OUTPUT FEEDBACK NONLINEAR MODEL PREDICTIVE CONTROL FOR A CLASS OF NONLINEAR SYSTEMS

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"Recite, and your Lord is the most Generous- Who taught by the pen- Taught men that which he knew not." (The Qur'an, Al-'Alaq 30.3-5)

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Muhammad Noman Hasan

DEDICATION

"To my Parents and my wife"

ABSTRACT

The main focus of this thesis is on the implementation of an output feedback control scheme with a nonlinear model predictive controller (NMPC) for a class of nonlinear systems. The class of the nonlinear system may include nonminimum phase systems structured in standard normal form having relative degree ρ and the corresponding internal system has full relative degree. The sample data NMPC scheme is applied as feedback controller. The observer proposed involves an extended high gain observer (EHGO) to estimate the output derivatives in combination with a high gain observer (HGO) to estimate the internal states. The main contribution of this thesis is the framework for the semi global practical stability of the proposed output feedback scheme. The necessary theoretical foundation has been laid to prove and accomplish the practical stability results of output feedback control. It is shown for the output feedback scheme that there exists an observer parameter and a sampling time such that for any desired region which is subset of the state feedback controller's region of attraction and for any small area around the origin that contains the origin, the trajectories starting in that subset will converge to that small area around the origin in finite time. With the aid of simulations, it is shown for the example of single link flexible joint manipulator system that the output feedback state trajectories converge to state feedback trajectories in relatively short time when the observer and NMPC controller parameters are set in accordance with the mentioned stability conditions.

Keywords: NMPC, EHGO, HGO, Nonminimum Phase Systems, Nonlinear Systems, Observer.

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Chapter 1
INTRODUCTION

1.1 Motivation

In the existing world, optimization of the process is considered as one of the fundamental objective of control engineers to incorporate efficient use of resources and to curtail the energy utilization and cost. In achieving this, many constraints should be consider in designing process control such as energy, quality, legal and safety requirements. In recent years, control strategies based on mathematical model of the plant have been successfully implemented to deal with these requirements as they can also equally incorporate the MIMO plants in the presence of constraints on control effort as well on process states.

Among these, one of the most popular control strategies is Model Predictive Control (MPC). In Model Predictive control, the mathematical model of the plant is used to predict the future behavior of the plant over a prediction horizon. It involves solution of the optimization problem to minimize the pre-defined cost function and to compute future control inputs over that prediction horizon. As the mathematical model of the plant is the main core of this controller, the degree of accuracy of mathematical model of plant greatly determine the performance of controller.

Generally, one can easily recognize the difference between linear and nonlinear model predictive control (NMPC). Linear Model Predictive Control refers to those MPC schemes in which linear mathematical plant models is used to predict the system's future performance. It incorporates linear constraints on the system states and control inputs and a quadratic pre-defined cost function which is to be curtailed. Even if the mathematical plant model is linear, the closed-loop dynamics are in general nonlinear due to the presence of constraints. Nonlinear Model Predictive Control refers to those MPC schemes that are based on nonlinear mathematical plant models and/or general nonlinear constraints on the states and inputs. During the last decades, the Linear MPC is widely used in industries [1, 2, 3, 4 and 5]. The NMPC also gained much interest of engineers in the last 15 years [2, 6, 7, 8, 9, 10, 11, 12 and 13].

In many circumstances, all the system states are not measured or measurable due to hardware or financial constraints. These system states are then estimated by an efficient observer. Such arrangements in which the states are estimated by observer are referred as Output feedback. Output Feedback NMPC gained a lot of interest of researches. NMPC scheme employing moving horizon observer is discussed in [14] and its closed loop semi global stability is proved in [15]. The local asymptotic stability of NMPC with sampled estimation of states is proved in [16]. The stability of weakly detectable discrete time systems with NMPC is presented in [17, 18 and 19]. The practical semi global stability for instantaneous NMPC with high gain observer is accomplished in [20]. The results are further illustrated for single input single output (SISO) system sampled data NMPC with high gain observer in [21] and for multi input multi output (MIMO) systems in [22]. The research work presented in this thesis is greatly inspired by the work of Boker and Findeisen presented in [21 and 22]. The results are primarily based on general nonlinear separation principle presented in [24, 25]. It is assumed in this thesis that a state feedback NMPC controller is available that asymptotically stabilizes the system and thus NMPC stability schemes are not discussed here. Some of these schemes are quasi infinite horizon NMPC (QIH-NMPC) [9], stability using control Lyapunov function [26] and zero terminal constraint NMPC [27]. Further insight on stability of state feedback NMPC is presented in [6, 7 and 8].

Presently, the high gain observers are widely used in a variety of control applications due to the numeral characteristics provided by them which might not be provided by other observers [23]. The major characteristic lies in the simplicity of their design by avoiding complex gain formulas and solution of partial differential equations linear matrix inequality (LMI). In addition to it, output feedback involving high gain observers can fully recover the state feedback performance if correctly tuned. Another important feature of high gain observers is the robustness in the estimation of states in the presence of model uncertainties and disturbances. The output feedback control of nonminimum phase system using extended high gain and high gain observer is presented in [23].

1.2 Organization of Thesis

The organization of this thesis is as follows.

In chapter 2, a brief background of nonlinear observers is presented with the explanation of high gain observers (HGO) and extended high gain observers (EHGO). A brief overview on output feedback control of nonlinear nonminimum phase system is presented.

In chapter 3, the history, basic principle mathematical formulation, nominal stability along with the advantages and disadvantages of Nonlinear Model Predictive Control (NMPC) is presented. It is assumed in the chapter that all the systems states are measured hence the NMPC is referred as State feedback NMPC. The NMPC controller presented is simulated on an example of nonlinear nonminimum phase single link flexible joint manipulator system and its stabilization at the origin is shown.

Chapter 4 constitutes the core of this thesis in which the output feedback nonlinear model predictive control of nonlinear nonminimum phases system is presented. The proposed observer formulation is presented and the semi global practical stabilization of output feedback scheme with NMPC controller is proved. The output feedback controller is then simulated on the same example of single link flexible joint manipulator system. The convergence of output feedback trajectories to those of state feedback is achieved and shown.

In chapter 5, the conclusion of the thesis along with the areas for the future work and advancements is presented.

Chapter 2
BRIEF BACKGROUND

2.1 Non minimum Phase Systems

After the advancement in the theory of the minimum phase systems, nonminimum phase systems gained a lot of interest and attention of control engineers. This is due to the fact that a number of physical systems by nature lies in the category of nonminimum phase system that have unstable internal or zero dynamics. Some examples of these systems are electromechanical systems, chemical reactors, flexible joint manipulator system, inverted pendulum, under actuated systems and sideways motion of aircraft and ships.
Some dominant features of nonminimum phase systems are:

- Inverse step response.
- Time delay.
- Instability of internal states.
- Bandwidth limitations.

2.2 Observers for nonlinear systems

A lot of work has been done and published in literature on linear observers. But the case is not similar for the nonlinear observers. In actual a uniform formulation for nonlinear observers is yet to emerge [23]. Additionally, the enlargement of region of attraction of stability of observer is one of the main obstacles. A number of different approaches are present in literature. The earliest methodology relies on the Leunberger observers [28] and Kalman filters [29 and 30] for nonlinear systems. These involve linearization of nonlinear systems to implement linear observers which results in local stability results which is a major drawback. The second method involves state transformation that results in the linearization of error dynamics so as to make the nonlinearities dependent on input and output [31, 32, and 34]. The third method involves the use of LMI techniques [34, 35 and 36]. The forth method involves the use of sliding mode observers [37] and high gain observers [38]. These observers depend on the systems that are in standard normal form and gained a lot of fame because of its robustness properties. The nonlinear observers are not limited to these approaches. For example observer based on extended Kalman filter and high gain observer is presented in [39] and some others in [40 and 41].

2.2.1 Extended High Gain and High Gain Observers

High gain observer is the major element of this thesis and the basic idea of high gain observer is presented here. Consider a following nonlinear system

$$\dot{z}_1 = z_2 \tag{2.1}$$

$$\dot{z}_2 = f(z_1, z_2) + u \tag{2.2}$$

$$y = z_1 \tag{2.3}$$

where u is the input and y is the output. The function $f(z_1, z_2)$ is continuous and could be unknown which satisfies

$$f(0,0) = 0$$

The high gain observer for above system can be given as,

$$\dot{\hat{z}}_1 = \hat{z}_2 + \frac{\alpha_1}{e}(y - \hat{z}_1) \tag{2.4}$$

$$\dot{\hat{z}}_2 = f(\hat{z}_1, \hat{z}_2) + u + \frac{\alpha_2}{e^2}(y - \hat{z}_1) \tag{2.5}$$

The constants α_1 and α_2 are chosen such that the roots of

$$s^2 + \alpha_1 s + \alpha_2 = 0$$

are negative and e is the small observer parameter.

Additionally one can employ the high gain observers to estimate the output signal and its derivatives [42]. So it can also estimate the right side of equation (2.2) and consequently can also estimate the unknown function f. Such an observer is referred as Extended High

Gain Observer (EHGO). It increases the dimension of observer by adding an extra state variable σ.

Let,

$$\sigma = f(z_1, z_2)$$

Now the observer becomes,

$$\dot{z}_1 = z_2$$

$$\dot{z}_2 = \sigma + u$$

$$\dot{\sigma} = f_1(z_1, z_2, u)$$

where,

$$f_1 = \frac{df}{dt} = \frac{\partial f}{\partial z_1} z_2 + \frac{\partial f}{\partial z_2}(f(z_1, z_2) + u)$$

So, the extended high gain observer becomes,

$$\dot{\hat{z}}_1 = \hat{z}_2 + \frac{\alpha_1}{e}(y - \hat{z}_1)$$

$$\dot{\hat{z}}_2 = \hat{\sigma} + u + \frac{\alpha_2}{e^2}(y - \hat{z}_1)$$

$$\dot{\hat{\sigma}} = f_1(\hat{z}_1, \hat{z}_2, u) + \frac{\alpha_3}{e^3}(y - \hat{z}_1)$$

Also

$$\hat{f} = \hat{\sigma}$$

It is also important to mention that the effect of function f_1 is attenuated as e approaches to zero. So the observer can neglect the unknown function f_1 and can accomplish the estimation error of order $O(e)$.

High gain observer evolves as a popular observer in nonlinear literature. A comprehensive survey over the last two decades on its development is presented in [43]. Previously high gain observers were restricted to minimum phase systems as in [44] where the open loop observer is employed for the internal dynamics. In [39] high gain observer in combination with the extended Kalman filter is implemented for minimum phase system to estimate the states of the system having linear internal dynamics that are driven by output signal.

On the other hand, the extended high gain observer is used for variety of objectives in the literature. One of the objectives is to estimate the uncertainties in the mathematical model represented by unknown signals or external inputs and thus enabling the controller to cancel the effect of these [45]. Similar methodology is adopted in [46] in which the match uncertainties are estimated by a full order high gain observer and then canceled by controller. A remarkable feature of desired transient performance by the nonlinear control in the presence disturbances and matched model uncertainties was accomplished in [46, 47 and 48]. Extended high gain observers also assist in the development of the switching control strategy methods relying on Lyapunov function [48, 49]. In [50] the extended high gain observer is used for the nonminimum phase system to estimate the signal which is observable to internal dynamics which allows the designing of a stabilizing controller. In [23] a combination of extended high gain observer and a high gain observer is presented. The extended high gain observer estimates the observable signal for the internal dynamics and a high gain observer is used to estimate the full order nonlinear unstable internal dynamics.

2.3 Control of Observer Based Nonminimum Phase Systems

The biggest hurdle in designing the observer for nonlinear systems is the absence of general separation principle for nonlinear systems. In contrast to the linear case in which the separation principle guarantees the global stabilization of output feedback control when the states in the state feedback control are replaced by their estimates. Many of the observers design reported in [24, 51 and 52] are not usable for nonlinear observer design. High gain observer is one of the observers that fulfill this principle.

The earliest result is presented by Isidori in [53] for control of nonminimum phase system in which the semi global stabilization for a class of nonlinear nonminimum phase system is proved. It was assumed in [53] that a stabilizing controller is present for the auxiliary system. Similarly the robust semi global stabilization of output feedback control involving extended high gain observer in presented in [50]. This shows the prospective of extended high gain observer to be used as a substitute of high gain feedback scheme presented in [53]. A successful semi global stabilization of output feedback control for a class of nonlinear nonminimum phase system in which the internal dynamics has full relative degree is shown in [23].

Chapter 3
STATE FEEDBACK NONLINEAR MODEL PREDICTIVE CONTROL (NMPC)

3.1 Introduction

In many control applications, the key idea is to implement such a stabilizing feedback control law which satisfies the constraints on states and control input and to minimize the performance criterion. The ideal solution for this is to find a closed solution for feedback control law which optimizes the performance of the system keeping the solutions within the constraints on input and states. As the optimal feedback control law involves the solution of Hamilton Jacobi-Bellman partial differential equations. Even for the unconstrained case, it cannot be obtained analytically. One way of avoiding such problem is to obtain repeated open loop optimal solution for a specified state. Only the first element of the obtained open loop control input is applied. This process is then repeated again. . The control techniques which employ such scheme are discussed as Model Predictive Control (MPC), receding or moving horizon control. The Model predictive control that utilizes and incorporates the nonlinear model of the system is referred as Nonlinear Model Predictive Control (NMPC).

3.2 History of Nonlinear Model Predictive Control

This section gives the brief history of NMPC in the light of literature references explaining the NMPC control technique. The model predictive control technique emerges in the middle of 20^{th} century from the optimal control theory with inspirational contributions by researchers. Some of which includes the work done by Pontryagin, Boltyanskii, Gamkrelidze, Mishchenko and Bellman [54 and 55] on maximum principle and the dynamic programming method. In 1960's, the first research paper exploiting the idea of model predictive control was presented by Propoi [56] for discrete time linear systems. It is worthy to note that to solve optimal control problem in this paper neither the dynamic problem nor the maximum principle of Pontryagin is used. Instead, the paper proposed a method which is very popular in NMPC nowadays. It transformed the optimal control problem into linear static optimization problem. This idea for nonlinear systems is presented in the book by Lee and Markus.

To achieve feedback controller from the knowledge of open loop controller requires the measurement of control process state and compute the open loop controller function fast enough. The first element of the control action is applied for the short interval (sampling time). The process state is re measured at the next sampling time and the open loop controller function is recalculated with this new state. This procedure is then repeated again. Due to unavailability of software and the computer hardware for fast enough computations at that time, this control technique had little practical impact.

As the progress and development in solving constrained linear algorithms and quadratic optimization problems was attained in the later 1970's. The linear MPC became much popular control technique. This method was proposed by Richalet, Rault, Testud and Papon [57] and Cutler and Ramaker [58] for rather slow systems so that online optimization is done even with the technology available at that time. The MPC technique was implemented for years without having strong theoretical background and Stability proofs which later become the part of literature. Some early results for MPC can be found in the Survey paper [3]. Many of the NMPC useful practices like stabilizing terminal constraints and stability proofs based on Lyapunov functions was first developed for Linear MPC and later developed for nonlinear systems.

The article by Chen and Shaw [59] is among the earliest paper that analyzes the NMPC technique similar to the one used today. In this paper, Lyapunov function technique is used in continuous time to prove the stability of NMPC with equilibrium terminal constraints. However, the paper proposed the application of whole optimal control function on the optimization horizon rather than the only first element (receding horizon) as in today's NMPC model. On today's NMPC model, Keerthi and Gilbert [60] gave the ample stability analysis for NMPC with equilibrium terminal constraints for discrete and systems and Mayne and Michalska [27] for continuous time systems. For nonlinear systems, the equilibrium terminal constraints impose severe numerical complications. This prompts the researchers for the alternative NMPC methods. The combination of suitable terminal and regional terminal constraints showed rapid development in the later 1990's. This forms the basis of self-evident stability framework for NMPC with stabilizing terminal constraints. For discrete time systems, the survey paper by Mayne [6] outline the comprehensive information on history of different NMPC schemes which are not mentioned here.

3.3 Model Predictive Control

In the past years, the model predictive control (MPC) has gained significant attention. MPC also refers as moving horizon control comprise of class of algorithms that uses mathematical model, of the process to be controlled, to predict the optimum future behavior of the process. One of the main reasons of its application in industries is the ease of handling constraints with in the controller formulation with optimum control effort. It is worthy to mention that despite of lack of theoretical results related to stability and robustness in the beginning years, it gained a lot of attention and popularity. The theoretical basis and results of this control technique emerged almost after 15 years of its appearance in industries [61]. Model predictive control, which had originally its applications in power plants and petroleum industries, is currently successfully applied to a variety of processes not only in process industries but also to Biomedical and automotive applications. Till date, many publications related to theoretical and practical issues of MPC have been reported. See e.g. the books [62, 63 and 64] and the survey papers [2, 4, 6, 10, 13 and 65].

The major reason of the success of the MPC is that it addresses the control problem intuitively. The sets of algorithms that estimate (predict) the future behavior of the plant by using the mathematical model of the plant.

The main ingredients of model predictive control are:

- The mathematical model of plant.
- The performance index (cost function) which is to be minimized that reflects the applied control effort and the error between the actual state and desired state value.
- Optimization algorithm that estimates the future control effort subject to constraints on input and states by minimizing the performance index.
- Receding horizon strategy in which only the first element of the optimum control sequence is applied to the system.

In model predictive control, the optimal control input applied to the system is obtained by the repeated solution of a (finite) horizon open-loop optimal control problem (FHOCP) subject to the dynamics of the system, state and input constraints. At each sampling instant 'δ', using the current (measured) state value 'x' as the initial value of the process, a finite horizon optimal control problem (FHOCP) is solved over a prediction horizon 'T_p' and for the control horizon 'T_c' to predict the future dynamic behavior of the system. This optimization which is online takes account of the system dynamics, constraints (both on control input and states) such that the open loop performance objective function is minimized. This optimization results in an optimal control sequence.

If it is assumed that the prediction horizon is infinite and the system model is exactly known (there is no system model mismatch) and in the absence of any external disturbance, we can apply the open loop control input to the system and under certain further assumptions we can achieve the state convergence of the system to the origin. However, as it is impossible to find a perfect model and due to presence of disturbance and the use of finite prediction horizons (due to calculation problems) instead of infinite horizons, the predicted and the actual system states differ which may results in poor performance of the controller. To overcome and counteract this difference the feedback is introduced. The feedback is achieved by applying the first element of the control sequence to the system which is for the current time and the remaining sequence is discarded. The optimization problem is recalculated at the next sampling instant and the new optimal control input is applied to the system for that sampling instant. This process is repeated at every sampling instant. This strategy is referred as Receding Horizon Control (RHC). Figure 3.1 shows the pictorial illustration of the model predictive control strategy.

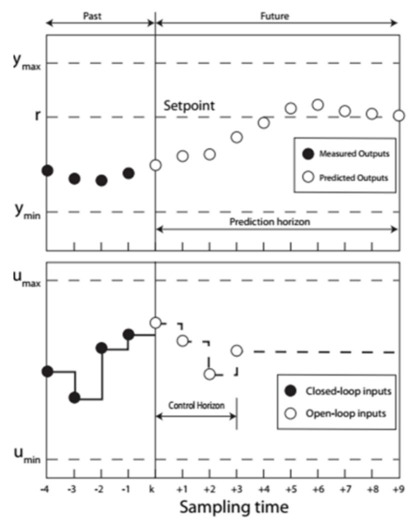

Figure 3.1: Basic principle of Model Predictive Control

3.3.1 Mathematical Formulation

Suppose we have a following nonlinear system

$$\dot{x} = f\big(x(t), u(t)\big), \qquad x(0) \in \mathcal{X}_0 \tag{3.1}$$

with input and state constraints

$$u(t) \in \mathcal{U}, \qquad \forall t \geq 0$$

$$x(t) \in \mathcal{X}, \qquad \forall t \geq 0$$

where $x(t) \in \mathcal{X} \subseteq \mathcal{R}^n$ are the states of the system and $u(t) \in \mathcal{U} \subset \mathcal{R}^m$ is the control input applied to the system. The set $\mathcal{U}$ denotes the set of all possible admissible inputs that can be applied to the system. The set $\mathcal{X}$ denotes the set of feasible states of the system and the initial states of the system is denoted by the set $\mathcal{X}_0 \subseteq \mathcal{R}^n$. It is assumed that the function f is continuous and locally Lipschitz in its arguments with $f : \mathcal{R}^n \times \mathcal{R}^m \to \mathcal{R}^n$ and it satisfies $f(0,0) = 0$. It is also assumed that the set $\mathcal{X} \subseteq \mathcal{R}^n$ is simply connected and the set $\mathcal{U} \subset \mathcal{R}^m$ is compact. The initial conditions set $\mathcal{X}_0 \subseteq \mathcal{X}$ and the origin is contained in the set $(0,0) \in \mathcal{X} \times \mathcal{U}$ and it is stationary point for the system (3.1). Typically the sets $\mathcal{X}$ and $\mathcal{U}$ are convex of the form,

$$\mathcal{X} := \{x \in \mathcal{R}^n | x_{min} \leq x \leq x_{max}\}$$

$$\mathcal{U} := \{u \in \mathcal{R}^m | u_{min} \leq u \leq u_{max}\}$$

where x_{min}, x_{max} , u_{min} and u_{max} are constant vectors.

Furthermore, the solution of (3.1) (if the solution exists) for the time $t_1$with the system state $x(t_1)$ with the application of input $u : [t_1, t_2] \to \mathcal{R}^m$ is denoted by

$$x\big(\tau; x(t_1), u(\cdot)\big), \tau \in [t_1, t_2],$$

$$x\big(\tau; x(t_1), u(\cdot)\big) = x(t_1) + \int_{t_1}^{\tau} f\big(x(p), u(p)\big) dp \qquad \forall t \in [t_1, t_2]$$

In NMPC, the feedback is achieved by application of repeated optimal control input at every sampling instant by solving open loop optimal control program. The open loop optimal control program is obtained by,

$$\min_{\bar{u}(\cdot)} J\big(\bar{x}(\cdot), \bar{u}(\cdot)\big) \tag{3.2a}$$

Subject to

$$\dot{\bar{x}}(\tau) = f\big(\bar{x}(\tau), u(\tau)\big), \qquad \bar{x}(t) = x(t) \tag{3.2b}$$

$$\bar{u}(\tau) \in \mathcal{U}, \quad \tau \in \big[t, t + T_p\big] \tag{3.2c}$$

$$\bar{x}(\tau) \in \mathcal{X}, \quad \tau \in \big[t, t + T_p\big] \tag{3.2d}$$

$$\bar{x}\big(t + T_p\big) \in \Omega \tag{3.2e}$$

Here, the bar denotes the internal variables of the controller. This distinction in the variables notation is necessary as the predicted values and the actual close loop system values are different even in the nominal case. The variation in values is due to finite horizon. The cost function, defined over the prediction horizon T_p in terms of stage cost S and terminal penalty term E_f, is given as,

$$J\big(\bar{x}(\cdot), \bar{u}(\cdot)\big) = \int_{t}^{t+T_p} S\big(x(\tau), u(\tau)\big) d\tau + E_f\left(x\big(t + T_p\big)\right) \tag{3.2f}$$

The stage cost S reflects the economical, energy or safety concerns. The stage cost is often chosen as the quadratic.

$$S(x,u) = x^T Qx + u^T Ru$$

Where the weighing matrix Q is the positive definite matrix $Q > 0$ and R is semi definite matrix $R < 0$. The term E_f which might or might not be present in the cost function along with (3.2e) which is terminal region constraint is included in the optimal control problem to enforce stability of the NMPC controller. The optimal solution of the problem (3.2) denoted by $\bar{u}^*$, which is obtained from the dynamic optimization problem. This optimal control input which is obtained at the time interval t_i is applied open loop till the next sampling interval $t_i + \delta$.

$$u\big(t, x(t_i)\big) = \bar{u}^*\big(t, x(t_i)\big), \quad \forall t \in [t_i, t_i + \delta]$$

The optimal control input $u := u\big(x(t)\big) = u\big(t, x(t_i)\big)$ is feedback input as it is calculated at each sampling interval with new measurement of states obtained.

Assumption 3.1: For an acceptable control input function $u(\cdot) : [0, T_p] \to \mathcal{U}$ and for all initial conditions of system states in the region of attraction $\mathfrak{R}$, the nonlinear system (3.1) has the unique continuous solution.

The optimal solution of (3.2) obtained at time t_i for the system states $x(t_i)$ with optimal control input u between sampling interval is given by,

$$x\big(\tau; u(\cdot), x(t_i)\big) \qquad \forall \tau \in [t_i, t_i + \delta]$$

It is important to mention that the optimal control must be admissible according to performance specification. The admissible input is defined as,

Definition 3.1: (Admissible Input) [13]

An optimal input $u: [0, T_p] \to \mathcal{R}^m$ obtained from the optimization problem is considered as admissible input for the state x_i if:

- It is piecewise continuous
- $u(\tau) \in \mathcal{U}, \quad \forall \tau \in [0, T_p]$
- $x(\tau; u(\cdot), x_i) \in \mathcal{X}, \quad \forall \tau \in [0, T_p]$
- $x\big(T_p; u(\cdot), x_i\big) \in \Omega$

Assumption 3.2: In the region of attraction $\Re$, the optimal control effort $u(\tau; x)$ is locally Lipschitz in the system states $x(t)$.

$$\|u(\tau; x_1) - u(x_2)\| \leq L_u \|x_1 - x_2\|, \quad \forall \tau \in [0, T_p)$$

This assumption reflects that we obtain two "close" optimal control input trajectories for two "close" initial conditions. Furthermore, the value function which is the optimal cost of (3.2) is define by,

Definition 3.2: (Value Function)

The minimal value of the cost for a given state x is referred as the value function $V(x(t))$ for the open loop optimization problem (3.2). The value function is given by,

$$V(x(t)) = J\left(x\left(\cdot; x(t), \bar{u}^*(\cdot; x(t))\right), \bar{u}^*(\cdot; x(t))\right)$$

The value function is very important as it plays a vital role in the stability investigation of the NMPC. The value function often served as Lyapunov function candidate [6 and 7]. The decrease in value function with the passage of time reflects the stability of NMPC.

Proposition 3.1: [66] For all the system states in the region of attraction, the value function $V(x) = V(x(t))$ is locally Lipschitz.

Lemma 3.1: The value function $V(x)$ has following properties

- $V(x)$ is positive definite and has zero value for $x = 0$

$$V(0) = 0; \quad V(x) > 0, \quad \forall x \neq 0$$

- For any system state in the region of attraction $\Re$ and for admissible optimal control input u, the difference in value function along the trajectories from a given starting point t_0 is given as,

$$V(x(t_1)) - V(x(t_2)) \leq -\int_{t_1}^{t_2} S(x(\tau), u(\tau; x_1))\, d\tau, \quad t_0 \leq t_1 \leq t_2 \leq \infty$$

There exist two major NMPC versions depending on the frequency of calculation of optimal control problem (3.2). If the open loop optimal control problem is solved at every instant, it is referred to as Instantaneous NMPC [6 and 27]. However if it is solved at certain time intervals and the resulting optimal control input is applied between the intervals, then it is referred as Sampled Data NMPC. This thesis relies on the sampled data NMPC technique. The different versions of NMPC scheme reported in literature is not the scope of thesis. For further details on different NMPC techniques see [67].

3.4 Nominal Stability of NMPC

Due to the use of finite horizon, the questions arise regarding the guarantee of stability of the closed loop. This is mainly due to difference in the trajectories of the predicted and actual closed loop trajectories. This is due to the fact that the infinite horizon cannot be approximated to finite horizon without altering the optimization problem. Then it is a natural need to adopt such an NMPC technique which guarantees stability of the closed loop irrespective of the choice of parameters and approximate the infinite horizon NMPC arrangement as close as possible. Such an NMPC approach which guarantees stability regardless of the choice of parameters is referred to as NMPC with guaranteed stability. There are different approaches which guarantees the stability of the closed loop are exist. The basic idea of some technique is presented here with simple illustration and most of the technical detail is not included. It is important to mention that not all the techniques have been discussed here, as it is out of scope of this thesis. For further insight on these, see [67].

Furthermore, it is also assumed that the origin $x = 0, u = 0$ is the steady state point which is to be stabilized.

3.4.1 NMPC with Infinite Horizon

It is perhaps the most basic way of achieving the close loop stability by setting the prediction horizon $T_p = \infty$ in (3.2f). In this setting, the open loop input and the state values obtained from the solution of the optimization problem (3.2) of NMPC are equal to nonlinear closed loop trajectories. This is due to Bellman's Optimality principle [54]. In the light of this, at the next sampling interval, the remaining portion of the trajectories is also optimal. This is due to the fact that the end part of optimal trajectory is also optimal. This entails the convergence of the trajectories of the closed loop. The further details and derivations can be found in the references [6, 27, 60 and 68].

3.4.2 Guaranteed stability with Finite Horizon NMPC

Different schemes that guarantees the closed loop stability with finite horizon exists. Infact most of such techniques modify the standard NMPC optimization problem, independently of system and performance specifications, to guarantee the stability of the closed loop. This is done by adding appropriate penalty terms and equality and/or inequality constraints in the standard optimization problem. The sole purpose of these addition is due to the enforce stability and it has nothing to do with the system and its performance requirements. So, these constraints are also referred as “Stability constraints”. One possibility of such stability constraint which enforces the stability is the zero terminal equality constraint. This forces the trajectories at the end of each prediction horizon to origin, i.e.

$$\bar{x}\left(t+T_p\right)=0$$

The zero terminal constraint is added to the standard optimization problem (3.2) [27, 59, 60 and 69]. This yields the stability of the closed loop if its solution is feasible at time $t=0$. As in the case of infinite horizon, if the solution is feasible at one time interval, it implies the solution to be feasible at following time intervals and will also decrease the value function. The major disadvantage of the zero terminal constraint is that it always forces the trajectories at the end of each interval to reach the origin. This may cause the feasibility problems if the prediction horizon length is short enough. Furthermore, the exact achievement of zero terminal constraint requires infinite number of iterations in the computations which is undesirable. However its advantage lies in the simplicity of its concept and rather straight forward application.

There are several schemes which try to overcome the use of zero terminal constraint. Infact they employ terminal region stability constraint rather than zero terminal constraint. In this setting, at the end of each interval the trajectories are forced to lie in a small and compact set around the origin. This relaxes the computational problem of enforcing the trajectories to exact zero and it is more practically realizable. The terminal region constraint is of the form,

$$\bar{x}\left(t+T_p\right)\in\Omega$$

This constraint together with terminal penalty term E_f enforces the stability and feasibility of the closed loop. The penalty term E_f penalized the deviation of the origin and the final predicted state. Usually the terminal region set Ω and terminal penalty term E_f are computed offline which gives an upper bound on the infinite horizon cost.

$$J(\bar{x}(\cdot), \bar{u}(\cdot)) = \int_t^{t+T_p} S(x(\tau), u(\tau))d\tau + E_f\left(x(t+T_p)\right)$$

This also ensures the decrease in the value function with the passage of time. If the terminal region Ω, terminal penalty term E_f and prediction horizon T_p are chosen appropriately, the stability and feasibility of the closed loop is guaranteed. Similar stability setups are discussed in [6 and 7]. The details of these setups are not provided here. The stability setup similar to [13] which is slight modification of [70, 71 and 72] is shown here.

Theorem 3.1: [21] (stability of finite horizon NMPC)

If we suppose,

- The terminal region Ω contains the origin and it is closed, i.e.

$$0 \in \Omega, \quad \Omega \subseteq \mathcal{X}$$

  The terminal penalty term E_f is positive semi definite, $E_f(x) \in C^1$.

- The terminal region is positively invariant. The terminal region Ω and penalty term E_f is set in a way that for all system states in the set Ω, there exist an admissible input $u_\Omega: [0, \delta] \to \mathcal{U}$ such that the states $x(\tau)$ remain in Ω in that time interval, i.e.

$$x(\tau) \in \Omega, \quad \forall \tau \in [0, \delta]$$

and

$$\frac{\partial E_f}{\partial x} f\left(x(\tau), u_\Omega(\tau)\right) + S\left(x(\tau), u_\Omega(\tau)\right) \leq 0, \quad \forall \tau \in [0, \delta]$$

- At time $t = 0$, the open loop finite horizon optimal control problem has a feasible solution.

If above assumptions are satisfied. Then for a given sampling interval δ, the trajectories of closed loop system $x(t)$ with optimal input u converge to origin having the region of attraction $\Re$. The set $\Re$ contains the possible system states for which it has feasible open loop solution.

$$x(t) \to 0 \quad t \to \infty$$

The proof of this theorem can be found in [13].

3.5 NMPC Advantages & Disadvantages

The performance of NMPC depends on the prediction horizon T_p. Setting the prediction horizon large enough results in good performance of NMPC. Naturally, one would assume the prediction horizon to be infinity (set $T_p = \infty$ in 3.2f) to minimize the overall cost. But this is not feasible in terms of computations. Therefore, a finite prediction horizon is always used. In case of finite prediction horizon, the actual close loop trajectories of input and states differ from the predicted ones even in the nominal case and in the absence of disturbance. This can be understood by an example of chess. If someone plans his future moves at the current situation of the chess game, and moves one step according to his future moves. The scenario might change as the opponent plays his move. Then one has to rebuild his future moves taking account of the changes occurred due to the opponent move. The new future moves might differ from the previous predicted future moves.

The same approach is applied to the finite horizon optimal control problem (FHOCP) strategy. At every sampling instant the optimal control input is recalculated by solving the optimal control problem and the future behavior of input and states is predicted over the prediction horizon. At next sampling interval, the prediction horizon moves forward which allows more information and re-planning. The variation in the actual and the predicted states has two major significances. Firstly, the minimization of the objective function of the close loop over the infinite horizon is not achieved. It is a fact that repeated finite horizon optimization of objective function does not lead to the optimal solution of corresponding infinite horizon optimization problem. The solutions of both will vary drastically if the prediction horizon if shorter prediction horizon is used.

Secondly if the actual and predicted solutions vary, the stability of closed loop is not guaranteed. It is not difficult to construct examples where the close loop becomes unstable if short prediction horizon is used [73 and 74]. The finite horizon optimization control problem must be modified in order to guarantee stability.

The overall control loop of NMPC is shown in fig 3.2.

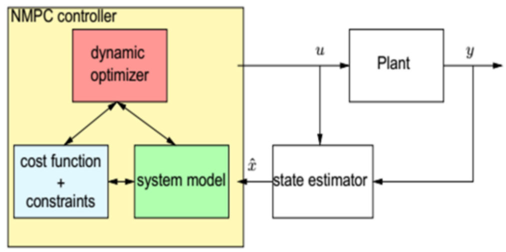

Figure 3.2: Control loop of NMPC

The states are estimated based on application of current input and measured system states. Using these estimates the NMPC controller computes the next optimal control input to be applied to the system.

NMPC has following properties:

- NMPC allows the nonlinear model of the plant to incorporate directly in the controller.
- NMPC allows us to consider the constraints on input and states explicitly.
- In NMPC, the objective function is minimized online.
- The closed loop behavior is usually different from the predicted behavior.
- The system's states must be measured or estimated (by observer) for the future prediction of plant.

The above properties can be viewed as advantages or may be disadvantages of NMPC apart from the explicit consideration of constraints on input and states. NMPC requires good and fast enough computation platform to obtain the solution of optimal control problem in relatively short finite time. Such computation platforms are available with the advancement of microcontrollers and microprocessors. In some cases, it is difficult to obtain the accurate mathematical model of the plant. The performance of NMPC varies drastically if the mathematical model is not accurate as all the NMPC computations rely on the accuracy of mathematical model.

3.6 Simulation Example

In this section, the NMPC controller is applied and simulated on a nonlinear non-minimum phase systems that have unstable internal dynamics such as n-link flexible joint manipulator system.

We consider a single link flexible joint manipulator system, in which the actuator is linked to a load with a torsional spring. The differential mathematical model is given by [75].

$$J_l\ddot{\theta}_l + MglSin\theta_l + k(\theta_l - \theta_r) = 0 \quad (3.3)$$

$$J_r\ddot{\theta}_r - k(\theta_l - \theta_r) = u \quad (3.4)$$

Where, θ_l, θ_r are the angular positions of link and rotor and J_l, J_r are the inertias of the link and the rotor. The load mass is represented by M, distance by l, gravity by g, joint stiffness by k and u represents the torque input. The output is the angular position of the rotor θ_r.

We can transform the above system into normal form by change of variables,

$$z_1 = \theta_l$$

$$z_2 = \dot{\theta}_l$$

$$x_1 = \theta_r$$

$$x_2 = \dot{\theta}_r$$

The transformed model becomes,

$$\dot{z}_1 = z_2 \quad (3.4)$$

$$\dot{z}_2 = -\frac{Mgl}{J_l} sinz_1 - \frac{k}{J_l}(z_1 - x_1) \quad (3.5)$$

$$\dot{x}_1 = x_2 \quad (3.6)$$

$$\dot{x}_2 = \frac{k}{J_r}(z_1 - x_1) + \frac{1}{J_r}u \quad (3.7)$$

$$y = x_1 \quad (3.8)$$

The relative degree of the above system is $\rho = 2$. The Internal (zero) dynamics are given by setting$x_1 = 0$.

$$\dot{z}_1 = z_2$$

$$\dot{z}_2 = -\frac{Mgl}{J_l} sinz_1 - \frac{k}{J_l} z_1$$

To check the stability of the zero dynamics, let's assume the Lyapunov function.

$$V = \int_0^{z_1} \frac{Mgl}{J_l} sinw\, dw + \frac{1}{2}\frac{k}{J_l} z_1^2 + \frac{1}{2} z_2^2$$

So,

$$V = \frac{Mgl}{J_l}(1 - \cos z_1) + \frac{1}{2}\frac{k}{J_l} z_1^2 + \frac{1}{2} z_2^2$$

Calculating the derivative of the Lyapunov function

$$\dot{V} = \frac{Mgl}{J_l} \sin z_1 \,(\dot{z}_1) + \frac{k}{J_l} z_1 \dot{z}_1 + z_2 \dot{z}_2$$

$$\dot{V} = \frac{Mgl}{J_l} z_2 \sin z_1 + \frac{k}{J_l} z_1 z_2 + z_2 \left(-\frac{Mgl}{J_l} sinz_1 - \frac{k}{J_l} z_1 \right)$$

$$\dot{V} = \frac{Mgl}{J_l} z_2 \sin z_1 + \frac{k}{J_l} z_1 z_2 - \frac{Mgl}{J_l} z_2 \sin z_1 - \frac{k}{J_l} z_1 z_2$$

$$\dot{V} = 0$$

As $\dot{V}$ is zero, the system is not asymptotically stable. Thus the above system is a non-minimum phase system. It is desired to stabilize the angular position of rotor $x_1$to the origin. We will apply nonlinear model predictive controller to stabilize the above system. The stage cost is chosen to be quadratic of the form,

$$S(x, u) = x^T Q x + u^T R u$$

Where, the weights in the stage cost are chosen as unity for simplicity. The state feedback controller is employed as Quasi-Infinite horizon NMPC [9 and 12]. The terminal penalty term E_f and the terminal region set Ω are obtained by LMI technique.

The system parameters as given in [75] are tabulated in table 3.1.

Table 3.1: System parameter values

S.No	Parameter	Value
1	Inertia of link J_l	$1kg \cdot m^2$
2	Inertia of rotor J_r	$1kg \cdot m^2$
3	Mgl	$9.8N \cdot m$
4	Joint stiffness k	$100\, Nm/rad$

The NMPC controller and system parameters are tabulated in table 3.2.

Table 3.2: NMPC controller and system initial values

S.No	Parameter	Value
1	T_p	$0.2s$
2	N	20
3	Sampling time δ	$0.01s$
4	$z_1(0)$	0.9
5	$z_2(0)$	0
6	$x_1(0)$	0.9
7	$x_2(0)$	0

The input and state constraints are given by,

$$-20 \leq u \leq 20$$

$$-1.5 \leq x \leq 1.5$$

$$-1.5 \leq z \leq 1.5$$

The terminal inequality constraint which also forces the internal dynamics of the system to be stable is given as,

$$\begin{bmatrix} -0.5 \\ -1 \end{bmatrix} \leq z \leq \begin{bmatrix} 0.5 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} -0.5 \\ -1 \end{bmatrix} \leq x \leq \begin{bmatrix} 0.5 \\ 1 \end{bmatrix}$$

Figure 3.3 shows the response of the flexible joint manipulator with NMPC controller. The output (angular position of rotor) approaches to origin as the time progresses.

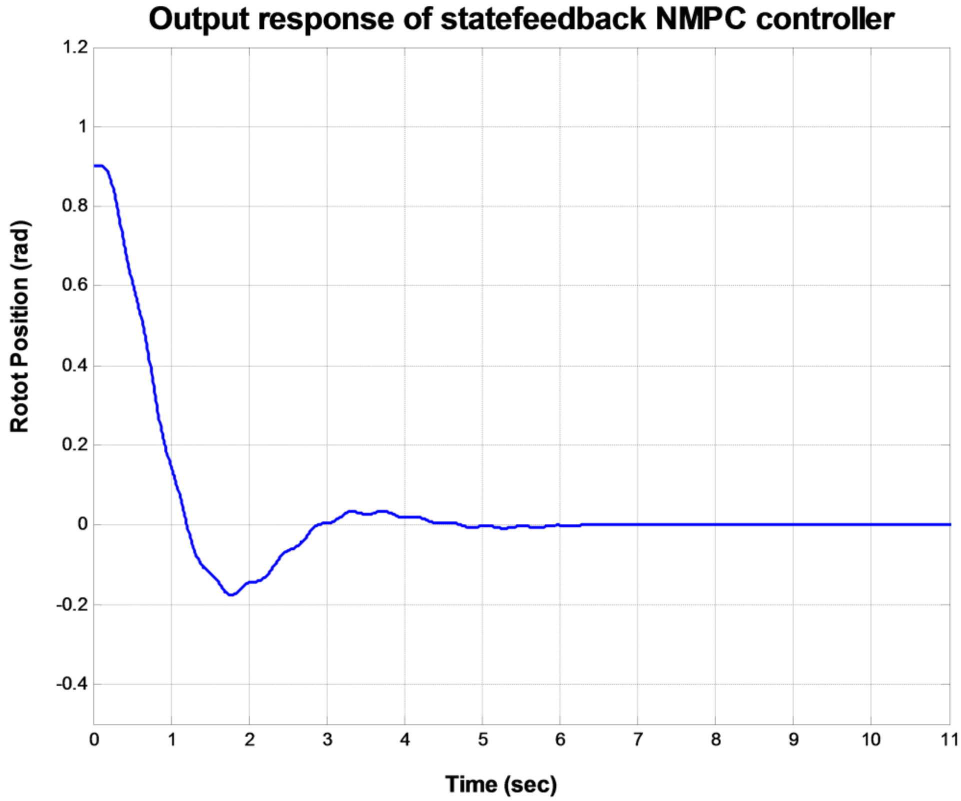

Figure 3.3: Output response of rotor angle with NMPC controller.

The angular position of the rotor is successfully stabilize to origin with NMPC controller. The response of the control input is shown in the figure 3.4.

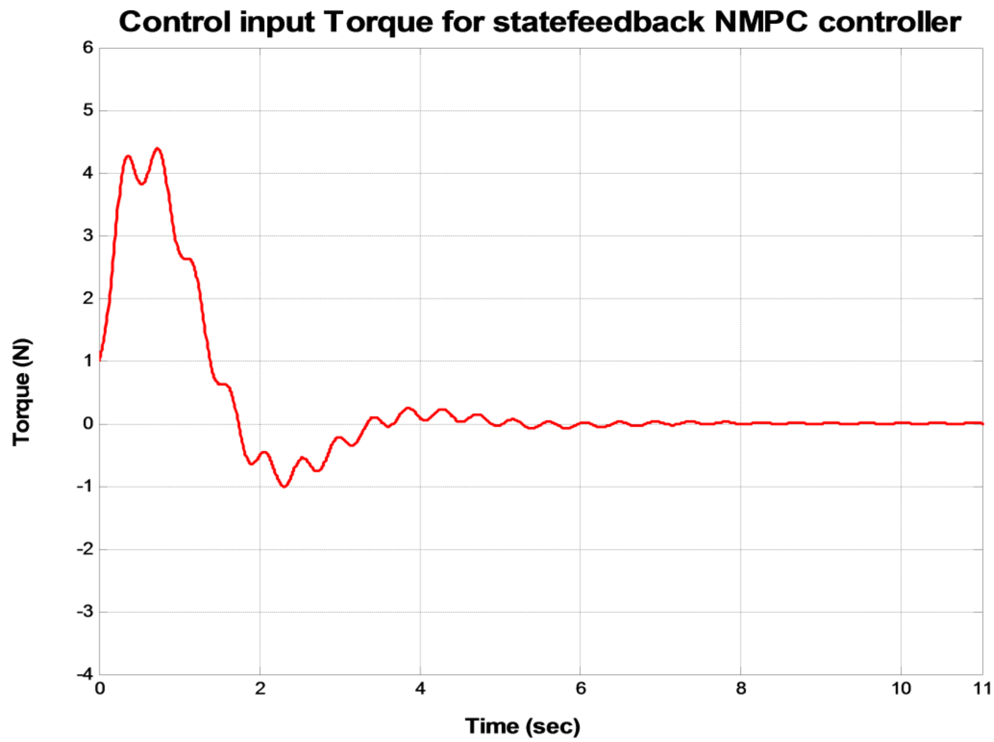

Figure 3.4: Control input Torque for state feedback NMPC controller.

As it is expected, the control input remains in the boundary and hence obeying the constraints. The NMPC controller successfully stabilizes the single link flexible joint manipulator system to the origin.

Chapter 4
OUTPUT FEEDBACK NONLINEAR MODEL PREDICTIVE CONTROL (NMPC)

4.1 Introduction

In the previous chapter, the NMPC Stability results and its nominal stability was dependent on the assumption that full state information is available, i.e. all the states are measured. In many practical scenarios, all the states cannot be measured due to hardware, practical and economic constraints. The states are estimated via suitable observer. So in the output feedback problem, the true states are replaced by their estimates in the feedback law. This often leads to performance nearer to the state feedback controller. Since there isn’t any separation principle for the nonlinear system the closed loop stability cannot be realized from the stability of state feedback controller and stability of observer explicitly. In this chapter, the output feedback controller is obtained that ensures the semi-global practical stability of the closed loop system for nonminimum phase systems.

The obtained results are encouraged by the work of Boker [23] on nonlinear observers that utilizes a combination of high gain and extended high gain observer for nonminimum phase systems and the findings of Findeisen on output feedback nonlinear model predictive control. The key idea of employing high gain observers for feedback is inspired by the nonlinear singular separation principles by Atassi and Khalil [24, 25, 76, 77, 78, 79 and 80]. It is shown that if the state estimation error, which is primarily considered as disturbance on the nominal closed loop, converge sufficiently fast enough, the closed loop semi global stability is achieved.

In this thesis, we consider a class of nonlinear nonminimum phase systems represented in the normal form having nonlinear internal dynamics with full relative degree. The implemented observer is motivated from the work of Boker [23] that comprises combination of extended high gain observer and high gain observer. The extended high gain observer is employed on the outer loop to estimates the output and its derivatives and the high gain observer estimates the internal states. The main challenge in this observer is to ensure that the extended high gain observer that estimates output and its derivatives works faster than the internal high gain observer. This is realizable in the sense that as both the observers forms high gain observer. It must be faster than the system dynamics. This scheme allows us to achieve several properties for the considered class of nonlinear systems. This observer allows us to recover the performance of any globally stabilizing state feedback controller [23].

4.2 Observer Design

Consider the stabilization of class of nonlinear system represented by,

$$\dot{x} = f\big(x(t), u(t)\big), \qquad x(0) = x_0 \tag{4.1}$$

$$y = h\big(x(t), u(t)\big) \tag{4.2}$$

where $x(t) \in \mathcal{X} \subseteq \mathcal{R}^n$ are states of the system, $u(t) \in \mathcal{U} \subset \mathcal{R}^m$ is the control input, and y is the measured output. The objective besides the stabilization is to satisfy the constraints on input and states.

$$u(t) \in \mathcal{U}, \qquad \forall t \geq 0$$

$$x(t) \in \mathcal{X}, \qquad \forall t \geq 0$$

Normally the sets $\mathcal{X}$ and $\mathcal{U}$ are convex of the form,

$$\mathcal{X} := \{x \in \mathcal{R}^n | x_{min} \leq x \leq x_{max}\}$$
$$\mathcal{U} := \{u \in \mathcal{R}^m | u_{min} \leq u \leq u_{max}\}$$

where x_{min}, x_{max} , u_{min} and u_{max} are specified constant vectors according to specifications.

Assumption 4.1: The function f is continuous and locally Lipschitz in its arguments with $f : \mathcal{R}^n \times \mathcal{R}^m \to \mathcal{R}^n$ and it satisfies $f(0,0) = 0$.

Assumption 4.2: The origin is contained in the set $(0,0) \in \mathcal{X} \times \mathcal{U}$ and it is stationary point for the nonlinear system (4.1)

The above system can be written in normal compact form as,

$$\dot{z} = A_0 z + B_0 \varphi(x, z)$$
$$\dot{x} = A_1 x + B_1[C_a x + a(x, u)]$$
$$y = Cx$$
$$y_a = C_a z$$

where, y is the measured output and y_a is the output for the internal system. Also $(z, x) \in \mathcal{R}^n$ where $n = q + r,\ \ q = m \times 2$.

$$z = [z_1, z_2]^T, \quad x = [x_1, x_2, \dots x_r]^T$$
$$A_0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ \ B_0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ \ C_a = [1 \quad 0]$$

The internal system has full relative degree. The above class of system could represent many under actuated mechanical and electro mechanical systems. These systems may have unstable internal dynamics or zero dynamics. The internal dynamics z are obtained by setting $x = 0$ in above equations.

The first $r + 1$ derivatives of the output signal y are estimated by an extended high-gain observer in which the first r derivatives of the output signal contains x states, while the $(r + 1)^{th}$ derivative is used to calculate the internal state z_1. We contemplate the internal system that is based on z dynamics with z_1 used as an output signal. This internal system has a full relative degree viewing x_1 as input and it is in standard normal form. Thus, we can imply a high-gain observer to estimate the internal state vector z. The most important consideration here is to design an extended high-gain observer (EHGO) for the output signal and its derivatives and making it faster than that of high-gain observer for the internal system. This can be achieved by setting the eigenvalues of the high-gain observer (HGO) of the order $O(1/e)$ and the eigenvalues of the extended high-gain observer of order $O(1/e^2)$ [23].

Consequently, the full order observer is given by,

$$\dot{\hat{z}} = A_0\hat{z} + B_0\hat{\varphi}(\hat{x},\hat{z}) + H_0(sat(\hat{\sigma}) - C_0\hat{z})$$

$$\dot{\hat{x}} = A_1\hat{x} + B_1[\hat{\sigma} + \hat{a}(\hat{x},u)] + H_1(y - C_1\hat{x})$$

$$\dot{\hat{\sigma}} = sat(\hat{z}_2) + H_2(y - C_1\hat{x})$$

Where,

$$H_0 = \begin{bmatrix} \alpha_1/e \\ \alpha_2/e^2 \end{bmatrix}$$

$$H_1 = \begin{bmatrix} \beta_1/e^2 \\ \beta_2/e^4 \\ \vdots \\ \beta_r/e^{2r} \end{bmatrix}$$

$$H_2 = diag\left[\beta_{r+1}/e^{2(r+1)}\right]_{m\times m}$$

Inside the region of interest, the functions $\hat{\varphi}(.,.)$ and $\hat{a}(.,.)$ are identical to $\varphi(.,.)$ and $a(.,.)$. The positive constants $\alpha_j$are fixed such that the roots of $s^2 + \alpha_1 s + \alpha_2 = 0$ lies in the left half plane. Similarly, the constants β_k are fixed to make the roots of $s^{r+1} + \beta_1 s^r + \cdots + \beta_r s + \beta_{r+1} = 0$ to lie in left half plane. The saturation function is used to prevent the states from peaking. The saturation of $\hat{\sigma}$ and $\hat{z}$ is done outside the domain of z_1 and z_2 under state feedback control.

4.3 Output Feedback Control

By replacing the states in the state feedback control law with their estimates, we get,

$$u = \gamma(\hat{x},\hat{z}) \tag{4.3}$$

Assumption 4.3: The control input u should obey:

- The control input u is admissible with respect to the state and input constraint sets $\mathcal{X}$ and $\mathcal{U}$ for all the states in the set $\Re$.
- For all $X \notin \Re$, the control input is saturated to a constant value.

$$u = u_{sat} \quad \forall X \notin \Re$$

The mentioned full order observer is employed to provide the estimates of the states. We will show that the output feedback control system recuperates the stability properties of the state feedback control. Furthermore, it is also possible to recover the trajectories of the system under state feedback. To obtain these results, we combine the dynamics of system and the estimation error $\tilde{\eta}$ of the observer. The scaled estimation error is given as,

$$\tilde{\eta}_j = \frac{z - \hat{z}}{e^{2-j}}$$

$$\tilde{\eta}_1 = \frac{z_1 - \hat{z}_1}{e}$$

$$\tilde{\eta}_2 = \frac{z_2 - \hat{z}_2}{e^2}$$

$$\hat{z} = z - D(e)\tilde{\eta}$$

$$D(e) = diag[e^{r-1}, \dots, 1]$$

$$E_k = \frac{x - \hat{x}}{e^{2r+3-2k}}$$

$$E_{r+1} = \frac{z_1 - \hat{\sigma}}{e}$$

where $j = 1, 2$ and $k = 1, \dots, r$.

Assumption 4.4: After a fixed number of freely chosen sampling instants k_{obs}, there exists observer parameter such that the desired maximum estimation error satisfies

$$\|X(t) - X_s(t)\| \leq e_{max}, \quad \forall t \geq t_{k_{obs}}$$

It is important to mention that the scaling for $\tilde{\eta}$ is usual for high-gain observer effects. But, the scaling E for the extended high gain observer is selected in a way that it is depends on the z dynamics dimensions. This is done to preserve the two time scale structures as there is connection and coupling through $\hat{\sigma}$. Therefore, we will analyze the closed loop system in a multi-time scale arrangement.

Applying this, we get

$$\phi = \begin{bmatrix} E_1 \\ \vdots \\ E_r \end{bmatrix}$$

$$E = \begin{bmatrix} \phi \\ \vdots \\ E_{r+1} \end{bmatrix}_{[(r+1)\times 1}$$

$$Q(e) = diag[e^{2r+1}, \dots, e^3]_{r\times r}$$

$$S(e) = diag[e^{2r+1}, \dots, e^3, e]_{(r+1)\times(r+1)}$$

$$Q(e)\phi = x - \hat{x}$$

$$D(e)\tilde{\eta} = z - \hat{z}$$

$$S(e)E = \begin{bmatrix} x - \hat{x} \\ z_1 - \hat{\sigma} \end{bmatrix}$$

Accordingly, the closed loop system under the output feedback scheme takes the form,

$$\begin{aligned} \dot{X} = f_1\big(X, \gamma(\hat{z}, \hat{x})\big) = f_1\big(X, \gamma(z - D(e)\tilde{\eta}, x - Q(e)\phi)\big) \\ \triangleq f_c(X, D(e)\tilde{\eta}, Q(e)\phi) \end{aligned} \tag{4.4}$$

$$e\dot{\tilde{\eta}} = A_0\tilde{\eta} + eB_0\Delta g + FE_{r+1} \tag{4.5}$$

$$e^2\dot{E} = A_1E + e\left[B_1g_1 + B_2\frac{\Delta a}{e^2}\right] \tag{4.6}$$

where,

$$F = \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0 \\ b_1 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} b_2 \\ 0 \end{bmatrix}$$

$$\Delta g = \varphi(z, x) - \hat{\varphi}(\hat{z} - \hat{x})$$

$$\varphi_1 = z_2 - sat(\hat{z}_2)$$

$$\Delta a = a(x, \gamma(\hat{x}, \hat{z}) - \hat{a}(\hat{x}, \gamma(\hat{x}, \hat{z})))$$

Because of the smoothness characteristic of $a(.,.)$, we comprehend that, for

$$(z, x, D\tilde{\eta}, Q\phi) \in Z \subset \mathcal{R}^{2n}$$

where Z is a compact set, Δa is locally Lipschitz in its arguments, uniformly in e. In addition, for any $\tilde{e} \leq 1$and for all

$$(z, x, D\tilde{\eta}, Q\phi) \in Z \subset \mathcal{R}^{2n}$$

there exists, $0 \leq e \leq \tilde{e}$ such that,

$$\left\|\frac{1}{e^3}\left[a\left(x,\gamma(\hat{x},\hat{z})-\hat{a}\big(\hat{x},\gamma(\hat{x},\hat{z})\big)\right)\right]\right\| \le \tilde{L}e^{-3}\|x-\hat{x}\| \le \tilde{L}e^{-3}\|Q\|\|E\| \le \tilde{L}\|E\|$$

where $\tilde{L}$ is the Lipschitz constant of nonlinear function $a(.,.)$ over the domain Z. It is important that to obtain the above inequality, we utilize the facts

$$\|Q\| \le e^3\,, \quad \|\phi\| \le \|E\|$$

From now on, we will always consider that the observer parameter $e \le \tilde{e}$.

The matrices $A_0$and A_1 are Hurwitz by scheme and given by

$$A_0 = \begin{bmatrix} -\alpha_1 & 1 \\ -\alpha_2 & 0 \end{bmatrix}_{2\times 2}$$

$$A_1 = \begin{bmatrix} -\beta_1 & 1 & 0 & \cdots & 0 \\ -\beta_2 & 0 & 1 & \ddots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots \\ -\beta_r & 0 & 0 & \ddots & 1 \\ -\beta_{r+1} & 0 & 0 & \cdots & 0 \end{bmatrix}_{(r+1)\times(r+1)}$$

The above system in the standard singularly perturbed form with three time scales structure. The slow variable of this structure is $X(t)$ and the fast variables are $\big(\tilde{\eta}(t), E(t)\big)$. Moreover, the dynamics of $E(t)$ is of order $O(1/e^2)$ and faster than $\tilde{\eta}(t)$.

By setting $e = 0$ in (4.5)-(4.6) we get $\tilde{\eta} = 0$ and $E = 0$, so that the system is reduced to

$$\dot{X} = f_s\big(X, \gamma(z,x)\big)$$

which is equivalent to the closed-loop system under state feedback. Representing the solutions of output feedback as $X(t)$ and that of state feedback as $X_s(t)$ starting from some initial condition $X_s(0)$. The initial states for output feedback are

$$X(0) \in \Theta$$

where Θ is any compact set in the interior of $\mathcal{R}$. The observer initial states are in

$$\big(\hat{z}(0), \hat{x}(0), \hat{\sigma}(0)\big) \in \mathcal{O}$$

where $\mathcal{O}$ is any compact subset of $\mathcal{R}^{n+1}$.

Before we proceed to our main results, we led the foundation of the presence of an invariant region for the proposed observer for all the states of system belongs to $\Omega(\Theta)$. Moreover it is shown that by decreasing observer parameter e and sampling time t_s, the incorporated error between the state feedback scheme and output feedback scheme can be made adequately small provided that the observer error begins in adequately small region. This indicates that the variation in the value functions obtained can be rendered adequately small.

4.4 Practical Stability of Output Feedback

To establish the practical stability, we begin by exploring the boundary layer model. For this reason, we apply the new time scale $\tau = t/e$ for the system to get,

$$\frac{dX}{d\tau} = ef_1\big(X, \gamma(z - D(e)\tilde{\eta}, x - Q(e)\phi)\big) \quad (4.7)$$

$$\frac{d\tilde{\eta}}{d\tau} = A_0\tilde{\eta} + eB_0\Delta g + FE_{r+1} \quad (4.8)$$

$$e\frac{d\dot{E}}{d\tau} = A_1E + e\left[B_1g_1 + B_2\frac{\Delta a}{e^2}\right] \quad (4.9)$$

Here we observe that for the original system, the subsystem (4.8)-(4.9) serves as the boundary layer subsystem. Additionally for (4.7)-(4.8), the subsystem (4.9) serves as the boundary layer subsystem.

By substituting $e = 0$ in this time scale results in $E = 0$ and

$$\frac{d\tilde{\eta}}{d\tau} = A_0\tilde{\eta}$$

Let us propose the Lyapunov function candidate,

$$W_1(\tilde{\eta}) = \tilde{\eta}^T P_0 \tilde{\eta}$$

where P_0 is the positive definite matrix and obtained from the solution of Algebraic Riccati equation

$$P_0A_0 + A_o^T P_0 = -I$$

This Lyapunov function fulfills

$$\lambda_{min}(P_0)\|\tilde{\eta}\|^2 \le W_1(\tilde{\eta}) \le \lambda_{max}(P_0)\|\tilde{\eta}\|^2$$

$$\frac{\partial W_1}{\partial \tilde{\eta}} A_0\tilde{\eta} \le -\|\tilde{\eta}\|^2$$

Where $\lambda_{min}(P_0)$ and $\lambda_{max}(P_0)$ are the corresponding minimum and maximum eigenvalues of the matrix P_0 respectively.

In a similar fashion, let us investigate the stability of this boundary layer. For this, we use the time Scale

$$\omega = \tau/e$$

This gives,

$$\frac{dX}{d\omega} = e^2 f_1\big(X, \gamma(z - D(e)\tilde{\eta}, x - Q(e)\phi)\big)$$

$$\frac{d\tilde{\eta}}{d\omega} = e[A_0\tilde{\eta} + eB_0\Delta g + FE_{r+1}]$$

$$\frac{d\dot{E}}{d\omega} = A_1 E + e\left[B_1 g_1 + B_2 \frac{\Delta a}{e^2}\right]$$

Substituting $e = 0$ in above equation, we get

$$\frac{d\dot{E}}{d\omega} = A_1 E$$

To analyze this boundary layer, we establish the Lyapunov function candidate

$$W_2(E) = E^T P_1 E$$

where P_1 is also a positive definite matrix and obtained similarly from the solution of Algebraic Riccati equation,

$$P_1 A_1 + A_1^T P_1 = -I$$

This Lyapunov function fulfills,

$$\lambda_{min}(P_1)\|E\|^2 \leq W_2(E) \leq \lambda_{max}(P_1)\|E\|^2$$

$$\frac{\partial W_2}{\partial \tilde{\eta}} A_1 \tilde{\eta} \leq -\|E\|^2$$

where $\lambda_{min}(P_1)$ and $\lambda_{max}(P_1)$ are the corresponding minimum and maximum eigenvalues of the matrix P_1respectively.

Let us define the sets

$$\mathrm{M} = \{\tilde{\eta} | W_1(\tilde{\eta}) \leq \rho_0 e^2\}$$

$$\mathrm{N} = \{E | W_2(E) \leq \rho_1 e^2\}$$

Where ρ_0 and ρ_1 are some positive constants which will be specified later. For all

$$(X_s, \tilde{\eta}, E) \in \mathrm{M} \times \mathrm{N}$$

it can be shown in a similar fashion as in [23],

$$\|f_1(X, D(e)\tilde{\eta}, Q(e)E) - f_1(X, 0,0)\| \leq L_1\|\tilde{\eta}\| + L_2\|E\|$$

Where L_1 and L_1 are positive constants independent of the observer parameter e. In the light of the fact that the continuous functions are confined over compact sets.

For all states $X \in \Omega(\Theta)$ and $(\tilde{\eta}, E) \in \mathcal{R}^{n+m}$, we can deduce [23],

$$\|\Delta g(z, x, D\tilde{\eta}, QE)\| \leq \mu_1$$

$$\|g_1(z_2, \hat{z}_2)\| \le \mu_2$$
$$\left\|f_1\big(X, \gamma(z - D(e)\tilde{\eta}, x - Q(e)\phi)\big)\right\| \le \mu_3$$

where μ_1 , μ_2,and μ_3 are positive constants independent of observer parameter e.

Lemma 4.1:

For all the system states $X \in \Omega(\Theta)$, the set $\mathrm{M} \times \mathrm{N}$ is invariant for the observer error. The proof is similar to shown in [21, 23 and 24]

Lemma 4.2:

Consider $\vartheta_1 > 0$, then there exist a sampling time $\bar{t}_s > 0$ and observer parameter $\bar{e}_1 > 0$ such that for all $0 < e \le \bar{e}_1$and $0 < t_s \le \bar{t}_s$ and for $\tilde{\eta} \in \mathrm{M}$ and $E \in \mathrm{N}$, $X \in \Theta$ the inequality

$$| \int_t^{t+T} F\big(X(\varrho), u(\varrho)\big) d\varrho - \int_t^{t+T} F(X_s(\varrho), u_s(\varrho)) d\varrho \le T\,\vartheta_1$$

Is valid for any time interval $T \in (0, \bar{T}]$ where $\bar{T} > t_s$ is finite, such that

$X(t+\varrho) \in \mathcal{R}, \quad \forall \varrho \in (0, \bar{T}]$

Lemma 4.3:

Consider time interval $T \in (0, \bar{T}]$ where $\bar{T} > t_s$ is finite and such that

$$X(t+\varrho) \in \mathcal{R}, \quad X_s(t+\varrho) \in \mathcal{R} \quad \forall \varrho \in [t, t+\bar{T})$$

both starting from $X(t) \in \Omega(\Theta)$. Then for every $\vartheta_2 > 0$, there exist a sampling time $\check{t}_s > 0$ and $\check{e}_1 > 0$ such that for all $0 < e \le \bar{\check{e}}_1$and $0 < t_s \le \bar{\check{t}}_s$, $\tilde{\eta} \in \mathrm{M}$ and $E \in \mathrm{N}$.

$$\left|V\big(X(t+T)\big) - V(X_s(t+T))\right| \le T\,\vartheta_2$$

4.4.1 Boundedness of Observer States

To prove the boundedness of observer states we state and prove the theorems which are modification of theorems presented in [21, 23 and 24] for the mentioned output feedback scheme.

Theorem 4.1:

Under assumptions, there exists an $\check{e} > 0$ such that for every $0 < e \le \check{e}$, the trajectories $\big(X(\varrho), \tilde{\eta}(\varrho), E(\varrho)\big)$ starting at

$$\big(X(t), \tilde{\eta}(t), E(t)\big) \in \Theta \times \mathcal{O}$$

are bounded for all $\varrho \ge t$.

Proof:

The retrieval of boundedness can be proved in following two steps.

1. The set $\Pi = \Omega(\Theta) \times \mathrm{M} \times \mathrm{N}$ is compact and positively invariant for some small enough e and t_s.
2. The close loop trajectories starting in $\Theta \times \mathcal{O}$ enter in set Π in finite time.

4.4.2 Invariance of Set Π

We define the boundary of $\Omega(\Theta)$ as $\delta\Omega(\Theta)$. For all

$$\big(X(t), \tilde{\eta}(t), E(t)\big) \in \delta\Omega(\Theta) \times \mathrm{M} \times \mathrm{N}$$

and for all time interval $T \in (0, \bar{T}]$ where $\bar{T} > t_s$ is finite, we have,

$$\begin{aligned}
&V\big(X(t+T)\big) - V\big(X(t)\big) \\
&\le V\big(X_s(t+T)\big) - V\big(X(t)\big) + \big|V\big(X(t+T)\big) - V(X_s(t+T))\big| \\
&\le -\int_t^{t+T} F\big(X_s(\varrho), u_s(\varrho)\big)d\varrho + \big|V\big(X(t+T)\big) - V\big(X_s(t+T)\big)\big| \\
&\le -\int_t^{t+T} F\big(X(\varrho), u(\varrho)\big)d\varrho \\
&+ \left|\int_t^{t+T} F\big(X(\varrho), u(\varrho)\big) - F\big(X_s(\varrho), u_s(\varrho)\big)d\varrho\right| \\
&+ \big|V\big(X(t+T)\big) - V\big(X_s(t+T)\big)\big|
\end{aligned}$$

We notice that as

$$\Omega(\Theta) \subset \mathcal{R}$$

there exists $\bar{T}$ such that

$$\forall T \le \bar{T}, \quad X(t+\varrho) \in \mathcal{R}$$

so that we can use Lemma 4.2 and Lemma 4.3,

$$V\big(X(t+T)\big) - V(X(t)) \le -\int_t^{t+T} F\big(X(\varrho), u(\varrho)\big)d\varrho + T\,\vartheta_1 + T\,\vartheta_2$$

As $X(t) \neq 0$ and $T \geq 0$ is finite, we identify that

$$\int_t^{t+T} F\big(X(\varrho), u(\varrho)\big)d\varrho \ge T\varsigma$$

for some positive constant $\varsigma > 0$. It follows that there exist observer parameter $\bar{e} > 0$, $\check{e} > 0$ and $\bar{t}_s > 0$, such that for $0 < e < \min(\bar{e}, \check{e})$ and $0 < t_s < \bar{t}_s$, we have,

$$-\varsigma + \vartheta_1 + \vartheta_2 \leq 0$$

$$V\big(X(t+T)\big) - V(X(t)) \leq 0$$

This proves that $X(t+T) \in \Omega(\Theta) \subset \mathcal{R}$ and therefore it holds for all $T \in (0, \bar{T}]$.

If for any finite time interval $t > \bar{T}$, the states $X(t)$ again reach the border of set $\Omega(\Theta)$, we can again apply the above mentioned procedure.

Further, for all

$$\big(X(t), \tilde{\eta}(t), E(t)\big) \in \Omega(\Theta) \times \mathrm{M} \times \mathrm{N}$$

we have from Lemma 4.1,

$$\dot{W}_1 \leq 0$$

And

$$\dot{W}_2 \leq 0$$

If

$$\rho_0 = (16\lambda_{max}^4(P_0)/\lambda_{min}(P_0))\left[\mu_1 + \mu_6\sqrt{\rho_1/\lambda_{min}(P_1)}\right]^2$$

and

$$\rho_1 = (64\mu_2^2\lambda_{max}^4(P_1)/\lambda_{min}(P_1))$$

We accomplish that for observer parameter $0 < e < \min(\bar{e}, \check{e})$ and $0 < t_s \leq \bar{t}_s$, the set Π is positively invariant.

4.4.3 Attractivity of Set Π in Finite Time

This is achieved in similar fashion as in [21, 23 and 24].

Consider

$$X(0), \tilde{\eta}(0), E(0) \in \Theta \times \mathcal{O}$$

It can be verified that the resultant initial errors $\tilde{\eta}(0)$ and $E(0)$ satisfy

$$\|\tilde{\eta}(0)\| \leq \mu_7/e$$

and

$$\|E(0)\| \leq \mu_8/e^{2r+1}$$

for some positive constants μ_7 and μ_8 dependent on the sets Θ *and* $\mathcal{O}$. It can also be presented that, since $X(0)$ is in the interior of $\Omega(\Theta)$ and as long as $X(t) \in \Omega(\Theta)$, we have

$$\|X(t) - X(0)\| \leq \mu_3 t$$

Hence there exist a finite time interval $\acute{T}$, independent of observer parameter e such that,

$$X(t) \in \Omega(\Theta) \quad \forall T \in \left(0, \acute{T}\right]$$

During this interval, we can indicate that,

$$\dot{W}_2(E) \leq -\frac{1}{2e^2}\|E\|^2$$

For $W_2 \geq \rho_1 e^2$ and $e \leq \bar{e}$.

Established on this point we can show that,

$$\dot{W}_1(\tilde{\eta}) \leq -\frac{1}{2e}\|\tilde{\eta}\|^2$$

For $W_1 \geq \rho_0 e^2$ and $W_2 \leq \rho_1 e^2$, the above inequalities reflects that the variable E first becomes of the order $O(e)$. This sequentially allows the variable $\tilde{\eta}$ to become $O(e)$.

We can choose small enough $\acute{e}$ such that $0 < e \leq \acute{e}$, we have

$$\bar{\bar{T}} \triangleq \tilde{T} + \breve{T} \leq \frac{1}{2}\acute{T}$$

Where,

$$\tilde{T} \triangleq \frac{2e}{\|P_0\|}\ln\left(\frac{\|P_0\|\mu_7^2}{\rho_0 e^4}\right)$$

$$\breve{T} \triangleq \frac{2e^2}{\|P_1\|}\ln\left(\frac{\|P_1\|\mu_8^2}{\rho_1 e^{4r+4}}\right)$$

We found that such observer parameter $\acute{e}$ exists, since the time intervals $\tilde{T}$ and $\breve{T}$ approaches to zero as e approaches to zero. It follows that, for every

$$0 < e \leq \acute{e}$$

We have,

$$W_1(\tilde{\eta}(\bar{\bar{T}})) \leq \rho_0 e^2$$

$$W_2(E(\bar{\bar{T}})) \leq \rho_1 e^2$$

Setting,

$$\bar{\bar{e}} = min\{\bar{e}, \breve{e}, \acute{e}\}$$

guarantees that for every $0 < e \leq \bar{\bar{e}}$, the trajectories $\left(X(t), \tilde{\eta}(t), E(t)\right)$ enters the set Π during the time interval $\left[0, \bar{\bar{T}}\right]$ and remains inside for all time interval $t > \bar{\bar{T}}$. We also observe that, for all time $t \in \left[0, \bar{\bar{T}}\right]$, the trajectories are bounded by virtue of above inequalities.

4.4.4 Semi Global Practical Stability of Output Feedback

Now we present the fact that for any sufficiently small area or ball around the origin, there exist an observer parameter e and sampling time interval t_s, such that the close loop trajectories will reach that small ball in finite time and will remain there and thus making the ball invariant.

Theorem 4.2:

Under the assumptions of theorem 4.1, then for any $\zeta > 0$ there exists $e^\star > 0$ and $t_s^\star > 0$ and $T^\star$ such that for every observer parameter $0 < e < e^\star$ and sampling time $0 < t_s < t_s^\star$, we have,

$$\|X(t)\| + \|\tilde{\eta}(t)\| + \|E(t)\| \leq \zeta, \quad \forall t > T^\star$$

Proof:

For a given positive constant ζ as in [21 and 24], we can find $e^\star \leq \bar{e}$ such that for all $0 < e < e^\star$ we have,

$$\|\tilde{\eta}(t)\| + \|E(t)\| \leq \frac{\zeta}{2}, \quad \forall t \geq T(e^\star)$$

Since the value function $V(X)$ is continuous around the origin and

$$V(0) = 0$$

it follows that it is thinkable to find a constant c such that,

$$\{X | V(X) \leq c\} \subset \{X | \|X\| \leq \zeta/2\}$$

Using the results of theorem 4.1, we can show that for

$$0 < e < e^\star$$

and

$$0 < t_s < t_s^\star$$

We have,

$$V\big(X(t + \hat{T}\big) - V(X(t)) \leq -\int_t^{t+\hat{T}} F\big(X(\varrho), u(\varrho)\big) d\varrho + \hat{T}\,\vartheta_1 + \hat{T}\,\vartheta_2$$

So, there exists a $\bar{\varsigma} > 0$ such that

$$F(X, u) \geq \bar{\varsigma}, \;\; X \in \Omega(\Theta) - \{X | V(X) \leq c\}, \;\; u \in \mathcal{U}$$

Suppose that states

$$X(\varrho), \;\; t \leq \varrho \leq t + \hat{T}$$

does not contain in the set

$$\{X|V(X) \leq c\}$$

then we get,

$$V(X(t+\hat{T}) - V(X(t)) \leq -\bar{\varsigma}\hat{T} + \hat{T}\,\vartheta_1 + \hat{T}\,\vartheta_2$$

We have to choose observer parameter e and sampling time t_s such that

$$\vartheta_1 + \vartheta_2 - \bar{\varsigma} < 0$$

If $\hat{T}$ is big enough (we can set its value as big till Lemma 4.2 and Lemma 4.3 is valid as the states are in the invariant set Π), we can ensure that

$$V(X(t+T) \leq c$$

for some $T \leq \hat{T}$. Then by contradiction, we ought to have

$$X(t+T) \in \{X|V(X) \leq c\}$$

By the similar reasoning as presented in theorem 4.1, this set is invariant and we can set

$$T^{\star} = \max(T(e^{\star}), \hat{T})$$

This institutes our key result. The semi global practical stability of closed loop system is achieved in a manner, that for given system trajectories $\Theta \subset \mathcal{R}$ and for any small ball around the origin there exists an observer parameter e and sampling time t_s, such that the trajectories approaches the ball from any starting point in the set Θ in finite time and will remain inside the ball and making it positively invariant.

Furthermore we can also show, that the close loop trajectories of the output feedback scheme converges to the trajectories of the state feedback as the observer parameter e and sampling time t_s approaches to zero

$$e \to 0, \;\; t_s \to 0$$

Theorem 4.3:

Under the assumptions of theorem 4.1, then for any $\zeta > 0$ there exists $e^{\diamond} > 0$ and $t_s^{\diamond} > 0$ such that for every $0 < e \leq e^{\diamond}$ and $0 < t_s < t_s^{\diamond}$, we have,

$$\|X(t) - X_s(t)\| \leq \zeta, \quad \forall t \geq 0$$

Proof:

This can be proved on the similar lines as shown in [23 and 24],

To prove this, we divide the time interval $[0, \infty]$ into three time intervals $[0, T^{\star}]$, $[T^{\star}, T^{\diamond}]$ and $[T^{\diamond}, \infty]$ where the interval $T^{\diamond} > 0$ is to be defined.

Interval $[\mathbf{0}, \boldsymbol{T}^\star]$:

Using similar reasoning as in the inequality (4.81), we can conclude that

$$\|X_s(t) - X(0)\| \le \mu_3 t$$

Therefore,

$$\|X(t) - X_s(t)\| \le 2\mu_3 T^\star, \quad \forall t \in [0, T^\star]$$

Since $T^\star \to 0$ as $e \to 0$ given any $\zeta > 0$ there exists observer parameter $0 < e^* \le e^\diamond$ and sampling time $t_s^\diamond > 0$ such that for every

$$0 < e \le e^*$$

and

$$0 < t_s < t_s^\diamond$$

we have,

$$\|X(t) - X_s(t)\| \le \zeta, \quad \forall t \in [0, T^\star]$$

Interval $[\boldsymbol{T}^\star, \boldsymbol{T}^\diamond]$:

During this interval the trajectories $X(t)$ satisfies

$$\dot{X} = f_c(X, D(e)\tilde{\eta}, Q(e)\phi)$$

with initial condition $X(T^\star)$ and $D(e)\tilde{\eta}$ and $Q(e)\phi$ are of order $O(e)$ and the state feedback trajectories $X_s(t)$ satisfies

$$\dot{X} = f_c(X, 0,0)$$

with initial condition $X_s(T^\star)$. We know that from [23 and 24]

$$\|X(t) - X_s(t)\| \le 2\mu_3 T^\star \triangleq \varpi, \quad \forall t \in [0, T^\star]$$

where, $\varpi \to 0$ as $e \to 0$. Consequently, by using (Theorem 3.5, [82]), we accomplish that, for any $\zeta > 0$, there exists observer parameter

$$0 < e^\blacksquare \le e^\diamond$$

such that for every

$$0 < e \le e^\blacksquare$$

we can ensure

$$\|X(t) - X_s(t)\| \le \zeta, \quad \forall t \in [T^\star, T^\diamond]$$

Interval $[\boldsymbol{T}^\diamond, \infty]$:

From the Theorem 4.2 and as the origin of the state feedback system is asymptotically stable, we establish that there exists a finite time interval $T^\diamond \ge T^\star$, independent of observer parameter e, such that for every

$$0 < e < e^{\star}$$

we get,

$$\|X(t) - X_s(t)\| \leq \zeta, \quad \forall t \geq T^{\diamond}$$

Taking,

$$e = min(e^{\star}, e^{\blacksquare})$$

This proves the theorem 4.3.

It is now evident that if the sampling time t_s and the observer parameter e are chosen in accordance with the above stability conditions, the semi global practical stability of output feedback i.e. observer with nonlinear model predictive controller can be achieved. So, the trajectories starting from the set Θ which is the subset of the region of attraction $\mathcal{R}$ of the state feedback will reach the small set around the origin in finite time and will remain thereafter.

4.5 Simulation Example

In this section we will apply the proposed observer and Nonlinear Model Predictive Controller (NMPC) to a non-minimum phase systems having unstable internal dynamics.

We consider the same nonlinear system of single link manipulator with a flexible joint as considered in chapter 3. The mathematical dynamics are given by [75].

$$J_l\ddot{\theta}_l + MglSin\theta_l + k(\theta_l - \theta_r) = 0 \tag{4.10}$$

$$J_r\ddot{\theta}_r - k(\theta_l - \theta_r) = u \tag{4.11}$$

In which the angular positions of link and rotor are denoted by θ_l, θ_r, the inertias of link and rotor are denoted by J_l, J_r . The load mass is denoted by M, gravity by g, distance by l, joint stiffness by k and u gives the input torque and the rotor angular position θ_r is the measured output.

We can convert the above system into standard normal form by change of variables,

$$z_1 = \theta_l$$

$$z_2 = \dot{\theta}_l$$

$$x_1 = \theta_r$$

$$x_2 = \dot{\theta}_r$$

The new transformed model becomes

$$\dot{z}_1 = z_2 \tag{4.12}$$

$$\dot{z}_2 = -\frac{Mgl}{J_l} sinz_1 - \frac{k}{J_l}(z_1 - x_1) \tag{4.13}$$

$$\dot{x}_1 = x_2 \tag{4.14}$$

$$\dot{x}_2 = \frac{k}{J_r}(z_1 - x_1) + \frac{1}{J_r}u \tag{4.15}$$

$$y = x_1 \tag{4.16}$$

We can see that the relative degree ρ of the system is 2 and the Internal (zero) dynamics are given by substituting $x_1 = 0$.

The states of the above system are estimated by an observer described in section 4.2. The first two derivatives of output are estimated by an extended high gain observer (EHGO). This makes the x states available so we can use it in computing the internal states z the second derivative of the output is the right hand side of equation (4.66). We can estimates the internal states z with the aid of it. The internal (auxiliary) system which is represented by,

$$\dot{z}_1 = z_2$$

$$\dot{z}_2 = -\frac{Mgl}{J_l} sinz_1 - \frac{k}{J_l}(z_1 - x_1)$$

in which the signal $\sigma = k/J_r\, z_1$ serves as the output signal of the internal (auxiliary) system. The internal system is already in standard normal form by viewing x_1 as input and it has full relative degree. So, we can use a high gain observer to estimate the internal states. The most important consideration here is to ensure that the extended high gain observer which estimates the first two derivatives of output must be fast enough than the high gain observer which estimates the internal states. This is achieved by making the extended high gain observer of order $O(1/e^2)$ and that of high gain observer of order $O(1/e)$.

Following the same procedure described in section 4.2, the full order observer takes the form

$$\dot{\hat{z}}_1 = \hat{z}_2 + \frac{\alpha_1}{e}\left(\frac{J_r}{k}\hat{\sigma} - \hat{z}_1\right)$$

$$\dot{\hat{z}}_2 = -\frac{Mgl}{J_l} sin\hat{z}_1 - \frac{k}{J_l}(\hat{z}_1 - \hat{x}_1) + \frac{\alpha_2}{e^2}\left(\frac{J_r}{k}\hat{\sigma} - \hat{z}_1\right)$$

$$\dot{\hat{x}}_1 = \hat{x}_2 + \frac{\beta_1}{e^2}(y - \hat{x}_1)$$

$$\dot{\hat{x}}_2 = \hat{\sigma} + \frac{k}{J_r}\hat{x}_1 + \frac{1}{J_r}u + \frac{\beta_2}{e^4}(y - \hat{x}_1)$$

$$\dot{\hat{\sigma}} = \frac{k}{J_r}\hat{z}_2 + \frac{\beta_3}{e^6}(y - \hat{x}_1)$$

For the simulation, the system parameters are fixed to same values as in chapter 3 and in [75]. The NMPC controller parameters, observer parameters, initial values of system and observer are tabulated in table 4.1.

Table 4.1: Observer and controller parameters for output feedback scheme

S.No	**Parameter**	**Value**
1	e	0.01
2	α_1	15
3	α_2	6
4	β_1	12
5	β_2	14
6	β_3	1
7	$z_1(0)$	0.5
8	$\hat{z}_1(0)$	0.9
9	$z_2(0)$	0
10	$\hat{z}_2(0)$	0
11	$x_1(0)$	0.5
12	$\hat{x}_1(0)$	0.9
13	$x_2(0)$	0
14	$\hat{x}_2(0)$	0
15	$\hat{\sigma}(0)$	0
16	T_p	$0.2s$
17	N	20
18	Sampling time δ	$0.01s$
19	Q	8
20	R	0.35

It must be assured that the control input u, $\hat{z}_1$, $\hat{z}_2$ and $\hat{\sigma}$ should be saturated to avoid the peaking phenomenon. The saturation was performed after we investigated that control input is in the range $[-20,20]$, z_1 is in the range $[-1.5,1.5]$, z_2 is in the range $[-1.5,1.5]$ and σ is in the range $[-60,60]$ under state feedback control. In the light of this the saturation level set for these parameters are as under,

$$-20 \leq u \leq 20$$
$$-5 \leq \hat{z}_1 \leq 5$$
$$-5 \leq \hat{z}_2 \leq 5$$
$$-70 \leq \hat{\sigma} \leq 70$$

Figure 4.1 shows the output response of the flexible joint manipulator for output feedback scheme. As expected, the angular position of rotor which is the output reaches the origin as the time progresses.

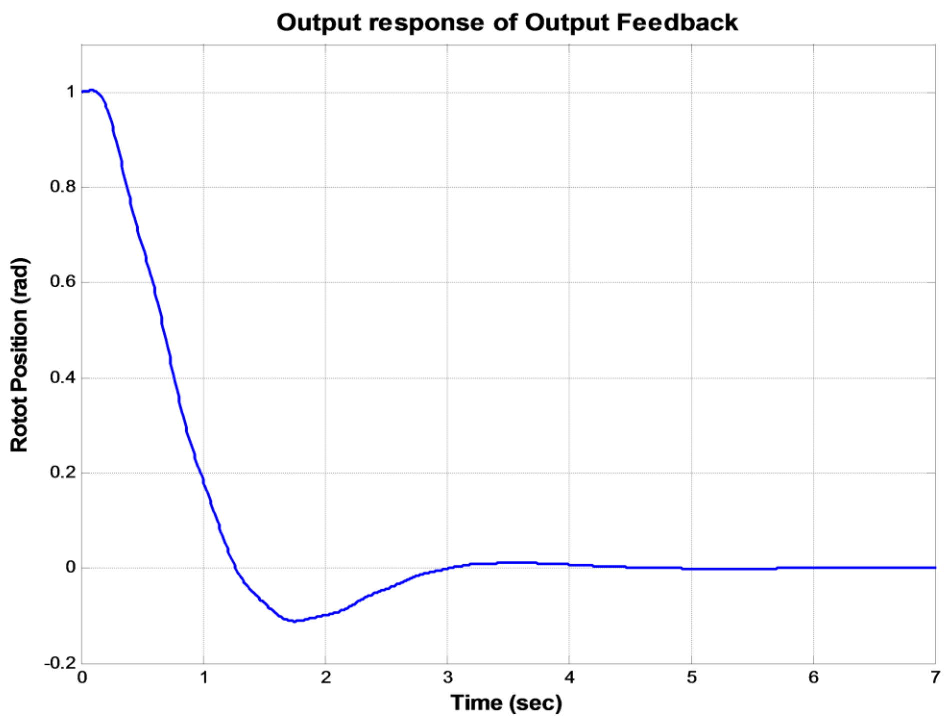

Figure 4.1: Output response of rotor angle for Output Feedback Scheme.

The angular position of rotor is successfully stabilized at the origin in relatively short time with good transient response. The transient response may further be tuned according to performance specifications by carrying the controller parameters. The response of the control input effort which is the input torque is shown in the figure 4.2.

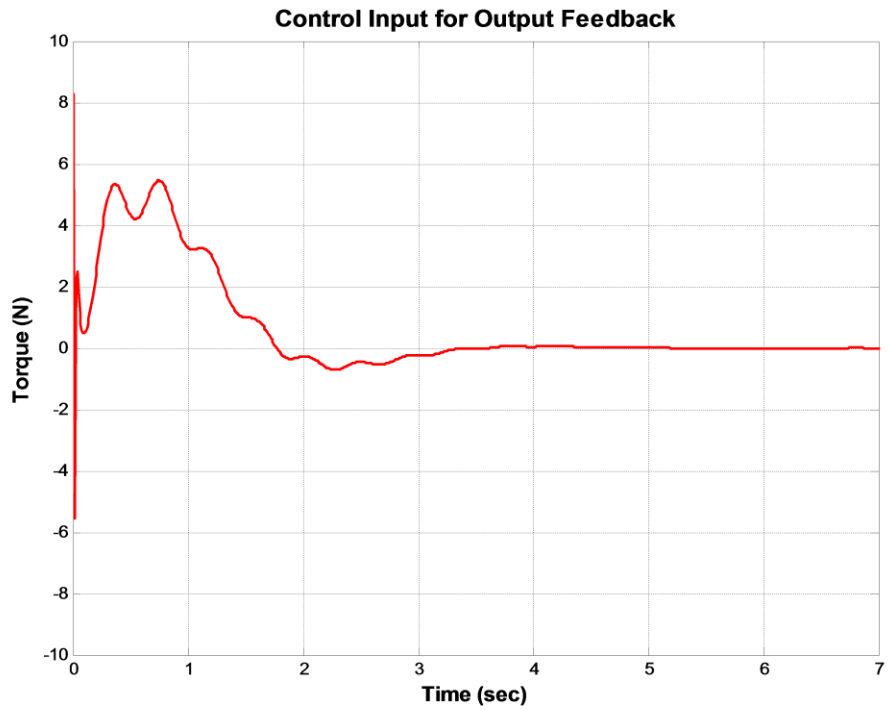

Figure 4.2: Control input Torque for Output Feedback scheme.

It is evident from the figure 4.2 that the optimal control input passes the saturation period in rapid time and the control input remains in the specified boundary and therefore obeying the input constraints. Moreover the figure 4.3 shows the estimation error which is the difference between the output feedback controller and state feedback controller for the internal state (link angular position). The estimation error for the output signal is shown in figure 4.4.

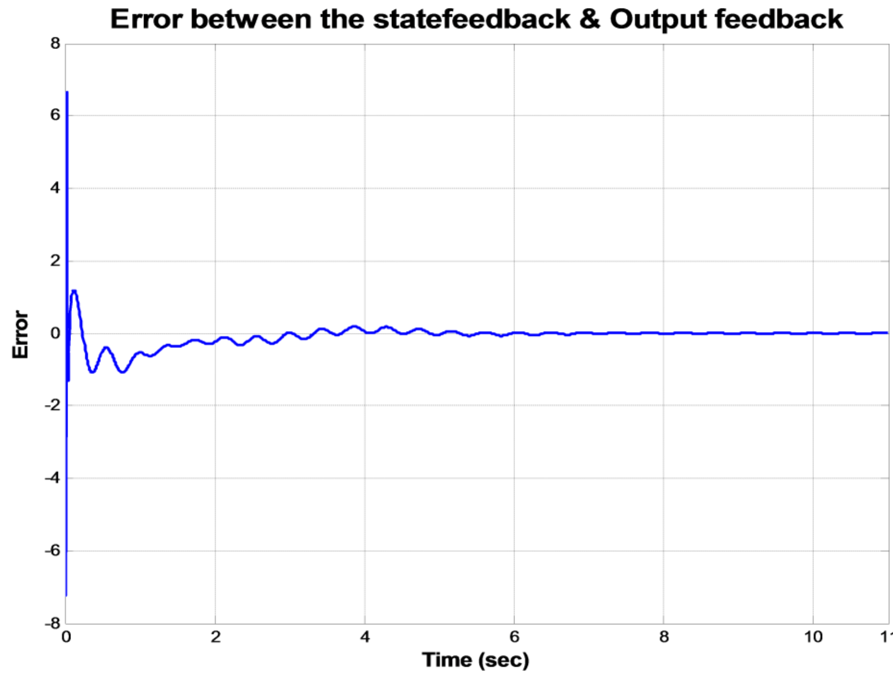

Figure 4.3: Error between the state feedback & output feedback for link angular position.

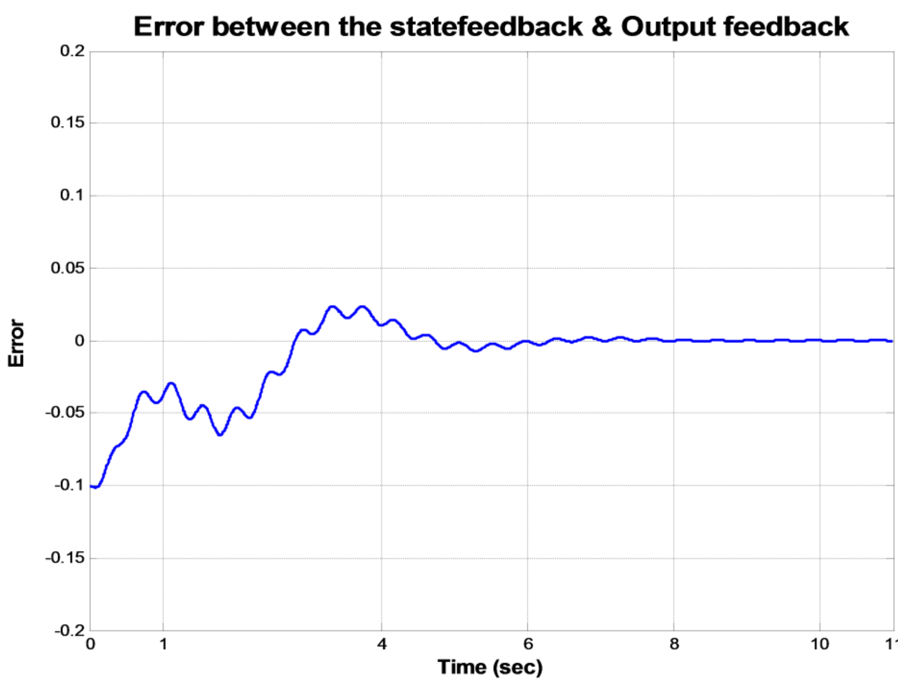

Figure 4.4: Error between the state feedback & output feedback for rotor angular position.

The observer estimates the system states efficiently. The internal states which were made observable with respect to virtual output by the observer are also satisfactorily estimated. Thus, the output feedback trajectories approach to the state feedback trajectories as the time passes. The proposed output feedback controller successfully stabilizes the single link flexible joint manipulator system to the origin.

Chapter 5
CONCLUSION AND FUTURE WORK

5.1 Conclusion

The output feedback control for nonlinear nonminimum phase systems is presented in this thesis that involves nonlinear model predictive controller (NMPC). The sampled data NMPC scheme is employed in which the control input is applied open loop between the sampling intervals. Here, a finite horizon optimal control problem (FHOCP) scheme is used and the stability of NMPC controller using a terminal constraint is briefly presented. However, the stability schemes for NMPC is not limited to one described here but can be adopted from a variety of available NMPC controller techniques reported in literature [67]. The observer used for the output feedback scheme includes combination of extended high gain observer and high gain observer. The extended high gain observer estimates the output and its derivatives and extends the dimension of observer by one state. This allows us to estimate the unknown state which is considered as virtual output to the remaining states of the system which are not estimated. The new estimated signal makes the remaining system states (internal dynamics) observable with respect to virtual output. The high gain observer is then applied to full order internal system to provide estimates of these internal states. The vital point here is to ensure that the extended high gain observer is fast enough to estimate the virtual output. This is achieved by setting the

observer parameter of order $O(1/e^2)$ for extended high gain observer and $O(1/e)$ for high gain observer. So, the extended high gain observer and high gain observer for internal dynamics together with the original close loop makes the output feedback in three time scale structure. The closed loop is then analyzed in these three time scales on the similar lines as in [23]. The required mathematical and theoretical foundation is established to achieve semi global practical stability of overall output feedback scheme.
The output feedback is simulated on an example of single link flexible joint manipulator system. When the sampling time and the observer parameter is set according to theoretical formulation, the semi global practical stability is achieved for a desired region which is the subset of the state feedback control's region of attraction. The close loop trajectories are made to converge to small area around the origin accomplishing practical stability.

5.2 Future Work

The research work covered in this thesis can be extended in the following directions:

- The scheme for the output feedback presented here is for single input single output (SISO) systems and it can be further extended to multi input multi output (MIMO) systems.
- The robustness analysis of the proposed output feedback scheme can be investigated in the presence of uncertainties and disturbance. As the observer used contains high gain observer which is generally considered as robust observer, further investigations can be done when it combined with sample data NMPC which also possess inherently some robustness properties as reported in [81].
- The efficient implementation and realization of output feedback scheme with NMPC can be investigated that involves real time implementation constraints such as computational delays, sub optimal solutions.
- Further investigation of semi global stability can be done for different NMPC techniques where the control input is not constant between the sampling instants.

- Another area could be the investigation of stability analysis of output feedback scheme for nonlinear systems not in normal form.

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