# A Waypoint Tracking Controller for an Autonomous Underwater Vehicle

by

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# ABSTRACT

## A Waypoint Tracking Controller for an Autonomous Underwater Vehicle

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This research deals with the design of nonlinear controller for the tracking of an autonomous underwater vehicle (AUV) in the steering plane. A state feedback sliding mode controller is designed for the path tracking, then the state feedback design is extended to an output feedback sliding mode controller by using a Luenberger observer of $3^{rd}$ order. The observer based control will minimize the need of measurement of states and avoids the complexities in the measurement due to noise and sensor dynamics. In path tracking, a complete path is given to the AUV as a reference trajectory and only the heading angle of AUV can be controlled. A better method to design the reference trajectory is to use waypoint tracking. In waypoint tracking, instead of a complete path, only the coordinates of selected points are given and the line of sight (LOS) and cross track error (CTE) controllers controls the orientation and position of AUV respectively by operating alternately. Both the methods of control design for path tracking and the waypoint tracking are robust and can handle parametric uncertainties. The mathematical model and control design along with the stability analysis is shown using Lyapunov's stability theorem. Finally, the performance of both the control design methods is discussed using simulation results.

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# Chapter 1 : INTRODUCTION

## 1.1 History & Background

The concept of Autonomous underwater vehicles (AUVs) arose decades ago specially in the field of naval warfare. First AUV was developed in 1957 by Stan Murphy, Bob Francois at the University of Washington. AUVs are usually deployed and operated in the areas of less shallow water but due to the advancements in the control technology AUVs can now be operated in shallow waters ranging more than the average depth of sea i.e. 3.7 Km to the maximum depth of 10.9 Km, in which ARIES (Acoustic Radar Interactive Exploratory Server) AUV has the ability to maneuver underwater upto 1.5 Km, HUGIN (High Precision Untethered Geosurvey and Inspection System) upto 4.5 Km and REMUS - 6000 (Remote Environmental Measuring Units) AUV upto a maximum of 6.0 Km. This ability makes an AUV suitable for underwater searches, explorations, spying and also for naval warfare operations.

Our country Pakistan is having a long coastal belt of Arabian Sea extending from India to Iran. Besides military needs for Pakistan Navy, Pakistan also needs to explore for oil and other minerals within its boundary inside the Arabian ocean. The underwater exploration can be done by AUVs which not only can explore for natural resources inside the sea, but can also measure the behavior of sea currents and sea environment which can warn and prevent from certain natural disasters.

While cruising underwater, AUVs experience different types of complexities. These complexities are due to undeterministic nature of the sea. Many models and techniques are presented in [3] which describe the behavior of sea under different weather and sea conditions but they depend on some measurements and assumptions. So the accurate measurement of sea behavior is not possible. Moreover the hydrodynamic coefficients of AUVs at higher speeds are very much nonlinear which severely affects the motion of AUVs [14]. Thus, a robust nonlinear controller is required which can handle the parametric uncertainties, model perturbations and the effect of hydrodynamic damping and friction.

## 1.2 Scope of the Work

This research focuses on the tracking of AUVs in steering plane in the presence of parametric uncertainties and environmental disturbances using nonlinear control techniques. The objectives are as

1) Derivation of 6 - DOF equations of motion and choosing the ones which are responsible for the motion in steering plane.

2) Design and stability analysis of a state feedback sliding mode controller for the path tracking of ARIES AUV.

3) Then extend the state feedback design to an output feedback observer based sliding mode controller for the path tracking along with the stability analysis. The purpose of

observer is to minimize the need of measurement of states. The controller designed must be capable of handling perturbations upto 20% of the actual parametric values.

4) Path tracking involves a complete path for traversing along the track line, which is not a good way to give reference trajectory as the AUV requires a complete path. To design the reference trajectory in a better way the technique used is called waypoint tracking. In waypoint tracking two controllers to be designed. One for correcting the orientation of AUV and is called line of sight (LOS) controller and other for correcting the position of AUV and is called cross track error (CTE) controller. LOS and CTE are both nonlinear controllers and work alternately to keeps the AUV along the track line.

## 1.3 Thesis Organization

Chapter II focus on the equations of motion for an AUV and the associated steering control laws. Chapter III discuss the different control design techniques for the path tracking using state feedback sliding mode control, observer based sliding mode control and then the waypoint tracking of AUV using line of sight (LOS) and cross track error (CTE) error control design. Chapter IV presents the simulation results and finally chapter V offer conclusions and recommendations for future study.

# Chapter 2 : LITERATURE REVIEW & MATHEMATICAL MODELING OF AUV

This section describes the literature review and modeling of AUV and focuses on the tracking in steering – horizontal plane so the kinetic and kinematic equations related to the horizontal plane will be discussed. To construct the mathematical model, the physical parameters of ARIES AUV are considered.

Control and navigation of AUVs is always a difficult and challenging task for the researchers as the AUV has to maneuver in sea waters and the behavior of sea is dependent on the environmental conditions. The precise measurement of the hydrodynamic terms of AUV due to the uncertain behavior of sea is thus not possible. So a robust controller is needed which can handle the parametric uncertainties and the effect of environmental conditions affecting the motion of AUV. In [1], $H_\infty$ design is presented which is effective in dealing with multivariable linear systems. In [2], fuzzy logic controller is presented, but the complication with these types of controllers is in their implementation. Thor l. Fossen and Anthony J. Healy have contributed a lot in marine vehicles and provided detailed mathematical model and control techniques in [3] and [4] for AUVs. These all control designs needs all the states of the system for control purpose, thus these states are needed to be measured via sensors. From the control design perspective, the measurement of all the states of the system is challenging task and it may increase the overall cost of the system as it involves different sensors. The problem with measurement via sensors is that it induces noise and time delays due to sensor dynamics [6]. In practical scenario, measurement causes noise which affects the control performance of the system. The purpose of using the observer is to drive the control law with the observed states which has benefits in cost as well as in the performance as presented in [6] and [7]. In path tracking, a complete path is given to the AUV as a reference trajectory and only the heading angle of AUV can be controlled. A better method to design the reference trajectory is to use waypoint tracking. In waypoint tracking, instead of a complete path, only the coordinates of selected points are given and the line of sight (LOS) and cross track error (CTE) controllers controls the orientation and position of AUV respectively by operating alternately [8]. To proceed further, first we need to develop the mathematical model and then further analysis will be done.

## 2.1 Six Degrees of Freedom Equations of Motion

Thor I. Fossen [3] has presented the generalized form of 6 degrees of freedom (6 - dof) equations of motion for underwater vehicles which is as

$$M\dot{v} + C(v)v + D(v)v + g(\eta) = \tau \tag{2.1}$$

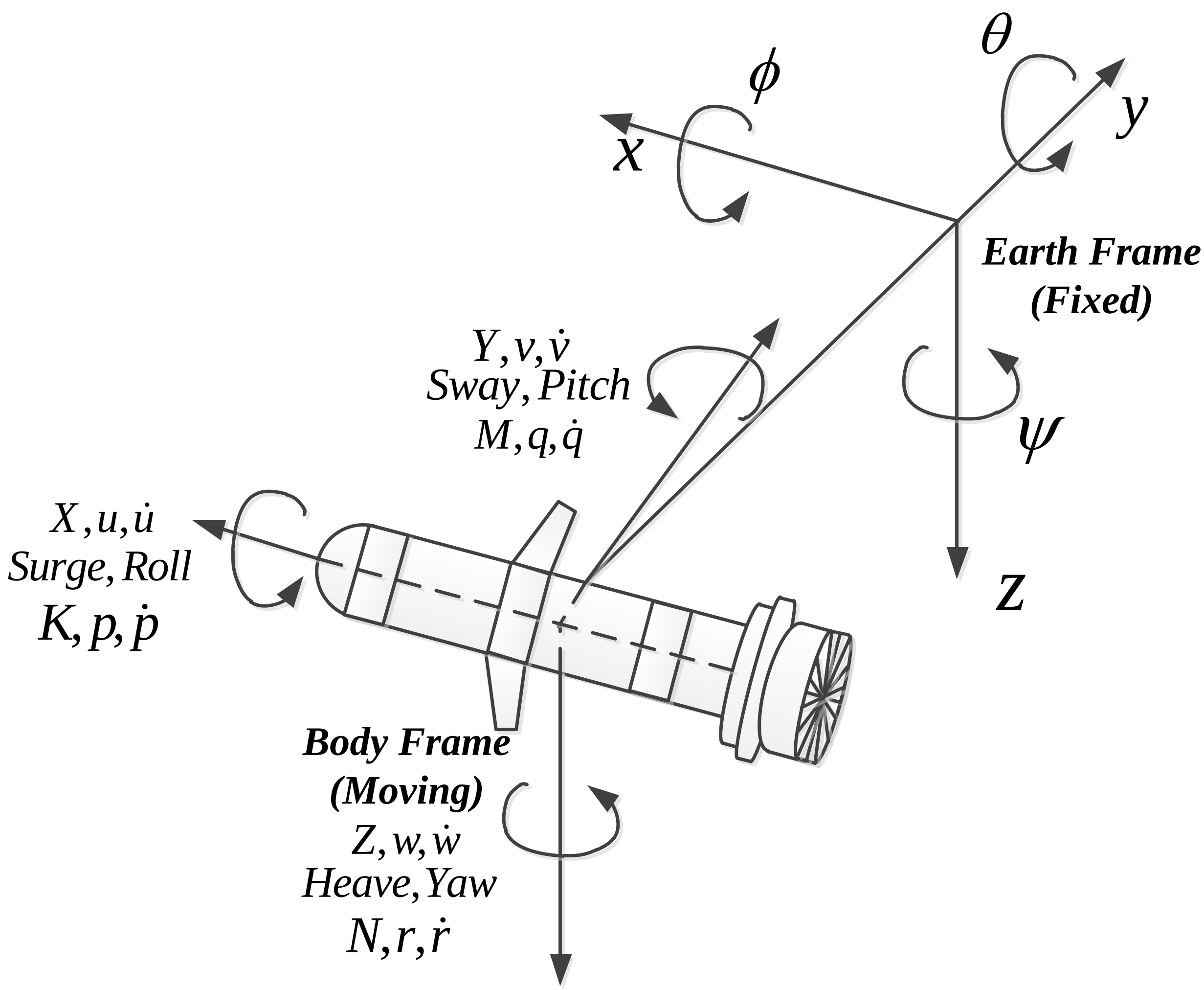


**Figure 2-1 : Earth Fixed Frame & Body Fixed Frame of AUV**

Where;

$M$ = Matrix of mass and added mass

$C(v)$ = Matrix of coriolis and centripetal terms

$D(v)$ = Matrix of hydrodynamic damping terms

$g(\eta) =$ Vector of gravitational moments and restoring forces

$\tau = [X \quad Y \quad Z \quad K \quad M \quad N]$ Vector of control inputs and external forces

$v = \begin{bmatrix} u & v & w & p & q & r \end{bmatrix}^T$ Vector of linear and angular velocities

$u, v, w$ are the linear velocities of the vehicle i.e. surge, sway and heave and is measured with respect to earth fixed frame or can be measured with respect to the displacement in water $p, q, r$ are the angular velocities of the vehicle i.e. roll, pitch and yaw and are measured with respect to body fixed frame of the vehicle

$\eta = \begin{bmatrix} x & y & z & \phi & \theta & \psi \end{bmatrix}^T$ Vector of positions and orientations

$x, y, z$ are the positions of the vehicle corresponding to surge, sway and heave and are measured with respect to earth fixed frame.

$\phi, \theta, \psi$ are the orientations corresponding to roll, pitch and yaw and are measured with respect to earth fixed frame.

### 2.1.1 Matrix of Mass & Inertia

We now proceed with the analysis of individual matrices. The matrix of mass and inertia is given by

$$M = M_{RB} + M_A \tag{2.2}$$

Where, $M_{RB}$ show the mass-inertia terms of the rigid body and can be termed as the actual mass of the body. $M_A$ show the added mass-inertia terms. Usually, it is considered that the added mass is the amount of water connected to the outer surface of water which results in a new system having considerably larger mass than the actual rigid body mass of the underwater vechicle. This is a misconception and the added mass-inertia or virtual mass is the pressure-induced forces and moments due to a forced harmonic motion of the body which are proportional to the acceleration of the body [3]. The mass-inertia matrix for the rigid body is given as

$$M_{RB} = \begin{bmatrix} m & 0 & 0 & 0 & mz_G & -my_G \\ 0 & m & 0 & -mz_G & 0 & mx_G \\ 0 & 0 & m & my_G & -mx_G & 0 \\ 0 & -mz_G & my_G & I_x & -I_{xy} & -I_{xz} \\ mz_G & 0 & -mx_G & -I_{yx} & I_y & -I_{yz} \\ -my_G & mx_G & 0 & -I_{zx} & -I_{zy} & I_z \end{bmatrix}$$

and the added mass inertia matrix is given as

$$M_A = -\begin{bmatrix} \frac{\partial X}{\partial \dot{u}} & \frac{\partial X}{\partial \dot{v}} & \frac{\partial X}{\partial \dot{w}} & \frac{\partial X}{\partial \dot{p}} & \frac{\partial X}{\partial \dot{q}} & \frac{\partial X}{\partial \dot{r}} \\ \frac{\partial Y}{\partial \dot{u}} & \frac{\partial Y}{\partial \dot{v}} & \frac{\partial Y}{\partial \dot{w}} & \frac{\partial Y}{\partial \dot{p}} & \frac{\partial Y}{\partial \dot{q}} & \frac{\partial Y}{\partial \dot{r}} \\ \frac{\partial Z}{\partial \dot{u}} & \frac{\partial Z}{\partial \dot{v}} & \frac{\partial Z}{\partial \dot{w}} & \frac{\partial Z}{\partial \dot{p}} & \frac{\partial Z}{\partial \dot{q}} & \frac{\partial Z}{\partial \dot{r}} \\ \frac{\partial K}{\partial \dot{u}} & \frac{\partial K}{\partial \dot{v}} & \frac{\partial K}{\partial \dot{w}} & \frac{\partial K}{\partial \dot{p}} & \frac{\partial K}{\partial \dot{q}} & \frac{\partial K}{\partial \dot{r}} \\ \frac{\partial M}{\partial \dot{u}} & \frac{\partial M}{\partial \dot{v}} & \frac{\partial M}{\partial \dot{w}} & \frac{\partial M}{\partial \dot{p}} & \frac{\partial M}{\partial \dot{q}} & \frac{\partial M}{\partial \dot{r}} \\ \frac{\partial N}{\partial \dot{u}} & \frac{\partial N}{\partial \dot{v}} & \frac{\partial N}{\partial \dot{w}} & \frac{\partial N}{\partial \dot{p}} & \frac{\partial N}{\partial \dot{q}} & \frac{\partial N}{\partial \dot{r}} \end{bmatrix}$$

According to the notation used in SNAME (Society of Naval Architects and Marine Engineers,1950) [9], $X_{\dot{u}} = \frac{\partial X}{\partial \dot{u}}$.........$N_{\dot{r}} = \frac{\partial N}{\partial \dot{r}}$, we can write

$$M_A = -\begin{bmatrix} X_{\dot{u}} & X_{\dot{v}} & X_{\dot{w}} & X_{\dot{p}} & X_{\dot{q}} & X_{\dot{r}} \\ Y_{\dot{u}} & Y_{\dot{v}} & Y_{\dot{w}} & Y_{\dot{p}} & Y_{\dot{q}} & Y_{\dot{r}} \\ Z_{\dot{u}} & Z_{\dot{v}} & Z_{\dot{w}} & Z_{\dot{p}} & Z_{\dot{q}} & Z_{\dot{r}} \\ K_{\dot{u}} & K_{\dot{v}} & K_{\dot{w}} & K_{\dot{p}} & K_{\dot{q}} & K_{\dot{r}} \\ M_{\dot{u}} & M_{\dot{v}} & M_{\dot{w}} & M_{\dot{p}} & M_{\dot{q}} & M_{\dot{r}} \\ N_{\dot{u}} & N_{\dot{v}} & N_{\dot{w}} & N_{\dot{p}} & N_{\dot{q}} & N_{\dot{r}} \end{bmatrix}$$

Adding the mass-inertia matrix of rigid body and the added mass-inertia matrix yields

$$M = \begin{bmatrix} m - X_{\dot{u}} & -X_{\dot{v}} & -X_{\dot{w}} & -X_{\dot{p}} & m\ z_G - X_{\dot{q}} & -m\ y_G - X_{\dot{q}} \\ -Y_{\dot{u}} & m - Y_{\dot{v}} & -Y_{\dot{w}} & -m\ z_G - Y_{\dot{p}} & -Y_{\dot{q}} & m\ x_G - Y_{\dot{r}} \\ -Z_{\dot{u}} & -Z_{\dot{v}} & m - Z_{\dot{w}} & m\ y_G - Z_{\dot{p}} & -m\ x_G - Z_{\dot{q}} & -Z_{\dot{r}} \\ -K_{\dot{u}} & -m\ z_G - K_{\dot{v}} & m\ y_G - K_{\dot{w}} & I_x - K_{\dot{p}} & -I_{xy} - K_{\dot{p}} & -I_{xz} - K_{\dot{r}} \\ m\ z_G - M_{\dot{u}} & -M_{\dot{v}} & -m\ x_G - M_{\dot{w}} & -I_{yx} - M_{\dot{p}} & I_y - M_{\dot{p}} & -I_{yz} - M_{\dot{r}} \\ -m\ y_G - N_{\dot{u}} & m\ x_G - N_{\dot{v}} & -N_{\dot{w}} & -I_{zx} - N_{\dot{p}} & -I_{zy} - N_{\dot{p}} & I_z - N_{\dot{r}} \end{bmatrix} \quad (2.3)$$

## 2.1.2 Coriolis & Centripetal Force

The coriolis effect is defined as the deflection of moving objects from their actual path when the motion is described relative to a rotating reference frame. In case of AUV the, the vector of position and altitude i.e. $\eta$ is measured with respect to earth frame. Since, due to the motion of earth, it has coriolis effect on the motion of AUV, thus according to [3] the matrix of coriolis and centripetal terms is given as

$$C = C_{RB} + C_A \tag{2.4}$$

Where, $C_{RB}$ shows coriolis and centripetal terms of the rigid body and $C_A$ shows the added terms and are given as.

$$C_{RB} = \begin{bmatrix} 0 & 0 & 0 & m\,(y_G q + z_G r) & -m\,(x_G q - w) & -m\,(x_G r + v) \\ 0 & 0 & 0 & -m\,(y_G p + w) & m\,(z_G r + x_G p) & -m\,(y_G r - u) \\ 0 & 0 & 0 & -m\,(z_G p - v) & -m\,(z_G q + u) & m\,(x_G p + y_G q) \\ -m\,(y_G q + z_G r) & m\,(y_G p + w) & m\,(z_G p - v) & 0 & -I_{yx} q - I_{xx} p + I_z r & I_{yz} r + I_{xy} p - I_y q \\ m\,(x_G q - w) & -m\,(z_G r + x_G p) & m\,(z_G q + u) & I_{yx} q + I_{xx} p - I_z r & 0 & -I_{xx} r - I_{xy} q + I_x p \\ m\,(x_G r + v) & m\,(y_G r - u) & -m\,(x_G p + y_G q) & -I_{yz} r - I_{xy} p + I_y q & I_{xx} r + I_{xy} q - I_x p & 0 \end{bmatrix}$$

$$C_A = \begin{bmatrix} 0 & 0 & 0 & 0 & -\gamma_3 & \gamma_2 \\ 0 & 0 & 0 & \gamma_3 & 0 & -\gamma_1 \\ 0 & 0 & 0 & -\gamma_2 & \gamma_1 & 0 \\ 0 & -\gamma_3 & \gamma_2 & 0 & -\gamma_6 & \gamma_5 \\ \gamma_3 & 0 & -\gamma_1 & \gamma_6 & 0 & -\gamma_4 \\ -\gamma_2 & \gamma_1 & 0 & -\gamma_5 & \gamma_4 & 0 \end{bmatrix}$$

Where;

$$\begin{aligned} \gamma_1 &= X_{\dot u} u + X_{\dot v} v + X_{\dot w} w + X_{\dot p} p + X_{\dot q} q + X_{\dot r} r \\ \gamma_2 &= X_{\dot v} u + Y_{\dot v} v + Y_{\dot w} w + Y_{\dot p} p + Y_{\dot q} q + Y_{\dot r} r \\ \gamma_3 &= X_{\dot w} u + Y_{\dot w} v + Z_{\dot w} w + Z_{\dot p} p + Z_{\dot q} q + Z_{\dot r} r \\ \gamma_4 &= X_{\dot p} u + Y_{\dot p} v + Z_{\dot p} w + K_{\dot p} p + K_{\dot q} q + M_{\dot r} r \\ \gamma_5 &= X_{\dot q} u + Y_{\dot q} v + Z_{\dot q} w + K_{\dot q} p + M_{\dot q} q + M_{\dot r} r \\ \gamma_6 &= X_{\dot r} u + Y_{\dot r} v + Z_{\dot r} w + K_{\dot r} p + M_{\dot r} q + N_{\dot r} r \end{aligned}$$

Adding both the matrices yields

$$C = \begin{bmatrix} 0 & 0 & 0 & m(y_G q + z_G r) & -m(x_G q - w) - \gamma_3 & -m(x_G r + v) + \gamma_2 \\ 0 & 0 & 0 & -m(y_G p + w) + \gamma_3 & m(z_G r + x_G p) & -m(y_G r - u) - \gamma_1 \\ 0 & 0 & 0 & -m(z_G p - v) - \gamma_2 & -m(z_G q + u) + \gamma_1 & m(x_G p + y_G q) \\ -m(y_G q + z_G r) & m(y_G p + w) - \gamma_3 & m(z_G p - v) + \gamma_2 & 0 & -I_{yx} q - I_{xx} p + I_z r - \gamma_6 & I_{yz} r + I_{xy} p - I_y q + \gamma_5 \\ m(x_G q - w) + \gamma_3 & -m(z_G r + x_G p) & m(z_G q + u) - \gamma_1 & I_{yx} q + I_{xx} p - I_z r + \gamma_6 & 0 & -I_{xx} r - I_{xy} q + I_x p - \gamma_4 \\ m(x_G r + v) - \gamma_2 & m(y_G r - u) + \gamma_1 & -m(x_G p + y_G q) & -I_{yz} r - I_{xy} p + I_y q - \gamma_5 & I_{xx} r + I_{xy} q - I_x p + \gamma_4 & 0 \end{bmatrix} \tag{2.5}$$

### 2.1.3 Hydrodynamic Damping

The hydrodynamic damping terms depends upon the behavior of sea water and it severely affects the movement of vehicle at higher speeds and its effect can’t be neglected. It mainly consists of drag and lift forces.

$$D = D_{Drag} + D_{Lift} \tag{2.6}$$

At lower speeds the effect of lift force is negligible as compared to the drag force, so it can be neglected. The drag force is consists of two parts, the linear drag force and the quadratic or nonlinear drag force

$$D_{Drag} = D_{Linear} + D_{Quadratic} \tag{2.7}$$

[14] has presented the matrices for $D_{Linear}$ and $D_{Quadratic}$ which are as

$$D_{Linear} = \begin{bmatrix} X_u & 0 & 0 & 0 & 0 & 0 \\ 0 & Y_v & 0 & 0 & 0 & 0 \\ 0 & 0 & Z_w & 0 & 0 & 0 \\ 0 & 0 & 0 & K_p & 0 & 0 \\ 0 & 0 & 0 & 0 & M_q & 0 \\ 0 & 0 & 0 & 0 & 0 & N_r \end{bmatrix}$$

$$D_{Quadratic} = -\begin{bmatrix} X_{u|u|}|u| & 0 & 0 & 0 & 0 & 0 \\ 0 & Y_{v|v|}|v| & 0 & 0 & 0 & 0 \\ 0 & 0 & Z_{w|w|}|w| & 0 & 0 & 0 \\ 0 & 0 & 0 & K_{p|p|}|p| & 0 & 0 \\ 0 & 0 & 0 & 0 & M_{q|q|}|q| & 0 \\ 0 & 0 & 0 & 0 & 0 & N_{r|r|}|r| \end{bmatrix}$$

(2.7) can be written as

$$D = \begin{bmatrix} X_u - X_{u|u|}|u| & 0 & 0 & 0 & 0 & 0 \\ 0 & Y_v - Y_{v|v|}|v| & 0 & 0 & 0 & 0 \\ 0 & 0 & Z_w - Z_{w|w|}|w| & 0 & 0 & 0 \\ 0 & 0 & 0 & K_p - K_{p|p|}|p| & 0 & 0 \\ 0 & 0 & 0 & 0 & M_q - M_{q|q|}|q| & 0 \\ 0 & 0 & 0 & 0 & 0 & N_r - N_{r|r|}|r| \end{bmatrix} \quad (2.8)$$

### 2.1.4 Gravitational Moments & Restoring Forces

In the terminology of hydrodynamics, restoring force consists of gravitational and buoyant forces. Let $m$ be the mass of the submerged vehicle, $\nabla$ be the fluid volume displaced by the vehicle, $g$ be the gravitational acceleration taken positive for downward movement and negative for upward movement and $\rho$ be the density of the fluid. Since the weight is defined as $W = mg$ and the buoyancy as $B = \rho g \nabla$. Then according to SNAME [9], the vector of gravitational moments and restoring forces is given as

$$g(\eta)=\begin{bmatrix} (W-B)\sin\theta \\ -(W-B)\cos\theta\sin\phi \\ -(W-B)\cos\theta\cos\phi \\ -(y_G W-y_B B)\cos\theta\cos\phi+(z_G W-z_B B)\cos\theta\sin\phi \\ (z_G W-z_B B)\sin\theta+(x_G W-x_B B)\cos\theta\cos\phi \\ -(x_G W-x_B B)\cos\theta\sin\phi-(y_G W-y_B B)\sin\theta \end{bmatrix} \tag{2.9}$$

For any vehicle to be neutrally buoyant, its weight and buoyancy must be equal. In this case, the vehicle will not move up or down as in the case of positive or negative buoyancy respectively. If the buoyancy and the weight are equal, also the geometric center lying at the gravitational center of the vehicle, then $g(\eta)$ can be neglected for obvious reasons.

### 2.1.5 Simplifications due to Symmetry in Geometrical Shape

While cruising at higher speeds the AUVs have very high degree of coupling between the added mass terms, coriolis terms and hydrodynamic damping terms. In the scope of this work, it is considered that the ARIES will maneuver at relatively slower speed with maximum speed of 10 knots per hour and it has gravitational and geometric symmetry in all three planes i.e. x, y and z. This assumption suggests that the off-diagonal terms in the matrix of added mass $M_A$, added coriolis terms $C_A$ and matrix of damping terms can be neglected.

The simplified matrix of added mass will thus become

$$M_A = -\begin{bmatrix} X_{\dot{u}} & 0 & 0 & 0 & 0 & 0 \\ 0 & Y_{\dot{v}} & 0 & 0 & 0 & 0 \\ 0 & 0 & Z_{\dot{w}} & 0 & 0 & 0 \\ 0 & 0 & 0 & K_{\dot{p}} & 0 & 0 \\ 0 & 0 & 0 & 0 & M_{\dot{q}} & 0 \\ 0 & 0 & 0 & 0 & 0 & N_{\dot{r}} \end{bmatrix}$$

Or $$M_A = -diag\{X_{\dot{u}}, Y_{\dot{v}}, Z_{\dot{w}}, K_{\dot{p}}, M_{\dot{q}}, N_{\dot{r}}\} \tag{2.10}$$

Using (2.10) in (2.2), (2.3) can be written as

$$M = \begin{bmatrix} m - X_{\dot{u}} & 0 & 0 & 0 & mz_G & -my_G \\ 0 & m - Y_{\dot{v}} & 0 & -mz_G & 0 & mx_G \\ 0 & 0 & m - Z_{\dot{w}} & my_G & -mx_G & 0 \\ 0 & -mz_G & my_G & I_x - K_{\dot{p}} & -I_{xy} & -I_{xz} \\ mz_G & 0 & -mx_G & -I_{yx} & I_y - M_{\dot{p}} & -I_{yz} \\ -my_G & mx_G & 0 & -I_{zx} & -I_{zy} & I_z - N_{\dot{r}} \end{bmatrix} \tag{2.11}$$

The simplified matrix of added coriolis and centripetal force terms will thus become

$$C_A(v) = \begin{bmatrix} 0 & 0 & 0 & 0 & -Z_{\dot{w}}w & Y_{\dot{v}}v \\ 0 & 0 & 0 & Z_{\dot{w}}w & 0 & -X_{\dot{u}}u \\ 0 & 0 & 0 & -Y_{\dot{v}}v & X_{\dot{u}}u & 0 \\ 0 & -Z_{\dot{w}}w & Y_{\dot{v}}v & 0 & -N_{\dot{r}}r & M_{\dot{q}}q \\ Z_{\dot{w}}w & 0 & -X_{\dot{u}}u & N_{\dot{r}}r & 0 & -K_{\dot{p}}p \\ -Y_{\dot{v}}v & X_{\dot{u}}u & 0 & -M_{\dot{q}}q & K_{\dot{p}}p & 0 \end{bmatrix} \tag{2.12}$$

Using (2.12) in(2.4), (2.5) can be written as

$$C = \begin{bmatrix} c_{1(6x3)} & c_{2(6x3)} \end{bmatrix} \tag{2.13}$$

Where, $c_1$ and $c_2$ are given as

$$c_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -m(y_G q + z_G r) & m(y_G p + w) - Z_{\dot{w}} w & m(z_G p - v) + Y_{\dot{v}} v \\ m(x_G q - w) + Z_{\dot{w}} w & -m(z_G r + x_G p) & m(z_G q + u) - X_{\dot{u}} u \\ m(x_G r + v) - Y_{\dot{v}} v & m(y_G r - u) + X_{\dot{u}} u & -m(x_G p + y_G q) \end{bmatrix}$$

$$c_2 = \begin{bmatrix} m(y_G q + z_G r) & -m(x_G q - w) - Z_{\dot{w}} w & -m(x_G r + v) + Y_{\dot{v}} v \\ -m(y_G p + w) + Z_{\dot{w}} w & m(z_G r + x_G p) & -m(y_G r - u) - X_{\dot{u}} u \\ -m(z_G p - v) - Y_{\dot{v}} v & -m(z_G q + u) + X_{\dot{u}} u & m(x_G p + y_G q) \\ 0 & -I_{yx} q - I_{xx} p + I_z r - N_{\dot{r}} r & I_{yz} r + I_{xy} p - I_y q + M_{\dot{q}} q \\ I_{yx} q + I_{xx} p - I_z r + N_{\dot{r}} r & 0 & -I_{xx} r - I_{xy} q + I_x p - K_{\dot{p}} p \\ -I_{yz} r - I_{xy} p + I_y q - M_{\dot{q}} q & I_{xx} r + I_{xy} q - I_x p K_{\dot{p}} p & 0 \end{bmatrix}$$

The matrix for the hydrodynamic damping terms matrix (2.8) while considering only the linear drag force, will thus become

$$D = \begin{bmatrix} X_u & 0 & 0 & 0 & 0 & 0 \\ 0 & Y_v & 0 & 0 & 0 & 0 \\ 0 & 0 & Z_w & 0 & 0 & 0 \\ 0 & 0 & 0 & K_p & 0 & 0 \\ 0 & 0 & 0 & 0 & M_q & 0 \\ 0 & 0 & 0 & 0 & 0 & N_r \end{bmatrix} \tag{2.14}$$

These simplified matrices of mass, coriolis and hydrodynamic damping along with the vector of gravitational moments will be used in the derivation of state space equations for steering plane.

### 2.1.6 Derivation of Dynamic & Kinematic Model for AUV

This section presents the mathematical model of dynamic and kinematic model of AUV. Thor I. Fossen [3] has presented a well-defined general form for mathematical model of marine vehicles which includes the autonomous underwater vehicles (AUVs), remotely operated vehicles (ROVs) and surface vehicles. The mathematical model is obtained as

#### 2.1.6.1 Dynamic Model of AUV

Using (2.11), (2.13), (2.14) and putting the values in (2.1) will results in the 6 degrees of freedom dynamic equations of motion for surge , sway, heave, roll, pitch and yaw respectively

$$m[\dot{u}_r - v_r r + w_r q - x_G(q^2 + r^2) + y_G(pq - \dot{r}) + z_G(pr + \dot{q})] + (W - B)\sin\theta = X \tag{2.15}$$

$$m[\dot{v}_r + u_r r - w_r p + x_G(pq + \dot{r}) - y_G(p^2 + r^2) + z_G(qr - \dot{p})] - (W - B)\cos\theta\sin\phi = Y \tag{2.16}$$

$$m[\dot{w}_r - u_r q + v_r p + x_G(pr - \dot{q}) + y_G(qr + \dot{p}) - z_G(p^2 + q^2)] + (W - B)\cos\theta\cos\phi = Z \tag{2.17}$$

$$\begin{gathered} I_x\dot{p} + (I_z - I_y)qr + I_{xy}(pr - \dot{q}) - I_{yz}(q^2 - r^2) - I_{xz}(pq + \dot{r}) + m[y_G(\dot{w} - u_r q + v_r p) \\ -z_G(\dot{v}_r + u_r r - w_r p)] - (y_G W - y_B B)\cos\theta\cos\phi + (z_G W - z_B B)\cos\theta\sin\phi = K \end{gathered} \tag{2.18}$$

$$\begin{gathered} I_y\dot{q} + (I_z - I_x)pr - I_{xy}(qr + \dot{p}) + I_{yz}(pq - \dot{r}) + I_{xz}(p^2 - r^2) - m[x_G(\dot{w} - u_r q + v_r p) \\ -z_G(\dot{u}_r - v_r r + w_r q)] + (x_G W - x_B B)\cos\theta\cos\phi = M \end{gathered} \tag{2.19}$$

$$\begin{gathered} I_z\dot{r} + (I_y - I_x)pq - I_{xy}(p^2 - q^2) - I_{yz}(pr + \dot{q}) + I_{xz}(qr - \dot{p}) + m[x_G(\dot{v}_r + u_r r - w_r p) \\ -y_G(\dot{u}_r - v_r r + w_r q)] - (x_G W - x_B B)\cos\theta\sin\phi - (y_G W - y_B B)\sin\theta = N \end{gathered} \tag{2.20}$$

In the above equation set (2.15) - (2.20)

$W =$ Weight of AUV

$B =$ Buoyancy (positive, negative or neutral)

$x_B, y_B, z_B =$ Difference of position between the center of buoyancy and the center of geometry with respect to $x, y, z$ axis respectively

$x_G, y_G, z_G =$ Difference of position between the center of gravity and the center of geometry with respect to $x, y, z$ axis respectively

$X,\ Y,\ Z,\ K,\ M,\ N =$ Sum of all the external forces acting on AUV in the particular body fixed direction

#### 2.1.6.2 Kinematic Model of AUV

Since there are two reference frames, earth fixed frame and body fixed frame, to measure the motion of AUV. So the motion with respect to earth fixed frame and body fixed frame is must be studied separately. In order to translate the inertial frame (earth fixed frame) coordinates to body fixed coordinates and to have a single frame of reference to measure the motion of AUV a transformation is defined which is as

$$\dot{\eta} = J(\eta)v \tag{2.21}$$

Where, $J$ is the kinematic transformation matrix and it is used to translate the earth frame to body frame, $v$ is the vector of linear and angular velocities and $\eta$ is vector of position and altitudes. The mathematical description of $J$ is given below

$$J = \begin{bmatrix} J_1 & 0 \\ 0 & J_2 \end{bmatrix} \tag{2.22}$$

Where;

$$J_1(\phi,\theta,\psi) = \begin{bmatrix} \cos\psi\cos\theta & -\sin\psi\cos\phi + \cos\psi\sin\theta\sin\phi & \sin\psi\sin\phi + \cos\psi\cos\phi\sin\theta \\ \sin\psi\cos\theta & \cos\psi\cos\phi + \sin\phi\sin\theta\sin\psi & -\cos\psi\sin\phi + \sin\theta\sin\psi\cos\phi \\ -\sin\theta & \cos\theta\sin\phi & \cos\theta\cos\phi \end{bmatrix}$$

$$J_2(\phi,\theta,\psi) = \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi / \cos\theta & \cos\phi / \cos\theta \end{bmatrix}$$

For the steady state analysis, consider that the vehicle is in equilibrium so that roll $(\phi)$ and pitch $(\theta)$ angles can be considered as zero, equation (2.21) will become

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} \cos\psi\cos\theta & -\sin\psi\cos\phi + \cos\psi\sin\theta\sin\phi & \sin\psi\sin\phi + \cos\psi\cos\phi\sin\theta & 0 & 0 & 0 \\ \sin\psi\cos\theta & \cos\psi\cos\phi + \sin\phi\sin\theta\sin\psi & -\cos\psi\sin\phi + \sin\theta\sin\psi\cos\phi & 0 & 0 & 0 \\ -\sin\theta & \cos\theta\sin\phi & \cos\theta\cos\phi & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & 0 & 0 & 0 & \cos\phi & -\sin\phi \\ 0 & 0 & 0 & 0 & \sin\phi / \cos\theta & \cos\phi / \cos\theta \end{bmatrix} \begin{bmatrix} u \\ v \\ w \\ p \\ q \\ r \end{bmatrix}$$

The kinematics equations of AUV will thus become

$$\dot{x} = u\cos\psi\cos\theta - v(\sin\psi\cos\phi + \cos\psi\sin\theta\sin\phi) + w(\sin\psi\sin\phi + \cos\psi\cos\phi\sin\theta) \tag{2.23}$$

$$\dot{y} = u\sin\psi\cos\theta + v(\cos\psi\cos\phi + \sin\phi\sin\theta\sin\psi) - w(\cos\psi\sin\phi + \sin\theta\sin\psi\cos\phi) \tag{2.24}$$

$$\dot{z} = -u\sin\theta + v\cos\theta\sin\phi + w\cos\theta\cos\phi \tag{2.25}$$

$$\dot{\phi} = p + q\sin\phi\tan\theta + r\cos\phi\tan\theta \tag{2.26}$$

$$\dot{\theta} = q\cos\phi - r\sin\phi \tag{2.27}$$

$$\dot{\psi} = q\frac{\sin\phi}{\cos\theta} + r\frac{\cos\phi}{\cos\theta} \tag{2.28}$$

Usually the motion and control of an AUV is studied separately in the steering and diving planes. In order to decouple the equations of motion let the terms which are responsible for motion in diving plane to be zero and study the equations of motion in steering plane.

Anthony J. Healy [10] have simplified the aforementioned 6 - dof equations of motions by considering that the AUV is symmetrical in vertical and horizontal axis of the body and the center of gravity coincides with the center of geometry of the vehicle i.e. $x_B, y_B, z_B$ and $x_G, y_G, z_G$ can be considered zero.

### 2.1.7 Steering Plane Equations of Motion

From (2.15) - (2.20), consider the terms which are responsible for the motion in steering plane and neglect the terms which are associated with the motion in diving plane i.e. $w = q = \theta = z = p = \phi = 0$. Here the forward speed i.e. the surge $u$ is considered to be constant i.e. $u = \mu_0$. Considering the above assumption the equations of motion described in (2.15) - (2.20) can be simplified for the steering plane as

$$u = \mu_0$$

$$m\dot{v} = -m\mu_0 r + \Delta Y$$

$$I_z \dot{r} = \Delta N$$

$$\dot{\psi} = r$$

After the aforementioned assumptions, the kinematic equations of the ARIES AUV in the steering plane described in (2.23) and (2.24) will thus become

$$\dot{x} = \mu_0 \cos\psi - v\sin\psi \qquad (2.29)$$

$$\dot{y} = \mu_0 \sin\psi - v\cos\psi \qquad (2.30)$$

In the above equations, the terms $\Delta Y$ and $\Delta N$ defines the forces that are functions of AUVs dynamic parameters. In the scope of the work, it is assumed that the AUV is maneuvering at relatively low speed so that hydrodynamic terms can be defined relative to the individual components of motion. The components of hydrodynamic terms relative to the individual components of motion in the transverse direction are as

$$\begin{aligned} Y_v &= \frac{\partial Y}{\partial v} \Rightarrow Y_{\dot{v}} = \frac{\partial Y}{\partial \dot{v}} \\ Y_r &= \frac{\partial Y}{\partial r} \Rightarrow Y_{\dot{r}} = \frac{\partial Y}{\partial \dot{r}} \end{aligned} \qquad (2.31)$$

Combining all the terms in (2.31) leads to the expression for the transverse force which is as under

$$Y = Y_v v + Y_{\dot{v}} \dot{v} + Y_r r + Y_{\dot{r}} \dot{r}$$

The components of hydrodynamic terms relative to the individual components of motion in the rotational direction are as

$$\begin{aligned} N_v &= \frac{\partial N}{\partial v_r} \Rightarrow N_{\dot{v}} = \frac{\partial N}{\partial \dot{v}} \\ N_r &= \frac{\partial N}{\partial r} \Rightarrow N_{\dot{r}} = \frac{\partial N}{\partial \dot{r}} \end{aligned} \tag{2.32}$$

Combining all the terms in (2.32) leads to the expression for the rotational force which is as under

$$N = N_v v + N_{\dot{v}} \dot{v} + N_r r + N_{\dot{r}} \dot{r} \tag{2.33}$$

$Y_v =$ Coefficient of sway force induced by side slip

$Y_{\dot{v}} =$ Mass added in the coefficient of sway

$Y_r =$ Coefficient of sway force induced by yaw

$Y_{\dot{r}} =$ Mass added in the coefficient of yaw

$N_v =$ Coefficient of sway moment obtained from side slip moment

$N_{\dot{v}} =$ Mass moment of inertia added in the coefficient of sway

$N_r =$ Coefficient of sway moment obtained from yaw moment

$N_{\dot{r}} =$ Mass moment of inertia added in the coefficient of yaw

According to [11], the action of both the rudders is given by $Y_\delta \delta_r(t)$ and $N_\delta \delta_r(t)$, where $Y_\delta$ is the gain in transverse direction of force and $N_\delta$ is the gain in the rotational direction of force and $\delta_r(t)$ is the control input to the each of the rudder.

The vehicle dynamics in the steering plane are thus;

$$m\dot{v} = -m\mu_0 r + Y_{\dot{v}}\dot{v} + Y_{\dot{v}}v + Y_{\dot{r}}\dot{r} + Y_r r + Y_\delta \delta_r(t) \tag{2.34}$$

$$I_z \dot{r} = N_{\dot{v}}\dot{v} + N_v v + N_{\dot{r}}\dot{r} + N_r r + N_\delta \delta_r(t) \tag{2.35}$$

$$\dot{\psi} = r \tag{2.36}$$

In the above equations,

$\Delta Y = Y_{\dot{v}}\dot{v} + Y_{\dot{v}}v + Y_{\dot{r}}\dot{r} + Y_r r + Y_\delta \delta_r(t)$, $\Delta N = N_{\dot{v}}\dot{v} + N_v v + N_{\dot{r}}\dot{r} + N_r r + N_\delta \delta_r(t)$, $v$ is the side slip velocity, $r$ is the yaw angular velocity and $\psi$ is yaw (vehicle heading angle).

From (2.34) - (2.36) the overall steering plane dynamics of the ARIES AUV in the matrix form can be written as

$$\begin{bmatrix} m - Y_{\dot{v}} & -Y_{\dot{r}} & 0 \\ -N_{\dot{v}} & I_z - N_{\dot{r}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{v} \\ \dot{r} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} Y_v & Y_r - m\mu_0 & 0 \\ N_v & N_r & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} v \\ r \\ \psi \end{bmatrix} + \begin{bmatrix} Y_\delta \\ N_\delta \\ 0 \end{bmatrix} \delta_r(t) \tag{2.37}$$

By considering the design symmetry of AUV in vertical and horizontal planes and the rudders are kept equidistant from the center of body, cross coupling terms $Y_{\dot{r}}$ and $N_{\dot{v}}$ in the matrix form of model can be considered null. (2.37) will thus become

$$\begin{bmatrix} m-Y_{\dot{v}} & 0 & 0 \\ 0 & I_z - N_{\dot{r}} & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \dot{v} \\ \dot{r} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} Y_v & Y_r - m\mu_0 & 0 \\ N_v & N_r & 0 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} v \\ r \\ \psi \end{bmatrix} + \begin{bmatrix} Y_\delta \\ N_\delta \\ 0 \end{bmatrix}\delta_r(t) \tag{2.38}$$

The state equations (2.38) can be written as

$$\begin{bmatrix} \dot{v} \\ \dot{r} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} m-Y_{\dot{v}} & 0 & 0 \\ 0 & I_z - N_{\dot{r}} & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1}\begin{bmatrix} Y_v & Y_r - m\mu_0 & 0 \\ N_v & N_r & 0 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} v \\ r \\ \psi \end{bmatrix} + \begin{bmatrix} m-Y_{\dot{v}} & 0 & 0 \\ 0 & I_z - N_{\dot{r}} & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1}\begin{bmatrix} Y_\delta \\ N_\delta \\ 0 \end{bmatrix}\delta_r(t)$$

$$\begin{bmatrix} \dot{v} \\ \dot{r} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} \frac{1}{m-Y_{\dot{v}}} & 0 & 0 \\ 0 & \frac{1}{I_z - N_{\dot{r}}} & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} Y_v & Y_r - m\mu_0 & 0 \\ N_v & N_r & 0 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} v \\ r \\ \psi \end{bmatrix} + \begin{bmatrix} \frac{1}{m-Y_{\dot{v}}} & 0 & 0 \\ 0 & \frac{1}{I_z - N_{\dot{r}}} & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} Y_\delta \\ N_\delta \\ 0 \end{bmatrix}\delta_r(t)$$

$$\begin{bmatrix} \dot{v} \\ \dot{r} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} \frac{Y_v}{m-Y_{\dot{v}}} & \frac{Y_r - m\mu_0}{m-Y_{\dot{v}}} & 0 \\ \frac{N_v}{I_z - N_{\dot{r}}} & \frac{N_r}{I_z - N_{\dot{r}}} & 0 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} v \\ r \\ \psi \end{bmatrix} + \begin{bmatrix} \frac{Y_\delta}{m-Y_{\dot{v}}} \\ \frac{N_\delta}{I_z - N_{\dot{r}}} \\ 0 \end{bmatrix}\delta_r(t)$$

$$\begin{bmatrix} \dot{v} \\ \dot{r} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} \frac{Y_v}{m-Y_{\dot{v}}} & \frac{Y_r - m\mu_0}{m-Y_{\dot{v}}} & 0 \\ \frac{N_v}{I_z - N_{\dot{r}}} & \frac{N_r}{I_z - N_{\dot{r}}} & 0 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} v \\ r \\ \psi \end{bmatrix} + \begin{bmatrix} \frac{Y_\delta}{m-Y_{\dot{v}}} \\ \frac{N_\delta}{I_z - N_{\dot{r}}} \\ 0 \end{bmatrix}\delta_r(t) \tag{2.39}$$

Thus finally the state equations in the steering plane will become

$$\dot{v} = \frac{Y_v}{m - Y_{\dot{v}}} v + \frac{Y_r - m\mu_0}{m - Y_{\dot{v}}} r + \frac{Y_\delta}{m - Y_{\dot{v}}} \delta_r(t) \tag{2.40}$$

$$\dot{r} = \frac{N_v}{I_z - N_{\dot{r}}} v + \frac{N_r}{I_z - N_{\dot{r}}} r + \frac{N_\delta}{I_z - N_{\dot{r}}} \delta_r(t) \tag{2.41}$$

$$\dot{\psi} = r \tag{2.42}$$

The values of parameters in (2.40) - (2.42) are determined in [11] and are given in appendix.

# Chapter 3 : CONTROL DESIGN & ANALYSIS

An AUV is needed to be controlled in steering-horizontal and diving-vertical planes. These two planes can be decoupled considering the symmetry and other aforementioned parameters and control law can be applied individually to both the planes. In this section, the control design and analysis for the path tracking and the waypoint tracking is discussed. First a state feedback sliding mode control is developed and then the state feedback control design is extended to an output feedback sliding mode controller. For the purpose waypoint tracking, two nonlinear controllers are used, one for the line of sight (LOS) error correction and other for the cross track (CTE) error correction. The control designs are tested for the actual and perturbed parameters. The detailed design and analysis is discussed below

## 3.1 Control Design for Path Tracking

This section describes the design of a nonlinear controller for the path tracking of ARIES AUV. First a sliding mode controller is designed using the feedback of actual states in the control law.

Then the controller design is extended to an output feedback sliding mode control in which observed states are fed to the controller. Controller design and stability analysis of the proposed control is presented in the following section.

### 3.1.1 State Feedback Sliding Mode Control Design

In this section the design of a state feedback sliding mode control along with its stability analysis is discussed. Lyapunov stability theory is used for the closed loop stability analysis of the system. The control law discussed here is based on the methodology presented in [6]. The objective of the controller is to track a reference heading angle $\psi_{com}$ under actual and perturbed system parameters. Consider the system (2.40) - (2.42), we can write the given system in the error coordinates as

$$\tilde{X} = X - X_d = \begin{bmatrix} v - v_d \\ r - r_d \\ \psi - \psi_d \end{bmatrix} \tag{3.1}$$

Where, $X_d$ is the reference state vector.

The sliding surface for the sliding mode control can be defined as

$$\sigma = S^T X \tag{3.2}$$

Where, $S = \begin{bmatrix} S_1 & S_2 & S_3 \end{bmatrix}$ is the vector of coefficients of the sliding surface. Rewriting (3.2) in the error coordinates, we get the sliding surface as;

$$\sigma = S^T \tilde{X} \tag{3.3}$$

$$\sigma = \begin{bmatrix} v - v_d \\ r - r_d \\ \psi - \psi_d \end{bmatrix}$$

S, must be chosen such that the condition $\lim_{t\to\infty} \sigma \to 0 \Rightarrow \lim_{t\to\infty} \left(X - X_d\right) \to 0$ must be fulfilled. The Lyapunov function candidate chosen to find S and prove stability is taken as the function of sliding surface as

$$V\left(\sigma\right) = \frac{1}{2}\sigma^2 \tag{3.4}$$

According to Lyapunov's theory, if

$$\begin{aligned} &V(\sigma) \geq 0 && \forall \;\; \sigma > 0 \\ &\dot{V}(\sigma) \leq 0 && \forall \;\; \sigma \end{aligned}$$

Then the system trajectories will converge to some equilibrium point in finite time and the system is stable

From (3.4),

$$\dot{V}(\sigma) = \sigma\dot{\sigma}$$

Or we can write as

$$\dot{V}(\sigma) = \sigma\dot{\sigma} \leq -\eta^2 \left|\sigma\right|^2 \leq 0 \tag{3.5}$$

For the nonlinear gain $\eta > 0$, (3.5) can be written as

$$\dot{\sigma} \leq -\eta^2 \left|\sigma\right|$$

$$\dot{\sigma} \le -\eta^2 \operatorname{sgn}(\sigma)\ \dot{\sigma} \le -\eta^2 \operatorname{sgn}(\sigma) \tag{3.6}$$

Under the condition, $\dot{\sigma} < 0 \Rightarrow \sigma \to 0 \Rightarrow \lim_{t\to\infty} \tilde{X} \to 0$, stability of the system can be ensured. To proceed for the design of control law, consider (3.3) and differentiate along the trajectories of the system, we get

$$\dot{\sigma} = S^T \dot{\tilde{X}}$$

$$\dot{\sigma} = S^T \left( \dot{X} - \dot{X}_d \right)$$

$$\dot{\sigma} = S^T \left( AX + Bu - \dot{X}_d \right)$$

Consider (3.6), we get

$$S^T \left( AX + Bu - \dot{X}_d \right) \le -\eta^2 \operatorname{sgn}(\sigma)$$

$$S^T AX + S^T Bu - S^T \dot{X}_d \le -\eta^2 \operatorname{sgn}(\sigma)$$

$$u \le \frac{-S^T AX + S^T \dot{X}_d - \eta^2 \operatorname{sgn}(\sigma)}{S^T B}$$

$$u \le -\left(S^T B\right)^{-1} S^T AX + \left(S^T B\right)^{-1} S^T \dot{X}_d - \left(S^T B\right)^{-1} \eta^2 \operatorname{sgn}(\sigma) \tag{3.7}$$

In (3.7), it can be seen that A, B and S are the linear terms, while $\eta^2 \operatorname{sgn}(\sigma)$ is a nonlinear term. Thus we can separate (3.7) in two parts, one having the linear terms and the other having the nonlinear terms as

$$u \leq \breve{u} + \bar{u} \tag{3.8}$$

Where, the linear term $\breve{u}$ is the state feedback – stabilizing control and the nonlinear term $\bar{u}$ is the tracking – nonlinear control. For a constant desired reference, we can define

$$\breve{u} \leq -\left(S^T B\right)^{-1} S^T AX = -KX \tag{3.9}$$

$$\bar{u} \leq -\left(S^T B\right)^{-1} \eta^2 \operatorname{sgn}(\sigma) \tag{3.10}$$

Now (3.7) can be rewritten as

$$u = -KX - \left(S^T B\right)^{-1} \eta^2 \operatorname{sgn}(\sigma) \tag{3.11}$$

Where, K is the state feedback gain vector.

To find the closed loop system, consider (3.1)

$$\dot{\tilde{X}} = \dot{X} - \dot{X}_d$$

$$\dot{\tilde{X}} = AX + Bu - \dot{X}_d$$

Putting the value of control signal $u$ from (3.11) and considering a constant reference signal, we get

$$\dot{\tilde{X}} = AX + B\left(-\left(S^T B\right)^{-1} S^T AX - \left(S^T B\right)^{-1} \eta^2 \operatorname{sgn}(\sigma)\right)$$

From (3.9), $K = \left(S^T B\right)^{-1} S^T AX$ , above equation can be written as

$$\dot{\tilde{X}} = AX + B\left(-KX - \left(S^T B\right)^{-1} \eta^2 \operatorname{sgn}(\sigma)\right)$$

$$\dot{\tilde{X}} = (A - BK)X - B\left(S^T B\right)^{-1} \eta^2 \operatorname{sgn}(\sigma) \tag{3.12}$$

(3.12) is the closed loop equation of the ARIES AUV in the steering plane.

In (3.7), the terms still to be determined are K, S and $\sigma$. Since, $\sigma$ is dependent on S, and S is dependent on K, so first we have to find K. The feedback gain matrix K can be found by placing the closed loop poles of the system at suitable location in the open left half of the s-plane as $P = \begin{bmatrix} 0 & p_1 & p_2 \end{bmatrix}$

Consider (3.9), for any constant reference signal, we get

$$-\left(S^T B\right)^{-1} S^T AX = -KX$$

Or $$K = \left(S^T B\right)^{-1} S^T A$$

$$\begin{bmatrix} k_1 & k_2 & k_3 \end{bmatrix} = \left( \begin{bmatrix} s_1 \\ s_2 \\ s_3 \end{bmatrix}^T \begin{bmatrix} \dfrac{Y_\delta}{m - Y_{\dot v}} \\ \dfrac{N_\delta}{I_z - N_{\dot r}} \\ 0 \end{bmatrix} \right)^{-1} \begin{bmatrix} s_1 \\ s_2 \\ s_3 \end{bmatrix}^T \begin{bmatrix} \dfrac{Y_v}{m - Y_{\dot v}} & \dfrac{Y_r - m\mu_0}{m - Y_{\dot v}} & 0 \\ \dfrac{N_v}{I_z - N_{\dot r}} & \dfrac{N_r}{I_z - N_{\dot r}} & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} k_1 & k_2 & k_3 \end{bmatrix} = \left( \begin{bmatrix} s_1 & s_2 & s_3 \end{bmatrix} \begin{bmatrix} \dfrac{Y_\delta}{m - Y_{\dot v}} \\ \dfrac{N_\delta}{I_z - N_{\dot r}} \\ 0 \end{bmatrix} \right)^{-1} \begin{bmatrix} s_1 & s_2 & s_3 \end{bmatrix} \begin{bmatrix} \dfrac{Y_v}{m - Y_{\dot v}} & \dfrac{Y_r - m\mu_0}{m - Y_{\dot v}} & 0 \\ \dfrac{N_v}{I_z - N_{\dot r}} & \dfrac{N_r}{I_z - N_{\dot r}} & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} k_1 & k_2 & k_3 \end{bmatrix} = \frac{1}{s_1\left(\frac{Y_\delta}{m-Y_{\dot{v}}}\right)+s_2\left(\frac{N_\delta}{I_z-N_{\dot{r}}}\right)}\begin{bmatrix} s_1\left(\frac{Y_v}{m-Y_{\dot{v}}}\right)+s_2\left(\frac{N_v}{I_z-N_{\dot{r}}}\right) & s_1\left(\frac{Y_r-m\mu_0}{m-Y_{\dot{v}}}\right)+s_2\left(\frac{N_r}{I_z-N_{\dot{r}}}\right)+s_3 & 0 \end{bmatrix}$$

Taking the transpose of both sides, we can write as

$$\begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix}^T = \begin{bmatrix} \left(s_1\left(\frac{Y_\delta}{m-Y_{\dot{v}}}\right)+s_2\left(\frac{N_\delta}{I_z-N_{\dot{r}}}\right)\right)^{-1}\left(s_1\left(\frac{Y_v}{m-Y_{\dot{v}}}\right)+s_2\left(\frac{N_v}{I_z-N_{\dot{r}}}\right)\right) \\ \left(s_1\left(\frac{Y_\delta}{m-Y_{\dot{v}}}\right)+s_2\left(\frac{N_\delta}{I_z-N_{\dot{r}}}\right)\right)^{-1}\left(s_1\left(\frac{Y_r-m\mu_0}{m-Y_{\dot{v}}}\right)+s_2\left(\frac{N_r}{I_z-N_{\dot{r}}}\right)+s_3\right) \\ 0 \end{bmatrix}^T$$

Where,

$$k_1 = \left(s_1\left(\frac{Y_\delta}{m-Y_{\dot{v}}}\right)+s_2\left(\frac{N_\delta}{I_z-N_{\dot{r}}}\right)\right)^{-1}\left(s_1\left(\frac{Y_v}{m-Y_{\dot{v}}}\right)+s_2\left(\frac{N_v}{I_z-N_{\dot{r}}}\right)\right) \tag{3.13}$$

$$k_2 = \left(s_1\left(\frac{Y_\delta}{m-Y_{\dot{v}}}\right)+s_2\left(\frac{N_\delta}{I_z-N_{\dot{r}}}\right)\right)^{-1}\left(s_1\left(\frac{Y_r-m\mu_0}{m-Y_{\dot{v}}}\right)+s_2\left(\frac{N_r}{I_z-N_{\dot{r}}}\right)+s_3\right) \tag{3.14}$$

$$k_3 = 0 \tag{3.15}$$

Solving (3.13) - (3.15) simultaneously, we can find the values of S. It can be seen that the solution found by solving simultaneously require a lot of computations; a rather simpler method is by finding the Eigen values of the closed loop system and then finding the Eigen vector of the matrix $(A-BK)^T$ corresponding to the null Eigen value.

Consider (3.2), we get

$$\dot{\sigma} = S^T \dot{X} = 0$$

$$\dot{\sigma} = S^T (AX + Bu) = 0$$

Since according to state feedback design, $u = -KX$ , we get

$$\dot{\sigma} = S^T (AX - BKX) = 0$$

$$\dot{\sigma} = S^T (A - BK)X = 0$$

$$S^T (A - BK) = 0$$

$$(A - BK)^T S = 0 \tag{3.16}$$

In (3.16), S is the Eigen vector corresponding to the null Eigen values of the matrix $(A - BK)^T$. Now the sliding surface, $\sigma$ can be found by putting the values of S in (3.2).

The nonlinear term $\mathrm{sgn}(\sigma)$ in the nonlinear control element (3.10) cause chattering due to switching in the control signal with larger peaks. These peaks can create undesired harmonics in the system and increases the control effort. The effect of chattering can't be eliminated completely in sliding mode control but can be reduced. $sat(\sigma)$, $sig(\sigma)$ and $\tanh(\sigma)$ are well known alternatives for $\mathrm{sgn}(\sigma)$ which can reduce the chattering effect and minimizes the control effort.

Now the control law (3.11) and the closed loop system (3.12) using $\tanh(\sigma)$ instead of $\mathrm{sgn}(\sigma)$ to reduce chattering, will become

$$u = -KX - \left(S^T B\right)^{-1} \eta^2 \tanh\left(\sigma\right) \tag{3.17}$$

and the closed loop system will thus become

$$\dot{X} = \left(A - BK\right) X - B\left(S^T B\right)^{-1} \eta^2 \tanh\left(\sigma\right) \tag{3.18}$$

### 3.1.2 Observer Based Sliding Mode Control Design

This section presents the design along with the stability analysis of an output feedback observer based sliding mode control law. The structure of observer is based on the linear Luenberger observer. Observer designed is of the same order as that of system model in steering i.e. $3^{rd}$ observer. The driving term for the observer is the output of the system i.e. the heading angle $\psi(t)$. In the previous section the system was controlled by state feedback sliding mode control which was driven by the actual measured states of the system. In this case, the control law will be driven by the observed states. The only state which is needed to be measured is $\psi(t)$. The design of proposed control law along with its stability analysis will proceed as follows.

Consider the system

$$\dot{X} = AX + Bu \tag{3.19}$$

$$Y = CX \tag{3.20}$$

Where, $C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$ is the output matrix and thus $y = \psi$ is the output equation. To proceed with the observer design first we need to check the set of matrices $(A, C)$ must be observable. In this case

$$O = \begin{bmatrix} C & CA & CA^2 \end{bmatrix}^T \tag{3.21}$$

$$O = \begin{bmatrix} \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{Y_v}{m - Y_{\dot{v}}} & \frac{Y_r - m\mu_0}{m - Y_{\dot{v}}} & 0 \\ \frac{N_v}{I_z - N_{\dot{r}}} & \frac{N_r}{I_z - N_{\dot{r}}} & 0 \\ 0 & 1 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{Y_v}{m - Y_{\dot{v}}} & \frac{Y_r - m\mu_0}{m - Y_{\dot{v}}} & 0 \\ \frac{N_v}{I_z - N_{\dot{r}}} & \frac{N_r}{I_z - N_{\dot{r}}} & 0 \\ 0 & 1 & 0 \end{bmatrix}^2 \end{bmatrix}^T$$

$$O = \begin{bmatrix} \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} & \begin{bmatrix} \frac{N_v}{I_z - N_{\dot{r}}} & \frac{N_r}{I_z - N_{\dot{r}}} & 0 \end{bmatrix} \end{bmatrix}^T$$

$$O = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ \frac{N_v}{I_z - N_{\dot{r}}} & \frac{N_r}{I_z - N_{\dot{r}}} & 0 \end{bmatrix}$$

By putting the values it can be seen that $\det(O)$ has a nonsingular solution. Thus the matrix ‘O’ has rank n and hence the pair $(A, B)$ is observable. From (3.17) the control law driven by the observed states will take the form as

$$\hat{u} = -K\hat{X} + (S^T B)^{-1} \eta^2 \tanh(\hat{\sigma}/\varepsilon) \tag{3.22}$$

In (3.22), $\hat{X}$ is the state vector of the observer and is defined as

$$\hat{X} = \begin{bmatrix} \hat{v} \\ \hat{r} \\ \hat{\psi} \end{bmatrix} = \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \hat{x}_3 \end{bmatrix} \tag{3.23}$$

and $\hat{\sigma}$ is the sliding surface of the observer and is defined as $\hat{\sigma} = S^T \hat{X}$ .

The general form of the Luenberger observer is as

$$\dot{\hat{X}}(t) = A\hat{X}(t) + B\hat{u}(t) + L\left[y(t) - \hat{y}(t)\right] \tag{3.24}$$

From (2.40) - (2.42), the system equations can be written as

$$\dot{x}_1 = \frac{Y_v}{m - Y_{\dot{v}}} x_1 + \frac{Y_r - m\mu_0}{m - Y_{\dot{v}}} x_2 + \frac{Y_\delta}{m - Y_{\dot{v}}} \hat{u}$$

$$\dot{x}_2 = \frac{N_v}{I_z - N_{\dot{r}}} x_1 + \frac{N_r}{I_z - N_{\dot{r}}} x_2 + \frac{N_\delta}{I_z - N_{\dot{r}}} \hat{u}$$

$$\dot{x}_3 = x_2$$

Where the control input $u$ is replaced by $\hat{u}$ i.e. the control signal which is driven by the observed states. Expanding (3.24), the Luenberger observer equations can be written as

$$\dot{\hat{x}}_1 = \frac{Y_v}{m - Y_{\dot{v}}} \hat{x}_1 + \frac{Y_r - m\mu_0}{m - Y_{\dot{v}}} \hat{x}_2 + \frac{Y_\delta}{m - Y_{\dot{v}}} \hat{u} + l_1\left(x_3 - \hat{x}_3\right) \tag{3.25}$$

$$\dot{\hat{x}}_2 = \frac{N_v}{I_z - N_{\dot{r}}} \hat{x}_1 + \frac{N_r}{I_z - N_{\dot{r}}} \hat{x}_2 + \frac{N_\delta}{I_z - N_{\dot{r}}} \hat{u} + l_2(x_3 - \hat{x}_3) \tag{3.26}$$

$$\dot{\hat{x}}_3 = \hat{x}_2 + l_3(x_3 - \hat{x}_3) \tag{3.27}$$

From (3.17) and (3.25) - (3.27), we can write the control law driven by the observed states as

$$\hat{u} = -K\hat{X} + (S^T B)^{-1}\eta^2 \tanh(\hat{\sigma}/\varepsilon) \tag{3.28}$$

To proceed with the stability analysis of the output feedback observer based sliding mode control, we will transform the system (2.40) - (2.42) and (3.25) - (3.27) in the error coordinates. The estimation error can be defined as

$$e_i = x_i - \hat{x}_i \tag{3.29}$$

$$\dot{e}_i = \dot{x}_i - \dot{\hat{x}}_i \tag{3.30}$$

Where, $i = 1, 2, 3$.

From (2.40) and (3.25), the side slip velocity in the estimation error coordinates can be defined as

$$\dot{e}_1 = \frac{Y_v}{m - Y_{\dot{v}}} x_1 + \frac{Y_r - m\mu_0}{m - Y_{\dot{v}}} x_2 + \frac{Y_\delta}{m - Y_{\dot{v}}} \hat{u} - \left( \frac{Y_v}{m - Y_{\dot{v}}} \hat{x}_1 + \frac{Y_r - m\mu_0}{m - Y_{\dot{v}}} \hat{x}_2 + \frac{Y_\delta}{m - Y_{\dot{v}}} \hat{u} + l_1 \left( x_3 - \hat{x}_3 \right) \right)$$

$$\dot{e}_1 = \frac{Y_v}{m - Y_{\dot{v}}} x_1 - \frac{Y_v}{m - Y_{\dot{v}}} \hat{x}_1 + \frac{Y_r - m\mu_0}{m - Y_{\dot{v}}} x_2 - \frac{Y_r - m\mu_0}{m - Y_{\dot{v}}} \hat{x}_2 + \frac{Y_\delta}{m - Y_{\dot{v}}} \hat{u} - \frac{Y_\delta}{m - Y_{\dot{v}}} \hat{u} - l_1 \left( x_3 - \hat{x}_3 \right)$$

$$\dot{e}_1 = \frac{Y_v}{m - Y_{\dot{v}}} e_1 + \frac{Y_r - m\mu_0}{m - Y_{\dot{v}}} e_2 - l_1 e_3 \tag{3.31}$$

From (2.41) and (3.26), the yaw angular velocity in the estimation error coordinates can be defined as

$$\dot{e}_2 = \frac{N_v}{I_z - N_{\dot{r}}} x_1 + \frac{N_r}{I_z - N_{\dot{r}}} x_2 + \frac{N_\delta}{I_z - N_{\dot{r}}} \hat{u} - \left( \frac{N_v}{I_z - N_{\dot{r}}} \hat{x}_1 + \frac{N_r}{I_z - N_{\dot{r}}} \hat{x}_2 + \frac{N_\delta}{I_z - N_{\dot{r}}} \hat{u} + l_2 (x_3 - \hat{x}_3) \right)$$

$$\dot{e}_2 = \frac{N_v}{I_z - N_{\dot{r}}} x_1 - \frac{N_v}{I_z - N_{\dot{r}}} \hat{x}_1 + \frac{N_r}{I_z - N_{\dot{r}}} x_2 - \frac{N_r}{I_z - N_{\dot{r}}} \hat{x}_2 + \frac{N_\delta}{I_z - N_{\dot{r}}} \hat{u} - \frac{N_\delta}{I_z - N_{\dot{r}}} \hat{u} - l_2 (x_3 - \hat{x}_3)$$

$$\dot{e}_2 = \frac{N_v}{I_z - N_{\dot{r}}} e_1 - \frac{N_r}{I_z - N_{\dot{r}}} e_2 - l_2 e_3 \tag{3.32}$$

From (2.42) and (3.27), the yawing or heading angle in the estimation error coordinates can be defined as

$$\dot{e}_3 = x_2 - \left( \hat{x}_2 + l_3 (x_3 - \hat{x}_3) \right)$$

$$\dot{e}_3 = x_2 - \hat{x}_2 - l_3 (x_3 - \hat{x}_3)$$

$$\dot{e}_3 = e_2 - l_3 e_3 \tag{3.33}$$

To show the convergence of the trajectories of the system in finite time Lyapunov function candidate in the error coordinates is as

$$V(e) = \sum_{i=1}^{3} \frac{1}{2} e_i^{\,2} \tag{3.34}$$

Taking the time derivative of (3.34) along the trajectories of the estimation error, we get

$$\dot{V}(e) = e_1 \dot{e}_1 + e_2 \dot{e}_2 + e_3 \dot{e}_3$$

$$\dot{V}(e) = e_1\left(\frac{Y_v}{m-Y_{\dot{v}}}e_1 + \frac{Y_r - m\mu_0}{m-Y_{\dot{v}}}e_2 - l_1e_3\right) + e_2\left(\frac{N_v}{I_z - N_{\dot{r}}}e_1 - \frac{N_r}{I_z - N_{\dot{r}}}e_2 - l_2e_3\right) + e_3\left(e_2 - l_3e_3\right)$$

Applying the property of mod, we get

$$\dot{V}(e) \leq -\left(-\frac{Y_v}{m-Y_{\dot{v}}} + \frac{Y_r - m\mu_0}{2\left(m-Y_{\dot{v}}\right)} + \frac{N_v}{2\left(I_z - N_{\dot{r}}\right)} + \frac{l_1}{2}\right)\left|e_1\right|^2$$

$$-\left(-\frac{N_r}{I_z - N_{\dot{r}}} + \frac{Y_r - m\mu_0}{2\left(m-Y_{\dot{v}}\right)} + \frac{N_v}{2\left(I_z - N_{\dot{r}}\right)} - \frac{l_1}{2}\right)\left|e_2\right|^2 - \left(\frac{1}{2} - \frac{l_1}{2} - \frac{l_2}{2} + l_3\right)\left|e_3\right|^2$$

$$\dot{V}(e) \leq -\beta_1\left|e_1\right|^2 - \beta_2\left|e_2\right|^2 - \beta_3\left|e_3\right|^2 \tag{3.35}$$

From (3.35), $\dot{V}(e) \leq 0, \forall \beta_1, \beta_2, \beta_3 > 0$.

Thus, by choosing the suitable gains $l_1, l_2$ and $l_3$ of the observer which make the terms $\beta_1, \beta_2, \beta_3 > 0$, $\dot{V}(e)$ can be rendered negative definite. The selection of observer gains allows us to find a region in which the combination of body parameters of AUV and gains give us $\beta_1, \beta_2, \beta_3 > 0$. The calculations show that the body parameters can be perturbed upto 20% of their actual values, which the observer gains can handle. Thus, according to Lyapunov stability theory the trajectories of the system will converge at equilibrium point in finite time and the stability as well as the robustness is ensured.

## 3.2 Controller Design for Waypoint Tracking

This section describes the design of nonlinear controllers for the waypoint tracking of ARIES AUV. The need for the waypoint tracking is that, in path tracking the control variable is the output of the system i.e. $\psi$ and it only contains the information of heading angle and it needs a complete path to follow as a reference trajectory. While in waypoint tracking the position of AUV in the earth fixed frame can be controlled by providing a set of waypoints instead of a complete path like path tracking. The orientation and position is controlled by the line of sight (LOS) and cross tack error (CTE) controllers respectively. Design of both the controllers is provided in the following section.

### 3.2.1 Line of Sight & Cross Track Error Controllers

An AUV is needed to be controlled in steering-horizontal and diving-vertical planes. These two planes can be decoupled considering the symmetry and other aforementioned parameters and control law can be applied individually to both the planes. For waypoint tracking, steering plane includes the line of sight and cross track error controllers, while the diving plane includes the depth controller. Speed controller is common in both the planes. The controllers discussed here are based on the theory of sliding mode control presented in [12]. Sliding mode control is a powerful and commonly used technique in controls and is practically suitable in most of the cases. Marco and Healy in [13] has designed a second order model for the steering control of AUV in which the sideslip $(v)$ is treated as disturbance in the system. This sideslip is fed to the sliding mode controller as disturbance.

From (2.40) - (2.42) we get the system model for the steering plane control design as under in which the effect of side slip velocity $(v)$ is treated as disturbance to the system

$$\dot{r}(t) = \frac{N_r}{I_z - N_{\dot{r}}} r(t) + \frac{N_\delta}{I_z - N_{\dot{r}}} \delta_r(t) + \delta_{Disturbance} \tag{3.36}$$

$$\dot{\psi}(t) = r(t) \tag{3.37}$$

In the above equations, the side slip is treated as a disturbance $\left(\delta_{Disturbance}\right)$ to the yaw velocity (angular velocity) and heading angle $(\psi)$ is the output which is needed to be controlled.

In this scope of work, two controllers are used for the waypoint tracking of ARIES AUV i.e. line of sight error correction controller and cross track error correction controller. The need for two controllers is that whenever the line of sight error or heading error is large enough, cross track error cannot be guaranteed stable and vice versa. Thus, alternating between both the controllers will help in minimizing both the errors simultaneously and thus the AUV will follow a guided path. The velocity vector diagram and the geometry of the track are explained in figure (2).

In order to achieve the goal of waypoint tracking in steering plane, the position and orientation of the AUV must be kept within a certain limit from the waypoints. The position is controlled by the cross track error and the orientation is controlled by the line of sight controller. Complete description of above figure with respect to waypoint tracking is as

$(x(t), y(t)) =$ Current position of AUV

$\left(X_{wp(i-1)}, Y_{wp(i-1)}\right) =$ Previous waypoint

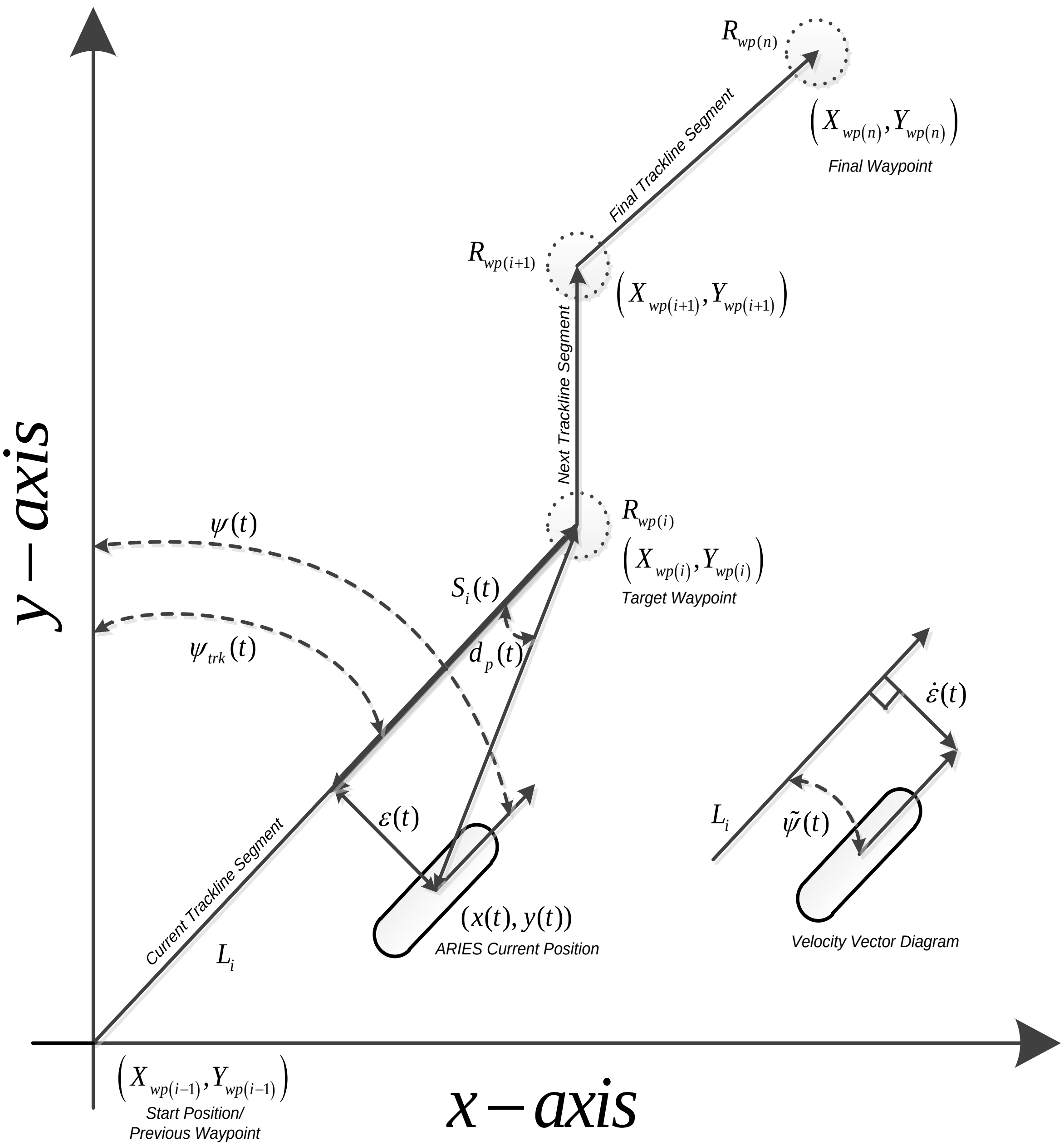


**Figure 3-1: Track Geometry & Velocity Vector Diagram of ARIES in Earth Fixed Coordinate Frame**

$\left(X_{wp(i)}, Y_{wp(i)}\right) =$ Target waypoint

$\left(X_{wp(i+1)}, Y_{wp(i+1)}\right) =$ Next waypoint to target waypoint

$(\tilde{X}(t)_{wp(i)}, \tilde{Y}(t)_{wp(i)}) =$ Difference between the current position of AUV and the next waypoint

$$\tilde{X}(t)_{wp(i)} = X_{wp(i)} - X(t) \tag{3.38}$$

$$\tilde{Y}(t)_{wp(i)} = Y_{wp(i)} - Y(t) \tag{3.39}$$

$L_i =$ Total length of the $i^{th}$ track segment. It is measured as the shortest distance between two consecutive waypoints

$$L_i = \sqrt{\left(X_{wp(i)} - X_{wp(i-1)}\right)^2 + \left(Y_{wp(i)} - Y_{wp(i-1)}\right)^2} \tag{3.40}$$

$S_i(t) =$ Distance to be covered by AUV to reach the target waypoint. It is measured as the distance between the next waypoint and the projection of current position of AUV on the track line

$$S_i(t) = \frac{\left[\tilde{X}(t)_{wp(i)} \quad \tilde{Y}(t)_{wp(i)}\right]\left[X_{wp(i)} - X_{wp(i-1)} \quad Y_{wp(i)} - Y_{wp(i-1)}\right]^T}{L_i} \tag{3.41}$$

Thus, from the above relation it can be seen that $S_i(t)$ lies between $0-100\%$ of $L_i$. 0 shows that no distance is covered on $i^{th}$ track segment and 100 shows that $i^{th}$ track segment has been traveresed completely.

$\psi(t) =$ Current heading angle of AUV. It is measured as the angle between the vector normal to the front of AUV and the x - axis with respect to earth fixed frame

$\psi_{trc(i)} = \psi_{com}$ is the angle of the $i^{th}$ track line segment, $L_i$. It is measured with respect to Cartesian coordinate system or earth fixed frame. It is the commanded heading angle for AUV

$$\psi_{trk(i)} = \psi_{com} = \tan^{-1}(Y_{wp(i)} - Y_{wp(i-1)}, X_{wp(i)} - X_{wp(i-1)}) \tag{3.42}$$

$d_p(t) =$ Angle between the line of sight of AUV to the next waypoint and the current track line segment $\angle\left(x(t), y(t)\right)\left(X_{wp(i)}, Y_{wp(i)}\right)\left(X_{wp(i-1)}, Y_{wp(i-1)}\right)$

$$d_p(t) = \tan^{-1}\left(\frac{Y_{wp(i)} - Y_{wp(i-1)}}{X_{wp(i)} - Y_{wp(i-1)}}\right) - \tan^{-1}\left(\frac{X_{wp(i)} - X(t)}{Y_{wp(i)} - Y(t)}\right) \tag{3.43}$$

This angle is normalized between $\pm 180^\circ$.

$\varepsilon(t) =$ Magnitude of cross track error (CTE). It is the perpendicular distance between the track line segment and the current position of AUV. The magnitude of the cross track error is defined as

$$\varepsilon(t) = S_i(t)\sin(d_p(t)) \tag{3.44}$$

$R_{wp(i)} =$ Watch radius of the $i^{th}$ waypoint

While cruising underwater an AUV can hardly navigate to the waypoint exactly due to disturbances and environmental factors acting on it, so a region across the waypoint is defined called as watch radius, which assumes the complete region as the current target waypoint. If the AUV reaches within this region the mission for the segment $L_i$ is assumed to be completed and next waypoint will be activated. Mathematically, the condition for watch radius is defined as, if

$\sqrt{\left(\tilde{X}(t)_{wp(i)}\right)^2+\left(\tilde{Y}(t)_{wp(i)}\right)^2} \le R_{wp(i)}$, then the tracking for the current track line segment $L_i$ is completed and the next waypoint is activated.

Since, there are two controllers used for the waypoint tracking, the line of sight (LOS) and cross track error (CTE), there must be two control signals. The control design methodology for both the controllers is given in the following section.

### 3.2.1.1 Line of Sight (LOS) Control Design

As mentioned earlier, the line of sight controller controls the orientation (heading angle) of AUV. The commanded heading for the line of sight controller is the angle between the current position of AUV and the target waypoint

$$\psi_{com(LOS(i))} = \tan^{-1}\left(\frac{\tilde{Y}(t)_{wp(i)}}{\tilde{X}(t)_{wp(i)}}\right)$$

Where, $\tilde{X}(t)_{wp(i)} = X_{wp(i)} - X(t)$ and $\tilde{Y}(t)_{wp(i)} = Y_{wp(i)} - Y(t)$

The line of sight heading angle error defines the orientation of AUV needed to correct and is given as

$$\tilde{\psi}_{LOS(i)} = \psi_{com(LOS(i))} - \psi(t)$$

The sliding surface $\sigma$ for the tracking control and the command signal to the rudder actuator is defined in [13] is given as

$$\sigma(t) = \alpha_1 r(t) + \alpha_2 \tilde{\psi}_{LOS} \tag{3.45}$$

Differentiating $\sigma(t)$, we get

$$\dot{\sigma}(t) = \alpha_3 r(t) + \alpha_4 \delta_r(t)$$

To prove the stability of the control law, choose a Lyapunov function candidate as a function of sliding surface as

$$V(\sigma) = \frac{1}{2}\sigma^2$$

To ensure stability of the control law, the condition must be fulfilled

$$\dot{V}(\sigma) = \sigma\dot{\sigma} \le -\eta^2 \left|\sigma\right|^2 \le 0$$

Solving $\dot{V}(\sigma)$ for $\delta_r(t)$, the equation for the line of sight controller will be given as

$$\delta_r(t) = \alpha_3(\alpha_4 r(t)) + \eta \tanh(\sigma(t)/\phi)) \quad (3.46)$$

It can be seen that the sliding surface and the control signal is a function of the states of the system i.e. the yaw rate $\dot{r}$ and the yaw angle $\psi(t)$. The driving term for the controller equation is the error of the yawing angle $\tilde{\psi}_{LOS}$.

In (3.45) - (3.46), $\alpha_1 = -0.9499, \alpha_2 = 0.1701, \alpha_3 = -1.543, \alpha_1 = 2.5394$ are the design parameters obtained by experimental results [13], $\eta = 2.0$ is the gain for nonlinear term, $\psi_{com(LOS)} - \psi(t)$ is the heading angle error, $\phi = 0.5$ is the width of the boundary layer for the sliding mode controller and $\tanh$ function is used to reduce the effect of chattering in the nonlinear control.

### 3.2.1.2 Cross Track Error (CTE) Control Design

The cross track error controller controls the position of AUV and keep the motion of AUV along the track line. The commanded heading angle for the cross track error controller is the angle between the previous waypoint and the target waypoint of AUV and is given as

$$\psi_{com(CTE(i))} = \tan^{-1}\left(\frac{\breve{Y}_{wp(i)}}{\breve{X}_{wp(i)}}\right)$$

Where, $\breve{Y}_{wp(i)} = Y_{wp(i)} - Y_{wp(i-1)}$ and $\breve{X}_{wp(i)} = X_{wp(i)} - X_{wp(i-1)}$

$\tilde{\psi}(t)_{CTE(i)} =$ Cross track heading angle error for $i^{th}$ segment. It is measured as the difference between the current orientation of AUV and the commanded cross track angle

$$\tilde{\psi}(t)_{CTE(i)} = \psi(t) - \psi_{com(CTE(i))} \tag{3.47}$$

The sliding surface chosen for the cross track error controller is defined as the function of derivatives of the magnitude of cross track error, $\varepsilon(t)$.

$$\sigma(t) = \ddot{\varepsilon}(t) + \lambda_1\dot{\varepsilon}(t) + \lambda_2\varepsilon(t)$$

Taking the derivative of the sliding surface along the trajectories of the system, we get

$$\dot{\sigma}(t) = \dddot{\varepsilon}(t) + \lambda_1\ddot{\varepsilon}(t) + \lambda_2\dot{\varepsilon}(t)$$

To prove the stability of the control law, choose a Lyapunov function candidate as a function of sliding surface as

$$V(\sigma) = \frac{1}{2}\sigma^2$$

To ensure stability of the control law, the condition must be fulfilled

$$\dot{V}(\sigma) = \sigma\dot{\sigma} \le -\eta^2 \left|\sigma\right|^2 \le 0$$

$$\Rightarrow \qquad \dddot{\varepsilon}(t) + \lambda_1 \ddot{\varepsilon}(t) + \lambda_2 \dot{\varepsilon}(t) \le -\eta^2 \operatorname{sgn}(\sigma) \qquad (3.48)$$

Since, $\varepsilon(t) = S_i(t)\sin(d_p(t)) \Rightarrow \varepsilon(t) = S_i(t)\sin(\tilde{\psi}(t)_{CTE(i)}) \Rightarrow \varepsilon(t) = S_i(t)\sin(\psi(t) - \psi_{com(CTE(i))})$,

taking the first, second and third derivatives (using product rule of differentiation), we get

$$\dot{\varepsilon}(t) = \mu_0 \sin(\psi(t) - \psi_{com(CTE(i))})$$

Where, $\dot{S}_i(t) = \mu_0$ is the speed of the vehicle.

Taking the second and third derivative for better precision in control and putting the value of $\dot{\psi}(t) = r(t)$, we get

$$\ddot{\varepsilon}(t) = \mu_0 r(t)\cos(\psi(t) - \psi_{com(CTE(i))})$$

$$\dddot{\varepsilon}(t) = \mu_0 \dot{r}(t)\cos(\psi(t) - \psi_{com(CTE(i))}) - \mu_0 r^2(t)\sin(\psi(t) - \psi_{com(CTE(i))})$$

Since, $\dot{r}(t) = \dfrac{N_r}{I_z - N_{\dot{r}}} r(t) + \dfrac{N_\delta}{I_z - N_{\dot{r}}} \delta_r(t)$, the above equation can be rewritten as

$$\dddot{\varepsilon}(t) = \mu_0 \frac{N_r}{I_z - N_{\dot{r}}} r(t) + \frac{N_\delta}{I_z - N_{\dot{r}}} \delta_r(t)\cos(\psi(t) - \psi_{com(CTE(i))}) - \mu_0 r^2(t)\sin(\psi(t) - \psi_{com(CTE(i))})$$

Putting the values of $\dot{\varepsilon}, \ddot{\varepsilon}$ and $\dddot{\varepsilon}$ in equation (3.48), we get

$$\mu_0 \frac{N_r}{I_z - N_{\dot{r}}} r(t) + \frac{N_\delta}{I_z - N_{\dot{r}}} \delta_r(t)\cos(\psi(t) - \psi_{com(CTE(i))}) - \mu_0 r^2(t)\sin(\psi(t) - \psi_{com(CTE(i))})$$

$$+ \lambda_1 \mu_0 r(t)\cos(\psi(t) - \psi_{com(CTE(i))}) + \lambda_2 \mu_0 \sin(\psi(t) - \psi_{com(CTE(i))}) = -\eta^2 \operatorname{sgn}(\sigma)$$

Since, $\tilde{\psi}(t)_{CTE(i)} = \psi(t) - \psi_{com(CTE(i))}$, the above equation can be rewritten as

$$-\eta^2 \operatorname{sgn}(\sigma) = \mu_0 \frac{N_r}{I_z - N_{\dot{r}}} r(t) + \frac{N_\delta}{I_z - N_{\dot{r}}} \delta_r(t) \cos(\tilde{\psi}(t)_{CTE(i)}) - \mu_0 r^2(t) \sin(\tilde{\psi}(t)_{CTE(i)})$$
$$+ \lambda_1 \mu_0 r(t) \cos(\tilde{\psi}(t)_{CTE(i)}) + \lambda_2 \mu_0 \sin(\tilde{\psi}(t)_{CTE(i)}) \tag{3.49}$$

Let, $\gamma_1 = \dfrac{N_r}{I_z - N_{\dot{r}}}$ and $\gamma_2 = \dfrac{N_\delta}{I_z - N_{\dot{r}}}$

Solving (3.49) for $\delta_r(t)$, we get

$$\delta_r(t) = \frac{1}{(\mu_0 \gamma_2 \cos(\tilde{\psi}(t)_{CTE(i)})} \{-\mu_0 \gamma_1 r(t) \cos(\tilde{\psi}(t)_{CTE(i)}) + \mu_0 r^2(t) \sin(\tilde{\psi}(t)_{CTE(i)})$$
$$-\lambda_1 \mu_0 r(t) \cos(\tilde{\psi}(t)_{CTE(i)}) - \lambda_2 \mu_0 \sin(\tilde{\psi}(t)_{CTE(i)}) - \eta \operatorname{sgn}(\sigma(t))\}$$

Hence by fulfilling the Lyapunov stability condition (3.48) the control law derived will ensure the stability of the system in finite time. To reduce the effect of chattering, $\operatorname{sgn}(\sigma)$ is replaced by its traditional substitute $\tanh(\sigma(t)/\phi)$ is used, thus finally we get the control signal for the cross track error controller as

$$\delta_r(t) = \frac{1}{(\mu_0 \gamma_2 \cos(\tilde{\psi}(t)_{CTE(i)})} \{-\mu_0 \gamma_1 r(t) \cos(\tilde{\psi}(t)_{CTE(i)}) + \mu_0 r^2(t) \sin(\tilde{\psi}(t)_{CTE(i)})$$
$$\forall;\ 0 \le \tilde{\psi}(t)_{CTE(i)} < \frac{\pi}{2}$$
$$-\lambda_1 \mu_0 r(t) \cos(\tilde{\psi}(t)_{CTE(i)}) - \lambda_2 \mu_0 \sin(\tilde{\psi}(t)_{CTE(i)}) - \eta \tan(\sigma(t)/\phi)\} \tag{3.50}$$

Where, $\phi$ is the width of the boundary layer.

$\tilde{\psi}(t)_{CTE(i)}$ is normalized between $\pm 180^{\circ}$ so that AUV rotate the shortest path instead of having a rotation more than $180^{\circ}$. For any rotation of $180^{\circ}+\theta$, instead of rotating $180^{\circ}+\theta$, AUV will rotate $180^{\circ}-\theta$ due to normalization.

From (3.50), it can be seen that if $\cos(\tilde{\psi}(t)_{CTE(i)})=\dfrac{\pi}{2}$, then there occurs a division by zero in the control signal and the control become infinite. Intuitively, when $\cos(\tilde{\psi}(t)_{CTE(i)})=\dfrac{\pi}{2}$, then the track line and the vehicle heading are perpendicular to each other. In this case, the rudder command $\delta_r(t)$ is set to zero for the reason that if $\left|\tilde{\psi}(t)_{CTE(i)}\right|=\left|\psi(t)-\psi_{com(i)}\right|>\pi/2$, then the AUV will continue to track the commanded line but will head back to the previous waypoint. To avoid this unwanted condition a bound on the line of sight heading is incorporated which is less than $40^{\circ}$. When the magnitude of the cross track angle error exceeds the limit of $40^{\circ}$, line of sight error controller is activated.

From the above discussion we can summarize as the controller equations (3.45) - (3.46) are used for heading (LOS) control. The need for the LOS controller is

1) The cross track error have no information of orientation, to avoid the maneuver of the AUV in the opposite direction, a line of sight heading control is needed

2) When the mission is started and the heading of the vehicle from the initial waypoint is greater than the limit of $40^{\circ}$.

3) While cruising when the angle between the current orientation of vehicle and the track line exceeds the limit of $40^{\circ}$ then orientation is needed to be correct again to avoid loss of tracking of AUV

When LOS controller reduces the heading less than $40^{\circ}$ then CTE controller is activated and thus both these controllers keep on alternating until the AUV reaches the watch radius of the final target waypoint.

# Chapter 4 : SIMULATION RESULTS

In this section the results of simulations and their analysis along with the controller performance are shown. To carry out simulations, the tool used is Matlab. It is considered that the AIRES AUV is maneuvering at a constant speed.

For the path tracking, a complete path is given to the AUV which defines the desired heading $\psi_d$ of the vehicle. First the results of state feedback sliding mode controller are presented, then the results of output feedback observer based sliding mode controller are discussed. The robustness of the controller is also discussed by adding 20% perturbations in the hydrodynamic terms of the system parameters.

For waypoint tracking, the $(x, y)$ coordinates of the desired points instead of a complete path are given in the earth fixed frame. Based on the initial value or start of the mission, the desired heading angle $\psi_d$ between the previous and the target waypoint is calculated itself by the controller. AUV keeps on maintaining its orientation to $\psi_d$ with the help of line of sight (LOS) controller while the position is corrected by the cross track error (CTE) controller. Each point has its own watch radius which gives a tolerance to AUV for tracking as in practical scenarios reaching at the waypoint completely is not possible. Once the AUV has reached in the area of watch radius, the next waypoint will be activated and the current waypoint will become the previous waypoint for AUV.

The controller and ARIES body parameters are defined in appendix.

# 4.1 Simulation Results for Path Tracking

## 4.1.1 State Feedback Sliding Mode Control Design

### 4.1.1.1 Tracking for Constant Heading Angle - Actual Parameters

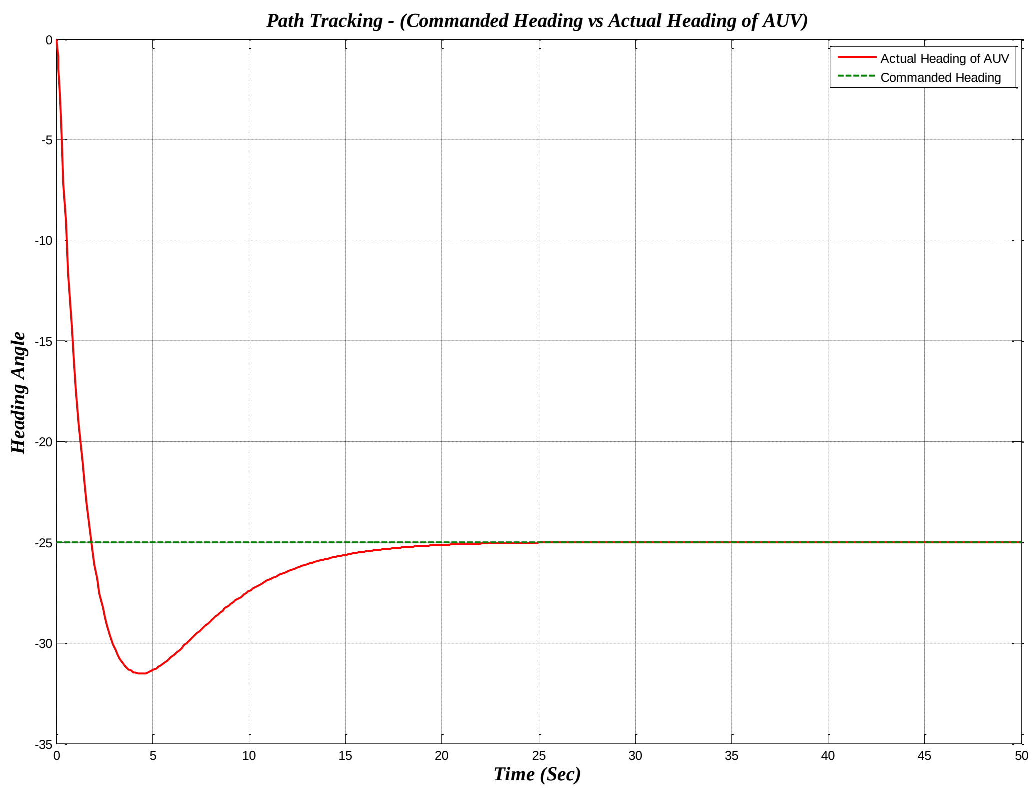


**Figure 4-1 : Tracking for Constant Heading Angle for State Feedback Control - Actual Parameters**

In figure (4-1), AUV is commanded to track a constant heading angle of $25^{\circ}$. At the start of the mission, the initial heading of AUV is $0^{\circ}$. It can be seen that the state feedback sliding mode controller tracks the desired reference trajectory in very short time. To generate this figure, actual parameters of the system are taken in this simulation.

### 4.1.1.2 Response of States for constant Heading Angle - Actual Parameters

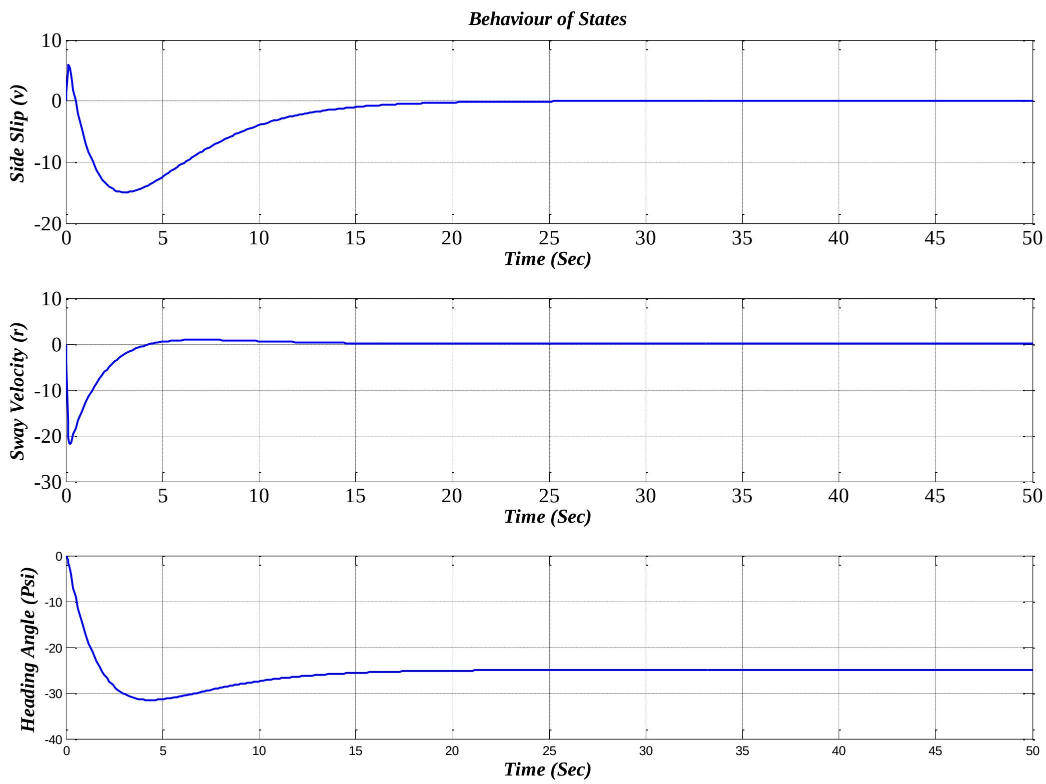


**Figure 4-2 : Response of States for constant Heading Angle for State Feedback Control - Actual Parameters**

For constant reference heading angle as shown in figure (4-1), the response of the states i.e. side slip velocity $(v)$, yaw velocity $(r)$ and the heading angle yaw $(\psi)$ is shown in figure (4-2). It can be noticed that the side slip and the yaw velocity goes to zero as the AUV has obtained the desired orientation.

#### 4.1.1.3 Control Effort for Tracking at Constant Heading Angle - Actual Parameters

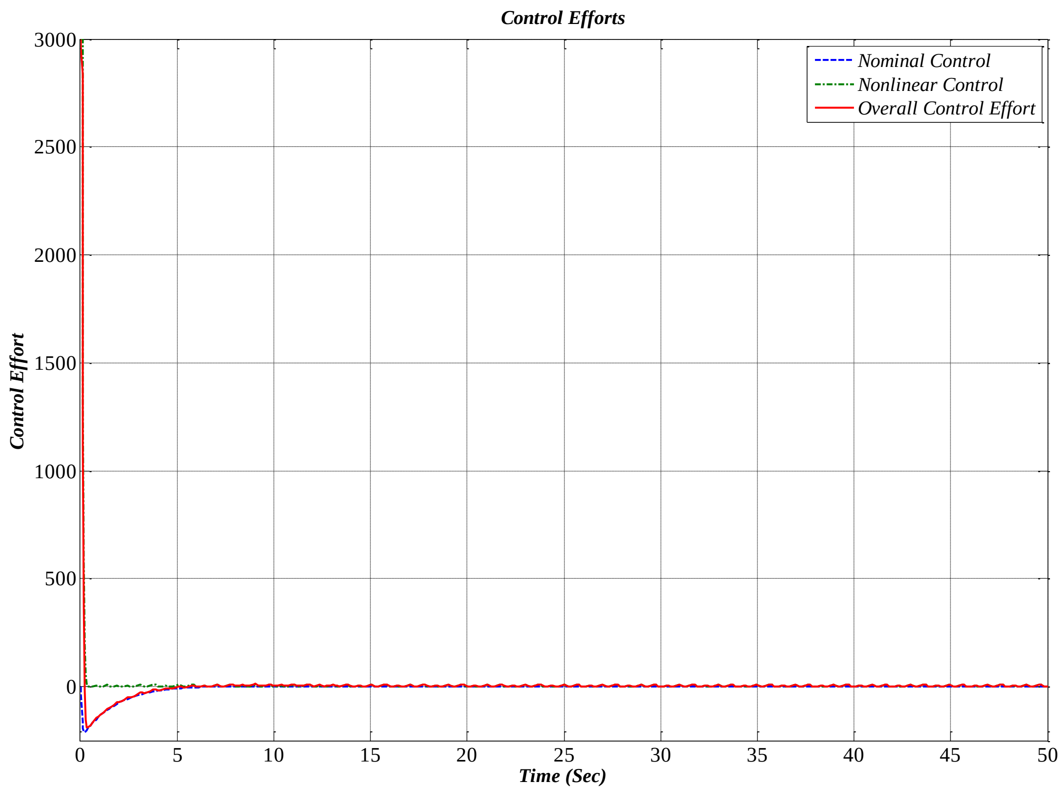


**Figure 4-3 : Control effort for Tracking at Constant Heading Angle for State Feedback Control - Actual Parameters**

In figure (4-3) the comparison of the nominal, nonlinear and overall control is shown. It can be seen that there is a peak in the control effort in the beginning. This peak is due to the fact that AUV was heading initially at $0^{\circ}$ and suddenly it is commanded to go to $25^{\circ}$. It can be seen that when the AUV has achieved the desired orientation and no further change in reference is commanded, then the control effort goes to zero.

#### 4.1.1.4 Tracking for Varying Heading Angle - Actual Parameters

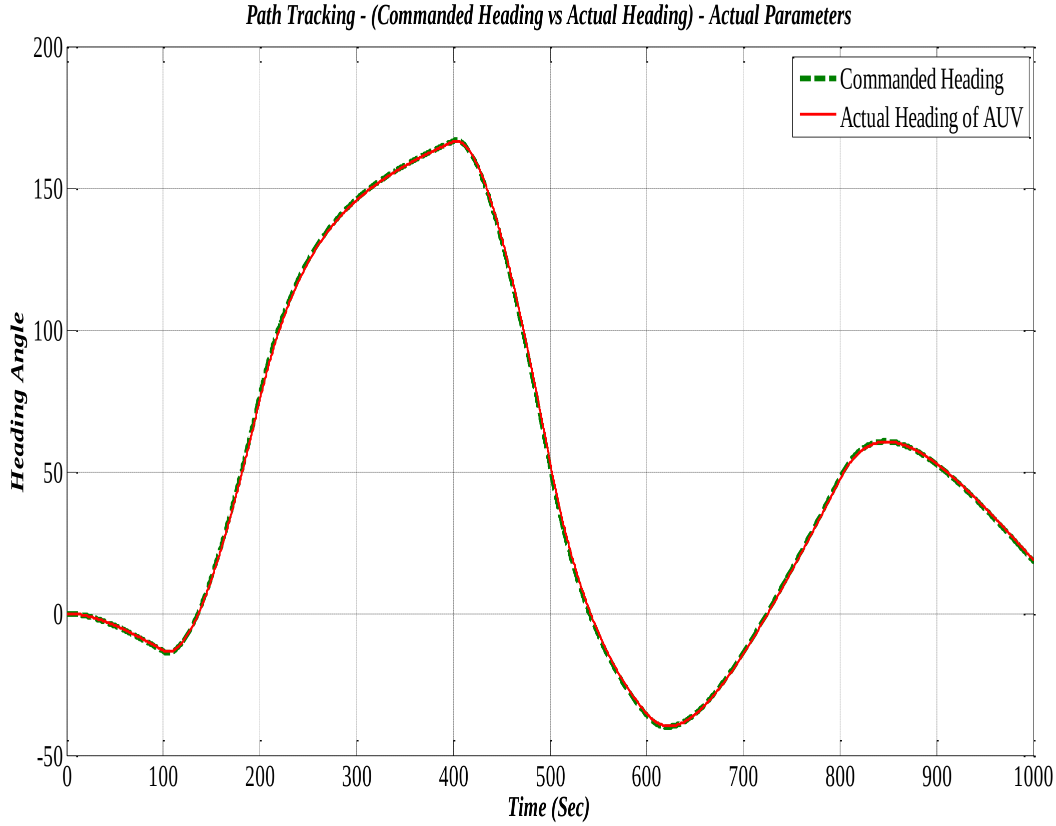


**Figure 4-4 : Tracking for Varying Heading Angle for State Feedback Control - Actual Parameters**

In the above figure, the initial heading of AUV is $0^\circ$. It is then commanded to track a varying heading angle. It can be seen that the state feedback sliding mode control tracks the desired orientation with a least of error. This is due to the fact that the sliding mode control is robust and no perturbations are added in the system parameters. In practical implementation, this design may induce noise and may degrade the performance of system. Also with the passage of time due to sensor dynamics the overall performance of the system might degrade.

### 4.1.1.5 Control Effort for Tracking at Varying Heading Angle - Actual Parameters

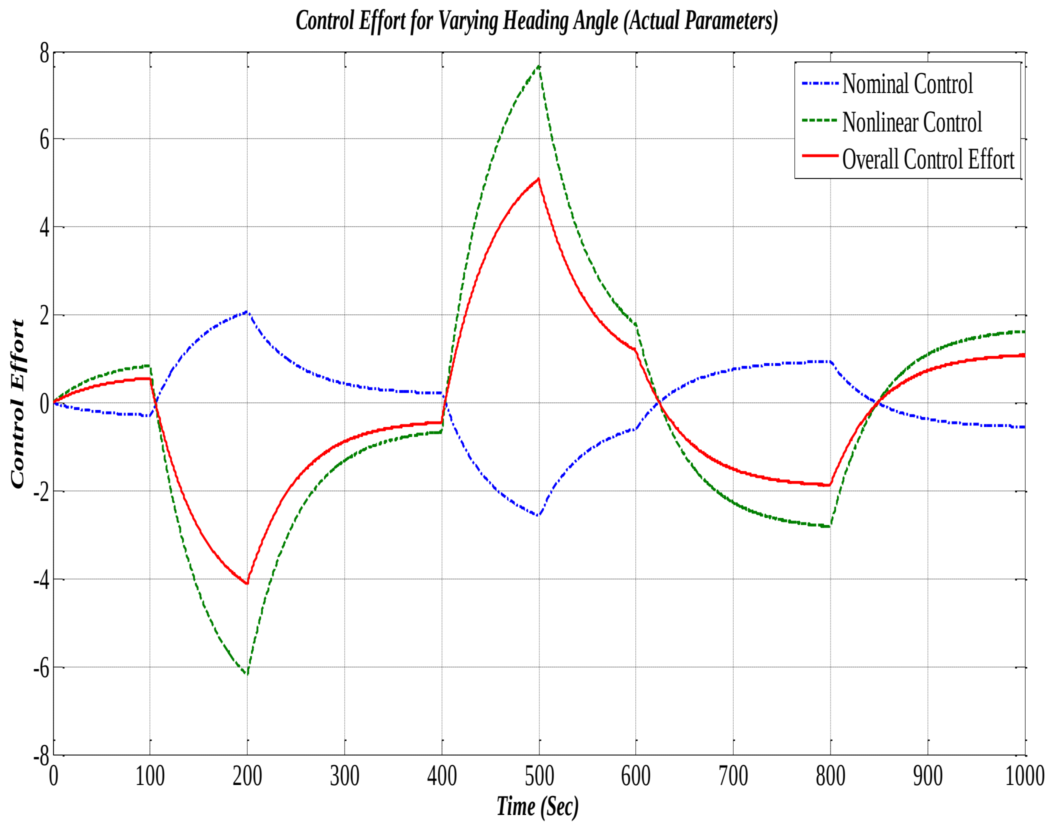


**Figure 4-5 : Control effort for Tracking at Varying Heading Angle for State Feedback Control - Actual Parameters**

Comparison of nominal, nonlinear and overall control is shown in the above figure for a reference trajectory shown in figure (4-6). In this plot, actual parameters of ARIES are used in simulations.

#### 4.1.1.6 Tracking for Varying Heading Angle - Perturbed Parameters

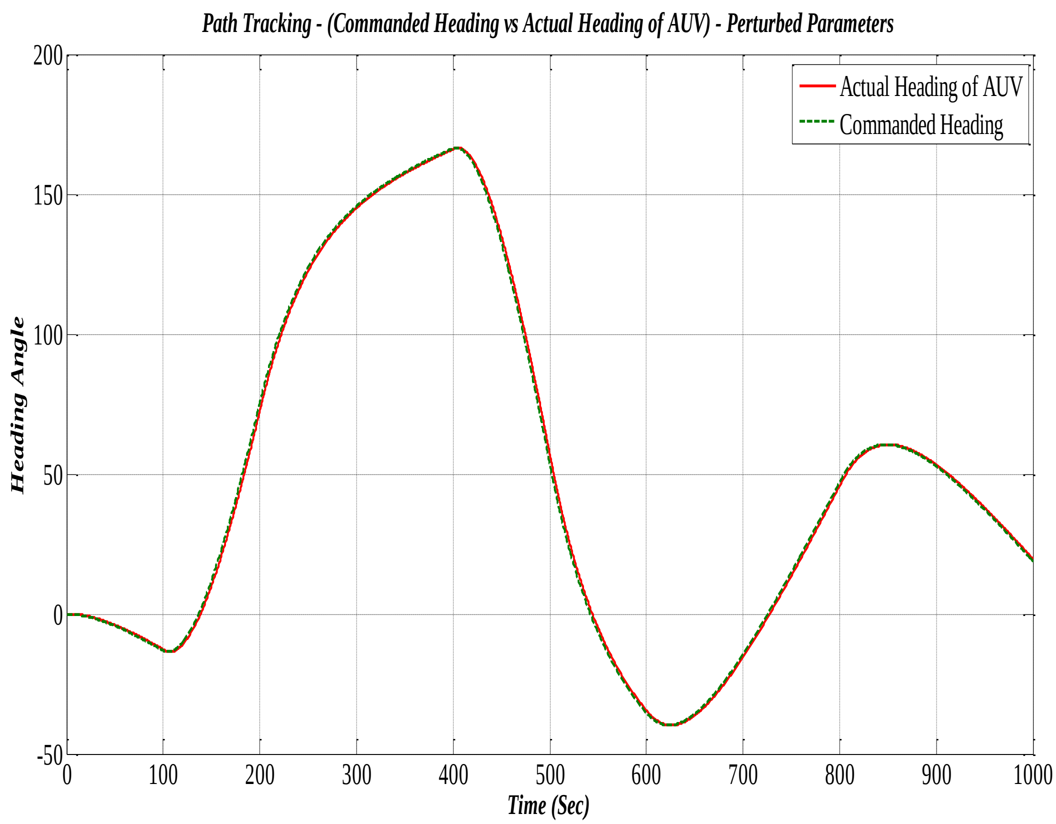


**Figure 4-6 : Tracking for Varying Heading Angle for State Feedback Control - Perturbed Parameters**

In the above figure, for a varying commanded heading angle, the tracking of AUV is shown by using the perturbed parameters of AUV. For this purpose, a maximum of 20% variation is added in the hydrodynamic parameters of AUV. It can be seen that the sliding mode control tracks the reference signal even in the presence of perturbations in the system which proves the robustness of the control design.

### 4.1.1.7 Control Effort at Varying Heading Angle - Perturbed Parameters

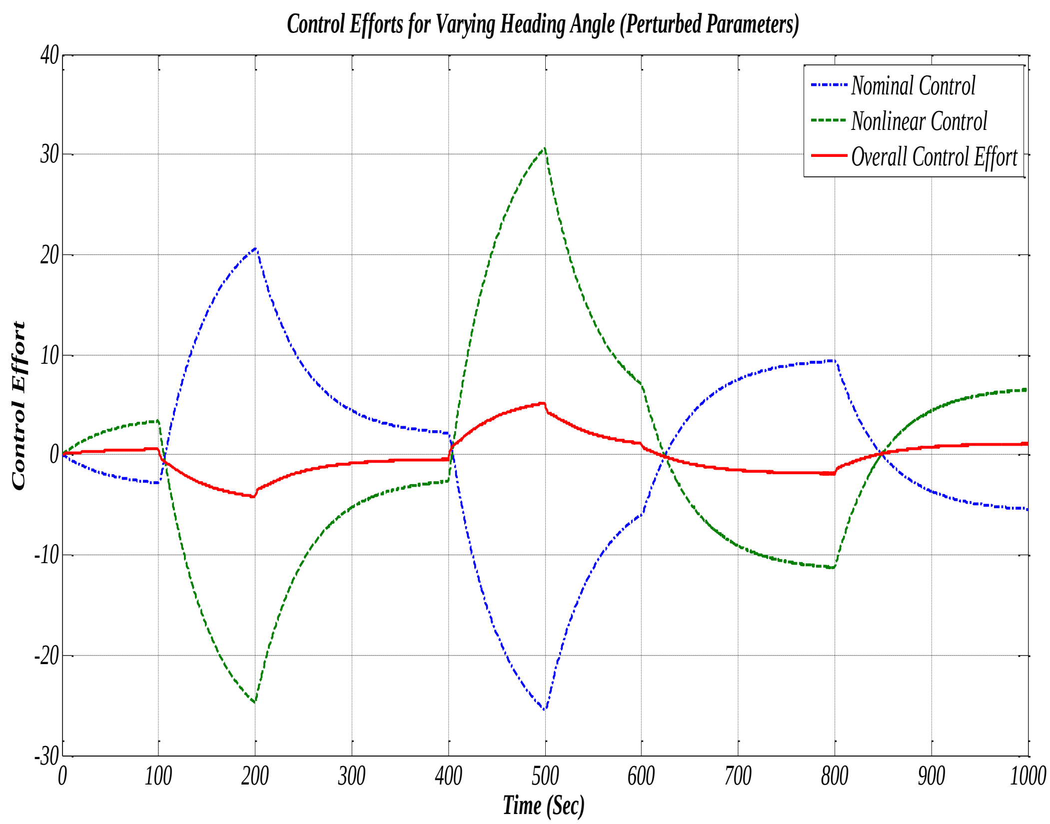


**Figure 4-7 : Control Effort at Varying Heading Angle for State Feedback Control - Perturbed Parameters**

In figure (4-7), the comaparison of control efforts for the perturbed parameters is shown. By comparing figure (4-5) and figure (4-7), it can be seen that the control effort with actual parameters is lesser than the control effort using perturbed paramerters. It is concluded that the sliding mode control tracks the reference signal with and without perturabations but there is a slight increase in the control effort with perturbations in the system.

## 4.1.2 Simulation Results for Output Feedback Observer Based Sliding Mode Control Design

### 4.1.2.1 Tracking for Varying Heading Angle - Actual Parameters

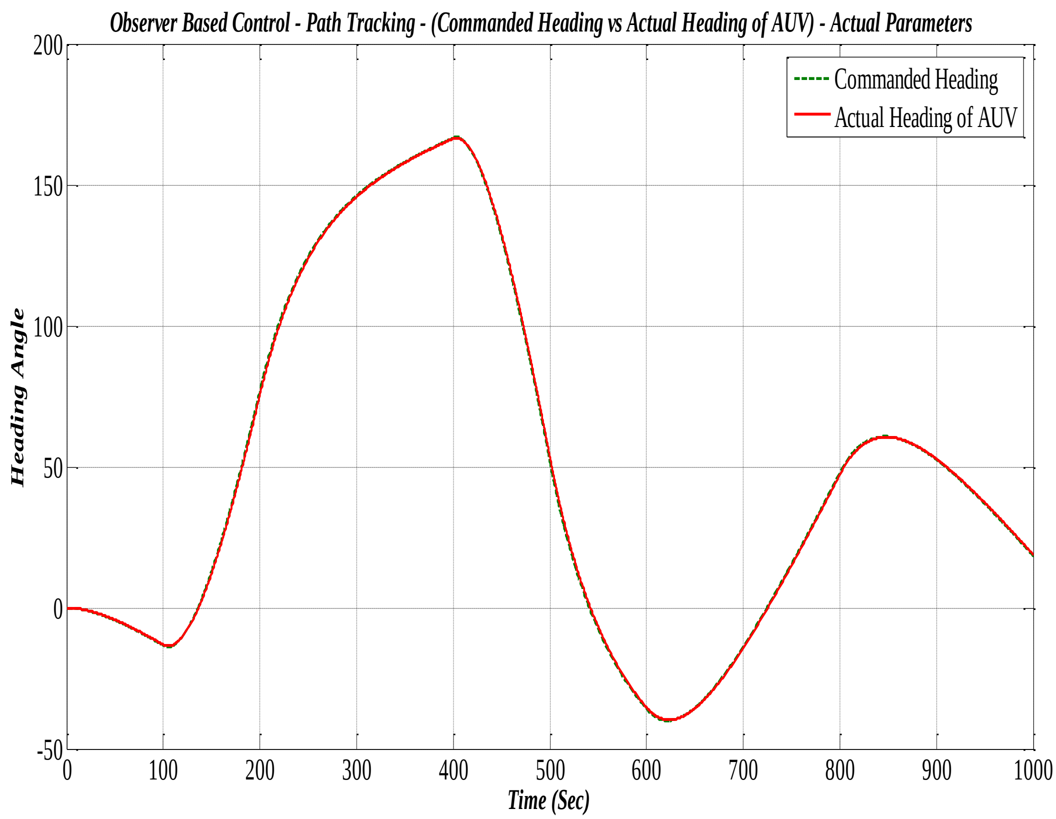


**Figure 4-8 : Tracking for Varying Heading Angle for Observer Based Control - Actual Parameters**

In figure (4-4), tracking for a changing reference signal for actual parameters using state feedback sliding mode control was shown. In this figure, the same reference signal is applied to the observer based control and the observer based control tracks the reference arbitrarily fast. The benefit of this control design is that only the output variable is measured and the control law is driven by the observed states.

#### 4.1.2.2 Control effort for Varying Heading Angle - Actual Parameters

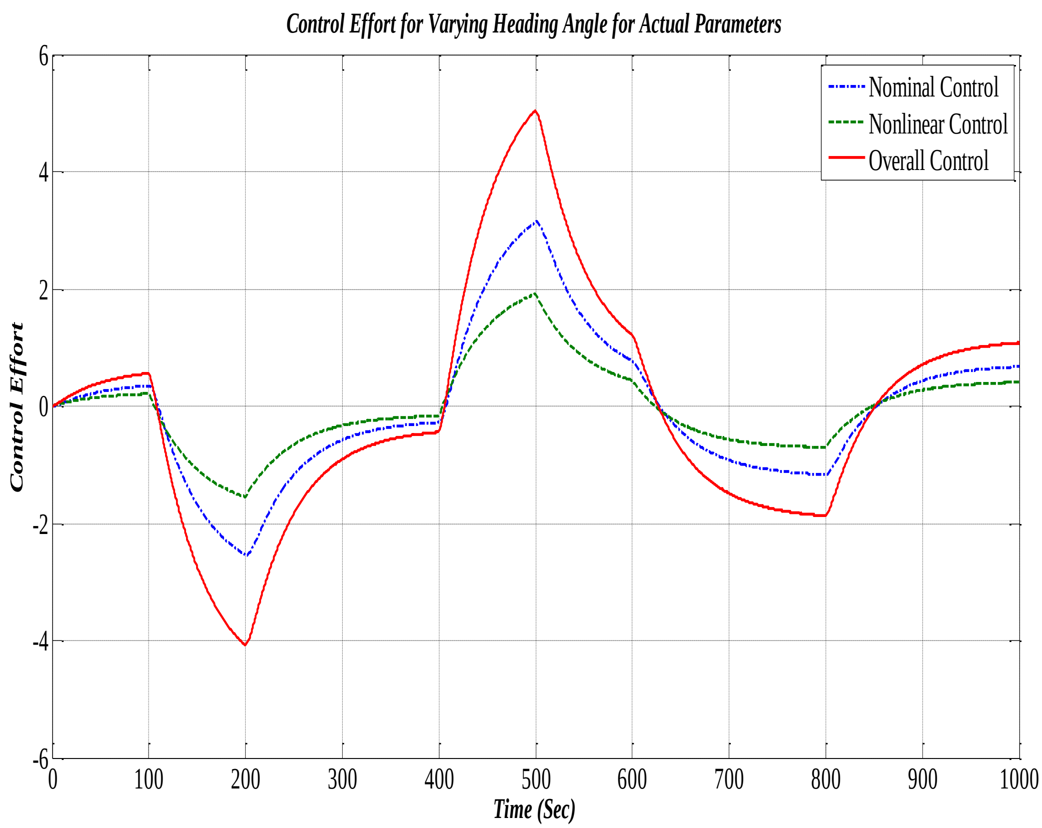


**Figure 4-9 : Control effort for Varying Heading Angle for Observer Based Control - Actual Parameters**

Above figure show the comparison of control efforts for tracking a varying reference heading angle at acutual parameters. The benefit of this control method is that only the output $\psi$ is measured and fed to the controller and the controller is driven by the observed states.

### 4.1.2.3 Tracking for Varying Heading Angle - Perturbed Parameters

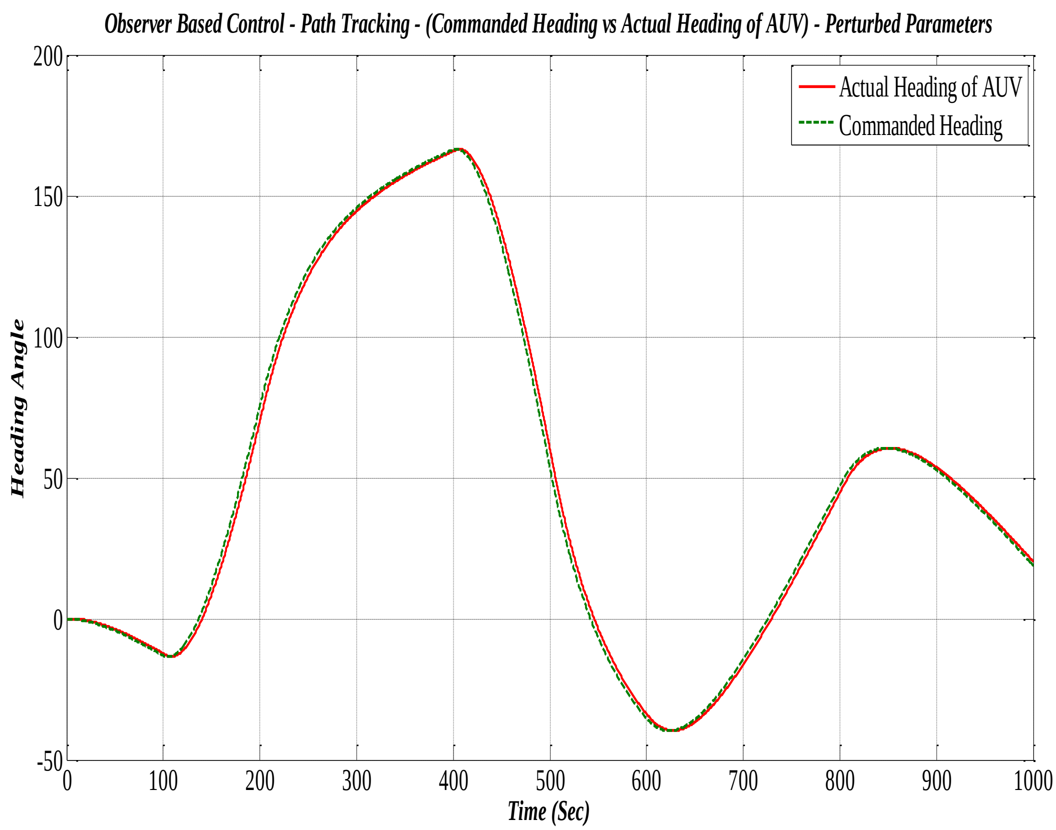


**Figure 4-10 : Tracking for Varying Heading Angle for Observer Based Control - Perturbed Parameters**

In the above figure, a maximum of 20% variation is added in the hydrodynamic parameters of AUV. It can be seen that the performance of the observer based control law is slightly degraded as compared to the state feedback sliding mode control. But in practical implementation, the result of figure (4-4) might degrade due to measurement of all states via sensors. While in this case only one state is measured.

### 4.1.2.4 Control effort for Varying Heading Angle - Perturbed Parameters

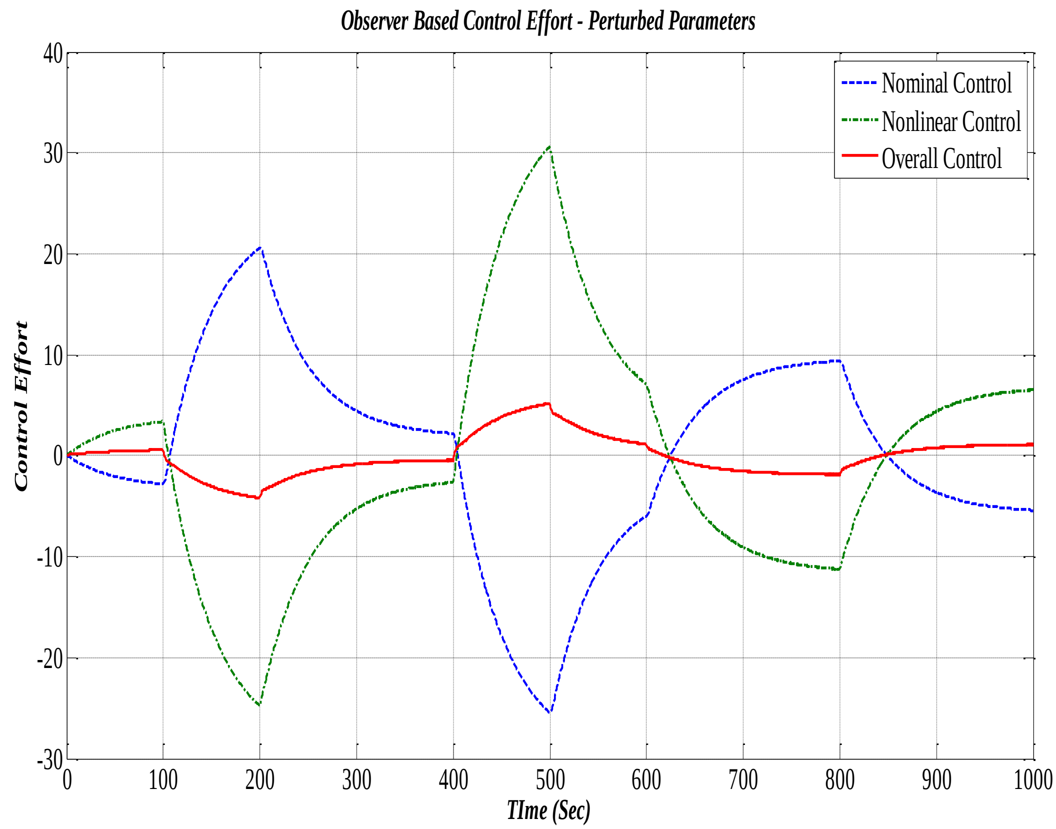


**Figure 4-11 : Control effort for Varying Heading Angle for Observer Based Control - Perturbed Parameters**

In the above figure, the comparison of control efforts for varying heading angle at perturbed parameters is shown. It is nearly the same as the control effort for the state feedback design shown in figure (4-7). Hence robustness of the current scheme is proved via simulations, since the controller performance is appreciable in presence of perturbations as well.

### 4.1.2.5 Estimation Error for Constant Heading Angle

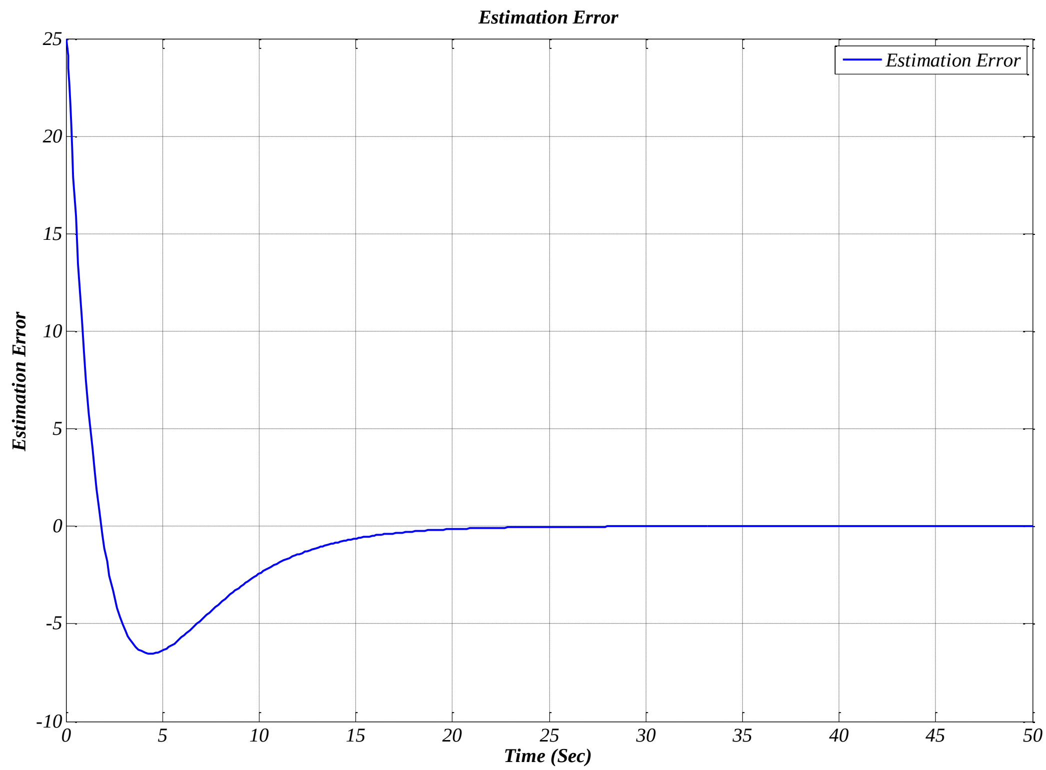


**Figure 4-12 : Estimation Error for Observer Based Control for Constant Heading Angle**

In the above figure the estimation error $\psi - \hat{\psi}$ is shown. The initial heading of AUV is $0^{\circ}$ and it is commanded to go to $25^{\circ}$. It can be seen that the estimation error is maximum at the start of the mission. By the time the observer based sliding mode control reduced and the estimation error and it approaches zero when the AUV completely tracks its commanded heading.

# 4.2 Simulation Results for Waypoint Tracking

## 4.2.1 Waypoint Tracking – Constant Reference

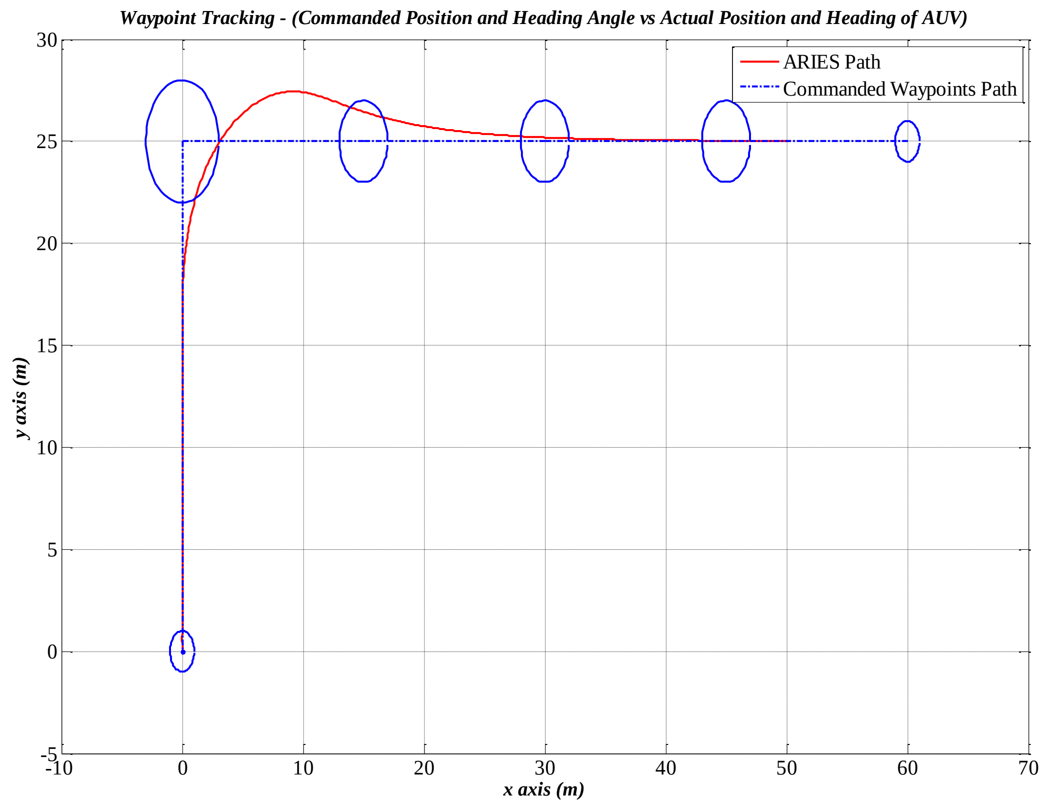


**Figure 4-13 : Waypoint Tracking – Constant Reference**

In figure (4-13), waypoint tracking method is shown for the tracking of AUV. The initial heading of the vehicle is $0^{\circ}$ and the start point is $(0,0)$ in the earth fixed frame. AUV is commanded to go to 25 meters on the y-axis and be there. The desired set of reference waypoints are given for this purpose. The circles in the above figure represent the watch radius for each waypoint. It can be seen that after reaching the vicinity of the watch radius of target waypoint, next waypoint is activated until the track is traversed completely.

### 4.2.2 Response of States – Constant Reference

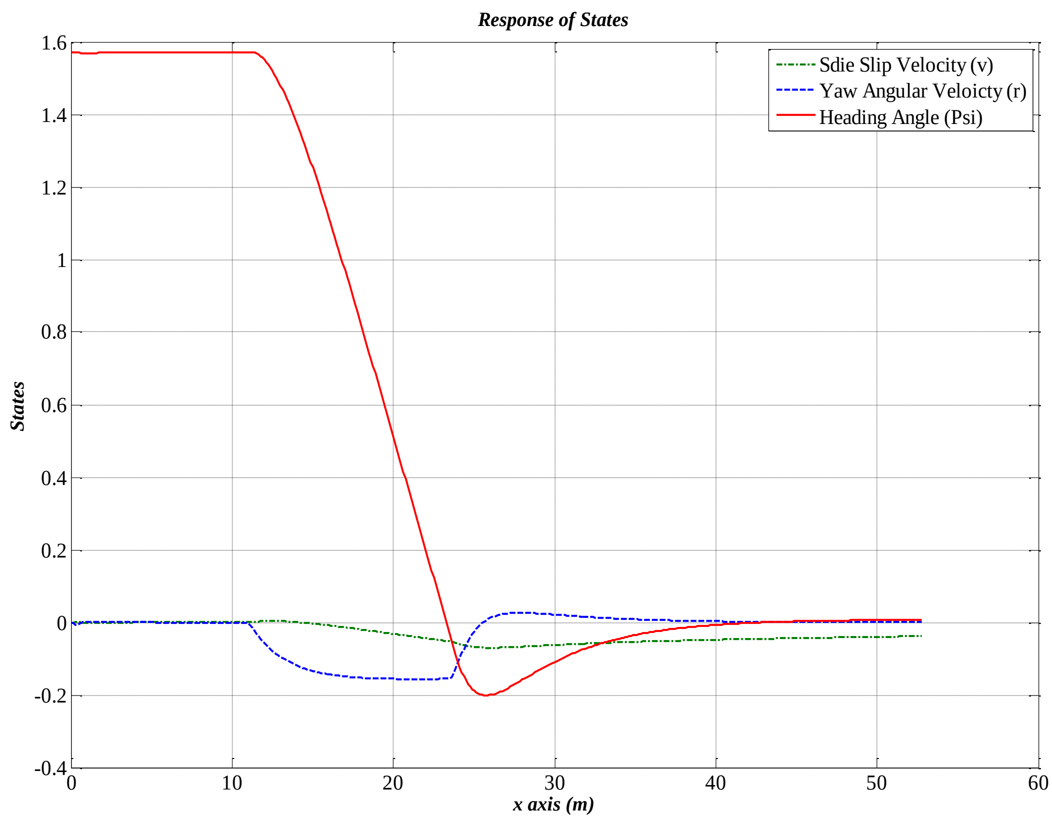


**Figure 4-14 : Response of States for Waypoint Tracking – Constant Reference**

Figure (4-14) show the response of the states i.e. side slip $(v)$, yaw rate $(r)$ and yawing angle $(\psi)$. It can be seen that when the AUV achieves its desired reference the value of states (side slip velocity and yawing rate) goes to zero.

### 4.2.3 LOS Control vs CTE – Constant Reference

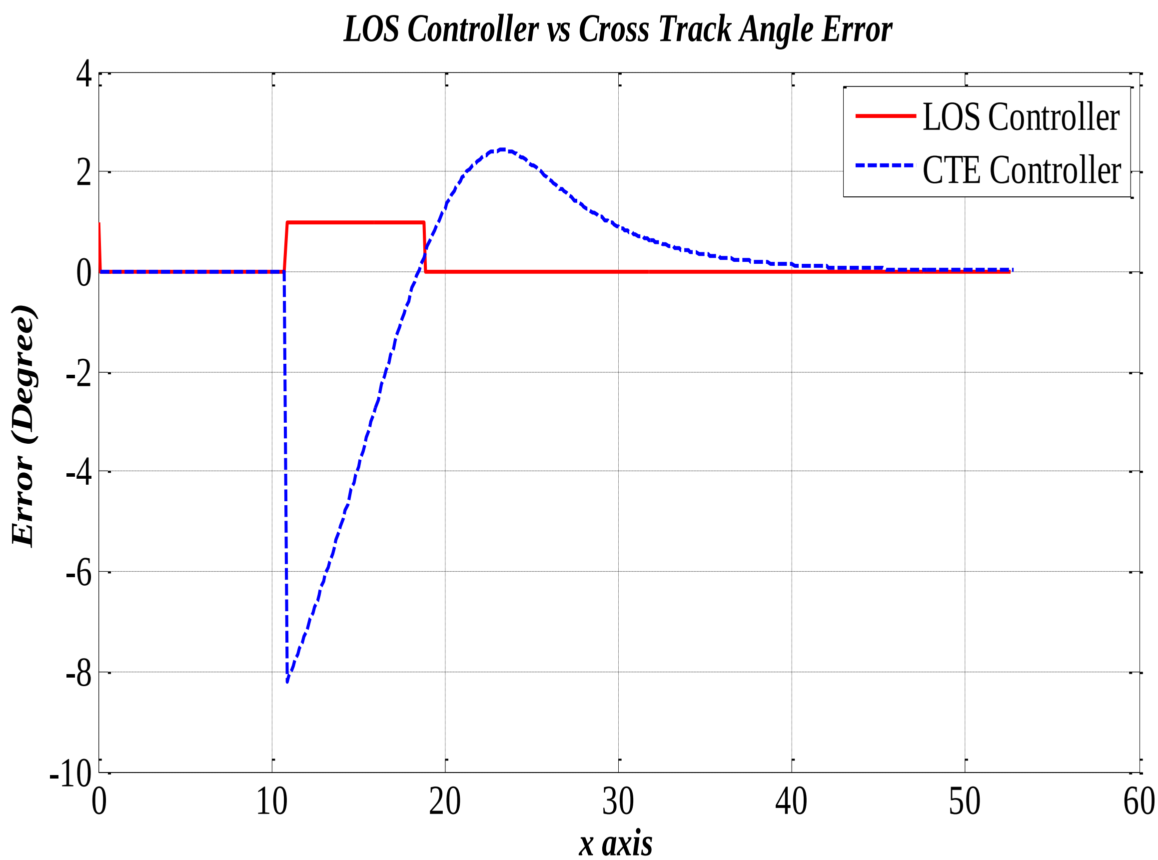


**Figure 4-15 : Line of Sight & Cross Track Error Control Efforts for Waypoint Tracking – Constant Reference**

In figure (4-15), the comparison of line of sight controller activation and cross track error control effort is shown. From figure (4-13), it can be seen that the position and orientation of AUV has larger errors at the point of sharp turnings. Thus, LOS and CTE controllers operate alternately to minimize the error.

### 4.2.4 Rudder Angle – Constant Reference

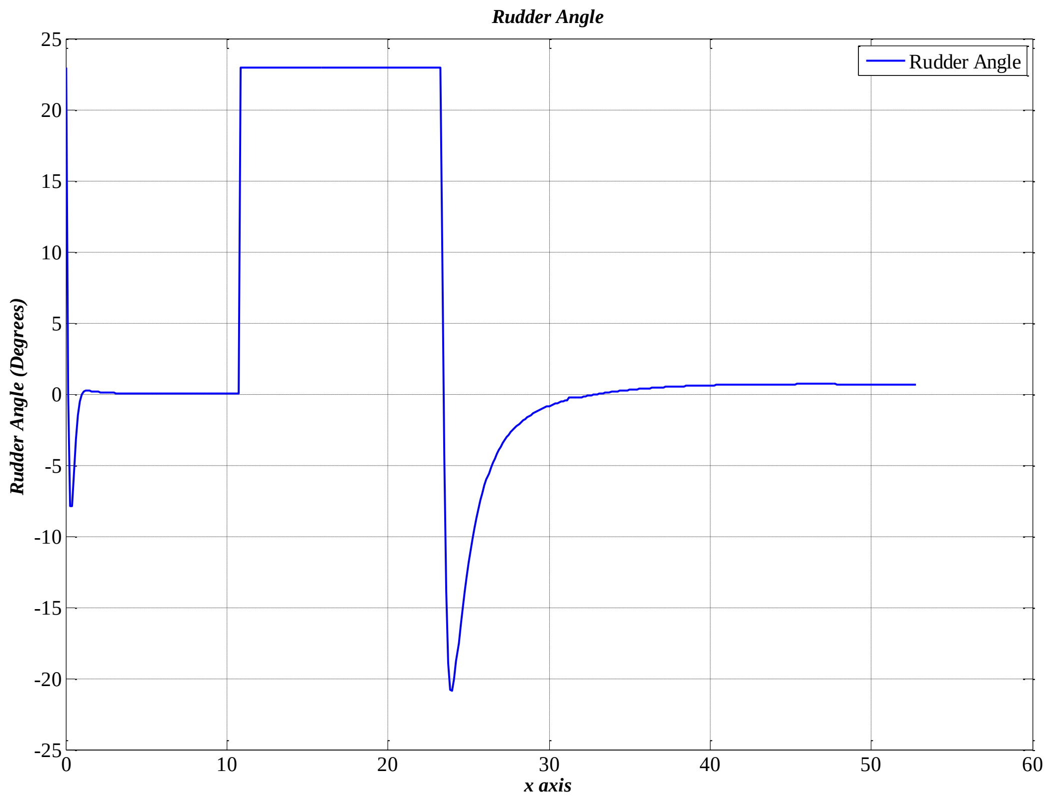


**Figure 4-16 : Rudder Angle for Waypoint Tracking – Constant Reference**

In an AUV the position and orientation is controlled by the rudder. Figure () show the action of rudder in degrees. It can be seen that the rudder turn at maximum angle for sharp turnings.

## 4.2.5 Waypoint Tracking – Varying Reference

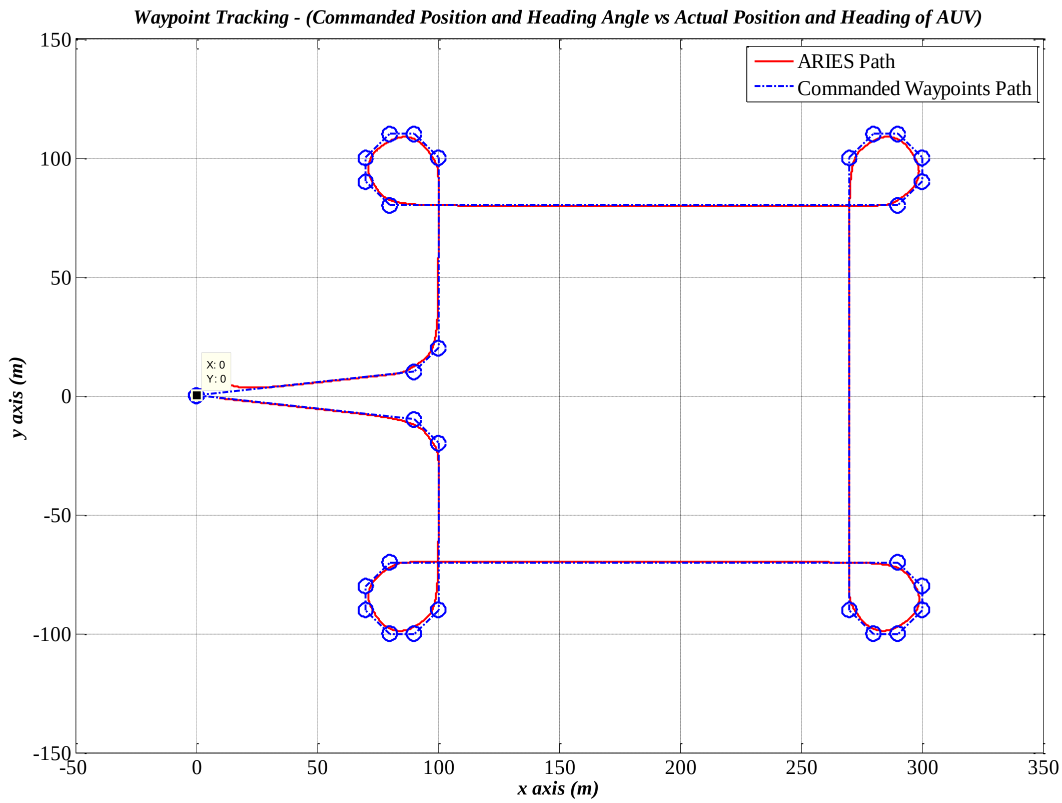


**Figure 4-17 : Waypoint Tracking – Varying Reference**

Figure (4-17) show the waypoint tracking of AUV for a varying commanded reference. The mission starts from $(0,0)$ in the earth fixed frame and travel through the set of waypoints taking circular paths at $(85,95)$, $(285,95)$, $(285,-85)$ and $(85,-85)$. It then head back to the initial position $(0,0)$. The circles shown in the figure are the watch radius of waypoints. It can be seen that the performance of controller is appreciable as the controller tracks the track line even on the sharp edges and circular paths.

# 4.2.6 LOS Control vs CTE – Varying Reference

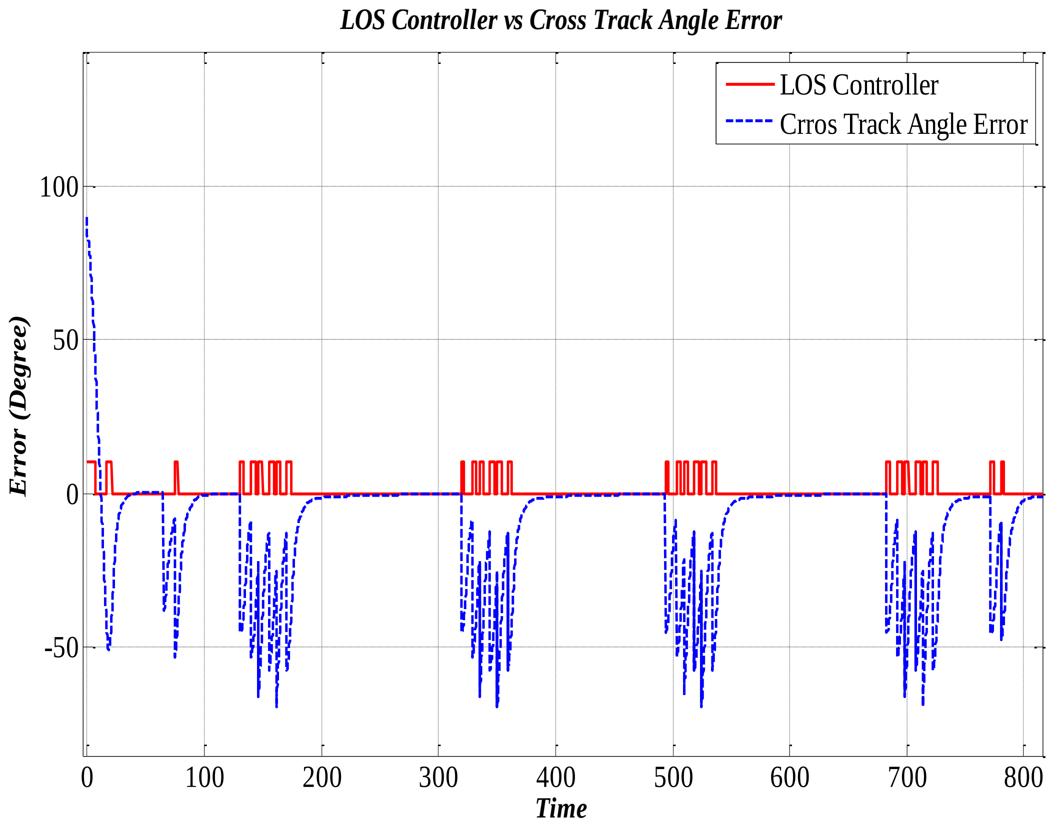


**Figure 4-18 : Line of Sight & Cross Track Error Control Efforts for Waypoint Tracking – Varying Reference**

In figure (4-18), the relation of line of sight controller activation and cross track angle error for the commanded reference trajectory shown in figure (4-17) is shown. It can be seen that when the AUV is traversing a circular path then the controllers become active and when the AUV traverses on a straight path and the desired orientation and position are achieved, then the controllers become inactive.

## 4.2.7 Rudder Angle – Varying Reference

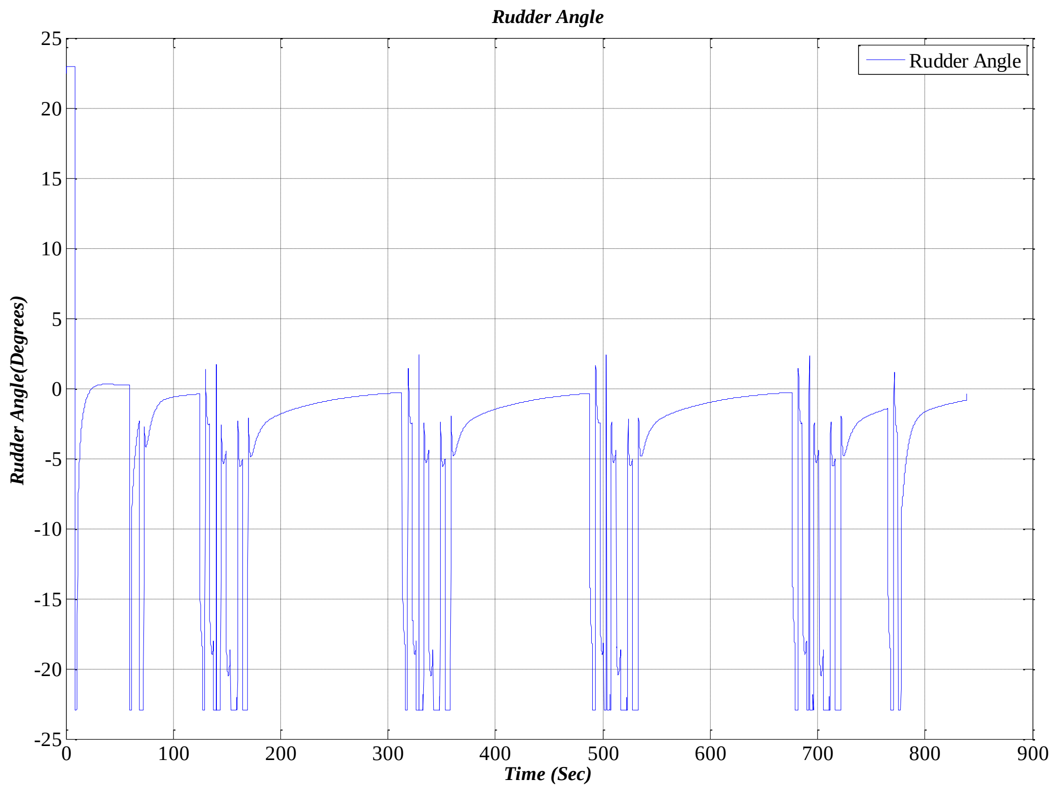


**Figure 4-19 : Rudder Angle for Waypoint Tracking – Varying Reference**

Figure (4-19) show the action of rudder when the AUV is traversing the reference trajectory show in figure (4-17). From figure (4-17) and (4-19), it can be seen that the control effort on the rudder actuator is applied whenever there is some variation in the reference trajectory. When the orientation and position of AUV is according to the reference trajectory then the control effort on the rudder actuator becomes zero.

# Chapter 5 : CONCLUSIONS & FUTURE WORKS

In this work, the tracking problem for AUVs are discussed and for this purpose AIRES AUV was considered. First a path tracking controller was designed in which only the heading angle of AUV is controlled. The control design method was based on the state feedback sliding mode control design. In state feedback design, the controller is driven by the actual states of the system which needs sensors to measure the states. In measurement of states, due to sensor dynamics, noise is induced in the system and the performance of the system can be degraded. To minimize the need of measurement of states, the state feedback sliding mode control design was extended to an output feedback sliding mode control design using a Luenberger observer of $3^{rd}$ order. In observer based design, the observer is driven by the estimation error $\psi - \hat{\psi}$ and the controller is driven by the observed states $\hat{X}$ and only the output i.e. heading angle $\psi$ was measured. Complete stability analysis using Lyapunov's stability theorem was also provided. It was shown in the simulations and the contributions [6] and [7] that the performance of the proposed control law approaches as that of state feedback design and is capable of handling parametric uncertainties and model perturbations.

In path tracking, a complete path is given to the AUV as a reference trajectory and only a single nonlinear controller keeps the AUV on that path which corrects its heading angle. A better method to design the reference trajectory is to use waypoint tracking. In waypoint tracking, instead of a complete path, only the coordinates of selected points are given and the line of sight (LOS) and cross track error (CTE) controllers controls the orientation and position of AUV respectively by operating alternately.

In this work, the sliding mode controller chosen is of first order, forward speed $(u)$ is considered constant, it is assumed that the AUV is heading a slower speeds so that coupling between hydrodynamic terms can be neglected. Areas still needed to be addressed for further improvements are as

1) Other nonlinear control design like Lyapunov redesign, back stepping, higher order sliding mode etc. can be applied on the AUV model and performance can be compared

2) Speed controller can be used to define the feasibility of the mission between the maximum and minimum speeds of the AUV

3) Obstacle avoidance can also be incorporated in the tracking of AUV

4) Mannering of AUV at higher speeds can be considered so that a more realistic model is obtained in which the effect of hydrodynamics at higher speed is also considered

5) It can be combined with diving plane to for complete 6-DOF tracking of the system in steering and diving planes

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# APPENDIX

This appendix contain the body parameters of ARIES AUV and the controller parameters.

Mass = 20 Kg

Length = 1.6256 m

Diameter = 0.191262 m

$W = 600$

$m = W / g$

$m - Y_{\dot{v}} = 456.76$

$Y_{\dot{r}} = 69.90$

$Y_{v} = -68.16$

$Y_{r} = 406.30$

$N_{\dot{r}} = -35.47$

$N_{v} = -10.89$

$N_{r} = -88.34$

Surge, $u = \mu_{0} = 5\ Knots / hr$

The system matrix, $A = \begin{bmatrix} -0.1492 & 0.6854 & 0 \\ -0.0132 & -0.1069 & 0 \\ 0 & 1.0000 & 0 \end{bmatrix}$

The control matrix, $B = \begin{bmatrix} 0.0194 \\ -0.0572 \\ 0 \end{bmatrix}$

The output matrix, $C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$

Gain of nonlinear term, $\eta = 2$

Boundary layer parameter, $\varepsilon = 1$

Poles of the system, $P = \begin{bmatrix} 0 & -0.3 & -0.5 \end{bmatrix}$

The feedback gain vector, $K = \begin{bmatrix} -1.0327 & -9.8582 & 0 \end{bmatrix}$

Coefficients of the sliding surface, $S = \begin{bmatrix} -0.3428 & 0.6130 & 0.7118 \end{bmatrix}$

The gains of observer, $L = \begin{bmatrix} -10.3271 & -98.5824 & 0 \end{bmatrix}$