

**Effect of Longitudinal Tensile Reinforcement Ratio on  
Minimum Shear Reinforcement in Reinforced  
Concrete Beams - II**

**By**

**Muhammad Usman Hanif**

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Submitted by

**Muhammad Usman Hanif**

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**Brigadier (Retired) Dr. Khaliq-ur-Rashid Kayani**  
**National Institute of Civil Engineering**  
**School of Civil and Environmental Engineering**

**National University of Sciences and Technology Islamabad**

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**DEDICATED**  
**TO**  
**MY PARENTS, TEACHERS**  
**AND**  
**WELL WISHERS**

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## **ABSTRACT**

To avoid abrupt shear failure, ACI 318 – 08 specifies the requirements for the minimum amount of shear reinforcement for Reinforced Concrete Beams. The equation given for minimum shear reinforcement is based on previous experimental data from beams of normal and high strength concrete without any consideration for the effects of longitudinal reinforcement.

Minimum amount of shear reinforcement corresponds to a value that restrains the growth of inclined cracking and prevents a sudden shear failure. The growth and width of inclined cracks, beside other factors, is also influenced by longitudinal reinforcement. Therefore, with the change in longitudinal reinforcement ratio, requirement of minimum shear reinforcement should also change.

The influence of longitudinal tensile reinforcement ratio on minimum shear reinforcement in Reinforced Concrete Beams has been investigated through literature review and laboratory tests. The 6 x specimen beams having cross section 8" x 18" were divided into two groups and investigated for influence of longitudinal tensile reinforcement.

The test results indicated that reserve strength (shear strength of beam with minimum shear reinforcement / shear strength of beam without shear reinforcement) of tested beams increased as the longitudinal tensile reinforcement ratio increased. Therefore inclusion of factor  $\rho_l$  in the ACI code for determining minimum shear reinforcement should be considered.

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## LIST OF ABBREVIATIONS / NOTATIONS

$f'_c$	Specified compressive strength of concrete, psi
$V_{cz}$	The shear in the compression zone
$V_{ay}$	The vertical component of the shear transferred across the crack by interlock of aggregate particles on the two faces of the crack
$V_d$	The dowel action of the longitudinal reinforcement
$V_c$	Shear carried by concrete, lb
$b_w$	Web width, in.
$d$	Distance from extreme compression fiber to centroid of longitudinal tension reinforcement, in.
$A_v$	Area of shear reinforcement, in <sup>2</sup> .
$A_{v, \min}$	Minimum area of shear reinforcement, in <sup>2</sup> .
$f_y$	Specified yield strength of reinforcement, psi.
$f_{yt}$	Specified yield strength $f_y$ of transverse reinforcement, psi.
$\rho_\ell$	Longitudinal Tensile Reinforcement Ratio
$\rho_t$	Transverse Tensile Reinforcement Ratio
$s$	Center to center spacing of transverse reinforcement, in.
MCFT	Modified Compression Field Theory

## **INTRODUCTION**

### **1.1 GENERAL**

A beam resists loads primarily by means of internal moments,  $M$ , and shear,  $V$ . In the design of reinforced concrete member, flexure is addressed first, leading to the size of the section and arrangement of reinforcement to provide the necessary moment resistance. Limits are placed on the amounts of flexural reinforcement which can be used, to ensure that if failure was to occur, it would develop gradually, giving warning of the impending failure. The beam is then proportioned for shear. Because a shear failure is frequently sudden and brittle, the design for shear must ensure that the shear strength equals or exceeds the flexural strength at all points in the beam. The manner in which shear failures can occur varies widely with the dimensions, geometry, loading, and properties of the members. For this reason, there is no unique way to design for shear (McGregor 2005).

The prime objective of shear design is to prevent an abrupt shear failure by identifying where shear reinforcement is required and how much is it required. Shear reinforcement is provided in the form of stirrups. These stirrups link flexural tension and flexural compression sides of a member and ensures that the two sides act as a unit. Shear failures involve breakdown of this linkage and the opening of major diagonal crack in members without shear reinforcement.

## **1.2 BRIEF OVERVIEW OF DEVELOPMENT OF SHEAR DESIGN PROVISIONS**

ACI 318 – 51, based on allowable stresses, specified that web reinforcement must be provided for the excess shear if the shear stress at service loads exceeded  $0.03f'_c$ . Calculation of the area of shear reinforcement was based on 45 degree truss analogy. The 1955 shear failure of beams in two Air Force warehouses initiated the major review of traditional ACI design procedures. Followed by a thorough research the study of these failures clearly indicated that shear and diagonal tension was a complex problem (ACI 445 R – 99).

In the 25 years after 1960, considerable research was conducted to understand the beam action and arch action - mechanisms by which cracked reinforced concrete beams transmitted shear. Shear force in a region indicates that the moment is changing along the length of the member. “Moment can change either by the tension in the reinforcement changing, which is called beam action, or by the internal lever arm changing along the length, which is called arch action. If arch action is to carry the entire shear in a region that has constant shear, then the concrete compression zone must form an inclined strut going from the load to the support. It was shown that, because of the geometric incompatibility of the two mechanisms, with beam action typically being much stiffer than arch action, nearly all of the shear would be carried by beam action until this mechanism failed. After failure of the beam mechanism, an internal redistribution of stresses could occur and the remaining arch mechanism could then carry even higher shears if the distance between the applied load and the support was sufficiently short” (Fenwick / Paulay 1968).



Using detailed measurements of cracked beams and direct measurements on subassemblies, Fenwick / Paulay (1968) concluded that approximately 70% of the vertical shear at a flexurally cracked section is carried across the flexural cracks. For slender beams without stirrups, the breakdown of aggregate interlock action triggers the shear failure. The key role played by aggregate interlock in transferring shear stresses across inclined flexural cracks was confirmed by Taylor (1970), Kani / Huggins / Wittkopp (1979) and Sherwood / Bentz / Collins (2007). Walraven (1981) showed that the shear stress that can be transmitted across a crack by aggregate interlock decreases as crack width increases and as aggregate size decreases. Fig 1.1 – Appendix I shows how experimental shear research has varied intensely over the period (Collins / Bentz / Sherwood 2008).

### **1.3 MINIMUM SHEAR REINFORCEMENT**

Minimum amount of shear reinforcement corresponds to a value that restrains the growth of inclined cracking (substantially the growth of critical diagonal crack), providing required ductility and preventing a sudden shear failure. A number of factors that influence the determination of the required minimum amount of shear reinforcement have not yet been fully explored and understood. As a consequence, the current ACI code provisions are still based on semi empirical considerations (Zarais 2003).

To avoid abrupt shear failure, ACI 318 – 08 specifies that minimum amount of shear reinforcement must be provided in Reinforced Concrete Beams. The equation given by ACI for minimum shear reinforcement is based on previous experimental data from beams of normal and high strength concrete with little consideration for the effects of longitudinal reinforcement (Lee / Kim 2008).

The ultimate goal of specifying minimum shear reinforcement is to ensure that total shear is transferred across cracks by aggregate interlock and prevent excessive crack opening that can cause reduction in interface shear transfer. The growth and width of inclined cracks, beside other factors is also influenced by longitudinal reinforcement. Therefore, with the change in longitudinal reinforcement ratio, requirement of minimum shear reinforcement should also change.

#### **1.4 OBJECTIVE**

The objective of this research is to study the effect of longitudinal tensile reinforcement ratio on minimum shear reinforcement in Reinforced Concrete Beams.

## **MINIMUM SHEAR REINFORCEMENT**

### **2.1 SHEAR STRENGTH (V<sub>c</sub>) PROVIDED BY CONCRETE**

In beam without stirrups, shear is transferred across inclined crack by  $V_{cz}$ , the shear in the compression zone, by  $V_{ay}$ , the vertical component of the shear transferred across the crack by interlock of aggregate particles, and by  $V_d$ , the dowel action of the longitudinal reinforcement. Diagrammatically the detail of these forces is given in Figure 2.1 – Appendix II. In the ACI Code  $V_{cz}$ ,  $V_{ay}$  and  $V_d$  are lumped together as  $V_c$ , which is referred to as “shear carried by concrete” and for normal weight concrete is given by following equation (ACI Equation 11-3).

$$V_c = 2\sqrt{f'_c} b_w d \quad (2.1)$$

The above mentioned ACI equation for  $V_c$  becomes less conservative as member depth  $d$  increases; as concrete strength  $f'_c$  increases; as maximum aggregate size decreases; and as stress in the longitudinal reinforcement  $f_s$  increases. The ACI requirement of minimum shear reinforcement where  $V_u$  exceeds  $0.5\phi V_c$  mitigates the consequences as a result of these trends (Collins / Bentz / Sherwood 2008).

### **2.2 MINIMUM SHEAR REINFORCEMENT**

In beams with at least minimum shear reinforcement, the stirrups holds the crack faces together so that the shear transfer across the cracks by aggregate interlock is not lost. Where required, the minimum shear reinforcement shall be computed by the equations (ACI Section 11.4.6.3) reproduced below. Equation 2.2 is new in the code and

was introduced in ACI 318-05 to account for the influence of compressive strength of concrete.

$$A_{v, \min} = 0.75 \sqrt{f'_c} \frac{b_w s}{f_{yt}} \quad (2.2)$$

But not less than

$$A_{v, \min} = \frac{50 b_w s}{f_{yt}} \quad (2.3)$$

Spacing of shear reinforcement placed perpendicular to axis of member has been restricted to  $d/2$  in non-prestressed members, or 24 in. This requirement ensures that any potential diagonal crack that may develop will be intercepted by a vertical stirrup.

## **2.3 DIAGONAL CRACK WIDTH AND MINIMUM SHEAR REINFORCEMENT**

### **2.3.1 Diagonal Cracking**

A crack will form in concrete when the principal tensile stress at some location reaches the cracking strength of concrete. The crack will form normal to the direction of principal tensile stress. For members subjected to pure axial tension or to pure flexure, the principal tensile stresses are parallel to the longitudinal axis of the member and cracks form perpendicular to the member axis. If a cross section of a member is subjected to shear stresses, the principal tensile stress directions are inclined to the longitudinal axis of the member. A crack forms at a location where significant shear stresses exist, and is inclined to the member axis. Such cracks are called diagonal cracks. Diagrammatically such a crack is shown in Figure 2.2 – Appendix II (Collins / Mitchell 1997).

### 2.3.2 Diagonal Crack Width

“The inclined crack width cannot be predicted by calculating principal stresses in an uncracked beam. Their slope, spacing and width depends on many factors like flexural and shear reinforcing steel areas, shape and dimension of cross section, shear stresses and mechanical properties of concrete and steel. Hence the control of inclined cracking width can be exercised using empirical equations, based on empirical works only.” (Jensen / Lapko 2009). Over the period, various research studies as mentioned below have been carried out to find the empirical expression for determining crack widths.

- Borishansky (1964) assumed that mean inclined crack width is directly proportional to strain in stirrups and crack spacing.
- Placas and Regan (1971) concluded that maximum crack width is directly proportional to spacing of stirrups and inversely proportional to  $A_v$ ,  $(f'_c)^{1/3}$  and  $d$ .
- Bentz, Vecchio and Collins (2006), reasoned out in MCFT that crack width is equal to the product of crack spacing and principal tensile strain.
- More recently, Muttoni and Ruiz (2008) stated that critical crack width is proportional to the product of longitudinal strain in the control depth  $(0.6d)$  times effective depth of element.

### 2.3.3 Minimum Shear Reinforcement and Interface Shear Transfer

In the light of detailed discussions on the subject by joint ASCE-ACI Task Committee 426, it has been shown that in beams without web reinforcement, 20 to 30 percent of the total shear is resisted by the compression zone, 15 to 25 percent of the

shear resistance is provided by the dowel action, and 30 to 60 percent is contributed by interface shear resistance across a crack. These resistance ranges demonstrate that interface shear contributes a significant share of shear resistance; therefore, minimum shear reinforcement must ensure full shear transfer across cracks by aggregate interlock. Also, such requirements should prevent excessive crack opening that can cause a reduction in interface shear transfer (Karuthammer 1992).

## **2.4 CONCLUSIONS FROM PREVIOUS EXPERIMENTAL PROGRAM**

Lee and Kim (2008) have presented test results of 26 reinforced concrete beams having minimum shear reinforcement. Three parameters were considered in the investigation: longitudinal tensile reinforcement ratio, shear span-depth ratio ( $a/d$ ) and compressive strength of concrete. The influence of three parameters was investigated by considering the reserve strength and the reserve deflection beyond diagonal cracking. In the study, **reserve strength** is defined as the ratio between the ultimate shear capacity of the beams with the minimum shear reinforcement and that of the beams without shear reinforcement. Likewise, **reserve deflection** is defined as the ratio between the deflection corresponding to the ultimate load of beams with minimum shear reinforcement and the deflection of beams without shear reinforcement. Salient of the conclusions drawn by Lee and Kim are appended below

- Reserve strength and reserve deflection are related with the longitudinal ratio  $\rho_\ell$ .
- The existing design codes contain an expression for the minimum shear reinforcement that depends on neither the longitudinal tensile

reinforcement ratio nor the  $a/d$ . This may lead to the reserve strength and the reserve deflection of a reinforced concrete beam with a small  $\rho_\ell f_y$  being lower than that of a reinforced concrete beam with a large  $\rho_\ell f_y$ , resulting in an inadequate shear behavior after cracking.

- The test result of reinforced concrete beams having the minimum shear reinforcements showed a 16% difference in the reserve strengths between beams having  $\rho_\ell$  equal to 0.0093 and 0.0279. Thus, inclusion of factor  $\rho_\ell$  in the design codes for determining minimum shear reinforcement should be considered.
- The amount of minimum shear reinforcement needs to increase (decrease) as  $\rho_\ell$  decreases (increases) to achieve uniform reserve strength and deflection.

## **EXPERIMENTAL PROGRAM**

### **3.1 GENERAL**

A brief on the materials used and experimental / testing procedures followed for the research are summarized in the succeeding paragraphs.

### **3.2 MIX DESIGN**

Concrete strength in conjunction with member size, were so selected that minimum shear reinforcement requirement is governed by the empirical equation 2.2 and not by the spacing requirements (equation 2.4). In the study,  $f'_c$  was selected as 6000 psi. The mix design (Table 3.1 – Appendix III), was used from another study carried out in NICE, NUST.

### **3.3 MATERIALS**

#### **3.3.1 Cement**

The Type I cement conforming to ASTM C 150–04 was used. Results of the tests carried out to ascertain the properties of cement are presented in Table 3.2 – Appendix III. Variation in the chemical composition and physical properties of the cement affect concrete compressive strength more than variations in any other single material.

#### **3.3.2 Fine Aggregate**

Sand from Qibla Bandi was used. Results of the tests conducted for verification of properties of sand are tabulated in Table 3.3 – Appendix III. The gradation of the fine



aggregate is tabulated in Table 3.4 -Appendix III, and graphically shown in Figure 3.1 – Appendix III.

### **3.3.3 Coarse Aggregate**

Aggregate from Margalla site was used in this research. Maximum size for the aggregate was kept as 12.7 mm. For gradation purpose, only three sizes were considered i.e.12.7 mm, 9.5 mm, 4.75 mm. The laboratory test results are tabulated in Table 3.5 – Appendix III. The gradation and sieve analysis was determined in accordance with ASTM C 136 – 01 and tabulated in Table 3.6 – Appendix III, and graphically illustrated in Figure 3.2 – Appendix III.

### **3.3.4 Reinforcing Steel**

# 8 and # 6 bars were used as longitudinal tensile and compressive reinforcement respectively. # 2 bar was used as shear reinforcement. The grade 60 and grade 40 steel was used for longitudinal and shear reinforcement respectively. Stress-strain diagram of #8 bar is shown in Figure 3.4 – Appendix III.

### **3.3.5 High Range Water Reducing Agent**

Glenium 51 (product of BASF Chemical Company), is a high performance concrete superplasticizer based on modified polycarboxylic ether, was used in the research. The dosage was kept constant throughout the research work as 1.2 % by weight of cement. The technical data of Glenium 51 is tabulated in Table 3.7 – Appendix III.

### **3.3.6 Water**

Fresh water available in the university was used as mixing water and for curing of concrete.

### **3.4 CASTING OF SPECIMEN**

Casting of specimens was carried out as per ASTM C 192 – 02. Six beams were prepared along with 5 cylinders (two 6" x 12" and three 4" x 8") for each beam.

#### **3.4.1 Description of Specimens**

Six reinforced concrete beams were cast to investigate the influence of longitudinal tensile steel reinforcement ratio  $\rho_\ell$ . These beams having different longitudinal tensile steel reinforcement ratios  $\rho_\ell = 0.978\%$ ,  $1.769\%$  and  $2.431\%$ , were divided into two groups depending upon the amount of shear reinforcement ratio. Group-A did not have any shear reinforcement, whereas Group-B had minimum amount of shear reinforcement ratio as required by ACI 318 – 08. The cross-sectional dimension of all the six beams was 8" x 18". The specification of specimens and material properties are shown in Table 3.8 – Appendix III, and diagrammatically illustrated in Figure 3.3 – Appendix III. Test results of concrete cylinder strength are shown in Table 3.9 – Appendix III.

#### **3.4.2 Fabrication of Specimens**

The specimens were cast in 3/4" thick plywood shuttering. The shuttering was prepared in such a manner that it could be dismantled easily. The steel reinforcement cage was placed in the shuttering over the 0.5" spacers and tied up with the bars. The concrete for the beams was mixed in a rotary mixer. In one batch, 1 bag of cement was used. Three standard cylinders (6" x 12") and two standard cylinders (4" x 8") were cast for each beam. The shuttering was removed from beams after 48 hours. Beams (B4A to B6A) were cast on 12 January 2011 and Beams (B4B to B6B) were cast on 19 January

2011. Hessian cloth was placed on the beams and cured under shade along with the test cylinders.

## **3.5 TESTING OF SPECIMENS**

### **3.5.1 Test Setup**

The specimens were transported to Structural Laboratory of University of Engineering and Technology Lahore, where a Reaction Frame made from I-Girders having 100 ton load capacity is installed. The load was applied through a hydraulic jack and pump having 100 ton capacity. A pressure gauge having maximum reading of 400 kg/cm<sup>2</sup> and least count of 10 kg/cm<sup>2</sup>, was attached to hydraulic pump to control the loading. The load was applied at centre point in 10 kg/cm<sup>2</sup> interval. Three deflection gauges were placed under the beam at mid span and at quarter points. The beams were placed on the supports with the help of a gantry crane. Diagrammatically the test setup is shown in Figure 3.5 to Figure 3.8 Appendix III.

### **3.5.2 Testing Procedure**

The beams were planned to be tested under one point load. The load was applied after centering and aligning the specimens on the test setup and making all necessary arrangements for recording the load and deflection. The load was applied in increments of 10 kg/cm<sup>2</sup>, and deflections recorded at each load increment. During the application of load, the cracks were observed and marked on the beams.

## **EXPERIMENTAL RESULTS**

### **4.1 GENERAL**

The load was applied at midspan of beams in increments of 10 kg/cm<sup>2</sup>. Each test was carried out in approximately one hour. At each load increment, deflections were recorded and cracks observed / marked before applying next load increment. Five cylinders were cast for each beam during casting of the specimens. Three cylinders (6" x 12") were tested after 28 x days and two cylinders (4" x 8") were tested on the day of testing of beams (Table 4.1 – Appendix IV).

### **4.2 TEST BEHAVIOR OF SPECIMENS**

#### **4.2.1 Specimen B4A**

The beam was loaded on 1 March 2011. The initial flexural cracks appeared at 11.43 Ton load. Increased load widened the cracks and additional cracks appeared between 13.33 Ton and 17.14 Ton loads. Inclined cracks appeared at 19.05 Ton load and the beam failed abruptly at 20 Ton load. Load deflection data plot are shown in Table 4.2 and Figure 4.1 – Appendix IV, respectively. The crack pattern of beam is shown in the Figure 4.4 – Appendix IV.

#### **4.2.2 Specimen B4B**

The beam was loaded on 28 February 2011. The initial flexural cracks appeared at 13.33 Ton load. Increased load widened the cracks and additional cracks appeared at 19.05 Ton load. First inclined crack appeared at 22.86 Ton load after which

more flexural cracks were observed and old cracks also propagated. Inclined cracks kept on appearing along with propagation of few flexural cracks. The beam developed several cracks before failing at 31.43 Ton load. Load deflection data plot are shown in Table 4.3 and Figure 4.1 – Appendix IV, respectively. The crack pattern of beam is shown in the Figure 4.5 – Appendix IV.

#### **4.2.3 Specimen B5A**

The beam was loaded on 1 March 2011. The initial flexural cracks appeared at 13.33 Ton load. Inclined cracks appeared at 19.05 Ton load and the beam failed abruptly at 26.67 Ton load. Load deflection data plot are shown in Table 4.4 and Figure 4.2 – Appendix IV, respectively. The crack pattern of beam is shown in the Figure 4.6 – Appendix IV.

#### **4.2.4 Specimen B5B**

The beam was loaded on 27 February 2011. The initial flexural cracks appeared at 13.33 Ton load. Increased load widened the cracks and additional cracks appeared at 19.05 Ton load. First inclined crack appeared at 22.86 Ton load after which more flexural cracks were observed and old cracks also propagated. Inclined cracks kept on appearing along with propagation of few flexural cracks. The beam developed several cracks before failing at 49.52 Ton load. Load deflection data plot are shown in Table 4.5 and Figure 4.2 – Appendix IV, respectively. The crack pattern of beam is shown in the Figure 4.7 – Appendix IV.

#### **4.2.5 Specimen B6A**

The beam was loaded on 28 February 2011. The initial flexural cracks appeared at 15.24 Ton load. Increased load widened the cracks and additional cracks appeared between 19.05 Ton and 22.86 Ton loads. Inclined cracks appeared at 24.76 Ton load and the beam failed abruptly at 27.24 Ton load. Load deflection data plot are shown in Table 4.6 and Figure 4.3 – Appendix IV, respectively. The crack pattern of beam is shown in the Figure 4.8 – Appendix IV.

#### **4.2.6 Specimen B6B**

The beam was loaded on 27 February 2011. The initial flexural cracks appeared at 17.14 Ton load. Increased load widened the cracks and additional cracks appeared between 19.05 Ton and 22.86 Ton load. First inclined crack appeared at 26.67 Ton load after which more flexural cracks were observed and old cracks also propagated. Inclined cracks kept on appearing along with propagation of few flexural cracks. The beam developed several cracks before failing at 49.52 Ton load. Load deflection data plot are shown in Table 4.7 and Figure 4.3 – Appendix IV, respectively. The crack pattern of beam is shown in the Figure 4.9 – Appendix IV.

#### **4.2.7 Summary**

Behavior of beams can be briefly described as under:

- Initial flexural cracks appeared between 11.43 Ton and 17.14 Ton load.
- As the load increased, some of these cracks were gradually inclined towards the loading point.

- The beams without shear reinforcement (the beams in Group-A) exhibited a sudden failure induced by diagonal cracking.
- The beams having minimum shear reinforcement (the beams in Group-B) showed several flexural diagonal cracks.
- The location of significant diagonal crack was between load and support points.
- The crack angle was observed to be between  $35^\circ$  and  $45^\circ$ .
- A drop in the applied load was observed in Group-B beams, when cracks approached the loading point.

## **ANALYSIS AND INTERPRETATION OF TEST**

### **RESULTS**

#### **5.1 GENERAL**

The objective of the tests was to evaluate the influence of the longitudinal tensile reinforcement ratio on the response of beams having minimum shear reinforcement. The influence of this parameter was investigated by considering the reserve strength and the reserve deflection of the beams.

Beams in Group – A (beams without shear reinforcement) exhibited sudden failure induced by diagonal cracking. The location of significant diagonal crack was between load and support points. A sudden drop in the applied load was observed after reaching the maximum load and beams failed due to extended diagonal crack widths.

Beams in Group – B (beams having minimum shear reinforcement) developed several diagonal cracks before failure. Small flexural cracks developed at or near the mid span during the early stages of the loading. These cracks appeared at the bottom of the beam where the flexural stresses were maximum. The flexural cracks developed over the entire length of the beam with increase in load and changed their orientation as they propagated above the longitudinal reinforcement. After reaching the maximum load, there was significant increase in deflection accompanied by a small increase / decrease in the load.



## 5.2 INTERPRETATION OF TEST RESULTS

### 5.2.1 Load – Deflection Relationship

The load – deflection curves at mid span are shown in Figure 4.1 to Figure 4.3 - Appendix IV. After reaching the maximum load, the beams showed sudden drop in the load followed by increase in deflection for Group B Beams and sudden failure in Group A Beams. Since, the readings were recorded manually, only two to three readings were taken just prior to failure. This trend is reflected in the aforementioned graphs in the descending portion of the curve shown as dotted lines. The beams without shear reinforcement (Group – A) exhibited sudden failure induced by diagonal cracking. The ultimate load and the midspan deflection of Group – B beams was much greater than those of beams without shear reinforcement (Group – A) and increased as the longitudinal tensile reinforcement ratio increased.

### 5.2.2 Moment – Curvature Relationship

Moment – curvature ( $M - \emptyset$ ) curve is shown from Figure 5.1 to Figure 5.3 - Appendix V and details given in Table 5.1 - Appendix V. These curves illustrate theoretical and experimental  $M - \emptyset$  relationships of the six beams. The experimental  $M - \emptyset$  curve has been developed from load – deflection data as under.

- For each load point, moment is determined from the load.
- From the three deflection gauges placed under the beams at third points, the deflected shape has been developed for each load increment and shown from Figure 5.4 to Figure 5.9 - Appendix V.
- From the deflected shape, the equation of curve is determined.

- For curvature values at a given load point, second order derivative of deflection equations are calculated.

### 5.2.3 Reserve Strength and Reserve Deflection

In the study, reserve strength is defined as the ratio between the ultimate shear capacity of the beams with the minimum shear reinforcement and that of the beams without shear reinforcement i.e.  $V_b / V_a$ . Likewise, reserve deflection is defined as the ratio between the deflection corresponding to the ultimate load of beams with shear reinforcement and the deflection of beams without shear reinforcement i.e.  $\Delta_b / \Delta_a$ .

### 5.2.4 Behavior of Beams

The reserve strength and deflections are given in detail in Table 5.2 – Appendix V. Figure 5.10 – Appendix V illustrates the relationship between longitudinal tensile reinforcement ratio and reserve strength. Clearly, the reserve strength increases as the amount of longitudinal tensile reinforcement ratio increases. Beams (B4B to B6B) designed according to ACI 318-08 shared the same amount of minimum shear reinforcement ratio  $\rho_t = 0.00154$ . However, the reserve strength of beams B4B, B5B and B6B increased with the increase in  $\rho_\ell$ . The reserve strength ratios of beams B4B, B5B and B6B were 1.51, 1.77 and 1.80 respectively.

The increase in the amount of longitudinal tensile reinforcement ratio also increased the reserve deflection of the beams. The reserve deflections of beams B4B, B5B and B6B are 1.82, 1.80 and 2.62 respectively. Figure 5.11 - Appendix V illustrates the relationship between longitudinal tensile reinforcement ratio and reserve deflection.

### 5.2.5 Analysis of Test Results

Based on the test results of the six reinforced concrete beams, following inferences can be clearly drawn:-

- The load – deflection plot shows that the beams having minimum amount of shear reinforcement (Group – B beams) resist more loads and hence exhibit more deflections before failure as compared to the beams without shear reinforcement
- The moment – curvature plot indicates that the resisting moment capacity and curvature of beam is more in beams having minimum amount of shear reinforcement than beams without shear reinforcement.
- With the increase in longitudinal tensile reinforcement ratio, there is less propagation as well as widening of cracks which results in increase in shear strength of the member.
- With the increase in longitudinal tensile reinforcement ratio, reserve strength and reserve deflections also increase. Thus the reserve strength and the reserve deflection of a reinforced concrete beam with a small  $\rho_\ell$  tends to be lower than that of a reinforced concrete beam with a large  $\rho_\ell$ .
- In ACI 318 – 05, the equation for calculating minimum shear reinforcement was modified to include the effect of concrete compressive strength; however the influence of longitudinal tensile reinforcement ratio still needs to be incorporated in the provisions for minimum shear reinforcement.

## 5.3 MINIMUM SHEAR REINFORCEMENT PARAMETERS

### 5.3.1 Other Test Programs

Lee and Kim (2008) presented test results of 26 reinforced concrete beams having minimum shear reinforcement. In the study, 12 beams with different longitudinal tensile reinforcement ratios were tested to study the effect of longitudinal tensile reinforcement ratio on minimum shear reinforcement as described in Chapter 2. Similarly, Javaid (2011) has presented test results of 6 reinforced concrete beams having minimum shear reinforcement and different longitudinal tensile reinforcement.

These available test results have been recorded as in Table 5.3 – Appendix V and for analysis the rudimentary evaluation of these data leads to the influencing factors for establishing requirements of minimum shear reinforcement.

These factors i.e.  $f'_c$ ,  $a/d$ ,  $f_{yt}$ ,  $f_{y\ell}$  and  $\rho_\ell$  must be investigated for their individual as well as collective relationships with Reserve Strength ( $V_b/V_a$ ).

### 5.3.2 Concrete Compressive Strength ( $f'_c$ )

The Reserve Strength and Reserve Deflection seem to be proportional to  $\sqrt{f'_c}$  as shown in Figure 5.12 – Appendix V and Figure 5.13 – Appendix V respectively. Though, the scatter in both the graphs does not follow any regular trend or pattern.

### 5.3.3 Shear Span to Depth Ratio ( $a/d$ )

The Reserve Strength and reserve deflection seem to be inversely proportional to Shear Span to Depth Ratio ( $a/d$ ) as shown in Figure 5.14 – Appendix V and Figure 5.15 – Appendix V respectively. The trend in the graphs show a pattern of decreasing trend of reserve strength and reserve deflection with increase in shear span to depth ratio.

### 5.3.4 Longitudinal and Transverse Reinforcement Tensile Strength Ratio ( $f_{y\ell}/f_{yt}$ )

The Reserve Strength and reserve deflection seem to have not any significant effect on Longitudinal and Transverse Reinforcement tensile strength ratio ( $f_{y\ell}/f_{yt}$ ) as shown in Figure 5.16 – Appendix V and Figure 5.17 – Appendix V respectively.

### 5.3.5 Longitudinal Tensile Reinforcement Ratio ( $\rho_\ell$ )

The Reserve Strength and reserve deflection seem to be proportional to Longitudinal Tensile Reinforcement Ratio ( $\rho_\ell$ ) as shown in Figure 5.18 – Appendix V and Figure 5.19 – Appendix V respectively.

### 5.3.6 Minimum Shear Reinforcement Parameters in Combinations

As the scattered pattern in the aforementioned paras does not give any clear picture of correlation between different tests so these parameters were incorporated together in combinations to find a correlation. Different combinations were tried to reach at some correlation between the reserve strength and the related parameters (as illustrated in Figure 5.20 - Appendix V and Figure 5.21 - Appendix V). Resultantly the plot of  $\rho_\ell$  vs.  $\frac{V_b}{V_a} \cdot \frac{a}{d} \cdot \frac{f_{y\ell}}{f_{yt}} \cdot \sqrt{f'_c}$  and  $\frac{\Delta_b}{\Delta_a} \cdot \frac{a}{d} \cdot \frac{f_{y\ell}}{f_{yt}} \cdot \sqrt{f'_c}$  presented in Figure 5.22 – Appendix V and Figure 5.23 – Appendix V respectively show a more closer pattern towards finding a correlation between these variables.

## 5.4 COMPARISON WITH ACI CODE 318 - 08

ACI 318 – 08 provide the expression for minimum shear reinforcement ratio that takes into account the effect of concrete compressive strength and yield stress of shear

reinforcement. However, the test results indicate that the amount of minimum shear reinforcement needs to increase (decrease) as  $\rho_\ell$  decreases (increases).

## 5.5 CONCLUSIONS

Six reinforced concrete beams were tested to verify the influence of longitudinal tensile reinforcement ratio on minimum shear reinforcement. Based on the test results, following conclusions are drawn:-

- Reserve strength and reserve deflection are positively related with the longitudinal ratio  $\rho_\ell$ .
- The ACI 318 - 08 contains an expression for the minimum shear reinforcement that does not take into account the effect of longitudinal tensile reinforcement ratio. This may lead to the reserve strength and the reserve deflection of a reinforced concrete beam with a small  $\rho_\ell f_y$  being lower than that of a reinforced concrete beam with a large  $\rho_\ell f_y$ .
- In order to have uniform reserve strength, inclusion of factor  $\rho_\ell$  in the ACI code for determining minimum shear reinforcement should be considered.

## 5.6 RECOMMENDATIONS FOR FUTURE WORK

Mechanism of shear resistance is yet to be understood. Thus, it is imperative to undertake extensive research on the subject. Following is recommended for future studies:-

- Further experimental work is needed to confirm the test results of this study.
- Relation of inclined cracks and crack widths with the longitudinal steel.
- Effect of  $a/d$  on the minimum shear reinforcement.

# **APPENDIX – I**

# APPENDIX I

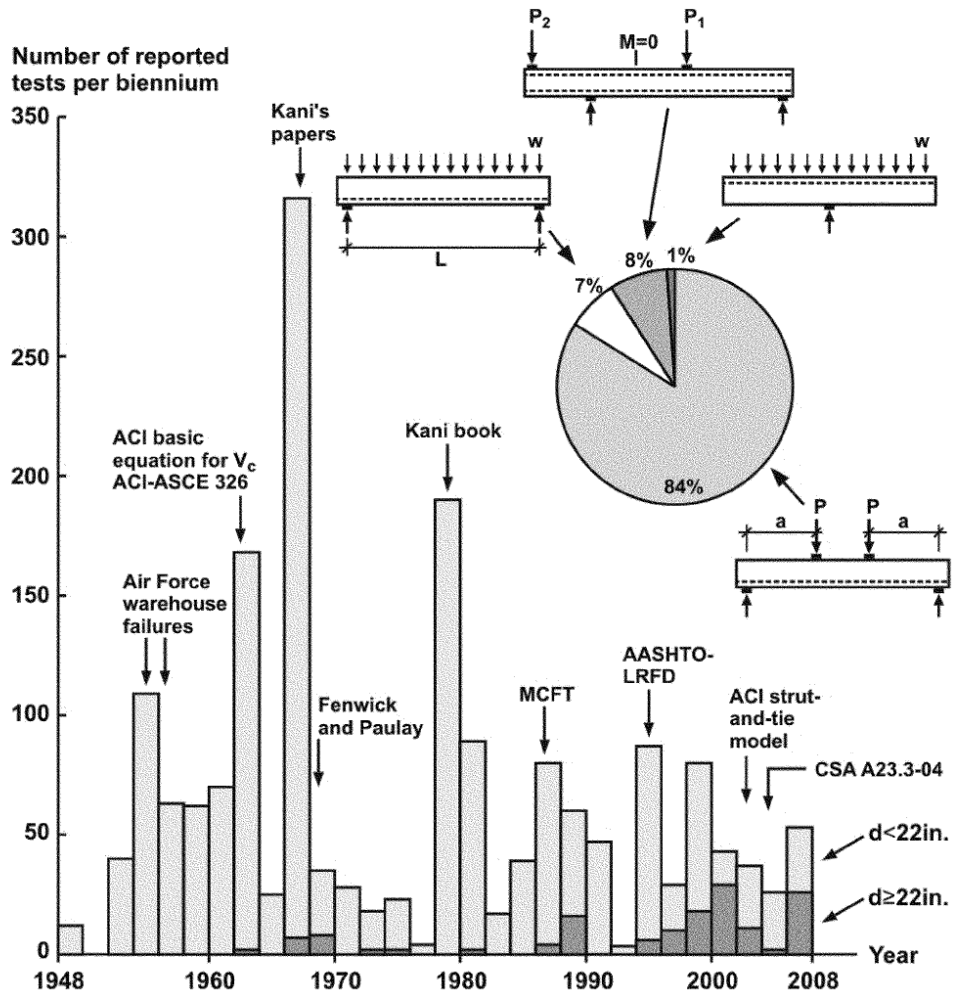


Fig 1.1 Sixty Years of Shear Research



## **APPENDIX – II**

## APPENDIX II

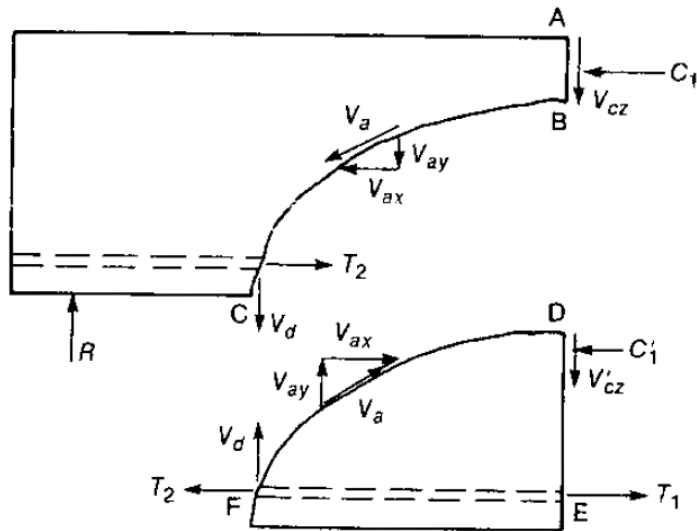


Fig 2.1 Internal Forces in a Cracked Beam without Stirrups

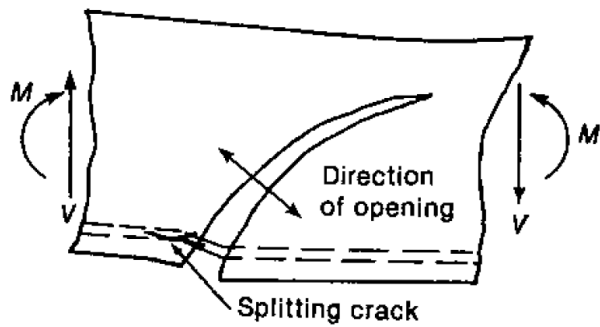


Fig 2.2 Opening of Diagonal Crack

## **Appendix-III**

## APPENDIX III

**Table 3.1 Mix Design**

Description	Details
Cement	455 Kg / m <sup>3</sup>
Fine Aggregate	724 Kg / m <sup>3</sup>
Coarse Aggregate	1063 Kg / m <sup>3</sup>
W/C ratio	0.37
Super-plasticizer	1.2 % by weight of cement
Mix ratio	1:1.59:2.34

**Table 3.2 Properties of Cement**

Tests	Test results	Specifications
Specific gravity	3.10	ASTM C 188 – 95
Initial setting time	150 minutes at 17 <sup>0</sup> C	ASTM C 191 – 01
Final setting time	390 minutes at 17 <sup>0</sup> C	ASTM C 191 – 01

**Table 3.3 Properties of Fine Aggregate**

Tests	Test results	Specifications
Specific gravity	2.65	ASTM C 128 – 01
Absorption	1.1%	ASTM C 128 – 01
FM	2.45	ASTM C 33 – 02

**Table 3.4 Gradation of Fine Aggregate**

Sieve No	Weight Retained (gm)	Percent Retained	Cumulative Percent Retained	Percent Passing	
				Actual	ASTM C 33 - 02
#4	2	0.2	0.2	99.8	95 - 100
#8	18	1.8	2	98	80 - 100
#16	130	13	15	85	50 - 85
#30	300	30	45	55	25 - 60
#50	413	41.3	86.3	13.7	5 - 30
#100	100	10	96.3	3.7	0 - 10
#200	34	3.4	99.7	0.3	
Pan	3	0.3	100	0	

**Table 3.5 Properties of Coarse Aggregate**

Detail of tests	Test results
Impact value (percent)	13.9
Crushing value(percent)	20.2
Abrasion value(percent)	15.2
Specific gravity	2.67

**Table 3.6 Gradation of Coarse Aggregate**

Sieve Size (mm)	Weight Retained (gm)	Percent Retained	Cumulative Percent Retained	Percent Passing	
				Actual	ASTM C 33 - 02
19	0	0	0	100	100
12.5	80	8	8	92	90 - 100
9.5	401	40.1	48.1	51.9	40 - 70
4.75	494	49.4	97.5	2.5	0 - 15
2.36	25	2.5	100	0	0 - 5

**Table 3.7 Technical Data – High Range Water Reducing Agent**

Description	Details
Name	Glenium 51
Form	Viscous liquid
Color	Light brown
Specific gravity	$1.08 \pm 0.02 \text{ g/cm}^3$
pH-value	$7.0 \pm 1$
Alkali content (%)	Less than or equal to 5.0
Chloride content (%)	Less than or equal to 0.10

**Table 3.8 Specification of Specimens and Material Properties**

Beams	$f'_c$ (psi)	Longitudinal tensile bars			Shear steel bars		a/d	d (in)
		No	$\rho_l$ (%)	$f_{ty}$ (ksi)	No.	$f_{ty}$ (ksi)		
Group-A								
B4A	5409	3 # 6	0.98	64	-	-	2.8	16.9
B5A	5409	3 # 8	1.77	64	-	-	2.9	16.8
B6A	5409	4 # 8	2.43	64	-	-	3	16.3
Group-B								
B4B	5409	3 # 6	0.98	64	# 2 @ 8.5" c/c	40	2.8	16.9
B5B	5409	3 # 8	1.77	64	# 2 @ 8.5" c/c	40	2.9	16.8
B6B	5409	4 # 8	2.43	64	# 2 @ 8.5" c/c	40	3	16.3

**Table 3.9 28 Days Compressive Strength of Cylinders**

<b>Specimen</b>	<b>Size of cylinders (in)</b>	<b>28 days compressive strength (psi)</b>
B 4 A – 1	6 x 12	4670
B 4 A – 2	6 x 12	5496.5
B 4 A – 3	6 x 12	4481.5
B 4 B – 1	6 x 12	3880
B 4 B – 2	6 x 12	5264.5
B 4 B – 3	6 x 12	5409.5
B 5 A – 1	6 x 12	4903.5
B 5 A – 2	6 x 12	5572
B 5 A – 3	6 x 12	6655.5
B 5 B – 1	6 x 12	4670
B 5 B – 2	6 x 12	5872.5
B 5 B – 3	6 x 12	5510
B 6 A – 1	6 x 12	4526.5
B 6 A – 2	6 x 12	4655.5
B 6 A – 3	6 x 12	4916.5
B 6 B – 1	6 x 12	4945.5
B 6 B – 2	6 x 12	6061
B 6 B – 3	6 x 12	4080
<b>Average</b>		<b>5087.25</b>

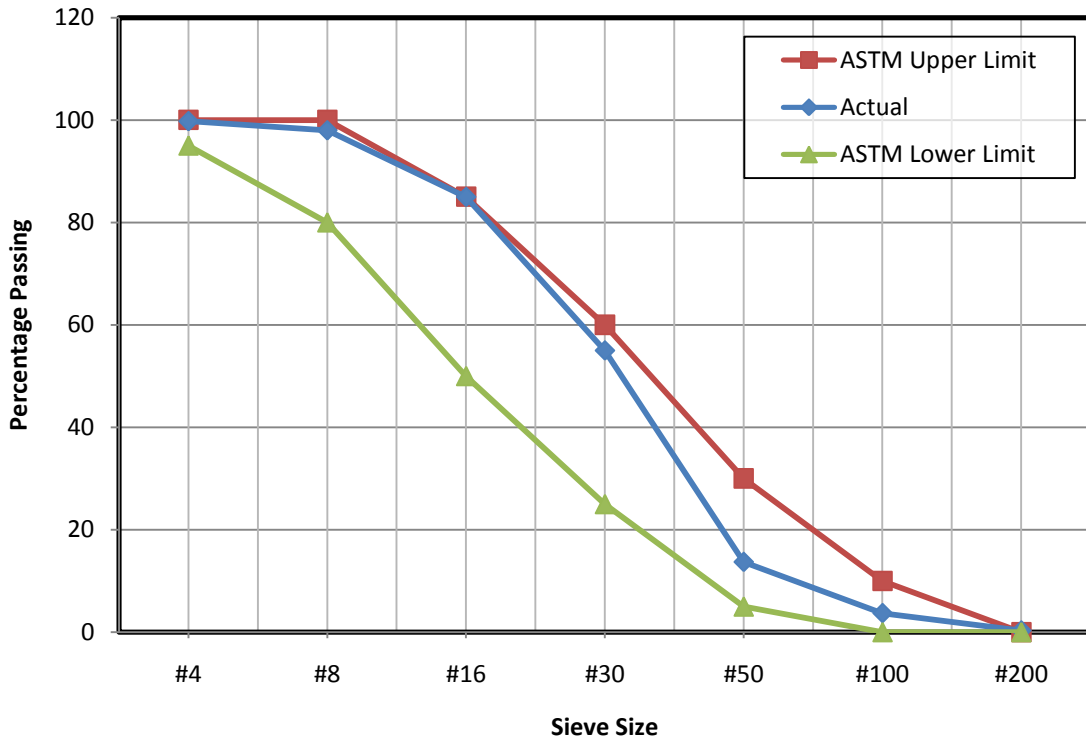


Fig 3.1 Particle Size Distribution of Fine Aggregate

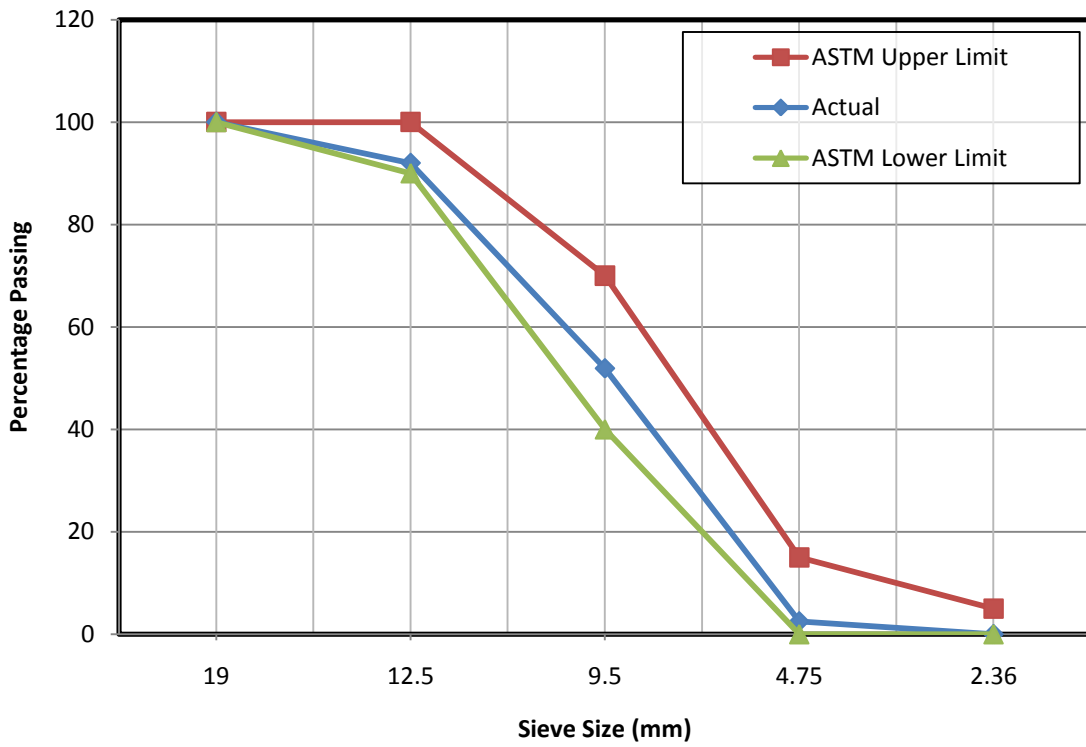
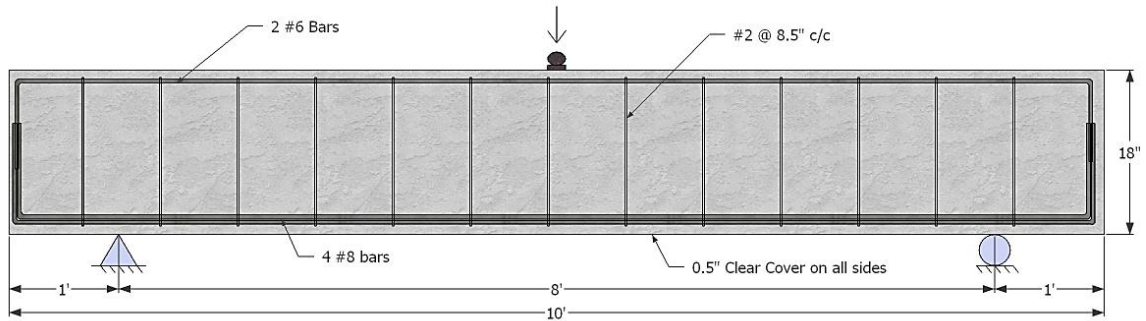


Fig 3.2 Particle Size Distribution of Coarse Aggregate





Longitudinal Section of Test Beam B6B

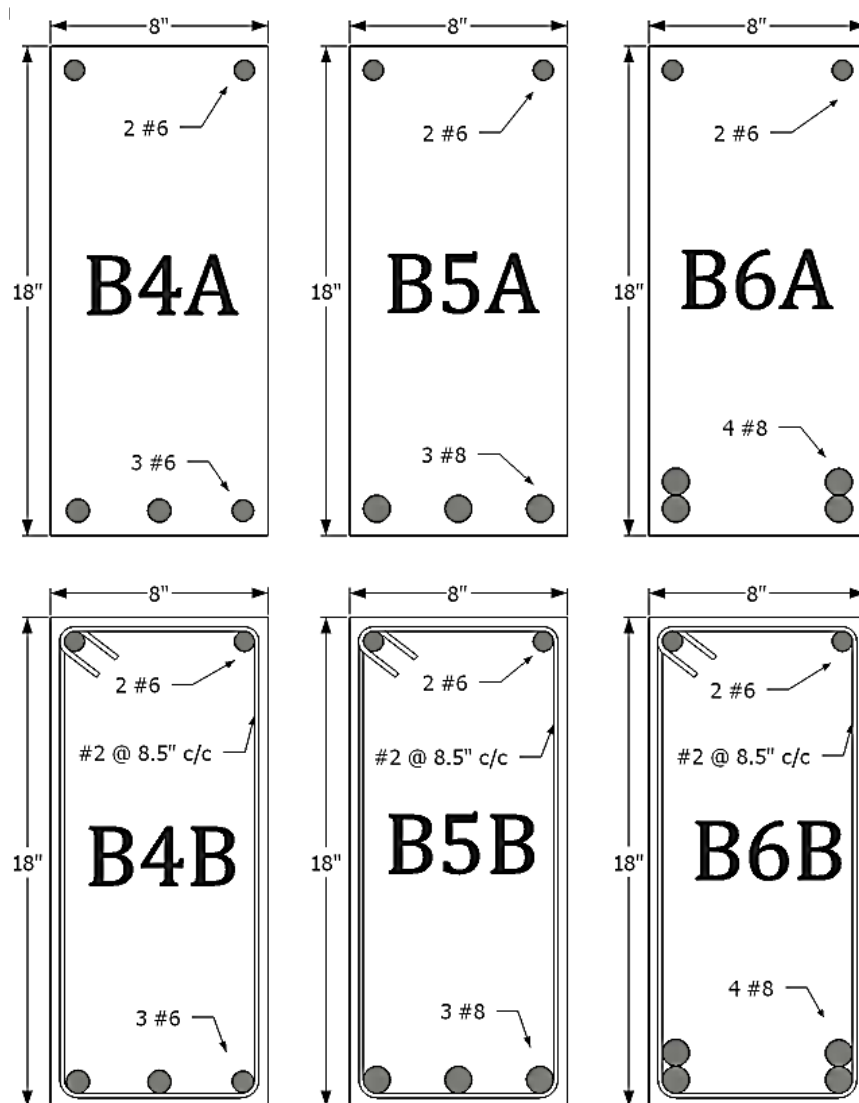


Fig 3.3 Dimensions and Reinforcement Detail of Test Beams

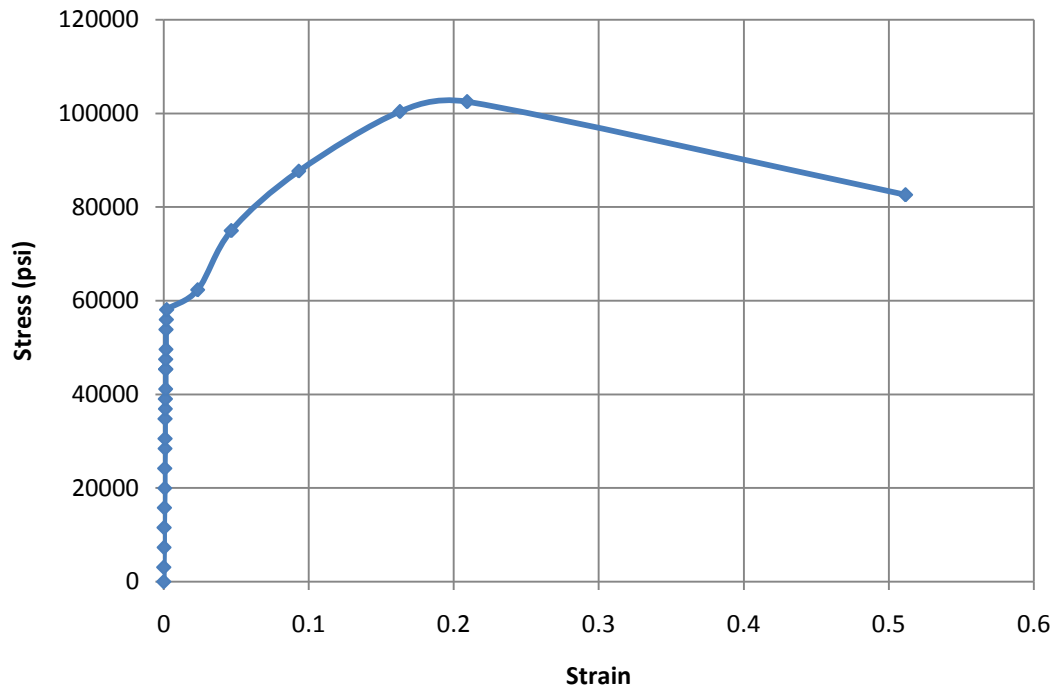


Fig 3.4 Stress-Strain Curve of Steel



Fig 3.5 Test Setup

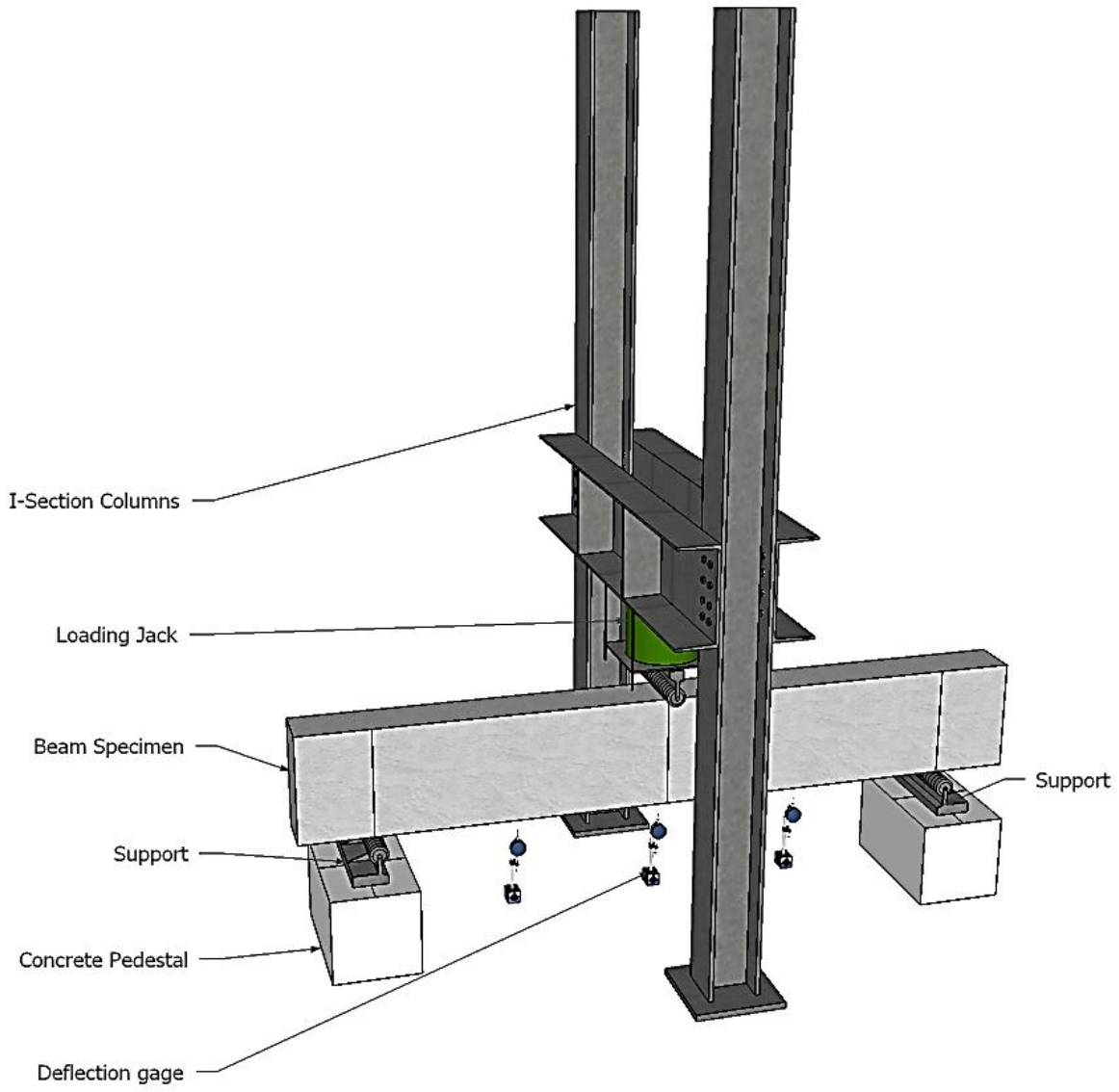


Fig 3.6 Test Setup



Fig 3.6 Test Setup

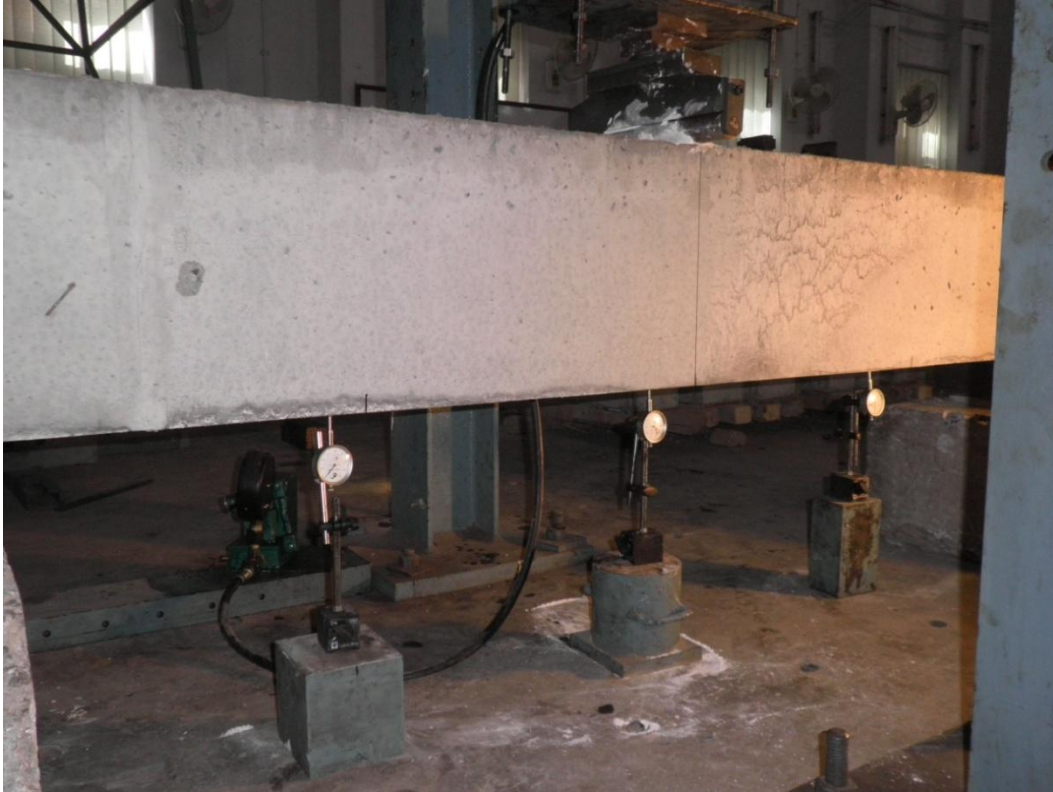


Fig 3.7 Test Setup (Deflection Gauges placed at 3<sup>rd</sup> points)



Fig 3.5 Test Setup (Supports)

## **APPENDIX – IV**

## APPENDIX IV

**Table 4.1 Compressive Strength on Day of Test**

<b>Specimen</b>	<b>Size of cylinders (in)</b>	<b>Compressive Strength (psi)</b>
B 4 A – 4	4 x 8	5476
B 4 A – 5	4 x 8	5689.5
B 4 B – 4	4 x 8	5394
B 4 B – 5	4 x 8	4987.5
B 5 A – 4	4 x 8	5205.5
B 5 A – 5	4 x 8	5640.5
B 5 B – 4	4 x 8	4861.5
B 5 B – 5	4 x 8	5205.5
B 6 A – 4	4 x 8	5678
B 6 A – 5	4 x 8	4889
B 6 B – 4	4 x 8	6481.5
B 6 B – 5	4 x 8	5399.5
<b>Average</b>		<b>5409</b>

**Table 4.2 Load Deflection Data - Beam B4A**

<b>Pressure Gauge Reading (Kg/cm<sup>2</sup>)</b>	<b>Load (Tons)</b>	<b>Deflection (mm)</b>		
		<b>Quarter</b>	<b>Midpoint</b>	<b>Quarter</b>
0	0.00	0.000	0.000	0.000
10	1.90	0.051	0.127	0.152
20	3.81	0.178	0.267	0.279
30	5.71	0.330	0.305	0.381
40	7.62	0.508	0.635	0.546
50	9.52	0.737	0.914	0.737
60	11.43	0.991	1.270	0.940
70	13.33	1.321	1.753	1.245
80	15.24	1.600	2.159	1.499
90	17.14	1.981	2.667	1.803
100	19.05	2.286	3.124	2.083
105	20.00	4.369	4.420	2.718
85	16.19	9.195	6.934	3.734
0	0.00	6.350	4.699	2.540



**Table 4.3 Load Deflection Data - Beam B4B**

<b>Pressure Gauge Reading  (Kg/cm<sup>2</sup>)</b>	<b>Load  (Tons)</b>	<b>Deflection (mm)</b>		
		<b>Quarter</b>	<b>Midpoint</b>	<b>Quarter</b>
0	0.00	0.000	0.000	0.000
10	1.90	0.203	0.152	0.051
20	3.81	0.254	0.279	0.203
30	5.71	0.406	0.483	0.381
40	7.62	0.508	0.635	0.533
50	9.52	0.584	0.762	0.635
60	11.43	0.787	1.067	0.889
70	13.33	1.041	1.499	1.168
80	15.24	1.422	2.134	1.651
90	17.14	1.753	2.692	2.057
100	19.05	2.057	3.200	2.413
110	20.95	2.540	3.835	2.870
120	22.86	2.997	4.547	3.429
130	24.76	3.404	5.182	3.937
140	26.67	4.039	6.096	4.661
150	28.57	4.420	6.706	5.220
160	30.48	5.283	8.052	6.223
165	31.43	22.073	16.205	23.241
150	28.57	0.000	17.018	27.051
140	26.67	14.224	25.324	-
0	0.00	-	16.002	-

**Table 4.4 Load Deflection Data - Beam B5A**

<b>Pressure Guage Reading (Kg/cm<sup>2</sup>)</b>	<b>Load (Tons)</b>	<b>Deflection (mm)</b>		
		<b>Quarter</b>	<b>Midpoint</b>	<b>Quarter</b>
0	0.00	0.000	0.000	0.000
10	1.90	0.076	0.102	0.127
20	3.81	0.178	0.254	0.267
30	5.71	0.330	0.508	0.533
40	7.62	0.381	0.559	0.572
50	9.52	0.483	0.711	0.699
60	11.43	0.584	0.864	0.813
70	13.33	0.787	1.194	1.041
80	15.24	0.940	1.422	1.194
90	17.14	1.118	1.727	1.397
100	19.05	1.321	2.007	1.575
110	20.95	1.524	2.311	1.778
120	22.86	1.803	2.565	2.032
130	24.76	2.159	2.921	2.311
140	26.67	6.579	5.232	3.048
0	0.00	3.073	2.540	1.486

**Table 4.5 Load Deflection Data - Beam B5B**

Pressure Guage Reading  (Kg/cm <sup>2</sup> )	Load  (Tons)	Deflection (mm)		
		Quarter	Midpoint	Quarter
0	0.00	0.000	0.000	0.000
10	1.90	0.127	0.152	0.051
20	3.81	0.229	0.279	0.229
30	5.71	0.330	0.432	0.292
40	7.62	0.432	0.610	0.508
50	9.52	0.584	0.813	0.673
60	11.43	0.737	1.041	0.838
70	13.33	0.914	1.346	1.041
80	15.24	1.118	1.651	1.245
90	17.14	1.346	2.007	1.499
100	19.05	1.524	2.286	1.702
110	20.95	1.727	2.591	1.905
120	22.86	1.956	2.972	2.159
130	24.76	2.261	3.531	2.616
140	26.67	2.616	4.064	3.023
150	28.57	3.073	4.597	3.429
160	30.48	3.632	5.207	3.823
170	32.38	3.988	5.817	4.191
180	34.28	4.293	6.071	4.496
190	36.19	4.597	6.502	4.851
200	38.09	4.953	6.985	5.207
210	40.00	5.385	7.493	5.588
220	41.90	5.842	8.077	6.096
230	43.81	6.299	8.712	6.604
240	45.71	6.807	9.398	7.112
250	47.62	9.652	15.037	10.693
260	49.52	13.335	22.200	13.589
250	47.62	13.716	22.606	-
220	41.90	14.351	19.050	-
100	19.05	-	15.748	-

**Table 4.6 Load Deflection Data - Beam B6A**

<b>Pressure Guage Reading (Kg/cm<sup>2</sup>)</b>	<b>Load (Tons)</b>	<b>Deflection (mm)</b>		
		<b>Quarter</b>	<b>Midpoint</b>	<b>Quarter</b>
0	0.00	0.000	0.000	0.000
10	1.90	0.203	0.152	0.102
20	3.81	0.406	0.330	0.229
30	5.71	0.610	0.521	0.330
40	7.62	0.787	0.686	0.483
50	9.52	0.940	0.876	0.597
60	11.43	1.118	1.067	0.737
70	13.33	1.295	1.295	0.889
80	15.24	1.473	1.575	1.054
90	17.14	1.702	1.854	1.245
100	19.05	1.930	2.159	1.422
110	20.95	2.134	2.413	1.575
120	22.86	2.311	2.667	1.753
130	24.76	2.565	3.023	2.032
140	26.67	2.819	3.327	2.286
143	27.24	6.096	3.861	6.477
0	0.00	-	3.302	-

**Table 4.7 Load Deflection Data - Beam B6B**

<b>Pressure Guage Reading</b>  <i>(Kg/cm<sup>2</sup>)</i>	<b>Load</b>  <i>(Tons)</i>	<b>Deflection (mm)</b>		
		<b>Quarter</b>	<b>Midpoint</b>	<b>Quarter</b>
0	0.00	0.000	0.000	0.000
10	1.90	0.127	0.178	0.152
20	3.81	0.203	0.254	0.229
30	5.71	0.279	0.394	0.318
40	7.62	0.381	0.559	0.495
50	9.52	0.508	0.724	0.635
60	11.43	0.610	0.889	0.762
70	13.33	0.737	1.118	0.902
80	15.24	0.889	1.372	1.067
90	17.14	1.067	1.626	1.232
100	19.05	1.219	1.829	1.372
110	20.95	1.372	2.057	1.524
120	22.86	1.524	2.311	1.689
130	24.76	1.702	2.591	1.880
140	26.67	1.905	2.896	2.083
150	28.57	2.210	3.251	2.337
160	30.48	2.502	3.658	2.591
170	32.38	2.769	4.064	2.946
180	34.28	3.023	4.420	3.175
190	36.19	3.353	4.877	3.581
200	38.09	3.708	5.309	3.937
210	40.00	4.064	5.715	4.267
220	41.90	4.699	6.375	4.674
230	43.81	5.715	7.442	5.398
240	45.71	6.604	8.179	5.969
250	47.62	7.620	9.144	6.655
260	49.52	8.636	10.109	-
200	38.09	10.744	16.510	9.906

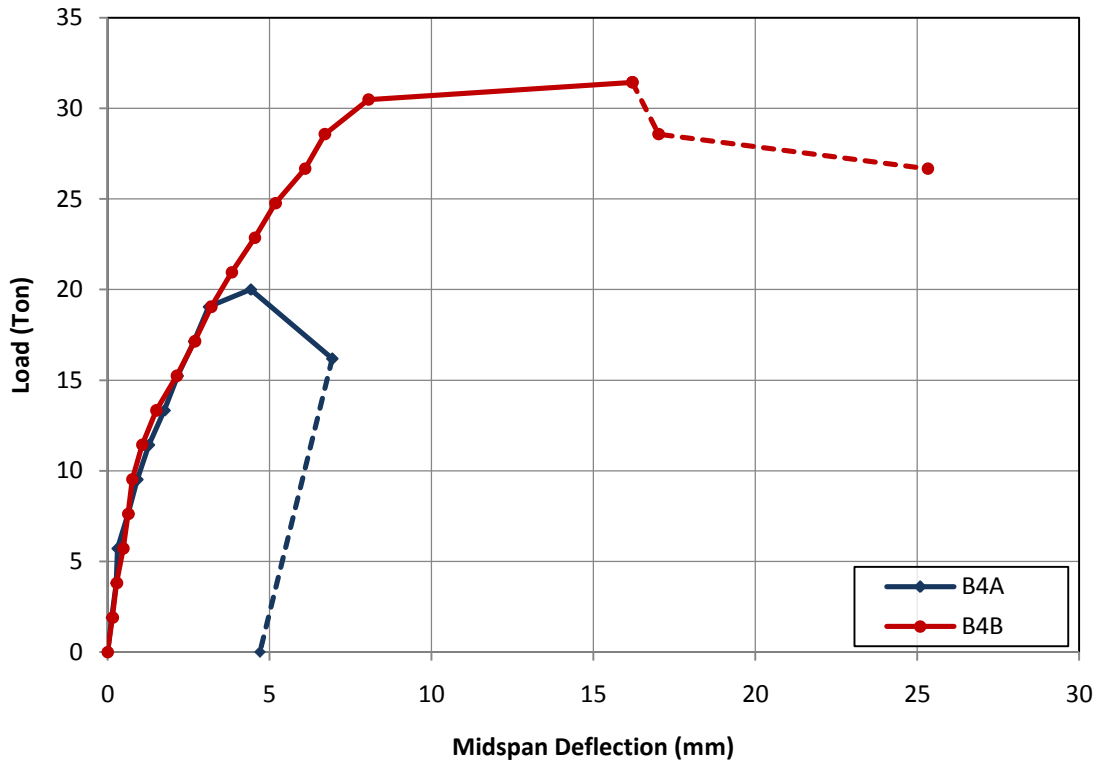


Fig 4.1 Load – Deflection Curves of Beams B4A and B4B

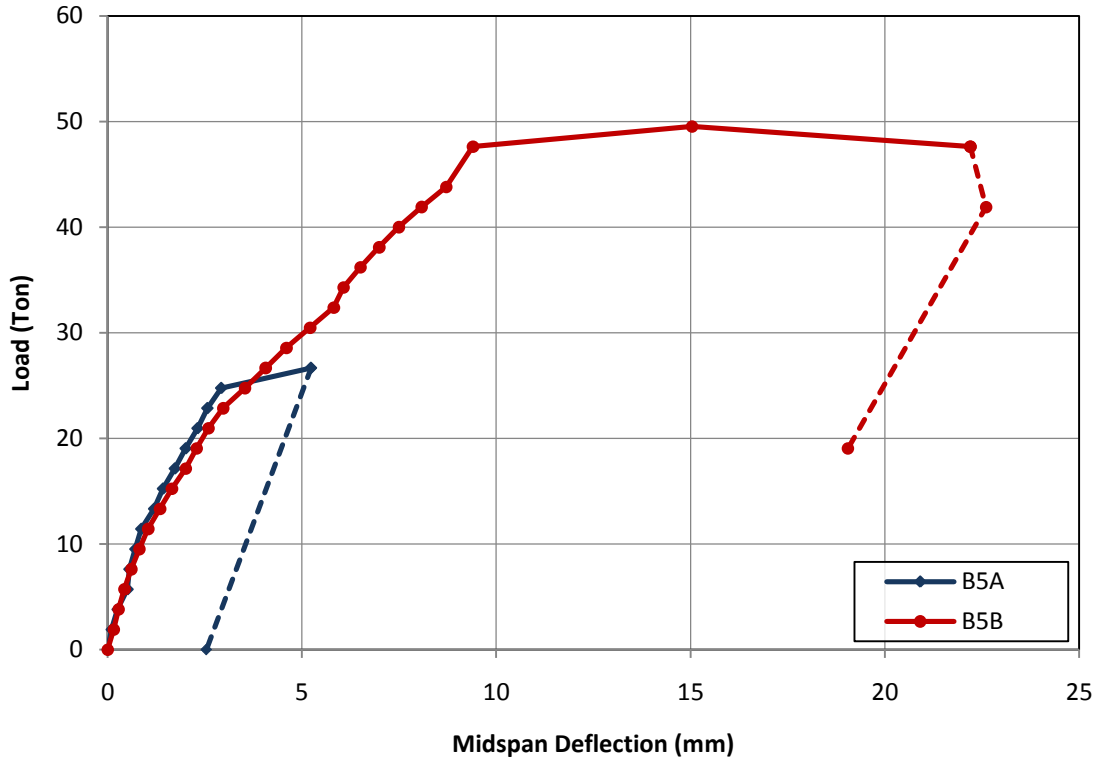


Fig 4.2 Load – Deflection Curves of Beams B5A and B5B

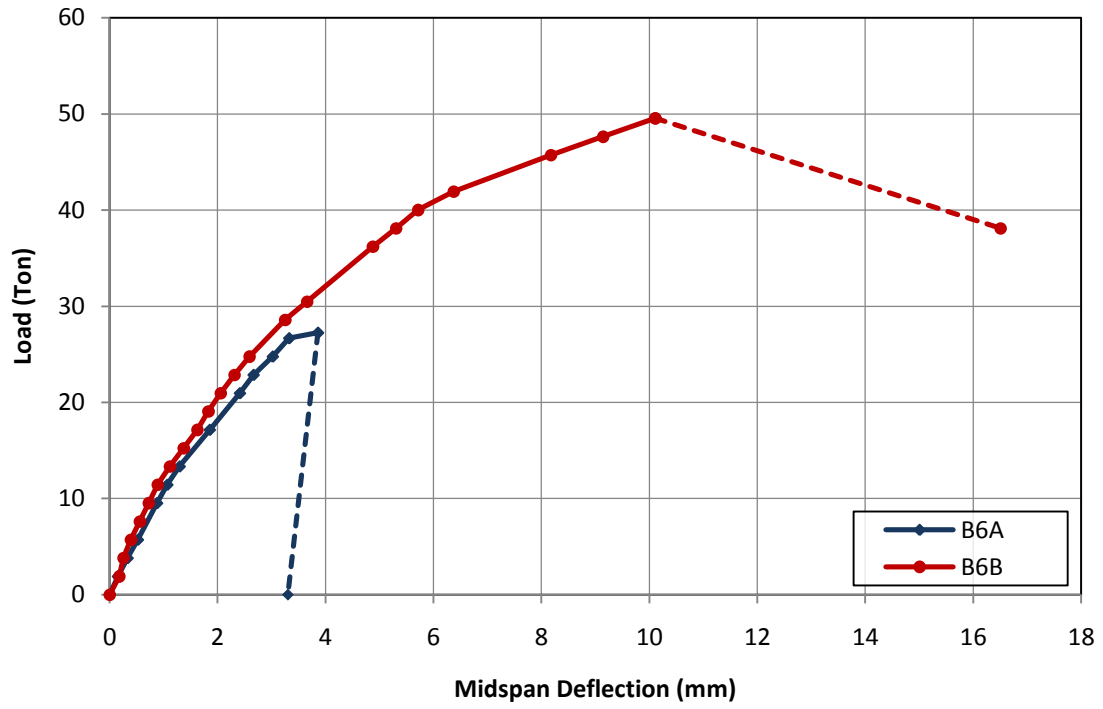


Fig 4.3 Load – Deflection Curves of Beams B6A and B6B

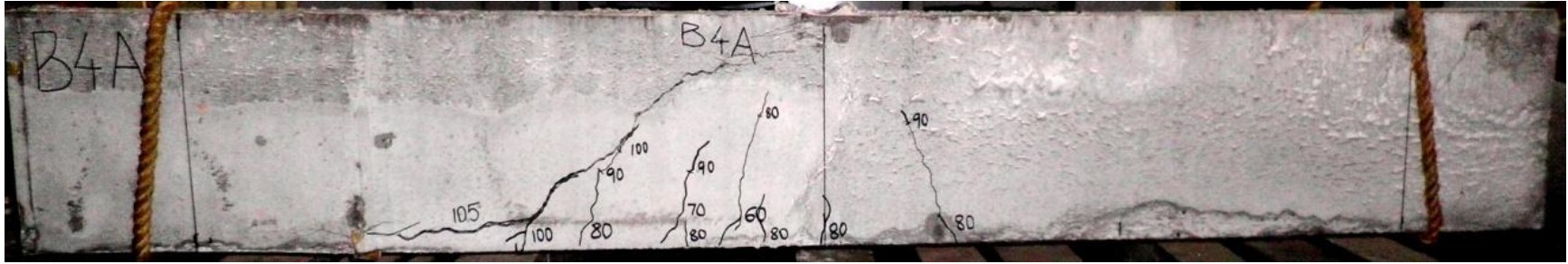


Fig 4.4 Crack Pattern of Beam B4A

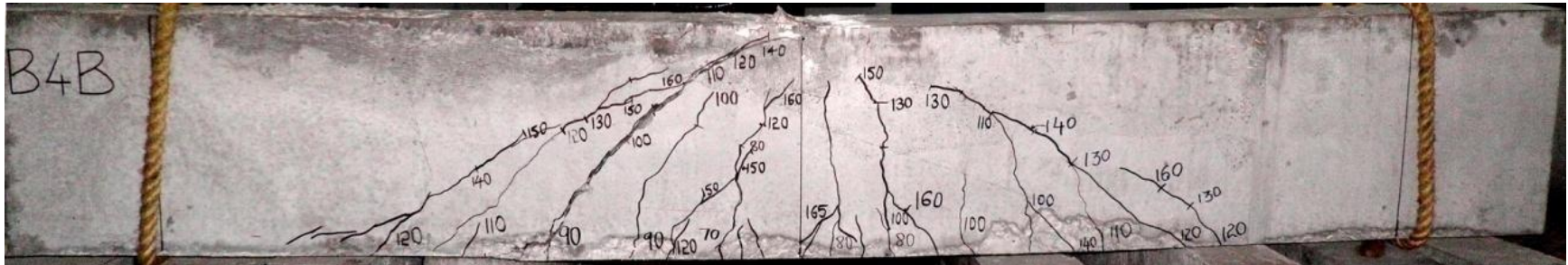


Fig 4.5 Crack Pattern of Beam B4B





Fig 4.6 Crack Pattern of Beam B5A

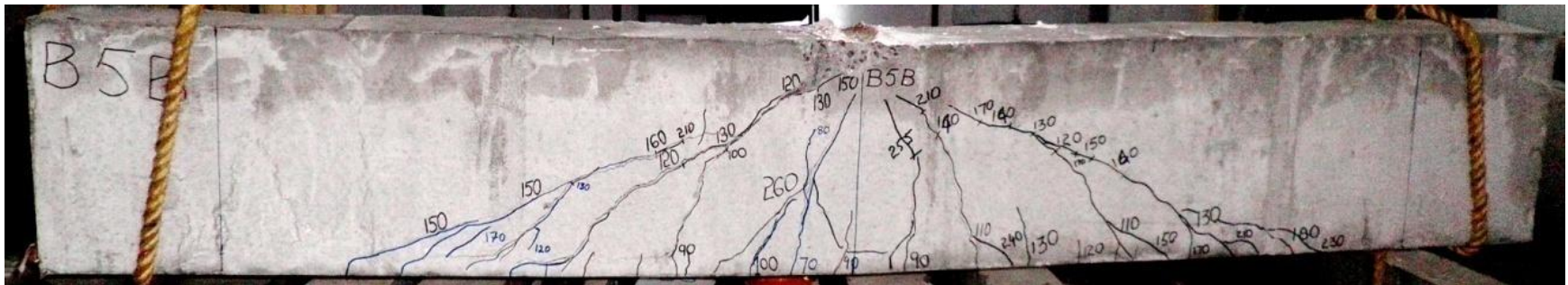


Fig 4.7 Crack Pattern of Beam B5B

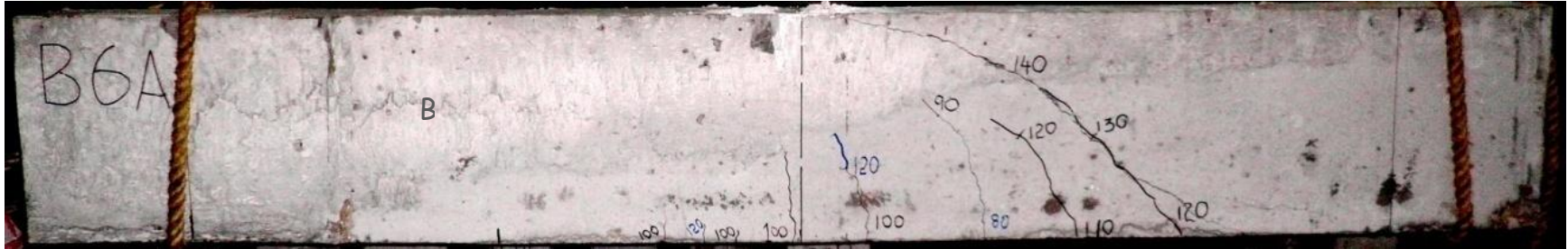


Fig 4.8 Crack Pattern of Beam B6A

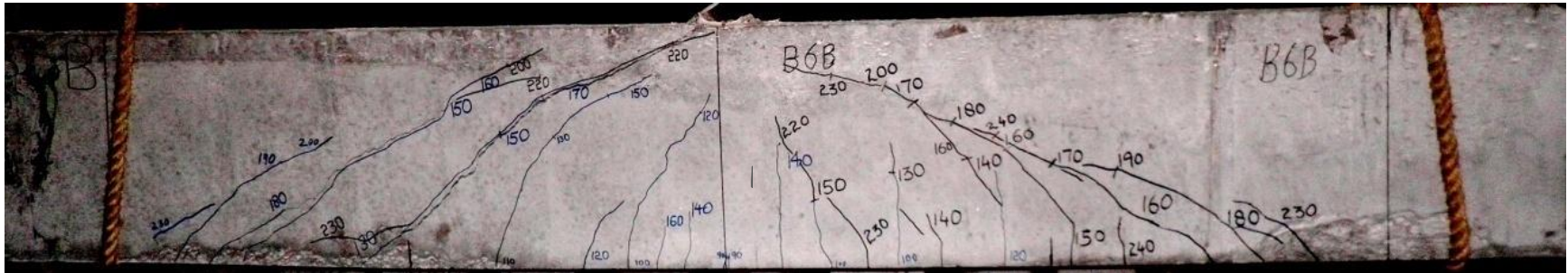


Fig 4.9 Crack Pattern of Beam B6B

## **APPENDIX – V**

## APPENDIX V

Table 5.1 Moment – Curvature Relationship

Theoretical		B4A		B4B		Theoretical		B5A		B5B		Theoretical		B6A		B6B	
M (Kip-in)	$\phi(x10^{-6})$ (rad/in)	M (Kip-in)	$\phi(x10^{-6})$ (rad/in)	M (Kip-in)	$\phi(x10^{-6})$ (rad/in)	M (Kip-in)	$\phi(x10^{-6})$ (rad/in)	M (Kip-in)	$\phi(x10^{-6})$ (rad/in)	M (Kip-in)	$\phi(x10^{-6})$ (rad/in)	M (Kip-in)	$\phi(x10^{-6})$ (rad/in)	M (Kip-in)	$\phi(x10^{-6})$ (rad/in)	M (Kip-in)	$\phi(x10^{-6})$ (rad/in)
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
285.84	15.32	14.40	0.00	14.40	0.00	313.48	15.70	14.40	0.00	14.40	0.00	337.48	16.10	14.40	0.00	14.40	0.00
386.59	56.85	115.18	4.40	115.18	0.00	492.60	44.82	115.18	3.76	115.18	4.48	517.35	41.13	115.18	5.60	115.18	4.88
520.75	78.17	215.96	10.40	215.96	10.24	658.55	61.49	215.96	9.52	215.96	10.00	688.40	56.35	215.96	10.14	215.96	8.88
651.83	98.72	316.74	11.76	316.74	17.12	821.73	77.60	316.74	18.80	316.74	14.24	857.30	71.07	316.74	13.44	316.74	13.76
779.60	118.76	417.51	23.76	417.51	22.80	981.08	93.34	417.51	19.92	417.51	21.25	1022.27	85.48	518.29	25.92	417.51	18.40
881.98	139.67	518.29	40.00	518.29	25.76	1136.42	108.80	518.29	24.80	518.29	27.52	1183.16	99.64	619.07	42.40	518.29	25.28
1025.87	157.73	619.07	45.02	619.07	34.40	1287.84	124.00	619.07	28.60	619.07	34.40	1339.83	113.58	719.85	50.24	619.07	30.16
1260.10	195.43	719.85	48.48	719.85	57.28	1579.01	153.73	719.85	34.80	719.85	37.28	1640.95	140.85	921.41	62.40	719.85	39.04
1245.59	193.12	820.63	65.60	820.63	73.86	2184.12	218.34	820.63	42.80	820.63	57.28	2743.24	250.15	1122.96	73.60	820.63	43.04
1363.93	772.73	921.41	96.96	921.41	89.76	2260.36	450.75	921.41	48.80	921.41	60.16	2797.60	344.62	1223.74	73.60	921.41	49.86
1415.47	878.79	1022.18	111.20	1022.18	117.60	2275.54	531.34	1022.18	68.80	1022.18	83.04	2810.58	400.00	1324.52	83.65	1022.18	60.16
1447.05	996.97	1072.57	169.60	1122.96	137.60	2316.92	597.01	1122.96	78.66	1122.96	89.76	2820.28	458.46	1425.30	93.60	1122.96	79.81
1467.49	1090.91	871.02	702.40	1223.74	157.60	2348.87	662.69	1223.74	88.80	1223.74	108.80	2825.18	516.92	1455.53	197.60	1223.74	83.04
1511.50	1272.73	14.40	0.00	1324.52	186.40	2383.19	776.12	1324.52	100.16	1324.52	117.60	2835.82	615.38			1324.52	85.92
				1425.30	206.40			1425.30	201.92	1425.30	137.60					1526.08	97.28
				1526.08	235.20			14.40	275.52	1526.08	157.60					1626.86	111.52
				1626.86	255.20					1626.86	165.92					1929.19	168.80
				1677.24	772.80					1727.63	188.80					2029.97	182.11
				1526.08	6280.00					1929.19	208.80					2130.75	188.80
				1425.30						2029.97	228.80					2533.86	256.00
										2130.75	249.76					2634.64	312.00
										2231.53	279.81					2029.97	484.80
										2332.30	327.46						
										2533.86	544.00						
										2634.64	657.60						
										2533.86	744.00						
										2231.53	582.72						
										1022.18	-						

Table 5.2 Test Results of Reinforced Concrete Beams

<b>Beam</b>	<b>V<sub>a</sub></b> <b>(Kip)</b>	<b>V<sub>b</sub></b> <b>(Kip)</b>	<b>Δ<sub>a</sub></b> <b>(in)</b>	<b>Δ<sub>b</sub></b> <b>(in)</b>	<b>V<sub>b</sub>/V<sub>a</sub></b>	<b>Δ<sub>b</sub>/Δ<sub>a</sub></b>
B4A	22.65		0.17		1.51	1.82
B4B		34.19		0.32		
B5A	29.99		0.21		1.77	1.80
B5B		53.09		0.37		
B6A	30.62		0.15		1.80	2.62
B6B		55.19		0.40		

Table 5.3 Comparison of Results with Other Test Programs

Details	f <sub>c</sub> psi	b <sub>w</sub> in	h in	d in	w <sub>(dead)</sub> k/ft	A <sub>s</sub> in <sup>2</sup>	ρ <sub>L</sub>	f <sub>yL</sub> ksi	s in	A <sub>v</sub> in <sup>2</sup>	ρ <sub>T</sub>	L ft	a in	a/d	f <sub>yT</sub> ksi	A Series			B Series			V <sub>b</sub> /V <sub>a</sub>	D <sub>b</sub> /D <sub>a</sub>
																P (Kip)	V (Kip)	D(in)	P (Kip)	V (Kip)	D(in)		
<b>Usman</b>																							
B4	5409	8	18	16.88	0.150	1.32	0.0098	64	8.50	0.10	0.0015	8.00	48.00	2.84	40.00	45.29	22.65	0.17	68.39	34.19	0.32	1.51	1.82
B5	5409	8	18	16.75	0.150	2.37	0.0177	64	8.50	0.10	0.0015	8.00	48.00	2.87	40.00	59.99	29.99	0.21	106.18	53.09	0.37	1.77	1.80
B6	5409	8	18	16.25	0.150	3.16	0.0243	64	8.50	0.10	0.0015	8.00	48.00	2.95	40.00	61.25	30.62	0.15	110.38	55.19	0.40	1.80	2.62
<b>Javaid</b>																							
B1	5554	10	16	14.75	0.167	1.58	0.0107	64	6.50	0.10	0.0015	8.00	48.00	3.25	40.00	51.71	25.85	0.19	64.31	32.15	0.28	1.24	1.50
B2	5554	10	16	14.25	0.167	3.16	0.0222	64	6.50	0.10	0.0015	8.00	48.00	3.37	40.00	72.70	36.35	0.17	110.48	55.24	0.38	1.52	2.28
B3	5554	10	16	14.35	0.167	3.95	0.0275	64	6.50	0.10	0.0015	8.00	48.00	3.34	40.00	72.70	36.35	0.14	131.47	65.74	0.53	1.81	3.71
<b>Lee and Kim (L1-L3)</b>																							
L1	5916	13.8	17.7	16.14	0.254	3.99	0.0179	76.13	3.15	0.08	0.0018	7.58	48.43	3.00	31.18	88.55	44.28	0.16	98.23	49.11	0.30	1.11	1.82
L2	5916	13.8	17.7	15.75	0.254	6.98	0.0321	76.13	3.15	0.08	0.0018	7.58	47.24	3.00	31.18	101.15	50.58	0.14	116.45	58.23	0.25	1.15	1.74
L3	5916	13.8	17.7	15.16	0.254	9.96	0.0476	76.13	3.15	0.08	0.0018	7.58	45.47	3.00	31.18	101.38	50.69	0.12	144.35	72.18	0.47	1.42	3.98
<b>Lee and Kim (L4-L6)</b>																							
L4	4423	8.7	12.6	11.02	0.114	0.89	0.0093	79.75	5.51	0.08	0.0016	7.88	33.07	3.00	31.18	32.62	16.31	0.14	35.88	17.94	0.21	1.10	1.56
L5	4423	8.7	12.6	11.02	0.114	1.78	0.0186	79.75	5.51	0.08	0.0016	7.88	33.07	3.00	31.18	42.07	21.04	0.11	47.25	23.62	0.26	1.12	2.42
L6	4423	8.7	12.6	10.24	0.114	2.48	0.0279	79.75	5.51	0.08	0.0016	7.88	30.71	3.00	31.18	46.35	23.17	0.10	58.05	29.02	0.24	1.25	2.55

Details	f <sub>c</sub>	b <sub>w</sub>	h	d	w <sub>(dead)</sub>	A <sub>s</sub>	ρ <sub>L</sub>	f <sub>y</sub> L	s	A <sub>v</sub>	ρ <sub>T</sub>	L	a	a/d	f <sub>yT</sub>	A Series			B Series			V <sub>b</sub> /V <sub>a</sub>	D <sub>b</sub> /D <sub>a</sub>
	psi	in	in	in	k/ft	in <sup>2</sup>		ksi	in	in <sup>2</sup>		ft	in		ksi	P (Kip)	V (Kip)	D(in)	P (Kip)	V (Kip)	D(in)		
<b>Lee and Kim (S1 – S3) 2008</b>																							
1	916	3.8	7.7	6.14	.254	.99	.0224	6.13	.03	.08	.0018	.38	2.28	.00	1.18	8.44	4.22	.06	58.19	9.10	.17	.79	.95
2	916	3.8	7.7	6.14	.254	.99	.0224	6.13	.03	.08	.0018	.07	8.42	.00	1.18	1.70	0.85	.14	01.73	0.86	.33	.25	.29
3	916	3.8	7.7	6.14	.254	.99	.0224	6.13	.03	.08	.0018	0.7	4.56	.00	1.18	7.89	8.94	.28	06.69	3.34	.62	.37	.20
<b>Lee and Kim (S4 – S6) 2008</b>																							
4	422.5	.7	2.6	1.02	.114	.34	.0140	9.75	.51	.08	.0016	.51	3.07	.00	1.18	9.10	9.55	.12	6.30	3.15	.25	.18	.07
5	422.5	.7	2.6	1.02	.114	.34	.0140	9.75	.51	.08	.0016	.35	4.09	.00	1.18	7.74	8.87	.28	2.46	1.23	.51	.13	.84
6	422.5	.7	2.6	1.02	.114	.34	.0140	9.75	.51	.08	.0016	.18	5.10	.00	1.18	4.57	7.29	.50	8.17	9.09	.86	.10	.73

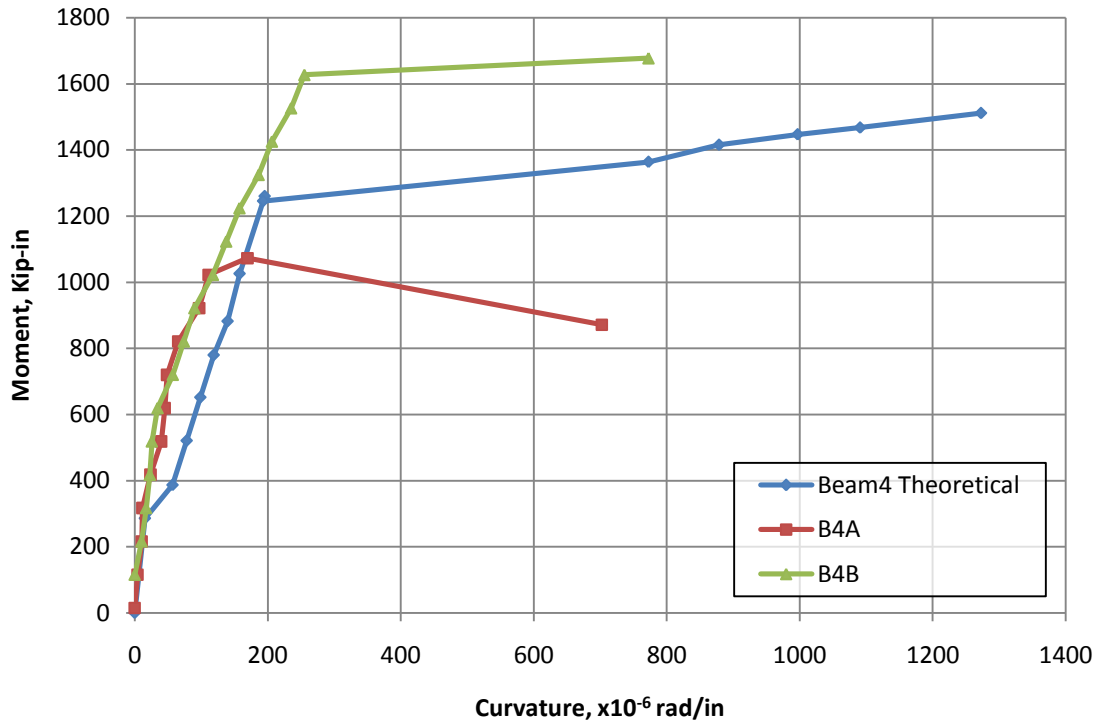


Fig 5.1 Moment – Curvature Relationship (B4A and B4B)

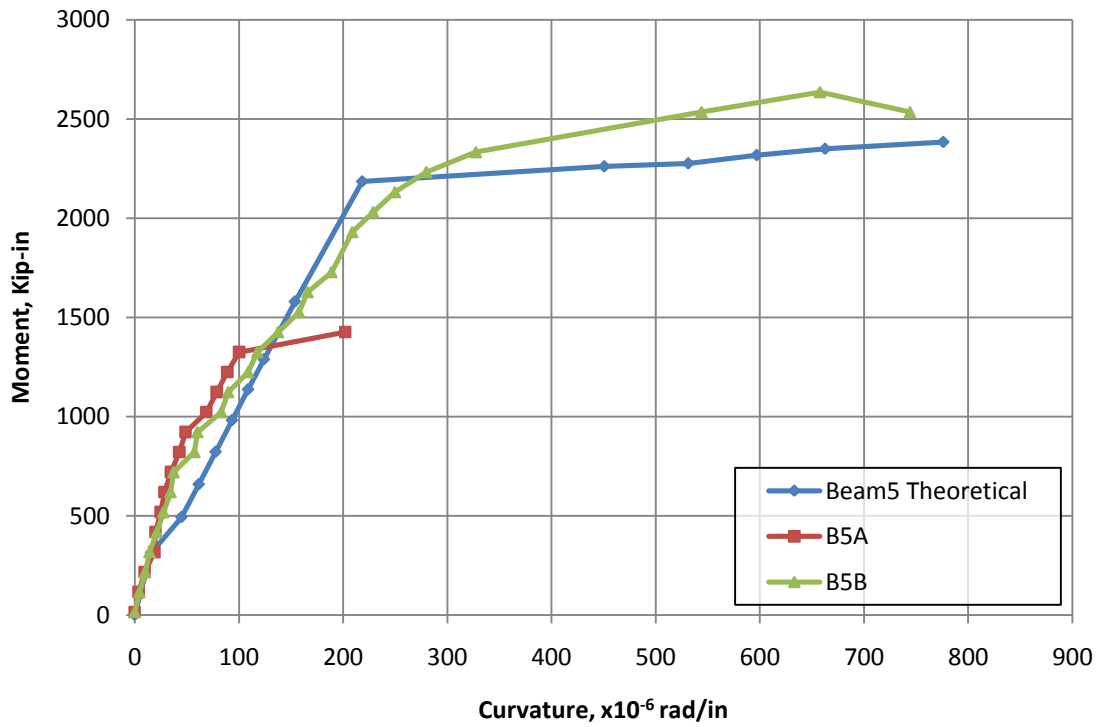


Fig 5.2 Moment – Curvature Relationship (B5A and B5B)



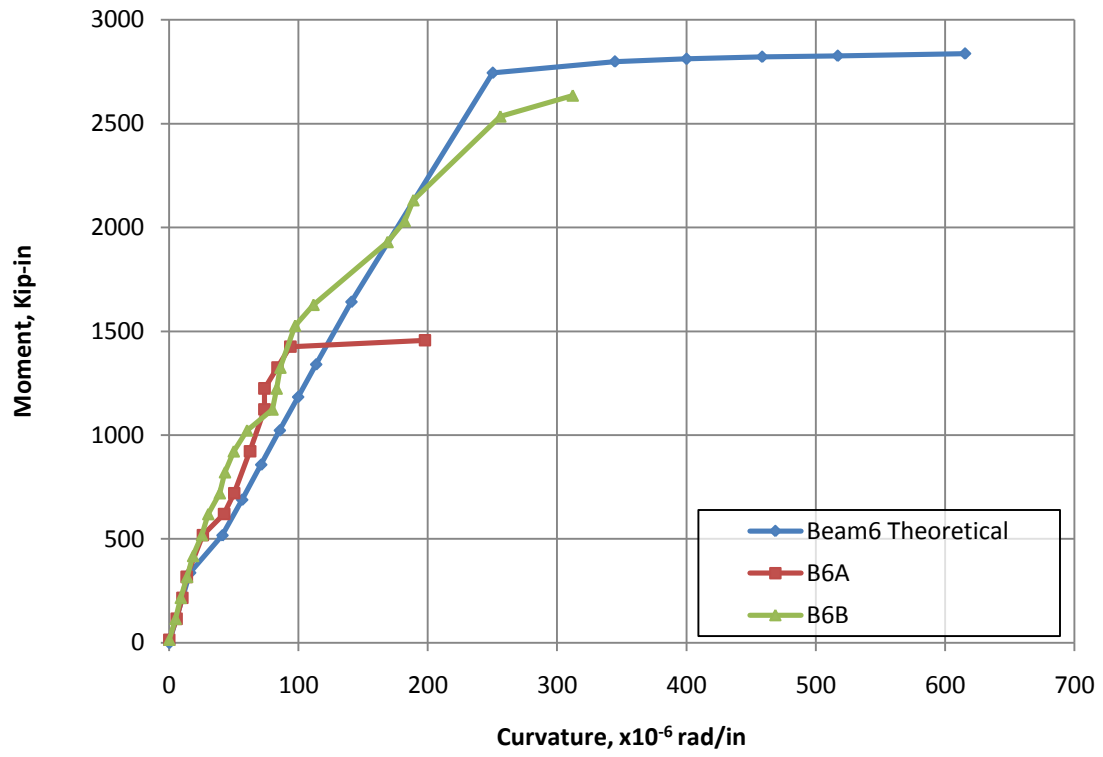


Fig 5.3 Moment – Curvature Relationship (B6A and B6B)

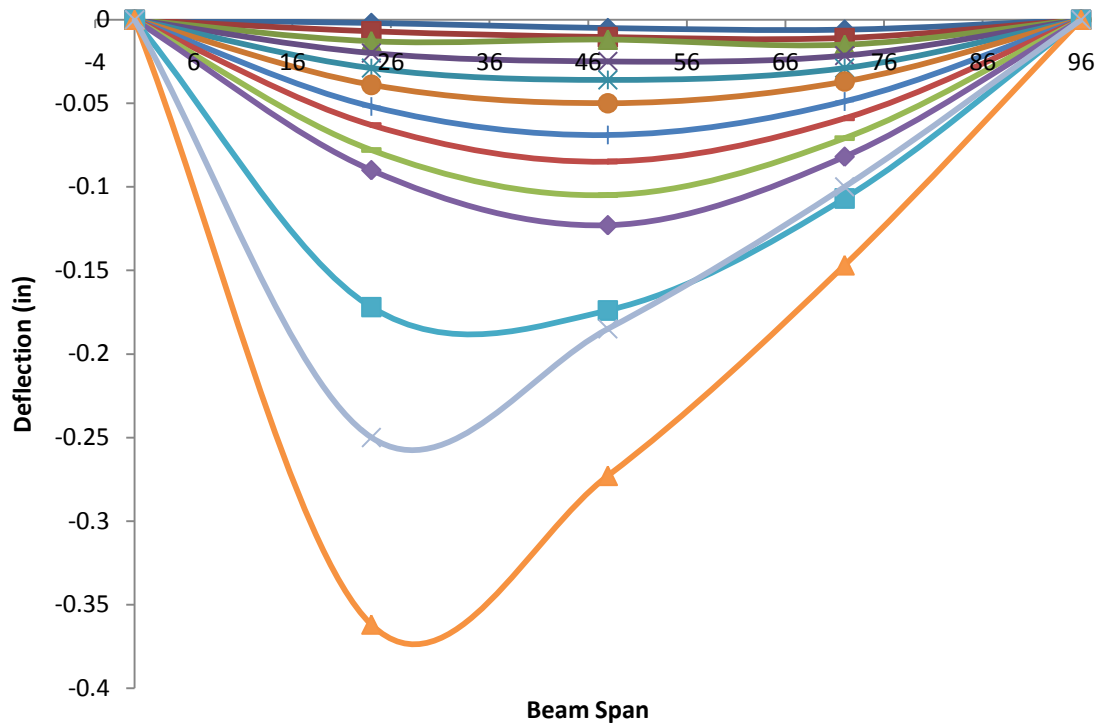


Fig 5.4 Deflected Shape of Beam at Different Load Levels (B4A)

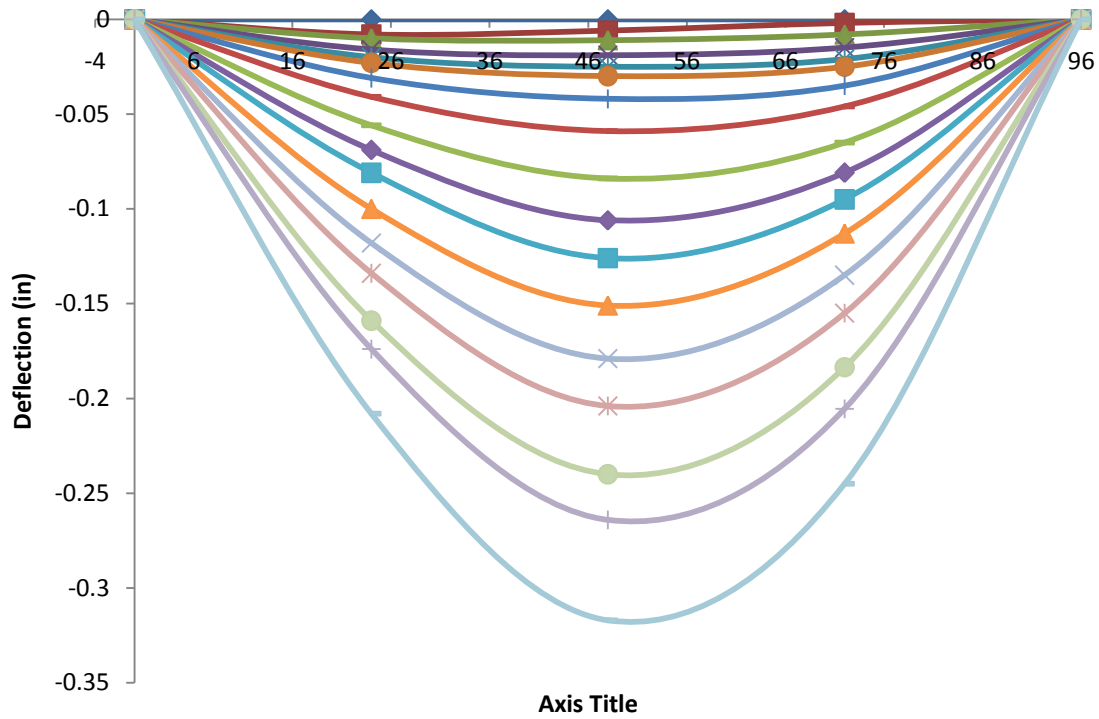


Fig 5.5 Deflected Shape of Beam at Different Load Levels (B4B)

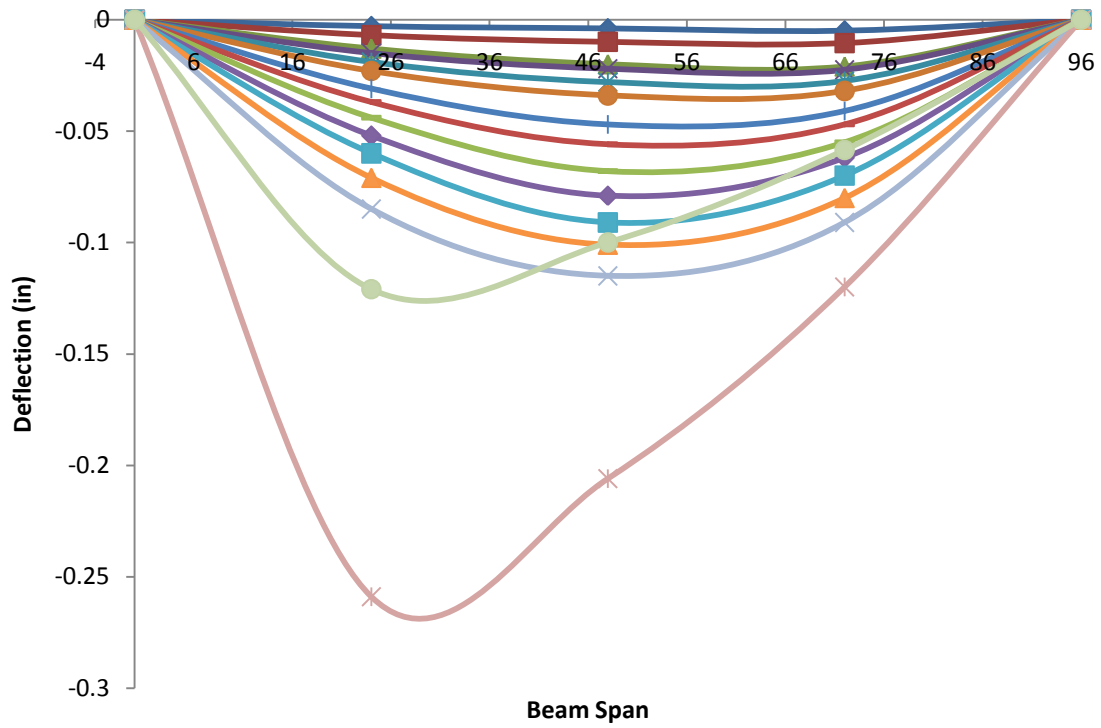


Fig 5.6 Deflected Shape of Beam at Different Load Levels (B5A)

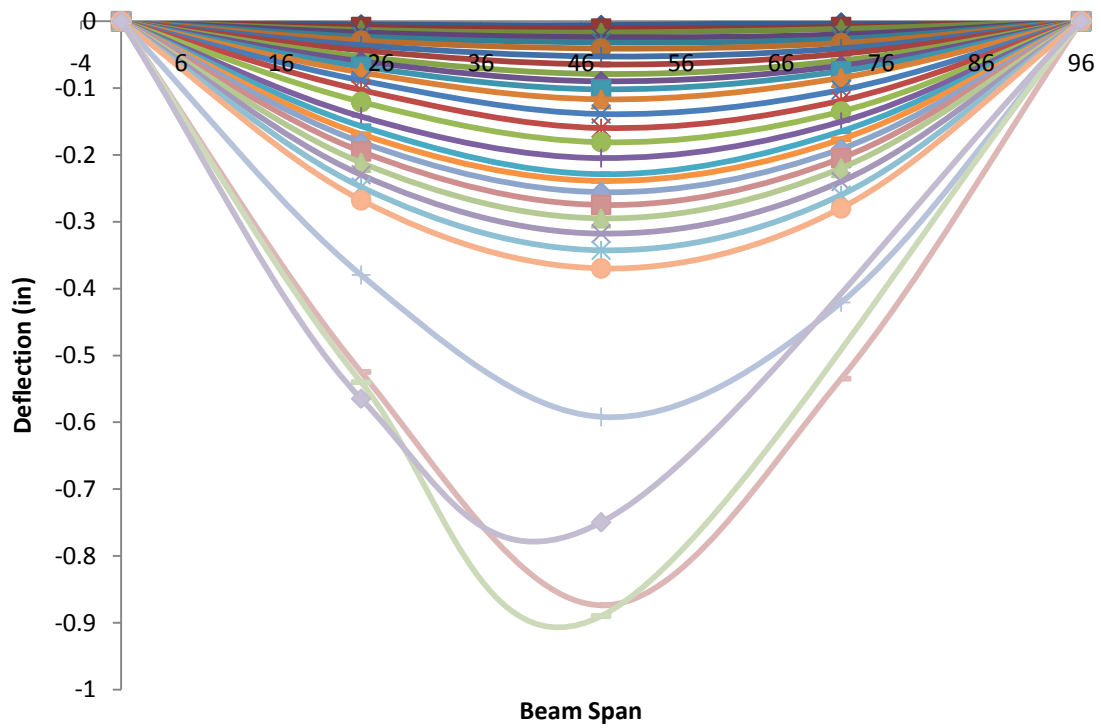


Fig 5.7 Deflected Shape of Beam at Different Load Levels (B5B)

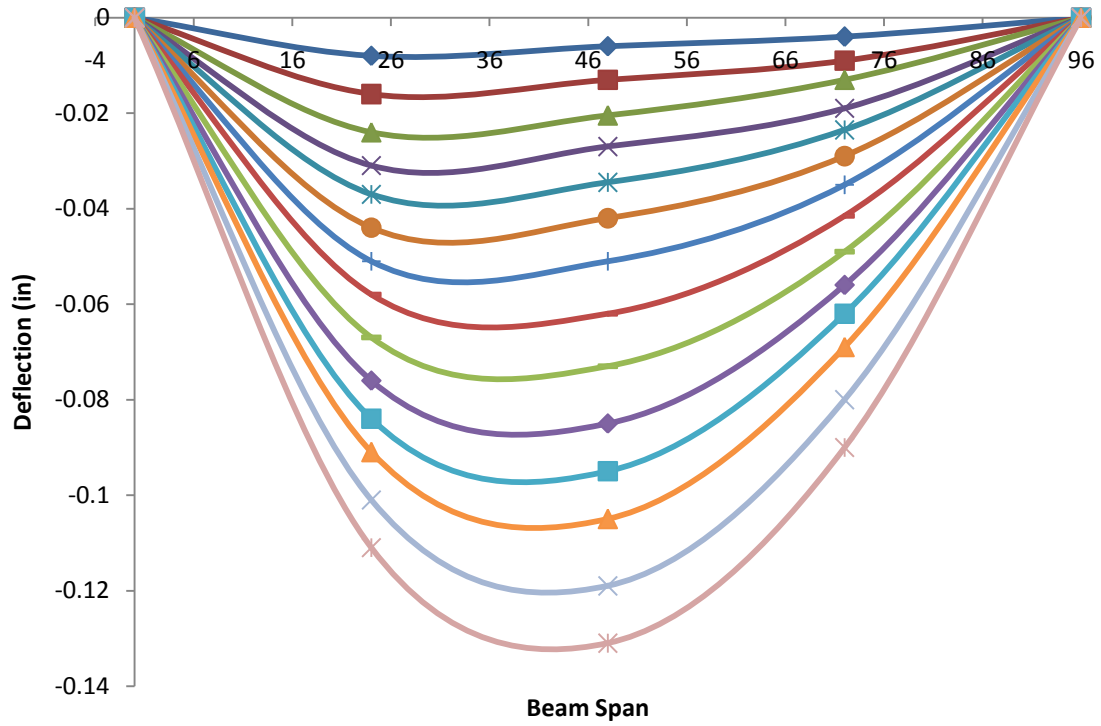


Fig 5.8 Deflected Shape of Beam at Different Load Levels (B6A)

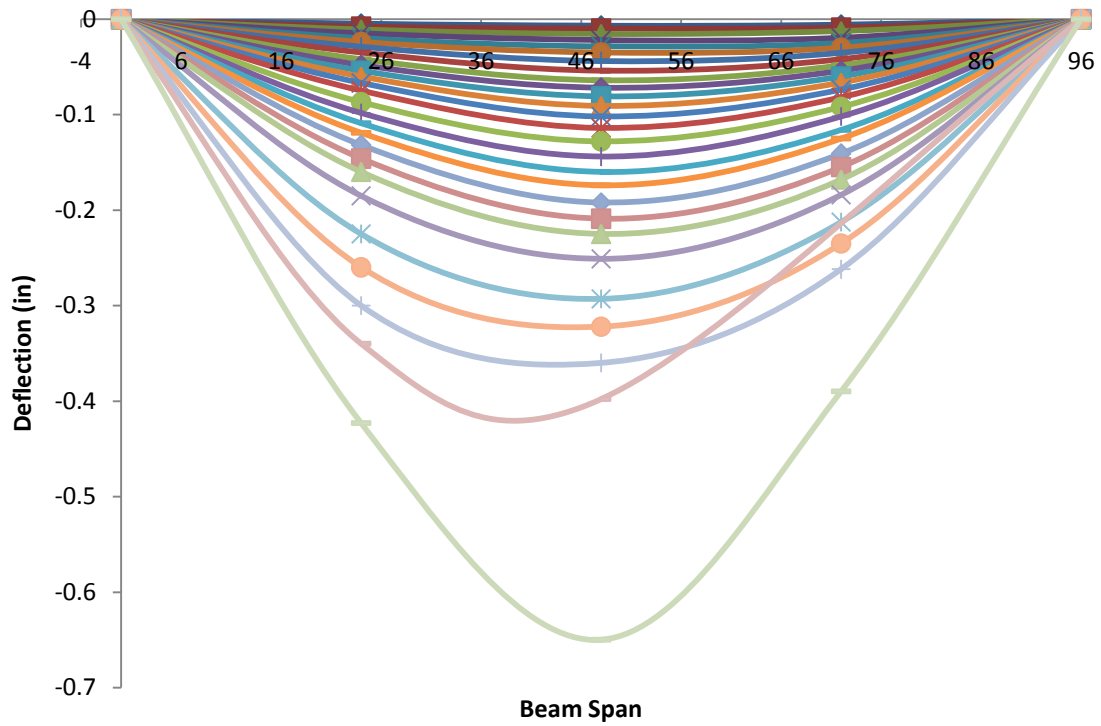


Fig 5.9 Deflected Shape of Beam at Different Load Levels (B6B)

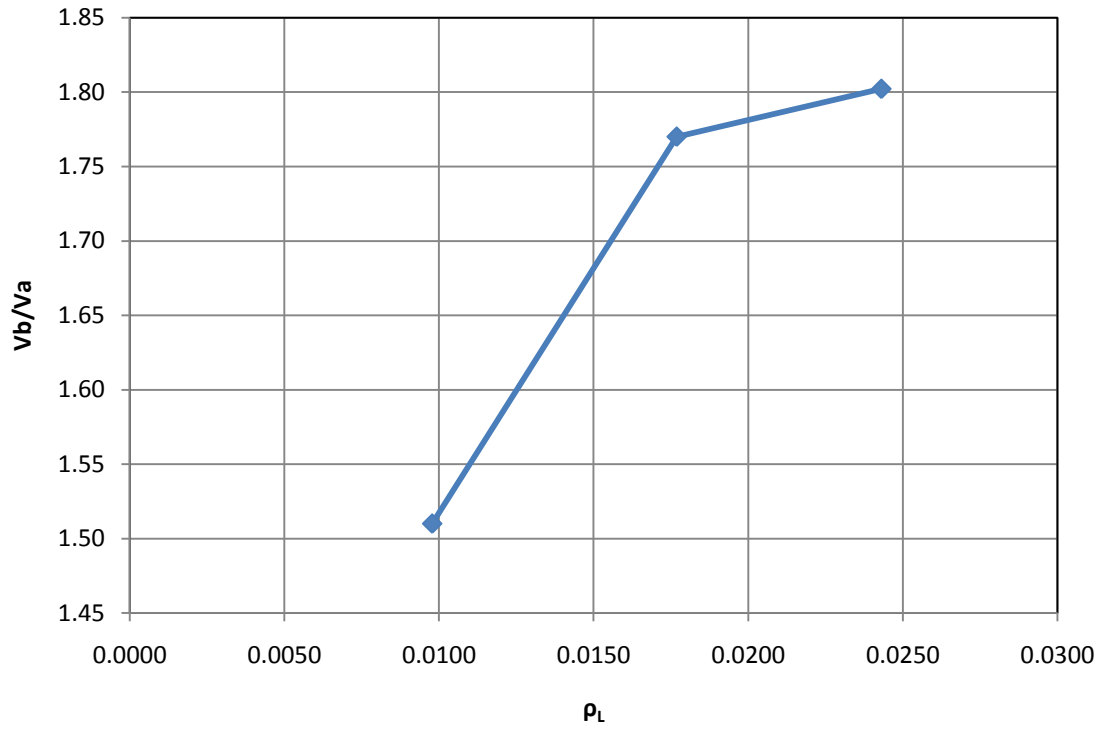


Fig 5.10 Reserve Strength vs Longitudinal Tensile Reinforcement Ratio

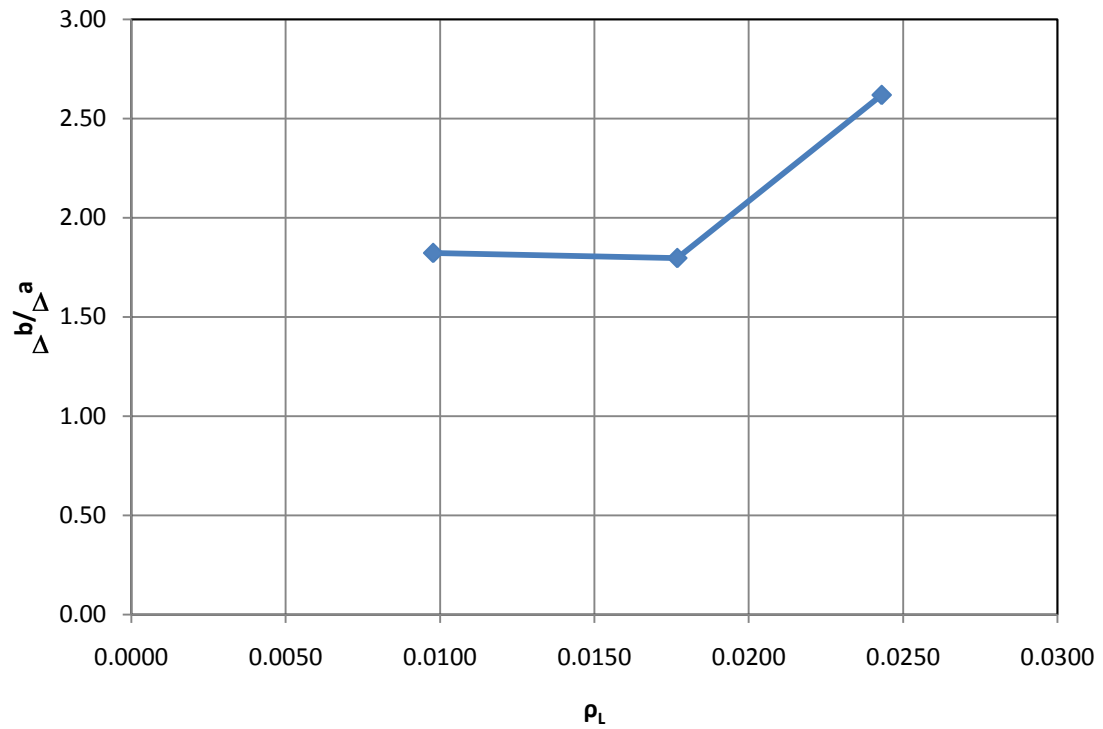


Fig 5.11 Reserve Deflection vs Longitudinal Tensile Reinforcement Ratio

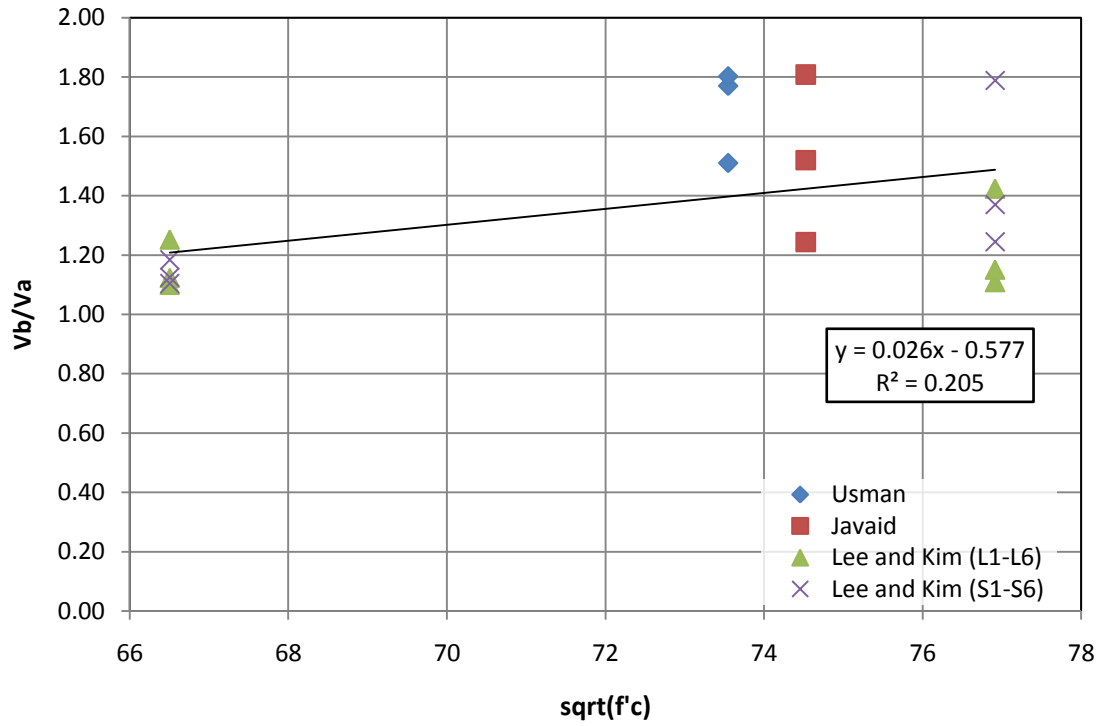


Fig 5.12 Reserve Strength vs Concrete Compressive Strength

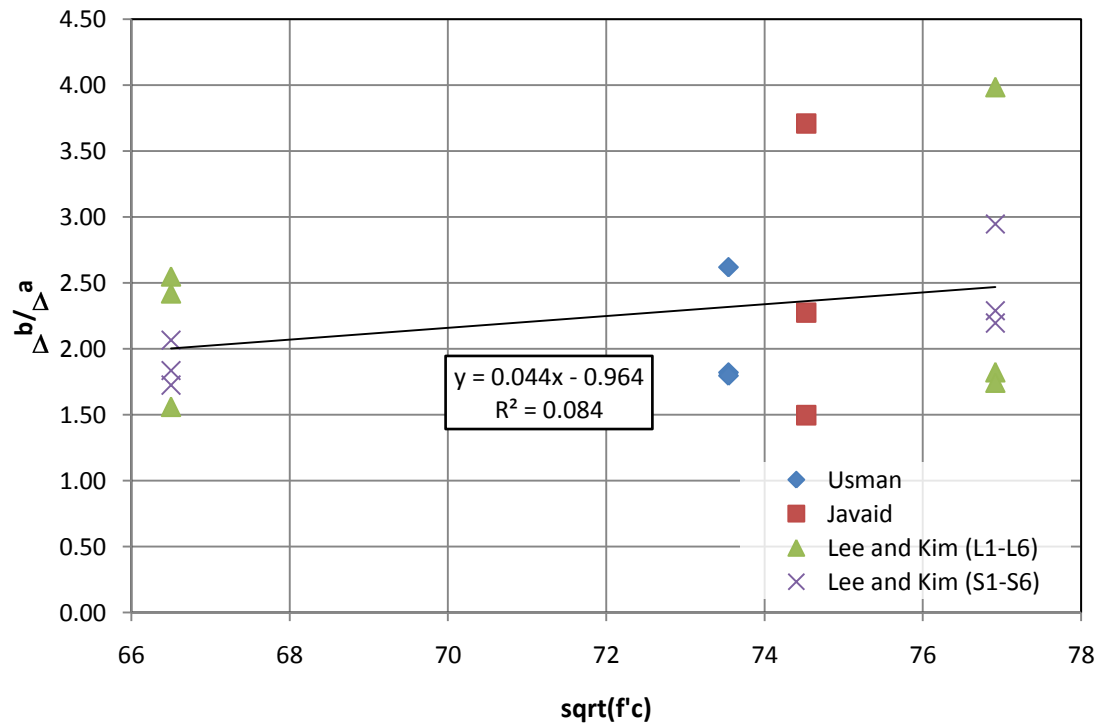


Fig 5.13 Reserve Deflection vs Concrete Compressive Strength

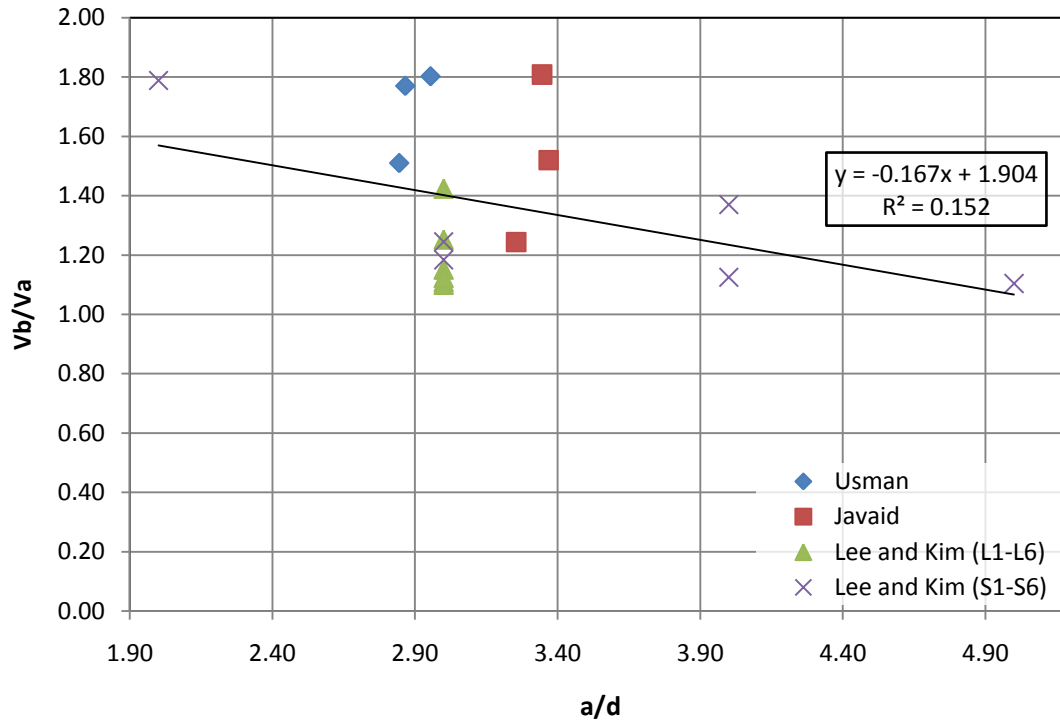


Fig 5.14 Reserve Strength vs Shear Span to Depth Ratio

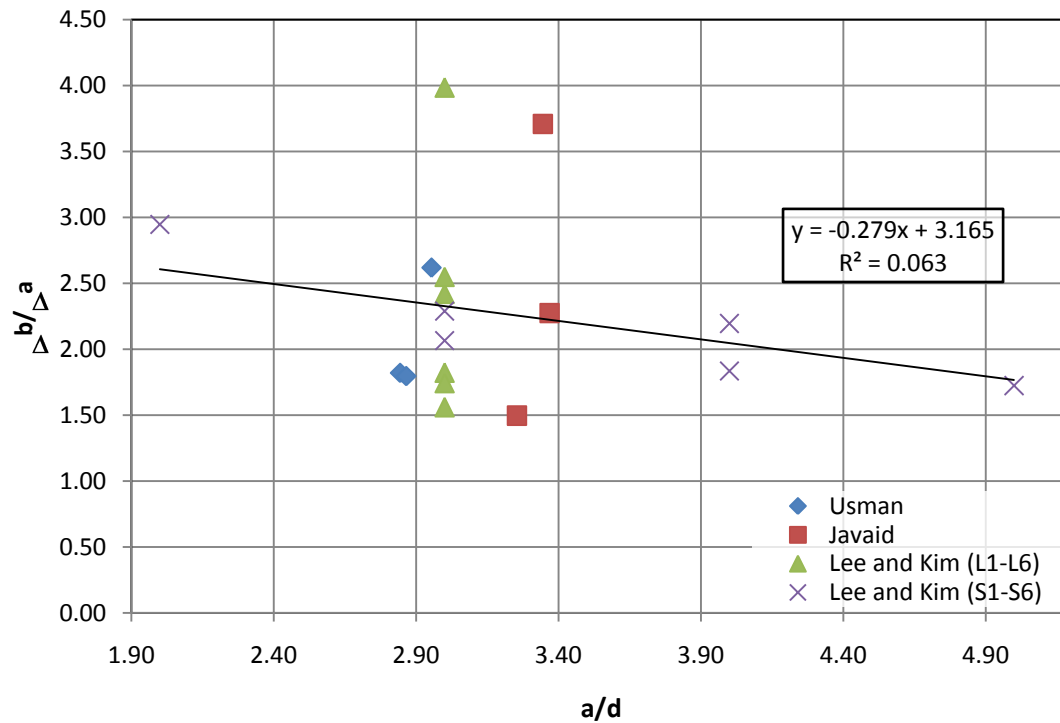


Fig 5.15 Reserve Deflection vs Shear Span to Depth Ratio

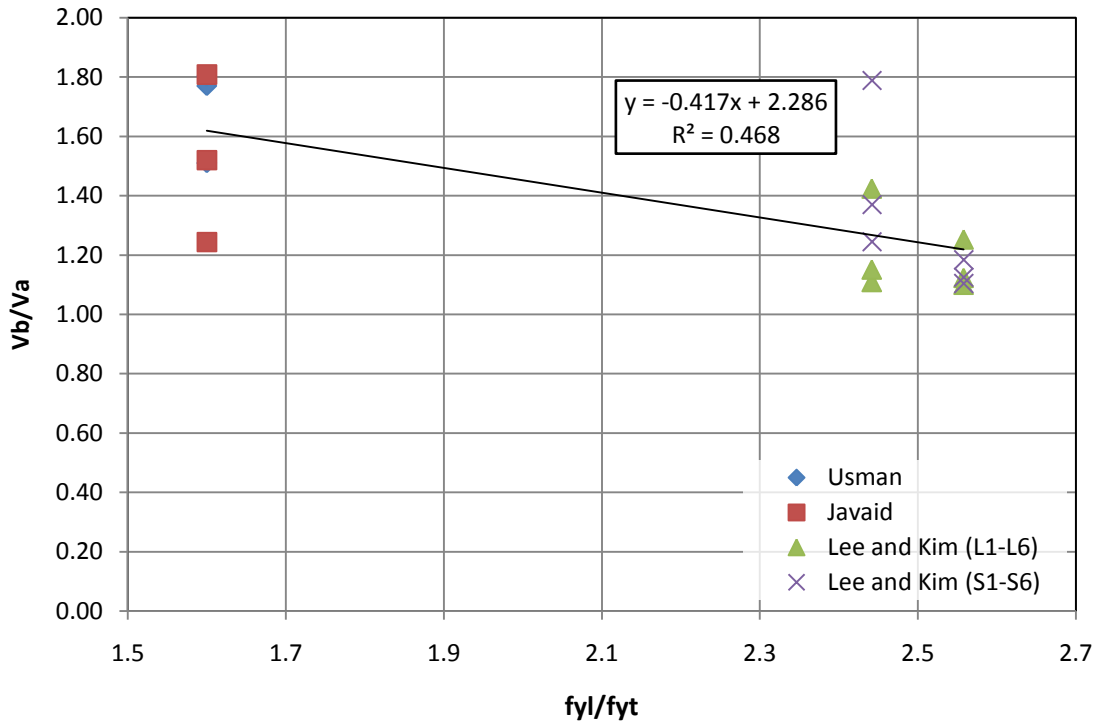


Fig 5.16 Reserve Deflection vs Longitudinal and Transverse Reinforcement Tensile Strength Ratio ( $f_{y\ell}/f_{yt}$ )

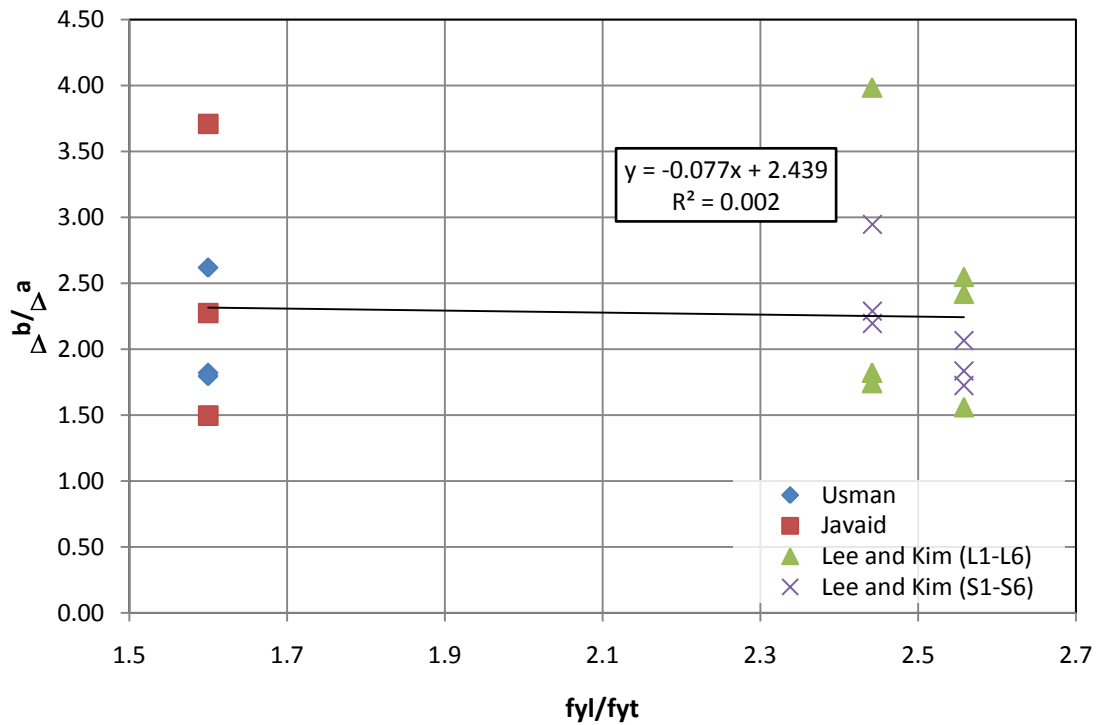


Fig 5.17 Reserve Deflection vs Longitudinal and Transverse Reinforcement Tensile Strength Ratio ( $f_{y\ell}/f_{yt}$ )



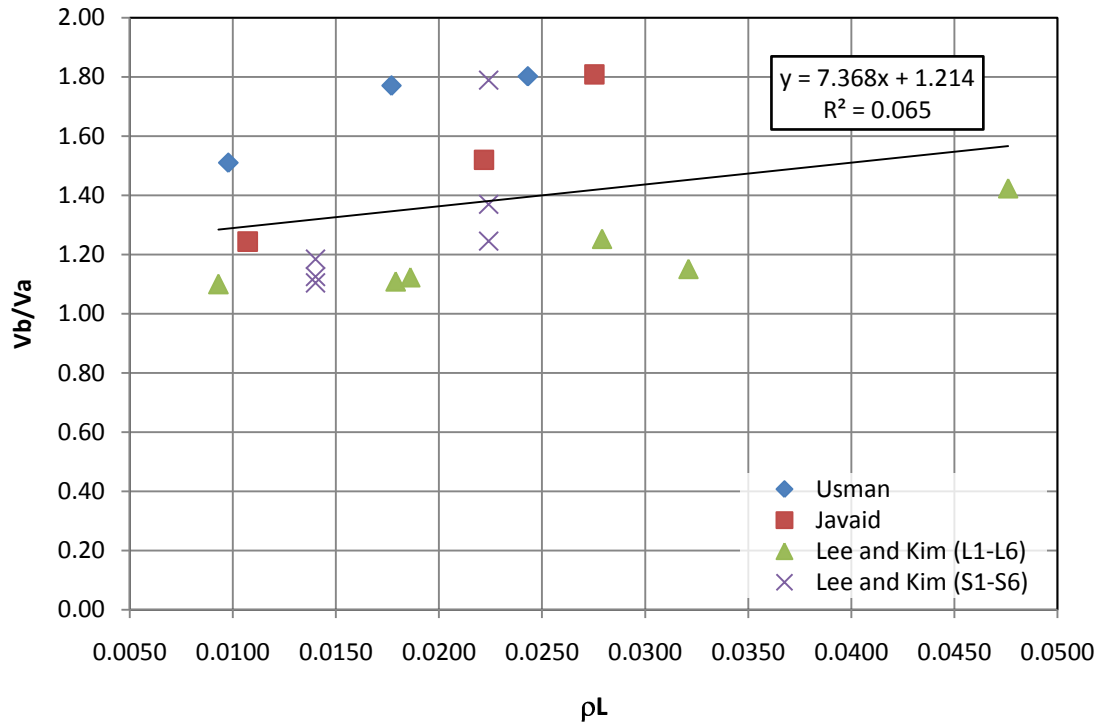


Fig 5.18 Reserve Strength vs Longitudinal Tensile Reinforcement Ratio

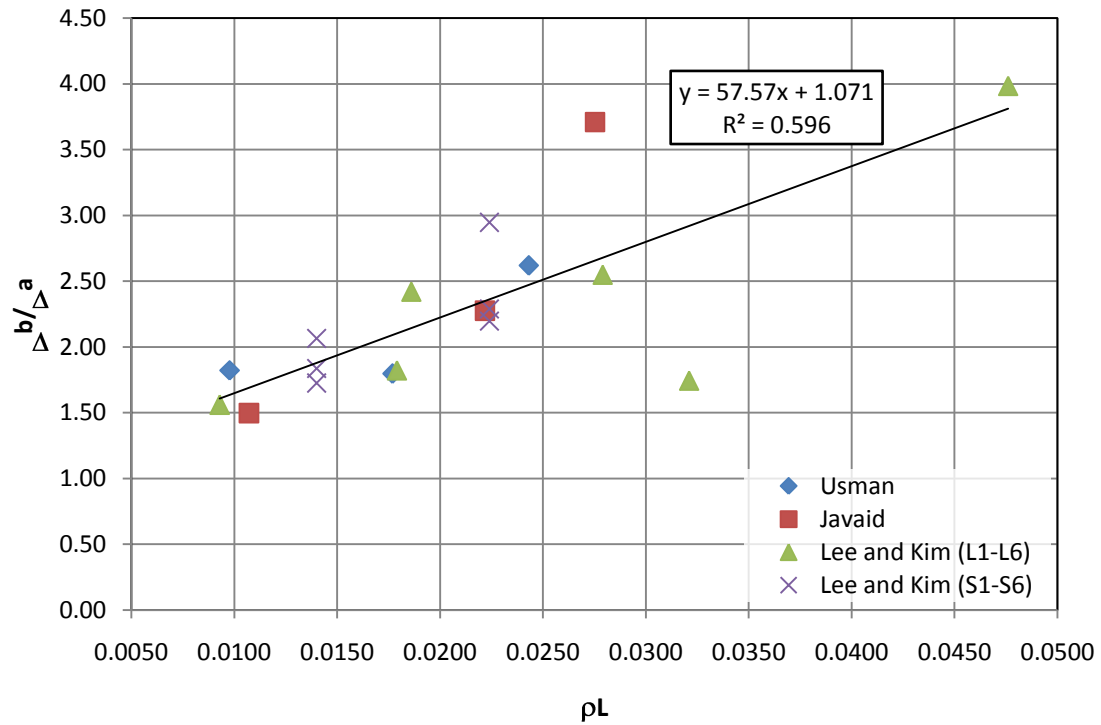


Fig 5.19 Reserve Deflection vs Longitudinal Tensile Reinforcement Ratio

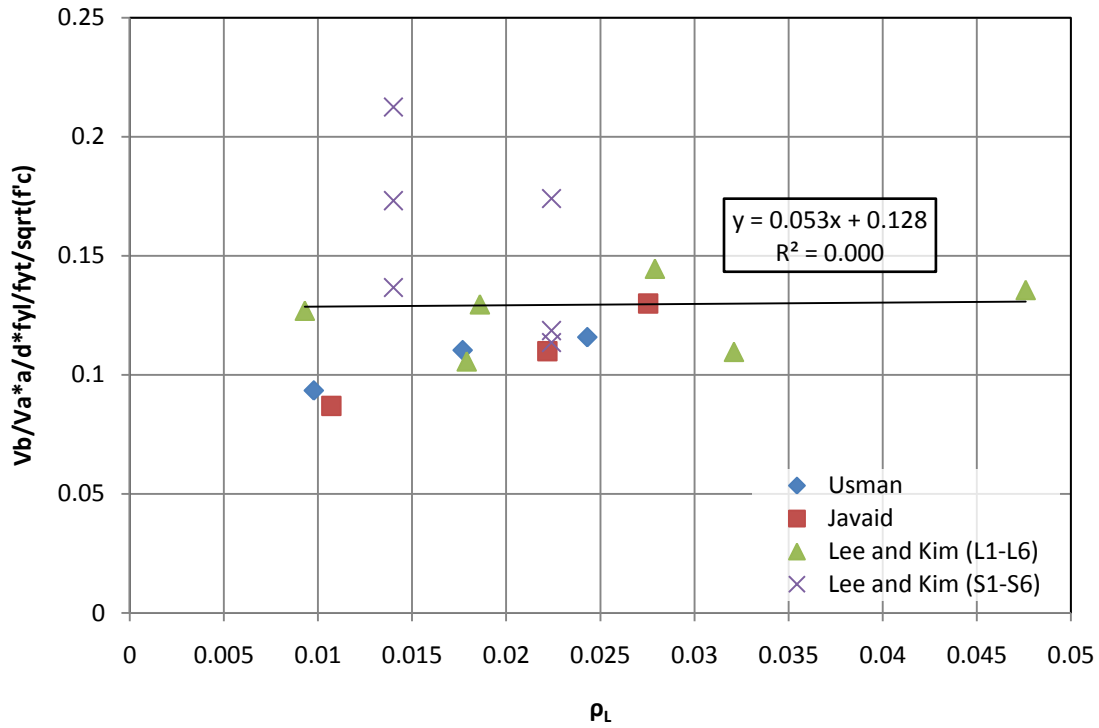


Fig 5.20 Reserve Strength vs Longitudinal Tensile Reinforcement Ratio (Effect of Parameters in Combination - I)

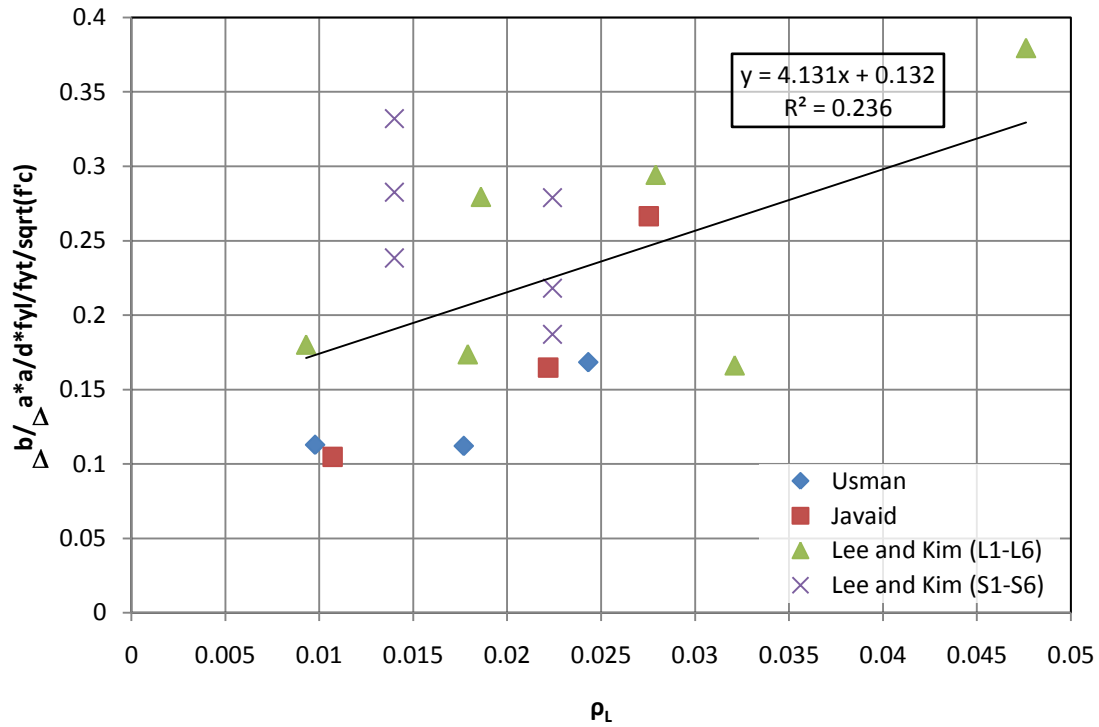


Fig 5.21 Reserve Deflection vs Longitudinal Tensile Reinforcement Ratio (Effect of Parameters in Combination - I)

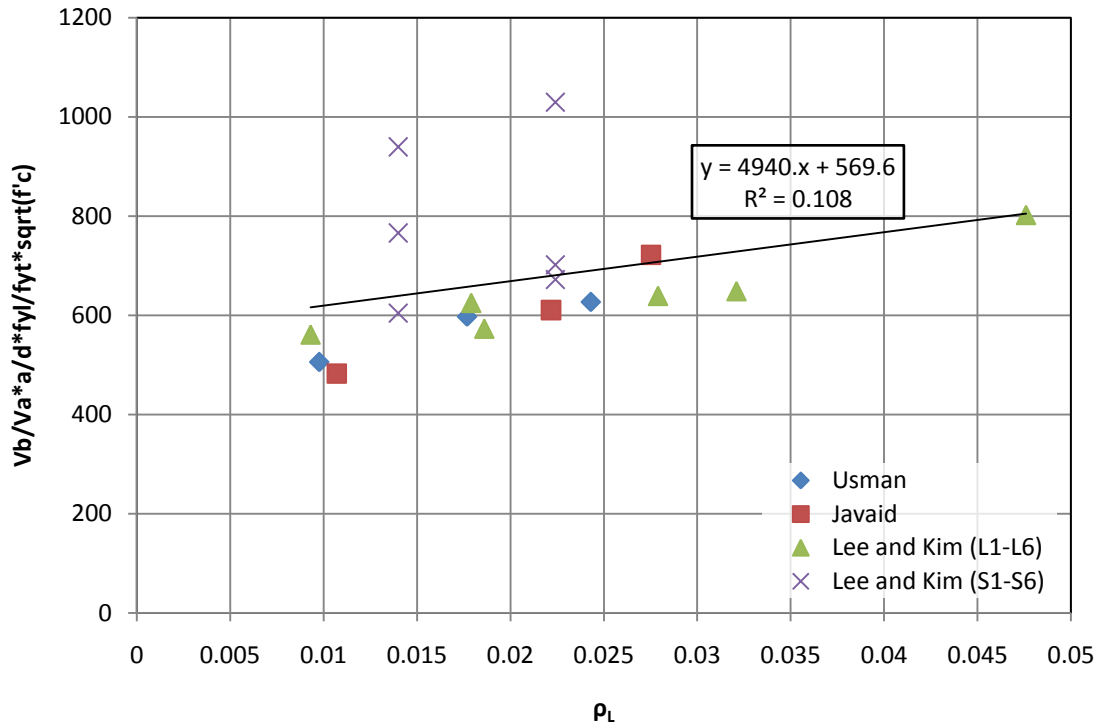


Fig 5.22 Reserve Strength vs Longitudinal Tensile Reinforcement Ratio (Effect of Parameters in Combination - II)

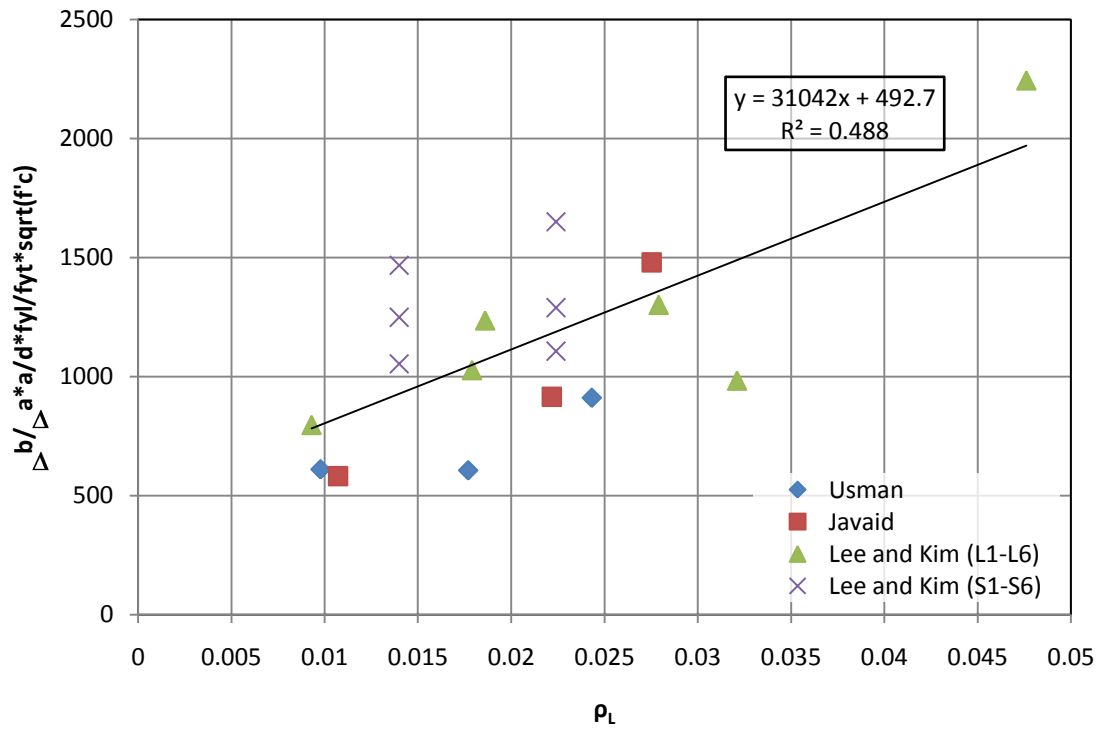


Fig 5.23 Reserve Deflection vs Longitudinal Tensile Reinforcement Ratio (Effect of Parameters in Combination - II)

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