OUTPUT REGULATION OF A MULTI-AGENT SYSTEM USING CONDITIONAL SERVOCOMPENSATORS

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Abstract

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Multiple agents – sometimes referred to as swarm of agents – and their control have been seeking interest significantly over the course of recent years. Their ability to move in desired formations and perform synchronized task has been the key arena in their development. In this research work, design of the continuous sliding mode controller for the output regulation of a multi agent system is studied. The idea of using conditional servocompensators for improving transient performance while achieving steady-state accuracy was introduced in the literature and has been shown to be a useful tool to achieve regulation of minimum phase nonlinear systems. We extend the use of the same approach to a class of multi-agent systems comprising of finite number of agents. The control scheme presented in this work is based on sliding mode control technique and incorporates a conditional servocompensator. Closed-loop analysis under the proposed control scheme for the multi-agent system is provided. Simulation results in the form of output trajectories of individual agents and error convergence are provided to illustrate the discussed approach.

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GEC Member: Dr. Bilal Khan.

To my beloved family ...

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Chapter 1

Introduction

Multiple agents – sometimes referred to as swarm of agents – and their control have been seeking interest significantly over the course of recent years. Their ability to move in desired formations and perform synchronized task has been the key arena in their development. Such a system bears a lot of benefits over a single robot. Tasks which may be left unaddressed by a single robot can be accomplished using this kind of system. These systems offer a huge range of environmental adaptability and are flexible to a very huge extent. They are impervious to failures as compared to others.

1.1 Why Multi-Agent Systems

Among all the emerging technologies for application problems, Multi-agent robot systems stand out as one of the most plausible platforms for providing solutions in different areas such as perturbed sensing, temporary mobile communication networks and RSR (robotic search and rescue). Any system can be labeled as a multi-agent system if at least two of its autonomous units work together so as to accomplish a particular goal; for instance formation control. Broadly speaking, MAS (multi-agent system) can be described as a network of autonomous agents loosely coupled interacting with each other and providing a solution to problems that lie outside the circle of the individual capacities.

1.2 Application of Multi-Agent Systems

MAS applications cover a variety of domains, including

- Satellite formation flying
- Distributed computing and problem solving
- Multi-robot systems and Robotic clusters
- Surveillance and Reconnaissance systems
- Electric power systems
- Missile systems
- Intelligent Transport Systems(ITS)

1.3 Limitations in Multi-Agent Systems

The main objective in solving a control problem involving mutual interaction between agents is designing the agent controllers so as to achieve the mutual objective. The feasibility of MAS is often questioned because of the high expenses involved in addition to the following limitations:

- High quantity of agents involved
- Actuators spatial distribution
- Possession of incomplete information by each agent (This is because sensors have restricted sensing capability)
- Limited ranges of wireless communication
- Data decentralization
- Asynchronous computation

Therefore it becomes very challenging to design a distributed controller that uses the individual information of the agents and couples them. Problems involving complex system dynamics are the most challenging because of the involvement of cooperative control law, agent interaction graph and mutual agent dynamics.

1.4 Advantages of Multi-Agent Approach

Even after possessing all the aforementioned limitations, MAS still have huge advantages over other systems and approaches. Some of them are stated below:

- MAS are impervious to robotic or communication failures. The overall objective can be achieved even after certain agents or communication links stop working due to any malfunctioning.

- MAS approach can also be utilized effectively for scaled-up or scaled-down versions of a system thereby making it independent of the system size. Therefore MAS approach is equally effective for any system whether it possesses just two agent or a million agents.

- Computational burden gets distributed to the interconnected agents throughout a network. This decentralization of computation and resources gets rid of the SPF problem (single point of failure) that is ever-present in centralized systems.

- Any problem is modeled by MAS with respect to each autonomous agent involved in the network. This has proven to be the most natural way of task distribution, agent interaction, user inputs and so on.

- Information is MAS is competently gathered from the sources, filtered and then transferred globally to other agents.

- By enhancing the computational efficacy, the net system performance of MAS gets enhanced. MAS approach is more reliable, extendible, robust, responsive, flexible and maintainable as compared to other approaches.

1.5 Scope of the Thesis

This work is an extension of the approach discussed in [15] for the output regulation of nonlinear systems using conditional servocompensators. The goal here is to design a conditional servocompensator in sliding mode control framework for formation control of nonlinear multi-agent system comprising of finite number fo agents agents to achieve the desired steady state accuracy as well as the transient performance. The analysis of the overall closed loop system for three agent system is also carried out that provides a better understanding for formation control problem of multi-agent system in both the directed and undirected graph topologies. The simulation results clearly indicate a better transient performance of all the agents while maintaining the desired steady state accuracy.

Chapter 2

Preliminaries

2.1 Mathematical Preliminaries

2.1.1 Notation

The following notations will be used throughout the thesis for representation of different terminologies;

- The set of all real number is represented by $\mathbb{R}$; whereas $\mathbb{R}^n$ denotes the set of all vectors of length n.
- $f(x)$ denotes the function in variable x.
- $\lambda_{min}(A)$ and $\lambda_{max}(A)$ denote the smallest and largest eigenvalues of the matrix A.
- A^T denotes the matrix transpose.
- Norm of a scalar will be denoted by $|x|$ whereas 2-norm of a vector will be denoted by $\|x\| = \sqrt{x^T x}$

2.1.2 Algebraic Graph Theory

A balanced graph is a graph whose ingoing edges and outgoing edges at every node are equal in number. Edge $(v_i.v_j)$ is outgoing and incoming at the same time with respect to v_i and v_j respectively, in case, if a graph has a property $(v_i, v_j) \in E$, this states that for any value of $v_i, v_j \in V, (v_j, v_i) \in E$. Such a graph is undirected and the edge (v_i, v_j) shows that there is mutual exchange of information between the nodes v_i and v_j. The undirected graph can clearly be called a special balanced graph.

Paths between nodes can either be direct or indirect. If the path between nodes v_{i1} and v_{il} is direct, then it can be represented in the form of a sequence of ordered edges (v_{ik}, v_{ik+1}) where $k = 1, \ldots, l-1$. On the other hand, there exists an analogous definition for an undirected path. A directed path for which a same node acts as the starting as well as ending point is referred as a cycle. If each node in a graph has a direct path to every other node, then the directed graph is said to be strongly connected (Figure 2.1(a)). A similar connection pattern in an undirected graph is referred as connectedness. If each node in a directed graph has an edge to every other node, then the directed graph is said to be complete. If all the nodes in an undirected graph connect via single undirected path, then such an undirected graph is called an undirected tree. If all the nodes in a directed graph have a single parent each apart from one node which does not possess any parent, then such a directed graph is called a (rooted) directed tree. The parent-less node is called the root and it is linked to every other node via directed paths. If every node in a directed tree is connected, then it is said to be spanning as shown in Figure 2.1(b). This means that there exists one root node at least that is connected to every other node via a simple path. A portion or subset of the graph can also contain a directed spanning tree. This is even true for undirected graphs too as they may also contain a directed spanning tree. Such undirected graphs are in fact connected. If directed graphs contain directed spanning tree, then this is indicative of a weaker condition as compared to being strongly connected. There exists a minimum of one directed spanning tree in a strongly connected graph.

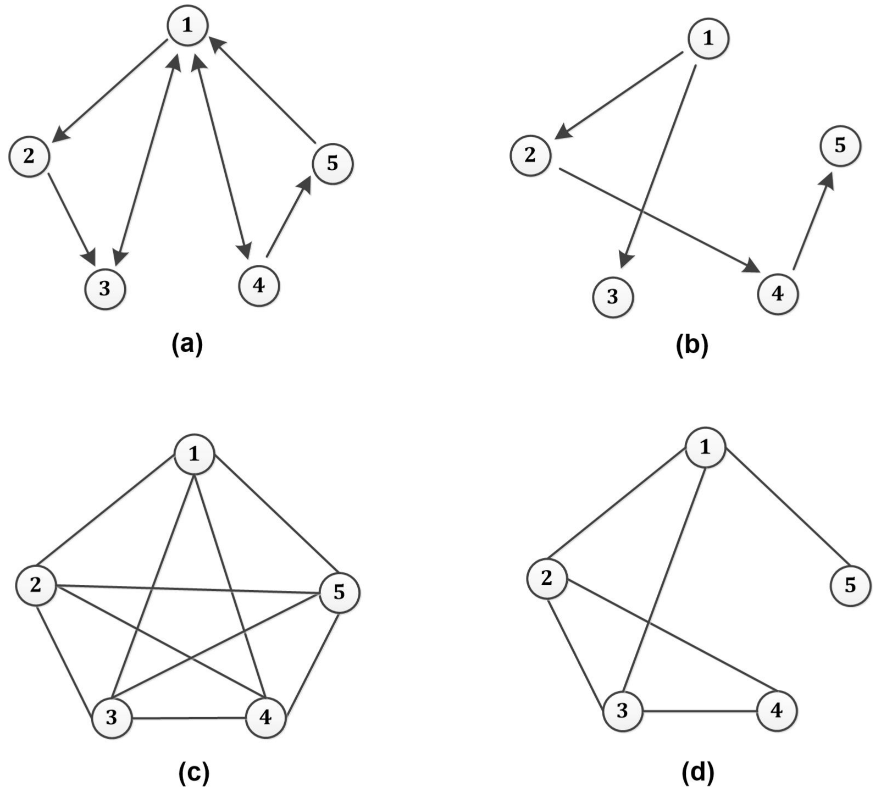

Figure 2.1: Different types of directed graphs with five nodes: (a) A strongly directed graph, (b) A directed spanning tree (c)an undirected balanced graph (d)an undirected connected graph

Hence limitations in communication network do not always allow each agent to communicate with all other agents. These limitations can be modeled with the introduction of some graph terminologies mentioned in [17].

- $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is a digraph that consists of a nodes $\mathcal{V} = \{0, 1, 2, 3, ., N\}$ and edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. (i, j) denotes an edge from ith node to jth node. Set N_i is the subset of $\mathcal{V}$ that contains all the nodes which are the neighbors of node i. In the set $\mathcal{V}$, node 0 represents the exosystem or a leader whereas all other nodes represent the subsystems or agents defined in (2.1).

- A non-negative matrix known as the adjacency matrix can be defined as $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N \times N}, i, j = 0, 1, ., N$ where for $i = 1, 2, ., N, a_{i0} > 0$ if the local control input u_i of the agent i can access the exogenous signal η.

Other elements in $\mathcal{A}$ satisfies $a_{ii} = 0$ and $a_{ij} \geq 0$.

- We can define a subgraph $\mathcal{G}_s = (\mathcal{V}_s, \mathcal{E}_s)$ of $\mathcal{G}$, where $\mathcal{V}_s = \{1, 2, 3, ., N\}, \mathcal{E}_s \subseteq \mathcal{V}_s \times \mathcal{V}_s$ by excluding all edges between exosystem (leader) and all other agents in set $\mathcal{V}$.

- The Laplacian matrix $\mathcal{L} = [l_i j] \in \mathbb{R}^{N \times N}$ of a digraph $\mathcal{G}$ corresponding to matrix $\mathcal{A}$ contains elements in which $l_{ii} = \sum_{j=1}^{N} a_{ij}$ and $l_{ii} = \sum_{j=1}^{N} -a_{ij}$ if $i \neq j$

2.1.3 Functions

Definition 2.1.1. For a vector $x_i \in \mathbb{R}^n$, sign(x_i) is defined as;

$$\text{sign}(x_i) = \begin{cases} -1 & : x_i < 0 \\ 0 & : x_i = 0 \\ 1 & : x_i > 0 \end{cases}$$

Definition 2.1.2. For a vector $x_i \in \mathbb{R}^n$, sat(x_i) is defined as;

$$\text{sat}(x_i) = \begin{cases} -1 & : x_i < -1 \\ x_i & : |x_i| \leq 1 \\ 1 & : x_i > 1 \end{cases}$$

Definition 2.1.3. A continuous function $\alpha : [0, a) \to [0, \infty)$ is said to belong to class $\mathcal{K}$ if it is strictly increasing and $\alpha(0) = 0$. It is said to belong to class $\mathcal{K}_\infty$ if $a = \infty$ and $\alpha(r) \to \infty$ as $r \to \infty$.

2.2 Literature Review

2.2.1 Multi-Agent System

Motion dynamics of a multi-agent system whose output regulation problem is under consideration here can be found in literature [16] is given by the following form;

$$
\begin{aligned}
\dot{x}_i =& f_i(x_i, \eta_i, u_i) \\
y_i =& g_i(x_i, \eta_i)
\end{aligned} \tag{2.1}
$$

$x_i \in \mathbb{R}^{n_i}, u_i \in \mathbb{R}^{m_i}$ and $y_i \in \mathbb{R}^{m_i}$ are the local state, local control input and local output of the ith subsystem. Functions f_i and g_i for all i agents are assumed to be known and smooth.

Assumption 2.1. *$\eta_i \in \mathbb{R}^{r_i}$ are the local exogenous signals supposed to be generated by a known system which is both local neutrally stable and autonomous, referred here as exosystem.*

$$
\dot{\eta}_i = h_i(\eta_i), \qquad i = 1, ..., M \tag{2.2}
$$

These local exogenous signals represent the reference inputs to be tracked where function h_i for all i agents are also known.

Collectively one can view the multi-agent system (2.1) and exosystem (2.2) as the network of M + 1 agents with exosystem as the leader (or virtual leader) and all the agents of (1) as followers. Furthermore, assumption of the local neutral stability for exosystem indicates that Lyapunov stability of the system in some neighborhood of the origin is preserved.

Assumption 2.2. *There exist locally Lipchitz functions $\varphi_i(x_i, \eta_i)$ with $\varphi_i(0, \eta_i) = 0$ and continuously differentiable Lyapunov functions $V_i(x_i, \eta_i)$, such that*

$$
\alpha_{1i}(\|x_i\|) \le V_i(x_i, \eta_i) \le \alpha_{2i}(\|x_i\|) \tag{2.3}
$$

$$
\frac{\partial V_i}{\partial \eta_i} h_i(\eta_i) + \frac{\partial V_i}{\partial x_i} f_i(x_i, \eta_i, \varphi_i(x_i, \eta_i)) \le -W_i(x_i) \tag{2.4}
$$

$\forall x_i \in \mathbb{R}^{n_i}$ and $\eta_i \in \mathbb{R}^{r_i}$ where α_{1i} and α_{2i} are class $\mathcal{K}$ functions and $W_i(x_i)$ are continuous positive definite functions.

Formation Control

Formation Control has one core objective. It makes the relative distances or positions among the agents stabilize to prescribed values. Application arenas of formation control are abundant. From formation flight of satellites to network of sensors and coordinated transportation; it can be utilized in many synchronized applications. Formation Control can roughly be classified into two categories.

- Formation Producing
- Formation Tracking

For a group of agents, if the algorithm design is such that the agents are made to converge at a pre-determined positional pattern – and that too without following any group reference – is Formation Producing. If the aforementioned task is gone through but a pre-desired group reference is followed, then this is referred as Formation Tracking [20]. The challenging of these two categories is formation tracking because of the involvement of group reference. Only the formation tracking problem will be discussed here.

Formation Control problems are firmly based on consensus algorithms. The required formation structures are produces by modifying consensus protocols and these protocols are modified by the addition suitable of offset variables {[21],[22] and [23]}. In an attempt to offset the control inputs – in single integrator kinematic systems and double integrator dynamic systems – by some angles, introduction of coupling matrices was done. These matrices are created by extending the consensus algorithms {[24] and [25]}. The research in {[28], [29] and [30]} – inspired from the graph rigidity – has been conducted in order to mobilize a bunch of agents in such a way that the required configuration among the agents is met. This can only be ensured by maintaining some of the critical edge-distances equivalent to the required values. The Formation Control Algorithm based only on the rigidity – when compared to other formation algorithms that make use of the edge-vector information – only needs the information regarding edge-distances. But this comes with a tradeoff as there might exist some unstable equilibria. [20] reports some more research on Formation Control.

2.2.2 Sliding Mode Control

One of the subset of the non-linear control is the Sliding Mode Control or SMC. It makes use of the discontinuous control signal in order to push the system trajectories so as to drive them towards the sliding manifold. The whole process can be divided into two stages. These are referred as[31]

- Reaching Phase
- Sliding Phase

In the first stage, the trajectories start off the manifold, then drive towards it and finally reaching it and all of this happens in a finite time. In the second stage, control law is used to restrict these trajectories to that very sliding manifold. Control law can be mathematically described as

$$u = -\gamma \text{sgn}(s) \tag{2.5}$$

The constant γ has a known positive value and it is dependent on the systems upper bound. SMC may introduce positive and negative properties to the system. Positivity:

- Bounded control signal
- Well-understood tuning values

Negativity:

- Control Signal Chattering

Control signal chattering is the rapid change in the signal that may decrease actuators life, decrease the control accuracy and decrease performance by increasing heat losses. Zig-zag motion is induced due to this process which is depicted in the figure below [31]. In order to get rid of this chattering, there exist many approaches in the literature. Some of the techniques include

- Breaking down the control into two components; continuous and switching
- Replacing a sign function for a continuous complement [32] (this research work has made use of this approach)
- Utilizing a boundary layer around the sliding surface [13]
- Bringing higher order sliding modes into use [27]

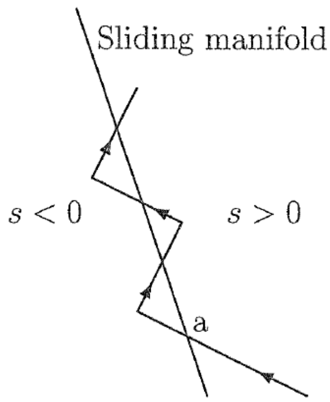

Figure 2.2: Zig-zag motion due to chattering of control signal [31]

2.2.3 Servomechanism

The problem of achieving asymptotic tracking of pre-defined trajectories and/or asymptotic rejection of disturbances lies at the very core in control theory. One of the various possibilities to handle this type of problem is called "Tracking via Internal Models" or "Servomechanism". This technique is robust to handle unknown uncertainties in plant as well as unknown reference trajectories to be tracked simultaneously. [33], [18] and [12] provide a very detailed discussion about servomechanism approach. Further details of servomechanism are explained in Chapter 3.

Chapter 3

Previous Work

3.1 Introduction

In this chapter, the previous work presented in [15] is reviewed in which the term *conditional servocompensator* was introduced by Seshagiri and Khalil. Basic purpose of this approach is to alleviate the degradation in transient performance by embedding the anti-windup scheme while achieving steady-state accuracy. References [12], [18] and [19] provide basis for the said previous work.

3.2 Conditional Integrators

In [12], Khalil and his companions presented an integral action based continuous sliding mode control for minimum phase non-linear systems in which the integral action only works conditionally to avoid the degradation in required performance. Later the same idea was extended for designing of conditional servocompensator.

Another work related to formation control of multiple vehicles using conditional integrators is explained in [**?**]. In this research work, author devised an application of conditional integrators while maintaining the path following and trajectory tracking of multiple vehicles.

3.3 Conditional Servocompensators

In [18], a robust servomechanism was discussed for minimum phase non-linear single-input-single-output (SISO) system. In the absence of disturbances, SISO system can be mathematically written as;

$$\begin{aligned}\dot{x} &= f(x,\nu) + g(x,\nu)u \\ y &= h(x,\nu)\end{aligned} \tag{3.1}$$

where $x \in \mathbb{R}^n$ is the state , $u \in \mathbb{R}$ is the control input and $y \in \mathbb{R}$ is the measured output. The functions f, g and h are smooth and continuously dependent on $\nu \in \Theta \subset \mathbb{R}^l$, a vector with unknown constant parameters. The output y is to track a time-varying reference signal y_R.

Assuming that the system (3.1) has a uniform relative degree $r \leq n$ that is;

$$L_g y = L_g L_f y = \cdots = L_g L_f^{r-2} y = 0 \quad \text{and} \quad |L_g L_f^{r-1} y| \geq k_0 \geq 0 \tag{3.2}$$

where k_0 is independent of θ.

Assumption 3.1. *A known exosystem generates the exogenous signal represented by $\nu(t)$*

$$\dot{\nu} = S_0 \nu \tag{3.3}$$

where eigenvalues of S_0 are distinct and lies on the imaginary axis.

Assumption 3.2. *There exist unique continuous differentiable mappings $x = \pi(\nu)$ with $\pi(0) = 0$ and $u = c(\nu)$ that solves the equation*

$$
\begin{aligned}
\frac{\partial \pi}{\partial \nu} S_0 \nu =& f(\pi, \nu) + g(\pi, \nu) c(\nu) \\
0 =& h(\pi, \nu)
\end{aligned}
\tag{3.4}
$$

Further assuming that there exist a set of real numbers $a_0, a_1, \ldots, a_{q-1}$ independent of θ, such that the steady state value of the control input represented by $c(\eta, \theta)$ satisfies the identity;

$$
L_\eta^q c = a_0 c + a_1 L_\eta c + \cdots + c_{q-1} L_\eta^{q-1} c \tag{3.5}
$$

$\forall\ (\eta, \theta) \in \mathcal{W} \times \Theta$, where $L_\eta c = \frac{\partial c}{\partial \eta} \dot{\eta}$ and the characteristic polynomial

$$
p^q - a_{q-1} p^{q-1} - \cdots - a_1 p - a_0 = 0
$$

have negative real parts, then following control law can be used;

$$
u = -k \ \mathrm{sign}(L_g L_f^{r-1} y) \ \mathrm{sat}\left(\frac{s}{\mu}\right) \tag{3.6}
$$

$$
\dot{\sigma} = S\sigma + J(y(t) - y_R(t)) \tag{3.7}
$$

$$
s = K_1 \sigma + k_1 e_1 + \sum_{i=2}^{r-1} k_i \hat{e}_i + \hat{e}_r \tag{3.8}
$$

where,

$$
S = \begin{bmatrix} 0 & 1 & \cdots & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & & & & \vdots \\ 0 & \cdots & \cdots & 0 & 1 \\ a_0 & \cdots & \cdots & \cdots & a_{q-1} \end{bmatrix} \quad \text{and} \quad J = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}
$$

K_1 and K_2 are chosen to make the matrix

$$\begin{bmatrix} S & JC_0 \\ -B_0K_1 & A_0 - B_0K_2 \end{bmatrix}$$

Hurwitz in which $\{A_0, B_0, C_0\}$ indicates the canonical form representation of chain of $r-1$ integrators. $k > 0$ and $\mu > 0$ are design parameters, and $\hat{e}_i \in \{2, \ldots, r\}$ provides estimates of tracking error.

Thus incorporating a robust servocompensator by above mentioned technique helps in achieving asymptotic tracking and disturbance rejection using continuous sliding mode control; this is applicable for both linear and non-linear system. But this technique suffers a degradation in transient performance of the system [18] which can be overcome by using a conditional servocompensator explained below.

3.3.1 Introduction to conditional servocompensators

Seshagiri and Khalil in their reserach work [15] have shown that the servocompensator can be designed as a conditional one just like conditional integrator [12] to recover the performance of ideal (discontinuous) SMC by avoiding the problem of chattering. The basic principle behind the conditional servocompensator is that it activates only inside the boundary layer to provide asymptotic output regulation with better transient performance. Taking σ as the output of the conditional servocompensator and μ is the size of the boundary layer, the new control law can be written as;

$$u = -k \ \text{sign}(L_g L_f^{r-1} y) \ \text{sat}\left(\frac{s}{\mu}\right) \tag{3.9}$$

$$\dot{\sigma} = (S - JK_1)\sigma + \mu J sat(s/\mu) \tag{3.10}$$

$$s = K_1\sigma + k_1 e_1 + \sum_{i=2}^{r-1} k_i \hat{e}_i + \hat{e}_r \tag{3.11}$$

The matrix K_1 is chosen such that $(S - JK_1)$ is Hurwitz (since pair (S,J) is controllable) and $[k_1 \;\; k_2 \;\; \cdots \;\; k_p]$ are chosen to make the polynomial $\lambda^{\rho-1} + k_{\rho-1}\lambda^{\rho-2} + \cdots + k_2\lambda + k_1$ Hurwitz. Inside the boundary layer, equation (**??**) reduces to (3.7). Below is given the basic example of single-input-single output (SISO) system for better understanding of conditional servocompensation technique.

3.3.2 Example

Consider a second-order SISO system of the form;

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -ax + u \\ y &= x_1 \end{aligned} \tag{3.12}$$

where $x \in \mathbb{R}$ is the system state, $u \in \mathbb{R}$ is the control input and $y \in \mathbb{R}$ is the measured output signal. The constant $a > 0$ is a known scalar system parameter. The reference signal r is the sinusoidal signal generated by the exosystem given as;

$$\dot{\nu} = \begin{bmatrix} 0 & -\omega \\ \omega & 0 \end{bmatrix} \nu, \qquad \nu^T(0) = [0, r_0], \qquad r(t) = \nu_1 \tag{3.13}$$

The system parameters are taken as;

$a = 1, \omega = 0.5; r_0 = 1$

Here, the control objective is to make sure that measured output signal tracks the reference signal $r(t)$.

Steady state value of the control input can be computed as;

$$c = (-\omega^2 + a)\nu_1$$

satisfying the identity $L_\eta^2 c = a_0 c + a_1 L_\eta c$ with $c_0 = -\omega^2$ and $c_1 = 0$.

Ideal SMC: For this design, the sliding surface is chosen as;

$s = k_1 e_1 + \dot{e}_1$, where $k_1 = 5 > 0$.

Continuous SMC: In this design, the sliding surface is also chosen as;

$s = k_1 e_1 + \dot{e}_1$, where $k_1 = 5 > 0$.

CSMC with Conventional Servocompensator:

In this design, a second-order conventional servocompensator $\dot{\sigma} = S\sigma + Je_1$ is introduced while sliding surface is taken as; $s = K_1\sigma + k_1 e_1 + \dot{e}_1$, where K_1 and k_1 are chosen to assign the eigenvalues of $\begin{bmatrix} S & J \\ -K_1 & -k_1 \end{bmatrix}$ at $-1, -2$ and -3. [18].

CSMC with Conditional Servocompensator:

In this design, a second-order conditional servocompensator of the form (3.10) is incorporated while sliding surface is taken as; $s = K_1\sigma + k_1 e_1 + \dot{e}_1$, where $k_1 = 5 > 0$ and K_1 is chosen to assign the eigenvalues of $(S - JK_1)$ at -1 and -2 [15].

And the control law is taken as;

- For ideal SMC case, $u_{smc} = -k_s sgn(s)$
- For other three cases, $u_{csmc} = -k_s sat(\frac{s}{\mu})$

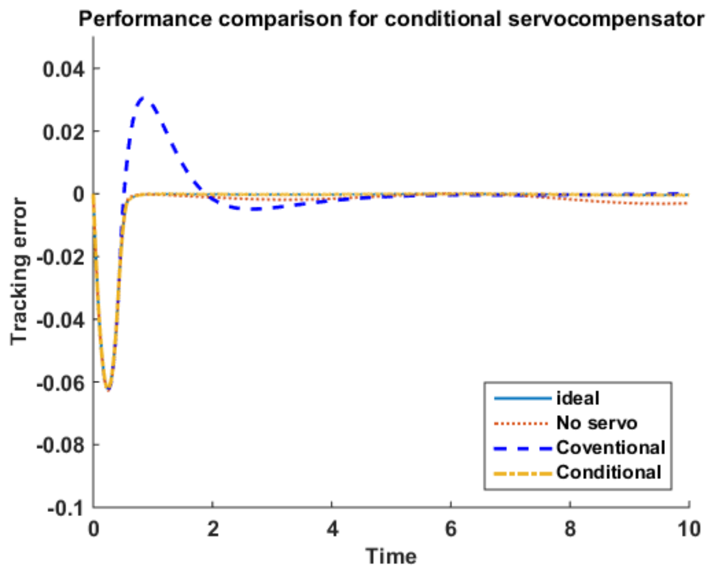

Figure 3.1: Performance comparison of conditional servocompensator response with other three designs

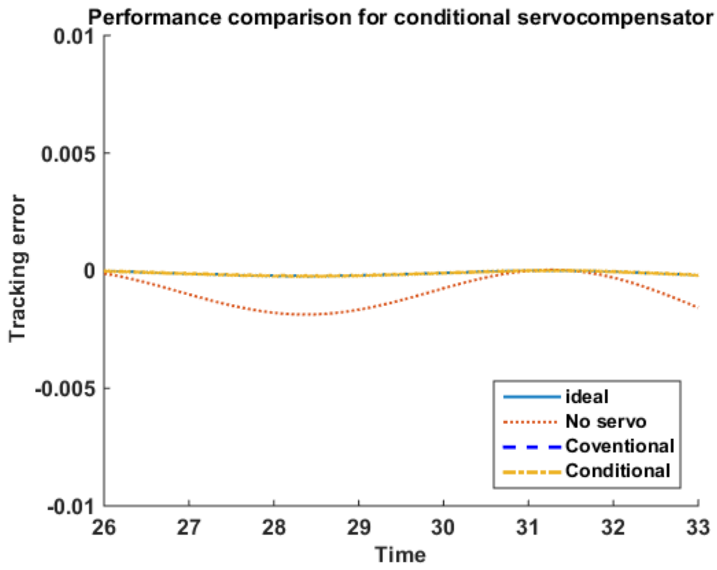

Figure 3.2: Performance comparison of conditional servocompensator response with other three designs at steady state

Results of the simulations are shown in Figure 3.1 and 3.2. The improvement in the transient response with conditional servocompensator by using continuous sliding mode control can be clearly seen in Figure 3.1 which is very similar to ideal SMC design. The transient response of continuous sliding mode control design (case 2) is also very similar to ideal SMC design but Figure 3.2 suggests the non-convergence of the asymptotic error using CSMC without servocompensator. Furthermore, conventional servocompensator design also produces zero steady state error but at the expense of degraded transient performance.

Chapter 4

Conditional Servocompensators for Output regulation of Multi-Agent System

4.1 Control Design

4.1.1 Directed Graph

The design of conditional servocompensator for multi-agent system is very similar to the conditional servocompensator in Reference [15]. Basically, this design involves the extension of separate servocompensators to each individual in a multi-agent system. The objective under consideration here is that agents with the dynamics given in (2.1) are required to achieve and maintain coordinated motion with respect to the leader. The relative position of ith agent with respect to jth agent is uniquely defined by a know vector r_{ij} where $i, j = L, 1, ...M$.

Assuming that for each agent, state e needs to be driven to zero is given by

$$e_{ij} = x_i - x_j - r_{ij}, \quad i,j = L, 1, ...M \tag{4.1}$$

where $e_{ij} \in \mathbb{R}^n$ is the position of the ith agent with respect to jth agent. Based on the observations made by each agent, we define the generalized error z_i as

$$z_i = \sum_{j \in \mathbb{J}} e_{ij}, \quad i,j = L, 1, ...M \tag{4.2}$$

where $\mathbb{J} = \{L, 1, ...M\}$. We define $\zeta_i^T = [z_i^1 \;\; z_i^2 \;\; \ldots \;\; z_i^{p-1}]$ and $K_i^2 = [k_i^1 \;\; k_i^2 \;\; \ldots \;\; k_i^{p-1}]$. Servocompensation for multi-agent system is then introduced by defining the sliding surface for each individual agent.

$$s_i = K_i^1 \sigma_i + K_i^2 \zeta_i + z_i^\rho \tag{4.3}$$

where σ_i is the output of the conditional servocompensator and is defined as

$$\dot{\sigma}_i = (S_i - J_i K_i^1)\sigma_i + \mu J_i sat(s_i/\mu) \tag{4.4}$$

The control is taken as;

$$u_i = -k_i sat(s_i/\mu) \tag{4.5}$$

where u_i is the control for ith agent and k_i is the upper bound on the control signal of the respective agent. We define $A_{\sigma_i} = (S_i - J_i K_i^1)$ and the Lyapunov function

$$V_{\sigma_i}(\sigma_i) = \sigma_i^T P_{\sigma_i} \sigma_i \tag{4.6}$$

where the symmetric positive definite matrix P_{σ_i} is the solution of $P_{\sigma_i} A_{\sigma_i} + A_{\sigma_i}^T P_{\sigma_i} = -I$. Defining the compact set Ω_{σ_i} by

$$\Omega_{\sigma_i} = \sigma_i : V_{\sigma_i}(\sigma_i) \leq \mu^2 \rho_i$$

where ρ_i are the positive constants. Using the inequality

$$\dot{V}_{\sigma_i} \leq -\|\sigma_i\|^2 + 2\mu\|\sigma_i\|\|P_{\sigma_i} J_i\|$$

it is easy to show that $\dot{V}_{\sigma_i} \leq 0$ on the boundary $V_{\sigma_i} = \mu^2 \rho_i$ for the choice $\rho_i = 4\|P_{\sigma_i} J_i\|^2 \lambda_{max}(P_{\sigma_i})$. Hence, Ω_{σ_i} is positively invariant

4.1.2 Undirected Graph

Undirected Graph can be treated as a special case of directed graph in which all the agents in a multi-agent system can access the exogenous signal, so for this case errors defined in (4.1) and (4.2) can be modified as;

$$\begin{aligned} e_i =& x_i - x_L - r_i, \\ z_i =& e_i \end{aligned} \tag{4.7}$$

where r_i is the relative position of ith agent with respect to leader or exogenous system. Equation (4.7) decreases the complexity of the system.

4.2 THREE AGENT SYSTEM IN DIRECTED GRAPH TOPOLOGY

4.2.1 Description

To show the performance with conditional servocompensator, we consider formation control of three-agent system consisting of two double integrator agents with a leader (exosystem) represented by node 0. Dynamics of the agents moving in two dimensional space are given by;

$$\begin{aligned} \dot{x}_i =& v_i \\ v_i =& u_i \\ y_i =& x_i \end{aligned} \tag{4.8}$$

where $x_i \in \mathbb{R}^2$ is the position, $v_i \in \mathbb{R}^2$ is the velocity and $u_i \in \mathbb{R}^2$ is the control signal for each individual agent. Agents are required to move along a circle in a desired fixed formation as shown in figure 4.1. This implies that the trajectories of the leader are generated by the linear exosystem.

$$\dot{\eta} = \begin{bmatrix} 0 & -\beta & 0 & 0 \\ \beta & 0 & 0 & 0 \\ 0 & 0 & 0 & -\beta \\ 0 & 0 & \beta & 0 \end{bmatrix} \eta, \quad y_l = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \eta \tag{4.9}$$

where $\eta \in \mathbb{R}^4$ and $\eta^T(0) = [10, 0, 10, 10]$. Formation control of multi-agent system with such dynamics can be found in [5], [6] and [16]. Note that the directed graphs of the above topology satisfy the required property that there exists at least one directed path from the leader to any agent. Let the required fixed relative position (with respect to the leader) for the agents be $r_1 = [-1, 0]^T$, and $r_2 = [-2, 0]^T$. Note that the generalized error state z can be found by the equation (4.2).

Figure 4.1: Network topolgy for directed graph

Furthermore, let us consider that there exist mappings (invariant manifold)

$x_i = \pi_i(\eta_i)$ and $u_i = c_i(\eta_i)$, $1 \le i \le M$ with $\pi_i(0) = 0$ and $c_i(0) = 0$, such that

$$\begin{aligned}\frac{\partial \pi_i(\eta_i)}{\partial \eta_i}\dot{\eta}_i =& f_i(\pi_i(\eta_i), \eta_i, c_i(\eta_i)) \\ 0 =& g_i(\pi_i(\eta_i), \eta_i) - q_i(\eta_i), \quad 1 \le i \le M\end{aligned} \tag{4.10}$$

For Agent 1, (4.10) can be written as;

$$\begin{bmatrix} \frac{\partial \pi_{11}}{\partial \eta_1} & \frac{\partial \pi_{11}}{\partial \eta_2} & \frac{\partial \pi_{11}}{\partial \eta_3} & \frac{\partial \pi_{11}}{\partial \eta_4} \\ \frac{\partial \pi_{21}}{\partial \eta_1} & \frac{\partial \pi_{21}}{\partial \eta_2} & \frac{\partial \pi_{21}}{\partial \eta_3} & \frac{\partial \pi_{21}}{\partial \eta_4} \end{bmatrix} \begin{bmatrix} -\beta\eta_2 \\ \beta\eta_1 \\ -\beta\eta_4 \\ \beta\eta_3 \end{bmatrix} = \begin{bmatrix} \pi_{21} \\ c_1 \end{bmatrix}$$

$$0 = (\pi_{11} - (y_L + r_{1L})) + (\pi_{11} - (\pi_{12} + r_{12})) \tag{4.11}$$

Solving above equation yields following results;

$$-\beta\eta_2 \frac{\partial \pi_{11}}{\partial \eta_1} + \beta\eta_1 \frac{\partial \pi_{11}}{\partial \eta_2} - \beta\eta_4 \frac{\partial \pi_{11}}{\partial \eta_3} + \beta\eta_3 \frac{\partial \pi_{11}}{\partial \eta_4} = \pi_{21} \tag{4.12}$$

$$-\beta\eta_2 \frac{\partial \pi_{21}}{\partial \eta_1} + \beta\eta_1 \frac{\partial \pi_{21}}{\partial \eta_2} - \beta\eta_4 \frac{\partial \pi_{21}}{\partial \eta_3} + \beta\eta_3 \frac{\partial \pi_{21}}{\partial \eta_4} = c_1 \tag{4.13}$$

$$y_L + \pi_{12} + r_{1L} + r_{12} = 2\pi_{11} \tag{4.14}$$

Now similar results can be obtained for Agent 2 using (4.10)

$$\begin{bmatrix} \frac{\partial \pi_{12}}{\partial \eta_1} & \frac{\partial \pi_{12}}{\partial \eta_2} & \frac{\partial \pi_{12}}{\partial \eta_3} & \frac{\partial \pi_{12}}{\partial \eta_4} \\ \frac{\partial \pi_{22}}{\partial \eta_1} & \frac{\partial \pi_{22}}{\partial \eta_2} & \frac{\partial \pi_{22}}{\partial \eta_3} & \frac{\partial \pi_{22}}{\partial \eta_4} \end{bmatrix} \begin{bmatrix} -\beta\eta_2 \\ \beta\eta_1 \\ -\beta\eta_4 \\ \beta\eta_3 \end{bmatrix} = \begin{bmatrix} \pi_{22} \\ c_2 \end{bmatrix}$$

$$0 = \pi_{12} - (\pi_{11} + r_{21}) \tag{4.15}$$

Solving above equation, following results can be obtained;

$$-\beta\eta_2\frac{\partial\pi_{12}}{\partial\eta_1}+\beta\eta_1\frac{\partial\pi_{12}}{\partial\eta_2}-\beta\eta_4\frac{\partial\pi_{12}}{\partial\eta_3}+\beta\eta_3\frac{\partial\pi_{12}}{\partial\eta_4}=\pi_{22} \tag{4.16}$$

$$-\beta\eta_2\frac{\partial\pi_{22}}{\partial\eta_1}+\beta\eta_1\frac{\partial\pi_{22}}{\partial\eta_2}-\beta\eta_4\frac{\partial\pi_{22}}{\partial\eta_3}+\beta\eta_3\frac{\partial\pi_{22}}{\partial\eta_4}=c_2 \tag{4.17}$$

$$\pi_{12}=\pi_{11}+r_{21} \tag{4.18}$$

Putting the value of π_{12} from (4.18) into (4.16), we get

$$\pi_{21}=\pi_{22} \tag{4.19}$$

Now, putting the value of π_{12} from (4.18) into (4.14), we get

$$\begin{aligned} 2\pi_{11} = &\quad y_L+\pi_{11}+r_{21}+r_{1L}+r_{12} \\ \pi_{11} = &\qquad y_L+r_{21}+r_{1L}+r_{12} \end{aligned}$$

or we can rewrite it as;

$$\pi_{11}=\begin{bmatrix}\eta_1\\ \eta_3\end{bmatrix}+r_{21}+r_{1L}+r_{12}$$

$$\pi_{11}=\eta^*+r_{21}+r_{1L}+r_{12} \tag{4.20}$$

where $\eta^*=\begin{bmatrix}\eta_1\\ \eta_3\end{bmatrix}=y_L$.

Putting the value of π_{11} from above equation into (4.12), we get

$$\pi_{21}=-\beta\eta_2\begin{bmatrix}1\\0\end{bmatrix}-\beta\eta_4\begin{bmatrix}0\\1\end{bmatrix}=\beta\begin{bmatrix}-\eta_2\\-\eta_4\end{bmatrix}=-\beta\eta^{**} \tag{4.21}$$

where $\eta^{**} = \begin{bmatrix} \eta_2 \\ \eta_4 \end{bmatrix}$. As $\pi_{21} = \pi_{22}$, so we can say that

$$\pi_{22} = \beta \begin{bmatrix} -\eta_2 \\ -\eta_4 \end{bmatrix} = -\beta\eta^{**} \tag{4.22}$$

Putting (4.20) in (4.18),

$$\pi_{12} = \begin{bmatrix} \eta_1 \\ \eta_3 \end{bmatrix} + 2r_{21} + r_{1L} + r_{12}$$

$$\pi_{12} = \eta^* + 2r_{21} + r_{1L} + r_{12} \tag{4.23}$$

The constant term r_{1L} in above equation shows that although second agent is not connected to the leader directly but it can still get the information (relative position) of the leader (Node 0) through agent 1. We have mapped our states to $\pi_i(\eta_i)$; next step is to map the control signals of each agent to $c_i(\eta_i)$.

Now solving equations (4.13) and (4.17) using the results of π_{11}, π_{12}, π_{21} and π_{22};

$$c_1 = \beta\eta_1 \begin{bmatrix} -\beta \\ 0 \end{bmatrix} + \beta\eta_3 \begin{bmatrix} 0 \\ -\beta \end{bmatrix} = -\beta^2 \begin{bmatrix} \eta_1 \\ \eta_3 \end{bmatrix} = -\beta^2\eta^* \tag{4.24}$$

and

$$c_2 = \beta\eta_1 \begin{bmatrix} -\beta \\ 0 \end{bmatrix} + \beta\eta_3 \begin{bmatrix} 0 \\ -\beta \end{bmatrix} = -\beta^2 \begin{bmatrix} \eta_1 \\ \eta_3 \end{bmatrix} = -\beta^2\eta^* \tag{4.25}$$

which employs that $c_1 = c_2 = c_i(\eta)$. Hence,

$$c_i(\eta) = -\beta^2 \begin{bmatrix} \eta_1 \\ \eta_3 \end{bmatrix}$$

Taking the first Lie derivative of $c_i(\eta)$ employs that;

$$L_\eta c_i(\eta) = \frac{\partial c_i(\eta)}{\partial \eta}\dot{\eta}$$

$$\Rightarrow L_\eta c_i(\eta) = \begin{bmatrix} \frac{\partial c_1}{\partial \eta_1} & \frac{\partial c_1}{\partial \eta_2} & \frac{\partial c_1}{\partial \eta_3} & \frac{\partial c_1}{\partial \eta_4} \\ \\ \frac{\partial c_2}{\partial \eta_1} & \frac{\partial c_2}{\partial \eta_2} & \frac{\partial c_2}{\partial \eta_3} & \frac{\partial c_2}{\partial \eta_4} \end{bmatrix} \begin{bmatrix} -\beta\eta_2 \\ \beta\eta_1 \\ -\beta\eta_4 \\ \beta\eta_3 \end{bmatrix}$$

$$\Rightarrow L_\eta c_i(\eta) = \begin{bmatrix} -\beta^2 & 0 & 0 & 0 \\ \\ 0 & 0 & -\beta^2 & 0 \end{bmatrix} \begin{bmatrix} -\beta\eta_2 \\ \beta\eta_1 \\ -\beta\eta_4 \\ \beta\eta_3 \end{bmatrix} = \beta^3 \begin{bmatrix} \eta_2 \\ \\ \eta_4 \end{bmatrix} \tag{4.26}$$

Similarly the second Lie derivative of $c_i(\eta)$ is given by;

$$L_\eta^2 c_i(\eta) = \frac{\partial L_\eta c_i(\eta)}{\partial \eta}\dot{\eta}$$

$$\Rightarrow L_\eta c_i(\eta) = \begin{bmatrix} 0 & \beta^3 & 0 & 0 \\ \\ 0 & 0 & 0 & \beta^3 \end{bmatrix} \begin{bmatrix} -\beta\eta_2 \\ \beta\eta_1 \\ -\beta\eta_4 \\ \beta\eta_3 \end{bmatrix} = \beta^4 \begin{bmatrix} \eta_1 \\ \\ \eta_3 \end{bmatrix} \tag{4.27}$$

So it can be verified that $c_i(\eta)$ satisfies the identity

$$L_\eta^2 c_i(\eta) = a_0 c_i(\eta) + a_1 L_\eta c_i(\eta) \tag{4.28}$$

with $a_0 = -\beta^2,\ \ a_1 = 0$.

4.2.2 Control Design

The performance of three designs are under consideration here:

1. The first is a continuous approximation of sliding mode without servo-

compensator. In this design, surface is taken as

$$
\begin{aligned}
s_i &= k_i^1 z_i^1 + z_i^2 \\
\Rightarrow s_i &= k_i^1 z_i + \dot{z}_i
\end{aligned}
\tag{4.29}
$$

2. Second design uses the second-order conventional servocompensator $\dot{\sigma}_i = S_i\sigma_i + J_i(k_i^1 z_i + \dot{z}_i)$ whereas the sliding surface is taken as;

$$
\begin{aligned}
s_i &= K_i^1\sigma_i + k_i^1 z_i^1 + z_i^2 \\
\Rightarrow s_i &= K_i^1\sigma_i + k_i^1 z_i + \dot{z}_i
\end{aligned}
\tag{4.30}
$$

where $k_i^1 > 0$ (any positive constant). For this design, K_i^1 and k_i^1 are chosen to make ψ_i Hurwitz where

$$
\psi_i = \begin{bmatrix} S_i & J_i \\ -K_i^1 & -k_i^1 \end{bmatrix}
$$

3. The third design uses second-order conditional servocompensator of (4.4) whereas the sliding surface is again taken as;

$$
s_i = K_i^1\sigma_i + k_i^1 z_i + \dot{z}_i \tag{4.31}
$$

In conditional servocompensator design K_i^1 is chosen to make $(S_i - J_iK_i^1)$ Hurwitz whereas in first and third design k_i^1 is chosen as any positive constant.

4.2.3 Analysis

Analysis of the agents following a leader is shown below. Depending upon the neighboring nodes of both the agents, error z_i is defined on the basis of set N_i.

Outside the boundary layers

Agent 1: For agent 1, $N_1 = \{L, 2\}$ so error z_1 and its derivatives are defined as;

$$z_1 = (x_1 - x_l - r_{1L}) + (x_1 - x_2 - r_{12}) \tag{4.32}$$

$$\dot{z}_1 = (v_1 + \eta^{**}) + (v_1 - v_2) \tag{4.33}$$

$$\ddot{z}_1 = (u_1 + \eta^{*}) + (u_1 - u_2) \tag{4.34}$$

and the sliding surfaces for all the three designs are already defined in (4.29), (4.30) and (4.31).

Agent 2: As from the network topology, agent 2 is directly connected to Agent 1 only, hence getting information of exosystem through agent 1. In this case, $N_2 = \{1\}$ so error z_2 and its derivatives are defined as;

$$z_2 = (x_2 - x_1 - r_{21}) \tag{4.35}$$

$$\dot{z}_1 = (v_2 - v_1) \tag{4.36}$$

$$\ddot{z}_1 = (u_2 - u_1) \tag{4.37}$$

and the sliding surfaces for all the three designs are already defined in (4.29), (4.30) and (4.31).

Assumption 4.1. *There exist known functions $\varrho_1(v, \eta)$ and $\varrho_2(v, \eta)$ for agent 1 and agent 2 for which the following inequalities are always satisfied;*

$$|k_1^1(2v_1 + \eta^{**} - v_2) + \eta^*| \leq \varrho_1(v, \eta), \;\; \forall (v) \in \mathbb{R}^2 \tag{4.38}$$

$$|k_2^1(v_2 - v_1)| \leq \varrho_2(v, \eta), \;\; \forall (v) \in \mathbb{R}^2 \tag{4.39}$$

Choosing the lyapunov candidate $V_{s_1}(s_1) = \frac{1}{2} s_1^T s_1$ for agent 1 and $V_{s_2}(s_2) = \frac{1}{2} s_2^T s_2$ for agent 2 to show that trajectories reach the boundary layers $\|s_1\| \leq \mu_1$ and $\|s_2\| \leq \mu_2$ respectively in finite time.

$$s_1^T \dot{s}_1 \leq \|s_1\| \big\{ \varrho_1(v) + (2u_1 - u_2) + (\|\sigma_1\| \|K_1^1\| \|A_{\sigma_1}\| + \mu_1 \|K_1^1\| \|J_1\|) \big\} \tag{4.40}$$

and

$$s_2^T \dot{s}_2 \leq \|s_2\| \big\{ \varrho_2(v,\eta) + (u_2 - u_1) + (\|\sigma_2\| \|K_2^1\| \|A_{\sigma_2}\| + \mu_2 \|K_2^1\| \|J_2\|) \big\} \tag{4.41}$$

Let $k_i \geq \max\{\varrho_1 + \beta_1, \varrho_2 + \beta_2\}$, $i = 1$ and 2, where $\beta_1 > 0$ and $\beta_2 > 0$. So now (4.40) and (4.41) can be written as;

$$s_1^T \dot{s}_1 \leq -\varrho_1(v,\eta)\|s_1\| - \beta_1\|s_1\| - \left((\varrho(v,\eta) + \beta_1) \frac{|s_2|}{\|s_2\|} \right) \|s_1\| + \left(\|\sigma_1\| \|K_1^1\| \|A_{\sigma_1}\| + \mu_1 \|K_1^1\| \|J_1\| \right) \|s_1\|$$

and (4.42)

$$s_2^T \dot{s}_2 \leq -\beta_2\|s_2\| - \left((\varrho_2(v,\eta) + \beta_2) \frac{|s_1|}{\|s_1\|} \right) \|s_2\| + \left(\|\sigma_2\| \|K_2^1\| \|A_{\sigma_2}\| + \mu_2 \|K_2^1\| \|J_2\| \right) \|s_2\| \tag{4.43}$$

The norms $\|\sigma_1\| \|K_1^1\| \|A_{\sigma_1}\|$ and $\mu_1 \|K_1^1\| \|J_1\|$ can be bounded by a class κ function $\delta_1(\mu_1)$. Similarly, the norms $\|\sigma_2\| \|K_2^1\| \|A_{\sigma_2}\|$ and $\mu_2 \|K_2^1\| \|J_2\|$ can also be bounded by another class κ function $\delta_2(\mu_2)$. Hence

$$s_1^T \dot{s}_1 \leq -\varrho_1(v,\eta)\|s_1\| - \beta_1\|s_1\| - \left((\varrho_1(v,\eta) + \beta_1) \frac{|s_2|}{\|s_2\|} \right) \|s_1\| + \delta_1(\mu_1)\|s_1\|$$

and

$$s_2^T \dot{s}_2 \leq -\beta_2\|s_2\| - \left((\varrho_2(v,\eta) + \beta_2) \frac{|s_1|}{\|s_1\|} \right) \|s_2\| + \delta_2(\mu_2)\|s_2\| \tag{4.44}$$

or we can write above equations as;

$$s_1^T \dot{s}_1 \leq -\left[\varrho_1(v,\eta)\|s_1\| + \beta_1\|s_1\| + (\varrho(v,\eta)+\beta_1)\frac{|s_2|}{\|s_2\|} - \delta_1(\mu_1)\right]\|s_1\| \quad (4.45)$$

and

$$s_2^T \dot{s}_2 \leq -\left[\beta_2\|s_2\| + (\varrho_2(v,\eta)+\beta_2)\frac{|s_1|}{\|s_1\|} - \delta_2(\mu_2)\right]\|s_2\| \quad (4.46)$$

which shows that for sufficiently small μ_1 and μ_2, all trajectories will reach the boundary layers $\{\|s_1\| \leq \mu_1\}$ and $\{\|s_2\| \leq \mu_2\}$ respectively. Furthermore, the terms $\frac{|s_2|}{\|s_2\|}\|s_1\|$ in $s_1^T \dot{s}_1$ and $\frac{|s_1|}{\|s_1\|}\|s_2\|$ in $s_2^T \dot{s}_2$ clearly show that both agents do experience the control impact of each other because they are directly connected to each other.

Inside the boundary layers

Agent1: Inside the boundary layer, the closed loop system for Agent 1 is given by;

$$\dot{\eta} = S_0\eta \quad (4.47)$$

$$\dot{z_1^1} = (v_1 + \eta^{**}) + (v_1 - v_2) \quad (4.48)$$

$$\dot{z_1^2} = 2u_1 - u_2 + \eta^* \quad (4.49)$$

$$\dot{\sigma_1} = S_1\sigma_1 + J_1(k_1 z_1^1 + z_1^2) \quad (4.50)$$

We define,

$$M_{\mu_1} = \{\sigma_1 = \bar{\sigma}_1,\ \ z_1^1 = 0\}$$

From (4.49), $\bar{\sigma}_1$ can be found as;

$$\begin{aligned} 0 &= 2u_1 - u_2 + \eta^* \\ \Rightarrow 0 &= 2\Big(-k_1K_1\frac{\bar{\sigma}_1}{\mu_1}\Big) - \Big(-k_2K_2\frac{\bar{\sigma}_2}{\mu_2}\Big) - \eta^* \\ \Rightarrow \bar{\sigma}_1 &= \frac{\mu_1}{2}\Big\{\Big(\frac{k_2}{k_1}\frac{K_2}{K_1}\frac{\bar{\sigma}_2}{\mu_2}\Big) + \frac{1}{k_1K_1}\eta^*\Big\} \end{aligned} \quad (4.51)$$

Define $\tilde{\sigma}_1 = \sigma_1 - \bar{\sigma}_1$, $\tilde{s}_1 = K_1\tilde{\sigma}_1 + k_1 z_1^1 + z_1^2$ and $\tilde{s}_2 = K_2\tilde{\sigma}_2 + k_2 z_2^1 + z_2^2$ Again taking eq (4.49),

$$\begin{aligned}
\dot{z_1^2} =& 2u_1 - u_2 + \eta^* \\
\Rightarrow \dot{z_1^2} =& -2k_1\frac{s_1}{\mu_1} + k_2\frac{s_2}{\mu_2} + \eta^* \\
\Rightarrow \dot{z_1^2} =& -2k_1\frac{K_1\sigma_1 + k_1 z_1^1 + z_1^2}{\mu_1} + k_2\frac{K_2\sigma_2 + k_2 z_2^1 + z_2^2}{\mu_2} + \eta^* \\
\Rightarrow \dot{z_1^2} =& \frac{-2k_1}{\mu_1}\bigg((K_1\sigma_1 + k_1 z_1^1 + z_1^2) + (K_1\bar{\sigma}_1 + k_1 z_1^1 + z_1^2) - (K_1\bar{\sigma}_1 + k_1 z_1^1 + z_1^2)\bigg) + \\
& \frac{k_2}{\mu_2}\bigg((K_2\sigma_2 + k_2 z_2^1 + z_2^2) + (K_2\bar{\sigma}_2 + k_2 z_2^1 + z_2^2) - (K_2\bar{\sigma}_2 + k_2 z_2^1 + z_2^2)\bigg) + \eta^* \\
\Rightarrow \dot{z_1^2} =& \frac{-2k_1}{\mu_1}\bigg((K_1\tilde{\sigma}_1 + k_1 z_1^1 + z_1^2) + K_1\bar{\sigma}_1\bigg) + \frac{k_2}{\mu_2}\bigg((K_2\tilde{\sigma}_2 + k_2 z_2^1 + z_2^2) + K_2\bar{\sigma}_2\bigg) + \eta^*
\end{aligned}$$

By putting the values of $\bar{\sigma}_1$ and $\bar{\sigma}_2$, we get

$$\Rightarrow \dot{z_1^2} = -\frac{2k_1}{\mu_1}\tilde{s}_1 - \frac{k_2K_2}{\mu_2}\bar{\sigma}_2 + \frac{k_2}{\mu_2}\tilde{s}_2 + \frac{k_1K_1}{\mu_1}\bar{\sigma}_1 \tag{4.52}$$

Next we can find $\tilde{s}_1^T\dot{\tilde{s}}_1$ as,

$$\tilde{s}_1^T\dot{\tilde{s}}_1 = \tilde{s}_1^T(K_1\dot{\tilde{\sigma}}_1 + k_1\dot{z}_1^1 + \dot{z}_1^2)$$

By putting the values, we get

$$\begin{aligned}
\Rightarrow \tilde{s}_1^T\dot{\tilde{s}}_1 =& \tilde{s}_1^T\Big\{K_1(S_1\tilde{\sigma}_1 + J_1(k_1 z_1^1 + z_1^2))\Big\} + \tilde{s}_1^T(k_1 z_1^2) + \\
& \tilde{s}_1^T\Big\{-\frac{2k_1}{\mu_1}\tilde{s}_1 - \frac{k_2K_2}{\mu_2}\bar{\sigma}_2 + \frac{k_2}{\mu_2}\tilde{s}_2 + \frac{k_1K_1}{\mu_1}\bar{\sigma}_1\Big\}
\end{aligned}$$

By putting the value of $\bar{\sigma}_2$, above equation can be rewritten as;

$$\begin{aligned}
\Rightarrow \tilde{s}_1^T \dot{\tilde{s}}_1 =& \tilde{s}_1^T K_1 S_1 \tilde{\sigma}_1 + \tilde{s}_1^T K_1 J_1 k_1 z_1^1 + \tilde{s}_1^T K_1 J_1 z_1^2 + \tilde{s}_1^T k_1 z_1^2 \\
& - \tilde{s}_1^T \frac{2k_1}{\mu_1} \tilde{s}_1 + \tilde{s}_1^T \frac{k_2}{\mu_2} \tilde{s}_2 \\
\Rightarrow \tilde{s}_1^T \dot{\tilde{s}}_1 \leq & - \frac{2k_1}{\mu_1} \|\tilde{s}_1\|^2 + \frac{k_2}{\mu_2} \|\tilde{s}_2\| \|\tilde{s}_1\| + K_1 \lambda_{max}(S_1) \|\tilde{s}_1\| \|\tilde{\sigma}_1\| \\
& + K_1 k_1 \|\tilde{s}_1\| \|z_1^1\| + (K_1 + k_1) \|\tilde{s}_1\| \|z_1^2\|
\end{aligned}$$

Substituting the value of $\|z_1^2\|$ in above equation, we get

$$\begin{aligned}
\Rightarrow \tilde{s}_1^T \dot{\tilde{s}}_1 \leq & - \frac{2k_1}{\mu_1} \|\tilde{s}_1\|^2 + \frac{k_2}{\mu_2} \|\tilde{s}_2\| \|\tilde{s}_1\| + K_1 \lambda_{max}(S_1) \|\tilde{s}_1\| \|\tilde{\sigma}_1\| + K_1 k_1 \|\tilde{s}_1\| \|z_1^1\| \\
& + (K_1 + k_1) \|\tilde{s}_1\| \Big(- k_1 \|z_1^1\| + \|\tilde{s}_1\| - K_1 \|\tilde{\sigma}_1\| \Big) \\
\Rightarrow \tilde{s}_1^T \dot{\tilde{s}}_1 \leq & - \left(\frac{2k_1}{\mu_1} - (K_1 + k_1) \right) \|\tilde{s}_1\|^2 + \frac{k_2}{\mu_2} \|\tilde{s}_2\| \|\tilde{s}_1\| - {k_1}^2 \|\tilde{s}_1\| \|z_1^1\| \\
& + K_1 \Big(\lambda_{max}(S_1) - (K_1 + k_1) \Big) \|\tilde{s}_1\| \|\tilde{\sigma}_1\| \qquad (4.53)
\end{aligned}$$

or

$$\begin{aligned}
\Rightarrow \tilde{s}_1^T \dot{\tilde{s}}_1 \leq & - \left(\frac{2k_1}{\mu_1} - k_{13} \right) \|\tilde{s}_1\|^2 + \frac{k_2}{\mu_2} \|\tilde{s}_2\| \|\tilde{s}_1\| - {k_1}^2 \|\tilde{s}_1\| \|z_1^1\| \\
& + K_1 \Big(k_{14} - k_{13} \Big) \|\tilde{s}_1\| \|\tilde{\sigma}_1\| \qquad (4.54)
\end{aligned}$$

where

$$k_{13} = K_1 + k_1, \qquad k_{14} = \lambda_{max}(S_1)$$

Agent 2: Inside the boundary layer, the closed loop system for agent 2 is given by;

$$\dot{\eta} = S_0 \eta \qquad (4.55)$$

$$\dot{z_2^1} = (v_2 - v_1) \qquad (4.56)$$

$$\dot{z_2^2} = u_2 - u_1 \qquad (4.57)$$

$$\dot{\sigma_2} = S_2 \sigma_2 + J_2 (k_2 z_2^1 + z_2^2) \qquad (4.58)$$

We define,

$$M_{\mu_2} = \{\sigma_2 = \bar{\sigma}_2,\ \ z_2^1 = 0\}$$

From (4.57), $\bar{\sigma}_2$ can be found as;

$$\begin{aligned} 0 &= u_2 - u_1 \\ \Rightarrow 0 &= -k_2 K_2 \frac{\bar{\sigma}_2}{\mu_2} + k_1 K_1 \frac{\bar{\sigma}_1}{\mu_1} \\ \Rightarrow \bar{\sigma}_2 &= \mu_2 \left(\frac{k_1}{k_2} \frac{K_1}{K_2} \frac{\bar{\sigma}_1}{\mu_1} \right) \end{aligned} \tag{4.59}$$

Define $\tilde{\sigma}_2 = \sigma_2 - \bar{\sigma}_2$, $\tilde{s}_1 = K_1\tilde{\sigma}_1 + k_1 z_1^1 + z_1^2$ and $\tilde{s}_2 = K_2\tilde{\sigma}_2 + k_2 z_2^1 + z_2^2$ Again taking eq (4.57),

$$\begin{aligned} \dot{z_2^2} =& u_2 - u_1 \\ \Rightarrow \dot{z_2^2} =& -k_2 \frac{s_2}{\mu_2} + k_1 \frac{s_1}{\mu_1} \\ \Rightarrow \dot{z_2^2} =& -k_2 \frac{K_2\sigma_2 + k_2 z_2^1 + z_2^2}{\mu_2} + k_1 \frac{K_1\sigma_1 + k_1 z_1^1 + z_1^2}{\mu_1} \\ \Rightarrow \dot{z_2^2} =& \frac{-k_2}{\mu_2}\Bigg((K_2\sigma_2 + k_2 z_2^1 + z_2^2) + (K_2\bar{\sigma}_2 + k_2 z_2^1 + z_2^2) - (K_2\bar{\sigma}_2 + k_2 z_2^1 + z_2^2) \Bigg) + \\ & \frac{k_1}{\mu_1}\Bigg((K_1\sigma_1 + k_1 z_1^1 + z_1^2) + (K_1\bar{\sigma}_1 + k_1 z_1^1 + z_1^2) - (K_1\bar{\sigma}_1 + k_1 z_1^1 + z_1^2) \Bigg) \\ \Rightarrow \dot{z_2^2} =& \frac{-k_2}{\mu_2}\Bigg((K_2\tilde{\sigma}_2 + k_2 z_2^1 + z_2^2) + K_2\bar{\sigma}_2 \Bigg) + \frac{k_1}{\mu_1}\Bigg((K_1\tilde{\sigma}_1 + k_1 z_1^1 + z_1^2) + K_1\bar{\sigma}_1 \Bigg) \end{aligned}$$

By putting the values of $\bar{\sigma}_1$ and $\bar{\sigma}_2$, we get

$$\Rightarrow \dot{z_2^2} = -\frac{k_2}{\mu_2}\tilde{s}_2 - \frac{k_1 K_1}{\mu_1}\bar{\sigma}_1 + \frac{k_1}{\mu_1}\tilde{s}_1 + \frac{k_2 K_2}{2\mu_2}\bar{\sigma}_2 + \frac{\eta^*}{2} \tag{4.60}$$

Next we can find $\tilde{s}_2^T \dot{\tilde{s}}_2$ as,

$$\tilde{s}_2^T \dot{\tilde{s}}_2 = \tilde{s}_2^T (K_2 \dot{\tilde{\sigma}}_2 + k_2 \dot{z}_2^1 + \dot{z}_2^2)$$

By putting the values, we get

$$
\begin{aligned}
\Rightarrow \tilde{s}_2^T \dot{\tilde{s}}_2 =& \tilde{s}_2^T \Big\{ K_2 (S_2 \tilde{\sigma}_2 + J_2 (k_2 z_2^1 + z_2^2)) \Big\} + \tilde{s}_2^T (k_2 z_2^2) + \\
& \tilde{s}_2^T \Big\{ -\frac{k_2}{\mu_2} \tilde{s}_2 - \frac{k_1 K_1}{\mu_1} \bar{\sigma}_1 + \frac{k_1}{\mu_1} \tilde{s}_1 + \frac{k_2 K_2}{2\mu_2} \bar{\sigma}_2 + \frac{\eta^*}{2} \Big\}
\end{aligned}
$$

By putting the value of $\bar{\sigma}_1$, above equation can be rewritten as

$$
\begin{aligned}
\Rightarrow \tilde{s}_2^T \dot{\tilde{s}}_2 =& \tilde{s}_2^T K_2 S_2 \tilde{\sigma}_2 + \tilde{s}_2^T K_2 J_2 k_2 z_2^1 + \tilde{s}_2^T K_2 J_2 z_2^2 + \tilde{s}_2^T k_2 z_2^2 \\
& - \tilde{s}_2^T \frac{k_2}{\mu_2} \tilde{s}_2 + \tilde{s}_2^T \frac{k_1}{\mu_1} \tilde{s}_1 \\
\Rightarrow \tilde{s}_2^T \dot{\tilde{s}}_2 \leq & -\frac{k_2}{\mu_2} \|\tilde{s}_2\|^2 + \frac{k_1}{\mu_1} \|\tilde{s}_1\| \|\tilde{s}_2\| + K_2 \lambda_{max}(S_2) \|\tilde{s}_2\| \|\tilde{\sigma}_2\| \\
& + K_2 k_2 \|\tilde{s}_2\| \|z_2^1\| + (K_2 + k_2) \|\tilde{s}_2\| \|z_2^2\|
\end{aligned}
$$

Substituting the value of $\|z_2^2\|$ in above equation, we get

$$
\begin{aligned}
\Rightarrow \tilde{s}_2^T \dot{\tilde{s}}_2 \leq & -\frac{k_2}{\mu_2} \|\tilde{s}_2\|^2 + \frac{k_1}{\mu_1} \|\tilde{s}_1\| \|\tilde{s}_2\| + K_2 \lambda_{max}(S_2) \|\tilde{s}_2\| \|\tilde{\sigma}_2\| \\
& + K_2 k_2 \|\tilde{s}_2\| \|z_2^1\| + (K_2 + k_2) \|\tilde{s}_2\| \Big(-k_2 \|z_2^1\| + \|\tilde{s}_2\| - K_2 \|\tilde{\sigma}_2\| \Big) \\
\Rightarrow \tilde{s}_2^T \dot{\tilde{s}}_2 \leq & -\Big(\frac{k_2}{\mu_2} - (K_2 + k_2) \Big) \|\tilde{s}_2\|^2 + \frac{k_1}{\mu_1} \|\tilde{s}_1\| \|\tilde{s}_2\| - {k_2}^2 \|\tilde{s}_2\| \|z_2^1\| \\
& + K_2 \Big(\lambda_{max}(S_2) - (K_2 + k_2) \Big) \|\tilde{s}_2\| \|\tilde{\sigma}_2\| \qquad (4.61)
\end{aligned}
$$

or

$$
\begin{aligned}
\Rightarrow \tilde{s}_2^T \dot{\tilde{s}}_2 \leq & -\Big(\frac{k_2}{\mu_2} - k_{23} \Big) \|\tilde{s}_2\|^2 + \frac{k_1}{\mu_1} \|\tilde{s}_1\| \|\tilde{s}_2\| - {k_2}^2 \|\tilde{s}_2\| \|z_2^1\| \\
& + K_2 \Big(k_{24} - k_{23} \Big) \|\tilde{s}_2\| \|\tilde{\sigma}_2\| \qquad (4.62)
\end{aligned}
$$

where

$$k_{23} = K_2 + k_2, \qquad k_{24} = \lambda_{max}(S_2)$$

Quadratic Form

Consider the Lyapunov function candidate;

$$\begin{aligned} V =& \frac{1}{2} z_1^T z_1 + \frac{d_1}{\mu_1} \tilde{\sigma}_1^T P_1 \tilde{\sigma}_1 + \frac{e_1}{2} \tilde{s}_1^T \tilde{s}_1 + \\ & \frac{1}{2} z_2^T z_2 + \frac{d_2}{\mu_2} \tilde{\sigma}_2^T P_2 \tilde{\sigma}_2 + \frac{e_2}{2} \tilde{s}_2^T \tilde{s}_2 \end{aligned} \tag{4.63}$$

where d_1, e_1, d_2 and e_2 are positive constants to be chosen. By Calculating $\dot{V}$, we obtain

$$\begin{aligned} \dot{V} =& z_1^T \dot{z}_1 + \frac{d_1}{\mu_1} \Big[\tilde{\sigma}_1^T P_1 \dot{\tilde{\sigma}}_1 + \dot{\tilde{\sigma}}_1^T P_1 \tilde{\sigma}_1 \Big] + e_1 \tilde{s}_1^T \dot{\tilde{s}}_1 + \\ & z_2^T \dot{z}_2 + \frac{d_2}{\mu_2} \Big[\tilde{\sigma}_2^T P_2 \dot{\tilde{\sigma}}_2 + \dot{\tilde{\sigma}}_2^T P_2 \tilde{\sigma}_2 \Big] + e_2 \tilde{s}_2^T \dot{\tilde{s}}_2 \end{aligned} \tag{4.64}$$

Values of the remaining terms in above equation are;

$$z_1^T \dot{z}_1 \leq - k_1 \|z_1\|^2 - K_1 \|\tilde{\sigma}_1\| \|z_1\| + \|z_1\| \|\tilde{s}_1\| \tag{4.65}$$

$$z_2^T \dot{z}_2 \leq - k_2 \|z_2\|^2 - K_2 \|\tilde{\sigma}_2\| \|z_2\| + \|z_2\| \|\tilde{s}_2\| \tag{4.66}$$

$$\begin{aligned} \Rightarrow e_1 \tilde{s}_1^T \dot{\tilde{s}}_1 \leq & - e_1 \Big(\frac{2k_1}{\mu_1} - k_{13} \Big) \|\tilde{s}_1\|^2 + \frac{e_1 k_2}{\mu_2} \|\tilde{s}_2\| \|\tilde{s}_1\| \\ & - e_1 {k_1}^2 \|\tilde{s}_1\| \|z_1^1\| + e_1 K_1 \Big(k_{14} - k_{13} \Big) \|\tilde{s}_1\| \|\tilde{\sigma}_1\| \end{aligned} \tag{4.67}$$

$$\begin{aligned} \Rightarrow e_2 \tilde{s}_2^T \dot{\tilde{s}}_2 \leq & - e_2 \Big(\frac{k_2}{\mu_2} - k_{23} \Big) \|\tilde{s}_2\|^2 + \frac{e_2 k_1}{\mu_1} \|\tilde{s}_1\| \|\tilde{s}_2\| \\ & - e_2 {k_2}^2 \|\tilde{s}_2\| \|z_2^1\| + e_2 K_2 \Big(k_{24} - k_{23} \Big) \|\tilde{s}_2\| \|\tilde{\sigma}_2\| \end{aligned} \tag{4.68}$$

$$\frac{d_1}{\mu_1} \Big[\tilde{\sigma}_1^T P_1 \dot{\tilde{\sigma}}_1 + \dot{\tilde{\sigma}}_1^T P_1 \tilde{\sigma}_1 \Big] \leq - \frac{d_1}{\mu_1} \|\tilde{\sigma}_1\|^2 + \frac{2 d_1 k_{15}}{\mu_1} \|\tilde{\sigma}_1\| \|\tilde{s}_1\| \tag{4.69}$$

$$\frac{d_2}{\mu_2} \Big[\tilde{\sigma}_2^T P_2 \dot{\tilde{\sigma}}_2 + \dot{\tilde{\sigma}}_2^T P_2 \tilde{\sigma}_2 \Big] \leq - \frac{d_2}{\mu_2} \|\tilde{\sigma}_2\|^2 + \frac{2 d_2 k_{25}}{\mu_2} \|\tilde{\sigma}_2\| \|\tilde{s}_2\| \tag{4.70}$$

where,

$k_{15} = \lambda_{max}(P_1)$ and $k_{25} = \lambda_{max}(P_2)$

By putting the values of equations (4.65) - (4.70) in (4.64), it can be seen that the right hand side of the equation can be arranged in the following quadratic form $\Pi = [\|z_1\| \quad \|z_2\| \quad \|\tilde{\sigma}_1\| \quad \|\tilde{\sigma}_2\| \quad \|\tilde{s}_1\| \quad \|\tilde{s}_2\|]^T$;

$$\dot{V} \leq -\Pi^T \Delta \Pi \tag{4.71}$$

where the symmetric matrix Δ has the form

$$\Delta = \begin{bmatrix} k_1 & 0 & \frac{K_1}{2} & 0 & -\frac{k_{16}}{2} & 0 \\ 0 & k_2 & 0 & \frac{K_2}{2} & 0 & -\frac{k_{26}}{2} \\ \frac{K_1}{2} & 0 & \frac{d_1}{\mu_1} & 0 & -\frac{e_1 k_{17}}{2} - \frac{d_1 k_{15}}{\mu_1} & 0 \\ 0 & \frac{K_2}{2} & 0 & \frac{d_2}{\mu_2} & 0 & -\frac{e_2 k_{27}}{2} - \frac{d_2 k_{25}}{\mu_2} \\ -\frac{k_{16}}{2} & 0 & -\frac{e_1 k_{17}}{2} - \frac{d_1 k_{15}}{\mu_1} & 0 & e_1(\frac{2k_1}{\mu_1} - k_{13}) & -(\frac{e_1 k_2}{2\mu_2} + \frac{e_2 k_1}{2\mu_1}) \\ 0 & -\frac{k_{26}}{2} & 0 & -\frac{e_2 k_{27}}{2} - \frac{d_2 k_{25}}{\mu_2} & -(\frac{e_1 k_2}{2\mu_2} + \frac{e_2 k_1}{2\mu_1}) & e_2(\frac{k_2}{\mu_2} - k_{23}) \end{bmatrix}$$

where,

$$k_{16} = (1 - e_1 {k_1}^2), \quad k_{17} = K_1(k_{14} - k_{13})$$
$$k_{26} = (1 - e_2 {k_2}^2), \quad k_{27} = K_2(k_{24} - k_{23})$$

Choosing the appropriate values of d_1, e_1, d_2 and e_2, principal leading minors of Δ can be made positive, thus making $\dot{V}$ negative definite. This implies that the trajectories of the closed loop sub-systems for agent 1 and agent 2, inside the boundary layers, will asymptotically approach their respective invariant manifolds M_{μ_1} and M_{μ_2} as $t \to \infty$.

4.2.4 Simulation

Following numerical values are used to obtain the simulation results of Three Agent System.

- $\beta = 1\text{rad/sec}$, $k_i^1 = 15$ and $\mu = 0.1$
- In the first and third control design K_i^1 is chosen to assign the eigenvalues of $(S_i - J_i K_i^1)$ at $-1 \pm i$.
- For conventional servocompensator, values of K_i^1 and k_i^1 are chosen to assign the eigenvalues of ψ_i at $-1 \pm i$ and -1.

Figure 4.2 and 4.3 shows the results of the simulation for two agents in which the improvement in transient performance using conditional servocompensator can be clearly seen while converging the asymptotic error to zero steady-state.

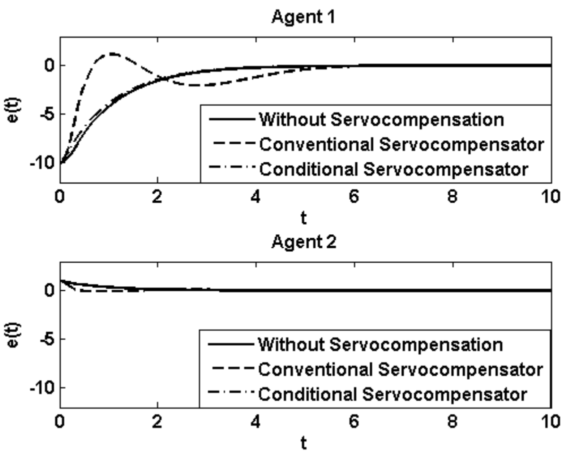

Figure 4.2: Error in x-axis for three agent system in directed graph topology

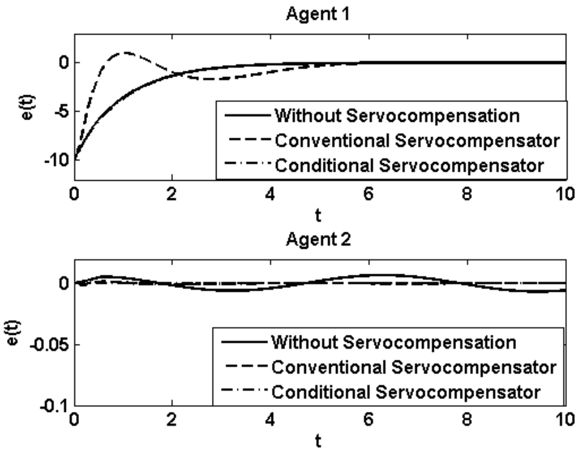

Figure 4.3: Error in y-axis for three agent system in directed graph topology

4.3 THREE AGENT SYSTEM IN UNDIRECTED GRAPH TOPOLOGY

As undirected graph topology can be considered as a special case of directed graph topology because exogenous signal is accessible to all the agents in a system. Dynamics of the system (agents and leader) are same as mentioned in (4.8) and (4.9). Network topology of three agent system in undirected graph topology is shown in figure 4.4. Node 0 represents the position of the leader. Let the required fixed relative position (with respect to the leader) for the agents be $r_1 = [\sqrt{3}, 3]^T$ and $r_2 = [-\sqrt{3}, 3]^T$

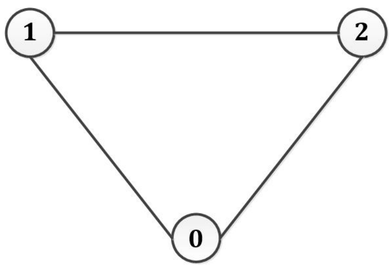

Figure 4.4: Network topology for undirected graph

For this formation control problem, we can define the generalized error using

(4.7). Using equations (4.10), it can easily be verified that,

$$c_i(\eta) = -\beta^2 \begin{bmatrix} \\ \eta_1 \\ \\ \\ \eta_3 \end{bmatrix}$$

and that $c_i(\eta)$ satisfies the identity $L_s^q c_i(\eta) = a_0 c_i(\eta) + a_1 L_s c_i(\eta) + \cdots + a_{q-1} L_s^{q-1} c_i(\eta)$ with $q = 2,\ a_0 = -\beta^2,\ a_1 = 0$. The performance of three designs are shown here. In the first design, surface is $s_i = k_i^1 z_i^1 + z_i^2$ and in the last two designs silding surface is $s_i = K_i^1 \sigma_i + k_i^1 e_i^1 + e_i^2$. For conventional servocompensator design K_i^1 and k_i^1 are chosen to make ψ_i Hurwitz.

Numerical values used in the simulation are:

- $\beta = 1$rad/sec, $k = 15$ and $\mu = 0.1$
- In first and last design where K_1^i chosen to assign the eigenvalues of $(S_i - J_i K_1^i)$ at $-1 \pm i$ whereas k_1^i is chosen as any positive constant.
- For conventional servocompensator, values of K_1^i and k_1^i are chosen to assign the eigenvalues of ψ_h^i at $-1 \pm i$ and -1.

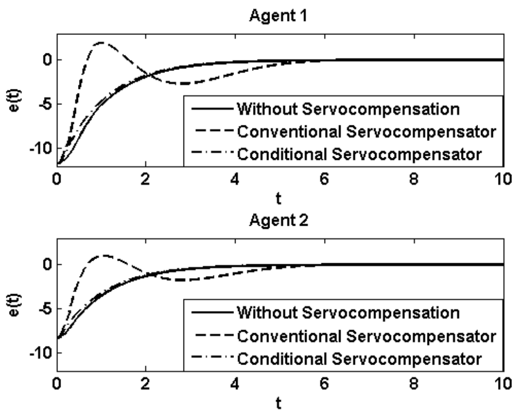

Figure 4.5: Error in x-axis for three agent system in undirected graph topology

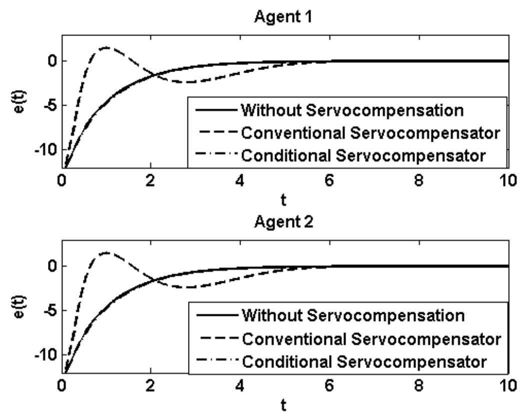

Figure 4.6: Error in y-axis for three agent system in undirected graph topology

Figures 4.5 and 4.6 show the results of the simulation for two agents following a leader in which the improvement in transient performance using conditional servocompensator can be clearly seen while converging the asymptotic error to zero steady-state. Response of the system using conditional servocompensator for undirected graph topology is shown in figure 4.7

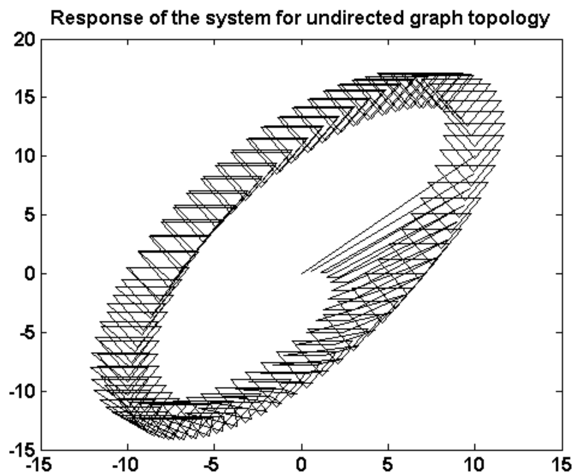

Figure 4.7: System response for undirected graph topology for first 15 seconds

Chapter 5

Conclusions

An approach of regulating the non linear systems using conditional servocompensators have been extended and applied to a class of multi-agent system. The approach is special in this regard that it provides conditional servocompensation that is servocompensation only inside the boundary layer of continuous sliding mode control for all agents following a leader (exosystem). The technique has been designed and analyzed for a Three Agent System to show that the technique is independent of the directed or undirected network topology. In directed network topology case, an agent computes its control signal using the information received from neighboring nodes whereas in undirected network topology, the agent computes its control signal depending upon the information received from the leader or exosystem. Results are shown analytically for asymptotic tracking to show the improvement in transient performance of all agents directly or undirectly connected to the leader. It is also shown that the steady state error (relative position of each agent in this case) converges while tracking the signal generated by exosystem is also converging for all the agents present in a system.

The proposed approach is then specialized for a Three Agent System in directed and undirected network graph topologies. Closed-loop analysis under the proposed control scheme for the multi-agent systems is provided. Simulation results in the form of output trajectories of individual agents and error convergence are provided to illustrate the discussed approach. Extension of this work includes development of a general approach applicable to all network topologies.

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