

Phase²

2D finite element program for stress analysis and support design around excavations in soil and rock

Stress Analysis Verification Manual

Part II

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Introduction

This manual contains a series of example problems which have been solved using *Phase²*. The verification problems are compared to the corresponding analytical solutions. For all examples, a short statement of the problem is given first, followed by the presentation of the analytical solution and a description of the *Phase²* model. Some typical output plots to demonstrate the field values are presented along with a discussion of the results. Finally, plots of stresses and displacements are included.

23 Bearing Capacity of Foundation on a Slope of Cohesive Soil Material

23.1 Problem Description

The problem involves determination of bearing capacity of a flexible footing resting on a slope, made of purely cohesive material (clay). The footing is a simple strip footing, flexible in nature, and of unit width ($B=1$). The typical geometry of the problem is as shown in figure below:

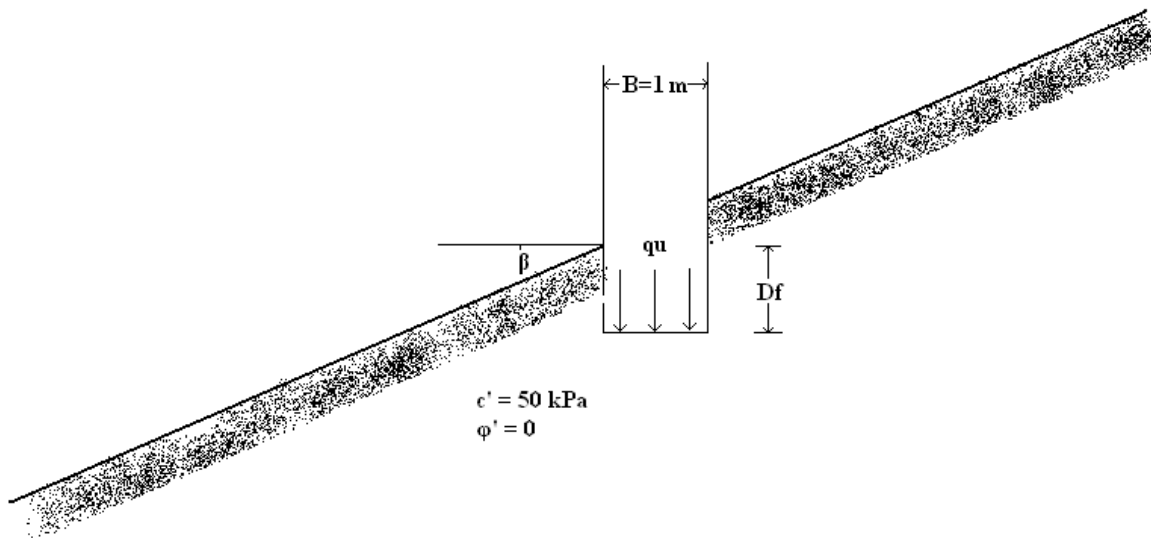


Figure 1: Model Geometry

23.2 Analytical Solution

A theoretical solution for the ultimate bearing capacity of a shallow foundation located on the face of a slope was developed by Meyerhof (1957). Based on this solution, the ultimate bearing capacity can be expressed as:

$$q_u = cN_c \text{ (for purely cohesive soil, that is, } \phi = 0) \quad (1)$$

and,

$$q_u = 0.5\gamma BN_\gamma \text{ (for granular soil, that is } c' = 0) \quad (2)$$

where, c is the undrained cohesion, and N_c , N_γ are the bearing capacity factors of soil. Another term, called stability number (N_s), is a dimensionless number which is defined as:

$$N_s = \gamma H/c \quad (3)$$

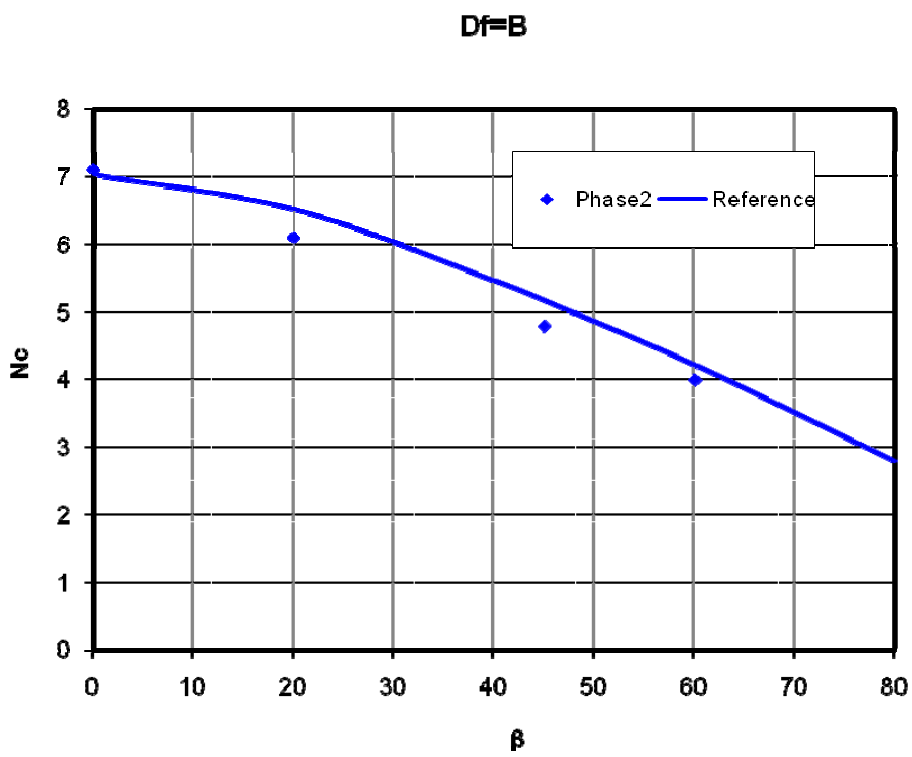
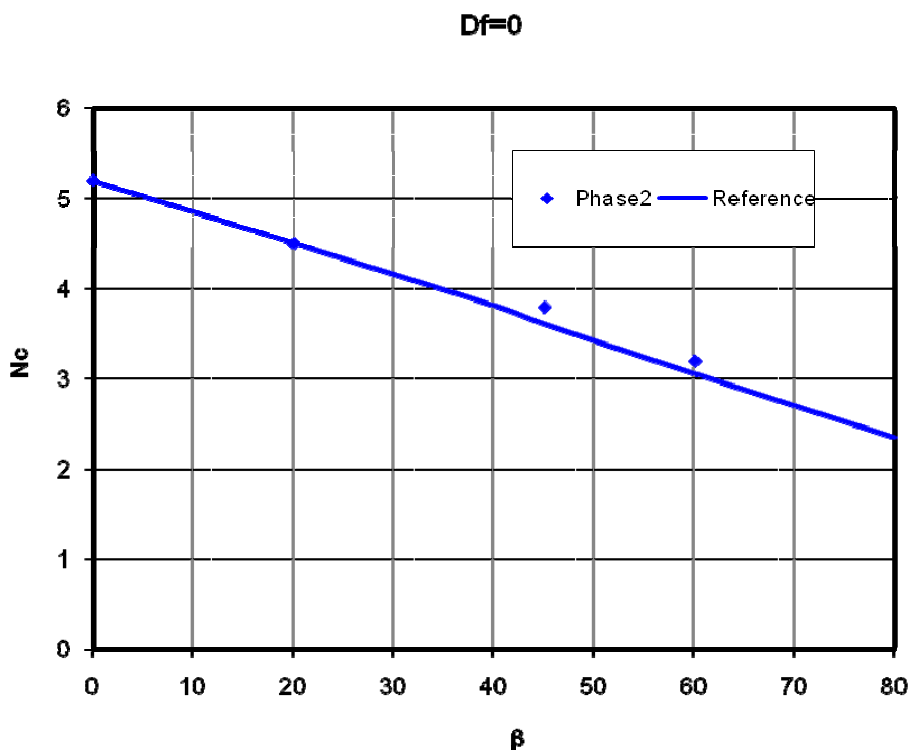
In the present problem we have not considered the effect of unit weight of soil on the bearing capacity of soil, i.e. the effect of γ is neglected, and thus, the bearing capacity factors so obtained correspond to $N_s = 0$.

23.3 Phase² Model

To determine the bearing capacity of the slope with footing on it, a multi-staged Phase² model is prepared in which the load stress over the footing is increased gradually until failure. The slope angle (β) is varied through out the subsequent cases and the bearing capacity factor (N_c) is determined using eq.1 once the ultimate bearing capacity (q_u) is known. Each case discussed below is sub-divided into two cases, one in which the footing is resting on the ground surface, and the other in which the footing is embedded into the soil to a depth equal to the width of footing i.e. $D_f = B = 1\text{m}$.

23.4 Phase² Results

Beta	Ns=0; Df=0			Ns=0; Df=B		
	Qu	Nc (Phase2)	Nc (Given)	Qu	Nc (Phase2)	Nc (Given)
0	260	5.2	5.19	355	7.1	7.02
20	225	4.5	4.50	300	6.1	6.51
45	190	3.8	3.61	240	4.8	5.17
60	160	3.2	3.06	200	4.0	4.22
40			3.81			5.47
80			2.35			2.80



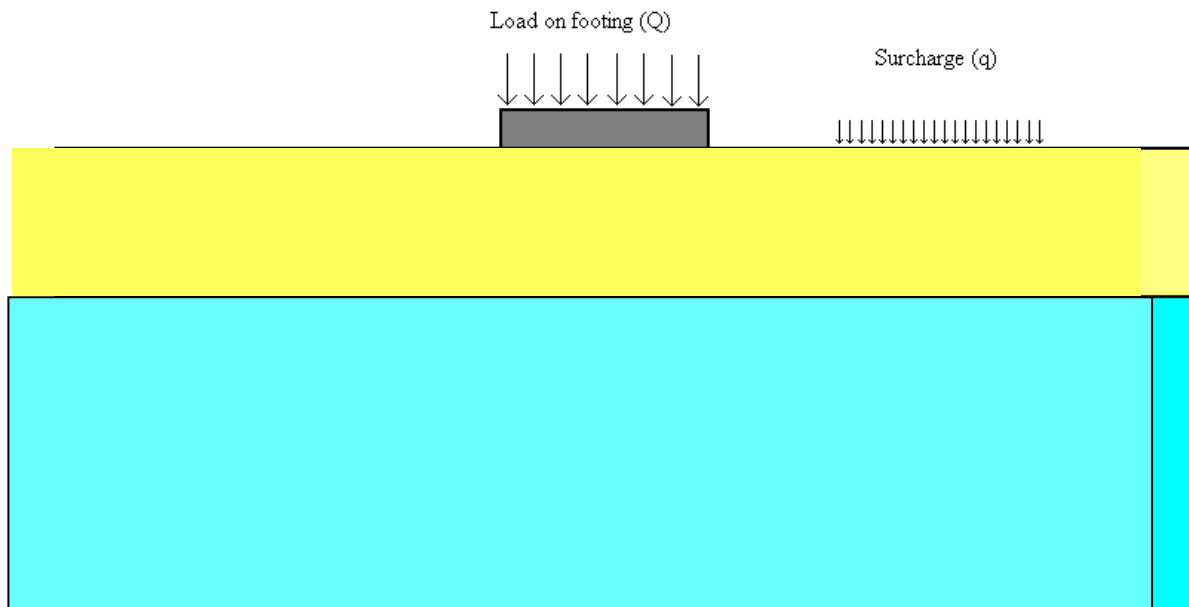
23.5 Reference

1. Principles of Foundation Engineering (sixth edition) by Braja M. Das.

24 Bearing capacity of Undrained Footing on Clayey Soil

24.1 Problem Description

The problem involves determination of bearing capacity of a flexible footing resting on a clayey stratum. The stratum is homogeneous in case 1 and layered in case 2 and 3.



24.2 Analytical solution

The bearing capacity of a shallow strip footing on a clay layer can be written in the form

$$q_u = c_u N_c + q \quad (1)$$

where N_c is a bearing capacity factor and q is a surcharge. For a surface strip footing without a surcharge, this equation reduces to

$$q_u = c_u N_c \quad (2)$$

Note that the ultimate bearing capacity for undrained loading of a footing is independent of the soil unit weight. This follows from the fact that the undrained strength is assumed to be independent of the mean normal stress.

For the case of a layered soil profile, it is convenient to rewrite equation (2) in the form

$$N_c^* = \frac{q_u}{c_{u1}} \quad (3)$$

Where c_{u1} is the undrained shear strength of the top layer, and N_c^* is the modified bearing capacity factor which is a function of both H/B and c_{u1}/c_{u2} . The value of N_c^* is computed using the result from both upper and lower bound analyses for each ratio of H/B and c_{u1}/c_{u2} . For a homogeneous profile where $c_{u1}=c_{u2}$, N_c^* equals the well-known Prantl's Wedge solution of $(2+\pi)$.

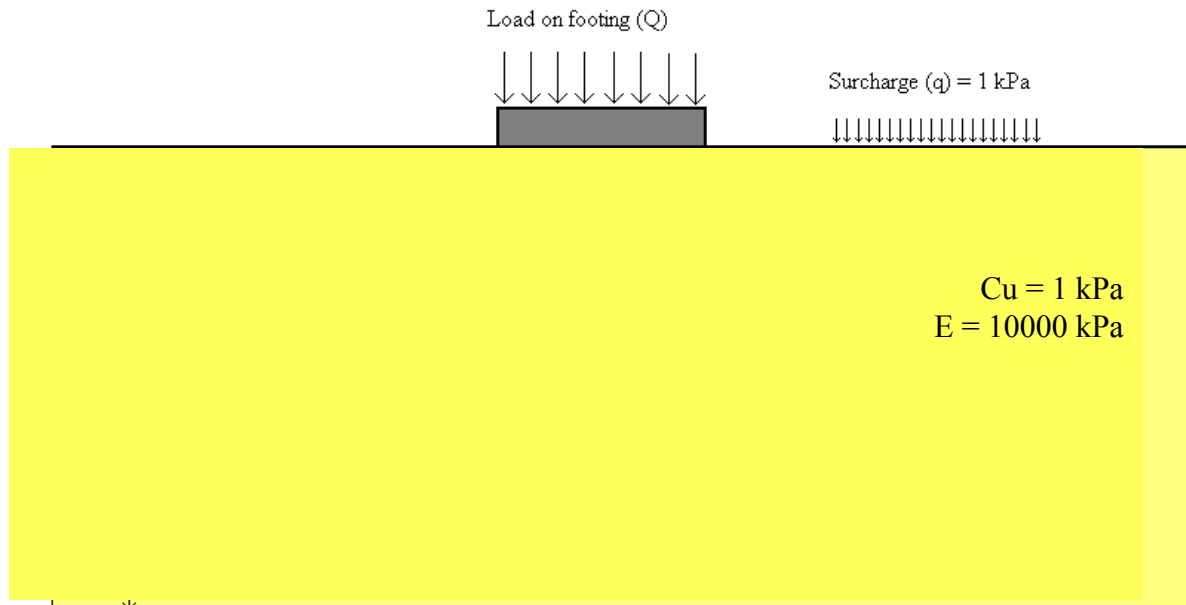
The lower bound solution is obtained by modeling a statically admissible stress field using finite elements with stress nodal variables, where stress discontinuities can occur at the interface between adjacent elements. Application of the stress boundary conditions, equilibrium equations and yield criterion leads to an expression of the collapse load which is maximized subjected to a set of linear constraint on the stresses.

An upper bound on the exact collapse load can be obtained by modeling a kinematically admissible velocity field. To be kinematically admissible, such a velocity field must satisfy the set of constraints imposed by compatibility, velocity boundary conditions and the flow rule. By prescribing a set of the velocities along a specified boundary segment, we can equate the power dissipated internally, due to plastic yielding within the soil mass and sliding of the velocity discontinuities, with the power dissipated by the external loads to yield a strict upper bound on the true limit load.

24.3 Phase² model

To determine the bearing capacity of clayey strata a multi-staged Phase² model is prepared in which the load stress over the footing is increased gradually until failure. Maximum displacement is recorded for each load stress value, and a graph is plotted between the stress and displacement values for each case. The symmetry of the problem is used while preparing the Phase² model. Since we have recorded displacement values to determine the failure of the strata so the results obtained from Phase² (for multilayer cases) are more likely to match the upper bound analytical results, but however the obtained results are compared with the mean results for each case.

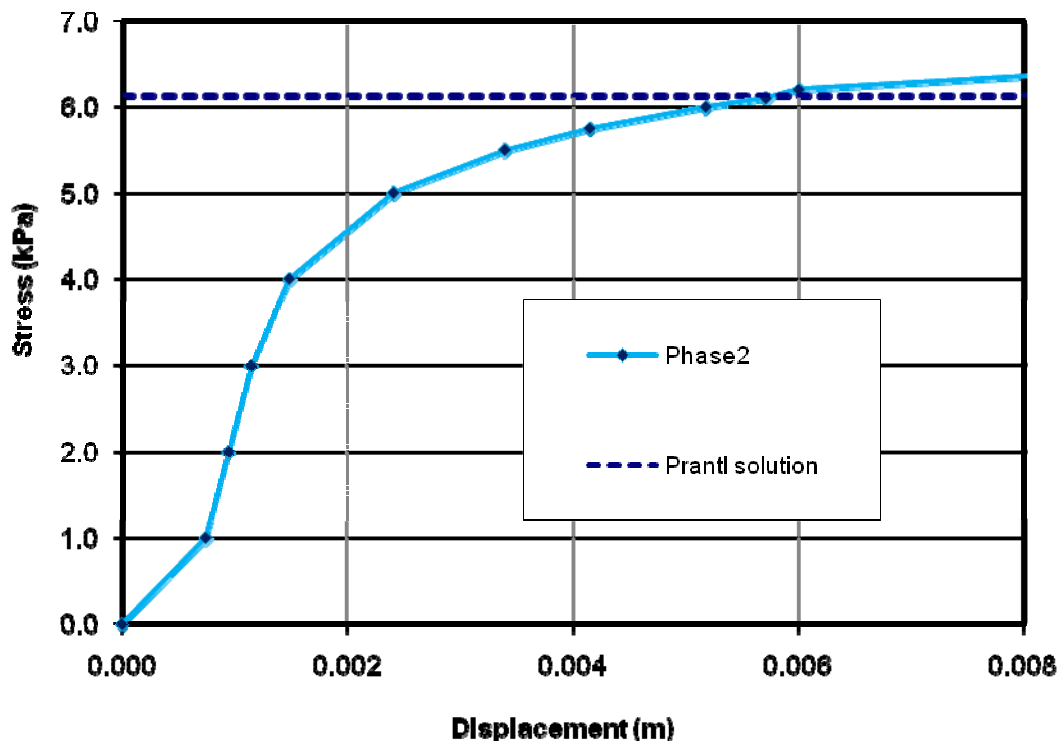
Case 1:



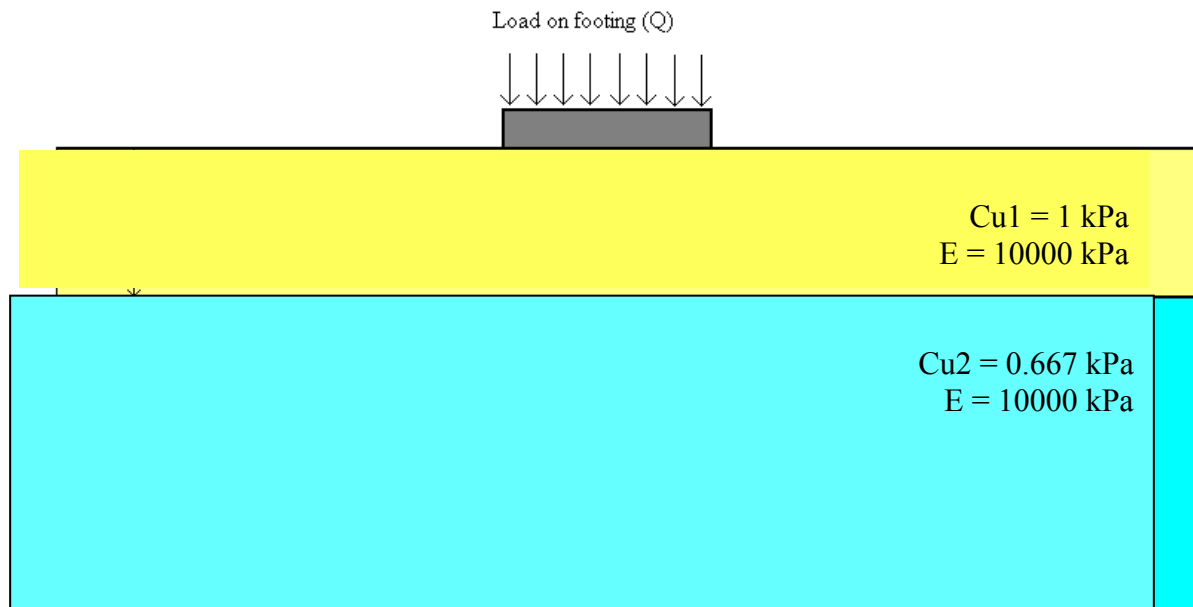
Prantl solution $N_c = 2 + \pi$

Using eq (1): $q_u = 2 + \pi + 1 = 6.14 \text{ kN/m}^2$

PHASE² SOLUTION: $q_u = 6.15 \text{ kN/m}^2$

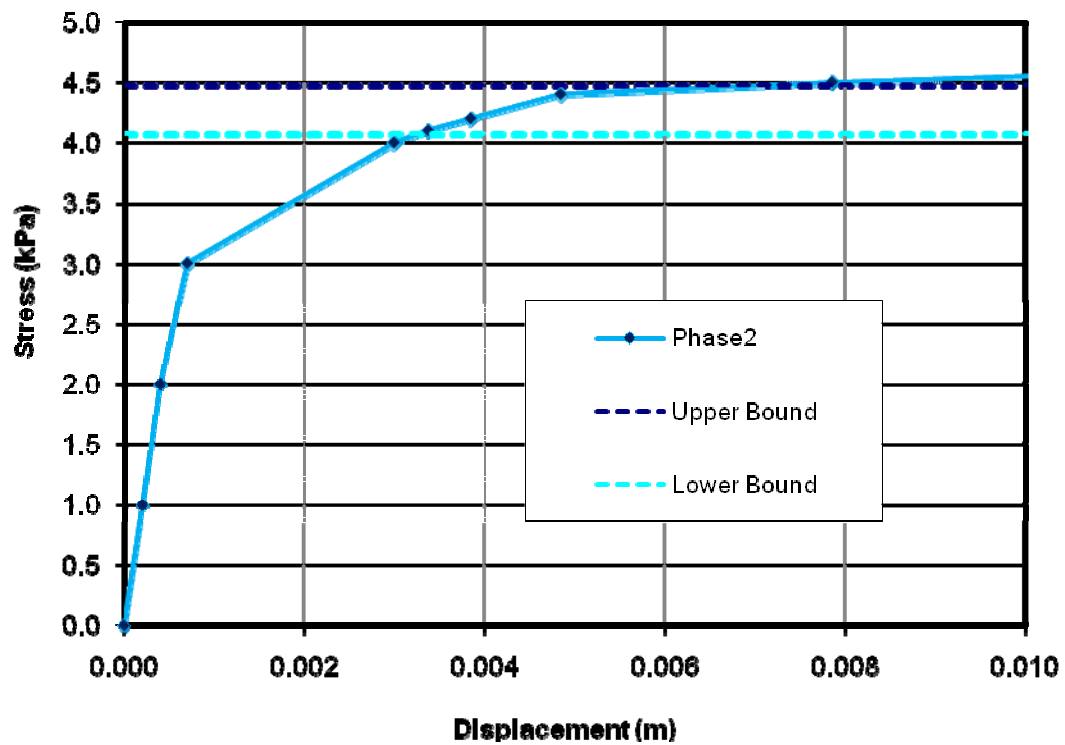


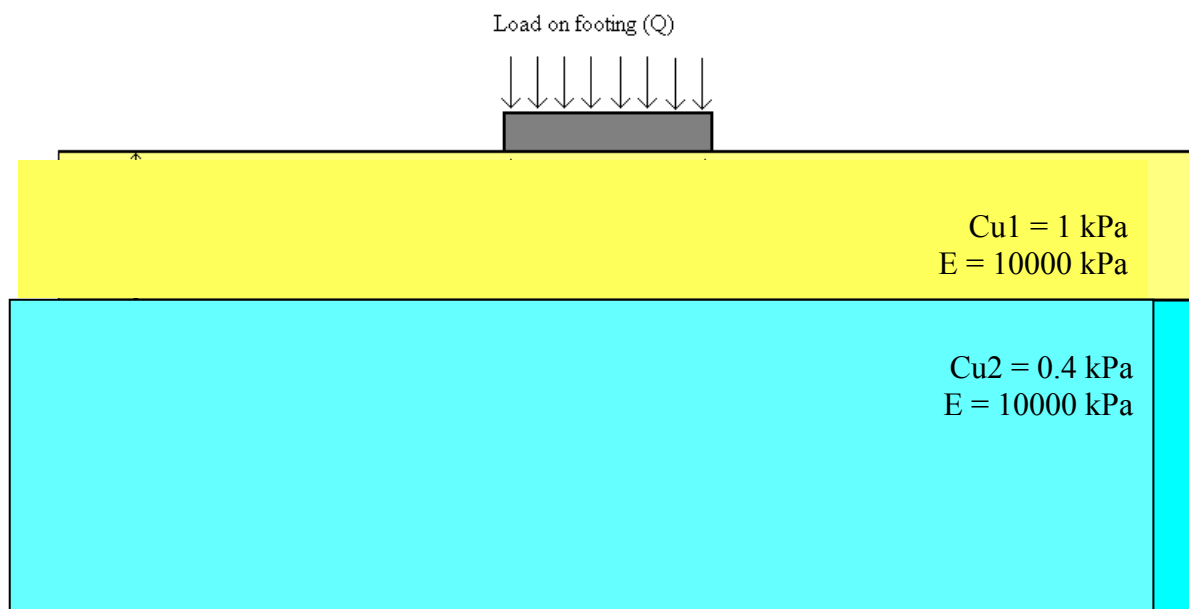
Case 2:



Upper bound = 4.48 kN/m², Lower bound = 4.07 kN/m², mean=4.275 kN/m²

PHASE² SOLUTION: $q_u=4.40\text{kN/m}^2$

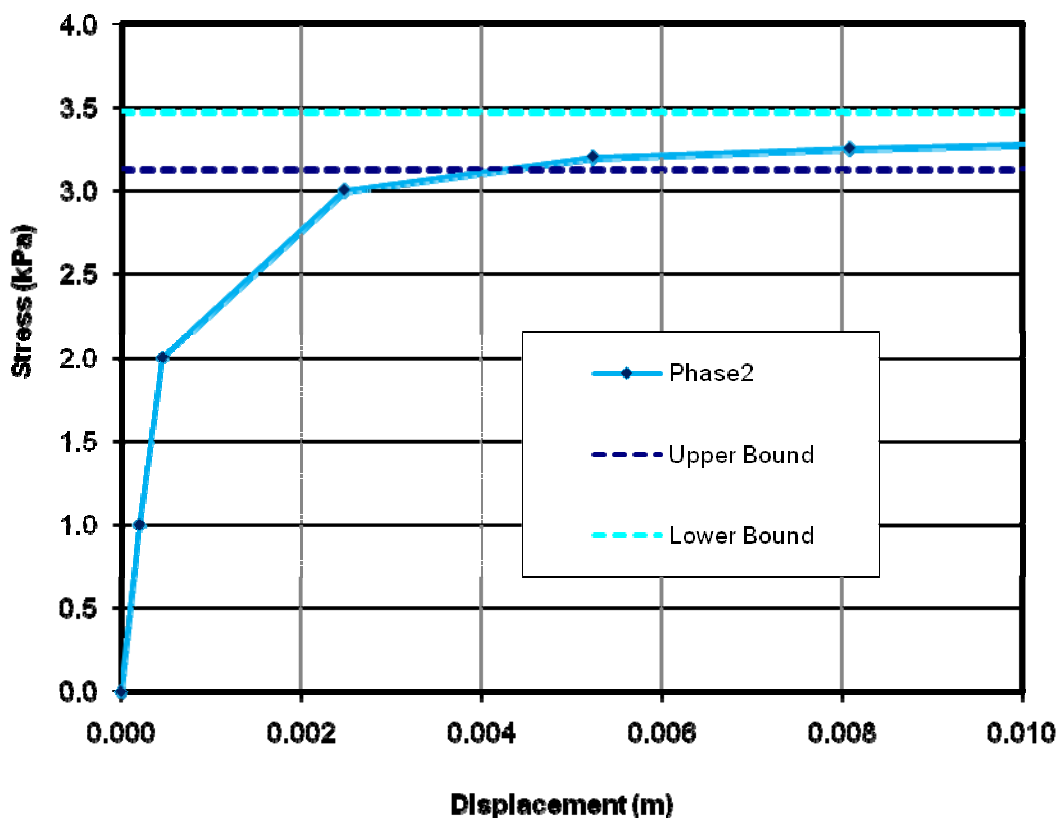


Case 3:

Upper bound = 3.47 kN/m², Lower bound = 3.13 kN/m², mean=3.30 kN/m²

PHASE² SOLUTION:

$$q_u = 3.40 \text{ kN/m}^2$$



24.4 References

1. Chen (2007), *Limit Analysis and Soil Plasticity*, J. Ross Publishing.
2. Merifield, R.S, Sloan, S.W., Yu, H.S. (1999) Rigorous plasticity solutions for the bearing capacity of two-layered clays. *Geotechnique*, Vol. 49, No. 4.

25 Passive Load Bearing Capacity of a Simple Retaining Wall

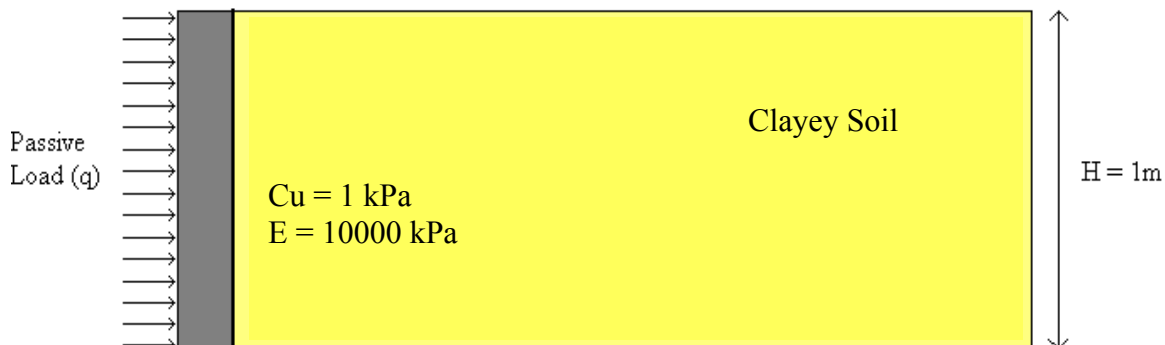
25.1 Problem Description

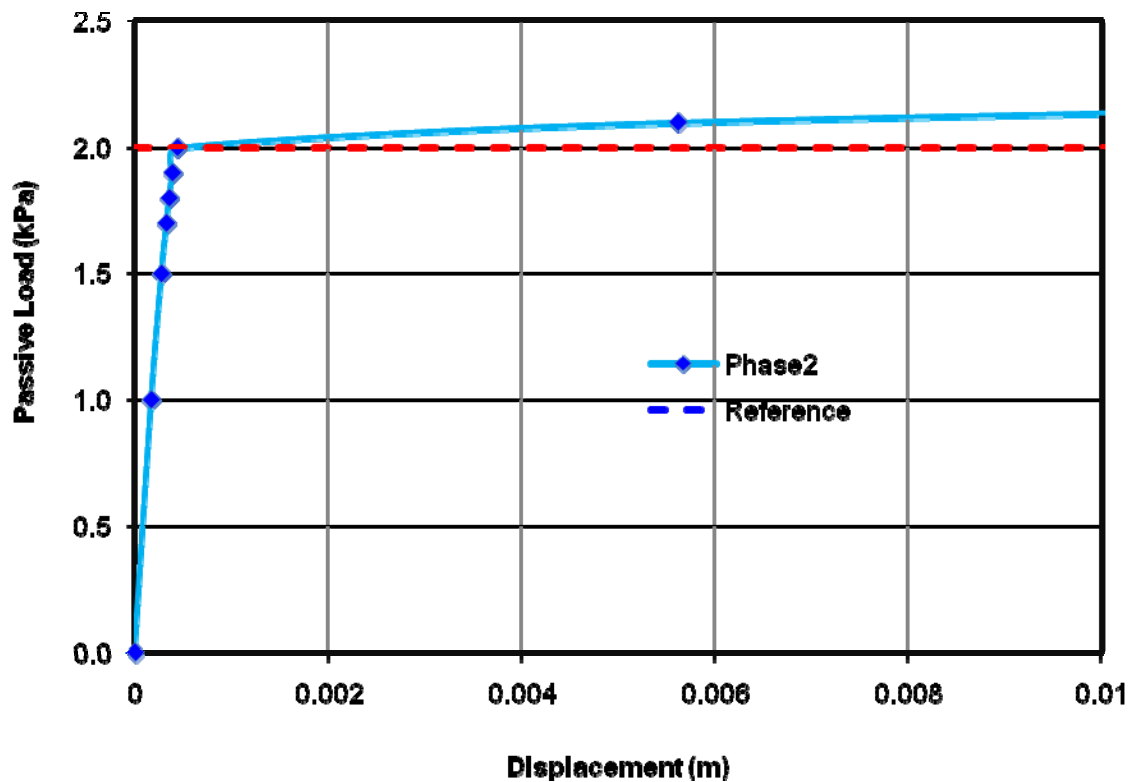
The problem involves determination of the passive load capacity of a simple retaining wall. The soil being retained is a clayey mass. The height of the wall is kept constant at 1 m while the various strength parameters of retained clay is varied in each cases, and the results so obtained from Phase² are compared with the analytical ones.

25.2 Phase² model

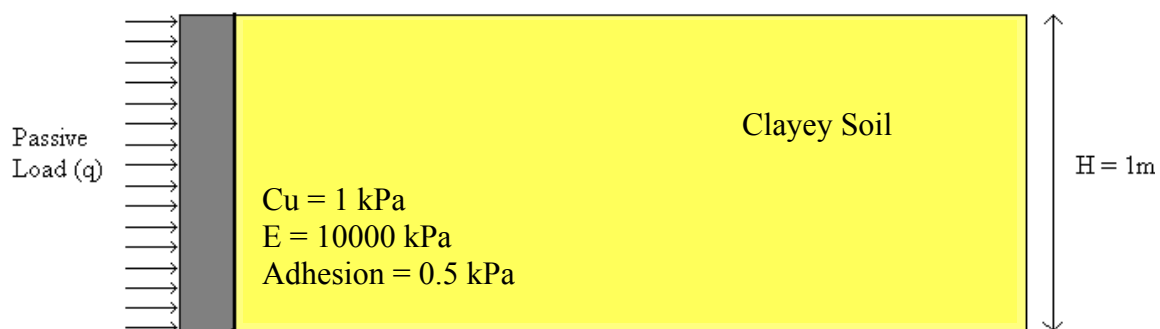
To determine the passive load retaining capacity of retaining wall a multi-staged Phase² model is prepared in which the passive load over the wall is increased gradually until failure. Maximum displacement is recorded for each load value, and a graph is plotted between the load and displacement values for each case.

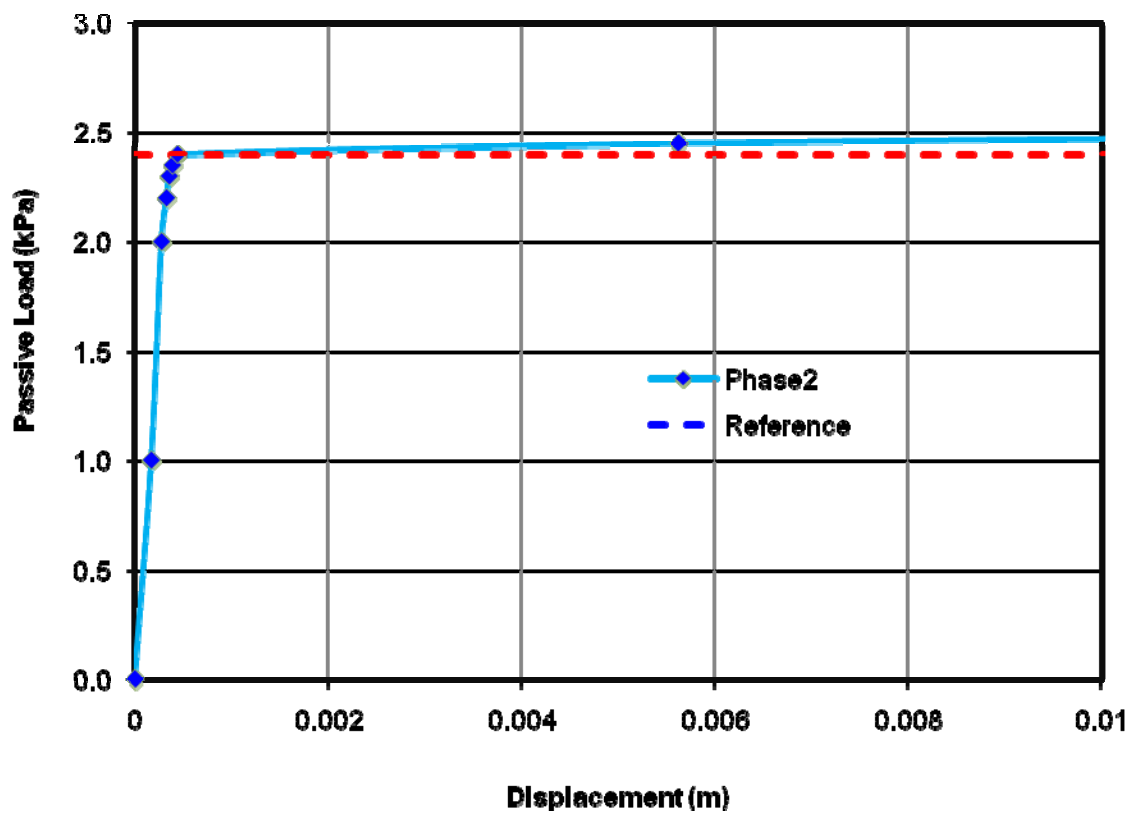
Case 1:



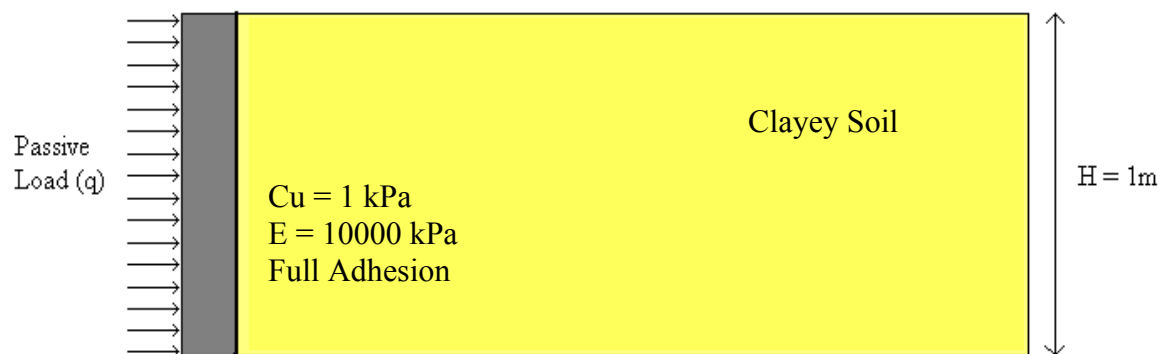


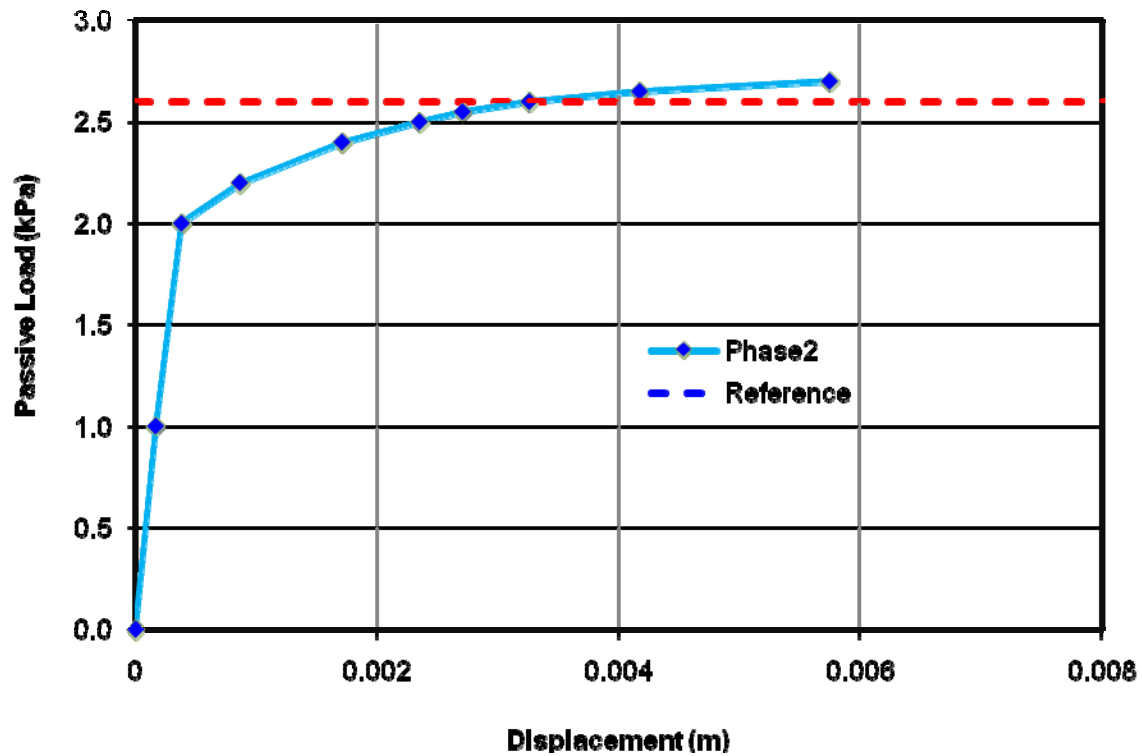
Case 2:





Case 3:





Case	Phase2	Reference
1	2.00	2.00
2	2.40	2.40
3	2.65	2.60

25.3 References

1. Chen (2007). Limit Analysis and Soil Plasticity, J. Ross Publishing.