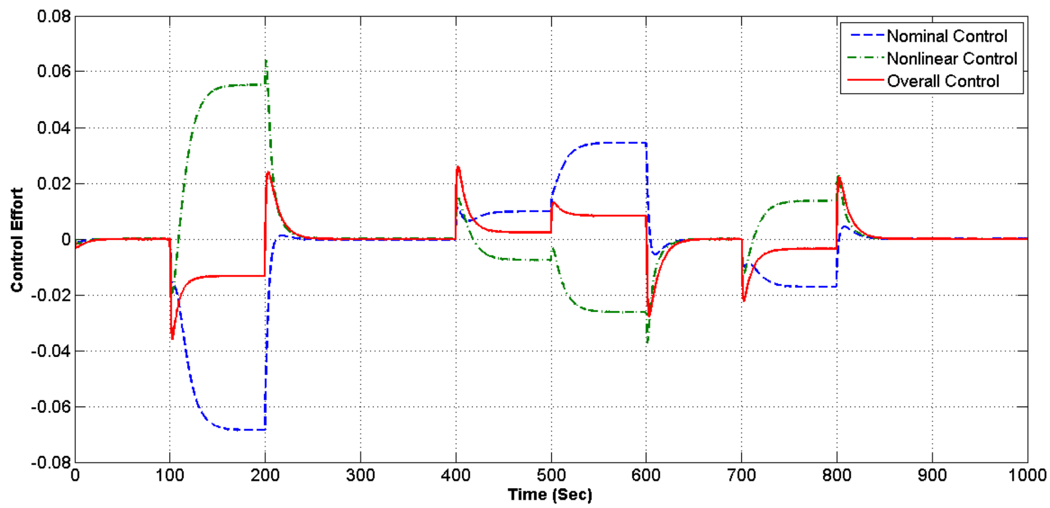

Figure 4.12: Control Effort for Varying Depth - Normal Form Actual Parameters

Figure (4.12) shows the control effort of the state feedback sliding mode control (3.41) applied to normal form of nonlinear system (3.38) with nominal parameters. A significant reduction in the control effort can be noticed with the transformation of the system to normal form in comparison with the control effort of state feedback control for actual nonlinear system (Figure 4.5).

4.6 Varying Depth Tracking - Normal Form Parameters Perturbation

Figure (4.13) shows the performance of state feedback sliding mode control (3.41) when the parameters of the normal form of the system (3.38) are perturbed to 20%

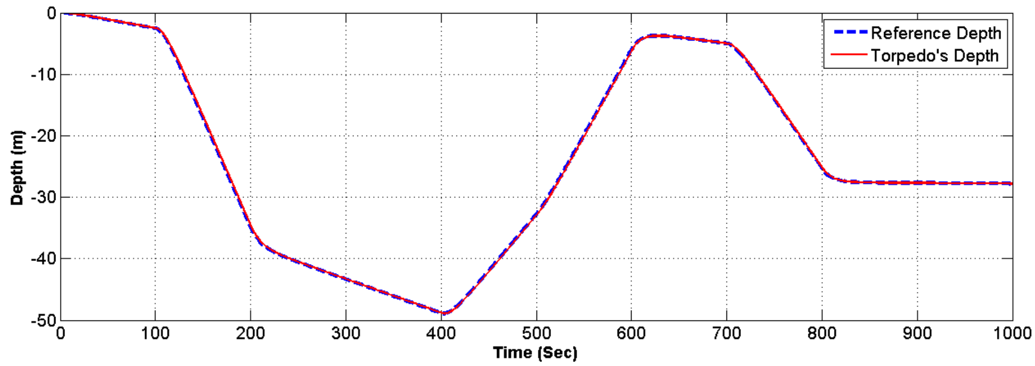

Figure 4.13: Tracking for Varying Depth - Normal Form Perturbed Parameters

of the actual values. Torpedo trajectory is successfully tracking the varying reference depth. This shows the robustness of proposed sliding mode control law.

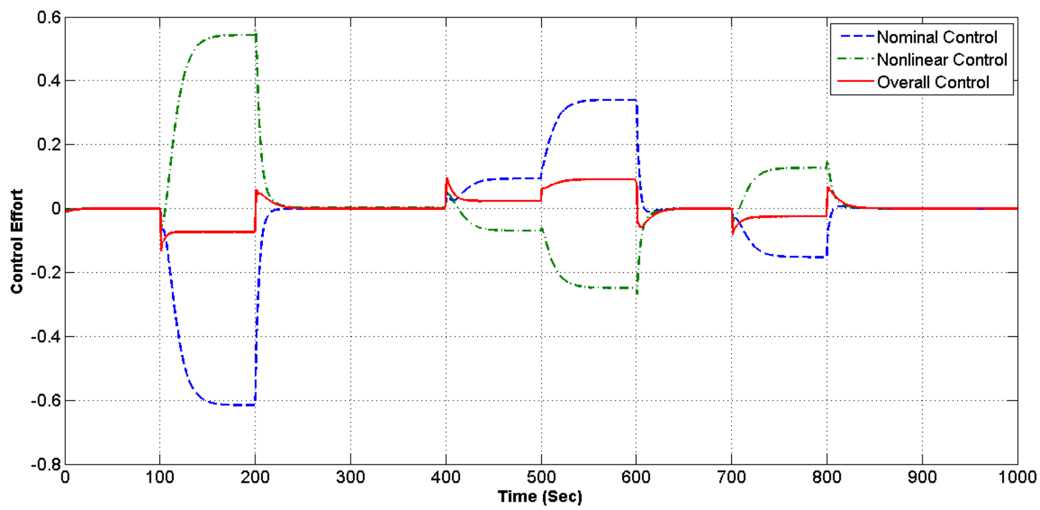

Figure 4.14: Control Effort for Varying Depth - Normal Form Perturbed Parameters

Figure (4.14) shows the control effort of the state feedback sliding mode control (3.41) applied to normal form of nonlinear system (3.38) when parameters of the system have been perturbed to 20% of the nominal values. Although, there is slight increase in control effort when parameters are perturbed in comparison of control effort for nominal parameters (Figure 4.12), however, a significant reduction in the control effort can be noticed with the transformation of the system to normal form in comparison with the control effort of state feedback control for actual nonlinear system (Figure 4.7).

Chapter 5

CONCLUSION

5.1 Conclusion

In this work we presented a nonlinear robust output feedback control for a class of autonomous underwater vehicles. In specific, we considered the nonlinear mathematical model of a heavy-weight torpedo and designed a sliding mode control to achieve robust stabilization in the presence of parametric uncertaintiesand model perturbations. First, a linearized model of the torpedo is developed from the nonlinear system equations. Then, using this linearized model, a robust sliding mode control law is proposed for the system. The closed-loop analysis using Lyapunov methods was also provided. Later, the state feedback sliding mode control was extended to an output feedback design by using a fourth order linear observer for diving plane. It was shown that the proposed output feedback controller recovers the performance of the state feedback in the presence of parametric uncertainties and model perturbations. Then the system nonlinear model is transformed in to normal form and nonlinear control based on state feedback and sliding mode control is proposed. Simulation results were provided to show the performance of the proposed control design.

5.2 Future Research

The future work will focus on extending the control design to achieve stabilization in waypoint tracking of the torpedo and path tracking of torpedo in steering plan. Various other nonlinear control design techniques like back stepping, higher order sliding mode control, high gain observer (HGO) and extended high gain observer (EHGO) can be used and system performance can be campared.

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