

**Cracking behavior of two way reinforced concrete base slabs of
telecommunication towers in flexure with varying reinforcement details**



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DEDICATED

TO

MY PARENTS, SIBLINGS AND BETTER HALF

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In the name of Allah, the most merciful, the most compassionate all praises be to Allah, the lord of the worlds and prayers and Peace be upon Muhammad his servant and messenger.

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ABSTRACT

Limited space availability leads to installation of telecommunication towers at roof slabs of residential and commercial buildings. Excessive and early cracking is envisaged in reinforced concrete slabs due to application of 4 points concentrated loads by towers. This affects the serviceability of the reinforced concrete structure. In order to evaluate the flexural behavior of reinforced concrete (RC) slabs, numerical study is carried out by using finite element approach. Four two way RC slabs towers with same geometry and loading conditions (four point concentrated loading corresponding to telecommunication tower load) but different steel reinforcement detailing are analyzed on computer software ANSYS 16.0. Distribution of steel reinforcement is in accordance with the expected bending moment of slabs against the gravity load. Geometric parameters of proposed slab models are applied on crack controlling equation proposed by Nawy and Blair. Results depict that well detailed reinforcement exhibits much better ductile response compared to the one that lacks in detail. Moreover, cracking load has been delayed successfully due to accurate distribution and detailing of reinforcements. It is found that crack width in RC slab is directly influenced by steel bars size and spacing between them. The study also reveals that crack widths in all slabs are within permissible limit recommended by ACI 224R (2001).

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INTRODUCTION

1.1 General

Since last three decades, telecommunications have become the central nervous system of all major components of human activities such as economy, health, travel, education and communications etc. This is done through cellular communications by transfer of data using cellular networks. Cellular communications have become an essential part of everyday life and use of this medium is growing exponentially. To provide full and uninterrupted communication networks coverage, telecommunication towers must be erected in specified closed vicinity to each other. Provision of closed proximity of communication towers is easy in country side, however it becomes challenging in densely populated urban areas.

Due to lack of construction space in built-up areas, telecommunication towers are usually erected on roof tops of residential and commercial buildings. Many times these towers are directly attached to the roof slabs of the buildings which are not designed to take such loading. As a result, these communication towers pose severe hazards to the occupants of the buildings. These roof slabs are designed for common loading conditions, whereas, communication towers have four point concentrated loading, which is hazardous and unsafe for the occupants. Four point loading can result in early cracking in conventionally designed slabs leading to their failure, thus posing a design challenge to structural engineers.

Serviceability of a structure system is equally important as strength of that structure. A designer must give special attention to efficient performance of structure at service load. If only strength parameter is taken into consideration, then although there will be no danger of getting fracture but efficient performance at service load before collapse becomes suspicious. This may result into excessively large deflection under full service load or sustained load causing long term deflection may lead to collapse. The same case is possible with slab system under telecommunication towers that, under service load of tower through four point concentrated loading condition, the slabs designed for normal loading conditions may show excessive cracking and deflection in the slab may be unacceptably large. Therefore it is necessary to

design a structure under consideration of many limit states (*limit state*: the state at which a structure does not remain serviceable). These states are as below:

- a. *Ultimate Limit State*: the state which leads a structure to collapse
- b. *Serviceability Limit State*: the state which disrupts performance of structure but does not lead to damage or fracture

The most important serviceability limit states to be considered in design of a structure are excessive deflection and excessive crack width at service load. The term *service load* or *work load* states the load exerted on a structure in daily life use. In the proposed research, RC slabs are studied with special consideration of serviceability limit states i.e. deflection and cracks, to enhance its serviceability and mitigate cracking under telecommunication towers concentrated load.

1.2 Deflection

Working Stress Design or Allowable Strength Design used earlier to 1970s reduced compressive strength of concrete up to 45% and tensile strength of steel to smaller than 50% of its yield strength. For reduced or allowable material strengths, the structure could be assumed to act perfectly within the elastic range for the most severe loadings. However, inelastic behavior, ultimate failure modes and redistribution of moment effects were not considered in this method of design. Consequently, heavier sections showed larger spare strength than that of attained by the current strength design approach.

In strength design, high compressive strength concrete i.e. more than 12000 psi, and high tensile strength steel are being used and growing development in material properties has led to lower of factor of safety and reduced reserve strength. Hence, more efficient and slender structural members are designed, with deflection becoming a more prominent controlling criteria, (Nawy, 1985).

Calculation of deflection in two way slab is very much complex and one must consider boundary conditions along with the edge of slab panel, live load pattern, loading history, cracking due to flexure and increase in deflection due to creep, during its calculation. The long term deflection coefficient λ_{Δ} is given as 2.00 in the ACI Code. (Coefficient λ_{Δ} equals to the coefficient ξ due to non-provision of compression reinforcement at midspan of two way slab).

MacGregor et al. (1997) recommended that its value is not probably sufficient and be increased to 3.00 for two way slab.

The general form of Elastic Deflections can be as under:

$$\Delta = \frac{f \text{ (loads, spans, supports)}}{EI}$$

Where EI defines flexural rigidity and f is a function of load, span and support arrangement. Flexural tension cracks will never occur, if the maximum moment is so less in a member that the tensile stress does not exceed the modulus of rupture f_r in the concrete. The member will resist stresses and provide rigidity as a complete uncracked section. Correspondingly, for this load range,

$$\Delta_{iu} = \frac{f}{E_c I_{ut}}$$

In addition to limitations on cracking, it is generally essential to apply controls on deflection of reinforced concrete (RC) structures to guarantee serviceability. The most suitable and usual approach to deflection control is fixing some appropriate higher limits on the span-depth ratio.

The ACI 435 (2003) and ACI 318 (2011), Sec 9.5 show two approaches for the deflection control. These are the limiting thickness and the limiting computed deflections.

Limiting Thickness The minimum thickness stipulated in Table 1.1 [Table 9.5 (a) of ACI 318 (2011)] applies for beams, one way slab and composite members. This minimum thickness criteria is applicable only on members not attached / supporting to nonstructural members expected to be affected by deflection, until calculation of deflection indicating smaller thickness may be taken into account without hostile effects.

Values of minimum thickness provided in table will be used for structural members having normal weight concrete ($w_c = 145\text{lb/ft}^3$) and Gde-60 steel and if circumstances change, the values will also be revised. For the members with light weight concrete, w_c in the series of 90 lb/ft³ to 115 lb/ft³, the minimum thickness answers will be multiplied by $(1.65 - 0.005 w_c)$ but not less than 1.09 and for the structural members with reinforcement of yield strength f_y except 60 ksi, the answers will be multiplied by $(0.4 + f_y/100,000)$.

	Minimum thickness, h			
	Simply supported	One end continuous	Both ends continuous	Cantilever
Member	Members not supporting or attached to partitions or other construction likely to be damaged by large deflections			
Solid one-way slabs	$l/20$	$l/24$	$l/28$	$l/10$
Beams or ribbed one-way slabs	$l/16$	$l/18.5$	$l/21$	$l/8$

Table 1.1 - Minimum Thickness of Beams or One-Way Slabs

For two way nonprestressed slab construction, the ACI 318 (2011) gives a table of minimum thickness of slab. The provisions take into account, panel shape, yield strength of reinforcement, relative stiffness of beams and whether the panel in an exterior or interior panel. The provisions do not take into account concrete weight or Young's modulus or the applied load intensity. The thickness given may be insufficient for heavily loaded slabs, especially for slabs without beams. Table 1.2 [Table 9.5 (c) of the ACI 318 (2011)] stipulates the minimum thickness for flat slabs.

f_y , psi [†]	Without drop panels [‡]			With drop panels [‡]		
	Exterior panels		Interior panels	Exterior panels		Interior panels
	Without edge beams	With edge beams [§]		Without edge beams	With edge beams [§]	
40,000	$l_n/33$	$l_n/36$	$l_n/36$	$l_n/36$	$l_n/40$	$l_n/40$
60,000	$l_n/30$	$l_n/33$	$l_n/33$	$l_n/33$	$l_n/36$	$l_n/36$
75,000	$l_n/28$	$l_n/31$	$l_n/31$	$l_n/31$	$l_n/34$	$l_n/34$

* For two-way construction, l_n is the length of clear span in the long direction, measured face-to-face of supports in slabs without beams and face-to-face of beams or other supports in other cases.
[†] For f_y between the values given in the table, minimum thickness shall be determined by linear interpolation.
[‡] Drop panels as defined in 13.2.5.
[§] Slabs with beams between columns along exterior edges. The value of α_f for the edge beam shall not be less than 0.8.

Table 1.2 - Minimum Thickness of Slabs Without Interior Beams

ACI Code gives two equations of minimum thickness for slabs having beams all along the edges. The formula of minimum thickness for interior panels of slabs with beam stiffness $0.2 \leq \alpha_m \leq 2.0$, is

$$h = \frac{l_n(0.8 + \frac{f_y}{200,000})}{36 + 5\beta (\alpha_{fm} - 0.2)}$$

where, h is the slab thickness; l_n the clear span in longer side of the panel, measured inner to inner sides of the columns in flat slabs or inner to inner sides of beams; f_y the yield strength; α_m the average of the α values of the four beams at the edges of panel, where α is taken as ratio between EI of the beam and the slab width and β is ratio between clear spans in long and short direction of slab.

For the slabs with beam stiffness, $\alpha_m \geq 2.0$, the minimum thickness is

$$h = \frac{l_n(0.8 + \frac{f_y}{200,000})}{36 + 9\beta}$$

It is necessary for edge and corner panels to have edge beams with stiffness ratio α_f greater than 0.8 or minimum thickness be increased by 10% of that calculated in above equations. A number of other limits on thickness of two way slabs are also stipulated in ACI Code including some absolute values, as followed;

Minimum thickness of slabs without beams or drop panel	=	5 in
Minimum thickness of slabs with drop panel	=	4 in

Limiting Computed Deflections

Lesser thickness of flexural member than those stipulated in ACI 318 (2011) can only be used when the computed deflections are found within acceptable bounds. Immediate deflections occurred after application of load will be computed by using an elastic analysis in view of influence of cracking and steel reinforcement of slab. Deflections calculated in accordance with 9.5.2.2 through 9.5.2.5 of ACI Code for one way slab and with 9.5.3.1 through 9.5.3.3 of

ACI Code for two way slab must not go beyond the limits set out in Table 1.3 [Table 9.5 (b) of the ACI Code].

Type of Member	Deflection to be considered	Deflection Limitation
Flat roofs not supporting or attached to nonstructural elements likely to be damaged by large deflections	Immediate Deflection due to live load L	$l/180^{\dagger}$
Floors not supporting or attached to nonstructural elements likely to be damaged by large deflections	Immediate Deflection due to live load L	$l/360$
Roof or floor construction supporting or attached to nonstructural elements likely to be damaged by large deflections	That part of the total deflection occurring after attachment of nonstructural elements (sum of the long-term deflection due to all sustained loads and the immediate deflection due to any additional live load). ^{&}	$l/480^{\%}$
Roof or floor construction supporting or attached to nonstructural elements not likely to be damaged by large deflections		$l/240^{\S}$
<p>[†]Limit not intended to safeguard against ponding. Ponding should be checked by suitable calculations of deflections, including added deflections due to ponded water and considering long term effects of all sustained loads, camber, construction tolerances and reliability of provisions for drainage.</p> <p>[%]Long term deflection shall be determined in accordance with 9.5.2.5 or 9.5.4.3, but may be reduced by amount of deflection calculated to occur before attachment of nonstructural elements. This amount shall be determined on basis of accepted engineering data relating to time-deflection characteristics of members similar to those being considered.</p> <p>[§]Limit may be exceeded if adequate measures are taken to prevent damage to supported or attached elements.</p> <p>^{&}Limit shall not be greater than tolerance provided for nonstructural elements. Limit may be exceeded if camber is provided so that total deflection minus camber does not exceed limit.</p>		

Table 1.3 – Maximum Permissible Computed Deflections

1.3 Cracking

There are many causes of occurrence of cracks in concrete. Besides, poor appearance, they directly affect structural strength and also serviceability. ACI 224.1R-07 (2007) describes a brief summary of the causes of cracks but none of these are due to flexural load, whereas, excessive cracking in RC slabs is envisaged due to telecommunication towers depicting four point concentrated loading. Syed Mohd Mehndi et al (2014) also published detailed notes on causes of cracks in plastic as well as elastic stage and evaluation of crack sizes including spacing and width through different types of instruments. This paper was very much helpful for knowing and eradication of root cause of cracks but it lacks in elaboration of the kinds of cracks produced in concrete members subjected to external load causing tensile stresses at the tension zone of concrete.

As concrete is weak in tension, consequently, it gets cracked in flexure even at small load. Therefore, it is necessary to study cracking behavior and crack width control. Longitudinal and lateral stresses act on the tension face of concrete member as illustrated in Fig 1.1 (a). At time of flexural cracks, biaxial lateral compression produced between concrete and reinforcing bars has to disappear at the crack because tension in concrete becomes zero. The concrete can no longer bear any tension due to high stress concentration at the moment of emerging fracture and it splits as shown in Fig 1.1 (b). Consequently, stress dynamically transfer to reinforcing bars from concrete as illustrated in Fig 1.1 (c) and at the same time, tensile stress in concrete becoming zero at the crack as seen in Fig 1.1 (d). Ultimately, position of neutral axis rises at the location of cracked section for the purpose of maintenance of equilibrium at that section.

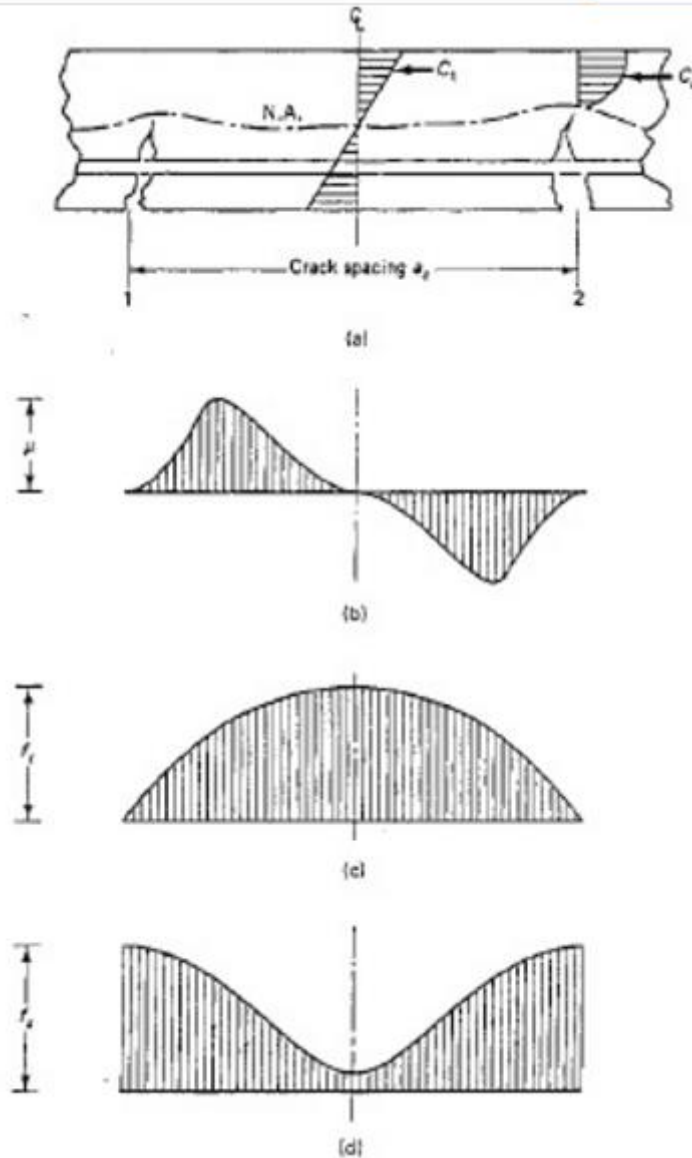


Figure 1.1 – Longitudinal stress distribution between adjacent cracks; (a) crack development geometry; (b) ultimate bond stress; (c) longitudinal tensile stress in the concrete; (d) longitudinal tensile stress in the steel

In two way slab, flexural behavior is totally different from one way slab or beams. In fact, equation of crack control developed for one way slab or beam underestimates the crack width established in two way slab. The basic parameters to control the cracks in two way slabs are spacing of steel reinforcement and steel stress level in orthogonal directions, whereas, concrete clear cover is almost constant in two way slab as per ACI Code, but it leaves a dominant effect

in beam. Nawy (1985) carried out extensive experiments on two way slabs. Results of these tests demonstrate the difference in flexural behavior in a fracture hypothesis on crack development. When load is applied on slab, stress concentration initially develops at the intersection points of steel reinforcement i.e. grid nodal points A_1 , A_2 , B_1 , B_2 and then producing fracture lines along the path of least resistance i.e. A_1A_2 , A_2B_2 , B_2B_1 , A_1B_1 . If the spacing of steel reinforcement is large, the stress concentration and energy absorbed per unit grid is too low to generate orthogonal cracks along the path of steel reinforcement. Resultantly, cracks follow diagonal path away from reinforcement, as shown in Fig 1.2. Width of these cracks are larger than that of orthogonal cracks, therefore, latter pattern of cracks are preferred.

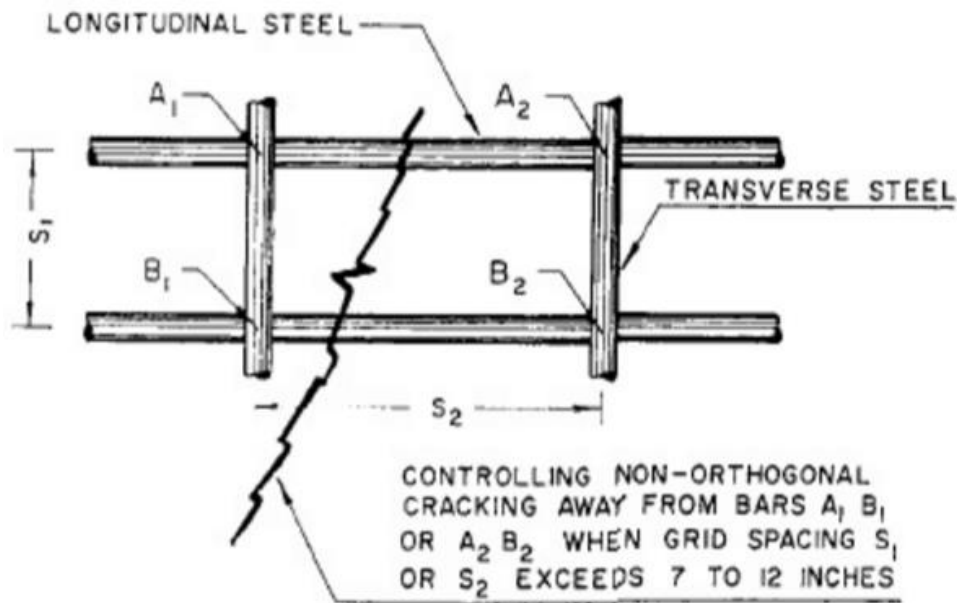


Figure 1.2 – Grid unit in two way action reinforcement

From these experiments, Nawy (1985) formulated an equation for computation of width of flexural cracks in two way slabs. A grid index factor was used for the determination of effect of reinforcement spacing, diameter and concrete cover on the crack width. Nawy-Orenstein equations for crack width for plates and two way slabs is as;

$$w_{max} = K\beta f_s \sqrt{M_1}$$

where,

$$K = 2.1 \times 10^{-5} \text{ (depends on boundary as well as loading conditions)}$$

$B = h_2/h_1 =$ ratio of distance to the neutral axis from extreme tension face and from centroid of reinforcement

$f_s =$ steel stress

$M_1 =$ grid index, given by

$$M_1 = \frac{d_{b1}s_2}{\rho_{t1}}$$

where,

$d_{b1} =$ diameter of bar along direction 1, in which crack width is measured

$s_2 =$ spacing of bars along direction 2 (perpendicular to direction 1)

$\rho_{t1} =$ ratio of area of bars along direction 1 and area of concrete in tension perpendicular to these bars

Nawy suggests, if no transverse reinforcement is there in slab, use $s_2 = 12$ in, otherwise grid index will be undefined. Steel reinforcement grid is shown as Fig 1.3

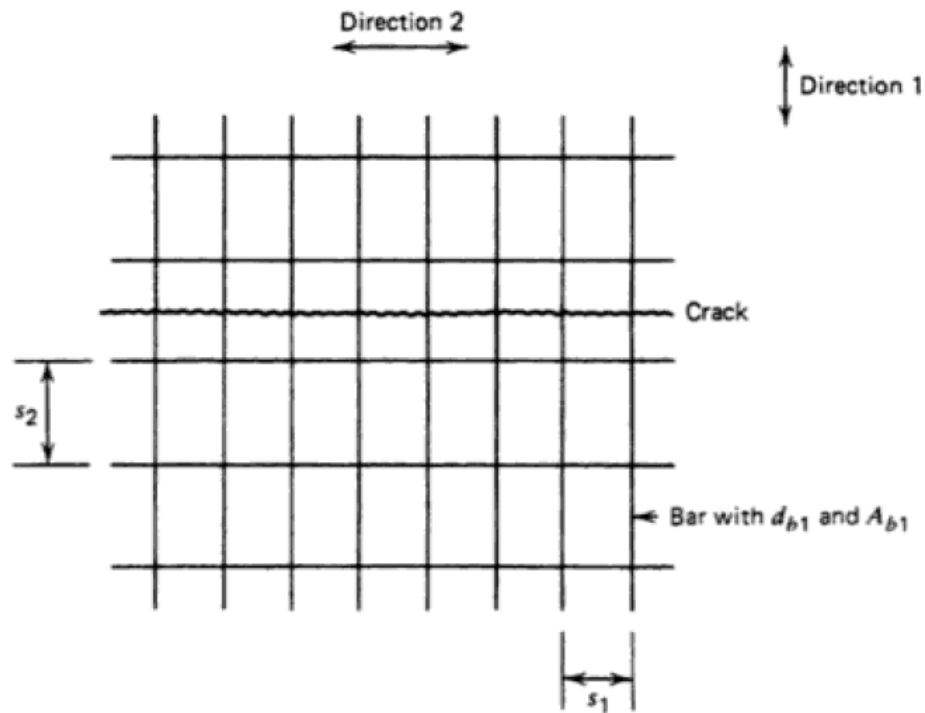


Fig 1.3 – Steel Arrangement for Determination of Grid Index

The report ACI 224R (2001) defines the tolerable crack widths on tension surface of concrete structure in different exposure conditions as a guideline. These tolerable crack widths are given in Table 1.3.

Exposure Condition	Tolerable Crack Widths	
	in.	mm
Dry air or protective membrane	0.016	0.41
Humidity, moist air, soil	0.012	0.30
Deicing chemicals	0.007	0.18
Seawater and seawater spray: Wetting and drying	0.006	0.15
Water retaining structures (excluding <u>nonpressure pipes</u>)	0.004	0.10

Table 1.4 – Tolerable crack widths

There is no exact reference in ACI Code for the crack control in two way slab. The best way to control cracks in slabs is only possible, if reinforcement is closely spaced. ACI 318 (2011) clause 13.3.2 indicates a signal for spacing of steel reinforcement to control the cracks in two way slabs i.e. steel reinforcement spacing should not exceed double of the slab thickness.

1.4 Scope of Project

Prime scope of the research includes

1. Safety of occupants
2. Stability of telecommunication towers
3. Formulation of new steel reinforcement detailing design
4. Mitigation of early cracking in RC slabs
5. Economization of RC slab construction

1.5 Current State of the Art

Current state of the art practice to mitigate cracking in RC slabs under normal loading conditions is the result of enormous research activities that have been carried out for the past many years with prime focus on increasing tensile strength of concrete by using steel fibers or

admixtures, successively by replacing steel reinforcement with FRP and RC members with pre-stressed members. Furthermore, avoidance of cracking has also been exercised by pronouncedly taking into account the ruling crack controlling parameters in steel reinforcement detailing like concrete cover, reinforcement spacing, steel diameter etc. But, unfortunately, the same techniques are being used for designing of RC slabs under telecommunication towers load, which results into early cracking in slabs.

1.6 Challenges

1. Structural safety for building occupants by formulating a new steel reinforcement detailing in RC slabs.
2. Durable erection of telecommunication towers on RC slabs designed under towers concentrated point loading conditions.
3. Proposing workable design method / solutions to minimize occurrence and propagation of cracks in reinforced concrete slabs.
4. Enhancement in flexural strength of slabs with indirect decrease in steel reinforcement resulting in economizing of the structure.

1.7 Motivation and Needs

1. Structural failure is a major cause of loss of human lives. Structural safety is very important issue to be considered in engineering design and construction.
2. In construction industry, design of steel reinforcement detailing to mitigate cracking in RC slabs under telecommunication towers is an important need of the time keeping in view the hazards the towers pose in urban areas.
3. By adapting the guidelines, resulting from this research, safe and economical structures can be erected with significantly enhanced flexural strength in slabs.
4. With this state-of-the-art-research, efforts will be made to open up new venues by suggesting an innovative flexural design of RC slabs.

1.8 Objectives of the Project

Research Objectives

1. To enhance structural safety for residential and commercial buildings occupants.

2. To formulate new steel reinforcement detailing design to mitigate early cracking in RC slabs under telecommunication towers load.
3. To enhance flexural strength of RC slabs vide proposed guidelines.
4. To economize the design with reevaluation of steel reinforcement through steel detailing for crack control in RC slabs.

Academic Objectives

1. To propose workable design method / solutions to minimize occurrence and propagation of cracks in RC slabs under four points loading such as loading from telecommunication towers.
2. To analyze the behavior of cracks in two way slabs by experimental investigation on life size specimens with the help of a reaction frame.
3. To enhance the tensile capacity of structural member when subjected to concentrated point loads.
4. Evaluate and understand flexural response of two-way slabs under four-point concentrated loading and compare its behavior to uniformly distributed loads.

Industrial Objectives

1. To enhance flexural strength of the RC slabs under four-point concentrated loading depicting telecommunication towers load vide proposed guidelines.
2. Improved safety measures through increase in strength and ductility of RC slabs used for telecommunication towers.
3. To minimize repair cost of RC slabs damaged due to loading from towers.
4. To economize provision of steel reinforcement in RC slabs.

1.9 Beneficiaries of the Project

1. Construction industry and structural design firms.
2. Telecommunication industry through safe provision of telecommunication towers.
3. General public through safe construction practices.
4. Scientific community through state of the art research.

1.10 Outputs Expected from the Project

Structural Safety: The prime output will be enhanced structural safety and security for the occupants by provision of safe design of two-way slab systems through design detailing of steel reinforcement in RC slabs under four-point loading depicting telecommunication towers load condition.

Strength & Durability: In construction industry, strength of structure is of vital importance. As a result of guidelines emerging from this study, higher performance and durability is expected from structural slab systems that could be utilized for safe erection of telecommunication towers and other such structures. An effort will be made to keep the steel reinforcement to minimum in RC slabs with more sustainable and secure design for occupants.

Economy: It is expected that from the guidelines obtained from this study, more economical structures are constructed with substantially increasing the flexural strength and serviceability.

Research: Cracking pattern, flexural behavior, first crack load, crack width and other parameters regarding cracking of RC structures will be more improved in RC slabs under four-point concentrated loading depicting telecommunication tower loads. State-of-the-art research will be carried out by studying the behavior of two-way slabs structural systems and effort will be made to add knowledge to flexural performance of RC slabs recognized around the research community.

Design: Review and formulation of steel reinforcement detailing guidelines for RC slab systems to mitigate early cracking in RC structures.

LITERATURE REVIEW

2.1 General

When designing the RC structural member's geometry and steel reinforcement detailing, the engineers normally consider strength requirements and serviceability criteria. But in the life of structures, crack occurrence becomes one of serious challenges to engineers. Cracks in reinforced concrete structures are source of concern for both design and construction. As a result of experimental as well as analytical studies, engineers categorize different types of cracks in RC slabs. Flexural cracks in RC slabs are significantly different from other structural cracks and therefore, need extensive understanding. This chapter contains notable studies carried out on flexural behavior and crack control in RC members. These research works have taken into account different variables contributing towards crack mitigation.

2.2 Previous Studies on Flexural Crack Control

A number of experimental as well as analytical research studies have been carried out on flexural crack mitigation in reinforced concrete structural members. Few of important studies are presented here:

Gilbert and Nejadi (2004) carried out an experimentation to study the development of flexural cracking in concrete members in The University of New South Wales, Australia. They cast six concrete beams and six slabs and cured for a period of 28 days. All specimens were tested under simply supported conditions to failure using two point loading. Test setup used by researchers is shown in Figure 2.1. In these specimens, reinforcement detailing was changed with different steel areas. Two fundamental parameters of steel reinforcement detailing; area of steel and spacing, were varied in all specimens for the purpose of controlling flexural cracks on increasing short term loading. Details of parameters used in specimens are given in Table 2.1 and Table 2.2. Two same specimens "a" and "b" were cast for each set of parameters. Initial crack produced at heavier load with small width in specimens B3 and S3 due to over design of specimens. Early cracking in concrete was satisfactorily controlled by use of extra steel but it turned out to be economically unviable.

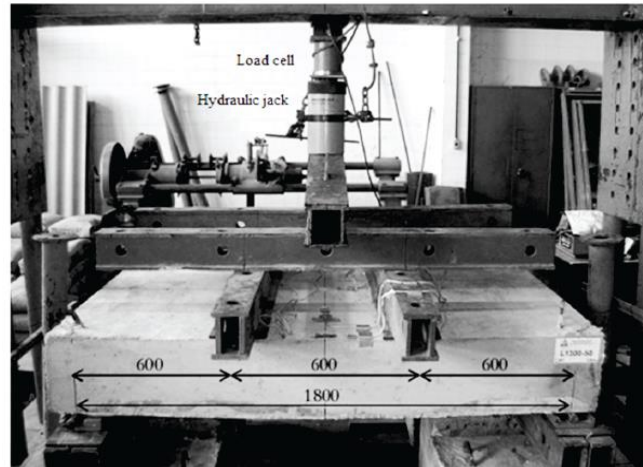


Figure 2.1 – General view of two way slab test set-up

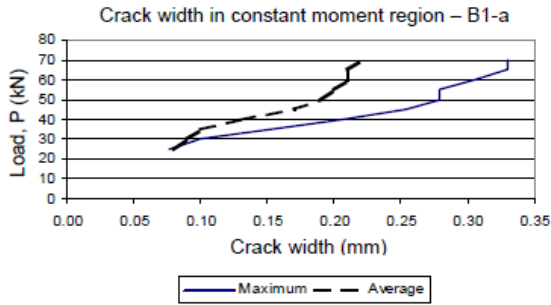
Specimen	No of Bars	Bar Dia (mm)	Steel Area (mm ²)	Spacing (mm)
B1-a	2	16	400	150
B1-b	2	16	400	150
B2-a	2	16	400	180
B2-b	2	16	400	180
B3-a	3	16	600	90
B3-b	3	16	600	90

Table 2.1 – Details of beams for short-term tests

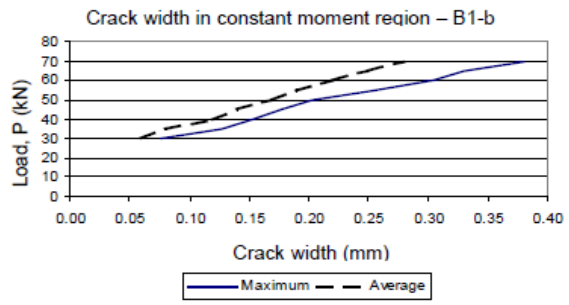
Specimen	No of Bars	Bar Dia (mm)	Steel Area (mm ²)	Spacing (mm)
S1-a	2	12	226	308
S1-b	2	12	226	308
S2-a	3	12	339	154
S2-b	3	12	339	154
S3-a	4	12	452	103
S3-b	4	12	452	103

Table 2.2 – Details of slabs for short-term tests

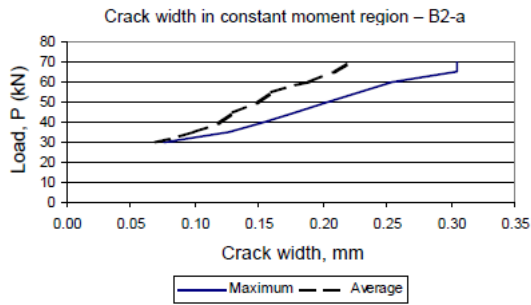
Results of all six beams and slabs are illustrated in Figure 2.2. Results show that by increasing area of steel in RC slab, strength and cracking load increases, whereas, crack width and deflection increases.



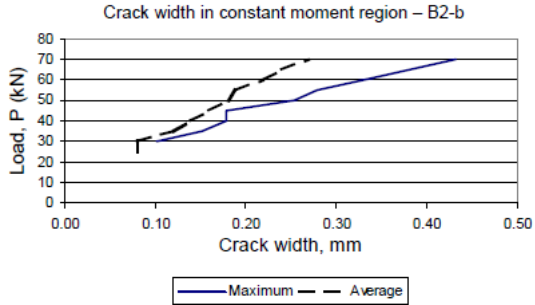
(a) Max and avg crack width for beam 1-a



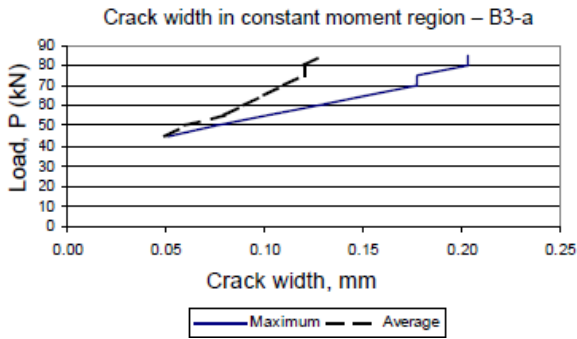
(b) Max and avg crack width for beam 1-b



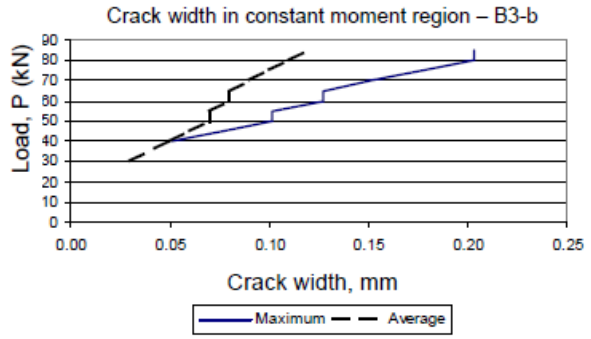
(c) Max and avg crack width for beam 2-a



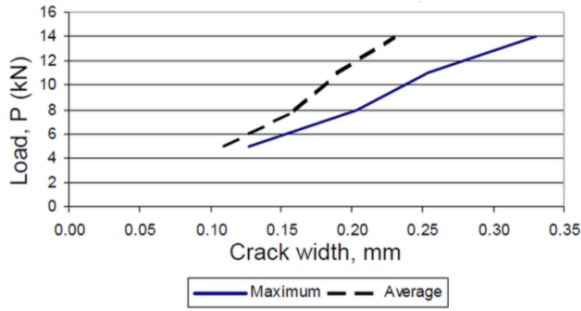
(d) Max and avg crack width for beam 2-b



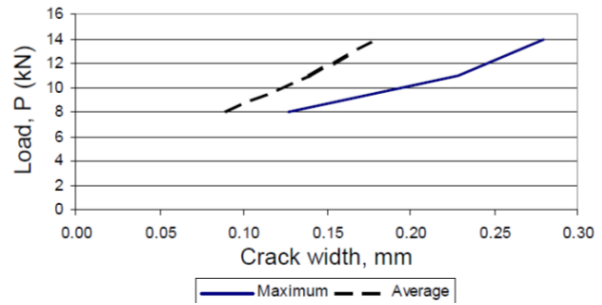
(e) Max and avg crack width for slab 3-a



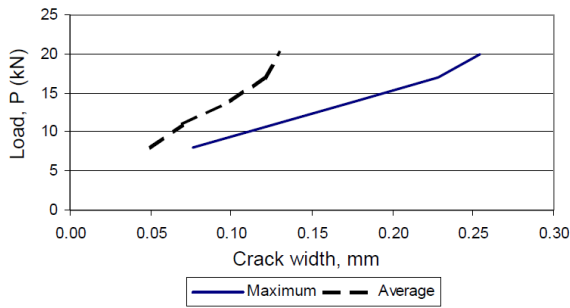
(f) Max and avg crack width for slab 3-b



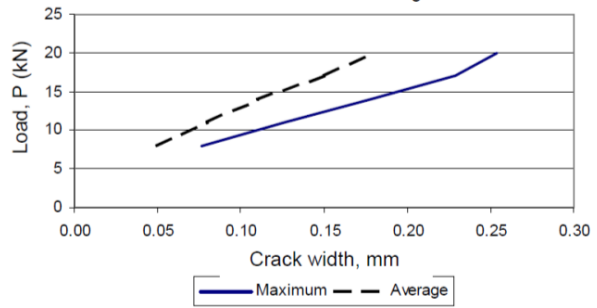
(g) Max and avg crack width for slab 1-a



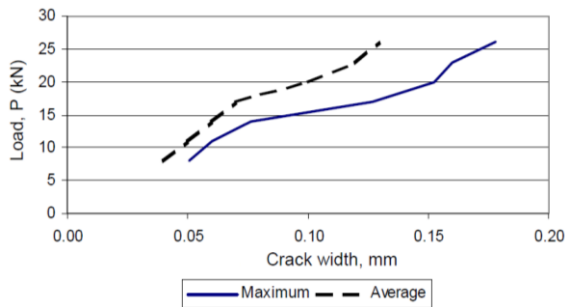
(h) Max and avg crack width for slab 1-b



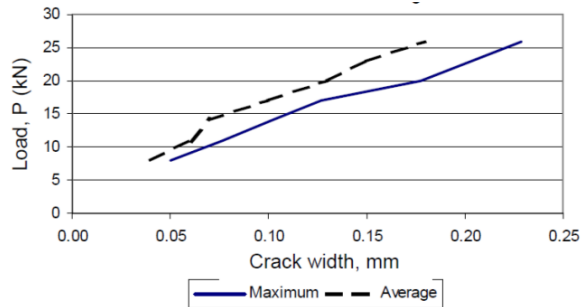
(i) Max and avg crack width for slab 2-a



(j) Max and avg crack width for slab 2-b



(k) Max and avg crack width for slab 3-a



(l) Max and avg crack width for slab 3-b

Figure 2.2 – Results of Twelve Specimens

From the extensive literature review and study of various flexural cracking models, Borosnyóí and Balázs (2005) found that cracking of concrete is always expected under tensile stresses. Along with other basic parameters, the tensile strength of concrete also depends upon maturity of concrete and reinforcement ratio. Cracks stabilize at the maximum tensile strength of reinforcement under the load corresponding to the crack stabilization, which is the same as maximum tensile strength of concrete due to assumption of accumulated bond slip. Crack stabilization in reinforced concrete structural member is shown in Figure 2.3. Controlling of concrete cracking is possible with the implementation of crack width formulae but available formulae of crack width in literature are normally based on simplifications. They suggested

that a rigorous calculation has to be carried out for the formulation of crack width considering integration of concrete between cracks and strain in steel reinforcement due to the accumulated slips. They declared that the cracking depends on two basic parameters; average crack spacing and average strain in steel reinforcement.

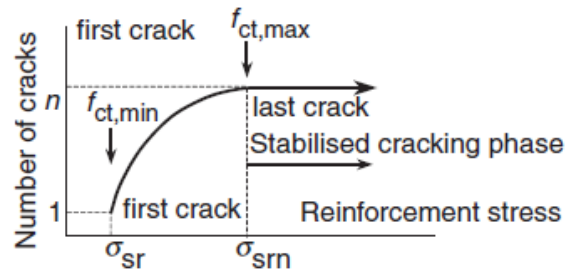


Figure 2.3 – Number of cracks and crack stabilization

Florian (2008), who was president of American Concrete Institute, investigated some slabs at Village Serramonte, California. The two way slabs were designed with dimensions 188' x 88' and post tensioned in both directions. As the cracks produced in transverse direction along the width of slab, the precompression stresses produced due to tendons in longitudinal direction was dissipated into the supporting walls. Layout of slab depicting cracks along the width of slab is shown in Figure 2.4. Florian found that the basic reason of cracking was the restraining effect of the supporting walls. If the slab is allowed to move freely, then the precompression produced in slab will balance the tendon force (F) as shown in Figure 2.5 (a). And if the slab is restrained through stiff walls or other supports, then a portion of tendon force (F) will divert to the supporting members, as shown in Figure 2.5 (b) and resultantly, cracks will produce in slab.

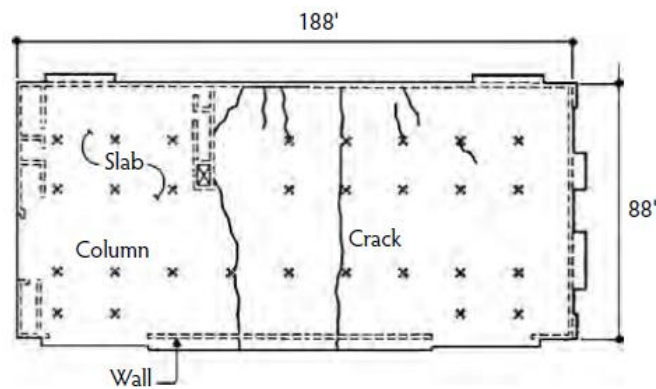


Figure 2.4 - Reflected ceiling showing cracks in slab

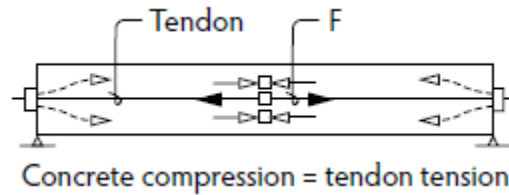


Figure 2.5 (a) – Slab free to move

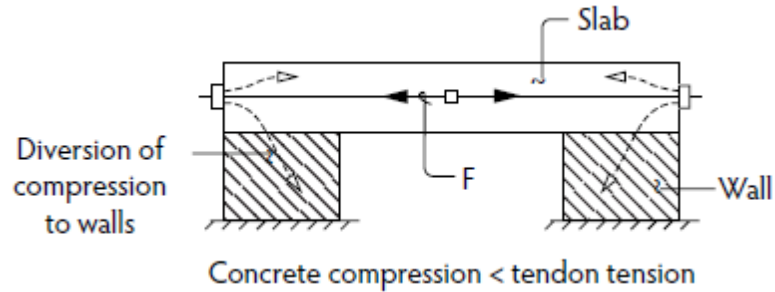


Figure 2.5 (b) – Slab restrained against movement

Ospina and Bakis (2007) validated the new procedure of indirect flexural crack controlling in concrete proposed by Frosch (1999), based on the fundamental crack control concepts, introduced originally by Broms (1965). He established the following formula for the calculation of crack width at the tension face of concrete member:

$$s = 2 \sqrt{\left(\frac{W E_r}{2f_r \beta k_b} \right)^2 - d_c^2}$$

Ospina and Bakis (2007) presented that crack width and crack spacing are the function of steel reinforcement spacing. The above equation was declared as “indirect flexural crack control” because maximum steel reinforcement spacing is constrained by a limiting crack width w . For the case of steel reinforcement, Figure 2.6 depicts the recommended maximum steel bar spacing according to ACI Committee 318 (2005) and above equation in terms of concrete cover, d_c . The spacing forecasts have been computed considering $f_r = 0.67f_y$ and $k_b = 1.0$. Although the above equation was derived to envisage only steel reinforced concrete structures but it may be applied irrespective of whether the reinforcement is steel or FRP.

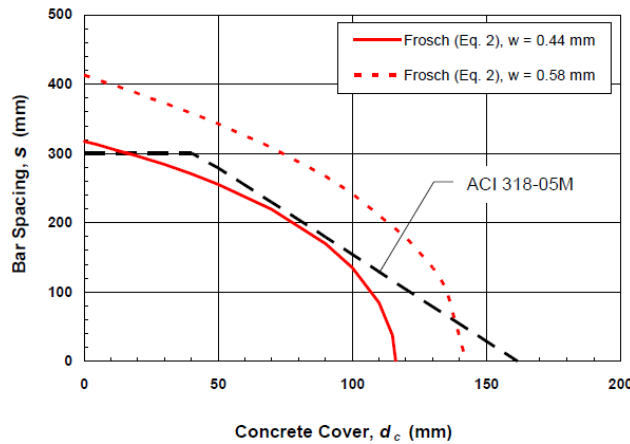


Figure 2.6 – ACI Committee 318 (2005) flexural crack control provision for steel reinforcement

Byrne and Bull (2012) gave way out of crack mitigation by presenting the idea of design and testing of steel reinforced concrete frames incorporating the slotted beam detail as shown in Figure 2.7.

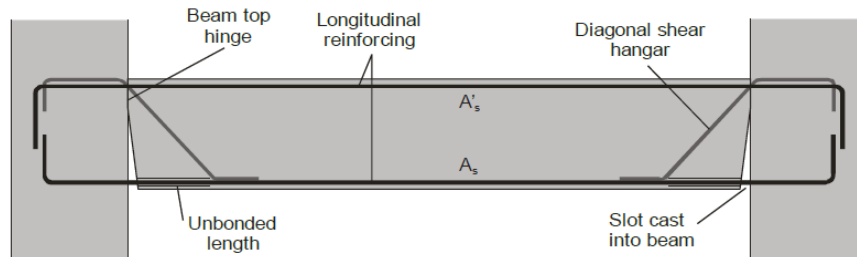


Figure 2.7 – Diagram of a slotted reinforced concrete beam

For this purpose, two beam-column samples, of cruciform configuration, were cast and tested at the University of Canterbury under quasi-static cyclic loading as per the ACI guidelines ACI 374 (2005). Reactions of specimens were measured, when the load was applied at the top of column. A huge data was recorded with the help of strain gauges fixed at several points on steel cage and linear potentiometers. The experimental setup is shown in Figure 2.8.

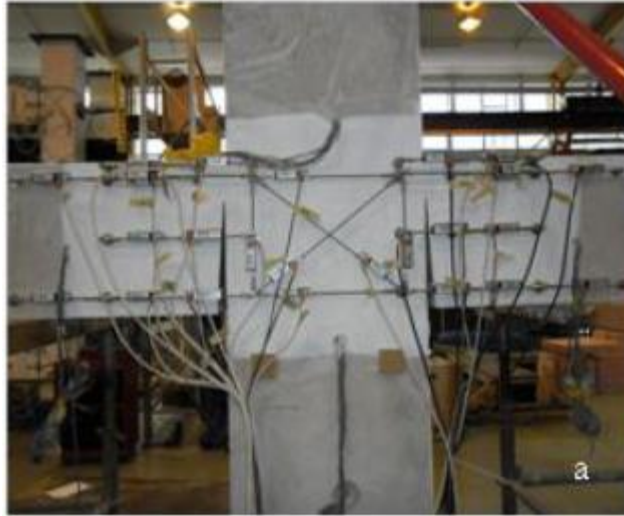


Figure 2.8 (a) – Linear potentiometer layout

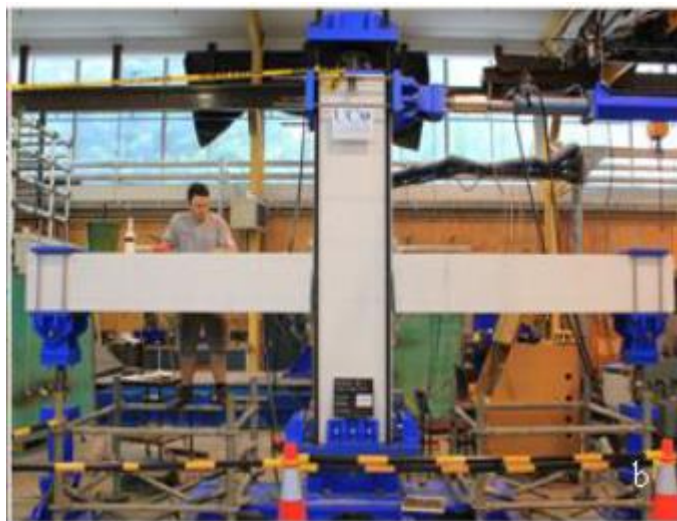


Figure 2.8 (b) – Specimen in testing ring

The hysteretic response of both of specimens is illustrated in Figure 2.9. Specimen A and B yielded at 0.65 and 0.67% drift, respectively, slightly greater than the predicted value i.e. 0.57%. The corresponding yield force was measured as 118 and 108 kN, respectively, presenting good agreement with the predicted value i.e. 113 kN from monolithic theory. Crack mitigation was achieved successfully up to the extent of only hairline flexural cracks on the tension face of beams and on hinges. Through this research, they concluded that validation of capability of slotted beams to reduce beam elongation, concrete cracks occurrence and damage coupled with a stable response up to 5.5% drift depicts that they are a viable alternate to

conventional beams. During flexure, tensile stresses generate at the bottom face of beam but due to slot cast at both ends of the beam, longitudinal forces in steel does not allow cracking in concrete. Slots offer steel, some gap to elongate along the longitudinal direction without making concrete crack. However, this idea of slotted beams is not so much courage able from the structural durability point of view. Due to slots at the ends of beams, there are firm chances of steel reinforcement corrosion.

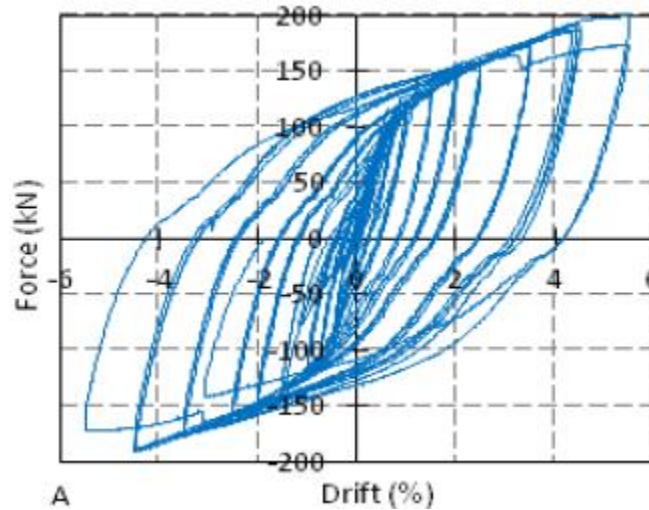


Figure 2.9 (a) – Lateral force vs inter storey drift for specimen A

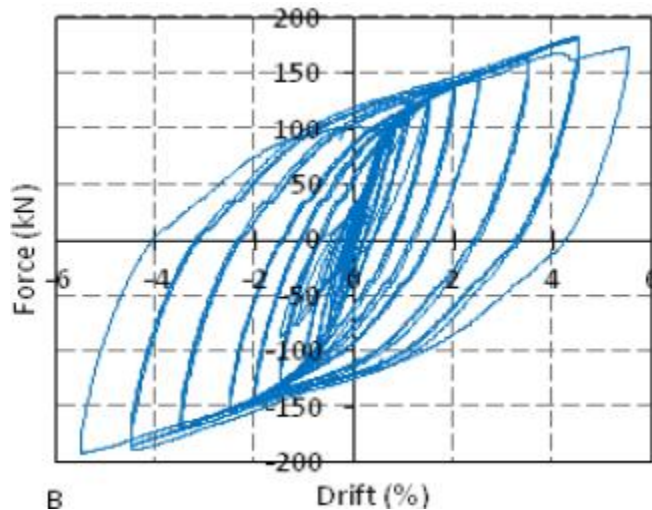


Figure 2.9 (b) – Lateral force vs inter storey drift for specimen B

Vanderbilt et al. (1961) carried out an extensive experimental research on steel reinforced concrete two way floor slabs to study serviceability of the structure and bending moments. For

this purpose, they carried out 39 tests on the structure. In all tests, a bulk data was recorded, which include strain measurement in steel reinforcement and concrete, deflection of slab and concrete cracking. Prototype structure was nine panel of reinforced concrete two way floor slab with each panel of size 20' x 20' but the test structure was a quarter scaled model of the same prototype structure. Similarly, all the materials including steel reinforcement and aggregate was of a rather small size. The structures were rested on 16 reaction piers which were concrete blocks with dimensions of 18 in square and 5 ft high. The reaction piers were tied together at the top by steel beams cast in concrete as shown in Figure 2.10. Load was applied on all panels through a hydraulic jack. Equal distribution of load was ensured with the help of pyramidal system of bars as illustrated in Figure 2.11. The structural floor slabs were designed for the load of 75 psf (dead load = 75 psf and live load = 70 psf). All of nine panels were in order of 3 x 3 with each of span of length 5 ft center to center of columns. Thickness of slab was kept 1.5 in and it was reinforced with 1/8 in square bars, whereas, reinforcement of beams and columns were 1/4 in round plain bar and 3/8 in round deformed bar respectively. Strains in concrete as well as steel reinforcement were measured using SR-4 electrical resistance strain gages at 341 different locations, whereas, deflections were recorded using dial gages at 33 different locations. Out of total 39 tests, only 5 tests were analyzed and deflections, strains and cracking were evaluated.

In Test 307, the first test, the structural slab was loaded to the full design load. Maximum deflection measured was 0.035 in a corner panel of the slab at the load of 108 psf. The maximum steel stress recorded was 4650 psi, at the center of interior beams. No cracks were observed anywhere on the structure even at the maximum load. Second test was Test 314, in which structural slab was loaded to 1.0DL+2.0LL. The maximum deflection occurred was 0.065 in at the central panel of the structure. Maximum steel stress was at the center of interior beams, which was 13,600 psi. At maximum load, only four cracks were observed on the upper face of slab, whereas, no crack was found at the bottom of the slab or anywhere on beams. In third test, Test 335, structure was loaded to 1.0DL+4.0LL. Deflection in a corner panel was found maximum, which was 0.23 in. Largest steel stress was observed in a corner panel edge beam, which was 38,000 psi. At maximum loading, crack pattern developed was almost along the diagonals of slab panels. At center of panels, in areas of constant moments in slabs, a rectangular pattern of cracks was observed following the steel reinforcing bars.

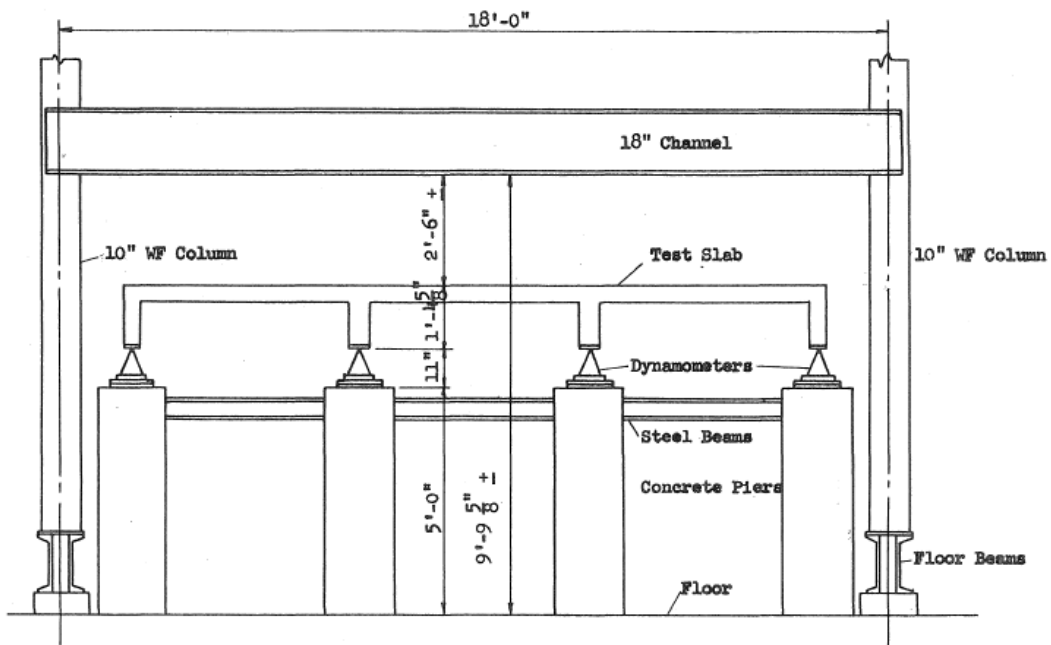


Figure 2.10 – Elevation of reaction frame and loading frame

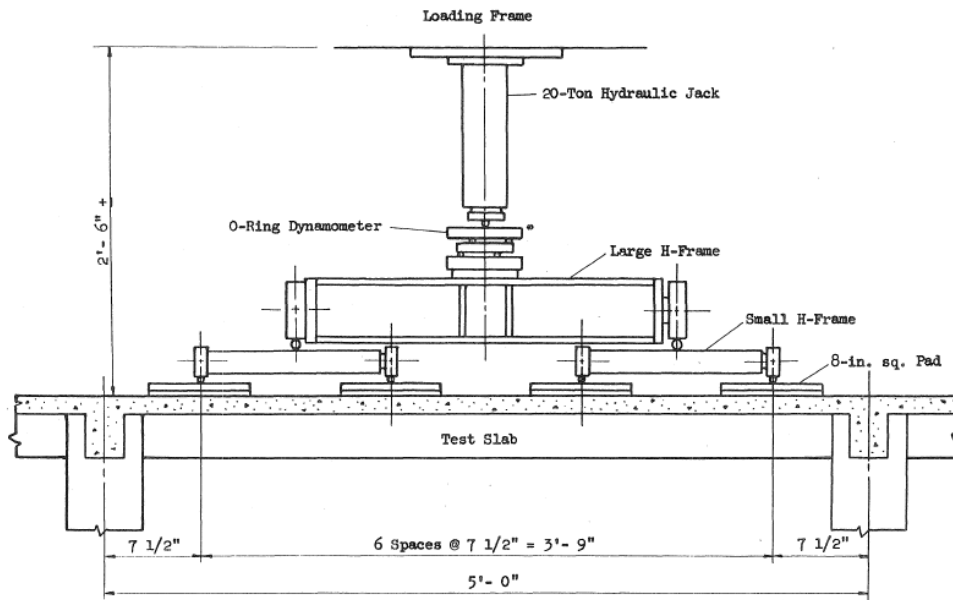


Figure 2.11 – Elevation of load distribution system

In Test 335, the fourth test, structural slab was loaded to failure load, which was measured as 537 psf. When maximum load was reached, the maximum deflection measured by dial gage was 1.15 in at edge panel. The maximum steel stress produced in edge beam, which was 24.6 ksi. The structure totally cracked from the bottom of slab, whereas, a rushed pattern of cracks

was also developed at the top surface of slab. In this test, only three panels in north row were damaged, while the other six panels were remained more or less intact. Fifth test, Test 339, was carried out on those six panels and these are loaded to collapse. Dial gages recorded collapse load as 829 psf. The residual deflection in six panels from previous test was 0.73 in and at failure load, maximum cumulative deflection was 2.5 in. The largest steel stress found was 7200 psi against the compressive strain 0.00024. Cracking pattern of slab was similar to that developed in previous test, Test 338.

Carino and Clifton (1995) presented a report on design and flexural behavior of concrete members and the factors affecting the concrete cracking developed due to applied load. According to this report, steel reinforced concrete structural members, which are subjected to external load, will develop the cracks, when flexural stresses produced in tension zone of concrete exceed its tensile strength. After the development of cracks, steel carries all the tensile forces. These tensile forces, then, transfer to the concrete present between the cracks through bond between steel and concrete. In this way force redistributes in concrete and it keeps happening until the ultimate strength of concrete reaches and it get failed due to compression. There are many important factors to control the occurrence and width of flexural cracks at the tension face of structural member, which were discussed in the report. It is obvious that the increase in steel reinforcement area decreases the distance from tension face to neutral axis, which subsequently, decreases the depth of tensile cracks. However, ACI Code believes in underdesign of structural member or limits the area of steel in tension zone, so that, steel yields prior to compression failure of concrete. In this way, the member will show some deflection warnings before collapse. For this purpose, ACI validates the following probable crack width formula, which is based on Gergely-Lutz equation:

$$w = 2.2 \beta \epsilon_s \sqrt[3]{d_c A}$$

Where w is crack width, β is the ratio of distance between tension face and neutral axis to distance between centroid of steel and neutral axis, ϵ_s is the strain in steel, d_c is the concrete cover and A is the effective area of concrete surrounding each bar. Geometrical factors involved in the above formula are illustrated in Figure 2.12. From the above equation, it can be observed that crack width can be controlled by reducing steel strain or stress under external

loads, concrete cover and the effective cross sectional area of concrete surrounding each steel bar. Decrease in area of concrete can be achieved successfully using more steel bars of smaller diameter rather than fewer steel bars of larger diameter. The other models of crack mitigation, presented in report were CEB/FIP Model Code (1990). The approach used by these models was totally different from ACI Code. These believes in the mechanism of transfer of tensile stress between steel and concrete for the estimation of crack width. Figure 3.13 depicting tensile zone of beam with single steel bar describes the CEB/FIP approach. When a load is applied on a steel reinforced concrete member, an equal strain produces in steel and concrete. This strain goes on increasing with the increase in applied load until the strain capacity of concrete reaches and a crack develops at that spot as shown in Figure 3.13 (a). At the cracked section, whole of tensile forces are resisted by steel reinforcement. There is a slip between steel and concrete, adjacent to the crack. This slip is the governing factor controlling crack width in concrete. In fact, the mechanism works in a way that tensile forces transfer from steel to concrete through this slip and the concrete present between cracks also play a role in carrying the tensile load. The bond-slip mechanism causes the strains in steel and concrete to have a periodic variation along the length of the member, as shown in Figure 3.13 (b). According to CEB/FIP approach of crack controlling, the crack width is dependent on difference between strains of steel and concrete in slip zone on both sides of crack and on the distance over which slip happens. The basic formula for computation of characteristic crack width is as followed:

$$w_k = l_{s,max} (\epsilon_{sm} - \epsilon_{cm} - \epsilon_{cs})$$

Where w_k is the characteristic crack width, $l_{s,max}$ is the maximum distance over which slip between steel and concrete occurs, ϵ_{sm} is the average strain in steel within $l_{s,max}$, ϵ_{cm} is the average strain in concrete within $l_{s,max}$ and ϵ_{cs} is the strain due to concrete shrinkage. This approach of crack width controlling was claimed that slip mechanism is not the only governing factor for crack width control. If it were the case, then there must be a difference in crack width and spacing of a structural member with plain bar as compared to that with deformed bar but nothing was like this. Then it was proposed that strain release in concrete surrounding steel bars, is also an important factor affecting flexural crack width. This idea gives birth to a concept of no-slip model for crack width as illustrated in Figure 3.14 (a). In this type of model, crack width is zero at steel-concrete interface and it increases with distance from the steel bar,

whereas, the crack spacing is the function of concrete cover distance. Normally crack spacing is two times the concrete cover distance in no-slip model. Crack width is due likely to a combination of these two models and some design codes try to accommodate this by using the following formula for computing crack width Beeby (1979):

$$w_m = \left(K_1 c + K_2 \frac{\phi}{\rho} \right) \epsilon_m$$

Where, w_m is the mean crack width, c is the concrete cover, K_1 and K_2 are the empirical constants, ϕ is the steel bar diameter, ρ is the reinforcement ratio and ϵ_m is the average strain at level of cracking being considered

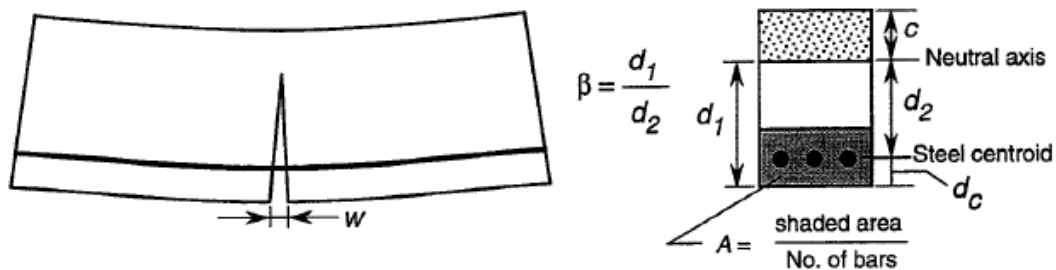


Figure 2.12 – Geometrical factors in ACI approach

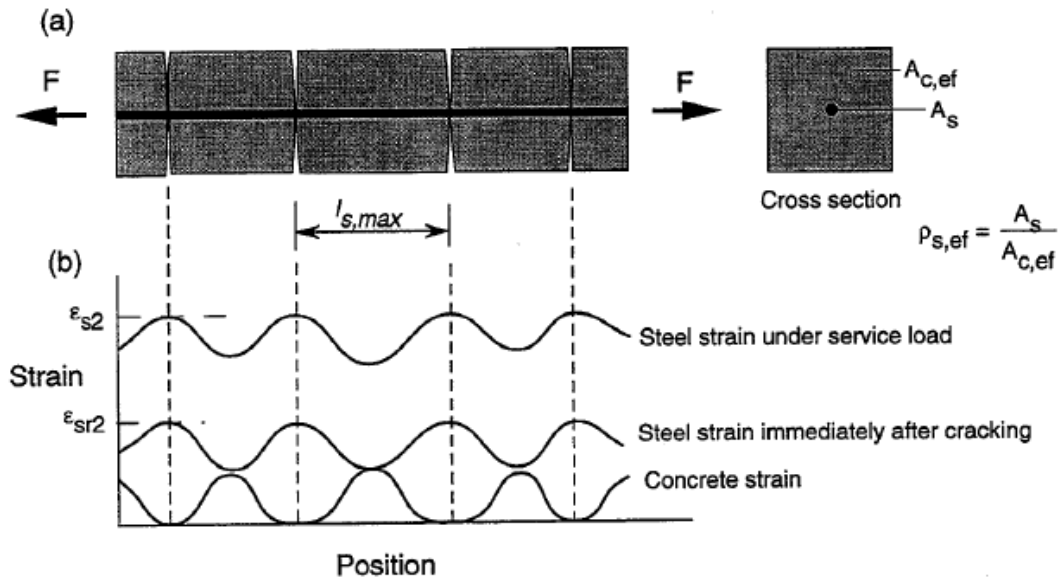


Figure 2.13 (a) – Multiple cracks in tensile zone of member; (b) strain variation along length of member

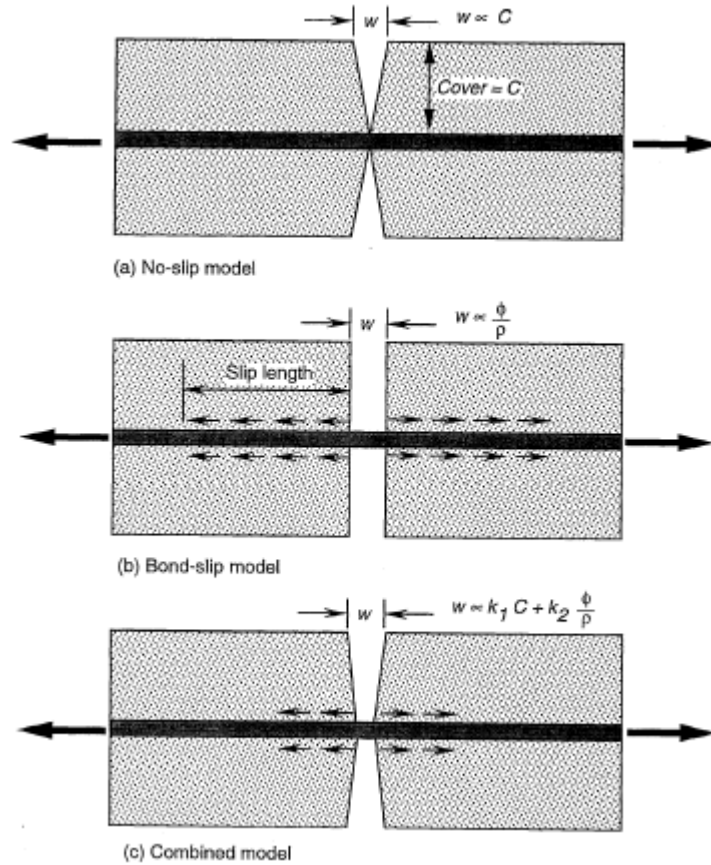


Figure 2.14 – Comparison of crack width model

Chowdhury and Loo (2001) derived an explicit formula for estimating the average crack width of concrete members and prestressed concrete members at same time. Through wide testing, the performance of proposed formula was checked and found good agreement with the test results. An experimental study of flexural cracking behavior of 18 steel reinforced and 12 prestressed concrete beams was carried out. All of them were subjected to static loading symmetrically at two points as shown in Figure 2.15. Crack widths were measured with the help of crack detection microscope and crack spacing were calculated at 60 to 70% of the ultimate strength of member. They found that development of crack at tension face of concrete is because of the difference between elongation of concrete and that of steel bar. This can be written in the following form:

$$W_{cr} = \epsilon_s l_{cr} - \epsilon_c l_{cr}$$

Where W_{cr} is the average crack width, l_{cr} is the average crack spacing, ϵ_c is the average strain in concrete, ϵ_s is the average strain in steel reinforcement. As concrete is weak in tension, therefore, tensile strain in concrete is also ignorable. The above equation may be written as under:

$$W_{cr} = \epsilon_s l_{cr}$$

Or in terms of average steel stress

$$w_{cr} = (f_s/E_s) l_{cr}$$

But crack width is not only function of stress or strain in steel reinforcement. Based on the findings of different researchers including Nawy (1968), Clark, (1956) and Watstein and Parsons (1943), the authors incorporated three governing factors for development of new formula to predict crack width in reinforced concrete members. These variables are ratio of average bar diameter to steel reinforcement ratio, concrete cover and spacing of steel bars. The regression equation gets the form

$$l_{cr} = C_1 c + C_2 s + C_3 (\Phi/\rho)$$

Where C_1 , C_2 and C_3 are the regression coefficients and can be calculated from the statistical analysis and for this analysis, the relevant data was collected from four reinforced and four prestressed concrete beams in such a way that all of them were having different lengths, reinforcement ratio and degrees of prestressing. The authors derived following equation for prediction of crack width and spacing after determination of solution for regression coefficients:

$$w_{cr} = (f_s/E_s) [0.6(c - s) + 0.1 (\Phi/\rho)]$$

This formula was counter checked with the data of author's own 30 tests as well as 76 other tests carried out by Clark (1956), Chi and Kirstein (1958) and Nawy (1984). It was found efficient and good agreement with not only test results but also with ACI Building Code (1995), British Standards (BS 1985; BS1987) and Eurocode (1992). The proposed formula of crack width is more versatile as it is workable for steel reinforced and prestressed at the same time.

Allam et al. (2012) carried out an investigation study of crack width evaluation formulae suggested by different building codes and researchers. Then all of these codes and formulae were applied to five steel reinforced concrete members having different values of parameters; steel reinforcement ratio, steel bar distribution, steel grade, section dimensions etc and the values of crack widths found from all equations drawn in a comparison table, Table 2.1. Cross sections of testing models along with salient are illustrated in Figure 2.15. Test models 2 and 4 have same characteristics, the only difference is surface condition and strength of steel bar. In model 2, deformed steel was used, whereas, plain mild steel was used in model 4. From the table, it is revealed that the values of crack width obtained from code equations show a large scatter among all formulae. British Standards BS 8110-97 underestimates the values of crack widths, whereas, Oh and Kang overestimate the values. For test models 1, 2 and 3, according to ECP-2007, ACI 318-95 and equations by Gergely and Lutz (1968) and Frosch (1999), values of crack widths are directly proportional to steel reinforcement ratio, whereas, Eurocode (1992) does not show any effect of steel reinforcement ratio on value of crack width and formula by Oh and Kang (1987) give inverse response. Increasing of steel ratio reduces the role of concrete in bearing tension and average steel strain increases, subsequently crack width increases. Therefore, limiting of steel reinforcement ratio is better than limiting steel stress. Steel reinforcement detailing is found an important factor playing a vital role in mitigation of cracks. Test results of model 2 and 5 clearly indicate that it is better to use more no of steel bars with less diameter instead of less no of bars with large diameters so that better and stronger bond between steel bars and concrete may produce. In comparison of test model 2 and 4 having identical properties except steel bar surface and strength, less value of crack width is obtained in case of model 4, which had lesser steel stress. Bar surface deformation does not influence only crack spacing, but also affects average strain. The stronger is the bond between steel bars and concrete, the more tensile forces transfer from steel bars to concrete. Increase in contribution of concrete towards tension leads to less slip between both of the materials. Subsequently, less elongation between them occurs i.e. $\epsilon_{sm}-\epsilon_{cm}$ developing less value of crack width. However, the effect of bar surface deformation is not considered in most codes' formulas except the Egyptian code and Eurocode (1992).

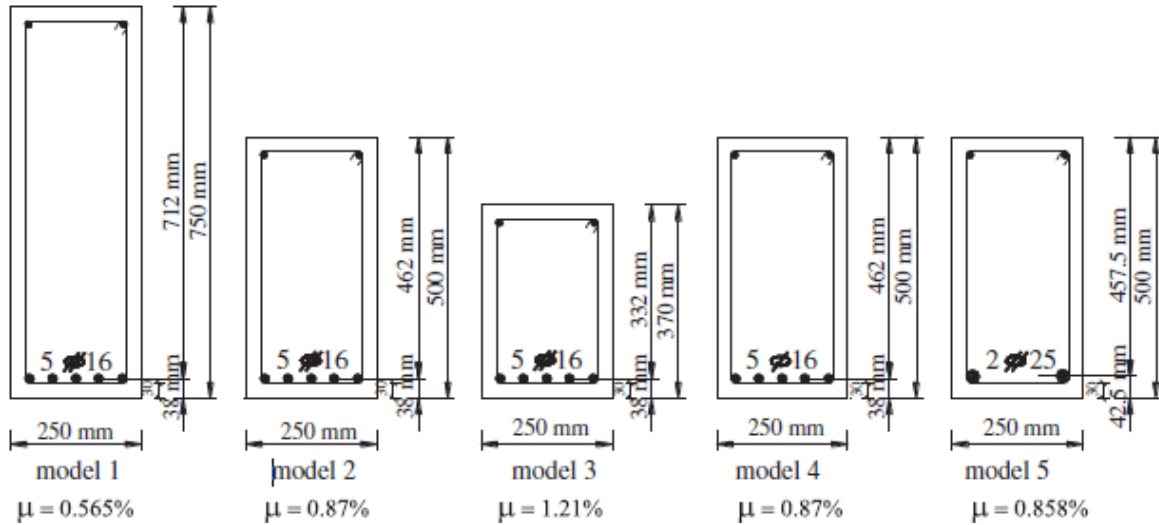


Figure 2.15 – Dimensions and reinforcement of studied models

Equation	Crack Width (mm)				
	Model 1	Model 2	Model 3	Model 4	Model 5
Eurocode2	0.127	0.127	0.127	0.107	0.160
ECP-2007	0.107	0.129	0.137	0.123	0.170
ACI 318-95	0.124	0.129	0.136	0.119	0.191
BS 8110-97	0.079	0.087	0.090	0.056	0.162
Gergely & Lutz	0.102	0.107	0.113	0.068	0.158
Frosch	0.101	0.106	0.111	0.0742	0.231
Oh & Kang	0.147	0.135	0.131	0.084	0.218

Table 2.3 – Value of the crack width for the studied models

Rizkalla et al. (1982) conducted an experimental work to study the flexural behavior of reinforced concrete members. Measured values of crack width and spacing were compared to those obtained from the equations proposed by other researchers. Then a simple and easy expression for computation of crack spacing was proposed based on the comparison. This research was of unique style in a way that flexural behavior of concrete members was studied in the presence of transverse reinforcement. In first phase, 18 concrete segments reinforced in two direction were designed to only observe the applicability of formulae proposed by other researchers for predicting crack width. In second phase, 9 steel reinforced concrete segments were designed having identical parameters except spacing of steel bars in transverse direction, so that effect of variable spacing of transverse reinforcement on cracking behavior could be studied. In all test specimens, spacing of longitudinal steel reinforcement was kept as 3 in

center to center and bars were extended 11 in beyond each end as illustrated in Figure 2.16. Transverse steel reinforcement for all test specimens in first phase was spaced at 3 in center to center. Segments in second phase, were divided into three sets of three test specimens each so that variable spacing of transverse steel bars could be tested. The spacing selected for three sets of specimens were 2 in, 4 in and 6 in. The test specimens were tested on a 600 kip universal testing machine (UTM) through specially designed end fittings as illustrated in Figure 2.17. All reinforcement bars are attached to a separate load cell to ensure uniform transfer of load to test specimen. Deformation over complete specimens was measured with the help of linear variable differential transducer (LVTD) and cracks width and spacing were measured using travelling micrometer microscope. The measured cracks spacing were, then, compared with those obtained from the proposed expressions of Leonhardt (1977) and Beeby (1972) as a function of ratio of bar size and reinforcement ratio. Comparison of cracks spacing with Beeby's expression resulted an underestimated value, since the mean value of ratio of calculated and measured values was 0.70, as shown in Figure 2.18 (a), whereas, Leonhardt (1977) expression for crack spacing suggested values closer to test results, since the mean ratio of calculated and measured values was 1.13, as shown in Figure 2.18 (b). Based on the comparative study, a refined and simplified formula for predicting the crack spacing was proposed, which is given as:

$$S_m = 5(d-0.28) + 1.33c + 0.08 d/p$$

Where, S_m is the average crack spacing, d is diameter of steel reinforcement bar, c is the concrete cover and p is the steel reinforcement ratio. Comparison of cracks width with Leonhardt (1977) expression resulted an overestimated value, since the mean value of ratio of computed and measured values was 2.38, as shown in Figure 2.19 (a), whereas, Beeby's expression for cracks width proposed values closed to test results, since the mean ratio of computed and measured values was 1.16, as shown in Figure 2.19 (b). Spacing of transverse steel bars directly affect the cracks spacing parallel to the direction of steel bars. Specimens with 6 in spacing in transverse steel bars have least cracks spacing. No of cracks increases with the increase in strain. At maximum strain 0.0011, full pattern of cracks developed.

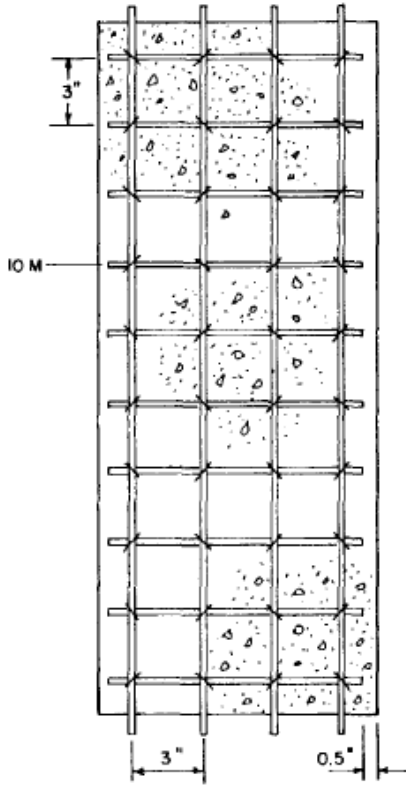


Figure 2.16 – Reinforcement details of a typical specimen

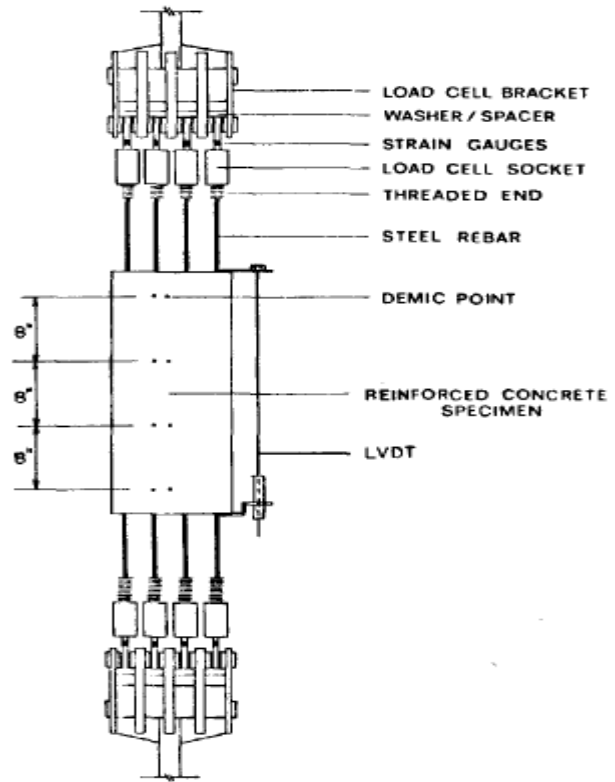


Figure 2.17 – Test set up and instrumentation of a typical specimen

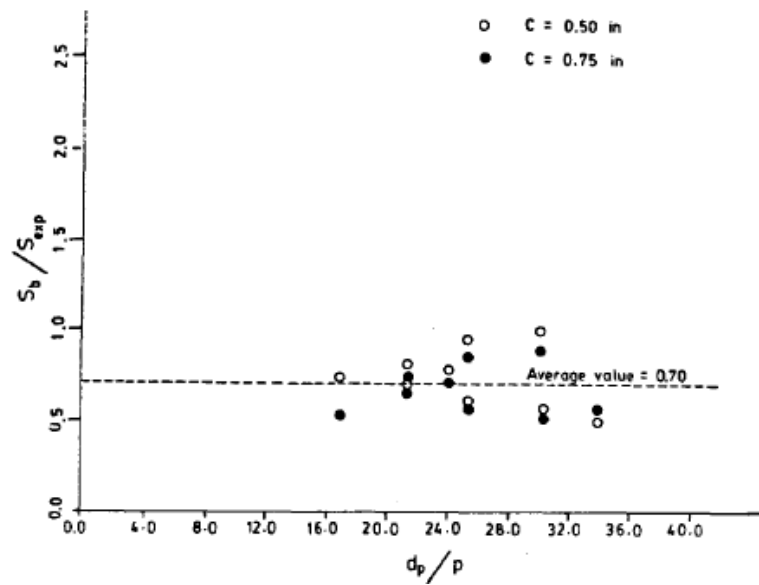


Figure 2.18 (a) – Comparison of crack spacing based on Beeby's expression

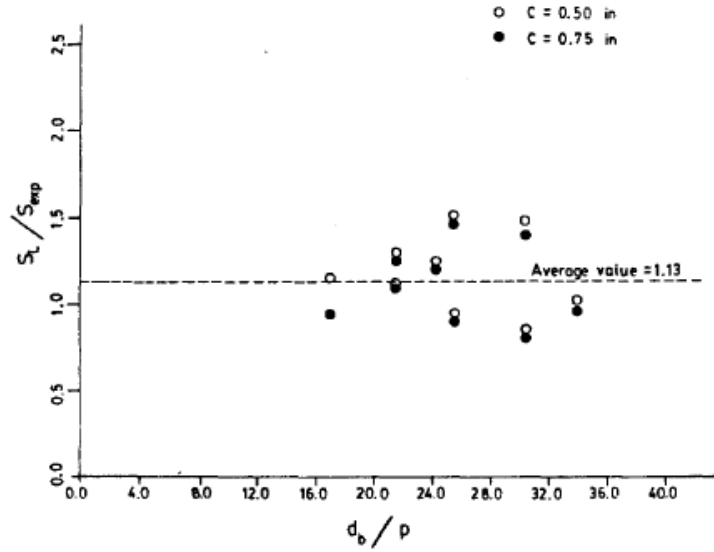


Figure 2.18 (b) – Comparison of crack spacing based on Leonhardt’s expression

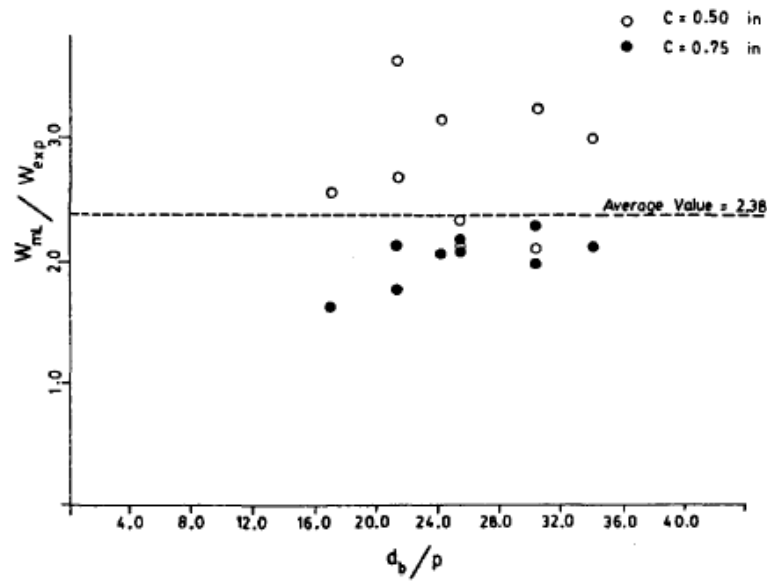


Figure 2.19 (a) – Comparison of crack width based on Leonhardt’s expression

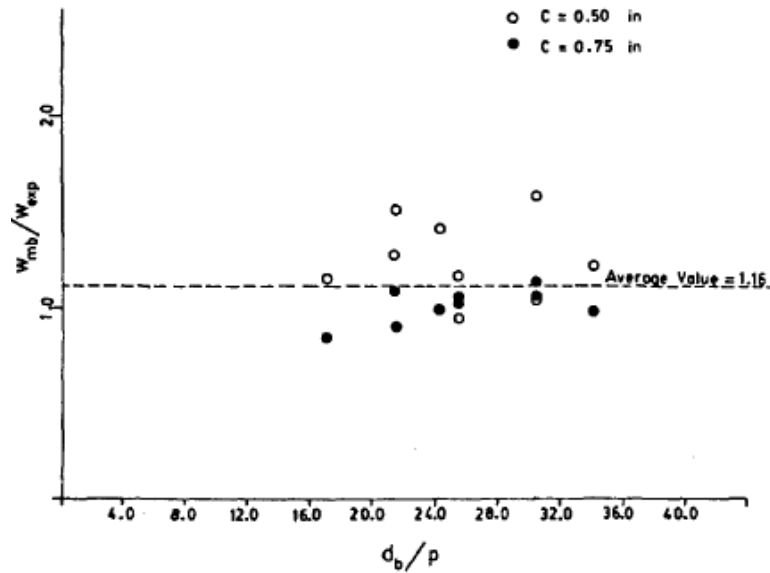


Figure 2.19 (b) – Comparison of crack width based on Beeby’s expression

Mickleborough et al. (1999) proposed a method for the improvement of evaluating the effective stiffness of steel reinforced concrete members considering the ratio of area of moment diagram of a structural member to the total area of moment diagram, as governing parameter. For this purpose, the authors carried out an experimental work. Midspan concentrated loading, two point concentrated loading and uniformly distributed loading was applied on nine steel reinforced concrete beams of dimensions of 300mm x 450mm x 3000mm and having different reinforcement ratio. Each column was subjected to vertical load of 200 kN, whereas, lateral load was applied on a frame at its second level. The dimensions of beams of two storey test frame was 250mm x 375mm x 3000mm and inter storey height was 2000mm, whereas, size of column was 250mm x 375mm. In view of already published data and data collected from experimentation, this analytical model provides improved accuracy for the prediction of effective stiffness of structural member as compared to the conventional methods. During life of a structure, decrease in flexural stiffness due to crack development is a normal occurrence, which results in nonlinear load deformation response of structural member. Based on analytical result data and experimental result data, it is definite that occurrence of cracks is the major cause of sudden decrease in member stiffness at low load, about 15% of ultimate load. Normally, the stiffness of flexural member decreases to approximately 50% of the total moment of inertia at 50% ultimate load, with further decrease as the load approaches 70% of

ultimate load. The most noteworthy attribute of the suggested model is its vast applicability to the steel reinforced concrete members subjected to numerous types of loading.

Gilbert (2001) proposed an analytical procedure of computing final crack width and spacing for direct tension as well as flexural cracks considering the shrinkage effect an important factor. Through analytical and experimental investigation, he concluded that Australian Standard (1994) considers only spacing of steel reinforcement bars and concrete cover only for the computation of crack width in a flexural member. It does not include stress of steel bars in tension region and results in unreliable outcomes in case of steel stress beyond 240 MPa (34 ksi). With the move to high strength concrete structural members having steel strength more than 500 MPa (72 ksi), there is an immense need of review the design rule of Australian Standard (1994). This design rule was compared with those given in other concrete codes i.e. BS 8110 (1997) ACI318 and Eurocode 2. In this comparison, crack controlling parameters including steel spacing and stress, concrete cover, size of bar and strength of concrete were also evaluated. Results, then, scrutinized in the light of local crack width measurements. Aftermaths of whole investigation led the author to conclude that design rule of Eurocode 2 is more satisfactory and reliable than that of either BS 8110 (1997) or ACI 318 but none of these design codes considers inclusion of time dependent parameter affecting the increase in crack width. An indirect approach of crack control has been proposed, which is computing and limiting design crack width. This method is similar to that of Eurocode 2 with some modification of inclusion of shrinkage effect on intact concrete between cracks. The proposed expression of calculating design crack width is as followed:

$$w = b_m s_{rm} (\epsilon_{sm} + \epsilon_{cst})$$

Where, w is design crack width, b_m is the coefficient relating the average crack width to design value and can be considered as $b_m = 1.0 + 0.025c \geq 1.7$; c is concrete cover, ϵ_{sm} is average strain permitting for the tension stiffening effect and can be considered as $\epsilon_{sm} = (s_s/E_s)[1 - b_1 b_2 (s_{sr}/s_s)^2]$; s_s is the stress in steel based on cracked section, s_{sr} is the stress in steel based on cracked section under load resulting first crack, b_1 is the coefficient of bond properties of steel bar and its value is 0.5 for plain bars and 1.0 for high bond bars and b_2 is the coefficient of time duration of loading and its value is 1.0 for short time and 0.5 for long term

loading. Mean crack spacing is given as $s_{rm} = 50 + 0.25 k_1 k_2 d_b / \rho$, where, d_b is the bar diameter, k_1 is the coefficient of bond properties and its value is 0.8 for high bond bars and 1.6 for plain bars, k_2 is the coefficient of strain distribution and its value is 0.5, ρ is the effective steel reinforcement ratio. ϵ_{cst} is the mean strain in intact concrete in between cracks due to shrinkage effect and is zero for short term loading, whereas, for long term loading, its value can be taken as $\epsilon_{cst} = \epsilon_{cs} / (1 + 3 p n)$, where, p is the reinforcement ratio and n is the age adjusted modular ratio.

The proposed approach of crack width computation shows a good agreement with the measured values from laboratory and field experimentation. Flexural crack widths for slabs of 200 mm thickness and beams of 400 mm x 400 mm dimensions are analyzed using the proposed approach, as given in Table 2.2 and 2.3. The calculated values of crack width are larger than those proposed by either ACI or Eurocode 2 but unlike these codes, the suggested approach will indicate serviceability issues to the designer, if any.

Effective depth, d (mm)	Bar size d_b (mm)	Area of tensile steel, A_{st} (mm ² /m)	Bar spacing, s (mm)	Crack width (mm)		
				Steel stress, s_s (MPa)		
				200	250	300
174	12	1044	108	0.226	0.279	0.330
172	16	1032	195	0.267	0.331	0.392
170	20	1020	308	0.309	0.384	0.455
168	24	1008	449	0.352	0.438	0.519
166	28	996	618	0.394	0.492	0.585

Table 2.4 – Flexural crack widths in 200 mm thick slab

Bar dia, d_b (mm)	No. of bars	A_{st} (mm ²)	$p = A_{st}/bd$	Crack width (mm)					
				Cover = 25 mm			Cover = 50 mm		
				Steel stress, s_s (MPa)			Steel stress, s_s (MPa)		
				200	250	300	200	250	300
20	2	620	.0039	0.309	0.397	0.479	0.488	0.646	0.791
20	3	930	.0058	0.267	0.326	0.384	0.414	0.513	0.607
20	4	1240	.0078	0.231	0.280	0.327	0.349	0.425	0.498
24	2	900	.0056	0.314	0.386	0.455	0.480	0.596	0.707
24	3	1350	.0084	0.251	0.304	0.355	0.369	0.449	0.526
24	4	1800	.0113	0.214	0.258	0.301	0.304	0.367	0.430
28	2	1240	.0078	0.299	0.362	0.424	0.434	0.529	0.621
28	3	1860	.0116	0.234	0.281	0.329	0.325	0.393	0.459
32	2	1600	.0100	0.285	0.344	0.402	0.394	0.477	0.558

Table 2.5 – Flexural crack widths in 400 mm x 400 mm beam

In general AS 3600 (2000) describes two methods of crack controlling in steel reinforced concrete structures in accordance to Eurocode 2. The first approach of crack controlling is to calculate crack width from the following expression:

$$w_k = \beta s_{cm} \epsilon_{sm}$$

Where, w_k is the design crack width with only 5% possibility of being exceeded, β is the coefficient of relating crack width to design value and it is 1.7 for cracking under direct load, s_{cm} is the mean crack spacing and depends on mean value of bond stress for deformed bar, minimum tensile strength of concrete, diameter of steel bar and effective steel reinforcement ratio, ϵ_{sm} is the mean strain in steel and it depends on tensile stress of steel under service load, tensile stress of steel causing concrete cracked and bond properties of steel reinforcement ratio.

The second approach of crack controlling is Simplified Design Method. This is indirect limitation of cracking in steel reinforced concrete members, in which minimum area of steel reinforcement is calculated and limit is subjected on steel stress depending on steel bar size and spacing. The basic design rules recommended by Eurocode 2 and the same in Australian Standard (1994) are as followed:

1. Minimum value of area of steel reinforcement bonded with concrete
2. During crack development, steel reinforcement must not yield
3. Limiting the steel reinforcement stress depending on steel bar size and spacing

In second approach, “Simplified Design Method” crack controlling by limiting the value of steel reinforcement bonded with concrete is given as followed:

$$A_{st.min} = \frac{f_t Z_t}{f_s z}$$

Where, $A_{st,min}$ is the minimum steel reinforcement area bonded with concrete, f_t is the tensile strength of concrete at time of cracking and its value can be taken as 3.0 MPa, Z_t is the section modulus on tension region, f_s is the stress allowed in steel reinforcement after cracking, z is the lever arm of internal force couple.

The second necessary design rule of simplified design method is to limit the steel reinforcement stress depending on steel bar size and spacing. The maximum steel bar size and spacing in accordance with steel stress are given in Table 2.4 and Table 2.5 respectively.

Steel Stress (f_s) (MPa)	Maximum Bar Diameter (d_b) (mm)
450 (375)	6
400 (345)	8
360 (320)	10
320 (300)	12
280 (265)	16
240	20
200	25
160	32

Table 2.6 – Maximum bar diameters

Note: the values of steel stress given in brackets are not proposed by Eurocode 2. From the parametric study, AS3600 recommended the decrease steel stresses for bar diameter less than 20 mm, for slabs having depth up to 300 mm.

Steel Stress (f_s) (MPa)	Maximum Bar Spacing – Pure Bending (mm)	Maximum Bar Spacing – Pure Tension (mm)
360	50	-
320	100	-
280	150	75
240	200	125
200	250	150
160	300	200

Table 2.7 – Maximum bar spacing

Realizing the need of crack controlling, ACI Report 224.4R (2013) proposed steel reinforcement detailing and structural framing guidelines for limiting the cracks developed in structural members. The scope of the report included crack controlling taking the effect of geometry and additional cross sectional stresses in members because of restraint caused by

structural system, into consideration. This document discussed mitigation and controlling of all types of cracks, may be developed in a structural member. This committee keep on revising its recommendations from time to time to incorporate any new innovations to cater for crack controlling in concrete.

2.3 Summary

The limited available researches and studies help us understand behavior of cracks in two way slabs, however, no rationales are presented to effectively control cracks in RC slabs. The lack of knowledge on crack behavior of two way slabs as well as crack mitigation is of significant importance to enhance strength of slabs against rare occurring type of loads such as concentrated point's load of telecommunication towers. Moreover, there is a need to make safe slab structures under four point's concentrated loading depicting telecommunication towers' load. This proposed research will help identify causes of crack initiation and propagation/growth and lead to design detailing to mitigate cracking in two way slab structures. This will result in structural integrity as well as safety for occupants by provision of slabs that are safe against loading of telecommunication towers.

Research Methodology and Modeling of RC Slabs

3.1 General

In dense populated urban areas, telecommunication towers are erected on RC roof slabs of residential and commercial buildings as illustrated in Figure 3.1. Installation of towers having 4 point concentrated loading on conventionally designed RC slabs is hazardous and threat to human lives. In the proposed research, structural design and safety will be enhanced by formulating new and updated design of steel reinforcement detailing for four two way RC slabs in frame structures with dimensions as 12'x12'.

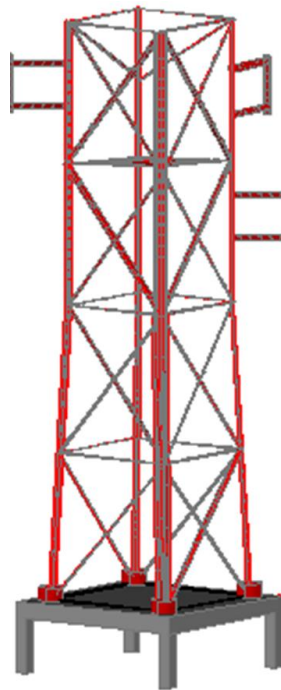


Figure 3.1 – Telecommunication tower erected on RC roof slab specimen

In first, second, and third structures, steel detailing will be varied after each 1 ft, 2 ft and 4 ft interval respectively. Variation of steel detailing will conform the bending moment in slabs, in such a manner that high concentration of steel will be provided at maximum moment and low concentration of steel at low moment with same bar bending quantum in all slab specimens. In

fourth specimen, steel detailing is designed to be provided uniformly throughout the slab. Then 4 point concentrated loading depicting telecommunication tower loads is applied on slab. There are many types of towers regarding length and function. Telecommunication tower, radar tower, electric tower etc are the kinds of towers and these are erected with the length normally 30 m, 40 m and 50 m depending on the environmental conditions. In this study, 50 m high telecommunication tower of 48 kips total weight is considered to be applying load on prototype slabs through four point loading i.e. 12 kips on each point. The proposed research will uniquely investigate full life size RC two-way slabs for the first time, under four-point concentrated load conditions with the help of ANSYS 16. The block diagram, illustrated as Figure 3.2, shows the test specimens detail planned in the proposed analytical research.

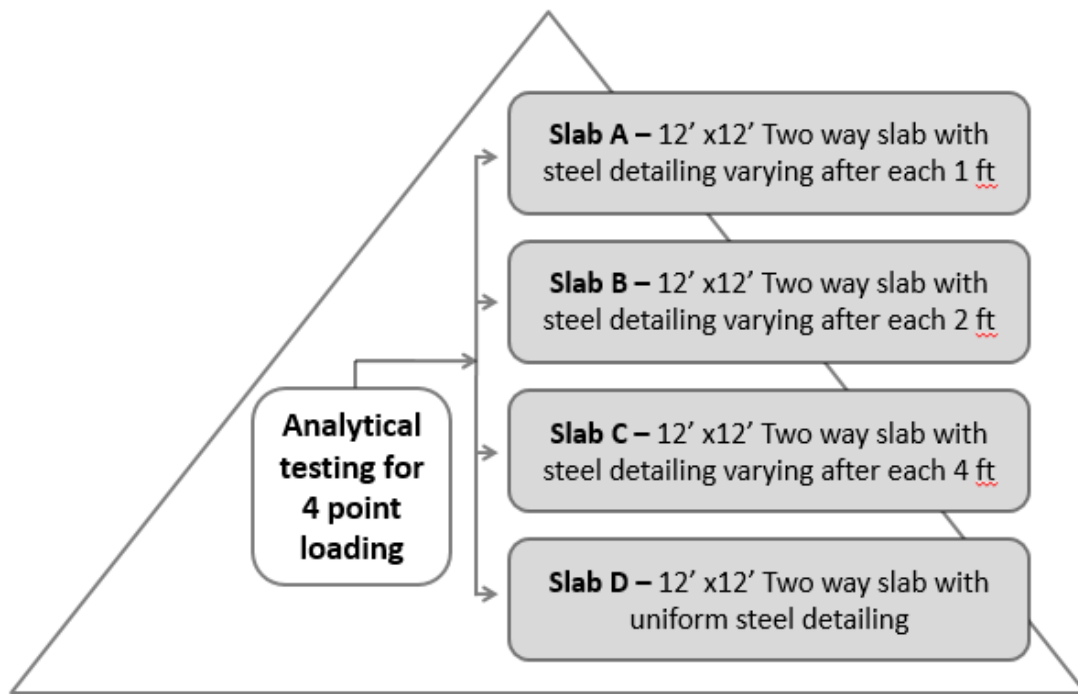


Figure 3.2 – Block diagram depicting test models specimens

3.2 Concrete Material Matrix

Constitutive relation for plane stress problems is presented below for concrete model. For stress combinations inside the initial yield surface, concrete is assumed to be a homogeneous, linear isotropic material. Stress strain relation for plane stress problems has the simple form:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{1-\nu^2} * \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} * \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

where E is the elastic modulus of concrete and ν is Poisson's ratio.

3.3 Steel Material Matrix

One dimensional truss element is widely used as reinforcement steel. Slab element can also be used with six degrees of freedom connected with concrete element. In both cases one dimensional reinforcing bar elements can be easily superimposed on the three-dimensional concrete element mesh. Stiffness matrix for one dimensional truss element is given by

$$\begin{Bmatrix} P1 \\ P2 \end{Bmatrix} = \frac{AE}{L} * \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} * \begin{Bmatrix} d1 \\ d2 \end{Bmatrix}$$

Where, A is the cross-sectional area, L is the length of the bar element and E is the modulus of elasticity. P1 and P2 are axial end forces and d1 and d2 are axial end displacements of the reinforcing bar.

3.4 Finite Element Modeling and Analysis of RC Slabs

Following four phases are involved from slab modeling and analysis of results

- Pre-processor phase
- Solution phase
- General post processor phase
- Time history post processor phase

3.5 Pre-Processor Phase

In pre-processor phase nonlinear finite element modeling of four numbers steel reinforced concrete two way slabs, referred in Table 3.1, were carried out. For this purpose ANSYS 16 multifunction finite element package was used. To create the finite element model, ANSYS 16 involves multiple tasks which have to be completed to run the model properly. This section describes the different tasks to create finite element models of slabs. Different label numbers

are marked to represent element type, real constants and material model as is used in actual ANSYS modeling.

Slab Type	Interval (No)	Distance from Origin (in)	No of Steel Bar (#)	Steel Spacing (in)
Slab A (1 ft Interval)	1	12	3	3
	2	24	3	3
	3	36	3	4
	4	48	3	4
	5	60	3	4
	6	72	3	4
	7	84	3	4
	8	96	3	4
	9	108	3	4
	10	120	3	4
	11	132	3	3
	12	144	3	3
Slab B (2 ft Interval)	1	24	4	6
	2	48	4	6
	3	72	4	7
	4	96	4	7
	5	120	4	6
	6	144	4	6
Slab C (4 ft Interval)	1	48	3	4
	2	96	3	4
	3	144	3	4
Slab D (Uniform Detailing)	1	144	4	7

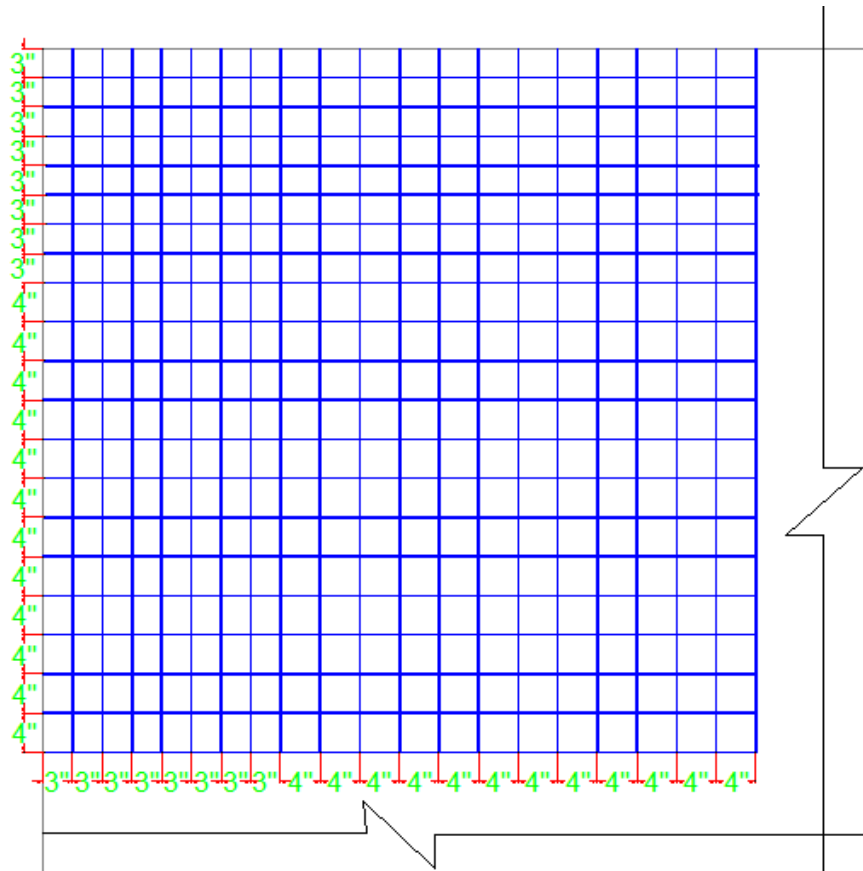
Table 3.1 – Slab models with reinforcement detailing

For simplification, square RC slab is cut from half of both orthogonal x and z directions to produce a quarter model of slab due to symmetric geometric dimensions and reinforcement detailing. Steel reinforcement detailing plan and cross sections of all types of RC slabs are shown in figures from Figure 3.3 to 3.6.

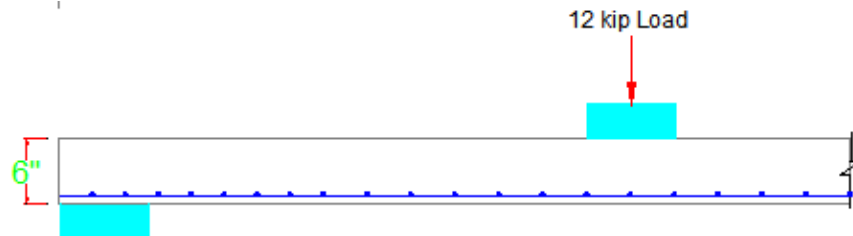
Geometric and strength specifications of all slab models are as followed:

Ser No	Slab	Geometry	Thickness (in)	fc' (psi)	fy (psi)	Dia of Steel bar	Reinforcement Spacing
1.	Slab A	12' x 12'	6	3000	60000	#3	3" x 4"
2.	Slab B	12' x 12'	6	3000	60000	#4	6" x 7"
3.	Slab C	12' x 12'	6	3000	60000	#3	4" x 4"
4.	Slab D	12' x 12'	6	3000	60000	#4	7" x 7"

Table 3.2 – Properties of all slabs

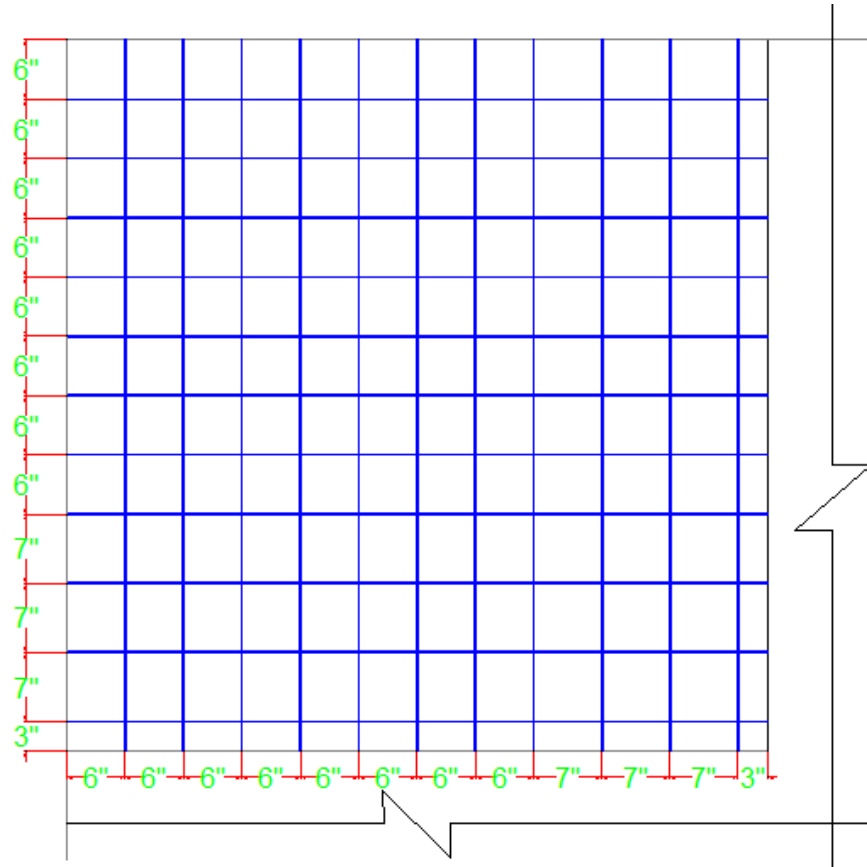


3.3 (a) – Reinforcement detailing layout

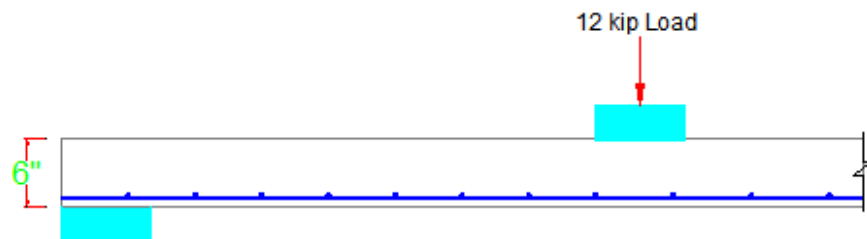


3.3 (b) – Cross section of slab

Figure 3.3 – Slab A (1 ft interval)

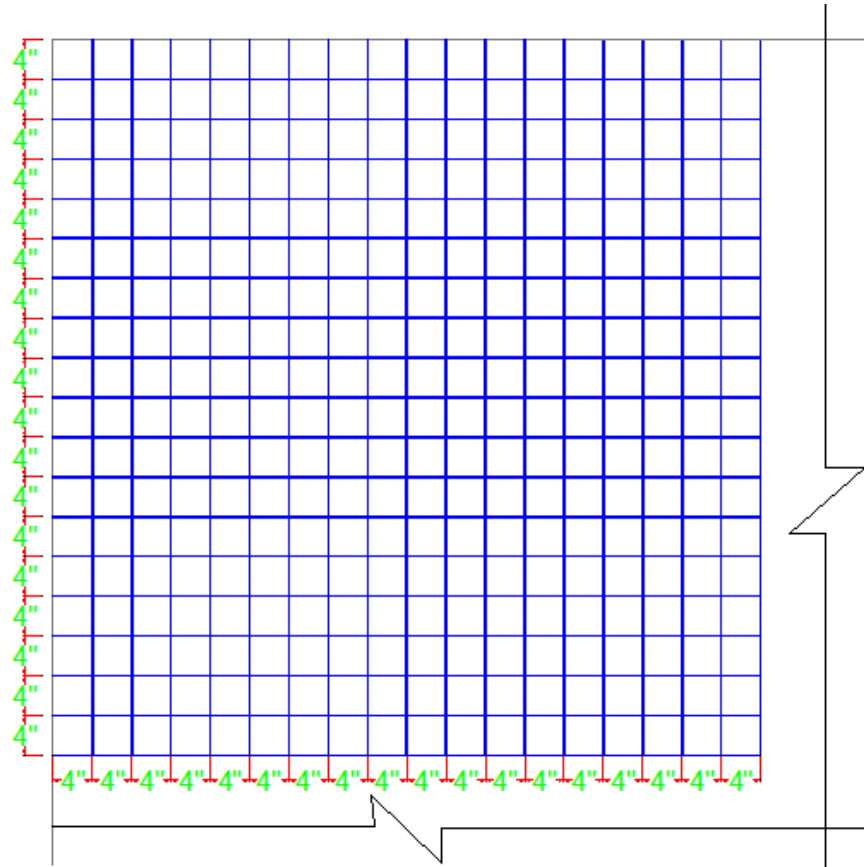


3.4 (a) – Reinforcement detailing layout

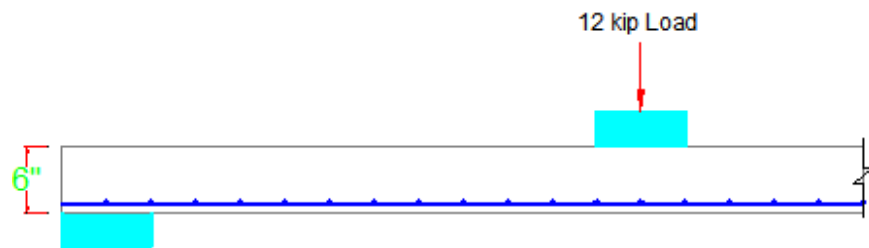


3.4 (b) – Cross section of slab

Figure 3.4 – Slab B (2 ft interval)

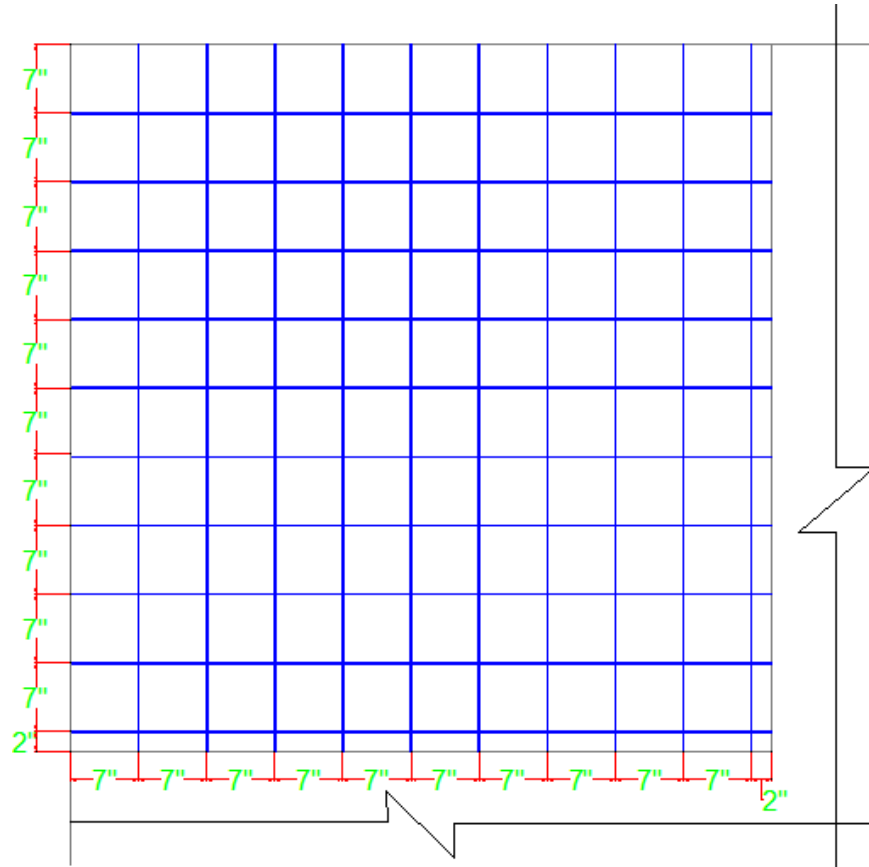


3.5 (a) – Reinforcement detailing layout

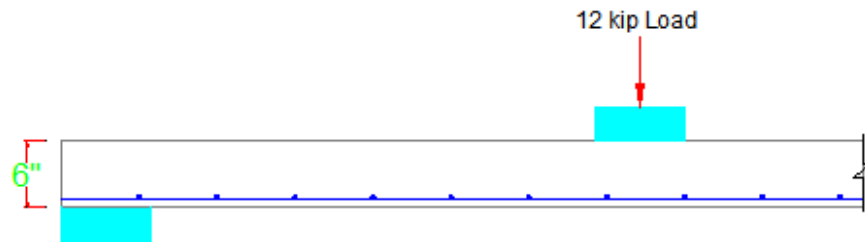


3.5 (b) – Cross section of slab

Figure 3.5 – Slab C (4 ft interval)



3.6 (a) – Reinforcement detailing layout



3.6 (b) – Cross section of slab

Figure 3.6 – Slab D (uniform detailing)

3.5.1 Element Types

More than 180 elements with different formulations and properties are the part of ANSYS element library. Each element is identified by its name, such as Solid65; which has a group label (Solid) and a unique number (65) that identify the particular element.

Element Type for Concrete: For three dimensional modeling of concrete, Solid65 element was used. Eight noded solid65 element has three translational degrees of freedom U_x , U_y and U_z in orthogonal X, Y and Z directions respectively. This element has capability of cracking, plastic deformation, and crushing in all orthogonal directions. A diagram of element representing concrete used in ANSYS is shown in Figure 3.7.

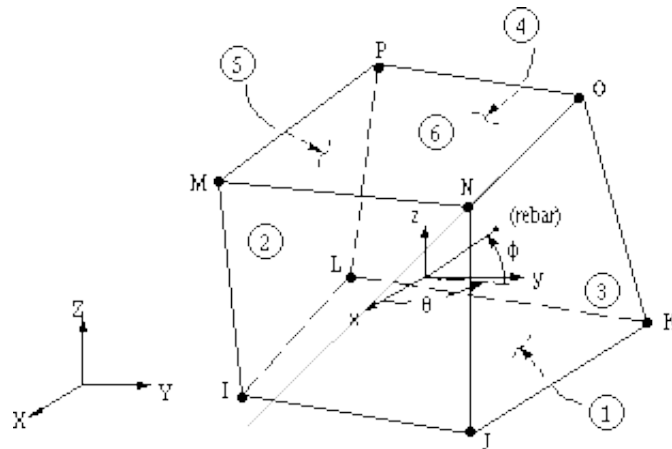


Figure 3.7 – Solid65 element representing concrete (ANSYS-Multiphysics, 2011)

Element Type for Steel Plates: Like solid65 element used for concrete modelling, Solid185 is also a three dimensional element with eight nodes having three translational degrees of freedom at each node in x, y and z directions. This was used for steel plates at the supports and loading positions of slabs. The node location and geometry of element is shown in Figure 3.8.

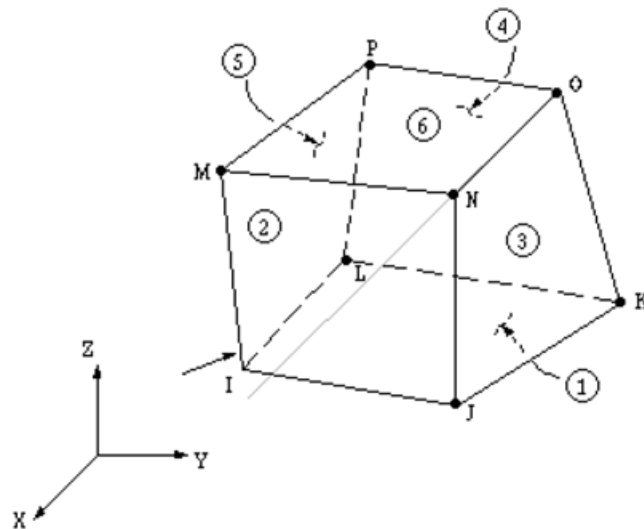


Figure 3.8 – Solid185 element representing steel plates (ANSYS-Multiphysics, 2011)

Element Type for Rebar: LINK180 is a uniaxial tension compression spar element which can be used in many of the engineering applications such as to model trusses, sagging cables, springs, etc. This element has single translational degree of freedom in x direction. In a slab this element was used to model longitudinal and transverse steel rebar. Bending resistance of the element is neglected, whereas, creep plasticity, rotation, large deflection and large strain properties are taken into consideration. Sketch of this spar element is shown in Figure 3.9

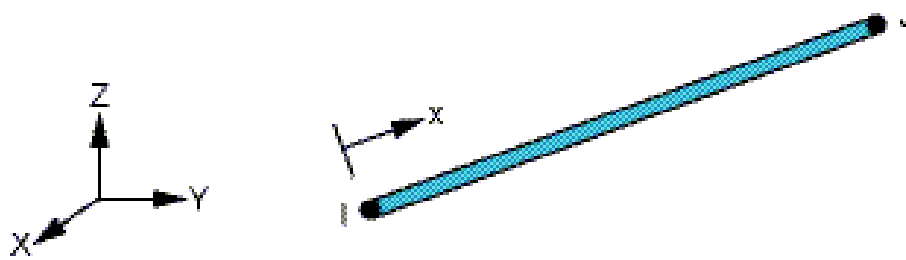


Figure 3.9 – Link180 element representing steel rebar (ANSYS-Multiphysics, 2011)

3.5.2 Real Constants

Real constants are the data which is required for element stiffness matrix calculation, but cannot be determined from the material properties or node locations. Properties such as cross sectional area of the steel rebar are input as real constant of Link180. Typical real constants include area, thickness, inner diameter, outer diameter, etc. The set of real constants used for slabs in proposed research are given in Table 3.3.

Type	Real Constants	Element Type	Constants				
			Properties	Slab A	Slab B	Slab C	Slab D
Concrete	1	Solid65	Material Number	65	65	65	65
			Young's Modulus of Elasticity	3.11E6	3.11E6	3.11E6	3.11E6
			Poisson's Ratio	0.2	0.2	0.2	0.2
#3 Bar	3	Link180	Cross Sectional Area	0.11	-	0.11	-
#4 Bar	4	Link180	Cross Sectional Area	-	0.20	-	0.20

Table 3.3 – Real constants for all type of slabs

For solid 65 elements, real constant 1 is used and values can be entered for material number, young's modulus of elasticity and poisson's ratio. Similarly, for link180 element, real constants are set with number 3 and 4 corresponding to dia of rebars.

3.5.3 Material Properties

Depending on the application, most of the element types require material properties. These properties may be:

- Linear or nonlinear
- Isotropic, orthotropic, or anisotropic
- Constant temperature or temperature-dependent etc

Each set of material properties has a material reference number. The table of material reference numbers versus material property sets is called the Material Table. Multiple sets of material properties can be used within one analysis (to correspond with multiple materials used in the model). ANSYS classifies each set with a particular reference number. Parameters defining the material models can be found in Table 3.4. There are multiple parts of the material model for each element.

Material Number	Element Type	Material Properties			
65	Solid65	Linear isotropic			
		EX	3.60E+06		
		PRXY	0.2		
		Multi-linear isotropic			
			Stress	Strain	
		Point 1	0.0001	311	
		Point 2	0.0002	618	
		Point 3	0.0004	1197	
		Point 4	0.0008	2129	
		Point 5	0.0010	2457	
		Point 6	0.0012	2696	
		Point 7	0.0014	2855	
		Point 8	0.0016	2950	
		Point 9	0.0018	2994	
		Point 10	0.0020	3000	
		Point 11	0.0030	3000	
		Concrete			
		Open Shear Coef.	0.5		
		Close Shear Coef.	0.8		
		Uniaxial Cracking Stress	410		
		Uniaxial Crushing Stress	-1		
Biaxial Crushing Stress	0				
Hydrostatic Pressure	0				
Hydro Biax Crush Stress	0				
Hydro Uniax Crush Stress	0				
Tensile Crack Factor	1				
185	Solid185	Linear isotropic			
		EX	29E+6		
		PRXY	0.3		
180	Link180	Linear isotropic			
		EX	29E+6		
		PRXY	0.3		
		Bilinear Isotropic			
		Yield Stress	60,000 psi		
Tang Mod	0				

Table 3.4 – Material model for slabs

The uniaxial compressive stress-strain relationship for the concrete model was obtained using the following equations to compute the multilinear isotropic stress-strain curve for the concrete (James K. Wight and Macgregor, 2012).

$$f_c = \frac{E_c \varepsilon}{1 + \left[\frac{\varepsilon}{\varepsilon_o} \right]^2}, \quad \varepsilon_o = \frac{2 f_c}{E_c} \quad \text{and} \quad E_c = \frac{f}{\varepsilon}$$

Where f_c = Stress at any strain

ε = Strain at any stress f_c

ε_o = Strain at ultimate compressive stress

The multilinear isotropic constituent relation of the material i.e. stress-strain data requires that first point of the curve to be defined by the user and it must obey Hooke's Law. Plot of stress-strain curve for concrete and steel are shown in Figure 3.10 and Figure 3.11 respectively.

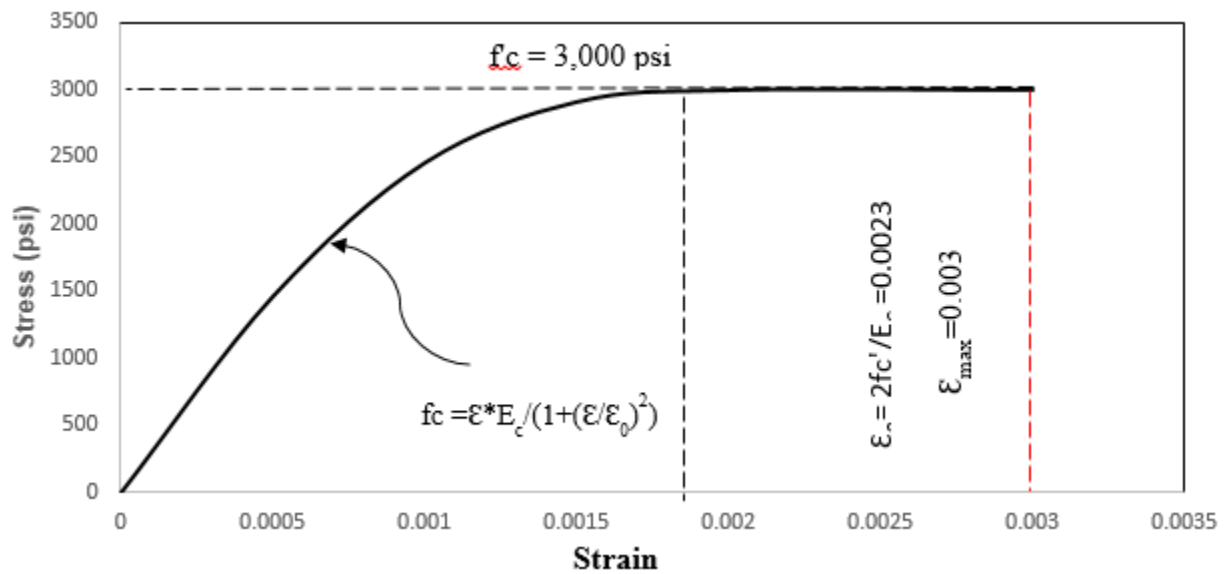


Figure 3.10 – Plot of stress strain relations of concrete

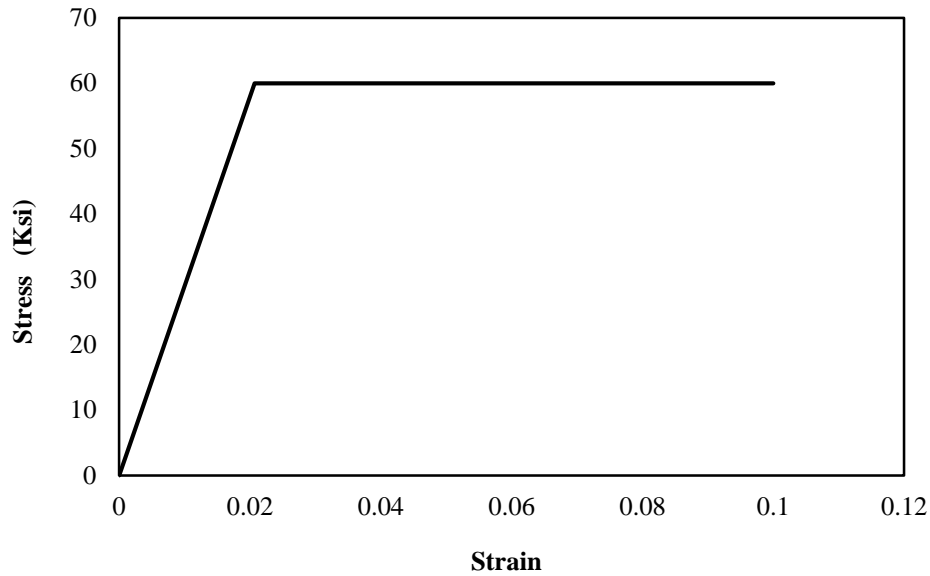


Figure 3.11 – Plot of stress strain relations of steel rebar

In Figure 3.10, the maximum concrete strain at failure, $\epsilon_{(\max)} = 0.003$, indicates traditional crushing strain for unconfined concrete, as recommended by ACI code. Shear transfer coefficient values represent the part of shear resistance provided by aggregate interlock at crack. Shear transfer coefficients range from 0.0 to 1.0. Bottom value of 0.0 represents a smooth development of crack that means full loss of shear resistance at crack location and similarly maximum value of 1.0 represents a rough crack which means no loss of shear resistance at crack location. Convergence problems may arise when the shear transfer coefficient for the open crack falls below 0.2. In present research work, a value of 0.25 has been used for all slabs.

The uniaxial tensile cracking stress that is based upon the modulus of rupture is determined using formula recommended by ACI 318 (2011). From here, value of f_{cr} comes out to be 410 psi. In the model, uniaxial tensile crushing stress is based on the uniaxial compressive strength (f_c'). The value of uniaxial crushing strength is put equal to -1, which turned off the crushing capability of the soild65 concrete element. Convergence problems have been frequently occurred when the crushing capability was turned on.

Material model number 185 refers to the Solid185 element and the same was used for steel plates at loading and support positions. The material properties of this element is modeled as

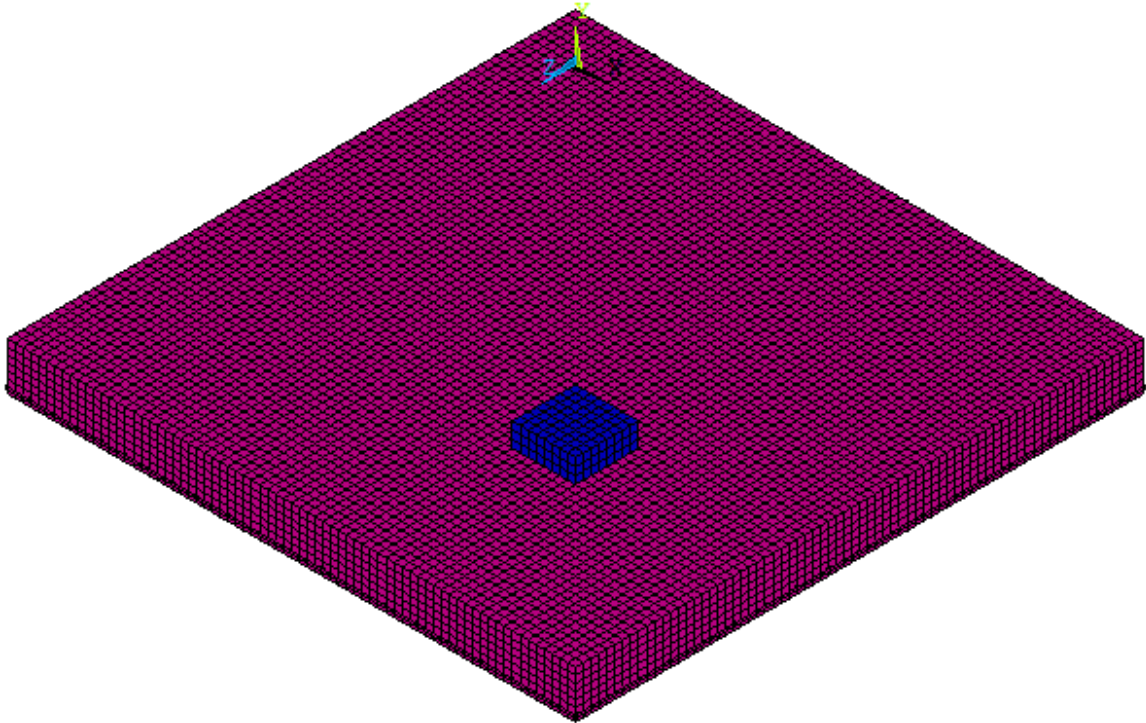
a linear isotropic with a modulus of elasticity same as that of steel ($E_s=29E+6$ psi), and poisson's ratio equal to 0.3.

Similarly, for the representation of steel reinforcement in the slabs, material model number 180 is defined that refers to the Link180 element. Material model for Link180 is a bilinear isotropic element and is used for all the steel reinforcement in the slabs. Failure criteria of bilinear isotropic material is based on the Von Misses Theory. The bilinear material model requires the yield stress f_y , as well as the hardening modulus of the steel to be defined. The yield stress was defined as 40,000 psi, and the hardening modulus as 0.

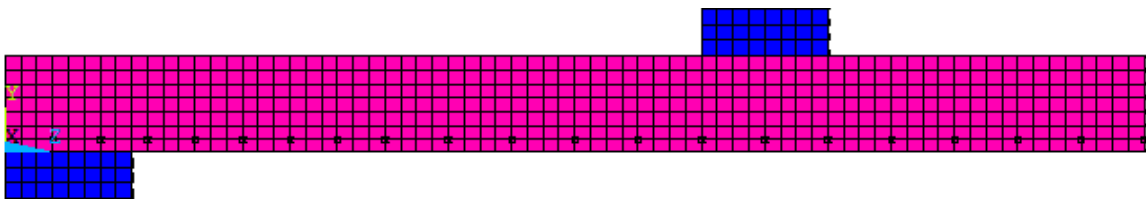
3.5.4 Modeling

The slabs and plates were modeled as volumes. Taking advantage of symmetry only quarter of the slabs were modeled; all slabs are of dimensions 12' x 12' x 0.5'. The zero values for the X, Y and Z coordinates coincide top left of the steel reinforcement layout of concrete slab.

Steel Rebar: As discussed above, Link180 elements were used to model all flexural reinforcement. For this purpose, volumes were so divided to place all reinforcement bars at desired spacing. Three dimensional finite element models of concrete slabs considered for analysis in the proposed study are presented in figures from Figure 3.12 to Figure 3.15.

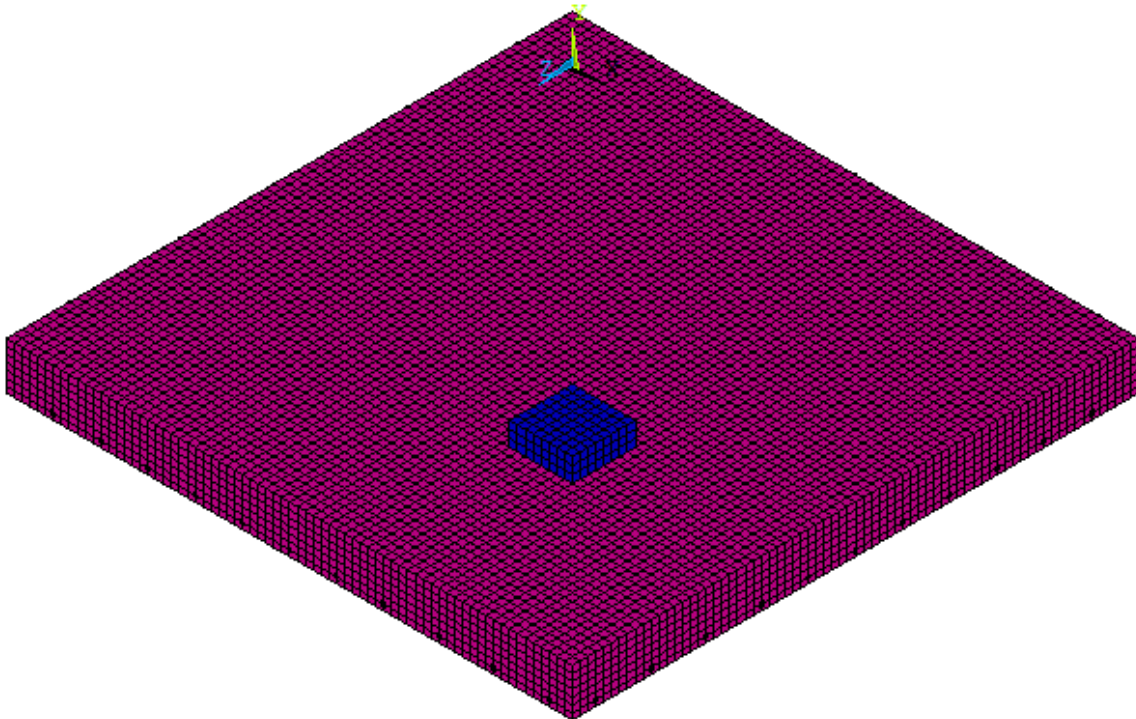


3.12 (a) – Isometric view of slab

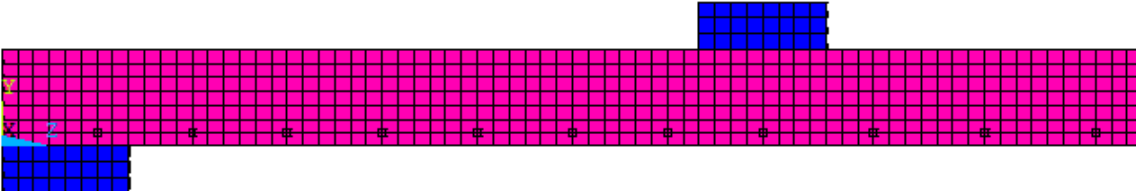


3.12 (b) – Cross section of slab

Figure 3.12 – Model of Slab A

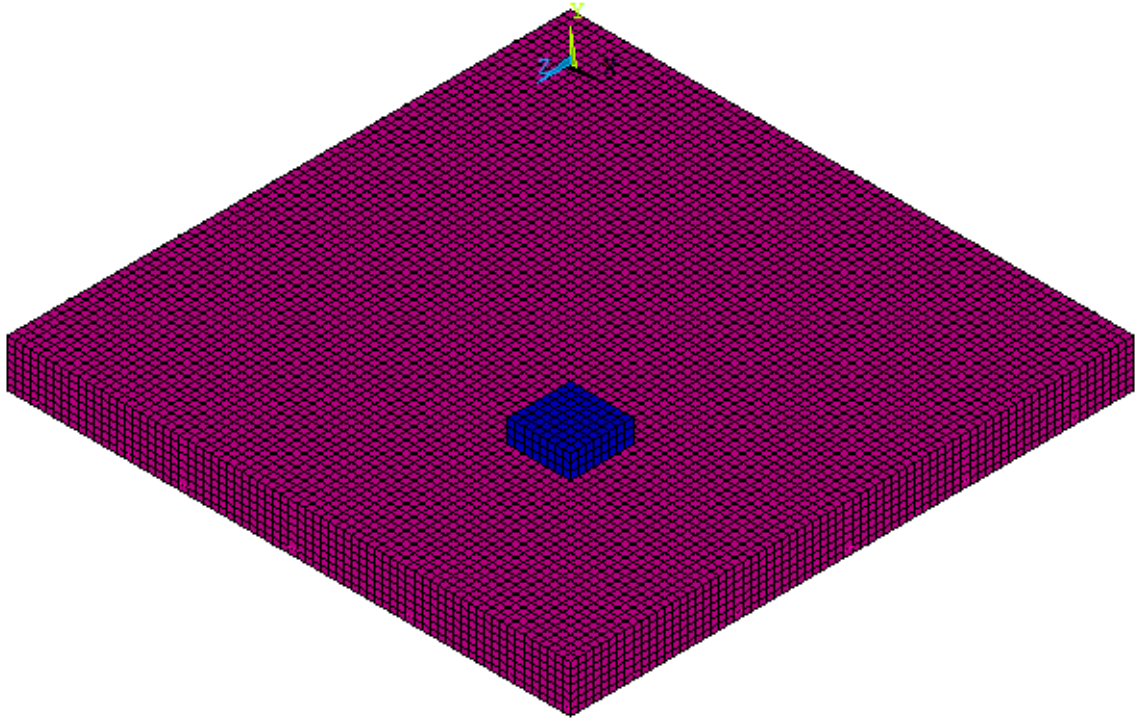


3.13 (a) – Isometric view of slab

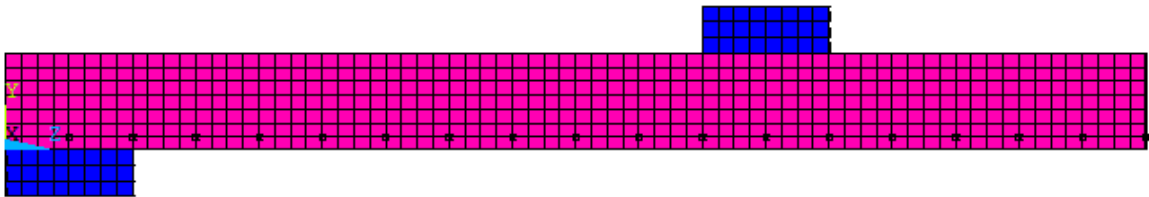


3.13 (b) – Cross section of slab

Figure 3.13 – Model of Slab B

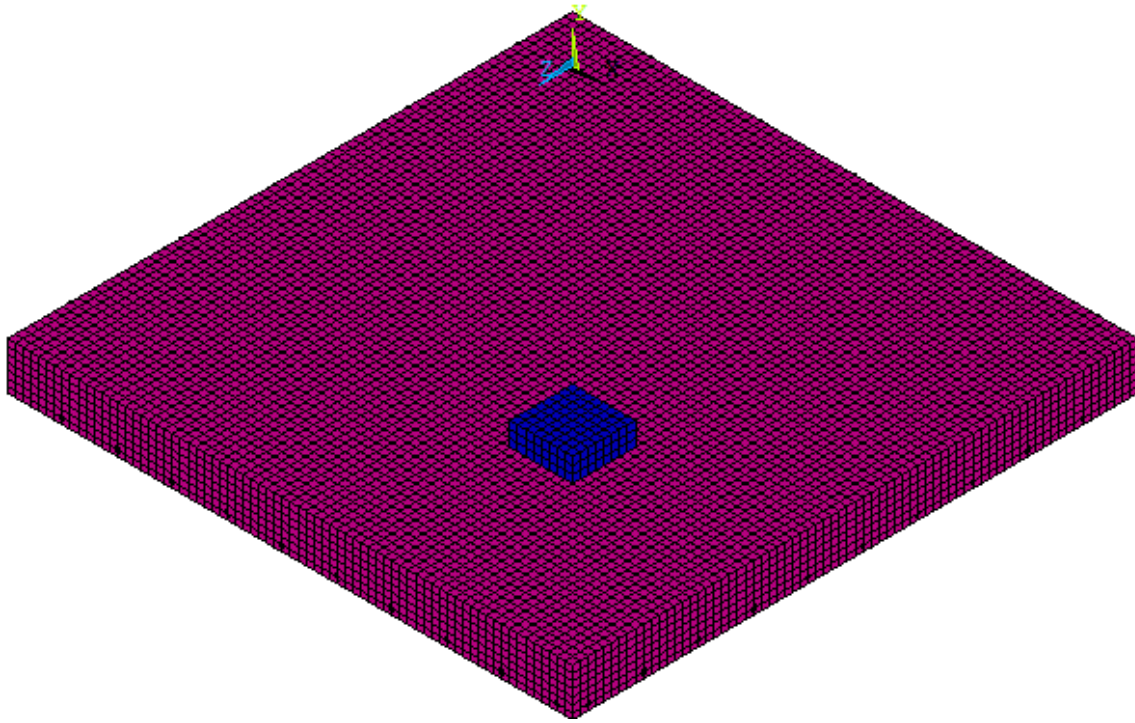


3.14 (a) – Isometric view of slab

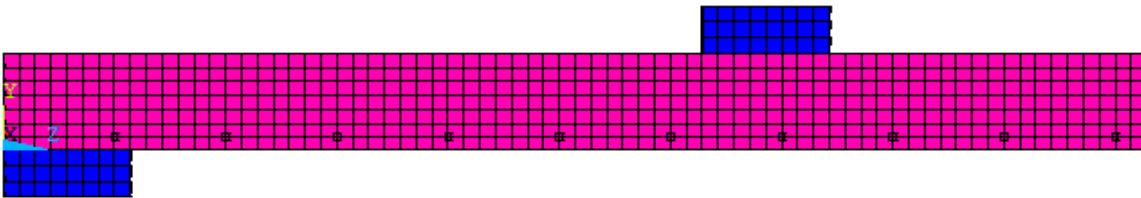


3.14 (b) – Cross section of slab

Figure 3.14 – Model of Slab C



3.15 (a) – Isometric view of slab



3.15 (b) – Cross section of slab

Figure 3.15 – Model of Slab D

3.5.5 Meshing

All steel and concrete volumes were so divided that the mesh was set up in such a manner that square or rectangular elements were formed. For this purpose the volume sweep command was used. This option sets the length and width of finite elements in the plates to be consistent with the elements and nodes in the concrete portions of the model. The meshed volume of the concrete and support and loading plates are shown in figures from Figure 3.16 to 3.19. After meshing volumes, all lines which have been attributed rebar properties were selected and meshed. The meshing of the reinforcement is a special case as compared to the volumes.

3.5.6 Numbering Controls

Separate entities that have the same location are merged by numbering control command. By using this command items will be merged into single entity. Merging key points before nodes can result in some of the nodes becoming “orphaned”, that means, the nodes lose their association with the solid model. The orphaned nodes can cause certain operations (such as surface load transfers, boundary condition transfers etc) failed. One must be careful, while merging entities in already meshed model because the merging order is very much significant step and ensure that all entities are merged properly.

3.6 Solution Phase

In this phase, loads at loading plates, boundary conditions at support, at symmetric locations, analysis types and solutions controlling parameters are selected.

3.6.1 Loads and Boundary Conditions

To obtain unique solution, displacement boundary conditions are needed to constrain the model. The boundary conditions need to be applied at support and at the plane of symmetry to ensure that the model must behave in the same way as the full life size slab. In the present case there are two symmetric locations in slabs parallel to x and z axis. The boundary conditions for the slabs at planes of symmetry and at both loading plates are shown in figure from Figure 3.20 to figure 3.23.

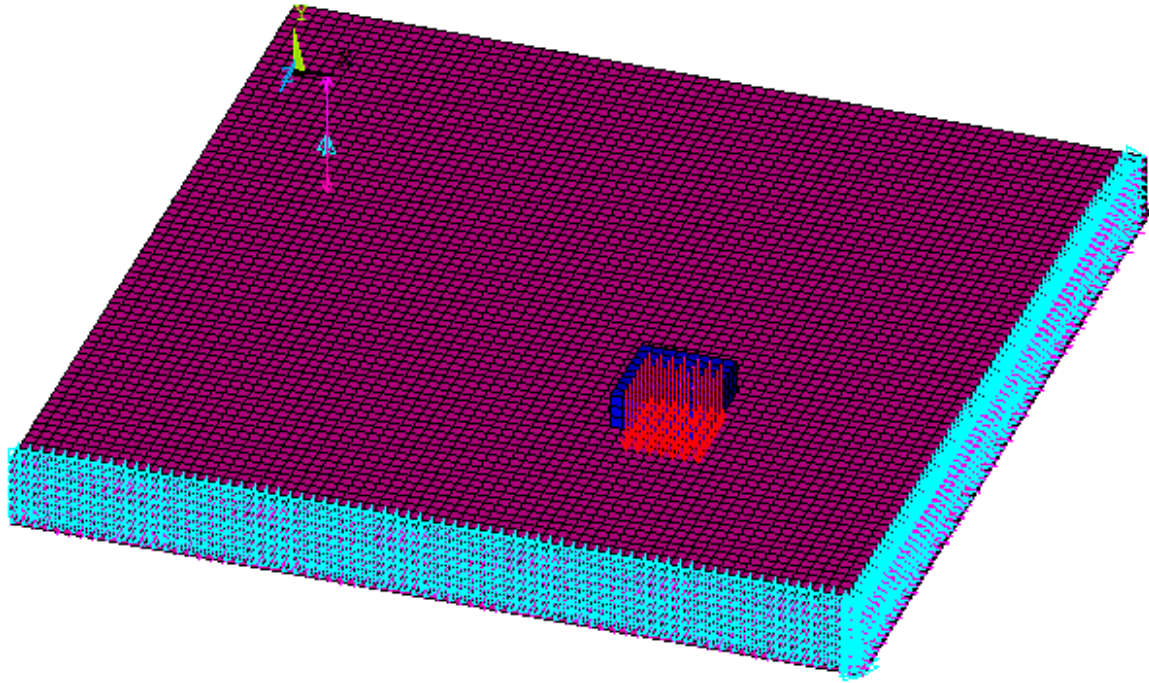


Figure 3.16 – Loading and boundary conditions on Slab A

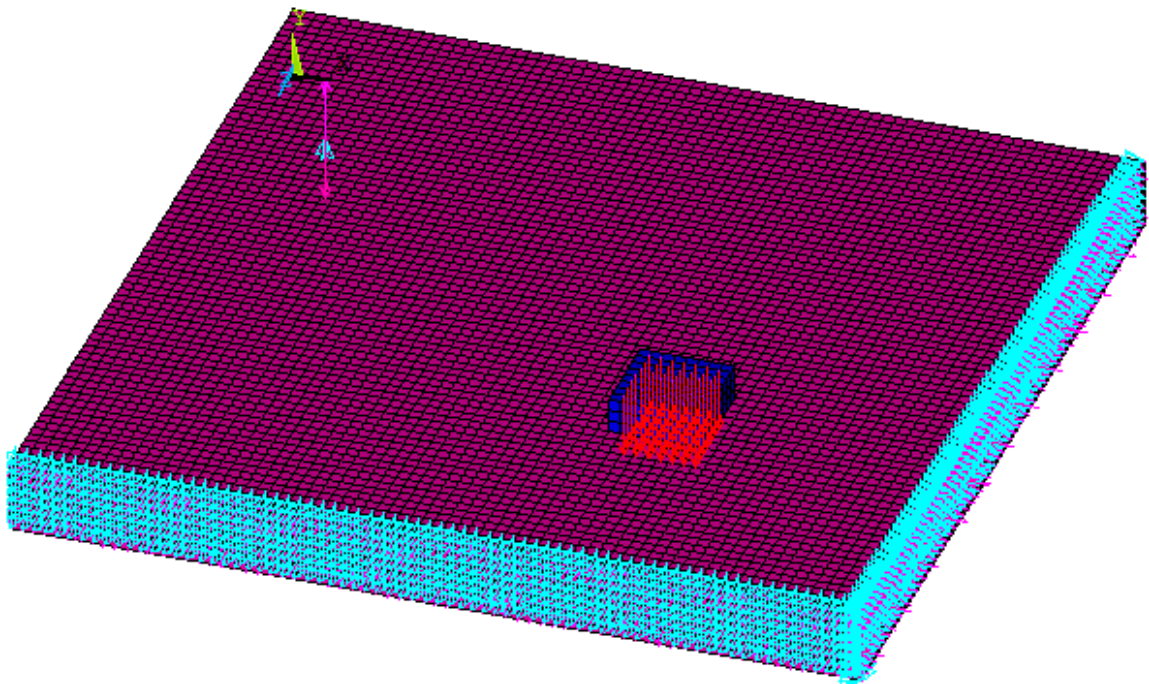


Figure 3.17 – Loading and boundary conditions on Slab B

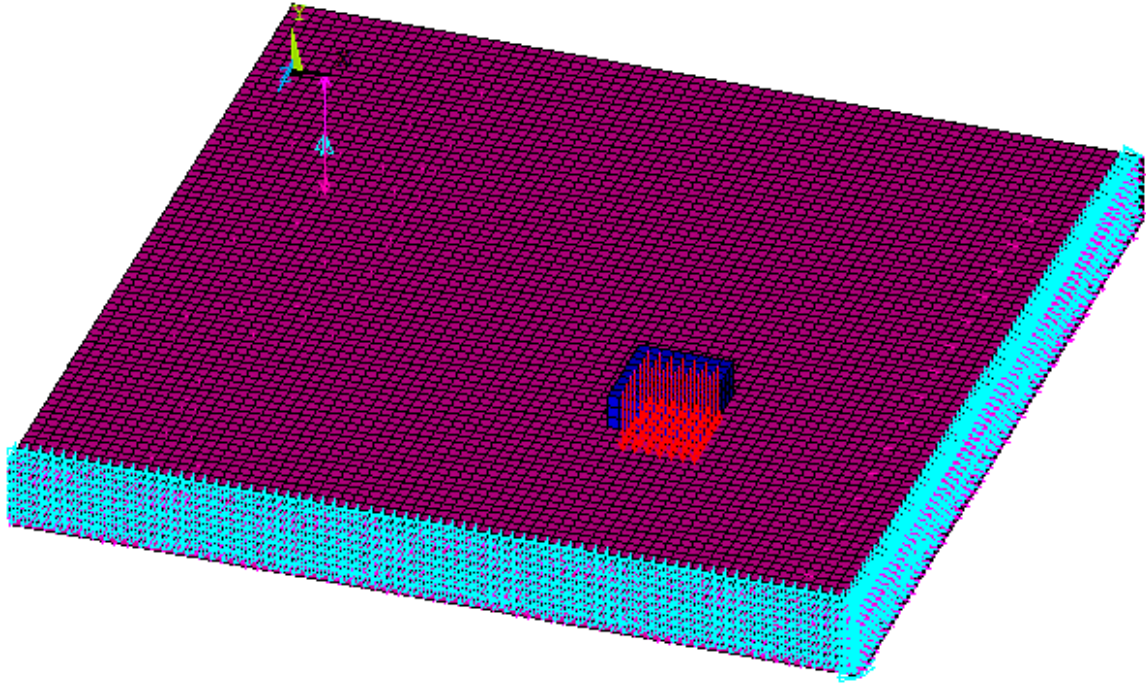


Figure 3.18 – Loading and boundary conditions on Slab C

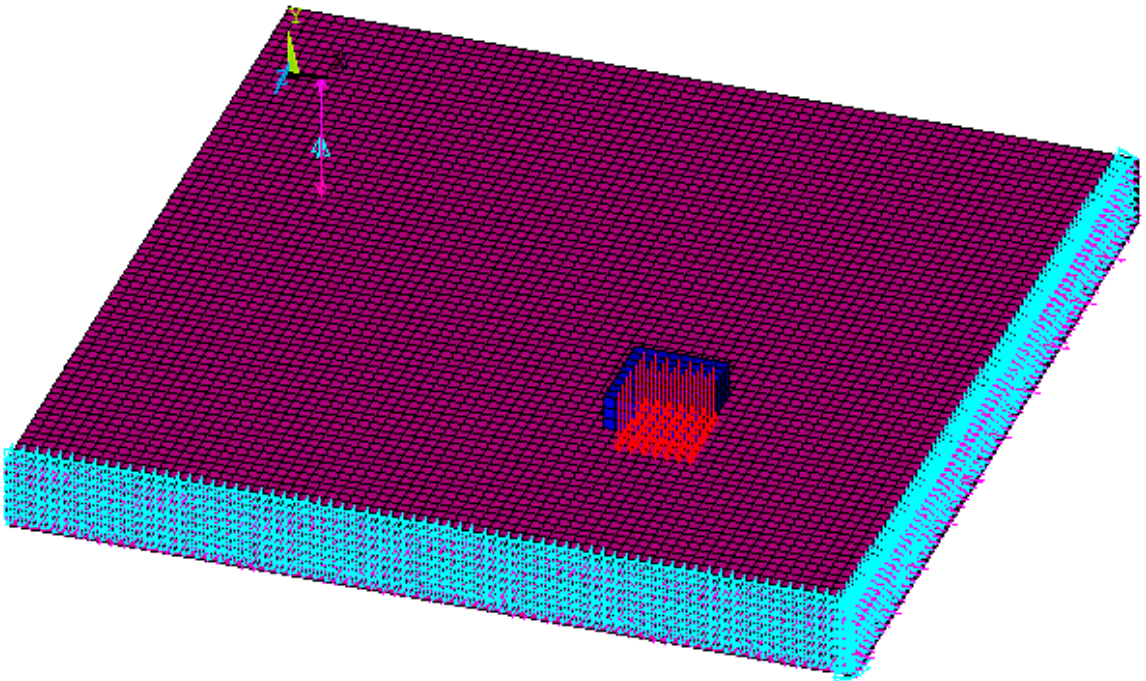


Figure 3.19 – Loading and boundary conditions on Slab D

Boundary conditions at planes of symmetry constrain the slabs in x and z directions. The nodes on planes of symmetry must be constrained in the direction perpendicular to the plane. For this

purpose, values of U_x and U_z is kept equal to zero for the plane parallel to z-axis and x-axis respectively. Similarly, the displacement along y-axis is put equal to zero at support plate i.e. $U_y = 0$. Application of boundary conditions at support plate will allow the slab to rotate and stable at support. The design load is applied along (-) y-axis at the loading plate to analyze the slab.

3.6.2 Analysis Type and Process

In the proposed study, nonlinear finite element analysis of simple slabs are being performed under flexural loading. Static analysis command is used for running of analysis of test model. The Newton-Raphson method of analysis is used to compute the nonlinear response. The load is applied in different steps with small range of load for smooth run of analysis by avoiding convergence issues. Every time after first range of load, restart command is used to start next load step. Static analysis is performed until failure of slab or yielding of steel, whatever occurs first. The convergence criterion is set to default for each load step except value of tolerance. The tolerance value is set equal to 0.25 for complete analysis, which is five times the default value. With this value of tolerance, analysis results seem in good co-relation with elastic theory analysis.

3.7 General Postprocessor

Post processor which includes results are discussed in chapter 4.

Results and Discussions

4.1 General

After applying boundary conditions and designed load on slabs model having steel reinforcement detailing as given in Table 3.1, the analysis is run for the results. General Postprocessing contains different structural and mechanical parameters presenting analyzed values for the slab models. But as, the main objective of the proposed research is to study flexural cracking behavior of two way RC slabs by using various steel reinforcement detailing, so, the only required structural parameters are analyzed, which are as followed:

- a. Cracking load
- b. Deflection of slab
- c. Stresses in concrete slab
- d. Stresses in steel reinforcement
- e. Crack widths

Among available parameters in ANSYS 16, the above said are sufficient to understand flexural behavior of two way RC slabs, when only gravity load is applied.

4.2 Cracking Load

The resulted cracking loads of all models are shown in graphical representation of Figure 4.1, after integration to whole slab by multiplying with 4. The cracking loads are observed on generation of first crack at the bottom of slabs. Cracking stress in concrete can be calculated using the formula mentioned in ACI 318 (2011).

All the slabs are subjected to same loading conditions and also the steel reinforcement area is kept same in each slab but the only major difference is the steel reinforcement detailing. During analysis, it is observed that Slab C cracked at the earliest, whereas, Slab A is found the best model to delay the cracks and cracking load on an RC slab. The potential reasons are the proper reinforcement detailing with the bending moment produced against the gravity load and the distribution of steel reinforcement as per requirements. In this way, the technique fulfills the

need of reinforcement detailing, it also economizes the structural member by reducing unnecessary steel reinforcement, where low bending moment generates.

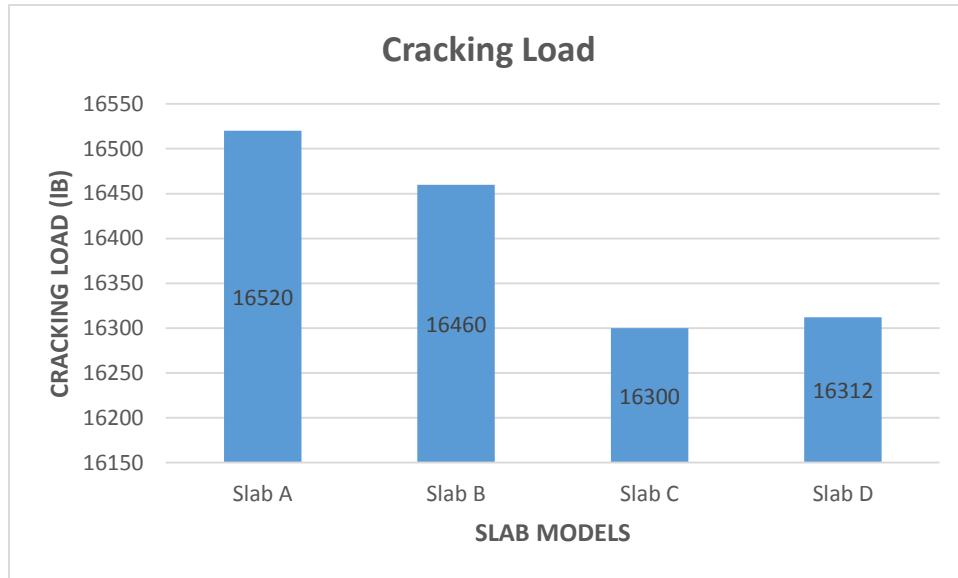


Figure 4.1 – Cracking loads of all slab models

4.3 Deflection of Slab

Deflection of a structural member is definite under a gravity load and the same is observed in case of all slab models at the time of first crack development. Although, the values of deflection are in permissible limit at cracking load, but differ in all cases. The graphical representation of deflection values at cracking load, obtained from ANSYS models is given in Figure 4.2.

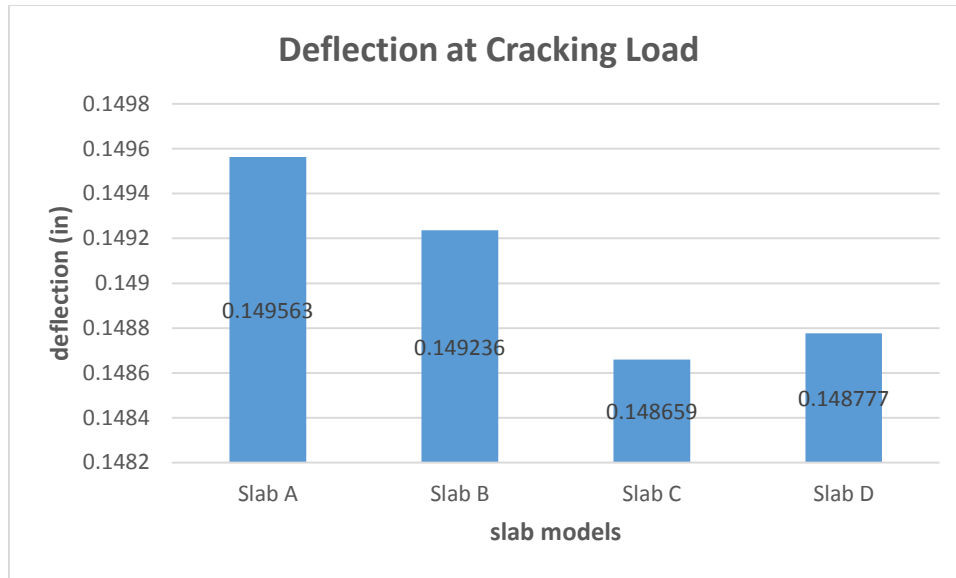


Figure 4.2 – Values of deflection at cracking load for all

From the figure 4.2, it is evident that the deflection of all slab models exactly corresponds to the cracking loads mentioned in Figure 4.1. As, slab A and C have the maximum and minimum cracking loads respectively, so the same case is with their deflection values. By taking the lowest value of cracking load i.e. 16300 lbs for Slab C, as a reference case, then the deflection values of all slab models for the said reference load can be found as shown in Figure 4.3.

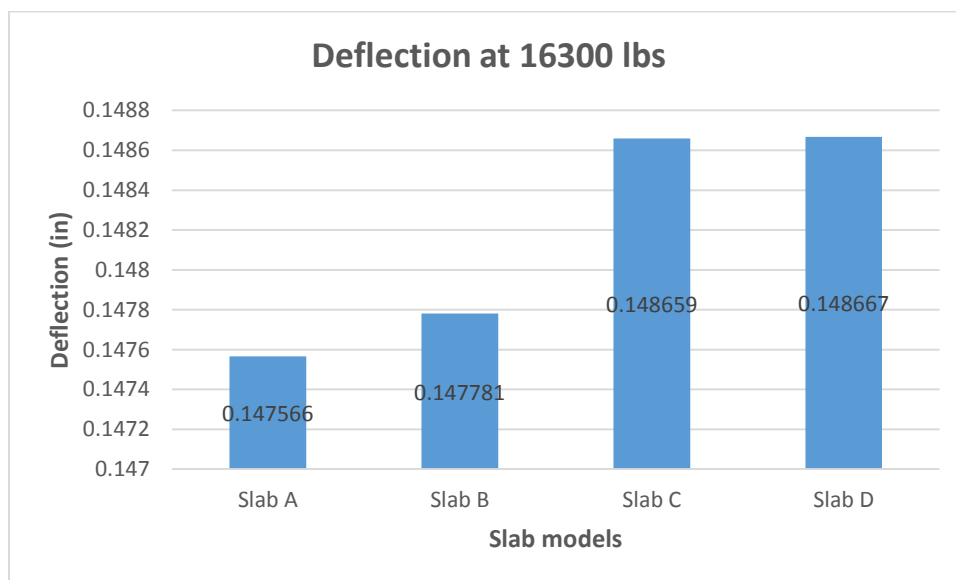


Figure 4.3 – Values of deflection at 16300 lbs for all slab

From here, it is clearly illustrated that Slab A has the lowest value of deflection among all slab models at the same reference load level i.e. 16300 lbs. This reveals that the steel reinforcement detailing is exactly in accordance with the bending moment and controls the deflection in slab positively and enhances the serviceability of structural member. Deflections of all slab models are shown in figure from Figure 4.4 to 4.7.

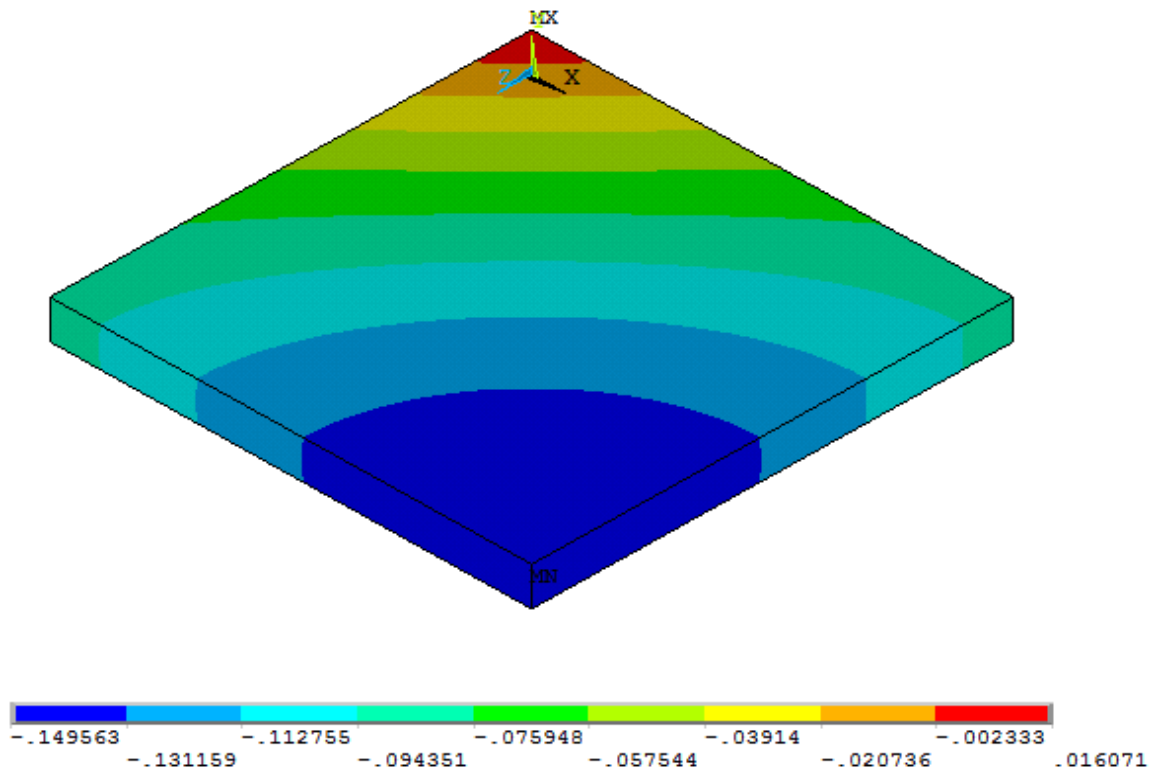


Figure 4.4 – Deflection of Slab A

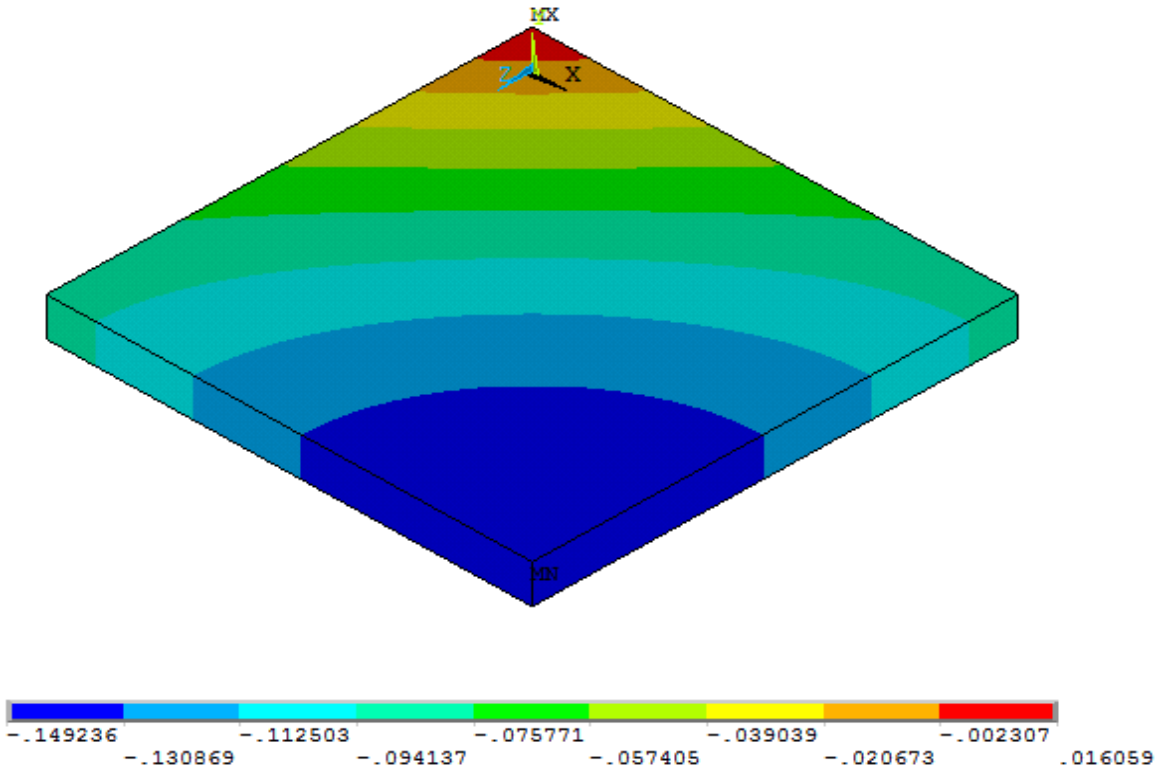


Figure 4.5 – Deflection of Slab B

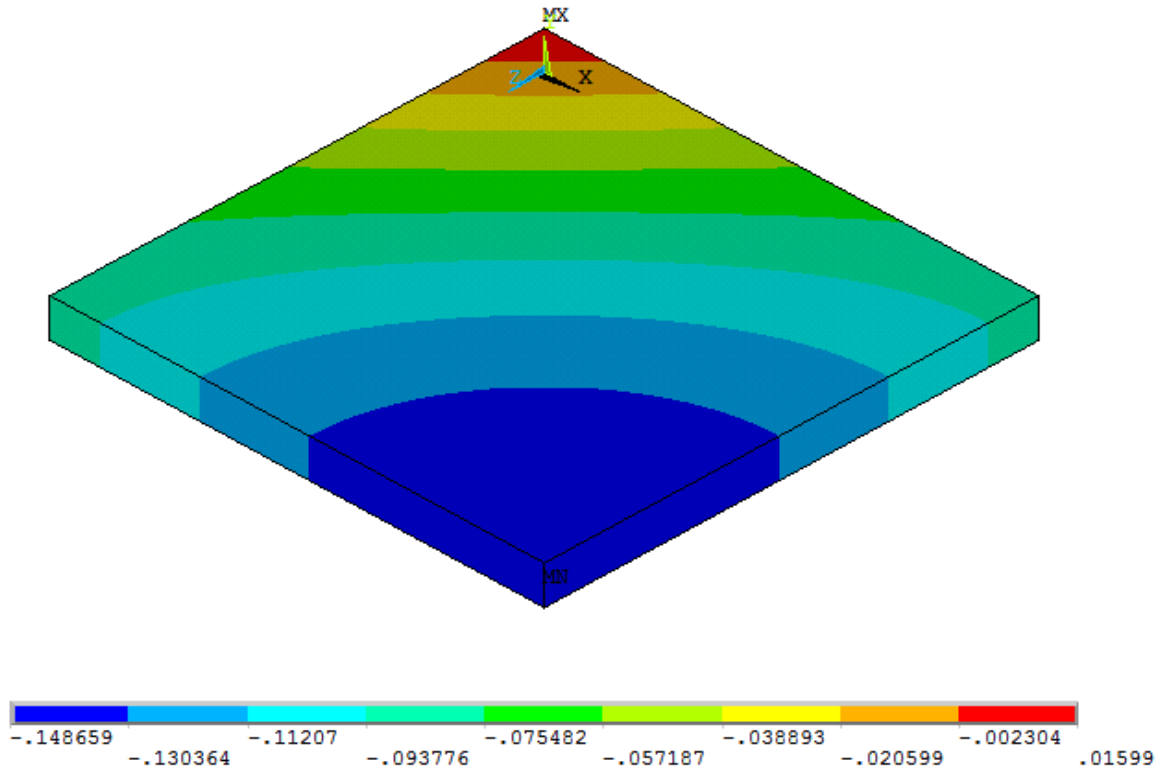


Figure 4.6 – Deflection of Slab C

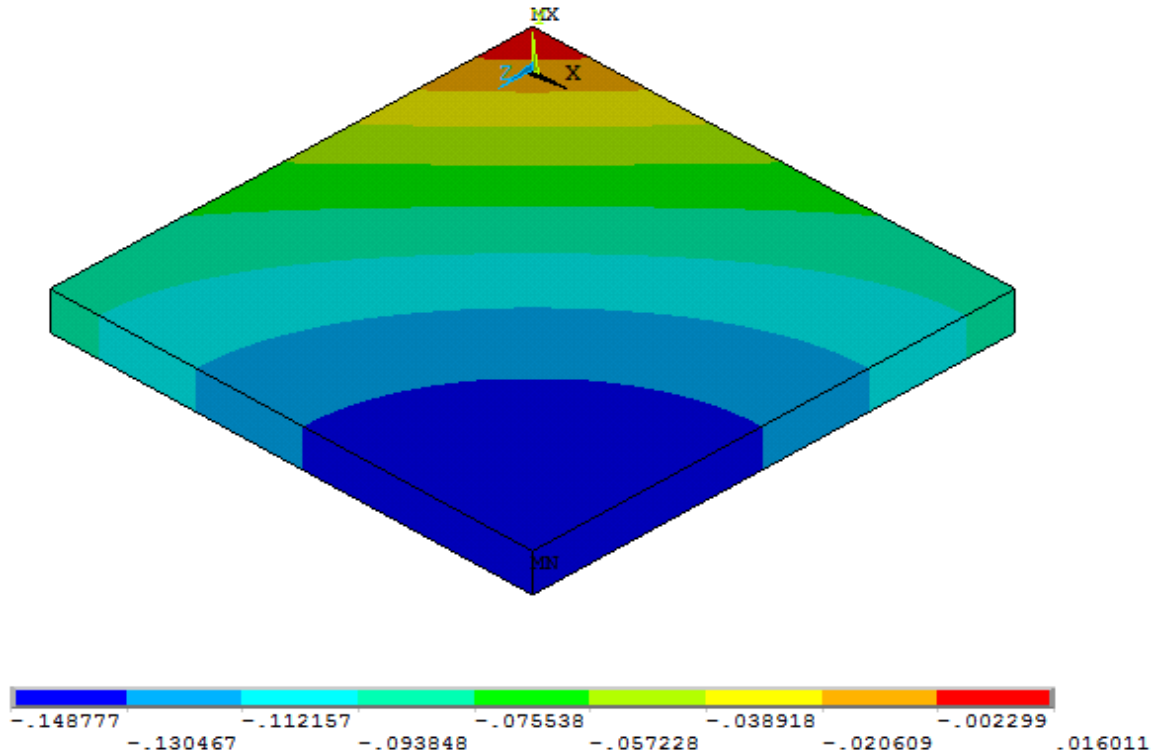


Figure 4.7 – Deflection of Slab D

4.4 Stresses in Concrete

Cracking strength of concrete is computed as 410 psi using the formula mentioned in ACI 318 (2011) and provided in ANSYS while assigning the Material Properties of concrete. As a result of the analysis, it is found that all slabs are cracked as soon as their stresses reach cracking strength when subjected to external load. In this analytical study, all the concrete slabs have the same cracking strengths but different minimum stresses, because, models are being analyzed theoretically, whereas, in experimentation, the scenario may be changed due to numerous reasons. Cracking strengths of concrete for all slab models are shown in Figure 4.8.

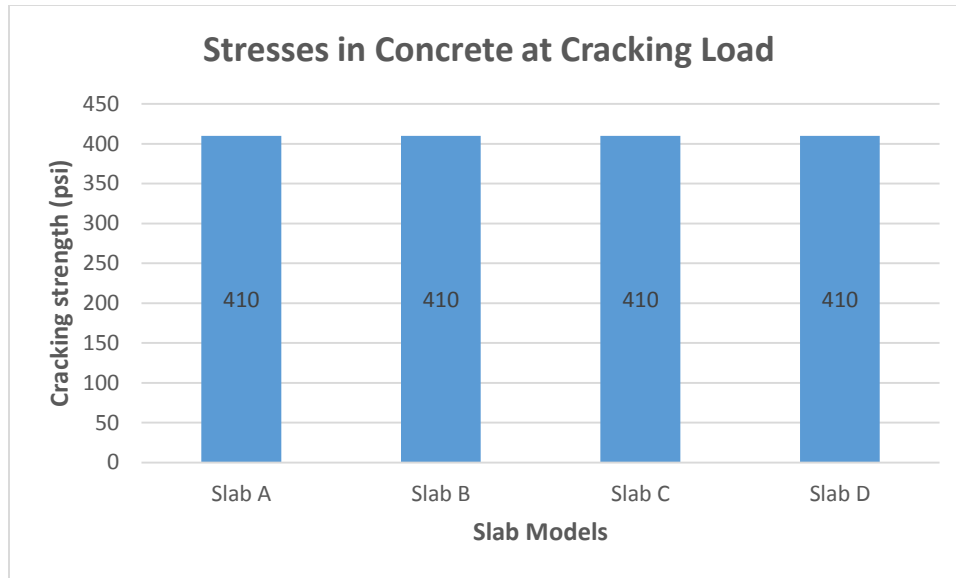


Figure 4.8 – Values of stresses in concrete for all slab

Stresses in concrete for all slab models are shown in figure from Figure 4.9 to 4.12.

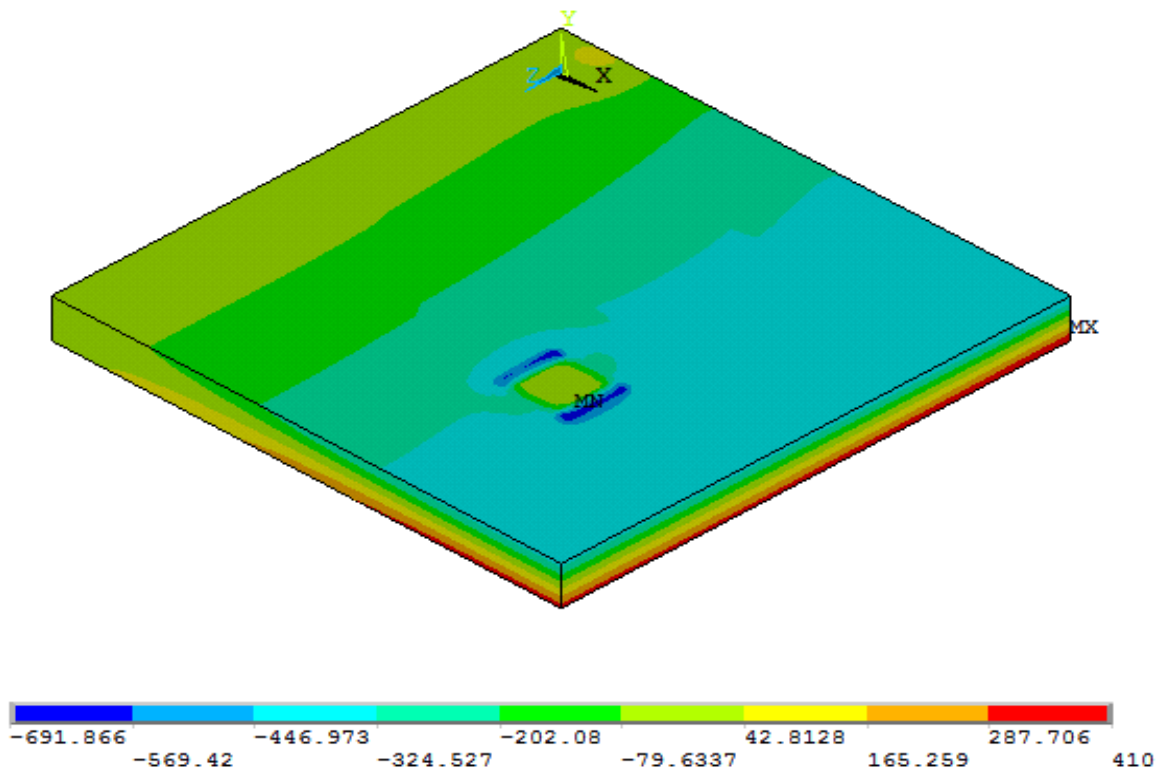


Figure 4.9 – Cracking strength of Slab A

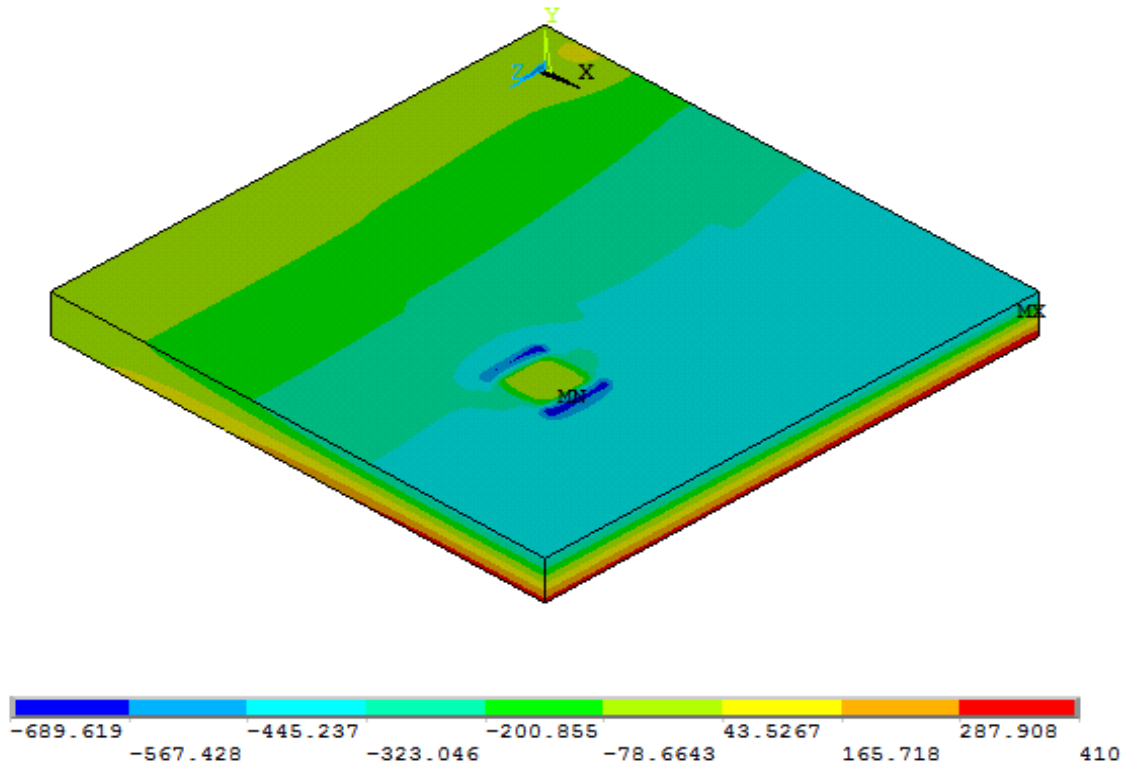


Figure 4.10 – Cracking strength of Slab B

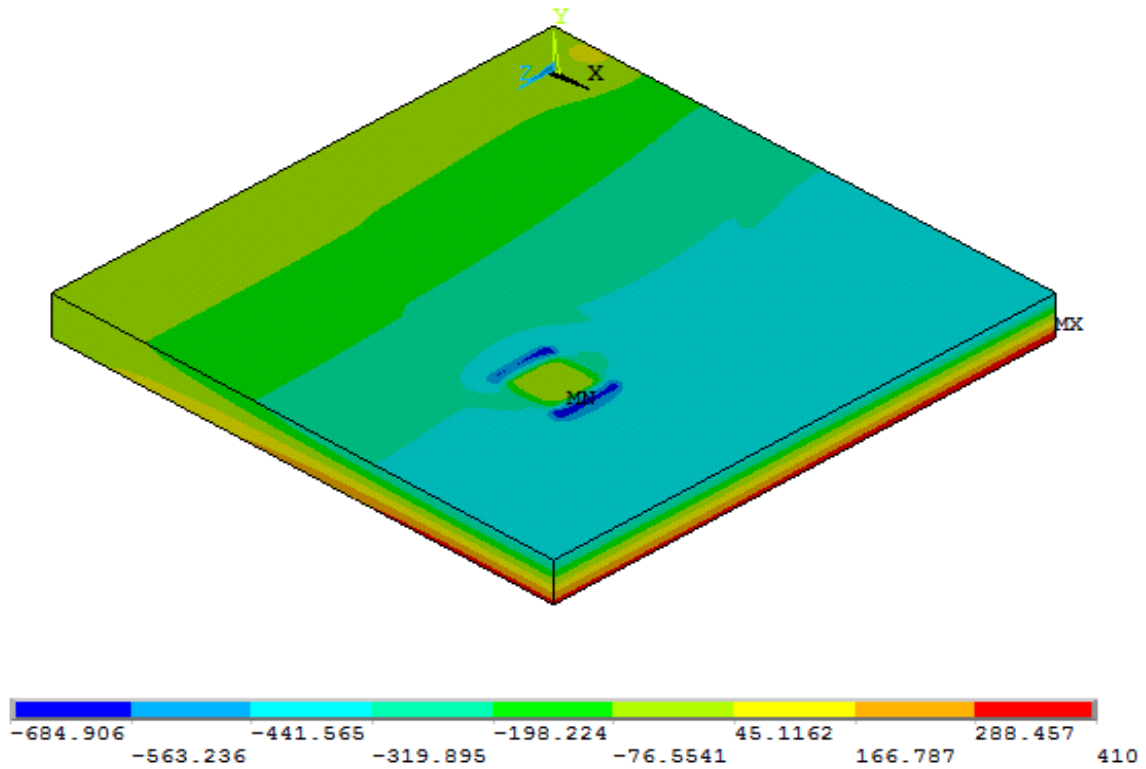


Figure 4.11 – Cracking strength of Slab C

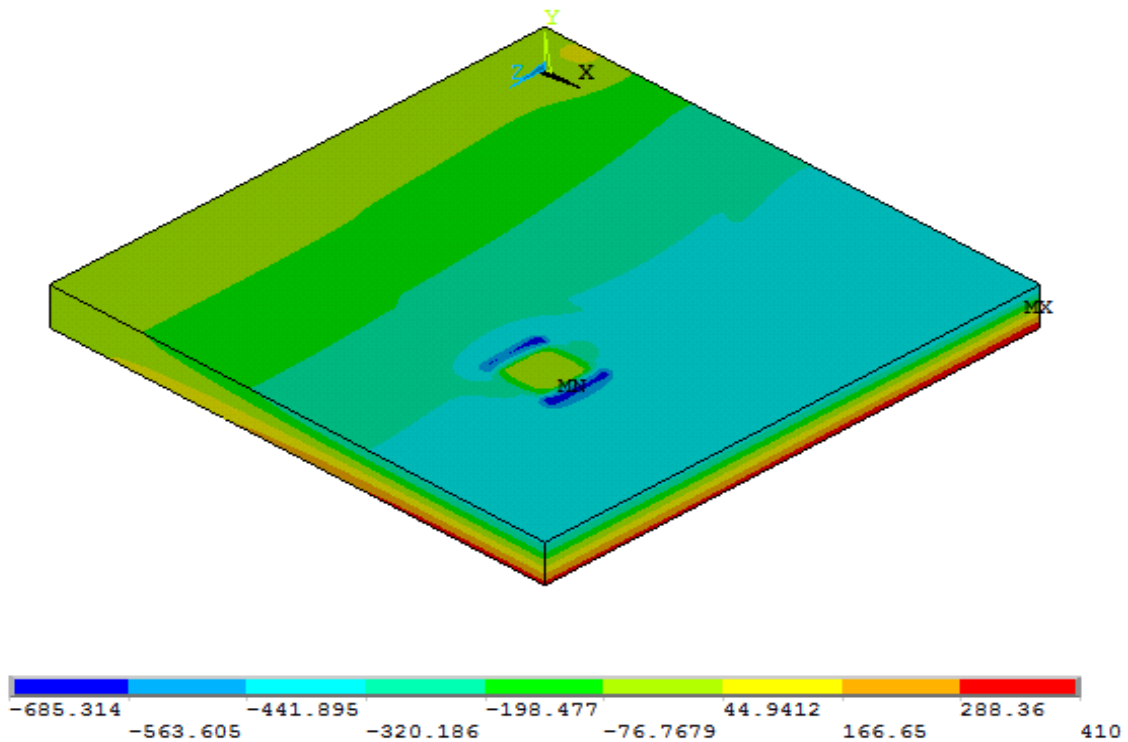


Figure 4.12 – Cracking strength of Slab D

4.5 Stresses in Steel Reinforcement

It is observed that tensile stresses are produced at the bottom of slab and are carried by concrete, when subjected to a gravity load. By increasing load, the stresses also increase until reach the cracking strength of concrete. On the development of crack, stresses start adjusting themselves by transferring from cracked section to uncracked section and tensile forces are taken by steel reinforcement at cracked section. Stresses in steel reinforcement for all slab models are shown in Figure 4.13 to 4.17. Values of the stresses are within the elastic range of steel reinforcement. It can be seen that slab A has the highest values of steel stresses, whereas, slab D shows lowest steel stresses on development of cracks.

Further, the order of the steel stresses is different from the trends of cracking loads. The abrupt sequence directly relates to the bar sizes and spacing taken into account for all slab models. Slab A and C have higher values of stresses resisted by tension steel due to lesser diameter of steel bars and center to center spacing. On contrary to that, steel stresses of Slab B and D do

have lower stress values due to larger sizes of steel bars taken in concrete slabs and likewise center to center spacing.

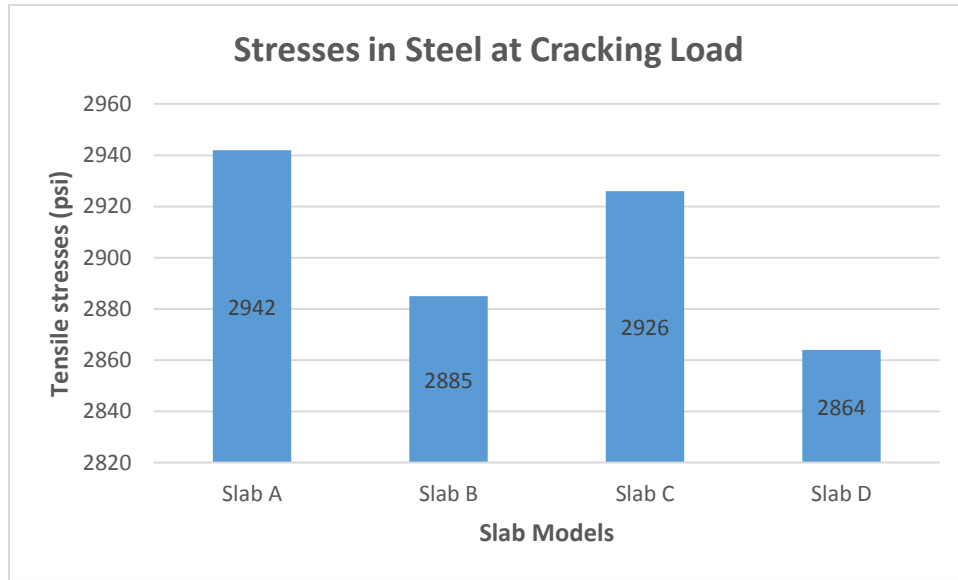


Figure 4.13 – Stresses in steel reinforcement for all slab

From the result of the analysis, it is found that that stress carrying capacity of tension steel can be enhanced by steel reinforcement detailing exactly in accordance with the bending moment produced as well as using steel bars of lesser dia, as provided in Slab A.

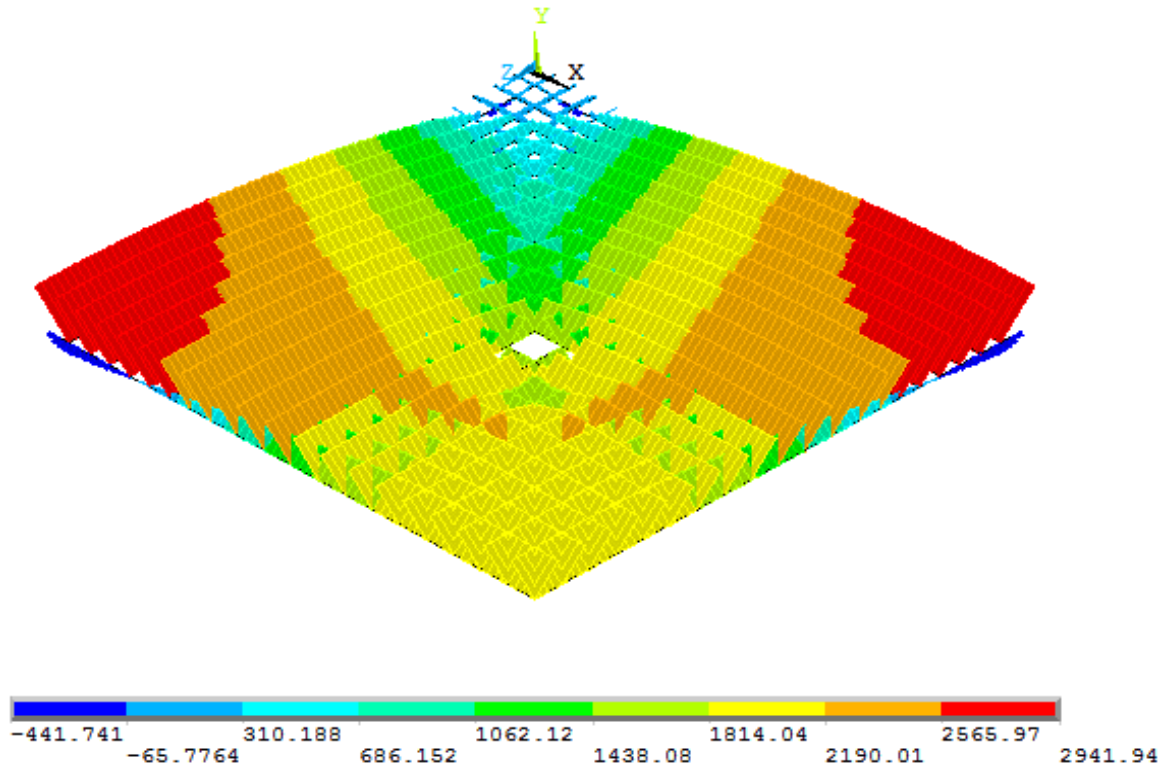


Figure 4.14 – Steel stresses in Slab A

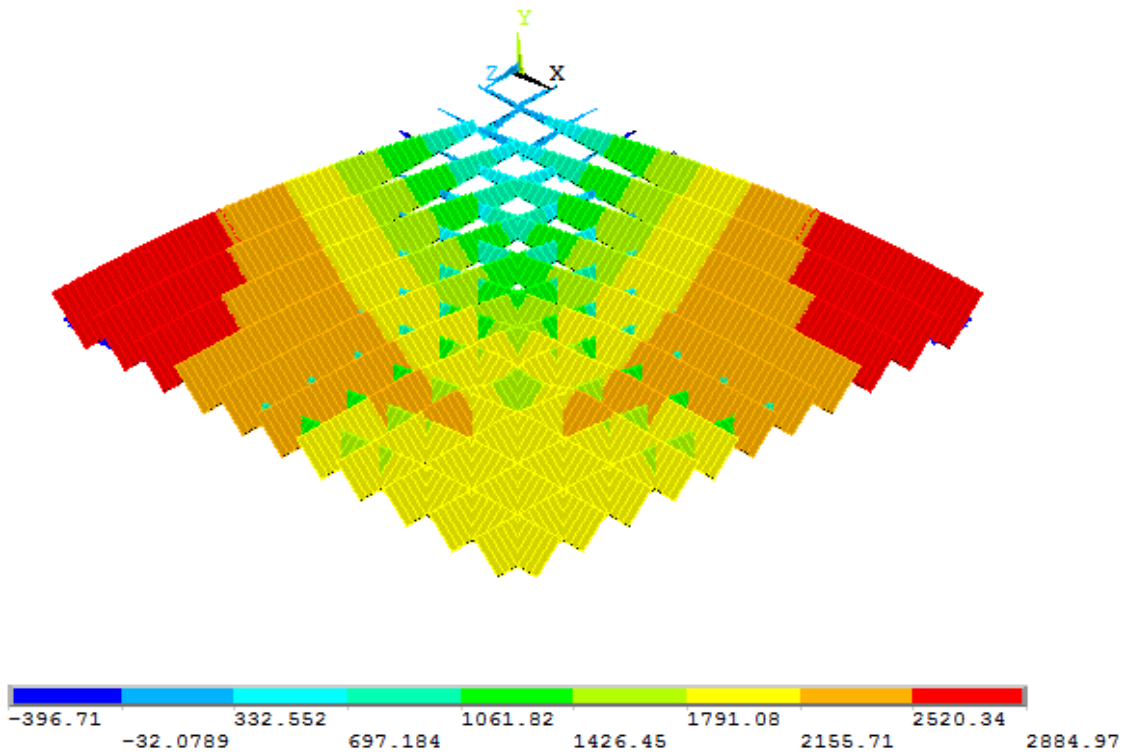


Figure 4.15 – Steel stresses in Slab B

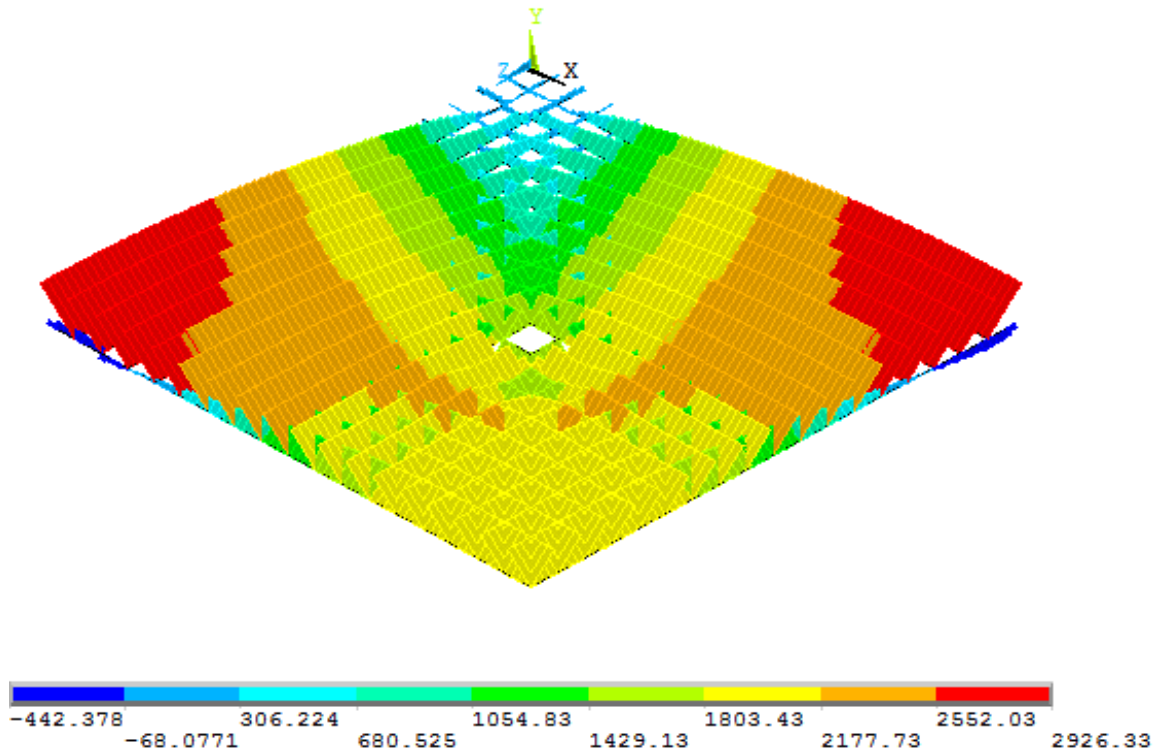


Figure 4.16 – Steel stresses in Slab C

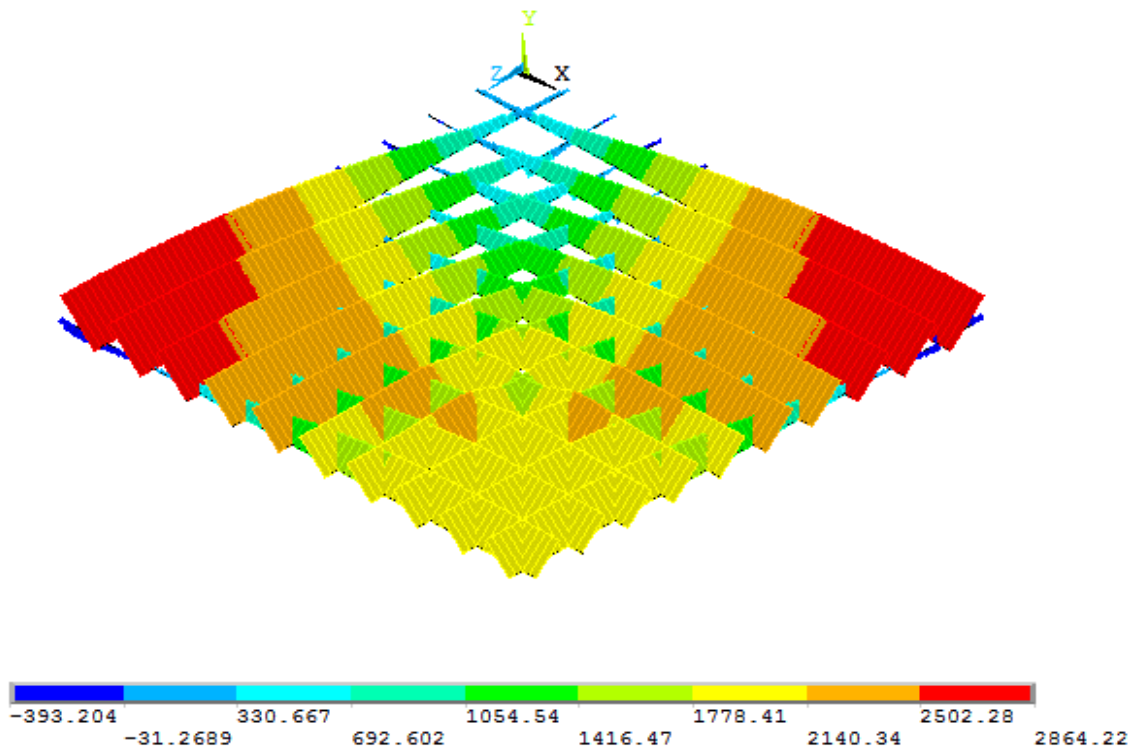


Figure 4.17 – Steel stresses in Slab D

4.6 Crack Widths

We cannot compute crack widths in a structural member directly from ANSYS, therefore, manual calculation has to be carried out. For this purpose, crack control equation formulated by Nawy and Blair is given in ACI report, “Control of Cracking in Concrete Structures”. This equation is as under:

$$w_{max} = K\beta f_s \sqrt{I}$$

where,

$K = 2.1 \times 10^{-5}$ (depends on boundary as well as loading conditions)

$\beta = h_2/h_1 =$ ratio of distance to the neutral axis from extreme tension face and from centroid of reinforcement. (for simplification $\beta=1.25$)

$f_s =$ steel stress

$I =$ grid index, given by

$$I = \frac{s_1 s_2 d_c}{d_{b1}} \frac{8}{\pi}$$

where,

$s_1 =$ spacing of reinforcement in direction 1

$s_2 =$ spacing of bars along direction 2 (perpendicular to direction 1)

$d_c =$ concrete cover to center of steel bar

$d_{b1} =$ diameter of steel bars in direction 1

The above equation depicts that, in two way slabs, crack widths are controlled basically by steel spacing, stress level of steel and also concrete cover. Although concrete cover is a major parameter in crack control equation for beams but in slabs, it is nearly constant (3/4 in).

Basing on the above equation, crack widths of all slab models have been computed and shown in a graphical representation as Figure 4.18.

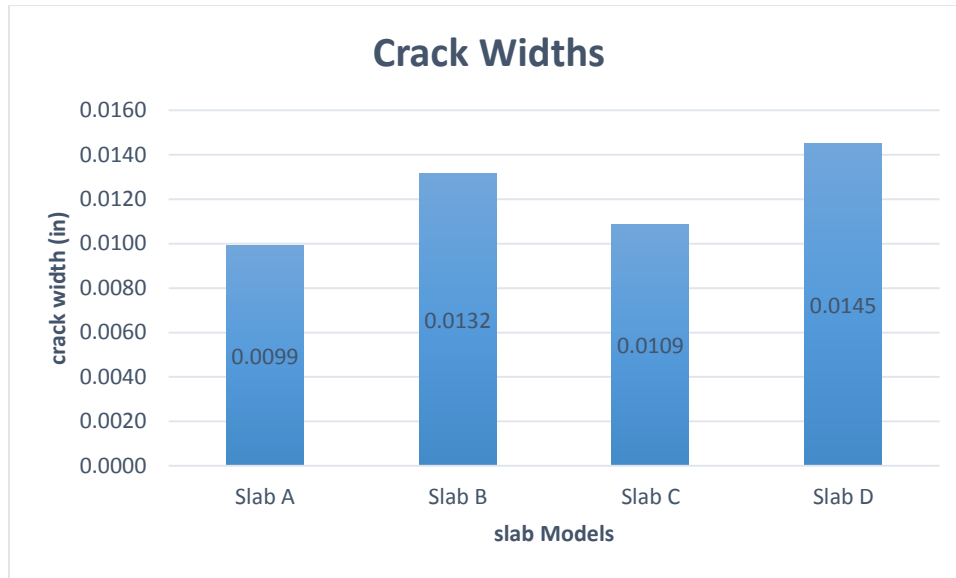


Figure 4.18 – Crack widths of all slab models

ACI defines tolerable crack width for a structural member in dry air exposure condition as 0.016 in and the crack widths of all slabs are within permissible limit. Slab A and C have lower values of crack width, whereas, Slab B and D have higher values of crack width. This sequence of crack widths matches exactly with that of steel stresses and the judgement behind both sequences is the bar size and center to center spacing. Due to using lesser dia of steel bars and spacing in Slab A and C causes low value of crack widths and in contrast to that, Slab B and D show higher values of crack widths as a result of using larger dia of steel bars and spacing accordingly. This indicates direct effect of steel reinforcement detailing on crack widths.

Conclusions and Recommendations

5.1 General

The aim of proposed research is the analytical study of flexural cracking behavior of two way RC slab used as base structure of telecommunication towers by using various steel reinforcement detailing. Four life size slabs having different steel reinforcement detailing are modeled in ANSYS 16 and different parameters like cracking load, concrete stresses, deflection of slab and steel stresses are analyzed to study effect of steel reinforcement detailing on flexural behavior of slabs. In addition, crack widths of all slabs are calculated manually basing on Nawy and Blair equation. Results obtained from analytical research is used to develop the relationships of different parameters of two way slabs with steel reinforcement detailing.

5.2 Conclusions

As a result of analytical study, following conclusions can be drawn.

- ⇒ As the value of bending moment produced as a result of gravity load varies all over the slab depending on loading and boundary conditions, therefore, provision of steel reinforcement also varies throughout the slab in such a manner that maximum steel reinforcement is provided at highest bending moment and minimum steel reinforcement at lowest bending moment. This leads to delay in concrete cracking.
- ⇒ Variable steel reinforcement detailing conforming to bending moment effectively controls deflection in concrete slab making the structure more serviceable.
- ⇒ Smaller diameter of steel bars and center to center spacing result in higher values of stresses resisted by tension steel and vice versa.
- ⇒ Crack width in an RC slab is directly influenced by steel bars size and their spacing. Higher the diameter of steel bars and spacing, larger is the value of crack width and vice versa.

- ⇒ Comparison of different crack models reveals that Eurocode (1992) gives the lowest values of crack widths in a slab, whereas, Gergely and Lutz (1968) portrays opposite behavior.
- ⇒ Since the variable steel reinforcement detailing as per the requirement reduces the unnecessary steel, hence it economizes the structure.

5.3 Recommendations

The research has provided significant understanding of flexural cracking behavior of two way RC slab used as base structure of telecommunication towers and the effects of different steel reinforcement detailing on crack mitigation. Now further research is required to extend this study and also validate the results through experimentation. Given below are some of the recommendations for further research in this area.

- ⇒ There is still a need of investigation of parameters proposed by other researchers to understand further flexural cracking behavior of two way slab under tower.
- ⇒ Further research is required by altering loading and boundary conditions to extend study of crack mitigation.
- ⇒ Results of proposed research need to be validated through experimentation with the same loading and boundary conditions.
- ⇒ Towers load be analyzed against wind and earthquake loading.

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