Chapter 2

Introduction to Engineering Calculations

2.1 Unit conversion

(a)

From Table A.9 (Appendix A): 1 cP = 10^{-3} kg m^{-1} s^{-1}

1 m = 100 cm

Therefore:

$$1.5\times10^{-6}\ \text{cP} = 1.5\times10^{-6}\ \text{cP}\,.\left|\frac{10^{-3}\ \text{kg m}^{-1}\ \text{s}^{-1}}{1\ \text{cP}}\right|.\left|\frac{1\ \text{m}}{100\ \text{cm}}\right| = 1.5\times10^{-11}\ \text{kg s}^{-1}\ \text{cm}^{-1}$$

Answer: 1.5 × 10^{-11} kg s^{-1} cm^{-1}

(b)

From Table A.8 (Appendix A): 1 hp (British) = 42.41 Btu min^{-1}

Therefore:

$$0.122\ \text{hp} = 0.122\ \text{hp}\,.\left|\frac{42.41\ \text{Btu min}^{-1}}{1\ \text{hp}}\right| = 5.17\ \text{Btu min}^{-1}$$

Answer: 5.17 Btu min^{-1}

(c)

1 min = 60 s

rpm means revolutions per minute. As revolutions is a non-dimensional quantity (Section 2.1.2), the units of rpm are min^{-1}. Therefore:

$$10{,}000\ \text{min}^{-1} = 10{,}000\ \text{min}^{-1}\,.\left|\frac{1\ \text{min}}{60\ \text{s}}\right| = 167\ \text{s}^{-1}$$

Answer: 167 s^{-1}

(d)

From Table A.8 (Appendix A): 1 W = 1 J s^{-1}

From Table A.7 (Appendix A): 1 J = 9.869 × 10^{-3} l atm

From Table A.1 (Appendix A): 1 m = 3.281 ft

1 min = 60 s

As explained in Section 2.4.6, $°C^{-1}$ is the same as K^{-1}. Therefore:

$$4335\ \text{W m}^{-2}\ °\text{C}^{-1} = 4335\ \text{W m}^{-2}\ °\text{C}^{-1}\,.\left|\frac{1\ \text{J s}^{-1}}{1\ \text{W}}\right|.\left|\frac{9.869\times10^{-3}\ \text{l atm}}{1\ \text{J}}\right|.\left|\frac{60\ \text{s}}{1\ \text{min}}\right|.\left|\frac{1\ \text{m}}{3.281\ \text{ft}}\right|^{2}$$

$$= 238.45\ \text{l atm min}^{-1}\text{ft}^{-2}\ \text{K}^{-1}$$

Answer: 238 l atm min^{-1} ft^{-2} K^{-1}

2.2 Unit conversion

(a)

From Table A.7 (Appendix A): 1 Btu = 0.2520 kcal

From Table A.3 (Appendix A): 1 lb = 453.6 g

Therefore:

$$345\ \text{Btu lb}^{-1} = 345\ \text{Btu lb}^{-1} . \left|\frac{0.2520\ \text{kcal}}{1\ \text{Btu}}\right| . \left|\frac{1\ \text{lb}}{453.6\ \text{g}}\right| = 0.192\ \text{kcal g}^{-1}$$

Answer: 0.192 kcal g^{-1}

(b)

From Table A.5 (Appendix A): 1 mmHg = 1.316×10^{-3} atm

From Table A.1 (Appendix A): 1 ft = 0.3048 m

From Table A.7 (Appendix A): 1 l atm = 9.604×10^{-2} Btu

From Table A.8 (Appendix A): 1 Btu min^{-1} = 2.391×10^{-2} metric horsepower

1 m = 100 cm

1 l = 1000 cm^3

1 h = 60 min

Therefore:

$$670\ \text{mmHg ft}^3 = 670\ \text{mmHg ft}^3 . \left|\frac{1.316\times10^{-3}\ \text{atm}}{1\ \text{mmHg}}\right| . \left|\frac{9.604\times10^{-2}\ \text{Btu}}{1\ \text{l atm}}\right| . \left|\frac{0.3048\ \text{m}}{1\ \text{ft}}\right|^3 . \left|\frac{100\ \text{cm}}{1\ \text{m}}\right|^3 .$$

$$\left|\frac{1\ \text{l}}{1000\ \text{cm}^3}\right| . \left|\frac{2.391\times10^{-2}\ \text{metric horsepower}}{1\ \text{Btu min}^{-1}}\right| . \left|\frac{1\ \text{h}}{60\ \text{min}}\right|$$

$$= 9.56\times10^{-4}\ \text{metric horsepower h}$$

Answer: 9.56×10^{-4} metric horsepower h

(c)

From Table A.7 (Appendix A): 1 kcal = 4.187×10^3 J

1 kcal = 1000 cal

1 kg = 1000 g

As explained in Section 2.4.6, $°C^{-1}$ is the same as K^{-1}. Therefore:

$$0.554\ \text{cal g}^{-1}\ °\text{C}^{-1} = 0.554\ \text{cal g}^{-1}\ °\text{C}^{-1} . \left|\frac{1\ \text{kcal}}{1000\ \text{cal}}\right| . \left|\frac{4.187\times10^3\ \text{J}}{1\ \text{kcal}}\right| . \left|\frac{1000\ \text{g}}{1\ \text{kg}}\right|$$

$$= 2319.6\ \text{J kg}^{-1}\ \text{K}^{-1}$$

Answer: 2320 J kg^{-1} K^{-1}

(d)

From Table A.2 (Appendix A): 1 m^3 = 10^3 l

1 kg = 1000 g

Therefore:

$$10^3\ \text{g l}^{-1} = 10^3\ \text{g l}^{-1} . \left| \frac{1\ \text{kg}}{1000\ \text{g}} \right| . \left| \frac{10^3\ \text{l}}{1\ \text{m}^3} \right| = 10^3\ \text{kg m}^{-3}$$

Answer: 10^3 kg m^{-3}

2.3 Unit conversion

(a)

From Table A.2 (Appendix A): 1 m^3 = 10^3 l

1 g = 10^6 μg

1 l = 1000 ml

Therefore:

$$10^6\ \mu\text{g ml}^{-1} = 10^6\ \mu\text{g ml}^{-1} . \left| \frac{1\ \text{g}}{10^6\ \mu\text{g}} \right| . \left| \frac{1000\ \text{ml}}{1\ \text{l}} \right| . \left| \frac{10^3\ \text{l}}{1\ \text{m}^3} \right| = 10^6\ \text{g m}^{-3}$$

Answer: 10^6 g m^{-3}

(b)

From Table A.9 (Appendix A): 1 cP = 10^{-3} Pa s

1 Pa s = 1000 mPa s

Therefore:

$$3.2\ \text{cP} = 3.2\ \text{cP} . \left| \frac{10^{-3}\ \text{Pa s}}{1\ \text{cP}} \right| . \left| \frac{1000\ \text{mPa s}}{1\ \text{Pa s}} \right| = 3.2\ \text{mPa s}$$

Answer: 3.2 mPa s

(c)

From Table A.7 (Appendix A): 1 Btu = 1.055 × 10^3 J

From Table A.8 (Appendix A): 1 W = 1 J s^{-1}

From Table A.1 (Appendix A): 1 ft = 0.3048 m

1 h = 3600 s

From Section 2.4.6, a temperature difference of 1 K corresponds to a temperature difference of 1.8 °F. Therefore:

$$150\ \text{Btu h}^{-1}\ \text{ft}^{-2}\ (^\circ\text{F ft}^{-1})^{-1} = 150\ \text{Btu h}^{-1}\ \text{ft}^{-1}\ ^\circ\text{F}^{-1} . \left| \frac{1.055 \times 10^3\ \text{J}}{1\ \text{Btu}} \right| . \left| \frac{1\ \text{h}}{3600\ \text{s}} \right| . \left| \frac{1\ \text{ft}}{0.3048\ \text{m}} \right| . \left| \frac{1.8\ ^\circ\text{F}}{1\ \text{K}} \right| . \left| \frac{1\ \text{W}}{1\ \text{J s}^{-1}} \right|$$

$$= 259.6\ \text{W m}^{-1}\ \text{K}^{-1}$$

Answer: 260 W m^{-1} K^{-1}

(d)

1 h = 3600 s

rph means revolutions per hour. As revolutions is a non-dimensional quantity (Section 2.1.2), the units of rph are h^{-1}. Therefore:

$$66 \text{ rph} = 66 \text{ h}^{-1} . \left| \frac{1 \text{ h}}{3600 \text{ s}} \right| = 1.83 \times 10^{-2} \text{ s}^{-1}$$

Answer: 1.83×10^{-2} s^{-1}

2.4 Unit conversion and calculation

Convert to units of kg, m, s.

From Table A.2 (Appendix A): 1 ft^3 = 2.832×10^{-2} m^3

From Table A.2 (Appendix A): 1 m^3 = 10^3 l

From Table A.3 (Appendix A): 1 lb = 0.4536 kg

From Table A.8 (Appendix A): 1 metric horsepower = 7.355×10^2 kg m^2 s^{-3}

From Table A.1 (Appendix A): 1 m = 39.37 in.

Therefore:

$$t_m = 5.9 \, (2.3 \text{ m})^{2/3} \left(\frac{65 \text{ lb}}{\text{ft}^3} \frac{10{,}000 \text{ l} . \left| \frac{1 \text{ m}^3}{10^3 \text{ l}} \right|}{0.70 \text{ metric hp} . \left| \frac{7.355 \times 10^2 \text{ kg m}^2 \text{ s}^{-3}}{1 \text{ metric hp}} \right|} . \left| \frac{1 \text{ ft}^3}{2.832 \times 10^{-2} \text{ m}^3} \right| . \left| \frac{0.4536 \text{ kg}}{1 \text{ lb}} \right| \right)^{1/3} \left(\frac{2.3 \text{ m}}{45 \text{ in.}} . \left| \frac{39.37 \text{ in.}}{1 \text{ m}} \right| \right)^{1/3}$$

$$= 5.9 \, (2.3)^{2/3} \text{ m}^{2/3} \, (2.724 \text{ m}^{-2/3} \text{ s}) \, 1.262$$

$$= 35.3 \text{ s}$$

Answer: 35 s

2.5 Unit conversion and dimensionless numbers

Case 1

Convert to units of kg, m, s.

From Table A.3 (Appendix A): 1 lb = 0.4536 kg

From Table A.2 (Appendix A): 1 ft^3 = 2.832×10^{-2} m^3

From Table A.9 (Appendix A): 1 cP = 10^{-3} kg m^{-1} s^{-1}

1 m = 100 cm = 1000 mm

Using Eq. (2.1):

$$Re = \frac{\left(2\ \text{mm} \cdot \left|\frac{1\ \text{m}}{1000\ \text{mm}}\right|\right)\left(3\ \text{cm s}^{-1} \cdot \left|\frac{1\ \text{m}}{100\ \text{cm}}\right|\right)\left(25\ \text{lb ft}^{-3} \cdot \left|\frac{0.4536\ \text{kg}}{1\ \text{lb}}\right| \cdot \left|\frac{1\ \text{ft}^3}{2.832\times10^{-2}\ \text{m}^3}\right|\right)}{10^{-6}\ \text{cP} \cdot \left|\frac{10^{-3}\ \text{kg m}^{-1}\ \text{s}^{-1}}{1\ \text{cP}}\right|}$$

$$= 2.4\times10^{7}$$

Answer: 2.4×10^7

Case 2

Convert to units of kg, m, s.

From Table A.1 (Appendix A): 1 in. = 2.54×10^{-2} m

From Table A.9 (Appendix A): 1 lb ft^{-1} h^{-1} = 4.134×10^{-4} kg m^{-1} s^{-1}

1 h = 3600 s

Using Eq. (2.1):

$$Re = \frac{\left(1\ \text{in.} \cdot \left|\frac{2.54\times10^{-2}\ \text{m}}{1\ \text{in.}}\right|\right)(1\ \text{m s}^{-1})\ (12.5\ \text{kg m}^{-3})}{0.14\times10^{-4}\ \text{lb}_m\ \text{s}^{-1}\ \text{ft}^{-1} \cdot \left|\frac{4.134\times10^{-4}\ \text{kg m}^{-1}\ \text{s}^{-1}}{1\ \text{lb ft}^{-1}\ \text{h}^{-1}}\right| \cdot \left|\frac{3600\ \text{s}}{\text{h}}\right|} = 1.5\times10^{4}$$

Answer: 1.5×10^4

2.6 Property data

Values were obtained from *Perry's Chemical Engineers' Handbook*, 8th edition, McGraw-Hill. Other sources may also be used.

(a)

The viscosity of ethanol can be evaluated as a function of temperature using the coefficients listed in Table 2-313 of *Perry's Chemical Engineers' Handbook*. Converting from °C to K using Eq. (2.27), T(K) = 40 + 273.15 = 313.15 K. Using the equation provided, the viscosity at 40°C is calculated as 8.18×10^{-4} Pa s. Alternatively, using the nomograph of Figure 2-32 in *Perry's Chemical Engineers' Handbook*, the viscosity of ethanol at 40°C is estimated at about 0.82 cP, which is consistent with the previous result.

Answer: 0.82 cP

(b)

From Table 2-325 in *Perry's Chemical Engineers' Handbook*, at 25°C and 1 atm pressure, the diffusivity of oxygen in water is 2.5×10^{-5} cm^2 s^{-1}.

Answer: 2.5×10^{-5} cm^2 s^{-1}

(c)

The thermal conductivity of borosilicate-type glass is listed in Table 2-326 of *Perry's Chemical Engineers' Handbook*. For temperatures between 30°C and 75°C, the thermal conductivity is 0.63 Btu h^{-1} ft^{-2} (°F/ft)$^{-1}$.

Answer: 0.63 Btu h^{-1} ft^{-2} (°F/ft)$^{-1}$

(d)

The density of 100% acetic acid at 20°C is listed in Table 2-109 of *Perry's Chemical Engineers' Handbook* as 1.0498 g cm^{-3}.

Answer: 1.0498 g cm^{-3}

(e)

The specific heat capacity of liquid water at 80°C can be calculated using the values listed in Table 2-305 of *Perry's Chemical Engineers' Handbook*. Converting from °C to K using Eq. (2.27), T(K) = 80 + 273.15 = 353.15 K. From the table, C_p = 0.075567 kJ mol^{-1} K^{-1} at 350 K and C_p = 0.075708 kJ mol^{-1} K^{-1} at 360 K. Assuming that C_p changes linearly with temperature between 350 K and 360 K, the value at 353.15 K can be interpolated as:

$$C_p(353.15\text{ K}) = 0.075567\text{ kJ mol}^{-1}\text{ K}^{-1} + \frac{(353.15-350)\text{ K}}{(360-350)\text{ K}}(0.075708-0.075567)\text{ kJ mol}^{-1}\text{ K}^{-1}$$

$$= 0.075611\text{ kJ mol}^{-1}\text{ K}^{-1}$$

Answer: 0.07561 kJ mol^{-1} K^{-1}

2.7 Dimensionless groups and property data

From *Perry's Chemical Engineers' Handbook*, the diffusivity of oxygen in water at 25°C and 1 atm pressure is 2.5 × 10^{-5} cm^2 s^{-1}. Assuming this is the same at 28°C, $\mathscr{D}$ = 2.5 × 10^{-5} cm^2 s^{-1}. As fermentation medium is mostly water, it is reasonable to assume that the density of liquid in the fermenter is the same as that of water. Unless the culture produces a highly viscous extracellular product such as gum, it is also reasonable to assume that the viscosity of liquid in the fermenter is the same as that of water. From *Perry's Chemical Engineers' Handbook*, the density of water at 28°C ρ_L = 0.9962652 g cm^{-3} and the viscosity of water at 28°C μ_L = 0.87 cP. The density of oxygen at 28°C and 1 atm pressure can be calculated using the ideal gas law. As molar density is the same as n/V, from Eq. (2.35):

$$\rho_G = \frac{n}{V} = \frac{p}{RT}$$

Temperature in the ideal gas equation is absolute temperature; therefore, from Eq. (2.27):

$$T(\text{K}) = (28 + 273.15)\text{ K} = 301.15\text{ K}$$

From Appendix B, R = 82.057 cm^3 atm K^{-1} $gmol^{-1}$. Substituting parameter values into the above equation for gas density gives:

$$\rho_G = \frac{1\text{ atm}}{(82.057\text{ cm}^3\text{ atm K}^{-1}\text{ gmol}^{-1})\,(301.15\text{ K})} = 4.05\times10^{-5}\text{ gmol cm}^{-3}$$

Using the atomic weights in Table C.1 (Appendix C), the molecular weight of oxygen is 32.0. Converting the result for ρ_G to mass terms:

$$\rho_G = 4.05\times10^{-5}\text{ gmol cm}^{-3}\,.\left|\frac{32.0\text{ g}}{1\text{ gmol}}\right| = 1.30\times10^{-3}\text{ g cm}^{-3}$$

From Table A.9 (Appendix A), 1 cP = 10^{-2} g cm^{-1} s^{-1}. From Eq. (2.17), g = 980.66 cm s^{-2}. 1 cm = 10 mm. The parameter values and conversion factors can now be used to calculate the dimensionless groups in the equation for the Sherwood number.

$$Gr = \frac{(2\text{ mm})^3\,(1.30\times10^{-3}\text{ g cm}^{-3})(0.9962652-1.30\times10^{-3})\text{ g cm}^{-3}\,(980.66\text{ cm s}^{-2})\,.\left|\dfrac{1\text{ cm}}{10\text{ mm}}\right|^3}{(0.87\text{ cP})^2\,.\left|\dfrac{10^{-2}\text{ g cm}^{-1}\text{ s}^{-1}}{1\text{ cP}}\right|^2}$$

$$= 134$$

Similarly for the Schmidt number:

$$Sc = \frac{0.87\text{ cP}\,.\left|\dfrac{10^{-2}\text{ g cm}^{-1}\text{ s}^{-1}}{1\text{ cP}}\right|}{(0.9962652\text{ g cm}^{-3})\,(2.5\times10^{-5}\text{ cm}^2\text{ s}^{-1})} = 349$$

Therefore:

$$Sh = 0.31\,(134)^{1/3}\,(349)^{1/3} = 11.2$$

From the equation for *Sh*:

$$k_L = \frac{Sh\,\mathscr{D}}{D_b} = \frac{(11.2)(2.5\times10^{-5}\text{ cm}^2\text{ s}^{-1})}{2\text{ mm}\,.\left|\dfrac{1\text{ cm}}{10\text{ mm}}\right|} = 1.40\times10^{-3}\text{ cm s}^{-1}$$

Answer: 1.40×10^{-3} cm s^{-1}

2.8 Dimensionless numbers and dimensional homogeneity

First, evaluate the units of the groups ($C_p\mu/k$) and (DG/μ):

$$\text{Units of}\left(\frac{C_p\mu}{k}\right) = \frac{(\text{Btu lb}^{-1}\,°\text{F}^{-1})\text{ lb h}^{-1}\text{ ft}^{-1}}{\text{Btu h}^{-1}\text{ ft}^{-2}\,(°\text{F ft}^{-1})^{-1}} = 1$$

$$\text{Units of}\left(\frac{DG}{\mu}\right) = \frac{(\text{ft})\text{ lb h}^{-1}\text{ ft}^{-2}}{\text{lb h}^{-1}\text{ ft}^{-1}} = 1$$

Therefore, these groups are dimensionless. For the equation to be dimensionally homogeneous, (h/C_pG) must also be dimensionless; the units of *h* must therefore cancel the units of C_pG.

$$\text{Units of } h = \text{units of } C_pG = (\text{Btu lb}^{-1}\,°\text{F}^{-1})\,(\text{lb h}^{-1}\text{ ft}^{-2}) = \text{Btu }°\text{F}^{-1}\text{ h}^{-1}\text{ ft}^{-2}$$

The dimensions of *h* can be deduced from its units. From Table A.7 (Appendix A), Btu is a unit of energy with dimensions = L^2MT^{-2}. °F is a unit of temperature which, from Table 2.1, has the dimensional symbol Θ. h (hour) is a unit of time with dimension T; ft is a unit of length with dimension L. Therefore:

$$\text{Dimensions of } h = L^2MT^{-2}\Theta^{-1}T^{-1}L^{-2} = MT^{-3}\Theta^{-1}$$

Answer: Units = Btu $°F^{-1}$ h^{-1} ft^{-2}; dimensions = $MT^{-3}\Theta^{-1}$

2.9 Dimensional homogeneity

λ has dimensions L. ε has units W kg^{-1}; therefore, from Tables A.8 and A.3 in Appendix A, the dimensions of ε are $L^2MT^{-3}M^{-1} = L^2T^{-3}$. Substituting this information into the equation for λ, for dimensional homogeneity:

$$L = \left(\frac{(\text{dimensions of } v)^3}{L^2T^{-3}}\right)^{1/4} = \frac{(\text{dimensions of } v)^{3/4}}{L^{1/2}T^{-3/4}}$$

Solving for the dimensions of v:

$$(\text{dimensions of } \nu)^{3/4} = \mathrm{L}^{3/2}\mathrm{T}^{-3/4}$$

or

$$\text{dimensions of } \nu = \mathrm{L}^{2}\mathrm{T}^{-1}$$

Answer: $\mathrm{L}^{2}\mathrm{T}^{-1}$

2.10 Dimensional homogeneity and g_c

The dimensions of D_i (length) = L. From Table 2.2, the dimensions of $P = \mathrm{L}^{2}\mathrm{MT}^{-3}$ and the dimensions of $\rho = \mathrm{L}^{-3}\mathrm{M}$. From Section 2.3, the dimensions of $g = \mathrm{LT}^{-2}$. From Section 2.1.2, the dimensions of rotational speed $N_i = \mathrm{T}^{-1}$. Therefore:

$$\text{Dimensions of } \frac{P\,g}{\rho\,N_i^3\,D_i^5} = \frac{(\mathrm{L}^{2}\mathrm{MT}^{-3})\,(\mathrm{LT}^{-2})}{(\mathrm{L}^{-3}\mathrm{M})\,(\mathrm{T}^{-1})^3\,\mathrm{L}^{5}} = \mathrm{LT}^{-2}$$

As N_P is a dimensionless number, equation (i) is not dimensionally homogeneous and therefore cannot be correct. From Section 2.3, the dimensions of $g_c = 1$. Therefore:

$$\text{Dimensions of } \frac{P\,g_c}{\rho\,N_i^3\,D_i^5} = \frac{(\mathrm{L}^{2}\mathrm{MT}^{-3})\,(1)}{(\mathrm{L}^{-3}\mathrm{M})\,(\mathrm{T}^{-1})^3\,\mathrm{L}^{5}} = 1$$

Equation (ii) is dimensionally homogeneous and therefore likely to be correct.

Answer: (ii)

2.11 Mass and weight

From the definition of density in Section 2.4.1, mass is equal to density multiplied by volume. Therefore:

$$\text{Mass of water} = (10\ \mathrm{ft}^3)(62.4\ \mathrm{lb}_m\ \mathrm{ft}^{-3}) = 624\ \mathrm{lb}_m$$

From Section 2.3, weight is the force with which a body is attracted to the centre of the earth by gravity. According to Newton's law (Section 2.3), this force is the mass of the body multiplied by gravitational acceleration.

(a)

From Eq. (2.18), at sea level and 45° latitude, gravitational acceleration $g = 32.174\ \mathrm{ft\ s}^{-2}$. Therefore:

$$\text{Weight} = 624\ \mathrm{lb}_m\ (32.174\ \mathrm{ft\ s}^{-2}) = 2.008 \times 10^4\ \mathrm{lb}_m\ \mathrm{ft\ s}^{-2}$$

Converting these units to lb_f using Eq. (2.19):

$$\text{Weight} = 2.008\times10^4\ \mathrm{lb}_m\ \mathrm{ft\ s}^{-2}\ .\ \left|\frac{1\ \mathrm{lb}_f}{32.174\ \mathrm{lb}_m\ \mathrm{ft\ s}^{-2}}\right| = 624\ \mathrm{lb}_f$$

Answer: 624 lb_f. *When $g = 32.174\ \mathrm{ft\ s}^{-2}$, lb mass is equal to lb force.*

(b)

From Table A.1 (Appendix A), 1 m = 3.281 ft. Using the same procedure as in **(a)**:

$$\text{Weight} = 624\ \mathrm{lb}_m\ (9.76\ \mathrm{m\ s}^{-2})\ .\ \left|\frac{3.281\ \mathrm{ft}}{1\ \mathrm{m}}\right| = 1.998\times10^4\ \mathrm{lb}_m\ \mathrm{ft\ s}^{-2}$$

Converting to lb_f using Eq. (2.19):

$$\text{Weight} = 1.998 \times 10^4 \text{ lb}_m \text{ ft s}^{-2} \,.\, \left| \frac{1 \text{ lb}_f}{32.174 \text{ lb}_m \text{ ft s}^{-2}} \right| = 621 \text{ lb}_f$$

Answer: 621 lb_f

2.12 Molar units

From the atomic weights in Table C.1 (Appendix C), the molecular weight of NaOH is 40.0.

(a)

From Eq. (2.22):

$$\text{lb-moles NaOH} = \frac{20.0 \text{ lb}}{40.0 \text{ lb lbmol}^{-1}} = 0.50 \text{ lbmol}$$

Answer: 0.50 lbmol

(b)

From Table A.3 (Appendix A), 1 lb = 453.6 g. Therefore:

$$20.0 \text{ lb} = 20.0 \text{ lb} \,.\, \left| \frac{453.6 \text{ g}}{1 \text{ lb}} \right| = 9072 \text{ g}$$

From Eq. (2.21):

$$\text{gram-moles NaOH} = \frac{9072 \text{ g}}{40.0 \text{ g gmol}^{-1}} = 227 \text{ gmol}$$

Answer: 227 gmol

(c)

From Section 2.4.4, 1 kgmol = 1000 gmol. Therefore, from **(b)**:

$$\text{kg-moles NaOH} = 227 \text{ gmol} \,.\, \left| \frac{1 \text{ kgmol}}{1000 \text{ gmol}} \right| = 0.227 \text{ kgmol}$$

Answer: 0.227 kgmol

2.13 Density and specific gravity

(a)

From Section 2.4.1, the density of water at 4°C is 1.0000 g cm^{-3}. Therefore, from Section 2.4.2, for a substance with specific gravity $1.5129^{20°C}_{4°C}$, the density at 20°C is 1.5129 g cm^{-3}.

(i)

1 kg = 1000 g

1 m = 100 cm

Therefore:

$$\text{Density} = 1.5129 \text{ g cm}^{-3} \,.\, \left| \frac{1 \text{ kg}}{1000 \text{ g}} \right| . \left| \frac{100 \text{ cm}}{1 \text{ m}} \right|^3 = 1512.9 \text{ kg m}^{-3}$$

Answer: 1512.9 kg m^{-3}

(ii)

From the atomic weights in Table C.1 (Appendix C), the molecular weight of nitric acid (HNO_3) is 63.0. If the density of HNO_3 at 20°C is 1.5129 g cm^{-3}, in 1 cm^3 HNO_3, from Eq. (2.21):

$$\text{gram-moles} = \frac{1.5129\ \text{g}}{63.0\ \text{g gmol}^{-1}} = 0.024\ \text{gmol}$$

Therefore, the molar density is 0.024 gmol cm^{-3}. From the definition of specific volume in Section 2.4.3:

$$\text{Molar specific volume} = \frac{1}{\text{molar density}} = \frac{1}{0.024\ \text{gmol cm}^{-3}} = 41.67\ \text{cm}^3\ \text{gmol}^{-1}$$

Answer: 41.67 cm^3 $gmol^{-1}$

(b)

(i)

Density is defined as mass per unit volume (Section 2.4.1). Therefore, the mass flow rate is equal to the volumetric flow rate multiplied by the density:

$$\text{Mass flow rate} = (50\ \text{cm}^3\ \text{min}^{-1})(1.6\ \text{g cm}^{-3}) = 80\ \text{g min}^{-1}$$

Answer: 80 g min^{-1}

(ii)

From the atomic weights in Table C.1 (Appendix C), the molecular weight of carbon tetrachloride is 153.8. Using the mass flow rate from **(a)**:

$$\text{Molar flow rate} = 80\ \text{g min}^{-1} . \left| \frac{1\ \text{gmol}}{153.8\ \text{g}} \right| = 0.52\ \text{gmol min}^{-1}$$

Answer: 0.52 gmol min^{-1}

2.14 Molecular weight

From Section 2.4.5, the composition of air is approximately 21% oxygen and 79% nitrogen. For gases at low pressures, this means 21 mol% O_2 and 79 mol% N_2. Therefore, in 1 gmol of air, there are 0.21 gmol O_2 and 0.79 gmol N_2 From the atomic weights in Table C.1 (Appendix C), the molecular weights of O_2 and N_2 are 32.0 and 28.0, respectively. The molecular weight of air is equal to the number of grams in 1 gmol of air:

$$1\ \text{gmol air} = 0.21\ \text{gmol O}_2 . \left| \frac{32.0\ \text{g}}{1\ \text{gmol}} \right| + 0.79\ \text{gmol N}_2 . \left| \frac{28.0\ \text{g}}{1\ \text{gmol}} \right| = 28.8\ \text{g}$$

Answer: 28.8

2.15 Mole fraction

The molecular weights are listed in Table C.7 (Appendix C): water 18.0; ethanol 46.1; methanol 32.0; glycerol 92.1; acetic acid 60.1; benzaldehyde 106.1. In 100 g solution, there are 30 g water, 25 g ethanol, 15 g methanol, 12 g glycerol, 10 g acetic acid, 8 g benzaldehyde, and no other components. Therefore:

$$\text{Moles water} = 30\ \text{g} . \left| \frac{1\ \text{gmol}}{18.0\ \text{g}} \right| = 1.67\ \text{gmol}$$

$$\text{Moles ethanol} = 25\ \text{g} \cdot \left|\frac{1\ \text{gmol}}{46.1\ \text{g}}\right| = 0.54\ \text{gmol}$$

$$\text{Moles methanol} = 15\ \text{g} \cdot \left|\frac{1\ \text{gmol}}{32.0\ \text{g}}\right| = 0.47\ \text{gmol}$$

$$\text{Moles glycerol} = 12\ \text{g} \cdot \left|\frac{1\ \text{gmol}}{92.1\ \text{g}}\right| = 0.13\ \text{gmol}$$

$$\text{Moles acetic acid} = 10\ \text{g} \cdot \left|\frac{1\ \text{gmol}}{60.1\ \text{g}}\right| = 0.17\ \text{gmol}$$

$$\text{Moles benzaldehyde} = 8\ \text{g} \cdot \left|\frac{1\ \text{gmol}}{106.1\ \text{g}}\right| = 0.075\ \text{gmol}$$

The total number of moles is 1.67 + 0.54 + 0.47 + 0.13 + 0.17 + 0.075 = 3.055 gmol. From Eq. (2.23):

$$\text{Mole fraction water} = \frac{1.67}{3.055} = 0.55$$

$$\text{Mole fraction ethanol} = \frac{0.54}{3.055} = 0.18$$

$$\text{Mole fraction methanol} = \frac{0.47}{3.055} = 0.15$$

$$\text{Mole fraction glycerol} = \frac{0.13}{3.055} = 0.043$$

$$\text{Mole fraction acetic acid} = \frac{0.17}{3.055} = 0.056$$

$$\text{Mole fraction benzaldehyde} = \frac{0.075}{3.055} = 0.025$$

Answer: 0.55 water; 0.18 ethanol; 0.15 methanol; 0.043 glycerol; 0.056 acetic acid; 0.025 benzaldehyde

2.16 Solution preparation

A 6% w/v solution means 6 g of $MgSO_4 \cdot 7H_2O$ in 100 ml of solution (Section 2.4.5). To make up 100 ml of solution, weigh out 6 g of $MgSO_4 \cdot 7H_2O$ using the balance and place the solid in the measuring cylinder. Add water to dissolve the solid and make up to the 100 ml mark on the measuring cylinder.

2.17 Moles, molarity, and composition

(a)

From the atomic weights in Table C.1 (Appendix C), the molecular weight of $C_{12}H_{22}O_4$ is 230. 1 kg = 1000 g. Therefore:

$$21.2\ \text{kg} = 21.2\ \text{kg} \cdot \left|\frac{1000\ \text{g}}{1\ \text{kg}}\right| \cdot \left|\frac{1\ \text{gmol}}{230\ \text{g}}\right| = 92.2\ \text{gmol}$$

Answer: 92.2 gmol

(b)

From the atomic weights in Table C.1 (Appendix C), the molecular weight of $C_{12}H_{22}O_{11}$ is 342. 1 kg = 1000 g. Therefore:

$$4.5 \text{ kg s}^{-1} = 4.5 \text{ kg} \,.\, \left| \frac{1000 \text{ g}}{1 \text{ kg}} \right| \,.\, \left| \frac{1 \text{ gmol}}{342 \text{ g}} \right| = 13.2 \text{ gmol s}^{-1}$$

1 min = 60 s. The amount of sucrose transferred in 30 min is:

$$13.2 \text{ gmol s}^{-1} (30 \text{ min}) \,.\, \left| \frac{60 \text{ s}}{1 \text{ min}} \right| = 2.38 \times 10^4 \text{ gmol}$$

Answer: 2.38×10^4 gmol

(c)

75 mM means 75×10^{-3} gmol per litre of solution (Section 2.4.5). 1 l = 1000 ml; therefore, in 10 ml or 1/100 of a litre, a 75 mM solution contains (75×10^{-3}/100) gmol = 75×10^{-5} gmol. From the atomic weights in Table C.1 (Appendix C), the molecular weight of $C_4H_6O_6 \cdot H_2O$ is 168. Therefore:

$$75 \times 10^{-5} \text{ gmol} = 75 \times 10^{-5} \text{ gmol} \,.\, \left| \frac{168 \text{ g}}{1 \text{ gmol}} \right| = 0.126 \text{ g}$$

Answer: 0.126 g

(d)

60 μM salicylaldehyde means 60×10^{-6} gmol per litre of solution (Section 2.4.5). 1 l = 1000 ml; therefore, in 250 ml or 1/4 of a litre, a 60 μM solution contains ($60/4 \times 10^{-6}$) gmol = 15×10^{-6} gmol. From the atomic weights in Table C.1 (Appendix C), the molecular weight of $C_7H_6O_2$ is 122. Therefore:

$$15 \times 10^{-6} \text{ gmol} = 15 \times 10^{-6} \text{ gmol} \,.\, \left| \frac{122 \text{ g}}{1 \text{ gmol}} \right| = 1.83 \times 10^{-3} \text{ g}$$

330 ppm dichloroacetic acid means 330 g per 10^6 g of solution (Section 2.4.5). Because the solution being considered contains only very dilute concentrations of components in water, we can assume that the density of the solution is the same as the density of water or 1 g ml^{-1} (Section 2.4.1). Therefore, the concentration of dichloroacetic acid is 330 g per 10^6 ml of solution, and the mass of dichloroacetic acid in 250 ml of solution is 330 g × (250 ml/10^6 ml) = 0.083 g.

Answer: 1.83×10^{-3} g of salicylaldehyde and 0.083 g of dichloroacetic acid

2.18 Concentration

(a)

If the concentration of NaCl added is the same as that already in the tank, the concentration remains constant at 25 mM NaCl.

Answer: 25 mM

(b)

25 mM NaCl means 25×10^{-3} gmol NaCl per litre (Section 2.4.5). Therefore, in the original 1500 l, there are 25×10^{-3} gmol $\text{l}^{-1} \times 1500$ l = 37.5 gmol NaCl. After addition of the water, there are 37.5 gmol NaCl in 4500 litres of solution. Therefore, the concentration is 37.5 gmol/4500 l = 8.33×10^{-3} gmol l^{-1} or 8.33 mM.

Answer: 8.3 mM

(c)

From the calculation in **(b)**, the tank originally contains 37.5 gmol of NaCl. Adding 500 l of 25 mM NaCl increases the amount of NaCl present by 25×10^{-3} gmol $l^{-1} \times 500$ l = 12.5 gmol, making the total amount of NaCl equal to (37.5 gmol + 12.5 gmol) = 50 gmol. After addition of 3000 l of water, these 50 gmol of NaCl are present in (1500 l + 500 l + 3000 l) = 5000 l of solution.

(i)

There are 50 gmol of NaCl present in 5000 l of solution. Therefore, the concentration is 50 gmol/5000 l = 0.01 gmol l^{-1}. As 1 gmol l^{-1} is the same as 1 M (Section 2.4.5), the concentration is 0.01 M.

Answer: 0.01 M

(ii)

% w/v means g per 100 ml of solution (Section 2.4.5) From the atomic weights in Table C.1 (Appendix C), the molecular weight of NaCl is 58.44. Therefore, the mass corresponding to 50 gmol of NaCl is:

$$50 \text{ gmol} = 50 \text{ gmol} . \left| \frac{58.44 \text{ g}}{1 \text{ gmol}} \right| = 2922 \text{ g}$$

This mass of NaCl is present in 5000 l of solution. Therefore, the concentration is 2922 g/5000 l = 0.584 g l^{-1}. 1 l = 1000 ml. In 100 ml or 1/10 of a litre of solution, the mass of NaCl is 0.584 g/10 = 0.0584 g. Therefore, the concentration is 0.0584 g per 100 ml, or 0.0584% w/v.

Answer: 0.0584% w/v

(iii)

From **(ii)**, the concentration of NaCl is 0.584 g l^{-1}. 1 l = 1000 cm^3. Therefore:

$$0.584 \text{ g } l^{-1} = 0.584 \text{ g } l^{-1} . \left| \frac{1 l}{1000 \text{ cm}^3} \right| = 5.84 \times 10^{-4} \text{ g cm}^{-3}$$

Answer: 5.84×10^{-4} g cm^{-3}

(iv)

ppm means g per 10^6 g of solution (Section 2.4.5) Because the solution of NaCl is very dilute, we can assume that the density of the solution is the same as the density of water = 1 g cm^{-3} (Section 2.4.1). Therefore, the concentration of NaCl calculated in **(iii)**, 5.84×10^{-4} g cm^{-3}, can be expressed as 5.84×10^{-4} g per g of solution. The mass of NaCl present in 10^6 g of solution is (5.84×10^{-4} g g^{-1}) $\times 10^6$ g = 584 g, so the concentration is 584 g per 10^6 g of solution, or 584 ppm.

Answer: 584 ppm, assuming that the density of the solution is equal to the density of water.

2.19 Gas composition

Gas compositions are expressed as volume % (Section 2.4.5). For relatively light gases such as oxygen, carbon dioxide, ammonia and nitrogen, we can assume that the ideal gas law is valid over the range of conditions applying to bioreactor operation (Section 2.5). This means that the relative partial volumes of the component gases will not change with temperature and pressure, so that the gas composition will be unaffected.

Answer: The composition is unaffected.

2.20 Specific gravity and composition

A pharmaceutical concentration of 38.6% w/w means 38.6 g in 100 g of solution (Section 2.4.5). If the specific gravity of the solution is 1.036, the density referenced against water at 4°C is 1.036 g cm^{-3} (Section 2.4.2).

(a)

From the definition of density as mass per unit volume (Section 2.4.1), the volume of 100 g of solution = 100 g/(1.036 g cm^{-3}) = 96.53 cm^3. Therefore, the concentration of pharmaceutical is 38.6 g per 96.53 cm^3 of solution, or 38.6 g/(96.53 cm^3) = 0.40 g cm^{-3}. 1 l = 1000 cm^3 and 1 kg = 1000 g. Converting units gives:

$$0.40 \text{ g cm}^{-3} = 0.40 \text{ g cm}^{-3} . \left| \frac{1000 \text{ cm}^3}{1 \text{ l}} \right| . \left| \frac{1 \text{ kg}}{1000 \text{ g}} \right| = 0.40 \text{ kg l}^{-1}$$

Answer: 0.40 kg l^{-1}

(b)

The flow rate of pharmaceutical is found by multiplying the solution flow rate by the pharmaceutical concentration:

$$\text{Pharmaceutical flow rate} = 8.6 \text{ l min}^{-1} \times 0.40 \text{ kg l}^{-1} = 3.44 \text{ kg min}^{-1}$$

1 kg = 1000 g. Converting to molar units using the pharmaceutical molecular weight:

$$\text{Pharmaceutical flow rate} = 3.44 \text{ kg min}^{-1} . \left| \frac{1000 \text{ g}}{1 \text{ kg}} \right| . \left| \frac{1 \text{ gmol}}{1421 \text{ g}} \right| = 2.42 \text{ gmol min}^{-1}$$

Answer: 2.42 gmol min^{-1}

2.21 Temperature scales

From Eq. (2.30):

$$-40 = 1.8\, T(°\text{C}) + 32$$

$$T(°\text{C}) = -40$$

From Eq. (2.28):

$$T(°\text{R}) = -40 + 459.67$$

$$T(°\text{R}) = 420$$

From Eq. (2.27) and the result for T(°C):

$$T(\text{K}) = -40 + 273.15$$

$$T(\text{K}) = 233$$

Answer: 40°C, 420°R, 233 K

2.22 Pressure scales

(a)

Assume that the atmospheric (barometric) pressure is 14.7 psi (Section 2.4.7). From Eq. (2.31):

$$\text{Absolute pressure} = 15 \text{ psi} + 14.7 \text{ psi} = 29.7 \text{ psi}$$

From Table A.5 (Appendix A), 1 psi = 6.805×10^{-2} atm. Therefore:

$$\text{Absolute pressure} = 29.7 \text{ psi} \,.\, \left| \frac{6.805 \times 10^{-2} \text{ atm}}{1 \text{ psi}} \right| = 2.02 \text{ atm}$$

Answer: 29.7 psi, 2.02 atm

(b)

Vacuum pressure is pressure below barometric (Section 2.4.7). If the barometric pressure is 14.7 psi (Section 2.4.7):

$$\text{Absolute pressure} = 14.7 \text{ psi} - 3 \text{ psi} = 11.7 \text{ psi}$$

Answer: 11.7 psi

2.23 Gas leak

The volume of the cylinder does not change as a result of the leak, so $V_1 = V_2 = 48$ l. $P_1 = 0.35$ MPa = 0.35×10^6 Pa. When the cylinder is left open to the atmosphere, $P_2 = 1$ atm. $T_1 = 22$°C; $T_2 = 33$°C. Let us assume that the ideal gas law applies to compressed air under the prevailing conditions (Section 2.5). Temperature in the ideal gas equation is absolute temperature; therefore, using Eq. (2.27), $T_1 = (22 + 273.15)$ K = 295.15 K and $T_2 = (33 + 273.15)$ K = 306.15 K. The ratio of the amounts of air in the tank before and after the leak can be determined using Eq. (2.35):

$$\frac{n_1}{n_2} = \frac{\dfrac{p_1 V_1}{R T_1}}{\dfrac{p_2 V_2}{R T_2}}$$

Because $V_1 = V_2$, these terms and R can be cancelled to give:

$$\frac{n_1}{n_2} = \frac{p_1 T_2}{p_2 T_1}$$

From Table A.5 (Appendix A), 1 atm = 1.013×10^5 Pa. Substituting values into the equation:

$$\frac{n_1}{n_2} = \frac{p_1 T_2}{p_2 T_1} = \frac{0.35 \times 10^6 \text{ Pa} \,.\, \left| \dfrac{1 \text{ atm}}{1.013 \times 10^5 \text{ Pa}} \right| .\, (306.15 \text{ K})}{1 \text{ atm } (295.15 \text{ K})} = 3.58$$

or

$$n_2 = 0.28 \, n_1$$

The amount of air in the tank after the leak is 28% of that present before the leak. The amount lost is therefore 72%.

Answer: 72%, assuming that the ideal gas law applies

2.24 Gas supply

The flow rate of air required to operate the bioreactor is $0.8 \times 1.5 = 1.2$ l min^{-1}. The amount of air provided to the bioreactor between 4 pm Friday and 9 am Monday, i.e. over a period of 65 hours, can be determined using the ideal gas law with $T = 25$°C and $P = 1$ atm. Temperature in the ideal gas equation is absolute temperature; therefore, using Eq. (2.27), $T = (25 + 273.15)$ K = 298.15 K. From Appendix B, R = 0.082057 l atm gmol^{-1} K^{-1}. Using Eq. (2.35), the amount of air required to operate the bioreactor is:

$$n = \frac{pV}{RT} = \frac{1\,\text{atm}\,(1.2\,\text{l min}^{-1})\,(65\,\text{h})\,.\left|\frac{60\,\text{min}}{1\,\text{h}}\right|}{0.082057\,\text{l atm gmol}^{-1}\,\text{K}^{-1}\,(298.15\,\text{K})} = 191\,\text{gmol}$$

This amount of air can be compared with the amount available in the gas cylinder. V = 48 l. T = 20°C; using Eq. (2.27), T = (20 + 273.15) K = 293.15 K. If the gauge pressure is 800 psi, the absolute pressure P in the cylinder is (800 + 14.7) psi = 814.7 psi (Section 2.4.7). From Table A.5 (Appendix A), 1 psi = 6.805 × 10^{-2} atm. Applying Eq. (2.35):

$$n = \frac{pV}{RT} = \frac{814.7\,\text{psi}\,.\left|\frac{6.805\times10^{-2}\,\text{atm}}{1\,\text{psi}}\right|.(48\,\text{l})}{0.082057\,\text{l atm gmol}^{-1}\,\text{K}^{-1}(293.15\,\text{K})} = 111\,\text{gmol}$$

Even if the uncertainty factor of 5% is taken into account with respect to the values of n calculated, the amount of air in the cylinder is less than that required to operate the bioreactor over the weekend.

Answer: No

2.25 Stoichiometry and incomplete reaction

(a)

The molecular weights are calculated from Table C.1 (Appendix C): penicillin = 334.4; glucose = 180.2. The maximum theoretical yield from the stoichiometric equation is 1 gmol of penicillin for every 1.67 gmol of glucose. This is equivalent to 334.4 g of penicillin per 1.67 × 180.2 = 300.9 g of glucose, or 334.4 g/300.9 g = 1.1 g g^{-1}.

Answer: 1.1 g g^{-1}

(b)

The maximum theoretical yield in **(a)** is obtained when all the glucose consumed is directed into penicillin production according to the stoichiometric equation. If only 6% of the glucose is used in this way, the amount of penicillin produced for every 300.9 g of glucose consumed is much lower at 334.4 g × 0.06 = 20.06 g. Therefore, the actual yield of penicillin from glucose is 20.06 g/300.9 g = 0.067 g g^{-1}.

Answer: 0.067 g g^{-1}

(c)

From the atomic weights in Table C.1 (Appendix C), the molecular weight of phenylacetic acid is 136.2.

(i)

The only possible limiting substrates are glucose and phenylacetic acid. Using a basis of 1 l of medium, if (50 – 5.5) = 44.5 g of glucose are consumed but only 6% is available for penicillin synthesis, the mass of glucose used in the penicillin reaction is 44.5 × 0.06 = 2.67 g. Converting to gmol of glucose, this is equivalent to 2.67 g/(180.2 g $gmol^{-1}$) = 1.48 × 10^{-2} gmol of glucose available for penicillin synthesis. At the same time, 4 g or 4 g/(136.2 g $gmol^{-1}$) = 2.94 × 10^{-2} gmol of phenylacetic acid is available which, according to the stoichiometric equation, requires 1.67 × (2.94 × 10^{-2}) = 4.91 × 10^{-2} gmol of glucose for complete reaction. As the amount of glucose (4.91 × 10^{-2} gmol) required for complete reaction of the phenylacetic acid is greater than the amount of glucose (1.48 × 10^{-2} gmol) available after growth and maintenance activities, glucose is the limiting substrate.

Answer: Glucose

(ii)

Of the 44.5 g l^{-1} glucose consumed, 24% or 10.7 g l^{-1} is used for growth. In a 100-litre tank, the total mass of glucose consumed for growth is therefore 10.7 g l^{-1} × 100 l = 1070 g, or 1.07 kg.

Answer: 1.07 kg

(iii)

From **(i)**, 1.48×10^{-2} gmol of glucose is used in the penicillin reaction per litre. According to the stoichiometry, this produces $(1.48 \times 10^{-2})/1.67 = 8.86 \times 10^{-3}$ gmol of penicillin per litre. Therefore, in a 100-litre tank, (8.86×10^{-3} gmol l^{-1}) × 100 l = 0.886 gmol of penicillin is formed. Converting to mass, 0.886 gmol × 334.4 g $gmol^{-1}$ = 296 g penicillin are formed.

Answer: 296 g

(iv)

If, from **(i)**, 1.48×10^{-2} gmol glucose is used in the penicillin reaction per litre, from stoichiometry $(1.48 \times 10^{-2})/1.67 = 8.86 \times 10^{-3}$ gmol l^{-1} phenylacetic acid must also be used. Converting from moles to mass, this is equivalent to 8.86×10^{-3} gmol l^{-1} × 136.2 g $gmol^{-1}$ = 1.21 g 1^{-1} phenylacetic acid. As 4 g l^{-1} phenylacetic acid is provided, (4 – 1.21) g l^{-1} = 2.79 g l^{-1} phenylacetic acid must remain.

Answer: 2.79 g l^{-1}

2.26 Stoichiometry, yield, and the ideal gas law

(a)

Adding up the numbers of C, H, O and N atoms on both sides of the equation shows that the equation is balanced: C (16 = 16), H (38.3 = 38.3), O (32.6 = 32.6), N (1.42 = 1.42).

Answer: Yes

(b)

The molecular weights are calculated from Table C.1 (Appendix C): cells = 91.5; hexadecane = 226.4. From the stoichiometry, as 1 gmol of hexadecane is required to produce 1.65 gmol of cells, the maximum yield is 1.65 gmol × 91.5 g $gmol^{-1}$ = 151 g cells per 226.4 g hexadecane, or 151 g/226.4 g = 0.67 g g^{-1}.

Answer: 0.67 g g^{-1}

(c)

From the atomic weights in Table C.1 (Appendix C), the molecular weight of oxygen is 32.0. From the stoichiometry, 16.28 gmol of oxygen is required to produce 1.65 gmol of cells which, from **(b)**, is equal to 151 g of cells. The maximum yield is therefore 151 g of cells per (16.28 gmol × 32.0 g $gmol^{-1}$) = 521 g oxygen, or 151 g/521 g = 0.29 g g^{-1}.

Answer: 0.29 g g^{-1}

(d)

2.5 kg = 2500 g. Converting to molar terms using the cell molecular weight from **(b)**, 2500 g = 2500 g/(91.5 g $gmol^{-1}$) = 27.3 gmol cells. The minimum amounts of substrates are required when 100% of the hexadecane is converted according to the stoichiometric equation.

(i)

From the stoichiometry, production of 27.3 gmol of cells requires 27.3/1.65 = 16.5 gmol of hexadecane. Converting to mass terms using the molecular weight of hexadecane, 16.5 gmol = 16.5 gmol × 226.4 g $gmol^{-1}$ = 3736 g = 3.74 kg of hexadecane.

Answer: 3.74 kg

(ii)

From the answer in **(d)(i)**, the concentration of hexadecane required is 3.74 kg in 3 m^3, or 1.25 kg m^{-3}.

Answer: 1.25 kg m^{-3}

(iii)

According to the stoichiometric equation, production of 27.3 gmol of cells requires 27.3 × 16.28/1.65 = 269.4 gmol of oxygen. As air contains approximately 21 mol% oxygen (Section 2.4.5), the moles of air required is 269.4/0.21 = 1282.9 gmol. The corresponding volume of air is calculated using the ideal gas law. From Eq. (2.35):

$$V = \frac{nRT}{p}$$

P = 1 atm; T = 20°C. As temperature in the ideal gas equation is absolute temperature, from Eq. (2.27):

$$T = (20 + 273.15)\ \text{K} = 293.15\ \text{K}$$

From Appendix B, R = 82.057 cm^3 atm K^{-1} $gmol^{-1}$. 1 m = 100 cm. Substituting these values into the equation for V:

$$V = \frac{(1282.9\ \text{gmol})(82.057\ \text{cm}^3\ \text{atm}\ \text{K}^{-1}\ \text{gmol}^{-1})(293.15\ \text{K})}{1\ \text{atm}} \cdot \left|\frac{1\ \text{m}}{100\ \text{cm}}\right|^3 = 31\ \text{m}^3$$

Answer: 31 m^3

2.27 Stoichiometry and the ideal gas law

(a)

Adding up the numbers of C, H, O and N atoms on both sides of the equation shows that the equation is balanced: C (6 = 6), H (12.6 ≈ 12.7), O (13.3 = 13.3), N (0.33 = 0.33)

Answer: Yes

(b)

From Table C.1 (Appendix C), the molecular weights are: glucose = 180.2; HNO_3 = 63.0; biomass 28.3. Converting to molar units, the concentration of glucose is 30 g l^{-1}/(180.2 g $gmol^{-1}$) = 0.166 gmol l^{-1}. From the stoichiometry, this requires 0.18 × 0.166 gmol l^{-1} = 0.030 gmol l^{-1} of HNO_3 for complete conversion.

Answer: 0.030 gmol l^{-1}

(c)

From the glucose concentration calculated in **(b)**, the gmol of glucose present in 50 l is 0.166 gmol l^{-1} × 50 l = 8.30 gmol. According to the stoichiometry, complete conversion of 8.30 gmol of glucose produces 2.5 × 8.30 gmol = 20.75 gmol of biomass. Converting to mass units using the molecular weight of the biomass, 20.75 gmol × 28.3 g $gmol^{-1}$ = 587 g of biomass are produced.

Answer: 587 g

(d)

According to the stoichiometric equation, complete conversion of 8.30 gmol of glucose requires 3.4 × 8.30 = 28.2 gmol of oxygen. The volume of oxygen required can be calculated using the ideal gas law with P = 1 atm. As temperature in the ideal gas equation is absolute temperature, 20°C is converted to K using Eq. (2.27):

$$T = (20 + 273.15)\ \text{K} = 293.15\ \text{K}$$

From Appendix B, $R = 0.000082057\ \text{m}^3\ \text{atm}\ \text{K}^{-1}\ \text{gmol}^{-1}$. Using Eq. (2.35), the volume of oxygen required is:

$$V = \frac{nRT}{p} = \frac{(28.2\ \text{gmol})(0.000082057\ \text{m}^3\ \text{atm}\ \text{K}^{-1}\ \text{gmol}^{-1})(293.15\ \text{K})}{1\ \text{atm}} = 0.678\ \text{m}^3$$

Air contains approximately 21 volume % oxygen (Section 2.4.5). Therefore, the volume of air required is $0.678\ \text{m}^3/0.21 = 3.2\ \text{m}^3$.

Answer: 3.2 m^3

2.28 Stoichiometry, yield, and limiting substrate

(a)

Adding up the numbers of C, H and O atoms and the charges on both sides of the equation shows that the equation is balanced: C (10 = 10), H (20 = 20), O (34 = 34), charge (–8 = –8).

Answer: Yes

(b)

From Table C.1 (Appendix C), the molecular weights are N_2 = 28.0 and acetate (≈ acetic acid) = 60.1. According to the stoichiometry, the yield is 4 gmol of N_2 for every 5 gmol of acetate consumed. Converting to mass units using the molecular weights, the yield is (4 × 28.0 g N_2)/(5 × 60.1 g acetate) = 0.37 g g^{-1}.

Answer: 0.37 g g^{-1}

(c)

Effectively, 0.75 × 6 mM = 4.5 mM of acetic acid and 0.85 × 7 mM = 5.95 mM of nitrate are available for the denitrification reaction. From the reaction stoichiometry, 8 mol of nitrate are required for every 5 mol of acetic acid consumed. Therefore, complete reaction of 4.5 mM acetic acid requires 4.5 mM × 8/5 = 7.2 mM nitrate. As only 5.95 mM nitrate is available, nitrate is the limiting substrate.

Answer: Nitrate

(d)

The conversion is based on the amount of limiting substrate available. From **(c)**, nitrate is the limiting substrate and 5.95 mM of nitrate is available for the denitrification reaction. Therefore, in 5000 l, the total amount of nitrate converted in the reaction is 5.95×10^{-3} gmol l^{-1} × 5000 l = 29.75 gmol. According to the stoichiometry, for each 8 gmol of nitrate reacted, 4 gmol of N_2 are formed. Therefore, conversion of 29.75 gmol of nitrate produces 29.75 × 4/8 = 14.88 gmol of N_2. Converting to mass units using the molecular weight of N_2, the mass of N_2 produced is 14.88 gmol × 28.0 g gmol^{-1} = 416.6 g.

Answer: 417 g

2.29 Order-of-magnitude calculation

rpm means revolutions per minute. As revolutions is a non-dimensional quantity (Section 2.1.2), the units of rpm are min^{-1}. Therefore:

$$12{,}000\ \text{min}^{-1} = 12{,}000\ \text{min}^{-1} . \left| \frac{1\ \text{min}}{60\ \text{s}} \right| = 200\ \text{s}^{-1}$$

π = 3.14159. From Eq. (2.16), g = 9.8066 m s^{-2}. The equation for Σ using the actual data is:

$$\Sigma = \frac{3.14159\ (200\ \text{s}^{-1})^2\ (1.25\ \text{m})\ (0.37\ \text{m})^2}{2\ (9.8066\ \text{m s}^{-2})}$$

Using approximate values, this becomes:

$$\Sigma = \frac{3\ (200)^2\ (1)\ (0.4)^2\ \text{m}^3\ \text{s}^{-2}}{2\ (10)\ \text{m s}^{-2}} = 960\ \text{m}^2$$

This result is closer to 1000 m^2 than to 100 m^2.

Answer: 1000 m^2

2.30 Order-of-magnitude calculation

π = 3.14159. The equation for r^*_{As} using the actual data is:

$$r^*_{\text{As}} = \frac{4}{3}(3.14159)\ (3.2\times10^{-3})^3\ \text{m}^3\ \frac{(0.12\ \text{gmol s}^{-1}\ \text{m}^{-3})(41\ \text{gmol m}^{-3})}{(0.8+41)\ \text{gmol m}^{-3}}$$

Using approximate values, this becomes:

$$r^*_{\text{As}} = \frac{4}{3}(3)(30\times10^{-9})\frac{(4)}{(40)}\ \text{gmol s}^{-1} = 120\times10^{-10}\ \text{gmol s}^{-1} = 1.2\times10^{-8}\ \text{gmol s}^{-1}$$

This result indicates that the student reporting the value 1.6×10^{-8} gmol s^{-1} is more likely to be correct.

Answer: 1.6×10^{-8} gmol s^{-1}

Chapter 3

Presentation and Analysis of Data

3.1 Combination of errors

C^*_{AL} = 0.25 mol m^{-3} ± 4% = (0.25 + 0.010) mol m^{-3}

C_{AL} = 0.183 mol m^{-3} ± 4% = (0.183 ± 0.0073) mol m^{-3}

OTR = 0.011 mol m^{-3} s^{-1} ± 5%

For subtraction, absolute errors are added (Section 3.1.3). Therefore:

$$C^*_{AL} - C_{AL} = (0.25 - 0.183) \pm (0.010 + 0.0073) \text{ mol m}^{-3} = (0.067 \pm 0.0173) \text{ mol m}^{-3}$$
$$= 0.067 \text{ mol m}^{-3} \pm 25.8\%$$

For division, relative errors are added (Section 3.1.3). Therefore:

$$k_L a = \frac{0.011 \text{ mol m}^{-3} \text{ s}^{-3}}{0.067 \text{ mol m}^{-3}} \pm (25.8 + 5)\% = 0.16 \text{ s}^{-1} \pm 31\%$$

Answer: 31%. *This example illustrates how a combination of small measurement errors can result in a relatively large uncertainty in the final result.*

3.2 Accuracy requirement

Let us use z to denote the absolute error associated with the cell concentrations. The calculation for Y_{XS} involves adding absolute errors for the terms that are subtracted, and adding relative errors in the division operation (Section 3.1.3):

$$Y_{XS} = \frac{[(12.7 - 0.66) \pm (z + z)] \text{ g l}^{-1}}{[(30 - 0.85) \pm (0.1 + 0.1)] \text{ g l}^{-1}} = \frac{12.04 \text{ g l}^{-1} \pm \frac{2z}{12.04} \times 100\%}{29.15 \text{ g l}^{-1} \pm 0.686\%}$$
$$= 0.413 \pm \left(\frac{2z}{12.04} \times 100\% + 0.686\% \right)$$

where z has units of g l^{-1}. If the error term in brackets is equivalent to 5%:

$$\frac{2z}{12.04} \times 100 + 0.686 = 5$$

Solving for z gives z = 0.26. Therefore, the maximum error in cell concentration is 0.26 g l^{-1}.

Answer: 0.26 g l^{-1}

3.3 Mean and standard error

The calculations shown can also be performed using computer or calculator software.

(a)

The best estimate is given by the sample mean. Calculating the mean using Eq. (3.1):

$$\bar{x} = \frac{5.15 + 5.25 + 5.45 + 5.20 + 5.50 + 5.35}{6} = 5.32$$

Answer: 5.32

(b)

Calculating the standard error of the mean using Eqs (3.2) and (3.3):

$$\sigma = \sqrt{\frac{(5.15-5.32)^2 + (5.25-5.32)^2 + (5.45-5.32)^2 + (5.20-5.32)^2 + (5.50-5.32)^2 + (5.35-5.32)^2}{6-1}} = 0.14$$

$$\sigma_m = \frac{0.14}{\sqrt{6}} = 0.057$$

The standard error is 0.057/5.32 × 100% = 1.1% of the mean value. This small standard error indicates that the reliability of the mean is high. From Section 3.1.5, there is a 95% chance that the true population mean falls within the interval $\bar{x} \pm 2\sigma_m = 5.32 \pm 0.11$.

Answer: Standard error = 1.1% of the mean; reliability is high

(c)

Calculating the mean for the first three measurements using Eq. (3.1):

$$\bar{x} = \frac{5.15 + 5.25 + 5.45}{3} = 5.28$$

Calculating the standard error of the mean using Eqs (3.2) and (3.3):

$$\sigma = \sqrt{\frac{(5.15-5.28)^2 + (5.25-5.28)^2 + (5.45-5.28)^2}{3-1}} = 0.15$$

$$\sigma_m = \frac{0.15}{\sqrt{3}} = 0.088$$

Answer: 5.28 ± 0.088, or 5.28 ± 1.7%

(d)

Calculating the mean using the results of 12 measurements and Eq. (3.1):

$$\bar{x} = \frac{5.15 + 5.25 + 5.45 + 5.20 + 5.50 + 5.35 + 5.15 + 5.25 + 5.45 + 5.20 + 5.50 + 5.35}{12} = 5.32$$

The mean is the same as in **(a)**. Calculating the standard error of the mean using Eqs (3.2) and (3.3):

$$\sigma = \sqrt{\frac{\begin{array}{l}(5.15-5.32)^2 + (5.25-5.32)^2 + (5.45-5.32)^2 + (5.20-5.32)^2 + \\ (5.50-5.32)^2 + (5.35-5.32)^2 + (5.15-5.32)^2 + (5.25-5.32)^2 + \\ (5.45-5.32)^2 + (5.20-5.32)^2 + (5.50-5.32)^2 + (5.35-5.32)^2\end{array}}{12-1}} = 0.13$$

$$\sigma_m = \frac{0.13}{\sqrt{12}} = 0.039$$

Taking more measurements reduces the standard error of the mean.

Answer: 5.32 ± 0.039, or 5.32 ± 0.7%

3.4 Confidence limits for the sample mean

The calculations shown can also be performed using computer or calculator software. The sample mean of the measured volumes is calculated using Eq. (3.1):

$$\bar{x} = \frac{(1.4+1.2+4.1+3.3+2.6+1.6+1.4+3.0+2.0+1.1)\text{ litres}}{10} = 2.17\text{ litres}$$

Calculating the standard error of the mean using Eqs (3.2) and (3.3):

$$\sigma = \sqrt{\frac{\begin{array}{l}(1.4-2.17)^2+(1.2-2.17)^2+(4.1-2.17)^2+(3.3-2.17)^2+\\(2.6-2.17)^2+(1.6-2.17)^2+(1.4-2.17)^2+(3.0-2.17)^2+\\(2.0-2.17)^2+(1.1-2.17)^2\end{array}}{10-1}} = 1.03$$

and

$$\sigma_m = \frac{1.03}{\sqrt{10}} = 0.32$$

From Section 3.1.5, the 95% confidence interval for the antifoam volume is $\bar{x} \pm 2\sigma_m = 2.17 \pm 0.64$, which corresponds to the range 1.53 to 2.81 litres. As 2.9 litres lies outside of this range, it is not regarded as a reasonable estimate of the antifoam volume.

Answer: No, it falls outside of the 95% confidence interval

3.5 Measurement accuracy

(a)

The calculations shown can also be performed using computer or calculator software. The sample mean is calculated using Eq. (3.1). For Probe 1:

$$\bar{x} = \frac{(51.7+52.6+52.9+49.5+50.2)\ \%}{5} = 51.38\%\text{ air saturation}$$

For Probe 2:

$$\bar{x} = \frac{(49.0+48.9+50.1+53.3+53.6)\ \%}{5} = 50.98\%\text{ air saturation}$$

Standard deviation is calculated using Eq. (3.2). For Probe 1:

$$\sigma = \sqrt{\frac{(51.7-51.38)^2+(52.6-51.38)^2+(52.9-51.38)^2+(49.5-51.38)^2+(50.2-51.38)^2}{5-1}} = 1.49$$

For Probe 2:

$$\sigma = \sqrt{\frac{(49.0-50.98)^2+(48.9-50.98)^2+(50.1-50.98)^2+(53.3-50.98)^2+(53.6-50.98)^2}{5-1}} = 2.31$$

Standard error is calculated using Eq. (3.3). For Probe 1:

$$\sigma_m = \frac{1.49}{\sqrt{5}} = 0.66$$

For Probe 2:

$$\sigma_m = \frac{2.31}{\sqrt{5}} = 1.03$$

Answer: Probe 1: the sample mean is 51.4% air saturation, the standard deviation is 1.5% air saturation and the standard error is 0.66% air saturation. Probe 2: the sample mean is 51.0% air saturation, the standard deviation is 2.3% air saturation and the standard error is 1.0% air saturation.

(b)

Because the standard deviation is higher for Probe 2, this probe exhibits the greater degree of measurement scatter.

Answer: Probe 2, because the standard deviation is higher

(c)

Because the standard error is smaller for Probe 1, this probe exhibits greater precision. If systematic errors are eliminated (Section 3.1.4), we can also say that Probe 1 is the more accurate probe.

Answer: Assuming no systematic error, Probe 1 because the standard error is smaller

3.6 Linear and nonlinear models

(a)

$x_1 = 1$; $y_1 = 10$

$x_2 = 8$; $y_2 = 0.5$

A straight line plot of y versus x on linear coordinates means that the data can be represented using Eq. (3.7). From Eqs (3.8) and (3.9):

$$A = \frac{0.5 - 10}{8 - 1} = -1.36$$

$$B = y_1 - Ax_1 = 10 - (-1.36)\,1 = 11.4$$

Answer: $y = -1.36\,x + 11.4$

(b)

$x_1 = 3.2$; $y_1 = 14.5$

$x_2 = 8.9$; $y_2 = 38.5$

A straight line plot of y versus $x^{1/2}$ on linear coordinates means that the data can be represented using the equation:

$$y = Ax^{1/2} + B$$

with A and B given by the equations:

$$A = \frac{y_2 - y_1}{x_2^{1/2} - x_1^{1/2}} = \frac{38.5 - 14.5}{8.9^{1/2} - 3.2^{1/2}} = 20.1$$

$$B = y_1 - Ax_1^{1/2} = 14.5 - 20.1\,(3.2^{1/2}) = -21.5$$

Answer: $y = 20.1x^{1/2} - 21.5$

(c)

$x_1 = 5$; $y_1 = 6$

$x_2 = 1$; $y_2 = 3$

A straight line plot of $1/y$ versus x^2 on linear coordinates means that the data can be represented using the equation:

$$1/y = Ax^2 + B$$

with A and B given by the equations:

$$A = \frac{1/y_2 - 1/y_1}{x_2^2 - x_1^2} = \frac{1/3 - 1/6}{1^2 - 5^2} = -6.9 \times 10^{-3}$$

$$B = 1/y_1 - Ax_1^2 = 1/6 - (-6.9 \times 10^{-3})(5^2) = 0.34$$

Answer: $1/y = -6.9 \times 10^{-3} x^2 + 0.34$

(d)

$x_1 = 0.5$; $y_1 = 25$

$x_2 = 550$; $y_2 = 2600$

A straight line plot of y versus x on log–log coordinates means that the data can be represented using Eq. (3.11). From Eqs (3.13) and (3.15):

$$A = \frac{(\ln y_2 - \ln y_1)}{(\ln x_2 - \ln x_1)} = \frac{\ln 2600 - \ln 25}{\ln 550 - \ln 0.5} = 0.663$$

and

$$\ln B = \ln y_1 - A \ln x_1 = \ln 25 - (0.663) \ln 0.5 = 3.678$$

or

$$B = e^{3.678} = 39.6$$

Answer: $y = 39.6x^{0.663}$

(e)

$x_1 = 1.5$; $y_1 = 2.5$

$x_2 = 10$; $y_2 = 0.036$

A straight line plot of y versus x on semi-log coordinates means that the data can be represented using Eq. (3.16). From Eqs (3.18) and (3.19):

$$A = \frac{(\ln y_2 - \ln y_1)}{(x_2 - x_1)} = \frac{\ln 0.036 - \ln 2.5}{10 - 1.5} = -0.50$$

$$\ln B = \ln y_1 - A x_1 = \ln 2.5 - (-0.50)\, 1.5 = 1.666$$

$$B = e^{1.666} = 5.29$$

Answer: $y = 5.29e^{-0.50x}$

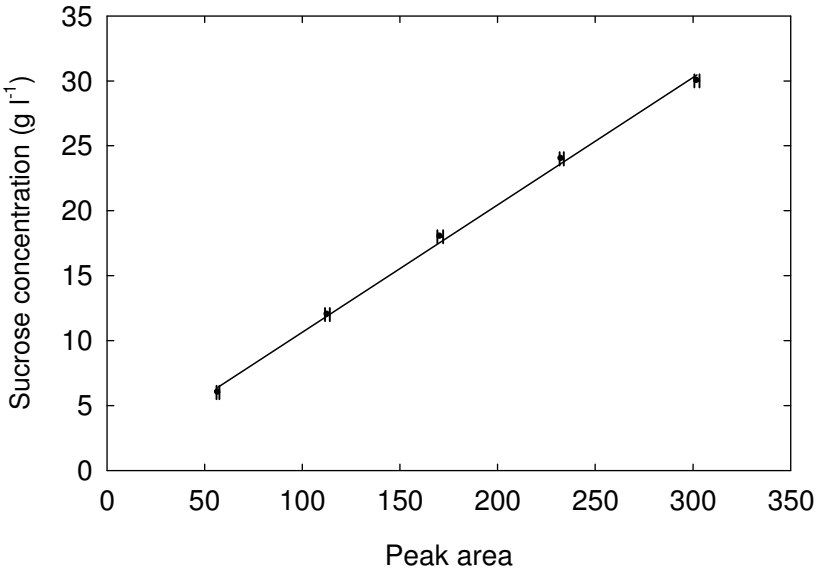
3.7 Calibration curve

(a)

The results determined using Eqs (3.1)–(3.3) are listed below.

Sucrose concentration (g l^{-1})	Mean peak area	Standard deviation	Standard error
6.0	56.84	1.21	0.70
12.0	112.82	2.06	1.19
18.0	170.63	2.54	1.47
24.0	232.74	1.80	1.04
30.0	302.04	2.21	1.27

(b)



(c)

The linear least-squares fit of the data is:

$$y = 0.098x + 0.83$$

Answer: $y = 0.098x + 0.83$, where y is sucrose concentration in g 1^{-1} and x is peak area.

(d)

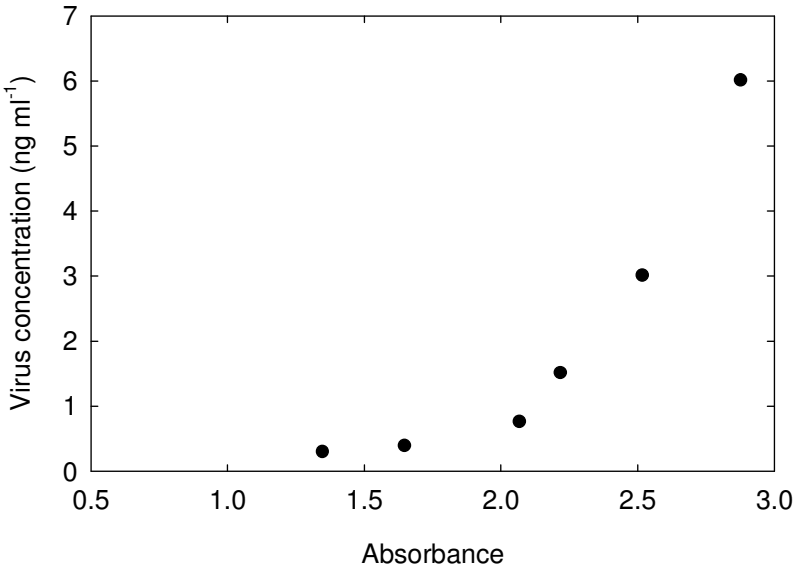
For $x = 209.86$, the equation in **(c)** gives a sucrose concentration of $y = 21.4$ g 1^{-1}.

Answer: 21.4 g 1^{-1}

3.8 Linear-range calibration

(a)

First, plot all the raw data using linear coordinates.

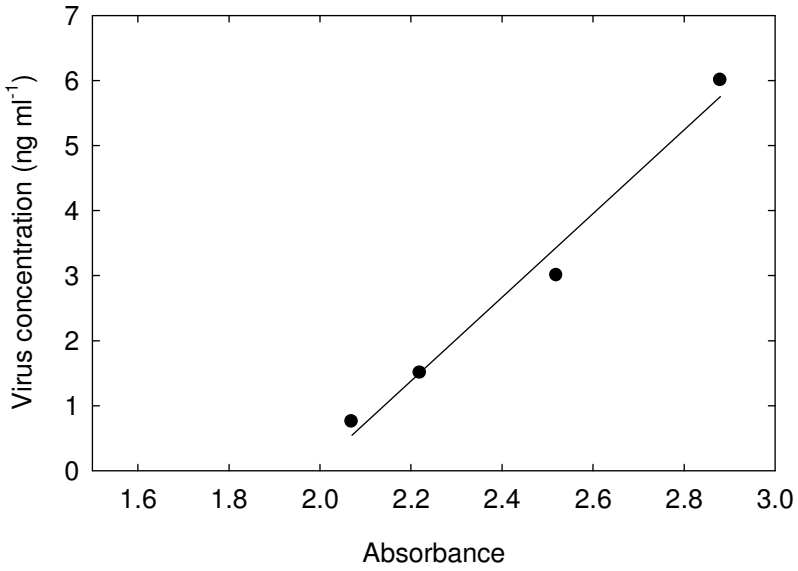


From the graph, there is an approximate linear relationship between virus concentration and absorbance for virus concentrations between 0.75 and 6.0 ng ml^{-1}.

Answer: Between 0.75 and 6.0 ng ml^{-1}

(b)

The plot and linear regression fit of the data for virus concentrations between 0.75 and 6.0 ng ml^{-1} is shown below.



The linear least-squares fit for these data is:

$$y = 6.43x - 12.76$$

Answer: $y = 6.43x - 12.76$, where y is virus concentration in ng $m1^{-1}$ and x is absorbance.

(c), (d)

From the equation in **(b)**, for $x = 2.02$, the virus concentration $y = 0.23$ ng m1^{-1}. However, this result falls outside of the linear range determined using the measured data. Therefore, the linear correlation equation derived in **(b)** cannot be considered valid for this sample.

For $x = 2.66$, the virus concentration $y = 4.3$ ng m1^{-1}.

For $x = 2.75$, the virus concentration $y = 4.9$ ng m1^{-1}.

Answer: The correlation curve can be applied with most confidence to the samples with absorbance values 2.66 and 2.75. For these samples, the virus concentrations are 4.3 ng m1^{-1} and 4.9 ng m1^{-1}, respectively. Because the absorbance value for the remaining sample lies outside of the linear range used to derive the correlation equation, the equation cannot be used with confidence for that sample.

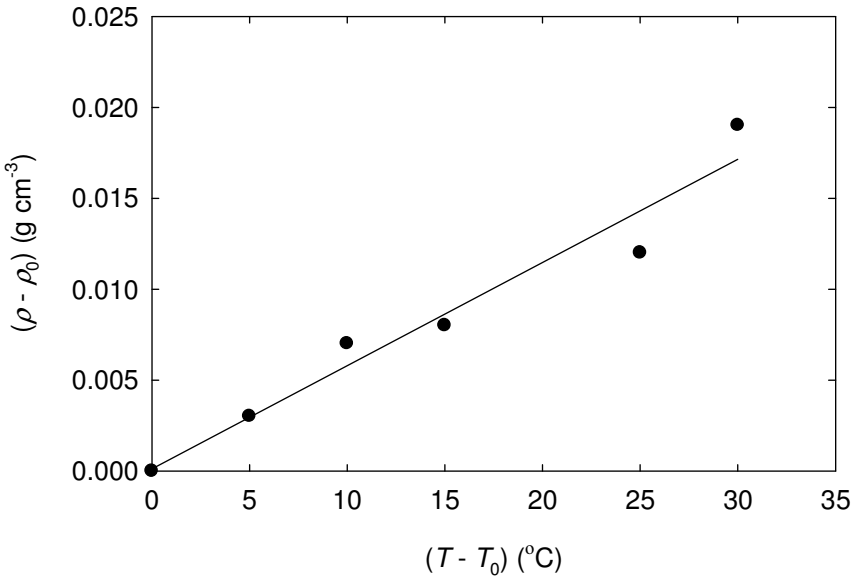
3.9 Data correlation

The equation for density as a function of temperature can be rearranged as follows:

$$(\rho - \rho_0) = A(T - T_0)$$

Therefore, a plot of $(\rho - \rho_0)$ versus $(T - T_0)$ using linear coordinates should give a straight line with slope A. Taking the measured values (0.0°C, 0.665 g cm^{-3}) as (T_0, ρ_0), calculated results for $(T - T_0)$ and $(\rho - \rho_0)$ are listed and plotted below.

$T - T_0$ (°C)	$\rho - \rho_0$ (g cm^{-3})
0.0	0.000
5.0	0.003
10.0	0.007
15.0	0.008
25.0	0.012
30.0	0.019



The linear least-squares fit for the data through the origin is:

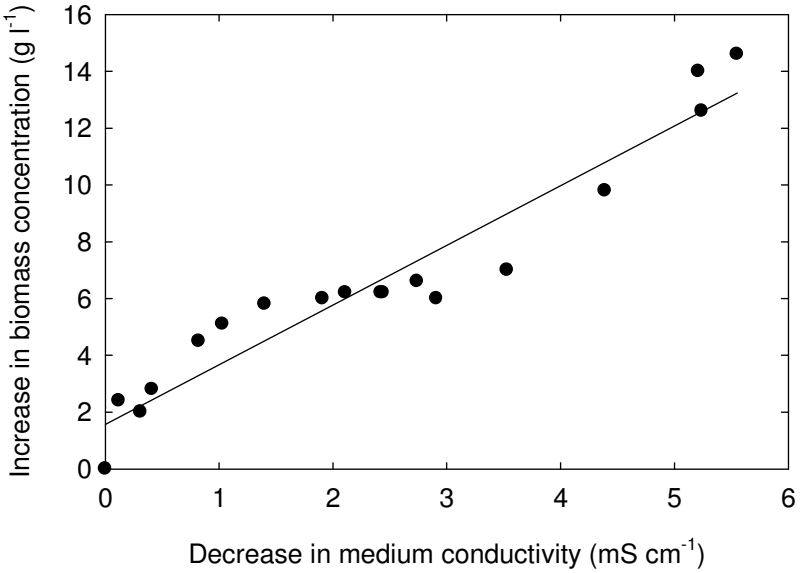
$$(\rho - \rho_0) = 5.73 \times 10^{-4} (T - T_0)$$

Therefore, $A = 5.73 \times 10^{-4}$ g cm^{-3} °C^{-1}.

Answer: $A = 5.73 \times 10^{-4}$ g cm^{-3} °C^{-1}

3.10 Linear regression: distribution of residuals

(a)



The linear least-squares fit of the data is:

$$y = 2.10x + 1.58$$

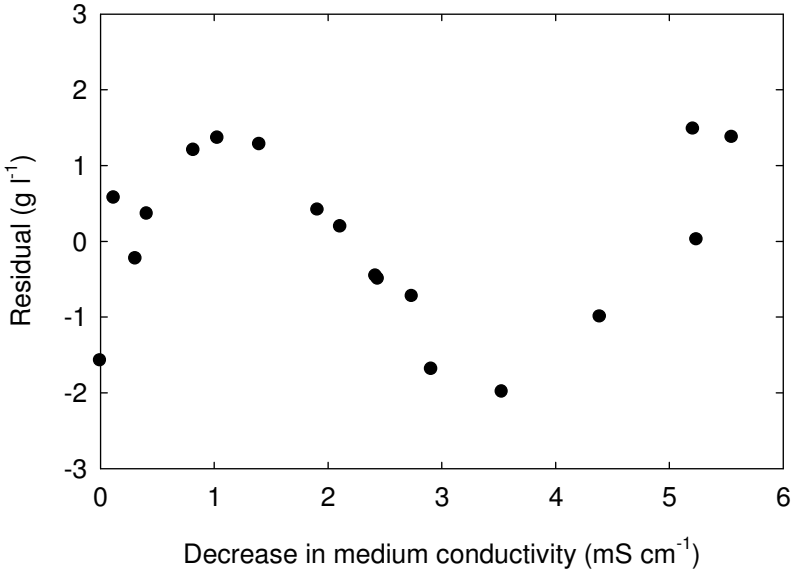
Answer: $y = 2.10x + 1.58$, where y is increase in biomass concentration in g l^{-1} and x is decrease in medium conductivity in mS cm^{-1}.

(b)

The residuals are calculated as the difference between the measured values for increase in biomass concentration and the values for y obtained from the equation in **(a)**. The results are listed and plotted below.

Decrease in medium conductivity (mS cm^{-1})	Residual in increase in biomass concentration (g l^{-1})
0	–1.58
0.12	0.57
0.31	–0.23
0.41	0.36
0.82	1.20
1.03	1.36
1.40	1.28
1.91	0.41

2.11	0.19
2.42	–0.46
2.44	–0.50
2.74	–0.73
2.91	–1.69
3.53	–1.99
4.39	–1.00
5.21	1.48
5.24	0.02
5.55	1.37



The residuals are not randomly distributed: they are mainly positive at low values of decrease in medium conductivity, then negative, then positive again. Therefore, the straight line fit of the data cannot be considered a very good one.

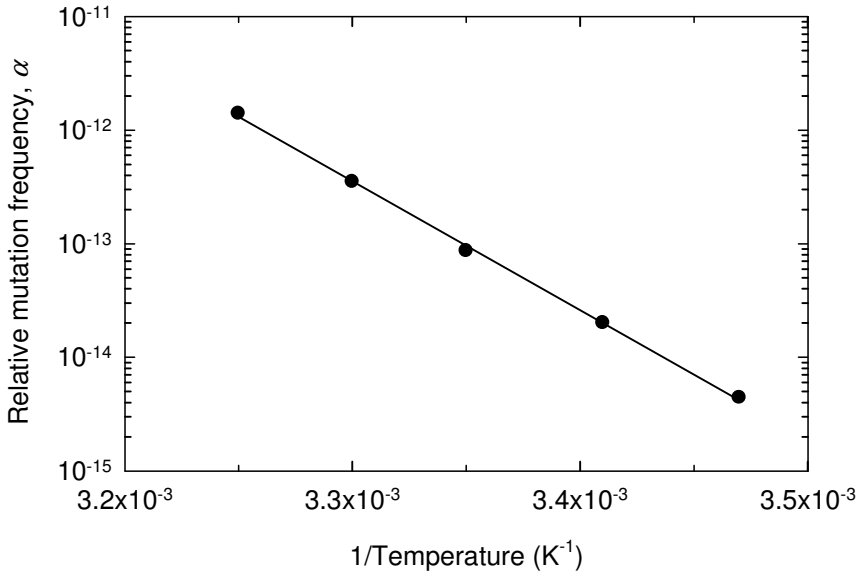
Answer: The straight line data fit is not very good because the residuals are not randomly distributed.

3.11 Nonlinear model: calculation of parameters

(a)

The proposed model equation has the general form of Eq. (3.16); therefore, if the model is suitable, a plot of α versus $1/T$ on semi-logarithmic coordinates will give a straight line. As T in the equation is absolute temperature, °C must first be converted to degrees Kelvin using Eq. (2.27). The data are listed and plotted below.

Temperature (°C)	Temperature (K)	$1/T$ (K^{-1})	Relative mutation frequency, α
15	288.15	3.47×10^{-3}	4.4×10^{-15}
20	293.15	3.41×10^{-3}	2.0×10^{-14}
25	298.15	3.35×10^{-3}	8.6×10^{-14}
30	303.15	3.30×10^{-3}	3.5×10^{-13}
35	308.15	3.25×10^{-3}	1.4×10^{-12}



As the data give a straight line on semi-logarithmic coordinates, the model can be considered to fit the data well.

Answer: The model fits the data well

(b)

The equation for the straight line in **(a)** is:

$$y = 9.66 \times 10^{24}\, e^{-26{,}121x}$$

where y is relative mutation frequency and x is reciprocal temperature in units of K^{-1}. For dimensional homogeneity, the exponent must be dimensionless (Section 2.1.3) so that –26,121 has units of K. Therefore, E/R in the model equation is equal to 26,121 K. Calculating E using R = 8.3144 J K^{-1} $gmol^{-1}$ (Appendix B):

$$E = (26{,}121\ \text{K})(8.3144\ \text{J K}^{-1}\,\text{gmol}^{-1}) = 217{,}180.4\ \text{J gmol}^{-1} = 217.2\ \text{kJ gmol}^{-1}$$

Answer: 217.2 kJ $gmol^{-1}$

(c)

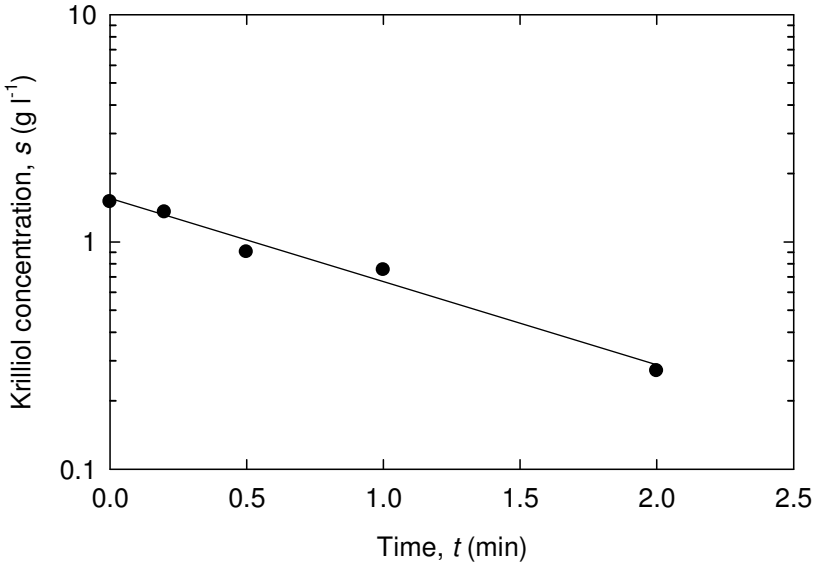
From the equation in **(b)**, α_0 is equal to 9.66×10^{24}.

Answer: 9.66×10^{24}

3.12 Nonlinear kinetics

(a)

The proposed model equation has the general form of Eq. (3.16); therefore, if the model is suitable, a plot of krilliol concentration s versus time t on semi-logarithmic coordinates will give a straight line. The data are plotted below.



As the data give a straight line on semi-logarithmic coordinates, the model can be considered to fit the data well.

Answer: The model fits the data well

(b)

The equation for the straight line in **(a)** is:

$$y = 1.53e^{-0.847x}$$

where y is krilliol concentration in units of g l^{-1} and x is time in units of min. For dimensional homogeneity, the exponent must be dimensionless (Section 2.1.3) so that –0.847 has units of min^{-1}. Therefore, $k = 0.847$ min^{-1}.

Answer: 0.847 min^{-1}

(c)

From the equation in **(b)**, at $y = 0.05$ g l^{-1}:

$$0.05 = 1.53e^{-0.847x}$$

or

$$0.0327 = e^{-0.847x}$$

Using Eqs (E.1) and (E.2) in Appendix E to solve for x:

$$\ln 0.0327 = -0.847x$$

$$-3.420 = -0.847x$$

$x = 4.0$

The time required is calculated as 4.0 min. However, as this value falls beyond the range of data values used to determine the correlation in **(b)**, we cannot be sure that first-order kinetics continues to be a valid model for the reaction at this time.

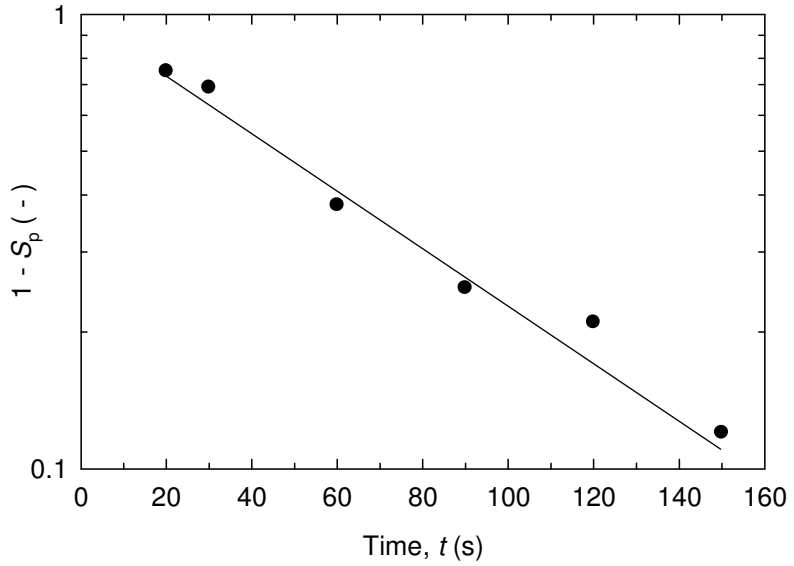
Answer: 4.0 min, assuming that the reaction continues to follow first-order kinetics at this time

3.13 Nonlinear data correlation

(a)

The proposed model equation has the general form of Eq. (3.16); therefore, if the model is suitable, a plot of $1 - S_p$ versus time t on semi-logarithmic coordinates will give a straight line. The data are listed and plotted below.

Time (s)	$1 - S_p$ (–)
20	0.75
30	0.69
60	0.38
90	0.25
120	0.21
150	0.12



As the data give a straight line on semi-logarithmic coordinates, the model can be considered to fit the data well.

Answer: The model fits the data well

(b)

The equation for the straight line in **(a)** is:

$$y = 0.958e^{-0.0136x}$$

where y is the dimensionless variable $(1 - S_p)$ and x is time in units of s. For dimensional homogeneity, the exponent must be dimensionless (Section 2.1.3) so that –0.0136 has units of s^{-1}. Therefore, $k = 0.0136\ s^{-1}$.

Answer: $0.0136\ s^{-1}$

(c)

If $S_p = 0.5$, $(1 - S_p) = 0.5$. From the equation in **(b)**, at $y = 0.5$:

$$0.5 = 0.958\,e^{-0.0136x}$$

or

$$0.5219 = e^{-0.0136x}$$

Using Eqs (E.1) and (E.2) in Appendix E to solve for x:

$$\ln 0.5219 = -0.0136x$$
$$-0.6503 = -0.0136x$$
$$x = 47.8$$

The time required is 48 s.

Answer: 48 s

(d)

If $S_p = 0.95$, $(1 - S_p) = 0.05$. From the equation in **(b)**, at $y = 0.05$:

$$0.05 = 0.958\,e^{-0.0136x}$$

or

$$0.0522 = e^{-0.0136x}$$

Using Eqs (E.1) and (E.2) in Appendix E to solve for x:

$$\ln 0.0522 = -0.0136x$$
$$-2.953 = -0.0136x$$
$$x = 217$$

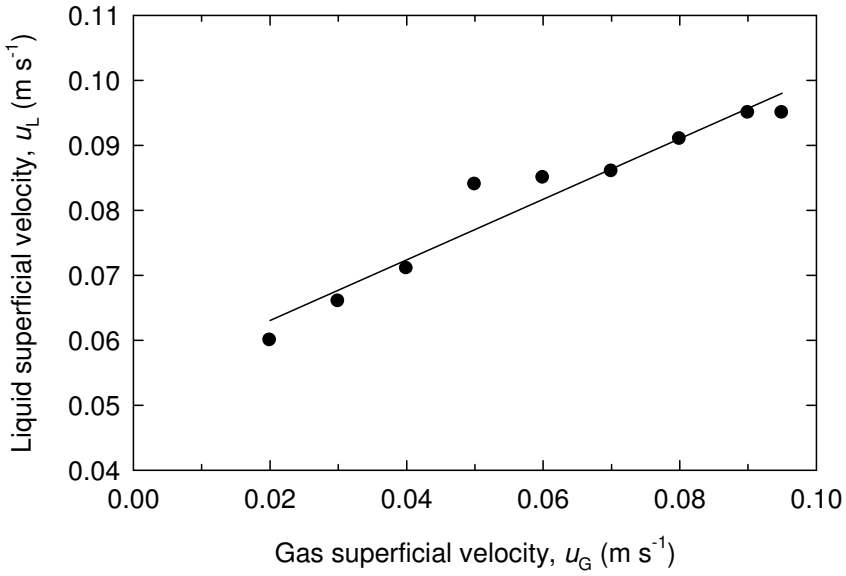
The time required is calculated as 217 s. However, as this value falls beyond the range of data values used to determine the correlation in **(b)**, we cannot be sure that the model equation is valid at this time.

Answer: 217 s, assuming that the model equation for homogenisation remains valid at this time

3.14 Discriminating between rival models

(a)

The results are plotted below using linear coordinates.



The data are reasonably well fitted using a linear model. The linear least-squares equation for the straight line is:

$$y = 0.466x + 0.054$$

where y is liquid superficial velocity in units of m s^{-1} and x is gas superficial velocity in units of m s^{-1}. The residuals between the measured values for liquid superficial velocity and the values for y obtained using the above equation are calculated as follows:

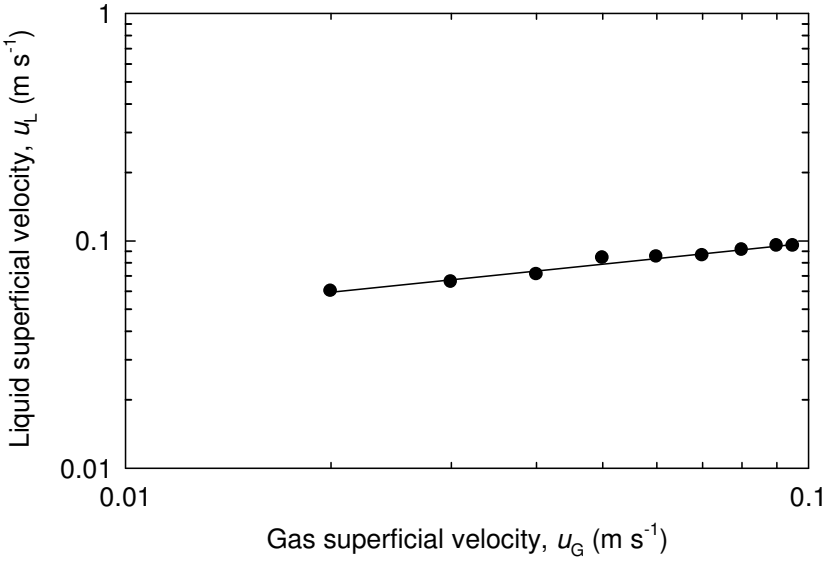
Gas superficial velocity (m s^{-1})	Residual in u_L (m s^{-1})
0.02	–0.0033
0.03	–0.0020
0.04	–0.0016
0.05	0.0067
0.06	0.0030
0.07	–0.0006
0.08	–0.0003
0.09	–0.0009
0.095	–0.0033

The sum of the squares of these residuals is 8.4×10^{-5}.

Answer: The linear model $y = 0.466x + 0.054$ fits the data reasonably well, where y is liquid superficial velocity in m s^{-1} and x is gas superficial velocity in m s^{-1}. The sum of squares of the residuals is 8.4×10^{-5}.

(b)

The proposed power-law equation has the same form as Eq. (3.11). Therefore, if the power-law model is suitable, the data will give a straight line when plotted on log–log coordinates. The data are plotted below.



The data are reasonably well fitted using the power-law model. The equation for the straight line in the plot is:

$$y = 0.199x^{0.309}$$

where y is liquid superficial velocity in m s^{-1} and x is gas superficial velocity in m s^{-1}. The residuals between the measured values for liquid superficial velocity and the values for y obtained from the above equation are calculated as follows:

Gas superficial velocity (m s^{-1})	Residual in u_L (m s^{-1})
0.02	0.0006
0.03	–0.0013
0.04	–0.0026
0.05	0.0051
0.06	0.0016
0.07	–0.0015
0.08	–0.0002
0.09	0.0004
0.095	–0.0012

The sum of squares of these residuals is 4.2×10^{-5}.

Answer: The power-law model fits the data reasonably well with the sum of squares of the residuals equal to 4.2×10^{-5}.

(c)

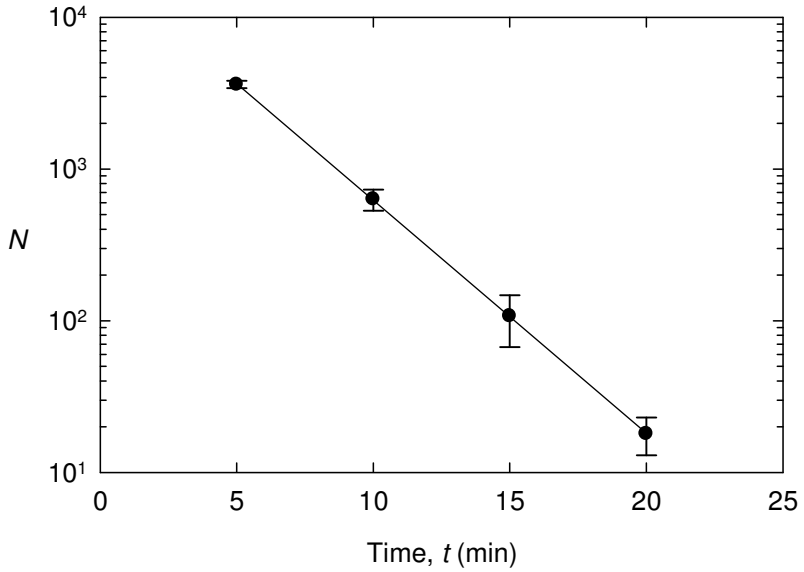
Both the linear and non-linear models fit the data reasonably well. However, the non-linear model can be considered a better fit because the sum of squares of the residuals is smaller than for the linear model.

Answer: Non-linear

3.15 Nonlinear model: calculation of parameters

(a), (b)

The proposed model equation has the general form of Eq. (3.16); therefore, if the model is suitable, a plot of N versus t on semi-logarithmic coordinates will give a straight line. The data are plotted below with the standard errors shown using error bars.



Note the distortion in the relative error values as a consequence of linearisation using semi-log coordinates.

(c), (d)

The equation for the straight line is:

$$y = 2.13\times10^{4}\, e^{-0.353x}$$

where y is the number of viable cells (dimensionless) and x is time in units of min. For dimensional homogeneity, the exponent must be dimensionless (Section 2.1.3) so that –0.353 has units of min^{-1}. Because number of cells is a dimensionless variable, the term 2.13×10^4 in the equation is also dimensionless. Therefore, $k_d = 0.353\ \text{min}^{-1}$ and $N_0 = 2.13 \times 10^4$.

Answer: $k_d = 0.353\ \text{min}^{-1}$ and $N_0 = 2.13 \times 10^4$; k_d has dimensions T^{-1} and units min^{-1}; N_0 is dimensionless with no units

3.16 Nonlinear model

(a)

The Langmuir equation has the general form:

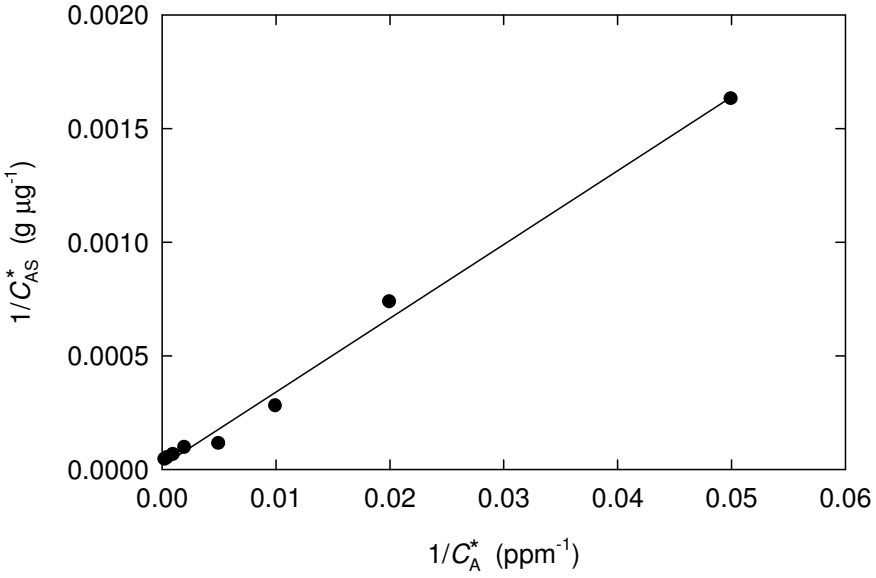
$$y = \frac{x}{A + Bx}$$

where x and y are variables and A and B are constants. As noted in Table 3.1, a linear form of this equation is produced by plotting $1/y$ versus $1/x$. Inverting both sides of the Langmuir equation gives:

$$\frac{1}{C_{AS}^*} = \frac{1}{C_{ASm} K_A C_A^*} + \frac{1}{C_{ASm}}$$

In this equation, C_{AS}^* and C_A^* are the two variables; C_{ASm} and K_A are model constants. Therefore, a plot of $1/C_{AS}^*$ versus $1/C_A^*$ on linear coordinates should give a straight line. Using the data provided, values of $1/C_A^*$ and $1/C_{AS}^*$ are listed and plotted below.

$1/C_A^*$ (ppm^{-1})	$1/C_{AS}^*$ (g μg^{-1})
0.050	1.63×10^{-3}
0.020	7.35×10^{-4}
0.010	2.79×10^{-4}
0.005	1.12×10^{-4}
0.002	9.52×10^{-5}
0.001	6.41×10^{-5}
0.0005	5.08×10^{-5}
0.000286	4.39×10^{-5}



(b), (c)

The linear least-squares fit for these data is:

$$y = 0.0325x + 1.58 \times 10^{-5}$$

where y is $1/C_{AS}^*$ in units of g μg^{-1} and x is $1/C_A^*$ in units of ppm^{-1}. Comparison of this equation with the linearised form of the Langmuir equation shows that the intercept 1.58×10^{-5} is equal to $1/C_{ASm}$, and the slope 0.0325 is equal to $1/(C_{ASm}K_A)$. From the value for the intercept:

$$C_{ASm} = \frac{1}{1.58 \times 10^{-5}} = 6.33 \times 10^4$$

For dimensional homogeneity in the Langmuir equation, the dimensions and units of C_{ASm} must be the same as those of C^*_{AS}; therefore C_{ASm} is dimensionless with units μg g^{-1}. Similarly, from the value for the slope:

$$C_{ASm}K_A = \frac{1}{0.0325} = 30.8$$

Applying the above result for C_{ASm}:

$$K_A = \frac{30.8}{6.33\times10^4} = 4.87\times10^{-4}$$

For dimensional homogeneity in the Langmuir equation, the dimensions and units of $C_{ASm}K_AC^*_A$ must be the same as those of C^*_{AS}; therefore K_A is dimensionless with units ppm^{-1}.

Answer: $K_A = 4.87 \times 10^{-4}$ ppm^{-1}, K_A is dimensionless; $C_{ASm} = 6.33 \times 10^4$ μg g^{-1}, C_{ASm} is dimensionless

(d)

ppm for solids and liquids represents a mass fraction equivalent to μg g^{-1} or mg per 10^3 g of solution (Section 2.4.5). For dilute aqueous solutions, we can assume that the density of the solution is the same as that of water, i.e. 1 g cm^{-3} (Section 2.4.1) or 10^3 g l^{-1}. Therefore, ppm is equivalent to mg per l of solution.

The liquid-phase Cd concentration is reduced from 120 ppm to 25 ppm. Therefore, the change in Cd concentration in the liquid is (120 – 25) mg l^{-1} = 95 mg l^{-1}. To achieve this reduction in Cd concentration in 5000 l of waste water, the mass of Cd that must be adsorbed by the algae is 95 mg l^{-1} × 5000 l = 4.75×10^5 mg = 4.75×10^8 μg.

From the equation derived for this algal system, when the residual liquid-phase Cd concentration C^*_A is 25 ppm or $1/C^*_A$ is 0.04 ppm^{-1}, $1/C^*_{AS}$ is 1.32×10^{-3} g μg^{-1}. Therefore, the equilibrium value of C^*_{AS} is 760 μg g^{-1} dry weight. If a total of 4.75×10^8 μg of Cd must be adsorbed by the biomass, the mass of algae required is (4.75×10^8 μg)/(760 μg g^{-1} dry weight) = 6.25×10^5 g dry weight = 625 kg dry weight.

Answer: 625 kg dry weight

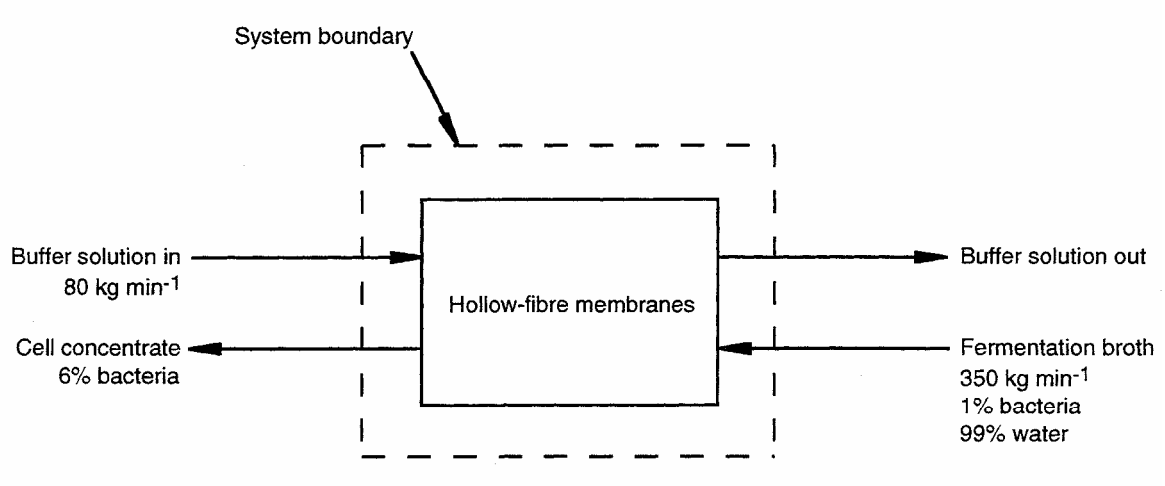
Chapter 4

Material Balances

4.1 Cell concentration using membranes

1. Assemble

(i) Flow sheet



(ii) System boundary

The system boundary is shown on the flow sheet.

(iii) Reaction equation

No reaction occurs.

2. Analyse

(i) Assumptions

– steady state

– no leaks

– only water passes across the membrane

(ii) Extra data

No extra data are required.

(iii) Basis

1 min, or 350 kg of fermentation broth

(iv) Compounds involved in reaction

No compounds are involved in reaction.

(v) Mass balance equation

As there is no reaction, the appropriate mass balance equation is Eq. (4.3):

mass in = mass out

3. Calculate

(i) Calculation table

The calculation table below shows all given quantities in kg. The total mass of cell concentrate is denoted C; the total mass of buffer out is denoted B. The columns for water refer to water originating in the fermentation broth only: water in the buffer solution entering remains in the buffer solution leaving.

Stream	*In*				*Out*			
	Water	*Bacteria*	*Buffer*	*Total*	*Water*	*Bacteria*	*Buffer*	*Total*
Fermentation broth	346.5	3.5	0	350	–	–	–	–
Buffer solution in	0	0	80	80	–	–	–	–
Cell concentrate	–	–	–	–	?	$0.06C$	0	C
Buffer solution out	–	–	–	–	?	0	80	B
Total	346.5	3.5	80	430	?	$0.06C$	80	$C + B$

(ii) Mass balance calculations

Bacteria balance

$$3.5 \text{ kg bacteria in} = 0.06C \text{ kg bacteria out}$$

$$C = 58.3 \text{ kg}$$

Total mass balance

$$430 \text{ kg total mass in} = (C + B) \text{ kg total mass out}$$

Using the result for C:

$$B = 371.7 \text{ kg}$$

Water balance

346.5 kg water in = water out

Water out = 346.5 kg

These calculations allow completion of the mass balance table with all quantities in kg.

Stream	*In*				*Out*			
	Water	*Bacteria*	*Buffer*	*Total*	*Water*	*Bacteria*	*Buffer*	*Total*
Fermentation broth	346.5	3.5	0	350	–	–	–	–
Buffer solution in	0	0	80	80	–	–	–	–
Cell concentrate	–	–	–	–	54.8	3.50	0	58.3
Buffer solution out	–	–	–	–	291.7	0	80	371.7
Total	346.5	3.5	80	430	346.5	3.50	80	430

(iii) Check the results

All columns and rows of the completed table add up correctly.

4. Finalise

(a)

After rounding to three significant figures, the total flow rate of buffer solution out of the annulus is 372 kg min^{-1}.

Answer: 372 kg min^{-1}

(b)

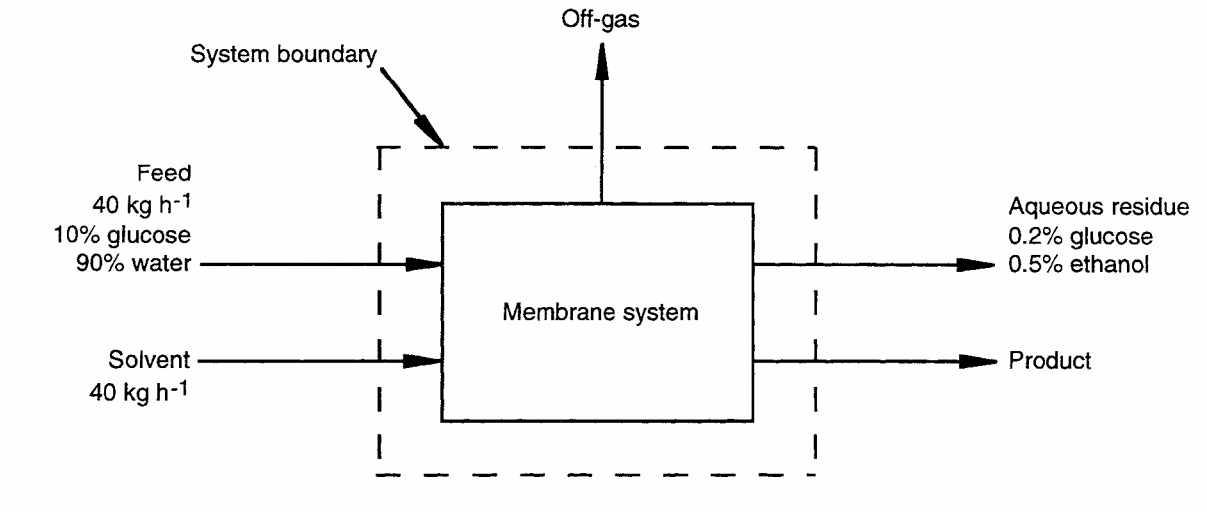
From the completed mass balance table, the total flow rate of cell concentrate from the membrane tubes is 58.3 kg min^{-1}.

Answer: 58.3 kg min^{-1}

4.2 Membrane reactor

1. Assemble

(i) Flow sheet



(ii) System boundary

The system boundary is shown on the flow sheet.

(iii) Reaction equation

$$C_6H_{12}O_6 \rightarrow 2C_2H_6O + 2CO_2$$

2. Analyse

(i) Assumptions

– steady state

– no leaks

– yeast cells do not grow or dislodge from the membrane

– no evaporation

– all CO_2 produced leaves in the off-gas

– no side reactions

(ii) Extra data

Molecular weights calculated from Table C.1 (Appendix C):

– Glucose = 180.2

– Ethanol = 46.1

– CO_2 = 44.0

(iii) Basis

1 h, or 40 kg of feed solution

(iv) Compounds involved in reaction

Glucose, ethanol and carbon dioxide are involved in the reaction.

(v) Mass balance equations

For glucose, ethanol and carbon dioxide, the appropriate mass balance equation is Eq. (4.2):

mass in + mass generated = mass out + mass consumed

For water, solvent and total mass, the appropriate mass balance equation is Eq. (4.3):

mass in = mass out

3. Calculate

(i) Calculation table

The calculation table below shows all given quantities in kg. The total mass of aqueous residue is denoted *R*; the total mass of product out is denoted *P*; the total mass of carbon dioxide out is denoted *G*.

Stream	*In*						*Out*					
	Glucose	*Ethanol*	*CO_2*	*Solvent*	*H_2O*	*Total*	*Glucose*	*Ethanol*	*CO_2*	*Solvent*	*H_2O*	*Total*
Feed	4	0	0	0	36	40	–	–	–	–	–	–
Solvent	0	0	0	40	0	40	–	–	–	–	–	–
Aqueous residue	–	–	–	–	–	–	0.002*R*	0.005*R*	0	0	?	*R*
Product	–	–	–	–	–	–	0	?	0	?	0	*P*
Off-gas	–	–	–	–	–	–	0	0	*G*	0	0	*G*
Total	4	0	0	40	36	80	0.002*R*	?	*G*	?	?	*R* + *P* + *G*

(ii) Mass balance calculations

Solvent balance

Solvent is a tie component.

40 kg solvent in = solvent out

Solvent out = 40 kg

Water balance

Water is a tie component.

36 kg water in = water out

Water out = 36 kg

As water appears on the Out side of the table only in the aqueous residue stream:

$$0.002R + 0.005R + 36 = R$$

$$R = 36.25 \text{ kg}$$

Therefore, the mass of residual glucose in the aqueous residue stream = $0.002R$ = 0.073 kg, and the mass of ethanol in the aqueous residue stream = $0.005R$ = 0.181 kg.

Glucose balance

4 kg glucose in + 0 kg glucose generated = 0.073 kg glucose out + glucose consumed

Glucose consumed = 3.927 kg

Converting the glucose consumed to molar terms:

$$3.927 \text{ kg glucose} = 3.927 \text{ kg} \,.\, \left| \frac{1 \text{ kgmol}}{180.2 \text{ kg}} \right| = 0.0218 \text{ kgmol}$$

From the reaction stoichiometry, conversion of this amount of glucose generates 2 × 0.0218 = 0.0436 kgmol ethanol and 2 × 0.0218 = 0.0436 kgmol CO_2. Converting these molar quantities to mass:

$$0.0436 \text{ kgmol ethanol} = 0.0436 \text{ kgmol} \,.\, \left| \frac{46.1 \text{ kg}}{1 \text{ kgmol}} \right| = 2.010 \text{ kg}$$

$$0.0436 \text{ kgmol } CO_2 = 0.0436 \text{ kgmol} \,.\, \left| \frac{44.0 \text{ kg}}{1 \text{ kgmol}} \right| = 1.92 \text{ kg}$$

CO_2 balance

0 kg CO_2 in + 1.92 kg CO_2 generated = CO_2 out + 0 kg CO_2 consumed

CO_2 out = 1.92 kg = G

Ethanol balance

0 kg ethanol in + 2.010 kg ethanol generated = ethanol out + 0 kg ethanol consumed

Ethanol out = 2.010 kg

Ethanol leaves the system only in the product and aqueous residue streams. Therefore:

Ethanol out in the product stream = (2.010 − 0.181) kg = 1.829 kg

As the product stream consists of ethanol and solvent only:

$$P = (1.829 + 40) \text{ kg} = 41.83 \text{ kg}$$

These calculations allow completion of the mass balance table with all quantities in kg.

Stream	*In*						*Out*					
	Glucose	*Ethanol*	*CO_2*	*Solvent*	*H_2O*	*Total*	*Glucose*	*Ethanol*	*CO_2*	*Solvent*	*H_2O*	*Total*
Feed	4	0	0	0	36	40	–	–	–	–	–	–
Solvent	0	0	0	40	0	40	–	–	–	–	–	–
Aqueous residue	–	–	–	–	–	–	0.073	0.181	0	0	36	36.25
Product	–	–	–	–	–	–	0	1.829	0	40	0	41.83
Off-gas	–	–	–	–	–	–	0	0	1.92	0	0	1.92
Total	4	0	0	40	36	80	0.073	2.010	1.92	40	36	80.00

(iii) Check the results

All columns and rows of the completed table add up correctly.

4. Finalise

(a)

1.829 kg of ethanol are contained in 41.83 kg of product stream. The ethanol concentration is therefore $1.829/41.83 \times 100\% = 4.4\%$.

Answer: 4.4%

(b)

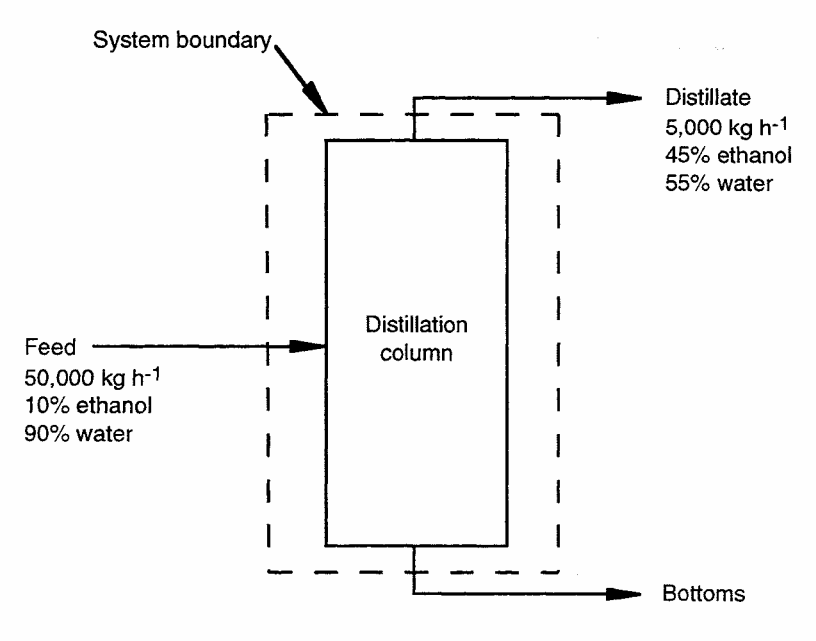
From the completed mass balance table, the mass flow rate of CO_2 is 1.92 kg h^{-1}.

Answer: 1.92 kg h^{-1}

4.3 Ethanol distillation

1. Assemble

(i) Flow sheet



(ii) System boundary

The system boundary is shown on the flow sheet.

(iii) Reaction equation

No reaction occurs.

2. Analyse

(i) Assumptions

– steady state

– no leaks

(ii) Extra data

No extra data are required.

(iii) Basis

1 h, or 50,000 kg of feed

(iv) Compounds involved in reaction

No compounds are involved in reaction.

(v) Mass balance equation

As there is no reaction, the appropriate mass balance equation is Eq. (4.3):

mass in = mass out

3. Calculate

(i) Calculation table

The calculation table shows all given quantities in kg.

Stream	*In*			*Out*		
	Ethanol	*Water*	*Total*	*Ethanol*	*Water*	*Total*
Feed	5000	45,000	50,000	–	–	–
Distillate	–	–	–	2250	2750	5000
Bottoms	–	–	–	?	?	?
Total	5000	45,000	50,000	?	?	?

(ii) Mass balance calculations

Total mass balance

$50,000$ kg total mass in = total mass out

Total mass out = $50,000$ kg

Therefore, from the Total column on the Out side of the table:

Bottoms out = $(50,000 - 5000)$ kg = $45,000$ kg

Ethanol balance

5000 kg ethanol in = ethanol out

Ethanol out = 5000 kg

From the Ethanol column of the Out side of the table:

Ethanol out in the bottoms = $(5000 - 2250)$ kg = 2750 kg

Water balance

$45,000$ kg water in = water out

Water out = $45,000$ kg

From the Water column of the Out side of the table:

Water out in the bottoms = $(45,000 - 2750)$ kg = $42,250$ kg

These calculations allow completion of the mass balance table with all quantities in kg.

Stream	*In*			*Out*		
	Ethanol	*Water*	*Total*	*Ethanol*	*Water*	*Total*
Feed	5000	45,000	50,000	–	–	–
Distillate	–	–	–	2250	2750	5000
Bottoms	–	–	–	2750	42,250	45,000
Total	5000	45,000	50,000	5000	45,000	50,000

(iii) Check the results

All columns and rows of the completed table add up correctly.

4. Finalise

(a)

The bottoms contains 2750 kg ethanol and 42,250 kg water in a total of 45,000 kg. Therefore, the composition is 2750/45,000 × 100% = 6.1% ethanol, and 42,250/45,000 × 100% = 93.9% water.

Answer: 6.1% ethanol, 93.9% water

(b)

Directly from the table, the rate of alcohol loss in the bottoms is 2750 kg h^{-1}.

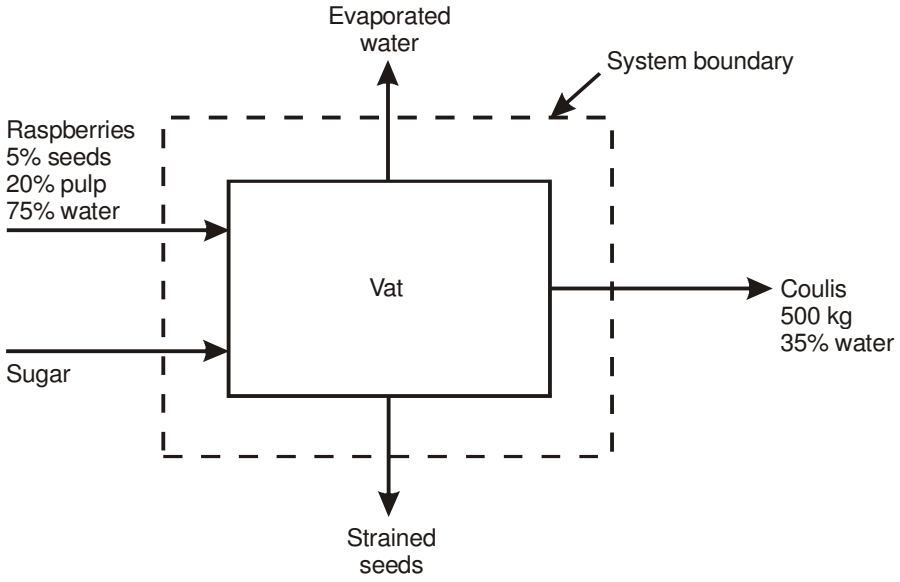
Answer: 2750 kg h^{-1}

4.4 Raspberry coulis manufacture

1. Assemble

(i) Flow sheet

From Table A.3 (Appendix A), 1 tonne = 10^3 kg.



(ii) System boundary

The system boundary is shown on the flow sheet.

(iii) Reaction equation

No reaction occurs.

2. Analyse

(i) Assumptions

– no leaks

– all seeds are removed in the straining process

(ii) Extra data

No extra data are required.

(iii) Basis

1 day, or 500 kg of coulis

(iv) Compounds involved in reaction

No compounds are involved in reaction.

(v) Mass balance equation

As there is no reaction, the appropriate mass balance equation is Eq. (4.3):

mass in = mass out

3. Calculate

(i) Calculation table

The calculation table below shows all given quantities in kg. The total mass of raspberries in is denoted R; the total mass of evaporated water out is denoted W; the total mass of strained seeds out is denoted S. As the inlet raspberry:sugar mass ratio is 3.5:1, the mass of sugar added is (R kg)/3.5 = 0.286R kg.

Stream	*In*					*Out*				
	Seeds	*Pulp*	*Water*	*Sugar*	*Total*	*Seeds*	*Pulp*	*Water*	*Sugar*	*Total*
Raspberries	0.05R	0.20R	0.75R	0	R	–	–	–	–	–
Sugar	0	0	0	0.286R	0.286R	–	–	–	–	–
Strained seeds	–	–	–	–	–	S	0	0	0	S
Evaporated water	–	–	–	–	–	0	0	W	0	W
Coulis	–	–	–	–	–	0	?	175	?	500
Total	0.05R	0.20R	0.75R	0.286R	1.286R	S	?	175 + W	?	500 + S + W

(ii) Mass balance calculations

Seeds balance

Seeds is a tie component.

$$0.05R \text{ kg seeds in} = S \text{ kg seeds out}$$

$$0.05R = S \tag{1}$$

Total mass balance

$$1.286R \text{ kg total mass in} = (500 + S + W) \text{ kg total mass out}$$

Using the result from (1), the total mass balance becomes:

$$1.286R \text{ kg total mass in} = (500 + 0.05R + W) \text{ kg total mass out}$$

$$W = 1.236R - 500 \tag{2}$$

Water balance

$$0.75R \text{ kg water in} = (W + 175) \text{ kg water out}$$

Using the result from (2):

$$0.75R \text{ kg water in} = (1.236R - 500 + 175) \text{ kg water out}$$

$$325 = 0.486R$$

$$R = 668.7 \text{ kg} \tag{3}$$

Combining the results from (3) and (2) gives:

$$W = 326.5 \text{ kg}$$

Combining the results from (3) and (1) gives:

$$S = 33.4 \text{ kg}$$

Pulp balance

Pulp is a tie component.

$$0.20R \text{ kg pulp in} = \text{Pulp out}$$

Using the result from (3):

$$\text{Pulp out} = 133.7 \text{ kg}$$

Sugar balance

Sugar is a tie component.

$$0.286R \text{ kg sugar in} = \text{Sugar out}$$

Using the result from (3):

$$\text{Sugar out} = 191.2 \text{ kg}$$

These calculations allow completion of the mass balance table with all quantities in kg.

Stream	*In*					*Out*				
	Seeds	*Pulp*	*Water*	*Sugar*	*Total*	*Seeds*	*Pulp*	*Water*	*Sugar*	*Total*
Raspberries	33.4	133.7	501.5	0	668.7	–	–	–	–	–
Sugar	0	0	0	191.2	191.2	–	–	–	–	–
Strained seeds	–	–	–	–	–	33.4	0	0	0	33.4
Evaporated water	–	–	–	–	–	0	0	326.5	0	326.5
Coulis	–	–	–	–	–	0	133.7	175	191.2	500
Total	33.4	133.7	501.5	191.2	859.9	33.4	133.7	501.5	191.2	859.9

(iii) Check the results

All columns and rows of the completed table add up correctly.

4. Finalise

(a)

The mass of raspberries processed each day is 668.7 kg. Therefore, per week, 7 × 668.7 kg = 4681 kg = 4.7 tonnes of raspberries are required.

Answer: 4.7 tonnes

(b)

The mass of sugar required per week is 7 × 191.2 kg = 1338.4 kg = 1.3 tonnes.

Answer: 1.3 tonnes

(c)

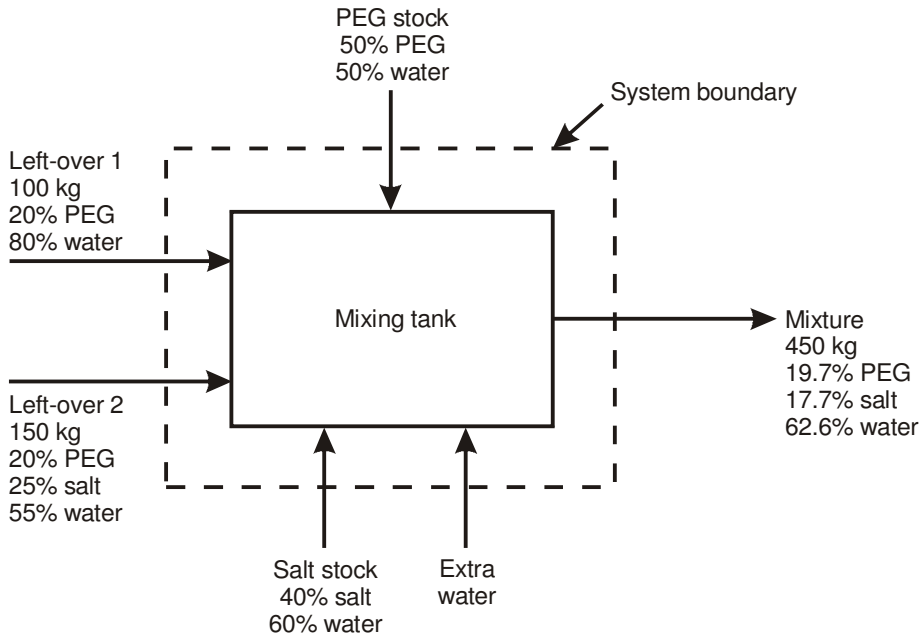
There are 191.2 kg of sugar in 500 kg of coulis. Therefore, the sugar content is (191.2 kg)/(500 kg) × 100% = 38.2%.

Answer: 38%

4.5 Polyethylene glycol–salt mixture

1. Assemble

(i) Flow sheet



(ii) System boundary

The system boundary is shown on the flow sheet.

(iii) Reaction equation

No reaction occurs.

2. Analyse

(i) Assumptions

– no leaks

(ii) Extra data

No extra data are required.

(iii) Basis

450 kg of PEG–salt mixture

(iv) Compounds involved in reaction

No compounds are involved in reaction.

(v) Mass balance equation

As there is no reaction, the appropriate mass balance equation is Eq. (4.3):

mass in = mass out

3. Calculate

(i) Calculation table

The calculation table below shows all given quantities in kg. The total mass of PEG stock in is denoted P; the total mass of salt stock in is denoted S; the total mass of extra water in is denoted W.

Stream	*In*				*Out*			
	PEG	*Salt*	*Water*	*Total*	*PEG*	*Salt*	*Water*	*Total*
Left-over 1	20	0	80	100	–	–	–	–
Left-over 2	30	37.5	82.5	150	–	–	–	–
PEG stock	$0.5P$	0	$0.5P$	P	–	–	–	–
Salt stock	0	$0.4S$	$0.6S$	S	–	–	–	–
Extra water	0	0	W	W	–	–	–	–
Mixture	–	–	–	–	88.65	79.65	281.7	450
Total	$50 + 0.5P$	$37.5 + 0.4S$	$162.5 + 0.5P + 0.6S + W$	$250 + P + S + W$	88.65	79.65	281.7	450

(ii) Mass balance calculations

Total mass balance

$(250 + P + S + W)$ kg total mass in $= 450$ kg total mass out

$$W = 200 - P - S \tag{1}$$

PEG balance

$(50 + 0.5P)$ kg PEG in $= 88.65$ kg PEG out

$$P = 77.3 \tag{2}$$

Salt balance

$(37.5 + 0.4S)$ kg salt in $= 79.65$ kg salt out

$$S = 105.4 \tag{3}$$

Applying (2) and (3) to (1):

$$W = 200 - 77.3 - 105.4 = 17.3$$

These calculations allow completion of the mass balance table with all quantities in kg.

Stream	*In*				*Out*			
	PEG	*Salt*	*Water*	*Total*	*PEG*	*Salt*	*Water*	*Total*
Left-over 1	20	0	80	100	–	–	–	–
Left-over 2	30	37.5	82.5	150	–	–	–	–
PEG stock	38.65	0	38.65	77.3	–	–	–	–
Salt stock	0	42.16	63.24	105.4	–	–	–	–
Extra water	0	0	17.3	17.3	–	–	–	–
Mixture	–	–	–	–	88.65	79.65	281.7	450
Total	88.65	79.66	281.69	450	88.65	79.65	281.7	450

(iii) Check the results

All columns and rows of the completed table add up correctly to within round-off error.

4. Finalise

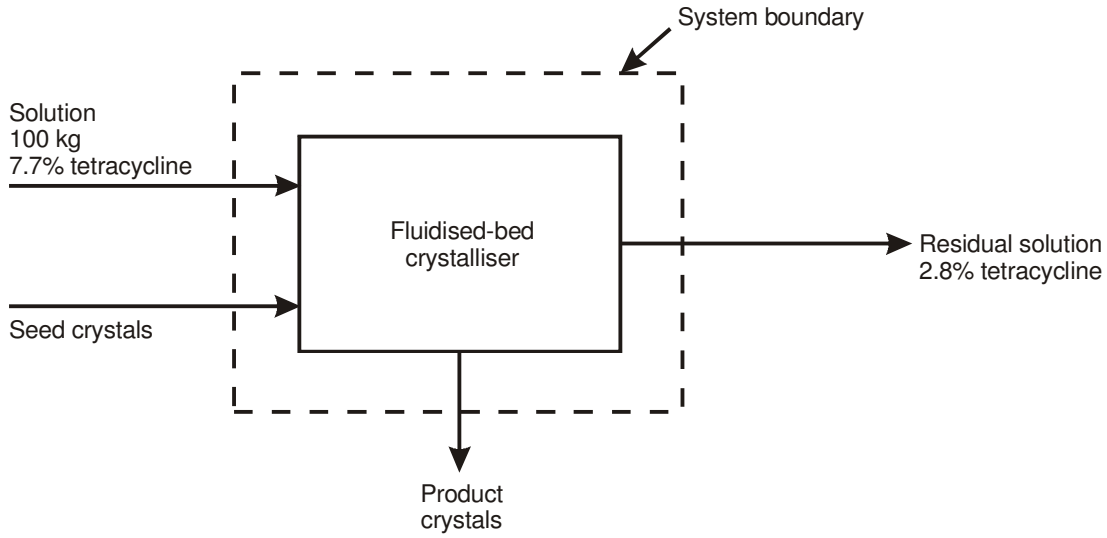
From the completed mass balance table, 77.3 kg of PEG stock, 105.4 kg of salt stock, and 17.3 kg of extra water are required.

Answer: 77 kg of PEG stock, 105 kg of salt stock, 17 kg of additional water

4.6 Tetracycline crystallisation

1. Assemble

(i) Flow sheet



(ii) System boundary

The system boundary is shown on the flow sheet.

(iii) Reaction equation

No reaction occurs.

2. Analyse

(i) Assumptions

– no leaks

– the solvent is water

– seed and product crystals contain no solvent or other impurities

(ii) Extra data

No extra data are required.

(iii) Basis

100 kg of supersaturated solution

(iv) Compounds involved in reaction

No compounds are involved in reaction.

(v) Mass balance equation

As there is no reaction, the appropriate mass balance equation is Eq. (4.3):

mass in = mass out

3. Calculate

(i) Calculation table

The calculation table below shows all given quantities in kg. The total mass of product crystals out is denoted C; the total mass of residual solution out is denoted R. Seed crystals are added to give a concentration of 40 ppm; from Section 2.4.5, this means 40 kg of crystals per 10^6 kg of crystal–solution mixture. As the concentration of seed crystals in the mixture is very low, we can say that seed crystals are added to 100 kg of mixture, so that $(100/10^6) \times 40$ kg = 0.004 kg of seed crystals are used.

Stream	*In*			*Out*		
	Tetracycline	*Water*	*Total*	*Tetracycline*	*Water*	*Total*
Solution	7.7	92.3	100	–	–	–
Seed crystals	0.004	0	0.004	–	–	–
Product crystals	–	–	–	C	0	C
Residual solution	–	–	–	$0.028R$	$0.972R$	R
Total	7.704	92.3	100.004	$C + 0.028R$	$0.972R$	$C + R$

(ii) Mass balance calculations

Tetracycline balance

$$7.704 \text{ kg tetracycline in} = (C + 0.028R) \text{ kg tetracycline out}$$

$$C = 7.704 - 0.028R \tag{1}$$

Total mass balance

$$100.004 \text{ kg total mass in} = (C + R) \text{ kg total mass out}$$

Applying the result from (1):

$$100.004 = 7.704 - 0.028R + R$$

$$R = 94.959 \qquad (2)$$

Using this result in (1) gives:

$$C = 5.045$$

These calculations allow completion of the mass balance table with all quantities in kg.

Stream	*In*			*Out*		
	Tetracycline	*Water*	*Total*	*Tetracycline*	*Water*	*Total*
Solution	7.7	92.3	100	–	–	–
Seed crystals	0.004	0	0.004	–	–	–
Product crystals	–	–	–	5.045	0	5.045
Residual solution	–	–	–	2.659	92.30	94.959
Total	7.704	92.3	100.004	7.704	92.30	100.004

(iii) Check the results

All columns and rows of the completed table add up correctly.

4. Finalise

(a)

From the completed mass balance table, the mass of the residual solution is 94.959 kg.

Answer: 95 kg

(b)

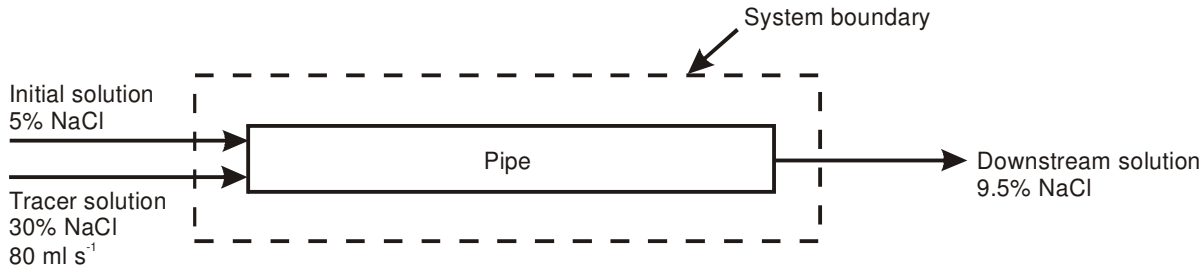
From the completed mass balance table, the mass of product crystals is 5.045 kg. (We could subtract the mass of seed crystals added to give the mass of crystals produced by the crystalliser = (5.045 – 0.004) kg = 5.041 kg.)

Answer: 5.0 kg

4.7 Flow rate calculation

1. Assemble

(i) Flow sheet



(ii) System boundary

The system boundary is shown on the flow sheet.

(iii) Reaction equation

No reaction occurs.

2. Analyse

(i) Assumptions

– steady state

– no leaks

– solutions are well mixed in the pipe

– temperature is 25°C

(ii) Extra data

The density of 30% NaCl in water is taken to be approximately 1.2 g ml^{-1}, based on the value of 1.19443 g ml^{-1} for a 26% NaCl solution at 25°C (Table 2-90, *Perry's Chemical Engineers' Handbook*, 8th edition, McGraw-Hill).

(iii) Basis

1 s

(iv) Compounds involved in reaction

No compounds are involved in reaction.

(v) Mass balance equation

As there is no reaction, the appropriate mass balance equation is Eq. (4.3):

mass in = mass out

3. Calculate

(i) Calculation table

The flow rate of the 30% NaCl tracer solution is given as a volumetric rate. For mass balancing, we need to convert this to a mass flow rate. From the definition of density (Section 2.4.1), mass = density × volume; therefore the mass flow rate of tracer solution = 1.2 g ml^{-1} × 80 ml s^{-1} = 96.0 g s^{-1}.

The calculation table below shows all given quantities in g. The total mass of initial solution is denoted M; the total mass of downstream solution is denoted D.

Stream	*In*			*Out*		
	NaCl	*Water*	*Total*	*NaCl*	*Water*	*Total*
Initial solution	0.05M	0.95M	M	–	–	–
Tracer solution	28.8	67.2	96.0	–	–	–
Downstream solution	–	–	–	0.095D	0.905D	D
Total	28.8 + 0.05M	67.2 + 0.95M	96.0 + M	0.095D	0.905D	D

(ii) Mass balance calculations

Total mass balance

$(96.0 + M)$ g total mass in = D g total mass out

$$D = 96.0 + M \quad (1)$$

NaCl balance

$$(28.8 + 0.05M) \text{ g NaCl in} = 0.095D \text{ g NaCl out}$$

Applying the result from (1):

$$28.8 + 0.05M = 0.095\,(96.0 + M)$$

$$19.68 = 0.045M$$

$$M = 437.3 \quad (2)$$

Applying this result in (1):

$$D = 533.3$$

These calculations allow completion of the mass balance table with all quantities in g.

Stream	*In*			*Out*		
	NaCl	*Water*	*Total*	*NaCl*	*Water*	*Total*
Initial solution	21.9	415.4	437.3	–	–	–
Tracer solution	28.8	67.2	96.0	–	–	–
Downstream solution	–	–	–	50.7	482.6	533.3
Total	50.7	482.6	533.3	50.7	482.6	533.3

(iii) Check the results

All columns and rows of the completed table add up correctly.

4. Finalise

From the completed mass balance table, the mass flow rate of the initial 5% NaCl solution is 437.3 g s^{-1}. This can be converted to a volumetric flow rate using the density of 5% NaCl in water = 1.0325 g ml^{-1} (Table 2-90, *Perry's Chemical Engineers' Handbook*):

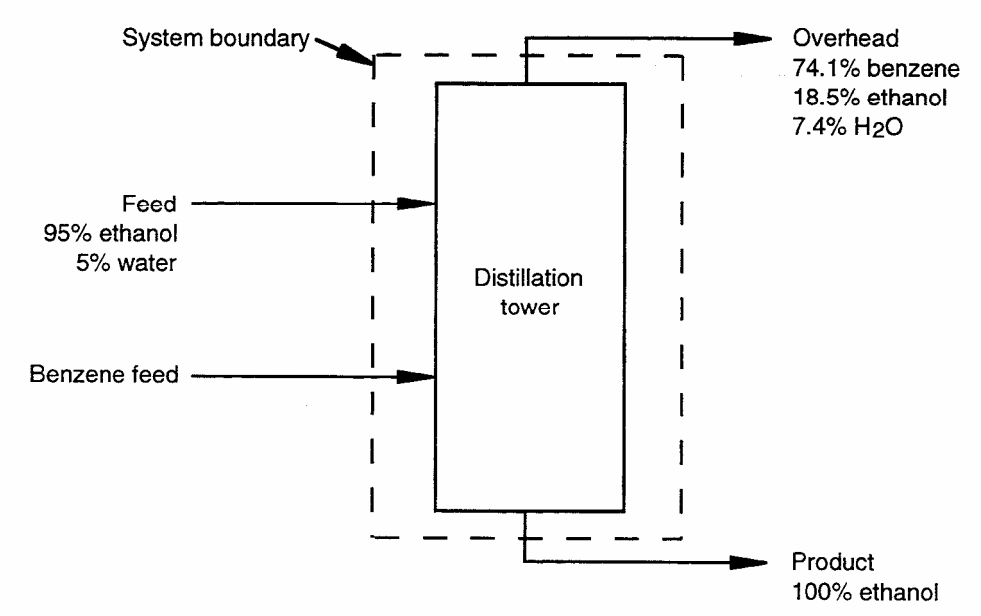
$$\text{Volumetric flow rate} = \frac{437.3 \text{ g s}^{-1}}{1.0325 \text{ g ml}^{-1}} = 423.5 \text{ ml s}^{-1}$$

Answer: 437 g s^{-1} or 424 ml s^{-1}

4.8 Azeotropic distillation

1. Assemble

(i) Flow sheet



(ii) System boundary

The system boundary is shown on the flow sheet.

(iii) Reaction equation

No reaction occurs.

2. Analyse

(i) Assumptions

– steady state

– no leaks

(ii) Extra data

1000 cm^3 = 1 l

1000 g = 1 kg

(iii) Basis

250 l of ethanol product

(iv) Compounds involved in reaction

No compounds are involved in reaction.

(v) Mass balance equation

As there is no reaction, the appropriate mass balance equation is Eq. (4.3):

mass in = mass out

3. Calculate

(i) Calculation table

As all quantities in mass balance calculations must be masses (rather than volumes), 250 1 of absolute ethanol must first be converted to mass. From the definition of density (Section 2.4.1), mass is equal to volume multiplied by density:

$$250\,\text{l absolute ethanol} = 250\,\text{l} \times 0.785\ \text{g cm}^{-3} \cdot \left|\frac{1000\ \text{cm}^3}{1\,\text{l}}\right| \cdot \left|\frac{1\ \text{kg}}{1000\ \text{g}}\right| = 196.25\ \text{kg}$$

The calculation table shows all given quantities in kg. The total mass of feed in is denoted *F*; the total mass of benzene feed in is denoted *B*; the total mass of overhead out is denoted *V*.

Stream	*In*				*Out*			
	Ethanol	*Water*	*Benzene*	*Total*	*Ethanol*	*Water*	*Benzene*	*Total*
Feed	$0.95F$	$0.05F$	0	F	–	–	–	–
Benzene feed	0	0	B	B	–	–	–	–
Product	–	–	–	–	196.25	0	0	196.25
Overhead	–	–	–	–	$0.185V$	$0.074V$	$0.741V$	V
Total	$0.95F$	$0.05F$	B	$F + B$	196.25 + $0.185V$	$0.074V$	$0.741V$	196.25 + V

(ii) Mass balance calculations

Ethanol balance

$0.95F$ kg ethanol in $= (196.25 + 0.185V)$ kg ethanol out

$F = (206.58 + 0.195V)$ kg

Benzene balance

B kg benzene in $= 0.741V$ kg benzene out

$B = 0.741V$

Total mass balance

$(F + B)$ kg total mass in $= (196.25 + V)$ kg total mass out

Substituting for *F* and *B* from the ethanol and benzene balances:

$(206.58 + 0.195V + 0.741V)$ kg $= (196.25 + V)$ kg

$10.33 = 0.064V$

$V = 161.4$ kg

Using this result in the ethanol and benzene balances gives:

$F = 238.1$ kg

$B = 119.6$ kg

These calculations allow completion of the mass balance table with all quantities in kg.

Stream	*In*				*Out*			
	Ethanol	*Water*	*Benzene*	*Total*	*Ethanol*	*Water*	*Benzene*	*Total*
Feed	226.2	11.9	0	238.1	–	–	–	–
Benzene feed	0	0	119.6	119.6	–	–	–	–
Product	–	–	–	–	196.25	0	0	196.25
Overhead	–	–	–	–	29.9	11.9	119.6	161.4
Total	226.2	11.9	119.6	357.7	226.2	11.9	119.6	357.7

(iii) Check the results

All columns and rows of the completed table add up correctly to within round-off error.

4. Finalise

From the completed mass balance table, the mass of benzene required is 119.6 kg. Using the definition of density (Section 2.4.1):

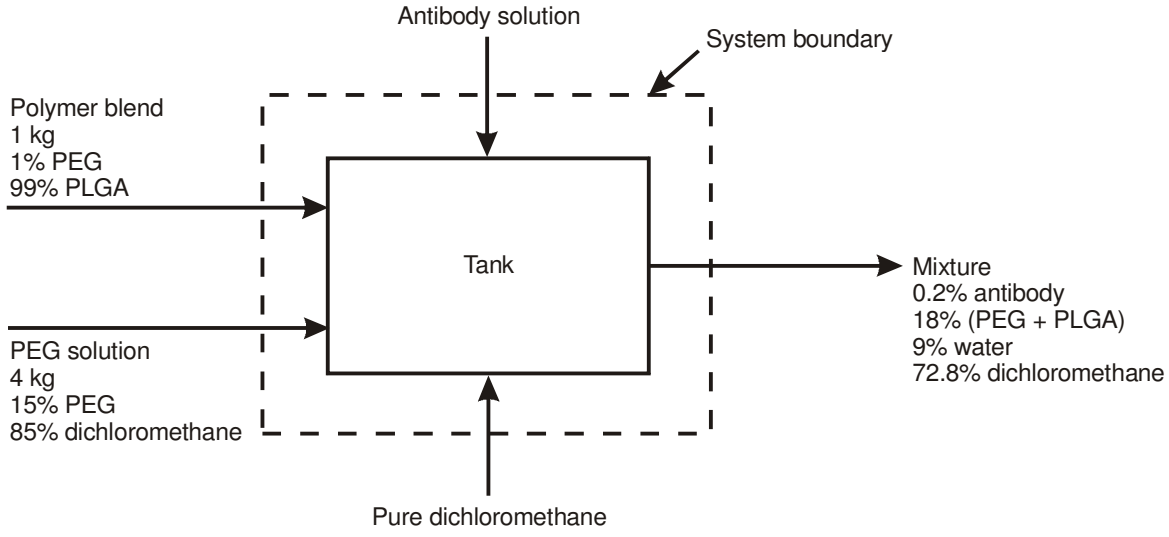
$$\text{Volume of benzene} = \frac{119.6\ \text{kg}}{0.872\ \text{g cm}^{-3}} \cdot \left|\frac{1000\ \text{g}}{1\ \text{kg}}\right| \cdot \left|\frac{1\ \text{l}}{1000\ \text{cm}^3}\right| = 137\ \text{l}$$

Answer: 137 litres

4.9 Microparticles for drug release

1. Assemble

(i) Flow sheet



(ii) System boundary

The system boundary is shown on the flow sheet.

(iii) Reaction equation

No reaction occurs.

2. Analyse

(i) Assumptions

– no leaks

– water is present in the antibody solution and final mixture only

(ii) Extra data

No extra data are required.

(iii) Basis

1 kg of polymer blend and 4 kg of PEG/dichloromethane solution

(iv) Compounds involved in reaction

No compounds are involved in reaction.

(v) Mass balance equation

As there is no reaction, the appropriate mass balance equation is Eq. (4.3):

mass in = mass out

3. Calculate

(i) Calculation table

The calculation table below shows all given quantities in kg. Dichloromethane is abbreviated as DCM. The total mass of antibody solution in is denoted A; the total mass of pure dichloromethane in is denoted D; the total mass of mixture out is denoted M. Within M, let M_{PEG} be the mass of PEG in the mixture stream.

Stream	*In*						*Out*					
	PEG	*PLGA*	*Antibody*	*DCM*	*H_2O*	*Total*	*PEG*	*PLGA*	*Antibody*	*DCM*	*H_2O*	*Total*
Polymer blend	0.01	0.99	0	0	0	1	–	–	–	–	–	–
PEG solution	0.6	0	0	3.4	0	4	–	–	–	–	–	–
Antibody solution	0	0	?	0	?	A	–	–	–	–	–	–
Pure DCM	0	0	0	D	0	D	–	–	–	–	–	–
Mixture	–	–	–	–	–	–	M_{PEG}	$0.18M - M_{PEG}$	$0.002M$	$0.728M$	$0.09M$	M
Total	0.61	0.99	?	$3.4 + D$	?	$5 + A + D$	M_{PEG}	$0.18M - M_{PEG}$	$0.002M$	$0.728M$	$0.09M$	M

(ii) Mass balance calculations

Total mass balance

$(5 + A + D)$ kg total mass in = M kg total mass out

$$A = M - 5 - D \quad (1)$$

PEG balance

$$0.61 \text{ kg PEG in} = M_{\text{PEG}} \text{ kg PEG out}$$

$$M_{\text{PEG}} = 0.61 \tag{2}$$

PLGA balance

$$0.99 \text{ kg PLGA in} = (0.18M - M_{\text{PEG}}) \text{ kg PLGA out}$$

Using the result from (2):

$$0.99 = 0.18M - 0.61$$

$$M = 8.89 \tag{3}$$

DCM balance

$$(3.4 + D) \text{ kg DCM in} = 0.728M \text{ kg DCM out}$$

Using the result from (3):

$$3.4 + D = 6.47$$

$$D = 3.07 \tag{4}$$

Applying (3) and (4) in (1):

$$A = 8.89 - 5 - 3.07 = 0.82 \tag{5}$$

Antibody balance

$$\text{Antibody in} = 0.002M \text{ kg antibody out}$$

From (3):

$$\text{Antibody in} = 0.018 \text{ kg}$$

These calculations allow completion of the mass balance table with all quantities in kg.

Stream	*In*						*Out*					
	PEG	*PLGA*	*Antibody*	*DCM*	*H_2O*	*Total*	*PEG*	*PLGA*	*Antibody*	*DCM*	*H_2O*	*Total*
Polymer blend	0.01	0.99	0	0	0	1	–	–	–	–	–	–
PEG solution	0.6	0	0	3.4	0	4	–	–	–	–	–	–
Antibody solution	0	0	0.018	0	0.802	0.82	–	–	–	–	–	–
Pure DCM	0	0	0	3.07	0	3.07	–	–	–	–	–	–
Mixture	–	–	–	–	–	–	0.61	0.99	0.018	6.47	0.800	8.89
Total	0.61	0.99	0.018	6.47	0.802	8.89	0.61	0.99	0.018	6.47	0.800	8.89

(iii) Check the results

All columns and rows of the completed table add up correctly to within round-off error.

4. Finalise

(a)

From the completed mass balance table, 3.07 kg of pure DCM are required.

Answer: 3.1 kg

(b)

From the mass balance table, 0.018 kg of antibody is required in 0.82 kg of antibody solution. The composition of the antibody solution is therefore (0.018/0.82) × 100% = 2.2%.

Answer: 0.018 kg as a 2.2% solution in water

(c)

From the mass balance table, the mass of mixture produced is 8.89 kg.

Answer: 8.9 kg

(d)

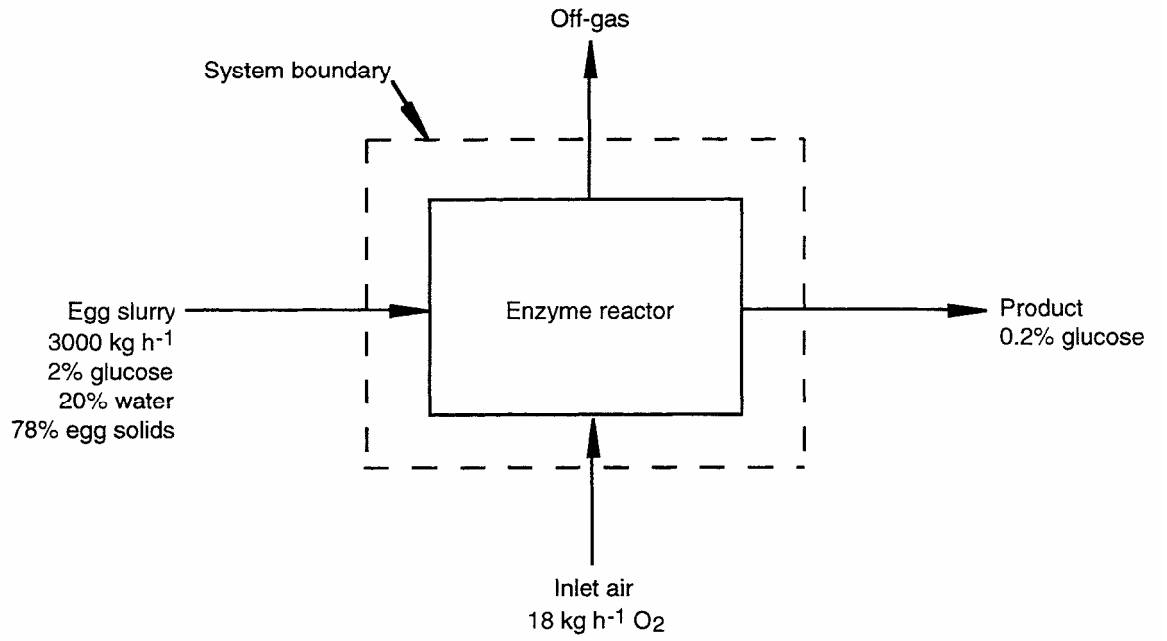
From the mass balance table, 0.61 kg of PEG is present in 8.89 kg of mixture. Therefore, the PEG composition is (0.61/8.89) × 100% = 6.9%.

Answer: 6.9%

4.10 Removal of glucose from dried egg

1. Assemble

(i) Flow sheet



(ii) System boundary

The system boundary is shown on the flow sheet.

(iii) Reaction equation

$$C_6H_{12}O_6 + O_2 + H_2O \rightarrow C_6H_{12}O_7 + H_2O_2$$

2. Analyse

(i) Assumptions

– steady state

– no leaks

– air and off-gas are dry

– gases are at low pressure so vol% = mol% (Section 2.4.5)

– H_2O_2 remains in the liquid phase

(ii) Extra data

Molecular weights calculated from Table C.1 (Appendix C):

– Glucose = 180.2

– O_2 = 32.0

– N_2 = 28.0

– H_2O = 18.0

– Gluconic acid = 196.2

– H_2O_2 = 34.0

Composition of air (Section 2.4.5): 21% O_2, 79% N_2 by volume

(iii) Basis

1 h, or 3000 kg of egg slurry

(iv) Compounds involved in reaction

Glucose, O_2, water, gluconic acid and H_2O_2 are involved in the reaction.

(v) Mass balance equations

For glucose, O_2, water, gluconic acid and H_2O_2, the appropriate mass balance equation is Eq. (4.2):

mass in + mass generated = mass out + mass consumed

For egg solids, N_2 and total mass, the appropriate mass balance equation is Eq. (4.3):

mass in = mass out

3. Calculate

(i) Calculation table

The mass of N_2 accompanying 18 kg of O_2 in air can be calculated from the known composition of air. Converting 18 kg O_2 to molar units:

$$18 \text{ kg } O_2 = 18 \text{ kg } O_2 . \left| \frac{1 \text{ kgmol}}{32.0 \text{ kg}} \right| = 0.563 \text{ kgmol } O_2$$

Therefore, 79/21 × 0.563 kgmol = 2.118 kgmol N_2 enter in the air stream. Converting this to mass units:

$$2.118 \text{ kgmol } N_2 = 2.118 \text{ kgmol } N_2 . \left| \frac{28.0 \text{ kg}}{1 \text{ kgmol}} \right| = 59.30 \text{ kg } N_2$$

The calculation tables below show all known quantities in kg. The total mass of off-gas is denoted *G*; the total mass of product is denoted *P*. The In side of the mass balance table is complete.

Stream	*In*							
	Glucose	*Water*	*Egg solids*	O_2	N_2	*Gluconic acid*	H_2O_2	*Total*
Egg slurry	60	600	2340	0	0	0	0	3000
Air	0	0	0	18	59.30	0	0	77.3
Off-gas	–	–	–	–	–	–	–	–
Product	–	–	–	–	–	–	–	–
Total	60	600	2340	18	59.30	0	0	3077.3

Stream	*Out*							
	Glucose	*Water*	*Egg solids*	O_2	N_2	*Gluconic acid*	H_2O_2	*Total*
Egg slurry	–	–	–	–	–	–	–	–
Air	–	–	–	–	–	–	–	–
Off-gas	0	0	0	?	?	0	0	G
Product	$0.002P$	?	?	0	0	?	?	P
Total	$0.002P$	?	?	?	?	?	?	$G + P$

(ii) Mass balance calculations

Egg solids balance

Egg solids is a tie component.

$$2340 \text{ kg egg solids in} = \text{egg solids out}$$

$$\text{Egg solids out} = 2340 \text{ kg}$$

N_2 balance

N_2 is a tie component.

$$59.30 \text{ kg } N_2 \text{ in} = N_2 \text{ out}$$

$$N_2 \text{ out} = 59.30 \text{ kg}$$

Glucose balance

$$60 \text{ kg glucose in} + 0 \text{ kg glucose generated} = 0.002P \text{ kg glucose out} + \text{glucose consumed}$$

$$\text{Glucose consumed} = (60 - 0.002P) \text{ kg}$$

Converting the glucose consumed to molar terms:

$$\text{Glucose consumed} = (60 - 0.002P) \text{ kg} \cdot \left| \frac{1 \text{ kgmol}}{180.2 \text{ kg}} \right| = \frac{(60 - 0.002P)}{180.2} \text{ kgmol}$$

$$= (0.333 - 1.11 \times 10^{-5} P) \text{ kgmol}$$

From the reaction stoichiometry, conversion of this amount of glucose requires the same number of kgmol of O_2. Converting this molar quantity to mass:

$$(0.333 - 1.11\times10^{-5}P)\ \text{kgmol O}_2 = (0.333 - 1.11\times10^{-5}P)\ \text{kgmol} \left.\cdot\right| \frac{32.0\ \text{kg}}{1\ \text{kgmol}} \left.\right|$$
$$= (10.656 - 3.552\times10^{-4}P)\ \text{kg O}_2$$

O_2 balance

$$18\ \text{kg O}_2\ \text{in} + 0\ \text{kg O}_2\ \text{generated} = \text{O}_2\ \text{out} + (10.656 - 3.552\times10^{-4}P)\ \text{kg O}_2\ \text{consumed}$$

$$\text{O}_2\ \text{out} = (18 - (10.656 - 3.552\times10^{-4}P))\ \text{kg}$$

$$\text{O}_2\ \text{out} = (7.344 + 3.552\times10^{-4}P)\ \text{kg}$$

Adding this mass of O_2 to the mass of N_2 in the off-gas:

$$G = 59.30 + (7.344 + 3.552\times10^{-4}P)\ \text{kg}$$

$$G = (66.64 + 3.552\times10^{-4}P)\ \text{kg}$$

Total mass balance

$$3077.3\ \text{kg total mass in} = (G + P)\ \text{kg total mass out}$$

Substituting the expression for G into the total mass balance:

$$3077.3\ \text{kg} = (66.64 + 3.552\times10^{-4}P + P)\ \text{kg}$$

$$3010.7\ \text{kg} = 1.0004P\ \text{kg}$$

$$P = 3009.6\ \text{kg}$$

Therefore, from the above expressions for G and O_2 out:

$$G = (66.64 + 3.552\times10^{-4}\times3009.6)\ \text{kg}$$
$$= 67.71\ \text{kg}$$

and

$$\text{O}_2\ \text{out} = (7.344 + 3.552\times10^{-4}\times3009.6)\ \text{kg}$$
$$= 8.41\ \text{kg}$$

The mass of glucose out is $0.002 \times 3009.6 = 6.02$ kg. The moles of glucose consumed is:

$$\text{Glucose consumed} = (0.333 - 1.11\times10^{-5}\times3009.6)\ \text{kgmol} = 0.300\ \text{kgmol}$$

Therefore, from stoichiometry and the molecular weights:

$$\text{Water consumed} = 0.300\ \text{kgmol} \left.\cdot\right| \frac{18.0\ \text{kg}}{1\ \text{kgmol}} \left.\right| = 5.40\ \text{kg}$$

$$\text{Gluconic acid generated} = 0.300\ \text{kgmol} \left.\cdot\right| \frac{196.2\ \text{kg}}{1\ \text{kgmol}} \left.\right| = 58.86\ \text{kg}$$

$$\text{H}_2\text{O}_2\ \text{generated} = 0.300\ \text{kgmol} \left.\cdot\right| \frac{34\ \text{kg}}{1\ \text{kgmol}} \left.\right| = 10.20\ \text{kg}$$

Water balance

$$600\ \text{kg water in} + 0\ \text{kg water generated} = \text{water out} + 5.40\ \text{kg water consumed}$$

$$\text{Water out} = 594.6\ \text{kg}$$

Gluconic acid balance

0 kg gluconic acid in + 58.86 kg gluconic acid generated =

gluconic acid out + 0 kg gluconic acid consumed

Gluconic acid out = 58.86 kg

H_2O_2 balance

0 kg H_2O_2 in + 10.20 kg H_2O_2 generated = H_2O_2 out + 0 kg H_2O_2 consumed

H_2O_2 out = 10.20 kg

These calculations allow completion of the Out side of the mass balance table with all quantities in kg.

Stream	*Out*							
	Glucose	*Water*	*Egg solids*	*O_2*	*N_2*	*Gluconic acid*	*H_2O_2*	*Total*
Egg slurry	–	–	–	–	–	–	–	–
Air	–	–	–	–	–	–	–	–
Off-gas	0	0	0	8.41	59.30	0	0	67.71
Product	6.02	594.6	2340	0	0	58.86	10.20	3009.6
Total	6.02	594.6	2340	8.41	59.30	58.86	10.20	3077.3

(iii) Check the results

All columns and rows of the completed table add up correctly to within round-off error.

4. Finalise

(a)

To determine which is the limiting substrate, the available number of moles of each substrate involved in the reaction must be determined. From the In side of the mass balance table:

$$\text{Moles glucose} = 60 \text{ kg} \cdot \left| \frac{1 \text{ kgmol}}{180.2 \text{ kg}} \right| = 0.333 \text{ kgmol}$$

$$\text{Moles water} = 600 \text{ kg} \cdot \left| \frac{1 \text{ kgmol}}{18.0 \text{ kg}} \right| = 33.3 \text{ kgmol}$$

$$\text{Moles } O_2 = 18 \text{ kg} \cdot \left| \frac{1 \text{ kgmol}}{32.0 \text{ kg}} \right| = 0.563 \text{ kgmol}$$

For reaction, these substrates are required in the molar stoichiometric ratio of 1:1:1. As glucose is available in the smallest molar quantity, the extent of reaction is limited by the availability of glucose.

Answer: Glucose

(b)

From **(a)**, water and O_2 are available in excess. As only 0.333 kgmol of each will be used if the reaction proceeds to completion with consumption of all of the glucose, from Eq. (2.37):

$$\% \text{ excess water} = \frac{(33.3 - 0.333) \text{ kgmol}}{0.333 \text{ kgmol}} \times 100\% = 9900\%$$

$$\% \text{ excess } O_2 = \frac{(0.563 - 0.333) \text{ kgmol}}{0.333 \text{ kgmol}} \times 100\% = 69\%$$

Answer: 9900% excess water; 69% excess O_2

(c)

From the completed mass balance table, the reactor off-gas contains 8.41 kg O_2 and 59.30 kg N_2. As gas compositions are normally expressed in molar or volumetric terms (Section 2.4.5), these masses must be converted to moles:

$$8.41 \text{ kg } O_2 = 8.41 \text{ kg } O_2 \,.\, \left| \frac{1 \text{ kgmol}}{32.0 \text{ kg}} \right| = 0.263 \text{ kgmol } O_2$$

The number of kgmol of N_2 was determined in Section 3(i) of the preliminary calculations to be 2.118. As N_2 is a tie component, the total number of moles of off-gas is (0.263 + 2.118) = 2.381 kgmol. Therefore, the composition of the off-gas is (0.263/2.381) × 100% = 11% O_2 and (2.118/2.381) × 100% = 89% N_2.

Answer. 11% O_2, 89% N_2

(d)

From the completed mass balance table, the product stream has a total mass of 3009.6 kg and contains 6.02 kg glucose, 594.6 kg water, 2340 kg egg solids, 58.86 kg gluconic acid and 10.20 kg H_2O_2. Therefore, the composition is:

$$\frac{6.02}{3009.6} \times 100\% = 0.20\% \text{ w/w glucose}$$

$$\frac{594.6}{3009.6} \times 100\% = 19.8\% \text{ w/w water}$$

$$\frac{2340}{3009.6} \times 100\% = 77.8\% \text{ w/w egg solids}$$

$$\frac{58.86}{3009.6} \times 100\% = 2.0\% \text{ w/w gluconic acid}$$

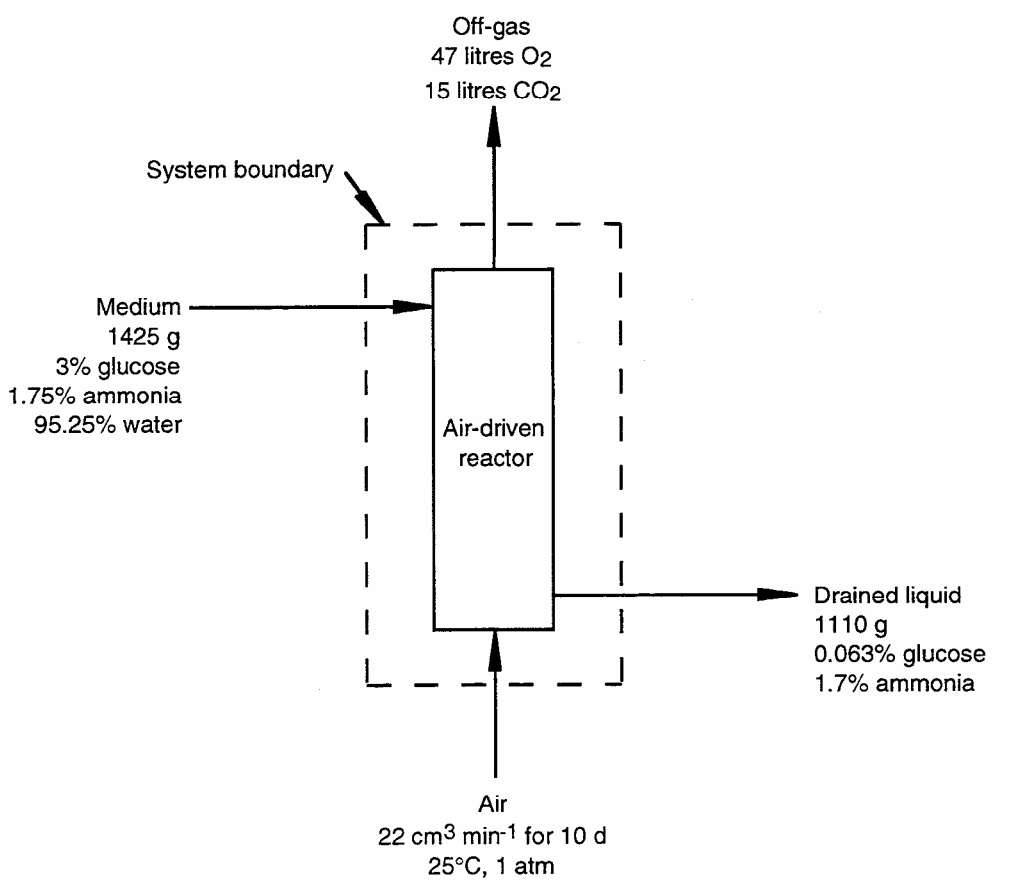
$$\frac{10.20}{3009.6} \times 100\% = 0.34\% \text{ w/w } H_2O_2$$

Answer. 0.20% w/w glucose, 20% w/w water, 78% w/w egg solids, 2.0% w/w gluconic acid, 0.34% w/w H_2O_2

4.11 Culture of plant roots

1. Assemble

(i) Flow sheet



(ii) System boundary

The system boundary is shown on the flow sheet.

(iii) Reaction equation

From Table C.2 (Appendix C), the molecular formula for glucose is $C_6H_{12}O_6$. The reaction equation is based on the general stoichiometric equation for aerobic growth, Eq. (4.4):

$$C_6H_{12}O_6 + a\,O_2 + b\,NH_3 \rightarrow c\,CH_\alpha O_\beta N_\delta + d\,CO_2 + e\,H_2O$$

2. Analyse

(i) Assumptions

– steady state

– no leaks

– air and off-gas are dry

– all the CO_2 produced leaves in the off-gas

– gases are at low pressure so vol% = mol% (Section 2.4.5)

(ii) Extra data

1 l = 1000 cm^3

Molecular weights calculated from Table C.1 (Appendix C):

– Glucose = 180.2

– O_2 = 32.0

– N_2 = 28.0

– NH_3 = 17.0

– CO_2 = 44.0

– H_2O = 18.0

Composition of air (Section 2.4.5): 21% O_2, 79% N_2 by volume

Ideal gas constant (Appendix B): R = 82.057 cm^3 atm K^{-1} $gmol^{-1}$

(iii) Basis

10 days, or 1425 g of nutrient medium

(iv) Compounds involved in reaction

Glucose, O_2, NH_3, biomass, CO_2 and H_2O are involved in the reaction.

(v) Mass balance equations

For glucose, O_2, NH_3, biomass, CO_2 and H_2O, the appropriate mass balance equation is Eq. (4.2):

mass in + mass generated = mass out + mass consumed

For N_2 and total mass, the appropriate mass balance equation is Eq. (4.3):

mass in = mass out

3. Calculate

(i) Calculation table

Over 10 days, the volume of air sparged into the fermenter is:

$$\text{Volume of air in} = 22\ \text{cm}^3\ \text{min}^{-1} \times 10\ \text{days} \left| \frac{60\ \text{min}}{1\ \text{h}} \right| \left| \frac{24\ \text{h}}{1\ \text{day}} \right| = 3.168 \times 10^5\ \text{cm}^3$$

Converting this gas volume to moles using the ideal gas law, Eq. (2.35), with the temperature converted from °C to Kelvin using Eq. (2.27):

$$\text{Moles of air in} = n = \frac{pV}{RT} = \frac{1\ \text{atm}\ (3.168 \times 10^5\ \text{cm}^3)}{82.057\ \text{cm}^3\ \text{atm}\ \text{K}^{-1}\ \text{gmol}^{-1}\ (25 + 273.15)\ \text{K}} = 12.95\ \text{gmol}$$

From the known composition of air, the moles of O_2 in the incoming air is 0.21 × 12.95 gmol = 2.72 gmol, and the moles of N_2 is 0.79 × 12.95 gmol = 10.23 gmol. Converting these values to masses:

$$\text{Mass of } O_2 \text{ in} = 2.72\ \text{gmol} \left| \frac{32.0\ \text{g}}{1\ \text{gmol}} \right| = 87.04\ \text{g}$$

$$\text{Mass of } N_2 \text{ in} = 10.23\ \text{gmol} \left| \frac{28.0\ \text{g}}{1\ \text{gmol}} \right| = 286.4\ \text{g}$$

The total mass of air in is therefore (87.04 + 286.4) g = 373.44 g.

The gas volumes in the off-gas must also be converted to masses. First, convert the volumes of O_2 and CO_2 to moles using Eq. (2.35) with the temperature converted using Eq. (2.27):

$$\text{Moles of } O_2 \text{ out} = n = \frac{pV}{RT} = \frac{1 \text{ atm } (47 \text{ l}) \left| \frac{1000 \text{ cm}^3}{1 \text{ l}} \right|}{82.057 \text{ cm}^3 \text{ atm K}^{-1} \text{ gmol}^{-1} (25 + 273.15) \text{ K}} = 1.92 \text{ gmol}$$

$$\text{Moles of } CO_2 \text{ out} = n = \frac{pV}{RT} = \frac{1 \text{ atm } (15 \text{ l}) \left| \frac{1000 \text{ cm}^3}{1 \text{ l}} \right|}{82.057 \text{ cm}^3 \text{ atm K}^{-1} \text{ gmol}^{-1} (25 + 273.15) \text{ K}} = 0.613 \text{ gmol}$$

Calculate the corresponding masses:

$$\text{Mass of } O_2 \text{ out} = 1.92 \text{ gmol} \left| \frac{32.0 \text{ g}}{1 \text{ gmol}} \right| = 61.44 \text{ g}$$

$$\text{Mass of } CO_2 \text{ out} = 0.613 \text{ gmol} \left| \frac{44.0 \text{ g}}{1 \text{ gmol}} \right| = 26.97 \text{ g}$$

The calculation tables below show all known quantities in g. The In side of the mass balance table is complete. The total mass of off-gas out is denoted *G*; the total biomass harvested is denoted *B*. As the ratio of biomass fresh weight to dry weight is 14:1, dry biomass comprises 1/15 = 0.0667 of the total biomass. Because this problem requires an integral mass balance, the biomass remaining in the fermenter after 10 days of culture must also be included in the table even though it is not contained in any of the streams flowing into or out of the vessel.

Stream	*In*							
	Glucose	*O_2*	*N_2*	*NH_3*	*Dry biomass*	*CO_2*	*H_2O*	*Total*
Medium	42.75	0	0	24.94	0	0	1357.31	1425
Air	0	87.04	286.4	0	0	0	0	373.44
Drained liquid	–	–	–	–	–	–	–	–
Off-gas	–	–	–	–	–	–	–	–
Harvested biomass	–	–	–	–	–	–	–	–
Total	42.75	87.04	286.4	24.94	0	0	1357.31	1798.44

Stream	*Out*							
	Glucose	O_2	N_2	NH_3	*Dry biomass*	CO_2	H_2O	*Total*
Medium	–	–	–	–	–	–	–	–
Air	–	–	–	–	–	–	–	–
Drained liquid	0.699	0	0	18.87	0	0	1090.43	1110
Off-gas	0	61.44	?	0	0	26.97	0	*G*
Harvested biomass	0	0	0	0	0.0667*B*	0	0.9333*B*	*B*
Total	0.699	61.44	?	18.87	0.0667*B*	26.97	1090.43 + 0.9333*B*	1110 + *G* + *B*

(ii) Mass balance calculations

N_2 balance

N_2 is a tie component.

$$286.4 \text{ g } N_2 \text{ in} = N_2 \text{ out}$$

$$N_2 \text{ out} = 286.4 \text{ g}$$

Using this result and adding up the row for off-gas on the Out side of the table:

$$G = (61.44 + 286.4 + 26.97) \text{ g} = 374.81 \text{ g}$$

Total mass balance

$$1798.44 \text{ g total mass in} = (1110 + G + B) \text{ g total mass out}$$

Using the result for *G*:

$$B = 313.63 \text{ g}$$

Therefore, the dry biomass produced is 0.0667×313.63 g = 20.92 g, and the mass of water in the biomass is 0.9333×313.63 g = 292.71 g.

These calculations allow completion of the Out side of the mass balance table with all quantities in g.

Stream	*Out*							
	Glucose	*O_2*	*N_2*	*NH_3*	*Dry biomass*	*CO_2*	*H_2O*	*Total*
Medium	–	–	–	–	–	–	–	–
Air	–	–	–	–	–	–	–	–
Drained liquid	0.699	0	0	18.87	0	0	1090.43	1110
Off-gas	0	61.44	286.4	0	0	26.97	0	374.81
Harvested biomass	0	0	0	0	20.92	0	292.71	313.63
Total	0.699	61.44	286.4	18.87	20.92	26.97	1383.14	1798.44

Further mass balance calculations allow evaluation of the masses of components consumed or generated in the reaction.

Glucose balance

$$42.75 \text{ g glucose in} + 0 \text{ g glucose generated} = 0.699 \text{ g glucose out} + \text{glucose consumed}$$

$$\text{Glucose consumed} = 42.05 \text{ g}$$

O_2 balance

$$87.04 \text{ g } O_2 \text{ in} + 0 \text{ g } O_2 \text{ generated} = 61.44 \text{ g } O_2 \text{ out} + O_2 \text{ consumed}$$

$$O_2 \text{ consumed} = 25.60 \text{ g}$$

NH_3 balance

$$24.94 \text{ g } NH_3 \text{ in} + 0 \text{ g } NH_3 \text{ generated} = 18.87 \text{ g } NH_3 \text{ out} + NH_3 \text{ consumed}$$

$$NH_3 \text{ consumed} = 6.07 \text{ g}$$

CO_2 balance

$$0 \text{ g } CO_2 \text{ in} + CO_2 \text{ generated} = 26.97 \text{ g } CO_2 \text{ out} + 0 \text{ g } CO_2 \text{ consumed}$$

$$CO_2 \text{ generated} = 26.97 \text{ g}$$

H_2O balance

$$1357.31 \text{ g } H_2O \text{ in} + H_2O \text{ generated} = (1090.43 + 0.9333B) \text{ g } H_2O \text{ out} + 0 \text{ g } H_2O \text{ consumed}$$

Substituting the value for *B* from the total mass balance:

$$H_2O \text{ generated} = 25.83 \text{ g}$$

(iii) Check the results

All columns and rows of the completed mass balance table add up correctly.

4. Finalise

(a)

Rounding to three significant figures, the mass of dry roots produced is 20.9 g.

Answer: 20.9 g

(b)

To determine the stoichiometry, the calculated masses of components consumed or generated in the reaction must be converted to molar quantities:

$$\text{Moles of glucose consumed} = 42.05\ \text{g} \cdot \left|\frac{1\ \text{gmol}}{180.2\ \text{g}}\right| = 0.233\ \text{gmol}$$

$$\text{Moles of } O_2 \text{ consumed} = 25.60\ \text{g} \cdot \left|\frac{1\ \text{gmol}}{32.0\ \text{g}}\right| = 0.800\ \text{gmol}$$

$$\text{Moles of } NH_3 \text{ consumed} = 6.07\ \text{g} \cdot \left|\frac{1\ \text{gmol}}{17.0\ \text{g}}\right| = 0.357\ \text{gmol}$$

$$\text{Moles of } CO_2 \text{ generated} = 26.97\ \text{g} \cdot \left|\frac{1\ \text{gmol}}{44.0\ \text{g}}\right| = 0.613\ \text{gmol}$$

$$\text{Moles of } H_2O \text{ generated} = 25.83\ \text{g} \cdot \left|\frac{1\ \text{gmol}}{18.0\ \text{g}}\right| = 1.435\ \text{gmol}$$

$$\text{Moles of biomass generated} = \frac{20.92\ \text{g}}{\text{MW biomass}}$$

The amount of biomass generated is not yet known explicitly because the molecular formula for the dry biomass is unknown. The above molar quantities can be used as coefficients in the reaction equation:

$$0.233\,C_6H_{12}O_6 + 0.800\,O_2 + 0.357\,NH_3 \rightarrow \frac{20.92}{\text{MW biomass}}\,CH_\alpha O_\beta N_\delta + 0.613\,CO_2 + 1.435\,H_2O$$

Dividing each coefficient by 0.233 to obtain the stoichiometry per gmol of glucose:

$$C_6H_{12}O_6 + 3.43\,O_2 + 1.53\,NH_3 \rightarrow \frac{89.79}{\text{MW biomass}}\,CH_\alpha O_\beta N_\delta + 2.63\,CO_2 + 6.16\,H_2O$$

The values of α, β and δ and the molecular formula for the biomass can obtained using elemental balances.

$$\text{C balance}:\ 6 = \frac{89.79}{\text{MW biomass}} + 2.63$$

Therefore:

$$\frac{89.79}{\text{MW biomass}} = 3.37$$

This result can be used in the remaining elemental balances for completion of the stoichiometric equation:

H balance: $12 + 3 \times 1.53 = 3.37\alpha + 2 \times 6.16 \rightarrow \alpha = 1.27$

O balance: $6 + 2 \times 3.43 = 3.37\beta + 2 \times 2.63 + 6.16 \rightarrow \beta = 0.43$

N balance: $1.53 = 3.37\delta \rightarrow \delta = 0.45$

Answer: The chemical formula for the dry roots is $CH_{1.27}O_{0.43}N_{0.45}$ and the complete stoichiometric equation is:

$$C_6H_{12}O_6 + 3.43\,O_2 + 1.53\,NH_3 \rightarrow 3.37\,CH_{1.27}O_{0.43}N_{0.45} + 2.63\,CO_2 + 6.16\,H_2O$$

(c)

Converting to moles the mass quantities of glucose, O_2 and NH_3 available for reaction from the In side of the mass balance table:

$$\text{Moles of glucose in} = 42.75\ \text{g} \cdot \left|\frac{1\ \text{gmol}}{180.2\ \text{g}}\right| = 0.24\ \text{gmol}$$

$$\text{Moles of } O_2 \text{ in} = 87.04\ \text{g} \cdot \left|\frac{1\ \text{gmol}}{32.0\ \text{g}}\right| = 2.72\ \text{gmol}$$

$$\text{Moles of } NH_3 \text{ in} = 24.93\ \text{g} \cdot \left|\frac{1\ \text{gmol}}{17.0\ \text{g}}\right| = 1.47\ \text{gmol}$$

From the stoichiometric equation, reaction of 0.24 gmol glucose requires 0.24 × 3.43 = 0.82 gmol of O_2 and 0.24 × 1.53 = 0.37 gmol of NH_3. As the molar quantities of O_2 and NH_3 available for reaction are in excess of these values, glucose must be the limiting substrate.

Answer: Glucose

(d)

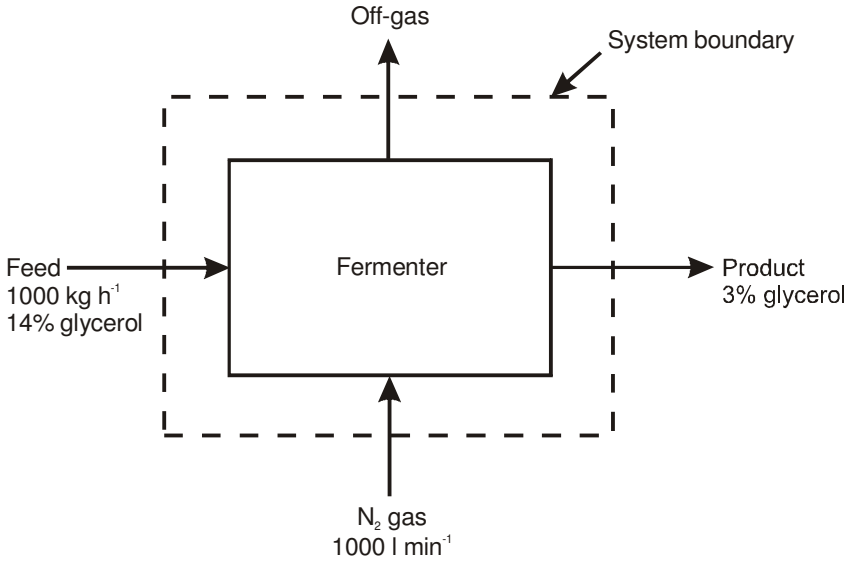
The mass of glucose consumed is 42.05 g and the mass of dry biomass produced is 20.92 g. Therefore, the biomass yield from glucose is (20.92 g)/(42.05 g) = 0.50 g g^{-1} dry weight.

Answer. 0.50 g g^{-1} dry weight

4.12 Production of 1,3-propanediol

1. Assemble

(i) Flow sheet



(ii) System boundary

The system boundary is shown on the flow sheet.

(iii) Reaction equation

$$68\,C_3H_8O_3 + 3\,NH_3 \rightarrow 3C_4H_7O_2N + 49\,C_3H_8O_2 + 15\,C_2H_4O_2 + 15\,CO_2 + 40\,H_2O$$

2. Analyse

(i) Assumptions

– steady state

– no leaks

– off-gas is dry

– all the CO_2 produced leaves in the off-gas

– gases are at low pressure so vol% = mol% (Section 2.4.5)

– the feed solvent is water

– all the NH_3 provided is consumed, i.e. the calculation gives the minimum NH_3 requirement

(ii) Extra data

Molecular weights calculated from Table C.1 (Appendix C):

– Glycerol = 92

– NH_3 = 17

– Biomass = 101

– 1,3-Propanediol = 76

– Acetic acid = 60

– CO_2 = 44

– H_2O = 18

– N_2 = 28

Ideal gas constant (Appendix B): $R = 0.082057\ \text{l atm K}^{-1}\ \text{gmol}^{-1}$

(iii) Basis

1 h, or 1000 kg of feed

(iv) Compounds involved in reaction

Glycerol, NH_3, biomass, 1,3-propanediol, acetic acid, CO_2 and H_2O are involved in the reaction.

(v) Mass balance equations

For glycerol, NH_3, biomass, 1,3-propanediol, acetic acid, CO_2 and H_2O, the appropriate mass balance equation is Eq. (4.2):

mass in + mass generated = mass out + mass consumed

For N_2 and total mass, the appropriate mass balance equation is Eq. (4.3):

mass in = mass out

3. Calculate

(i) Calculation table

In 1 h, the volume of N_2 gas sparged into the fermenter is:

$$\text{Volume of } N_2 \text{ in} = 1000\ \text{l min}^{-1} \times 1\ \text{h} \left| \frac{60\ \text{min}}{1\ \text{h}} \right| = 60{,}000\ \text{l}$$

Converting this gas volume to moles using the ideal gas law, Eq. (2.35), with the temperature converted from °C to Kelvin using Eq. (2.27):

$$\text{Moles of } N_2 \text{ in} = n = \frac{pV}{RT} = \frac{1 \text{ atm } (60,000 \text{ l})}{0.082057 \text{ l atm K}^{-1} \text{ gmol}^{-1} (37 + 273.15) \text{ K}} = 2357.6 \text{ gmol}$$

Converting this to mass using the molecular weight of N_2:

$$\text{Mass of } N_2 \text{ in} = 2357.6 \text{ gmol} \cdot \left| \frac{28 \text{ g}}{1 \text{ gmol}} \right| \cdot \left| \frac{1 \text{ kg}}{1000 \text{ g}} \right| = 66.0 \text{ kg}$$

The calculation tables below show all known quantities in kg. The total mass of product out is denoted *P*; the total mass of off-gas out is denoted *G*.

Stream	*In*								
	Glycerol	*NH_3*	*N_2*	*Dry biomass*	*1,3-Propanediol*	*Acetic acid*	*CO_2*	*H_2O*	*Total*
Feed	140	?	0	0	0	0	0	?	1000
N_2 gas	0	0	66.0	0	0	0	0	0	66.0
Product	–	–	–	–	–	–	–	–	–
Off-gas	–	–	–	–	–	–	–	–	–
Total	140	?	66.0	0	0	0	0	?	1066.0

Stream	*Out*								
	Glycerol	*NH_3*	*N_2*	*Dry biomass*	*1,3-Propanediol*	*Acetic acid*	*CO_2*	*H_2O*	*Total*
Feed	–	–	–	–	–	–	–	–	–
N_2 gas	–	–	–	–	–	–	–	–	–
Product	0.03*P*	0	0	?	?	?	0	?	*P*
Off-gas	0	0	?	0	0	0	?	0	*G*
Total	0.03*P*	0	?	?	?	?	?	?	*P* + *G*

(ii) Mass balance calculations

N_2 balance

N_2 is a tie component.

$$66.0 \text{ kg } N_2 \text{ in} = N_2 \text{ out}$$

$$N_2 \text{ out} = 66.0 \text{ kg}$$

Using this result in the mass balance table for off-gas:

$$CO_2 \text{ out} = (G - 66.0) \text{ kg} \quad (1)$$

Total mass balance

$$1066.0 \text{ kg total mass in} = (P + G) \text{ kg total mass out}$$

$$G = 1066.0 - P \quad (2)$$

Glycerol balance

$$140 \text{ kg glycerol in} + 0 \text{ g glycerol generated} = 0.03P \text{ kg glycerol out} + \text{glycerol consumed}$$

$$\text{Glycerol consumed} = (140 - 0.03P)\ \text{kg}$$

Converting this expression for glycerol consumed to kgmoles using the molecular weight of glycerol:

$$\text{Glycerol consumed} = (140 - 0.03P)\ \text{kg} \cdot \left|\frac{1\ \text{kgmol}}{92\ \text{kg}}\right| = (1.522 - 3.26\times10^{-4}P)\ \text{kgmol}$$

From the reaction stoichiometry, conversion of this amount of glycerol requires 3/68 ($1.522 - 3.26 \times 10^{-4}$ P) kgmol NH_3. Converting this molar quantity to mass of NH_3:

$$\text{Mass of NH}_3\text{ consumed} = \frac{3}{68}(1.522 - 3.26\times10^{-4}P)\ \text{kgmol} \cdot \left|\frac{17\ \text{kg}}{1\ \text{kgmol}}\right|$$
$$= (1.142 - 2.45\times10^{-4}P)\ \text{kg}$$

Expressions for the masses of compounds produced by reaction are similarly derived from stoichiometry in terms of the unknown quantity, P.

$$\text{Mass of biomass generated} = \frac{3}{68}(1.522 - 3.26\times10^{-4}P)\ \text{kgmol} \cdot \left|\frac{101\ \text{kg}}{1\ \text{kgmol}}\right|$$
$$= (6.782 - 1.45\times10^{-3}P)\ \text{kg}$$

$$\text{Mass of 1,3-propanediol generated} = \frac{49}{68}(1.522 - 3.26\times10^{-4}P)\ \text{kgmol} \cdot \left|\frac{76\ \text{kg}}{1\ \text{kgmol}}\right|$$
$$= (83.35 - 0.018P)\ \text{kg}$$

$$\text{Mass of acetic acid generated} = \frac{15}{68}(1.522 - 3.26\times10^{-4}P)\ \text{kgmol} \cdot \left|\frac{60\ \text{kg}}{1\ \text{kgmol}}\right|$$
$$= (20.14 - 4.31\times10^{-3}P)\ \text{kg}$$

$$\text{Mass of CO}_2\text{ generated} = \frac{15}{68}(1.522 - 3.26\times10^{-4}P)\ \text{kgmol} \cdot \left|\frac{44\ \text{kg}}{1\ \text{kgmol}}\right|$$
$$= (14.77 - 3.16\times10^{-3}P)\ \text{kg}$$

$$\text{Mass of H}_2\text{O generated} = \frac{40}{68}(1.522 - 3.26\times10^{-4}P)\ \text{kgmol} \cdot \left|\frac{18\ \text{kg}}{1\ \text{kgmol}}\right|$$
$$= (16.12 - 3.45\times10^{-3}P)\ \text{kg}$$

CO_2 balance

Applying (1) and the result for the mass of CO_2 generated:

$$0\ \text{kg CO}_2\text{ in} + (14.77 - 3.16\times10^{-3}P)\ \text{kg CO}_2\text{ generated} =$$
$$(G - 66.0)\ \text{kg CO}_2\text{ out} + 0\ \text{g CO}_2\text{ consumed}$$

$$14.77 - 3.16\times10^{-3}P = G - 66.0$$

Applying (2) to this equation gives:

$$14.77 - 3.16\times10^{-3}P = 1066.0 - P - 66.0$$

$$0.997P = 985.23$$

$$P = 988.35 \qquad (3)$$

Using this result in (2):

$$G = 1066.0 - 988.35 = 77.65$$

Combining this result with (1):

$$\text{CO}_2 \text{ out} = (77.65 - 66.0) \text{ kg} = 11.65 \text{ kg}$$

Biomass balance

$$0 \text{ kg biomass in} + (6.782 - 1.45 \times 10^{-3} P) \text{ kg biomass generated} = \text{biomass out} + 0 \text{ kg biomass consumed}$$

Applying (3):

$$\text{Biomass out} = 5.35 \text{ kg}$$

1,3-Propanediol balance

$$0 \text{ kg 1,3-propanediol in} + (83.35 - 0.018P) \text{ kg 1,3-propanediol generated} = \text{1,3-propanediol out} + 0 \text{ kg 1,3-propanediol consumed}$$

Applying (3):

$$\text{1,3-Propanediol out} = 65.56 \text{ kg}$$

Acetic acid balance

$$0 \text{ kg acetic acid in} + (20.14 - 4.31 \times 10^{-3} P) \text{ kg acetic acid generated} = \text{acetic acid out} + 0 \text{ kg acetic acid consumed}$$

Applying (3):

$$\text{Acetic acid out} = 15.88 \text{ kg}$$

NH_3 balance

If all the NH_3 provided is consumed:

$$\text{NH}_3 \text{ in} + 0 \text{ kg NH}_3 \text{ generated} = 0 \text{ kg NH}_3 \text{ out} + (1.142 - 2.45 \times 10^{-4} P) \text{ kg NH}_3 \text{ consumed}$$

Applying (3):

$$\text{NH}_3 \text{ in} = 0.90 \text{ kg}$$

H_2O balance

$$\text{H}_2\text{O in} + (16.12 - 3.45 \times 10^{-3} P) \text{ kg H}_2\text{O generated} = \text{H}_2\text{O out} + 0 \text{ kg H}_2\text{O consumed}$$

Applying (3):

$$\text{H}_2\text{O in} + 12.71 \text{ kg} = \text{H}_2\text{O out} \qquad (4)$$

The mass of H_2O in is obtained from the mass balance table by subtracting the kg of glycerol and NH_3 in from the total kg of feed:

$$\text{H}_2\text{O in} = (1000 - 140 - 0.90) \text{ kg} = 859.1 \text{ kg}$$

Applying this result in (4) gives:

$$\text{H}_2\text{O out} = 871.8 \text{ kg}$$

These calculations allow completion of the mass balance tables with all quantities in kg.

Stream	*In*								
	Glycerol	*NH_3*	*N_2*	*Dry biomass*	*1,3-Propanediol*	*Acetic acid*	*CO_2*	*H_2O*	*Total*
Feed	140	0.90	0	0	0	0	0	859.1	1000
N_2 gas	0	0	66.0	0	0	0	0	0	66.0
Product	–	–	–	–	–	–	–	–	–
Off-gas	–	–	–	–	–	–	–	–	–
Total	140	0.90	66.0	0	0	0	0	859.1	1066.0

Stream	*Out*								
	Glycerol	*NH_3*	*N_2*	*Dry biomass*	*1,3-Propanediol*	*Acetic acid*	*CO_2*	*H_2O*	*Total*
Feed	–	–	–	–	–	–	–	–	–
N_2 gas	–	–	–	–	–	–	–	–	–
Product	29.65	0	0	5.35	65.56	15.88	0	871.8	988.35
Off-gas	0	0	66.0	0	0	0	11.65	0	77.65
Total	29.65	0	66.0	5.35	65.56	15.88	11.65	871.8	1066.0

(iii) Check the results

All columns and rows of the completed mass balance table add up correctly to within round-off error.

4. Finalise

(a)

The off-gas contains 66.0 kg N_2 + 11.65 kg CO_2. Converting to gmoles:

$$\begin{aligned} n &= 66.0\ \text{kg N}_2 \cdot \left|\frac{1000\ \text{g}}{1\ \text{kg}}\right| \cdot \left|\frac{1\ \text{gmol}}{28\ \text{g}}\right| + 11.65\ \text{kg CO}_2 \cdot \left|\frac{1000\ \text{g}}{1\ \text{kg}}\right| \cdot \left|\frac{1\ \text{gmol}}{44\ \text{g}}\right| \\ &= 2357.1\ \text{gmol N}_2 + 264.8\ \text{gmol CO}_2 \\ &= 2621.9\ \text{gmol} \end{aligned}$$

Gas compositions are commonly given as volume percent; at low pressures, this is the same as mole percent (Section 2.4.5). Therefore, the off-gas composition is (2357.1 gmol)/(2621.9 gmol) × 100% = 89.9% N_2 and (264.8 gmol)/(2621.9 gmol) × 100% = 10.1% CO_2. The off-gas temperature is 37°C and the pressure is 1 atm. Using the ideal gas law, Eq. (2.35), to calculate the off-gas volume with the temperature converted from °C to Kelvin using Eq. (2.27):

$$V = \frac{nRT}{p} = \frac{2621.9\ \text{gmol}\ (0.082057\ \text{l atm K}^{-1}\ \text{gmol}^{-1})\ (37 + 273.15)\ \text{K}}{1\ \text{atm}} = 6.67 \times 10^4\ \text{l}$$

As the basis used for the mass balance calculation is 1 h, the volumetric flow rate of off-gas is 6.67×10^4 l h^{-1}.

Answer: 6.67×10^4 l h^{-1}; 89.9% N_2 and 10.1% CO_2

(b)

The mass balance was performed for complete NH_3 consumption; therefore, from the mass balance table, the minimum mass of NH_3 required in the feed stream is 0.90 kg. The concentration of NH_3 in the feed is (0.90 kg)/(1000 kg) × 100% = 0.09% NH_3.

Answer: 0.09%

(c)

The concentration of 1,3-propanediol in the product stream is (65.56 kg)/(988.35 kg) × 100% = 6.6%.

Answer: 6.6%

4.13 Cell culture using whey

(a)

The reaction can be represented using the general stoichiometric equation for aerobic growth, Eq. (4.4), with lactose as the substrate:

$$C_{12}H_{22}O_{11} + a\,O_2 + b\,NH_3 \rightarrow c\,CH_{1.63}O_{0.54}N_{0.16} + d\,CO_2 + e\,H_2O$$

This equation must be solved before starting the mass balance calculations. From Table C.1 (Appendix C), the molecular weight of lactose is 342 and the biomass formula weight is 24.51. Taking into account the 7.5% ash in the biomass:

$$\text{Biomass molecular weight} = \frac{24.51}{0.925} = 26.50$$

For Y_{XS} = 0.25 g g^{-1}, from Eq. (4.15):

$$c = \frac{Y_{XS}\,(\text{MW substrate})}{\text{MW cells}} = \frac{0.25\ \text{g g}^{-1}(342)}{26.50} = 3.23 \tag{1}$$

The degree of reduction of lactose relative to NH_3 is:

$$\gamma_S = \frac{12\times4 + 22\times1 - 11\times2}{12} = 4.00$$

The degree of reduction of the biomass relative to NH_3 is:

$$\gamma_B = \frac{1\times4 + 1.63\times1 - 0.54\times2 - 0.16\times3}{1} = 4.07$$

The stoichiometric coefficient a is obtained from Eq. (4.20) with $f = 0$ and $w = 12$ for lactose:

$$a = \frac{1}{4}(w\gamma_S - c\gamma_B) = \frac{1}{4}(12\times4.00 - 3.23\times4.07) = 8.71 \tag{2}$$

The stoichiometric coefficient b is obtained from an elemental balance on N.

N balance: $b = 0.16c$

Using the result from (1):

$$b = 0.52$$

d is obtained from an elemental balance on C.

C balance: $12 = c + d$

Using the result from (1):

$$d = 12 - 3.23 = 8.77 \tag{3}$$

e is obtained from an elemental balance on O.

O balance: $11 + 2a = 0.54c + 2d + e$

Substituting in the results from (1), (2) and (3):

$$11 + 2(8.71) = 0.54(3.23) + 2(8.77) + e$$

$$e = 9.14$$

Therefore, the completed reaction equation is:

$$C_{12}H_{22}O_{11} + 8.71O_2 + 0.52NH_3 \rightarrow 3.23CH_{1.63}O_{0.54}N_{0.16} + 8.77CO_2 + 9.14H_2O$$

This result can be checked using an elemental balance on H.

H balance: $22 + 3b = 1.63c + 2e$

$$22 + 3(0.52) = 1.63(3.23) + 2(9.14)$$

$$23.56 = 23.54$$

We can conclude that the reaction equation is correct to within round-off error. Calculating *RQ* using Eq. (4.9):

$$RQ = \frac{8.77}{8.71} = 1.0$$

Answer: 1.0

(b)

We need to calculate the amount of O_2 required for complete conversion of lactose. Using a basis of 1 h, the feed stream contains (0.04 × 200) kg = 8 kg of lactose. Converting this to kgmoles using the molecular weight of lactose:

$$\text{Lactose consumed} = 8 \text{ kg} \cdot \left| \frac{1 \text{ kgmol}}{342 \text{ kg}} \right| = 2.34 \times 10^{-2} \text{ kgmol}$$

From the reaction stoichiometry in **(a)**, conversion of this amount of lactose requires $8.71 \times 2.34 \times 10^{-2}$ = 0.204 kgmol O_2. If air is sparged into the bioreactor at a rate of 305 l min^{-1}, in 1 h the volume of air entering the fermenter is:

$$\text{Volume of air in} = 305 \text{ l min}^{-1} \times 1 \text{ h} \cdot \left| \frac{60 \text{ min}}{1 \text{ h}} \right| = 18{,}300 \text{ l}$$

Converting this to moles using the ideal gas law, Eq. (2.35), with R = 0.082057 l atm K^{-1} $gmol^{-1}$ (Appendix B) and the temperature converted from °C to Kelvin using Eq. (2.27):

$$\text{Moles of air in} = n = \frac{pV}{RT} = \frac{1 \text{ atm } (18{,}300 \text{ l})}{0.082057 \text{ l atm K}^{-1} \text{ gmol}^{-1} (30 + 273.15) \text{ K}} \cdot \left| \frac{1 \text{ kgmol}}{1000 \text{ gmol}} \right| = 0.736 \text{ kgmol}$$

From the known composition of air (Section 2.4.5), the moles of O_2 in the incoming air is 0.21 × 0.736 gmol = 0.155 kgmol. As this is less than the 0.204 kgmol O_2 required for complete conversion of lactose, oxygen enrichment of the air is needed.

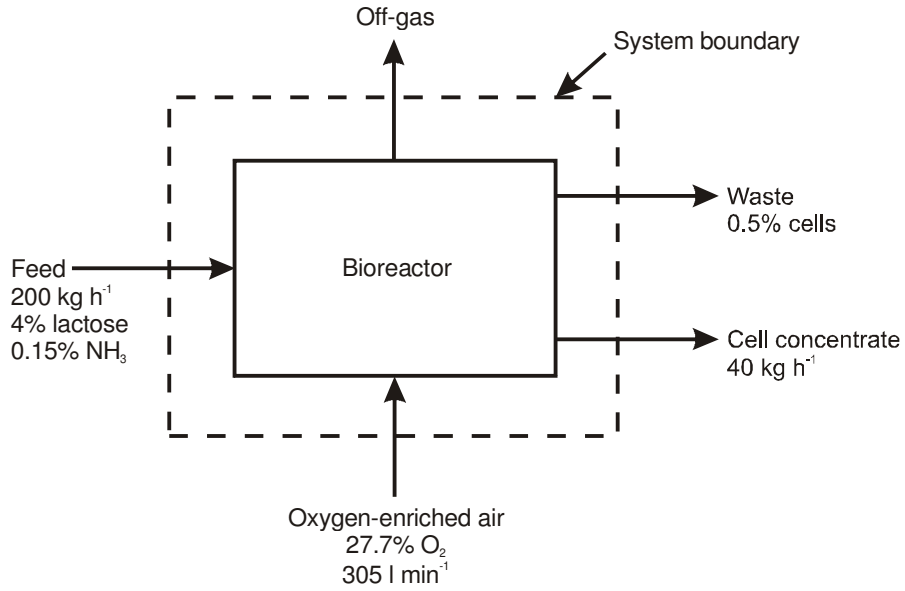
If the flow rate of oxygen-enriched air is 305 l min^{-1}, 0.736 kgmol of gas enters the bioreactor per hour. If this includes the 0.204 kgmol O_2 required for the reaction, the O_2 concentration in the gas stream is (0.204 kgmol)/(0.736 kgmol) × 100% = 27.7%.

Answer: No, the air needs to be enriched to 27.7% O_2

(c), (d)

1. Assemble

(i) Flow sheet



(ii) System boundary

The system boundary is shown on the flow sheet.

(iii) Reaction equation

From **(a)**, the reaction equation is:

$$C_{12}H_{22}O_{11} + 8.71O_2 + 0.52\,NH_3 \rightarrow 3.23CH_{1.63}O_{0.54}N_{0.16} + 8.77\,CO_2 + 9.14\,H_2O$$

2. Analyse

(i) Assumptions

– steady state

– no leaks

– off-gas is dry

– all the CO_2 produced leaves in the off-gas

– gases are at low pressure so vol% = mol% (Section 2.4.5)

– oxygen-enriched air enters the bioreactor at 30°C and 1 atm pressure

– the feed solvent is water

– all the lactose provided is consumed

(ii) Extra data

Molecular weights (Table C.1, Appendix C):

– Lactose = 342

– O_2 = 32

– NH_3 = 17

– CO_2 = 44

– H_2O = 18

– N_2 = 28

– Biomass = 26.50

(iii) Basis

1 h, or 200 kg of feed

(iv) Compounds involved in reaction

Lactose, O_2, NH_3, biomass, CO_2 and H_2O are involved in the reaction.

(v) Mass balance equations

For lactose, O_2, NH_3, biomass, CO_2 and H_2O, the appropriate mass balance equation is Eq. (4.2):

mass in + mass generated = mass out + mass consumed

For N_2 and total mass, the appropriate mass balance equation is Eq. (4.3):

mass in = mass out

3. Calculate

(i) Calculation table

From **(b)**, the gas stream in contains 0.736 kgmol of gas, including 0.204 kgmol O_2. The remaining (0.736 – 0.204) kgmol = 0.532 kgmol is N_2. Converting these molar amounts to mass:

$$\text{Mass of } O_2 \text{ in} = 0.204 \text{ kgmol} \cdot \left| \frac{32.0 \text{ kg}}{1 \text{ kgmol}} \right| = 6.53 \text{ kg}$$

$$\text{Mass of } N_2 \text{ in} = 0.532 \text{ kgmol} \cdot \left| \frac{28.0 \text{ kg}}{1 \text{ kgmol}} \right| = 14.9 \text{ kg}$$

Therefore, the total mass of gas in is (6.53 + 14.9) kg = 21.4 kg. The calculation tables below show all known quantities in kg. The In side of the mass balance table is complete. The total mass of waste out is denoted *W*; the total mass of off-gas out is denoted *G*.

Stream	*In*							
	Lactose	*O_2*	*N_2*	*NH_3*	*Biomass*	*CO_2*	*H_2O*	*Total*
Feed	8.0	0	0	0.3	0	0	191.7	200
Enriched air	0	6.53	14.9	0	0	0	0	21.4
Cell concentrate	–	–	–	–	–	–	–	–
Waste	–	–	–	–	–	–	–	–
Off-gas	–	–	–	–	–	–	–	–
Total	8.0	6.53	14.9	0.3	0	0	191.7	221.4

Stream	*Out*							
	Lactose	O_2	N_2	NH_3	*Biomass*	CO_2	H_2O	*Total*
Feed	–	–	–	–	–	–	–	–
Enriched air	–	–	–	–	–	–	–	–
Cell concentrate	0	0	0	?	?	0	?	40
Waste	0	0	0	?	?	0	?	W
Off-gas	0	0	?	0	0	?	0	G
Total	0	0	?	?	?	?	?	$40 + W + G$

(ii) Mass balance calculations

From **(b)**, 2.34×10^{-2} kgmol of lactose is consumed. From the stoichiometry:

$$\text{Mass of } O_2 \text{ consumed} = 8.71 \times 2.34 \times 10^{-2} \text{ kgmol} \cdot \left| \frac{32 \text{ kg}}{1 \text{ kgmol}} \right| = 6.52 \text{ kg}$$

$$\text{Mass of } NH_3 \text{ consumed} = 0.52 \times 2.34 \times 10^{-2} \text{ kgmol} \cdot \left| \frac{17 \text{ kg}}{1 \text{ kgmol}} \right| = 0.207 \text{ kg}$$

$$\text{Mass of biomass generated} = 3.23 \times 2.34 \times 10^{-2} \text{ kgmol} \cdot \left| \frac{26.50 \text{ kg}}{1 \text{ kgmol}} \right| = 2.00 \text{ kg}$$

$$\text{Mass of } CO_2 \text{ generated} = 8.77 \times 2.34 \times 10^{-2} \text{ kgmol} \cdot \left| \frac{44 \text{ kg}}{1 \text{ kgmol}} \right| = 9.03 \text{ kg}$$

$$\text{Mass of } H_2O \text{ generated} = 9.14 \times 2.34 \times 10^{-2} \text{ kgmol} \cdot \left| \frac{18 \text{ kg}}{1 \text{ kgmol}} \right| = 3.85 \text{ kg}$$

N_2 balance

N_2 is a tie component.

$$14.9 \text{ kg } N_2 \text{ in} = N_2 \text{ out}$$

$$N_2 \text{ out} = 14.9 \text{ kg}$$

Total mass balance

$$221.4 \text{ kg total mass in} = (40 + W + G) \text{ kg total mass out}$$

$$W = 181.4 - G \tag{4}$$

NH_3 balance

$$0.3 \text{ kg } NH_3 \text{ in} + 0 \text{ kg } NH_3 \text{ generated} = NH_3 \text{ out} + 0.207 \text{ kg } NH_3 \text{ consumed}$$

$$NH_3 \text{ out} = 0.093 \text{ kg} \tag{5}$$

Biomass balance

$$0 \text{ kg biomass in} + 2.00 \text{ kg biomass generated} = \text{biomass out} + 0 \text{ kg biomass consumed}$$

$$\text{Biomass out} = 2.00 \text{ kg} \tag{6}$$

CO_2 balance

$$0 \text{ kg } CO_2 \text{ in} + 9.03 \text{ kg } CO_2 \text{ generated} = CO_2 \text{ out} + 0 \text{ g } CO_2 \text{ consumed}$$

$$CO_2 \text{ out} = 9.03 \text{ kg}$$

H_2O balance

$$191.7 \text{ kg } H_2O \text{ in} + 3.85 \text{ kg } H_2O \text{ generated} = H_2O \text{ out} + 0 \text{ kg } H_2O \text{ consumed}$$

$$H_2O \text{ out} = 195.55 \text{ kg} \tag{7}$$

Adding together the masses of N_2 and CO_2 out gives G, the mass of off-gas out:

$$G = N_2 \text{ out} + CO_2 \text{ out} = (14.9 + 9.03) \text{ kg}$$
$$= 23.93 \text{ kg}$$

Using this result in (4) gives:

$$W = (181.4 - 23.93) \text{ kg}$$
$$= 157.5 \text{ kg}$$

The biomass concentration in the waste stream is 0.5%. Therefore, the biomass out in the waste is (0.5/100) × W = 0.005 × 157.5 kg = 0.788 kg. Subtracting this mass from the total mass of the waste stream, we can deduce that the mass of (NH_3 and H_2O) in the waste = (157.5 – 0.788) kg = 156.7 kg.

From (6), the total biomass out is 2.00 kg. If the biomass out in the waste stream is 0.788 kg, the biomass out in the cell concentrate must be (2.00 – 0.788) kg = 1.212 kg. Therefore, the mass of (NH_3 and H_2O) in the cell concentrate stream = (40 – 1.212) kg = 38.79 kg. Adding this value to the mass of (NH_3 and H_2O) in the waste, the total mass of (NH_3 and H_2O) out = (156.7 + 38.79) kg = 195.49 kg.

Although these calculations have given us the total mass of (NH_3 and H_2O) out, the separate NH_3 and H_2O contents of the cell concentrate and waste streams remain unknown. However, NH_3 and H_2O are partitioned between these two streams in the same proportions as the entire aqueous phase leaving the bioreactor is partitioned between the cell concentrate and waste streams. We can determine the separate masses of NH_3 and H_2O in these streams from the total masses of NH_3 and H_2O out given by (5) and (7):

$$NH_3 \text{ out in cell concentrate} = \frac{\text{total kg } NH_3 \text{ out}}{\text{total kg } (NH_3 + H_2O) \text{ out}} \times \text{kg } (NH_3 + H_2O) \text{ out in cell concentrate}$$
$$= \frac{0.093 \text{ kg}}{195.49 \text{ kg}} \times 38.79 \text{ kg}$$
$$= 0.0185 \text{ kg}$$

and

$$NH_3 \text{ out in waste} = \frac{\text{total kg } NH_3 \text{ out}}{\text{total kg } (NH_3 + H_2O) \text{ out}} \times \text{kg } (NH_3 + H_2O) \text{ out in waste}$$
$$= \frac{0.093 \text{ kg}}{195.49 \text{ kg}} \times 156.7 \text{ kg}$$
$$= 0.0745 \text{ kg}$$

Similarly for H_2O:

$$H_2O \text{ out in cell concentrate} = \frac{195.55 \text{ kg}}{195.49 \text{ kg}} \times 38.79 \text{ kg} = 38.80 \text{ kg}$$

and

$$\text{H}_2\text{O out in waste} = \frac{195.55\ \text{kg}}{195.49\ \text{kg}} \times 156.7\ \text{kg} = 156.75\ \text{kg}$$

These calculations allow completion of the Out mass balance table with all quantities in kg.

Stream	*Out*							
	Lactose	*O_2*	*N_2*	*NH_3*	*Biomass*	*CO_2*	*H_2O*	*Total*
Feed	–	–	–	–	–	–	–	–
Enriched air	–	–	–	–	–	–	–	–
Cell concentrate	0	0	0	0.0185	1.212	0	38.80	40
Waste	0	0	0	0.0745	0.788	0	156.75	157.5
Off-gas	0	0	14.9	0	0	9.03	0	23.93
Total	0	0	14.9	0.093	2.00	9.03	195.55	221.4

(iii) Check the results

All columns and rows of the completed mass balance table add up correctly to within round-off error.

4. Finalise

From the completed mass balance table, the concentration of residual NH_3 in the waste stream = (0.0745 kg)/(157.5 kg) × 100% = 0.047%. The concentration of cells in the cell concentrate = (1.212 kg)/(40 kg) × 100% = 3.0%.

Answer: 0.047% NH_3 in the waste stream; 3.0% cells in the cell concentrate

4.14 Oxygen requirement for growth on glycerol

From Table C.2 (Appendix C), the molecular formula for glycerol is $C_3H_8O_3$. From Table 4.11, the chemical formula for *Klebsiella aerogenes* can be taken as $CH_{1.75}O_{0.43}N_{0.22}$. Substituting these formulae into the general stoichiometric equation for growth, Eq. (4.4), gives:

$$\text{C}_3\text{H}_8\text{O}_3 + a\text{O}_2 + b\text{NH}_3 \rightarrow c\text{CH}_{1.75}\text{O}_{0.43}\text{N}_{0.22} + d\text{CO}_2 + e\text{H}_2\text{O}$$

From Table C.8 (Appendix C), the molecular weight of glycerol is 92.1; from Table C.1 (Appendix C), the molecular weight of oxygen is 32.0. The biomass formula weight calculated using Table C.1 is 23.74. Taking into account the 8% ash:

$$\text{Biomass molecular weight} = \frac{23.74}{0.92} = 25.8$$

The value of the stoichiometric coefficient c is determined from the yield Y_{XS} = 0.4 g g^{-1} and Eq. (4.15):

$$c = \frac{Y_{XS}\,(\text{MW substrate})}{\text{MW cells}} = \frac{0.40\ \text{g g}^{-1}\,(92.1)}{25.8} = 1.43$$

From Table C.2 (Appendix C), the degree of reduction of glycerol relative to NH_3 is γ_S = 4.67. The degree of reduction of the biomass relative to NH_3 is:

$$\gamma_B = \frac{1\times 4 + 1.75\times 1 - 0.43\times 2 - 0.22\times 3}{1} = 4.23$$

(This result is also listed in Table 4.11.) The theoretical oxygen demand is determined using Eq. (4.20) with w = 3 for glycerol and f = 0:

$$a = \frac{1}{4}(w\gamma_S - c\gamma_B) = \frac{1}{4}(3\times 4.67 - 1.43\times 4.23) = 1.99$$

Therefore, 1.99 gmol of oxygen are required per gmol of glycerol. Converting a to mass terms using the molecular weights of glycerol and oxygen:

$$a = 1.99 \text{ gmol } O_2 \text{ (gmol glycerol)}^{-1} . \left| \frac{32.0 \text{ g}}{1 \text{ gmol } O_2} \right| . \left| \frac{1 \text{ gmol glycerol}}{92.1 \text{ g}} \right| = 0.69 \text{ g g}^{-1}$$

Answer: 0.69 g per g of glycerol consumed

4.15 Product yield in anaerobic digestion

From Eq. (4.16), the stoichiometric equation for anaerobic growth and product formation by methane bacteria is:

$$CH_3COOH + bNH_3 \rightarrow cCH_{1.4}O_{0.40}N_{0.20} + dCO_2 + eH_2O + fCH_4$$

From Table C.8 (Appendix C), the molecular weight of acetic acid is 60.1. From Table C.1 (Appendix C), the molecular weight of CO_2 is 44.0. The value of the stoichiometric coefficient d can be determined using an equation analogous to Eq. (4.17) with carbon dioxide as the product and the yield Y_{PS} = 0.67 kg kg^{-1} = 0.67 g g^{-1}:

$$d = \frac{Y_{PS}(\text{MW acetic acid})}{\text{MW } CO_2} = \frac{0.67 \text{ g g}^{-1}(60.1)}{44.0} = 0.915$$

The other coefficients can be determined using this result and elemental balances.

C balance: $2 = c + d + f = c + 0.915 + f \rightarrow f = 1.085 - c$

H balance: $4 + 3b = 1.4c + 2e + 4f$

O balance: $2 = 0.40c + 2d + e = 0.40c + 2 \times 0.915 + e = 0.40c + 1.83 + e \rightarrow e = 0.17 - 0.40c$

N balance: $b = 0.20c$

Substituting the expressions for f, e and b from the C, O and N balances, respectively, into the H balance:

$$4 + 3\times 0.20c = 1.4c + 2\times(0.17 - 0.40c) + 4\times(1.085 - c)$$

$$4c = 0.680$$

$$c = 0.170$$

Substituting this value for c into the expressions for the other coefficients gives b = 0.034, e = 0.102 and f = 0.915. The yield of methane is therefore 0.915 gmol per gmol of acetic acid.

The maximum possible methane yield is calculated using Eq. (4.24) with w = 2 for acetic acid and j = 1 for methane. From Table C.2 (Appendix C), the degree of reduction of acetic acid relative to NH_3 is γ_S = 4.00 and the degree of reduction of methane relative to NH_3 is γ_P = 8.00. Substituting these values into Eq. (4.24) gives:

$$f_{max} = \frac{2(4.00)}{1(8.00)} = 1.0$$

Therefore, the actual methane yield f = 0.915 gmol gmol^{-1} represents 91.5% of the theoretical maximum.

Answer: 91.5% of the theoretical maximum

4.16 Production of PHB

(a)

From Eq. (4.16), the stoichiometric equation for PHB production from starch can be written as:

$$C_6H_{10}O_5 + aO_2 + bNH_3 \rightarrow cCH_2O_{0.5}N_{0.25} + dCO_2 + eH_2O + fC_4H_6O_2$$

where PHB is considered a separate product from the biomass. From Table C.1 (Appendix C), the molecular weight of monomeric starch is 162, the molecular weight of the biomass is 25.5, and the molecular weight of PHB is 86. If the concentration of PHB in the cells is 44%, 44 g of PHB is produced for every 56 g of PHB-free biomass. Therefore, based on the stoichiometric equation and by analogy with Eq. (4.17), the yield of PHB from biomass, Y_{PX}, is:

$$Y_{PX} = \frac{\text{g product formed}}{\text{g cells formed}} = \frac{f\,(\text{MW product})}{c\,(\text{MW cells})}$$

Substituting values:

$$Y_{PX} = \frac{44\text{ g}}{56\text{ g}} = \frac{86f}{25.5c}$$

$$0.786 = 3.373\frac{f}{c}$$

$$f = 0.233c \tag{1}$$

From Eq. (4.9):

$$d = RQ\,a = 1.3a \tag{2}$$

Other equations for the stoichiometric coefficients can be obtained from elemental balances.

C balance: $6 = c + d + 4f$

Applying (1) to the C-balance equation gives:

$$6 = c + d + 4(0.233c) = 1.932c + d$$

$$d = 6 - 1.932c \tag{3}$$

Combining (2) and (3) gives:

$$1.3a = 6 - 1.932c$$

$$a = 4.615 - 1.486c \tag{4}$$

O balance: $5 + 2a = 0.5c + 2d + e + 2f$

Expressing a, d and f in the O-balance equation in terms of c using (4), (3) and (1) gives:

$$5 + 2(4.615 - 1.486c) = 0.5c + 2(6 - 1.932c) + e + 2(0.233c)$$

$$5 + 9.230 - 2.972c = 0.5c + 12 - 3.864c + e + 0.466c$$

$$e = 2.230 - 0.074c \tag{5}$$

N balance: $b = 0.25c$ (6)

H balance: $10 + 3b = 2c + 2e + 6f$

Expressing b, e and f in the H-balance equation in terms of c using (6), (5) and (1) gives:

$$10 + 3(0.25c) = 2c + 2(2.230 - 0.074c) + 6(0.233c)$$

$$10 + 0.75c = 2c + 4.460 - 0.148c + 1.398c$$

$$5.540 = 2.500c$$

$$c = 2.22 \qquad (7)$$

Using this result for c in (1), (3), (4), (5) and (6) gives:

$$f = 0.52$$

$$d = 1.71$$

$$a = 1.32$$

$$e = 2.07$$

$$b = 0.56$$

The reaction equation can now be completed:

$$C_6H_{10}O_5 + 1.32O_2 + 0.56NH_3 \rightarrow 2.22CH_2O_{0.5}N_{0.25} + 1.71CO_2 + 2.07H_2O + 0.52C_4H_6O_2$$

This result can be checked by performing the elemental balances again.

C balance: $6 = 2.22 + 1.71 + 0.52 \times 4$

$$6 = 6.01$$

N balance: $0.56 = 2.22 \times 0.25$

$$0.56 = 0.56$$

O balance: $5 + 1.32 \times 2 = 2.22 \times 0.5 + 1.71 \times 2 + 2.07 + 0.52 \times 2$

$$7.64 = 7.64$$

H balance: $10 + 0.56 \times 3 = 2.22 \times 2 + 2.07 \times 2 + 0.52 \times 6$

$$11.68 = 11.70$$

Therefore, the stoichiometric equation and elemental balances are correct to within round-off error.

Answer: $C_6H_{10}O_5 + 1.32O_2 + 0.56NH_3 \rightarrow 2.22CH_2O_{0.5}N_{0.25} + 1.71CO_2 + 2.07H_2O + 0.52C_4H_6O_2$

(b)

From the completed stoichiometric equation, for each gmol of monomeric starch consumed, 2.22 gmol of cells and 0.52 gmol of PHB are formed. Converting to mass terms, the yield of PHB-containing cells from starch is:

$$\text{Yield} = \frac{2.22 \text{ gmol cells} \left| \frac{25.5 \text{ g}}{1 \text{ gmol}} \right| + 0.52 \text{ gmol PHB} \left| \frac{86 \text{ g}}{1 \text{ gmol}} \right|}{1 \text{ gmol starch} \left| \frac{162 \text{ g}}{1 \text{ gmol}} \right|} = 0.63 \text{ g g}^{-1}$$

Answer: 0.63 g g^{-1}

(c)

If the downstream recovery of PHB is 65%, production of 25 kg of PHB after losses requires (25 kg)/0.65 = 38.46 kg of PHB to be synthesised by the cells. Converting to kgmol:

$$38.46 \text{ kg PHB} = 38.46 \text{ kg} \left| \frac{1 \text{ kgmol}}{86 \text{ kg}} \right| = 0.447 \text{ kgmol}$$

From the completed stoichiometric equation, this requires (0.447 kgmol)/0.52 = 0.860 kgmol of monomeric starch. Converting to mass:

$$0.860 \text{ kgmol starch} = 0.860 \text{ kgmol} \,.\left|\frac{162 \text{ kg}}{1 \text{ kgmol}}\right| = 139.3 \text{ kg}$$

Answer: 139 kg

4.17 Oxygen consumption by suspended plant cells

From Eq. (4.16), the stoichiometric equation can be written as:

$$C_{12}H_{22}O_{11} + aO_2 + bNH_3 \rightarrow cCH_{1.63}O_{0.80}N_{0.13} + dCO_2 + eH_2O + fCH_{1.63}O_{0.80}N_{0.13}$$

where the excreted by-product has the same molecular formula as the biomass. From Table C.1 (Appendix C), the molecular weight of sucrose is 342 and the molecular weight of the biomass and by-product is 28.25. The value of the stoichiometric coefficient c is determined from the yield Y_{XS} = 0.32 g g^{-1} and Eq. (4.15):

$$c = \frac{Y_{XS}\,(\text{MW substrate})}{\text{MW cells}} = \frac{0.32 \text{ g g}^{-1}\,(342 \text{ g gmol}^{-1})}{28.25 \text{ g gmol}^{-1}} = 3.87 \qquad (1)$$

Based on the stoichiometric equation and by analogy with Eq. (4.17), the yield of by-product from biomass, Y_{PX}, is:

$$Y_{PX} = \frac{\text{g by-product formed}}{\text{g cells formed}} = \frac{f\,(\text{MW by-product})}{c\,(\text{MW cells})}$$

Substituting values:

$$Y_{PX} = \frac{0.2 \text{ g}}{1 \text{ g}} = \frac{28.25f}{28.25c}$$

$$0.2 = \frac{f}{c}$$

Using the result from (1):

$$f = 0.774$$

The oxygen demand is determined using an electron balance. The degree of reduction of sucrose relative to NH_3 is:

$$\gamma_S = \frac{12\times4 + 22\times1 - 11\times2}{12} = 4.00$$

The degree of reduction of cells and by-product relative to NH_3 is:

$$\gamma_B = \gamma_P = \frac{1\times4 + 1.63\times1 - 0.80\times2 - 0.13\times3}{1} = 3.64$$

w = 12 for sucrose and j = 1 for by-product. Substituting these values into Eq. (4.20):

$$a = \frac{1}{4}(12\times4.00 - 3.87\times3.64 - 0.774\times1\times3.64) = 7.77$$

Converting 10 kg of sucrose consumed per hour to gmol min^{-1}:

$$10 \text{ kg sucrose h}^{-1} = 10 \text{ kg h}^{-1} \,.\left|\frac{1000 \text{ g}}{1 \text{ kg}}\right| .\left|\frac{1 \text{ gmol}}{342 \text{ g}}\right| .\left|\frac{1 \text{ h}}{60 \text{ min}}\right| = 0.487 \text{ gmol min}^{-1}$$

From the result for a, the oxygen requirement is 7.77 gmol per gmol of sucrose consumed. Therefore, the oxygen requirement is 7.77×0.487 gmol $min^{-1} = 3.78$ gmol min^{-1}.

Answer: 3.8 gmol min^{-1}

4.18 Substrate requirements for continuous culture

(a)

When biomass is the only major product, the stoichiometric equation is based on Eq. (4.4). Using the general formula $CH_{1.8}O_{0.5}N_{0.2}$ for the biomass (Section 4.6.1), the stoichiometric equation is:

$$C_2H_6O + aO_2 + bNH_3 \rightarrow cCH_{1.8}O_{0.5}N_{0.2} + dCO_2 + eH_2O$$

From Table C.1 (Appendix C), the molecular weight of ethanol is 46, the molecular weight of NH_3 is 17, and the molecular weight of the biomass is 24.6. Using a basis of 1 h, the biomass yield from substrate Y_{XS} = (45 g)/(150 g) = 0.30 g g^{-1}. The stoichiometric coefficient c is evaluated from Eq. (4.15):

$$c = \frac{Y_{XS}\,(\text{MW substrate})}{\text{MW cells}} = \frac{0.30 \text{ g g}^{-1}\,(46)}{24.6} = 0.561 \qquad (1)$$

The oxygen demand is determined using an electron balance. From Table C.2 (Appendix C), the degree of reduction of ethanol relative to NH_3 is $\gamma_S = 6.00$. The degree of reduction of the biomass relative to NH_3 is:

$$\gamma_B = \frac{1\times4 + 1.8\times1 - 0.5\times2 - 0.2\times3}{1} = 4.20$$

(This value for γ_B is also given in Table C.2.) $w = 2$ for ethanol. Substituting values into Eq. (4.20) with $f = 0$:

$$a = \frac{1}{4}(w\gamma_S - c\gamma_B) = \frac{1}{4}(2\times6.00 - 0.561\times4.20) = 2.41$$

Therefore, the oxygen requirement is 2.41 gmol per gmol of ethanol consumed. Converting the rate of ethanol consumption to gmol h^{-1}:

$$150 \text{ g ethanol h}^{-1} = 150 \text{ g h}^{-1}.\left|\frac{1 \text{ gmol}}{46 \text{ g}}\right| = 3.26 \text{ gmol h}^{-1}$$

From the result for a, the oxygen requirement is 2.41×3.26 gmol $h^{-1} = 7.86$ gmol h^{-1}. This rate of oxygen consumption can be converted to a mass basis using the molecular weight of O_2 = 32 (Table C.1, Appendix C):

$$7.86 \text{ gmol O}_2 \text{ h}^{-1} = 7.86 \text{ gmol h}^{-1}.\left|\frac{32 \text{ g}}{1 \text{ gmol}}\right| = 251.5 \text{ g h}^{-1}$$

Answer: 7.9 gmol h^{-1} or 252 g h^{-1}

(b)

The stoichiometric coefficient b is evaluated using a nitrogen balance:

N balance: $b = 0.2c$

Substituting the value for c evaluated in **(a)** gives $b = 0.112$ gmol NH_3 per gmol of ethanol consumed. From **(a)**, the rate of ethanol consumption is 3.26 gmol h^{-1}; therefore, the rate of NH_3 consumption is 0.112×3.26 gmol $h^{-1} = 0.365$ gmol h^{-1}. Converting this rate to a mass basis:

$$0.365 \text{ gmol NH}_3 \text{ h}^{-1} = 0.365 \text{ gmol h}^{-1}.\left|\frac{17 \text{ g}}{1 \text{ gmol}}\right| = 6.21 \text{ g h}^{-1}$$

If 20 g h^{-1} NH_3 is provided, the rate at which unreacted NH_3 leaves the reactor is (20 – 6.21) g h^{-1} = 13.79 g h^{-1}.

Answer: 13.8 g h^{-1}

4.19 Oxygen and sulphur requirements for bacterial culture

(a)

From Table C.2 (Appendix C), the molecular formula for glucose is $C_6H_{12}O_6$. When biomass is the only major product, the stoichiometric equation is based on Eq. (4.4) with an additional term for H_2SO_4 to represent the sulphur substrate:

$$C_6H_{12}O_6 + aO_2 + bNH_3 + gH_2SO_4 \rightarrow cCH_{1.73}O_{0.52}N_{0.17}S_{0.0032} + dCO_2 + eH_2O$$

From Table C.1 (Appendix C), the molecular weight of glucose is 180 and the molecular weight of the biomass is 24.56. The biomass yield from substrate Y_{XS} = 0.29 g g^{-1}. The stoichiometric coefficient c is evaluated from Eq. (4.15):

$$c = \frac{Y_{XS}\,(\text{MW substrate})}{\text{MW cells}} = \frac{0.29\text{ g g}^{-1}(180)}{24.56} = 2.13 \quad (1)$$

The oxygen demand is determined using an electron balance. From Table C.2 (Appendix C), the degree of reduction of glucose relative to NH_3 is γ_S = 4.00. The degree of reduction of the biomass relative to NH_3 is:

$$\gamma_B = \frac{1\times4 + 1.73\times1 - 0.52\times2 - 0.17\times3 + 0.0032\times6}{1} = 4.20$$

As the degree of reduction of H_2SO_4 = 2 × 1 + 1 × 6 – 4 × 2 = 0, this component does not appear in the electron balance and the oxygen requirement can be determined using Eq. (4.20) with f = 0 and w = 6 for glucose:

$$a = \frac{1}{4}(w\gamma_S - c\gamma_B) = \frac{1}{4}(6\times4.00 - 2.13\times4.20) = 3.76$$

Therefore, the oxygen requirement is 3.76 gmol per gmol of glucose consumed.

Answer: 3.8 gmol per gmol of glucose

(b)

The stoichiometric coefficient g is evaluated using a sulphur balance.

S balance: $g = 0.0032c$

Substituting the value for c evaluated in **(a)** gives $g = 0.0032 \times 2.13 = 6.82 \times 10^{-3}$ gmol H_2SO_4 per gmol of glucose consumed. Converting the mass concentration of glucose consumed to gmol l^{-1}:

$$20\text{ g glucose l}^{-1} = 20\text{ g l}^{-1}.\left|\frac{1\text{ gmol}}{180\text{ g}}\right| = 0.111\text{ gmol l}^{-1}$$

Therefore, the H_2SO_4 concentration required is $6.82 \times 10^{-3} \times 0.111$ gmol l^{-1} = 7.57×10^{-4} gmol l^{-1}.

Answer: 7.6×10^{-4} gmol l^{-1}

4.20 Stoichiometry of single-cell protein synthesis

(a)

From Table C.2 (Appendix C), the molecular formula for glucose is $C_6H_{12}O_6$. If all the carbon in the substrate is converted into biomass, production of carbon dioxide is zero. Therefore, from Eq. (4.4), the stoichiometric equation for anaerobic growth of *Cellulomonas* is:

$$C_6H_{12}O_6 + bNH_3 \rightarrow cCH_{1.56}O_{0.54}N_{0.16} + eH_2O$$

The stoichiometric coefficients can be determined using elemental balances.

C balance: $6 = c$

H balance: $12 + 3b = 1.56c + 2e$

O balance: $6 = 0.54c + e$

N balance: $b = 0.16c$

The yield of biomass from substrate in molar terms is therefore $c = 6$ gmol gmol^{-1}. Substituting the value for c from the C balance into the O and N balances gives $e = 2.76$ and $b = 0.96$, respectively.

From Table C.1 (Appendix C), the molecular weight of glucose is 180.2 and the biomass formula weight is 24.46. Taking into account the 5% ash:

$$\text{Biomass molecular weight} = \frac{24.46}{0.95} = 25.75$$

Therefore, in mass terms, the molar biomass yield of 6 gmol gmol^{-1} = (6 × 25.75) g biomass per 180.2 g of substrate = 0.86 g g^{-1}.

The maximum possible biomass yield is calculated using Eq. (4.23) with $w = 6$ for glucose. From Table C.2 (Appendix C), the degree of reduction of glucose relative to NH_3 is $\gamma_S = 4.00$. The degree of reduction of the biomass relative to NH_3 is:

$$\gamma_B = \frac{1\times 4 + 1.56\times 1 - 0.54\times 2 - 0.16\times 3}{1} = 4.00$$

Substituting these values into Eq. (4.23):

$$c_{max} = \frac{6(4.00)}{4.00} = 6.0$$

The theoretical maximum biomass yield c_{max} is therefore the same as the actual biomass yield, c.

Answer: The biomass yield from substrate of 0.86 g g^{-1} is 100% of the theoretical maximum. *When there is no product formation and no oxygen for electron transfer, all the available electrons from the substrate must go to the biomass.*

(b)

(i)

From Table C.2 (Appendix C), the molecular formula for methanol is CH_4O and the degree of reduction of methanol relative to NH_3 is $\gamma_S = 6.00$. The degree of reduction of *Methylophilus methylotrophus* biomass relative to NH_3 is:

$$\gamma_B = \frac{1\times 4 + 1.68\times 1 - 0.36\times 2 - 0.22\times 3}{1} = 4.30$$

$w = 1$ for methanol. Substituting values into Eq. (4.23):

$$c_{max} = \frac{1(6.00)}{4.30} = 1.40$$

From Table C.8 (Appendix C), the molecular weight of methanol is 32.0. The biomass formula weight calculated from the atomic weights in Table C.1 (Appendix C) is 22.55. With 6% ash:

$$\text{Biomass molecular weight} = \frac{22.55}{0.94} = 23.99$$

In mass terms, the maximum possible biomass yield c_{max} is equal to (1.40 × 23.99) g of biomass per 32.0 g of methanol = 1.05 g g^{-1}.

Answer: The maximum possible biomass yield from methanol is 1.05 g g^{-1}. In terms of C atoms, the biomass yield is 1.40 gmol $gmol^{-1}$ as both biomass and substrate have 1 C atom each. In comparison, the C-atom biomass yield from glucose in **(a)** is 1 gmol $gmol^{-1}$. The main reason for the increased yield in **(b)** is the high degree of reduction of methanol compared with glucose.

(ii)

The actual yield of biomass from methanol is c = 0.42 × 1.40 = 0.59 gmol $gmol^{-1}$. The oxygen demand can be determined from Eq. (4.20) if biomass remains the only major product so that f = 0. Using the parameter values determined in **(b) (i)**:

$$a = \frac{1}{4}(w\gamma_S - c\gamma_B) = \frac{1}{4}(1\times 6.00 - 0.59\times 4.30) = 0.87$$

Therefore, 0.87 gmol of oxygen is required per gmol of methanol. As the molecular weights of methanol and oxygen are the same, the oxygen demand is 0.87 g g^{-1} methanol.

Answer: 0.87 g per g of methanol consumed

4.21 Ethanol production by yeast and bacteria

(a)

From Table C.2 (Appendix C), the molecular formula for glucose is $C_6H_{12}O_6$ and the molecular formula for ethanol is C_2H_6O. From Eq. (4.16), the stoichiometric equation for anaerobic growth and product formation is:

$$C_6H_{12}O_6 + bNH_3 \rightarrow cCH_{1.8}O_{0.5}N_{0.2} + dCO_2 + eH_2O + fC_2H_6O$$

From Table C.8 (Appendix C), the molecular weight of ethanol is 46.1. Using the atomic weights in Table C.1 (Appendix C), the molecular weight of glucose is 180.2 and the biomass molecular weight is 24.6. The values of the stoichiometric coefficients c can be determined from Eq. (4.15) and the yields Y_{XS} = 0.11 g g^{-1} for yeast and Y_{XS} = 0.05 g g^{-1} for bacteria.

$$\text{For yeast, } c = \frac{Y_{XS}\,(\text{MW substrate})}{\text{MW cells}} = \frac{0.11 \text{ g g}^{-1}\,(180.2)}{24.6} = 0.81$$

$$\text{For bacteria, } c = \frac{Y_{XS}\,(\text{MW substrate})}{\text{MW cells}} = \frac{0.05 \text{ g g}^{-1}\,(180.2)}{24.6} = 0.37$$

The other coefficients can be determined using elemental balances.

Yeast

C balance: $6 = c + d + 2f = 0.81 + d + 2f \rightarrow d = 5.19 - 2f$

H balance: $12 + 3b = 1.8c + 2e + 6f = 1.8 \times 0.81 + 2e + 6f \rightarrow 10.54 + 3b = 2e + 6f$

O balance: $6 = 0.5c + 2d + e + f = 0.5 \times 0.81 + 2d + e + f \rightarrow 5.595 = 2d + e + f$

N balance: $b = 0.2c = 0.2 \times 0.81 = 0.16$

Substituting the result for b from the N balance into the H balance gives:

$$10.54 + 3 \times 0.16 = 2e + 6f$$

$$e = 5.51 - 3f$$

Substituting this and the expression for d from the C balance into the O balance gives:

$$5.595 = 2 \times (5.19 - 2f) + (5.51 - 3f) + f$$

$$6f = 10.295$$

$$f = 1.72$$

Therefore, for yeast, the yield of ethanol from glucose is 1.72 gmol gmol^{-1}.

Bacteria

C balance: $6 = c + d + 2f = 0.37 + d + 2f \rightarrow d = 5.63 - 2f$

H balance: $12 + 3b = 1.8c + 2e + 6f = 1.8 \times 0.37 + 2e + 6f \rightarrow 11.33 + 3b = 2e + 6f$

O balance: $6 = 0.5c + 2d + e + f = 0.5 \times 0.37 + 2d + e + f \rightarrow 5.815 = 2d + e + f$

N balance: $b = 0.2c = 0.2 \times 0.37 = 0.074$

Using the same solution procedure as for yeast, substituting the result for b from the N balance into the H balance gives:

$$11.33 + 3 \times 0.074 = 2e + 6f$$

$$e = 5.78 - 3f$$

Substituting this and the expression for d from the C balance into the O balance gives:

$$5.815 = 2 \times (5.63 - 2f) + (5.78 - 3f) + f$$

$$6f = 11.225$$

$$f = 1.87$$

Therefore, for bacteria, the yield of ethanol from glucose is 1.87 gmol gmol^{-1}.

Answer: 1.72 gmol gmol^{-1} for yeast; 1.87 gmol gmol^{-1} for bacteria

(b)

The maximum possible ethanol yield can be calculated using Eq. (4.24) with $w = 6$ for glucose and $j = 2$ for ethanol. From Table C.2 (Appendix C), the degree of reduction of glucose relative to NH_3 is $\gamma_S = 4.00$ and the degree of reduction of ethanol relative to NH_3 is $\gamma_P = 6.00$. Using these values in Eq. (4.24) gives:

$$f_{max} = \frac{6(4.00)}{2(6.00)} = 2.0$$

Therefore, the actual ethanol yield of 1.72 gmol gmol^{-1} obtained in **(a)** for yeast represents 1.72/2.0 × 100% = 86% of the theoretical maximum. For bacteria, the actual yield of 1.87 gmol gmol^{-1} represents 1.87/2.0 × 100% = 94% of the theoretical maximum.

Answer: 86% of the theoretical maximum for yeast; 94% of the theoretical maximum for bacteria

4.22 Detecting unknown products

From Table C.2 (Appendix C), the molecular formula for glucose is $C_6H_{12}O_6$. Assuming that no products other than biomass are formed, from Eq. (4.4), the stoichiometric equation for growth is:

$$C_6H_{12}O_6 + aO_2 + bNH_3 \rightarrow cCH_{1.79}O_{0.56}N_{0.17} + dCO_2 + eH_2O$$

From Table C.1 (Appendix C), the molecular weight of glucose is 180.2, the molecular weight of oxygen is 32.0, and the biomass molecular weight is 25.16. The value of the stoichiometric coefficient c is determined from the yield $Y_{XS} = 0.37$ g g^{-1} and Eq. (4.15):

$$c = \frac{Y_{XS}\,(\text{MW substrate})}{\text{MW cells}} = \frac{0.37\ \text{g g}^{-1}\,(180.2)}{25.16} = 2.65$$

Therefore, 2.65 gmol of cells are produced per gmol of glucose consumed. Converting the oxygen demand to a molar basis:

$$0.88\ \text{g O}_2\ \text{per g cells} = \frac{0.88\ \text{g O}_2}{1\ \text{g cells}} \cdot \left|\frac{25.16\ \text{g cells}}{1\ \text{gmol cells}}\right| \cdot \left|\frac{1\ \text{gmol O}_2}{32.0\ \text{g O}_2}\right| = 0.69\ \text{gmol O}_2\ (\text{gmol cells})^{-1}$$

Combining this with the result for c, the observed oxygen demand a is:

$$a = \frac{0.69\ \text{gmol O}_2}{1\ \text{gmol cells}} \times \frac{2.65\ \text{gmol cells}}{1\ \text{gmol glucose}} = 1.83$$

From Table C.2 (Appendix C), the degree of reduction of glucose relative to NH_3 is $\gamma_S = 4.00$. The degree of reduction of the biomass relative to NH_3 is:

$$\gamma_B = \frac{1\times 4 + 1.79\times 1 - 0.56\times 2 - 0.17\times 3}{1} = 4.16$$

If no products are formed other than biomass, the theoretical oxygen demand can be determined from Eq. (4.20) with $w = 6$ for glucose and $f = 0$:

$$a = \frac{1}{4}(w\gamma_S - c\gamma_B) = \frac{1}{4}(6\times 4.00 - 2.65\times 4.16) = 3.24$$

As the theoretical oxygen demand is significantly higher than that observed, formation of other products acting as electron acceptors is likely to have occurred in the culture.

Answer: Yes

4.23 Medium formulation

A basis of 1 litre is used for the calculations. From Table C.1 (Appendix C), the molecular weight of $(NH_4)_2SO_4$ is 132.1 and the biomass molecular weight is 26.16. Production of 25 g of cells corresponds to 25/26.16 = 0.956 gmol of cells produced. As each gmol of cells contains 0.25 gmol N, (0.956 × 0.25) = 0.239 gmol of N are needed from the medium for biomass synthesis. As $(NH_4)_2SO_4$ is the sole N source and each gmol of $(NH_4)_2SO_4$ contains 2 gmol of N, 0.239/2= 0.120 gmol of $(NH_4)_2SO_4$ is required. Multiplying this by the molecular weight, 0.120 × 132.1 = 15.9 g of $(NH_4)_2SO_4$ are required. The minimum concentration of $(NH_4)_2SO_4$ is therefore 15.9 g 1^{-1}.

Answer: 15.9 g 1^{-1}

4.24 Oxygen demand for production of recombinant protein

(a)

From Table 4.11, the chemical formula for wild-type *E. coli* can be taken as $CH_{1.77}O_{0.49}N_{0.24}$. From Table C.2 (Appendix C), the molecular formula for glucose is $C_6H_{12}O_6$. Substituting these formulae into Eq. (4.16) gives:

$$C_6H_{12}O_6 + aO_2 + bNH_3 \rightarrow cCH_{1.77}O_{0.49}N_{0.24} + dCO_2 + eH_2O + fCH_{1.55}O_{0.31}N_{0.25}$$

From Table C.1 (Appendix C), the molecular weight of glucose is 180.2, the biomass molecular weight is 25.00, and the recombinant protein molecular weight is 22.03. The stoichiometric coefficient c can be determined from the yield $Y_{XS} = 0.48$ g g^{-1} and Eq. (4.15):

$$c = \frac{Y_{XS}\,(\text{MW substrate})}{\text{MW cells}} = \frac{0.48\text{ g g}^{-1}\,(180.2)}{25.00} = 3.46$$

The value of the stoichiometric coefficient f can be determined from the yield $Y_{PS} = 0.20 \times 0.48 = 0.096$ g g^{-1} and Eq. (4.17):

$$f = \frac{Y_{PS}\,(\text{MW substrate})}{\text{MW product}} = \frac{0.096\text{ g g}^{-1}\,(180.2)}{22.03} = 0.79$$

The ammonia requirement can be determined using an elemental balance for N.

N balance: $b = 0.24c + 0.25f$

Substituting the above values for c and f into the N balance gives $b = 0.24 \times 3.46 + 0.25 \times 0.79 = 1.03$.

Answer: 1.03 gmol per gmol of glucose

(b)

The oxygen demand can be determined using an electron balance. From Table C.2 (Appendix C), the degree of reduction of glucose relative to NH_3 is $\gamma_S = 4.00$; from Table 4.11, the degree of reduction of *E. coli* biomass relative to NH_3 is $\gamma_B = 4.07$. The degree of reduction of the recombinant protein relative to NH_3 is:

$$\gamma_P = \frac{1\times 4 + 1.55\times 1 - 0.31\times 2 - 0.25\times 3}{1} = 4.18$$

$w = 6$ for glucose and $j = 1$ for recombinant protein. Substituting these values into Eq. (4.20) for the theoretical oxygen demand gives:

$$a = \frac{1}{4}(6\times 4.00 - 3.46\times 4.07 - 0.79\times 1\times 4.18) = 1.65$$

Answer: 1.65 gmol per gmol of glucose

(c)

If f in the stoichiometric equation is zero but c remains equal to 3.46, the elemental balance for N is:

N balance: $b = 0.24c = 0.24 \times 3.46 = 0.83$

Therefore, in wild-type *E. coli*, the ammonia requirement is reduced from 1.03 to 0.83 gmol gmol^{-1} glucose, a decrease of 19%. For $f = 0$, Eq. (4.20) becomes:

$$a = \frac{1}{4}(w\gamma_S - c\gamma_B) = \frac{1}{4}(6\times 4.00 - 3.46\times 4.07) = 2.48$$

The oxygen demand is increased from 1.65 to 2.48 gmol gmol^{-1}, a rise of 50%.

Answer: The ammonia and oxygen requirements for wild-type *E. coli* are 0.83 gmol and 2.48 gmol per gmol of glucose, respectively. These values represent a 19% reduction and a 50% increase, respectively, compared with the genetically engineered strain.

4.25 Effect of growth on oxygen demand

The stoichiometric equation for acetic acid production using cell culture must include terms for growth. From Eq. (4.16), the stoichiometric equation for growth and product formation is:

$$C_2H_6O + aO_2 + bNH_3 \rightarrow cCH_{1.8}O_{0.5}N_{0.2} + dCO_2 + eH_2O + fC_2H_4O_2$$

From Table C.8 (Appendix C), the molecular weights of ethanol and acetic acid are 46.1 and 60.1, respectively. From the atomic weights in Table C.1 (Appendix C), the biomass molecular weight is 24.63. The value of the stoichiometric coefficient c is determined from the biomass yield Y_{XS} = 0.14 g g^{-1} and Eq. (4.15):

$$c = \frac{Y_{XS}\,(\text{MW substrate})}{\text{MW cells}} = \frac{0.14\text{ g g}^{-1}(46.1)}{24.63} = 0.26$$

The value of the stoichiometric coefficient f is determined from the yield Y_{PS} = 0.92 g g^{-1} and Eq. (4.17):

$$f = \frac{Y_{PS}\,(\text{MW substrate})}{\text{MW product}} = \frac{0.92\text{ g g}^{-1}(46.1)}{60.1} = 0.71$$

From Table C.2 (Appendix C), the degree of reduction of ethanol relative to NH_3 is γ_S = 6.00 and the degree of reduction of acetic acid relative to NH_3 is γ_P = 4.00. The degree of reduction of the biomass relative to NH_3 is:

$$\gamma_B = \frac{1\times4 + 1.8\times1 - 0.5\times2 - 0.2\times3}{1} = 4.20$$

w = 2 for ethanol and j = 2 for acetic acid. Substituting these values into Eq. (4.20) gives:

$$a = \frac{1}{4}(2\times6.00 - 0.26\times4.20 - 0.71\times2\times4.00) = 1.31$$

With growth, 1.31 gmol of oxygen are required per gmol of glucose consumed, compared with 1 gmol of oxygen per gmol of glucose without growth. Therefore, with growth, the oxygen demand for acetic acid production is increased by 31%.

Answer: The oxygen demand is increased by 31%

4.26 Aerobic sugar metabolism

(a)

Based on Eq. (4.16), the stoichiometric equation for growth and formation of two different products is:

$$C_6H_{12}O_6 + aO_2 + bNH_3 \rightarrow cCH_{1.8}O_{0.5}N_{0.2} + dCO_2 + eH_2O + f_1C_2H_6O + f_2C_3H_8O_3$$

where f_1 and f_2 are stoichiometric coefficients for ethanol and glycerol, respectively, and $CH_{1.8}O_{0.5}N_{0.2}$ is the general formula for biomass (Section 4.6.1). Ammonium phosphate is represented as NH_3 as phosphorous does not occur elsewhere in the reaction equation. From Table C.1 (Appendix C), the molecular weights of fructose, ethanol, glycerol and biomass are 180, 46, 92 and 24.6, respectively. The stoichiometric coefficient c is determined from the biomass yield Y_{XS} = 0.025 g g^{-1} and Eq. (4.15):

$$c = \frac{Y_{XS}\,(\text{MW substrate})}{\text{MW cells}} = \frac{0.025\text{ g g}^{-1}(180)}{24.6} = 0.18$$

The stoichiometric coefficient f_1 is determined from the ethanol yield Y_{PS1} = 0.21 g g^{-1} and Eq. (4.17):

$$f_1 = \frac{Y_{PS1}\,(\text{MW substrate})}{\text{MW product}} = \frac{0.21\text{ g g}^{-1}(180)}{46} = 0.82$$

Similarly for f_2 using the glycerol yield Y_{PS2} = 0.07 g g^{-1}:

$$f_2 = \frac{Y_{PS2}\,(\text{MW substrate})}{\text{MW product}} = \frac{0.07\text{ g g}^{-1}(180)}{92} = 0.14$$

Because fructose has the same molecular formula as glucose, from Table C.2 (Appendix C), we can say that the degree of reduction of fructose relative to NH_3 is γ_S = 4.00. Also from Table C.2, the degrees of reduction of ethanol and glycerol relative to NH_3 are γ_{P1} = 6.00 and γ_{P2} = 4.67, respectively. The degree of reduction of the biomass relative to NH_3 is:

$$\gamma_B = \frac{1\times4 + 1.8\times1 - 0.5\times2 - 0.2\times3}{1} = 4.20$$

(This value for γ_B is also given in Table C.2.) The oxygen demand is calculated using a modified form of Eq. (4.20) to account for transfer of electrons to two separate products, with w = 6 for fructose, j_1 = 2 for ethanol and j_2 = 3 for glycerol:

$$\begin{aligned} a &= \frac{1}{4}(w\gamma_S - c\gamma_B - f_1 j_1 \gamma_{P1} - f_2 j_2 \gamma_{P2}) \\ &= \frac{1}{4}(6\times4.00 - 0.18\times4.20 - 0.82\times2\times6.00 - 0.14\times3\times4.67) \\ &= 2.86 \end{aligned}$$

Therefore, 2.86 gmol of oxygen are required per gmol of fructose consumed. Converting the rate of fructose consumption to gmol h^{-1}:

$$190\text{ g fructose h}^{-1} = 190\text{ g h}^{-1}.\left|\frac{1\text{ gmol}}{180\text{ g}}\right| = 1.056\text{ gmol h}^{-1}$$

From the result for a, the oxygen requirement is 2.86 × 1.056 gmol h^{-1} = 3.02 gmol h^{-1}. Converting this to a mass basis using the molecular weight of O_2 = 32 (Table C.1, Appendix C):

$$3.02\text{ gmol O}_2\text{ h}^{-1} = 3.02\text{ gmol h}^{-1}.\left|\frac{32\text{ g}}{1\text{ gmol}}\right| = 96.6\text{ g h}^{-1}$$

Answer: 97 g h^{-1}

(b)

Calculation of *RQ* using Eq. (4.9) requires the stoichiometric coefficient d, which can be obtained from an elemental balance on C.

C balance: $6 = c + d + 2f_1 + 3f_2$

Substituting values for c, f_1 and f_2 from **(a)**:

$$d = 6 - 0.18 - 2\times0.82 - 3\times0.14 = 3.76$$

Applying Eq. (4.9) with the value of a from **(a)**:

$$RQ = \frac{3.76}{2.86} = 1.31$$

Answer: 1.3

4.27 Stoichiometry of animal cell growth

(a)

The stoichiometric equation is:

$$C_6H_{12}O_6 + pC_5H_{10}O_3N_2 + qO_2 + rCO_2 \rightarrow sCH_{1.82}O_{0.84}N_{0.25} + tC_3H_6O_3 + uNH_3 + vCO_2 + wH_2O$$

From Table C.1 (Appendix C), the molecular weights of glucose, glutamine, lactic acid, CO_2, NH_3 and biomass are 180, 146, 90, 44, 17 and 30.8, respectively. The value of the stoichiometric coefficient s can be determined from the biomass yield from glucose $Y_{XS} = 0.26$ g g^{-1} and Eq. (4.15):

$$s = \frac{Y_{XS}\,(\text{MW substrate})}{\text{MW cells}} = \frac{0.26\text{ g g}^{-1}(180)}{30.8} = 1.52$$

The stoichiometric coefficients p and t are determined from analogous equations using the yields from glucose of 0.42 g g^{-1} for glutamine and 0.90 g g^{-1} for lactic acid:

$$p = \frac{0.42\text{ g g}^{-1}(180)}{146} = 0.52$$

$$t = \frac{0.90\text{ g g}^{-1}(180)}{90} = 1.80$$

(i)

CO_2 appears on both sides of the stoichiometric equation. The net production of CO_2 can be determined using an elemental balance on C.

C balance: $6 + 5p = s + 3t + (v - r)$

Substituting values for p, s and t gives:

$$(v - r) = 6 + 5\times 0.52 - 1.52 - 3\times 1.80 = 1.68$$

The net CO_2 production is 1.68 gmol per gmol of glucose. Converting 100 g of glucose to gmol:

$$100\text{ g glucose} = 100\text{ g}\,.\left|\frac{1\text{ gmol}}{180\text{ g}}\right| = 0.556\text{ gmol}$$

Therefore, the net CO_2 production is 1.68 × 0.556 gmol = 0.934 gmol. Converting this to a mass basis using the molecular weight of CO_2:

$$0.934\text{ gmol CO}_2 = 0.934\text{ gmol}\,.\left|\frac{44\text{ g}}{1\text{ gmol}}\right| = 41.1\text{ g}$$

Answer: 41 g

(ii)

The oxygen demand can be evaluated using a modified version of Eq. (4.20) for culture with two substrates:

$$a = \frac{1}{4}(w_1\gamma_{S1} + pw_2\gamma_{S2} - s\gamma_B - tj\gamma_P)$$

From Table C.2 (Appendix C), degrees of reduction relative to NH_3 are $\gamma_{S1} = 4.00$ for glucose, $\gamma_{S2} = 3.60$ for glutamine and $\gamma_P = 4.00$ for lactic acid. The degree of reduction of the biomass relative to NH_3 is:

$$\gamma_B = \frac{1\times 4 + 1.82\times 1 - 0.84\times 2 - 0.25\times 3}{1} = 3.39$$

w_1 = 6 for glucose, w_2 = 5 for glutamine and j = 3 for lactic acid. Substituting values into the above equation for a:

$$a = \frac{1}{4}(6\times 4.00 + 0.52\times 5\times 3.60 - 1.52\times 3.39 - 1.80\times 3\times 4.00) = 1.65$$

Therefore, 1.65 gmol of oxygen are required per gmol of glucose consumed.

Answer: 1.65 gmol per gmol of glucose

(iii)

From the stoichiometry, for every mole of glucose consumed, p = 0.52 moles of glutamine are required. In the culture medium, for every mole of glucose provided, 2/11 = 0.18 moles of glutamine are present. As the culture medium provides less glutamine than is required for consumption of all the glucose, glutamine is the limiting reactant and glucose is present in excess.

Answer: Glucose

(iv)

Converting the maximum concentrations of lactic acid and ammonia to gmol l^{-1}:

$$1\text{ g lactic acid } l^{-1} = 1\text{ g } l^{-1}.\left|\frac{1\text{ gmol}}{90\text{ g}}\right| = 0.011\text{ gmol } l^{-1}$$

$$0.07\text{ g } NH_3\, l^{-1} = 0.07\text{ g } l^{-1}.\left|\frac{1\text{ gmol}}{17\text{ g}}\right| = 0.0041\text{ gmol } l^{-1}$$

From the stoichiometry, production of 0.011 gmol l^{-1} of lactic acid requires 0.011/t gmol l^{-1} of glucose = (0.011/1.80) gmol l^{-1} = 0.0061 gmol l^{-1} of glucose, and 0.011p/t gmol l^{-1} of glutamine = (0.011 × 0.52)/1.80 gmol l^{-1} = 0.0032 gmol l^{-1} of glutamine. Similarly, production of 0.0041 gmol l^{-1} of NH_3 requires 0.0041/u gmol l^{-1} of glucose and 0.0041p/u gmol l^{-1} of glutamine. The value of u can be determined from an elemental balance on N.

N balance: $2p = 0.25s + u$

Substituting values for p and s gives:

$$u = 2\times 0.52 - 0.25\times 1.52 = 0.66$$

Therefore, production of 0.0041 gmol l^{-1} of NH_3 requires (0.0041/0.66) gmol l^{-1} = 0.0062 gmol l^{-1} of glucose and (0.0041× 0.52)/0.66 gmol l^{-1} = 0.0032 gmol l^{-1} of glutamine. These calculations show that the glucose and glutamine requirements for production of the maximum concentration of lactic acid are similar to those required for production of the maximum concentration of NH_3.

Therefore, the maximum allowable concentration of glucose in the medium is 0.0062 gmol l^{-1} and the maximum allowable concentration of glutamine is 0.0032 gmol l^{-1}. From the definition of molarity in Section 2.4.5, the maximum allowable concentrations of glucose and glutamine in the medium are 0.0062 M = 6.2 mM and 0.0032 M = 3.2 mM, respectively.

Answer: 6.2 mM glucose and 3.2 mM glutamine

(b)

The stoichiometric equation for anabolic metabolism is:

$$C_6H_{12}O_6 + pC_5H_{10}O_3N_2 + rCO_2 \rightarrow sCH_{1.82}O_{0.84}N_{0.25}$$

The coefficients p, r and s for this equation can be found from elemental balances.

C balance: $6 + 5p + r = s$ (1)

N balance: $2p = 0.25s \rightarrow p = 0.125s$ (2)

O balance: $6 + 3p + 2r = 0.84s$

Substituting for p from (2) and rearranging gives an expression for r:

$$2r = 0.84s - 6 - 3(0.125s) = 0.465s - 6$$

$$r = 0.233s - 3 \quad (3)$$

Substituting for p and r from (2) and (3) into (1) gives:

$$6 + 5(0.125s) + (0.233s - 3) = s$$

$$3 = 0.142s$$

$$s = 21.1 \quad (5)$$

Applying (5) to (2) and (3) gives:

$$p = 2.64$$

$$r = 1.92$$

These results can be checked using an elemental balance on H.

H balance: $12 + 10p = 1.82s$

Substituting for p and r gives:

$$12 + 10 \times 2.64 = 1.82 \times 21.1$$

$$38.4 = 38.4$$

Therefore, the elemental balances and stoichiometric coefficients are correct and the completed stoichiometric equation for anabolic metabolism is:

$$C_6H_{12}O_6 + 2.64\,C_5H_{10}O_3N_2 + 1.92\,CO_2 \rightarrow 21.1\,CH_{1.82}O_{0.84}N_{0.25}$$

(i)

According to the reaction equation, 6 C atoms in glucose combine with 2.64 × 5 = 13.2 C atoms in glutamine to give 21.1 C atoms in the biomass. Therefore, the proportion of C in the biomass derived from glucose is (6/21.1) × 100% = 28.4% and the proportion of C in the biomass derived from glutamine is (13.2/21.1) × 100% = 62.6%%

Answer: 28% from glucose and 63% from glutamine

(ii)

The reaction equation in **(b)** represents the stoichiometry of biomass synthesis only, i.e. it assumes that all the C atoms in glucose and glutamine are used for biomass production. However, as indicated in the reaction equation in **(a)** for the overall metabolism of hybridoma cells, C from glucose and glutamine is also used for the production of by-products. From **(a)**, the overall carbon balance can be written as:

$$6\text{ C in glucose} + 5p\text{ C in glutamine} \rightarrow s\text{ C in biomass} + 3t\text{ C in lactic acid} + (v - r)\text{ C in } CO_2$$

Substituting values for p, s, t and $(v - r)$ from **(a)** gives:

$$6\text{ C in glucose} + 2.60\text{ C in glutamine} \rightarrow 1.52\text{ C in biomass} + 5.40\text{ C in lactic acid} + 1.68\text{ C in } CO_2$$

The completed reaction equation in **(b)** allows us to estimate how many of the C atoms provided by glucose and glutamine are used to yield 1.52 C atoms in the biomass during hybridoma culture:

$$1.52\text{ C in the biomass uses } \frac{1.52}{21.1} \times 6 = 0.43\text{ C from glucose}$$

1.52 C in the biomass uses $\frac{1.52}{21.1} \times (5 \times 2.64) = 0.95$ C from glutamine

Therefore, during hybridoma culture, the proportion of the 6 C atoms from glucose used for biomass production = 0.43/6 × 100% = 7.2%, and the proportion of the 2.60 C atoms from glutamine used for biomass production = 0.95/2.60 × 100% = 36.5%.

Answer: 7.2% of the C in glucose and 37% of the C in glutamine

4.28 pH as an indicator of yeast growth

Using $CH_{1.66}O_{0.50}N_{0.15}$ as the molecular formula for the biomass, we can perform an elemental balance on N.

N balance: $b = 0.15c$

If there is a 1:1 molar ratio between H^+ production and N consumption, $f = b$ in the stoichiometric equation. Therefore, using the result from the N balance, $f = 0.15c$. This means that the molar rate of H^+ production must be equal to 0.15 times the molar rate of biomass production.

Answer: QED

Chapter 5

Energy Balances

5.1 Sensible energy change

(a)

From Table C.5 (Appendix C), C_p for *m*-cresol between 25°C and 100°C is 0.551 cal g^{-1} °C^{-1}. The specific enthalpy change calculated using Eq. (5.13) is:

$$\Delta h = C_p \Delta T = 0.551 \text{ cal g}^{-1} \text{°C}^{-1} (100 - 25)\text{°C}$$

$$= 41.3 \text{ cal g}^{-1}$$

Answer: 41.3 cal g^{-1}

(b)

From Table C.5 (Appendix C), C_p for ethylene glycol between 10°C and 20°C can be taken as approximately 0.569 cal g^{-1} °C^{-1}. The specific enthalpy change calculated using Eq. (5.13) is:

$$\Delta h = C_p \Delta T = 0.569 \text{ cal g}^{-1} \text{°C}^{-1} (10 - 20)\text{°C}$$

$$= -5.69 \text{ cal g}^{-1}$$

Answer: –5.69 cal g^{-1}

(c)

From Table C.6 (Appendix C), C_p for succinic acid between 15°C and 120°C is given by the expression C_p = 0.248 + 0.00153T, where T is in °C and C_p is in cal g^{-1} °C^{-1}. The sensible energy change is best determined from the integral of this equation between the limits T = 15°C and T = 120°C:

$$\Delta h = \int_{15°C}^{120°C} C_p \, dT = \int_{15°C}^{120°C} (0.248 + 0.00153T) \, dT \text{ cal g}^{-1} = \left(0.248T + \frac{0.00153}{2} T^2 \right) \Bigg|_{15°C}^{120°C} \text{cal g}^{-1}$$

$$= 36.9 \text{ cal g}^{-1}$$

Answer: 36.9 cal g^{-1}

(d)

From Table C.3 (Appendix C), the heat capacity of air between 65°C and 150°C is given by the equation:

$$C_p = 28.94 + 0.4147 \times 10^{-2} T + 0.3191 \times 10^{-5} T^2 - 1.965 \times 10^{-9} T^3$$

where C_p is heat capacity in J $gmol^{-1}$ °C^{-1} and T is temperature in °C. The sensible energy change can be determined by evaluating the integral of this expression between the limits T = 150°C and T = 65°C:

$$\Delta h = \int_{150°C}^{65°C} C_p \, dT = \int_{150°C}^{65°C} (28.94 + 0.4147 \times 10^{-2} T + 0.3191 \times 10^{-5} T^2 - 1.965 \times 10^{-9} T^3) \, dT \text{ J gmol}^{-1}$$

$$= \left(28.94T + \frac{0.4147 \times 10^{-2}}{2} T^2 + \frac{0.3191 \times 10^{-5}}{3} T^3 - \frac{1.965 \times 10^{-9}}{4} T^4 \right) \Bigg|_{150°C}^{65°C} \text{J gmol}^{-1}$$

$$= -2500.9 \text{ J gmol}^{-1} = -2.50 \text{ kJ gmol}^{-1}$$

Answer: –2.50 kJ $gmol^{-1}$

5.2 Heat of vaporisation

The latent heat of vaporisation of water at 33°C is obtained from Table D.1 (Appendix D). Taking the average of the values at 32°C and 34°C, Δh_v = 2423.55 kJ kg^{-1} at 33°C. From Eq. (5.16):

$$\Delta H = 20 \text{ g h}^{-1} (2423.55 \text{ kJ kg}^{-1}) . \left| \frac{1 \text{ kg}}{1000 \text{ g}} \right|$$

$$= 48.5 \text{ kJ h}^{-1}$$

Answer: 48.5 kJ h^{-1}

5.3 Steam tables

(a)

The heat of vaporisation of water at 85°C is obtained from Table D.1 (Appendix D). Taking the average of the values at 84°C and 86°C, Δh_v = 2296.05 kJ kg^{-1} at 85°C.

Answer: 2296.05 kJ kg^{-1}

(b)

From Table D.1 (Appendix D), the enthalpy of liquid water at 10°C relative to the triple point is 42.0 kJ kg^{-1}. The enthalpy of liquid water at 35°C relative to the triple point can be estimated as the average of the values in Table D.1 for 34°C and 36°C = 146.55 kJ kg^{-1}. Using the relationship derived in Section 5.3.1, the enthalpy of water at 35°C relative to 10°C is therefore (146.55 – 42.0) kJ kg^{-1} = 104.55 kJ kg^{-1}.

Answer: 104.55 kJ kg^{-1}

(c)

The enthalpy of saturated water vapour at 40°C relative to the triple point can be read directly from Table D.1 (Appendix D) as 2574.4 kJ kg^{-1}.

Answer: 2574.4 kJ kg^{-1}

(d)

The enthalpy of superheated steam at 2.5 atm and 275°C relative to the triple point can be obtained from Table D.3 (Appendix D). To convert the pressure to kPa, from Table A.5 (Appendix A), 1 atm = 1.013 × 10^5 Pa. Therefore:

$$2.5 \text{ atm} = 2.5 \text{ atm} . \left| \frac{1.013 \times 10^5 \text{ Pa}}{1 \text{ atm}} \right| . \left| \frac{1 \text{ kPa}}{1000 \text{ Pa}} \right| = 253.3 \text{ kPa}$$

From Table D.3, the enthalpy at 100 kPa and 275°C is 3024 kJ kg^{-1} and the enthalpy at 500 kPa and 275°C is 3013 kJ kg^{-1}. Interpolating between these values gives an enthalpy of 3019.8 kJ kg^{-1} at 253.3 kPa.

Answer: 3019.8 kJ kg^{-1}

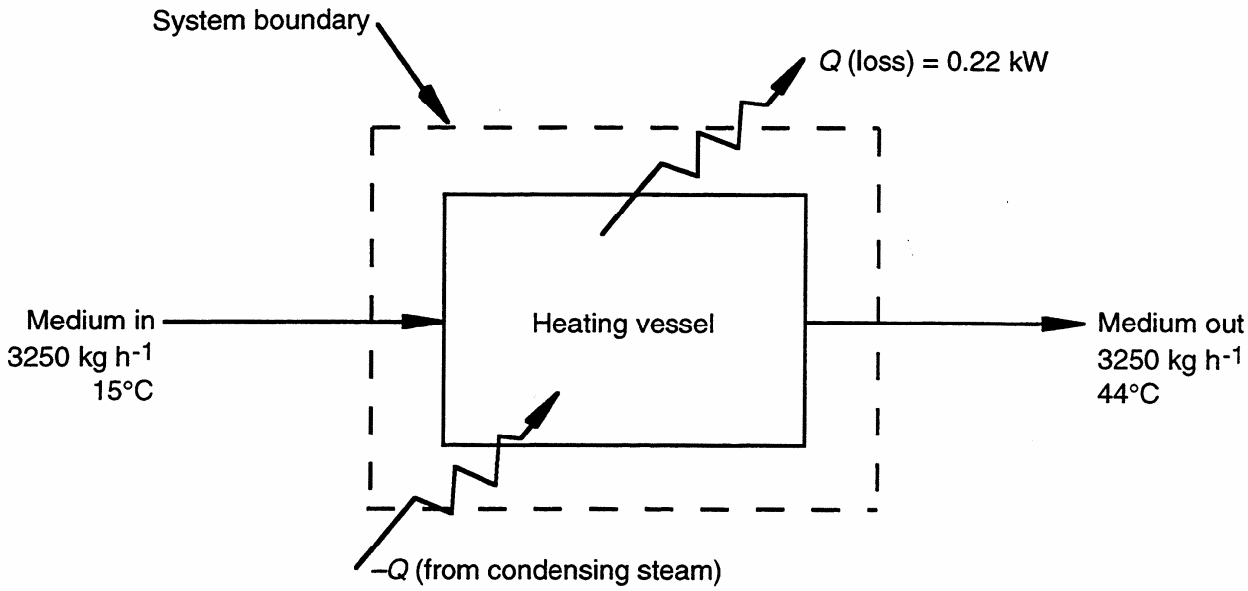
5.4 Preheating nutrient medium

1. Assemble

(i) Units

kg, h, kJ, °C

(ii) Flow sheet



(iii) System boundary

The system boundary is shown on the flow sheet.

2. Analyse

(i) Assumptions

– steady state

– no leaks

– system is homogenous

– condensate temperature is 150°C

– no shaft work

(ii) Basis

1 h, or 3250 kg of medium in

(iii) Reference state

H = 0 for water (steam) at its triple point

H = 0 for medium at 15°C

(iv) Extra data

C_p medium = 0.9 cal g^{-1} $°C^{-1}$ = 0.9 kcal kg^{-1} $°C^{-1}$

Δh_v water at 150°C = 2113.1 kJ kg^{-1} (interpolated from Table D.2, Appendix D)

1 kcal = 4.187×10^3 J (Table A.7, Appendix A)

1 W = 1 J s^{-1} (Table A.8, Appendix A); therefore, 1 kW = 1 kJ s^{-1}

(v) Mass balance

The mass balance is already complete.

(vi) Energy balance equation

At steady state, Eq. (5.9) applies:

$$\sum_{\substack{\text{input}\\ \text{streams}}} (Mh) - \sum_{\substack{\text{output}\\ \text{streams}}} (Mh) - Q + W_s = 0$$

3. Calculate

W_s = 0. There are two components for the heat term Q: Q_{loss} and Q from the condensing steam. Using subscript MD to denote the medium, the energy balance equation becomes:

$$(Mh)_{\text{MD in}} - (Mh)_{\text{MD out}} - Q - Q_{loss} = 0$$

$(Mh)_{\text{MD in}}$ = 0 (reference state)

$(Mh)_{\text{MD out}}$ at 44°C is calculated as a sensible energy change from H = 0 at 15°C using Eq. (5.13):

$$(Mh)_{\text{MD out}} = MC_p\Delta T = 3250 \text{ kg } (0.9 \text{ kcal kg}^{-1} \text{ °C}^{-1})\ (44 - 15)\text{°C} = 8.483\times10^4 \text{ kcal}$$

Converting to kJ:

$$(Mh)_{\text{MD out}} = 8.483\times10^4 \text{ kcal} \cdot \left|\frac{4.187\times10^3 \text{ J}}{1 \text{ kcal}}\right| \cdot \left|\frac{1 \text{ kJ}}{1000 \text{ J}}\right| = 3.55\times10^5 \text{ kJ}$$

The rate of heat loss is 0.22 kW. Converting to kJ h^{-1}:

$$0.22 \text{ kW} = 0.22 \text{ kW} \cdot \left|\frac{1 \text{ kJ s}^{-1}}{1 \text{ kW}}\right| \cdot \left|\frac{3600 \text{ s}}{1 \text{ h}}\right| = 792 \text{ kJ h}^{-1}$$

Therefore, on the basis of 1 h, Q_{loss} = 792 kJ. Substituting values into the energy balance equation gives:

$$0 - 3.55\times10^5 \text{ kJ} - Q - 792 \text{ kJ} = 0$$

$$Q = -3.56\times10^5 \text{ kJ}$$

Q has a negative value consistent with the sign conventions outlined in Section 5.2 for heat supplied to the system from the surroundings. This heat is provided as the latent heat of vaporisation as saturated steam condenses at 150°C. The enthalpy change from this change of phase is calculated using Eq. (5.16) and must be equal to $-Q$.

$$3.56\times10^5 \text{ kJ} = M_{\text{steam}}\Delta h_v = M_{\text{steam}}\ (2113.1 \text{ kJ kg}^{-1})$$

$$M_{\text{steam}} = 168 \text{ kg}$$

4. Finalise

Answer: 168 kg h^{-1}

5.5 Designer coffee mug

Calculate the enthalpy change of the coffee as it cools from 92°C to 74°C. Assuming that coffee has the thermal properties of water, from the steam tables (Table D.1, Appendix D):

$$h(92°C) = 385.4 \text{ J g}^{-1}$$

$$h(74°C) = 309.7 \text{ J g}^{-1}$$

Taking the density of coffee to be the same as that of water = 1 g cm^{-3} (Section 2.4.1) = 1 g ml^{-1}, the mass of coffee in the mug is 250 g. The enthalpy change of the coffee after it is poured into the mug is:

$$\Delta H = M\,\Delta h = 250 \text{ g } (385.4 - 309.7) \text{ J g}^{-1} = 1.89 \times 10^4 \text{ J}$$

If there is no loss of heat to the environment or to other mug components, the sensible heat released by the coffee must be taken up by the beeswax, which is initially in a solid state at 25°C. First, the temperature of the solid beeswax is raised to its melting point of 74°C by absorption of sensible heat. The enthalpy change associated with this process is determined using Eq. (5.12):

$$\Delta H \text{ for heating solid beeswax} = M_{\text{wax}} (1.6 \text{ J g}^{-1} \text{ °C}^{-1})(74 - 25)°C = 78.4 M_{\text{wax}} \text{ J}$$

where M_{wax} is the mass of beeswax in the jacket in units of g. Once the beeswax has reached its melting point, latent heat is applied to melt all of the beeswax present. The latent heat required is evaluated using an equation analogous to Eq. (5.16):

$$\Delta H \text{ for melting solid beeswax} = M_{\text{wax}} \Delta h_{\text{f}} = M_{\text{wax}} (190 \text{ J g}^{-1}) = 190 M_{\text{wax}} \text{ J}$$

We can equate the enthalpy change of the coffee to the total enthalpy change of the beeswax in units of J:

$$1.89 \times 10^4 = (78.4 M_{\text{wax}} + 190 M_{\text{wax}}) = 268.4 M_{\text{wax}}$$

$$M_{\text{wax}} = 70.4$$

Answer: 70.4 g

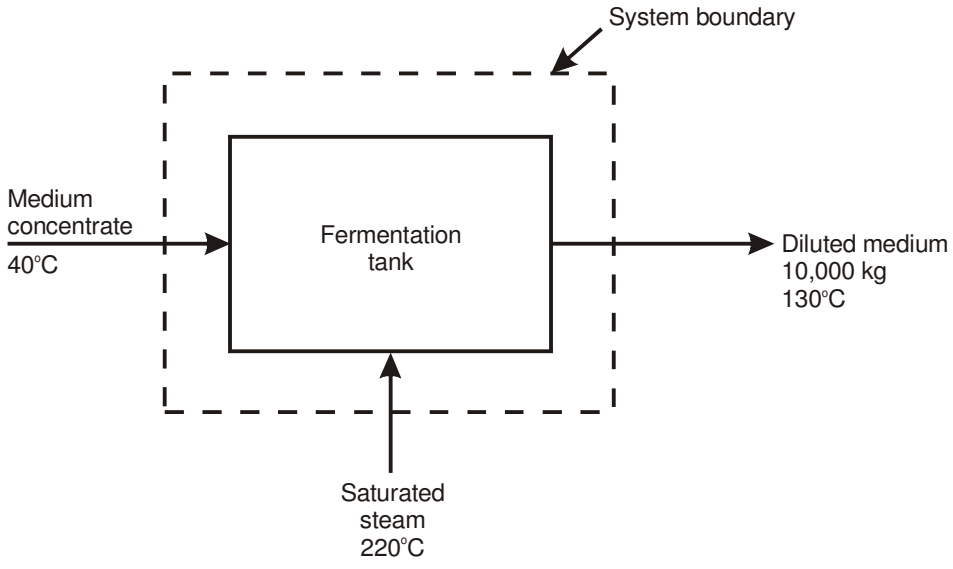
5.6 Medium preparation

1. Assemble

(i) Units

kg, kJ, °C

(ii) Flow sheet



(iii) System boundary

The system boundary is shown on the flow sheet.

2. Analyse

(i) Assumptions

– no leaks

– no heat losses

– no shaft work

– medium has the thermal properties of water

– system is homogeneous

(ii) Basis

10,000 kg of diluted medium out

(iii) Reference state

H = 0 for water (steam) at its triple point

H = 0 for medium at the triple point of water

(iv) Extra data

h (medium concentrate) at 40°C = 167.5 kJ kg^{-1} (Table D.1, Appendix D)

h for the diluted medium at 130°C can be interpolated from the values in the steam tables for liquid water at 128.7°C and 131.2°C (Table D.2, Appendix D):

$$h(\text{diluted medium) at } 130°\text{C} = \left(540.9 + \frac{130-128.7}{131.2-128.7}(551.4-540.9)\right) \text{kJ kg}^{-1} = 546.36 \text{ kJ kg}^{-1}$$

h for saturated stream at 220°C can be interpolated from the values in the steam tables at 217.2°C and 221.8°C (Table D.2, Appendix D):

$$h(\text{saturated steam) at } 220°\text{C} = \left(2799.1 + \frac{220-217.2}{221.8-217.2}(2800.4-2799.1)\right) \text{kJ kg}^{-1} = 2799.89 \text{ kJ kg}^{-1}$$

(v) Compounds involved in reaction

No compounds are involved in reaction.

(vi) Mass balance equation

As there is no reaction, the appropriate mass balance equation is Eq. (4.3):

$$\text{mass in} = \text{mass out}$$

(vii) Energy balance equation

Eq. (5.9) applies:

$$\sum_{\substack{\text{input}\\\text{streams}}} (Mh) - \sum_{\substack{\text{output}\\\text{streams}}} (Mh) - Q + W_s = 0$$

3. Calculate

(i) Mass balance

Total mass balance

Let S denote the total mass of steam in kg; let C denote the total mass of medium concentrate in kg. The mass balance equation for total mass is:

$$(S + C) \text{ kg total mass in} = 10{,}000 \text{ kg total mass out}$$

$$C = 10{,}000 - S \qquad (1)$$

(ii) Energy balance

As $Q = W_s = 0$, the energy balance equation becomes:

$$(Mh)_{\text{C in}} + (Mh)_{\text{S in}} - (Mh)_{\text{M out}} = 0$$

where subscripts C = medium concentrate, S = steam and M = diluted medium. For the medium concentrate:

$$(Mh)_{\text{C in}} = C \text{ kg } (167.5 \text{ kJ kg}^{-1}) = 167.5C \text{ kJ}$$

Applying the expression for C from (1):

$$(Mh)_{\text{C in}} = 167.5\,(10{,}000 - S) \text{ kJ} = (1.675\times10^{6} - 167.5S) \text{ kJ}$$

For the steam:

$$(Mh)_{\text{S in}} = S \text{ kg } (2799.89 \text{ kJ kg}^{-1}) = 2799.89S \text{ kJ}$$

For the diluted medium:

$$(Mh)_{\text{M out}} = 10{,}000 \text{ kg } (546.36 \text{ kJ kg}^{-1}) = 5.464\times10^{6} \text{ kJ}$$

Substituting these terms into the energy balance equation gives:

$$(1.675\times10^{6} - 167.5S) + 2799.89S - 5.464\times10^{6} = 0$$

$$2632.39S = 3.789\times10^{6}$$

$$S = 1439.4$$

Using this result in (1) gives:

$$C = 8560.6$$

4. Finalise

(a)

Answer: 1440 kg

(b)

Answer: 8560 kg

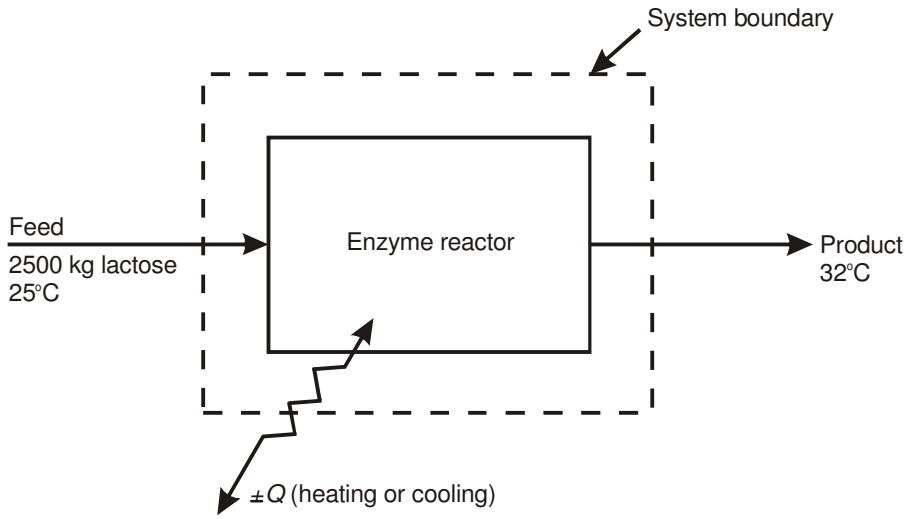
5.7 Enzyme conversion

1. Assemble

(i) Units

kg, day, kJ, °C

(ii) Flow sheet



(iii) System boundary

The system boundary is shown on the flow sheet.

(iv) Reaction equation

$$C_{12}H_{22}O_{11} + H_2O \rightarrow C_6H_{12}O_6 + C_6H_{12}O_6$$

2. Analyse

(i) Assumptions

– steady state

– no leaks

– system is homogenous

– all lactose is converted

– no evaporation

– no shaft work

(ii) Basis

1 day, or 2500 kg of lactose in

(iii) Reference state

h = 0 for lactose at 25°C

h = 0 for water at 25°C

h = 0 for glucose at 25°C

(iv) Extra data

Molecular weights calculated from Table C.1 (Appendix C):

– Lactose = 342.3

– Glucose = 180.2

– Galactose = 180.2

– H_2O = 18.0

1 kcal = 4.187×10^3 J (Table A.7, Appendix A) = 4.187 kJ

(v) Compounds involved in reaction

Lactose, water, glucose and galactose are involved in the reaction.

(vi) Mass balance equations

For lactose, water, glucose and galactose, the appropriate mass balance equation is Eq. (4.2):

mass in + mass generated = mass out + mass consumed

For total mass, the appropriate mass balance equation is Eq. (4.3):

mass in = mass out

(vii) Energy balance equation

At steady state, Eq. (5.9) applies:

$$\sum_{\substack{\text{input}\\ \text{streams}}} (Mh) - \sum_{\substack{\text{output}\\ \text{streams}}} (Mh) - Q + W_s = 0$$

3. Calculate

(i) Mass balance

The calculation table below show all known quantities in kg. The total mass of product is denoted *P*. The In side of the mass balance table is complete.

Stream	*In*					*Out*				
	Lactose	*H_2O*	*Glucose*	*Galactose*	*Total*	*Lactose*	*H_2O*	*Glucose*	*Galactose*	*Total*
Feed	2500	22,500	0	0	25,000	–	–	–	–	–
Product	–	–	–	–	–	0	?	?	?	*P*
Total	2500	22,500	0	0	25,000	0	?	?	?	*P*

Lactose balance

2500 kg lactose in + 0 kg lactose generated = 0 kg lactose out + lactose consumed

Lactose consumed = 2500 kg

Converting the lactose consumed to moles:

$$\text{Lactose consumed} = 2500 \text{ kg} \left| \frac{1 \text{ kgmol}}{342.3 \text{ kg}} \right| = 7.304 \text{ kgmol}$$

From the reaction stoichiometry, conversion of this number of kgmol of lactose requires the same number of kgmol of H_2O. Converting to mass of H_2O:

$$\text{H}_2\text{O consumed} = 7.304 \text{ kgmol} \left| \frac{18.0 \text{ kg}}{1 \text{ kgmol}} \right| = 131.5 \text{ kg}$$

Similarly, the masses of glucose and galactose generated can be determined from the stoichiometry:

$$\text{Glucose generated} = 7.304 \text{ kgmol} \left| \frac{180.2 \text{ kg}}{1 \text{ kgmol}} \right| = 1316.2 \text{ kg}$$

$$\text{Galactose generated} = 7.304 \text{ kgmol} \left| \frac{180.2 \text{ kg}}{1 \text{ kgmol}} \right| = 1316.2 \text{ kg}$$

Total mass balance

$$25{,}000 \text{ kg total mass in} = P \text{ kg total mass out}$$

$$P = 25{,}000 \text{ kg}$$

Glucose and galactose balance

$$0 \text{ kg glucose in} + 1316.2 \text{ kg glucose generated} = \text{glucose out} + 0 \text{ kg glucose consumed}$$

$$\text{Glucose out} = 1316.2 \text{ kg}$$

As the galactose balance is the same as that for glucose:

$$\text{Galactose out} = 1316.2 \text{ kg}$$

H_2O balance

$$22{,}500 \text{ kg water in} + 0 \text{ kg water generated} = \text{water out} + 131.5 \text{ kg water consumed}$$

$$\text{Water out} = 22{,}368.5 \text{ kg}$$

These calculations allow completion of the Out side of the mass balance table with all quantities in kg.

Stream	*Out*				
	Lactose	*H_2O*	*Glucose*	*Galactose*	*Total*
Feed	–	–	–	–	–
Product	0	22,368.5	1316.2	1316.2	25,000
Total	0	22,368.5	1316.2	1316.2	25,000

All columns and rows of the completed table add up correctly to within round-off error.

(ii) Energy balance

The standard heat of reaction is evaluated using Eq. (5.20). As the heat of combustion of H_2O is zero:

$$\Delta H^\circ_{\text{rxn}} = (n \Delta h^\circ_{\text{c}})_{\text{L}} - (n \Delta h^\circ_{\text{c}})_{\text{G}} - (n \Delta h^\circ_{\text{c}})_{\text{Gal}}$$

where subscripts L = lactose, G = glucose and Gal = galactose. Using a basis of 1 gmol of lactose, the n in this equation are the stoichiometric coefficients. Substituting values gives:

$$\Delta H^\circ_{\text{rxn}} = 1 \text{gmol} \, (-5652.5 \text{ kJ gmol}^{-1}) - 1 \text{gmol} \, (-2805.0 \text{ kJ gmol}^{-1}) - 1 \text{gmol} \, (-2805.7 \text{ kJ gmol}^{-1})$$

$$= -41.8 \text{ kJ}$$

As the value of $\Delta H^\circ_{\text{rxn}}$ is negative, the reaction is exothermic (Section 5.8). This $\Delta H^\circ_{\text{rxn}}$ applies at 25°C and can be used to determine the specific enthalpy h of galactose at 25°C using Eq. (5.19):

$$\Delta H^\circ_{\text{rxn}} = (nh)_{\text{G}} + (nh)_{\text{Gal}} - (nh)_{\text{L}} - (nh)_{\text{W}} \quad \text{at } 25^\circ\text{C}$$

where subscript W = water. As the reference states for lactose, water and glucose are defined as $h = 0$ at 25°C:

$$\Delta H^{\circ}_{\text{rxn}} = (nh)_{\text{Gal}} \quad \text{at } 25^{\circ}\text{C}$$

Therefore, h for galactose at 25°C = –41.8 kJ gmol^{-1}.

Converting units for the C_p and Δh_{m} values provided:

$$C_{p\text{G}} = C_{p\text{Gal}} = 0.30 \text{ cal g}^{-1}\,^{\circ}\text{C}^{-1} = 0.30 \text{ kcal kg}^{-1}\,^{\circ}\text{C}^{-1} . \left|\frac{4.187 \text{ kJ}}{1 \text{ kcal}}\right| = 1.26 \text{ kJ kg}^{-1}\,^{\circ}\text{C}^{-1}$$

$$C_{p\text{W}} = 1.0 \text{ cal g}^{-1}\,^{\circ}\text{C}^{-1} = 1.0 \text{ kcal kg}^{-1}\,^{\circ}\text{C}^{-1} . \left|\frac{4.187 \text{ kJ}}{1 \text{ kcal}}\right| = 4.187 \text{ kJ kg}^{-1}\,^{\circ}\text{C}^{-1}$$

$$\Delta h_{\text{mL}} = 3.7 \text{ kcal gmol}^{-1} = 3.7 \text{ kcal gmol}^{-1} . \left|\frac{4.187 \text{ kJ}}{1 \text{ kcal}}\right| . \left|\frac{1 \text{ gmol}}{342.3 \text{ g}}\right| . \left|\frac{1000 \text{ g}}{1 \text{ kg}}\right| = 45.26 \text{ kJ kg}^{-1}$$

$$\Delta h_{\text{mG}} = \Delta h_{\text{mGal}} = 5.6 \text{ kcal gmol}^{-1} = 5.6 \text{ kcal gmol}^{-1} . \left|\frac{4.187 \text{ kJ}}{1 \text{ kcal}}\right| . \left|\frac{1 \text{ gmol}}{180.2 \text{ g}}\right| . \left|\frac{1000 \text{ g}}{1 \text{ kg}}\right| = 130.1 \text{ kJ kg}^{-1}$$

As $W_{\text{s}} = 0$, the energy balance equation can be written as:

$$(Mh)_{\text{F in}} - (Mh)_{\text{P out}} - Q = 0$$

where subscripts F = feed and P = product. From Eq. (5.17), the feed and product enthalpies can be expanded as follows:

$$(Mh)_{\text{L in}} + (Mh)_{\text{W in}} + (M\Delta h_{\text{m}})_{\text{L}} - [(Mh)_{\text{G out}} + (Mh)_{\text{Gal out}} + (Mh)_{\text{W out}} + (M\Delta h_{\text{m}})_{\text{G}} + (M\Delta h_{\text{m}})_{\text{Gal}}] - Q = 0$$

h at 25°C for lactose in and water in are defined as zero. h for glucose, galactose and water out can be evaluated as sensible energy changes from 25°C to 32°C. Using Eq. (5.13) to express the changes in sensible heat, the energy balance equation becomes:

$$0 + 0 + M_{\text{L in}}\Delta h_{\text{mL}} - [M_{\text{G out}}C_{p\text{G}}\Delta T + M_{\text{Gal out}}h_{\text{Gal}}(\text{at } 25^{\circ}\text{C}) + M_{\text{Gal out}}C_{p\text{Gal}}\Delta T + M_{\text{W out}}C_{p\text{W}}\Delta T + M_{\text{G out}}\Delta h_{\text{mG}} + M_{\text{Gal out}}\Delta h_{\text{mGal}}] - Q = 0$$

Substituting values for each term using results from the completed mass balance table:

$$0 + 0 + 2500 \text{ kg } (45.26 \text{ kJ kg}^{-1}) - \Big[1316.2 \text{ kg } (1.26 \text{ kJ kg}^{-1}\,^{\circ}\text{C}^{-1})(32 - 25)^{\circ}\text{C} + 1316.2 \text{ kg} . \left|\frac{1000 \text{ g}}{1 \text{ kg}}\right| . \left|\frac{1 \text{ gmol}}{180.2 \text{ g}}\right| (-41.8 \text{ kJ gmol}^{-1}) + 1316.2 \text{ kg } (1.26 \text{ kJ kg}^{-1}\,^{\circ}\text{C}^{-1})(32 - 25)^{\circ}\text{C} + 22{,}368.5 \text{ kg } (4.187 \text{ kJ kg}^{-1}\,^{\circ}\text{C}^{-1})(32 - 25)^{\circ}\text{C} + 1316.2 \text{ kg } (130.1 \text{ kJ kg}^{-1}) + 1316.2 \text{ kg } (130.1 \text{ kJ kg}^{-1})\Big] - Q = 0$$

$$Q = -6.03 \times 10^{5} \text{ kJ}$$

From the sign conventions outlined in Section 5.2, negative Q indicates that heat must be added to the system.

4. Finalise

(a)

Answer: –41.8 kJ per gmol of lactose

(b)

Answer: Heating is required at a rate of 6.03×10^5 kJ per day

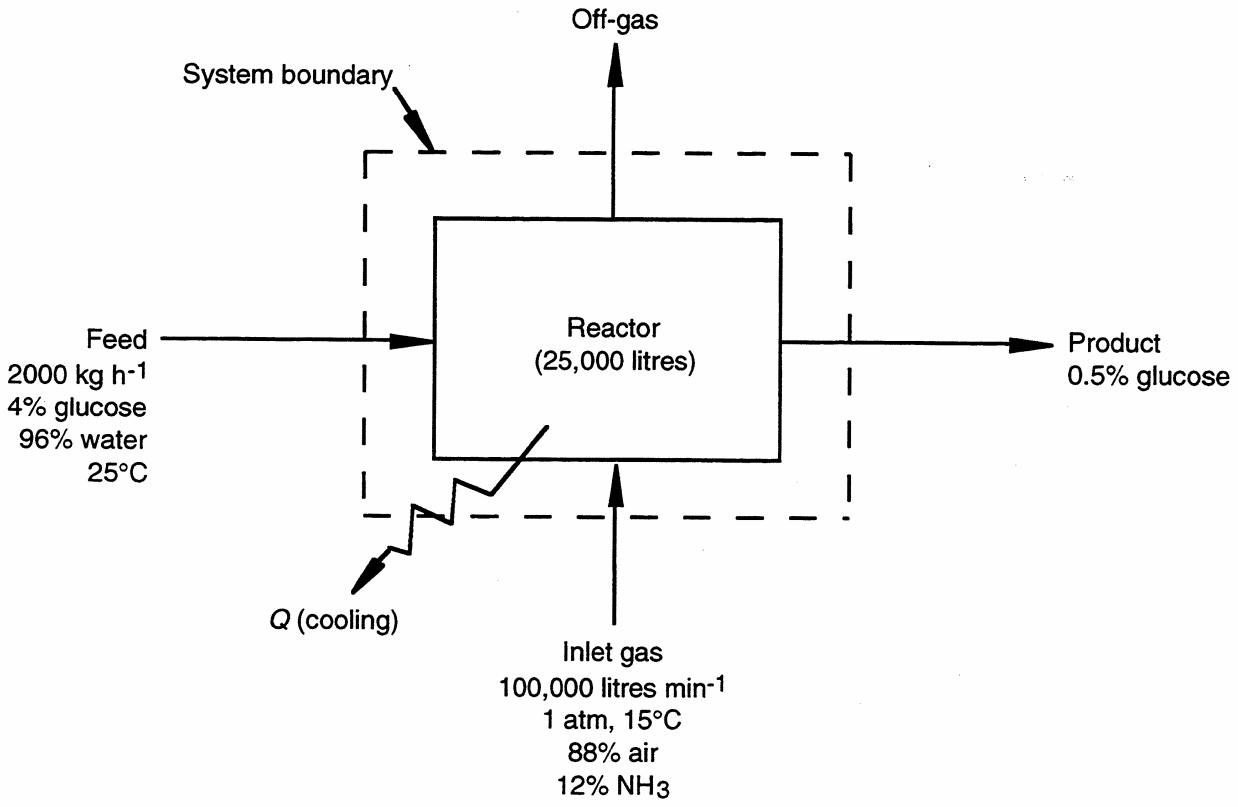
5.8 Production of glutamic acid

1. Assemble

(i) Units

kg, h, kJ, °C

(ii) Flow sheet



(iii) System boundary

The system boundary is shown on the flow sheet.

(iv) Reaction equation

$$C_6H_{12}O_6 + NH_3 + 1.5O_2 \rightarrow C_5H_9O_4N + CO_2 + 3H_2O$$

2. Analyse

(i) Assumptions

– steady state

– no leaks

– system is homogenous

– solutions are ideal

– inlet air and off-gas are dry

– all excess NH_3 is dissolved in the aqueous phase

– all CO_2 produced leaves in the off-gas

– negligible sensible heat change

– no evaporation

– no shaft work

(ii) Basis

1 h, or 2000 kg of feed

(iii) Reference state

$H = 0$ for water at its triple point

$H = 0$ for feed at 25°C

(iv) Extra data

Molecular weights calculated from Table C.1 (Appendix C):

– Glucose = 180.2

– NH_3 = 17.0

– O_2 = 32.0

– N_2 = 28.0

– Glutamic acid = 147.1

– CO_2 = 44.0

– H_2O = 18.0

– Air = 28.8 (see Problem 2.14, Chapter 2)

Ideal gas constant (Appendix B): $R = 0.082057$ l atm K^{-1} $gmol^{-1}$

Composition of air (Section 2.4.5): 21% O_2, 79% N_2 by volume

Heats of combustion (Table C.8, Appendix C):

Δh_c^o glucose = –2805.0 kJ $gmol^{-1}$

Δh_c^o NH_3 = –382.6 kJ $gmol^{-1}$

Δh_c^o glutamic acid = –2244.1 kJ $gmol^{-1}$

(v) Compounds involved in reaction

Glucose, ammonia, oxygen, glutamic acid, carbon dioxide and water are involved in the reaction.

(vi) Mass balance equations

For glucose, ammonia, oxygen, glutamic acid, carbon dioxide and water, the appropriate mass balance equation is Eq. (4.2):

mass in + mass generated = mass out + mass consumed

For N_2 and total mass, the appropriate mass balance equation is Eq. (4.3):

mass in = mass out

(vii) Energy balance equation

The modified energy balance equation, Eq. (5.29), applies to cell cultures:

$$-\Delta H_{rxn} - M_v \Delta h_v - Q + W_s = 0$$

3. Calculate

(i) Mass balance

Every minute, 88,000 litres of air and 12,000 litres of NH_3 enter the reactor. On a basis of 1 h, the volume of air in is (88,000 l min^{-1}) × (60 min) = 5.28×10^6 l. Similarly, the volume of NH_3 in is (12,000 l min^{-1}) × (60 min) = 7.20×10^5 l. Using the known composition of air in vol%, the volume of O_2 in = 0.21 × (5.28×10^6 l) = 1.11×10^6 l and the volume of N_2 in = 0.79 × (5.28×10^6 l) = 4.17×10^6 l. Converting these gas volumes to moles using the ideal gas law, Eq. (2.35), with the temperature converted from °C to Kelvin using Eq. (2.27):

$$\text{Moles of } O_2 \text{ in} = \frac{pV}{RT} = \frac{1\text{ atm } (1.11\times10^6\text{ l})}{0.082057\text{ l atm K}^{-1}\text{ gmol}^{-1}\ (15+273.15)\text{ K}} = 4.69\times10^4\text{ gmol}$$

$$\text{Moles of } N_2 \text{ in} = \frac{pV}{RT} = \frac{1\text{ atm } (4.17\times10^6\text{ l})}{0.082057\text{ l atm K}^{-1}\text{ gmol}^{-1}\ (15+273.15)\text{ K}} = 1.76\times10^5\text{ gmol}$$

$$\text{Moles of } NH_3 \text{ in} = \frac{pV}{RT} = \frac{1\text{ atm } (7.20\times10^5\text{ l})}{0.082057\text{ l atm K}^{-1}\text{ gmol}^{-1}\ (15+273.15)\text{ K}} = 3.05\times10^4\text{ gmol}$$

These molar quantities can now be converted to masses using the molecular weights:

$$\text{Mass of } O_2 \text{ in} = 4.69\times10^4\text{ gmol} \left.\frac{32.0\text{ g}}{1\text{ gmol}}\right| \left.\frac{1\text{ kg}}{1000\text{ g}}\right| = 1500.8\text{ kg}$$

$$\text{Mass of } N_2 \text{ in} = 1.76\times10^5\text{ gmol} \left.\frac{28.0\text{ g}}{1\text{ gmol}}\right| \left.\frac{1\text{ kg}}{1000\text{ g}}\right| = 4928.0\text{ kg}$$

$$\text{Mass of } NH_3 \text{ in} = 3.05\times10^4\text{ gmol} \left.\frac{17.0\text{ g}}{1\text{ gmol}}\right| \left.\frac{1\text{ kg}}{1000\text{ g}}\right| = 518.5\text{ kg}$$

Therefore, the total mass of inlet gas is 1500.8 + 4928.0 + 518.5 = 6947.3 kg.

The calculation tables below show all known quantities in kg. The total mass of off-gas is denoted *G*; the total mass of product is denoted *P*. The In side of the mass balance table is complete.

Stream	*In*							
	Glucose	*NH_3*	*O_2*	*N_2*	*Gluconic acid*	*CO_2*	*H_2O*	*Total*
Feed	80	0	0	0	0	0	1920	2000
Inlet gas	0	518.5	1500.8	4928.0	0	0	0	6947.3
Off-gas	–	–	–	–	–	–	–	–
Product	–	–	–	–	–	–	–	–
Total	80	518.5	1500.8	4928.0	0	0	1920	8947.3

Stream	*Out*							
	Glucose	*NH_3*	*O_2*	*N_2*	*Gluconic acid*	*CO_2*	*H_2O*	*Total*
Feed	–	–	–	–	–	–	–	–
Inlet gas	–	–	–	–	–	–	–	–
Off-gas	0	0	?	?	0	?	0	*G*
Product	0.005*P*	?	0	0	?	0	?	*P*
Total	0.005*P*	?	?	?	?	?	?	*G* + *P*

N_2 balance

N_2 is a tie component.

$$4928.0 \text{ kg } N_2 \text{ in} = N_2 \text{ out}$$

$$N_2 \text{ out} = 4928.0 \text{ kg}$$

Glucose balance

$$80 \text{ kg glucose in} + 0 \text{ kg glucose generated} = 0.005P \text{ kg glucose out} + \text{glucose consumed}$$

$$\text{Glucose consumed} = (80 - 0.005P) \text{ kg}$$

Converting the glucose consumed to molar terms:

$$\text{Glucose consumed} = (80 - 0.005P) \text{ kg} \,.\, \left| \frac{1 \text{ kgmol}}{180.2 \text{ kg}} \right| = (0.444 - 2.775 \times 10^{-5} P) \text{ kgmol}$$

From the reaction stoichiometry, conversion of this number of kgmol of glucose requires the same number of kgmol of NH_3 and 1.5 × the number of kgmol of O_2. Converting these molar quantities to masses:

$$NH_3 \text{ consumed} = (0.444 - 2.775 \times 10^{-5} P) \text{ kgmol} = (0.444 - 2.775 \times 10^{-5} P) \text{ kgmol} \,.\, \left| \frac{17.0 \text{ kg}}{1 \text{ kgmol}} \right|$$

$$= (7.548 - 4.718 \times 10^{-4} P) \text{ kg}$$

$$O_2 \text{ consumed} = 1.5 \times (0.444 - 2.775 \times 10^{-5} P) \text{kgmol} = 1.5 \times (0.444 - 2.775 \times 10^{-5} P) \text{ kgmol} \,.\, \left| \frac{32.0 \text{ kg}}{1 \text{ kgmol}} \right|$$

$$= (21.312 - 1.332 \times 10^{-3} P) \text{ kg}$$

Similarly, expressions for the masses of glutamic acid, CO_2 and water generated can be determined from the stoichiometry:

$$\text{Glutamic acid generated} = (0.444 - 2.775 \times 10^{-5} P) \text{ kgmol}$$

$$= (0.444 - 2.775 \times 10^{-5} P) \text{ kgmol} \,.\, \left| \frac{147.1 \text{ kg}}{1 \text{ kgmol}} \right|$$

$$= (65.312 - 4.082 \times 10^{-3} P) \text{ kg}$$

$$\text{CO}_2 \text{ generated} = (0.444 - 2.775\times10^{-5}\,P) \text{ kgmol} = (0.444 - 2.775\times10^{-5}\,P) \text{ kgmol} \,.\, \left|\frac{44.0 \text{ kg}}{1 \text{ kgmol}}\right|$$

$$= (19.536 - 1.221\times10^{-3}\,P) \text{ kg}$$

$$\text{Water generated} = 3\times(0.444 - 2.775\times10^{-5}\,P) \text{ kgmol} = 3\times(0.444 - 2.775\times10^{-5}\,P) \text{ kgmol} \,.\, \left|\frac{18.0 \text{ kg}}{1 \text{ kgmol}}\right|$$

$$= (23.976 - 1.499\times10^{-3}\,P) \text{ kg}$$

O_2 balance

$$1500.8 \text{ kg } \text{O}_2 \text{ in} + 0 \text{ kg } \text{O}_2 \text{ generated} = \text{O}_2 \text{ out} + (21.312 - 1.332\times10^{-3}P) \text{ kg } \text{O}_2 \text{ consumed}$$

$$\text{O}_2 \text{ out} = (1500.8 - (21.312 - 1.332\times10^{-3}\,P)) \text{ kg}$$

$$= (1479.5 + 1.332\times10^{-3}\,P) \text{ kg}$$

CO_2 balance

$$0 \text{ kg } \text{CO}_2 \text{ in} + (19.536 - 1.221\times10^{-3}\,P) \text{ kg } \text{CO}_2 \text{ generated} = \text{CO}_2 \text{ out} + 0 \text{ kg } \text{CO}_2 \text{ consumed}$$

$$\text{CO}_2 \text{ out} = (19.536 - 1.221\times10^{-3}\,P) \text{ kg}$$

Adding together the masses of N_2, O_2 and CO_2 out gives the total mass of off-gas, G:

$$G = 4928.0 \text{ kg } \text{N}_2 + (1479.5 + 1.332\times10^{-3}\,P) \text{ kg } \text{O}_2 + (19.536 - 1.221\times10^{-3}\,P) \text{ kg } \text{CO}_2$$

$$= (6427.0 + 1.11\times10^{-4}\,P) \text{ kg}$$

Total mass balance

$$8947.3 \text{ kg total mass in} = (G + P) \text{ kg total mass out}$$

Substituting the above expression for G into the total mass balance:

$$8947.3 \text{ kg} = (6427.0 + 1.11\times10^{-4}P + P) \text{ kg}$$

$$2520.3 \text{ kg} = 1.000P \text{ kg}$$

$$P = 2520.3 \text{ kg}$$

Therefore, from the total mass balance:

$$G = (8947.3 - 2520.3) \text{ kg}$$

$$G = 6427.0 \text{ kg}$$

Substituting the result for P into the glucose, O_2 and CO_2 balances gives:

$$\text{Glucose consumed} = (80 - 0.005P) \text{ kg} = 67.40 \text{ kg}$$

$$\text{Glucose out} = 0.005P \text{ kg} = 12.60 \text{ kg}$$

$$\text{O}_2 \text{ consumed} = (21.312 - 1.332\times10^{-3}\,P) \text{ kg} = 17.95 \text{ kg}$$

$$\text{O}_2 \text{ out} = (1479.5 + 1.332\times10^{-3}\,P) \text{ kg} = 1482.9 \text{ kg}$$

$$\text{CO}_2 \text{ out} = \text{CO}_2 \text{ generated} = (19.536 - 1.221\times10^{-3}\,P) \text{ kg} = 16.46 \text{ kg}$$

Using the result for P to evaluate the masses of the other reactants and products involved in the reaction:

$$\text{NH}_3 \text{ consumed} = (7.548 - 4.718\times10^{-4}\,P) \text{ kg} = 6.36 \text{ kg}$$

Glutamic acid generated = $(65.312 - 4.082\times10^{-3}P)$ kg = 55.02 kg

Water generated = $(23.976 - 1.499\times10^{-3}P)$ kg = 20.20 kg

These results can be used directly in the energy balance for evaluation of the cooling requirements. However, completion of the mass balance allows the calculations to be checked.

NH_3 balance

518.5 kg NH_3 in + 0 kg NH_3 generated = NH_3 out + 6.36 kg NH_3 consumed

NH_3 out = 512.14 kg

Water balance

1920 kg water in + 20.20 kg water generated = water out + 0 kg water consumed

Water out = 1940.20 kg

Glutamic acid balance

0 kg glutamic acid in + 55.02 kg glutamic acid generated =

glutamic acid out + 0 kg glutamic acid consumed

Glutamic acid out = 55.02 kg

The Out side of the mass balance table can now be completed with all quantities in kg.

Stream	*Out*							
	Glucose	*NH_3*	*O_2*	*N_2*	*Gluconic acid*	*CO_2*	*H_2O*	*Total*
Feed	–	–	–	–	–	–	–	–
Inlet gas	–	–	–	–	–	–	–	–
Off-gas	0	0	1482.9	4928.0	0	16.46	0	6427.4
Product	12.60	512.14	0	0	55.02	0	1940.20	2520
Total	12.60	512.14	1482.9	4928.0	55.02	16.46	1940.20	8947.3

All columns and rows of the completed table add up correctly to within round-off error.

(ii) Energy balance

$W_s = 0$; $M_v = 0$. Therefore, the energy balance equation becomes:

$$-\Delta H_{rxn} - Q = 0$$

The heat of reaction is evaluated using Eq. (5.20). As the heat of combustion of CO_2 and H_2O is zero:

$$\Delta H_{rxn} = (n\,\Delta h_c^o)_G + (n\,\Delta h_c^o)_A - (n\,\Delta h_c^o)_{GA}$$

where subscripts G = glucose, A = NH_3 and GA = glutamic acid. The n in this equation are the moles of reactant or product involved in the reaction. Converting the masses of reactants and products consumed or generated to moles:

$$\text{Glucose consumed} = 67.40\text{ kg} = 67.40\text{ kg}\,.\left|\frac{1000\text{ g}}{1\text{ kg}}\right|.\left|\frac{1\text{ gmol}}{180.2\text{ g}}\right| = 374\text{ gmol}$$

As glucose, NH_3 and glutamic acid are involved in the reaction in equal molar quantities, 374 gmol of NH_3 are consumed and 374 gmol of glutamic acid are produced. Substituting these quantities into the equation for heat of reaction gives:

$$\Delta H_{rxn} = 374 \text{ gmol } (-2805.0 \text{ kJ gmol}^{-1}) + 374 \text{ gmol } (-382.6 \text{ kJ gmol}^{-1})$$
$$- 374 \text{ gmol } (-2244.1 \text{ kJ gmol}^{-1})$$
$$= -3.53\times10^5 \text{ kJ}$$

Substituting this result into the energy balance equation:

$$3.53\times10^5 \text{ kJ} - Q = 0$$
$$Q = 3.53\times10^5 \text{ kJ}$$

From the sign conventions outlined in Section 5.2, positive Q indicates that heat must be removed from the system.

4. Finalise

(a)

Answer: 3.53×10^5 kJ h^{-1}

(b)

If cooling were not provided, the heat of reaction would be absorbed as sensible heat by the streams passing through the reactor. For a rough calculation of the effect of this heat on the temperature of the reactor, let us assume that the 3.53×10^5 kJ h^{-1} of heat from **(a)** is absorbed by 2000 kg h^{-1} of aqueous medium and 6947.3 kg h^{-1} of gas. The aqueous medium can be considered to have the properties of water so that its heat capacity = 75.4 J $gmol^{-1}$ $°C^{-1}$ (Table C.3, Appendix C). Converting this to a mass basis using the molecular weight of water:

$$(C_p)_{\text{liquid}} = 75.4 \text{ J gmol}^{-1}\,°\text{C}^{-1} \cdot \left|\frac{1 \text{ gmol}}{18.0 \text{ g}}\right| \cdot \left|\frac{1000 \text{ g}}{1 \text{ kg}}\right| \cdot \left|\frac{1 \text{ kJ}}{1000 \text{ J}}\right| = 4.189 \text{ kJ kg}^{-1}\,°\text{C}^{-1}$$

Assume that the heat capacity of the gas stream is equal to that of air = approx. 29 J $gmol^{-1}$ $°C^{-1}$ (Table C.3, Appendix C). Converting this to a mass basis using the molecular weight of air:

$$(C_p)_{\text{gas}} = 29 \text{ J gmol}^{-1}\,°\text{C}^{-1} \cdot \left|\frac{1 \text{ gmol}}{28.8 \text{ g}}\right| \cdot \left|\frac{1000 \text{ g}}{1 \text{ kg}}\right| \cdot \left|\frac{1 \text{ kJ}}{1000 \text{ J}}\right| = 1.007 \text{ kJ kg}^{-1}\,°\text{C}^{-1}$$

Applying Eq. (5.12):

$$\Delta T = \frac{\Delta H}{(MC_p)_{\text{liquid}} + (MC_p)_{\text{gas}}}$$
$$= \frac{3.53\times10^5 \text{ kJ h}^{-1}}{2000 \text{ kg h}^{-1}\,(4.189 \text{ kJ kg}^{-1}\,°\text{C}^{-1}) + 6947.3 \text{ kg h}^{-1}\,(1.007 \text{ kJ kg}^{-1}\,°\text{C}^{-1})}$$
$$= 23.0°\text{C}$$

As a temperature rise of 23°C in the reactor would not be well tolerated by most commercial organisms, provision of adequate cooling for this reaction is an important consideration. Assuming that the usual temperature for the reaction is 25°C, the temperature without cooling would increase to (25 + 23)°C = 48°C.

Answer: 48°C

5.9 Bacterial production of alginate

Alginate production at a rate of 5 kg h^{-1} requires:

$$\frac{5\ \text{kg h}^{-1}}{4\ \text{kg kg}^{-1}} = 1.25\ \text{kg h}^{-1}\ \text{of}\ O_2$$

Converting this quantity to gmol using the molecular weight of O_2 = 32.0 (Table C.1, Appendix C):

$$O_2\ \text{required} = 1.25\ \text{kg h}^{-1} . \left|\frac{1000\ \text{g}}{1\ \text{kg}}\right| . \left|\frac{1\ \text{gmol}}{32.0\ \text{g}}\right| = 39.06\ \text{gmol h}^{-1}$$

From Eq. (5.23), the heat of reaction for aerobic metabolism is approximately –460 kJ $gmol^{-1}$ of O_2 consumed. Therefore, the heat of reaction for alginate production is:

$$\Delta H_{rxn} = -460\ \text{kJ gmol}^{-1}\ (39.06\ \text{gmol h}^{-1}) = -1.80 \times 10^4\ \text{kJ h}^{-1}$$

This result can be used in the modified energy balance equation, Eq. (5.29) with $M_v = 0$ (no evaporation) and W_s = 1.5 kW. From Table A.8 (Appendix A), 1 W = 1 J s^{-1}; therefore W_s = 1.5 kJ s^{-1} and Eq. (5.29) becomes:

$$1.80 \times 10^4\ \text{kJ h}^{-1} - 0 - Q + 1.5\ \text{kJ s}^{-1} . \left|\frac{3600\ \text{s}}{1\ \text{h}}\right| = 0$$

$$Q = 2.3 \times 10^4\ \text{kJ h}^{-1}$$

From the sign conventions outlined in Section 5.2, positive Q indicates that heat must be removed from the system.

Answer: 2.3 × 10^4 kJ h^{-1}

5.10 Acid fermentation

From Tables C.8 and C.1 (Appendix C), the molecular formulae and molecular weights are:

Sucrose: $C_{12}H_{22}O_{11}$, MW = 342.3

Propionic acid: $C_3H_6O_2$, MW = 74.1

Acetic acid: $C_2H_4O_2$, MW = 60.1

Butyric acid: $C_4H_8O_2$, MW = 88.1

Lactic acid: $C_3H_6O_3$, MW = 90.1

As outlined in Section 4.6.1, $CH_{1.8}O_{0.5}N_{0.2}$ can be used as the molecular formula for biomass when composition analysis is not available. From the end of Table C.8 (Appendix C), the molecular weight of the biomass is 24.6. The reaction equation is obtained by modifying Eq. (4.16) for anaerobic growth and multiple product formation:

$$C_{12}H_{22}O_{11} + bNH_3 \rightarrow cCH_{1.8}O_{0.5}N_{0.2} + dCO_2 + eH_2O + f_1 C_3H_6O_2 + f_2 C_2H_4O_2 + f_3 C_4H_8O_2 + f_4 C_3H_6O_3$$

The biomass yield from substrate Y_{XS} = 0.12 g g^{-1}. This value can be used to determine the stoichiometric coefficient c using Eq. (4.15):

$$c = \frac{Y_{XS}\ (\text{MW substrate})}{\text{MW cells}} = \frac{0.12\ \text{g g}^{-1}\ (342.3)}{24.6} = 1.67$$

The coefficients f_1, f_2, f_3 and f_4 can be determined similarly using the product yields and Eq. (4.17):

$$f_1 = \frac{Y_{PS}\,(\text{MW substrate})}{\text{MW propionic acid}} = \frac{0.40\ \text{g g}^{-1}\,(342.3)}{74.1} = 1.85$$

$$f_2 = \frac{Y_{PS}\,(\text{MW substrate})}{\text{MW acetic acid}} = \frac{0.20\ \text{g g}^{-1}\,(342.3)}{60.1} = 1.14$$

$$f_3 = \frac{Y_{PS}\,(\text{MW substrate})}{\text{MW butyric acid}} = \frac{0.05\ \text{g g}^{-1}\,(342.3)}{88.1} = 0.19$$

$$f_4 = \frac{Y_{PS}\,(\text{MW substrate})}{\text{MW lactic acid}} = \frac{0.034\ \text{g g}^{-1}\,(342.3)}{90.1} = 0.13$$

Of the remaining coefficients b, d and e, because CO_2 and H_2O do not figure in heat of reaction calculations as their heat of combustion = 0, only b need be determined. This can be done using an elemental balance on N.

N balance: $b = 0.2c = 0.2 \times 1.67 = 0.33$

To calculate the heat of reaction, the heats of combustion of the reactants and products are obtained from Table C.8 (Appendix C):

$$\Delta h_c^o \text{ sucrose} = -5644.9\ \text{kJ gmol}^{-1}$$

$$\Delta h_c^o\ NH_3 = -382.6\ \text{kJ gmol}^{-1}$$

$$\Delta h_c^o \text{ biomass} = -552\ \text{kJ gmol}^{-1}$$

$$\Delta h_c^o \text{ propionic acid} = -1527.3\ \text{kJ gmol}^{-1}$$

$$\Delta h_c^o \text{ acetic acid} = -874.2\ \text{kJ gmol}^{-1}$$

$$\Delta h_c^o \text{ butyric acid} = -2183.6\ \text{kJ gmol}^{-1}$$

$$\Delta h_c^o \text{ lactic acid} = -1368.3\ \text{kJ gmol}^{-1}$$

The heat of reaction is determined using Eq. (5.20). As the heat of combustion of CO_2 and H_2O is zero:

$$\Delta H_{rxn} = (n\Delta h_c^o)_S + (n\Delta h_c^o)_A - (n\Delta h_c^o)_B - (n\Delta h_c^o)_{PA} - (n\Delta h_c^o)_{AA} - (n\Delta h_c^o)_{BA} - (n\Delta h_c^o)_{LA}$$

where subscripts S = sucrose, A = NH_3, B = biomass, PA = propionic acid, AA = acetic acid, BA = butyric acid and LA = lactic acid. Using a basis of 1 gmol of sucrose, the n in this equation are the stoichiometric coefficients. Substituting values:

$$\begin{aligned}\Delta H_{rxn} = {} & 1\ \text{gmol}\,(-5644.9\ \text{kJ gmol}^{-1}) + 0.33\ \text{gmol}\,(-382.6\ \text{kJ gmol}^{-1}) - 1.67\ \text{gmol}\,(-552\ \text{kJ gmol}^{-1}) \\ & - 1.85\ \text{gmol}\,(-1527.3\ \text{kJ gmol}^{-1}) - 1.14\ \text{gmol}\,(-874.2\ \text{kJ gmol}^{-1}) \\ & - 0.19\ \text{gmol}\,(-2183.6\ \text{kJ gmol}^{-1}) - 0.13\ \text{gmol}\,(-1368.3\ \text{kJ gmol}^{-1}) \\ = {} & -434.5\ \text{kJ}\end{aligned}$$

This ΔH_{rxn} was determined using a basis of 1 gmol of sucrose. If 30 kg of sucrose are consumed during the culture period:

$$\Delta H_{rxn} = 30\ \text{kg} \cdot \left|\frac{1000\ \text{g}}{1\ \text{kg}}\right| \cdot \left|\frac{1\ \text{gmol}}{342.3\ \text{g}}\right| \cdot (-434.5\ \text{kJ gmol}^{-1}) = -3.81\times10^4\ \text{kJ}$$

The cooling requirements are determined using the modified energy balance equation, Eq. (5.29). For no evaporation and no shaft work, $M_v = W_s = 0$, so that:

$$Q = -\Delta H_{rxn} = 3.81 \times 10^4 \text{ kJ}$$

From the sign conventions outlined in Section 5.2, positive Q means that heat must be removed from the system.

Answer: 3.81×10^4 kJ

5.11 Ethanol fermentation

From Table C.2 (Appendix C), the molecular formula for glucose is $C_6H_{12}O_6$ and the molecular formula for ethanol is C_2H_6O. From Eq. (4.16), the stoichiometric equation under anaerobic conditions is:

$$C_6H_{12}O_6 + bNH_3 \rightarrow cCH_{1.75}O_{0.58}N_{0.18} + dCO_2 + eH_2O + fC_2H_6O$$

From Table C.8 (Appendix C), the molecular weight of ethanol is 46.1. From Table C.1 (Appendix C), the molecular weight of glucose is 180.2 and the biomass formula weight is 25.58. Taking into account the 8% ash:

$$\text{Biomass molecular weight} = \frac{25.58}{0.92} = 27.80$$

The stoichiometric coefficient f can be determined from the yield $Y_{PS} = 0.45$ g g^{-1} and Eq. (4.17):

$$f = \frac{Y_{PS}\,(\text{MW substrate})}{\text{MW product}} = \frac{0.45 \text{ g g}^{-1}\,(180.2)}{46.1} = 1.76$$

The other coefficients can be determined using elemental balances.

C balance: $6 = c + d + 2f = c + d + 2 \times 1.76 \rightarrow c = 2.48 - d$

H balance: $12 + 3b = 1.75c + 2e + 6f = 1.75c + 2e + 6 \times 1.76 \rightarrow 1.44 + 3b = 1.75c + 2e$

O balance: $6 = 0.58c + 2d + e + f = 0.58c + 2d + e + 1.76 \rightarrow 4.24 = 0.58c + 2d + e$

N balance: $b = 0.18c$

Substituting the expression for c from the C balance into the N balance:

$$b = 0.18(2.48 - d) = 0.45 - 0.18d$$

Substituting this and the results from the C and N balances into the H balance:

$$1.44 + 3(0.45 - 0.18d) = 1.75(2.48 - d) + 2e$$

$$1.21d - 1.55 = 2e$$

$$e = 0.61d - 0.78$$

Substituting the expressions for c, b and e into the O balance:

$$4.24 = 0.58(2.48 - d) + 2d + (0.61d - 0.78)$$

$$3.58 = 2.03d$$

$$d = 1.76$$

Substituting this value for d into the expressions for the other coefficients gives $c = 0.72$, $b = 0.13$ and $e = 0.29$. The completed stoichiometric equation is therefore:

$$C_6H_{12}O_6 + 0.13NH_3 \rightarrow 0.72CH_{1.75}O_{0.58}N_{0.18} + 1.76CO_2 + 0.29H_2O + 1.76C_2H_6O$$

Using a basis of 1 h, 0.4 kg ethanol are produced. Converting this to moles:

$$\text{Moles of ethanol produced} = 0.4 \text{ kg} \,.\left|\frac{1000 \text{ g}}{1 \text{ kg}}\right| .\left|\frac{1 \text{ gmol}}{46.1 \text{ g}}\right| = 8.68 \text{ gmol}$$

From stoichiometry:

$$\text{Moles of glucose consumed} = 8.68 \text{ gmol} \times \frac{1}{1.76} = 4.93 \text{ gmol}$$

$$\text{Moles of } NH_3 \text{ consumed} = 8.68 \text{ gmol} \times \frac{0.13}{1.76} = 0.64 \text{ gmol}$$

$$\text{Moles of biomass produced} = 8.68 \text{ gmol} \times \frac{0.72}{1.76} = 3.55 \text{ gmol}$$

The heats of combustion from Table C.8 (Appendix C) are:

$$\Delta h_c^o \text{ glucose} = -2805.0 \text{ kJ gmol}^{-1}$$

$$\Delta h_c^o \text{ } NH_3 = -382.6 \text{ kJ gmol}^{-1}$$

$$\Delta h_c^o \text{ ethanol} = -1366.8 \text{ kJ gmol}^{-1}$$

From Eq. (5.28), the heat of combustion of yeast can be taken as –21.2 kJ g^{-1}. The heat of reaction is determined using Eq. (5.20). As the heat of combustion of CO_2 and H_2O is zero:

$$\Delta H_{rxn} = (n\Delta h_c^o)_G + (n\Delta h_c^o)_A - (n\Delta h_c^o)_B - (n\Delta h_c^o)_E$$

where subscripts G = glucose, A = NH_3, B = biomass and E = ethanol. The *n* in this equation can be taken as the actual moles of reactants and products consumed or produced. Substituting values gives:

$$\Delta H_{rxn} = 4.93 \text{ gmol } (-2805.0 \text{ kJ gmol}^{-1}) + 0.64 \text{ gmol } (-382.6 \text{ kJ gmol}^{-1})$$
$$- 3.55 \text{ gmol } (-21.2 \text{ kJ g}^{-1}) .\left|\frac{27.80 \text{ g}}{1 \text{ gmol}}\right| - 8.68 \text{ gmol } (-1366.8 \text{ kJ gmol}^{-1})$$
$$= -117.5 \text{ kJ}$$

Using the modified energy balance equation, Eq. (5.29), with $M_v = W_s = 0$:

$$Q = -\Delta H_{rxn} = 117.5 \text{ kJ}$$

From the sign conventions outlined in Section 5.2, positive *Q* means that heat must be removed from the system. In this case, 117.5 kJ h^{-1} is used to raise the temperature of 2.5 l h^{-1} water from 10°C. The sensible heat change for the water is calculated from Eq. (5.12). The density of water = 1 g cm^{-3} (Section 2.4.1) = 1 kg l^{-1}. Using C_p for water = 75.4 J $gmol^{-1}$ $°C^{-1}$ (Table C.3, Appendix C), and the molecular weight of water = 18.0 (Table C.1, Appendix C):

$$\Delta T = \frac{Q}{MC_p} = \frac{117.5 \text{ kJ h}^{-1}}{2.5 \text{ l h}^{-1} .\left|\frac{1 \text{ kg}}{1 \text{ l}}\right| \times 75.4 \text{ J gmol}^{-1} \text{°C}^{-1} .\left|\frac{1 \text{ gmol}}{18.0 \text{ g}}\right| .\left|\frac{1000 \text{ g}}{1 \text{ kg}}\right| .\left|\frac{1 \text{ kJ}}{1000 \text{ J}}\right|} = 11.2\text{°C}$$

The final temperature of the water is therefore 10°C + 11.2°C = 21.2°C.

Answer: 21.2°C

5.12 Production of bakers' yeast

From Table C.8 (Appendix C), the molecular formula for sucrose is $C_{12}H_{22}O_{11}$. Using Eq. (4.4), the stoichiometric equation for aerobic cell growth is:

$$C_{12}H_{22}O_{11} + aO_2 + bNH_3 \rightarrow cCH_{1.83}O_{0.55}N_{0.17} + dCO_2 + eH_2O$$

From Table C.1 (Appendix C), the molecular weight of sucrose is 342.3 and the biomass formula weight is 25.04. Taking into account the 5% ash:

$$\text{Biomass molecular weight} = \frac{25.04}{0.95} = 26.36$$

The degree of reduction of the biomass relative to NH_3 is:

$$\gamma_B = \frac{1\times4 + 1.83\times1 - 0.55\times2 - 0.17\times3}{1} = 4.22$$

The degree of reduction of sucrose relative to NH_3 is:

$$\gamma_S = \frac{12\times4 + 22\times1 - 11\times2}{12} = 4.00$$

The stoichiometric coefficient c is determined from the yield Y_{XS} = 0.5 g g^{-1} and Eq. (4.15):

$$c = \frac{Y_{XS}(\text{MW substrate})}{\text{MW cells}} = \frac{0.5 \text{ g g}^{-1}\,(342.3)}{26.36} = 6.49$$

The oxygen requirements can be determined from Eq. (4.20) with f = 0 and w = 12 for sucrose:

$$a = \frac{1}{4}(w\gamma_S - c\gamma_B) = \frac{1}{4}(12\times4.00 - 6.49\times4.22) = 5.15$$

Therefore, 5.15 gmol of O_2 are required for each 6.49 gmol of biomass produced; this corresponds to 5.15/6.49 = 0.79 gmol O_2 per gmol biomass. Converting this oxygen demand to mass terms using the molecular weight of oxygen = 32.0 (Table C.1, Appendix C):

$$0.79 \text{ gmol O}_2 \text{ per gmol biomass} = \frac{0.79 \text{ gmol O}_2}{1 \text{ gmol biomass}} \cdot \left|\frac{32.0 \text{ g O}_2}{1 \text{ gmol O}_2}\right| \cdot \left|\frac{1 \text{ gmol biomass}}{26.36 \text{ g biomass}}\right| = 0.96 \text{ g g}^{-1}$$

The specific growth rate means that 0.45 g of biomass are produced per g of biomass per h. As 0.96 g of O_2 are required per g of biomass produced, the specific rate of O_2 consumption is 0.45 × 0.96 = 0.43 g O_2 per g biomass per h. When the biomass concentration is 10 g l^{-1} in a 50,000-litre fermenter, the mass of cells is 10 g l^{-1} × 50,000 l = 5 × 10^5 g. Therefore, the total rate of O_2 consumption is 0.43 g g^{-1} h^{-1} × (5 × 10^5 g) = 2.15 × 10^5 g O_2 h^{-1}. Converting this to moles:

$$\text{Rate of } O_2 \text{ consumption} = 2.15\times10^5 \text{ g h}^{-1} \cdot \left|\frac{1 \text{ gmol}}{32.0 \text{ g}}\right| = 6.72\times10^3 \text{ gmol h}^{-1}$$

From Eq. (5.23), the heat of reaction for aerobic growth is approximately –460 kJ $gmol^{-1}$ O_2 consumed. Therefore:

$$\Delta H_{rxn} = -460 \text{ kJ gmol}^{-1}\,(6.72\times10^3 \text{ gmol h}^{-1}) = -3.09\times10^6 \text{ kJ h}^{-1}$$

The rate of heat removal from the fermenter is determined using the modified energy balance equation, Eq. (5.29), with $W_s = M_v = 0$:

$$Q = -\Delta H_{rxn} = 3.09\times10^6 \text{ kJ h}^{-1}$$

From the sign conventions outlined in Section 5.2, positive Q confirms that heat is removed from the system.

Answer. 3.09×10^6 kJ h^{-1}

5.13 Culture kinetic parameters from thermal properties

(a)

The temperature of the cooling water increases from 20°C at the inlet to 27.5°C at the outlet. The enthalpy change is evaluated using Eq. (5.12):

$$\Delta H = (100 \text{ kg h}^{-1})(4.2 \text{ kJ kg}^{-1}\,°\text{C}^{-1})(27.5 - 20)°\text{C} = 3.15 \times 10^3 \text{ kJ h}^{-1}$$

Answer: 3.15×10^3 kJ h^{-1}

(b)

The modified energy balance equation, Eq. (5.29), applies to this cell culture system. Airlift bioreactors have no mechanical agitation; therefore $W_s = 0$. Let us assume that no evaporation occurs so that $M_v = 0$. Under these conditions, Eq. (5.29) reduces to:

$$-\Delta H_{rxn} = Q$$

If there is no heat loss to the environment, Q in this equation is the heat transferred to the cooling water, resulting in the increase in enthalpy calculated in **(a)**. Therefore:

$$-\Delta H_{rxn} = 3.15 \times 10^3 \text{ kJ h}^{-1}$$

From Eq. (5.23), for aerobic cell cultures the heat of reaction is approximately –460 kJ gmol^{-1} of O_2 consumed. Therefore:

$$\text{Rate of } O_2 \text{ consumption} = \frac{\Delta H_{rxn}}{-460 \text{ kJ gmol}^{-1}} = \frac{-3.15 \times 10^3 \text{ kJ h}^{-1}}{-460 \text{ kJ gmol}^{-1}} = 6.85 \text{ gmol h}^{-1}$$

Answer: 6.85 gmol h^{-1}, assuming no evaporation and no heat loss to the environment

(c)

The reaction can be represented using the general stoichiometric equation for aerobic growth, Eq. (4.4), with succinic acid as the substrate and using $CH_{1.8}O_{0.5}N_{0.2}$ as the molecular formula for the biomass (Section 4.6.1):

$$C_4H_6O_4 + a\,O_2 + b\,NH_3 \rightarrow c\,CH_{1.8}O_{0.5}N_{0.2} + d\,CO_2 + e\,H_2O$$

From Table C.1 (Appendix C), the molecular weight of succinic acid is 118 and the molecular weight of the biomass is 24.6. Converting the rate of succinic acid consumption to a molar basis:

$$\text{Rate of substrate consumption} = 395 \text{ g h}^{-1} = 395 \text{ g h}^{-1} \cdot \left| \frac{1 \text{ gmol}}{118 \text{ g}} \right| = 3.35 \text{ gmol h}^{-1}$$

The stoichiometric coefficient a is equal to the molar ratio of the rates of O_2 and substrate consumption. Therefore, using the result in **(b)**:

$$a = \frac{\text{molar rate of } O_2 \text{ consumption}}{\text{molar rate of substrate consumption}} = \frac{6.85 \text{ gmol h}^{-1}}{3.35 \text{ gmol h}^{-1}} = 2.04$$

From Table C.2 (Appendix C), the degree of reduction of succinic acid relative to NH_3 is $\gamma_S = 3.50$ and the degree of reduction of the biomass relative to NH_3 is $\gamma_B = 4.20$. The stoichiometric coefficient c is obtained from Eq. (4.13) with $w = 4$ for succinic acid:

$$c = \frac{w\gamma_S - 4a}{\gamma_B} = \frac{4\times 3.50 - 4\times 2.04}{4.20} = 1.39 \quad (2)$$

Therefore, for every gmol of succinic acid consumed, 1.39 gmol of biomass is produced. If the rate of succinic acid consumption is 3.35 gmol h^{-1}, the rate of biomass production is 1.39 × 3.35 gmol h^{-1} = 4.66 gmol h^{-1}. Converting this to a mass rate using the biomass molecular weight:

$$\text{Rate of biomass production} = 4.66 \text{ gmol h}^{-1} = 4.66 \text{ gmol h}^{-1} . \left|\frac{24.6 \text{ g}}{1 \text{ gmol}}\right| = 114.6 \text{ g h}^{-1}$$

Answer: 115 g h^{-1}

(d)

The mass of cells in the bioreactor is equal to the cell concentration multiplied by the vessel volume = 4.5 g l^{-1} × 250 l = 1125 g. The specific growth rate is the biomass production rate obtained in **(c)** divided by the mass of cells present:

$$\text{Specific growth rate} = \frac{114.6 \text{ g h}^{-1}}{1125 \text{ g}} = 0.10 \text{ h}^{-1}$$

Answer: 0.10 h^{-1}

5.14 Production of snake antivenin

(a)

If n gmoles of O_2 are consumed by the cells, 5.5n gmoles of lactic acid are produced. Therefore, the equation for the heat of reaction can be written in units of kJ as:

$$-680n = -460n + \Delta h_{rxn}(\text{anaerobic}) \times 5.5n$$

where Δh_{rxn} (anaerobic) has units of kJ $gmol^{-1}$. Therefore:

$$\Delta h_{rxn}(\text{anaerobic}) = \frac{-680 + 460}{5.5} = -40 \text{ kJ gmol}^{-1}$$

As the value of Δh_{rxn} (anaerobic) is negative, anaerobic metabolism is exothermic (Section 5.8).

Answer: –40 kJ per gmol of lactic acid

(b)

(i)

The maximum rate of cooling is required when the cell density is at its maximum. The heat of reaction has fully oxidative and anaerobic components. For the fully oxidative component, the heat of reaction is approximately –460 kJ $gmol^{-1}$ of O_2 consumed. The rate of O_2 consumption in the 500-litre bioreactor is:

$$\text{Rate of } O_2 \text{ consumption} = 0.3\times 10^{-3} \text{ gmol } (10^9 \text{ cells})^{-1} \text{ h}^{-1} (7.5\times 10^6 \text{ cells ml}^{-1}) (500 \text{ l}) . \left|\frac{1000 \text{ ml}}{1 \text{ l}}\right|$$

$$= 1.125 \text{ gmol h}^{-1}$$

Therefore:

$$\Delta H_{rxn}(\text{fully oxidative}) = -460 \text{ kJ gmol}^{-1} (1.125 \text{ gmol h}^{-1})$$

$$= -517.5 \text{ kJ h}^{-1}$$

For the anaerobic component of metabolism, the heat of reaction is Δh_{rxn} (anaerobic) from **(a)** × the rate of lactic acid production:

$$\Delta H_{rxn}(\text{anaerobic}) = -40 \text{ kJ gmol}^{-1} \times 1.0 \times 10^{-3} \text{ gmol } (10^9 \text{ cells})^{-1} \text{ h}^{-1} (7.5 \times 10^6 \text{ cells ml}^{-1}) (500 \text{ l}) \left| \frac{1000 \text{ ml}}{1 \text{ l}} \right|$$

$$= -150.0 \text{ kJ h}^{-1}$$

Adding these components together:

$$\Delta H_{rxn} = -517.5 \text{ kJ h}^{-1} + (-150 \text{ kJ h}^{-1}) = -667.5 \text{ kJ h}^{-1}$$

The modified energy balance equation, Eq. (5.29), applies to this cell culture system. Assuming that shaft work and evaporation have a negligible effect on the energy balance, $W_s = M_v = 0$ and Eq. (5.29) reduces to:

$$-\Delta H_{rxn} = Q$$

Using the above result for ΔH_{rxn}:

$$Q = 667.5 \text{ kJ h}^{-1}$$

Answer: 668 kJ h^{-1}

(ii)

If the rate of O_2 consumption stays the same, ΔH_{rxn} for the fully oxidative component of metabolism remains at –517.5 kJ h^{-1}. Using the new rate of lactic acid production in the calculation for ΔH_{rxn} (anaerobic):

$$\Delta H_{rxn}(\text{anaerobic}) = -40 \text{ kJ gmol}^{-1} \times 0.05 \times 10^{-3} \text{ gmol } (10^9 \text{ cells})^{-1} \text{ h}^{-1} (7.5 \times 10^6 \text{ cells ml}^{-1}) (500 \text{ l}) \left| \frac{1000 \text{ ml}}{1 \text{ l}} \right|$$

$$= -7.50 \text{ kJ h}^{-1}$$

Adding the components together:

$$\Delta H_{rxn} = -517.5 \text{ kJ h}^{-1} + (-7.50 \text{ kJ h}^{-1}) = -525.0 \text{ kJ h}^{-1}$$

Therefore, from the energy balance equation:

$$Q = 525.0 \text{ kJ h}^{-1}$$

Answer: 525 kJ h^{-1}

(iii)

Q is transferred to the cooling water. The resulting increase in cooling water enthalpy can be evaluated using Eq. (5.12):

$$Q = \Delta H = MC_p \Delta T$$

Therefore:

$$M = \frac{Q}{C_p \Delta T}$$

Applying the value of Q from **(b) (ii)**:

$$M = \frac{525.0 \text{ kJ h}^{-1}}{4.19 \text{ kJ kg}^{-1} \text{°C}^{-1} (29 - 20)\text{°C}} = 13.9 \text{ kg h}^{-1}$$

Answer: 13.9 kg h^{-1}

5.15 Ginseng production

From Section 2.4.5, the composition of air is 21% O_2 and 79% N_2 by volume. Therefore, in 5.5×10^6 litres of inlet air, the volume of oxygen is $0.21 \times 5.5 \times 10^6$ l = 1.155×10^6 l. If the off-gas contains 5.1×10^5 l of oxygen at the same temperature and pressure, the difference $(1.155 \times 10^6 - 5.1 \times 10^5)$ l = 6.45×10^5 l is the volume of oxygen consumed by the cells. Converting this gas volume to moles using the ideal gas law, Eq. (2.35), with R = 0.082057 l atm K^{-1} $gmol^{-1}$ (Appendix B) and the temperature converted from °C to Kelvin using Eq. (2.27):

$$\text{Moles of } O_2 \text{ consumed} = \frac{pV}{RT} = \frac{1 \text{ atm } (6.45 \times 10^5 \text{ l})}{0.082057 \text{ l atm K}^{-1} \text{ gmol}^{-1} (25 + 273.15) \text{ K}} = 2.636 \times 10^4 \text{ gmol}$$

From Eq. (5.23), the heat of reaction for aerobic metabolism is approximately –460 kJ $gmol^{-1}$ of O_2 consumed. Therefore, the heat of reaction for the plant cell culture over the 12-day culture period is:

$$\Delta H_{rxn} = -460 \text{ kJ gmol}^{-1} (2.636 \times 10^4 \text{ gmol}) = -1.213 \times 10^7 \text{ kJ}$$

The modified energy balance equation, Eq. (5.29), can be applied to the culture. Assuming negligible shaft work:

$$Q = -\Delta H_{rxn} - M_v \Delta h_v$$

The latent heat of vaporisation Δh_v for water at 25°C is 2442.5 kJ kg^{-1} (Table D.1, Appendix D). Substituting values into the energy balance equation gives:

$$Q = 1.213 \times 10^7 \text{ kJ} - 135 \text{ kg } (2442.5 \text{ kJ kg}^{-1})$$

$$= 1.180 \times 10^7 \text{ kJ}$$

Answer: 1.18×10^7 kJ over the 12-day culture period

5.16 Evaporative cooling

The modified energy balance equation, Eq. (5.29), can be applied to the culture. Assuming that shaft work is negligible:

$$Q = -\Delta H_{rxn} - M_v \Delta h_v$$

From Eq. (5.23), the heat of reaction for aerobic metabolism is approximately –460 kJ $gmol^{-1}$ of O_2 consumed. Therefore, the heat of reaction for this culture is:

$$\Delta H_{rxn} = -460 \text{ kJ gmol}^{-1} (140 \times 10^{-3} \text{ gmol h}^{-1}) = -64.40 \text{ kJ h}^{-1}$$

The latent heat of vaporisation Δh_v for water at 30°C is 2430.7 kJ kg^{-1} (Table D.1, Appendix D). Substituting values into the energy balance equation gives:

$$Q = 64.40 \text{ kJ h}^{-1} - 0.5 \text{ kg day}^{-1} (2430.7 \text{ kJ kg}^{-1}) \cdot \left| \frac{1 \text{ day}}{24 \text{ h}} \right| = 13.8 \text{ kJ h}^{-1}$$

From the sign conventions outlined in Section 5.2, positive Q means that heat must be removed from the system. This result indicates that the cooling effect of evaporation is less than the heating effect due to reaction, so that additional cooling is required at a rate of 13.8 kJ h^{-1}.

Answer: No

5.17 Penicillin process

From Eq. (5.23), the heat of reaction for aerobic metabolism is approximately –460 kJ $gmol^{-1}$ of O_2 consumed. When the rate of O_2 consumption is at its lowest value, the heat of reaction is:

$$\Delta H_{rxn} = -460 \text{ kJ gmol}^{-1} (0.45\times10^{-3} \text{ gmol l}^{-1} \text{ min}^{-1})(90{,}000 \text{ l}) = -1.863\times10^{4} \text{ kJ min}^{-1}$$

When the rate of O_2 consumption is at its highest value, the heat of reaction is:

$$\Delta H_{rxn} = -460 \text{ kJ gmol}^{-1} (0.85\times10^{-3} \text{ gmol l}^{-1} \text{ min}^{-1})(90{,}000 \text{ l}) = -3.519\times10^{4} \text{ kJ min}^{-1}$$

1 W = 1 J s^{-1} (Table A.8, Appendix A). The power input by stirring is the rate at which shaft work is done in the system; therefore:

$$W_s = 2.9 \text{ W l}^{-1} = 2.9 \text{ W l}^{-1} . \left|\frac{1 \text{ J s}^{-1}}{1 \text{ W}}\right| . \left|\frac{60 \text{ s}}{1 \text{ min}}\right| . \left|\frac{1 \text{ kJ}}{1000 \text{ J}}\right| (90{,}000 \text{ l}) = 1.566\times10^{4} \text{ kJ min}^{-1}$$

The modified energy balance equation, Eq. (5.29), can be applied to this culture. Assuming that evaporation is negligible:

$$Q = -\Delta H_{rxn} + W_s$$

Substituting values into the energy balance equation for when the rate of O_2 consumption is at its lowest:

$$Q = 1.863\times10^{4} \text{ kJ min}^{-1} + 1.566\times10^{4} \text{ kJ min}^{-1} = 3.429\times10^{4} \text{ kJ min}^{-1}$$

Similarly, when the rate of O_2 consumption is at its highest:

$$Q = 3.519\times10^{4} \text{ kJ min}^{-1} + 1.566\times10^{4} \text{ kJ min}^{-1} = 5.085\times10^{4} \text{ kJ min}^{-1}$$

Answer: From 3.43 × 10^4 kJ min^{-1} to 5.09 × 10^4 kJ min^{-1}

5.18 Culture of methylotrophic yeast

(a)

When biomass is the only major product, the stoichiometric equation is based on Eq. (4.4). As outlined in Section 4.6.1, $CH_{1.8}O_{0.5}N_{0.2}$ can be used as the molecular formula for biomass when composition analysis is not available. Representing ammonium hydroxide as NH_3, Eq. (4.4) becomes:

$$C_3H_8O_3 + aO_2 + bNH_3 \rightarrow cCH_{1.8}O_{0.5}N_{0.2} + dCO_2 + eH_2O$$

From Table C.1 (Appendix C), the molecular weight of glycerol = 92 and the molecular weight of the biomass = 24.6. The stoichiometric coefficient c is evaluated from the biomass yield from substrate Y_{XS} = 0.57 g g^{-1} and Eq. (4.15):

$$c = \frac{Y_{XS} (\text{MW substrate})}{\text{MW cells}} = \frac{0.57 \text{ g g}^{-1} (92)}{24.6} = 2.13$$

The oxygen demand is determined using an electron balance. From Table C.2 (Appendix C), the degree of reduction of glycerol relative to NH_3 is γ_S = 4.67. The degree of reduction of the biomass relative to NH_3 is:

$$\gamma_B = \frac{1\times4 + 1.8\times1 - 0.5\times2 - 0.2\times3}{1} = 4.20$$

(This value for γ_B is also given in Table C.2.) w = 3 for glycerol. Substituting values into Eq. (4.20) with f = 0:

$$a = \frac{1}{4}(w\gamma_S - c\gamma_B) = \frac{1}{4}(3\times4.67 - 2.13\times4.20) = 1.27$$

The results for a and c indicate that 1.27 gmol of O_2 are consumed for every 2.13 gmol of biomass produced. Applying the molecular weight of the biomass, this is equivalent to 1.27 gmol O_2 per (2.13 × 24.6) g of biomass, or 0.024 gmol O_2 per g of biomass. From Eq. (5.23), the heat of reaction for aerobic metabolism is approximately –460 kJ $gmol^{-1}$ of O_2 consumed. Therefore, per g of biomass formed, the heat of reaction for this culture is:

$$\Delta H_{rxn} = -460 \text{ kJ gmol}^{-1} (0.024 \text{ gmol}) = -11.04 \text{ kJ}$$

The modified energy balance equation, Eq. (5.29), applies to this culture. Assuming that evaporation and shaft work are negligible:

$$Q = -\Delta H_{rxn} = 11.04 \text{ kJ}$$

Answer: 11.0 kJ per g of biomass produced

(b)

From Table C.1 (Appendix C), the molecular weight of methanol = 32. The stoichiometric coefficient c is determined for Y_{XS} = 0.44 g g^{-1} using Eq. (4.15):

$$c = \frac{Y_{XS} (\text{MW substrate})}{\text{MW cells}} = \frac{0.44 \text{ g g}^{-1} (32)}{24.6} = 0.572$$

From Table C.2 (Appendix C), the degree of reduction of methanol relative to NH_3 is γ_S = 6.00. The O_2 requirement is determined using Eq. (4.20) with f = 0 and w = 1 for methanol:

$$a = \frac{1}{4} (w\gamma_S - c\gamma_B) = \frac{1}{4} (1 \times 6.00 - 0.572 \times 4.20) = 0.899$$

This means that 0.899 gmol of O_2 are consumed for every 0.572 gmol of biomass produced. This is equivalent to 0.899 gmol O_2 per (0.572 × 24.6) g of biomass, or 0.064 gmol O_2 per g of biomass. From Eq. (5.23), the heat of reaction per g of biomass formed is:

$$\Delta H_{rxn} = -460 \text{ kJ gmol}^{-1} (0.064 \text{ gmol}) = -29.44 \text{ kJ}$$

From the modified energy balance equation, Eq. (5.29), with negligible evaporation and shaft work:

$$Q = -\Delta H_{rxn} = 29.44 \text{ kJ}$$

Answer: The cooling requirement of 29.4 kJ per g of biomass produced represents an increase of 2.7-fold relative to the cooling requirement with glycerol as substrate

5.19 Algal culture for carotenoid synthesis

Carotenoid is an intracellular product and is considered part of the biomass. From Table C.2 (Appendix C), the molecular formula for glucose is $C_6H_{12}O_6$. From Eq. (4.4), the stoichiometric equation for growth of algal cells is:

$$C_6H_{12}O_6 + aO_2 + bNH_3 \rightarrow cCH_{1.8}O_{0.6}N_{0.2} + dCO_2 + eH_2O$$

From Table C.1 (Appendix C), the molecular weight of glucose is 180 and the molecular weight of the biomass is 26.2. The stoichiometric coefficient c is evaluated from the biomass yield from substrate Y_{XS} = 0.45 g g^{-1} and Eq. (4.15):

$$c = \frac{Y_{XS} (\text{MW substrate})}{\text{MW cells}} = \frac{0.45 \text{ g g}^{-1} (180)}{26.2} = 3.092 \quad (1)$$

The oxygen demand is determined using an electron balance. From Table C.2 (Appendix C), the degree of reduction of glucose relative to NH_3 is γ_S = 4.00. The degree of reduction of the biomass relative to NH_3 is:

$$\gamma_B = \frac{1\times 4 + 1.8\times 1 - 0.6\times 2 - 0.2\times 3}{1} = 4.00$$

Applying Eq. (4.20) with $f = 0$ and $w = 6$ for glucose:

$$a = \frac{1}{4}(w\gamma_S - c\gamma_B) = \frac{1}{4}(6\times 4.00 - 3.092\times 4.00) = 2.91$$

Therefore, the oxygen requirement is 2.91 gmol per gmol of glucose consumed. Converting the rate of glucose consumption to a molar basis:

$$\text{Rate of substrate consumption} = 77 \text{ g h}^{-1} = 77 \text{ g h}^{-1} . \left|\frac{1 \text{ gmol}}{180 \text{ g}}\right| = 0.428 \text{ gmol h}^{-1}$$

Therefore, the rate of O_2 consumption is 2.91×0.428 gmol h^{-1} = 1.245 gmol O_2 h^{-1}. From Eq. (5.23), the heat of reaction for aerobic metabolism is approximately –460 kJ $gmol^{-1}$ of O_2 consumed. Therefore, the heat of reaction for this culture is:

$$\Delta H_{rxn} = -460 \text{ kJ gmol}^{-1} (1.245 \text{ gmol h}^{-1}) = -572.7 \text{ kJ h}^{-1}$$

The modified energy balance equation, Eq. (5.29), applies to this culture. Assuming that evaporation and shaft work are negligible:

$$Q = -\Delta H_{rxn} = 572.7 \text{ kJ h}^{-1}$$

Answer: 573 kJ h^{-1}, assuming that evaporation and shaft work in the bioreactor are negligible

5.20 Checking the consistency of measured culture data

Assume that the reducing sugars used as substrate for the culture are glucose and fructose. From Table C.8 (Appendix C), glucose and fructose have the same molecular formula, $C_6H_{12}O_6$, and the molecular formula for lysine is $C_6H_{14}O_2N_2$. As outlined in Section 4.6.1, $CH_{1.8}O_{0.5}N_{0.2}$ can be used as the molecular formula for biomass when composition analysis is not available. From Eq. (4.16), the stoichiometric equation is:

$$C_6H_{12}O_6 + aO_2 + bNH_3 \rightarrow cCH_{1.8}O_{0.5}N_{0.2} + dCO_2 + eH_2O + fC_6H_{14}O_2N_2$$

From Table C.1 (Appendix C), the molecular weight of glucose and fructose = 180, the molecular weight of the biomass is 24.6, the molecular weight of lysine is 146, and the molecular weight of O_2 is 32. The calculations are performed using a basis of 1 litre and 1 h.

(a)

Sugar consumption in molar terms for the 12–36-h fermentation period is:

$$\text{Sugar consumption} = 2.0 \text{ g} = 2.0 \text{ g} . \left|\frac{1 \text{ gmol}}{180 \text{ g}}\right| = 0.0111 \text{ gmol}$$

The moles of biomass and lysine produced per gmol of sugar consumed correspond to the stoichiometric coefficients c and f, respectively. Similarly, the moles of oxygen consumed per gmol of sugar corresponds to a. Using the data provided:

$$c = \frac{0.21 \text{ g}}{0.0111 \text{ gmol}} . \left|\frac{1 \text{ gmol}}{24.6 \text{ g}}\right| = 0.769$$

$$f = \frac{0.66 \text{ g}}{0.0111 \text{ gmol}} . \left|\frac{1 \text{ gmol}}{146 \text{ g}}\right| = 0.407$$

$$a = \frac{0.75\ \text{g}}{0.0111\ \text{gmol}} . \left| \frac{1\ \text{gmol}}{32\ \text{g}} \right| = 2.112$$

To check the consistency of these results, the oxygen demand can be determined using an electron balance. From Table C.2 (Appendix C), the degree of reduction of substrate relative to NH_3 is $\gamma_S = 4.00$ and the degree of reduction of lysine relative to NH_3 is $\gamma_P = 4.67$. The degree of reduction of the biomass relative to NH_3 is:

$$\gamma_B = \frac{1\times 4 + 1.8\times 1 - 0.5\times 2 - 0.2\times 3}{1} = 4.20$$

(This value for γ_B is also given in Table C.2.) $w = 6$ for glucose and fructose; $j = 6$ for lysine. Substituting values into Eq. (4.20):

$$a = \frac{1}{4}(6\times 4.00 - 0.769\times 4.20 - 0.407\times 6\times 4.67) = 2.342$$

As the value of a calculated directly from the measured data (2.112) is within 10% of the value determined using an electron balance (2.342), we can say that the data for the 12–36-h period of the fermentation are consistent.

We can also check the consistency of the measured heat evolution data. From the modified energy balance equation, Eq. (5.29), if evaporation and shaft work are negligible, the heat that must be removed from the culture at steady state is equal to the heat of reaction:

$$Q = -\Delta H_{\text{rxn}}$$

Also, from Eq. (5.23), the heat of reaction for aerobic metabolism is approximately –460 kJ gmol^{-1} of O_2 consumed. Therefore, using the measured data for O_2 consumption:

$$Q = 460\ \text{kJ gmol}^{-1}\ (0.75\ \text{g}) . \left| \frac{1\ \text{gmol}}{32\ \text{g}} \right| = 10.78\ \text{kJ}$$

This can be compared with 12.1 kJ measured directly: the measured value is 12% higher than the calculated value. Although this difference is greater than ±10%, given the approximate nature of the –460 kJ gmol^{-1} relationship between heat of reaction and O_2 consumption for aerobic cultures, the measured data for heat evolution can be considered reasonably consistent.

Answer: The data are reasonably consistent

(b)

Repeating the electron balance calculations carried out in **(a)** using data for the 0–12-h fermentation period:

$$\text{Sugar consumption} = 0.42\ \text{g} = 0.42\ \text{g} . \left| \frac{1\ \text{gmol}}{180\ \text{g}} \right| = 2.333\times 10^{-3}\ \text{gmol}$$

$$c = \frac{0.29\ \text{g}}{2.333\times 10^{-3}\ \text{gmol}} . \left| \frac{1\ \text{gmol}}{24.6\ \text{g}} \right| = 5.053$$

$$f = \frac{0.20\ \text{g}}{2.333\times 10^{-3}\ \text{gmol}} . \left| \frac{1\ \text{gmol}}{146\ \text{g}} \right| = 0.587$$

$$a = \frac{0.40\ \text{g}}{2.333\times 10^{-3}\ \text{gmol}} . \left| \frac{1\ \text{gmol}}{32\ \text{g}} \right| = 5.358$$

Applying Eq. (4.20) with values of γ_S, γ_P, γ_B, w and j from **(a)**:

$$a = \frac{1}{4}(6\times 4.00 - 5.053\times 4.20 - 0.587\times 6\times 4.67) = -3.42$$

A negative value for a cannot be correct and is significantly different from the value of 5.358 determined directly from the measured data for O_2 uptake. The electron balance indicates that there must be an additional source of electrons in the culture that has not been taken into account in the calculations. This result is consistent with amino acids being used as an additional substrate during the initial culture period.

Answer: Yes

(c)

Using the result for sugar consumption from **(b)**, the measured heat evolution per gmol of sugar consumed for the 0–12-h fermentation period is:

$$\text{Heat evolved per gmol sugar consumed} = \frac{2.5\text{ kJ}}{2.333\times 10^{-3}\text{ gmol}} = 1.072\times 10^{3}\text{ kJ gmol}^{-1}$$

From the energy balance in **(a)**, the heat evolved Q is equal to the heat of reaction, which can be estimated using Eq. (5.23). This allows calculation of the stoichiometric coefficient a, the gmol of O_2 consumed per gmol of sugar consumed:

$$a = \text{gmol } O_2 \text{ consumed per gmol sugar consumed} = \frac{\text{heat evolved per gmol sugar consumed}}{\text{heat evolved per gmol } O_2 \text{ consumed}}$$

$$= \frac{1.072\times 10^{3}\text{ kJ per gmol sugar}}{460\text{ kJ per gmol } O_2}$$

$$= 2.33$$

When glutamine is used as an additional substrate, the stoichiometric equation becomes:

$$C_6H_{12}O_6 + gC_5H_{10}O_3N_2 + aO_2 + bNH_3 \rightarrow cCH_{1.8}O_{0.5}N_{0.2} + dCO_2 + eH_2O + fC_6H_{14}O_2N_2$$

where $C_5H_{10}O_3N_2$ is the molecular formula for glutamine (Table C.8, Appendix C) and g is its stoichiometric coefficient. The molecular weight of glutamine is 146 (Table C.8) and the degree of reduction of glutamine relative to NH_3 is $\gamma_S = 3.60$ (Table C.2). Eq. (4.19) must be modified to include an additional substrate:

$$w_1\gamma_{S1} + g\,w_2\gamma_{S2} - 4a = c\gamma_B + fj\gamma_P$$

where subscript 1 refers to reducing sugar and subscript 2 refers to glutamine. Rearranging gives:

$$g = \frac{c\gamma_B + fj\gamma_P + 4a - w_1\gamma_{S1}}{w_2\gamma_{S2}}$$

Substituting values for the 0–12-h fermentation period from **(b)** with $w_2 = 5$ for glutamine and $a = 2.33$:

$$g = \frac{5.053\times 4.20 + 0.587\times 6\times 4.67 + 4\times 2.33 - 6\times 4.00}{5\times 3.60} = 1.277$$

Therefore, 1.277 gmol of glutamine are consumed per gmol of reducing sugar consumed. Using the measured result for sugar consumption from **(b)**, the gmol of glutamine consumed is $1.277 \times 2.333 \times 10^{-3}$ gmol = 2.979×10^{-3} gmol. Converting this to mass:

$$\text{Glutamine consumed} = 2.979\times 10^{-3}\text{ gmol} \cdot \left|\frac{146\text{ g}}{1\text{ gmol}}\right| = 0.435\text{ g}$$

Therefore, the rate of glutamine consumption is 0.435 g l^{-1} h^{-1}.

Answer: 0.43 g l^{-1} h^{-1}

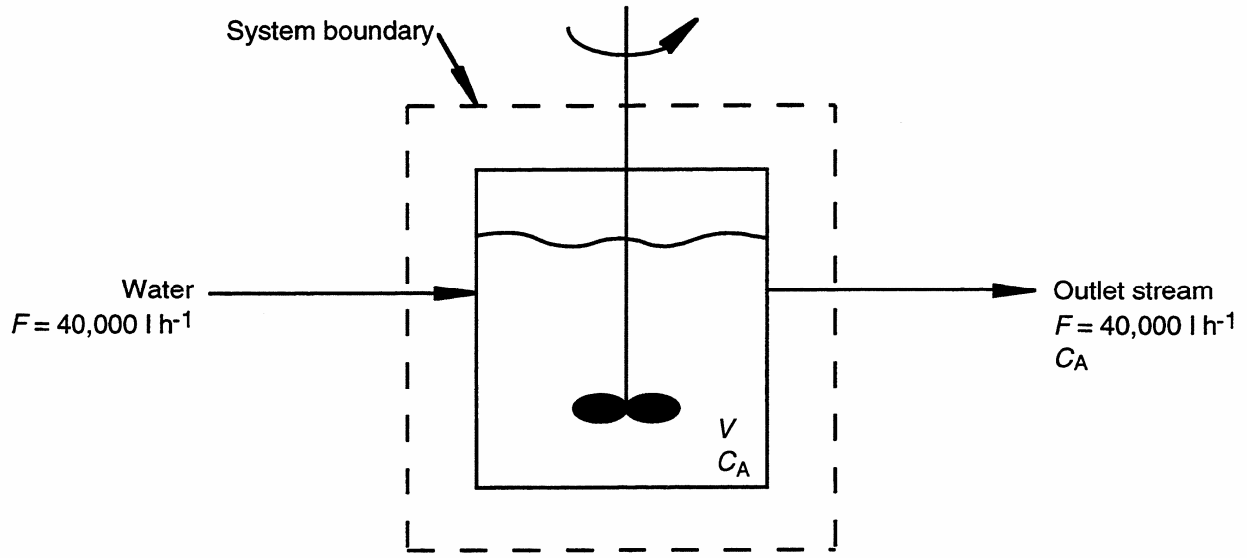
Chapter 6

Unsteady-State Material and Energy Balances

6.1 Dilution of sewage

1. Flow sheet and system boundary

These are shown in the figure below.



2. Define variables

V = volume of material in the tank; F = volumetric flow rate into and out of the vessel = 40,000 l h^{-1}; C_A = concentration of suspended solids

3. Assumptions

– no leaks

– tank is well mixed; therefore C_A in the outlet stream = C_A inside the tank

– density of the suspended solids is the same as that of water = 1 g cm^{-3} (Section 2.4.1) = 1 kg l^{-1}

4. Boundary conditions

At $t = 0$, $V = V_0$ = 440,000 litres + the volume of the solids. The initial mass of solids in the tank is 10,000 kg; from the definition of density (Section 2.4.1), the volume of solids = (10,000 kg)/(1 kg l^{-1}) = 10,000 l. Therefore, V_0 = (440,000 + 10,000) l = 450,000 l. At $t = 0$, $C_A = C_{A0}$:

$$C_{A0} = \frac{10{,}000\ \text{kg}}{450{,}000\,\text{l}} = 0.022\ \text{kg l}^{-1}$$

5. Mass balance

Total mass balance

As the volumetric flow rates of material in and out are the same and the density of the inlet and outlet streams are assumed to be equal, the volume of material in the tank is constant (see Example 6.2, Section 6.4) so that $V = V_0$ at all times.

Solids mass balance

The general unsteady-state mass balance equation is Eq. (6.5). As there is no reaction, $R_G = R_C = 0$. In this problem, $\hat{M}_i = 0$, $\hat{M}_o = FC_A$ and $M = VC_A$. Substituting into Eq. (6.5) gives:

$$\frac{\mathrm{d}(VC_A)}{\mathrm{d}t} = -FC_A$$

As V is constant, it can be taken outside of the differential:

$$V\frac{\mathrm{d}C_A}{\mathrm{d}t} = -FC_A$$

As F is also constant, the differential equation contains only two variables, C_A and t. Separating variables gives:

$$\frac{\mathrm{d}C_A}{C_A} = \frac{-F}{V}\mathrm{d}t$$

Integrating:

$$\int \frac{\mathrm{d}C_A}{C_A} = \int \frac{-F}{V}\,\mathrm{d}t$$

Using integration rules (E.27) and (E.24) from Appendix E and combining the constants of integration:

$$\ln C_A = \frac{-F}{V}t + K$$

Applying the initial condition for C_A gives $\ln C_{A0} = K$. Substituting this value of K into the equation gives:

$$\ln C_A = \frac{-F}{V}t + \ln C_{A0}$$

$$\ln \frac{C_A}{C_{A0}} = \frac{-F}{V}t$$

$$C_A = C_{A0}e^{(-F/V)t}$$

Substituting the known values for C_{A0}, F and V:

$$C_A = 0.022e^{-0.089t}$$

where C_A has units of kg l^{-1} and t has units of h. From this equation, at $t = 5$ h, $C_A = 0.014$ kg l^{-1}.

Answer: 0.014 kg 1^{-1}; the assumptions used are listed in 3. above

6.2 Production of fish protein concentrate

1. System

The system is the whole gutted fish placed in the batch drier. During drying, the mass of the system decreases as water is removed.

2. Assumptions

No additional assumptions are required.

3. Boundary conditions

At $t = 0$, the mass of water in the fish M is equal to M_0.

4. Mass balance

Water mass balance

The general unsteady-state mass balance equation is Eq. (6.5). As there is no reaction, $R_G = R_C = 0$. No water is added during drying; therefore $\hat{M}_i = 0$. At any time during drying, because the rate of water removal $\hat{M}_o$ is proportional to the moisture content M, $\hat{M}_o = kM$ where k is a constant. Substituting into Eq. (6.5) gives:

$$\frac{dM}{dt} = -kM$$

where M and t are the only variables. Separating variables and integrating:

$$\int \frac{dM}{M} = -\int k\, dt$$

Using integration rules (E.27) and (E.24) from Appendix E and combining the constants of integration:

$$\ln M = -kt + K$$

At $t = 0$, $M = M_0$; therefore, $K = \ln M_0$. Substituting this value of K into the equation gives:

$$\ln M = -kt + \ln M_0$$

$$\ln \frac{M}{M_0} = -kt$$

At $t = 20$ min, $M = 0.5M_0$. Substituting these values into the equation:

$$\ln \frac{0.5M_0}{M_0} = -k\,(20\text{ min})$$

$$\ln 0.5 = -k\,(20\text{ min})$$

$$k = 0.0347\text{ min}^{-1}$$

Therefore:

$$\ln \frac{M}{M_0} = -0.0347t$$

where t has units of min. When 95% of the water has been removed, $M = 0.05M_0$. Therefore:

$$\ln \frac{0.05M_0}{M_0} = -0.0347t$$

$$\ln 0.05 = -0.0347t$$

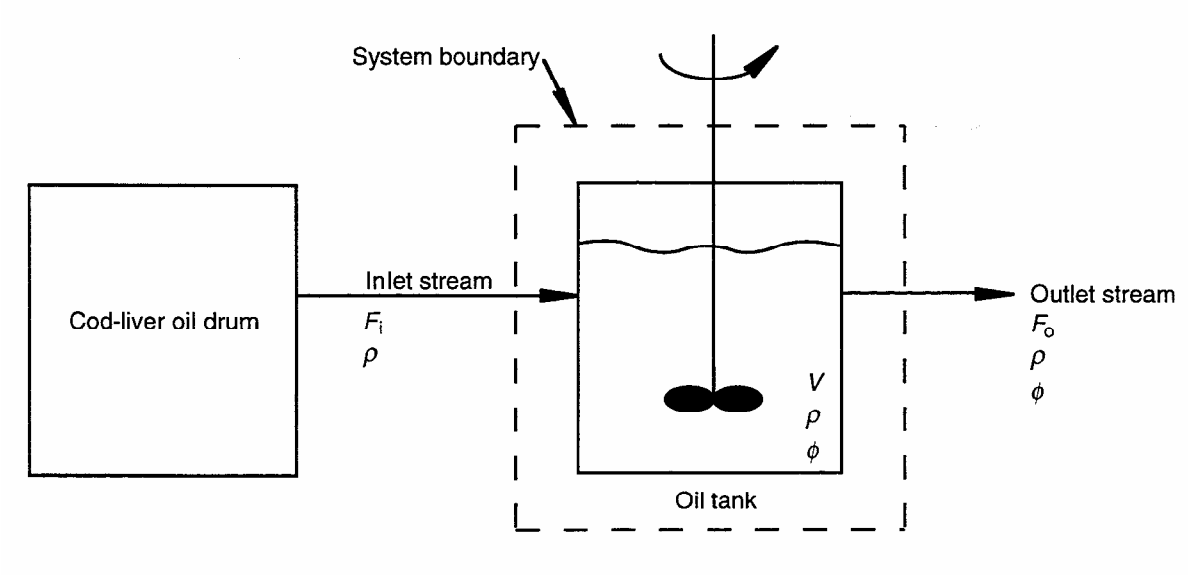
$$t = 86.3\text{ min}$$

Answer: 86 min

6.3 Contamination of vegetable oil

1. Flow sheet and system boundary

These are shown in the figure below.



2. Define variables

V = volume of oil in the tank; F_i = volumetric flow rate of cod-liver oil into the oil tank; F_o = volumetric flow rate of mixed oil out of the oil tank; ϕ = mass fraction of vegetable oil; ρ = density of vegetable and cod-liver oils

3. Assumptions

– no leaks

– oil tank is well mixed; therefore ϕ in the outlet stream = ϕ inside the tank

– densities of each oil and the oil mixture are the same

4. Boundary conditions

At $t = 0$ when cod-liver oil first enters the oil tank, $V = V_0 = 60$ litres. At $t = 0$, $\phi = \phi_0 = 1$.

5. Mass balance

(a)

Total mass balance

The general unsteady-state mass balance equation is Eq. (6.5). As there is no reaction, $R_G = R_C = 0$. From the definition of density (Section 2.4.1), the total mass flow rate into the tank is equal to the volumetric flow rate multiplied by the density of the cod-liver oil: $\hat{M}_i = F_i \rho$. Similarly, $\hat{M}_o = F_o \rho$. The total mass of oil in the tank is equal to the volume of oil multiplied by its density: $M = V\rho$. Substituting these terms into Eq. (6.5) gives:

$$\frac{\mathrm{d}(V\rho)}{\mathrm{d}t} = F_i\rho - F_o\rho$$

As ρ is constant it can be taken outside of the differential and cancelled:

$$\rho \frac{\mathrm{d}V}{\mathrm{d}t} = (F_\mathrm{i} - F_\mathrm{o})\rho$$

$$\frac{\mathrm{d}V}{\mathrm{d}t} = (F_\mathrm{i} - F_\mathrm{o})$$

The differential equation contains only two variables, V and t. Separating variables and integrating:

$$\mathrm{d}V = (F_\mathrm{i} - F_\mathrm{o})\,\mathrm{d}t$$

$$\int \mathrm{d}V = \int (F_\mathrm{i} - F_\mathrm{o})\,\mathrm{d}t$$

Using integration rule (E.24) from Appendix E and combining the constants of integration:

$$V = (F_\mathrm{i} - F_\mathrm{o})t + K$$

Applying the initial condition for V at $t = 0$, $K = V_0$. Substituting this value of K into the equation gives:

$$V = (F_\mathrm{i} - F_\mathrm{o})t + V_0$$

The time between 8 pm and 9 am the next morning is 13 h. For t = 13 h, F_i = 7.5 l h^{-1}, F_o = 4.8 l h^{-1} and V_0 = 60 l, from the above equation, V = 95.1 l. As this volume is less than the tank capacity of 100 l, the tank will not overflow.

Answer: No

(b)

Total mass balance

As the volumetric flow rates of oils into and out of the tank are the same and the density of the inlet and outlet streams are assumed to be equal, the volume of oil in the tank is constant (see Example 6.2, Section 6.4) so that $V = V_0$ = 60 l at all times.

Vegetable oil mass balance

The general unsteady-state mass balance equation is Eq. (6.5). As there is no reaction, $R_\mathrm{G} = R_\mathrm{C} = 0$. No vegetable oil enters the tank; therefore $\hat{M}_\mathrm{i} = 0$. The mass flow rate of vegetable oil out is $\hat{M}_\mathrm{o} = F_\mathrm{o}\rho\phi$. The mass of vegetable oil in the tank at any time is $M = V\rho\phi$. Substituting these terms into Eq. (6.5):

$$\frac{\mathrm{d}(V\rho\phi)}{\mathrm{d}t} = -F_\mathrm{o}\rho\phi$$

As both V and ρ are constants, they can be taken outside of the differential and ρ can be cancelled:

$$V\rho\frac{\mathrm{d}\phi}{\mathrm{d}t} = -F_\mathrm{o}\rho\phi$$

$$V\frac{\mathrm{d}\phi}{\mathrm{d}t} = -F_\mathrm{o}\phi$$

As F_o is also constant, the differential equation contains only two variables, ϕ and t. Separating variables and integrating:

$$\frac{\mathrm{d}\phi}{\phi} = \frac{-F_\mathrm{o}}{V}\mathrm{d}t$$

$$\int \frac{\mathrm{d}\phi}{\phi} = \int \frac{-F_\mathrm{o}}{V}\mathrm{d}t$$

Using integration rules (E.27) and (E.24) from Appendix E and combining the constants of integration:

$$\ln \phi = \frac{-F_o}{V} t + K$$

Applying the initial condition for ϕ at $t = 0$, $\ln \phi_0 = K$. Substituting this value of K into the equation gives:

$$\ln \phi = \frac{-F_o}{V} t + \ln \phi_0$$

$$\ln \frac{\phi}{\phi_0} = \frac{-F_o}{V} t$$

$$\phi = \phi_0 e^{(-F_o/V)t}$$

In this problem, $F_o = 4.8$ l h^{-1}. Substituting the known values for ϕ_0 and V:

$$\phi = 1 e^{-0.080t}$$

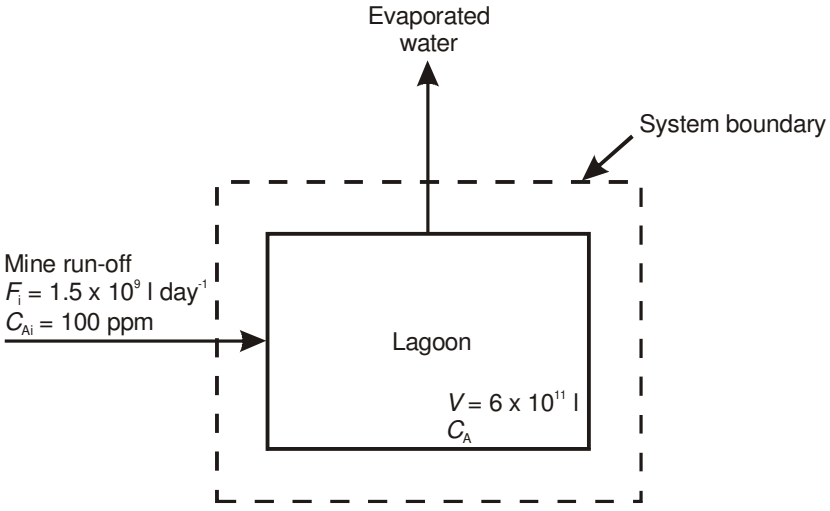
where ϕ is dimensionless and t has units of h. The time between 8 pm and midnight is 4 h. From the above equation, when t = 4 h, ϕ = 0.73. Therefore, at midnight, the composition of oil in the tank is 73% vegetable oil and 27% cod-liver oil.

Answer: 73% vegetable oil, 27% cod-liver oil

6.4 Drainage from mine tailings

1. Flow sheet and system boundary

These are shown in the figure below.



2. Define variables

V = volume of water in the lagoon = 6×10^{11} litres; C_A = concentration of arsenic in the lagoon; F_i = volumetric flow rate of mine run-off into the lagoon; C_{Ai} = concentration of arsenic in the mine run-off

3. Assumptions

– no ground seepage

– lagoon is well mixed

– density of the arsenic solution is the same as that of water = 1 g cm^{-3} (Section 2.4.1) = 1000 g l^{-1}

4. Boundary conditions

At $t = 0$ there is no arsenic in the lagoon; therefore, at $t = 0$, $C_A = 0$.

5. Mass balance

Arsenic mass balance

The general unsteady-state mass balance equation is Eq. (6.5). The mass flow rate of arsenic in is $\hat{M}_i = F_i C_{Ai}$; the mass flow rate of arsenic out $\hat{M}_o = 0$. The rate of arsenic generation $R_G = 0$ and the rate of arsenic consumption $R_C = 0$. The mass of arsenic in the fermenter M is equal to VC_A. Substituting these terms into Eq. (6.5) gives:

$$\frac{\mathrm{d}(VC_A)}{\mathrm{d}t} = F_i C_{Ai}$$

As V is constant, it can be taken outside of the differential:

$$V\frac{\mathrm{d}C_A}{\mathrm{d}t} = F_i C_{Ai}$$

As F_i and C_{Ai} are also constants, the differential equation contains only two variables, C_A and t. Separating variables and integrating gives:

$$\mathrm{d}C_A = \frac{F_i C_{Ai}}{V}\mathrm{d}t$$

$$\int \mathrm{d}C_A = \int \frac{F_i C_{Ai}}{V}\mathrm{d}t$$

Using integration rule (E.24) from Appendix E and combining the constants of integration:

$$C_A = \frac{F_i C_{Ai}}{V}t + K$$

Applying the initial condition for C_A at $t = 0$ gives $K = 0$. Substituting this value of K into the equation:

$$C_A = \frac{F_i C_{Ai}}{V}t$$

From Section 2.4.5, 100 ppm arsenic means 100 g per 10^6 g of solution. If the density of the arsenic solution is 1000 g l^{-1}, C_{Ai} = 100 ppm = (100 g/10^6 g) × (1000 g l^{-1}) = 0.1 g l^{-1}. Substituting values into the equation for C_A:

$$C_A = \frac{1.5\times10^9\,\mathrm{l\,day^{-1}}\,(0.1\,\mathrm{g\,l^{-1}})}{6\times10^{11}\,\mathrm{l}}t$$

$$= 2.5\times10^{-4}t$$

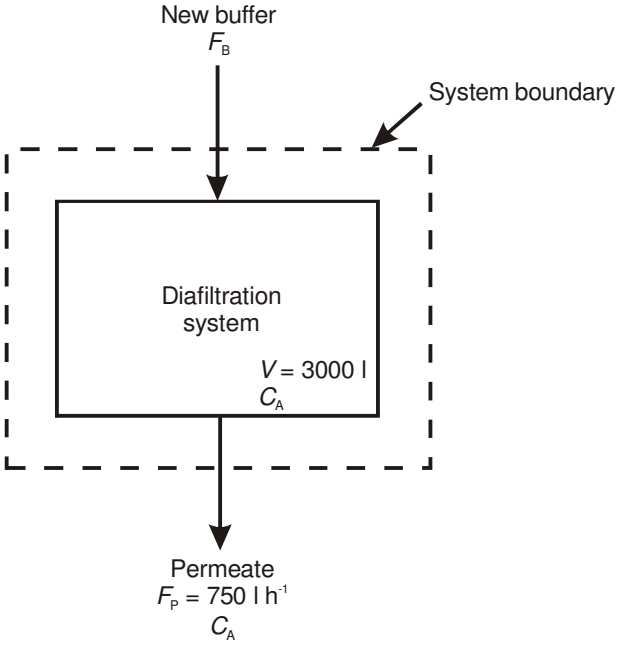
where C_A has units of g l^{-1} and t has units of days. From this equation, when t = 14 days, $C_A = 3.5 \times 10^{-3}$ g l^{-1} or 3.5 ppm.

Answer: 3.5 ppm

6.5 Diafiltration

1. Flow sheet and system boundary

These are shown in the figure below. As the mass balance is performed on the recycle system as a whole, new buffer and permeate are the only streams crossing the system boundary.



2. Define variables

V = volume of solution in the system = 3000 l; C_A = concentration of salt in the solution; F_B = volumetric flow rate of new buffer; F_P = volumetric flow rate of permeate = 750 l h^{-1}

3. Assumptions

– no leaks

– solution is well mixed

– salt and water pass equally well through the membrane, i.e. the salt concentration in the permeate is the same as that in the solution

– density of new buffer is the same as that of permeate

4. Boundary conditions

At $t = 0$, $C_A = C_{A0} = 45$ g l^{-1}.

5. Mass balance

Salt mass balance

The general unsteady-state mass balance equation is Eq. (6.5). The mass flow rate of salt in is $\hat{M}_i = 0$; the mass flow rate of salt out $\hat{M}_o = F_P C_A$. The rate of salt generation $R_G = 0$ and the rate of salt consumption $R_C = 0$. The mass of salt in the solution M is equal to VC_A. Substituting these terms into Eq. (6.5) gives:

$$\frac{\mathrm{d}(VC_A)}{\mathrm{d}t} = -F_P C_A$$

As V is constant, it can be taken outside of the differential:

$$V\frac{\mathrm{d}C_A}{\mathrm{d}t} = -F_P C_A$$

As F_P is also constant, the differential equation contains only two variables, C_A and t. Separating variables and integrating:

$$\frac{\mathrm{d}C_\mathrm{A}}{C_\mathrm{A}} = \frac{-F_\mathrm{P}}{V}\mathrm{d}t$$

$$\int \frac{\mathrm{d}C_\mathrm{A}}{C_\mathrm{A}} = \int \frac{-F_\mathrm{P}}{V}\mathrm{d}t$$

Using integration rules (E.27) and (E.24) from Appendix E and combining the constants of integration:

$$\ln C_\mathrm{A} = \frac{-F_\mathrm{P}}{V}t + K$$

Applying the initial condition for C_A at $t = 0$ gives $\ln C_{\mathrm{A}0} = K$. Substituting this value of K into the equation:

$$\ln C_\mathrm{A} = \frac{-F_\mathrm{P}}{V}t + \ln C_{\mathrm{A}0}$$

$$\ln \frac{C_\mathrm{A}}{C_{\mathrm{A}0}} = \frac{-F_\mathrm{P}}{V}t$$

(a)

For removal of 99.9% of the salt present in the initial solution, as the total volume of solution remains constant, the final C_A must be 0.1% of $C_{\mathrm{A}0}$. Therefore, $C_\mathrm{A}/C_{\mathrm{A}0} = 0.001$. Substituting parameter values into the equation and solving for t:

$$\ln 0.001 = \frac{-750\,\mathrm{l\,h^{-1}}}{3000\,\mathrm{l}}t$$

$$t = 27.6\ \mathrm{h}$$

Answer: 27.6 h

(b)

As the total volume of solution in the system is constant and the density of the new buffer and permeate streams are assumed equal, the volumetric flow rates into and out of the system must be the same. Therefore, the flow rate of new buffer $F_\mathrm{B} = F_\mathrm{P} = 750\,\mathrm{l\,h^{-1}}$. From **(a)**, as the process is operated for 27.6 h:

$$\text{Volume of new buffer} = 750\,\mathrm{l\,h^{-1}}\,(27.6\ \mathrm{h}) = 20{,}700\,\mathrm{l}$$

Answer: 20,700 litres

(c)

The final concentration of salt = 0.1% of 45 g $\mathrm{l^{-1}}$ = 0.045 g $\mathrm{l^{-1}}$. As the total volume of solution in the system remains constant at 3000 l, the mass of salt present in the final solution = 0.045 g $\mathrm{l^{-1}} \times 3000$ l = 135 g.

Answer: 135 g

6.6 Radioactive decay

The general unsteady-state mass balance equation is Eq. (6.5). In this problem, $\hat{M}_\mathrm{i} = \hat{M}_\mathrm{o} = 0$. For a mass balance on isotope, $R_\mathrm{G} = 0$. The rate of isotope decay R_C is proportional to the concentration of isotope present: $R_\mathrm{C} = -k_1 CV$, where k_1 is a constant, C is the isotope concentration and V is the solution volume. The total mass of isotope M is equal to the solution volume V multiplied by the isotope concentration C: $M = VC$. Substituting these terms into Eq. (6.5) gives:

$$\frac{\mathrm{d}(VC)}{\mathrm{d}t} = -k_1 CV$$

If we assume that the density of the solution is constant during isotope decay, V is constant and can be taken outside of the differential and cancelled:

$$V\frac{\mathrm{d}C}{\mathrm{d}t} = -k_1 CV$$

$$\frac{\mathrm{d}C}{\mathrm{d}t} = -k_1 C$$

The differential equation contains only two variables, C and t. Separating variables and integrating:

$$\frac{\mathrm{d}C}{C} = -k_1 \mathrm{d}t$$

$$\int \frac{\mathrm{d}C}{C} = \int -k_1 \mathrm{d}t$$

Using integration rules (E.27) and (E.24) from Appendix E and combining the constants of integration:

$$\ln C = -k_1 t + K$$

Assume an initial condition: at $t = 0$, $C = C_0$. Applying this condition at $t = 0$ gives $\ln C_0 = K$. Substituting this value of K into the equation:

$$\ln C = -k_1 t + \ln C_0$$

$$\ln \frac{C}{C_0} = -k_1 t$$

(a)

When $t = t_\mathrm{h}$, the half-life of the isotope, $C = 0.5C_0$. Substituting these values into the equation:

$$\ln \frac{0.5C_0}{C_0} = -k_1 t_\mathrm{h}$$

$$\ln 0.5 = -k_1 t_\mathrm{h}$$

$$t_\mathrm{h} = \frac{-\ln 0.5}{k_1}$$

Using mathematical rule (E.10) in Appendix E, $-\ln 0.5 = \ln (1/0.5) = \ln 2$. Therefore:

$$t_\mathrm{h} = \frac{\ln 2}{k_1}$$

Answer: QED

(b)

From the equation derived in **(a)**:

$$k_1 = \frac{\ln 2}{t_\mathrm{h}}$$

For $t_\mathrm{h} = 14.3$ days:

$$k_1 = 4.85\times10^{-2}\ \mathrm{day}^{-1}$$

Substituting this value of k_1 into the equation for isotope concentration as a function of time:

$$\ln\frac{C}{C_0} = -4.85\times10^{-2}\,t$$

where t has units of days. For $C = 0.01C_0$:

$$\ln\frac{0.01C_0}{C_0} = -4.85\times10^{-2}\,t$$

$$\ln 0.01 = -4.85\times10^{-2}\,t$$

$$t = 95 \text{ days}$$

Answer: 95 days

6.7 Batch growth of bacteria

The general unsteady-state mass balance equation is Eq. (6.5). For a batch culture, $\hat{M}_\text{i} = \hat{M}_\text{o} = 0$. For a mass balance on cells, assuming that there is no loss of cells from the system such as by lysis, $R_\text{C} = 0$. The rate of generation of cells R_G is proportional to the concentration of cells present: $R_\text{G} = \mu xV$ where μ is a constant, x is the cell concentration and V is the culture volume. The total mass of cells M is equal to the culture volume V multiplied by the cell concentration x: $M = Vx$. Substituting these terms into Eq. (6.5) gives:

$$\frac{\mathrm{d}(Vx)}{\mathrm{d}t} = \mu xV$$

Assuming that V is constant throughout the batch culture, it can be taken outside of the differential and cancelled:

$$V\frac{\mathrm{d}x}{\mathrm{d}t} = \mu xV$$

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \mu x$$

The differential equation contains only two variables, x and t. Separating variables and integrating:

$$\frac{\mathrm{d}x}{x} = \mu\,\mathrm{d}t$$

$$\int\frac{\mathrm{d}x}{x} = \int\mu\,\mathrm{d}t$$

Using integration rules (E.27) and (E.24) from Appendix E and combining the constants of integration:

$$\ln x = \mu t + K$$

Assume an initial condition: at $t = 0$ at the beginning of exponential growth, $x = x_0$. Applying this condition at $t = 0$ gives $\ln x_0 = K$. Substituting this value of K into the equation:

$$\ln x = \mu t + \ln x_0$$

$$\ln\frac{x}{x_0} = \mu t$$

When $t = 45$ min, $x = 2x_0$. Substituting these values into the equation:

$$\ln\frac{2x_0}{x_0} = \mu(45\text{ min})$$

$$\ln 2 = \mu(45\text{ min})$$

$$\mu = 0.0154\text{ min}^{-1}$$

Therefore:

$$\ln\frac{x}{x_0} = 0.0154t$$

or

$$x = x_0 e^{0.0154t}$$

where t has units of min. For t = 12 h = 12 × 60 = 720 min:

$$x = 6.54\times10^4 x_0$$

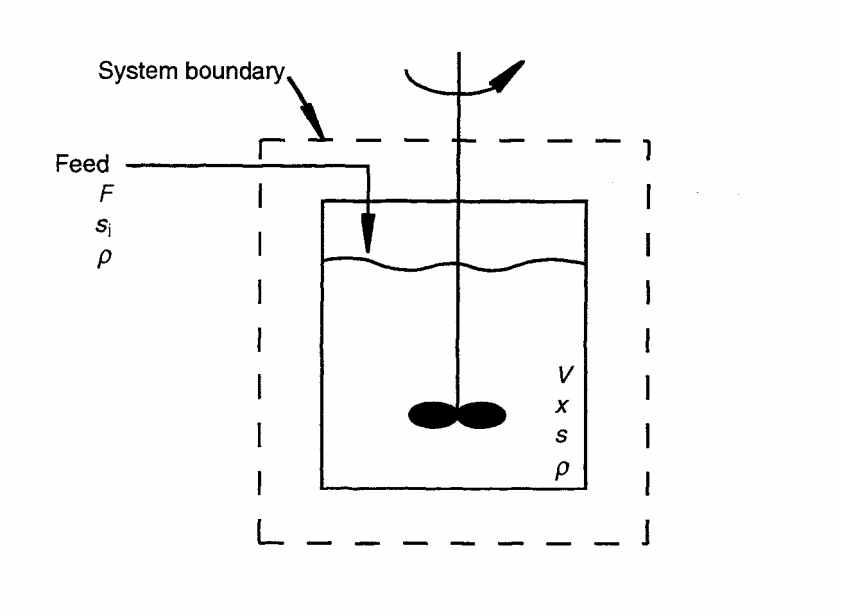
After 12 h, the cell concentration is 6.54×10^4 times the cell concentration at the beginning of exponential growth. Assuming that the cell concentration at the beginning of exponential growth is very close to that at inoculation, the cell concentration is about 6.54×10^4 times the inoculum level.

Answer. 6.54×10^4 times the inoculum level

6.8 Fed-batch fermentation

1. Flow sheet and system boundary

These are shown in the figure below.



2. Define variables

V = volume of broth in the fermenter; F = volumetric flow rate into the fermenter; x = concentration of cells in the fermenter; s = concentration of substrate in the fermenter; s_i = concentration of substrate in the feed; ρ = density of the feed and fermentation broth

3. Assumptions

– no leaks

– fermenter is well mixed

– density of the fermentation broth is the same as that of the feed

4. Boundary condition

At $t = 0$, $V = V_0$.

5. Mass balance

(a)

Total mass balance

The general unsteady-state mass balance equation is Eq. (6.5). As total mass cannot be generated or consumed, $R_G = R_C = 0$. No mass leaves the fermenter; therefore $\hat{M}_o = 0$. The mass flow rate in $\hat{M}_i = \rho F$. The total mass in the fermenter M is equal to $V\rho$. Substituting these terms into Eq. (6.5) gives:

$$\frac{\mathrm{d}(V\rho)}{\mathrm{d}t} = \rho F$$

As ρ is constant, it can be taken outside of the differential and cancelled:

$$\rho \frac{\mathrm{d}V}{\mathrm{d}t} = \rho F$$

$$\frac{\mathrm{d}V}{\mathrm{d}t} = F$$

As F is constant, the differential equation contains only two variables, V and t. Separating variables and integrating:

$$\mathrm{d}V = F\,\mathrm{d}t$$

$$\int \mathrm{d}V = \int F\,\mathrm{d}t$$

Using integration rule (E.24) from Appendix E and combining the constants of integration:

$$V = Ft + K$$

Applying the initial condition for V at $t = 0$, $V_0 = K$. Substituting this value of K into the equation:

$$V = Ft + V_0$$

Answer: $V = Ft + V_0$

(b)

Substrate mass balance

The general unsteady-state mass balance equation is Eq. (6.5). The mass flow rates of substrate in and out are $\hat{M}_i = Fs_i$ and $\hat{M}_o = 0$. The rate of substrate generation $R_G = 0$; the rate of substrate consumption $R_C = r_S V = k_1 s V$. The mass of substrate in the fermenter M is equal to Vs. Substituting these terms into Eq. (6.5) gives:

$$\frac{\mathrm{d}(Vs)}{\mathrm{d}t} = Fs_i - k_1 s V$$

As neither V nor s is constant, both must be kept in the differential as a product. Expanding the differential using the product rule (E.22) from Appendix E:

$$V\frac{\mathrm{d}s}{\mathrm{d}t}+s\frac{\mathrm{d}V}{\mathrm{d}t}=Fs_\mathrm{i}-k_1sV$$

Using the equation $\mathrm{d}V/\mathrm{d}t = F$ derived in **(a)**:

$$V\frac{\mathrm{d}s}{\mathrm{d}t}+Fs=Fs_\mathrm{i}-k_1sV$$

Grouping terms gives:

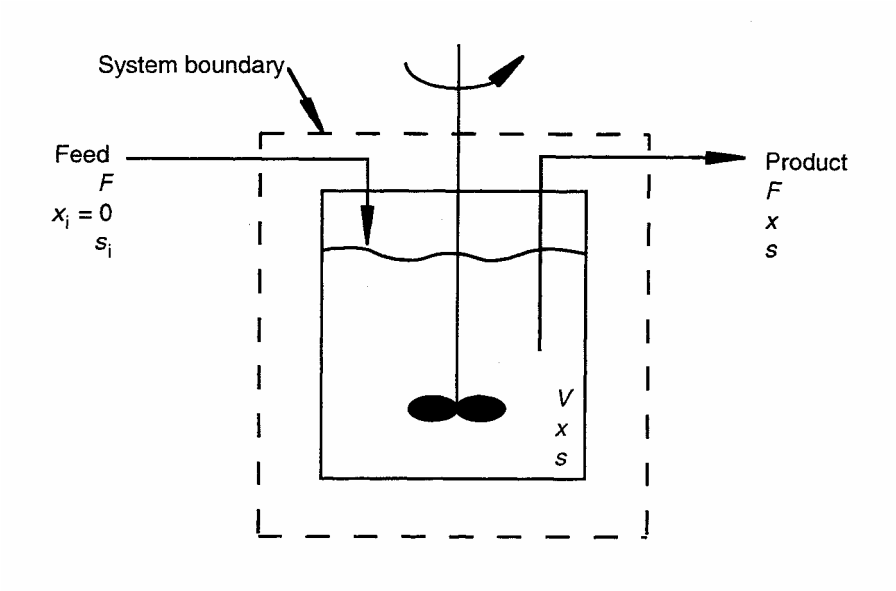
$$\frac{\mathrm{d}s}{\mathrm{d}t}=\frac{F}{V}(s_\mathrm{i}-s)-k_1s$$

Answer: $\dfrac{\mathrm{d}s}{\mathrm{d}t}=\dfrac{F}{V}(s_\mathrm{i}-s)-k_1s$

6.9 Continuous fermentation

1. Flow sheet and system boundary

These are shown in the figure below.



2. Define variables

V = volume of broth in the fermenter; F = volumetric flow rate into and out of the vessel; x = concentration of cells; s = concentration of substrate; x_i = concentration of cells in the feed; s_i = concentration of substrate in the feed

3. Assumptions

– no leaks

– fermenter is well mixed; therefore x and s in the outlet stream = x and s, respectively, inside the vessel

– density of the fermentation broth is the same as that of the feed

4. Boundary condition

At $t = 0$, $x = x_0$.

5. Mass balance

Total mass balance

As the volumetric flow rates into and out of the fermenter are the same and the density of the inlet and outlet streams are assumed equal, the volume of broth in the fermenter is constant (see Example 6.2, Section 6.4) and equal to V at all times.

(a)

Cell mass balance

The general unsteady-state mass balance equation is Eq. (6.5). As no cells enter in the feed stream, $\hat{M}_i = 0$. The mass flow rate of cells out $\hat{M}_o = Fx$. The rate of cell generation $R_G = r_X V = k_1 x V$. Assuming that there is no loss of cells from the system such as by lysis, $R_C = 0$. The mass of cells in the fermenter M is equal to Vx. Substituting these terms into Eq. (6.5) gives:

$$\frac{\mathrm{d}(Vx)}{\mathrm{d}t} = -Fx + k_1 x V$$

As V is constant, it can be taken outside of the differential:

$$V\frac{\mathrm{d}x}{\mathrm{d}t} = -Fx + k_1 x V$$

Dividing through by V and grouping terms:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = x\left(k_1 - \frac{F}{V}\right)$$

Answer: $\dfrac{\mathrm{d}x}{\mathrm{d}t} = x\left(k_1 - \dfrac{F}{V}\right)$

(b)

At steady state, $\mathrm{d}x/\mathrm{d}t = 0$. Therefore, from the equation derived in **(a)**, at steady state k_1 must be equal to F/V.

Answer: $k_1 = F/V$

(c)

As F, V and k_1 are constants, the differential equation derived in **(a)** contains only two variables, x and t. Separating variables and integrating:

$$\frac{\mathrm{d}x}{x} = \left(k_1 - \frac{F}{V}\right)\mathrm{d}t$$

$$\int \frac{\mathrm{d}x}{x} = \int \left(k_1 - \frac{F}{V}\right)\mathrm{d}t$$

Using integration rules (E.27) and (E.24) from Appendix E and combining the constants of integration:

$$\ln x = \left(k_1 - \frac{F}{V}\right)t + K$$

Applying the initial condition for x at $t = 0$, $\ln x_0 = K$. Substituting this value of K into the equation:

$$\ln x = \left(k_1 - \frac{F}{V} \right) t + \ln x_0$$

$$\ln \frac{x}{x_0} = \left(k_1 - \frac{F}{V} \right) t$$

$$x = x_0 e^{\left(k_1 - \frac{F}{V} \right) t}$$

Answer: $x = x_0 e^{\left(k_1 - \frac{F}{V} \right) t}$

(d)

Substituting parameter values into the equation derived in **(c)**:

$$x = (0.5 \text{ g l}^{-1}) e^{\left(0.33 \text{ h}^{-1} - \frac{2200 \text{ l h}^{-1}}{10{,}000 \text{ l}} \right) t}$$

$$x = 0.5 e^{0.110 t}$$

where x has units of g l^{-1} and t has units of h. For $x = 4.0$ g l^{-1}:

$$4 = 0.5 e^{0.110 t}$$

$$\ln 8 = 0.110 t$$

$$t = 18.9 \text{ h}$$

Answer: 18.9 h

(e)

Substrate mass balance

The general unsteady-state mass balance equation is Eq. (6.5). For substrate, the mass flow rates in and out are $\hat{M}_\text{i} = Fs_\text{i}$ and $\hat{M}_\text{o} = Fs$. The rate of substrate generation $R_\text{G} = 0$; the rate of substrate consumption $R_\text{C} = r_\text{S} V = k_2 x V$. The mass of substrate in the fermenter M is equal to Vs. Substituting these terms into Eq. (6.5) gives:

$$\frac{\text{d}(Vs)}{\text{d}t} = Fs_\text{i} - Fs - k_2 x V$$

As V is constant, it can be taken outside of the differential:

$$V \frac{\text{d}s}{\text{d}t} = Fs_\text{i} - Fs - k_2 x V$$

Dividing through by V and grouping terms:

$$\frac{\text{d}s}{\text{d}t} = \frac{F}{V}(s_\text{i} - s) - k_2 x$$

Substituting the expression for x from **(c)**:

$$\frac{\text{d}s}{\text{d}t} = \frac{F}{V}(s_\text{i} - s) - k_2 x_0 e^{\left(k_1 - \frac{F}{V} \right) t}$$

In this equation, F, V, k_2 and x_0 are constants and there are only two variables, s and t. However, the variables cannot be easily separated as in the previous problems, making algebraic solution difficult.

Answer: $\dfrac{\mathrm{d}s}{\mathrm{d}t} = \dfrac{F}{V}(s_\mathrm{i} - s) - k_2 x_0 e^{\left(k_1 - \frac{F}{V}\right)t}$

(f)

At steady state, the equation derived in **(e)** for d*s*/d*t* as a function of *x* becomes:

$$0 = \frac{F}{V}(s_\mathrm{i} - s) - k_2 x$$

$$s_\mathrm{i} - s = \frac{Vk_2}{F} x$$

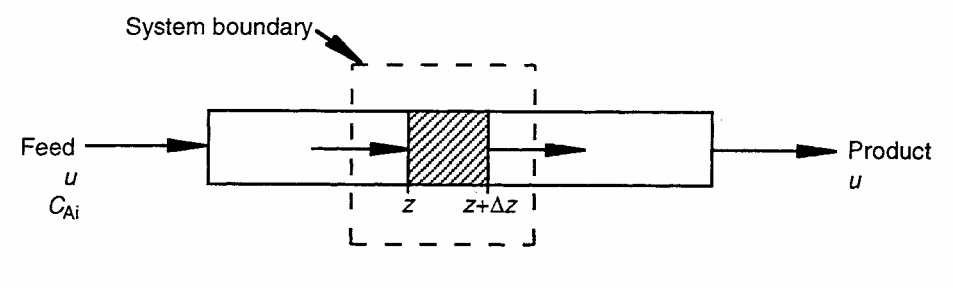
$$s = s_\mathrm{i} - \frac{Vk_2}{F} x$$

Answer: $s = s_\mathrm{i} - \dfrac{Vk_2}{F} x$

6.10 Plug-flow reactor

1. Flow sheet and system boundary

These are shown in the figure below.



2. Define variables

u = fluid linear velocity; z = distance along the reactor; C_A = concentration of reactant; C_Ai = concentration of reactant in the feed stream; A = reactor cross-sectional area

3. Assumptions

– no leaks

– plug flow

4. Mass balance

(a)

Reactant balance

Consider the system to be a small section of the reactor located between z and $z + \Delta z$. The general unsteady-state mass balance equation is Eq. (6.5). The rate of entry of reactant into the system is:

$$\hat{M}_\mathrm{i} = C_\mathrm{A} u A\big|_z$$

where $|_z$ means that the parameter values are those at distance z from the front of the reactor. Similarly, the rate at which reactant leaves the system is:

$$\hat{M}_{\mathrm{o}} = C_{\mathrm{A}} u\, A\big|_{z+\Delta z}$$

Reactant is not generated; therefore $R_{\mathrm{G}} = 0$. The rate of consumption of reactant is given by the equation:

$$R_{\mathrm{C}} = r_{\mathrm{C}} V = r_{\mathrm{C}} A \Delta z$$

where $A\Delta z$ is the volume of the system and r_{C} is the volumetric rate of reaction. At steady state, there is no accumulation in the system and $\mathrm{d}M/\mathrm{d}t = 0$. Substituting these terms into Eq. (6.5) gives:

$$0 = C_{\mathrm{A}} u\, A\big|_{z} - C_{\mathrm{A}} u\, A\big|_{z+\Delta z} - r_{\mathrm{C}}\, A \Delta z$$

As A is constant and does not depend on z, it can be cancelled from each of the terms:

$$0 = C_{\mathrm{A}} u\big|_{z} - C_{\mathrm{A}} u\big|_{z+\Delta z} - r_{\mathrm{C}}\, \Delta z$$

Dividing through by Δz:

$$0 = \frac{C_{\mathrm{A}} u\big|_{z} - C_{\mathrm{A}} u\big|_{z+\Delta z}}{\Delta z} - r_{\mathrm{C}}$$

Taking the limit as Δz approaches zero and applying the definition of the derivative (E.13) in Appendix E:

$$0 = \frac{-\mathrm{d}(C_{\mathrm{A}} u)}{\mathrm{d}z} - r_{\mathrm{C}}$$

or

$$\frac{\mathrm{d}(C_{\mathrm{A}} u)}{\mathrm{d}z} = -r_{\mathrm{C}}$$

As the fluid velocity is constant throughout the reactor, u can be taken outside of the differential:

$$u \frac{\mathrm{d}C_{\mathrm{A}}}{\mathrm{d}z} = -r_{\mathrm{C}}$$

Answer: $u \dfrac{\mathrm{d}C_{\mathrm{A}}}{\mathrm{d}z} = -r_{\mathrm{C}}$

(b)

As shown in the flow sheet, an appropriate boundary condition is $C_{\mathrm{A}} = C_{\mathrm{Ai}}$ at $z = 0$.

Answer: At $z = 0$, $C_{\mathrm{A}} = C_{\mathrm{Ai}}$

(c)

If the reaction is first-order, $r_{\mathrm{C}} = k_1 C_{\mathrm{A}}$ where k_1 is the first-order rate constant. The differential equation becomes:

$$u \frac{\mathrm{d}C_{\mathrm{A}}}{\mathrm{d}z} = -k_1 C_{\mathrm{A}}$$

As u and k_1 are constants, the differential equation contains only two variables, C_{A} and z. Separating variables and integrating:

$$\frac{\mathrm{d}C_{\mathrm{A}}}{C_{\mathrm{A}}} = \frac{-k_1}{u} \mathrm{d}z$$

$$\int \frac{\mathrm{d}C_{\mathrm{A}}}{C_{\mathrm{A}}} = \int \frac{-k_1}{u} \mathrm{d}z$$

Using integration rules (E.27) and (E.24) from Appendix E and combining the constants of integration:

$$\ln C_A = \frac{-k_1}{u} z + K$$

Applying the initial condition from **(b)** at $z = 0$, $\ln C_{Ai} = K$. Substituting this value of K into the equation:

$$\ln C_A = \frac{-k_1}{u} z + \ln C_{Ai}$$

$$\ln \frac{C_A}{C_{Ai}} = \frac{-k_1}{u} z$$

$$C_A = C_{Ai} e^{(-k_1/u)z}$$

Answer: $C_A = C_{Ai} e^{(-k_1/u)z}$

(d)

The equation derived in **(c)** is directly analogous to the equation for the reactant concentration in a batch reactor as a function of time. As $z = ut$ where t is the time taken for the fluid to travel distance z, the above equation can be written as:

$$C_A = C_{Ai} e^{-k_1 t}$$

which is the same as the equation for reactant concentration in a batch reactor where C_{Ai} is the concentration at time zero.

Answer: Essentially identical

6.11 Sequential batch reactors

The seed and production fermenters are operated as separate batch systems. The general unsteady-state mass balance equation for each fermenter is Eq. (6.5). For a batch culture, $\hat{M}_i = \hat{M}_o = 0$. For a mass balance on cells, assuming that there is no loss of cells from the system such as by lysis, $R_C = 0$. From the equation provided, the rate of generation of cells $R_G = r_X V = kxV$ where k is the rate constant, x is the cell concentration and V is the culture volume. The total mass of cells in the fermenter M is equal to the culture volume V multiplied by the cell concentration x: $M = Vx$. Substituting these terms into Eq. (6.5) gives:

$$\frac{\mathrm{d}(Vx)}{\mathrm{d}t} = kxV$$

Assuming that V is constant for each batch fermenter, it can be taken outside of the differential and cancelled:

$$V \frac{\mathrm{d}x}{\mathrm{d}t} = kxV$$

$$\frac{\mathrm{d}x}{\mathrm{d}t} = kx$$

The differential equation contains only two variables, x and t. Separating variables and integrating:

$$\frac{\mathrm{d}x}{x} = k\,\mathrm{d}t$$

$$\int \frac{\mathrm{d}x}{x} = \int k\,\mathrm{d}t$$

Using integration rules (E.27) and (E.24) from Appendix E and combining the constants of integration:

$$\ln x = kt + K$$

For each fermenter, the initial cell concentration $x = x_0$. Applying this initial condition at $t = 0$ gives $\ln x_0 = K$. Substituting this value of K into the equation:

$$\ln x = kt + \ln x_0$$

$$\ln \frac{x}{x_0} = kt$$

This equation applies to both the seed and production fermenters.

(a)

The above equation for cell growth can be rearranged to give an equation for t, the duration of the culture:

$$t = \frac{\ln \frac{x}{x_0}}{k}$$

k for the seed fermenter = 0.40 h^{-1}. The initial mass of cells inoculated into the seed vessel is 4.5 g l^{-1} × 3 l = 13.5 g. Therefore, the initial cell concentration x_0 = (13.5 g)/150 l = 0.09 g l^{-1}. The seed fermentation is completed when x = 6.5 g l^{-1}. Substituting values into the equation for t gives:

$$t = \frac{\ln \frac{6.5 \text{ g l}^{-1}}{0.09 \text{ g l}^{-1}}}{0.40 \text{ h}^{-1}} = 10.7 \text{ h}$$

Answer: 10.7 h

(b)

The production fermenter is inoculated with the contents of the seed fermenter. The mass of cells transferred from the seed fermenter is 6.5 g l^{-1} × 150 l = 975 g. Therefore, the initial cell concentration in the production fermenter x_0 = (975 g)/8000 l = 0.122 g l^{-1}. The production culture is carried out for 16 h with k = 0.28 h^{-1}. Substituting values into the equation for cell growth gives:

$$\ln \frac{x}{0.122 \text{ g l}^{-1}} = 0.28 \text{ h}^{-1}(16 \text{ h}) = 4.48$$

Solving for x:

$$\frac{x}{0.122 \text{ g l}^{-1}} = e^{4.48}$$

$$x = 10.76 \text{ g l}^{-1}$$

Answer: 10.8 g l^{-1}

6.12 Boiling water

1. System

The system is the beaker containing water.

2. Assumptions

– no evaporation

– water is well mixed

– no shaft work

– heat capacity is independent of temperature

– heat losses are negligible

– the density of water is constant between 18°C and 100°C

3. Extra data

Density of water = 1 g cm^{-3} (Section 2.4.1) = 1 kg l^{-1}

C_p water = 75.4 J $gmol^{-1}$ $°C^{-1}$ (Table C.3, Appendix C) = 75.4 kJ $kgmol^{-1}$ $°C^{-1}$

Molecular weight of water (Table C.1, Appendix C) = 18.0

1 W = 1 J s^{-1} (Table A.8, Appendix A); therefore, 1 kW = 1 kJ s^{-1}

4. Boundary conditions

At $t = 0$, $T = T_0 = 18°C$.

5. Energy balance

(a)

The general unsteady-state energy balance equation is Eq. (6.10). For a batch system, $\hat{M}_i = \hat{M}_o = 0$; also $\hat{W}_s = 0$. Energy is accumulated by the system in the form of sensible heat only; therefore:

$$\frac{dE}{dt} = MC_p \frac{dT}{dt}$$

where M is the mass of water in the beaker and T is its temperature. Substituting these terms into Eq. (6.10) gives:

$$MC_p \frac{dT}{dt} = -\hat{Q}$$

Answer: $MC_p \frac{dT}{dt} = -\hat{Q}$

(b)

If $\hat{Q}$, C_p and M are constant, T and t are the only variables in the differential equation derived in **(a)**. Separating variables and integrating:

$$dT = \frac{-\hat{Q}}{MC_p} dt$$

$$\int dT = \int \frac{-\hat{Q}}{MC_p} dt$$

Using integration rule (E.24) from Appendix E and combining the constants of integration:

$$T = \frac{-\hat{Q}}{MC_p} t + K$$

Applying the initial condition for T at $t = 0$, $T_0 = K$. Substituting this value of K into the equation:

$$T = \frac{-\hat{Q}}{MC_p} t + T_0$$

From the definition of density (Section 2.4.1), the mass of 2 litres of water M = 2 l × 1 kg l^{-1} = 2 kg. Converting the C_p for water to mass terms:

$$C_p = 75.4 \text{ kJ kgmol}^{-1}\,°\text{C}^{-1} . \left|\frac{1 \text{ kgmol}}{18.0 \text{ kg}}\right| = 4.189 \text{ kJ kg}^{-1}\,°\text{C}^{-1}$$

T = the boiling temperature of water = 100°C; t = 11 min. Substituting parameter values into the equation for T and solving for $\hat{Q}$:

$$100°\text{C} = \frac{-\hat{Q}}{2 \text{ kg } (4.189 \text{ kJ kg}^{-1}\,°\text{C}^{-1})}(11 \text{ min}) + 18°\text{C}$$

$$\hat{Q} = -62.45 \text{ kJ min}^{-1} = -62.45 \text{ kJ min}^{-1} . \left|\frac{1 \text{ min}}{60 \text{ s}}\right| . \left|\frac{1 \text{ kW}}{1 \text{ kJ s}^{-1}}\right| = -1.04 \text{ kW}$$

From the sign conventions outlined in Section 5.2, the negative value for $\hat{Q}$ confirms that heat is added to the system.

Answer: 1.04 kW

6.13 Heating glycerol solution

1. System

The system is the stirred tank containing the solution of glycerol in water.

2. Assumptions

– no leaks

– no evaporation

– tank is well mixed

– no shaft work

– heat capacities are independent of temperature between 15°C and 90°C

– ideal solution

– system is adiabatic; therefore heat losses are negligible

3. Extra data

C_p glycerol = 0.576 cal g^{-1} °C^{-1} (Table C.5, Appendix C) = 0.576 kcal kg^{-1} °C^{-1}

C_p water = 75.4 J $gmol^{-1}$ °C^{-1} (Table C.3, Appendix C) = 75.4 kJ $kgmol^{-1}$ °C^{-1}

Molecular weight of water (Table C.1, Appendix C) = 18.0

1 kcal = 4.187 × 10^3 J (Table A.7, Appendix A) = 4.187 kJ

1 W = 1 J s^{-1} (Table A.8, Appendix A); therefore, 1 kW = 1 kJ s^{-1}

4. Boundary conditions

At t = 0, $T = T_0$ = 15°C.

5. Energy balance

(a)

The general unsteady-state energy balance equation is Eq. (6.10). For a batch system, $\hat{M}_i = \hat{M}_o = 0$; also $\hat{W}_s = 0$. Energy is accumulated by the system in the form of sensible heat only; therefore:

$$\frac{dE}{dt} = MC_p \frac{dT}{dt}$$

where M is the mass of glycerol solution in the tank and T is its temperature. Substituting these terms into Eq. (6.10) gives:

$$MC_p \frac{dT}{dt} = -\hat{Q}$$

M has two components, glycerol and water, which have different heat capacities. Therefore, this equation can be written:

$$(M_W C_{pW} + M_G C_{pG}) \frac{dT}{dt} = -\hat{Q}$$

where M_W is the mass of water in the tank, M_G is the mass of glycerol, C_{pW} is the heat capacity of water, and C_{pG} is the heat capacity of glycerol.

Answer: $(M_W C_{pW} + M_G C_{pG}) \frac{dT}{dt} = -\hat{Q}$

(b)

If $\hat{Q}$, C_{pW}, C_{pG}, M_W and M_G are constant, T and t are the only variables in the differential equation derived in **(a)**. Separating variables and integrating:

$$dT = \frac{-\hat{Q}}{(M_W C_{pW} + M_G C_{pG})} dt$$

$$\int dT = \int \frac{-\hat{Q}}{(M_W C_{pW} + M_G C_{pG})} dt$$

Using integration rule (E.24) from Appendix E and combining the constants of integration:

$$T = \frac{-\hat{Q}}{(M_W C_{pW} + M_G C_{pG})} t + K$$

Applying the initial condition for T at $t = 0$, $T_0 = K$. Substituting this value of K into the equation:

$$T = \frac{-\hat{Q}}{(M_W C_{pW} + M_G C_{pG})} t + T_0$$

Answer: $T = \frac{-\hat{Q}}{(M_W C_{pW} + M_G C_{pG})} t + T_0$

(c)

The mass of glycerol in the tank $M_G = 45$ kg; the mass of water $M_W = 55$ kg. Converting C_p for water to mass terms:

$$C_{pW} = 75.4 \text{ kJ kgmol}^{-1}\,^{\circ}\text{C}^{-1} . \left| \frac{1 \text{ kgmol}}{18.0 \text{ kg}} \right| = 4.189 \text{ kJ kg}^{-1}\,^{\circ}\text{C}^{-1}$$

Converting the C_p for glycerol to kJ:

$$C_{pG} = 0.576 \text{ kcal kg}^{-1}\,^{\circ}\text{C}^{-1} . \left| \frac{4.187 \text{ kJ}}{1 \text{ kcal}} \right| = 2.412 \text{ kJ kg}^{-1}\,^{\circ}\text{C}^{-1}$$

The rate of heat input to the system is 0.88 × 2.5 kW= 2.2 kW = 2.2 kJ s^{-1}. From the sign conventions outlined in Section 5.2, $\hat{Q}$ must be negative as heat is added to the system; therefore, $\hat{Q}$ = –2.2 kJ s^{-1}. Substituting parameter values into the equation for T with T_0 = 15°C and T = 90°C:

$$90^{\circ}\text{C} = \frac{2.2 \text{ kJ s}^{-1}}{55 \text{ kg } (4.189 \text{ kJ kg}^{-1}\,^{\circ}\text{C}^{-1}) + 45 \text{ kg } (2.412 \text{ kJ kg}^{-1}\,^{\circ}\text{C}^{-1})} t + 15^{\circ}\text{C}$$

Calculating and solving for t:

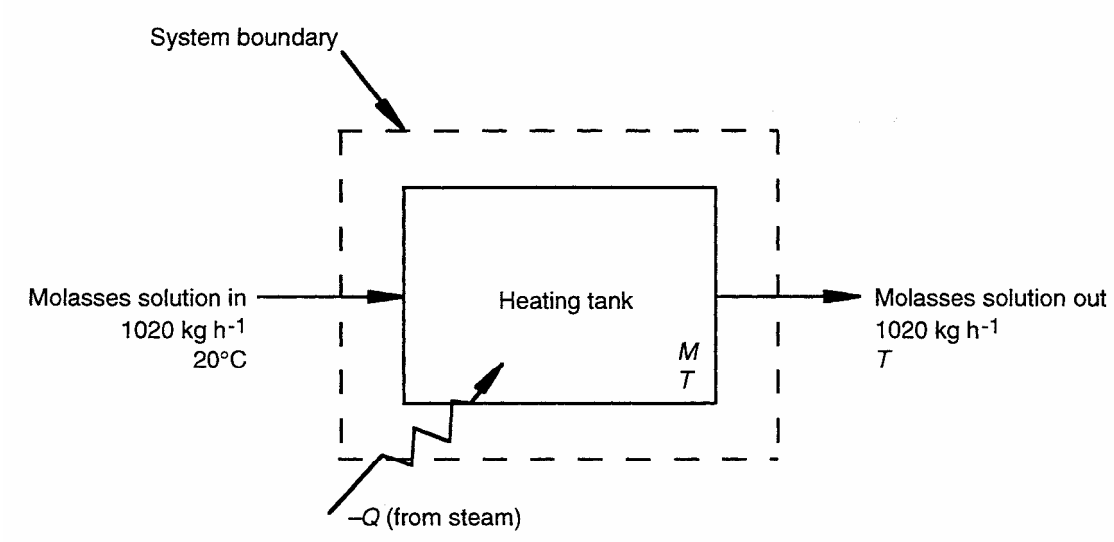
$$t = 1.16 \times 10^4 \text{ s} = 1.16 \times 10^4 \text{ s} . \left| \frac{1 \text{ h}}{3600 \text{ s}} \right| = 3.2 \text{ h}$$

Answer: 3.2 h

6.14 Heating molasses

1. Flow sheet and system boundary

These are shown in the figure below.



2. Assumptions

– no leaks

– no evaporation

– tank is well mixed; therefore the temperature of the molasses solution out is the same as that in the tank

– no shaft work

– heat capacity is independent of temperature

– negligible heat losses

– condensate from the steam leaves at saturation conditions

3. Reference state

T_{ref} = 20°C; H = 0 for molasses solution at 20°C

4. Extra data

1 psi = 6.895×10^3 Pa (Table A.5, Appendix A) = 6.895 kPa

Converting 40 psi to kPa:

$$40 \text{ psi} = 40 \text{ psi} \,.\left|\frac{6.895 \text{ kPa}}{1 \text{ psi}}\right| = 275.8 \text{ kPa}$$

The temperature of saturated steam at 275.8 kPa interpolated from Table D.2 (Appendix D) = 130.7°C.

5. Boundary conditions

At $t = 0$, $T = T_0$ = 20°C.

6. Mass balance

Total mass balance

As the mass flow rates into and out of the tank are the same, the mass of molasses solution in the tank M is constant and equal to 5000 kg at all times.

7. Energy balance

(a)

The general unsteady-state energy balance equation is Eq. (6.10). From the reference state, $h_i = 0$. The value of h_0 relative to the reference state is equal to the sensible heat absorbed by the molasses solution between T_{ref} and the exit temperature T. From Eq. (5.13):

$$h_o = \Delta h = C_p (T - T_{ref})$$

$\hat{W}_s = 0$. From the sign conventions outlined in Section 5.2, $\hat{Q}$ must be negative as heat is added to the system. Therefore, the rate at which the molasses solution is heated is:

$$\hat{Q} = -UA(T_{steam} - T)$$

Energy is accumulated in the form of sensible heat only; therefore:

$$\frac{dE}{dt} = MC_p \frac{dT}{dt}$$

Substituting these expressions into Eq. (6.10) gives:

$$MC_p \frac{dT}{dt} = -\hat{M}_o C_p (T - T_{ref}) + UA(T_{steam} - T)$$

After rearranging, the differential equation is:

$$\frac{dT}{dt} = \frac{UAT_{steam} + \hat{M}_o C_p T_{ref}}{MC_p} - \left(\frac{\hat{M}_o C_p + UA}{MC_p}\right) T$$

Answer: $\dfrac{\mathrm{d}T}{\mathrm{d}t} = \dfrac{UAT_{\text{steam}} + \hat{M}_{\text{o}} C_p T_{\text{ref}}}{MC_p} - \left(\dfrac{\hat{M}_{\text{o}} C_p + UA}{MC_p} \right) T$

(b)

As U, A, T_{steam}, $\hat{M}_{\text{o}}$, C_p, T_{ref} and M are all constant, T and t are the only variables in the differential equation derived in **(a)**. Substituting parameter values:

$$\frac{\mathrm{d}T}{\mathrm{d}t} = \frac{190 \text{ kcal m}^{-2} \text{ h}^{-1} \text{ °C}^{-1} (1.5 \text{ m}^2)(130.7\text{°C}) + 1020 \text{ kg h}^{-1} (0.85 \text{ kcal kg}^{-1} \text{ °C}^{-1})(20\text{°C})}{5000 \text{ kg } (0.85 \text{ kcal kg}^{-1} \text{ °C}^{-1})} - \left(\frac{1020 \text{ kg h}^{-1} (0.85 \text{ kcal kg}^{-1} \text{ °C}^{-1}) + 190 \text{ kcal m}^{-2} \text{ h}^{-1} \text{ °C}^{-1} (1.5 \text{ m}^2)}{5000 \text{ kg } (0.85 \text{ kcal kg}^{-1} \text{ °C}^{-1})} \right) T$$

$$\frac{\mathrm{d}T}{\mathrm{d}t} = 12.84 - 0.271T$$

where T has units of °C and t has units of h. Separating variables and integrating:

$$\frac{\mathrm{d}T}{12.84 - 0.271T} = \mathrm{d}t$$

$$\int \frac{\mathrm{d}T}{12.84 - 0.271T} = \int \mathrm{d}t$$

Using integration rules (E.28) and (E.24) from Appendix E and combining the constants of integration:

$$\frac{-1}{0.271} \ln (12.84 - 0.271T) = t + K$$

Applying the initial condition for T at $t = 0$:

$$K = \frac{-1}{0.271} \ln (12.84 - 0.271T_0)$$

As $T_0 = 20$°C, $K = -7.395$. Substituting this value for K into the equation:

$$\frac{-1}{0.271} \ln (12.84 - 0.271T) + 7.395 = t$$

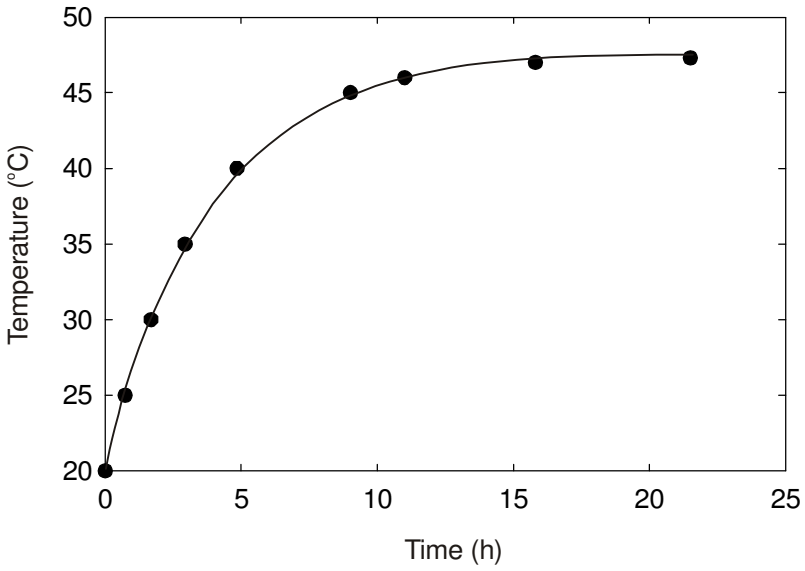
Answer: $\dfrac{-1}{0.271} \ln (12.84 - 0.271T) + 7.395 = t$

(c)

Values of t corresponding to various temperatures in the tank can be calculated from the equation derived in **(b)**.

Temperature, T (ºC)	Time, t (h)
20	0.00
25	0.74
30	1.68
35	2.93
40	4.84
45	9.01
46	11.0
47	15.8
47.3	21.5

These results are plotted in the figure below.



(d)

From the equation derived in **(b)**, as the logarithm of zero and negative numbers is not defined (Section E.1, Appendix E), the theoretical maximum temperature that can be achieved in the tank occurs when 12.84 just equals 0.271T, i.e. when T = 47.4°C.

Answer: 47.4°C

(e)

The temperature changes constantly with time; therefore, strictly speaking, there is no steady state. For practical purposes, however, from the figure in **(c)**, the temperature approaches a constant value after about 16 h.

Answer: About 16 h

(f)

From the calculation table in **(c)**, the temperature reaches 40°C after 4.84 h.

Answer: 4.84 h

6.15 Preheating culture medium

1. System

The system is the glass fermenter containing nutrient medium.

2. Assumptions

– no evaporation

– fermenter is well mixed

– no shaft work

– heat capacities are independent of temperature between 15°C and 36°C

– heat losses are negligible

3. Extra data

C_p glass vessel = 0.20 cal g^{-1} $°C^{-1}$ = 0.20 kcal kg^{-1} $°C^{-1}$

C_p medium = 0.92 cal g^{-1} $°C^{-1}$ = 0.92 kcal kg^{-1} $°C^{-1}$

1 W = 1.433×10^{-2} kcal min^{-1} (Table A.8, Appendix A)

4. Boundary conditions

At $t = 0$, $T = T_0 = 15°C$.

5. Energy balance

The general unsteady-state energy balance equation is Eq. (6.10). For a batch system, $\hat{M}_i = \hat{M}_o = 0$; also $\hat{W}_s = 0$. Energy is accumulated by the system in the form of sensible heat only; therefore:

$$\frac{dE}{dt} = MC_p \frac{dT}{dt}$$

M has two components, the glass vessel and the medium, which have different heat capacities. Therefore, this equation can be written as:

$$\frac{dE}{dt} = (M_V C_{pV} + M_M C_{pM}) \frac{dT}{dt}$$

where M_V is the mass of the glass vessel, M_M is the mass of the medium, C_{pV} is the heat capacity of the glass vessel, and C_{pM} is the heat capacity of the medium. Substituting terms into Eq. (6.10) gives:

$$(M_V C_{pV} + M_M C_{pM}) \frac{dT}{dt} = -\hat{Q}$$

As $\hat{Q}$, C_{pV}, C_{pM}, M_V and M_M are constant, T and t are the only variables in the differential equation. Separating variables and integrating:

$$dT = \frac{-\hat{Q}}{(M_V C_{pV} + M_M C_{pM})} dt$$

$$\int \mathrm{d}T = \int \frac{-\hat{Q}}{(M_{\mathrm{V}}C_{p\mathrm{V}} + M_{\mathrm{M}}C_{p\mathrm{M}})}\mathrm{d}t$$

Using integration rule (E.24) from Appendix E and combining the constants of integration:

$$T = \frac{-\hat{Q}}{(M_{\mathrm{V}}C_{p\mathrm{V}} + M_{\mathrm{M}}C_{p\mathrm{M}})}t + K$$

Applying the initial condition for T at $t = 0$, $T_0 = K$. Substituting this value of K into the equation:

$$T = \frac{-\hat{Q}}{(M_{\mathrm{V}}C_{p\mathrm{V}} + M_{\mathrm{M}}C_{p\mathrm{M}})}t + T_0$$

The rate of heat input to the system is 450 W. From the sign conventions outlined in Section 5.2, $\hat{Q}$ must be negative as heat is added to the system; therefore, $\hat{Q} = -450$ W. Converting this to kcal min^{-1}:

$$\hat{Q} = -450\ \mathrm{W} = -450\ \mathrm{W}.\left|\frac{1.433\times10^{-2}\ \mathrm{kcal\ min^{-1}}}{1\ \mathrm{W}}\right| = -6.45\ \mathrm{kcal\ min^{-1}}$$

Substituting parameter values into the equation for T with $T_0 = 15$°C and $T = 36$°C:

$$36°\mathrm{C} = \frac{6.45\ \mathrm{kcal\ min^{-1}}}{12.75\ \mathrm{kg}\ (0.20\ \mathrm{kcal\ kg^{-1}\ °C^{-1}}) + 7.5\ \mathrm{kg}\ (0.92\ \mathrm{kcal\ kg^{-1}\ °C^{-1}})}t + 15°\mathrm{C}$$

Calculating and solving for t gives:

$t = 30.8$ min

Answer: 30.8 min

6.16 Water heater

1. System

The system is the tank containing the water.

2. Assumptions

– no leaks

– no evaporation

– tank is well mixed

– no shaft work

– heat capacity is independent of temperature

– condensate from the steam leaves at saturation conditions

3. Extra data

C_p water = 75.4 J gmol^{-1} °C^{-1} (Table C.3, Appendix C) = 75.4 kJ kgmol^{-1} °C^{-1}

Molecular weight of water (Table C.1, Appendix C) = 18.0

1 kcal = 4.187×10^3 J (Table A.7, Appendix A) = 4.187 kJ

4. Boundary conditions

At $t = 0$, $T = T_0 = 24°C$.

5. Energy balance

The general unsteady-state energy balance equation is Eq. (6.10). For a batch system, $\hat{M}_i = \hat{M}_o = 0$; also $\hat{W}_s = 0$. Energy is accumulated by the system in the form of sensible heat only; therefore:

$$\frac{dE}{dt} = MC_p \frac{dT}{dt}$$

where M is the mass of water in the tank and T is its temperature. Substituting these terms into Eq. (6.10) gives:

$$MC_p \frac{dT}{dt} = -\hat{Q}$$

(a)

There are two components to $\hat{Q}$: the rate of heating from the steam, and the rate of heat loss to the surrounding air:

$$\hat{Q} = U_1 A_1 (T - T_{air}) - U_2 A_2 (T_{steam} - T)$$

This equation reflects the sign conventions outlined in Section 5.2: the term for the heat loss to the atmosphere is positive to indicate heat removal from the system, while the term for heat input from the steam is negative. Substituting into Eq. (6.10) gives:

$$MC_p \frac{dT}{dt} = U_2 A_2 (T_{steam} - T) - U_1 A_1 (T - T_{air})$$

$$\frac{dT}{dt} = \frac{U_2 A_2 (T_{steam} - T) - U_1 A_1 (T - T_{air})}{MC_p}$$

As U_1, A_1, U_2, A_2, T_{steam}, T_{air}, C_p and M are all constant, T and t are the only variables in the differential equation. Substituting parameter values gives:

$$\frac{dT}{dt} = \frac{220 \text{ kcal m}^{-2} \text{ h}^{-1} \text{ °C}^{-1} (0.3 \text{ m}^2)(130 - T)\text{°C} - 25 \text{ kcal m}^{-2} \text{ h}^{-1} \text{ °C}^{-1} (0.9 \text{ m}^2)(T - 20)\text{°C}}{1000 \text{ kg} \left(75.4 \text{ kJ kgmol}^{-1} \text{ °C}^{-1} . \left| \frac{1 \text{ kgmol}}{18.0 \text{ kg}} \right| . \left| \frac{1 \text{ kcal}}{4.187 \text{ kJ}} \right| \right)}$$

$$\frac{dT}{dt} = 9.026 - 0.088T$$

where T has units of °C and t has units of h. Separating variables and integrating:

$$\frac{dT}{9.026 - 0.088T} = dt$$

$$\int \frac{dT}{9.026 - 0.088T} = \int dt$$

Using integration rules (E.28) and (E.24) from Appendix E and combining the constants of integration:

$$\frac{-1}{0.088} \ln (9.026 - 0.088T) = t + K$$

Applying the initial condition for T at $t = 0$:

$$K = \frac{-1}{0.088}\ln(9.026 - 0.088T_0)$$

As $T_0 = 24°C$, $K = -21.97$. Substituting this value for K into the equation:

$$\frac{-1}{0.088}\ln(9.026 - 0.088T) + 21.97 = t$$

From this equation, when $T = 80°C$, $t = 14.2$ h.

Answer: 14.2 h

(b)

If heat losses can be neglected, $\hat{Q}$ has only one component and Eq. (6.10) becomes:

$$MC_p\frac{dT}{dt} = U_2A_2(T_{steam} - T)$$

$$\frac{dT}{dt} = \frac{U_2A_2(T_{steam} - T)}{MC_p}$$

Substituting parameter values:

$$\frac{dT}{dt} = \frac{220\ \text{kcal m}^{-2}\ \text{h}^{-1}\ °\text{C}^{-1}\ (0.3\ \text{m}^2)(130 - T)°\text{C}}{1000\ \text{kg}\left(75.4\ \text{kJ kgmol}^{-1}\ °\text{C}^{-1}\cdot\left|\frac{1\ \text{kgmol}}{18.0\ \text{kg}}\right|\cdot\left|\frac{1\ \text{kcal}}{4.187\ \text{kJ}}\right|\right)}$$

$$\frac{dT}{dt} = 8.576 - 0.066T$$

where T has units of °C and t has units of h. Separating variables and integrating:

$$\frac{dT}{8.576 - 0.066T} = dt$$

$$\int\frac{dT}{8.576 - 0.066T} = \int dt$$

Using integration rules (E.28) and (E.24) from Appendix E and combining the constants of integration:

$$\frac{-1}{0.066}\ln(8.576 - 0.066T) = t + K$$

Applying the initial condition for T at $t = 0$:

$$K = \frac{-1}{0.066}\ln(8.576 - 0.066T_0)$$

As $T_0 = 24°C$, $K = -29.47$. Substituting this value for K into the equation:

$$\frac{-1}{0.066}\ln(8.576 - 0.066T) + 29.47 = t$$

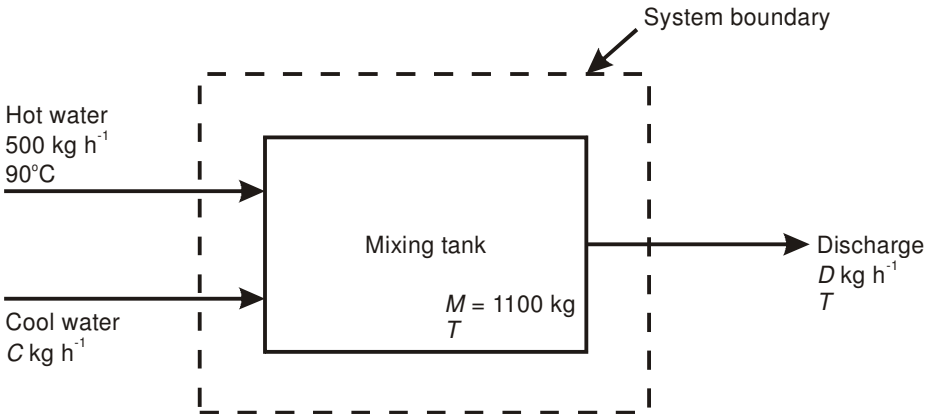
From this equation, when $T = 80°C$, $t = 11.4$ h. Therefore, using the result from **(a)**, the time saved is (14.2 − 11.4) h = 2.8 h, which corresponds to 20% of the time required when heat losses occur.

Answer: 2.8 h, or 20% of the time required when heat losses occur

6.17 Thermal mixing

1. Flow sheet and system boundary

These are shown in the figure below.



2. Assumptions

– no leaks

– no evaporation

– no heat loss to the environment

– no shaft work

– tank is well mixed; therefore the temperature of the discharge stream is the same as that in the tank

3. Define variables

C = mass flow rate of cool water into the tank; D = mass flow rate of discharge out of the tank; M = mass of water in the tank = 1100 kg; T = temperature in the tank

4. Reference state

$H = 0$ for water at its triple point

5. Extra data

From the steam tables (Table D.1, Appendix D) for liquid water:

$h(90°\text{C}) = 376.9 \text{ kJ kg}^{-1}$

$h(40°\text{C}) = 167.5 \text{ kJ kg}^{-1}$

$h(30°\text{C}) = 125.7 \text{ kJ kg}^{-1}$

$h(18°\text{C}) = 75.5 \text{ kJ kg}^{-1}$

6. Boundary conditions

At $t = 0$, $T = T_0 = 30°\text{C}$; therefore, at $t = 0$, $h = h_0 = 125.7 \text{ kJ kg}^{-1}$.

7. Mass balance

The system mass and mass flow conditions remain at steady state even though the temperature changes. The total mass balance at steady state is:

$$(500 + C)\ \text{kg h}^{-1}\ \text{total mass in} = D\ \text{kg h}^{-1}\ \text{total mass out}$$

$$500 + C = D \tag{1}$$

8. Energy balance

First, let us perform a steady-state energy balance representing the conditions in the system before the temperature of the cool water is changed. At steady state, Eq. (5.9) applies. As $Q = W_s = 0$, this equation becomes:

$$(Mh)_{HW} + (Mh)_{CW} - (Mh)_D = 0$$

where subscripts HW = hot water, CW = cool water and D = discharge. Substituting values gives:

$$500\ \text{kg h}^{-1}(376.9\ \text{kJ kg}^{-1}) + C\ \text{kg h}^{-1}(75.5\ \text{kJ kg}^{-1}) - D\ \text{kg h}^{-1}(125.7\ \text{kJ kg}^{-1}) = 0$$

Using the result from (1):

$$188{,}450 + 75.5C - (500 + C)(125.7) = 0$$

where C has units of kg h^{-1}. Solving for C:

$$125{,}600 = 50.2C$$

$$C = 2501.99\ \text{kg h}^{-1}$$

Applying this result in (1):

$$D = 3001.99\ \text{kg h}^{-1}$$

We can now perform an unsteady-state energy balance with the temperature of the cool water at 40°C to derive an equation for the rate of enthalpy change in the mixing tank. The general unsteady-state energy balance equation is Eq. (6.10). As $\hat{Q} = \hat{W}_s = 0$, this equation becomes:

$$\frac{\mathrm{d}(Mh)}{\mathrm{d}t} = \hat{M}_{HW} h_{HW} + C h_{CW} - D h$$

where h is the specific enthalpy of the water in the tank, which is the same as the specific enthalpy of the discharge stream. As the mass of water in the tank M remains constant, it can be taken out of the differential:

$$M\frac{\mathrm{d}h}{\mathrm{d}t} = \hat{M}_{HW} h_{HW} + C h_{CW} - D h$$

Rearranging gives:

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{\hat{M}_{HW} h_{HW}}{M} + \frac{C h_{CW}}{M} - \frac{D h}{M}$$

Substituting values:

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{500\ \text{kg h}^{-1}(376.9\ \text{kJ kg}^{-1})}{1100\ \text{kg}} + \frac{2501.99\ \text{kg h}^{-1}(167.5\ \text{kJ kg}^{-1})}{1100\ \text{kg}} - \frac{3001.99\ \text{kg h}^{-1}(h\ \text{kJ kg}^{-1})}{1100\ \text{kg}}$$

$$\frac{\mathrm{d}h}{\mathrm{d}t} = 552.3 - 2.729h$$

where h has units of kJ kg^{-1} and t has units of h. Separating variables and integrating:

$$\frac{\mathrm{d}h}{552.3 - 2.729h} = \mathrm{d}t$$

$$\int \frac{\mathrm{d}h}{552.3 - 2.729h} = \int \mathrm{d}t$$

Using integration rules (E.28) and (E.24) from Appendix E and combining the constants of integration:

$$\frac{-1}{2.729}\ln(552.3 - 2.729h) = t + K$$

Applying the initial condition for h at $t = 0$:

$$K = \frac{-1}{2.729}\ln(552.3 - 2.729h_0)$$

As $h_0 = 125.7$ kJ kg^{-1}, $K = -1.958$. Substituting this value for K into the equation:

$$\frac{-1}{2.729}\ln(552.3 - 2.729h) = t - 1.958$$

$$t = 1.958 - \frac{1}{2.729}\ln(552.3 - 2.729h)$$

The time required for the discharge stream to reach 40°C is the same as the time required for h to reach 167.5 kJ kg^{-1}. From the equation, when $h = 167.5$ kJ kg^{-1}, $t = 0.289$ h.

Answer: 0.29 h

6.18 Laboratory heating

1. System

The system is the one-room mobile laboratory.

2. Assumptions

– no leaks

– air in the room is well mixed

– heat capacity is independent of temperature

3. Boundary conditions

At $t = 0$, $T = T_0 = 5$°C.

4. Extra data

1 W = 1 J s^{-1} (Table A.8, Appendix A)

5. Energy balance

The general unsteady-state energy balance equation is Eq. (6.10). As there is no mass flow into or out of the system, $\hat{M}_\mathrm{i} = \hat{M}_\mathrm{o} = 0$; also $\hat{W}_\mathrm{s} = 0$. Energy is accumulated by the system in the form of sensible heat only; therefore:

$$\frac{\mathrm{d}E}{\mathrm{d}t} = MC_p \frac{\mathrm{d}T}{\mathrm{d}t}$$

where MC_p represents the heat capacity of the laboratory and T is its temperature. Substituting into Eq. (6.10) gives:

$$MC_p \frac{\mathrm{d}T}{\mathrm{d}t} = -\hat{Q}$$

$\hat{Q}$ has two components: the heat input to the system by the furnace $\hat{Q}_{\text{in}}$ and the heat lost to the surroundings $\hat{Q}_{\text{loss}}$. We can write the energy balance equation as:

$$MC_p \frac{\mathrm{d}T}{\mathrm{d}t} = -\hat{Q}_{\text{in}} - \hat{Q}_{\text{loss}}$$

According to the sign conventions outlined in Section 5.2, Q is negative when the system receives heat from the surroundings; therefore $\hat{Q}_{\text{in}}$ takes a negative value, $\hat{Q}_{\text{in}} = -2400$ W $= -2400$ J s^{-1}. Conversely, Q is positive when the system loses heat to the surroundings; therefore $\hat{Q}_{\text{loss}}$ takes a positive value. Substituting parameter values into the energy balance equation:

$$MC_p \frac{\mathrm{d}T}{\mathrm{d}t} = 2400 \text{ J s}^{-1} - 420 \text{ kJ °C}^{-1} \text{ h}^{-1} (T - 5)\text{°C} \left| \frac{1 \text{ h}}{3600 \text{ s}} \right| \left| \frac{1000 \text{ J}}{1 \text{ kJ}} \right|$$

Calculating gives:

$$\begin{aligned} MC_p \frac{\mathrm{d}T}{\mathrm{d}t} &= 2400 - 116.67\,(T - 5) \\ &= 2983.33 - 116.67T \end{aligned}$$

where T has units of °C, t has units of s and MC_p has units of J °C^{-1}. As MC_p is constant, this equation has only two variables, T and t. Separating variables and integrating:

$$\frac{\mathrm{d}T}{2983.33 - 116.67T} = \frac{\mathrm{d}t}{MC_p}$$

$$\int \frac{\mathrm{d}T}{2983.33 - 116.67T} = \int \frac{\mathrm{d}t}{MC_p}$$

Using integration rules (E.28) and (E.24) from Appendix E and combining the constants of integration:

$$\frac{-1}{116.67} \ln (2983.33 - 116.67T) = \frac{t}{MC_p} + K$$

Applying the initial condition for T at $t = 0$:

$$K = \frac{-1}{116.67} \ln (2983.33 - 116.67T_0)$$

As $T_0 = 5$°C, $K = -0.0667$. Substituting this value for K into the equation:

$$\frac{-1}{116.67} \ln (2983.33 - 116.67T) = \frac{t}{MC_p} - 0.0667$$

Rearranging to obtain an equation for MC_p:

$$MC_p = \frac{t}{0.0667 - \dfrac{1}{116.67} \ln (2983.33 - 116.67T)}$$

At t = 30 min = 1800 s, T = 20°C. Using this information, MC_p is evaluated as:

$$MC_p = \frac{1800}{0.0667 - \frac{1}{116.67}\ln(2983.33 - 116.67 \times 20)} = 160,921$$

Therefore, MC_p is 160,921 J °C^{-1} = 160.9 kJ °C^{-1}. This means that the energy required to raise the temperature of the laboratory by 1°C = 160.9 kJ.

Answer: 161 kJ

Chapter 7

Fluid Flow

7.1 Conditions for turbulence

(a)

To determine whether flow in the hose is laminar or turbulent, the Reynolds number for pipe flow is calculated using Eq. (7.1). $D = 15 \times 10^{-3}$ m, $\rho = 1000$ kg m^{-3}, $\mu = 1.0$ cP. Converting the units for viscosity (Table A.9, Appendix A), $\mu = 10^{-3}$ kg m^{-1} s^{-1}. The volumetric flow rate F is:

$$F = \frac{5\,\text{l}}{75\,\text{s}} \cdot \left| \frac{1\,\text{m}^3}{1000\,\text{l}} \right| = 6.67 \times 10^{-5}\ \text{m}^3\ \text{s}^{-1}$$

The linear velocity u of water in the hose is equal to the volumetric flow rate F divided by the cross-sectional area of the hose, πR^2, where R is the hose inner radius:

$$u = \frac{6.67 \times 10^{-5}\ \text{m}^3\ \text{s}^{-1}}{\pi \left(\frac{15 \times 10^{-3}\ \text{m}}{2} \right)^2} = 0.377\ \text{m s}^{-1}$$

Substituting values into Eq. (7.1) gives:

$$Re = \frac{15 \times 10^{-3}\ \text{m}\ (0.377\ \text{m s}^{-1})(1000\ \text{kg m}^{-3})}{10^{-3}\ \text{kg m}^{-1}\ \text{s}^{-1}} = 5655$$

As this value is greater than 4000, the critical Reynolds number for pipe flow (Section 7.2.3), flow in the hose is turbulent.

Answer: Turbulent

(b)

To determine whether flow in the saucepan is laminar or turbulent, the impeller Reynolds number for stirred vessels is calculated using Eq. (7.2). $N_i = 1.5$ s^{-1}, $D_i = 5 \times 10^{-2}$ m, $\rho = 970$ kg m^{-3}, $\mu = 0.45$ cP. Converting the units for viscosity (Table A.9, Appendix A), $\mu = 0.45 \times 10^{-3}$ kg m^{-1} s^{-1}. Substituting these values into Eq. (7.2) gives:

$$Re_i = \frac{1.5\,\text{s}^{-1}\ (5 \times 10^{-2}\ \text{m})^2\ (970\ \text{kg m}^{-3})}{0.45 \times 10^{-3}\ \text{kg m}^{-1}\ \text{s}^{-1}} = 8083$$

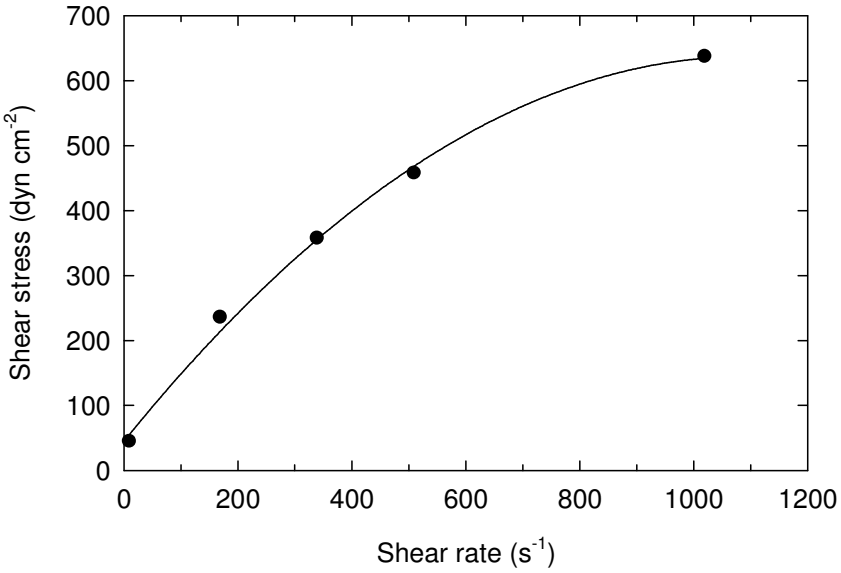
As this impeller Reynolds number is less than the typical transition value for many impellers of approximately 10^4 for turbulent flow (Section 7.2.3), flow in the saucepan is not likely to be turbulent.

Answer: No

7.2 Rheology of fermentation broth

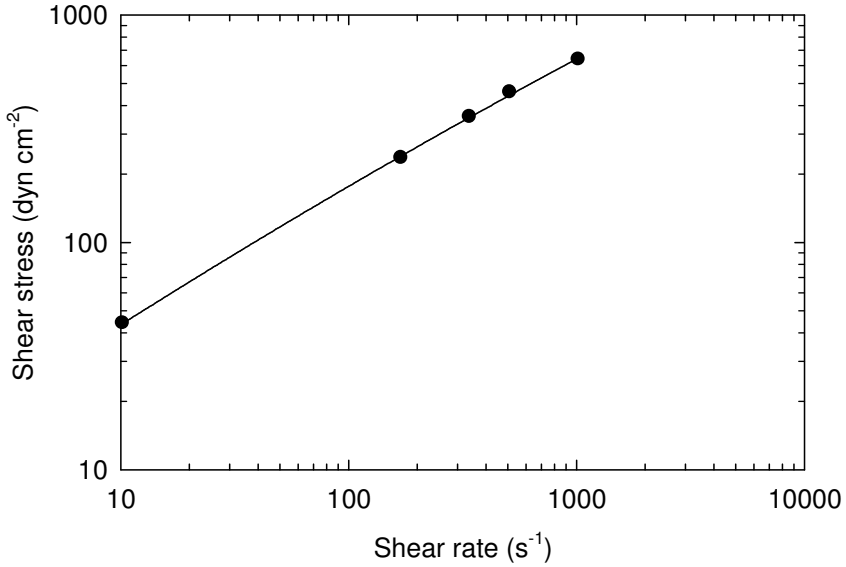
(a)

The rheogram is obtained by plotting shear stress τ against shear rate $\dot{\gamma}$.



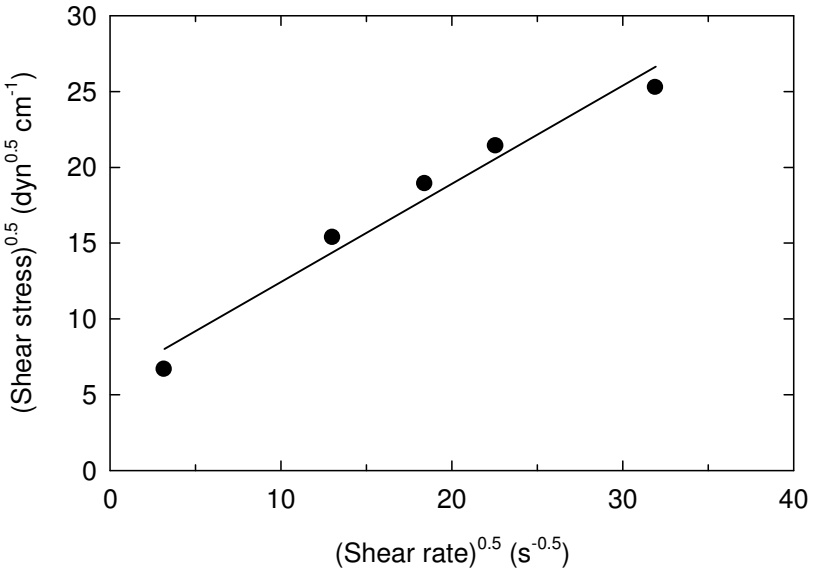
(b)

The shape of the rheogram in **(a)** is similar to those for pseudoplastic and Casson plastic fluids in Figure 7.8. As the fluid may exhibit a yield stress at $\dot{\gamma} = 0$, both these non-Newtonian models are worth checking. From the equation in Figure 7.8 for pseudoplastic fluids, a plot of shear stress versus shear rate on log–log coordinates gives a straight line if the fluid is pseudoplastic. This plot is shown below.



The equation for the straight line in the log–log plot is $\tau = 11.43\,\dot{\gamma}^{0.587}$. Therefore, the flow behaviour index $n = 0.587$ and the consistency index $K = 11.43$ dyn s^n cm^{-2}. The sum of squares of the residuals for this data fit is 1137.

From the equation in Figure 7.8 for Casson plastic fluids, a plot of the square root of shear stress versus the square root of shear rate on linear coordinates gives a straight line if the fluid is a Casson plastic. This plot is shown below.



The equation for the straight line in the plot is $\tau^{1/2} = 5.93 + 0.648\,\dot{\gamma}^{1/2}$. From the equation in Figure 7.8 for Casson plastic fluids, this means that K_p = 0.648 dyn$^{1/2}$ s$^{1/2}$ cm^{-1} and the yield stress τ_0 = 35.2 dyn cm^{-2}. The sum of squares of the residuals for this data fit is 8986.

Comparison of the residuals from the two models suggests that the equation for a pseudoplastic fluid is the better fit.

Answer: Pseudoplastic fluid: n = 0.587; K = 11.43 dyn s^n cm^{-2}

(c)

The apparent viscosity for a pseudoplastic fluid is given by Eq. (7.12) and can be calculated using the parameter values determined in **(b)**.

(i)

$$\mu_a = K\dot{\gamma}^{n-1} = 11.43\ \text{dyn s}^{0.587}\ \text{cm}^{-2}\ (15\ \text{s}^{-1})^{0.587-1} = 3.7\ \text{dyn s cm}^{-2}$$

Answer: 3.7 dyn s cm^{-2}

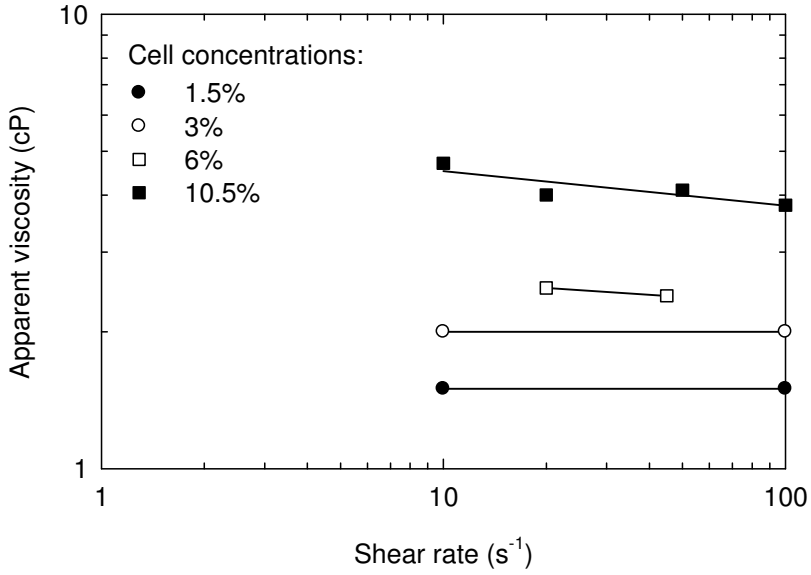
(ii)

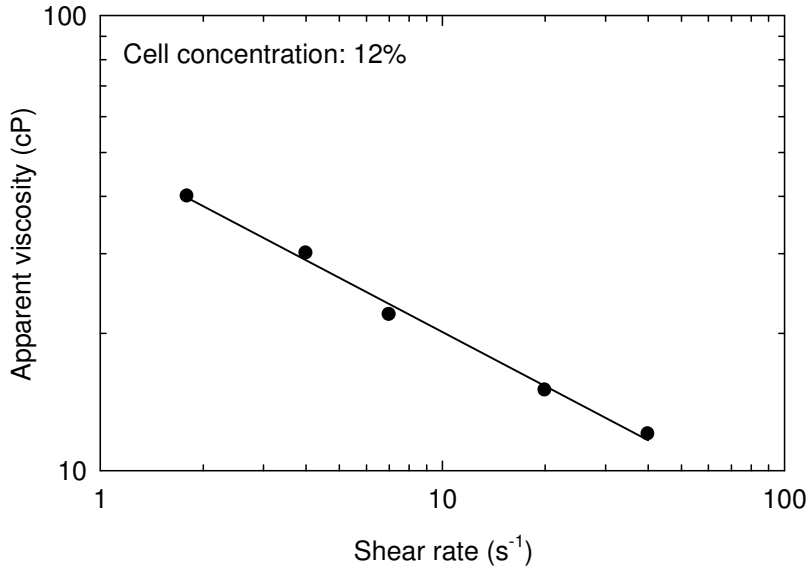
$$\mu_a = K\dot{\gamma}^{n-1} = 11.43\ \text{dyn s}^{0.587}\ \text{cm}^{-2}\ (200\ \text{s}^{-1})^{0.587-1} = 1.3\ \text{dyn s cm}^{-2}$$

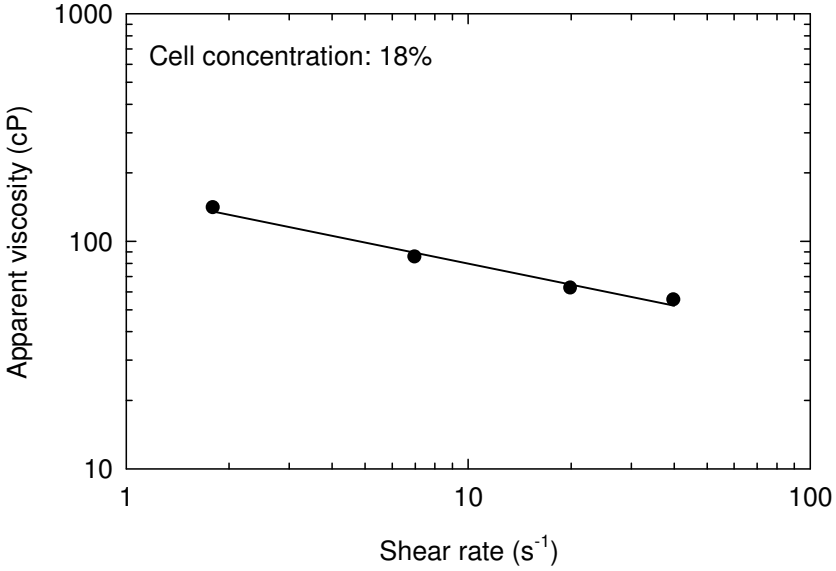
Answer: 1.3 dyn s cm^{-2}

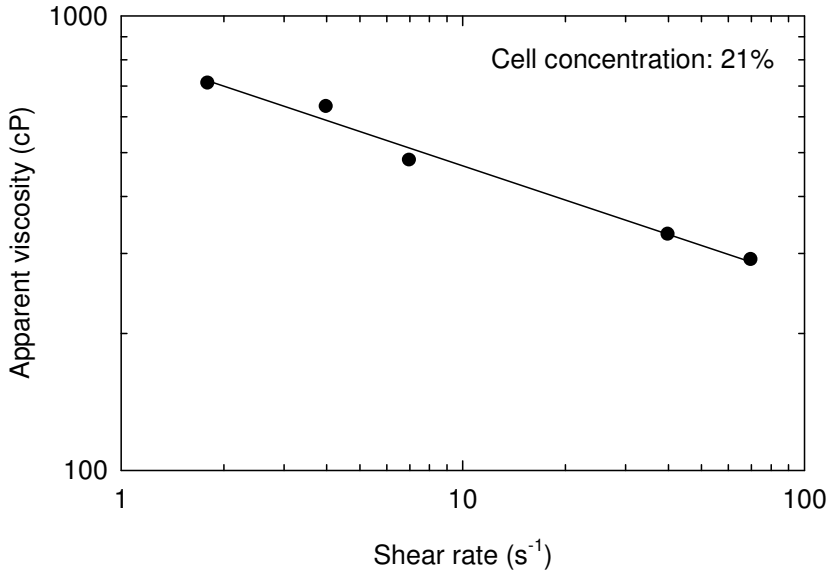
7.3 Rheology of yeast suspensions

From Eq. (7.12), for pseudoplastic fluids, a plot of apparent viscosity versus shear rate on log–log coordinates can be expected to give a straight line. Log–log plots for the different cell concentrations are shown below.


10
1
Cell concentrations:
1.5%
3%
6%
10.5%
Apparent viscosity (cP)
1
10
100
Shear rate (s-1)


100
10
Cell concentration: 12%
Apparent viscosity (cP)
1
10
100
Shear rate (s-1)

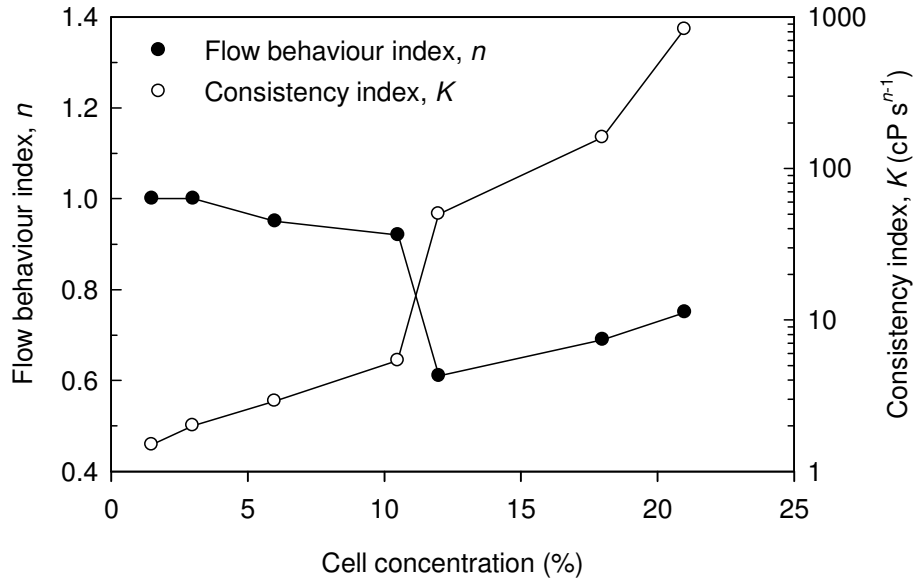




The equations and parameter values for the straight lines in each plot are listed below.

Cell concentration (%)	Equation	Flow behaviour index, n	Consistency index, K (cP s^{n-1})
1.5	$\mu_a = 1.5\dot{\gamma}^0 = 1.5$	1	1.5
3	$\mu_a = 2.0\dot{\gamma}^0 = 2.0$	1	2.0
6	$\mu_a = 2.91\dot{\gamma}^{-0.050}$	0.95	2.9
10.5	$\mu_a = 5.38\dot{\gamma}^{-0.076}$	0.92	5.4
12	$\mu_a = 50.1\dot{\gamma}^{-0.395}$	0.61	50
18	$\mu_a = 162\dot{\gamma}^{-0.307}$	0.69	160
21	$\mu_a = 833\dot{\gamma}^{-0.251}$	0.75	830

K and *n* are plotted as a function of cell concentration below. To represent the relatively wide range of *K* values obtained, *K* is plotted using logarithmic coordinates.

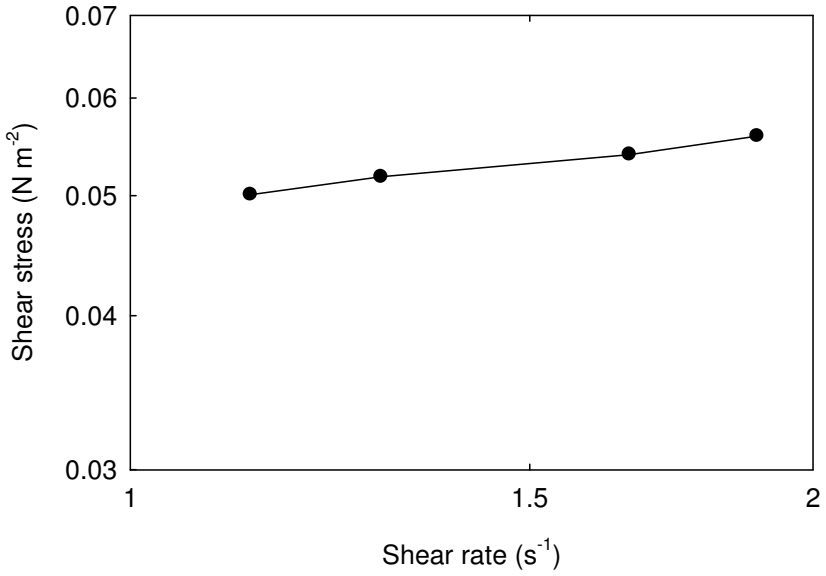


The cell broth is Newtonian ($n = 1$) up to a cell concentration of about 2%, then becomes pseudoplastic. The flow behaviour index continues to decrease until a cell concentration of about 12% is reached. The consistency index rises substantially throughout the culture with increasing cell concentration.

7.4 Impeller viscometer

If the rheology can be described using a power-law model, a plot of shear stress versus shear rate on log–log coordinates can be expected to give a straight line. Values of shear stress and shear rate can be determined from torque and stirrer speed data using Eqs (7.15) and (7.16) with $k = 10.2$ and $D_i = 4$ cm $= 4 \times 10^{-2}$ m. The results are listed and plotted below.

Stirrer speed (s^{-1})	Torque (N m)	Shear stress (N m^{-2})	Shear rate (s^{-1})
0.185	3.57×10^{-6}	0.0559	1.89
0.163	3.45×10^{-6}	0.0540	1.66
0.126	3.31×10^{-6}	0.0518	1.29
0.111	3.20×10^{-6}	0.0501	1.13



The equation for the straight line on the log–log plot is $\tau = 0.049\,\dot{\gamma}^{0.20}$. From Eq. (7.11), this means that the flow behaviour index n is 0.20 and the consistency index K is 0.049 N s^n m^{-2}.

Answer: Yes; n = 0.20, K = 0.049 N s^n m^{-2}

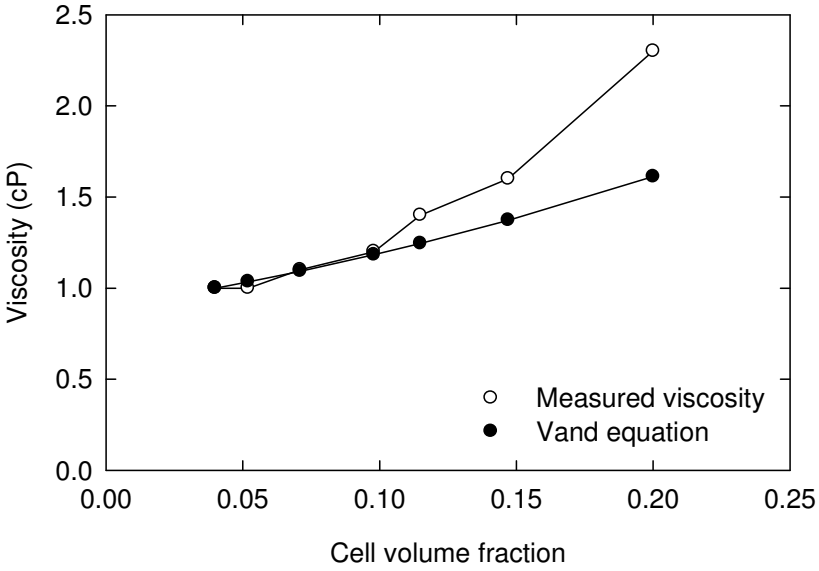
7.5 Vand equation

The Vand equation Eq. (7.17) is written in terms of the cell volume fraction. From the data provided, the volume fraction of cells is determined as:

$$\text{Volume fraction of cells} = \frac{\text{total volume of broth} - \text{volume of decanted liquid}}{\text{total volume of broth}}$$

The measured results for viscosity and viscosity calculated using the Vand equation with μ_L = 0.9 cP are listed and plotted below.

Cell volume fraction (–)	Measured viscosity (cP)	Viscosity calculated using Eq. (7.17) (cP)
0.040	1.0	1.000
0.052	1.0	1.035
0.071	1.1	1.093
0.098	1.2	1.183
0.115	1.4	1.245
0.147	1.6	1.372
0.200	2.3	1.611



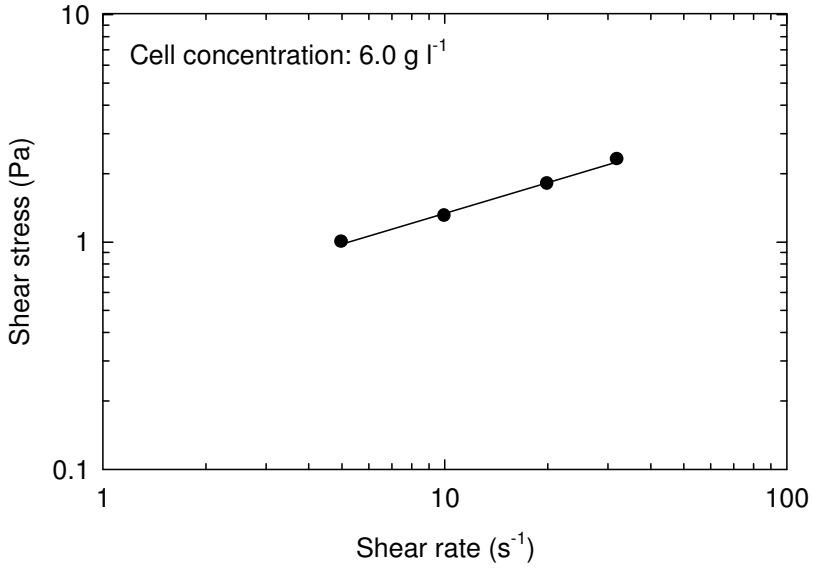
The Vand equation predicts the viscosity of this cell suspension reasonably well for cell volume fractions between about 0.04 and 0.10. Above this range, the equation underestimates the suspension viscosity, with the extent of underestimation increasing with increasing cell density.

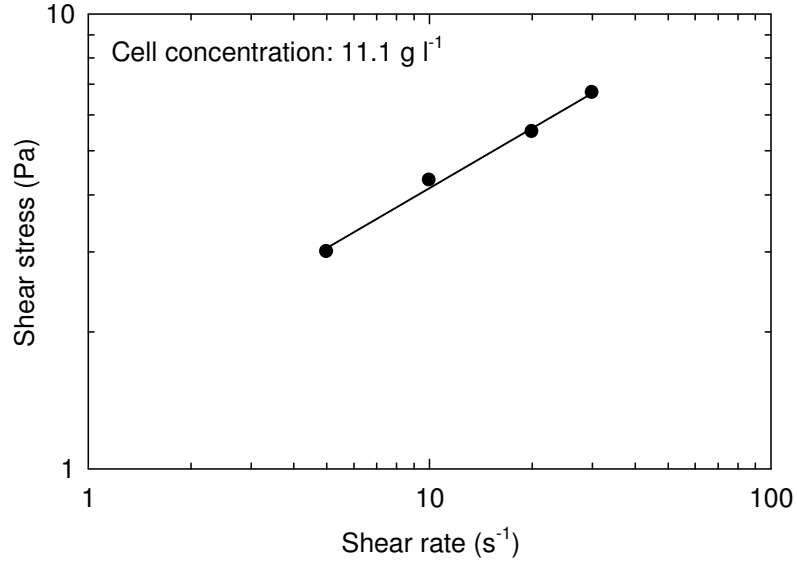
Answer: Yes, but only for suspensions with cell volume fractions in the range 0.04–0.10

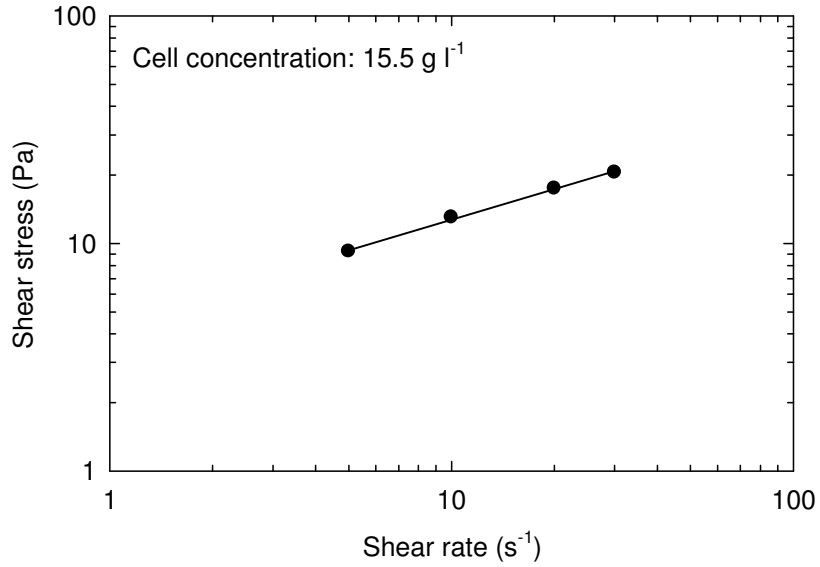
7.6 Viscosity and cell concentration

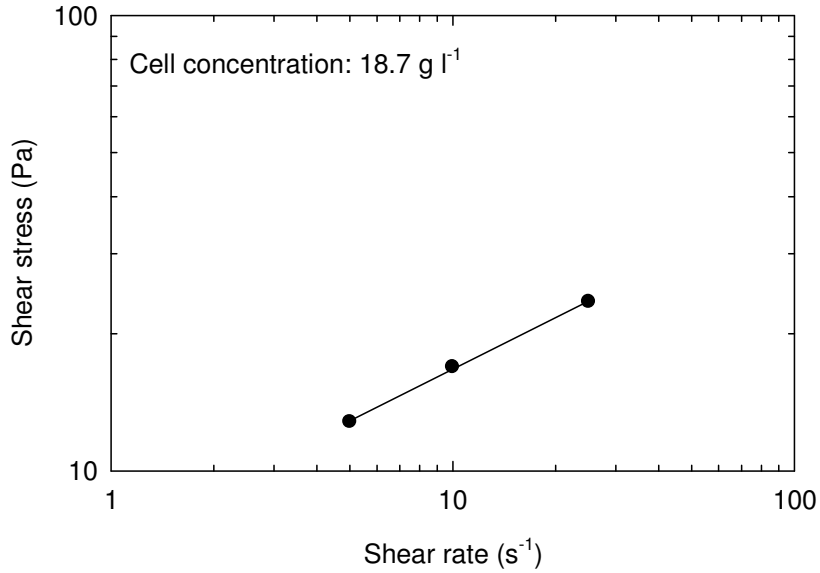
(a)

From Eq. (7.11), for pseudoplastic fluids, a plot of shear stress τ versus shear rate $\dot{\gamma}$ on log–log coordinates can be expected to give a straight line. Log–log plots for the different cell concentrations are shown below.




Cell concentration: 11.1 g l-1
Shear stress (Pa)
Shear rate (s-1)
10
1
1
10
100

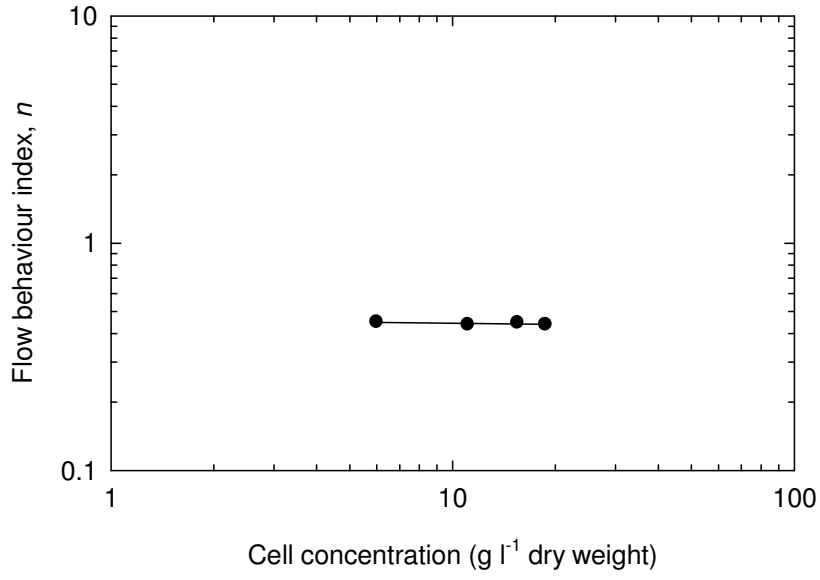

Cell concentration: 15.5 g l-1
Shear stress (Pa)
Shear rate (s-1)
100
10
1
1
10
100

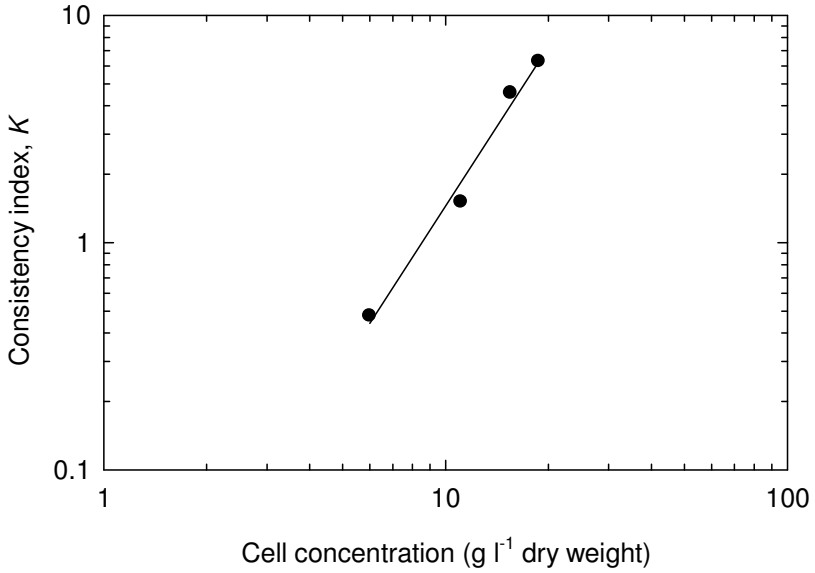


The equations and parameter values for the straight lines in each plot are listed below.

Cell concentration (g l^{-1} dry weight)	Equation	Flow behaviour index, n	Consistency index, K (Pa s^n)
6.0	$\tau = 0.476\dot{\gamma}^{0.449}$	0.449	0.476
11.1	$\tau = 1.511\dot{\gamma}^{0.438}$	0.438	1.511
15.5	$\tau = 4.556\dot{\gamma}^{0.446}$	0.446	4.556
18.7	$\tau = 6.268\dot{\gamma}^{0.438}$	0.438	6.268

If the proposed equation relating K and n to cell concentration is valid, log–log plots of K and n versus cell concentration should give straight lines. These plots are shown below.





The model fits the data reasonably well for both n and K.

Answer: Yes

(b)

The equations for the straight lines in the plots of n and K versus cell concentration are:

$$n = 0.460\,x^{-0.016}$$

and

$$K = 6.84\times10^{-3}\,x^{2.33}$$

where x is cell concentration in units of g l^{-1}, n is the flow behaviour index, and K is the consistency index in units of Pa s^n. The flow behaviour index could be considered independent of cell concentration; this is reflected in the low value of the power –0.016 in the equation for n.

Answer: $n = 0.460\,x^{-0.016}$ where x has units of g l^{-1}; $K = 6.84\times10^{-3}\,x^{2.33}$ where x has units of g l^{-1} and K has units of Pa s^n

(c)

For x = 12.3 g l^{-1}, the above equations for n and K give n = 0.442 and K = 2.369 Pa s^n. Substituting these values into Eq. (7.12) for the apparent viscosity of a pseudoplastic fluid:

$$\mu_a = 2.369\ \text{Pa s}^{0.442}\ (8.5\ \text{s}^{-1})^{0.442-1} = 0.718\ \text{Pa s}$$

Answer: 0.72 Pa s

7.7 Scale of turbulence dissipation

(a)

The scale of the smallest eddies is evaluated using Eq. (7.36). From Table A.9 (Appendix A), 1 N s m^{-2} = 1 kg m^{-1} s^{-1}. The kinematic viscosity ν is determined using Eq. (7.9):

$$\nu = \frac{2.3\times10^{-3}\ \text{N s m}^{-2}\,.\left|\dfrac{1\ \text{kg m}^{-1}\,\text{s}^{-1}}{1\ \text{N s m}^{-2}}\right|}{1000\ \text{kg m}^{-3}} = 2.3\times10^{-6}\ \text{m}^2\ \text{s}^{-1}$$

From Table A.8 (Appendix A), 1 W = 1 kg m^2 s^{-3}. From the definition of density (Section 2.4.1), the mass of fluid in the dissipation volume is equal to the dissipation volume × fluid density. Therefore, the local rate of dissipation of turbulence kinetic energy per unit mass of fluid ε is:

$$\varepsilon = \frac{0.011\ \text{W}\,.\left|\dfrac{1\ \text{kg m}^2\,\text{s}^{-3}}{1\ \text{W}}\right|}{200\ \text{cm}^3\,.\left|\dfrac{1\ \text{m}}{100\ \text{cm}}\right|^3\times1000\ \text{kg m}^{-3}} = 0.055\ \text{m}^2\ \text{s}^{-3}$$

Substituting these values into Eq. (7.36):

$$\lambda = \left(\frac{(2.3\times10^{-6}\ \text{m}^2\ \text{s}^{-1})^3}{0.055\ \text{m}^2\ \text{s}^{-3}}\right)^{1/4} = (2.212\times10^{-16}\ \text{m}^4)^{1/4} = 1.22\times10^{-4}\ \text{m}$$

From Table A.1 (Appendix A), 1 m = 10^6 μm; therefore, λ = 122 μm.

Answer: 122 μm

(b)

The kinematic viscosity ν is changed to:

$$\nu = \frac{1.5\times10^{-2}\ \text{N s m}^{-2}\,.\left|\dfrac{1\ \text{kg m}^{-1}\,\text{s}^{-1}}{1\ \text{N s m}^{-2}}\right|}{1000\ \text{kg m}^{-3}} = 1.5\times10^{-5}\ \text{m}^2\ \text{s}^{-1}$$

Assuming that dextran addition does not affect the local rate of dissipation of turbulence kinetic energy:

$$\lambda = \left(\frac{(1.5\times10^{-5}\ \text{m}^2\ \text{s}^{-1})^3}{0.055\ \text{m}^2\ \text{s}^{-3}}\right)^{1/4} = 4.98\times10^{-4}\ \text{m} = 498\ \mu\text{m}$$

Answer: 498 μm

7.8 Size of dissipating eddies

(a)

From Table C.3 (Appendix C), the heat capacity C_p of liquid water in the temperature range of interest is 75.4 J $gmol^{-1}$ °C^{-1}. Using the atomic weights in Table C.1 (Appendix C), the molecular weight of water is 18.0. Converting the units of C_p from gmol to kg:

$$C_p\,(\text{water}) = 75.4\ \text{J gmol}^{-1}\,^{\circ}\text{C}^{-1}\,.\left|\frac{1\ \text{gmol}}{18.0\ \text{g}}\right|.\left|\frac{1000\ \text{g}}{1\ \text{kg}}\right| = 4189\ \text{J kg}^{-1}\,^{\circ}\text{C}^{-1}$$

Taking the density of water to be 1.00 g cm^{-3} and calculating mass as volume × density (Section 2.4.1), the mass M of 5 litres of water is:

$$M = 5\,\text{l}\,(1.00\ \text{g cm}^{-3})\,.\left|\frac{1000\ \text{cm}^3}{1\,\text{l}}\right|.\left|\frac{1\ \text{kg}}{1000\ \text{g}}\right| = 5\ \text{kg}$$

The enthalpy change associated with raising the temperature of 5 litres of medium by 1°C is determined using Eq. (5.12):

$$\Delta H = 5\ \text{kg}\ (4189\ \text{J kg}^{-1}\ ^{\circ}\text{C}^{-1})\ (1^{\circ}\text{C}) = 2.095\times10^{4}\ \text{J}$$

When this occurs over a period Δt = 1 h, the power or rate of energy input is:

$$P = \frac{\Delta H}{\Delta t} = \frac{2.095\times10^{4}\ \text{J}}{1\ \text{h}} . \left| \frac{1\ \text{h}}{3600\ \text{s}} \right| = 5.82\ \text{J s}^{-1}$$

From Table A.8 (Appendix A), 1 J s^{-1} = 1 W. Therefore, the power input to the liquid is 5.82 W. Assuming that all the energy provided by the stirrer is dissipated as heat, the power output of the stirrer is also 5.82 W.

Answer: 5.82 W

(b)

From Table A.8 (Appendix A), 1 W = 1 kg m^2 s^{-3}; therefore, from **(a)**, the power input to the liquid is 5.82 kg m^2 s^{-3}. Let us assume that this power is dissipated over the entire fluid volume so that the local rate of dissipation of turbulence kinetic energy per unit mass of fluid ε is:

$$\varepsilon = \frac{P}{M} = \frac{5.82\ \text{kg m}^2\ \text{s}^{-3}}{5\ \text{kg}} = 1.16\ \text{m}^2\ \text{s}^{-3}$$

From Eq. (7.8), the viscosity of water is approximately 10^{-3} kg m^{-1} s^{-1}. The density of water is 1.00 g cm^{-3} (Section 2.4.1); therefore, the kinematic viscosity ν from Eq. (7.9) is:

$$\nu = \frac{10^{-3}\ \text{kg m}^{-1}\ \text{s}^{-1}}{1.00\ \text{g cm}^{-3} . \left| \frac{100\ \text{cm}}{1\ \text{m}} \right|^{3} . \left| \frac{1\ \text{kg}}{1000\ \text{g}} \right|} = 10^{-6}\ \text{m}^2\ \text{s}^{-1}$$

The scale of the smallest eddies is evaluated using Eq. (7.36):

$$\lambda = \left(\frac{(10^{-6}\ \text{m}^2\ \text{s}^{-1})^3}{1.16\ \text{m}^2\ \text{s}^{-3}} \right)^{1/4} = (8.62\times10^{-19}\ \text{m}^4)^{1/4} = 3.05\times10^{-5}\ \text{m}$$

From Table A.1 (Appendix A), 1 m = 10^6 μm; therefore, λ = 30.5 μm.

Answer: 30 μm, assuming that the stirrer power is dissipated over the entire fluid volume

Chapter 8

Mixing

8.1 Impeller loading and gas dispersion

D_T = 1.2 m; D_i = 0.4 m. N_i = 110 rpm = 110 min^{-1}. At the beginning of the culture, the gas phase is just dispersed. Assume that the impeller off-bottom clearance is such that this situation for a Rushton turbine is represented by Eq. (8.4). An equation for the gas flow rate F_g under these conditions is obtained by substituting the definitions of Eqs (8.1) and (8.2) into Eq. (8.4):

$$\frac{F_g}{N_i D_i^3} = 0.2\left(\frac{D_i}{D_T}\right)^{0.5}\frac{N_i D_i^{0.5}}{g^{0.5}}$$

Grouping terms gives:

$$F_g = 0.2\left(\frac{D_i}{D_T}\right)^{0.5}\frac{N_i D_i^{0.5}}{g^{0.5}} N_i D_i^3 = 0.2\frac{N_i^2 D_i^4}{D_T^{0.5} g^{0.5}} \quad (1)$$

Substituting values with g = 9.8066 m s^{-2} from Eq. (2.16):

$$F_g = 0.2\frac{\left(110\ \text{min}^{-1}.\left|\frac{1\ \text{min}}{60\ \text{s}}\right|\right)^2 (0.4\ \text{m})^4}{(1.2\ \text{m})^{0.5}(9.8066\ \text{m s}^{-2})^{0.5}} = 5.02\times10^{-3}\ \text{m}^3\ \text{s}^{-1}$$

This is the value of F_g at the beginning of the culture. Towards the end of the culture when the gas flow rate is doubled, $F_g = 2 \times 5.02 \times 10^{-3}$ m^3 s^{-1} = 1.00 ×10^{-2} m^3 s^{-1}.

(a)

An equation for the gas flow rate for a Rushton turbine at the flooding–loading transition is obtained by combining Eqs (8.1), (8.2) and Eq. (8.3):

$$\frac{F_g}{N_i D_i^3} = 30\left(\frac{D_i}{D_T}\right)^{3.5}\frac{N_i^2 D_i}{g}$$

Grouping terms gives:

$$F_g = 30\left(\frac{D_i}{D_T}\right)^{3.5}\frac{N_i^2 D_i}{g} N_i D_i^3 = 30\frac{N_i^3 D_i^{7.5}}{D_T^{3.5} g}$$

Substituting values:

$$F_g = 30\frac{\left(110\ \text{min}^{-1}.\left|\frac{1\ \text{min}}{60\ \text{s}}\right|\right)^3 (0.4\ \text{m})^{7.5}}{(1.2\ \text{m})^{3.5}(9.8066\ \text{m s}^{-2})} = 1.03\times10^{-2}\ \text{m}^3\ \text{s}^{-1}$$

As the operating gas flow rate of 1.00 ×10^{-2} m^3 s^{-1} towards the end of the culture is roughly equal to that at the flooding–loading transition, we can say that the impeller is just loaded.

Answer: Loaded

(b)

An expression for the stirrer speed required for complete gas dispersion is obtained by rearranging (1):

$$N_i^2 = \frac{F_g D_T^{0.5} g^{0.5}}{0.2 D_i^4}$$

Substituting values with $F_g = 1.00 \times 10^{-2}$ m^3 s^{-1}:

$$N_i^2 = \frac{1.00 \times 10^{-2}\ \text{m}^3\ \text{s}^{-1} (1.2\ \text{m})^{0.5} (9.8066\ \text{m s}^{-2})^{0.5}}{0.2 (0.4\ \text{m})^4} = 6.70\ \text{s}^{-2}$$

$$N_i = 2.59\ \text{s}^{-1}$$

Converting to rpm:

$$N_i = 2.59\ \text{s}^{-1} . \left| \frac{60\ \text{s}}{1\ \text{min}} \right| = 155\ \text{min}^{-1} = 155\ \text{rpm}$$

Answer: 155 rpm

8.2 Hydrodynamic conditions for animal cell culture

D_T = 1 m; V_L = 0.8 m^3; D_i = 0.4 m. N_i = 1.5 rps = 1.5 s^{-1}. ρ = 1000 kg m^{-3}. μ = 1.4 cP; therefore, from Table A.9 (Appendix A), $\mu = 1.4 \times 10^{-3}$ kg m^{-1} s^{-1}.

$$F_g = 0.3\ \text{vvm} = 0.3 V_L\ \text{min}^{-1} = (0.3 \times 0.8\ \text{m}^3)\ \text{min}^{-1} . \left| \frac{1\ \text{min}}{60\ \text{s}} \right| = 4.0 \times 10^{-3}\ \text{m}^3\ \text{s}^{-1}$$

(a)

Assume that the impeller off-bottom clearance is such that Eq. (8.4) applies. An equation for the gas flow rate for complete gas dispersion with a Rushton turbine is obtained by substituting the definitions of Eqs (8.1) and (8.2) into Eq. (8.4):

$$\frac{F_g}{N_i D_i^3} = 0.2 \left(\frac{D_i}{D_T} \right)^{0.5} \frac{N_i D_i^{0.5}}{g^{0.5}}$$

Grouping terms gives:

$$F_g = 0.2 \left(\frac{D_i}{D_T} \right)^{0.5} \frac{N_i D_i^{0.5}}{g^{0.5}} N_i D_i^3 = 0.2 \frac{N_i^2 D_i^4}{D_T^{0.5} g^{0.5}}$$

Substituting values with g = 9.8066 m s^{-2} from Eq. (2.16):

$$F_g = 0.2 \frac{(1.5\ \text{s}^{-1})^2 (0.4\ \text{m})^4}{(1\ \text{m})^{0.5} (9.8066\ \text{m s}^{-2})^{0.5}} = 3.68 \times 10^{-3}\ \text{m}^3\ \text{s}^{-1}$$

This is the value of F_g for complete gas dispersion. As the operating gas flow rate of 4.0×10^{-3} m^3 s^{-1} is greater than this value, the gas is not completely dispersed.

Answer: No

(b)

The impeller Reynolds number is evaluated using Eq. (7.2):

$$Re_i = \frac{1.5\ \text{s}^{-1}\ (0.4\ \text{m})^2\ (1000\ \text{kg m}^{-3})}{1.4\times10^{-3}\ \text{kg m}^{-1}\ \text{s}^{-1}} = 1.71\times10^5$$

As this Re_i is greater than the threshold of 10^4 for turbulent flow (Section 8.5.1 and Figure 8.29), we can say that flow is turbulent.

Answer: Yes

(c)

The power input by sparging is given by Eq. (8.13). The liquid height H_L is calculated from the equation for the volume of a cylindrical tank:

$$V_L = \pi\left(\frac{D_T}{2}\right)^2 H_L$$

Therefore:

$$H_L = \frac{V_L}{\pi\left(\frac{D_T}{2}\right)^2} = \frac{0.8\ \text{m}^3}{\pi\left(\frac{1\ \text{m}}{2}\right)^2} = 1.02\ \text{m}$$

Substituting values into Eq. (8.13) gives:

$$P_v = 4.0\times10^{-3}\ \text{m}^3\ \text{s}^{-1}\ (1000\ \text{kg m}^{-3})\ (9.8066\ \text{m s}^{-2})\ (1.02\ \text{m}) = 40.0\ \text{kg m}^2\ \text{s}^{-3}$$

From Table A.8 (Appendix A), 1 W = 1 kg m^2 s^{-3}; therefore, P_v = 40.0 W.

From **(b)**, we know that flow is turbulent. The power input by stirring without gassing can therefore be determined using Eq. (8.9). The value of D_i/D_T is (0.4 m)/(1 m) = 0.4. For a Rushton turbine, N_P' = 5.0 for D_i/D_T = 0.33 (Table 8.1) and N_P' = 5.9 for D_i/D_T = 0.50 (Table 8.2). Interpolating between these results, we can estimate that the value of N_P' for D_i/D_T = 0.4 is about 5.5. Substituting values into Eq. (8.9):

$$P = 5.5\ (1000\ \text{kg m}^{-3})\ (1.5\ \text{s}^{-1})^3\ (0.4\ \text{m})^5 = 190\ \text{kg m}^2\ \text{s}^{-3}$$

As 1 W = 1 kg m^2 s^{-3} (Table A.8, Appendix A):

$$P = 190\ \text{W}$$

If we assume there is no reduction to the stirrer power due to gassing, the total power input by stirring and sparging $P_T = P + P_v$ = 190 W + 40.0 W = 230 W. Therefore, the proportion of the power input by sparging is P_v/P_T = (40.0 W)/(230 W) = 0.17.

It is more realistic, however, that a reduction in stirrer power will occur due to gassing. If we assume there is a 50% reduction, the total power input by stirring and sparging $P_T = 0.5P + P_v$ = 0.5 (190 W) + 40.0 W = 135 W, and the proportion of the power input by sparging is P_v/P_T = (40.0 W)/(135 W) = 0.30.

Answer: 0.17, assuming no reduction in stirrer power due to gassing; or 0.30 assuming a 50% reduction in stirrer power due to gassing

8.3 Gas dispersion and power requirements

D_T = 1.1 m; H_L = 1.1 m. $D_i = 0.5D_T$ = 0.5 (1.1 m) = 0.55 m. ρ = 1000 kg m^{-3}. μ = 15 cP; therefore, from Table A.9 (Appendix A), $\mu = 15\times10^{-3}$ kg m^{-1} s^{-1}. For a cylindrical tank, the liquid volume V_L is:

$$V_L = \pi\left(\frac{D_T}{2}\right)^2 H_L$$

Substituting values:

$$V_L = \pi\left(\frac{1.1\text{ m}}{2}\right)^2 1.1\text{ m} = 1.05\text{ m}^3$$

The gas flow rate F_g is:

$$F_g = 0.66\text{ vvm} = 0.66V_L\text{ min}^{-1} = (0.66\times 1.05\text{ m}^3)\text{ min}^{-1}.\left|\frac{1\text{ min}}{60\text{ s}}\right| = 1.16\times 10^{-2}\text{ m}^3\text{ s}^{-1}$$

(a)

Assume that the impeller off-bottom clearance is such that Eq. (8.4) applies. An expression for the stirrer speed N_i required for complete gas dispersion with a Rushton turbine is obtained by substituting the definitions of Eqs (8.1) and (8.2) into Eq. (8.4):

$$\frac{F_g}{N_i D_i^3} = 0.2\left(\frac{D_i}{D_T}\right)^{0.5}\frac{N_i D_i^{0.5}}{g^{0.5}}$$

Rearranging and grouping terms gives:

$$F_g = 0.2\left(\frac{D_i}{D_T}\right)^{0.5}\frac{N_i D_i^{0.5}}{g^{0.5}} N_i D_i^3 = 0.2\,\frac{N_i^2 D_i^4}{D_T^{0.5} g^{0.5}}$$

$$N_i^2 = \frac{F_g D_T^{0.5} g^{0.5}}{0.2 D_i^4}$$

Substituting values with $g = 9.8066\text{ m s}^{-2}$ from Eq. (2.16):

$$N_i^2 = \frac{1.16\times 10^{-2}\text{ m}^3\text{ s}^{-1}(1.1\text{ m})^{0.5}(9.8066\text{ m s}^{-2})^{0.5}}{0.2(0.55\text{ m})^4} = 2.08\text{ s}^{-2}$$

$$N_i = 1.44\text{ s}^{-1}$$

Converting to rpm:

$$N_i = 1.44\text{ s}^{-1}.\left|\frac{60\text{ s}}{1\text{ min}}\right| = 86.4\text{ min}^{-1} = 86.4\text{ rpm}$$

Answer: 86 rpm

(b)

The impeller Reynolds number is evaluated using Eq. (7.2) and the result for N_i from **(a)**:

$$Re_i = \frac{1.44\text{ s}^{-1}(0.55\text{ m})^2(1000\text{ kg m}^{-3})}{15\times 10^{-3}\text{ kg m}^{-1}\text{ s}^{-1}} = 2.9\times 10^4$$

As this Re_i is greater than the threshold of 10^4 for turbulent flow (Section 8.5.1 and Figure 8.29), we can say that flow is turbulent. Therefore, the power requirements without gassing can be determined using Eq. (8.9). For $D_i/D_T = 0.5$, from Table 8.2, $N_P' = 5.9$. Substituting values into Eq. (8.9):

$$P = 5.9\,(1000\text{ kg m}^{-3})(1.44\text{ s}^{-1})^3(0.55\text{ m})^5 = 887\text{ kg m}^2\text{ s}^{-3}$$

As 1 W = 1 kg m^2 s^{-3} (Table A.8, Appendix A):

$$P = 887\text{ W}$$

The power input with gassing $P_g = 0.5P = 0.5 \times (887)\text{ W} = 443\text{ W}$.

Answer: 443 W

8.4 Electrical power required for mixing

D_i = 7 cm = 0.07 m. N_i = 900 rpm = 900 min^{-1}. From Section 2.4.1, ρ = 1 g cm^{-3} = 1000 kg m^{-3}. From Eq. (7.8), μ = 10^{-3} kg m^{-1} s^{-1}. Determine the flow regime by calculating the impeller Reynolds number. Substituting parameter values into Eq. (7.2):

$$Re_i = \frac{900\ \text{min}^{-1}.\left|\frac{1\ \text{min}}{60\ \text{s}}\right|(0.07\ \text{m})^2(1000\ \text{kg m}^{-3})}{10^{-3}\ \text{kg m}^{-1}\text{s}^{-1}} = 7.35\times10^4$$

From Figure 8.29 for a Rushton turbine, this value of Re_i corresponds to turbulent flow. Assume that D_i/D_T = 0.33 for the laboratory-scale fermenter so that N_P' can be taken as 5.0 (Table 8.1). If the fermenter is not gassed, the power required is evaluated using Eq. (8.9):

$$P = 5.0\ (1000\ \text{kg m}^{-3})\left(900\ \text{min}^{-1}.\left|\frac{1\ \text{min}}{60\ \text{s}}\right|\right)^3(0.07\ \text{m})^5 = 28.4\ \text{kg m}^2\ \text{s}^{-3}$$

From Table A.8 (Appendix A), 1 W = 1 kg m^2 s^{-3}; therefore:

$$P = 28.4\ \text{W}$$

This value is considerably lower that the electrical power consumed by the stirrer motor. Much of the remainder of the electrical power is converted into heat within the motor housing.

Answer: 28.4 W; a significant fraction of the electrical power is dissipated as heat within the motor housing

8.5 Effect of viscosity on power requirements

D_T = 3 m. $D_i = D_T/3$ = (3 m)/3 = 1 m. H_L = 3 m. ρ = 1 g cm^{-3} = 1000 kg m^{-3}.

(a)

N_i = 90 rpm = 90 min^{-1}.

(i)

From Eq. (7.8), μ = 10^{-3} kg m^{-1} s^{-1}. Determine the flow regime by calculating the impeller Reynolds number. Substituting parameter values into Eq. (7.2):

$$Re_i = \frac{90\ \text{min}^{-1}.\left|\frac{1\ \text{min}}{60\ \text{s}}\right|(1\ \text{m})^2(1000\ \text{kg m}^{-3})}{10^{-3}\ \text{kg m}^{-1}\text{s}^{-1}} = 1.5\times10^6$$

From Figure 8.29, this value of Re_i corresponds to turbulent flow and N_P' can be taken as 5.0 (Table 8.1). The power required without gassing is evaluated using Eq. (8.9):

$$P = 5.0\ (1000\ \text{kg m}^{-3})\left(90\ \text{min}^{-1}.\left|\frac{1\ \text{min}}{60\ \text{s}}\right|\right)^3(1\ \text{m})^5 = 1.69\times10^4\ \text{kg m}^2\ \text{s}^{-3}$$

From Table A.8 (Appendix A), 1 W = 1 kg m^2 s^{-3}; therefore:

$$P = 1.69\times10^4\ \text{W} = 16.9\ \text{kW}$$

Answer: 16.9 kW

(ii)

For a viscosity of 100 × 10^{-3} kg m^{-1} s^{-1} = 0.1 kg m^{-1} s^{-1}, Re_i is:

$$Re_i = \frac{90\ \text{min}^{-1} . \left| \frac{1\ \text{min}}{60\ \text{s}} \right| (1\ \text{m})^2\ (1000\ \text{kg m}^{-3})}{0.1\ \text{kg m}^{-1}\ \text{s}^{-1}} = 1.5 \times 10^4$$

From Figure 8.29, this value of Re_i is still within the turbulent regime so that N_P' is again 5.0. Therefore, the power required without gassing is the same as that calculated in **(a) (i)**: P = 16.9 kW.

Answer: 16.9 kW

(iii)

For a viscosity of $2 \times 10^5 \times 10^{-3}$ kg m^{-1} s^{-1} = 200 kg m^{-1} s^{-1}, Re_i is:

$$Re_i = \frac{90\ \text{min}^{-1} . \left| \frac{1\ \text{min}}{60\ \text{s}} \right| (1\ \text{m})^2\ (1000\ \text{kg m}^{-3})}{200\ \text{kg m}^{-1}\ \text{s}^{-1}} = 7.5$$

From Figure 8.29, this value of Re_i is within the laminar regime and N_P read from the graph is about 10. The power required without gassing is evaluated using Eq. (8.7):

$$P = 10\ (1000\ \text{kg m}^{-3}) \left(90\ \text{min}^{-1} . \left| \frac{1\ \text{min}}{60\ \text{s}} \right| \right)^3 (1\ \text{m})^5 = 3.38 \times 10^4\ \text{kg m}^2\ \text{s}^{-3}$$

From Table A.8 (Appendix A), 1 W = 1 kg m^2 s^{-3}; therefore:

$$P = 3.38 \times 10^4\ \text{W} = 33.8\ \text{kW}$$

Answer: 33.8 kW

(b)

The viscosity is 1000×10^{-3} kg m^{-1} s^{-1} = 1 kg m^{-1} s^{-1}.

(i)

From Figure 8.29, turbulence with N_P' = 5.0 is achieved at a minimum Reynolds number of about 10^4. The stirrer speed required can be determined by rearranging Eq. (7.2):

$$N_i = \frac{Re_i \mu}{D_i^2 \rho} = \frac{10^4\ (1\ \text{kg m}^{-1}\ \text{s}^{-1})}{(1\ \text{m})^2\ (1000\ \text{kg m}^{-3})} = 10\ \text{s}^{-1}$$

Converting to rpm:

$$N_i = 10\ \text{s}^{-1} . \left| \frac{60\ \text{s}}{1\ \text{min}} \right| = 600\ \text{min}^{-1} = 600\ \text{rpm}$$

Answer: 600 rpm

(ii)

Using the result from **(b) (i)** in Eq. (8.9), the power required to achieve turbulence is:

$$P = 5.0\ (1000\ \text{kg m}^{-3}) (10\ \text{s}^{-1})^3\ (1\ \text{m})^5 = 5.0 \times 10^6\ \text{kg m}^2\ \text{s}^{-3}$$

From Table A.8 (Appendix A), 1 W = 1 kg m^2 s^{-3}; therefore:

$$P = 5.0 \times 10^6\ \text{W} = 5000\ \text{kW}$$

Answer: 5000 kW

(iii)

For a cylindrical tank, the liquid volume V_L is:

$$V_L = \pi\left(\frac{D_T}{2}\right)^2 H_L$$

Substituting values:

$$V_L = \pi\left(\frac{3\text{ m}}{2}\right)^2 3\text{ m} = 21.2\text{ m}^3$$

Using the result for power from **(b) (ii)**, the power per unit volume is:

$$P / V_L = \frac{5000\text{ kW}}{21.2\text{ m}^3} = 236\text{ kW m}^{-3}$$

Typical values for power consumption in industrial bioreactors are given in Section 8.5. Relative to the 1–2 kW m^{-3} guideline for large fermenters, the value of 236 kW m^{-3} is extremely large. It is unreasonable to expect to be able to provide this amount of power; the size of the motor and stirrer assembly required is impractical. Therefore, achieving turbulence with viscosity 1000 times greater than that of water is not possible.

Answer: 236 kW m^{-3}; no; an impractically large stirrer motor would be required

8.6 Power and scale-up

ρ = 1000 kg m^{-3}. μ = 5 cP; therefore, from Table A.9 (Appendix A), $\mu = 5 \times 10^{-3}$ kg m^{-1} s^{-1}. Assume that both fermenters are operated without sparging.

For the pilot-scale vessel:

$D_T = H_L$ = 0.5 m. $D_i = D_T/3$ = (0.5 m)/3 = 0.167 m. N_i = 185 rpm = 185 min^{-1}. For a cylindrical tank, the liquid volume V_L is:

$$V_L = \pi\left(\frac{D_T}{2}\right)^2 H_L$$

When $D_T = H_L$, this equation can be written as:

$$V_L = \frac{\pi}{4} D_T^3 \qquad (1)$$

Therefore, the volume of the pilot-scale fermenter is:

$$V_L = \frac{\pi}{4}(0.5\text{ m})^3 = 0.098\text{ m}^3$$

For the large-scale vessel:

V_L = 6 m^3. After geometric scale-up, $D_T = H_L$ so (1) applies. Rearranging (1) gives:

$$D_T^3 = \frac{4V_L}{\pi}$$

Therefore, the diameter of the large-scale vessel is:

$$D_T^3 = \frac{4(6\text{ m}^3)}{\pi} = 7.64\text{ m}^3$$

$$D_T = 1.97 \text{ m}$$

After geometric scale-up, $D_i/D_T = 0.33$ for the large-scale vessel; therefore, $D_i = 0.33\ (1.97 \text{ m}) = 0.65$ m.

(a)

Determine the flow regime in the pilot-scale fermenter by calculating the impeller Reynolds number. Substituting parameter values into Eq. (7.2):

$$Re_i = \frac{185 \text{ min}^{-1} . \left| \frac{1 \text{ min}}{60 \text{ s}} \right| (0.167 \text{ m})^2 (1000 \text{ kg m}^{-3})}{5 \times 10^{-3} \text{ kg m}^{-1} \text{ s}^{-1}} = 1.72 \times 10^4$$

As this Re_i is greater than the threshold of 10^4 for turbulent flow (Section 8.5.1), the power required can be evaluated using Eq. (8.9). From Table 8.2, $N'_P = 1.5$ for a Scaba 6SRGT turbine in a tank with this geometry. Substituting values into Eq. (8.9) gives:

$$P = 1.5\ (1000 \text{ kg m}^{-3}) \left(185 \text{ min}^{-1} . \left| \frac{1 \text{ min}}{60 \text{ s}} \right| \right)^3 (0.167 \text{ m})^5 = 5.71 \text{ kg m}^2 \text{ s}^{-3}$$

From Table A.8 (Appendix A), 1 W = 1 kg m^2 s^{-3}; therefore:

$$P = 5.71 \text{ W}$$

Answer: 5.7 W

(b)

Using the result for P from **(a)**, the power per unit volume P/V_L in the pilot-scale vessel is (5.71 W)/(0.098 m^3) = 58.3 W m^{-3}. Applying this value for power per unit volume to the large-scale fermenter, P = (58.3 W m^{-3}) × (6 m^3) = 349.8 W.

Answer: 350 W

(c)

First, let us assume that flow in the large-scale vessel is turbulent: this is checked below. The stirrer speed under turbulent flow conditions is determined by rearranging Eq. (8.9):

$$N_i^3 = \frac{P}{N'_P \rho D_i^5}$$

Using the result for P from **(b)**:

$$N_i^3 = \frac{349.8 \text{ W} . \left| \frac{1 \text{ kg m}^2 \text{ s}^{-3}}{1 \text{ W}} \right|}{1.5\ (1000 \text{ kg m}^{-3})\ (0.65 \text{ m})^5} = 2.01 \text{ s}^{-3}$$

$$N_i = 1.26 \text{ s}^{-1}$$

Converting to rpm:

$$N_i = 1.26 \text{ s}^{-1} . \left| \frac{60 \text{ s}}{1 \text{ min}} \right| = 75.6 \text{ min}^{-1} = 75.6 \text{ rpm}$$

To check whether this stirrer speed corresponds to turbulent flow, calculate the impeller Reynolds number using Eq. (7.2):

$$Re_i = \frac{1.26\ \text{s}^{-1}\ (0.65\ \text{m})^2\ (1000\ \text{kg m}^{-3})}{5\times10^{-3}\ \text{kg m}^{-1}\ \text{s}^{-1}} = 1.06\times10^5$$

As this value is greater than the threshold of 10^4 for turbulent flow (Section 8.5.1), the use of Eq. (8.9) and N_P' is justified.

Answer: 76 rpm

(d)

The impeller tip speed in the pilot-scale vessel is:

$$\text{Impeller tip speed} = \pi N_i D_i = \pi\left(185\ \text{min}^{-1}.\left|\frac{1\ \text{min}}{60\ \text{s}}\right|\right)(0.167\ \text{m}) = 1.62\ \text{m s}^{-1}$$

Applying this tip speed to the large-scale fermenter to calculate the stirrer speed required:

$$N_i = \frac{\text{Impeller tip speed}}{\pi D_i} = \frac{1.62\ \text{m s}^{-1}}{\pi(0.65\ \text{m})} = 0.793\ \text{s}^{-1}$$

Converting to rpm:

$$N_i = 0.793\ \text{s}^{-1}.\left|\frac{60\ \text{s}}{1\ \text{min}}\right| = 47.6\ \text{min}^{-1} = 47.6\ \text{rpm}$$

Answer: 48 rpm

(e)

Determine the flow regime by calculating the impeller Reynolds number for the large-scale fermenter using Eq. (7.2) and the result for N_i from **(d)**:

$$Re_i = \frac{0.793\ \text{s}^{-1}\ (0.65\ \text{m})^2\ (1000\ \text{kg m}^{-3})}{5\times10^{-3}\ \text{kg m}^{-1}\ \text{s}^{-1}} = 6.70\times10^4$$

This value of Re_i indicates turbulent flow so that Eq. (8.9) can be applied with $N_P' = 1.5$:

$$P = 1.5\ (1000\ \text{kg m}^{-3})\ (0.793\ \text{s}^{-1})^3\ (0.65\ \text{m})^5 = 86.8\ \text{kg m}^2\ \text{s}^{-3}$$

Converting units using 1 W = 1 kg m^2 s^{-3} (Table A.8, Appendix A):

$$P = 86.8\ \text{W}$$

Answer: 87 W

8.7 Particle suspension and gas dispersion

ρ_L = 1000 kg m^{-3}. μ = 0.8 × 10^{-3} Pa s; therefore, from Table A.9 (Appendix A), μ = 0.8 × 10^{-3} kg m^{-1} s^{-1}. From Eq. (7.9):

$$\nu_L = \frac{0.8\times10^{-3}\ \text{kg m}^{-1}\ \text{s}^{-1}}{1000\ \text{kg m}^{-3}} = 8\times10^{-7}\ \text{m}^2\ \text{s}^{-1}$$

V_L = 400 l; therefore, from Table A.2 (Appendix A), V_L = 0.4 m^3. D_T = 0.8 m. $D_i = D_T/3$ = (0.8 m)/3 = 0.267 m. D_p = 250 μm; therefore, from Table A.1 (Appendix A), D_p = 250 × 10^{-6} m. ρ_p = 1.9 g cm^{-3} = 1900 kg m^{-3}. X = 15%.

$$F_g = 0.5\ \text{vvm} = 0.5V_L\ \text{min}^{-1} = (0.5\times0.4\ \text{m}^3)\ \text{min}^{-1}.\left|\frac{1\ \text{min}}{60\ \text{s}}\right| = 3.33\times10^{-3}\ \text{m}^3\ \text{s}^{-1}$$

For a Rushton turbine with $D_i/D_T = 0.33$ and $C_i/D_T = 0.25$, from Table 8.4, the value of S in the Zwietering equation is 6.7. Substituting values into Eq. (8.18) with $g = 9.8066\ \text{m s}^{-2}$ from Eq. (2.16):

$$N_{JS} = \frac{\left(6.7\,(8\times10^{-7}\ \text{m}^2\ \text{s}^{-1})^{0.1}\,(250\times10^{-6}\ \text{m})^{0.2} \left[9.8066\ \text{m s}^{-2}\,(1900\ \text{kg m}^{-3} - 1000\ \text{kg m}^{-3})\,/\,1000\ \text{kg m}^{-3} \right]^{0.45} 15^{0.13} \right)}{(0.267\ \text{m})^{0.85}}$$

$$= 3.65\ \text{s}^{-1}$$

Applying Eq. (8.20) to account for the effect of gassing:

$$N_{JSg} = [3.65 + 2.4\,(0.5)]\ \text{s}^{-1} = 4.85\ \text{s}^{-1}$$

This is the stirrer speed required for just complete suspension of particles with gassing. Eq. (8.4) for complete gas dispersion by a Rushton turbine is valid for $C_i/H_L = 0.25$ (Section 8.4.1, With Gassing subsection). H_L can be determined from the equation for the volume of a cylindrical tank:

$$V_L = \pi\left(\frac{D_T}{2}\right)^2 H_L$$

Therefore:

$$H_L = \frac{V_L}{\pi\left(\frac{D_T}{2}\right)^2}$$

Substituting values:

$$H_L = \frac{0.4\ \text{m}^3}{\pi\left(\frac{0.8\ \text{m}}{2}\right)^2} = 0.8\ \text{m}$$

As $H_L = D_T$, if $C_i/D_T = 0.25$ then $C_i/H_L = 0.25$ and Eq. (8.4) is valid. An equation for the gas flow rate F_g required to achieve complete gas dispersion with a Rushton turbine is obtained by substituting the definitions of Eqs (8.1) and (8.2) into Eq. (8.4):

$$\frac{F_g}{N_i D_i^3} = 0.2\left(\frac{D_i}{D_T}\right)^{0.5} \frac{N_i D_i^{0.5}}{g^{0.5}}$$

Grouping terms gives:

$$F_g = 0.2\left(\frac{D_i}{D_T}\right)^{0.5} \frac{N_i D_i^{0.5}}{g^{0.5}} N_i D_i^3 = 0.2\,\frac{N_i^2 D_i^4}{D_T^{0.5} g^{0.5}} \tag{1}$$

Substituting values with $N_i = N_{JSg}$ gives:

$$F_g = 0.2\frac{(4.85\ \text{s}^{-1})^2\,(0.267\ \text{m})^4}{(0.8\ \text{m})^{0.5}\,(9.8066\ \text{m s}^{-2})^{0.5}} = 8.54\times10^{-3}\ \text{m}^3\ \text{s}^{-1}$$

As the operating gas flow rate of $3.33 \times 10^{-3}\ \text{m}^3\ \text{s}^{-1}$ is less than that corresponding to complete gas dispersion at this stirrer speed, we can say that complete gas dispersion is achieved.

Answer: Yes

8.8 Impeller flooding and power requirements

$D_T = H_L = 2$ m. For a cylindrical tank, the liquid volume V_L is:

$$V_L = \pi\left(\frac{D_T}{2}\right)^2 H_L$$

Substituting values:

$$V_L = \pi\left(\frac{2\text{ m}}{2}\right)^2 2\text{ m} = 6.28\text{ m}^3$$

The gas flow rate F_g is:

$$F_g = 1.5\text{ vvm} = 1.5V_L\text{ min}^{-1} = (1.5\times 6.28\text{ m}^3)\text{ min}^{-1}.\left|\frac{1\text{ min}}{60\text{ s}}\right| = 0.157\text{ m}^3\text{ s}^{-1}$$

$\rho = 1$ g cm^{-3} = 1000 kg m^{-3}. $\mu = 0.9 \times 10^{-3}$ Pa s; therefore, from Table A.9 (Appendix A), $\mu = 0.9 \times 10^{-3}$ kg m^{-1} s^{-1}.

(a)

An equation for the stirrer speed at the flooding–loading transition with a Rushton turbine is obtained by substituting the definitions of Eqs (8.1) and (8.2) into Eq. (8.3):

$$\frac{F_g}{N_i D_i^3} = 30\left(\frac{D_i}{D_T}\right)^{3.5}\frac{N_i^2 D_i}{g}$$

Grouping terms and rearranging gives:

$$N_i^3 = \frac{F_g g}{30 D_i^4}\left(\frac{D_T}{D_i}\right)^{3.5}$$

$D_i = D_T/3 = (2\text{ m})/3 = 0.667$ m. Substituting values with $g = 9.8066$ m s^{-2} from Eq. (2.16):

$$N_i^3 = \frac{0.157\text{ m}^3\text{ s}^{-1}(9.8066\text{ m s}^{-2})}{30\ (0.667\text{ m})^4}(3)^{3.5} = 12.13\text{ s}^{-3}$$

$$N_i = 2.30\text{ s}^{-1}$$

Converting to rpm:

$$N_i = 2.30\text{ s}^{-1}.\left|\frac{60\text{ s}}{1\text{ min}}\right| = 138\text{ min}^{-1} = 138\text{ rpm}$$

Answer: 138 rpm

(b)

$D_i = D_T/2 = (2\text{ m})/2 = 1$ m. Substituting values into the equation for N_i^3 derived in **(a)**:

$$N_i^3 = \frac{0.157\text{ m}^3\text{ s}^{-1}(9.8066\text{ m s}^{-2})}{30\ (1\text{ m})^4}(2)^{3.5} = 0.581\text{ s}^{-3}$$

$$N_i = 0.834\text{ s}^{-1}$$

Converting to rpm:

$$N_i = 0.834\text{ s}^{-1}.\left|\frac{60\text{ s}}{1\text{ min}}\right| = 50\text{ min}^{-1} = 50\text{ rpm}$$

Answer: 50 rpm

(c)

For the smaller impeller, determine the flow regime at the stirrer speed determined in **(a)** by calculating the impeller Reynolds number. Substituting parameter values into Eq. (7.2):

$$Re_i = \frac{2.30\ \text{s}^{-1}\,(0.667\ \text{m})^2\,(1000\ \text{kg m}^{-3})}{0.9\times10^{-3}\ \text{kg m}^{-1}\,\text{s}^{-1}} = 1.14\times10^6$$

From Figure 8.29 for a Rushton turbine, this value of Re_i corresponds to turbulent flow. From Figure 8.29 and Table 8.1, N_P' for $D_i/D_T = 0.33$ is 5.0. The power required without gassing is evaluated using Eq. (8.9):

$$P = 5.0\,(1000\ \text{kg m}^{-3})\,(2.30\ \text{s}^{-1})^3\,(0.667\ \text{m})^5 = 8.03\times10^3\ \text{kg m}^2\,\text{s}^{-3}$$

From Table A.8 (Appendix A), 1 W = 1 kg m^2 s^{-3}; therefore:

$$P = 8.03\times10^3\ \text{W} = 8.03\ \text{kW}$$

For the larger impeller at the stirrer speed determined in **(b)**:

$$Re_i = \frac{0.834\ \text{s}^{-1}\,(1\ \text{m})^2\,1000\ \text{kg m}^{-3}}{0.9\times10^{-3}\ \text{kg m}^{-1}\,\text{s}^{-1}} = 9.27\times10^5$$

This value of Re_i corresponds to turbulent flow. From Table 8.2 for a Rushton turbine with $D_i/D_T = 0.5$, N_P' = 5.9. The power required without gassing is evaluated using Eq. (8.9):

$$P = 5.9\,(1000\ \text{kg m}^{-3})\,(0.834\ \text{s}^{-1})^3\,(1\ \text{m})^5 = 3.42\times10^3\ \text{kg m}^2\,\text{s}^{-3}$$

From Table A.8 (Appendix A), 1 W = 1 kg m^2 s^{-3}; therefore:

$$P = 3.42\times10^3\ \text{W} = 3.42\ \text{kW}$$

Without gassing, the larger impeller consumes only 43% of the power required by the smaller impeller to achieve impeller loading. If the power draw of both impellers is reduced by the same percentage with gassing, the results of this comparison remain valid with gassing.

Answer: The larger impeller. *Although N_P' and D_i are greater for the larger impeller, a higher value for D_i means that a much smaller N_i is required to avoid impeller flooding. This translates into a smaller overall power draw for operation of the larger impeller.*

8.9 Stirrer effectiveness with sparging

$D_T = H_L = 1.15$ m; $D_i = 0.36$ m. $N_i = 200$ rpm = 200 min^{-1}. $F_g = 0.036$ m^3 s^{-1}. $N_P' = 1.0$ with gassing. $\rho = 1$ g cm^{-3} = 1000 kg m^{-3}. $\mu = 1$ cP; therefore, from Table A.9 (Appendix A), $\mu = 10^{-3}$ kg m^{-1} s^{-1}.

(a)

We do not have equations for the flooding–loading transition for a four-blade pitched-blade turbine. However, if this turbine is operated with downward pumping, we can assume that it will flood at lower gas flow rates than a corresponding Rushton impeller (Section 8.4.3), all other geometric and operating parameters being equal. Therefore, we will perform the calculations for a Rushton turbine. An equation for the gas flow rate at the flooding–loading transition is obtained by substituting the definitions of Eqs (8.1) and (8.2) into Eq. (8.3):

$$\frac{F_g}{N_i D_i^3} = 30\left(\frac{D_i}{D_T}\right)^{3.5}\frac{N_i^2 D_i}{g}$$

Grouping terms gives:

$$F_g = 30\left(\frac{D_i}{D_T}\right)^{3.5}\frac{N_i^2 D_i}{g}N_i D_i^3 = 30\frac{N_i^3 D_i^{7.5}}{D_T^{3.5} g}$$

Substituting values with $g = 9.8066$ m s^{-2} from Eq. (2.16):

$$F_g = 30\frac{\left(200\ \text{min}^{-1}.\left|\frac{1\ \text{min}}{60\ \text{s}}\right|\right)^3 (0.36\ \text{m})^{7.5}}{(1.15\ \text{m})^{3.5}(9.8066\ \text{m s}^{-2})} = 0.033\ \text{m}^3\ \text{s}^{-1}$$

As the operating gas flow rate of 0.036 m^3 s^{-1} is greater than that at the flooding–loading transition for a Rushton turbine, we can say that a Rushton turbine of this geometry will be flooded under the prevailing operating conditions. Therefore, according to our assumption, the pitched-blade turbine will also be flooded.

Answer: Flooded, assuming that the pitched-blade turbine is operated with downward flow so that it is likely to be flooded at lower gas flow rates than a same-geometry Rushton turbine operated at the same stirrer speed

(b)

Determine the flow regime by calculating the impeller Reynolds number using Eq. (7.2):

$$Re_i = \frac{200\ \text{min}^{-1}.\left|\frac{1\ \text{min}}{60\ \text{s}}\right|(0.36\ \text{m})^2(1000\ \text{kg m}^{-3})}{10^{-3}\ \text{kg m}^{-1}\ \text{s}^{-1}} = 4.32\times10^5$$

As this value is greater than the threshold of 10^4 for turbulent flow for many impeller systems (Section 8.5.1 and Figure 8.29), we can say that flow is turbulent. Applying Eq. (8.9) with $N_P' = 1.0$ for the pitched-blade turbine with gassing:

$$P = 1.0\ (1000\ \text{kg m}^{-3})\left(200\ \text{min}^{-1}.\left|\frac{1\ \text{min}}{60\ \text{s}}\right|\right)^3 (0.36\ \text{m})^5 = 224\ \text{kg m}^2\ \text{s}^{-3}$$

Converting units using 1 W = 1 kg m^2 s^{-3} (Table A.8, Appendix A):

$$P = 224\ \text{W}$$

Answer: 224 W

(c)

The power input by sparging is given by Eq. (8.13):

$$P_v = 0.036\ \text{m}^3\ \text{s}^{-1}(1000\ \text{kg m}^{-3})(9.8066\ \text{m s}^{-2})(1.15\ \text{m}) = 406\ \text{kg m}^2\ \text{s}^{-3}$$

From Table A.8 (Appendix A), 1 W = 1 kg m^2 s^{-3}; therefore, $P_v = 406$ W.

Answer: 406 W

(d)

The impeller is flooded and there is a much higher rate of energy input by sparging than by stirring. The vessel is operating virtually as a bubble column with little benefit being obtained from stirring in terms of mixing or gas dispersion.

Answer: No

8.10 Cell suspension and power requirements

The following information is missing from this problem in the textbook: the diameter of the fermenter is 75 cm.

From Section 2.4.1, $\rho_L = 1$ g cm^{-3} = 1000 kg m^{-3}. From Eq. (7.8), $\mu = 10^{-3}$ kg m^{-1} s^{-1}. From Eq. (7.9):

$$\nu_L = \frac{10^{-3}\ \text{kg m}^{-1}\ \text{s}^{-1}}{1000\ \text{kg m}^{-3}} = 10^{-6}\ \text{m}^2\ \text{s}^{-1}$$

$D_i = 30$ cm = 0.3 m. For $D_T = 75$ cm = 0.75 m, $D_i/D_T = (0.3\ \text{m})/(0.75\ \text{m}) = 0.40$.

(a)

$X = 40\%$. $D_p = 10$ μm; therefore, from Table A.1 (Appendix A), $D_p = 10 \times 10^{-6}$ m. $\rho_p = 1.04$ g cm^{-3} = 1040 kg m^{-3}. From Table 8.4, $S = 6.6$ for a marine propeller with $D_i/D_T = 0.33$ and $C_i/D_T = 0.25$. We will assume that $C_i/D_T = 0.25$ for the fermenter in this problem. However, as $D_i/D_T = 0.40$ rather than 0.33, Eq. (8.19) is applied to estimate the corresponding change in S with $\alpha = 0.82$ for a propeller (Section 8.8.1):

$$\frac{S_2}{S_1} = \left[\frac{(D_T / D_i)_2}{(D_T / D_i)_1}\right]^{0.82}$$

where subscript 1 refers to $D_i/D_T = 0.33$ and subscript 2 refers to $D_i/D_T = 0.40$. Substituting values:

$$\frac{S_2}{6.6} = \left[\frac{2.5}{3.0}\right]^{0.82}$$

$$S_2 = 5.68$$

Substituting values into Eq. (8.18) with $g = 9.8066$ m s^{-2} from Eq. (2.16):

$$N_{JS} = \frac{\left(\begin{array}{c} 5.68\,(10^{-6}\ \text{m}^2\ \text{s}^{-1})^{0.1}\,(10\times10^{-6}\ \text{m})^{0.2} \\ \left[9.8066\ \text{m s}^{-2}\,(1040\ \text{kg m}^{-3} - 1000\ \text{kg m}^{-3})\,/\,1000\ \text{kg m}^{-3}\right]^{0.45}\,40^{0.13} \end{array}\right)}{(0.3\ \text{m})^{0.85}}$$

$$= 0.421\ \text{s}^{-1}$$

Converting to rpm:

$$N_{JS} = 0.421\ \text{s}^{-1} \cdot \left|\frac{60\ \text{s}}{1\ \text{min}}\right| = 25.3\ \text{min}^{-1} = 25.3\ \text{rpm}$$

Answer: 25 rpm

(b)

Determine the flow regime by calculating the impeller Reynolds number using Eq. (7.2) with $N_i = N_{JS}$ from **(a)**:

$$Re_i = \frac{0.421\ \text{s}^{-1}\,(0.3\ \text{m})^2\,(1000\ \text{kg m}^{-3})}{10^{-3}\ \text{kg m}^{-1}\ \text{s}^{-1}} = 3.79\times10^4$$

As this value is greater than the threshold of 10^4 for turbulent flow (Section 8.5.1 and Figure 8.29), we can say that flow is turbulent. N'_P for a marine propeller with $D_i/D_T = 0.33$ is given in Table 8.1 as 0.35; we will assume that this value also holds for $D_i/D_T = 0.40$. Calculating the power required without gassing from Eq. (8.9):

$$P = 0.35\,(1000\ \text{kg m}^{-3})\,(0.421\ \text{s}^{-1})^3\,(0.3\ \text{m})^5 = 0.063\ \text{kg m}^2\ \text{s}^{-3}$$

Converting units using 1 W = 1 kg m^2 s^{-3} (Table A.8, Appendix A):

$$P = 0.063\ \text{W}$$

Answer: 0.063 W

(c)

For the immobilised cell system, D_p = 2 mm = 2 × 10^{-3} m, ρ_p = 1.75 g cm^{-3} = 1750 kg m^{-3}, and X = 10%. Substituting these values into Eq. (8.18):

$$N_{JS} = \frac{\left(\begin{array}{c} 5.68\,(10^{-6}\ \text{m}^2\ \text{s}^{-1})^{0.1}\,(2\times10^{-3}\ \text{m})^{0.2} \\ \left[9.8066\ \text{m s}^{-2}\,(1750\ \text{kg m}^{-3} - 1000\ \text{kg m}^{-3})\,/\,1000\ \text{kg m}^{-3}\right]^{0.45}\,10^{0.13} \end{array}\right)}{(0.3\ \text{m})^{0.85}}$$

$$= 3.79\ \text{s}^{-1}$$

Converting to rpm:

$$N_{JS} = 3.79\ \text{s}^{-1} \cdot \left|\frac{60\ \text{s}}{1\ \text{min}}\right| = 227\ \text{min}^{-1} = 227\ \text{rpm}$$

As flow was shown in **(b)** to be turbulent at 25 rpm, the flow regime at the higher stirrer speed of 226 rpm will also be turbulent. From Eq. (8.9):

$$P = 0.35\,(1000\ \text{kg m}^{-3})\,(3.79\ \text{s}^{-1})^3\,(0.3\ \text{m})^5 = 46.3\ \text{kg m}^2\ \text{s}^{-3}$$

Converting units using 1 W = 1 kg m^2 s^{-3} (Table A.8, Appendix A):

$$P = 46.3\ \text{W}$$

Answer: The stirrer speed required is increased 9-fold to 227 rpm and the power required is increased by a factor of about 730 to 46 W.

8.11 Particle suspension and scale-up

The equation for the volume of a cylindrical tank is:

$$V_L = \pi\left(\frac{D_T}{2}\right)^2 H_L$$

When the liquid height is equal to the tank diameter, this equation becomes:

$$V_L = \frac{\pi}{4} D_T^3 \tag{1}$$

The volume change on scale-up is $V_{L2} = 90V_{L1}$, where subscript 1 refers to the small-scale vessel and subscript 2 refers to the vessel after scale-up. From (1), this can be expressed as:

$$90 = \frac{V_{L2}}{V_{L1}} = \frac{\frac{\pi}{4} D_{T2}^3}{\frac{\pi}{4} D_{T1}^3} = \frac{D_{T2}^3}{D_{T1}^3}$$

$$D_{T2}^3 = 90 D_{T1}^3$$

$$D_{T2} = 4.48 D_{T1}$$

As the scaled-up vessel is geometrically similar to the small-scale tank, this relationship applies also to the impeller diameter:

$$D_{i2} = 4.48 D_{i1}$$

$$\frac{D_{i1}}{D_{i2}} = 0.223$$

As the process is carried out under anaerobic conditions, the system is not gassed.

(a)

If Eq. (8.18) were applied to each vessel, the only term that would change is the denominator. Therefore, we can say that:

$$\frac{N_{JS2}}{N_{JS1}} = \left(\frac{D_{i1}}{D_{i2}}\right)^{0.85} = (0.223)^{0.85} = 0.279$$

Therefore, $N_{JS2} = 0.279\ N_{JS1}$.

Answer: The stirrer speed required after scale-up is 28% of that required in the smaller-scale vessel

(b)

(i)

From Eq. (8.9), because N_P' and ρ do not change with scale-up:

$$\frac{P_2}{P_1} = \frac{N_{i2}^3 D_{i2}^5}{N_{i1}^3 D_{i1}^5} \tag{2}$$

Using $N_i = N_{JS}$, substituting values gives:

$$\frac{P_2}{P_1} = (0.279)^3\,(4.48)^5 = 39.2$$

Answer: The power required is increased by a factor of 39

(ii)

An equation for power per unit volume can be obtained from (2):

$$\frac{P_2 / V_2}{P_1 / V_1} = \frac{N_{i2}^3 D_{i2}^5 V_1}{N_{i1}^3 D_{i1}^5 V_2}$$

Substituting values:

$$\frac{P_2 / V_2}{P_1 / V_1} = (0.279)^3\,(4.48)^5\,\frac{1}{90} = 0.435$$

Answer: The power per unit volume required after scale-up is 44% of that required in the smaller-scale vessel

8.12 Impeller diameter, mixing, and power requirements

(a)

From Eq. (8.18), the dependence of N_{JS} on D_i is:

$$N_{JS} \propto S D_i^{-0.85}$$

S must be included in this relationship because S depends on D_i. From Eq. (8.19) with $\alpha = 1.5$ for a Rushton turbine:

$$S \propto D_i^{-1.5}$$

Combining these equations gives:

$$N_{JS} \propto D_i^{-1.5} D_i^{-0.85}$$

$$N_{JS} \propto D_i^{-2.35} \tag{1}$$

In the turbulent regime, the relationship between P and D_i is given by Eq. (8.9). For just complete solids suspension, the stirrer speed is equal to N_{JS}. As N_P' depends on D_i, we can write:

$$P \propto N_P' N_{JS}^3 D_i^5$$

Substituting (1) into this equation:

$$P \propto N_P' (D_i^{-2.35})^3 D_i^5$$

$$P \propto N_P' D_i^{-2.05}$$

Answer: The power required is proportional to $N_P' D_i^{-2.05}$

(b)

From **(a)**, the ratio of power requirements for the two different impeller diameters is:

$$\frac{P_{0.50}}{P_{0.33}} = \frac{(N_P')_{0.50}\left(\frac{D_T}{2}\right)^{-2.05}}{(N_P')_{0.33}\left(\frac{D_T}{3}\right)^{-2.05}} = \frac{(N_P')_{0.50}}{(N_P')_{0.33}}\left(\frac{3}{2}\right)^{-2.05}$$

where subscripts 0.50 and 0.33 refer to impellers with diameter 1/2 and 1/3 the tank diameter, respectively. For a Rushton turbine, $N_P' = 5.0$ for $D_i/D_T = 0.33$ (Table 8.1) and $N_P' = 5.9$ for $D_i/D_T = 0.50$ (Table 8.2). Substituting these values into the above equation:

$$\frac{P_{0.50}}{P_{0.33}} = \frac{5.9}{5.0}\left(\frac{3}{2}\right)^{-2.05} = 0.51$$

Answer: The power required using an impeller diameter of one-half the tank diameter is 51% of that required using an impeller diameter of one-third the tank diameter

(c)

Assume that the impeller off-bottom clearance is such that complete gas dispersion for a Rushton turbine is represented by Eq. (8.4). An expression for the gas flow rate F_g under these conditions is obtained by substituting the definitions of Eqs (8.1) and (8.2) into Eq. (8.4):

$$\frac{F_g}{N_i D_i^3} = 0.2\left(\frac{D_i}{D_T}\right)^{0.5} \frac{N_i D_i^{0.5}}{g^{0.5}}$$

Grouping terms gives:

$$\frac{F_g D_T^{0.5} g^{0.5}}{0.2} D_i^{-4} = N_i^2$$

When F_g, D_T and g remain constant, this relationship can be written as:

$$N_i^2 \propto D_i^{-4}$$

$$N_i \propto D_i^{-2} \tag{2}$$

where N_i is the stirrer speed required for complete gas dispersion. In the turbulent regime, the relationship between P and D_i is given by Eq. (8.9). Assuming that the percentage reduction in power with sparging is the same for Rushton turbines of different diameter, we can write:

$$P \propto N_P' N_i^3 D_i^5$$

For complete gas dispersion, N_i depends on D_i according to (2). Therefore:

$$P \propto N_P' D_i^{-6} D_i^5$$

$$P \propto N_P' D_i^{-1}$$

Answer: The power required is proportional to $N_P' D_i^{-1}$

(d)

From the result in **(c)**, we can write:

$$\frac{P_{0.50}}{P_{0.33}} = \frac{(N_P')_{0.50}\left(\frac{D_T}{2}\right)^{-1}}{(N_P')_{0.33}\left(\frac{D_T}{3}\right)^{-1}} = \frac{(N_P')_{0.50}}{(N_P')_{0.33}}\left(\frac{3}{2}\right)^{-1}$$

where subscripts 0.50 and 0.33 refer to impellers with diameter 1/2 and 1/3 the tank diameter, respectively. As $N_P' = 5.0$ for $D_i/D_T = 0.33$ (Table 8.1) and $N_P' = 5.9$ for $D_i/D_T = 0.50$ (Table 8.2):

$$\frac{P_{0.50}}{P_{0.33}} = \frac{5.9}{5.0}\left(\frac{3}{2}\right)^{-1} = 0.79$$

Answer: Assuming that the percentage reduction in power with sparging is the same for both impellers, the power required using an impeller diameter of one-half the tank diameter is 79% of that required using an impeller diameter of one-third the tank diameter

8.13 Efficiency of different impellers for solids suspension

Assume that the tank is not sparged. The stirrer speed required for complete solids suspension is given by Eq. (8.18). For different impellers of the same size, all the terms in Eq. (8.18) remain constant except for S. Therefore, we can write:

$$N_{JS} \propto S$$

In the turbulent regime without gassing, the equation for power as a function of operating conditions is Eq. (8.9). From Eq. (8.9), for complete solids suspension using different impellers of the same diameter we can write:

$$P \propto N_P' N_{JS}^3$$

Combining these equations gives:

$$P \propto N_P' S^3$$

Therefore, the ratio of power requirements for the pitched-blade and Rushton turbines is:

$$\frac{P_{PB}}{P_R} = \frac{(N_P')_{PB} S_{PB}^3}{(N_P')_R S_R^3}$$

where subscripts PB and R refer to the pitched-blade and Rushton turbines, respectively. From Table 8.2, $N_P' = 1.6$ for a downward-pumping, six-blade pitched-blade turbine with $D_i/D_T = 0.50$, and $N_P' = 5.9$ for a Rushton turbine with $D_i/D_T = 0.50$. Let us assume that $C_i/D_T = 0.25$ for both impeller systems. From Table 8.4, $S = 5.7$ for the downward-pumping, six-blade pitched-blade turbine, and $S = 4.25$ for the Rushton turbine. Substituting these values into the above equation:

$$\frac{P_{PB}}{P_R} = \frac{1.6\,(5.7)^3}{5.9\,(4.25)^3} = 0.65$$

Answer: The power required by the pitched-blade turbine is 65% of that required by the Rushton turbine

8.14 Power and mixing time with aeration

$D_T = H_L = 2$ m. $D_i = D_T/3 = (2\text{ m})/3 = 0.667$ m. From Figure 8.29 and Table 8.1, without gassing, $N_P' = 5.0$ for a Rushton turbine of this geometry. From Section 2.4.1, $\rho = 1$ g cm^{-3} = 1000 kg m^{-3}. $\mu = 4$ cP; therefore, from Table A.9 (Appendix A), $\mu = 4 \times 10^{-3}$ kg m^{-1} s^{-1}.

(a)

For a cylindrical tank, the liquid volume V_L is:

$$V_L = \pi\left(\frac{D_T}{2}\right)^2 H_L$$

Substituting values:

$$V_L = \pi\left(\frac{2\text{ m}}{2}\right)^2 2\text{ m} = 6.28\text{ m}^3$$

Therefore, if P/V_L must not exceed 1.5 kW m^{-3}, the maximum P = 1.5 kW m^{-3} × 6.28 m^3 = 9.42 kW = 9420 W. From Table A.8 (Appendix A), 1 W = 1 kg m^2 s^{-3}; therefore, P = 9420 kg m^2 s^{-3}. Let us assume that flow is turbulent: this is checked below. The stirrer speed is determined by rearranging Eq. (8.9):

$$N_i^3 = \frac{P}{N_P'\rho D_i^5}$$

Substituting values:

$$N_i^3 = \frac{9420\text{ kg m}^2\text{ s}^{-3}}{5.0\,(1000\text{ kg m}^{-3})\,(0.667\text{ m})^5} = 14.27\text{ s}^{-3}$$

$$N_i = 2.43\text{ s}^{-1}$$

Converting to rpm:

$$N_i = 2.43\text{ s}^{-1}.\left|\frac{60\text{ s}}{1\text{ min}}\right| = 146\text{ min}^{-1} = 146\text{ rpm}$$

To check whether this stirrer speed corresponds to turbulent flow, calculate the impeller Reynolds number using Eq. (7.2):

$$Re_i = \frac{2.43\text{ s}^{-1}\,(0.667\text{ m})^2\,(1000\text{ kg m}^{-3})}{4\times10^{-3}\text{ kg m}^{-1}\text{ s}^{-1}} = 2.70\times10^5$$

As this value is greater than the threshold of 10^4 for turbulent flow (Section 8.5.1 and Figure 8.29), we can say that flow is turbulent. This justifies the above application of Eq. (8.9) and N_P'.

Answer: 146 rpm

(b)

The mixing time is calculated using Eq. (8.24):

$$t_m = \frac{5.4}{2.43\ \text{s}^{-1}} \left(\frac{1}{5.0} \right)^{1/3} (3)^2 = 11.7\ \text{s}$$

Answer: 12 s

(c)

With gassing, $N_P' = 0.5 \times 5.0 = 2.5$. Repeating the calculation in **(a)** with $N_P' = 2.5$:

$$N_i^3 = \frac{9420\ \text{kg m}^2\ \text{s}^{-3}}{2.5\ (1000\ \text{kg m}^{-3})\ (0.667\ \text{m})^5} = 28.54\ \text{s}^{-3}$$

$$N_i = 3.06\ \text{s}^{-1}$$

Converting to rpm:

$$N_i = 3.06\ \text{s}^{-1} . \left| \frac{60\ \text{s}}{1\ \text{min}} \right| = 184\ \text{min}^{-1} = 184\ \text{rpm}$$

As this stirrer speed is higher than that found in **(a)**, flow must be turbulent, justifying the application of Eq. (8.9) and N_P'.

Answer: 184 rpm

(d)

Eq. (8.24) applies under aerated conditions provided the gas is dispersed effectively (Section 8.10). Therefore, the mixing time under the conditions determined in **(c)** is:

$$t_m = \frac{5.4}{3.06\ \text{s}^{-1}} \left(\frac{1}{2.5} \right)^{1/3} (3)^2 = 11.7\ \text{s}$$

Answer: 12 s. *This is the same answer as in **(b)**. The power P in Eq. (8.22) did not change with gassing because the operating stirrer speed was increased. Therefore, the effects of increased stirrer speed and decreased N_P' cancelled each other exactly.*

8.15 Scale-up of mixing system

(a)

$D_T = H_L = 15$ cm = 0.15 m. $D_i = 5$ cm = 0.05 m. $N_i = 800$ rpm = 800 min^{-1}. From Section 2.4.1, $\rho = 1$ g cm^{-3} = 1000 kg m^{-3}. From Eq. (7.2), an equation for μ is:

$$\mu = \frac{N_i D_i^2 \rho}{Re_i} \tag{1}$$

Substituting values with $Re_i = 10^4$ for turbulence:

$$\mu = \frac{\left(800\ \text{min}^{-1} . \left| \frac{1\ \text{min}}{60\ \text{s}} \right| \right) (0.05\ \text{m})^2\ (1000\ \text{kg m}^{-3})}{10^4} = 3.33 \times 10^{-3}\ \text{kg m}^{-1}\ \text{s}^{-1}$$

From Table A.9 (Appendix A), 1 cP = 10^{-3} kg m^{-1} s^{-1}; therefore, $\mu = 3.33$ cP.

Answer: 3.3 cP

(b)

D_i/D_T = (5 cm)/(15 cm) = 0.33. From Figure 8.29 and Table 8.1, $N_P' = 5.0$ for a Rushton turbine of this geometry. The mixing time is calculated using Eq. (8.24):

$$t_m = \frac{5.4}{800\ \text{min}^{-1}.\left|\frac{1\ \text{min}}{60\ \text{s}}\right|}\left(\frac{1}{5.0}\right)^{1/3}(3)^2 = 2.13\ \text{s}$$

Answer: 2.1 s

(c)

After scale-up, $D_T = H_L = 2.25$ m and $D_i = 0.75$ m. The impeller tip speed in the laboratory vessel is:

$$\text{Impeller tip speed} = \pi N_i D_i = \pi\left(800\ \text{min}^{-1}.\left|\frac{1\ \text{min}}{60\ \text{s}}\right|\right)(0.05\ \text{m}) = 2.09\ \text{m s}^{-1}$$

Applying this tip speed to the large fermenter to calculate the stirrer speed:

$$N_i = \frac{\text{Impeller tip speed}}{\pi D_i} = \frac{2.09\ \text{m s}^{-1}}{\pi(0.75\ \text{m})} = 0.89\ \text{s}^{-1}$$

From (1), this tip speed can be used to determine the viscosity for $Re_i = 10^4$ in the large fermenter:

$$\mu = \frac{0.89\ \text{s}^{-1}\,(0.75\ \text{m})^2\,(1000\ \text{kg m}^{-3})}{10^4} = 0.05\ \text{kg m}^{-1}\,\text{s}^{-1}$$

From Table A.9 (Appendix A), 1 cP = 10^{-3} kg m^{-1} s^{-1}; therefore, μ = 50 cP.

Answer: With scale-up, the maximum allowable viscosity for turbulence is increased 15-fold to 50 cP

(d)

With scale-up, D_i/D_T remains equal to 0.33 so that N_P' remains at 5.0. The mixing time is calculated using Eq. (8.24):

$$t_m = \frac{5.4}{0.89\ \text{s}^{-1}}\left(\frac{1}{5.0}\right)^{1/3}(3)^2 = 31.9\ \text{s}$$

Answer: With scale-up, the mixing time is increased 15-fold to 32 s

8.16 Effect on mixing of scale-up at constant power per unit volume

$V_{L2} = 50V_{L1}$, where subscripts 1 and 2 refer to the pilot- and production-scale fermenters, respectively. The equation for the volume of a cylindrical tank is:

$$V_L = \pi\left(\frac{D_T}{2}\right)^2 H_L$$

When $H_L = D_T$, rearrangement of this equation gives an expression for D_T as a function of V_L:

$$D_T = \left(\frac{4V_L}{\pi}\right)^{1/3}$$

For the pilot-scale vessel, this equation can be written as:

$$D_{T1} = \left(\frac{4V_{L1}}{\pi}\right)^{1/3} = 1.08V_{L1}^{1/3} \qquad (1)$$

For the production-scale vessel, the equation becomes:

$$D_{T2} = \left(\frac{4V_{L2}}{\pi}\right)^{1/3} = \left(\frac{4(50V_{L1})}{\pi}\right)^{1/3} = 3.99V_{L1}^{1/3} \quad (2)$$

Assume that flow is turbulent in both fermenters. Eq. (8.22) gives the mixing time as a function of operating conditions in the turbulent flow regime. Scale-up is carried out at constant power per unit volume P/V_L, the vessels are geometrically similar so that D_T/D_i remains constant, and ρ is the same for both vessels. Therefore, from Eq. (8.22), the ratio of mixing times in the two fermenters is:

$$\frac{t_{m2}}{t_{m1}} = \left(\frac{D_{T2}}{D_{T1}}\right)^{2/3}$$

Substituting the expressions from (1) and (2):

$$\frac{t_{m2}}{t_{m1}} = \left(\frac{3.99V_{L1}^{1/3}}{1.08V_{L1}^{1/3}}\right)^{2/3} = \left(\frac{3.99}{1.08}\right)^{2/3} = 2.39$$

Answer: The mixing time in the production-scale fermenter is 2.4-fold higher than in the pilot-scale vessel

8.17 Alternative impellers

$D_T = 2.3$ m; $V_L = 10$ m^3. $\rho = 1000$ kg m^{-3}. $\mu = 1$ cP; therefore, from Table A.9 (Appendix A), $\mu = 1 \times 10^{-3}$ kg m^{-1} s^{-1}.

(a)

$D_i = D_T/3 = (2.3\text{ m})/3 = 0.767$ m. $N_i = 60$ rpm $= 60$ min^{-1}. $P_g = 0.6P$.

(i)

Determine the flow regime by calculating the impeller Reynolds number using Eq. (7.2):

$$Re_i = \frac{60\text{ min}^{-1}.\left|\frac{1\text{ min}}{60\text{ s}}\right|(0.767\text{ m})^2(1000\text{ kg m}^{-3})}{1\times10^{-3}\text{ kg m}^{-1}\text{ s}^{-1}} = 5.88\times10^5$$

As this Re_i is greater than the threshold of 10^4 for turbulent flow (Section 8.5.1 and Figure 8.29), we can say that flow is turbulent. $N_P' = 5.0$ for a Rushton turbine of this geometry (Table 8.1). The power required without gassing is evaluated using Eq. (8.9):

$$P = 5.0\,(1000\text{ kg m}^{-3})\left(60\text{ min}^{-1}.\left|\frac{1\text{ min}}{60\text{ s}}\right|\right)^3(0.767\text{ m})^5 = 1.33\times10^3\text{ kg m}^2\text{ s}^{-3}$$

From Table A.8 (Appendix A), 1 W = 1 kg m^2 s^{-3}; therefore:

$$P = 1.33\times10^3\text{ W}$$

The power draw with gassing $P_g = 0.6 \times 1.33 \times 10^3$ W $= 798$ W.

Answer: 798 W

(ii)

Eq. (8.22) applies under aerated conditions provided the gas is dispersed effectively (Section 8.10). Substituting values into Eq. (8.22):

$$t_m = 5.9\,(2.3\text{ m})^{2/3}\left(\frac{1000\text{ kg m}^{-3}\times 10\text{ m}^3}{798\text{ W}\,.\left|\frac{1\text{ kg m}^2\text{ s}^{-3}}{1\text{ W}}\right|}\right)^{1/3}(3)^{1/3} = 34.4\text{ s}$$

Answer: 34 s

(b)

(i)

$D_T/D_i = 2$. Assume that flow is turbulent: this is checked below in **(b) (iii)**. An expression for the power required to achieve a given mixing time is obtained by rearranging Eq. (8.22):

$$P^{1/3} = \frac{5.9\,D_T^{2/3}\left(\rho V_L\right)^{1/3}\left(\frac{D_T}{D_i}\right)^{1/3}}{t_m}$$

Substituting values using the mixing time determined in **(a) (ii)**:

$$P^{1/3} = \frac{5.9\,(2.3\text{ m})^{2/3}(1000\text{ kg m}^{-3}\times 10\text{ m}^3)^{1/3}(2)^{1/3}}{34.4\text{ s}} = 8.11\text{ kg}^{1/3}\text{ m}^{2/3}\text{ s}^{-1}$$

$$P = 534\text{ kg m}^2\text{ s}^{-3}$$

From Table A.8 (Appendix A), 1 W = 1 kg m^2 s^{-3}; therefore:

$$P = 534\text{ W}$$

The power savings compared with the result for the smaller Rushton turbine in **(a) (i)** is (798 – 534) W/(798 W) × 100% = 33%.

Answer: 33%

(ii)

Assume that flow is turbulent: this is checked below in **(b) (iii)**. Repeating the calculation in **(b) (i)** with $D_T/D_i = 2.5$:

$$P^{1/3} = \frac{5.9\,(2.3\text{ m})^{2/3}(1000\text{ kg m}^{-3}\times 10\text{ m}^3)^{1/3}(2.5)^{1/3}}{34.4\text{ s}} = 8.74\text{ kg}^{1/3}\text{ m}^{2/3}\text{ s}^{-1}$$

$$P = 667\text{ kg m}^2\text{ s}^{-3} = 667\text{ W}$$

The power savings compared with the result for the smaller Rushton turbine in **(a) (i)** is (798 – 667) W/(798 W) × 100% = 16%.

Answer: 16%

(iii)

The relationship between power and stirrer speed in the turbulent regime without gassing is given by Eq. (8.9). Rearranging this equation to derive an expression for N_i:

$$N_i^3 = \frac{P}{N_P'\rho D_i^5}$$

$$N_i = \left(\frac{P}{N_P'\rho D_i^5}\right)^{1/3} \qquad (1)$$

P in this equation is the ungassed power draw. For the larger Rushton turbine, from **(b) (i)**, if the power required under aerated conditions to achieve the desired mixing time is P_g = 534 W = 534 kg m^2 s^{-3}, the ungassed power corresponding to the required stirrer speed is P = 2 × 534 kg m^2 s^{-3} = 1068 kg m^2 s^{-3}. D_i = 0.5D_T = 0.5 × 2.3 m = 1.15 m. From Table 8.2, for a Rushton turbine with D_i/D_T = 0.50, N_P' = 5.9. Substituting values into (1) gives:

$$N_i = \left(\frac{1068 \text{ kg m}^2 \text{ s}^{-3}}{5.9\,(1000 \text{ kg m}^{-3})\,(1.15 \text{ m})^5} \right)^{1/3} = 0.45 \text{ s}^{-1}$$

Converting to rpm:

$$N_i = 0.45 \text{ s}^{-1} . \left| \frac{60 \text{ s}}{1 \text{ min}} \right| = 27 \text{ min}^{-1} = 27 \text{ rpm}$$

Check that this stirrer speed corresponds to turbulent flow by calculating the impeller Reynolds number using Eq. (7.2):

$$Re_i = \frac{0.45 \text{ s}^{-1}\,(1.15 \text{ m})^2\,(1000 \text{ kg m}^{-3})}{1\times10^{-3} \text{ kg m}^{-1} \text{ s}^{-1}} = 5.95\times10^5$$

As this value is greater than the threshold of 10^4 for turbulent flow (Section 8.5.1), we can say that flow is turbulent. This justifies application of Eqs (8.9) and (8.22) and N_P' for the Rushton turbine.

For the A315 hydrofoil, from **(b) (ii)**, if the power required under aerated conditions is P_g = 667 W = 667 kg m^2 s^{-3}, the ungassed power corresponding to the required stirrer speed is 2 × 667 kg m^2 s^{-3} = 1334 kg m^2 s^{-3}. D_i = 0.4D_T = 0.4 × 2.3 m = 0.92 m. From Table 8.2, for a A315 hydrofoil with D_i/D_T = 0.40, N_P' = 0.84. Substituting values into (1) gives:

$$N_i = \left(\frac{1334 \text{ kg m}^2 \text{ s}^{-3}}{0.84\,(1000 \text{ kg m}^{-3})\,(0.92 \text{ m})^5} \right)^{1/3} = 1.34 \text{ s}^{-1}$$

Converting to rpm:

$$N_i = 1.34 \text{ s}^{-1} . \left| \frac{60 \text{ s}}{1 \text{ min}} \right| = 80.4 \text{ min}^{-1} = 80.4 \text{ rpm}$$

Check that this stirrer speed corresponds to turbulent flow by calculating the impeller Reynolds number using Eq. (7.2):

$$Re_i = \frac{1.34 \text{ s}^{-1}\,(0.92 \text{ m})^2\,(1000 \text{ kg m}^{-3})}{1\times10^{-3} \text{ kg m}^{-1} \text{ s}^{-1}} = 1.13\times10^6$$

As this value is greater than the threshold of 10^4 for turbulent flow (Section 8.5.1), we can say that flow is turbulent. This justifies the application of Eqs (8.9) and (8.22) and N_P' for the A315 hydrofoil impeller.

Answer: 27 rpm for the Rushton turbine; 80 rpm for the A315 hydrofoil

8.18 Retrofitting

D_T = 1.8 m; V_L = 4.6 m^3. ρ = 1000 kg m^{-3}. μ = 20 cP; therefore, from Table A.9 (Appendix A), μ = 20 × 10^{-3} kg m^{-1} s^{-1}.

As the value of the power number for turbulent flow N_P' depends on the tank geometry (Section 8.5.1), we need to know the vessel height H_L. This can be calculated from the equation for the volume of a cylindrical tank:

$$V_L = \pi\left(\frac{D_T}{2}\right)^2 H_L$$

Rearranging terms gives:

$$H_L = \frac{V_L}{\pi\left(\frac{D_T}{2}\right)^2}$$

Substituting values:

$$H_L = \frac{4.6\ \text{m}^3}{\pi\left(\frac{1.8\ \text{m}}{2}\right)^2} = 1.81\ \text{m}$$

Therefore, $H_L/D_T = 1$.

(a)

For the Rushton turbine, $D_i = 0.6$ m; therefore, $D_i/D_T = (0.6\ \text{m})/(1.8\ \text{m}) = 0.33$. $N_i = 150$ rpm $= 150\ \text{min}^{-1}$. Converting the stirrer speed to s^{-1}:

$$N_i = 150\ \text{min}^{-1} . \left|\frac{1\ \text{min}}{60\ \text{s}}\right| = 2.5\ \text{s}^{-1}$$

To compare the performance of the impellers, we will assume that the system is ungassed. To avoid having to modify the stirrer motor or drive assembly, impeller retrofitting is carried out so that the power draw and stirrer speed remain the same (Section 8.14). First, calculate the power draw for operation of the Rushton turbine at 150 rpm before retrofitting. Check that this stirrer speed corresponds to turbulent flow by calculating the impeller Reynolds number using Eq. (7.2):

$$Re_i = \frac{2.5\ \text{s}^{-1}\ (0.6\ \text{m})^2\ (1000\ \text{kg m}^{-3})}{20\times10^{-3}\ \text{kg m}^{-1}\ \text{s}^{-1}} = 4.50\times10^4$$

As this value is greater than the threshold of 10^4 for turbulent flow (Section 8.5.1 and Figure 8.29), we can say that flow is turbulent. Therefore, the power requirements can be determined using Eq. (8.9). $N_P' = 5.0$ for a Rushton turbine with $D_i/D_T = 0.33$ (Table 8.1). Substituting values into Eq. (8.9):

$$P = 5.0\ (1000\ \text{kg m}^{-3})\ (2.5\ \text{s}^{-1})^3\ (0.6\ \text{m})^5 = 6075\ \text{kg m}^2\ \text{s}^{-3}$$

For the retrofitted curved-blade disc turbine, we will assume that $N_P' = 1.5$ (Table 8.2). For turbulent flow conditions, the impeller diameter can be evaluated by rearranging Eq. (8.9):

$$D_i^5 = \frac{P}{N_P'\rho N_i^3} \tag{1}$$

Substituting values using the same P and N_i as for the Rushton turbine:

$$D_i^5 = \frac{6075\ \text{kg m}^2\ \text{s}^{-3}}{1.5\ (1000\ \text{kg m}^{-3})\ (2.5\ \text{s}^{-1})^3} = 0.26\ \text{m}^5$$

$$D_i = 0.76\ \text{m}$$

As this diameter is greater than the diameter of the Rushton turbine, the value of the Re_i for this impeller will be greater than the 4.50×10^4 determined above, indicating that operation with the curved-blade disc turbine is also in the turbulent regime. This justifies the use of Eq. (8.9) and N_P' in the calculation; however, the value of 1.5 for N_P' applies to $D_i/D_T = 0.33$ (Table 8.2), whereas we have calculated an

impeller diameter giving D_i/D_T = (0.76 m)/(1.8 m) = 0.42. Therefore, an assumption in our calculation is that, for the curved-blade disc turbine, N_P' for $D_i/D_T = 0.42$ is the same as that for $D_i/D_T = 0.33$.

For the retrofitted hydrofoil impeller with characteristics similar to those of the Lightnin A315, we will assume that $N_P' = 0.84$ (Table 8.2). Substituting values into (1) using the same P and N_i as for the Rushton turbine:

$$D_i^5 = \frac{6075 \text{ kg m}^2 \text{ s}^{-3}}{0.84\,(1000 \text{ kg m}^{-3})\,(2.5 \text{ s}^{-1})^3} = 0.46 \text{ m}^5$$

$$D_i = 0.86 \text{ m}$$

As this diameter is even larger than for the other impellers, operation of the hydrofoil is in the turbulent regime. The value of 0.84 for N_P' applies to $D_i/D_T = 0.40$ (Table 8.2), whereas the diameter calculated gives D_i/D_T = (0.86 m)/(1.8 m) = 0.48. Therefore, we assume for the hydrofoil that N_P' for $D_i/D_T = 0.48$ is the same as that for $D_i/D_T = 0.40$.

Answer: 0.76 m for the curved-blade disc turbine and 0.86 m for the hydrofoil impeller, assuming that the system is not gassed and N_P' for these impellers is not a strong function of impeller diameter. *The second assumption is not required if additional N_P' values were available for different impeller:tank diameter ratios. An iterative solution using different N_P' values for impellers of different size could then be found.*

(b)

Eq. (8.24) for mixing time applies irrespective of the type of impeller used (Section 8.10). Substituting values into Eq. (8.24) for the Rushton turbine:

$$t_m = \frac{5.4}{2.5 \text{ s}^{-1}} \left(\frac{1}{5.0}\right)^{1/3} \left(\frac{1.8 \text{ m}}{0.6 \text{ m}}\right)^2 = 11.4 \text{ s}$$

For the curved-blade disc turbine:

$$t_m = \frac{5.4}{2.5 \text{ s}^{-1}} \left(\frac{1}{1.5}\right)^{1/3} \left(\frac{1.8 \text{ m}}{0.76 \text{ m}}\right)^2 = 10.6 \text{ s}$$

For the hydrofoil impeller:

$$t_m = \frac{5.4}{2.5 \text{ s}^{-1}} \left(\frac{1}{0.84}\right)^{1/3} \left(\frac{1.8 \text{ m}}{0.86 \text{ m}}\right)^2 = 10.0 \text{ s}$$

Answer: The mixing times are 11.4 s for the Rushton turbine, 10.6 s for the curved-blade disc turbine, and 10.0 s for the hydrofoil impeller. Although retrofitting reduces the mixing time, the percentage reduction is small at 7–12%. Therefore, the cost of retrofitting is difficult to justify based on the expected mixing times.

(c)

(i)

For future new fermenter installations without gassing, the hydrofoil impeller is likely to give the best mixing performance with strong axial velocities and low power consumption (Section 8.4.4, Hydrofoil Impellers subsection).

Answer: Hydrofoil impeller

(ii)

As downward-pumping hydrofoil impellers do not perform well with gassing (Section 8.4.4, Hydrofoil Impellers subsection), the curved-blade disc turbine is the best choice under aerated conditions. As

described in Section 8.4.4 (Curved-Blade Disc Turbines subsection), because large ventilated cavities do not form behind the blades, these impellers are more difficult to flood than Rushton turbines and do not suffer significant power loss with aeration.

Answer: Curved-blade disc turbine

8.19 Retrofitting multiple impellers

D_T = 1.9 m. $H_L = 3D_T$ = 5.7 m. D_i/D_T = 1/3; therefore, D_i = (1.9 m)/3 = 0.633 m. N_i = 1.2 rps = 1.2 s^{-1}. We will assume that the density and viscosity of the culture broth are the same as those of water; therefore, ρ = 1 g cm^{-3} (Section 2.4.1) = 1000 kg m^{-3} and, from Eq. (7.8), μ = 1 cP = 10^{-3} kg m^{-1} s^{-1}. To avoid having to modify the stirrer motor or drive assembly, impeller retrofitting is carried out so that the power draw and stirrer speed remain the same (Section 8.14).

First, calculate the power draw for operation of the three Rushton turbines at 1.2 rps before retrofitting. Because the flow patterns generated by each impeller do not interact significantly, we can estimate the power requirements for each turbine as if it were a single impeller in a tank with $H_L = D_T$. To check whether 1.2 rps corresponds to turbulent flow, calculate the impeller Reynolds number using Eq. (7.2):

$$Re_i = \frac{1.2\ \text{s}^{-1}\,(0.633\ \text{m})^2\,(1000\ \text{kg m}^{-3})}{10^{-3}\ \text{kg m}^{-1}\,\text{s}^{-1}} = 4.81\times10^5$$

As this value is greater than the threshold of 10^4 for turbulent flow (Section 8.5.1 and Figure 8.29), we can say that flow in the fermenter is turbulent. Therefore, the power requirements without gassing can be determined using Eq. (8.9). For a Rushton turbine with D_i/D_T = 0.33, N_P' = 5.0 (Table 8.1). Substituting values into Eq. (8.9):

$$P = 5.0\,(1000\ \text{kg m}^{-3})\,(1.2\ \text{s}^{-1})^3\,(0.633\ \text{m})^5 = 878.1\ \text{kg m}^2\,\text{s}^{-3}$$

Therefore, the total power draw for the three Rushton turbines = 3 × 878.1 kg m^2 s^{-3} = 2634.3 kg m^2 s^{-3}.

For the system after retrofitting, calculate the power drawn by the lower curved-blade disc turbine operated at 1.2 rps. As the diameter and operating speed of this turbine are the same as those of the Rushton turbines, Re_i is equal to 4.81 × 10^5. Therefore, the curved-blade disc turbine generates turbulent flow and Eq. (8.9) can be used to evaluate the power requirement. For a curved-blade disc turbine with D_i/D_T = 0.33, N_P' = 1.5 (Table 8.2). Substituting values into Eq. (8.9):

$$P = 1.5\,(1000\ \text{kg m}^{-3})\,(1.2\ \text{s}^{-1})^3\,(0.633\ \text{m})^5 = 263.4\ \text{kg m}^2\,\text{s}^{-3}$$

To keep the total power draw the same before and after retrofitting, the power used by the two hydrofoil impellers must be (2634.3 – 263.4) kg m^2 s^{-3} = 2370.9 kg m^2 s^{-3}. Therefore, for each hydrofoil, the power draw is (2370.9 ÷ 2) kg m^2 s^{-3} = 1185.5 kg m^2 s^{-3}. The diameter of the hydrofoil impellers is determined by rearranging Eq. (8.9):

$$D_i^5 = \frac{P}{N_P'\rho N_i^3}$$

Substituting values with N_P' = 0.9:

$$D_i^5 = \frac{1185.5\ \text{kg m}^2\,\text{s}^{-3}}{0.9\,(1000\ \text{kg m}^{-3})\,(1.2\ \text{s}^{-1})^3} = 0.76\ \text{m}^5$$

$$D_i = 0.95\ \text{m}$$

To check that this impeller diameter is realistic for the vessel size, D_i/D_T = (0.95 m)/(1.9 m) = 0.50, which is reasonable.

Answer: 0.95 m, assuming that the power requirements for each impeller are not affected by the presence of the other impellers and the density and viscosity of the culture both are the same as those of water

8.20 Impeller viscometer

ρ = 1000 kg m^{-3}. n = 0.2. K = 0.05 N s^n m^{-2} = 0.05 N s$^{0.2}$ m^{-2}. From Table A.4 (Appendix A), 1 N = 1 kg m s^{-2}; therefore, K = 0.05 kg s$^{-1.8}$ m^{-1}.

(a)

D_i = 4 cm = 4 × 10^{-2} m. The highest operating stirrer speed is 10 rpm = 10 min^{-1}. Converting this to s^{-1}:

$$N_i = 10 \text{ min}^{-1} . \left| \frac{1 \text{ min}}{60 \text{ s}} \right| = 0.167 \text{ s}^{-1}$$

The flow regime corresponding to this stirrer speed is determined by calculating the impeller Reynolds number. For pseudoplastic fluids, Re_i is given by Eq. (8.12); the geometric constant k for a Rushton turbine can be taken as approximately 10 (Table 8.5). Substituting values into Eq. (8.12):

$$Re_i = \frac{(0.167 \text{ s}^{-1})^{2-0.2} (4 \times 10^{-2} \text{ m})^2 (1000 \text{ kg m}^{-3})}{0.05 \text{ kg s}^{-1.8} \text{ m}^{-1} (10)^{0.2-1}} = 8.06$$

As Re_i < 10, from Figure 8.31 and Section 8.5.2, flow at 10 rpm is laminar. Operation at lower stirrer speeds between 2.5 and 10 rpm will also generate laminar flow.

Answer: Yes

(b)

The lowest operating stirrer speed is 2.5 rpm = 2.5 min^{-1}. Converting this to s^{-1}:

$$N_i = 2.5 \text{ min}^{-1} . \left| \frac{1 \text{ min}}{60 \text{ s}} \right| = 0.042 \text{ s}^{-1}$$

The average shear rate in a stirred vessel is calculated using Eq. (8.34). Using k = 10 for a Rushton turbine (Table 8.5), for operation at 2.5 rpm:

$$\dot{\gamma}_{av} = 10 (0.042 \text{ s}^{-1}) = 0.42 \text{ s}^{-1}$$

Similarly, for operation at 10 rpm:

$$\dot{\gamma}_{av} = 10 (0.167 \text{ s}^{-1}) = 1.67 \text{ s}^{-1}$$

Answer: From 0.42 s^{-1} to 1.7 s^{-1}

(c)

The shear stress generated during flow of a pseudoplastic fluid can be estimated using Eq. (7.11). For operation of the viscometer at 2.5 rpm:

$$\tau = (0.05 \text{ kg s}^{-1.8} \text{ m}^{-1}) (0.42 \text{ s}^{-1})^{0.2} = 0.042 \text{ kg m}^{-1} \text{ s}^{-2}$$

From Table A.5 (Appendix A), 1 kg m^{-1} s^{-2} = 1 Pa; therefore τ = 0.042 Pa. Similarly, for operation at 10 rpm:

$$\tau = (0.05 \text{ kg s}^{-1.8} \text{ m}^{-1}) (1.67 \text{ s}^{-1})^{0.2} = 0.055 \text{ kg m}^{-1} \text{ s}^{-2} = 0.055 \text{ Pa}$$

Answer: From 0.042 Pa to 0.055 Pa

(d)

For the helical ribbon impeller, D_i = 7.5 cm = 7.5 × 10^{-2} m. For this type of impeller, the upper boundary for laminar flow can be taken as Re_i = 100 (Figure 8.30) and the geometric constant k = 30 (Table 8.5). The highest operating stirrer speed for laminar flow is obtained by rearranging Eq. (8.12):

$$N_i^{2-n} = \frac{Re_i K k^{n-1}}{D_i^2 \rho}$$

Substituting values gives:

$$N_i^{2-0.2} = \frac{100\,(0.05\ \text{kg s}^{-1.8}\ \text{m}^{-1})\,(30)^{0.2-1}}{(7.5\times10^{-2}\ \text{m})^2\,(1000\ \text{kg m}^{-3})}$$

$$N_i^{1.8} = 0.058\ \text{s}^{-1.8}$$

$$N_i = 0.206\ \text{s}^{-1}$$

Converting to rpm:

$$N_i = 0.206\ \text{s}^{-1}.\left|\frac{60\ \text{s}}{1\ \text{min}}\right| = 12.4\ \text{min}^{-1} = 12.4\ \text{rpm}$$

Answer: 12.4 rpm

(e)

The lowest stirrer speed for operation of the helical ribbon impeller is 2.5 rpm = 2.5 min^{-1}. Converting this to s^{-1}:

$$N_i = 2.5\ \text{min}^{-1}.\left|\frac{1\ \text{min}}{60\ \text{s}}\right| = 0.042\ \text{s}^{-1}$$

Therefore, from **(d)**, the stirrer speed range for the helical ribbon impeller is 0.042 s^{-1} to 0.206 s^{-1}. Using k = 30, the range of average shear rates is calculated using Eq. (8.34):

$$\dot{\gamma}_{av} = 30\,(0.042\ \text{s}^{-1})\ \text{to}\ 30\,(0.206\ \text{s}^{-1}) = 1.26\ \text{s}^{-1}\ \text{to}\ 6.18\ \text{s}^{-1}$$

This shear rate range is considerably wider than that calculated in **(b)** for the Rushton turbine. From Eq. (7.11), the average shear stress at the lowest stirrer speed is:

$$\tau = (0.05\ \text{kg s}^{-1.8}\ \text{m}^{-1})\,(1.26\ \text{s}^{-1})^{0.2} = 0.052\ \text{kg m}^{-1}\ \text{s}^{-2}$$

and the average shear stress at the highest stirrer speed is:

$$\tau = (0.05\ \text{kg s}^{-1.8}\ \text{m}^{-1})\,(6.18\ \text{s}^{-1})^{0.2} = 0.072\ \text{kg m}^{-1}\ \text{s}^{-2}$$

Using 1 kg m^{-1} s^{-2} = 1 Pa (Table A.5, Appendix A), the range of shear stresses generated is 0.052 Pa to 0.072 Pa. This range of shear rates is also considerably broader than that calculated in **(c)** for the Rushton turbine.

Answer: 1.26 to 6.18 s^{-1} and 0.052 to 0.072 Pa. The helical ribbon viscometer allows rheological investigation of fluids over a wider range of shear rates and shear stresses than the Rushton turbine viscometer.

8.21 Turbulent shear damage

D_p = 120 μm; therefore, from Table A.1 (Appendix A), $D_p = 120 \times 10^{-6}$ m. D_i = 20 cm = 0.2 m. V_L = 200 l; therefore, from Table A.2 (Appendix A), V_L = 0.2 m^3. ρ = 1010 kg m^{-3}. $\mu = 10^{-3}$ Pa s; therefore, from Table A.9 (Appendix A), $\mu = 10^{-3}$ kg m^{-1} s^{-1}.

To avoid shear damage, the Kolmogorov scale λ should remain above 2/3 to 1/2 the diameter of the microcarrier beads (Section 8.16.1, Interaction Between Microcarriers and Turbulent Eddies subsection). Given the uncertainties associated with this criterion and the effective volume for dissipation of turbulence kinetic energy, we will use as a conservative approximation that λ should remain above $D_p = 120 \times 10^{-6}$ m. The kinematic viscosity is calculated using Eq. (7.9):

$$\nu = \frac{10^{-3}\ \text{kg m}^{-1}\ \text{s}^{-1}}{1010\ \text{kg m}^{-3}} = 9.90 \times 10^{-7}\ \text{m}^2\ \text{s}^{-1}$$

An equation for the rate of dissipation of turbulence kinetic energy per unit mass of fluid ε is obtained by rearranging Eq. (7.36):

$$\varepsilon = \frac{\nu^3}{\lambda^4}$$

Substituting values with $\lambda = 120 \times 10^{-6}$ m:

$$\varepsilon = \frac{(9.90 \times 10^{-7}\ \text{m}^2\ \text{s}^{-1})^3}{(120 \times 10^{-6}\ \text{m})^4} = 4.68 \times 10^{-3}\ \text{m}^2\ \text{s}^{-3}$$

(a)

If the stirrer power is dissipated uniformly throughout the vessel, ε is related to the power P using Eq. (8.35). Rearranging Eq. (8.35) gives:

$$P = \varepsilon \rho V_L$$

Substituting values:

$$P = 4.68 \times 10^{-3}\ \text{m}^2\ \text{s}^{-3}\ (1010\ \text{kg m}^{-3})(0.2\ \text{m}^3) = 0.945\ \text{kg m}^2\ \text{s}^{-3}$$

In the turbulent flow regime, Eq. (8.9) relates P to the operating stirrer speed N_i. As gassing is provided through the reactor headspace, the culture liquid in the bioreactor is not sparged; therefore, there is no reduction in stirrer power due to gassing. Let us assume that flow is turbulent: this is checked below. As the value of N_P' depends on the impeller:tank diameter ratio, we must first calculate D_T. For a cylindrical tank:

$$V_L = \pi \left(\frac{D_T}{2} \right)^2 H_L$$

Assuming that $D_T = H_L$ for the bioreactor, we can write:

$$D_T^3 = \frac{4V_L}{\pi}$$

Substituting values gives:

$$D_T^3 = \frac{4(0.2\ \text{m}^3)}{\pi} = 0.255\ \text{m}^3$$

$$D_T = 0.634\ \text{m}$$

Therefore, D_i/D_T = (0.2 m)/(0.634 m) = 0.32. As this is close to D_i/D_T = 0.33, for a Rushton turbine we can use N_P' = 5.0 (Table 8.1). The stirrer speed is determined by rearranging Eq. (8.9):

$$N_i^3 = \frac{P}{N_P' \rho D_i^5} \tag{1}$$

Using the result for P:

$$N_i^3 = \frac{0.945 \text{ kg m}^2 \text{ s}^{-3}}{5.0\,(1010 \text{ kg m}^{-3})\,(0.2 \text{ m})^5} = 0.585 \text{ s}^{-3}$$

$$N_i = 0.84 \text{ s}^{-1}$$

Converting to rpm:

$$N_i = 0.84 \text{ s}^{-1} . \left| \frac{60 \text{ s}}{1 \text{ min}} \right| = 50 \text{ min}^{-1} = 50 \text{ rpm}$$

This is the stirrer speed corresponding to $\lambda = 120 \times 10^{-6}$ m. For $\lambda > 120 \times 10^{-6}$ m, $N_i < 50$ rpm; therefore 50 rpm is the maximum allowable stirrer speed to avoid cell shear damage. To check whether this stirrer speed corresponds to turbulent flow, calculate the impeller Reynolds number using Eq. (7.2):

$$Re_i = \frac{0.84 \text{ s}^{-1}\,(0.2 \text{ m})^2\,(1010 \text{ kg m}^{-3})}{10^{-3} \text{ kg m}^{-1} \text{ s}^{-1}} = 3.39 \times 10^4$$

As this value is greater than the threshold of 10^4 for turbulent flow (Section 8.5.1 and Figure 8.29), we can say that flow is turbulent. This justifies the application of Eq. (8.9) and N_P'.

Answer: 50 rpm

(b)

If the stirrer power is dissipated close to the impeller rather than throughout the vessel volume, ε is related to P by Eq. (8.36). Rearranging Eq. (8.36) gives:

$$P = \varepsilon \rho D_i^3$$

Substituting values:

$$P = 4.68 \times 10^{-3} \text{ m}^2 \text{ s}^{-3}\,(1010 \text{ kg m}^{-3})\,(0.2 \text{ m})^3 = 0.038 \text{ kg m}^2 \text{ s}^{-3}$$

Calculating the stirrer speed using (1):

$$N_i^3 = \frac{0.038 \text{ kg m}^2 \text{ s}^{-3}}{5.0\,(1010 \text{ kg m}^{-3})\,(0.2 \text{ m})^5} = 0.024 \text{ s}^{-3}$$

$$N_i = 0.29 \text{ s}^{-1}$$

Converting to rpm:

$$N_i = 0.29 \text{ s}^{-1} . \left| \frac{60 \text{ s}}{1 \text{ min}} \right| = 17 \text{ min}^{-1} = 17 \text{ rpm}$$

To check whether this stirrer speed corresponds to turbulent flow, calculate the impeller Reynolds number using Eq. (7.2):

$$Re_i = \frac{0.29 \text{ s}^{-1}\,(0.2 \text{ m})^2\,(1010 \text{ kg m}^{-3})}{10^{-3} \text{ kg m}^{-1} \text{ s}^{-1}} = 1.17 \times 10^4$$

This value is greater than the threshold of 10^4 for turbulent flow (Section 8.5.1 and Figure 8.29); therefore, flow is turbulent and Eq. (8.9) and N_P' apply.

Answer: 17 rpm. This result is only about 1/3 of the value determined in **(a)**.

*There is a substantial difference between the answers obtained in **(a)** and **(b)**. This illustrates the shortcomings of the Kolmogorov-scale approach for estimating the operating conditions required to avoid shear damage. The calculation depends on the distribution of the rate of turbulence kinetic energy dissipation within the vessel, which is difficult to know accurately.*

8.22 Avoiding cell damage

$E = 10^5$ J m^{-3}; therefore, from Table A.7 (Appendix A), $E = 10^5$ kg m^{-1} s^{-2}. Converting $\tau = 2.9$ days to s:

$$\tau = 2.9 \text{ days} \,.\left|\frac{24 \text{ h}}{1 \text{ day}}\right| .\left|\frac{60 \text{ min}}{1 \text{ h}}\right| .\left|\frac{60 \text{ s}}{1 \text{ min}}\right| = 2.51\times10^5 \text{ s}$$

$D_T = H_L = 0.73$ m. For a cylindrical tank, the liquid volume V is:

$$V = \pi\left(\frac{D_T}{2}\right)^2 H_L$$

Substituting values:

$$V = \pi\left(\frac{0.73 \text{ m}}{2}\right)^2 0.73 \text{ m} = 0.306 \text{ m}^3$$

$D_i = 25$ cm $= 0.25$ m; therefore, $D_i/D_T = (0.25 \text{ m})/(0.73 \text{ m}) = 0.34$. For a curved-blade disc turbine of this geometry, $N_P' = 1.5$ (Table 8.2). $\phi = 0.24$. $\rho = 1$ g cm^{-3} $= 1000$ kg m^{-3}. $\mu = 3.3 \times 10^{-3}$ Pa s; from Table A.9 (Appendix A), $\mu = 3.3 \times 10^{-3}$ kg m^{-1} s^{-1}.

An equation for the power P is obtained by rearranging Eq. (8.38):

$$P = \frac{EV}{\phi\tau}$$

Substituting values gives:

$$P = \frac{10^5 \text{ kg m}^{-1} \text{ s}^{-2} (0.306 \text{ m}^3)}{0.24 (2.51\times10^5 \text{ s})} = 0.508 \text{ kg m}^2 \text{ s}^{-3}$$

This is the maximum allowable value of P to avoid cell damage. Let us assume that flow in the bioreactor is turbulent: this is checked below. As the loss of power with gassing is negligible, the stirrer speed can be determined by rearranging Eq. (8.9):

$$N_i^3 = \frac{P}{N_P'\rho D_i^5}$$

Substituting values gives:

$$N_i^3 = \frac{0.508 \text{ kg m}^2 \text{ s}^{-3}}{1.5\,(1000 \text{ kg m}^{-3})\,(0.25 \text{ m})^5} = 0.347 \text{ s}^{-3}$$

$$N_i = 0.703 \text{ s}^{-1}$$

Converting to rpm:

$$N_i = 0.703 \text{ s}^{-1} .\left|\frac{60 \text{ s}}{1 \text{ min}}\right| = 42 \text{ min}^{-1} = 42 \text{ rpm}$$

Check that this stirrer speed corresponds to turbulent flow by calculating the impeller Reynolds number using Eq. (7.2):

$$Re_i = \frac{0.703\ \text{s}^{-1}\,(0.25\ \text{m})^2\,(1000\ \text{kg m}^{-3})}{3.3\times10^{-3}\ \text{kg m}^{-1}\,\text{s}^{-1}} = 1.3\times10^4$$

As this Re_i is greater than the threshold of 10^4 for turbulent flow (Section 8.5.1), we can say that flow is turbulent. This justifies the use of Eq. (8.9) and N_P' in the calculation.

Answer: 42 rpm

Chapter 9

Heat Transfer

9.1 Rate of conduction

(a)

B = 15 cm = 0.15 m. ΔT = (700 – 80) = 620°C, which is equal to 620 K as temperature differences are the same on the Celsius and Kelvin scales (Section 2.4.6). The rate of heat conduction can be calculated using Eq. (9.10):

$$\hat{Q} = \frac{(0.3 \text{ W m}^{-1} \text{ K}^{-1})\, 1.5 \text{ m}^2}{0.15 \text{ m}} (620 \text{ K}) = 1860 \text{ W} = 1.86 \text{ kW}$$

Answer: 1.86 kW

(b)

In this case there are two thermal resistances in series. Their magnitudes are calculated using Eq. (9.15). For the firebrick:

$$R_1 = \frac{0.15 \text{ m}}{(0.3 \text{ W m}^{-1} \text{ K}^{-1})\, 1.5 \text{ m}^2} = 0.33 \text{ K W}^{-1}$$

For the asbestos, B_2 = 4 cm = 0.04 m, so that:

$$R_2 = \frac{0.04 \text{ m}}{(0.1 \text{ W m}^{-1} \text{ K}^{-1})\, 1.5 \text{ m}^2} = 0.27 \text{ K W}^{-1}$$

Therefore, the total wall resistance $R_W = R_1 + R_2$ = (0.33 + 0.27) K W^{-1} = 0.60 K W^{-1}. For thermal resistances in series, the rate of heat conduction is calculated using Eq. (9.14):

$$\hat{Q} = \frac{\Delta T}{R_w} = \frac{620 \text{ K}}{(0.60 \text{ K W}^{-1})} = 1033 \text{ W} = 1.03 \text{ kW}$$

Answer: 1.03 kW

9.2 Overall heat transfer coefficient

The overall heat transfer coefficient is calculated using Eq. (9.25) with h_{fh} = 830 W m^{-2} K^{-1}, h_h = 1.2 kW m^{-2} K^{-1}, B = 6 mm = 0.006 m, k = 19 W m^{-1} K^{-1}, h_c = 1.7 kW m^{-2} K^{-1} and h_{fc} = 0:

$$\frac{1}{U} = \frac{1}{830 \text{ W m}^{-2} \text{ K}^{-1}} + \frac{1}{1.2 \text{ kW m}^{-2} \text{ K}^{-1} . \left| \frac{1000 \text{ W}}{1 \text{ kW}} \right|} + \frac{0.006 \text{ m}}{19 \text{ W m}^{-1} \text{ K}^{-1}} + \frac{1}{1.7 \text{ kW m}^{-2} \text{ K}^{-1} . \left| \frac{1000 \text{ W}}{1 \text{ kW}} \right|} + 0$$

$$\frac{1}{U} = 2.94 \times 10^{-3} \text{ W}^{-1} \text{ m}^2 \text{ K}$$

$$U = 340 \text{ W m}^{-2} \text{ K}^{-1}$$

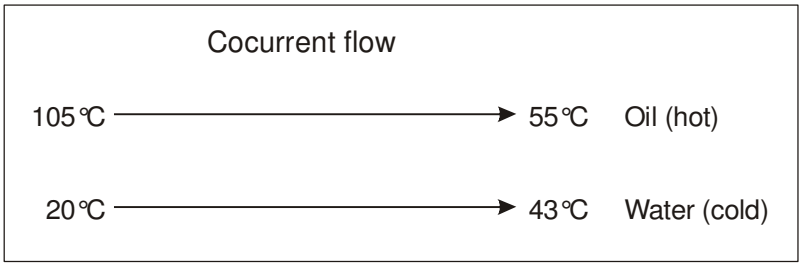
Answer: 340 W m^{-2} K^{-1}

9.3 Cocurrent versus countercurrent flow

The heat transfer area A is related to operating conditions by Eq. (9.19):

$$A = \frac{\hat{Q}}{U\Delta T} \tag{1}$$

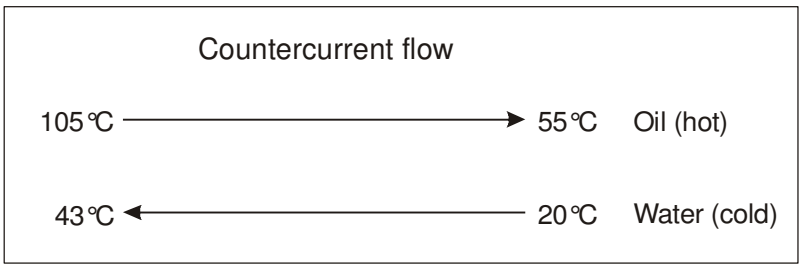
All else (i.e. $\hat{Q}$ and U) being equal, the difference in heat transfer area required for cocurrent and countercurrent flow depends on the temperature-difference driving force, ΔT, for the two different flow configurations. The flow diagram for cocurrent flow is shown below.



Applying Eq. (9.38) to evaluate ΔT for cocurrent flow:

$$\Delta T = \frac{(105-20)°\text{C} - (55-43)°\text{C}}{\ln\dfrac{(105-20)°\text{C}}{(55-43)°\text{C}}} = 37.3°\text{C}$$

The flow diagram for countercurrent flow is shown below.



Applying Eq. (9.38) in this case:

$$\Delta T = \frac{(105-43)°\text{C} - (55-20)°\text{C}}{\ln\dfrac{(105-43)°\text{C}}{(55-20)°\text{C}}} = 47.2°\text{C}$$

Because ΔT for countercurrent flow is (47.2 – 37.3)°C/(37.3°C) × 100% = 27% higher than for cocurrent flow, from (1), the heat transfer area required using cocurrent flow is 27% higher than that required for countercurrent flow.

Answer: The heat transfer area required using cocurrent flow is 27% higher than that required using countercurrent flow.

9.4 Kitchen hot water

(a)

The volume of water collected in 20 s is 2.5 l. From Table A.2 (Appendix A), 2.5 l = 2.5 × 10^{-3} m^3. Therefore, the volumetric flow rate of water in the pipe is:

$$\text{Volumetric flow rate} = \frac{2.5\times10^{-3}\ \text{m}^3}{20\ \text{s}} = 1.25\times10^{-4}\ \text{m}^3\ \text{s}^{-1}$$

The cross-sectional area of the pipe is πR^2, where R is the pipe radius = 5 mm = 5 × 10^{-3} m. The linear velocity of the water is equal to the volumetric flow rate divided by the cross-sectional area for flow:

$$\text{Flow velocity} = \frac{1.25\times10^{-4}\ \text{m}^3\ \text{s}^{-1}}{\pi(5\times10^{-3}\ \text{m})^2} = 1.59\ \text{m}\ \text{s}^{-1}$$

Therefore, in 20 s, the length of pipe traversed is (1.59 m s^{-1}) × 20 s = 31.8 m.

Answer: 32 m

(b)

The rate of heat loss to the atmosphere can be determined using Eq. (9.19). From energy balance considerations, this rate of heat loss is equal to the rate of reduction in sensible heat of the water as represented by Eq. (9.46). Combining these equations gives:

$$UA\Delta T = \hat{M}_\text{h} C_{ph} (T_\text{hi} - T_\text{ho}) \tag{1}$$

U = 90 W m^{-2} K^{-1}. As K^{-1} is the same as $°C^{-1}$ (Section 2.4.6) and using the unit conversion 1 W = 1 J s^{-1} (Table A.8, Appendix A), U = 90 J s^{-1} m^{-2} $°C^{-1}$. The heat transfer area A is the area of the wall of the pipe given by Eq. (9.23) with L equal to the result in **(a)**. The ambient temperature is constant at 12°C; therefore, ΔT is given by a modified form of Eq. (9.39):

$$\Delta T = \frac{T_\text{hi} - T_\text{ho}}{\ln\left(\dfrac{T_\text{hi} - T_\text{A}}{T_\text{ho} - T_\text{A}}\right)}$$

where T_A is the ambient temperature. From Section 2.4.1, the density of water is 1 g cm^{-3} = 1000 kg m^{-3}. From the definition of density (Section 2.4.1), the mass flow rate of the water is equal to the volumetric flow rate × density. Therefore, using the result for volumetric flow rate from **(a)**:

$$\hat{M}_\text{h} = 1.25\times10^{-4}\ \text{m}^3\ \text{s}^{-1} \times 1000\ \text{kg}\ \text{m}^{-3} = 0.125\ \text{kg}\ \text{s}^{-1}$$

Substituting values into (1):

$$90\ \text{J}\ \text{s}^{-1}\ \text{m}^{-2}\ °\text{C}^{-1}\left(2\pi(5\times10^{-3}\ \text{m})\ 31.8\ \text{m}\right)\frac{(75 - T_\text{ho})°\text{C}}{\ln\dfrac{(75-12)°\text{C}}{(T_\text{ho}-12)°\text{C}}}$$

$$= 0.125\ \text{kg}\ \text{s}^{-1}\ (4.2\times10^3\ \text{J}\ \text{kg}^{-1}\ °\text{C}^{-1})\ (75 - T_\text{ho})°\text{C}$$

where T_ho is in °C. Calculating gives:

$$89.91\frac{(75 - T_\text{ho})}{\ln\dfrac{63}{(T_\text{ho}-12)}} = 3.94\times10^4 - 525T_\text{ho}$$

This expression can be simplified using Eq. (E.9) in Appendix E and rearranging:

$$\frac{(75 - T_\text{ho})}{\ln 63 - \ln(T_\text{ho} - 12)} = 438.2 - 5.84T_\text{ho}$$

$$75 - T_\text{ho} = (438.2 - 5.84T_\text{ho})(4.14 - \ln(T_\text{ho} - 12))$$

$$= 1814.15 - 24.18T_\text{ho} - 438.2\ln(T_\text{ho} - 12) + 5.84T_\text{ho}\ln(T_\text{ho} - 12)$$

$$23.18T_{ho} + 438.2\ln(T_{ho} - 12) - 5.84T_{ho}\ln(T_{ho} - 12) = 1739.15$$

This equation cannot be solved analytically; therefore an iterative solution is required. Estimated values of T_{ho} are used to calculate the left-hand side; successive estimates are then determined depending on whether the calculated value deviates from 1739.15. Using 70°C as a first estimate for T_{ho}, the left-hand side of the equation = 1741.98. For T_{ho} = 65°C, the left-hand side = 1739.36. For T_{ho} = 64°C, the left-hand side = 1738.14. These results indicate that T_{ho} lies between 64°C and 65°C. For T_{ho} = 64.8°C, the left-hand side = 1739.13. This is close enough to 1739.15, so we can say that T_{ho} = 64.8°C.

Answer: 64.8°C

9.5 Heat losses from a steam pipe on a windy day

From Eq. (2.27), 330 K = 57°C. The temperature across the heat transfer boundary layer around the pipe varies from 118°C at the pipe surface to 5°C in the bulk air. Therefore, the average air film temperature is (118°C + 5°C)/2 = 61.5°C. The use of property data at 57°C for the calculations is therefore reasonable. Eq. (9.34) allows calculation of the heat transfer coefficient for flow across a single pipe without phase change. The dimensionless numbers in this equation are *Re, Pr* and *Nu. Re* is given by Eq. (9.28):

$$Re = \frac{(6\times10^{-2}\ \text{m})(7.5\ \text{m s}^{-1})(1.076\ \text{kg m}^{-3})}{1.99\times10^{-5}\ \text{kg m}^{-1}\,\text{s}^{-1}} = 2.43\times10^{4}$$

From Table 9.4, at this value of *Re*, C = 0.193 and n = 0.618. *Pr* is given by Eq. (9.30):

$$Pr = \frac{1007\ \text{J kg}^{-1}\,°\text{C}^{-1}\,(1.99\times10^{-5}\ \text{kg m}^{-1}\,\text{s}^{-1})}{0.0283\ \text{W m}^{-1}\,°\text{C}^{-1}\,.\left|\frac{1\ \text{J s}^{-1}}{1\ \text{W}}\right|} = 0.708$$

Applying Eq. (9.34):

$$Nu = 0.193\,(2.43\times10^{4})^{0.618}\,(0.708)^{0.33} = 88.38$$

From the definition of *Nu* in Eq. (9.27):

$$h = \frac{Nu\,k_{fb}}{D} = \frac{88.38\,(0.0283\ \text{W m}^{-1}\,°\text{C}^{-1})}{6\times10^{-2}\ \text{m}} = 41.7\ \text{W m}^{-2}\,°\text{C}^{-1}$$

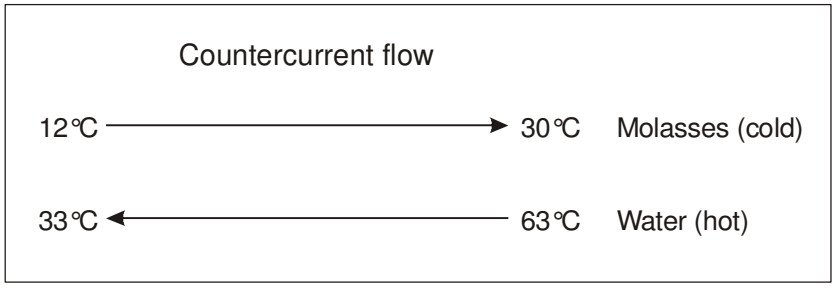
The rate of heat loss from the surface of the pipe is given by Eq. (9.16). The heat transfer area A is the area of the wall of the pipe given by Eq. (9.23). Substituting values into Eq. (9.16):

$$\hat{Q} = (41.7\ \text{W m}^{-2}\,°\text{C}^{-1})\left(\pi(6\times10^{-2}\ \text{m})(20\ \text{m})\right)(118-5)°\text{C} = 1.78\times10^{4}\ \text{W} = 17.8\ \text{kW}$$

Answer: 17.8 kW

9.6 Double-pipe heat exchanger

The flow diagram for the double-pipe heat exchanger with countercurrent flow is shown below.



$\hat{M}_c$ = 19 kg min^{-1}. C_{ph} = 4.2 kJ kg^{-1} °C^{-1}; C_{pc} = 3.7 kJ kg^{-1} °C^{-1}. U = 12 kW m^{-2} °C^{-1}; therefore, from Table A.8 (Appendix A), U = 12 kJ s^{-1} m^{-2} °C^{-1}.

(a)

The required rate of heat transfer is determined using Eq. (9.46) applied to the molasses stream:

$$\hat{Q} = \hat{M}_c C_{pc}(T_{co} - T_{ci}) = 19 \text{ kg min}^{-1} . \left| \frac{1 \text{ min}}{60 \text{ s}} \right| (3.7 \text{ kJ kg}^{-1} \text{°C}^{-1})\,(30 - 12)\text{°C} = 21.1 \text{ kJ s}^{-1}$$

The total heat transfer area required is obtained from Eq. (9.19):

$$A = \frac{\hat{Q}}{U \Delta T}$$

where ΔT is given by Eq. (9.38). Substituting values:

$$A = \frac{21.1 \text{ kJ s}^{-1}}{12 \text{ kJ s}^{-1} \text{ m}^{-2} \text{°C}^{-1} \left(\dfrac{(63 - 30)\text{°C} - (33 - 12)\text{°C}}{\ln \dfrac{(63 - 30)\text{°C}}{(33 - 12)\text{°C}}} \right)} = 0.0662 \text{ m}^2$$

If each unit contains 0.02 m^2, (0.0662 m^2)/(0.02 m^2) = 3.31 units are required. As fractional units are not available, four units must be purchased.

Answer: 4

(b)

A = 4 × 0.02 m^2 = 0.08 m^2. To obtain the same hot- and cold-fluid outlet temperatures as those corresponding to A = 0.0662 m^2, the fluid flow rates must be modified. The rate of heat transfer with A = 0.08 m^2 is determined using Eq. (9.19):

$$\hat{Q} = 12 \text{ kJ s}^{-1} \text{ m}^{-2} \text{°C}^{-1} (0.08 \text{ m}^2) \left(\frac{(63 - 30)\text{°C} - (33 - 12)\text{°C}}{\ln \dfrac{(63 - 30)\text{°C}}{(33 - 12)\text{°C}}} \right) = 25.5 \text{ kJ s}^{-1}$$

$\hat{M}_c$ is obtained from Eq. (9.46):

$$\hat{M}_c = \frac{\hat{Q}}{C_{pc}(T_{co} - T_{ci})} = \frac{25.5 \text{ kJ s}^{-1}}{3.7 \text{ kJ kg}^{-1} \text{°C}^{-1} (30 - 12)\text{°C}} = 0.38 \text{ kg s}^{-1}$$

Converting units to kg min^{-1}:

$$\hat{M}_c = 0.38 \text{ kg s}^{-1} . \left| \frac{60 \text{ s}}{1 \text{ min}} \right| = 23 \text{ kg min}^{-1}$$

Answer: 23 kg min^{-1}

(c)

$\hat{M}_h$ is obtained from Eq. (9.46):

$$\hat{M}_h = \frac{\hat{Q}}{C_{ph}(T_{hi} - T_{ho})} = \frac{25.5 \text{ kJ s}^{-1}}{4.2 \text{ kJ kg}^{-1} \text{°C}^{-1} (63 - 33)\text{°C}} = 0.20 \text{ kg s}^{-1}$$

Converting units to kg min^{-1}:

$$\hat{M}_h = 0.20 \text{ kg s}^{-1} . \left| \frac{60 \text{ s}}{1 \text{ min}} \right| = 12 \text{ kg min}^{-1}$$

Answer: 12 kg min^{-1}

9.7 Water heater on vacation

T_c = 5°C; T_h = 68°C. U = 2 W m^{-2} K^{-1}; as K^{-1} is the same as °C^{-1} (Section 2.4.6), U = 2 W m^{-2} °C^{-1}. A = 1.4 m^2. The rate of heat loss to the atmosphere is evaluated using Eq. (9.19):

$$\hat{Q} = 2 \text{ W m}^{-2} \text{ °C}^{-1} (1.4 \text{ m}^2) (68 - 5)\text{°C} = 176 \text{ W}$$

Therefore, from Table A.8 (Appendix A), $\hat{Q}$ = 176 J s^{-1}. The heat lost over a period of 4 days is:

$$Q = 176 \text{ J s}^{-1} \times 4 \text{ days} . \left| \frac{24 \text{ h}}{1 \text{ day}} \right| . \left| \frac{3600 \text{ s}}{1 \text{ h}} \right| . \left| \frac{1 \text{ kJ}}{1000 \text{ J}} \right| = 6.08 \times 10^4 \text{ kJ}$$

This can be compared with the heat required to raise the temperature of 110 litres of water from 5°C to 68°C. From Section 2.4.1, the density of water is 1 g cm^{-3} = 1 kg l^{-1}. Therefore, the mass of water in the storage heater is 110 kg. From Eq. (5.12), the sensible energy required to heat the water is:

$$\Delta H = 110 \text{ kg } (4.18 \text{ kJ kg}^{-1} \text{ °C}^{-1}) (68 - 5)\text{°C} = 2.90 \times 10^4 \text{ kJ}$$

As the energy required to heat the water from 5°C is less than that required to maintain the temperature at 68°C for 4 days, it is better to turn the heater off before leaving.

Answer: Turning off the heater is more economical

9.8 Fouling and pipe wall resistances

R = (3.5 cm)/2 = 1.75 × 10^{-2} m. B = 4 mm = 4 × 10^{-3} m. L = 60 m. From Section 2.4.1, the density of water = 1 g cm^{-3} = 1 kg l^{-1}; therefore, $\hat{M}_h$ = 20 kg s^{-1}. T_{hi} = 90°C; T_{ho} = 84°C. The ambient temperature T_A = 25°C. h_{fh} = 7500 W m^{-2} °C^{-1}. From Table C.3 (Appendix C), the heat capacity of water = 75.4 J gmol^{-1} °C^{-1} = 75.4 kJ kgmol^{-1} °C^{-1}. Converting this to a mass basis using the molecular weight of water = 18.0 (Table C.1, Appendix C):

$$C_{ph} = 75.4 \text{ kJ kgmol}^{-1} \text{ °C}^{-1} . \left| \frac{1 \text{ kgmol}}{18.0 \text{ kg}} \right| = 4.189 \text{ kJ kg}^{-1} \text{ °C}^{-1}$$

(a)

The rate of heat loss from the water is determined using Eq. (9.46):

$$\hat{Q} = \hat{M}_h C_{ph} (T_{hi} - T_{ho}) = 20 \text{ kg s}^{-1} (4.189 \text{ kJ kg}^{-1} \text{ °C}^{-1}) (90 - 84)\text{°C} = 503 \text{ kJ s}^{-1}$$

Answer: 503 kJ s^{-1}

(b)

U is determined using Eq. (9.19):

$$U = \frac{\hat{Q}}{A \Delta T} \tag{1}$$

where ΔT is the logarithmic-mean temperature difference. As the ambient temperature is constant at 25°C, ΔT is given by a modified form of Eq. (9.39):

$$\Delta T = \frac{T_{\text{hi}} - T_{\text{ho}}}{\ln\left(\dfrac{T_{\text{hi}} - T_{\text{A}}}{T_{\text{ho}} - T_{\text{A}}}\right)}$$

Substituting values gives:

$$\Delta T = \frac{(90-84)\text{°C}}{\ln\dfrac{(90-25)\text{°C}}{(84-25)\text{°C}}} = 61.95\text{°C}$$

The heat transfer area A is the area of the wall of the pipe given by Eq. (9.23):

$$A = 2\pi(1.75\times10^{-2}\text{ m})\,60\text{ m} = 6.60\text{ m}^2$$

Substituting values into (1) using $\hat{Q}$ from **(a)**:

$$U = \frac{503\text{ kJ s}^{-1}}{6.60\text{ m}^2\,(61.95\text{°C})} = 1.23\text{ kJ s}^{-1}\text{ m}^{-2}\text{ °C}^{-1}$$

Using the conversion factor 1 W = 1 J s^{-1} from Table A.8 (Appendix A), $U = 1.23 \times 10^3$ W m^{-2} °C^{-1}. From Eq. (9.20), the total resistance to heat transfer is equal to $1/(UA)$. Analogously, the resistance to heat transfer due to fouling is $1/(h_{\text{fh}}A)$. Assuming that A is roughly the same throughout the thickness of the film and fouling layers (Section 9.4.3), the proportion of the total resistance provided by fouling is:

$$\frac{1/(h_{\text{fh}}A)}{1/(UA)} = \frac{1/h_{\text{fh}}}{1/U} = \frac{1/(7500\text{ W m}^{-2}\text{ °C}^{-1})}{1/(1.23\times10^3\text{ W m}^{-2}\text{ °C}^{-1})}\times100\% = 16.4\%$$

Answer: 16%

(c)

Assuming that the thermal conductivity does not vary significantly with temperature, from Table 9.1, k for copper can be taken as 377 W m^{-1} °C^{-1}. Similarly, k for stainless steel can be taken as 16 W m^{-1} °C^{-1}. The results for the copper pipe with fouling are used to determine the hot- and cold-film resistances that apply also with the stainless steel pipe. From Eq. (9.25) with $h_{\text{fc}} = 0$:

$$\frac{1}{h_{\text{h}}} + \frac{1}{h_{\text{c}}} = \frac{1}{U} - \frac{1}{h_{\text{fh}}} - \frac{B}{k}$$

Substituting values:

$$\frac{1}{h_{\text{h}}} + \frac{1}{h_{\text{c}}} = \frac{1}{1.23\times10^3\text{ W m}^{-2}\text{ °C}^{-1}} - \frac{1}{7500\text{ W m}^{-2}\text{ °C}^{-1}} - \frac{4\times10^{-3}\text{ m}}{377\text{ W m}^{-1}\text{ °C}^{-1}} = 6.69\times10^{-4}\text{ W}^{-1}\text{ m}^2\text{ °C}$$

For the stainless steel pipe without fouling, from Eq. (9.24):

$$\frac{1}{U} = 6.69\times10^{-4}\text{ W}^{-1}\text{ m}^2\text{ °C} + \frac{4\times10^{-3}\text{ m}}{16\text{ W m}^{-1}\text{ °C}^{-1}} = 9.19\times10^{-4}\text{ W}^{-1}\text{ m}^2\text{ °C}$$

$$U = 1088\text{ W m}^{-2}\text{ °C}^{-1}$$

From Eq. (9.12), the resistance to heat transfer due to the pipe wall is $B/(kA)$. Therefore, the proportion of the total resistance provided by the stainless steel pipe is:

$$\frac{B/(kA)}{1/(UA)} = \frac{B/k}{1/U} = \frac{\dfrac{4\times10^{-3}\text{ m}}{16\text{ W m}^{-1}\text{ °C}^{-1}}}{9.19\times10^{-4}\text{ W}^{-1}\text{ m}^2\text{ °C}}\times100\% = 27.2\%$$

Answer: 27%

9.9 Effect of cooling-coil length on coolant requirements

$T_F = 35°C$; $T_{ci} = 8°C$; $C_{pc} = 4.18$ kJ kg^{-1} °C^{-1}.

(a)

$\hat{M}_c = 0.5$ kg s^{-1}; $T_{co} = 15°C$. The steady-state energy balance equation for the cooling coil is Eq. (9.47). Substituting values gives:

$$\hat{Q} = 0.5\,\text{kg s}^{-1}\,(4.18\,\text{kJ kg}^{-1}\,°\text{C}^{-1})\,(15-8)°\text{C} = 14.6\,\text{kJ s}^{-1}$$

Answer: 14.6 kJ s^{-1}

(b)

The temperature difference between the fermentation fluid and cooling water is calculated as the log-mean temperature difference using Eq. (9.39):

$$\Delta T = \frac{(15-8)°\text{C}}{\ln\dfrac{(35-8)°\text{C}}{(35-15)°\text{C}}} = 23.3°\text{C}$$

Answer: 23.3°C

(c)

UA is evaluated from Eq. (9.19):

$$UA = \frac{\hat{Q}}{\Delta T}$$

Substituting values:

$$UA = \frac{14.6\,\text{kJ s}^{-1}}{23.3°\text{C}} = 0.63\,\text{kJ s}^{-1}\,°\text{C}^{-1}$$

Answer: 0.63 kJ s^{-1} °C^{-1}

(d)

$$UA' = 1.5UA = 1.5 \times 0.63\,\text{kJ s}^{-1}\,°\text{C}^{-1} = 0.95\,\text{kJ s}^{-1}\,°\text{C}^{-1}$$

Answer: 0.95 kJ s^{-1} °C^{-1}

(e)

Applying Eq. (9.19) to determine $\Delta T'$ for the new coil:

$$\Delta T' = \frac{\hat{Q}}{UA'} = \frac{14.6\,\text{kJ s}^{-1}}{0.95\,\text{kJ s}^{-1}\,°\text{C}^{-1}} = 15.4°\text{C}$$

The new cooling water outlet temperature T'_{co} can be obtained from Eq. (9.39). However, this equation cannot be solved analytically; therefore an iterative solution is required. Substituting values into Eq. (9.39) gives:

$$15.4°\text{C} = \frac{(T'_{co}-8)°\text{C}}{\ln\dfrac{(35-8)°\text{C}}{(35-T'_{co})°\text{C}}} = \frac{(T'_{co}-8)°\text{C}}{\ln\dfrac{27°\text{C}}{(35-T'_{co})°\text{C}}}$$

After applying mathematical rule Eq. (E.9) from Appendix E, this equation becomes:

$$15.4 = \frac{(T'_{co} - 8)}{\ln 27 - \ln(35 - T'_{co})}$$

where T'_{co} is in °C. Rearranging gives:

$$15.4\left(3.296 - \ln(35 - T'_{co})\right) + 8 = T'_{co}$$

$$58.76 - 15.4 \ln(35 - T'_{co}) = T'_{co}$$

$$58.76 = T'_{co} + 15.4 \ln(35 - T'_{co})$$

To solve this equation iteratively, estimated values of T'_{co} are used to calculate the right-hand side; successive estimates are then determined depending on whether the right-hand side of the equation deviates from 58.76. Using 25°C as a first estimate for T'_{co}, the right-hand side of the equation = 60.46. For T'_{co} = 27°C, the right-hand side = 59.02. For T'_{co} = 28°C, the right-hand side = 57.97. These results indicate that T'_{co} is between 27°C and 28°C. For T'_{co} = 27.3°C, the right-hand side of the equation = 58.73. This is close enough to 58.76 to say that T'_{co} for the new cooling coil is 27.3°C.

Answer: 27.3°C

(f)

From Eq. (9.47), for the new coil with $T_{co} = T'_{co}$ = 27.3°C:

$$\hat{M}_c = \frac{\hat{Q}}{C_{pc}(T_{co} - T_{ci})} = \frac{14.6 \text{ kJ s}^{-1}}{4.18 \text{ kJ kg}^{-1} \text{°C}^{-1} (27.3 - 8)\text{°C}} = 0.18 \text{ kg s}^{-1}$$

Therefore, installation of the new coil allows a 64% reduction in cooling water requirements from 0.5 kg s^{-1} to 0.18 kg s^{-1}.

Answer: 64%

9.10 Fermenter cooling coil

T_F = 35°C; T_{ci} = 15°C; T_{co} = 25°C. $\hat{Q}$ = 15.5 kW = 15.5 × 10^3 W. U = 340 W m^{-2} K^{-1}; as K^{-1} is the same as $°C^{-1}$ (Section 2.4.6), U = 340 W m^{-2} $°C^{-1}$. R = (4 cm)/2 = 2 × 10^{-2} m. The heat transfer area required is obtained from Eq. (9.19):

$$A = \frac{\hat{Q}}{U \Delta T}$$

where ΔT is given by Eq. (9.39). Substituting values:

$$A = \frac{15.5 \times 10^3 \text{ W}}{340 \text{ W m}^{-2} \text{°C}^{-1} \left(\dfrac{(25 - 15)\text{°C}}{\ln \dfrac{(35 - 15)\text{°C}}{(35 - 25)\text{°C}}} \right)} = 3.16 \text{ m}^2$$

The cooling coil length is determined using Eq. (9.23):

$$L = \frac{A}{2\pi R}$$

Substituting values:

$$L = \frac{3.16 \text{ m}^2}{2\pi (2 \times 10^{-2} \text{ m})} = 25.1 \text{ m}$$

Answer: 25 m

9.11 Effect of fouling on heat transfer resistance

T_{ci} = 12°C; T_F = 37°C. L = 150 m; R = (12 cm)/2 = 6 × 10^{-2} m. From Table C.3 (Appendix C), the heat capacity of water = 75.4 J $gmol^{-1}$ $°C^{-1}$ = 75.4 kJ $kgmol^{-1}$ $°C^{-1}$. Converting this to a mass basis using the molecular weight of water = 18.0 (Table C.1, Appendix C):

$$C_{pc} = 75.4 \text{ kJ kgmol}^{-1}\,°\text{C}^{-1} . \left| \frac{1 \text{ kgmol}}{18.0 \text{ kg}} \right| = 4.189 \text{ kJ kg}^{-1}\,°\text{C}^{-1}$$

(a)

Before cleaning, $\hat{M}_c$ = 20 kg s^{-1} and T_{co} = 28°C. The rate of heat transfer before cleaning is evaluated using Eq. (9.47):

$$\hat{Q} = 20 \text{ kg s}^{-1} (4.189 \text{ kJ kg}^{-1}\,°\text{C}^{-1}) (28-12)°\text{C} = 1.34 \times 10^3 \text{ kJ s}^{-1}$$

U before cleaning is determined using Eq. (9.19):

$$U = \frac{\hat{Q}}{A \Delta T} \tag{1}$$

where ΔT is the logarithmic-mean temperature difference given by Eq. (9.39). A is the area of the wall of the pipe given by Eq. (9.23):

$$A = 2\pi (6 \times 10^{-2} \text{ m}) 150 \text{ m} = 56.55 \text{ m}^2$$

Substituting values into (1):

$$U = \frac{1.34 \times 10^3 \text{ kJ s}^{-1}}{56.55 \text{ m}^2 \left(\dfrac{(28-12)°\text{C}}{\ln \dfrac{(37-12)°\text{C}}{(37-28)°\text{C}}} \right)} = 1.51 \text{ kJ s}^{-1} \text{ m}^{-2}\,°\text{C}^{-1}$$

Answer: 1.51 kJ s^{-1} m^{-2} $°C^{-1}$

(b)

After cleaning, $\hat{M}_c$ = 13 kg s^{-1}. The outlet cooling water temperature after cleaning is determined from Eq. (9.47):

$$T_{co} = \frac{\hat{Q}}{\hat{M}_c C_{pc}} + T_{ci}$$

Substituting values gives:

$$T_{co} = \frac{1.34 \times 10^3 \text{ kJ s}^{-1}}{13 \text{ kg s}^{-1} (4.189 \text{ kJ kg}^{-1}\,°\text{C}^{-1})} + 12°\text{C}$$

$$= 36.6°\text{C}$$

Answer: 36.6°C

(c)

U after cleaning is determined using (1) and the result for T_{co} from **(b)**:

$$U = \frac{1.34\times10^{3}\ \text{kJ s}^{-1}}{56.55\ \text{m}^{2}\left(\dfrac{(36.6-12)^{\circ}\text{C}}{\ln\dfrac{(37-12)^{\circ}\text{C}}{(37-36.6)^{\circ}\text{C}}}\right)} = 3.98\ \text{kJ s}^{-1}\ \text{m}^{-2}\ ^{\circ}\text{C}^{-1}$$

From the equations in Sections 9.4.3 and 9.4.4, the relationship between the heat transfer resistances before and after cleaning can be represented as:

$$\frac{1}{U_{\text{bc}}} = \frac{1}{U_{\text{ac}}} + R_{\text{f}}$$

where subscript bc means before cleaning, subscript ac means after cleaning, and R_{f} is the resistance due to the fouling layer. Rearranging this equation and substituting values gives:

$$R_{\text{f}} = \frac{1}{U_{\text{bc}}} - \frac{1}{U_{\text{ac}}} = \frac{1}{1.51\ \text{kJ s}^{-1}\ \text{m}^{-2}\ ^{\circ}\text{C}^{-1}} - \frac{1}{3.98\ \text{kJ s}^{-1}\ \text{m}^{-2}\ ^{\circ}\text{C}^{-1}} = 0.41\ \text{kJ}^{-1}\ \text{s m}^{2}\ ^{\circ}\text{C}$$

Therefore, the fraction of the total resistance before cleaning due to the fouling deposits is:

$$\frac{R_{\text{f}}}{\dfrac{1}{U_{\text{bc}}}} = \frac{0.41\ \text{kJ}^{-1}\ \text{s m}^{2}\ ^{\circ}\text{C}}{\left(\dfrac{1}{1.51\ \text{kJ s}^{-1}\ \text{m}^{-2}\ ^{\circ}\text{C}^{-1}}\right)} = 0.62$$

Answer: 0.62

9.12 Preheating of nutrient medium

The medium in the tubes is the cold fluid and water in the shell is the hot fluid. $C_{ph} = C_{pc} = C_p$ water. From Table C.3 (Appendix C), the heat capacity of water = 75.4 J gmol^{-1} °C^{-1} = 75.4 kJ kgmol^{-1} °C^{-1}. Converting this to a mass basis using the molecular weight of water = 18.0 (Table C.1, Appendix C):

$$C_{pc} = C_{ph} = 75.4\ \text{kJ kgmol}^{-1}\ ^{\circ}\text{C}^{-1}.\left|\frac{1\ \text{kgmol}}{18.0\ \text{kg}}\right| = 4.189\ \text{kJ kg}^{-1}\ ^{\circ}\text{C}^{-1}$$

From Section 2.4.1, the density of water is 1 g cm^{-3} = 1000 kg m^{-3}; therefore ρ medium = ρ water = 1000 kg m^{-3}. From Eq. (7.8), μ water = 1 cP = 10^{-3} kg m^{-1} s^{-1}; therefore μ_{b} medium = μ_{b} water = 10^{-3} kg m^{-1} s^{-1}. k_{fb} medium = 0.54 W m^{-1} °C^{-1}; therefore, from Table A.8 (Appendix A), k_{fb} medium = 0.54 J s^{-1} m^{-1} °C^{-1} = 0.54 × 10^{-3} kJ s^{-1} m^{-1} °C^{-1}. From Table 9.1, k_{fb} water at 60°C = 0.66 W m^{-1} °C^{-1} = 0.66 J s^{-1} m^{-1} °C^{-1} = 0.66 × 10^{-3} kJ s^{-1} m^{-1} °C^{-1}. k pipe wall = 50 W m^{-1} °C^{-1} = 50 J s^{-1} m^{-1} °C^{-1} = 50 × 10^{-3} kJ s^{-1} m^{-1} °C^{-1}.

Pipe inner D = 5 cm = 5 × 10^{-2} m; therefore, R_{i} = (5 cm)/2 = 2.5 × 10^{-2} m. B = 5 mm = 5 × 10^{-3} m.

T_{ci} = 10°C; T_{co} = 28°C; T_{hi} = 60°C.

From the definition of density (Section 2.4.1), the mass flow rate of the medium is equal to its volumetric flow rate × density:

$$\hat{M}_{\text{c}} = 50\ \text{m}^{3}\ \text{h}^{-1}\times1000\ \text{kg m}^{-3}.\left|\frac{1\ \text{h}}{3600\ \text{s}}\right| = 13.89\ \text{kg s}^{-1}$$

u in the shell = 0.15 m s^{-1}. Converting units for the mass flow rate of water:

$$\hat{M}_{\text{h}} = 3\times10^{4}\ \text{kg h}^{-1}.\left|\frac{1\ \text{h}}{3600\ \text{s}}\right| = 8.33\ \text{kg s}^{-1}$$

(a)

The rate of heat transfer is evaluated using Eq. (9.46) for the medium:

$$\hat{Q} = \hat{M}_c C_{pc} (T_{co} - T_{ci}) = 13.89 \text{ kg s}^{-1} (4.189 \text{ kJ kg}^{-1} \text{°C}^{-1}) (28 - 10)\text{°C} = 1047.3 \text{ kJ s}^{-1}$$

Answer: 1047 kJ s^{-1}

(b)

The heat transfer coefficient for the medium in the tubes of the heat exchanger can be calculated using the empirical correlation, Eq. (9.32). The parameters in this equation are *Re, Pr* and *Nu. Re* is given by Eq. (9.28). The linear velocity of the fluid *u* is equal to the volumetric flow rate per tube divided by the inside cross-sectional area of the tube, πR_i^2:

$$u = \frac{\dfrac{50 \text{ m}^3 \text{ h}^{-1}}{30} \cdot \left| \dfrac{1 \text{ h}}{3600 \text{ s}} \right|}{\pi (2.5 \times 10^{-2} \text{ m})^2} = 0.236 \text{ m s}^{-1}$$

Substituting values into Eq. (9.28) using the inner pipe diameter for *D*:

$$Re = \frac{5 \times 10^{-2} \text{ m } (0.236 \text{ m s}^{-1}) 1000 \text{ kg m}^{-3}}{10^{-3} \text{ kg m}^{-1} \text{ s}^{-1}} = 1.18 \times 10^4$$

Eq. (9.32) is valid at this *Re*. *Pr* for the medium is given by Eq. (9.30):

$$Pr = \frac{4.189 \text{ kJ kg}^{-1} \text{°C}^{-1} (10^{-3} \text{ kg m}^{-1} \text{ s}^{-1})}{0.54 \times 10^{-3} \text{ kJ s}^{-1} \text{ m}^{-1} \text{°C}^{-1}} = 7.76$$

Substituting values into Eq. (9.32):

$$Nu = 0.023 (1.18 \times 10^4)^{0.8} (7.76)^{0.4} = 94.4$$

From the definition of *Nu* in Eq. (9.27):

$$h = \frac{Nu \, k_{fb}}{D} = \frac{94.4 (0.54 \times 10^{-3} \text{ kJ s}^{-1} \text{ m}^{-1} \text{°C}^{-1})}{5 \times 10^{-2} \text{ m}} = 1.02 \text{ kJ s}^{-1} \text{ m}^{-2} \text{°C}^{-1}$$

As the medium in the tubes is the cold fluid, h_c = 1.02 kJ s^{-1} m^{-2} °C^{-1}.

The heat transfer coefficient for water flowing in the shell of the heat exchanger is calculated using the empirical correlation, Eq. (9.35). The parameters in this equation are *F*, Re_{max}, *m*, *Pr* and *Nu*. Re_{max} is given by Eq. (9.28) with *D* equal to the outer pipe diameter (Section 9.5.1, Flow at Right Angles to a Bank of Tubes without Phase Change subsection). The outer pipe diameter is the sum of the inner pipe diameter and the pipe wall thicknesses: outer *D* = inner *D* + 2 × *B* = 5 × 10^{-2} m + 2 × 5 × 10^{-3} m = 6 × 10^{-2} m. Substituting parameter values into Eq. (9.28):

$$Re_{max} = \frac{6 \times 10^{-2} \text{ m } (0.15 \text{ m s}^{-1}) 1000 \text{ kg m}^{-3}}{10^{-3} \text{ kg m}^{-1} \text{ s}^{-1}} = 9.00 \times 10^3$$

For this value of Re_{max} and for tubes of the heat exchanger arranged in line, from Table 9.5, *F* = 0.211 and *m* = 0.651. *Pr* for the water is calculated using Eq. (9.30):

$$Pr = \frac{4.189 \text{ kJ kg}^{-1} \text{°C}^{-1} (10^{-3} \text{ kg m}^{-1} \text{ s}^{-1})}{0.66 \times 10^{-3} \text{ kJ s}^{-1} \text{ m}^{-1} \text{°C}^{-1}} = 6.35$$

Applying Eq. (9.35):

$$Nu = 0.211 (9.00 \times 10^3)^{0.651} (6.35)^{0.34} = 148.4$$

From the definition of *Nu* in Eq. (9.27) with *D* the outer tube diameter:

$$h = \frac{Nu\, k_{\text{fb}}}{D} = \frac{148.4\,(0.66\times10^{-3}\ \text{kJ s}^{-1}\ \text{m}^{-1}\ ^\circ\text{C}^{-1})}{6\times10^{-2}\ \text{m}} = 1.63\ \text{kJ s}^{-1}\ \text{m}^{-2}\ ^\circ\text{C}^{-1}$$

As water is the hot fluid, $h_{\text{h}} = 1.63\ \text{kJ s}^{-1}\ \text{m}^{-2}\ ^\circ\text{C}^{-1}$.

Answer: h_{c} (tube) = 1.02 kJ s^{-1} m^{-2} °C^{-1}; h_{h} (shell) = 1.63 kJ s^{-1} m^{-2} °C^{-1}

(c)

The overall heat transfer coefficient without fouling factors is calculated using Eq. (9.24):

$$\frac{1}{U} = \frac{1}{1.63\ \text{kJ s}^{-1}\ \text{m}^{-2}\ ^\circ\text{C}^{-1}} + \frac{5\times10^{-3}\ \text{m}}{50\times10^{-3}\ \text{kJ s}^{-1}\ \text{m}^{-1}\ ^\circ\text{C}^{-1}} + \frac{1}{1.02\ \text{kJ s}^{-1}\ \text{m}^{-2}\ ^\circ\text{C}^{-1}}$$

$$= 1.69\ \text{kJ}^{-1}\ \text{s m}^2\ ^\circ\text{C}$$

$$U = 0.590\ \text{kJ s}^{-1}\ \text{m}^{-2}\ ^\circ\text{C}^{-1}$$

Answer: 0.59 kJ s^{-1} m^{-2} °C^{-1}

(d)

The outlet temperature of the water from the shell is determined from Eq. (9.46):

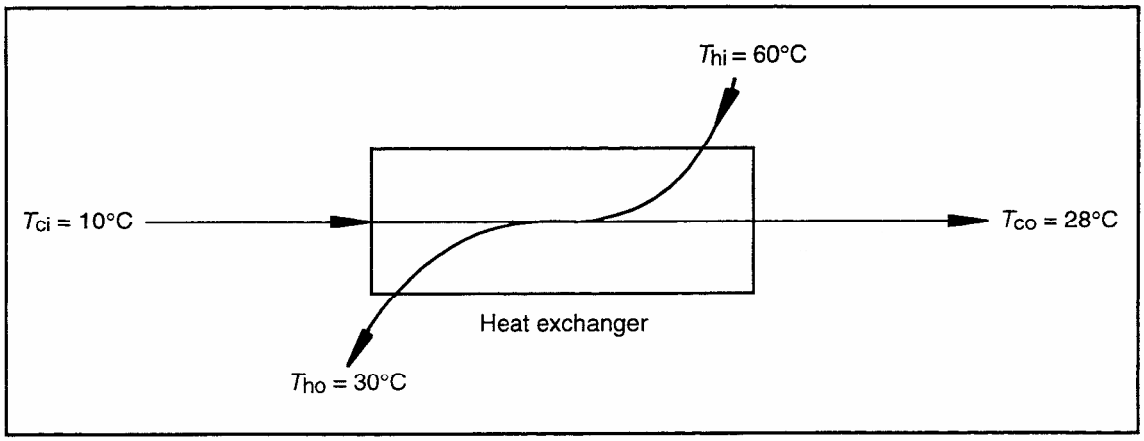
$$T_{\text{ho}} = T_{\text{hi}} - \frac{\hat{Q}}{\hat{M}_{\text{h}}\, C_{p\text{h}}}$$

Substituting values gives:

$$T_{\text{ho}} = 60^\circ\text{C} - \frac{1047.3\ \text{kJ s}^{-1}}{8.33\ \text{kg s}^{-1}\,(4.189\ \text{kJ kg}^{-1}\ ^\circ\text{C}^{-1})}$$

$$= 30.0^\circ\text{C}$$

The fluid flow directions and the inlet and outlet temperatures for a single-pass countercurrent shell-and-tube heat exchanger are represented graphically below.



The temperature differences at the two ends of the exchanger are $\Delta T_1 = T_{\text{ho}} - T_{\text{ci}} = (30 - 10) = 20^\circ\text{C}$ and $\Delta T_2 = T_{\text{hi}} - T_{\text{co}} = (60 - 28) = 32^\circ\text{C}$. Substituting these values into Eq. (9.38) for the log-mean temperature difference:

$$\Delta T = \frac{(32-20)^\circ\text{C}}{\ln\dfrac{32^\circ\text{C}}{20^\circ\text{C}}} = 25.5^\circ\text{C}$$

Answer: 25.5°C

(e)

The heat transfer area is determined from Eq. (9.19):

$$A = \frac{\hat{Q}}{U\Delta T}$$

Substituting values:

$$A = \frac{1047.3\ \text{kJ s}^{-1}}{0.590\ \text{kJ s}^{-1}\ \text{m}^{-2}\ ^\circ\text{C}^{-1}\ (25.5^\circ\text{C})} = 69.6\ \text{m}^2$$

Answer: 70 m^2

(f)

The total tube length is determined from Eq. (9.23):

$$L = \frac{A}{2\pi R}$$

For this calculation, we will use an average radius for the tubes to take into account the thickness of the tube wall: $R_{av} = (R_i + R_o)/2 = [(2.5 \times 10^{-2}\ \text{m}) + (3 \times 10^{-2}\ \text{m})]/2 = 2.75 \times 10^{-2}$ m. Substituting values gives:

$$L = \frac{69.6\ \text{m}^2}{2\pi(2.75\times10^{-2}\ \text{m})} = 402.8\ \text{m}$$

This total tube length is provided using 30 tubes; therefore, each tube must be of length (402.8 m)/30 = 13.4 m.

Answer: 13.4 m

9.13 Cooling coil design

$D_T = 4$ m; $H_L = 8$ m. For a cylindrical tank, the liquid volume V_L is:

$$V_L = \pi\left(\frac{D_T}{2}\right)^2 H_L$$

Substituting values:

$$V_L = \pi\left(\frac{4\ \text{m}}{2}\right)^2 8\ \text{m} = 100.5\ \text{m}^3$$

From Section 5.8, the heat of reaction for an exothermic reaction is negative; therefore, $\Delta\hat{H}_{rxn} = -520$ kW. $D_i = 1.9$ m. $N_i = 50$ rpm = 50 min^{-1}. The power input by stirring is 1 metric horsepower per 1000 l. Multiplying by the liquid volume and applying unit conversion factors from Tables A.2 and A.8 (Appendix A):

$$\hat{W}_s = \frac{1\ \text{metric horsepower}}{1000\ \text{l}} \times 100.5\ \text{m}^3 . \left|\frac{7.355\times10^2\ \text{W}}{1\ \text{metric horsepower}}\right| . \left|\frac{10^3\ \text{l}}{1\ \text{m}^3}\right| = 7.392\times10^4\ \text{W}$$

Therefore, $\hat{W}_s = 73.92$ kW.

For the broth: $T_F = 29^\circ$C; $C_p = 4.0$ kJ kg^{-1} $^\circ\text{C}^{-1}$; $\rho = 10^3$ kg m^{-3}. $\mu_b = 0.05$ Pa s; therefore, from Table A.9 (Appendix A), $\mu_b = 0.05$ kg m^{-1} s^{-1}. $k_{fb} = 0.62$ W m^{-1} $^\circ\text{C}^{-1}$; therefore, from Table A.8 (Appendix A), $k_{fb} = 0.62$ J s^{-1} m^{-1} $^\circ\text{C}^{-1}$ = 0.62×10^{-3} kJ s^{-1} m^{-1} $^\circ\text{C}^{-1}$ = 0.62×10^{-3} kW m^{-1} $^\circ\text{C}^{-1}$.

For the cooling water: $T_{ci} = 14°C$; $C_p = 4.2$ kJ kg^{-1} °C^{-1}; $\rho = 10^3$ kg m^{-3}; $\mu_b = 10^{-3}$ kg m^{-1} s^{-1}; $k_{fb} = 0.66$ W m^{-1} °C^{-1} = 0.66×10^{-3} kW m^{-1} °C^{-1} = 0.66×10^{-3} kJ s^{-1} m^{-1} °C^{-1}. From the definition of density (Section 2.4.1), the mass flow rate of cooling water is equal to the volumetric flow rate × density:

$$\hat{M}_c = 50 \text{ m}^3 \text{ h}^{-1} \times 10^3 \text{ kg m}^{-3} . \left| \frac{1 \text{ h}}{3600 \text{ s}} \right| = 13.89 \text{ kg s}^{-1}$$

For the pipe wall: $k = 17$ W m^{-1} °C^{-1} = 17×10^{-3} kW m^{-1} °C^{-1}. Inner $D = 4.2$ cm = 4.2×10^{-2} m. R_i = (4.2 cm)/2 = 2.1×10^{-2} m. $B = 7$ mm = 7×10^{-3} m. $h_f = 2.5$ kW m^{-2} °C^{-1}.

(a)

The rate of heat transfer is determined using Eq. (9.48). Assuming that evaporation is negligible:

$$\hat{Q} = -\Delta \hat{H}_{rxn} + \hat{W}_s = 520 \text{ kW} + 73.92 \text{ kW} = 593.92 \text{ kW}$$

Therefore, from Table A.8 (Appendix A), $\hat{Q}$ = 593.92 kJ s^{-1}. From the energy balance equation for the cooling water, Eq. (9.47), an expression for the outlet cooling-water temperature is:

$$T_{co} = \frac{\hat{Q}}{\hat{M}_c C_{pc}} + T_{ci}$$

Substituting values gives:

$$T_{co} = \frac{593.92 \text{ kJ s}^{-1}}{13.89 \text{ kg s}^{-1} (4.2 \text{ kJ kg}^{-1} \text{°C}^{-1})} + 14\text{°C}$$

$$= 24.2\text{°C}$$

Answer: 24.2°C

(b)

The fermenter-side heat transfer coefficient is evaluated using the empirical correlation, Eq. (9.36). The dimensionless numbers in this equation are Re_i, Pr and Nu. Re_i is given by Eq. (9.29):

$$Re_i = \frac{50 \text{ min}^{-1} . \left| \frac{1 \text{ min}}{60 \text{ s}} \right| (1.9 \text{ m})^2 (10^3 \text{ kg m}^{-3})}{0.05 \text{ kg m}^{-1} \text{ s}^{-1}} = 6.02 \times 10^4$$

Pr is given by Eq. (9.30):

$$Pr = \frac{4.0 \text{ kJ kg}^{-1} \text{°C}^{-1} (0.05 \text{ kg m}^{-1} \text{ s}^{-1})}{0.62 \times 10^{-3} \text{ kJ s}^{-1} \text{ m}^{-1} \text{°C}^{-1}} = 322.6$$

From Eq. (9.36):

$$Nu = 0.9 (6.02 \times 10^4)^{0.62} (322.6)^{0.33} \left(\frac{1}{1.15} \right)^{0.14} = 5.457 \times 10^3$$

From the definition of Nu in Eq. (9.27) with $D = D_T$ (Section 9.5.1, Stirred Liquids subsection):

$$h = \frac{Nu \, k_{fb}}{D} = \frac{5.457 \times 10^3 (0.62 \times 10^{-3} \text{ kW m}^{-1} \text{°C}^{-1})}{4 \text{ m}} = 0.846 \text{ kW m}^{-2} \text{°C}^{-1}$$

Therefore, as the fermenter fluid is the hot fluid, h_h is 0.846 kW m^{-2} °C^{-1}.

Answer: 0.846 kW m^{-2} °C^{-1}

(c)

The heat transfer coefficient for cooling water in the coil is calculated using the empirical correlation, Eq. (9.32). The parameters in this equation are *Re, Pr* and *Nu. Re* is given by Eq. (9.28). The linear velocity of the fluid *u* is equal to the volumetric flow rate in the cooling coil divided by the inside cross-sectional area of the pipe, πR_i^2:

$$u = \frac{50\ \text{m}^3\ \text{h}^{-1} . \left| \frac{1\ \text{h}}{3600\ \text{s}} \right|}{\pi (2.1\times10^{-2}\ \text{m})^2} = 10.0\ \text{m s}^{-1}$$

Substituting values into Eq. (9.28) using the inner pipe diameter for *D* (Section 9.5.1, Flow in Tubes without Phase Change subsection):

$$Re = \frac{4.2\times10^{-2}\ \text{m}\ (10.0\ \text{m s}^{-1})\ 10^3\ \text{kg m}^{-3}}{10^{-3}\ \text{kg m}^{-1}\ \text{s}^{-1}} = 4.20\times10^5$$

This value of *Re* is within the range of validity for Eq. (9.32). *Pr* is given by Eq. (9.30):

$$Pr = \frac{4.2\ \text{kJ kg}^{-1}\ ^\circ\text{C}^{-1}\ (10^{-3}\ \text{kg m}^{-1}\ \text{s}^{-1})}{0.66\times10^{-3}\ \text{kJ s}^{-1}\ \text{m}^{-1}\ ^\circ\text{C}^{-1}} = 6.36$$

Substituting values into Eq. (9.32):

$$Nu = 0.023\ (4.20\times10^5)^{0.8}\ (6.36)^{0.4} = 1.52\times10^3$$

From the definition of *Nu* in Eq. (9.27) with *D* equal to the inner tube diameter:

$$h = \frac{Nu\, k_{fb}}{D} = \frac{1.52\times10^3\ (0.66\times10^{-3}\ \text{kW m}^{-1}\ ^\circ\text{C}^{-1})}{4.2\times10^{-2}\ \text{m}} = 23.9\ \text{kW m}^{-2}\ ^\circ\text{C}^{-1}$$

As water is the cold fluid, h_c = 23.9 kW m^{-2} °C^{-1}.

Answer: 23.9 kW m^{-2} °C^{-1}

(d)

The overall heat transfer coefficient with fouling is calculated using a modified form of Eq. (9.25) to include only one fouling factor, h_f:

$$\frac{1}{U} = \frac{1}{h_f} + \frac{1}{h_h} + \frac{B}{k} + \frac{1}{h_c}$$

Substituting values gives:

$$\frac{1}{U} = \frac{1}{2.5\ \text{kW m}^{-2}\ ^\circ\text{C}^{-1}} + \frac{1}{0.846\ \text{kW m}^{-2}\ ^\circ\text{C}^{-1}} + \frac{7\times10^{-3}\ \text{m}}{17\times10^{-3}\ \text{kW m}^{-1}\ ^\circ\text{C}^{-1}} + \frac{1}{23.9\ \text{kW m}^{-2}\ ^\circ\text{C}^{-1}}$$

$$= 2.04\ \text{kW}^{-1}\ \text{m}^2\ ^\circ\text{C}$$

$$U = 0.491\ \text{kW m}^{-2}\ ^\circ\text{C}^{-1}$$

Answer: 0.491 kW m^{-2} °C^{-1}

(e)

As we have assumed that *A* remains the same throughout the thickness of the film, wall and fouling layers (Section 9.4.3), the fraction of the overall resistance provided by the tube-side liquid film is:

$$\frac{1/h_c}{1/U} = \frac{1/(23.9\ \text{kW m}^{-2}\ °\text{C}^{-1})}{2.04\ \text{kW}^{-1}\ \text{m}^2\ °\text{C}} = 0.021$$

Answer: 0.021

(f)

The fraction of the overall resistance provided by the stainless steel pipe is:

$$\frac{B/k}{1/U} = \frac{\left(\frac{7\times10^{-3}\ \text{m}}{17\times10^{-3}\ \text{kW m}^{-1}\ °\text{C}^{-1}}\right)}{2.04\ \text{kW}^{-1}\ \text{m}^2\ °\text{C}} = 0.202$$

Answer: 0.20

(g)

The fraction of the overall resistance provided by the fouling layer is:

$$\frac{1/h_f}{1/U} = \frac{1/(2.5\ \text{kW m}^{-2}\ °\text{C}^{-1})}{2.04\ \text{kW}^{-1}\ \text{m}^2\ °\text{C}} = 0.196$$

Answer: 0.20

(h)

The temperature-difference driving force is represented by the log-mean temperature difference between the fermentation fluid and cooling water. From Eq. (9.39) using the result for T_{co} from **(a)**:

$$\Delta T = \frac{(24.2-14)°\text{C}}{\ln\frac{(29-14)°\text{C}}{(29-24.2)°\text{C}}} = 8.95°\text{C}$$

Answer: 9.0°C

(i)

The heat transfer area is determined from Eq. (9.19):

$$A = \frac{\hat{Q}}{U\Delta T}$$

Substituting values:

$$A = \frac{593.92\ \text{kW}}{0.491\ \text{kW m}^{-2}\ °\text{C}^{-1}\ (8.95°\text{C})} = 135.2\ \text{m}^2$$

Answer: 135 m^2

(j)

The cooling coil length is determined from Eq. (9.23):

$$L = \frac{A}{2\pi R}$$

For this calculation, we will use an average radius for the cooling coil pipe to take into account the thickness of the tube wall: $R_{av} = (R_i + R_o)/2 = (R_i + R_i + B)/2 = [(2.1 \times 10^{-2}\ \text{m}) + (2.8 \times 10^{-2}\ \text{m})]/2 = 2.45 \times 10^{-2}$ m. Substituting values gives:

$$L = \frac{135.2\ \text{m}^2}{2\pi(2.45\times10^{-2}\ \text{m})} = 878.3\ \text{m}$$

Answer: 878 m

(k)

The length of pipe in one coil of the helix is approximately equal to the circumference of a circle with diameter 3.2 m:

$$\text{Length per coil} = \pi D = \pi(3.2\ \text{m}) = 10.05\ \text{m}$$

Therefore, to form 878.3 m of pipe into a helix, the number of coils required is:

$$\text{Number of coils} = \frac{\text{total pipe length}}{\text{length per coil}} = \frac{878.3\ \text{m}}{10.05\ \text{m}} = 87.4$$

Answer: 87.4

(l)

Each coil of the helix takes up vertical space equal to the outer diameter of the cooling water pipe plus spacing of 10 cm. The outer diameter of the pipe is equal to the inner diameter + 2 × the pipe wall thickness. Therefore:

$$\text{Height per coil} = (4.2\times10^{-2}\ \text{m} + 2\times7\times10^{-3}\ \text{m}) + 10\times10^{-2}\ \text{m} = 0.156\ \text{m}$$

From **(k)**, 87.4 coils are required; therefore the total height of the helix is roughly:

$$\text{Total height of helix} = 87.4\times0.156\ \text{m} = 13.6\ \text{m}$$

As the height of the fermenter is only 8 m, the cooling coil will not fit into the vessel.

Answer: No

9.14 Suitability of an existing cooling coil

V_L = 20 m^3. D = 7.5 cm = 7.5 × 10^{-2} m; therefore, $R = D/2$ = (7.5 × 10^{-2} m)/2 = 3.75 × 10^{-2} m. D_T = 3 m; D_i = 1 m; N_i = 50 rpm = 50 min^{-1}.

Q_O = 90 mol m^{-3} h^{-1}; T_F = 28°C; T_{ci} = 12°C.

From Eq. (7.8), μ water = 10^{-3} kg m^{-1} s^{-1}; therefore μ = 10^{-3} kg m^{-1} s^{-1}. From Section 2.4.1, the density of water is 1 g cm^{-3} = 1000 kg m^{-3}; therefore ρ = 1000 kg m^{-3}. Applying the definition of density (Section 2.4.1), the mass flow rate of cooling water is equal to the volumetric flow rate × density:

$$\hat{M}_c = 20\ \text{m}^3\ \text{h}^{-1}\times1000\ \text{kg m}^{-3}.\left|\frac{1\ \text{h}}{3600\ \text{s}}\right| = 5.56\ \text{kg s}^{-1}$$

From Table C.3 (Appendix C), the heat capacity of water = 75.4 J gmol^{-1} °C^{-1} = 75.4 kJ kgmol^{-1} °C^{-1}. Converting this to a mass basis using the molecular weight of water = 18.0 (Table C.1, Appendix C):

$$C_p = 75.4\ \text{kJ kgmol}^{-1}\ ^\circ\text{C}^{-1}.\left|\frac{1\ \text{kgmol}}{18.0\ \text{kg}}\right| = 4.189\ \text{kJ kg}^{-1}\ ^\circ\text{C}^{-1}$$

From Table 9.1, k_{fb} water at 30°C = 0.62 W m^{-1} °C^{-1}; therefore, from Table A.8 (Appendix A), k_{fb} = 0.62 J s^{-1} m^{-1} °C^{-1} = 0.62 × 10^{-3} kJ s^{-1} m^{-1} °C^{-1} = 0.62 × 10^{-3} kW m^{-1} °C^{-1}.

The cooling requirements are determined from the energy balance equation, Eq. (9.48). From Eq. (5.23), the heat of reaction for aerobic cultures is –460 kJ per gmol of oxygen consumed. Therefore:

$$\Delta\hat{H}_{rxn} = (-460\ \text{kJ mol}^{-1})Q_O V_L = (-460\ \text{kJ mol}^{-1})\ 90\ \text{mol m}^{-3}\ \text{h}^{-1}\ (20\ \text{m}^3)\ .\left|\frac{1\ \text{h}}{3600\ \text{s}}\right| = -230\ \text{kJ s}^{-1}$$

From Table A.8 (Appendix A), 1 W = 1 J s^{-1}; therefore:

$$\Delta\hat{H}_{rxn} = -230\ \text{kW}$$

Stirring provides an additional energy input to the system. The rate of shaft work $\hat{W}_s$ in Eq. (9.48) is the power dissipated by the impeller. First, check that flow in the fermenter is turbulent by calculating the impeller Reynolds number using Eq. (7.2):

$$Re_i = \frac{50\ \text{min}^{-1}\ .\left|\frac{1\ \text{min}}{60\ \text{s}}\right|\ (1\ \text{m})^2\ (1000\ \text{kg m}^{-3})}{10^{-3}\ \text{kg m}^{-1}\ \text{s}^{-1}} = 8.33\times10^5$$

As this Re_i is greater than the threshold of 10^4 for turbulent flow (Section 8.5.1), power can be evaluated using Eq. (8.9). D_i/D_T for the Rushton turbine = (1 m)/(3 m) = 0.33; therefore N'_P = 5.0 (Figure 8.29 and Table 8.1). Assume that the fermenter is not gassed. Substituting values into Eq. (8.9) gives:

$$P = 5.0\ (1000\ \text{kg m}^{-3})\left(50\ \text{min}^{-1}\ .\left|\frac{1\ \text{min}}{60\ \text{s}}\right|\right)^3\ (1\ \text{m})^5 = 2.89\times10^3\ \text{kg m}^2\ \text{s}^{-3}$$

From Table A.8 (Appendix A), 1 W = 1 kg m^2 s^{-3}; therefore:

$$P = \hat{W}_s = 2.89\ \text{kW}$$

Assuming that evaporation can be neglected, Eq. (9.48) becomes:

$$\hat{Q} = -\Delta\hat{H}_{rxn} + \hat{W}_s$$

Substituting values:

$$\hat{Q} = 230\ \text{kW} + 2.89\ \text{kW} = 232.89\ \text{kW}$$

The fermenter-side heat transfer coefficient is evaluated using the empirical correlation, Eq. (9.36). The dimensionless numbers in this equation are Re_i, Pr and Nu. Re_i has already been calculated; Pr is given by Eq. (9.30):

$$Pr = \frac{4.189\ \text{kJ kg}^{-1}\ ^\circ\text{C}^{-1}\ (10^{-3}\ \text{kg m}^{-1}\ \text{s}^{-1})}{0.62\times10^{-3}\ \text{kJ s}^{-1}\ \text{m}^{-1}\ ^\circ\text{C}^{-1}} = 6.76$$

From Eq. (9.36), assuming that the viscosity at the fermenter wall is the same as the bulk viscosity:

$$Nu = 0.9\ (8.33\times10^5)^{0.62}\ (6.76)^{0.33} = 7923.6$$

From the definition of Nu in Eq. (9.27) with $D = D_T$ (Section 9.5.1, Stirred Liquids subsection):

$$h = \frac{Nu\ k_{fb}}{D} = \frac{7923.6\ (0.62\times10^{-3}\ \text{kW m}^{-1}\ ^\circ\text{C}^{-1})}{3\ \text{m}} = 1.64\ \text{kW m}^{-2}\ ^\circ\text{C}^{-1}$$

Therefore, as the fermenter fluid is the hot fluid, h_h is 1.64 kW m^{-2} $°C^{-1}$. From Eq. (9.24), assuming that the tube-side and pipe-wall resistances to heat transfer are negligible compared with the fermenter-side film resistance (Section 9.6.1):

$$U = h_h = 1.64\ \text{kW m}^{-2}\ ^\circ\text{C}^{-1}$$

From the energy balance equation for the cooling water, Eq. (9.47), an expression for the outlet cooling-water temperature is:

$$T_{co} = \frac{\hat{Q}}{\hat{M}_c C_{pc}} + T_{ci}$$

Substituting values and applying the conversion factor 1 kW = 1 kJ s^{-1} derived from Table A.8 (Appendix A):

$$T_{co} = \frac{232.89 \text{ kW} \cdot \left| \frac{1 \text{ kJ s}^{-1}}{1 \text{ kW}} \right|}{5.56 \text{ kg s}^{-1} (4.189 \text{ kJ kg}^{-1} \text{°C}^{-1})} + 12\text{°C}$$

$$= 22.0\text{°C}$$

The temperature-difference driving force is represented by the log-mean temperature difference between the fermentation fluid and cooling water. Applying Eq. (9.39):

$$\Delta T = \frac{(22.0 - 12)\text{°C}}{\ln \frac{(28 - 12)\text{°C}}{(28 - 22.0)\text{°C}}} = 10.2\text{°C}$$

The heat transfer area is determined from Eq. (9.19):

$$A = \frac{\hat{Q}}{U \Delta T}$$

Substituting values:

$$A = \frac{232.89 \text{ kW}}{1.64 \text{ kW m}^{-2} \text{°C}^{-1} (10.2\text{°C})} = 13.92 \text{ m}^2$$

The cooling coil length is determined from Eq. (9.23):

$$L = \frac{A}{2\pi R}$$

Substituting values gives:

$$L = \frac{13.92 \text{ m}^2}{2\pi (3.75 \times 10^{-2} \text{ m})} = 59.1 \text{ m}$$

As the cooling coil in the second-hand fermenter is only 45 m long, it is inadequate for this application.

Answer: No

9.15 Heat transfer and cooling water in fermenter design

V_L = 100 m^3; D_T = 5 m; D_i = 1.7 m. N_i = 80 rpm = 80 min^{-1}. From Section 5.8, the heat of reaction for an exothermic reaction is negative; therefore, $\Delta \hat{H}_{rxn}$ = –2500 kW.

Culture fluid: C_p = 4.2 kJ kg^{-1} °C^{-1} = 4.2 × 10^3 J kg^{-1} °C^{-1}. k_{fb} = 0.6 W m^{-1} °C^{-1}; therefore, from Table A.8 (Appendix A), k_{fb} = 0.6 J s^{-1} m^{-1} °C^{-1}. ρ = 10^3 kg m^{-3}. μ_b = 10^{-3} N s m^{-2}; therefore, from Table A.9 (Appendix A), μ_b = 10^{-3} kg m^{-1} s^{-1}. T_F = 30°C.

Cooling coil: R_i = (12 cm)/2 = 6 cm = 6 × 10^{-2} m. B = 6 mm = 6 × 10^{-3} m. k = 20 W m^{-1} °C^{-1}.

Cooling water: T_{ci} = 10°C. From Table C.3 (Appendix C), C_{pc} = 75.4 J $gmol^{-1}$ °C^{-1} = 75.4 kJ $kgmol^{-1}$ °C^{-1}. Converting C_{pc} to a mass basis using the molecular weight of water = 18.0 (Table C.1, Appendix C):

$$C_{pc} = 75.4 \text{ kJ kgmol}^{-1} \text{°C}^{-1} \cdot \left| \frac{1 \text{ kgmol}}{18.0 \text{ kg}} \right| = 4.19 \text{ kJ kg}^{-1} \text{°C}^{-1}$$

(a)

The fermenter-side heat transfer coefficient is evaluated using the empirical correlation, Eq. (9.36). The dimensionless numbers in this equation are Re_i, Pr and Nu. Re_i is given by Eq. (9.29):

$$Re_i = \frac{80\ \text{min}^{-1} . \left| \frac{1\ \text{min}}{60\ \text{s}} \right| (1.7\ \text{m})^2 (10^3\ \text{kg m}^{-3})}{10^{-3}\ \text{kg m}^{-1}\ \text{s}^{-1}} = 3.85 \times 10^6$$

Pr is given by Eq. (9.30):

$$Pr = \frac{4.2 \times 10^3\ \text{J kg}^{-1}\ ^\circ\text{C}^{-1} (10^{-3}\ \text{kg m}^{-1}\ \text{s}^{-1})}{0.6\ \text{J s}^{-1}\ \text{m}^{-1}\ ^\circ\text{C}^{-1}} = 7.00$$

Applying Eq. (9.36):

$$Nu = 0.9\ (3.85 \times 10^6)^{0.62}\ (7.00)^{0.33}\ (1)^{0.14} = 2.07 \times 10^4$$

From the definition of Nu in Eq. (9.27) with $D = D_T$ (Section 9.5.1, Stirred Liquids subsection):

$$h = \frac{Nu\, k_{fb}}{D} = \frac{2.07 \times 10^4\ (0.6\ \text{W m}^{-1}\ ^\circ\text{C}^{-1})}{5\ \text{m}} = 2.48 \times 10^3\ \text{W m}^{-2}\ ^\circ\text{C}^{-1} = 2.48\ \text{kW m}^{-2}\ ^\circ\text{C}^{-1}$$

Therefore, as the fermenter fluid is the hot fluid, h_h is 2.48 kW m^{-2} °C^{-1}.

Answer: 2.48 kW m^{-2} °C^{-1}

(b)

The overall heat transfer coefficient in the absence of fouling layers is determined using Eq. (9.24). As the tube-side heat transfer coefficient h_c can be ignored, Eq. (9.24) becomes:

$$\frac{1}{U} = \frac{1}{h_h} + \frac{B}{k}$$

Substituting values:

$$\frac{1}{U} = \frac{1}{2.48 \times 10^3\ \text{W m}^{-2}\ ^\circ\text{C}^{-1}} + \frac{6 \times 10^{-3}\ \text{m}}{20\ \text{W m}^{-1}\ ^\circ\text{C}^{-1}} = 7.03 \times 10^{-4}\ \text{W}^{-1}\ \text{m}^2\ ^\circ\text{C}$$

Therefore:

$$U = 1.42 \times 10^3\ \text{W m}^{-2}\ ^\circ\text{C}^{-1} = 1.42\ \text{kW m}^{-2}\ ^\circ\text{C}^{-1}$$

Answer: 1.42 kW m^{-2} °C^{-1}

(c)

From Eqs (9.21) and (9.22), the heat transfer resistance due to the pipe wall is $B/(kA)$. From Eq. (9.20), the total resistance to heat transfer is $1/(UA)$. Therefore, the proportion of the total resistance due to the pipe wall is:

$$\frac{B/(kA)}{1/(UA)} = \frac{BU}{k} = \frac{6 \times 10^{-3}\ \text{m}\ (1.42 \times 10^3\ \text{W m}^{-2}\ ^\circ\text{C}^{-1})}{20\ \text{W m}^{-1}\ ^\circ\text{C}^{-1}} = 0.43$$

Answer: 0.43

(d)

The rate of shaft work $\hat{W}_s$ in Eq. (9.48) is the power dissipated by the impeller. From the calculation in **(a)**, Re_i is greater than the threshold of 10^4 for turbulent flow (Section 8.5.1); therefore, power can be evaluated using Eq. (8.9). D_i/D_T for the Rushton turbine is (1.7 m)/(5 m) = 0.34, which is close enough to

0.33; therefore, $N_P' = 5.0$ (Figure 8.29 and Table 8.1). Assume that the fermenter is not gassed. Substituting values into Eq. (8.9) gives:

$$P = 5.0\,(10^3\ \text{kg m}^{-3})\left(80\ \text{min}^{-1}.\left|\frac{1\ \text{min}}{60\ \text{s}}\right|\right)^3 (1.7\ \text{m})^5 = 1.68\times10^5\ \text{kg m}^2\ \text{s}^{-3}$$

From Table A.8 (Appendix A), 1 W = 1 kg m^2 s^{-3}; therefore:

$$P = \hat{W}_s = 168\ \text{kW}$$

Applying Eq. (9.48) to determine the cooling requirements, assuming that evaporation can be neglected:

$$\hat{Q} = -\Delta\hat{H}_{rxn} + \hat{W}_s$$
$$= 2500\ \text{kW} + 168\ \text{kW} = 2668\ \text{kW}$$

Therefore, the contribution of shaft work to $\hat{Q}$ is (168 kW)/(2668 kW) × 100% = 6.3%.

Answer: 6.3%

(e)

From the energy balance equation for the cooling water, Eq. (9.47), an expression for the outlet cooling-water temperature is:

$$T_{co} = \frac{\hat{Q}}{\hat{M}_c\, C_{pc}} + T_{ci} \tag{1}$$

Applying the conversion factor 1 W = 1 J s^{-1} from Table A.8 (Appendix A), $\hat{Q}$ from **(d)** is 2668 kJ s^{-1}. $\hat{M}_c$ = 1.5 × 10^5 kg h^{-1}. Substituting values into (1) gives:

$$T_{co} = \frac{2668\ \text{kJ s}^{-1}}{1.5\times10^5\ \text{kg h}^{-1}.\left|\frac{1\ \text{h}}{3600\ \text{s}}\right|\ (4.19\ \text{kJ kg}^{-1}\ ^\circ\text{C}^{-1})} + 10^\circ\text{C}$$

$$= 25.3^\circ\text{C}$$

Answer: 25.3°C

(f)

The log-mean temperature difference between the fermentation fluid and cooling water is calculated using Eq. (9.39) and the result for T_{co} from **(e)**:

$$\Delta T = \frac{(25.3-10)^\circ\text{C}}{\ln\frac{(30-10)^\circ\text{C}}{(30-25.3)^\circ\text{C}}} = 10.6^\circ\text{C}$$

The cooling coil area A is calculated using Eq. (9.19):

$$A = \frac{\hat{Q}}{U\,\Delta T} \tag{2}$$

Substituting values:

$$A = \frac{2668\ \text{kW}}{1.42\ \text{kW m}^{-2}\ ^\circ\text{C}^{-1}\,(10.6^\circ\text{C})} = 177.3\ \text{m}^2$$

This result is used to calculate the cooling coil length from Eq. (9.23):

$$L = \frac{A}{2\pi R} \tag{3}$$

The cooling coil outer radius $R_o = (R_i + B) = (6 \times 10^{-2} + 6 \times 10^{-3})$ m = 6.6×10^{-2} m. We will use an average radius $R_{av} = (R_i + R_o)/2 = (6 \times 10^{-2}\ \text{m} + 6.6 \times 10^{-3}\ \text{m})/2 = 6.3 \times 10^{-2}$ m in the above equation for L. Substituting values:

$$L = \frac{177.3\ \text{m}^2}{2\pi(6.3\times10^{-2}\ \text{m})} = 447.9\ \text{m}$$

Answer: 448 m. *This is a long cooling coil, representing a considerable expense when fabricated from stainless steel.*

(g)

After the cooling-water flow rate is increased by 50%:

$$\hat{M}_c = 1.5\times\left(1.5\times10^5\ \text{kg h}^{-1}.\left|\frac{1\ \text{h}}{3600\ \text{s}}\right|\right) = 62.5\ \text{kg s}^{-1}$$

Applying this value in (1):

$$T_{co} = \frac{2668\ \text{kJ s}^{-1}}{62.5\ \text{kg s}^{-1}(4.19\ \text{kJ kg}^{-1}\,°\text{C}^{-1})} + 10°\text{C}$$

$$= 20.2°\text{C}$$

Using this result in Eq. (9.39), the log-mean temperature difference is:

$$\Delta T = \frac{(20.2-10)°\text{C}}{\ln\dfrac{(30-10)°\text{C}}{(30-20.2)°\text{C}}} = 14.3°\text{C}$$

Substituting values into (2):

$$A = \frac{2668\ \text{kW}}{1.42\ \text{kW m}^{-2}\,°\text{C}^{-1}(14.3°\text{C})} = 131.4\ \text{m}^2$$

and (3):

$$L = \frac{131.4\ \text{m}^2}{2\pi(6.3\times10^{-2}\ \text{m})} = 332.0\ \text{m}$$

Relative to the result in **(f)**, the percentage reduction in cooling coil length is:

$$\frac{(447.9\ \text{m}-332.0\ \text{m})}{447.9\ \text{m}}\times100\% = 26\%$$

Answer: The length would be reduced by 26% to 332 m.

9.16 Test for heat transfer limitation

If the glucose provided is completely consumed, the cell concentration x produced is:

$$x = 12\ \text{g l}^{-1}\left(\frac{1.0\ \text{g}}{2.2\ \text{g}}\right) = 5.45\ \text{g l}^{-1}$$

This can be compared with the maximum cell concentration supported by the heat transfer system, which can be calculated using Eq. (9.54). Converting the units of q_O:

$$q_O = 7.5 \text{ mmol g}^{-1} \text{ h}^{-1} = 7.5\times10^{-3} \text{ gmol g}^{-1} \text{ h}^{-1} . \left| \frac{1 \text{ h}}{3600 \text{ s}} \right| = 2.08\times10^{-6} \text{ gmol g}^{-1} \text{ s}^{-1}$$

Using Eq. (9.23) to determine the heat transfer area:

$$A = 2\pi \left(\frac{5\times10^{-2} \text{ m}}{2} \right) 55 \text{ m} = 8.64 \text{ m}^2$$

Applying the conversion factor 1 W = 1 J s^{-1} (Table A.8, Appendix A), the overall heat transfer coefficient U is 250 J s^{-1} m^{-2} °C^{-1} = 0.25 kJ s^{-1} m^{-2} °C^{-1}. Substituting values into Eq. (9.54) gives:

$$x_{max} = \frac{0.25 \text{ kJ s}^{-1} \text{ m}^{-2} \text{ °C}^{-1} (8.64 \text{ m}^2) (32-15)\text{°C}}{460 \text{ kJ gmol}^{-1} (2.08\times10^{-6} \text{ gmol g}^{-1} \text{ s}^{-1}) (8 \text{ m}^3)} = 4.80\times10^3 \text{ g m}^{-3}$$

As 1 m^3 = 10^3 l (Table A.2, Appendix A), x_{max} = 4.80 g l^{-1}.

The maximum cell concentration supported by the heat transfer system (4.80 g l^{-1}) is less than the cell concentration corresponding to complete substrate consumption (5.45 g l^{-1}). Therefore, the heat transfer system does not allow complete consumption of the substrate.

Answer: No

9.17 Optimum stirrer speed for removal of heat from viscous broth

Culture broth: μ_b = 10,000 cP; therefore, from Table A.9 (Appendix A), μ_b = 10,000 × 10^{-3} kg m^{-1} s^{-1} = 10 kg m^{-1} s^{-1}. C_p = 2 kJ kg^{-1} °C^{-1}; ρ = 10^3 kg m^{-3}. k_{fb} = 2 W m^{-1} °C^{-1}; therefore, from Table A.8 (Appendix A), k_{fb} = 2 J s^{-1} m^{-1} °C^{-1} = 2 × 10^{-3} kJ s^{-1} m^{-1} °C^{-1}.

V_L = 10 m^3; D_T = 2.3 m; D_i = 0.78 m; A = 14 m^2; ΔT = 20°C.

(a), (b), (c)

The fermenter-side heat transfer coefficient is evaluated using the empirical correlation, Eq. (9.36). The dimensionless numbers in this equation are Re_i, Pr and Nu. The dependence of Re_i on stirrer speed is given by Eq. (9.29). Substituting available parameter values gives:

$$Re_i = \frac{N_i (0.78 \text{ m})^2 (10^3 \text{ kg m}^{-3})}{10 \text{ kg m}^{-1} \text{ s}^{-1}} = 60.84 N_i$$

where N_i has units of s^{-1} and Re_i is dimensionless. Pr is given by Eq. (9.30):

$$Pr = \frac{2 \text{ kJ kg}^{-1} \text{ °C}^{-1} (10 \text{ kg m}^{-1} \text{ s}^{-1})}{2\times10^{-3} \text{ kJ s}^{-1} \text{ m}^{-1} \text{ °C}^{-1}} = 10^4$$

From Eq. (9.36), as the viscosity at the wall is equal to the viscosity of the bulk fluid:

$$Nu = 0.9 (60.84 N_i)^{0.62} (10^4)^{0.33} (1)^{0.14} = 240.1 N_i^{0.62}$$

From the definition of Nu in Eq. (9.27) with $D = D_T$ (Section 9.5.1, Stirred Liquids subsection):

$$h = \frac{Nu\, k_{fb}}{D} = \frac{240.1 N_i^{0.62} (2\times10^{-3} \text{ kJ s}^{-1} \text{ m}^{-1} \text{ °C}^{-1})}{2.3 \text{ m}} = 0.21 N_i^{0.62} \text{ kJ s}^{-1} \text{ m}^{-2} \text{ °C}^{-1}$$

Therefore, as the fermentation broth is the hot fluid:

$$h_h = 0.21 N_i^{0.62} \text{ kJ s}^{-1} \text{ m}^{-2} \text{ °C}^{-1}$$

From Eq. (9.24), assuming that the tube-side and pipe wall resistances can be neglected (Section 9.6.1):

$$U = h_h = 0.21 N_i^{0.62} \text{ kJ s}^{-1} \text{ m}^{-2} \text{ °C}^{-1}$$

Substituting parameter values into Eq. (9.19):

$$\hat{Q} = 0.21 N_i^{0.62} \text{ kJ s}^{-1} \text{ m}^{-2} \text{ °C}^{-1} (14 \text{ m}^2) \, 20\text{°C} = 58.8 N_i^{0.62} \text{ kJ s}^{-1}$$

This equation is used to calculate $\hat{Q}$ as a function of stirrer speed N_i, as shown in the table below.

The rate of shaft work $\hat{W}_s$ is equal to the power dissipated from the stirrer. The range of stirrer speeds to be investigated is 0.5 s^{-1} to 10 s^{-1}. We can determine the flow regime corresponding to the maximum value of N_i by calculating the impeller Reynolds number using Eq. (7.2):

$$Re_i = \frac{10 \text{ s}^{-1} (0.78 \text{ m})^2 (10^3 \text{ kg m}^{-3})}{10 \text{ kg m}^{-1} \text{ s}^{-1}} = 608$$

This Re_i is considerably less than the threshold of 10^4 for turbulent flow (Section 8.5.1 and Figure 8.29). Therefore, flow for the stirrer speed range of interest is either laminar or in the transition regime. Under these conditions, the stirrer power without gassing is evaluated using Eq. (8.7) with the value of N_P dependent on Re_i. As the value of N_P with gassing (N_{Pg}) is 40% lower or 0.6 × the value of N_P read from Figure 8.29, from Eq. (8.7):

$$\hat{W}_s = P = N_{Pg} \rho N_i^3 D_i^5 = 0.6 N_P (10^3 \text{ kg m}^{-3}) N_i^3 (0.78 \text{ m})^5 = 173 N_P N_i^3 \text{ kg m}^2 \text{ s}^{-3}$$

where the value of N_P depends on Re_i and N_i has units of s^{-1}. From Table A.8 (Appendix A), 1 kg m^2 s^{-3} = 1 J s^{-1} = 10^{-3} kJ s^{-1}; therefore:

$$\hat{W}_s = 0.173 N_P N_i^3 \text{ kJ s}^{-1}$$

From the steady-state energy balance equation, Eq. (9.48), assuming that evaporation is negligible:

$$-\Delta \hat{H}_{rxn} = \hat{Q} - \hat{W}_s$$

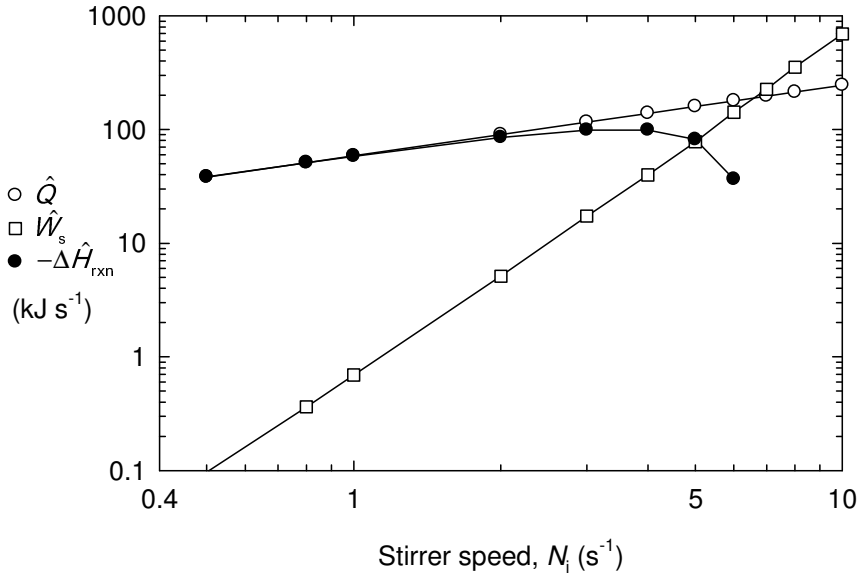
Substituting the above expressions for $\hat{Q}$ and $\hat{W}_s$:

$$-\Delta \hat{H}_{rxn} = (58.8 N_i^{0.62} - 0.173 N_P N_i^3) \text{ kJ s}^{-1}$$

Values of $\hat{Q}$, $\hat{W}_s$ and $-\Delta \hat{H}_{rxn}$ calculated using the equations derived above are listed below as a function of N_i. D_i/D_T for the Rushton turbine is (0.78 m)/(2.3 m) = 0.34; this is close enough to 0.33 so that values of N_P as a function of Re_i can be read from Curve 1 in Figure 8.29. Alternatively, N_P values can be obtained from the original reference (J.H. Rushton, E.W. Costich and H.J. Everett, 1950, Power characteristics of mixing impellers. *Chem. Eng. Prog*. **46,** 467–476) for more accurate interpolation of the power curve in Figure 8.29.

N_i (s^{-1})	Re_i	$\hat{Q}$ (kJ s^{-1})	N_P	$\hat{W}_s$ (kJ s^{-1})	$-\Delta \hat{H}_{rxn}$ (kJ s^{-1})
0.5	30.4	38.3	4.5	0.10	38.2
0.8	48.7	51.2	4.1	0.36	50.8
1.0	60.8	58.8	4.0	0.69	58.1
2.0	122	90.4	3.7	5.12	85.2
3.0	183	116	3.7	17.3	98.9
4.0	243	139	3.6	39.9	99.0
5.0	304	159	3.6	77.9	81.6
6.0	365	179	3.8	142	36.6
7.0	426	196	3.8	225	–29
8.0	487	213	4.0	354	–141
10.0	608	245	4.0	692	–447

The results for $\hat{Q}, \hat{W}_s$ and $-\Delta\hat{H}_{rxn}$ are plotted below on log–log coordinates as a function of N_i.



(d)

The rate of removal of metabolic heat (the $-\Delta\hat{H}_{rxn}$ component of $\hat{Q}$) reaches a maximum value at about N_i = 4 s^{-1}.

Answer: About 4 s^{-1}

(e)

From Eq. (5.23), the heat of reaction for aerobic cultures is –460 kJ per gmol of oxygen consumed. Therefore, if the rate of oxygen consumption is 6 mmol g^{-1} h^{-1} = 6 × 10^{-3} gmol g^{-1} h^{-1}, when the stirrer speed is 4 s^{-1} and $\Delta\hat{H}_{rxn}$ = –99.0 kJ s^{-1}:

$$\text{Biomass} = \frac{-99.0 \text{ kJ s}^{-1}}{6\times10^{-3} \text{ gmol g}^{-1} \text{ h}^{-1} \cdot \left|\frac{1 \text{ h}}{3600 \text{ s}}\right| (-460 \text{ kJ gmol}^{-1})} = 1.29\times10^{5} \text{ g}$$

The cell concentration is equal to the biomass divided by the fermenter volume. Using the volume conversion factor 1 m^3 = 10^3 l from Table A.2 (Appendix A):

$$\text{Cell concentration} = \frac{1.29\times10^{5} \text{ g}}{10 \text{ m}^3 \cdot \left|\frac{10^3 \text{ l}}{1 \text{ m}^3}\right|} = 12.9 \text{ g l}^{-1}$$

Answer: 12.9 g l^{-1}

(f)

As the stirrer speed is increased, the overall heat transfer coefficient increases and the rate of heat transfer through the boundary layer in the fermentation broth is improved. Therefore, the rate at which heat can be removed from the system $\hat{Q}$ is increased. However, the rate at which heat is dissipated by the stirrer $\hat{W}_s$ also increases with stirrer speed. When the curves for $\hat{Q}$ and $\hat{W}_s$ intersect, the entire heat transfer capacity of the fermenter cooling system is being used just to remove the heat generated by stirring; the system is unable to remove any of the heat generated from reaction. Accordingly, at high stirrer speeds, the system has limited capacity to handle exothermic reactions.

Chapter 10

Mass Transfer

10.1 Rate-controlling processes in fermentation

Converting the units of the maximum specific oxygen uptake rate using the molecular weight of oxygen = 32.0 (Table C.1, Appendix C):

$$q_O = 5 \text{ mmol g}^{-1} \text{ h}^{-1} . \left| \frac{1 \text{ gmol}}{1000 \text{ mmol}} \right| . \left| \frac{32.0 \text{ g}}{1 \text{ gmol}} \right| . \left| \frac{1 \text{ h}}{3600 \text{ s}} \right| = 4.44 \times 10^{-5} \text{ g g}^{-1} \text{ s}^{-1}$$

At a cell density of 40 g l^{-1}, the maximum oxygen requirement is:

$$q_O x = 4.44 \times 10^{-5} \text{ g g}^{-1} \text{ s}^{-1} \ (40 \text{ g l}^{-1}) . \left| \frac{1000 \text{ l}}{1 \text{ m}^3} \right| . \left| \frac{1 \text{ kg}}{1000 \text{ g}} \right| = 1.78 \times 10^{-3} \text{ kg m}^{-3} \text{ s}^{-1}$$

The rate of oxygen transfer is given by Eq. (10.39); N_A takes a maximum value when $C_{AL} = 0$:

$$N_A = k_L a \, C_{AL}^* = 0.15 \text{ s}^{-1} (8 \times 10^{-3} \text{ kg m}^{-3}) = 1.20 \times 10^{-3} \text{ kg m}^{-3} \text{ s}^{-1}$$

As the maximum oxygen demand of the culture (1.78×10^{-3} kg m^{-3} s^{-1}) is greater than the maximum oxygen transfer rate in the fermenter (1.20×10^{-3} kg m^{-3} s^{-1}), the system will be limited by mass transfer.

Answer: Limited by mass transfer

10.2 Test for oxygen limitation

q_O = 7.5 mmol g^{-1} h^{-1}. Converting the units of q_O to g g^{-1} s^{-1} using the molecular weight of oxygen = 32.0 (Table C.1, Appendix C):

$$q_O = 7.5 \text{ mmol g}^{-1} \text{ h}^{-1} . \left| \frac{1 \text{ gmol}}{1000 \text{ mmol}} \right| . \left| \frac{32 \text{ g}}{1 \text{ gmol}} \right| . \left| \frac{1 \text{ h}}{3600 \text{ s}} \right| = 6.67 \times 10^{-5} \text{ g g}^{-1} \text{ s}^{-1}$$

C_{AL}^* for oxygen in water at 1 atm air pressure and 32°C can be interpolated from the values for 30°C and 35°C listed in Table 10.2:

$$C_{AL}^* (\text{O}_2 \text{ in water at 1 atm air pressure and } 32^\circ\text{C})$$

$$= 8.05 \times 10^{-3} \text{ kg m}^{-3} - \frac{(32 - 30)^\circ\text{C}}{(35 - 30)^\circ\text{C}} (8.05 - 7.52) \times 10^{-3} \text{ kg m}^{-3}$$

$$= 7.84 \times 10^{-3} \text{ kg m}^{-3}$$

From Henry's law Eq. (10.45), C_{AL}^* is directly proportional to the total gas pressure. As the pressure in the fermenter is 1.5 atm rather than 1 atm:

$$C_{AL}^* (\text{O}_2 \text{ in water at 1.5 atm air pressure and } 32^\circ\text{C}) = 7.84 \times 10^{-3} \text{ kg m}^{-3} \left(\frac{1.5 \text{ atm}}{1 \text{ atm}} \right) = 1.18 \times 10^{-2} \text{ kg m}^{-3}$$

The solubility of oxygen in the fermentation broth is 15% lower than in water; therefore:

$$C_{AL}^* (\text{O}_2 \text{ in fermentation broth at 1.5 atm air pressure and } 32^\circ\text{C}) = 0.85 \ (1.18 \times 10^{-2} \text{ kg m}^{-3})$$

$$= 9.99 \times 10^{-3} \text{ kg m}^{-3}$$

Converting to units of g l^{-1}:

$$C_{AL}^* (\text{O}_2 \text{ in fermentation broth at 1.5 atm air pressure and } 32^\circ\text{C})$$

$$= 9.99\times10^{-3}\ \text{kg m}^{-3} \cdot \left|\frac{1000\ \text{g}}{1\ \text{kg}}\right| \cdot \left|\frac{1\ \text{m}^3}{1000\ \text{l}}\right|$$

$$= 9.99\times10^{-3}\ \text{g l}^{-1}$$

If 1.0 g of cells is produced for every 4.2 g of sucrose consumed, then complete consumption of 80 g l^{-1} of sucrose produces (80 g l^{-1})/(4.2 g g^{-1}) = 19.05 g l^{-1} of cells. The k_La required to achieve this cell concentration under conditions allowing the maximum rate of mass transfer is estimated using Eq. (10.42):

$$k_\text{L}a = \frac{x_\text{max}\, q_\text{O}}{C^*_\text{AL}} = \frac{19.05\ \text{g l}^{-1}\,(6.67\times10^{-5}\ \text{g g}^{-1}\ \text{s}^{-1})}{9.99\times10^{-3}\ \text{g l}^{-1}} = 0.13\ \text{s}^{-1}$$

As this value of k_La is greater than the maximum that can be achieved in the fermenter (0.10 s^{-1}), the fermenter's mass transfer capacity does not support complete consumption of substrate.

Answer: No

10.3 k_La required to maintain critical oxygen concentration

Q_O = 80 mmol l^{-1} h^{-1}. Converting the units of Q_O to kg m^{-3} s^{-1} using the molecular weight of oxygen = 32.0 (Table C.1, Appendix C) and 1 m^3 = 10^3 l (Table A.2, Appendix A):

$$Q_\text{O} = 80\ \text{mmol l}^{-1}\,\text{h}^{-1} \cdot \left|\frac{1\ \text{gmol}}{1000\ \text{mmol}}\right| \cdot \left|\frac{32.0\ \text{g}}{1\ \text{gmol}}\right| \cdot \left|\frac{1\ \text{kg}}{1000\ \text{g}}\right| \cdot \left|\frac{10^3\ \text{l}}{1\ \text{m}^3}\right| \cdot \left|\frac{1\ \text{h}}{3600\ \text{s}}\right| = 7.11\times10^{-4}\ \text{kg m}^{-3}\ \text{s}^{-1}$$

Converting the units of the critical oxygen concentration to kg m^{-3}:

$$C_\text{crit} = 0.004\ \text{mM} = 0.004\ \text{mmol l}^{-1} \cdot \left|\frac{1\ \text{gmol}}{1000\ \text{mmol}}\right| \cdot \left|\frac{32.0\ \text{g}}{1\ \text{gmol}}\right| \cdot \left|\frac{1\ \text{kg}}{1000\ \text{g}}\right| \cdot \left|\frac{10^3\ \text{l}}{1\ \text{m}^3}\right| = 1.28\times10^{-4}\ \text{kg m}^{-3}$$

(a)

From Table 10.2, the solubility of oxygen in water at 30°C under 1 atm air pressure is 8.05 × 10^{-3} kg m^{-3}. If the solubility in fermentation broth is 10% lower than this:

$$C^*_\text{AL} = 0.9\,(8.05\times10^{-3}\ \text{kg m}^{-3}) = 7.25\times10^{-3}\ \text{kg m}^{-3}$$

To maintain the oxygen concentration in the broth at the critical level, from Eqs (10.40) and (10.43):

$$(k_\text{L}a)_\text{crit} = \frac{Q_\text{O}}{(C^*_\text{AL} - C_\text{crit})} = \frac{7.11\times10^{-4}\ \text{kg m}^{-3}\ \text{s}^{-1}}{(7.25\times10^{-3} - 1.28\times10^{-4})\ \text{kg m}^{-3}} = 0.10\ \text{s}^{-1}$$

Answer: 0.10 s^{-1}

(b)

From Section 10.6.5, if pure oxygen is used instead of air, the solubility of oxygen in the fermentation broth is increased by a factor of 4.8:

$$C^*_\text{AL} = 4.8\,(7.25\times10^{-3}\ \text{kg m}^{-3}) = 3.48\times10^{-2}\ \text{kg m}^{-3}$$

Therefore, to maintain the critical oxygen concentration:

$$(k_\text{L}a)_\text{crit} = \frac{Q_\text{O}}{(C^*_\text{AL} - C_\text{crit})} = \frac{7.11\times10^{-4}\ \text{kg m}^{-3}\ \text{s}^{-1}}{(3.48\times10^{-2} - 1.28\times10^{-4})\ \text{kg m}^{-3}} = 0.021\ \text{s}^{-1}$$

Answer: 0.021 s^{-1}

10.4 Oxygen transfer with different impellers

V_L = 10 m^3; H_L = 2.3 m; ρ = 1000 kg m^{-3}; P_0 = 9 kW = 9000 W. The gas flow rate is 0.6 m^3 m^{-3} min^{-1}; therefore, for a tank with liquid volume 10 m^3, the volumetric gas flow rate F_g is:

$$F_g = 0.6\ \text{m}^3\ \text{m}^{-3}\ \text{min}^{-1}\ (10\ \text{m}^3)\ .\left|\frac{1\ \text{min}}{60\ \text{s}}\right| = 0.1\ \text{m}^3\ \text{s}^{-1}$$

The superficial gas velocity u_G is equal to F_g divided by the cross-sectional area of the fermenter (Section 10.9). The cross-sectional area of the tank A can be calculated from V_L and H_L, as V_L for a cylindrical tank is equal to AH_L. Therefore:

$$A = \frac{V_L}{H_L} = \frac{10\ \text{m}^3}{2.3\ \text{m}} = 4.35\ \text{m}^2$$

and

$$u_G = \frac{F_g}{A} = \frac{0.1\ \text{m}^3\ \text{s}^{-1}}{4.35\ \text{m}^2} = 0.023\ \text{m s}^{-1}$$

The power input by gassing P_v is evaluated using Eq. (8.13) with g = 9.8066 m s^{-2} (Section 2.3) and 1 W = 1 kg m^2 s^{-3} (Table A.8, Appendix A):

$$P_v = 0.1\ \text{m}^3\ \text{s}^{-1}\ (1000\ \text{kg m}^{-3})\ (9.8066\ \text{m s}^{-2})\ (2.3\ \text{m}) = 2256\ \text{kg m}^2\ \text{s}^{-3}\ .\left|\frac{1\ \text{W}}{1\ \text{kg m}^2\ \text{s}^{-3}}\right| = 2256\ \text{W}$$

(a)

For the Rushton turbine, the power input by the impeller is $0.5P_0$ = 4500 W. Adding this to the power input by gassing P_v, P_T = 4500 W + 2256 W = 6756 W. Substituting values for P_T, V_L and u_G into the equation for k_La using the units specified gives:

$$k_La = 2.5\times10^{-3}\left(\frac{6756}{10}\right)^{0.7}(0.023)^{0.3} = 0.077\ \text{s}^{-1}$$

For the curved-blade disc turbine, the power input by the impeller is $0.95P_0$ = 8550 W. Adding this to the power input by gassing P_v, P_T = 8550 W + 2256 W = 10,806 W. Therefore:

$$k_La = 2.5\times10^{-3}\left(\frac{10{,}806}{10}\right)^{0.7}(0.023)^{0.3} = 0.107\ \text{s}^{-1}$$

Answer: For the Rushton turbine, k_La is 0.077 s^{-1}; for the curved-blade disc turbine, k_La is 39% higher at 0.107 s^{-1}

(b)

For the Rushton turbine:

$$\frac{P_v}{P_T} = \frac{2256\ \text{W}}{6756\ \text{W}}\times100\% = 33\%$$

For the curved-blade disc turbine:

$$\frac{P_v}{P_T} = \frac{2256\ \text{W}}{10{,}806\ \text{W}}\times100\% = 21\%$$

Answer: 33% for the Rushton turbine, 21% for the curved-blade disc turbine

(c)

From Eq. (10.42), as C_{AL}^{*} and q_O do not vary with impeller design, x_{max} is directly proportional to k_La. Therefore, using the results from **(a)**, if x_{max} = 15 g l^{-1} with the Rushton turbine, for the curved-blade disc turbine:

$$x_{max} = \frac{0.107\ \text{s}^{-1}}{0.077\ \text{s}^{-1}}(15\ \text{g l}^{-1}) = 20.8\ \text{g l}^{-1}$$

Answer: 20.8 g l^{-1}

(d)

k_La depends on the gas flow rate through both u_G and P_T. P_T can be expressed in terms of u_G using Eq. (8.13) and the definition of superficial gas velocity (Section 10.9). If P is the power input by the impeller:

$$P_T = P + P_v = P + F_g \rho g H_L = P + u_G A \rho g H_L$$

Assuming that the fractional impeller power loss with gassing is independent of u_G, we can substitute values into this equation for the Rushton turbine:

$$P_T = 4500\ \text{W} + u_G(4.35\ \text{m}^2)(1000\ \text{kg m}^{-3})(9.8066\ \text{m s}^{-2})(2.3\ \text{m}).\left|\frac{1\ \text{W}}{1\ \text{kg m}^2\ \text{s}^{-3}}\right|$$

$$= (4500 + 9.812\times10^4 u_G)\ \text{W}$$

where u_G has units m s^{-1}. From Eq. (10.42) and the results from **(a)**, to achieve the same x_{max} using the Rushton turbine as for the curved-blade disc turbine, k_La must be equal to 0.107 s^{-1}. Using the above expression for P_T together with values for k_La and V_L in the appropriate units, the expression for k_La becomes:

$$0.107 = 2.5\times10^{-3}\left(\frac{4500 + 9.812\times10^4 u_G}{10}\right)^{0.7} u_G^{0.3}$$

This equation can be solved using iterative methods. Starting with an estimate for u_G of 0.06 m s^{-1}, the value of the right side of the equation is:

$$2.5\times10^{-3}\left(\frac{4500 + 9.812\times10^4(0.06)}{10}\right)^{0.7} 0.06^{0.3} = 0.139$$

As this value is greater than the left side of the equation (0.107), the estimated value of u_G was too high. Using u_G = 0.04 m s^{-1}, the value of the right side of the equation is:

$$2.5\times10^{-3}\left(\frac{4500 + 9.812\times10^4(0.04)}{10}\right)^{0.7} 0.04^{0.3} = 0.106$$

This is close to but slightly less than 0.107, indicating that u_G should be increased a little. Using u_G = 0.042 m s^{-1}, the value of the right side of the equation is:

$$2.5\times10^{-3}\left(\frac{4500 + 9.812\times10^4(0.042)}{10}\right)^{0.7} 0.042^{0.3} = 0.110$$

This result indicates that 0.042 m s^{-1} is too high. Using u_G = 0.0405 m s^{-1}, the value of the right side of the equation is:

$$2.5\times10^{-3}\left(\frac{4500 + 9.812\times10^4(0.0405)}{10}\right)^{0.7} 0.0405^{0.3} = 0.107$$

Therefore, a k_La value of 0.107 s^{-1} is achieved using the Rushton turbine by operating the fermenter with u_G = 0.0405 m s^{-1}. Converting this result to a volumetric gas flow rate using the definition of superficial gas velocity (Section 10.9):

$$F_g = u_G A = 0.0405 \text{ m s}^{-1}(4.35 \text{ m}^2).\left|\frac{60 \text{ s}}{1 \text{ min}}\right| = 10.57 \text{ m}^3 \text{ min}^{-1}$$

As V_L = 10 m^3, this gas flow rate can be expressed as (10.57 m^3 min^{-1})/10 m^3 = 1.06 vvm.

Answer: 1.06 vvm, assuming that the fractional power loss with gassing does not vary with gas flow rate

10.5 Foam control and oxygen transfer

D_T = 1.5 m. Calculating the cross-sectional area of the fermenter:

$$A = \frac{\pi}{4} D_T^2 = \frac{\pi}{4}(1.5 \text{ m})^2 = 1.767 \text{ m}^2$$

q_O = 2.6 mmol g^{-1} h^{-1}. Converting to units of g g^{-1} s^{-1} using the molecular weight of oxygen = 32.0 (Table C.1, Appendix C):

$$q_O = 2.6 \text{ mmol g}^{-1} \text{ h}^{-1}.\left|\frac{1 \text{ gmol}}{1000 \text{ mmol}}\right|.\left|\frac{32.0 \text{ g}}{1 \text{ gmol}}\right|.\left|\frac{1 \text{ h}}{3600 \text{ s}}\right| = 2.31\times10^{-5} \text{ g g}^{-1} \text{ s}^{-1}$$

s_0 = 20 g l^{-1}; Y_{XS} = 0.32 g g^{-1}; Y_{PX} = 0.055 g g^{-1}; C^*_{AL} = 7.8 g m^{-3}. Converting the units of C^*_{AL} using 1 m^3 = 10^3 l (Table A.2, Appendix A):

$$C^*_{AL} = 7.8 \text{ g m}^{-3}.\left|\frac{1 \text{ m}^3}{10^3 \text{ l}}\right| = 7.8\times10^{-3} \text{ g l}^{-1}$$

(a)

For the different liquid heights in the table, the maximum cell concentration is estimated using Eq. (10.42). For example, at H_L = 1.10 m, k_La = 0.016 s^{-1} and:

$$x_{max} = \frac{0.016 \text{ s}^{-1}\ (7.8\times10^{-3} \text{ g l}^{-1})}{2.31\times10^{-5} \text{ g g}^{-1} \text{ s}^{-1}} = 5.40 \text{ g l}^{-1}$$

Assuming that antifoam addition does not affect q_O or C^*_{AL}, the results for all operating liquid heights are listed below.

H_L (m)	x_{max} (g l^{-1})
1.10	5.40
1.29	4.39
1.37	4.05
1.52	4.05
1.64	3.17

(b)

The liquid volume in the fermenter V_L is equal to AH_L. Therefore, for H_L = 1.10 m, V_L = 1.94 m^3. Multiplying x_{max} by V_L gives the maximum mass of cells produced, X_{max}. Using the result in **(a)** for H_L = 1.10 m:

$$X_{max} = 5.40\ \text{g l}^{-1}\ (1.94\ \text{m}^3)\ .\left|\frac{10^3\ \text{l}}{1\ \text{m}^3}\right| .\left|\frac{1\ \text{kg}}{1000\ \text{g}}\right| = 10.5\ \text{kg}$$

As 0.055 g of protease is produced per g of biomass formed, for H_L = 1.10 m, the maximum mass of protease produced P_{max} is:

$$P_{max} = Y_{PX}\ X_{max} = 0.055\ \text{g g}^{-1}\ (10.5\ \text{kg}) = 0.58\ \text{kg}$$

Repeating these calculations gives results for V_L, X_{max} and P_{max} for all operating liquid heights.

H_L (m)	V_L (m^3)	X_{max} (kg)	P_{max} (kg)
1.10	1.94	10.5	0.58
1.29	2.28	10.0	0.55
1.37	2.42	9.81	0.54
1.52	2.69	10.9	0.60
1.64	2.90	9.19	0.51

The values of X_{max} and P_{max} vary depending on the balance of effects from the increase in liquid volume and decrease in k_La as H_L increases and more antifoam is required.

(c)

The values of X_{max} and P_{max} evaluated in **(b)** represent the maximum masses of cells and protease that can be supported by oxygen transfer in the fermenter. These results can be compared with the mass of cells X'_{max} and the mass of protease P'_{max} produced by complete conversion of substrate in the absence of mass transfer limitations:

$$X'_{max} = s_0 V_L Y_{XS}$$

and

$$P'_{max} = Y_{PX}\ X'_{max}$$

As an example, for H_L = 1.10 m:

$$X'_{max} = 20\ \text{g l}^{-1}\ (1.94\ \text{m}^3)\ (0.32\ \text{g g}^{-1}) .\left|\frac{10^3\ \text{l}}{1\ \text{m}^3}\right| .\left|\frac{1\ \text{kg}}{1000\ \text{g}}\right| = 12.4\ \text{kg}$$

and

$$P'_{max} = Y_{PX}\ X'_{max} = 0.055\ \text{g g}^{-1}\ (12.4\ \text{kg}) = 0.68\ \text{kg}$$

Because V_L increases as H_L increases, X'_{max} and P'_{max} at the other liquid heights must be greater than at H_L = 1.10 m. As all the P_{max} values in **(b)** are smaller than the corresponding P'_{max} values that would be achieved in the absence of mass transfer limitations, we can conclude that protease production is limited by oxygen transfer at all the liquid heights tested.

Answer: Yes, at all the liquid heights tested

(d)

From the table in **(b)**, X_{max} and P_{max} achieve maximum values at H_L = 1.52 m. Therefore, operation at this liquid height is recommended. In drawing this conclusion, we assume that the higher antifoam consumption and increased antifoam cost for operation at this liquid height is off-set by the greater mass of protease produced compared with operation at lower heights.

Answer: 1.52 m

10.6 Improving the rate of oxygen transfer

V_L = 17 m^3; C^*_{AL} = 10.7 g m^{-3}. Converting the units of C^*_{AL} to kg m^{-3}:

$$C^*_{AL} = 10.7 \text{ g m}^{-3} . \left| \frac{1 \text{ kg}}{1000 \text{ g}} \right| = 1.07 \times 10^{-2} \text{ kg m}^{-3}$$

(a)

From the time-course data, before addition of the vegetable oil, k_La = 250 h^{-1} and C_{AL} = 77% air saturation. The rate of oxygen transfer is determined using Eq. (10.39):

$$N_A = 250 \text{ h}^{-1} (1.07 \times 10^{-2} - 0.77 \times 1.07 \times 10^{-2}) \text{ kg m}^{-3} = 0.615 \text{ kg m}^{-3} \text{ h}^{-1}$$

After addition of the vegetable oil, the steady-state value of k_La = 100 h^{-1} and C_{AL} = 55% air saturation. The rate of oxygen transfer is:

$$N_A = 100 \text{ h}^{-1} (1.07 \times 10^{-2} - 0.55 \times 1.07 \times 10^{-2}) \text{ kg m}^{-3} = 0.482 \text{ kg m}^{-3} \text{ h}^{-1}$$

Answer: 0.62 kg m^{-3} h^{-1} before oil addition and 0.48 kg m^{-3} h^{-1} after oil addition

(b)

From the equation for k_La as a function of power input, if u_G is unchanged:

$$\frac{(k_La)_1}{(k_La)_2} = \left(\frac{(P_T)_1}{(P_T)_2} \right)^{0.5}$$

where subscript 1 refers to the conditions at k_La = 100 h^{-1} and subscript 2 refers to the conditions required to achieve k_La = 250 h^{-1}. Manipulating this equation gives:

$$\left(\frac{(k_La)_1}{(k_La)_2} \right)^2 = \frac{(P_T)_1}{(P_T)_2}$$

or

$$(P_T)_2 = (P_T)_1 \left(\frac{(k_La)_2}{(k_La)_1} \right)^2$$

As $(k_La)_2/(k_La)_1$ = 250/100 = 2.5:

$$(P_T)_2 = (P_T)_1 (2.5)^2 = 6.25 (P_T)_1$$

Therefore, if the contribution to P_T from gas sparging is negligible, the stirrer power must be increased by a factor of 6.25.

Answer: By a factor of 6.25

(c)

The use of oxygen-enriched air increases the value of C^*_{AL} but does not affect k_La. From **(a)**, the rate of oxygen transfer before oil addition is 0.615 kg m^{-3} h^{-1}. The value of C^*_{AL} needed to produce this transfer rate when k_La = 100 h^{-1} and C_{AL} = 6.2 × 10^{-3} kg m^{-3} is determined from Eq. (10.39):

$$C^*_{AL} = \frac{N_A}{k_La} + C_{AL} = \frac{0.615 \text{ kg m}^{-3} \text{ h}^{-1}}{100 \text{ h}^{-1}} + 6.2 \times 10^{-3} \text{ kg m}^{-3} = 1.24 \times 10^{-2} \text{ kg m}^{-3}$$

This value of C^*_{AL} can be compared with 1.07 × 10^{-2} kg m^{-3} calculated above for air containing 0.2099 mole fraction oxygen (Section 2.4.5). The mole fraction of oxygen y_{AG} in the gas phase corresponding to

C^*_{AL} = 1.24 × 10^{-2} kg m^{-3} is evaluated using Henry's law and Eq. (10.46). When the total pressure p_T is unchanging:

$$y_{AG2} = \frac{C^*_{AL2}}{C^*_{AL1}} y_{AG1} = \frac{1.24\times10^{-2}\ \text{kg m}^{-3}}{1.07\times10^{-2}\ \text{kg m}^{-3}} (0.2099) = 0.243$$

For gases at relatively low pressure, volume % = mole % (Section 2.4.5). Therefore, oxygen-enriched air containing 24.3 volume % oxygen is required to restore the rate of oxygen transfer.

Answer: 24.3 vol%

10.7 Oxygen transfer for different cell types

(a)

C^*_{AL} = 7.2 × 10^{-3} kg m^{-3}. Converting the units of C^*_{AL} to mmol l^{-1} using the molecular weight of oxygen = 32.0 (Table C.1, Appendix C) and 1 m^3 = 10^3 l (Table A.2, Appendix A):

$$C^*_{AL} = 7.2\times10^{-3}\ \text{kg m}^{-3} . \left|\frac{1000\ \text{g}}{1\ \text{kg}}\right| . \left|\frac{1\ \text{gmol}}{32\ \text{g}}\right| . \left|\frac{1000\ \text{mmol}}{1\ \text{gmol}}\right| . \left|\frac{1\ \text{m}^3}{10^3\ \text{l}}\right| = 0.225\ \text{mmol l}^{-1}$$

The k_La required to maintain the dissolved oxygen concentration at critical level is determined for each cell type using Eq. (10.43). For *E. coli*:

$$(k_La)_{crit} = \frac{8.5\ \text{mmol g}^{-1}\ \text{h}^{-1}\ (25\ \text{g l}^{-1})}{(0.225-0.0082)\ \text{mmol l}^{-1}} = 980\ \text{h}^{-1}$$

For grape cells:

$$(k_La)_{crit} = \frac{0.60\ \text{mmol g}^{-1}\ \text{h}^{-1}\ (25\ \text{g l}^{-1})}{(0.225-0.055)\ \text{mmol l}^{-1}} = 88\ \text{h}^{-1}$$

For CHO cells:

$$(k_La)_{crit} = \frac{3.0\times10^{-10}\ \text{mmol cell}^{-1}\ \text{h}^{-1}\ (3.0\times10^{9}\ \text{cell l}^{-1})}{(0.225-0.020)\ \text{mmol l}^{-1}} = 4.4\ \text{h}^{-1}$$

Answer: For *E. coli* 980 h^{-1}, for grape cells 88 h^{-1}, for CHO cells 4.4 h^{-1}. *This problem highlights the very high demands on oxygen transfer inherent in microbial cell culture relative to plant and animal cell systems.*

(b)

From the equation provided, when V_L remains constant:

$$\frac{(P_T)_{grape}}{(P_T)_{CHO}} = \left(\frac{(k_La)_{grape}}{(k_La)_{CHO}}\right)^2 = \left(\frac{88\ \text{h}^{-1}}{4.4\ \text{h}^{-1}}\right)^2 = 400$$

Therefore, the power required to maintain critical oxygen tension in the grape cell culture is 400 times that required for CHO cells. Similarly:

$$\frac{(P_T)_{E.\ coli}}{(P_T)_{CHO}} = \left(\frac{(k_La)_{E.\ coli}}{(k_La)_{CHO}}\right)^2 = \left(\frac{980\ \text{h}^{-1}}{4.4\ \text{h}^{-1}}\right)^2 = 4.96\times10^4$$

The power required to maintain critical oxygen tension in the *E. coli* culture is 5 × 10^4 times that required for CHO cells.

Answer: The power required for culture of grape cells is 400 times that required for culture of CHO cells; the power required for culture of *E. coli* is 5.0×10^4 times greater than that required for CHO cells

10.8 Single-point $k_L a$ determination using the oxygen balance method

(a)

V_L = 200 l. The oxygen transfer rate for $k_L a$ determination by the oxygen balance method is calculated using Eq. (10.53). Air contains 21 mole % oxygen (Section 2.4.5); therefore, from Henry's law Eq. (10.45), the partial pressure p_{AG} of oxygen in the inlet air at 1 atm is (0.21 × 1 atm) = 0.21 atm. Assuming that the exit gas leaves the fermenter at the fermentation conditions (1 atm pressure and 28°C), the partial pressure of oxygen in the exit gas is (0.201 × 1 atm) = 0.201 atm. The inlet air flow rate F_g is 1 vvm or 200 l min^{-1}. Using R = 0.000082057 m^3 atm K^{-1} $gmol^{-1}$ (Table B.1, Appendix B) and converting the temperatures to degrees Kelvin using Eq. (2.27), Eq. (10.53) becomes:

$$N_A = \frac{1}{0.000082057\ \text{m}^3\ \text{atm K}^{-1}\ \text{gmol}^{-1}\ (200\ \text{l})} \left[\left(\frac{200\ \text{l min}^{-1} . \left| \frac{1\ \text{min}}{60\ \text{s}} \right| . 0.21\ \text{atm}}{(20+273.15)\ \text{K}} \right) - \left(\frac{189\ \text{l min}^{-1} . \left| \frac{1\ \text{min}}{60\ \text{s}} \right| . 0.201\ \text{atm}}{(28+273.15)\ \text{K}} \right) \right]$$

$$= 0.0174\ \text{gmol m}^{-3}\ \text{s}^{-1}$$

Answer: 0.0174 gmol m^{-3} s^{-1}

(b)

Using Eq. (10.39) with the units of N_A from **(a)** converted to mass using the molecular weight of oxygen = 32.0 (Table C.1, Appendix C):

$$k_L a = \frac{N_A}{(C^*_{AL} - C_{AL})} = \frac{0.0174\ \text{gmol m}^{-3}\ \text{s}^{-1} . \left| \frac{32.0\ \text{g}}{1\ \text{gmol}} \right| . \left| \frac{1\ \text{kg}}{1000\ \text{g}} \right|}{(7.8\times10^{-3} - 0.52\times7.8\times10^{-3})\ \text{kg m}^{-3}} = 0.15\ \text{s}^{-1}$$

Answer: 0.15 s^{-1}

(c)

If the measured exit gas composition of 20.1% O_2 is an overestimation, the actual value is 20.1% × 1/1.1 = 18.3% O_2. Therefore, the partial pressure of oxygen in the exit gas is (0.183 × 1 atm) = 0.183 atm. From Eq. (10.53):

$$N_A = \frac{1}{0.000082057\ \text{m}^3\ \text{atm K}^{-1}\ \text{gmol}^{-1}\ (200\ \text{l})} \left[\left(\frac{200\ \text{l min}^{-1} . \left| \frac{1\ \text{min}}{60\ \text{s}} \right| . 0.21\ \text{atm}}{(20+273.15)\ \text{K}} \right) - \left(\frac{189\ \text{l min}^{-1} . \left| \frac{1\ \text{min}}{60\ \text{s}} \right| . 0.183\ \text{atm}}{(28+273.15)\ \text{K}} \right) \right]$$

$$= 0.0289\ \text{gmol m}^{-3}\ \text{s}^{-1}$$

Therefore, from Eq. (10.39):

$$k_L a = \frac{N_A}{(C^*_{AL} - C_{AL})} = \frac{0.0289 \text{ gmol m}^{-3} \text{ s}^{-1} . \left|\frac{32.0 \text{ g}}{1 \text{ gmol}}\right| . \left|\frac{1 \text{ kg}}{1000 \text{ g}}\right|}{(7.8\times10^{-3} - 0.52\times7.8\times10^{-3}) \text{ kg m}^{-3}} = 0.25 \text{ s}^{-1}$$

The $k_L a$ value obtained in **(b)** using the incorrectly calibrated oxygen analyser is 60% of the actual $k_L a$ value; the error is therefore 40%.

Answer: 40%. *This calculation illustrates the sensitivity of the oxygen balance method to the accuracy of the measured parameters used in Eq. (10.53). This sensitivity arises from the subtraction of two numbers of similar magnitude for the moles of oxygen in and out of the system. When errors in both F_g terms are taken into account, the error in the final $k_L a$ value can be very large.*

10.9 Steady-state $k_L a$ measurement

(a)

The effect of medium components on the solubility of oxygen is evaluated using the methods of Section 10.8.3. From Tables C.8 and C.1 (Appendix C), the molecular formulae for glucose and sucrose and the molecular weights of the medium components are: glucose ($C_6H_{12}O_6$) = 180.2, sucrose ($C_{12}H_{22}O_{11}$) = 342.3, $CaCO_3$ = 100.1, $(NH_4)_2SO_4$ = 132.1, Na_2HPO_4 = 142.0 and KH_2PO_4 = 136.1.

The parameter values for application in Eq. (10.49) are listed below. Values of H_i and K_j are taken from Table 10.5.

Medium component	H_i or K_j (m^3 mol^{-1})	z_i	C_{iL} or C_{jL} (mol m^{-3})
Glucose	0.119×10^{-3}	–	$20 \text{ g l}^{-1} . \left\|\frac{1 \text{ mol}}{180.2 \text{ g}}\right\| . \left\|\frac{1000 \text{ l}}{1 \text{ m}^3}\right\| = 111$
Sucrose	0.149×10^{-3}	–	$8.5 \text{ g l}^{-1} . \left\|\frac{1 \text{ mol}}{342.3 \text{ g}}\right\| . \left\|\frac{1000 \text{ l}}{1 \text{ m}^3}\right\| = 24.8$
Ca^{2+}	-0.303×10^{-3}	2	$1.3 \text{ g l}^{-1} . \left\|\frac{1 \text{ mol}}{100.1 \text{ g}}\right\| . \left\|\frac{1000 \text{ l}}{1 \text{ m}^3}\right\| = 13.0$
CO_3^{2-}	0.485×10^{-3}	2	$1.3 \text{ g l}^{-1} . \left\|\frac{1 \text{ mol}}{100.1 \text{ g}}\right\| . \left\|\frac{1000 \text{ l}}{1 \text{ m}^3}\right\| = 13.0$
NH_4^+	-0.720×10^{-3}	1	$2\times1.3 \text{ g l}^{-1} . \left\|\frac{1 \text{ mol}}{132.1 \text{ g}}\right\| . \left\|\frac{1000 \text{ l}}{1 \text{ m}^3}\right\| = 19.7$
SO_4^{2-}	0.453×10^{-3}	2	$1.3 \text{ g l}^{-1} . \left\|\frac{1 \text{ mol}}{132.1 \text{ g}}\right\| . \left\|\frac{1000 \text{ l}}{1 \text{ m}^3}\right\| = 9.8$
Na^+	-0.550×10^{-3}	1	$2\times0.09 \text{ g l}^{-1} . \left\|\frac{1 \text{ mol}}{142.0 \text{ g}}\right\| . \left\|\frac{1000 \text{ l}}{1 \text{ m}^3}\right\| = 1.3$
HPO_4^{2-}	0.485×10^{-3}	2	$0.09 \text{ g l}^{-1} . \left\|\frac{1 \text{ mol}}{142.0 \text{ g}}\right\| . \left\|\frac{1000 \text{ l}}{1 \text{ m}^3}\right\| = 0.63$
K^+	-0.596×10^{-3}	1	$0.12 \text{ g l}^{-1} . \left\|\frac{1 \text{ mol}}{136.1 \text{ g}}\right\| . \left\|\frac{1000 \text{ l}}{1 \text{ m}^3}\right\| = 0.88$
$H_2PO_4^-$	1.037×10^{-3}	1	$0.12 \text{ g l}^{-1} . \left\|\frac{1 \text{ mol}}{136.1 \text{ g}}\right\| . \left\|\frac{1000 \text{ l}}{1 \text{ m}^3}\right\| = 0.88$

Substituting these values into Eq. (10.49) gives:

$$\log_{10}\left(\frac{C^*_{AL0}}{C^*_{AL}}\right) = 0.5\left(\begin{array}{l}(-0.303\times10^{-3})(2)^2 13.0 \;+\; (0.485\times10^{-3})(2)^2 13.0 + (-0.720\times10^{-3})(1)^2 19.7 \;+ \\ (0.453\times10^{-3})(2)^2 9.8 + (-0.550\times10^{-3})(1)^2 1.3 + (0.485\times10^{-3})(2)^2 0.63 \;+ \\ (-0.596\times10^{-3})(1)^2 0.88 + (1.037\times10^{-3})(1)^2 0.88 \\ \qquad + \left((0.119\times10^{-3})111 + (0.149\times10^{-3})24.8\right)\end{array}\right)$$

$$= 2.39\times10^{-2}$$

Therefore:

$$\frac{C^*_{AL0}}{C^*_{AL}} = 10^{2.39\times10^{-2}} = 1.06$$

or

$$C^*_{AL} = 0.95\,C^*_{AL0}$$

This result indicates that solutes in the medium reduce the oxygen solubility by about 5% compared with the oxygen solubility at zero solute concentration. From Table 10.2, the solubility of oxygen in water at 35°C and 1 atm air pressure is 7.52 × 10^{-3} kg m^{-3}. However, if the gas phase in the fermenter is well mixed, the mole fraction of oxygen in the bubbles is equal to that in the off-gas, 0.197, which is less than that in air (0.2099, Section 2.4.5). Therefore, applying Henry's law Eq. (10.46):

$$C^*_{AL0} = \frac{0.197}{0.2099}(7.52\times10^{-3})\ \text{kg m}^{-3} = 7.06\times10^{-3}\ \text{kg m}^{-3}$$

so that C^*_{AL} in the fermentation medium at 1 atm and 35°C is 0.95 × (7.06 × 10^{-3} kg m^{-3}) = 6.70 × 10^{-3} kg m^{-3}.

Answer: 6.70 × 10^{-3} kg m^{-3}, assuming that the gas phase in the fermenter is well mixed, and that medium solutes but not other broth components such as cells affect oxygen solubility

(b)

The oxygen transfer rate is determined using Eq. (10.53). The mole fraction of oxygen in the incoming air is 0.2099 (Section 2.4.5); therefore, from Henry's law Eq. (10.45), the partial pressure of oxygen in the inlet air at 1 atm is 0.2099 × 1 atm = 0.2099 atm. The mole fraction of oxygen in the off-gas is 0.197; therefore, from Henry's law Eq. (10.45), at an off-gas pressure of 1 atm, the partial pressure of oxygen in the off-gas is 0.197 × 1 atm = 0.197 atm. Using R = 0.082057 l atm K^{-1} $gmol^{-1}$ (Table B.1, Appendix B) and converting the temperatures to degrees Kelvin using Eq. (2.27), Eq. (10.53) becomes:

$$N_A = \frac{1}{0.082057\ \text{l atm K}^{-1}\ \text{gmol}^{-1}\ (500\ \text{l})}$$

$$\left[\left(\frac{0.4\ \text{m}^3\ \text{min}^{-1}\,.\left|\frac{10^3\ \text{l}}{1\ \text{m}^3}\right|.\left|\frac{1\ \text{min}}{60\ \text{s}}\right|.0.2099\ \text{atm}}{(25+273.15)\ \text{K}}\right) - \left(\frac{6.3\,1\ \text{s}^{-1}\ (0.197\ \text{atm})}{(35+273.15)\ \text{K}}\right)\right]$$

$$= 1.623\times10^{-5}\ \text{gmol l}^{-1}\ \text{s}^{-1}$$

Answer: 1.62 × 10^{-5} gmol l^{-1} s^{-1}

(c)

k_La is determined from N_A using Eq. (10.39). C_{AL} = 45% air saturation; this oxygen tension must be converted to a dissolved oxygen concentration. From Table 10.2, the solubility of oxygen in water at 35°C and 1 atm air pressure is 7.52 × 10^{-3} kg m^{-3}. Using the result in **(a)** for the effect of medium solutes, air

saturation in fermentation medium corresponds to 0.95 × (7.52 × 10^{-3} kg m^{-3}) = 7.14 × 10^{-3} kg m^{-3}; therefore, C_{AL} = 0.45 × (7.14 × 10^{-3} kg m^{-3}) = 3.21 × 10^{-3} kg m^{-3}. Converting the units of N_A in **(b)** to mass using the molecular weight of oxygen = 32.0 (Table C.1, Appendix C) and applying the value of C^*_{AL} from **(a)**, Eq. (10.39) becomes:

$$k_L a = \frac{N_A}{(C^*_{AL} - C_{AL})} = \frac{1.623\times10^{-5}\ \text{gmol}\,\text{l}^{-1}\,\text{s}^{-1} \cdot \left|\frac{32.0\ \text{g}}{1\ \text{gmol}}\right| \cdot \left|\frac{1\ \text{kg}}{1000\ \text{g}}\right|}{(6.70-3.21)\times10^{-3}\ \text{kg}\,\text{m}^{-3} \cdot \left|\frac{1\ \text{m}^3}{1000\ \text{l}}\right|} = 0.15\ \text{s}^{-1}$$

Answer: 0.15 s^{-1}

(d)

q_O = 5.4 mmol g^{-1} h^{-1}. The maximum cell concentration supported by oxygen transfer is evaluated using Eq. (10.42). C^*_{AL} in this equation depends on the medium composition and the gas-phase oxygen partial pressure. We will assume that the oxygen partial pressure in the bubbles is 0.197, although this may vary with cell concentration, so that the value of C^*_{AL} is that determined in **(a)**. Using this result together with the value of $k_L a$ from **(c)** and converting q_O to mass units using the molecular weight of oxygen = 32.0 (Table C.1, Appendix C):

$$x_{max} = \frac{0.15\ \text{s}^{-1}\ (6.70\times10^{-3}\ \text{kg}\,\text{m}^{-3})}{5.4\ \text{mmol}\ \text{g}^{-1}\,\text{h}^{-1} \cdot \left|\frac{1\ \text{gmol}}{1000\ \text{mmol}}\right| \cdot \left|\frac{32.0\ \text{g}}{1\ \text{gmol}}\right| \cdot \left|\frac{1\ \text{h}}{3600\ \text{s}}\right|} = 20.9\ \text{g}\,\text{l}^{-1}$$

Answer: 20.9 g l^{-1}

(e)

Y_{XS} = 0.5 g g^{-1}. For complete conversion of 20 g l^{-1} glucose and 8.5 g l^{-1} sucrose:

$$x = 0.5\ \text{g}\ \text{g}^{-1}(20+8.5)\ \text{g}\,\text{l}^{-1} = 14.3\ \text{g}\,\text{l}^{-1}$$

As this value of x is less than the maximum cell concentration supported by oxygen transfer evaluated in **(d)**, growth is limited by substrate availability, not oxygen transfer.

Answer: Substrate availability

10.10 Oxygen transfer in a pressure vessel

(a)

D_T = 3.6 m; H_L = 6.1 m. The volume of liquid V_L in a cylindrical vessel of these dimensions is:

$$V_L = \frac{\pi}{4} D_T^2 H_L = \frac{\pi}{4}(3.6\ \text{m})^2\ (6.1\ \text{m}) = 62.09\ \text{m}^3$$

The oxygen transfer rate is determined using Eq. (10.53). The inlet air contains 20.99 mole % oxygen (Section 2.4.5); therefore, from Henry's law Eq. (10.45), the partial pressure of oxygen in the inlet air measured at 1 atm is (0.2099 × 1 atm) = 0.2099 atm. The mole fraction of oxygen in the off-gas leaving from the top of the vessel is 0.172 at a pressure of 1.4 atm; therefore the partial pressure of oxygen in the off-gas is (0.172 × 1.4 atm) = 0.241 atm. Using R = 0.000082057 m^3 atm K^{-1} $gmol^{-1}$ (Table B.1, Appendix B) and converting the temperatures to degrees Kelvin using Eq. (2.27), Eq. (10.53) becomes:

$$N_A = \frac{1}{0.000082057\ \text{m}^3\ \text{atm K}^{-1}\ \text{gmol}^{-1}\ (62.09\ \text{m}^3)} \left[\left(\frac{30\ \text{m}^3\ \text{min}^{-1} . \left| \frac{1\ \text{min}}{60\ \text{s}} \right| . 0.2099\ \text{atm}}{(20+273.15)\ \text{K}} \right) - \left(\frac{20.5\ \text{m}^3\ \text{min}^{-1} . \left| \frac{1\ \text{min}}{60\ \text{s}} \right| . 0.241\ \text{atm}}{(29+273.15)\ \text{K}} \right) \right]$$

$$= 0.0168\ \text{gmol m}^{-3}\ \text{s}^{-1}$$

Answer: 0.0168 gmol m^{-3} s^{-1}

(b)

The hydrostatic pressure at the bottom of the tank due to the weight of liquid is calculated using Eq. (10.58) with g = 9.8066 m s^{-2} (Section 2.3):

$$p_s = 1000\ \text{kg m}^{-3}\ (9.8066\ \text{m s}^{-2})(6.1\ \text{m}) = 5.98\times10^4\ \text{kg m}^{-1}\ \text{s}^{-2}$$

Converting units using 1 atm = 1.013×10^5 kg m^{-1} s^{-2} (Table A.5, Appendix A):

$$p_s = 5.98\times10^4\ \text{kg m}^{-1}\ \text{s}^{-2} . \left| \frac{1\ \text{atm}}{1.013\times10^5\ \text{kg m}^{-1}\ \text{s}^{-2}} \right| = 0.590\ \text{atm}$$

The pressure at the bottom of the tank is equal to the pressure at the top + p_s:

$$\text{Pressure at the bottom of the tank} = 1.4\ \text{atm} + 0.590\ \text{atm} = 1.99\ \text{atm}$$

Answer: 1.99 atm

(c)

From Table 10.2, the solubility of oxygen in water at 29°C and 1 atm air pressure is 8.17×10^{-3} kg m^{-3}. At the bottom of the tank, the composition of the gas phase is the same as air but, from **(b)**, the pressure is 1.99 atm rather than 1 atm. Using Henry's law Eq. (10.46) to evaluate C^*_{AL} at the bottom of the tank:

$$C^*_{AL2} = C^*_{AL1} \frac{p_{T2} y_{AG2}}{p_{T1} y_{AG1}} = C^*_{AL1} \frac{p_{T2}}{p_{T1}}$$

where subscript 1 refers to air at 1 atm and subscript 2 refers to air at 1.99 atm. Substituting values gives:

$$C^*_{AL2} = 8.17\times10^{-3}\ \text{kg m}^{-3} \left(\frac{1.99\ \text{atm}}{1\ \text{atm}} \right) = 1.63\times10^{-2}\ \text{kg m}^{-3}$$

At the top of the tank, the mole fraction of oxygen in the gas phase is 0.172 and the pressure is 1.4 atm. Using Eq. (10.46) to evaluate C^*_{AL} at the top of the tank:

$$C^*_{AL2} = C^*_{AL1} \frac{p_{T2} y_{AG2}}{p_{T1} y_{AG1}}$$

where subscript 1 refers to air at 1 atm and subscript 2 refers to off-gas at 1.4 atm. Substituting values gives:

$$C^*_{AL2} = 8.17\times10^{-3}\ \text{kg m}^{-3} \left(\frac{1.4\ \text{atm}\times0.172}{1\ \text{atm}\times0.2099} \right) = 9.37\times10^{-3}\ \text{kg m}^{-3}$$

Answer: 9.37×10^{-3} kg m^{-3} at the top of the tank and 1.63×10^{-2} kg m^{-3} at the bottom

(d)

$k_L a$ is determined using Eq. (10.39) modified for large vessels, where the logarithmic-mean concentration difference given by Eq. (10.59) is a better representation than ($C^*_{AL} - C_{AL}$) of the concentration-difference driving force for oxygen transfer.

At the top of the fermenter, C_{AL} = 50% air saturation. The measuring probe is calibrated *in situ*; therefore, air saturation at this location corresponds to the solubility of oxygen in water at 29°C under air at 1.4 atm pressure. From Table 10.2, the solubility of oxygen in water at 29°C under 1 atm air pressure is 8.17×10^{-3} kg m^{-3}; therefore, using Eq. (10.46), air saturation at the top of the fermenter means an oxygen concentration of (1.4 atm/1 atm) × 8.15×10^{-3} kg m^{-3} = 1.14×10^{-2} kg m^{-3}. If the dissolved oxygen tension is 50% air saturation, C_{AL} at the gas outlet = $0.5 \times (1.14 \times 10^{-2}$ kg m$^{-3}) = 5.71 \times 10^{-3}$ kg m^{-3}.

Similarly, at the bottom of the fermenter, C_{AL} = 65% air saturation. Air saturation at this location corresponds to the solubility of oxygen in water at 29°C under air at 1.99 atm pressure. Therefore, air saturation at the bottom of the fermenter means an oxygen concentration of (1.99 atm/1 atm) × 8.15×10^{-3} kg m^{-3} = 1.62×10^{-2} kg m^{-3}. If the dissolved oxygen tension is 65% air saturation, C_{AL} at the gas inlet = $0.65 \times (1.62 \times 10^{-2}$ kg m$^{-3}) = 1.05 \times 10^{-2}$ kg m^{-3}.

Substituting these values for C_{AL} and the results for C^*_{AL} from **(c)** into Eq. (10.59):

$$(C^*_{AL} - C_{AL})_{lm} = \frac{(9.37\times10^{-3} - 5.71\times10^{-3})\ \text{kg m}^{-3} - (1.63\times10^{-2} - 1.05\times10^{-2})\ \text{kg m}^{-3}}{\ln\left(\dfrac{(9.37\times10^{-3} - 5.71\times10^{-3})\ \text{kg m}^{-3}}{(1.63\times10^{-2} - 1.05\times10^{-2})\ \text{kg m}^{-3}}\right)}$$

$$= 4.65\times10^{-3}\ \text{kg m}^{-3}$$

Converting to mole units using the molecular weight of oxygen = 32.0 (Table C.1, Appendix C):

$$(C^*_{AL} - C_{AL})_{lm} = 4.65\times10^{-3}\ \text{kg m}^{-3} . \left|\frac{1\ \text{gmol}}{32\ \text{g}}\right| . \left|\frac{1000\ \text{g}}{1\ \text{kg}}\right| = 0.145\ \text{gmol m}^{-3}$$

Calculating $k_L a$ using Eq. (10.39) with the logarithmic-mean concentration difference and N_A from **(a)**:

$$k_L a = \frac{N_A}{(C^*_{AL} - C_{AL})_{lm}} = \frac{0.0168\ \text{gmol m}^{-3}\ \text{s}^{-1}}{0.145\ \text{gmol m}^{-3}} = 0.116\ \text{s}^{-1}$$

Answer: 0.12 s^{-1}

(e)

From Section (10.5.2), at steady state, the rate of oxygen transfer N_A is equal to the rate of oxygen uptake by the cells Q_O. Using the definition of Q_O in Eq. (10.40) and converting units using the molecular weight of oxygen = 32.0 (Table C.1, Appendix C) and 1 m^3 = 10^3 l (Table A.2, Appendix A):

$$q_O = \frac{Q_O}{x} = \frac{N_A}{x} = \frac{0.0168\ \text{gmol m}^{-3}\ \text{s}^{-1}}{16\ \text{g l}^{-1} . \left|\dfrac{10^3\ \text{l}}{1\ \text{m}^3}\right| . \left|\dfrac{1\ \text{gmol}}{32.0\ \text{g}}\right|} = 3.36\times10^{-5}\ \text{s}^{-1}$$

Answer: 3.4×10^{-5} s^{-1}

(f)

Because of the effect of the hydrostatic pressure p_s, the maximum pressure of 2.7 atm applies to the bottom of the tank. Therefore, using the result for p_s from **(b)**, the pressure at the top of the tank is (2.7–0.590) atm = 2.11 atm.

C^*_{AL} depends on the gas-phase oxygen partial pressure. To determine a maximum value for x, we will assume that the mole fraction of oxygen in the bubbles everywhere in the tank is the same as that in

air, 0.2099 (Section 2.4.5), although this may vary with cell concentration. Using the solubility of oxygen in water at 29°C and 1 atm air pressure as a basis (Table 10.2), applying Henry's law and Eq. (10.46) gives:

$$C^*_{AL} \text{ at the bottom of the tank} = 8.17\times10^{-3}\ \text{kg m}^{-3}\left(\frac{2.7\ \text{atm}}{1\ \text{atm}}\right) = 2.21\times10^{-2}\ \text{kg m}^{-3}$$

$$C^*_{AL} \text{ at the top of the tank} = 8.17\times10^{-3}\ \text{kg m}^{-3}\left(\frac{2.11\ \text{atm}}{1\ \text{atm}}\right) = 1.72\times10^{-2}\ \text{kg m}^{-3}$$

As explained in Section 10.5.2, the maximum cell concentration occurs when $C_{AL} = 0$. x_{max} can be determined using Eq. (10.41) with $C_{AL} = 0$; however, for large vessels, the logarithmic-mean concentration difference $(C^*_{AL} - C_{AL})_{lm}$ is more appropriate in Eq. (10.41) than $(C^*_{AL} - C_{AL})$. Evaluating $(C^*_{AL} - C_{AL})_{lm}$ from Eq. (10.59) with $C_{AL} = 0$:

$$(C^*_{AL} - C_{AL})_{lm} = \frac{(C^*_{AL})_o - (C^*_{AL})_i}{\ln\dfrac{(C^*_{AL})_o}{(C^*_{AL})_i}} = \frac{(1.72\times10^{-2} - 2.21\times10^{-2})\ \text{kg m}^{-3}}{\ln\left(\dfrac{1.72\times10^{-2}\ \text{kg m}^{-3}}{2.21\times10^{-2}\ \text{kg m}^{-3}}\right)} = 1.95\times10^{-2}\ \text{kg m}^{-3}$$

Using this result in Eq. (10.41) together with the values of k_La and q_O from **(d)** and **(e)**:

$$x_{max} = \frac{0.116\ \text{s}^{-1}\ (1.95\times10^{-2}\ \text{kg m}^{-3}).\left|\dfrac{1\ \text{m}^3}{10^3\ \text{l}}\right|.\left|\dfrac{1000\ \text{g}}{1\ \text{kg}}\right|}{3.36\times10^{-5}\ \text{s}^{-1}} = 67.3\ \text{g l}^{-1}$$

Answer: 67 g l^{-1}

10.11 Dynamic k_La measurement

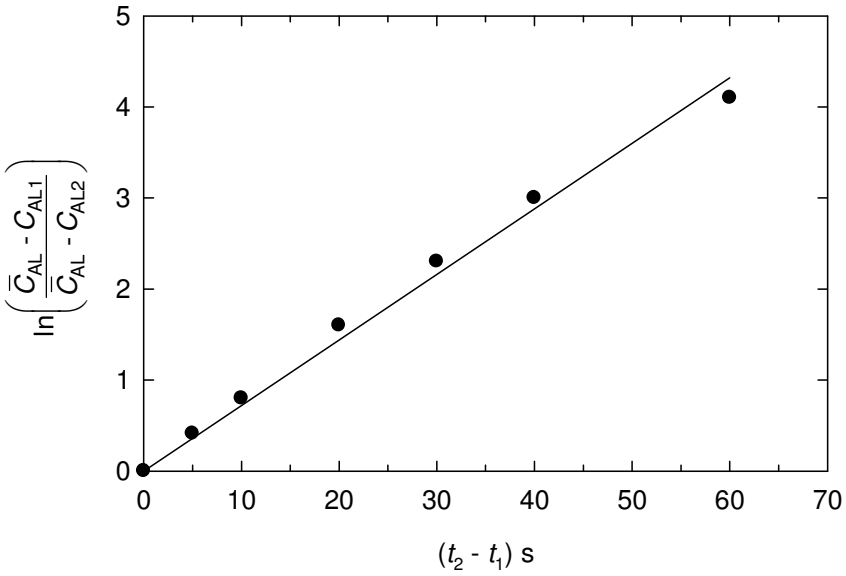
(a)

k_La is estimated using Eq. (10.57) and the methods described in Section 10.10.2. Let t_1 = 10 s and C_{AL1} = 43.5% air saturation. From the measured data, the steady-state dissolved oxygen tension $\bar{C}_{AL}$ = 73.5% air saturation. Calculated values of

$$\ln\left(\frac{\bar{C}_{AL} - C_{AL1}}{\bar{C}_{AL} - C_{AL2}}\right)$$

and corresponding values of $(t_2 - t_1)$ are listed and plotted below.

$\ln\left(\dfrac{\bar{C}_{AL} - C_{AL1}}{\bar{C}_{AL} - C_{AL2}}\right)$	$(t_2 - t_1)$ (s)
0	0
0.41	5
0.80	10
1.6	20
2.3	30
3.0	40
4.1	60



From Eq. (10.57), k_La is equal to the slope of the straight line in the plot through the origin = 0.072 s^{-1}.

Answer: 0.072 s^{-1}

(b)

The simple dynamic method cannot be considered to give reliable results for k_La unless further experimental checks are performed to show that the electrode, boundary layers and gas-phase dynamics do not influence the measurement system. The effect of the electrode response time and boundary layers should be determined at a range of stirrer speeds using the techniques described in Section 10.10.2 (Electrode Response Time and Liquid Boundary Layers subsection). The effect of gas-phase dynamics should be tested using different methods of deoxygenation as described in Section 10.10.2 (Gas-Phase Dynamics subsection).

10.12 k_La measurement using the dynamic pressure method

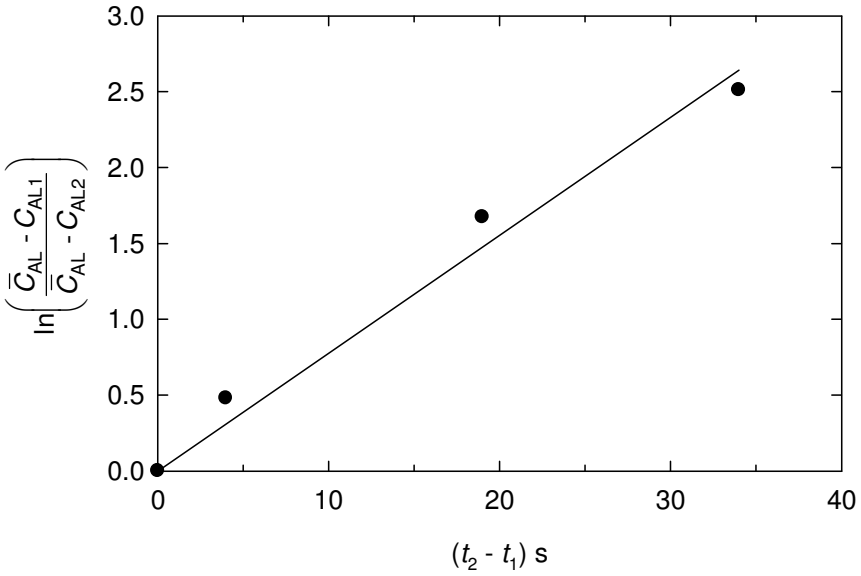
(a)

k_La is estimated using Eq. (10.57) and the methods described in Section 10.10.2. Let t_1 = 6 s and C_{AL1} = 50% air saturation. The steady-state dissolved oxygen tension $\bar{C}_{AL}$ = 66% air saturation. Calculated values of

$$\ln\left(\frac{\bar{C}_{AL} - C_{AL1}}{\bar{C}_{AL} - C_{AL2}}\right)$$

and corresponding values of $(t_2 - t_1)$ are listed and plotted below.

$\ln\left(\frac{\bar{C}_{AL} - C_{AL1}}{\bar{C}_{AL} - C_{AL2}}\right)$	$(t_2 - t_1)$ (s)
0	0
0.480	4
1.674	19
2.510	34



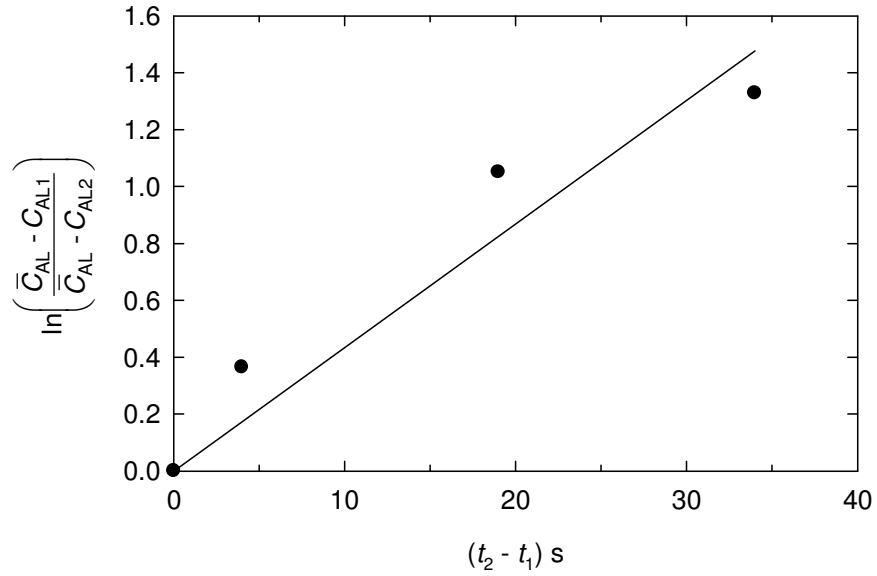
From Eq. (10.57), $k_L a$ is equal to the slope of the straight line in the plot through the origin = 0.078 s^{-1}. Although the dynamic pressure method reduces the influence of gas-phase dynamics on $k_L a$ measurement (Section 10.10.2, Modified Dynamic Methods subsection), before this result can be considered reliable the effects of the electrode response time and boundary layers should be checked using the techniques described in Section 10.10.2 (Electrode Response Time and Liquid Boundary Layers subsection).

Answer: 0.078 s^{-1}

(b)

Repeating the calculation with t_1 = 6 s, C_{AL1} = 50% air saturation and $\bar{C}_{AL}$ = 70% air saturation:

$\ln\left(\frac{\bar{C}_{AL} - C_{AL1}}{\bar{C}_{AL} - C_{AL2}}\right)$	$(t_2 - t_1)$ (s)
0	0
0.364	4
1.050	19
1.328	34



k_La = the slope of the straight line in the plot through the origin = 0.043 s^{-1}. Therefore, a 6% error in $\bar{C}_{AL}$ results in an error in k_La of:

$$\frac{0.078 - 0.043}{0.078} \times 100\% = 45\%$$

Answer: The measured k_La value is 0.043 s^{-1}, representing an error of 45%

(c)

The electrode response time is the same at 50 rpm and 60 rpm. This suggests that boundary layer effects were eliminated at 50 rpm. As the k_La measurements were conducted at 60 rpm, we can conclude that significant boundary layers were not present during the measurement procedure.

In the absence of boundary layers, the time taken for the electrode to record 63.2% of the step change from 0 to 100% air saturation is about 2.2 s; therefore, the electrode response time τ_E = 2.2 s (Figure 10.17). This result can be compared with $1/k_La$ and $0.2/k_La$ to determine whether the electrode influenced the measured value of k_La (Section 10.10.2, Electrode Response Time and Liquid Boundary Layers subsection). Using the result for k_La from **(a)**, $1/k_La = 1/(0.078\ s^{-1})$ = 12.8 s, and $0.2/k_La$ = 2.6 s. As $\tau_E < 0.2/k_La$, we can conclude that the error in k_La due to the electrode response is very small.

The test experiments show that there was a negligible effect on k_La due to the electrode response time and boundary layers. The influence of gas-phase dynamics was not tested; however, use of the dynamic pressure method for measuring k_La avoids many of the problems with transient gas-phase composition and the loss of gas hold-up that occur using the simple dynamic method.

Answer: We can have reasonable confidence in the k_La measurements, as the effects of the electrode response time, boundary layers and gas-phase dynamics are considered negligible

10.13 Surface versus bubble aeration

(a)

From Section 10.5.2, at steady state, the rate of oxygen transfer N_A is equal to the rate of oxygen uptake by the cells Q_O; therefore, k_La can be evaluated using Eq. (10.41). C_{AL} = 50% air saturation; this oxygen tension must be converted to a dissolved oxygen concentration. Assuming that the solubility of oxygen in

culture medium is the same as that in water, the dissolved oxygen concentration corresponding to air saturation at 37°C and 1 atm can be interpolated from the values for 35°C and 40°C in Table 10.2:

$$C^*_{AL}(\text{air at 1 atm and } 37\,^{\circ}\text{C}) = 7.52\times10^{-3}\ \text{kg m}^{-3} - \frac{(37-35)\,^{\circ}\text{C}}{(40-35)\,^{\circ}\text{C}}(7.52-7.07)\times10^{-3}\ \text{kg m}^{-3}$$
$$= 7.34\times10^{-3}\ \text{kg m}^{-3}$$

Therefore, $C_{AL} = 0.5 \times (7.34 \times 10^{-3}\ \text{kg m}^{-3}) = 3.67 \times 10^{-3}\ \text{kg m}^{-3}$. C^*_{AL} in the bioreactor must be evaluated for operation with a 50:20:30 mixture of air:oxygen:nitrogen at 1 atm. The oxygen mole fraction y_{AG} in the gas mixture is determined using 0.2099 for the mole fraction of oxygen in air (Section 2.4.5):

$$y_{AG} = \frac{20 + 0.2099\,(50)}{100} = 0.305$$

If gas with 0.305 mole fraction oxygen is used instead of air, the solubility of oxygen in the bioreactor is greater than the value for C^*_{AL} under air. From Henry's law and Eq. (10.46):

$$C^*_{AL}(\text{gas mixture at 1 atm and } 37\,^{\circ}\text{C}) = \frac{0.305}{0.2099}(7.34\times10^{-3}\ \text{kg m}^{-3}) = 1.07\times10^{-2}\ \text{kg m}^{-3}$$

Applying Eq. (10.41) with $q_O = 7.7 \times 10^{-12}$ g cell^{-1} h^{-1}, $x = 1.1 \times 10^9$ cells l^{-1} and the above values for C_{AL} and C^*_{AL}:

$$k_L a = \frac{q_O x}{(C^*_{AL} - C_{AL})} = \frac{7.7\times10^{-12}\ \text{g cell}^{-1}\,\text{h}^{-1}\,(1.1\times10^{9}\ \text{cell l}^{-1}).\left|\frac{1000\ \text{l}}{1\ \text{m}^3}\right|.\left|\frac{1\ \text{kg}}{1000\ \text{g}}\right|}{(1.07\times10^{-2} - 3.67\times10^{-3})\ \text{kg m}^{-3}}$$
$$= 1.20\ \text{h}^{-1}$$

Answer: 1.2 h^{-1}

(b)

$x = 3.9 \times 10^9$ cells l^{-1} when C_{AL} = 8% air saturation = $0.08 \times (7.34 \times 10^{-3}\ \text{kg m}^{-3}) = 5.87 \times 10^{-4}\ \text{kg m}^{-3}$. Recalculating $k_L a$ using Eq. (10.41) for air sparging:

$$k_L a = \frac{7.7\times10^{-12}\ \text{g cell}^{-1}\,\text{h}^{-1}\,(3.9\times10^{9}\ \text{cell l}^{-1}).\left|\frac{1000\ \text{l}}{1\ \text{m}^3}\right|.\left|\frac{1\ \text{kg}}{1000\ \text{g}}\right|}{(7.34\times10^{-3} - 5.87\times10^{-4})\ \text{kg m}^{-3}}$$
$$= 4.45\ \text{h}^{-1}$$

Answer: 4.5 h^{-1}

(c)

The $k_L a$ for surface aeration must be increased by a factor of 4.45/1.20 = 3.71 to match the $k_L a$ for bubble aeration. In surface aeration, the term a in $k_L a$ represents the surface area of liquid at the liquid–gas interface. Let us assume that a for a cylindrical vessel is the area of a circle with diameter equal to the tank diameter D_T:

$$a = \frac{\pi}{4} D_T^2$$

This assumption is valid when the liquid is stationary; however, if the liquid is agitated, the area available for oxygen transfer may be higher due to rippling and movement of the gas–liquid interface. Assuming that the value of k_L is unchanged, i.e. increasing the tank diameter does not affect the velocity of the liquid at the surface, a must be increased by a factor of 3.71 to achieve the required improvement in $k_L a$. This

can be achieved by increasing D_T by a factor of $\sqrt{3.71}$ = 1.93. Therefore, D_T must be increased from 8.5 cm to 1.93 × 8.5 cm = 16.4 cm while keeping the liquid volume constant at 500 ml.

Answer: 16.4 cm, assuming that the surface liquid velocity and the effect of liquid movement on the gas–liquid interfacial area are unaffected by the increase in tank diameter. *A vessel diameter of 16.4 cm for a liquid volume of 500 ml corresponds to a liquid height of only 2.4 cm. This calculation shows that increasing D_T is not a practical approach for obtaining surface aeration k_La values similar to those achieved using air sparging.*

10.14 Shake-flask aeration

(a)

By analogy with Eq. (10.7), the resistance to oxygen transfer is equal to $1/k_La$. k_La for the dynamic method is estimated using Eq. (10.57) with t_1 = 5 s, C_{AL1} = 65% air saturation, t_2 = 30 s, C_{AL2} = 75% air saturation and $\overline{C}_{AL}$ = 90% air saturation:

$$k_La = \frac{\ln\left(\frac{90-65}{90-75}\right)}{(30-5)\text{ s}} = 0.0204\text{ s}^{-1}$$

Therefore, $1/k_La$ = 48.9 s. The assumptions involved in the simple dynamic method apply as described in Section 10.10.2. The calculated value for k_La is not reliable unless electrode response and liquid boundary layer effects can be shown to be negligible. Because bubbles and gas hold-up are not involved in shake-flask aeration, gas-phase dynamics is not a significant issue in this case.

Answer: 49 s, assuming that electrode and boundary layer effects are negligible

(b)

The closures have cylindrical geometry. The cross-sectional area of the closure A_c is equal to:

$$A_c = \frac{\pi}{4}D_c^2$$

where D_c is the diameter of the closure. As D_c is equal to the width of the flask opening:

$$A_c = \frac{\pi}{4}(3.2\text{ cm})^2 = 8.04\text{ cm}^2$$

The volume of gas in the flask V_G can be estimated as the overall flask volume minus the liquid volume:

$$V_G = (300-100)\text{ cm}^3 = 200\text{ cm}^3$$

Substituting values into the equation for K_c:

$$K_c = \frac{20.8\text{ cm}^2\text{ s}^{-1}(8.04\text{ cm}^2)}{4\text{ cm }(200\text{ cm}^3)} = 0.209\text{ s}^{-1}$$

The resistance due to the flask closure is equal to $1/K_c = 1/(0.209\text{ s}^{-1})$ = 4.78 s.

Answer: 4.8 s

(c)

The total resistance to mass transfer is the sum of the resistance due to the flask closure and the resistance due to the liquid boundary layer = (4.78 + 48.9) s = 53.68 s. Therefore, the proportion of the total resistance due to the flask closure is (4.78 s)/(53.68 s) × 100% = 8.9%.

Answer: 8.9%

(d)

$k_L a$ must be increased by a factor of 0.209/0.0204 = 10.25 for the resistance within the flask to be equal to that of the flask closure. From the equation for $k_L a$ as a function of shake-flask operating parameters, increasing the shaker speed from 80 rpm to 150 rpm increases $k_L a$ by a factor of:

$$\left(\frac{150\ \text{rpm}}{80\ \text{rpm}}\right)^{1.2} = 2.13$$

As this is less than the 10.25-fold increase required, additional changes must be made to the shake-flask system to obtain a further 10.25/2.13 = 4.8-fold increase in $k_L a$.

(i)

From the equation for $k_L a$ as a function of system parameters, for the required additional improvement in $k_L a$ to be achieved:

$$\left(\frac{V_F}{V_L}\right)^{0.85}$$

must be increased by a factor of 4.8. Therefore, V_F/V_L must be increased by a factor of $(4.8)^{1/0.85} = 6.33$. If the liquid volume V_L remains constant, V_F must be increased 6.33-fold, from 300 ml to 6.33 × 300 ml = 1899 ml = 1.9 l.

Answer: 1.9 l

(ii)

From the calculation in **(i)**, if V_F/V_L must be increased by a factor of 6.33 while keeping V_F constant, V_L must be reduced 6.33-fold, from 100 ml to (100 ml)/6.33 = 15.8 ml.

Answer: 16 ml

Chapter 11

Unit Operations

11.1 Overall product recovery

The overall product recovery is obtained by multiplying together the fractional recoveries for each downstream processing step:

$$\text{Overall product recovery} = (0.89)\,(0.74)\,(0.71)\,(0.82)\,(0.68) = 0.26$$

Answer: 0.26

11.2 Product yield from transgenic goats' milk

(a)

The overall product recovery is obtained by multiplying together the fractional recoveries for each processing step:

$$\text{Overall product recovery} = (0.75)\,(0.60)\,(0.85)\,(0.85) = 0.325$$

Answer: 0.33

(b)

Before product recovery, the mass of anti-thrombin III produced per week is (20,000 l) × (10 g l^{-1}) = 200,000 g = 200 kg. Applying the result for the overall fractional product recovery from **(a)**, the mass of purified anti-thrombin III recovered per week is 0.325 × 200 kg = 65 kg.

Answer: 65 kg

(c)

To produce 80 kg of purified anti-thrombin III per week, the overall fractional product recovery required is (80 kg)/(200 kg) = 0.40. If the increase in overall recovery is achieved by improving the salt precipitation and filtration step only, the product yield for this step must be:

$$\text{Recovery for salt precipitation and filtration step} = \frac{0.40}{(0.75)\,(0.85)\,(0.85)} = 0.738$$

Therefore, the percentage increase in product yield required for the salt precipitation and filtration step is:

$$\frac{0.738 - 0.60}{0.60} \times 100\% = 23\%$$

Answer: 23%

11.3 Laboratory algal filtration

The filter area A is the area of a circle of diameter 6 cm = 6×10^{-2} m:

$$A = \pi R^2 = \pi\left(\frac{D}{2}\right)^2 = \pi\left(\frac{6\times10^{-2}\ \text{m}}{2}\right)^2 = 2.83\times10^{-3}\ \text{m}^2$$

Δp = 5 psi; therefore, from Table A.5 (Appendix A):

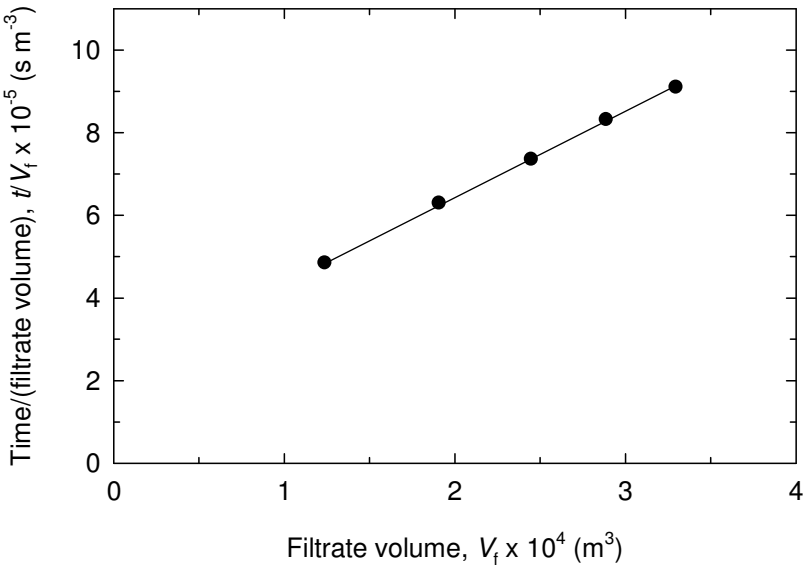
$$\Delta p = 5\ \text{psi} \,.\left|\frac{6.895\times10^{3}\ \text{kg m}^{-1}\,\text{s}^{-2}}{1\ \text{psi}}\right| = 3.45\times10^{4}\ \text{kg m}^{-1}\,\text{s}^{-2}$$

$\mu_f = 10^{-3}$ Pa s; therefore, from Table A.9 (Appendix A), $\mu_f = 10^{-3}$ kg m^{-1} s^{-1}.

(a)

The proportion of the total resistance to filtration due to the filter cake is given by Eq. (11.10). To use this equation, we must evaluate the terms α and R_m. According to Eqs (11.16)–(11.18), a plot of t/V_f versus V_f should yield a straight line for determination of the filtration parameters. The data after converting the units to s and m^3 are listed and plotted below.

Time, t (s)	Filtrate volume, V_f (m^3)	t/V_f (s m^{-3})
60	1.24×10^{-4}	4.839×10^5
120	1.91×10^{-4}	6.283×10^5
180	2.45×10^{-4}	7.347×10^5
240	2.89×10^{-4}	8.304×10^5
300	3.30×10^{-4}	9.091×10^5



The slope of the straight line in the plot $K_1 = 2.066 \times 10^9$ s m^{-6}; the intercept $K_2 = 2.301 \times 10^5$ s m^{-3}. From Eq. (11.17), the specific cake resistance α is:

$$\alpha = \frac{2A^2 \Delta p\, K_1}{\mu_f c}$$

As $c = 7.5$ g l^{-1} = 7.5 kg m^{-3}, substituting values gives:

$$\alpha = \frac{2\,(2.83\times10^{-3}\ \text{m}^2)^2\,(3.45\times10^4\ \text{kg m}^{-1}\,\text{s}^{-2})\,(2.066\times10^9\ \text{s m}^{-6})}{10^{-3}\ \text{kg m}^{-1}\,\text{s}^{-1}\,(7.5\ \text{kg m}^{-3})} = 1.522\times10^{11}\ \text{m kg}^{-1}$$

From Eq. (11.18), the filter medium resistance R_m is:

$$R_m = \frac{A \Delta p\, K_2}{\mu_f}$$

Substituting values gives:

$$R_m = \frac{2.83\times10^{-3}\ \text{m}^2\ (3.45\times10^{4}\ \text{kg m}^{-1}\ \text{s}^{-2})\ (2.301\times10^{5}\ \text{s m}^{-3})}{10^{-3}\ \text{kg m}^{-1}\ \text{s}^{-1}} = 2.247\times10^{10}\ \text{m}^{-1}$$

From the measured data provided, at t = 3 min = 180 s, $V_f = 2.45 \times 10^{-4}$ m^3. Calculating the numerator of Eq. (11.10):

$$\alpha\left(\frac{cV_f}{A}\right) = 1.522\times10^{11}\ \text{m kg}^{-1}\left(\frac{7.5\ \text{kg m}^{-3}\ (2.45\times10^{-4}\ \text{m}^3)}{2.83\times10^{-3}\ \text{m}^2}\right) = 9.882\times10^{10}\ \text{m}^{-1}$$

Therefore, Eq. (11.10) becomes:

$$\text{Proportion of the total resistance due to the filter cake} = \frac{9.882\times10^{10}\ \text{m}^{-1}}{9.882\times10^{10}\ \text{m}^{-1} + 2.247\times10^{10}\ \text{m}^{-1}}$$
$$= 0.81$$

Answer: 0.81

(b)

For the new filtration, V_f = 200 ml = 0.2 × 10^{-3} m^3 and t = 5 min = 300 s. K_2 is the same for the new filtration; however, K_1 is changed as the value of c is changed. The new value of K_1 is obtained from Eq. (11.16):

$$K_1 = \frac{t}{V_f^2} - \frac{K_2}{V_f}$$

Substituting values using the result for K_2 from **(a)**:

$$K_1 = \frac{300\ \text{s}}{(0.2\times10^{-3}\ \text{m}^3)^2} - \frac{2.301\times10^{5}\ \text{s m}^{-3}}{0.2\times10^{-3}\ \text{m}^3} = 6.35\times10^{9}\ \text{s m}^{-6}$$

From the definition of K_1 in Eq. (11.17):

$$c = \frac{2A^2\Delta p\, K_1}{\mu_f \alpha}$$

Substituting values using the result for α from **(a)**:

$$c = \frac{2\ (2.83\times10^{-3}\ \text{m}^2)^2\ (3.45\times10^{4}\ \text{kg m}^{-1}\ \text{s}^{-2})\ (6.35\times10^{9}\ \text{s m}^{-6})}{10^{-3}\ \text{kg m}^{-1}\ \text{s}^{-1}\ (1.522\times10^{11}\ \text{m kg}^{-1})} = 23.06\ \text{kg m}^{-3}$$

As 1 m^3 = 10^3 l (Table A.2, Appendix A) and 1 kg = 10^3 g, c = 23.06 g l^{-1}.

Answer: 23 g l^{-1}

11.4 Filtration of plant cells

Δp = 9 psi; therefore, from Table A.5 (Appendix A):

$$\Delta p = 9\ \text{psi}\ .\left|\frac{6.895\times10^{3}\ \text{kg m}^{-1}\ \text{s}^{-2}}{1\ \text{psi}}\right| = 6.21\times10^{4}\ \text{kg m}^{-1}\ \text{s}^{-2}$$

μ_f = 5 cP; therefore, from Table A.9 (Appendix A), $\mu_f = 5 \times 10^{-3}$ kg m^{-1} s^{-1}. c = 25 g l^{-1} = 25 kg m^{-3}.

(a)

Converting units for the filter area A:

$$A = 78\ \text{cm}^2 . \left| \frac{1\ \text{m}}{100\ \text{cm}} \right|^2 = 78 \times 10^{-4}\ \text{m}^2$$

Converting units for K_1 and K_2:

$$K_1 = 8.3 \times 10^{-6}\ \text{min cm}^{-6} . \left| \frac{60\ \text{s}}{1\ \text{min}} \right| . \left| \frac{100\ \text{cm}}{1\ \text{m}} \right|^6 = 4.98 \times 10^8\ \text{s m}^{-6}$$

$$K_2 = 4.2 \times 10^{-3}\ \text{min cm}^{-3} . \left| \frac{60\ \text{s}}{1\ \text{min}} \right| . \left| \frac{100\ \text{cm}}{1\ \text{m}} \right|^3 = 2.52 \times 10^5\ \text{s m}^{-3}$$

From Eq. (11.17), the specific cake resistance α is:

$$\alpha = \frac{2A^2 \Delta p\, K_1}{\mu_f\, c}$$

Substituting values gives:

$$\alpha = \frac{2\,(78 \times 10^{-4}\ \text{m}^2)^2\,(6.21 \times 10^4\ \text{kg m}^{-1}\,\text{s}^{-2})\,(4.98 \times 10^8\ \text{s m}^{-6})}{5 \times 10^{-3}\ \text{kg m}^{-1}\,\text{s}^{-1}\,(25\ \text{kg m}^{-3})} = 3.01 \times 10^{10}\ \text{m kg}^{-1}$$

From Eq. (11.18), the filter medium resistance R_m is:

$$R_m = \frac{A \Delta p\, K_2}{\mu_f}$$

Substituting values gives:

$$R_m = \frac{78 \times 10^{-4}\ \text{m}^2\,(6.21 \times 10^4\ \text{kg m}^{-1}\,\text{s}^{-2})\,(2.52 \times 10^5\ \text{s m}^{-3})}{5 \times 10^{-3}\ \text{kg m}^{-1}\,\text{s}^{-1}} = 2.44 \times 10^{10}\ \text{m}^{-1}$$

Answer: Specific cake resistance $\alpha = 3.01 \times 10^{10}$ m kg^{-1}; filter medium resistance $R_m = 2.44 \times 10^{10}$ m^{-1}

(b)

The values of α and R_m determined in **(a)** apply for operation at 9 psi. If A is increased by a factor of (4 m^2)/(78 × 10^{-4} m^2) = 512.8, from the definition of K_1 in Eq. (11.17), K_1 is reduced by a factor of $(512.8)^2$ = 2.63 × 10^5:

$$K_1 = \frac{4.98 \times 10^8\ \text{s m}^{-6}}{2.63 \times 10^5} = 1.89 \times 10^3\ \text{s m}^{-6}$$

Similarly, from Eq. (11.18), K_2 is reduced by a factor of 512.8:

$$K_2 = \frac{2.52 \times 10^5\ \text{s m}^{-3}}{512.8} = 4.91 \times 10^2\ \text{s m}^{-3}$$

V_f = 6000 l; therefore, from Table A.2 (Appendix A), V_f = 6 m^3. The time required to collect this volume of filtrate is determined from Eq. (11.16):

$$t = K_1 V_f^2 + K_2 V_f$$

Substituting values gives:

$$t = 1.89 \times 10^3\ \text{s m}^{-6}\,(6\ \text{m}^3)^2 + 4.91 \times 10^2\ \text{s m}^{-3}\,(6\ \text{m}^3) = 7.10 \times 10^4\ \text{s}$$

Converting units:

$$t = 7.10\times10^4 \text{ s} \,.\left|\frac{1 \text{ h}}{3600 \text{ s}}\right| = 19.7 \text{ h}$$

Answer: 19.7 h

11.5 Bacterial filtration

Δp = 360 mmHg; therefore, from Table A.5 (Appendix A):

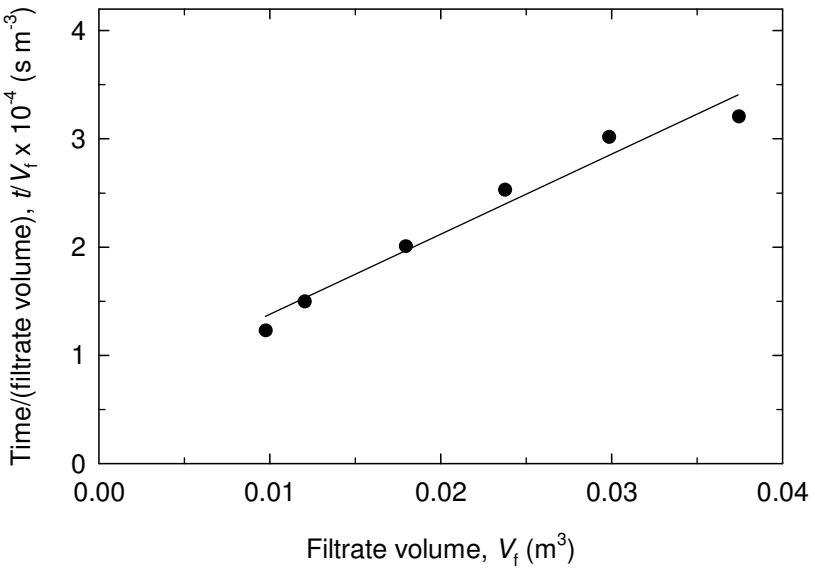
$$\Delta p = 360 \text{ mmHg} \,.\left|\frac{1.333\times10^2 \text{ kg m}^{-1} \text{ s}^{-2}}{1 \text{ mmHg}}\right| = 4.799\times10^4 \text{ kg m}^{-1} \text{ s}^{-2}$$

μ_f = 4.0 cP; therefore, from Table A.9 (Appendix A), $\mu_f = 4.0 \times 10^{-3}$ kg m^{-1} s^{-1}. c = 22 g l^{-1} = 22 kg m^{-3}.

(a)

A = 0.25 m^2. According to Eqs (11.16)–(11.18), a plot of t/V_f versus V_f should yield a straight line for determination of the filtration parameters. The data after converting the units to s and m^3 are listed and plotted below.

Time, t (s)	Filtrate volume, V_f (m^3)	t/V_f (s m^{-3})
120	0.0098	1.22×10^4
180	0.0121	1.49×10^4
360	0.0180	2.00×10^4
600	0.0238	2.52×10^4
900	0.0299	3.01×10^4
1200	0.0375	3.20×10^4



The slope of the straight line in the plot $K_1 = 7.438 \times 10^5$ s m^{-6}; the intercept K_2 = 6147 s m^{-3}. From Eq. (11.17), the specific cake resistance α is:

$$\alpha = \frac{2A^2 \Delta p \, K_1}{\mu_f \, c}$$

Substituting values gives:

$$\alpha = \frac{2\,(0.25\ \text{m}^2)^2\,(4.799\times10^4\ \text{kg m}^{-1}\,\text{s}^{-2})\,(7.438\times10^5\ \text{s m}^{-6})}{4.0\times10^{-3}\ \text{kg m}^{-1}\,\text{s}^{-1}\,(22\ \text{kg m}^{-3})} = 5.07\times10^{10}\ \text{m kg}^{-1}$$

From Eq. (11.18), the filter medium resistance R_m is:

$$R_m = \frac{A\Delta p\,K_2}{\mu_f}$$

Substituting values gives:

$$R_m = \frac{0.25\ \text{m}^2\,(4.799\times10^4\ \text{kg m}^{-1}\,\text{s}^{-2})\,(6147\ \text{s m}^{-3})}{4.0\times10^{-3}\ \text{kg m}^{-1}\,\text{s}^{-1}} = 1.84\times10^{10}\ \text{m}^{-1}$$

Answer: Specific cake resistance $\alpha = 5.07 \times 10^{10}$ m kg^{-1}; filter medium resistance $R_m = 1.84 \times 10^{10}$ m^{-1}

(b)

The values of α and R_m determined in **(a)** apply for operation at 360 mmHg. $V_f = 4000$ l = 4 m^3; $t = 30$ min = 1800 s. Therefore, the required $t/V_f = (1800\ \text{s})/(4\ \text{m}^3) = 450$ s m^{-3}. Applying Eq. (11.15) to the scaled-up filter of area A:

$$450\ \text{s m}^{-3} = \frac{4.0\times10^{-3}\ \text{kg m}^{-1}\,\text{s}^{-1}\,(5.07\times10^{10}\ \text{m kg}^{-1})\,(22\ \text{kg m}^{-3})}{2A^2\,(4.799\times10^4\ \text{kg m}^{-1}\,\text{s}^{-2})}\,(4\ \text{m}^3)$$

$$+\frac{4.0\times10^{-3}\ \text{kg m}^{-1}\,\text{s}^{-1}\,(1.84\times10^{10}\ \text{m}^{-1})}{A\,(4.799\times10^4\ \text{kg m}^{-1}\,\text{s}^{-2})}$$

Calculating terms gives:

$$450\ \text{s m}^{-3} = \frac{1.86\times10^5\ \text{s m}}{A^2} + \frac{1.53\times10^3\ \text{s m}^{-1}}{A}$$

Rearranging to form a quadratic equation:

$$450A^2 - 1.53\times10^3 A - 1.86\times10^5 = 0$$

where A has units of m^2. Solving for A:

$$A = \frac{1.53\times10^3 \pm \sqrt{(1.53\times10^3)^2 - 4\,(450)\,(-1.86\times10^5)}}{2\,(450)}$$

$$A = \frac{1.53\times10^3 \pm 1.84\times10^4}{2\,(450)}$$

A realistic (non-negative) answer is obtained by adding the two terms in the numerator:

$$A = \frac{1.53\times10^3 + 1.84\times10^4}{2\,(450)} = 22.1$$

Answer: 22.1 m^2

11.6 Filtration of mycelial suspensions

c for pelleted cells = 0.25 g ml^{-1} = 0.25 kg l^{-1} = 250 kg m^{-3}. Similarly, c for filamentous cells = 0.1 g ml^{-1} = 100 kg m^{-3}. $\mu_f = 1.4$ cP; therefore, from Table A.9 (Appendix A), $\mu_f = 1.4 \times 10^{-3}$ kg m^{-1} s^{-1}.

(a)

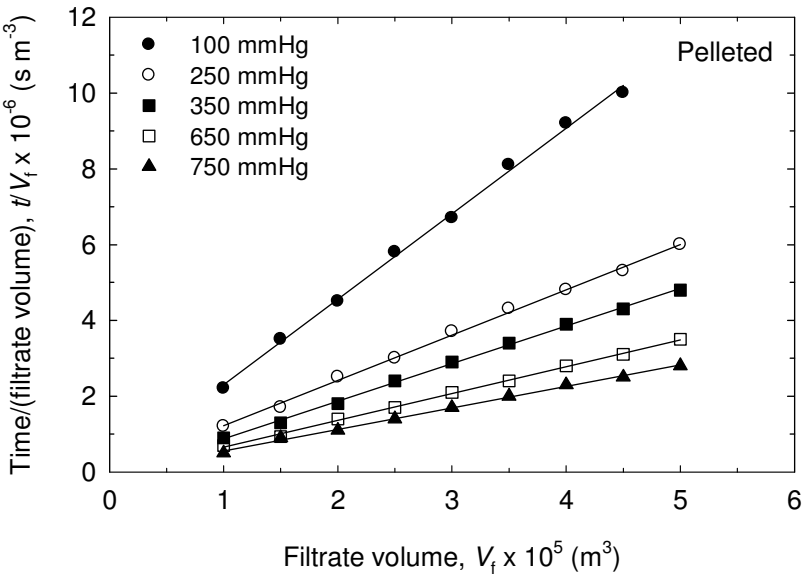
Converting units for the filter area A:

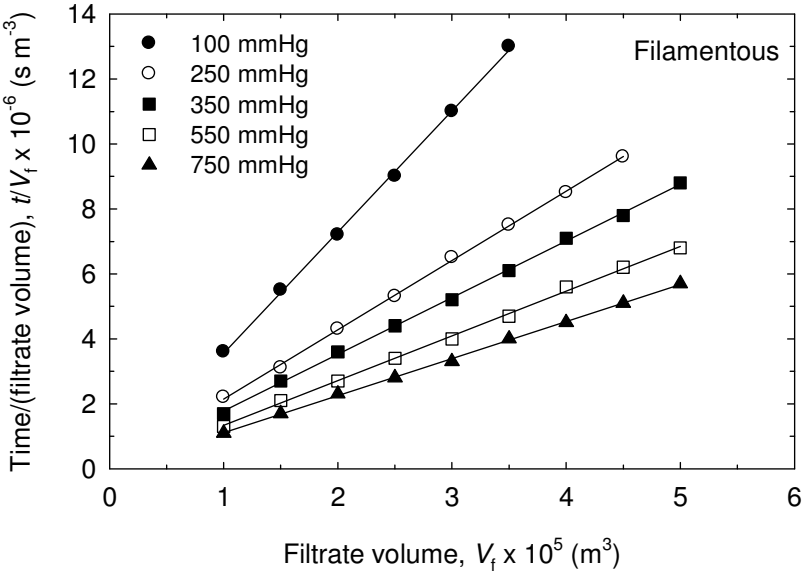
$$A = 1.8\ \text{cm}^2 \cdot \left| \frac{1\ \text{m}}{100\ \text{cm}} \right|^2 = 1.8 \times 10^{-4}\ \text{m}^2$$

After converting units using the conversion factor 1 m^3 = 10^6 ml, the data for V_f and t/V_f are listed and plotted below.

	Pressure drop (mmHg)				
	100	250	350	550	750
V_f (m^3) pelleted	t/V_f (s m^{-3})				
1.0×10^{-5}	2.2×10^6	1.2×10^6	9.0×10^5	7.0×10^5	5.0×10^5
1.5×10^{-5}	3.5×10^6	1.7×10^6	1.3×10^6	9.3×10^5	8.0×10^5
2.0×10^{-5}	4.5×10^6	2.5×10^6	1.8×10^6	1.4×10^6	1.1×10^6
2.5×10^{-5}	5.8×10^6	3.0×10^6	2.4×10^6	1.7×10^6	1.4×10^6
3.0×10^{-5}	6.7×10^6	3.7×10^6	2.9×10^6	2.1×10^6	1.7×10^6
3.5×10^{-5}	8.1×10^6	4.3×10^6	3.4×10^6	2.4×10^6	2.0×10^6
4.0×10^{-5}	9.2×10^6	4.8×10^6	3.9×10^6	2.8×10^6	2.3×10^6
4.5×10^{-5}	1.0×10^7	5.3×10^6	4.3×10^6	3.1×10^6	2.5×10^6
5.0×10^{-5}	–	6.0×10^6	4.8×10^6	3.5×10^6	2.8×10^6

	Pressure drop (mmHg)				
	100	250	350	550	750
V_f (m^3) filamentous	t/V_f (s m^{-3})				
1.0×10^{-5}	3.6×10^6	2.2×10^6	1.7×10^6	1.3×10^6	1.1×10^6
1.5×10^{-5}	5.5×10^6	3.1×10^6	2.7×10^6	2.1×10^6	1.7×10^6
2.0×10^{-5}	7.2×10^6	4.3×10^6	3.6×10^6	2.7×10^6	2.3×10^6
2.5×10^{-5}	9.0×10^6	5.3×10^6	4.4×10^6	3.4×10^6	2.8×10^6
3.0×10^{-5}	1.1×10^7	6.5×10^6	5.2×10^6	4.0×10^6	3.3×10^6
3.5×10^{-5}	1.3×10^7	7.5×10^6	6.1×10^6	4.7×10^6	4.0×10^6
4.0×10^{-5}	–	8.5×10^6	7.1×10^6	5.6×10^6	4.5×10^6
4.5×10^{-5}	–	9.6×10^6	7.8×10^6	6.2×10^6	5.1×10^6
5.0×10^{-5}	–	–	8.8×10^6	6.8×10^6	5.7×10^6





From Eq. (11.16), the slopes of the straight lines in the plots are equal to K_1 for each pressure drop. These values are listed below.

	Pressure drop (mmHg)				
	100	250	350	550	750
Pelleted: slope = K_1 (s m^{-6})	2.26×10^{11}	1.20×10^{11}	9.93×10^{10}	7.07×10^{10}	5.77×10^{10}
Filamentous: slope = K_1 (s m^{-6})	3.73×10^{11}	2.13×10^{11}	1.75×10^{11}	1.38×10^{11}	1.14×10^{11}

From Eq. (11.17), the specific cake resistance α is:

$$\alpha = \frac{2A^2 \Delta p\, K_1}{\mu_f\, c}$$

We can derive an empirical expression for α as a function of Δp and K_1 by substituting values into this equation and applying the pressure unit conversion factor 1 mmHg = 1.333×10^2 kg m^{-1} s^{-2} (Table A.5, Appendix A). For the pelleted cells:

$$\alpha = \frac{2\,(1.8\times10^{-4}\ \text{m}^2)^2\,\Delta p\,.\left|\dfrac{1.333\times10^2\ \text{kg m}^{-1}\,\text{s}^{-2}}{1\ \text{mmHg}}\right| K_1}{1.4\times10^{-3}\ \text{kg m}^{-1}\,\text{s}^{-1}\,(250\ \text{kg m}^{-3})} = 2.47\times10^{-5}\,\Delta p\, K_1\ \ \text{m kg}^{-1}$$

where Δp has units of mmHg and K_1 has units of s m^{-6}. Similarly, for the filamentous cells:

$$\alpha = \frac{2\,(1.8\times10^{-4}\ \text{m}^2)^2\,\Delta p\,.\left|\dfrac{1.333\times10^2\ \text{kg m}^{-1}\,\text{s}^{-2}}{1\ \text{mmHg}}\right| K_1}{1.4\times10^{-3}\ \text{kg m}^{-1}\,\text{s}^{-1}\,(100\ \text{kg m}^{-3})} = 6.17\times10^{-5}\,\Delta p\, K_1\ \ \text{m kg}^{-1}$$

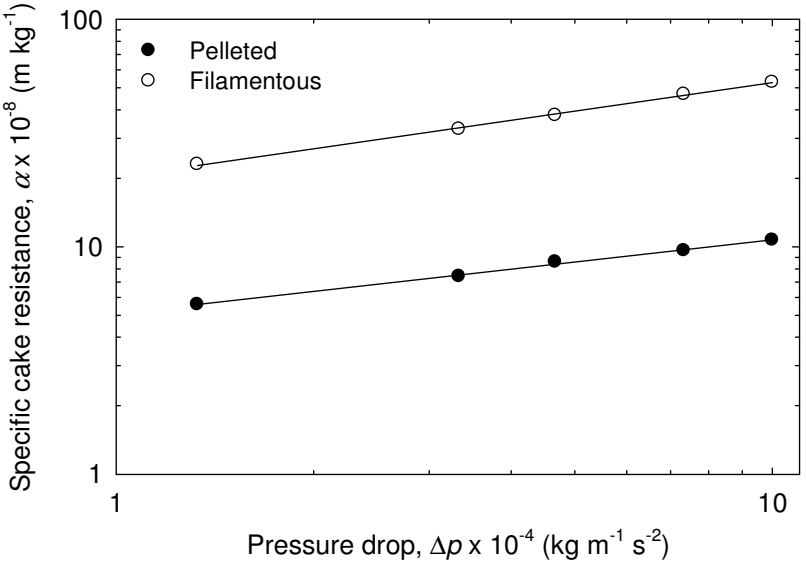
The results for specific cake resistance obtained after substituting the values for Δp and K_1 into the equations for α are listed below.

	Pressure drop (mmHg)				
	100	250	350	550	750
Pelleted: α (m kg^{-1})	5.58×10^8	7.41×10^8	8.58×10^8	9.60×10^8	1.07×10^9
Filamentous: α (m kg^{-1})	2.30×10^9	3.29×10^9	3.78×10^9	4.68×10^9	5.28×10^9

(b)

From Eq. (11.4), the compressibility s can be obtained by plotting the specific cake resistance α versus Δp on log–log coordinates. The data with the units of Δp converted to kg m^{-1} s^{-2} are listed and plotted below.

	Pressure drop (kg m^{-1} s^{-2})				
	1.33×10^4	3.33×10^4	4.67×10^4	7.33×10^4	1.00×10^5
Pelleted: α (m kg^{-1})	5.58×10^8	7.41×10^8	8.58×10^8	9.60×10^8	1.07×10^9
Filamentous: α (m kg^{-1})	2.30×10^9	3.29×10^9	3.78×10^9	4.68×10^9	5.28×10^9



From Eq. (11.4), the equations for the two lines in the figure have the form, $\alpha = \alpha'(\Delta p)^s$. For the pelleted suspension, $\alpha = 2.60 \times 10^7\ \Delta p^{0.323}$; for the filamentous suspension, $\alpha = 4.42 \times 10^7\ \Delta p^{0.415}$, where Δp has units kg m^{-1} s^{-2}, α has units m kg^{-1}, and α' has units kg$^{-(1+s)}$ m^{1+s} s^{2s}. Therefore, the compressibility is 0.323 for the pelleted suspension and 0.415 for the filamentous suspension.

Answer: 0.323 for the pelleted suspension; 0.415 for the filamentous suspension

(c)

$A = 15$ m^2. $V_f = 20$ m^3 and $t = 1$ h = 3600 s. The filtration equation for a compressible filter cake is Eq. (11.15) with $\alpha = \alpha'(\Delta p)^s$. Assuming that the filter medium resistance R_m is negligible, Eq. (11.15) can be written as:

$$\frac{t}{V_f} = \frac{\mu_f \alpha'(\Delta p)^s c}{2A^2 \Delta p} V_f = \frac{\mu_f \alpha'(\Delta p)^{s-1} c}{2A^2} V_f$$

Solving for $(\Delta p)^{s-1}$:

$$(\Delta p)^{s-1} = \frac{t}{V_f^2} \frac{2A^2}{\mu_f \alpha' c}$$

Substituting parameter values for the filamentous suspension using the results for α' and s from **(b)**:

$$\Delta p^{0.415-1} = \frac{3600\ \text{s}}{(20\ \text{m}^3)^2} \frac{2\,(15\ \text{m}^2)^2}{(1.4\times10^{-3}\ \text{kg m}^{-1}\ \text{s}^{-1})\ 4.42\times10^7\ \text{kg}^{-(1+0.415)}\ \text{m}^{1+0.415}\ \text{s}^{2\times0.415}\ (100\ \text{kg m}^{-3})}$$

$$\Delta p^{-0.585} = 6.54\times10^{-4}\ \text{kg}^{-0.585}\ \text{m}^{0.585}\ \text{s}^{1.170}$$

$$\Delta p = 2.78\times10^5\ \text{kg m}^{-1}\ \text{s}^{-2}$$

Converting units from Table A.5 (Appendix A):

$$\Delta p = 2.78\times10^5\ \text{kg m}^{-1}\ \text{s}^{-2} \left| \frac{1\ \text{mmHg}}{1.333\times10^2\ \text{kg m}^{-1}\ \text{s}^{-2}} \right| = 2086\ \text{mmHg}$$

Answer: 2086 mmHg. *We have assumed in this calculation that the filter medium resistance is negligible and that the filter cake characteristics measured at pressures between 100 and 750 mmHg apply at the much higher pressure of 2086 mmHg.*

11.7 Rotary drum vacuum filtration

(a)

For negligible filter medium resistance, the filtration equation is Eq. (11.15) with $R_m = 0$. From Eq. (11.4), for a compressible filter cake $\alpha = \alpha'(\Delta p)^s$. Therefore, Eq. (11.15) becomes:

$$\frac{t}{V_f} = \frac{\mu_f \alpha'(\Delta p)^s c}{2A^2 \Delta p} V_f = \frac{\mu_f \alpha'(\Delta p)^{s-1} c}{2A^2} V_f \tag{1}$$

Solving for $\mu_f \alpha' c$:

$$\mu_f \alpha' c = \frac{t}{V_f^2} \frac{2A^2}{\Delta p^{s-1}}$$

Substituting parameter values for the laboratory filter and using the conversion factor 1 psi = 6.895×10^3 kg m^{-1} s^{-2} (Table A.5, Appendix A):

$$\mu_f \alpha' c = \frac{23.5\ \text{min} \left| \frac{60\ \text{s}}{1\ \text{min}} \right|}{\left(500\ \text{ml} \left| \frac{1\ \text{m}^3}{10^6\ \text{ml}} \right| \right)^2} \frac{2\left(5\ \text{cm}^2 \left| \frac{1\ \text{m}}{100\ \text{cm}} \right|^2 \right)^2}{\left(12\ \text{psi} \left| \frac{6.895 \times 10^3\ \text{kg m}^{-1}\ \text{s}^{-2}}{1\ \text{psi}} \right| \right)^{0.57-1}} = 3.67 \times 10^5\ \text{kg}^{0.43}\ \text{m}^{-2.43}\ \text{s}^{0.14}$$

Answer: 3.67×10^5 kg$^{0.43}$ m$^{-2.43}$ s$^{0.14}$

(b)

The cycle time is 1/N hours per revolution.

Answer: 1/N h

(c)

As 30% of the rotating filter cloth is submerged at any time, each cm^2 of cloth is submerged for 0.3 × the cycle time = 0.3/N h.

Answer: 0.3/N h

(d)

The volume filtered per revolution is 20 m^3 h^{-1} × 1/N h = 20/N m^3.

Answer: 20/N m^3

(e)

If we apply (1) to a single revolution of the filter, t = 0.3/N h and V_f = 20/N m^3. Substituting the filter cake parameter values from **(a)**, evaluating the filter area $A = 2\pi rW$ where r is the drum radius = 0.75 m and W is the drum width = 1.2 m, and using the conversion factor 1 psi = 6.895×10^3 kg m^{-1} s^{-2} (Table A.5, Appendix A), (1) becomes:

$$\frac{\frac{0.3}{N}\ \text{h} \left| \frac{3600\ \text{s}}{1\ \text{h}} \right|}{\frac{20}{N}\ \text{m}^3} = \frac{3.67 \times 10^5\ \text{kg}^{0.43}\ \text{m}^{-2.43}\ \text{s}^{0.14} \left(4.5\ \text{psi} \left| \frac{6.895 \times 10^3\ \text{kg m}^{-1}\ \text{s}^{-2}}{1\ \text{psi}} \right| \right)^{0.57-1}}{2\left[2\pi\ (0.75\ \text{m})\ (1.2\ \text{m}) \right]^2} \left(\frac{20}{N}\ \text{m}^3 \right)$$

where N is the number of revolutions per h. Calculating both sides of the equation gives:

$$54 \text{ s m}^{-3} = \frac{1343.9}{N} \text{ s m}^{-3}$$

$$N = \frac{1343.9}{54} = 24.9$$

Answer: 24.9 revolutions per hour

(f)

The filtration time is increased from 0.3/*N* h to 0.5/*N* h. Substituting this value for *t* into the left-hand side of the equation in **(e)**:

$$\frac{\frac{0.5}{N} \text{ h} \cdot \left| \frac{3600 \text{ s}}{1 \text{ h}} \right|}{\frac{20}{N} \text{ m}^3} = \frac{1343.9}{N} \text{ s m}^{-3}$$

Solving for *N* gives:

$$N = 14.9$$

Answer: 14.9 revolutions per hour

11.8 Centrifugation of yeast

(a)

Convert the parameter values to units of kg, m, s:

$$\rho_p = 1.06 \text{ g cm}^{-3} = 1.06 \text{ g cm}^{-3} \cdot \left| \frac{1 \text{ kg}}{1000 \text{ g}} \right| \cdot \left| \frac{100 \text{ cm}}{1 \text{ m}} \right|^3 = 1060 \text{ kg m}^{-3}$$

$$\rho_L = 0.997 \text{ g cm}^{-3} = 0.997 \text{ g cm}^{-3} \cdot \left| \frac{1 \text{ kg}}{1000 \text{ g}} \right| \cdot \left| \frac{100 \text{ cm}}{1 \text{ m}} \right|^3 = 997 \text{ kg m}^{-3}$$

$$Q = 500 \text{ l h}^{-1} = 500 \text{ l h}^{-1} \cdot \left| \frac{1 \text{ m}^3}{1000 \text{ l}} \right| \cdot \left| \frac{1 \text{ h}}{3600 \text{ s}} \right| = 1.39 \times 10^{-4} \text{ m}^3 \text{ s}^{-1}$$

From Table A.9 (Appendix A), 1 N s m^{-2} = 1 kg m^{-1} s^{-1}; therefore $\mu = 1.36 \times 10^{-3}$ kg m^{-1} s^{-1}. D_p = 5 μm = 5×10^{-6} m. From Eq. (2.16), g = 9.8 m s^{-2}. The sedimentation velocity is determined using Eq. (11.19):

$$u_g = \frac{(1060 - 997) \text{ kg m}^{-3}}{18 \, (1.36 \times 10^{-3} \text{ kg m}^{-1} \text{ s}^{-1})} \, (5 \times 10^{-6} \text{ m})^2 \, (9.8 \text{ m s}^{-2}) = 6.31 \times 10^{-7} \text{ m s}^{-1}$$

Substituting this result into Eq. (11.22):

$$\Sigma = \frac{1.39 \times 10^{-4} \text{ m}^3 \text{ s}^{-1}}{2 \, (6.31 \times 10^{-7} \text{ m s}^{-1})} = 110 \text{ m}^2$$

Answer: 110 m^2

(b)

From Section 2.4.1, the density of water is 1 g cm^{-3} = 1000 kg m^{-3}. From Section 2.4.2, if a particle has specific gravity 2.0, its density is 2.0 × 1000 kg m^{-3} = 2000 kg m^{-3}. Therefore, ρ_p = 2000 kg m^{-3}. D_p = 0.1 mm = 0.1×10^{-3} m. From Eq. (11.19), the sedimentation velocity for the quartz particles is:

$$u_g = \frac{(2000-997)\ \text{kg m}^{-3}}{18\,(1.36\times10^{-3}\ \text{kg m}^{-1}\,\text{s}^{-1})}\ (0.1\times10^{-3}\ \text{m})^2\,(9.8\ \text{m s}^{-2}) = 4.02\times10^{-3}\ \text{m s}^{-1}$$

From Eq. (11.22), the sigma factor is:

$$\Sigma = \frac{1.39\times10^{-4}\ \text{m}^3\,\text{s}^{-1}}{2\,(4.02\times10^{-3}\ \text{m s}^{-1})} = 0.017\ \text{m}^2$$

From the result in **(a)**, Σ for the yeast cells is 110/0.017 = 6470 times that for the quartz particles.

Answer: By a factor of 6470.

11.9 Centrifugation of food particles

Convert the parameter values to units of kg, m, s:

$$\rho_p = 1.03\ \text{g cm}^{-3} = 1.03\ \text{g cm}^{-3}\,.\left|\frac{1\ \text{kg}}{1000\ \text{g}}\right|.\left|\frac{100\ \text{cm}}{1\ \text{m}}\right|^3 = 1030\ \text{kg m}^{-3}$$

$$\rho_L = 1.00\ \text{g cm}^{-3} = 1.00\ \text{g cm}^{-3}\,.\left|\frac{1\ \text{kg}}{1000\ \text{g}}\right|.\left|\frac{100\ \text{cm}}{1\ \text{m}}\right|^3 = 1000\ \text{kg m}^{-3}$$

From Table A.9 (Appendix A), 1 Pa s = 1 kg m^{-1} s^{-1}; therefore $\mu = 1.25 \times 10^{-3}$ kg m^{-1} s^{-1}. $D_p = 10^{-2}$ mm = 10^{-5} m. b = 70 cm = 0.70 m; r = 11.5 cm = 0.115 m. One revolution = 2π radians, where radians is a non-dimensional unit (Section 2.1.2). Converting the centrifuge speed to radians s^{-1}:

$$\omega = 10{,}000\ \text{rpm} = 2\pi\ (10{,}000\ \text{min}^{-1})\,.\left|\frac{1\ \text{min}}{60\ \text{s}}\right| = 1.047\times10^3\ \text{s}^{-1}$$

From Eq. (2.16), g = 9.8 m s^{-2}. From Eq. (11.26), the sigma factor for the tubular bowl centrifuge is:

$$\Sigma = \frac{2\pi\,(1.047\times10^3\ \text{s}^{-1})^2\,(0.70\ \text{m})\,(0.115\ \text{m})^2}{9.8\ \text{m s}^{-2}} = 6.506\times10^3\ \text{m}^2$$

Applying Eq. (11.19):

$$u_g = \frac{(1030-1000)\ \text{kg m}^{-3}}{18\,(1.25\times10^{-3}\ \text{kg m}^{-1}\,\text{s}^{-1})}\ (10^{-5}\ \text{m})^2\,(9.8\ \text{m s}^{-2}) = 1.307\times10^{-6}\ \text{m s}^{-1}$$

From Eq. (11.22):

$$Q = 2u_g\Sigma = 2\,(1.307\times10^{-6}\ \text{m s}^{-1})\,6.506\times10^3\ \text{m}^2 = 0.017\ \text{m}^3\,\text{s}^{-1}$$

Answer: 0.017 m^3 s^{-1}

11.10 Scale-up of disc stack centrifuge

The sigma factor for the pilot-scale disc stack bowl centrifuge Σ_1 is calculated using Eq. (11.24) with r_2 = (10 cm)/2 = 5 cm = 0.05 m and r_1 = (2 cm)/2 = 1 cm = 0.01 m. From Eq. (2.16), g = 9.8 m s^{-2}. ω is converted to rad s^{-1} using the conversion factor 1 revolution = 2π radians, where radians is a non-dimensional unit (Section 2.1.2). Applying Eq. (11.24):

$$\Sigma_1 = \frac{2\pi\left(2\pi\times3000\ \text{min}^{-1}.\left|\frac{1\ \text{min}}{60\ \text{s}}\right|\right)^2\,(25-1)}{3\,(9.8\ \text{m s}^{-2})\,(\tan 35°)}\left[(0.05\ \text{m})^3-(0.01\ \text{m})^3\right] = 89.6\ \text{m}^2$$

From Eq. (11.23), the sigma factor Σ_2 for the bigger centrifuge is:

$$\Sigma_2 = \frac{Q_2}{Q_1}\Sigma_1 = \frac{80\,\text{l min}^{-1}}{3.5\,\text{l min}^{-1}}\,(89.6\,\text{m}^2) = 2048\,\text{m}^2$$

From Eq. (11.24), for the bigger centrifuge with r_2 = (15 cm)/2 = 7.5 cm = 0.075 m and r_1 = (4.7 cm)/2 = 2.35 cm = 0.0235 m:

$$\omega^2 = \frac{(3g\tan\theta)\Sigma_2}{2\pi(N-1)(r_2^3 - r_1^3)} = \frac{3\,(9.8\,\text{m s}^{-2})\,(\tan 45°)\,2048\,\text{m}^2}{2\pi(55-1)\left[(0.075\,\text{m})^3 - (0.0235\,\text{m})^3\right]} = 4.34\times10^5\,\text{s}^{-2}$$

$$\omega = \sqrt{4.34\times10^5\,\text{s}^{-2}} = 659\,\text{rad s}^{-1}$$

Converting to rpm:

$$\omega = 659\,\text{rad s}^{-1}.\left|\frac{1\,\text{revolution}}{2\pi\,\text{rad}}\right|.\left|\frac{60\,\text{s}}{1\,\text{min}}\right| = 6290\,\text{min}^{-1} = 6290\,\text{rpm}$$

Answer: 6290 rpm

11.11 Centrifugation of yeast and cell debris

Combining Eqs (11.19) and (11.22):

$$\Sigma_1 = \frac{18\mu_1 Q_1}{2(\rho_\text{p} - \rho_\text{L})D_{\text{p1}}^2 g}$$

and

$$\Sigma_2 = \frac{18\mu_2 Q_2}{2(\rho_\text{p} - \rho_\text{L})D_{\text{p2}}^2 g}$$

where subscript 1 refers to the centrifugation conditions for the yeast suspension and subscript 2 refers to the centrifugation conditions for the cell debris. As the same centrifuge operated at the same speed is used in both applications, $\Sigma_1 = \Sigma_2$ and:

$$\frac{18\mu_1 Q_1}{2(\rho_\text{p} - \rho_\text{L})D_{\text{p1}}^2 g} = \frac{18\mu_2 Q_2}{2(\rho_\text{p} - \rho_\text{L})D_{\text{p2}}^2 g}$$

Assuming that the particle and liquid densities are the same in both applications, cancelling terms gives:

$$\frac{\mu_1 Q_1}{D_{\text{p1}}^2} = \frac{\mu_2 Q_2}{D_{\text{p2}}^2}$$

or

$$Q_2 = \frac{D_{\text{p2}}^2}{D_{\text{p1}}^2}\frac{\mu_1}{\mu_2}Q_1$$

As $D_{\text{p2}} = D_{\text{p1}}/3$, $\mu_2 = 5\mu_1$ and $Q_1 = 3\,\text{l min}^{-1}$:

$$Q_2 = \frac{(D_{\text{p1}}/3)^2}{D_{\text{p1}}^2}\frac{\mu_1}{5\mu_1}\,(3\,\text{l min}^{-1}) = \frac{(1/3)^2}{5}\,(3\,\text{l min}^{-1}) = 0.067\,\text{l min}^{-1}$$

Answer: 0.067 l min^{-1}

11.12 Centrifuge throughput

Convert the cell density to units of kg m^{-3}:

$$\rho_p = 1.06 \text{ g cm}^{-3} = 1.06 \text{ g cm}^{-3} . \left| \frac{1 \text{ kg}}{1000 \text{ g}} \right| . \left| \frac{100 \text{ cm}}{1 \text{ m}} \right|^3 = 1060 \text{ kg m}^{-3}$$

$D_p = 1.1$ μm $= 1.1 \times 10^{-6}$ m. $\mu_f = 1.25$ cP; therefore, from Table A.9 (Appendix A), $\mu_f = 1.25 \times 10^{-3}$ kg m^{-1} s^{-1}. The sigma factor for the disc stack centrifuge is calculated using Eq. (11.24) with $r_2 = (15 \text{ cm})/2 = 7.5$ cm $= 7.5 \times 10^{-2}$ m and $r_1 = (7.5 \text{ cm})/2 = 3.75$ cm $= 3.75 \times 10^{-2}$ m. From Eq. (2.16), $g = 9.81$ m s^{-2}. ω is converted to rad s^{-1} using the conversion factor 1 revolution $= 2\pi$ radians, where radian is a non-dimensional unit (Section 2.1.2). Applying Eq. (11.24):

$$\Sigma = \frac{2\pi \left(2\pi \times 7300 \text{ min}^{-1} . \left| \frac{1 \text{ min}}{60 \text{ s}} \right| \right)^2 (72 - 1)}{3 \, (9.81 \text{ m s}^{-2}) \, (\tan 36°)} \left[(7.5 \times 10^{-2} \text{ m})^3 - (3.75 \times 10^{-2} \text{ m})^3 \right] = 4500.7 \text{ m}^2$$

(a)

From Eq. (11.19):

$$u_g = \frac{(1060 - 1025) \text{ kg m}^{-3}}{18 \, (1.25 \times 10^{-3} \text{ kg m}^{-1} \text{ s}^{-1})} \, (1.1 \times 10^{-6} \text{ m})^2 \, (9.81 \text{ m s}^{-2}) = 1.85 \times 10^{-8} \text{ m s}^{-1}$$

From Eq. (11.22):

$$Q = 2 u_g \Sigma = 2 \, (1.85 \times 10^{-8} \text{ m s}^{-1}) \, 4500.7 \text{ m}^2 = 1.67 \times 10^{-4} \text{ m}^3 \text{ s}^{-1}$$

Converting to units of l h^{-1}:

$$Q = 1.67 \times 10^{-4} \text{ m}^3 \text{ s}^{-1} . \left| \frac{1000 \text{ l}}{1 \text{ m}^3} \right| . \left| \frac{3600 \text{ s}}{1 \text{ h}} \right| = 601.2 \text{ l h}^{-1}$$

Answer: 601 l h^{-1}

(b)

If the particle density is 0.98×1060 kg m^{-3} $= 1038.8$ kg m^{-3}, from Eq. (11.19):

$$u_g = \frac{(1038.8 - 1025) \text{ kg m}^{-3}}{18 \, (1.25 \times 10^{-3} \text{ kg m}^{-1} \text{ s}^{-1})} \, (1.1 \times 10^{-6} \text{ m})^2 \, (9.81 \text{ m s}^{-2}) = 7.28 \times 10^{-9} \text{ m s}^{-1}$$

From Eq. (11.22):

$$Q = 2 u_g \Sigma = 2 \, (7.28 \times 10^{-9} \text{ m s}^{-1}) \, 4500.7 \text{ m}^2 = 6.55 \times 10^{-5} \text{ m}^3 \text{ s}^{-1}$$

Converting units to l h^{-1}:

$$Q = 6.55 \times 10^{-5} \text{ m}^3 \text{ s}^{-1} . \left| \frac{1000 \text{ l}}{1 \text{ m}^3} \right| . \left| \frac{3600 \text{ s}}{1 \text{ h}} \right| = 235.8 \text{ l h}^{-1}$$

This result is (235.8 l h^{-1})/(601.2 l h^{-1}) × 100% = 39% of that evaluated in **(a)**. Therefore, a 2% error in measurement of the particle density results in a 61% error in the estimate of the maximum centrifuge throughput.

Answer: The estimated maximum centrifuge throughput has an error of 61%. Predicting centrifuge performance depends strongly on accurate measurement of the particle and fluid densities.

11.13 Cell disruption

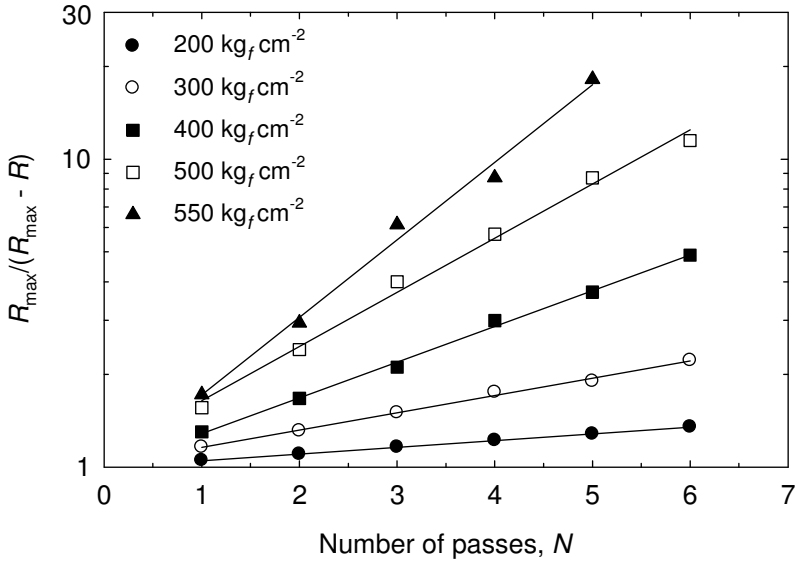
The relationship between pressure and protein release for a Manton-Gaulin homogeniser is given by Eq. (11.27). At constant pressure, a semi-log plot of $R_{max}/(R_{max} - R)$ versus N should yield a straight line, so that the value of kp^{α} can be determined from the slope.

The data for % protein release represent values of $(R/R_{max}) \times 100$. These data can be converted to $R_{max}/(R_{max} - R)$ as follows:

$$\frac{R_{max}}{R_{max} - R} = \frac{1}{1 - R/R_{max}} = \frac{100}{100 - (R/R_{max}) \times 100} = \frac{100}{100 - \%\ \text{protein release}} \tag{1}$$

The results are listed and plotted below.

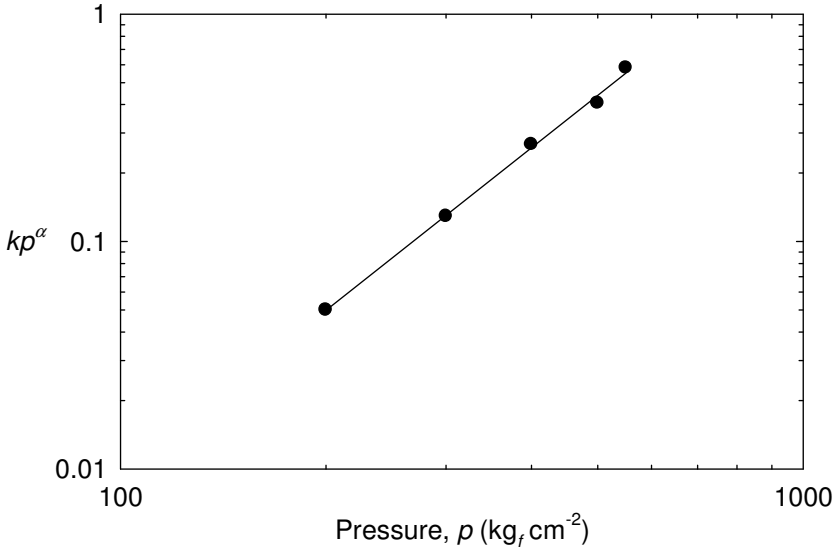
	Pressure ($kg_f\ cm^{-2}$)				
	200	300	400	500	550
N	$\frac{R_{max}}{R_{max} - R}$				
1	1.05	1.16	1.30	1.56	1.72
2	1.10	1.31	1.67	2.41	2.94
3	1.16	1.50	2.11	4.00	6.13
4	1.22	1.75	2.99	5.71	8.70
5	1.28	1.90	3.70	8.70	18.2
6	1.35	2.22	4.88	11.5	–



The values of kp^{α} obtained from the slopes of the straight lines for each pressure are listed below.

	Pressure ($kg_f\ cm^{-2}$)				
	200	300	400	500	550
kp^{α}	0.050	0.129	0.267	0.406	0.580

A log–log plot of these values versus p can be expected to give a straight line with the value of α obtained from the slope.



The equation to the straight line in the plot is $kp^{\alpha} = 1.71 \times 10^{-7}\, p^{2.37}$. Therefore $\alpha = 2.37$. For this system, Eq. (11.27) becomes:

$$\ln\left(\frac{R_{\max}}{R_{\max} - R}\right) = 1.71\times10^{-7}\, p^{2.37} N \tag{2}$$

where p has units of $kg_f\,cm^{-2}$.

(a)

From (1), for 80% protein release, $R_{\max}/(R_{\max} - R) = 5.0$. Rearranging (2) and using $p = 460$ $kg_f\,cm^{-2}$:

$$N = \frac{\ln\left(\frac{R_{\max}}{R_{\max} - R}\right)}{1.71\times10^{-7}\, p^{2.37}} = \frac{\ln 5.0}{1.71\times10^{-7}\,(460)^{2.37}} = 4.6$$

Therefore, 80% protein release is achieved within 5 passes through the homogeniser.

Answer: 5

(b)

From (1), for 70% protein release, $R_{\max}/(R_{\max} - R) = 3.33$. Rearranging (2) and substituting $N = 2$:

$$p^{2.37} = \frac{\ln\left(\frac{R_{\max}}{R_{\max} - R}\right)}{1.71\times10^{-7} N} = \frac{\ln 3.33}{1.71\times10^{-7}\,(2)} = 3.52\times10^{6}$$

Therefore:

$$p = 578\ kg_f\ cm^{-2}$$

Answer: 578 kg_f cm^{-2}. *An assumption in this calculation is that the homogenisation characteristics measured at pressures between 200 and 550 $kg_f\,cm^{-2}$ apply at the higher pressure of 578 $kg_f\,cm^{-2}$.*

11.14 Disruption of cells cultured under different conditions

The relationship between pressure and protein release for a Manton-Gaulin homogeniser is given by Eq. (11.27). The measured data for % protein release represent values of $(R/R_{max}) \times 100$. Rewriting Eq. (11.27) in terms of R/R_{max} after applying mathematical rule Eq. (E.10) from Appendix E:

$$-\ln\left(\frac{R_{max}-R}{R_{max}}\right) = kNp^{\alpha}$$

or

$$-\ln\left(1-\frac{R}{R_{max}}\right) = kNp^{\alpha} \tag{1}$$

Results for $-\ln(1 - R/R_{max})$ calculated from the measured data are listed below.

	Pressure (MPa)	
	57	89
Type of culture	$-\ln\left(1-\frac{R}{R_{max}}\right)$	
Repeated batch	1.079	1.833
Continuous	0.357	0.799

(a)

Applying (1) to cells from the repeated batch culture treated at 57 MPa with $N = 1$:

$$1.079 = k\,(1)\,(57)^{\alpha}$$

Therefore:

$$k = \frac{1.079}{(57)^{\alpha}} \tag{2}$$

Applying this result and (1) to cells from repeated batch culture treated at 89 MPa with $N = 1$:

$$1.833 = \frac{1.079}{(57)^{\alpha}}\,(1)\,(89)^{\alpha}$$

$$1.699 = \frac{(89)^{\alpha}}{(57)^{\alpha}} = (1.561)^{\alpha}$$

Solving for α by taking the logarithm of both sides and applying Eq. (E.11) from Appendix E:

$$\ln 1.699 = \alpha\,(\ln 1.561)$$

$$\alpha = \frac{\ln 1.699}{\ln 1.561} = 1.190$$

Applying this result in (2) gives:

$$k = \frac{1.079}{(57)^{1.190}} = 8.78\times10^{-3}$$

Applying the calculated values for k and α in (1) gives an empirical equation for cells from the repeated batch culture:

$$-\ln\left(1-\frac{R}{R_{\max}}\right)=8.78\times10^{-3}N\,p^{1.190}$$

where p has units of MPa. We can use this equation to determine the N required to achieve 90% protein release or $R/R_{\max} = 0.9$ at $p = 70$ MPa:

$$-\ln\left(1-0.9\right)=8.78\times10^{-3}N(70^{1.190})$$

$$N=1.67$$

Therefore, 90% protein release is achieved at 70 MPa within 2 passes through the homogeniser.

Answer: 2

(b)

Applying (1) to cells from the continuous culture treated at 57 MPa with $N = 1$:

$$0.357=k\,(57)^{\alpha}$$

Therefore:

$$k=\frac{0.357}{(57)^{\alpha}} \tag{3}$$

Applying this result and (1) to cells from the continuous culture treated at 89 MPa with $N = 1$:

$$0.799=\frac{0.357}{(57)^{\alpha}}(89)^{\alpha}$$

$$2.238=\frac{(89)^{\alpha}}{(57)^{\alpha}}=(1.561)^{\alpha}$$

Solving for α by taking the logarithm of both sides and applying Eq. (E.11) from Appendix E:

$$\ln 2.238=\alpha\,(\ln 1.561)$$

$$\alpha=\frac{\ln 2.238}{\ln 1.561}=1.809$$

Applying this result in (3) gives:

$$k=\frac{0.357}{(57)^{1.809}}=2.38\times10^{-4}$$

Therefore, the empirical equation for cells from the continuous culture is:

$$-\ln\left(1-\frac{R}{R_{\max}}\right)=2.38\times10^{-4}N\,p^{1.809} \tag{4}$$

where p has units of MPa. From the result in **(a)**, $N = 2$. Therefore, at $p = 70$ MPa for cells from the continuous culture:

$$-\ln\left(1-\frac{R}{R_{\max}}\right)=2.38\times10^{-4}\,(2)\,(70^{1.809})$$

$$\ln\left(1-\frac{R}{R_{\max}}\right)=-1.036$$

Solving this equation by raising both sides to the power e and applying Eq. (E.4) from Appendix E:

$$1-\frac{R}{R_{\max}}=e^{-1.036}=0.355$$

$$\frac{R}{R_{\max}}=1-0.355=0.645$$

Therefore, using 2 passes through the homogeniser at 70 MPa, the percentage protein release from the continuous-culture cells is 64.5%.

Answer: 65%

(c)

The empirical equation for cells from the continuous culture is (4), where p has units of MPa. If $R/R_{\max}$ = 0.9 at N = 5:

$$-\ln\left(1-0.9\right)=2.38\times10^{-4}\,(5)\,p^{1.809}$$

$$p^{1.809}=1934.95$$

$$p=65.6\text{ MPa}$$

Answer: 66 MPa

11.15 Enzyme purification using two-phase aqueous partitioning

(a)

From Eq. (11.28), for K = 3.5, the product partitions into the upper phase. Eq. (11.31) with K = 3.5 and Y_{Au} = 0.8 is:

$$0.8=\frac{V_{\text{u}}}{V_{\text{u}}+\dfrac{V_{\text{l}}}{3.5}}$$

Rearranging gives:

$$0.8V_{\text{u}}+0.23V_{\text{l}}=V_{\text{u}}$$

$$0.23V_{\text{l}}=0.2V_{\text{u}}$$

$$\frac{V_{\text{u}}}{V_{\text{l}}}=\frac{0.23}{0.2}=1.15$$

Answer: 1.15

(b)

The mass of enzyme in the two phases must be equal to the mass of enzyme in the original homogenate. The mass balance equation is:

$$C_{\text{Au}}V_{\text{u}}+C_{\text{Al}}V_{\text{l}}=C_{\text{A0}}V_0$$

From Eq. (11.28), for K = 3.5, C_{Au} = 3.5C_{Al}. From **(a)**, if V_{l} = 100 l, for 80% recovery, V_{u} = 115 l. Substituting these values into the mass balance equation with V_0 = 150 l and C_{A0} = 3.2 u ml^{-1} = 3.2 × 10^3 u l^{-1}:

$$3.5\,C_{\text{Al}}\,(115\text{ l})+C_{\text{Al}}\,(100\text{ l})=3.2\times10^{3}\text{ u l}^{-1}\,(150\text{ l})$$

Solving for C_{Al}:

$$502.5\,C_{Al} = 4.8\times10^{5}$$

$$C_{Al} = 955 \text{ u l}^{-1}$$

As $C_{Au} = 3.5C_{Al}$, $C_{Au} = 3.34 \times 10^3$ u l^{-1}. From Eq. (11.34), the concentration factor for product partitioning into the upper phase is:

$$\delta_c = \frac{3.34\times10^{3} \text{ u l}^{-1}}{3.2\times10^{3} \text{ u l}^{-1}} = 1.04$$

Answer: 1.04

11.16 Recovery of viral particles

From Eq. (11.28), for $K = 10^{-2}$, $C_{Au} = 10^{-2}C_{Al}$ and the product partitions into the lower phase. If 5 l of culture volume is added to 2 l of polymer solution and, after phase separation, the volume of the lower phase is 1 l, the volume of the upper phase must be ((5 + 2) – 1) l = 6 l.

(a)

The yield of virus in the lower phase is given by Eq. (11.32) with $V_l = 1$ l, $V_u = 6$ l and $K = 10^{-2}$:

$$Y_{Al} = \frac{11}{(6\,\text{l})\,(10^{-2}) + 11} = 0.94$$

Answer: 0.94

(b)

The mass of viral particles in the two phases must be equal to the mass of viral particles in the original culture broth. The mass balance equation is:

$$C_{Au}V_u + C_{Al}V_l = C_{A0}V_0$$

Answer: $C_{Au}V_u + C_{Al}V_l = C_{A0}V_0$

(c)

From Eq. (11.28):

$$C_{Al} = \frac{C_{Au}}{K}$$

Rearranging the mass balance equation in **(b)** and substituting for C_{Al} gives:

$$C_{A0} = \frac{C_{Au}V_u + C_{Al}V_l}{V_0} = \frac{C_{Au}V_u + \dfrac{C_{Au}}{K}V_l}{V_0}$$

The concentration factor for product partitioning into the lower phase is given by Eq. (11.33). Substituting expressions for C_{Al} and C_{A0} into this equation:

$$\delta_c = \frac{\dfrac{C_{Au}}{K}}{\left(\dfrac{C_{Au}V_u + \dfrac{C_{Au}}{K}V_l}{V_0}\right)}$$

Cancelling terms and rearranging gives:

$$\delta_c = \frac{\frac{V_0}{K}}{V_u + \frac{V_l}{K}} = \frac{V_0}{V_u K + V_l}$$

Answer: $\delta_c = \dfrac{V_0}{V_u K + V_l}$

(d)

Using the equation for δ_c derived in **(c)** with $V_0 = 5$ l, $V_l = 1$ l, $V_u = 6$ l and $K = 10^{-2}$:

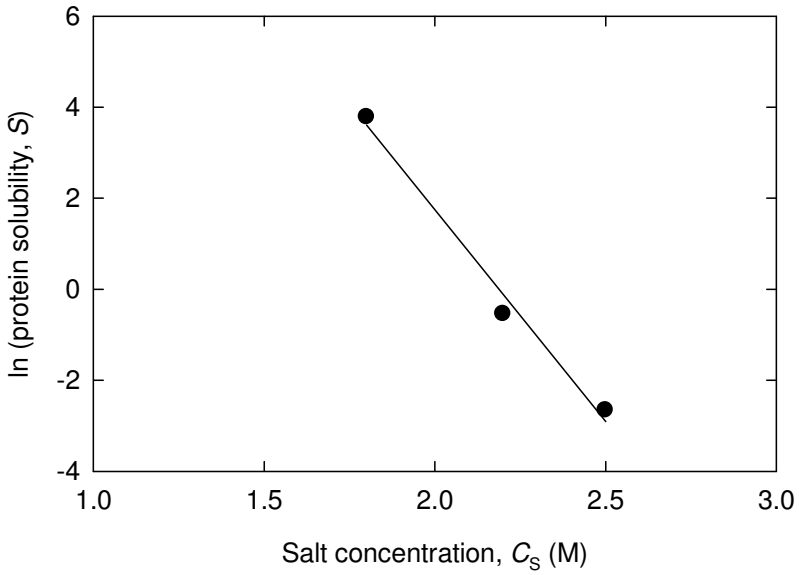
$$\delta_c = \frac{5\,\text{l}}{(6\,\text{l})\,(10^{-2}) + 1\,\text{l}} = 4.7$$

Answer: 4.7

11.17 Enzyme salting-out

The relationship between protein solubility S and salt concentration C_S is given by Eq. (11.37). According to this equation, a plot of ln S versus C_S should give a straight line. The measured solubility results are listed and plotted below.

Salt concentration, C_S (M)	Protein solubility, S (mg l^{-1})	ln S
1.8	44	3.784
2.2	0.58	–0.545
2.5	0.07	–2.659



The equation to the straight line through the data is:

$$\ln S = 20.33 - 9.292\,C_S \tag{1}$$

where C_S has units of M and S has units of mg l^{-1}. If a salt concentration of 2 M is applied, from the equation, ln S = 1.746. Applying Eq. (E.4) from Appendix E, S = 5.73 mg l^{-1}.

(a)

The initial mass of cellulase treated is (25 mg l^{-1}) × 800 l = 20,000 mg. Using a salt concentration of 2 M, S = 5.73 mg l^{-1} and the mass of cellulase remaining in the solution is (5.73 mg l^{-1}) × 930 l = 5328.9 mg. Therefore, the mass of cellulase recovered in the precipitate is (20,000 – 5328.9) mg = 14,671 mg = 14.67 g.

Answer: 14.7 g

(b)

The percentage recovery is:

$$\frac{14,671\text{ mg}}{20,000\text{ mg}} \times 100\% = 73.4\%$$

Answer: 73%

(c)

The mass of residual cellulase in solution is 5328.9 mg. If 90% of this material is recovered in the second treatment, the mass of cellulase remaining in solution after the second treatment is 0.1 × 5328.9 mg = 532.9 mg. Therefore, the solubility S after the second treatment is (532.9 mg)/(930 l) = 0.573 mg l^{-1}. Applying this result in (1) and solving for C_S:

$$\ln 0.573 = 20.33 - 9.292 C_S$$

$$C_S = \frac{20.33 - \ln 0.573}{9.292} = 2.25$$

Therefore, the salt concentration required is 2.25 M.

Answer: 2.25 M

11.18 Precipitation of monoclonal antibody

(a)

Almost all the antibody is recovered using 12% w/v polyethylene glycol; however, the precipitate is relatively impure as other substances co-precipitate with the desired product. Accordingly, considerable further processing of the precipitate is likely to be required to increase the product purity. In contrast, 8% w/v polyethylene glycol gives good precipitate purity and product selectivity, but significantly less antibody is recovered compared with that obtained using 12% w/v. Further precipitation steps would be required using 8% w/v polyethylene glycol to recover 95–100% of the product.

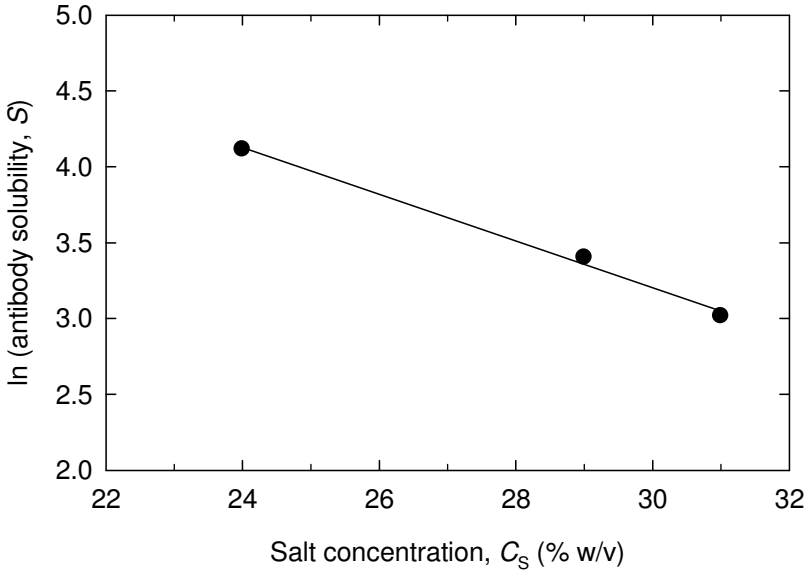
(b)

The relationship between protein solubility S and salt concentration C_S is given by Eq. (11.37). According to this equation, a plot of ln S versus C_S should give a straight line. As the density of the cell-free broth = 1000 kg m^{-3} = 1 g cm^{-3} = 1 g ml^{-1}, the initial antibody concentration of 120 μg ml^{-1} = 120 μg g^{-1}. The solubility of antibody in solution is obtained from the measurements of antibody yield in the precipitate:

$$S = \frac{100 - \%\text{ yield}}{100} \times 120\ \mu\text{g g}^{-1} \tag{1}$$

Applying this equation, results for antibody solubility as a function of ammonium sulphate concentration are listed and plotted below.

Salt concentration, C_S (% w/v)	Antibody solubility, S (μg g^{-1})	ln S
24	61.2	4.114
29	30.0	3.401
31	20.4	3.016



The equation to the straight line through the data is:

$$\ln S = 7.827 - 0.1542 C_S \tag{2}$$

where C_S has units of % w/v and S has units of μg g^{-1}.

Answer: $\ln S = 7.827 - 0.1542 C_S$

(c)

If a salt concentration of 27% w/v is used, from (2), ln S = 3.664. Applying Eq. (E.4) from Appendix E, S = 39.0 μg g^{-1}. Therefore, the antibody yield in the precipitate is:

$$\text{Yield} = \frac{(120 - 39.0)\ \mu\text{g g}^{-1}}{120\ \mu\text{g g}^{-1}} \times 100\% = 67.5\%$$

Answer: 67.5%

(d)

From the definition of density (Section 2.4.1), the mass of cell-free broth is equal to the volume × density. As the density = 1000 kg m^{-3} = 1000 g l^{-1}, the mass of cell-free broth is 100 l × 1000 g l^{-1} = 10^5 g. For S = 39.0 μg g^{-1}, the mass of antibody remaining in solution is 39.0 μg g^{-1} × 10^5 g = 3.9 × 10^6 μg = 3.9 g.

Answer: 3.9 g

(e)

(i)

Calculate S corresponding to a yield of 94% using (1):

$$S = \frac{100 - 94}{100} \times 120\ \mu g\ g^{-1} = 7.2\ \mu g\ g^{-1}$$

Applying this value of S in (2) and solving for C_S:

$$\ln 7.2 = 7.827 - 0.1542 C_S$$

$$C_S = \frac{7.827 - \ln 7.2}{0.1542} = 38.0$$

Therefore, the salt concentration required is 38.0% w/v.

Answer: 38% w/v

(ii)

From the calculations in **(d)** and **(e) (i)**, the mass of antibody remaining in solution after the second treatment step is 7.2 μg g^{-1} × 10^5 g = 7.2 × 10^5 μg = 0.72 g. As the mass of antibody in solution before the second treatment was 3.9 g, the mass of antibody recovered is (3.9 – 0.72) g = 3.18 g.

Answer: 3.2 g.

(iii)

The salt concentration of 38% w/v determined in **(e) (i)** is outside the range of 24–31% w/v used to derive the empirical equation for this salting-out process. This may be responsible for some deviation between the predicted and experimental antibody recoveries. In addition, solubility in protein mixtures such as culture broth is generally less than that predicted by the Cohn relationship, due to co-precipitation of other proteins (Section 11.8.2, Salting-Out subsection). If the antibody solubility is lower than that predicted by the equation, this would explain the higher than expected antibody content in the precipitate.

11.19 Enzyme ultrafiltration

C_0 = 0.1% w/v; C_R = 3% w/v; F_0 = 2500 l h^{-1}; C_P = 0; C_G = 45% w/v.

(a)

The solute mass balance equation for continuous membrane filtration is Eq. (11.99). When C_P = 0, this equation becomes:

$$F_0 C_0 = F_R C_R$$

or

$$\frac{F_0}{F_R} = \frac{C_R}{C_0}$$

Substituting values and solving for F_R:

$$\frac{2500\ l\ h^{-1}}{F_R} = \frac{3\%\ w/v}{0.1\%\ w/v} = 30$$

$$F_R = \frac{2500\ l\ h^{-1}}{30} = 83.31\ h^{-1}$$

From Eq. (11.98):

$$F_P = F_0 - F_R$$

Substituting values gives:

$$F_P = 2500\,\text{l h}^{-1} - 83.31\,\text{l h}^{-1} = 2416.71\,\text{l h}^{-1}$$

Answer: 2420 l h^{-1}

(b)

The steady-state value of the permeate flux J is calculated using Eq. (11.62). Because, for well-mixed operation, $C_B = C_R$ (Section 11.10.6, Continuous subsection), Eq. (11.62) becomes:

$$J = k \ln\left(\frac{C_G}{C_R}\right)$$

Substituting values for the hollow fibre unit with $k = 10\,\text{l m}^{-2}\,\text{h}^{-1}$:

$$J = (10\,\text{l m}^{-2}\,\text{h}^{-1}) \ln\left(\frac{45\%\ \text{w/v}}{3\%\ \text{w/v}}\right) = 27.11\,\text{l m}^{-2}\,\text{h}^{-1}$$

Similarly, for the plate-and-frame device with $k = 6.5\,\text{l m}^{-2}\,\text{h}^{-1}$:

$$J = (6.5\,\text{l m}^{-2}\,\text{h}^{-1}) \ln\left(\frac{45\%\ \text{w/v}}{3\%\ \text{w/v}}\right) = 17.6\,\text{l m}^{-2}\,\text{h}^{-1}$$

Answer: 27.1 l m^{-2} h^{-1} for the hollow fibre unit; 17.6 l m^{-2} h^{-1} for the plate-and-frame device

(c)

The membrane area is found from Eq. (11.48):

$$A = \frac{F_P}{J}$$

Using the result for F_P from **(a)**, for the hollow fibre unit:

$$A = \frac{2416.71\,\text{l h}^{-1}}{27.11\,\text{l m}^{-2}\,\text{h}^{-1}} = 89.2\,\text{m}^2$$

For the plate-and-frame device:

$$A = \frac{2416.71\,\text{l h}^{-1}}{17.6\,\text{l m}^{-2}\,\text{h}^{-1}} = 137.3\,\text{m}^2$$

Answer: 89 m^2 for the hollow fibre unit; 137 m^2 for the plate-and-frame device

11.20 Antibody concentration with membrane fouling

C_0 = 150 mg l^{-1}; C_R = 10 × 150 mg l^{-1} = 1500 mg l^{-1}. For well-mixed operation (Section 11.10.6, Continuous subsection), $C_B = C_R$ = 1500 mg l^{-1}. p_i = 450 kPa; p_o = 200 kPa. p_P = 1 atm; converting units using Table A.5 (Appendix A), $p_P = 1.013 \times 10^5$ Pa = 101.3 kPa. From Eq. (11.52), the average pressure on the feed side of the membrane is:

$$p_F = \frac{450\,\text{kPa} + 200\,\text{kPa}}{2} = 325\,\text{kPa}$$

Therefore, from Eq. (11.49), the transmembrane pressure difference is:

$$\Delta p = 325\,\text{kPa} - 101.3\,\text{kPa} = 223.7\,\text{kPa}$$

R_M = 0.24 kPa m^2 h l^{-1}. Applying the equation provided for the resistance due to concentration polarisation:

$$R_C = 4.5 + 4.33\times10^{-3}(1500) = 11.0$$

Therefore, R_C = 11.0 kPa m^2 h l^{-1}.

(a)

From Eq. (11.56), the water flux is:

$$J_{water} = \frac{\Delta p}{R_M}$$

Substituting values gives:

$$J_{water} = \frac{223.7\ \text{kPa}}{0.24\ \text{kPa m}^2\ \text{h l}^{-1}} = 932\ \text{l m}^{-2}\ \text{h}^{-1}$$

This can be compared with the permeate flux given by Eq. (11.55):

$$J = \frac{223.7\ \text{kPa}}{(0.24+11.0)\ \text{kPa m}^2\ \text{h l}^{-1}} = 19.9\ \text{l m}^{-2}\ \text{h}^{-1}$$

The percentage reduction in flux due to concentration polarisation is:

$$\frac{(932-19.9)\ \text{l m}^{-2}\ \text{h}^{-1}}{932\ \text{l m}^{-2}\ \text{h}^{-1}}\times100\% = 98\%$$

Answer: 98%

(b)

R_F = 5.3 kPa m^2 h l^{-1}. The flux with fouling is evaluated using Eq. (11.67):

$$J = \frac{223.7\ \text{kPa}}{(0.24+11.0+5.3)\ \text{kPa m}^2\ \text{h l}^{-1}} = 13.5\ \text{l m}^{-2}\ \text{h}^{-1}$$

Relative to the permeate flux without fouling determined in **(a)**, the percentage reduction in flux after fouling is:

$$\frac{(19.9-13.5)\ \text{l m}^{-2}\ \text{h}^{-1}}{19.9\ \text{l m}^{-2}\ \text{h}^{-1}}\times100\% = 32\%$$

Answer: 32%

11.21 Clarification of wine

C_0 = 7.5 g l^{-1}. Using a basis of V_0 = 1 m^3, V_R = 0.05 m^3. C_P = 0; therefore, from Eq. (11.47), R = 1.

(a)

For concentration operations with R = 1, from Eq. (11.74):

$$C_R = \frac{7.5\ \text{g l}^{-1}\ (1\ \text{m}^3)}{0.05\ \text{m}^3} = 150\ \text{g l}^{-1}$$

Answer: 150 g l^{-1}

(b)

The total volume of filtered wine or permeate generated per year is:

$$\text{Volume of filtered wine per year} = 1500 \times 12 \times 750 \text{ ml} \left| \frac{1\,\text{l}}{1000 \text{ ml}} \right| \left| \frac{1 \text{ m}^3}{1000\,\text{l}} \right| = 13.5 \text{ m}^3$$

From Eq. (11.68):

$$V_\text{P} = V_0 - V_\text{R}$$

so that the volume of filtered wine produced for every m^3 of unfiltered wine is:

$$V_\text{P} = 1 \text{ m}^3 - 0.05 \text{ m}^3 = 0.95 \text{ m}^3$$

Therefore, the volume of unfiltered wine that must be processed per year is:

$$\text{Volume of unfiltered wine per year} = \frac{13.5 \text{ m}^3}{0.95 \text{ m}^3 \text{ m}^{-3}} = 14.2 \text{ m}^3$$

The difference between the total volumes of filtered and unfiltered wine is the volume of concentrated cell retentate produced per year:

$$V_\text{R} = V_0 - V_\text{P} = 14.2 \text{ m}^3 - 13.5 \text{ m}^3 = 0.7 \text{ m}^3$$

Converting units using Table A.2 (Appendix A):

$$V_\text{R} = 0.7 \text{ m}^3 \left| \frac{10^3\,\text{l}}{1 \text{ m}^3} \right| = 700\,\text{l}$$

Answer: 700 l

11.22 Fractionation of cell homogenate

For the cell debris, $C_0 = 5\%$ w/w, $C_\text{R} = 5 \times C_0 = 25\%$ w/w, $R = 0.95$. For the proinsulin, $C_0 = 8\%$ w/w and $R = 0.90$. For the cleaved proteins, $C_0 = 5\%$ w/w and $R = 0.25$. For the sugar, $C_0 = 0.5\%$ w/w and $R = 0$.

(a)

For concentration operations, from Eq. (11.72):

$$(VCR)^R = \frac{C_\text{R}}{C_0}$$

Applying this equation to the cell debris:

$$(VCR)^{0.95} = \frac{25\% \text{ w/w}}{5\% \text{ w/w}} = 5$$

$$VCR = 5.44$$

This volume concentration ratio applies to the other components as well. Applying Eq. (11.72) to the proinsulin:

$$C_\text{R} = 8\% \text{ w/w } (5.44)^{0.90} = 36.74\% \text{ w/w}$$

Similarly, for the cleaved proteins:

$$C_\text{R} = 5\% \text{ w/w } (5.44)^{0.25} = 7.64\% \text{ w/w}$$

and sugar:

$$C_R = 0.5\%\text{ w/w } (5.44)^0 = 0.5\%\text{ w/w}$$

Answer: The composition of the retentate is 25% w/w cell debris, 37% w/w proinsulin, 7.6% w/w cleaved proteins and 0.5% w/w sugar.

(b)

The mass balance equations for this process are Eqs (11.68) and (11.69). Rearranging Eq. (11.68) gives:

$$1 = \frac{V_R}{V_0} + \frac{V_P}{V_0}$$

Incorporating the definition of *VCR* from Eq. (11.70):

$$1 = \frac{1}{VCR} + \frac{V_P}{V_0}$$

Substituting the result for *VCR* from **(a)**:

$$1 = \frac{1}{5.44} + \frac{V_P}{V_0}$$

Therefore:

$$\frac{V_P}{V_0} = 0.82$$

Rearranging Eq. (11.69) gives:

$$C_0 = \frac{V_R}{V_0} C_R + \frac{V_P}{V_0} C_P$$

$$C_0 = \frac{1}{VCR} C_R + \frac{V_P}{V_0} C_P$$

Substituting values for *VCR* and V_P/V_0:

$$C_0 = 0.18\,C_R + 0.82\,C_P$$

and solving for C_P:

$$C_P = \frac{C_0 - 0.18\,C_R}{0.82}$$

This equation can be applied to each component of the homogenate using the results from **(a)**. For the cell debris:

$$C_P = \frac{5\%\text{ w/w} - 0.18\ (25\%\text{ w/w})}{0.82} = 0.61\%\text{ w/w}$$

For the proinsulin:

$$C_P = \frac{8\%\text{ w/w} - 0.18\ (36.74\%\text{ w/w})}{0.82} = 1.69\%\text{ w/w}$$

For the cleaved proteins:

$$C_P = \frac{5\%\text{ w/w} - 0.18\ (7.64\%\text{ w/w})}{0.82} = 4.42\%\text{ w/w}$$

and sugar:

$$C_P = \frac{0.5\%\ \text{w/w} - 0.18\ (0.5\%\ \text{w/w})}{0.82} = 0.5\%\ \text{w/w}$$

Answer: The composition of the permeate is 0.61% w/w cell debris, 1.7% w/w proinsulin, 4.4% w/w cleaved proteins and 0.5% w/w sugar.

11.23 Ultrafiltration pressure

The membrane area per module = 58 m^2; therefore, the total filtration area $A = 3 \times 58\ m^2 = 174\ m^2$. $b = 3$ mm $= 3 \times 10^{-3}$ m; therefore, from Section 11.10.2 (Mass Transfer Model subsection), $d_h = 2b = 6 \times 10^{-3}$ m. $\mu = 1.0$ cP; therefore, from Table A.9 (Appendix A), $\mu = 10^{-3}$ kg m^{-1} s^{-1}. $\rho = 1000$ kg m^{-3}. Converting units for $\mathscr{D}$:

$$\mathscr{D} = 3.3 \times 10^{-7}\ \text{cm}^2\ \text{s}^{-1} . \left| \frac{1\ \text{m}}{100\ \text{cm}} \right|^2 = 3.3 \times 10^{-11}\ \text{m}^2\ \text{s}^{-1}$$

$C_R = 8$ g l^{-1}. For well-mixed operation, $C_B = C_R$ (Section 11.10.6, Continuous subsection); therefore, $C_B = 8$ g l^{-1}. $C_G = 300$ g l^{-1}. $u = 65$ cm $s^{-1} = 65 \times 10^{-2}$ m s^{-1}. $F_P = 8.7$ m^3 h^{-1}.

The mass transfer coefficient k in the plate-and-frame modules is determined using the empirical correlation, Eq. (11.66). *Re* is given by Eq. (11.64):

$$Re = \frac{6 \times 10^{-3}\ \text{m}\ (65 \times 10^{-2}\ \text{m s}^{-1})\ 1000\ \text{kg m}^{-3}}{10^{-3}\ \text{kg m}^{-1}\ \text{s}^{-1}} = 3900$$

From Table 11.5 for open channels without mesh spacers, at this *Re* the parameters in Eq. (11.66) are $M = 0.023$, $\alpha = 0.89$, $\beta = 0.3$ and $\omega = 0$. *Sc* is given by Eq. (11.65):

$$Sc = \frac{10^{-3}\ \text{kg m}^{-1}\ \text{s}^{-1}}{1000\ \text{kg m}^{-3}\ (3.3 \times 10^{-11}\ \text{m}^2\ \text{s}^{-1})} = 3.03 \times 10^4$$

Substituting values into Eq. (11.66) with $\omega = 0$:

$$Sh = 0.023\ (3900)^{0.89}\ (3.03 \times 10^4)^{0.3} = 798.4$$

From the definition of *Sh* in Eq. (11.63):

$$k = \frac{Sh\,\mathscr{D}}{d_h} = \frac{798.4\ (3.3 \times 10^{-11}\ \text{m}^2\ \text{s}^{-1})}{6 \times 10^{-3}\ \text{m}} = 4.4 \times 10^{-6}\ \text{m s}^{-1}$$

The permeate flux under gel polarisation conditions is determined using Eq. (11.62):

$$J = (4.4 \times 10^{-6}\ \text{m s}^{-1}) \ln \frac{300\ \text{g l}^{-1}}{8\ \text{g l}^{-1}} = 1.59 \times 10^{-5}\ \text{m s}^{-1}$$

This can be compared with the actual operating flux evaluated using Eq. (11.48):

$$J = \frac{8.7\ \text{m}^3\ \text{h}^{-1}}{174\ \text{m}^2} . \left| \frac{1\ \text{h}}{3600\ \text{s}} \right| = 1.39 \times 10^{-5}\ \text{m s}^{-1}$$

As the operating flux is less than that corresponding to gel polarisation, increasing the operating pressure drop can be expected to improve the rate of filtration.

Answer: Yes

11.24 Washing thawed red blood cells

For the glycerol, $C_0 = 1.7$ M, $C_R = 0.1$ M and $R = 0$. For the haemoglobin, $C_0 = 5.5$ g l^{-1} and $R = 0$. $V_0 = 500$ ml.

(a)

Applying Eq. (11.88) for diafiltration of glycerol with $R = 0$:

$$0.1\ \text{M} = (1.7\ \text{M})\, e^{-V_\text{D}/V_0}$$

$$0.0588 = e^{-V_\text{D}/V_0}$$

Taking the logarithm of both sides and applying Eq. (E.3) from Appendix E:

$$\ln 0.0588 = \frac{-V_\text{D}}{V_0}$$

$$-2.834 = \frac{-V_\text{D}}{500\ \text{ml}}$$

$$V_\text{D} = 1417\ \text{ml} = 1.42\ \text{l}$$

Answer: 1.42 l

(b)

Applying Eq. (11.88) to haemoglobin using the result for V_D from **(a)**:

$$C_\text{R} = (5.5\ \text{g l}^{-1})\, e^{(-1417\ \text{ml})/(500\ \text{ml})} = 0.32\ \text{g l}^{-1}$$

Answer: 0.32 g l^{-1}

(c)

Applying Eq. (11.88) to glycerol and haemoglobin using $V_\text{D} = 500$ ml:

$$C_\text{R}\ \text{glycerol} = (1.7\ \text{M})\, e^{(-500\ \text{ml})/(500\ \text{ml})} = 0.625\ \text{M}$$

$$C_\text{R}\ \text{haemoglobin} = (5.5\ \text{g l}^{-1})\, e^{(-500\ \text{ml})/(500\ \text{ml})} = 2.02\ \text{g l}^{-1}$$

Answer: 0.63 M glycerol, 2.0 g l^{-1} haemoglobin

11.25 Diafiltration of recombinant protein

For the recombinant protein, $R = 1$ and $C_\text{G} = 120$ mg l^{-1}. For the salt, $C_\text{R} = 0.1$ g l^{-1} and $R = 0$. $A = 2.5\ \text{m}^2$. $k = 1.4 \times 10^{-5}$ m s^{-1}.

(a)

$V_0 = 150$ l. For the recombinant protein, $C_0 = 18$ mg l^{-1}; for the salt, $C_0 = 50$ g l^{-1}. From Eq. (11.88) for diafiltration of salt with $R = 0$:

$$0.1\ \text{g l}^{-1} = (50\ \text{g l}^{-1})\, e^{-V_\text{D}/V_0}$$

$$0.002 = e^{-V_\text{D}/V_0}$$

Taking the logarithm of both sides and applying Eq. (E.3) from Appendix E:

$$\ln 0.002 = \frac{-V_\text{D}}{V_0}$$

$$-6.215 = \frac{-V_\text{D}}{150\ \text{l}}$$

$$V_D = 932.3\,\text{l}$$

Answer: 932 l

(b)

From Eq. (11.87), for solute such as recombinant protein with $R = 1$, $C_R = C_0$ for constant-volume diafiltration. For well-mixed operation, $C_B = C_R$ (Section 11.10.6, Batch subsection); therefore, for the recombinant protein, $C_B = C_0 = 18$ mg l^{-1}. The permeate flux under gel polarisation conditions is determined using Eq. (11.62):

$$J = (1.4\times10^{-5}\ \text{m s}^{-1}) \ln\frac{120\ \text{mg l}^{-1}}{18\ \text{mg l}^{-1}} = 2.66\times10^{-5}\ \text{m s}^{-1}$$

The permeate flow rate is calculated from Eq. (11.48):

$$F_P = JA$$

Substituting values gives:

$$F_P = 2.66\times10^{-5}\ \text{m s}^{-1}\,(2.5\ \text{m}^2) = 6.65\times10^{-5}\ \text{m}^3\ \text{s}^{-1}$$

Converting units from Table A.2 (Appendix A):

$$F_P = 6.65\times10^{-5}\ \text{m}^3\ \text{s}^{-1} . \left|\frac{10^3\,\text{l}}{1\ \text{m}^3}\right| = 6.65\times10^{-2}\ \text{l s}^{-1}$$

For constant-volume diafiltration, the flow rate of diafiltration solvent F_D must be equal to the permeate flow rate; therefore $F_D = F_P = 6.65 \times 10^{-2}$ l s^{-1}. The time required for constant-volume diafiltration is equal to the volume of diafiltration solvent divided by the diafiltration solvent flow rate:

$$t = \frac{V_D}{F_D} = \frac{932.3\,\text{l}}{6.65\times10^{-2}\ \text{l s}^{-1}} . \left|\frac{1\ \text{h}}{3600\ \text{s}}\right| = 3.89\ \text{h}$$

Answer: 3.9 h

(c)

$V_0 = 30$ l. For the recombinant protein, $C_0 = 90$ mg l^{-1}; for the salt, $C_0 = 250$ g l^{-1}.

(i)

Applying Eq. (11.88) for diafiltration of salt:

$$0.1\ \text{g l}^{-1} = (250\ \text{g l}^{-1})\, e^{-V_D/V_0}$$

$$4.0\times10^{-4} = e^{-V_D/V_0}$$

Taking the logarithm of both sides and applying Eq. (E.3) from Appendix E:

$$\ln 4.0\times10^{-4} = \frac{-V_D}{V_0}$$

$$-7.824 = \frac{-V_D}{30\,\text{l}}$$

$$V_D = 234.7\,\text{l}$$

The reduction in volume compared with the result in **(a)** is:

$$\frac{932.31 - 234.71}{932.31} \times 100\% = 74.8\%$$

Answer: 75%

(ii)

The permeate flux is determined from Eq. (11.62) with $C_B = C_0$ for the recombinant protein:

$$J = (1.4\times10^{-5}\ \text{m s}^{-1}) \ln\frac{120\ \text{mg l}^{-1}}{90\ \text{mg l}^{-1}} = 4.03\times10^{-6}\ \text{m s}^{-1}$$

The permeate flow rate calculated using Eq. (11.48) is:

$$F_P = JA = 4.03\times10^{-6}\ \text{m s}^{-1} . \left|\frac{10^3\ \text{l}}{1\ \text{m}^3}\right| (2.5\ \text{m}^2) = 1.01\times10^{-2}\ \text{l s}^{-1}$$

The reduction in permeate flow rate compared with the result for F_P in **(b)** is:

$$\frac{(6.65\times10^{-2} - 1.01\times10^{-2})\ \text{l s}^{-1}}{6.65\times10^{-2}\ \text{l s}^{-1}} \times 100\% = 84.8\%$$

Answer: The rate of diafiltration is reduced by 85%.

(iii)

$F_D = F_P = 1.01 \times 10^{-2}$ l s^{-1}. The time required is:

$$t = \frac{V_D}{F_D} = \frac{234.71}{1.01\times10^{-2}\ \text{l s}^{-1}} . \left|\frac{1\ \text{h}}{3600\ \text{s}}\right| = 6.45\ \text{h}$$

Answer: 6.5 h

11.26 Time for batch ultrafiltration

$\mu = 1.2$ mPa s $= 1.2 \times 10^{-3}$ Pa s; therefore, from Table A.9 (Appendix A), $\mu = 1.2 \times 10^{-3}$ kg m^{-1} s^{-1}. $\rho = 1.0$ g cm^{-3} $= 1000$ kg m^{-3}. $A = 250$ m^2. $b = 2.5$ mm $= 2.5 \times 10^{-3}$ m; therefore, from Section 11.10.2 (Mass Transfer Model subsection), $d_h = 2b = 5 \times 10^{-3}$ m. $u = 45$ cm s^{-1} $= 45 \times 10^{-2}$ m s^{-1}. Converting units for $\mathscr{D}$:

$$\mathscr{D} = 11.5\times10^{-7}\ \text{cm}^2\ \text{s}^{-1} . \left|\frac{1\ \text{m}}{100\ \text{cm}}\right|^2 = 11.5\times10^{-11}\ \text{m}^2\ \text{s}^{-1}$$

For interleukin, $C_0 = 5$ u ml^{-1}, $R = 1$ and $C_R = 20 \times 5$ u ml^{-1} $= 100$ u ml^{-1}. For well-mixed operation, $C_B = C_R$ (Section 11.10.6, Batch subsection); therefore, $C_B = 100$ u ml^{-1}. $C_G = 2000$ u ml^{-1}. $V_0 = 15$ m^3. From Eq. (11.72), to achieve a concentration factor $C_R/C_0 = 20$ with $R = 1$, $VCR = 20$. Therefore, from Eq. (11.70):

$$V_R = \frac{V_0}{VCR} = \frac{15\ \text{m}^3}{20} = 0.75\ \text{m}^3$$

The mass transfer coefficient k in the spiral-wound membrane modules is determined using the empirical correlation, Eq. (11.66). From Table 11.5, for channels with mesh spacers, $M = 0.0096$, $\alpha = 0.5$, $\beta = 0.6$ and $\omega = 0$. Re is given by Eq. (11.64):

$$Re = \frac{5\times10^{-3}\ \text{m}\ (45\times10^{-2}\ \text{m s}^{-1})\ 1000\ \text{kg m}^{-3}}{1.2\times10^{-3}\ \text{kg m}^{-1}\ \text{s}^{-1}} = 1875$$

Sc is given by Eq. (11.65):

$$Sc = \frac{1.2\times10^{-3}\ \text{kg m}^{-1}\ \text{s}^{-1}}{1000\ \text{kg m}^{-3}\ (11.5\times10^{-11}\ \text{m}^2\ \text{s}^{-1})} = 1.04\times10^4$$

Substituting values into Eq. (11.66) with $\omega = 0$:

$$Sh = 0.0096\ (1875)^{0.5}\ (1.04\times10^4)^{0.6} = 106.9$$

From the definition of *Sh* in Eq. (11.63):

$$k = \frac{Sh\mathscr{D}}{d_\text{h}} = \frac{106.9\ (11.5\times10^{-11}\ \text{m}^2\ \text{s}^{-1})}{5\times10^{-3}\ \text{m}} = 2.46\times10^{-6}\ \text{m s}^{-1}$$

For batch membrane filtration, the dimensionless time $(Akt)/V_0$ is obtained using Figure 11.39. The dimensionless variables in this figure are $C_\text{G}/C_0 = (2000\ \text{u ml}^{-1})/(5\ \text{u ml}^{-1}) = 400$ and $V_\text{R}/V_0 = (0.75\ \text{m}^3)/(15\ \text{m}^3) = 0.05$. Interpolating Figure 11.39 with these parameter values, $(Akt)/V_0$ is approximately 0.2. Therefore:

$$t = \frac{0.2V_0}{Ak} = \frac{0.2\ (15\ \text{m}^3)}{250\ \text{m}^2\ (2.46\times10^{-6}\ \text{m s}^{-1})} = 4878\ \text{s}$$

Converting units:

$$t = 4878\ \text{s} \left| \frac{1\ \text{h}}{3600\ \text{s}} \right| = 1.36\ \text{h}$$

Answer: 1.4 h

11.27 Combined ultrafiltration and diafiltration

For somatotropin, $C_0 = 5\%$, $R = 1$ and $C_\text{G} = 50\%$. For the detergent, $C_0 = 1.2\%$ and $R = 0$.

(a)

(i)

$V_0 = 5000\ \text{l} = 5\ \text{m}^3$; $V_\text{R} = 1000\ \text{l} = 1\ \text{m}^3$. $k = 2.2 \times 10^{-6}\ \text{m s}^{-1}$. $t = 2$ h. For batch membrane filtration, the dimensionless time $(Akt)/V_0$ is obtained using Figure 11.39. The dimensionless variables in this figure are $C_\text{G}/C_0 = (50\%)/(5\%) = 10$ and $V_\text{R}/V_0 = (1\ \text{m}^3)/(5\ \text{m}^3) = 0.2$. Interpolating Figure 11.39 with these parameter values, $(Akt)/V_0$ is approximately 0.5. Therefore:

$$A = \frac{0.5V_0}{kt} = \frac{0.5\ (5\ \text{m}^3)}{2.2\times10^{-6}\ \text{m s}^{-1} \left(2\ \text{h} \left| \frac{3600\ \text{s}}{1\ \text{h}} \right| \right)} = 158\ \text{m}^2$$

Answer: 158 m^2

(ii)

For the detergent, $C_\text{R} = 0.002\%$. After the protein concentration step, V_0 for diafiltration is 1 m^3. From Eq. (11.88), for diafiltration of detergent with $R = 0$:

$$\frac{C_\text{R}}{C_0} = e^{-V_\text{D}/V_0} \tag{1}$$

Substituting values:

$$\frac{0.002\%}{1.2\%} = e^{-V_\text{D}/(1\ \text{m}^3)}$$

$$1.67\times10^{-3} = e^{-V_D}$$

where V_D has units of m^3. Taking the logarithm of both sides and applying Eq. (E.3) from Appendix E:

$$-6.39 = -V_D$$

Therefore:

$$V_D = 6.4\ m^3$$

Answer: 6.4 m^3

(b)

In this case, $V_0 = 5\ m^3$. Applying (1):

$$\frac{0.002\%}{1.2\%} = e^{-V_D/(5\ m^3)}$$

$$1.67\times10^{-3} = e^{-0.2V_D}$$

where V_D has units of m^3. Solving this equation gives:

$$-6.39 = -0.2V_D$$

$$V_D = 32.0\ m^3$$

Answer: 32 m^3

11.28 Scale-up of virus ultrafiltration

For the pilot-scale filtration, $u = 0.45\ m\ s^{-1}$ and $J = 27\ l\ m^{-2}\ h^{-1}$. If $J = Cu^{0.66}$ where C is a proportionality constant, when J has units of $l\ m^{-2}\ h^{-1}$ and u has units of $m\ s^{-1}$:

$$C = \frac{J}{u^{0.66}} = \frac{27}{(0.45)^{0.66}} = 45.7$$

For the large-scale filtration, $u = 2.2\ m\ s^{-1}$. As $0.40 < u < 3.5\ m\ s^{-1}$:

$$J = Cu^{0.66} = 45.7\,(2.2)^{0.66} = 76.9\ l\ m^{-2}\ h^{-1}$$

$F_0 = (100\ m^3)/(1\ h) = 100\ m^3\ h^{-1}$. From Eq. (11.72), to achieve a concentration factor $C_R/C_0 = 2.5$ for the virus with $R = 1$, $VCR = 2.5$. Therefore, from Eq. (11.100):

$$F_R = \frac{F_0}{VCR} = \frac{100\ m^3\ h^{-1}}{2.5} = 40\ m^3\ h^{-1}$$

From Eq. (11.98):

$$F_P = F_0 - F_R$$

Therefore:

$$F_P = 100\ m^3\ h^{-1} - 40\ m^3\ h^{-1} = 60\ m^3\ h^{-1}$$

Applying Eq. (11.48):

$$A = \frac{F_P}{J}$$

Substituting values for the large-scale filtration gives:

$$A = \frac{60\ \text{m}^3\ \text{h}^{-1}}{76.9\ \text{l m}^{-2}\ \text{h}^{-1} . \left| \frac{1\ \text{m}^3}{10^3\ \text{l}} \right|} = 780\ \text{m}^2$$

Answer: 780 m^2

11.29 Scale-up of enzyme chromatography

For the laboratory-scale column, $D_1 = 4.5$ cm and $L_1 = 25$ cm.

$$Q_1 = \frac{0.6\ \text{g}}{35\ \text{min}} . \left| \frac{60\ \text{min}}{1\ \text{h}} \right| = 1.03\ \text{g h}^{-1}$$

After scale-up, $Q_2 = 15\ \text{g h}^{-1}$. If the column length, linear flow velocity and packing particle size are kept the same on scale-up (Section 11.11.5), from Eq. (11.117):

$$D_2 = D_1 \sqrt{\frac{Q_2}{Q_1}}$$

Substituting values gives:

$$D_2 = (4.5\ \text{cm}) \sqrt{\frac{15\ \text{g h}^{-1}}{1.03\ \text{g h}^{-1}}} = 17.2\ \text{cm}$$

Answer: 17 cm diameter, 25 cm length

11.30 Gel chromatography scale-up

(a)

The elution volume for the toxoid is lower than for the impurity. Therefore, as the toxoid stays in the column for the shorter time, it must be the larger molecule.

Answer: Toxoid

(b)

The internal pore volume in the gel in the laboratory reactor is calculated using Eq. (11.106):

$$V_\text{i} = 10\ \text{g} . \left| \frac{1\ \text{kg}}{1000\ \text{g}} \right| (0.0035\ \text{m}^3\ \text{kg}^{-1}) . \left| \frac{10^6\ \text{ml}}{1\ \text{m}^3} \right| = 35\ \text{ml}$$

As $V_\text{o} = 23$ ml, the partition coefficients for the toxoid and impurity can be determined using the measured elution volumes and Eq. (11.105):

$$\text{Toxoid } K_\text{p} = \frac{(29 - 23)\ \text{ml}}{35\ \text{ml}} = 0.171$$

$$\text{Impurity } K_\text{p} = \frac{(45 - 23)\ \text{ml}}{35\ \text{ml}} = 0.629$$

Answer: 0.171 for the toxoid; 0.629 for the impurity

(c)

Let subscripts 1 and 2 denote the small and large columns, respectively. The total volume of the laboratory column of inner diameter $D_{c1} = 1.5$ cm = 0.015 m and height $H_1 = 0.4$ m is:

$$V_{T1} = \pi\left(\frac{D_{c1}}{2}\right)^2 H_1 = \pi\left(\frac{0.015\text{ m}}{2}\right)^2 0.4\text{ m} = 7.07\times10^{-5}\text{ m}^3$$

The total volume of the large-scale column of diameter $D_{c2} = 0.5$ m and height $H_2 = 0.6$ m is:

$$V_{T2} = \pi\left(\frac{D_{c2}}{2}\right)^2 H_2 = \pi\left(\frac{0.5\text{ m}}{2}\right)^2 0.6\text{ m} = 0.118\text{ m}^3$$

If the void fraction in the large column is the same as that in the small column, V_{o2} in the large column is:

$$V_{o2} = \frac{V_{o1}}{V_{T1}} V_{T2}$$

Substituting values gives:

$$V_{o2} = \frac{23\text{ ml}.\left|\frac{1\text{ m}^3}{10^6\text{ ml}}\right|}{7.07\times10^{-5}\text{ m}^3}(0.118\text{ m}^3) = 0.0384\text{ m}^3$$

If the pore volume fraction is also the same:

$$V_{i2} = \frac{V_{i1}}{V_{T1}} V_{T2}$$

Therefore:

$$V_{i2} = \frac{35\text{ ml}.\left|\frac{1\text{ m}^3}{10^6\text{ ml}}\right|}{7.07\times10^{-5}\text{ m}^3}(0.118\text{ m}^3) = 0.0584\text{ m}^3$$

If the large-scale column is operated with the same packing and flow conditions, the partition coefficients can be assumed to be the same as those in the laboratory column. Therefore, for toxoid in the large column, from Eq. (11.104):

$$V_{e2} = V_{o2} + K_p V_{i2}$$

Substituting values using the result for K_p from **(b)**:

$$V_{e2} = 0.0384\text{ m}^3 + 0.171(0.0584\text{ m}^3) = 0.0484\text{ m}^3$$

Similarly, for the impurity:

$$V_{e2} = 0.0384\text{ m}^3 + 0.629(0.0584\text{ m}^3) = 0.0751\text{ m}^3$$

Answer: 0.0484 m^3 for the toxoid; 0.0751 m^3 for the impurity

(d)

The volumetric flow rate in the small column Q_1 = 14 ml min^{-1}. The liquid flow rate is scaled up in proportion to the column cross-sectional area. As the cross-sectional area = $\pi(D_c/2)^2$, the volumetric flow rate Q_2 in the large column is:

$$Q_2 = Q_1 \frac{\pi\left(\frac{D_{c2}}{2}\right)^2}{\pi\left(\frac{D_{c1}}{2}\right)^2} = Q_1\left(\frac{D_{c2}}{D_{c1}}\right)^2$$

Substituting values:

$$Q_2 = 14\ \text{ml min}^{-1} . \left| \frac{1\ \text{m}^3}{10^6\ \text{ml}} \right| \left(\frac{0.5\ \text{m}}{0.015\ \text{m}} \right)^2 = 0.0156\ \text{m}^3\ \text{min}^{-1}$$

Answer: 0.0156 $\text{m}^3\ \text{min}^{-1}$

(e)

The retention time t_R is equal to the elution volume divided by the volumetric flow rate. For the toxoid in the large column, using the results from **(c)** and **(d)**:

$$t_R = \frac{V_{e2}}{Q_2} = \frac{0.0484\ \text{m}^3}{0.0156\ \text{m}^3\ \text{min}^{-1}} = 3.1\ \text{min}$$

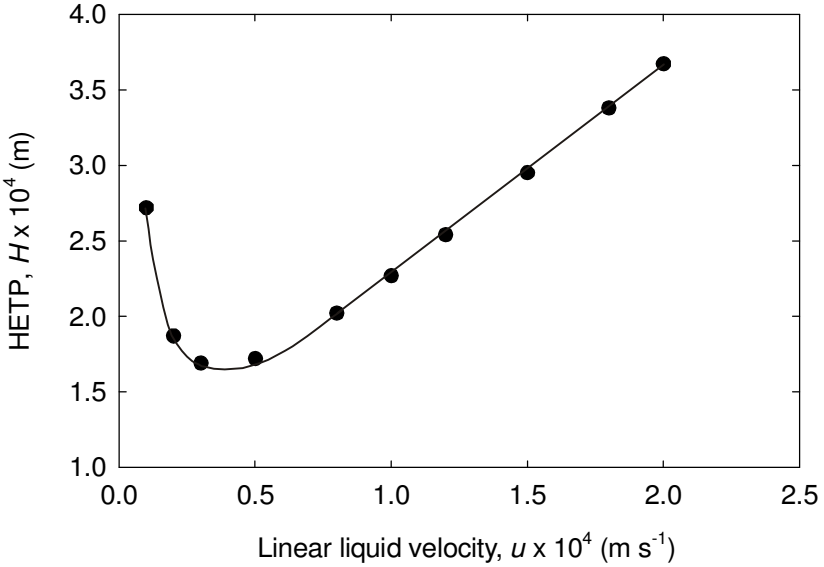
Answer: 3.1 min

11.31 Protein separation using affinity chromatography

(a)

Selected values of u and H calculated using the equation provided are listed and plotted below.

Linear liquid velocity, u (m s^{-1})	HETP, H (m)
0.1×10^{-4}	2.72×10^{-4}
0.2×10^{-4}	1.87×10^{-4}
0.3×10^{-4}	1.69×10^{-4}
0.5×10^{-4}	1.72×10^{-4}
0.8×10^{-4}	2.02×10^{-4}
1.0×10^{-4}	2.27×10^{-4}
1.2×10^{-4}	2.54×10^{-4}
1.5×10^{-4}	2.95×10^{-4}
1.8×10^{-4}	3.38×10^{-4}
2.0×10^{-4}	3.67×10^{-4}



(b)

From the graph in **(a)**, the minimum HETP is around 1.7×10^{-4} m. This can be confirmed by differentiating the equation for H and solving for $dH/du = 0$:

$$\frac{dH}{du} = \frac{-A}{u^2} + B = 0$$

Therefore:

$$u = \sqrt{\frac{A}{B}}$$

Substituting values:

$$u = \sqrt{\frac{2\times10^{-9}\ \text{m}^2\ \text{s}^{-1}}{1.5\ \text{s}}} = 3.65\times10^{-5}\ \text{m s}^{-1}$$

The value of H corresponding to this u is:

$$H = \frac{2\times10^{-9}\ \text{m}^2\ \text{s}^{-1}}{3.65\times10^{-5}\ \text{m s}^{-1}} + 1.5\ \text{s}\ (3.65\times10^{-5}\ \text{m s}^{-1}) + 5.7\times10^{-5}\ \text{m} = 1.67\times10^{-4}\ \text{m}$$

Answer: The minimum HETP is 1.67×10^{-4} m, obtained at a liquid velocity of 3.65×10^{-5} m s^{-1}.

(c)

The column diameter D_c = 25 cm = 0.25 m. The volumetric flow rate Q is 0.31 l min^{-1}. The linear flow rate u is obtained by dividing the volumetric flow rate by the column cross-sectional area:

$$u = \frac{Q}{\pi\left(\frac{D_c}{2}\right)^2} = \frac{0.311\ \text{min}^{-1}.\left|\frac{1\ \text{min}}{60\ \text{s}}\right|.\left|\frac{1\ \text{m}^3}{1000\ \text{l}}\right|}{\pi\left(\frac{0.25\ \text{m}}{2}\right)^2} = 1.05\times10^{-4}\ \text{m s}^{-1}$$

Substituting this result into the equation for H:

$$H = \frac{2\times10^{-9}\ \text{m}^2\ \text{s}^{-1}}{1.05\times10^{-4}\ \text{m s}^{-1}} + 1.5\ \text{s}\ (1.05\times10^{-4}\ \text{m s}^{-1}) + 5.7\times10^{-5}\ \text{m} = 2.34\times10^{-4}\ \text{m}$$

The capacity factors for the A and B chains are k_A = 0.85 and k_B = 1.05, respectively. From Eq. (11.102):

$$\delta = \frac{k_B}{k_A} = \frac{1.05}{0.85} = 1.235$$

Substituting parameter values into Eq. (11.115) with L = 1 m for the larger column and $k_2 = k_B$:

$$R_N = \frac{1}{4}\sqrt{\frac{1\ \text{m}}{2.34\times10^{-4}\ \text{m}}}\left(\frac{1.235-1}{1.235}\right)\left(\frac{1.05}{1.05+1}\right) = 1.59$$

From Section 11.11.4, as $R_N > 1.5$, the two peaks are completely separated.

Answer: Yes

(d)

L = 0.7 m for the smaller column. The value of H is determined by rearranging Eq. (11.115) with $k_2 = k_B$:

$$\sqrt{H} = \frac{1}{4}\frac{\sqrt{L}}{R_N}\left(\frac{\delta - 1}{\delta}\right)\left(\frac{k_B}{k_B + 1}\right)$$

Substituting values with $R_N = 1.5$ for virtual complete separation:

$$\sqrt{H} = \frac{1}{4}\frac{\sqrt{0.7\ \text{m}}}{1.5}\left(\frac{1.235 - 1}{1.235}\right)\left(\frac{1.05}{1.05 + 1}\right) = 0.0136\sqrt{\text{m}}$$

$$H = 1.85 \times 10^{-4}\ \text{m}$$

The linear liquid velocity corresponding to this value of H is obtained by rearranging the expression provided for H as a function of u and solving for u:

$$Hu = A + Bu^2 + Cu$$

$$Bu^2 + (C - H)u + A = 0$$

The solution to this quadratic equation is:

$$u = \frac{-(C - H) \pm \sqrt{(C - H)^2 - 4BA}}{2B} = \frac{H - C \pm \sqrt{(C - H)^2 - 4BA}}{2B}$$

Substituting parameter values:

$$u = \frac{(1.85 \times 10^{-4} - 5.7 \times 10^{-5})\ \text{m} \pm \sqrt{(5.7 \times 10^{-5}\ \text{m} - 1.85 \times 10^{-4}\ \text{m})^2 - 4(1.5\ \text{s})(2 \times 10^{-9}\ \text{m}^2\ \text{s}^{-1})}}{2\ (1.5\ \text{s})}$$

$$= 4.27 \times 10^{-5}\ \text{m s}^{-1} \pm 2.21 \times 10^{-5}\ \text{m s}^{-1}$$

Therefore, there are two possible solutions:

$$u = 6.48 \times 10^{-5}\ \text{m s}^{-1} \quad \text{or} \quad u = 2.06 \times 10^{-5}\ \text{m s}^{-1}$$

The maximum flow rate for complete separation is $u = 6.48 \times 10^{-5}$ m s^{-1}. Between $u = 2.06 \times 10^{-5}$ m s^{-1} and $u = 6.48 \times 10^{-5}$ m s^{-1}, $H < 1.85 \times 10^{-4}$ m so that $R_N > 1.5$ and complete separation is maintained. The volumetric flow rate Q is equal to u multiplied by the column cross-sectional area:

$$Q = u\pi\left(\frac{D_c}{2}\right)^2 = (6.48 \times 10^{-5}\ \text{m s}^{-1})\pi\left(\frac{0.25\ \text{m}}{2}\right)^2 = 3.18 \times 10^{-6}\ \text{m}^3\ \text{s}^{-1}$$

Converting units:

$$Q = 3.18 \times 10^{-6}\ \text{m}^3\ \text{s}^{-1} \cdot \left|\frac{1000\ \text{l}}{1\ \text{m}^3}\right| \cdot \left|\frac{60\ \text{s}}{1\ \text{min}}\right| = 0.19\ \text{l min}^{-1}$$

Answer: 0.19 l min^{-1}

11.32 Crystal size distribution from screen analysis

Aperture sizes for U.S. sieves are given in Table F.1 in Appendix F. The increment in aperture size from above ΔL is calculated from the aperture values. The average particle size L_{av} retained on each screen is calculated by taking the average of the aperture sizes of that screen and the one above it. Adding together the masses retained at each screen and in the pan gives the total sample mass M_T = 74.65 g. The mass density m is determined by combining Eqs (11.121) and (11.122):

$$m = \frac{\Delta M\ /\ M_T}{\Delta L}$$

As an example, the mass density of the particles on No. 50 sieve is:

$$m = \frac{3.05\text{ g} / 74.65\text{ g}}{0.123\text{ mm}} = 0.332\text{ mm}^{-1}$$

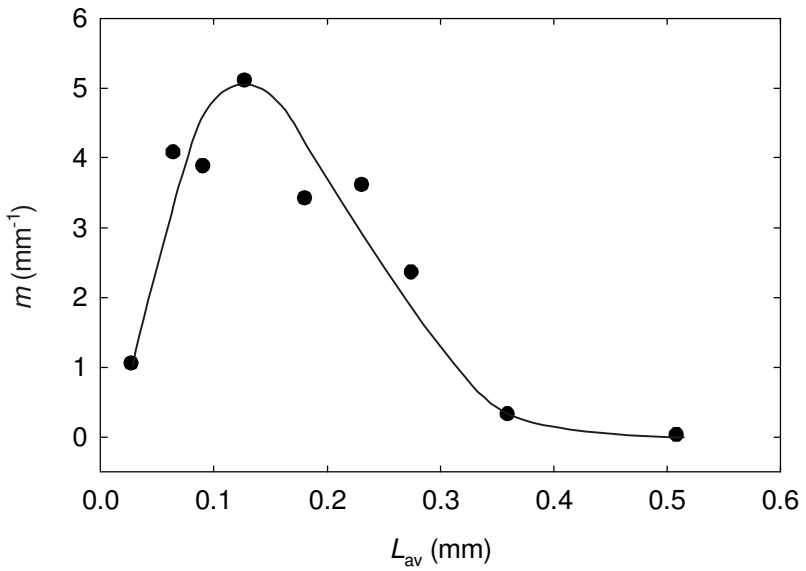
The mass density and average size of the particles collected in the pan are calculated as if the aperture size of the pan is zero.

Results calculated from the data provided are listed below.

U.S. sieve	Aperture size (mm)	Increment in aperture size from above, ΔL (mm)	Average particle size retained, L_{av} (mm)	Mass retained, ΔM (g)	Mass density, m (mm^{-1})
30	0.595	–	–	0	0
40	0.420	0.175	0.508	0.5	0.038
50	0.297	0.123	0.359	3.05	0.332
60	0.250	0.047	0.274	8.3	2.366
70	0.210	0.040	0.230	10.8	3.617
100	0.149	0.061	0.180	15.6	3.426
140	0.105	0.044	0.127	16.8	5.115
200	0.074	0.031	0.090	9.0	3.889
270	0.053	0.021	0.064	6.4	4.083
Pan	–	0.053	0.027	4.2	1.062
Total	–	–	–	74.65	–

(a)

The mass density is plotted as a function of average particle size below.



(b)

The dominant crystal size L_D corresponds to the maximum value of the mass density distribution (Section 11.12.1, Size Distribution subsection). From the graph above, $L_D \approx 0.13$ mm.

Answer: 0.13 mm

11.33 Crystal growth and nucleation rates

Aperture sizes for Tyler screens are given in Table F.2 in Appendix F. The increment in aperture size from above ΔL is calculated from the aperture values. The average particle size L_{av} retained on each screen is calculated by taking the average of the aperture sizes of that screen and the one above it. The mass of the crystals is represented as a slurry density (Section 11.12.1, Size Distribution subsection) to take into account the sample volume of 1 litre. As an example, ΔM for the 16 mesh screen is:

$$\Delta M = 0.77 \text{ g l}^{-1} = 0.77 \text{ g l}^{-1} . \left| \frac{10^3 \text{ l}}{1 \text{ m}^3} \right| . \left| \frac{1 \text{ kg}}{1000 \text{ g}} \right| = 0.77 \text{ kg m}^{-3}$$

The crystal population density n is determined from Eq. (11.124) with $\rho = 1.542$ g cm^{-3} = 1542 kg m^{-3} and $k_v = 0.775$. Therefore, for the 16 mesh screen example:

$$n = \frac{0.77 \text{ kg m}^{-3} / 1.77 \times 10^{-4} \text{ m}}{1542 \text{ kg m}^{-3} (0.775) (1.08 \times 10^{-3} \text{ m})^3} = 2.89 \times 10^9 \text{ m}^{-4}$$

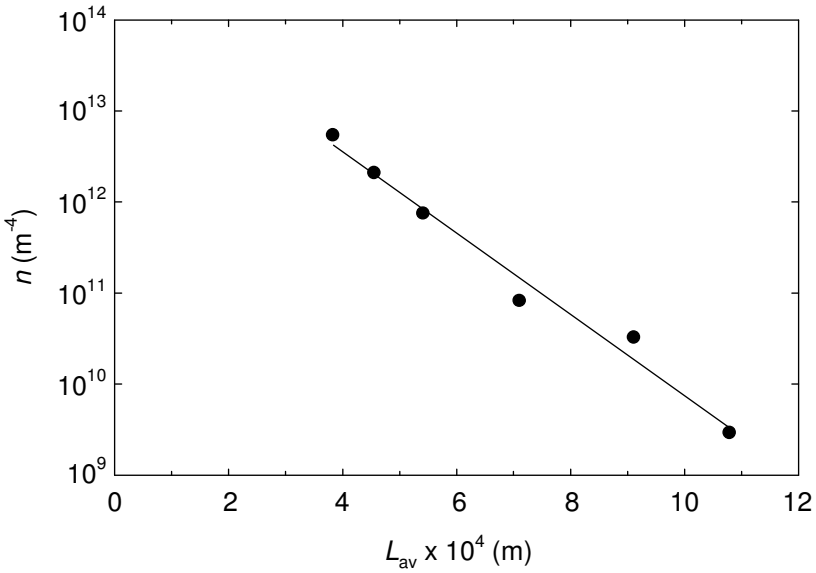
The crystal population density and average size of the particles collected in the pan are calculated as if the aperture size of the pan is zero.

Results calculated from the data provided are listed below.

Tyler screen	Aperture size (mm)	Increment in aperture size from above, ΔL (m)	Average particle size retained, L_{av} (m)	Mass retained, ΔM (kg m^{-3})	Crystal population density, n (m^{-4})
14 mesh	1.168	–	–	0	0
16 mesh	0.991	1.77×10^{-4}	1.08×10^{-3}	0.77	2.89×10^{9}
20 mesh	0.833	1.58×10^{-4}	9.12×10^{-4}	4.6	3.21×10^{10}
28 mesh	0.589	2.44×10^{-4}	7.11×10^{-4}	8.5	8.11×10^{10}
32 mesh	0.495	9.40×10^{-5}	5.42×10^{-4}	13.2	7.38×10^{11}
35 mesh	0.417	7.80×10^{-5}	4.56×10^{-4}	18.1	2.05×10^{12}
42 mesh	0.351	6.60×10^{-5}	3.84×10^{-4}	23.8	5.33×10^{12}
Pan	–	3.51×10^{-4}	1.76×10^{-4}	0	0
Total	–	–	–	68.97	

(a)

The crystal growth rate G is determined from Eq. (11.152) using L_{av} to represent the crystal size L. From the form of Eq. (11.152), a plot of n versus L_{av} on semi-logarithmic coordinates can be expected to give a straight line. The results are shown below.



The equation for the straight line through the data is:

$$n = 2.17\times10^{14}\, e^{-1.028\times10^{4} L_{av}} \tag{1}$$

where n has units of m^{-4} and L_{av} has units of m. Therefore, from Eq. (11.152):

$$\frac{-F}{VG} = -1.028\times10^{4}\ \text{m}^{-1} \tag{2}$$

As $V = 10\ \text{m}^3$ and $F = 40\ \text{l min}^{-1} = 40 \times 10^{-3}\ \text{m}^3\ \text{min}^{-1}$, from (2):

$$G = \frac{-40\times10^{-3}\ \text{m}^3\ \text{min}^{-1} . \left|\frac{1\ \text{min}}{60\ \text{s}}\right|}{10\ \text{m}^3\ (-1.028\times10^{4}\ \text{m}^{-1})} = 6.49\times10^{-9}\ \text{m s}^{-1}$$

Answer: $6.5 \times 10^{-9}\ \text{m s}^{-1}$

(b)

From (1) and Eq. (11.152):

$$n_0 = 2.17\times10^{14}\ \text{m}^{-4}$$

The nucleation rate B is determined from the values of n_0 and G using Eq. (11.153):

$$B = n_0 G = 2.17\times10^{14}\ \text{m}^{-4}\ (6.49\times10^{-9}\ \text{m s}^{-1}) = 1.41\times10^{6}\ \text{m}^{-3}\ \text{s}^{-1}$$

Answer: $1.41 \times 10^{6}\ \text{m}^{-3}\ \text{s}^{-1}$

(c)

The magma density in the crystalliser is the total mass of crystals per unit volume. If the crystalliser is well mixed, the total mass of crystals per unit volume in the sieved sample taken from the vessel outlet is equal to the operating magma density. Adding together the masses retained at each screen and in the pan gives a total crystal density in the sample of 68.97 kg m^{-3}. This result can be compared with that calculated using the equation in Table 11.9 for the total mass of crystals per unit volume in an MSMPR crystalliser. Substituting values into the equation and using the result from (2):

$$\text{Magma density} = 6\,(0.775)\,(1542\ \text{kg m}^{-3})\,(2.17\times10^{14}\ \text{m}^{-4})\left(\frac{1}{1.028\times10^{4}\ \text{m}^{-1}}\right)^{4} = 139\ \text{kg m}^{-3}$$

This result is about twice the value obtained directly from the sample. The discrepancy may be attributed to the strong dependence of the MSMPR equation on the fourth power of the slope obtained from the measured data. A relatively small variation in the slope obtained from fitting the data results in a large change in the calculated magma density.

Answer: 69 kg m^{-3}

11.34 Bench-scale continuous crystalliser

Aperture sizes for Tyler screens are given in Table F.2 in Appendix F. The increment in aperture size from above ΔL is calculated from the aperture values. The average particle size L_{av} retained on each screen is calculated by taking the average of the aperture sizes of that screen and the one above it. The mass of the crystals retained on each screen ΔM is represented as a slurry density (Section 11.12.1, Size Distribution subsection) and is determined from the data provided for the mass percent of crystals retained in a sample containing 280 g l^{-1} = 280 kg m^{-3} of crystals. For example, for the 100 mesh:

$$\Delta M = \frac{23.5\%}{100\%}\,(280\ \text{kg m}^{-3}) = 65.80\ \text{kg m}^{-3}$$

The crystal population density n is determined from Eq. (11.124) with ρ = 1.25 g cm^{-3} = 1250 kg m^{-3} and k_v = 0.60. Therefore, for the 100 mesh example:

$$n = \frac{65.80\ \text{kg m}^{-3} / 2.80\times10^{-5}\ \text{m}}{1250\ \text{kg m}^{-3}\,(0.60)\,(1.61\times10^{-4}\ \text{m})^{3}} = 7.51\times10^{14}\ \text{m}^{-4}$$

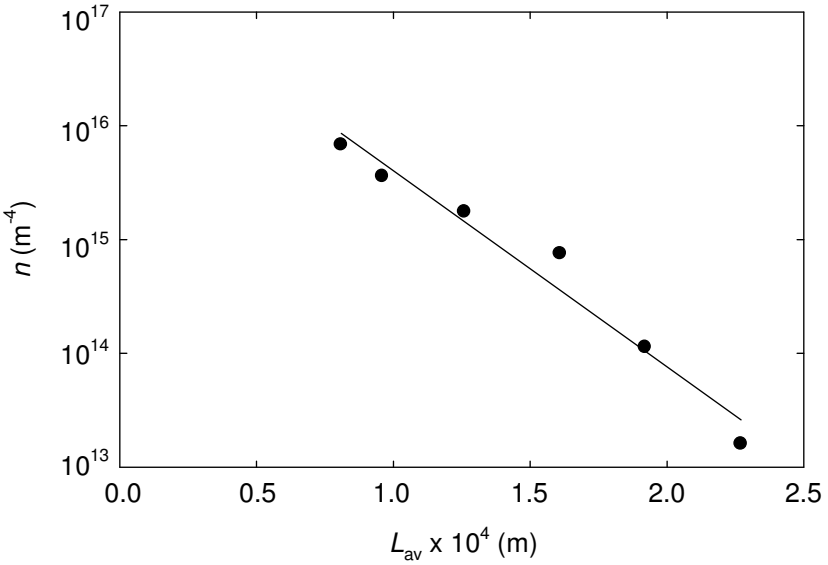
The crystal population density and average size of the particles collected in the pan are calculated as if the aperture size of the pan is zero.

Results calculated from the data provided are listed below.

Tyler screen	Aperture size (mm)	Increment in aperture size from above, ΔL (m)	Average particle size retained, L_{av} (m)	Mass retained, ΔM (kg m^{-3})	Crystal population density, n (m^{-4})
60 mesh	0.246	–	–	0	0
65 mesh	0.208	3.80×10^{-5}	2.27×10^{-4}	5.32	1.60×10^{13}
80 mesh	0.175	3.30×10^{-5}	1.92×10^{-4}	19.88	1.13×10^{14}
100 mesh	0.147	2.80×10^{-5}	1.61×10^{-4}	65.80	7.51×10^{14}
150 mesh	0.104	4.30×10^{-5}	1.26×10^{-4}	113.1	1.75×10^{15}
170 mesh	0.088	1.60×10^{-5}	9.60×10^{-5}	38.08	3.59×10^{15}
200 mesh	0.074	1.40×10^{-5}	8.10×10^{-5}	37.80	6.77×10^{15}
Pan	–	7.40×10^{-5}	3.70×10^{-5}	0	0
Total	–	–	–	280	

(a)

The crystal growth rate G is determined from Eq. (11.152) using L_{av} to represent the crystal size L. A plot of n versus L_{av} on semi-logarithmic coordinates is shown below.



The equation for the straight line through the data is:

$$n = 2.13\times10^{17}\, e^{-3.968\times10^{4} L_{av}} \tag{1}$$

where n has units of m^{-4} and L_{av} has units of m. Therefore, from Eq. (11.152):

$$\frac{-F}{VG} = -3.968\times10^{4}\ \text{m}^{-1} \tag{2}$$

V = 2.5 l; therefore, from Table A.2 (Appendix A), $V = 2.5 \times 10^{-3}$ m^3. $F = 0.4$ l $h^{-1} = 0.4 \times 10^{-3}$ m^3 h^{-1}. Solving for G gives:

$$G = \frac{-0.4\times10^{-3}\ \text{m}^3\ \text{h}^{-1} . \left|\frac{1\ \text{h}}{3600\ \text{s}}\right|}{2.5\times10^{-3}\ \text{m}^3\ (-3.968\times10^{4}\ \text{m}^{-1})} = 1.12\times10^{-9}\ \text{m s}^{-1}$$

Answer: 1.12×10^{-9} m s^{-1}

(b)

From (1) and Eq. (11.152):

$$n_0 = 2.13\times10^{17}\ \text{m}^{-4}$$

The nucleation rate B is determined from the values of n_0 and G using Eq. (11.153):

$$B = n_0 G = 2.13\times10^{17}\ \text{m}^{-4}\ (1.12\times10^{-9}\ \text{m s}^{-1}) = 2.39\times10^{8}\ \text{m}^{-3}\ \text{s}^{-1}$$

Answer: 2.39×10^{8} m^3 s^{-1}

11.35 Continuous crystalliser specification

ρ = 1690 kg m^{-3}; k_v = 0.5. $B = 4.5 \times 10^{7}$ m^{-3} s^{-1}; $G = 2.1 \times 10^{-9}$ m s^{-1}. $F = 1.8$ m^3 h^{-1}.

(a)

If the volumetric flow rate through the crystalliser is 1.8 m^3 h^{-1} and the crystal mass production rate is 350 kg h^{-1}, the slurry density is:

$$\text{Slurry density} = \frac{350\ \text{kg h}^{-1}}{1.8\ \text{m}^3\ \text{h}^{-1}} = 194.4\ \text{kg m}^{-3}$$

Answer: 194 kg m^{-3}

(b)

The crystalliser volume can be determined using the equation in Table 11.9 for the total mass of crystals per unit volume, or slurry density, in an MSMPR crystalliser. Rearranging the equation gives:

$$\left(\frac{VG}{F}\right)^4 = \frac{\text{slurry density}}{6k_v\rho n_0}$$

or

$$V^4 = \frac{\text{slurry density}}{6k_v\rho n_0}\left(\frac{F}{G}\right)^4 \tag{1}$$

The value of n_0 is obtained using Eq. (11.153):

$$n_0 = \frac{4.5\times10^7\ \text{m}^{-3}\ \text{s}^{-1}}{2.1\times10^{-9}\ \text{m s}^{-1}} = 2.14\times10^{16}\ \text{m}^{-4}$$

Substituting values into (1) gives:

$$V^4 = \frac{194.4\ \text{kg m}^{-3}}{6\ (0.5)\ (1690\ \text{kg m}^{-3})\ (2.14\times10^{16}\ \text{m}^{-4})}\left(\frac{1.8\ \text{m}^3\ \text{h}^{-1}.\left|\frac{1\ \text{h}}{3600\ \text{s}}\right|}{2.1\times10^{-9}\ \text{m s}^{-1}}\right)^4 = 5758.08\ \text{m}^{12}$$

Therefore:

$$V = 8.71\ \text{m}^3$$

Answer: 8.7 m^3

(c)

The dominant crystal size L_D is obtained using Eq. (11.154):

$$L_D = \frac{3\ (8.71\ \text{m}^3)\ (2.1\times10^{-9}\ \text{m s}^{-1})}{1.8\ \text{m}^3\ \text{h}^{-1}.\left|\frac{1\ \text{h}}{3600\ \text{s}}\right|} = 1.10\times10^{-4}\ \text{m}$$

Converting units from Table A.1 (Appendix A):

$$L_D = 1.10\times10^{-4}\ \text{m}.\left|\frac{10^6\ \mu\text{m}}{1\ \text{m}}\right| = 110\ \mu\text{m}$$

Answer: 110 μm

11.36 Drying of benzyl penicillin

(a)

The equilibrium water content after drying is:

$$20\%\text{ wet weight} = \frac{20\text{ g water}}{80\text{ g dry solid}} = 0.25\text{ g g}^{-1}\text{ dry solid} = 25\text{ g per }100\text{ g dry weight}$$

For sodium benzyl penicillin at 25°C, the relative humidity required to achieve this equilibrium moisture content can be read from Figure 11.56(a) as ~65%.

Answer: 65%

(b)

From Eq. (11.157):

$$p_w = \frac{\text{relative humidity} \times p_{sw}}{100\%}$$

The saturation vapour pressure of water p_{sw} at 25°C is obtained from the steam tables (Table D.1, Appendix D) as 3.17 kPa. Using the result for relative humidity from **(a)**:

$$p_w = \frac{65\% \times 3.17\text{ kPa}}{100\%} = 2.06\text{ kPa}$$

The water mole fraction y_w corresponding to this partial pressure is determined using Eq. (11.156) and the unit conversion factor for pressure from Table A.5 (Appendix A):

$$y_w = \frac{p_w}{p_T} = \frac{2.06\text{ kPa}}{1\text{ atm}.\left|\frac{1.013\times10^5\text{ Pa}}{1\text{ atm}}\right|.\left|\frac{1\text{ kPa}}{1000\text{ Pa}}\right|} = 0.020$$

Answer: 0.020

(c)

The humidity is determined using Eq. (11.155). In 1 mole of moist air, from the result in **(b)**, there are 0.020 moles of water and 0.980 moles of dry air. Converting these molar quantities to mass using the molecular weight of water = 18.0 (Table C.1, Appendix C) and the molecular weight of dry air = 28.8 (see solution to Problem 2.14), Eq. (11.155) becomes:

$$\text{Humidity} = \frac{0.020\times18.0\text{ g gmol}^{-1}}{0.980\times28.8\text{ g gmol}^{-1}} = 0.013$$

Answer: 0.013

11.37 Drying rate of filter cake

The cake diameter D = 30 cm = 0.3 m; the cake height H = 5 mm = 5 × 10^{-3} m. As drying occurs from both the top and bottom of the filter cake, the area available for drying A_h is equal to 2 × the area of the filter cake disc:

$$A_h = 2\times\pi\left(\frac{D}{2}\right)^2 = 2\times\pi\left(\frac{0.3\text{ m}}{2}\right)^2 = 0.141\text{ m}^2$$

X_c = 5% wet basis. ρ = 1380 kg m^{-3}. h_s = 45 W m^{-2} °C^{-1}; therefore, from Table A.8 (Appendix A), h_s = 45 J s^{-1} m^{-2} °C^{-1}. T_a = 35°C and T = 26°C.

(a)

We will assume that drying occurs at a constant rate: this applies to moisture contents in the cake of $\geq X_c$. The rate of drying during the constant rate period is given by Eq. (11.165). At the drying temperature of

26°C, from Table D.1 (Appendix D), $\Delta h_v = 2440.2$ kJ kg^{-1} = 2440.2 × 10^3 J kg^{-1}. Substituting values into Eq. (11.165):

$$N_c = \frac{h_s A_h (T_a - T)}{\Delta h_v} = \frac{45\ \text{J s}^{-1}\ \text{m}^{-2}\ °\text{C}^{-1}\ (0.141\ \text{m}^2)\ (35-26)°\text{C}}{2440.2\times10^3\ \text{J kg}^{-1}} = 2.34\times10^{-5}\ \text{kg s}^{-1}$$

Answer: 2.34 × 10^{-5} kg s^{-1}

(b)

The time required for constant-rate drying is determined using Eq. (11.168). X_0 = 15% wet basis; X_1 = 5% wet basis. Converting these values to a dry weight basis:

$$X_0 = 15\%\ \text{wet weight} = \frac{15\ \text{g water}}{85\ \text{g dry solid}} = 0.176\ \text{g g}^{-1}\ \text{dry solid}$$

$$X_1 = 5\%\ \text{wet weight} = \frac{5\ \text{g water}}{95\ \text{g dry solid}} = 0.053\ \text{g g}^{-1}\ \text{dry solid}$$

As the moisture content remains ≥ X_c during drying, constant-rate drying applies. The mass of completely dry solid M_s is obtained from the cake volume and the density of the dry material. From the definition of density (Section 2.4.1), mass = density × volume. The volume of the cylindrical filter cake V_s is:

$$V_s = \pi\left(\frac{D}{2}\right)^2 H$$

where D is the cake diameter and H is the cake thickness. Therefore:

$$M_s = \rho V_s = \rho\pi\left(\frac{D}{2}\right)^2 H$$

Substituting values gives:

$$M_s = 1380\ \text{kg m}^{-3}\ \pi\left(\frac{0.30\ \text{m}}{2}\right)^2 5\times10^{-3}\ \text{m} = 0.488\ \text{kg}$$

Applying Eq. (11.168) using the result for N_c from **(a)**:

$$\Delta t = \frac{0.488\ \text{kg}}{2.34\times10^{-5}\ \text{kg s}^{-1}}(0.176-0.053) = 2565\ \text{s}$$

Converting units:

$$\Delta t = 2565\ \text{s}\ .\left|\frac{1\ \text{min}}{60\ \text{s}}\right| = 42.75\ \text{min}$$

Answer: 43 min

11.38 Drying time for protein crystals

M_s = 20 kg. X_0 = 0.18; X_1 = 0.08; X_c = 0.05. Δt = 4.5 h. A_h = 0.5 m^2. T_a = 28°C and T = 20°C. As $X_1 > X_c$, constant-rate drying applies and the rate of drying N_c can be determined from Eq. (11.168):

$$N_c = \frac{M_s}{\Delta t}(X_0 - X_1)$$

Substituting values gives:

$$N_c = \frac{20\text{ kg}}{4.5\text{ h}}(0.18 - 0.08) = 0.444\text{ kg h}^{-1}$$

The heat transfer coefficient can be determined from Eq. (11.165):

$$h_s = \frac{N_c \Delta h_v}{A_h (T_a - T)}$$

At the drying temperature of 20°C, from Table D.1 (Appendix D), $\Delta h_v = 2454.3$ kJ kg^{-1} = 2454.3 × 10^3 J kg^{-1}. Substituting values gives:

$$h_s = \frac{0.444\text{ kg h}^{-1}.\left|\frac{1\text{ h}}{3600\text{ s}}\right|(2454.3\times10^3\text{ J kg}^{-1})}{0.5\text{ m}^2\,(28-20)°\text{C}} = 75.7\text{ J s}^{-1}\text{ m}^{-2}\,°\text{C}^{-1}$$

Answer: 75.7 J s^{-1} m^{-2} °C^{-1}

11.39 Drying time for solid precipitate

$X_0 = 0.35$; $X_c = 0.08$. When $X_1 = 0.15$, $\Delta t = 6$ h. To achieve $X_1 = 0.10$ under the same drying conditions, because X_1 remains > X_c, the same constant rate of drying applies. As the mass of the sample also remains the same, from Eq. (11.168):

$$\Delta t \propto (X_0 - X_1)$$

or

$$\frac{\Delta t_A}{\Delta t_B} = \frac{(X_0 - X_1)_A}{(X_0 - X_1)_B}$$

where subscript A refers to achieving $X_1 = 0.10$ and subscript B refers to achieving $X_1 = 0.15$. Substituting values gives:

$$\frac{\Delta t_A}{6\text{ h}} = \frac{(0.35 - 0.10)}{(0.35 - 0.15)} = 1.25$$

$$\Delta t_A = 1.25 \times 6\text{ h} = 7.5\text{ h}$$

Answer: 7.5 h

11.40 Drying of amino acid crystals

$M_s = 32$ kg. Converting $X_0 = 25\%$ wet basis to a dry weight basis:

$$X_0 = 25\%\text{ wet weight} = \frac{25\text{ g water}}{75\text{ g dry solid}} = 0.333\text{ g g}^{-1}\text{ dry solid}$$

$\Delta t = 3$ h. $T_a = 28$°C and $T = 24$°C. $X_c = 0.10$. $A_h = 12$ m^2. $h_s = 32$ W m^{-2} °C^{-1}; therefore, from Table A.8 (Appendix A), $h_s = 32$ J s^{-1} m^{-2} °C^{-1}. At the drying temperature of 24°C, from Table D.1 (Appendix D), $\Delta h_v = 2444.9$ kJ kg^{-1} = 2444.9 × 10^3 J kg^{-1}.

(a)

Let us assume that constant-rate drying occurs; this is checked later when we determine X_1. The rate of drying N_c is given by Eq. (11.165):

$$N_c = \frac{h_s A_h (T_a - T)}{\Delta h_v} = \frac{32\ \text{J s}^{-1}\ \text{m}^{-2}\ ^\circ\text{C}^{-1}\ (12\ \text{m}^2)\ (28-24)^\circ\text{C}}{2444.9\times10^3\ \text{J kg}^{-1}} = 6.28\times10^{-4}\ \text{kg s}^{-1}$$

Answer: 6.28×10^{-4} kg s^{-1}

(b)

The moisture content of the dried crystals X_1 is determined from Eq. (11.168):

$$X_1 = X_0 - \frac{\Delta t N_c}{M_s}$$

Substituting values gives:

$$X_1 = 0.333 - \frac{3\ \text{h}\ .\left|\frac{3600\ \text{s}}{1\ \text{h}}\right| 6.28\times10^{-4}\ \text{kg s}^{-1}}{32\ \text{kg}} = 0.12$$

$X_1 = 0.12$ or 12% dry basis. Because X_1 remains > X_c, the assumption of constant-rate drying is valid.

Answer: 12% dry basis

Chapter 12

Homogeneous Reactions

12.1 Reaction equilibrium

From Section 12.1.1, standard conditions can be taken as 25°C and 1 atm pressure. The temperature is converted from °C to kelvin using Eq. (2.27); from Table B.1 (Appendix B), R = 8.3144 J K^{-1} $gmol^{-1}$ = 8.3144×10^{-3} kJ K^{-1} $gmol^{-1}$.

(a)

From Eq. (12.3):

$$\ln K_{eq} = \frac{14.1 \text{ kJ mol}^{-1}}{8.3144 \times 10^{-3} \text{ kJ K}^{-1} \text{ gmol}^{-1} (25 + 273.15 \text{ K})} = 5.688$$

Solving this equation using Eq. (E.4) from Appendix E:

$$K_{eq} = e^{5.688} = 295$$

The large magnitude of K_{eq} indicates that the reaction can be considered irreversible.

Answer: 295; irreversible

(b)

From Eq. (12.3):

$$\ln K_{eq} = \frac{-3.2 \text{ kJ mol}^{-1}}{8.3144 \times 10^{-3} \text{ kJ K}^{-1} \text{ gmol}^{-1} (25 + 273.15 \text{ K})} = -1.291$$

Therefore:

$$K_{eq} = e^{-1.291} = 0.275$$

The relatively small magnitude of K_{eq} indicates that the reaction is reversible.

Answer: 0.275; reversible

12.2 Equilibrium yield

(a)

From Eq. (12.2) with G6P and G1P representing glucose 6-phosphate and glucose 1-phosphate, respectively:

$$K_{eq} = \frac{[\text{G6P}]_e}{[\text{G1P}]_e} = \frac{0.038 \text{ M}}{0.002 \text{ M}} = 19$$

Answer: 19

(b)

From Eq. (12.9) and the reaction stoichiometry:

$$\text{Theoretical yield} = \frac{\text{moles of G6P formed}}{\text{moles of G1P used to form G6P}} = \frac{1 \text{ mol}}{1 \text{ mol}} = 1 \text{ mol mol}^{-1}$$

Answer: 1 mol mol^{-1}

(c)

The gross yield is based on the amount of reactant supplied. From Eq. (12.12) and using a basis of 1 litre:

$$\text{Gross yield} = \frac{\text{moles of G6P formed}}{\text{moles of G1P supplied}} = \frac{0.038 \text{ mol}}{0.04 \text{ mol}} = 0.95 \text{ mol mol}^{-1}$$

Answer: 0.95 mol mol^{-1}

12.3 Reaction rate

(a)

The terms used to express reaction rate are outlined in Section 12.1.3.

(i)

Answer: The volumetric productivity is unaffected by change in volume

(ii)

Answer: The specific productivity is unaffected by change in volume

(iii)

Answer: The total productivity is doubled if the fermenter volume is doubled

(b)

Answer: The volumetric productivity is doubled, the specific productivity is unaffected, and the total productivity is doubled

(c)

(i)

R_A = 100 kg day^{-1}; r_A = 0.8 g l^{-1} h^{-1}. From Eq. (12.17):

$$V = \frac{R_A}{r_A} = \frac{100 \text{ kg day}^{-1}}{0.8 \text{ g l}^{-1} \text{ h}^{-1} . \left| \frac{24 \text{ h}}{1 \text{ day}} \right| . \left| \frac{1 \text{ kg}}{1000 \text{ g}} \right|} = 5208 \text{ l}$$

Answer: 5208 l

(ii)

Applying Eq. (12.98) to product A:

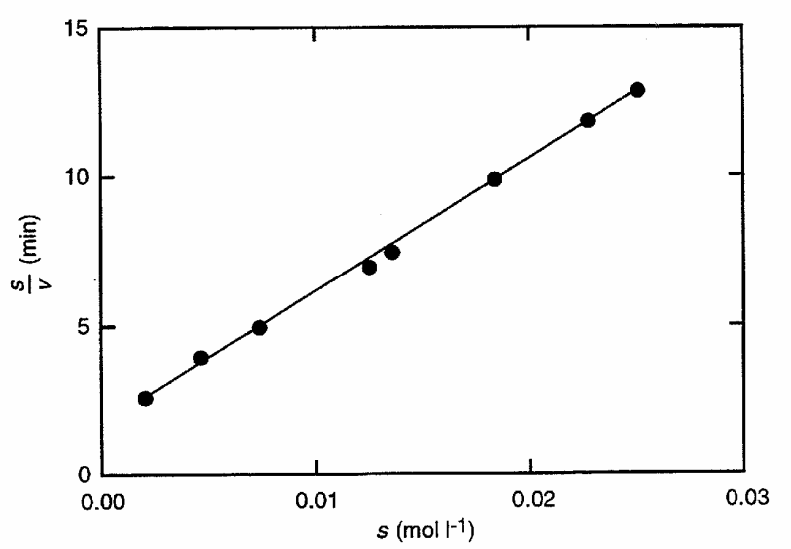
$$q_A = \frac{r_A}{x} = \frac{0.8 \text{ g l}^{-1} \text{ h}^{-1}}{20 \text{ g l}^{-1}} = 0.04 \text{ g g}^{-1} \text{ h}^{-1}$$

Answer: 0.04 g g^{-1} h^{-1}

12.4 Enzyme kinetics

The enzyme kinetic parameters are evaluated by plotting *s*/*v* versus *s* as a Langmuir plot (Section 12.4.4). The values are listed and plotted below.

s (mol l^{-1})	s/v (min)
0.0250	12.89
0.0227	11.88
0.0184	9.95
0.0135	7.50
0.0125	7.02
0.00730	5.00
0.00460	3.93
0.00204	2.62



The equation for the straight line in the plot is $s/v = 1.70 + 445s$, where s has units of mol l^{-1} and s/v has units of min. From Eq. (12.45), $1/v_{max} = 445$ mol^{-1} l min; therefore, $v_{max} = 2.25 \times 10^{-3}$ mol l^{-1} min^{-1}. Also from Eq. (12.45), $K_m/v_{max} = 1.70$ min. Multiplying this value by the result for v_{max} gives $K_m = 3.83 \times 10^{-3}$ mol l^{-1}.

Answer: $v_{max} = 2.25 \times 10^{-3}$ mol l^{-1} min^{-1}; $K_m = 3.83 \times 10^{-3}$ mol l^{-1}

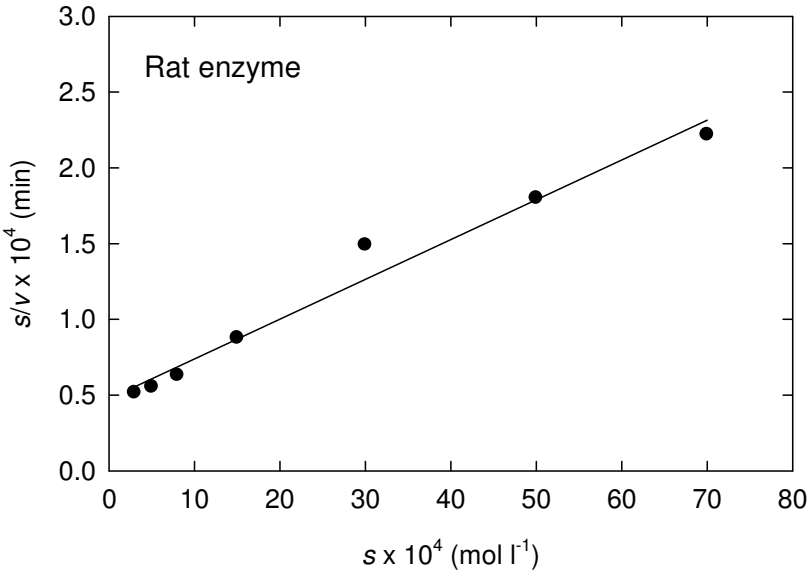
12.5 Enzyme kinetics after site-specific mutagenesis

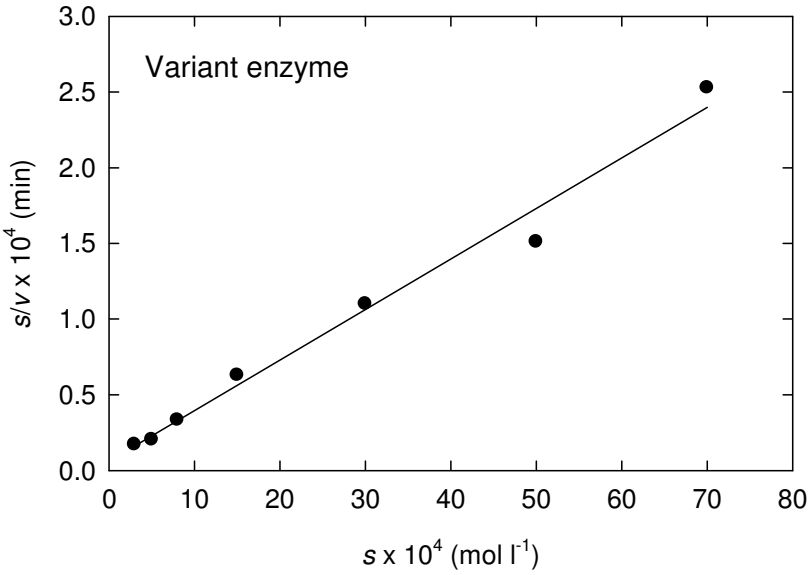
(a)

Use a basis of 1 mol l^{-1} of enzyme. The kinetic parameters for the rat and variant enzymes are evaluated by plotting s/v versus s as Langmuir plots (Section 12.4.4). The values are listed and plotted below.

	Rat enzyme		Variant enzyme	
s (mol l^{-1})	v (mol min^{-1} l^{-1})	s/v (min)	v (mol min^{-1} l^{-1})	s/v (min)
0.3×10^{-3}	5.82	5.15×10^{-5}	17.5	1.71×10^{-5}
0.5×10^{-3}	9.03	5.54×10^{-5}	24.5	2.04×10^{-5}
0.8×10^{-3}	12.7	6.30×10^{-5}	24.0	3.33×10^{-5}
1.5×10^{-3}	17.1	8.77×10^{-5}	23.9	6.28×10^{-5}

3×10^{-3}	20.2	1.49×10^{-4}	27.3	1.10×10^{-4}
5×10^{-3}	27.8	1.80×10^{-4}	33.1	1.51×10^{-4}
7×10^{-3}	31.5	2.22×10^{-4}	27.7	2.53×10^{-4}





The equation for the straight line in the plot for the rat enzyme is $s/v = 4.75 \times 10^{-5} + 2.63 \times 10^{-2}s$, where s has units of mol l^{-1} and s/v has units of min. Therefore, from Eq. (12.45), $1/v_{max} = 2.63 \times 10^{-2}$ mol^{-1} l min, so that $v_{max} = 38.0$ mol l^{-1} min^{-1}. Also from Eq. (12.45), $K_m/v_{max} = 4.75 \times 10^{-5}$ min. Multiplying this value by the result for v_{max}, $K_m = 1.81 \times 10^{-3}$ mol l^{-1}.

Similarly, the equation for the straight line in the plot for the variant enzyme is $s/v = 6.15 \times 10^{-6} + 3.34 \times 10^{-2}s$, where s has units of mol l^{-1} and s/v has units of min. From Eq. (12.45), $1/v_{max} = 3.34 \times 10^{-2}$ mol^{-1} l min; therefore, $v_{max} = 29.9$ mol l^{-1} min^{-1}. $K_m/v_{max} = 6.15 \times 10^{-6}$ min; therefore, $K_m = 1.84 \times 10^{-4}$ mol l^{-1}.

Answer: For the rat enzyme, $v_{max} = 38.0$ mol l^{-1} min^{-1} and $K_m = 1.81 \times 10^{-3}$ mol l^{-1}. For the variant enzyme, $v_{max} = 29.9$ mol l^{-1} min^{-1} and $K_m = 1.84 \times 10^{-4}$ mol l^{-1}.

(b)

If an enzyme has a relatively low K_m, it means that the reaction rate can be maintained at relatively high levels when the substrate concentration is low (Figure 12.7). Whereas the v_{max} values of the variant and rat enzymes are not very different, the variant enzyme K_m is an order of magnitude smaller than that of the rat enzyme. Therefore, the reaction at very low substrate concentrations can be expected to proceed at a significantly higher rate using the variant compared with the rat enzyme. The kinetic properties of the variant enzyme are therefore more suited for application to cancer patients.

Answer: Yes, because of its much lower K_m

(c)

The catalytic efficiency is defined as k_{cat}/K_m (Section 12.3.3). From Eq. (12.39):

$$k_{cat} = \frac{v_{max}}{e_a}$$

From the basis used in **(a)** to calculate the kinetic parameters, the concentration of active enzyme e_a = 1 mol l^{-1}. Therefore, k_{cat} = 38.0 min^{-1} for the rat enzyme and k_{cat} = 29.9 min^{-1} for the variant enzyme. Therefore, the catalytic efficiency or k_{cat}/K_m for the rat enzyme is (38.0 min^{-1})/(1.81 × 10^{-3} mol l^{-1}) = 2.1 × 10^4 mol^{-1} l min^{-1}. Similarly, the catalytic efficiency of the variant enzyme is (29.9 min^{-1})/(1.84 × 10^{-4} mol l^{-1}) = 1.6 × 10^5 mol^{-1} l min^{-1}. The efficiency of the variant enzyme is almost an order of magnitude greater than that of the rat enzyme.

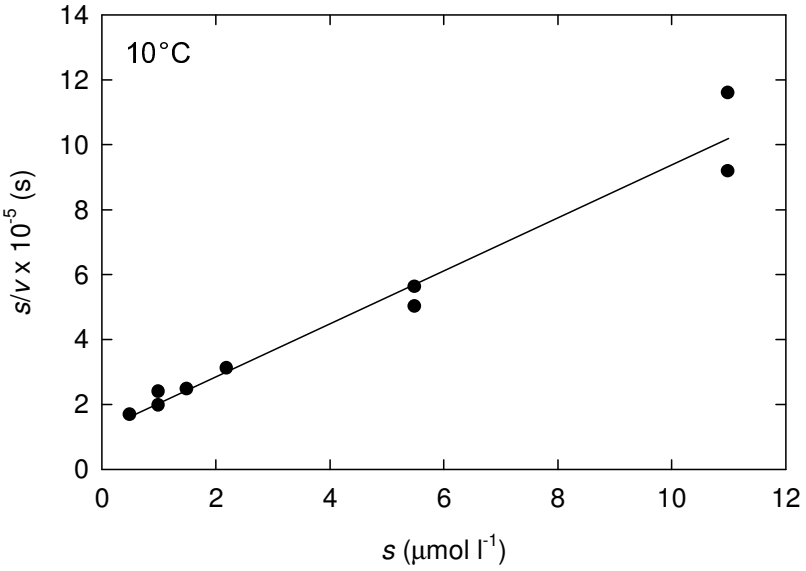
Answer: Yes

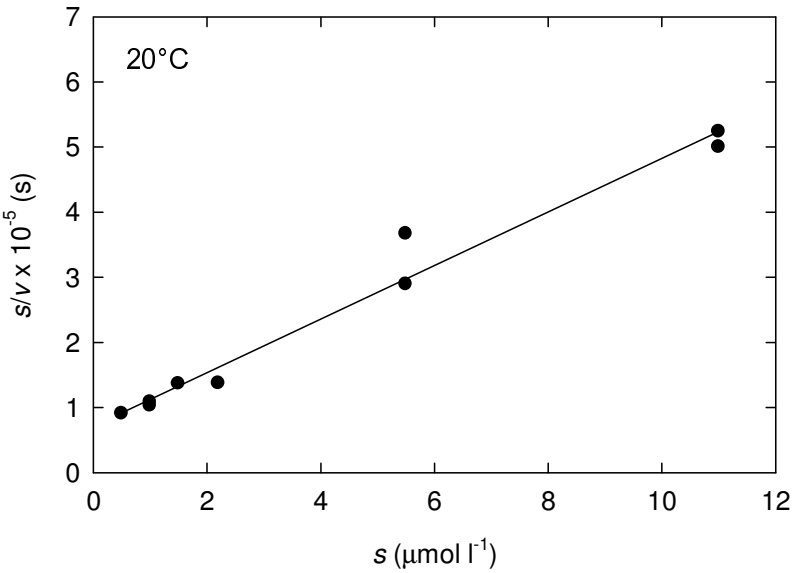
12.6 Kinetic properties of pheromone-degrading enzyme

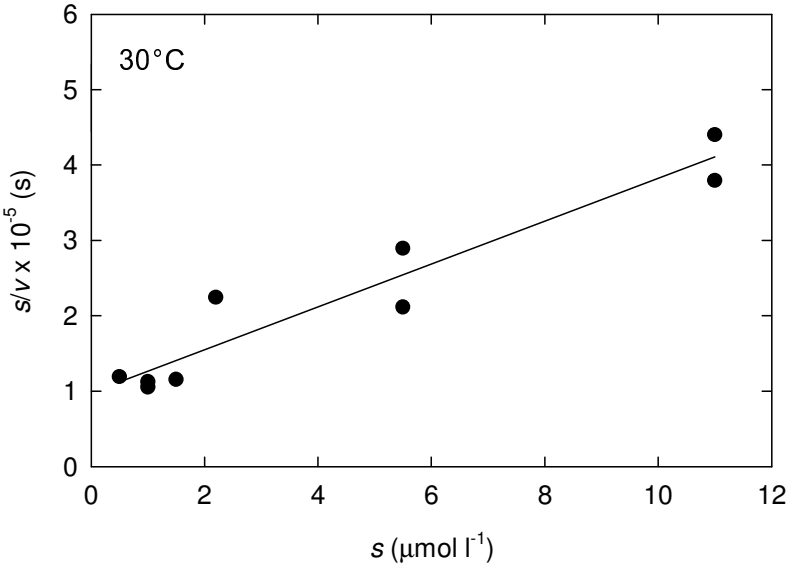
(a)

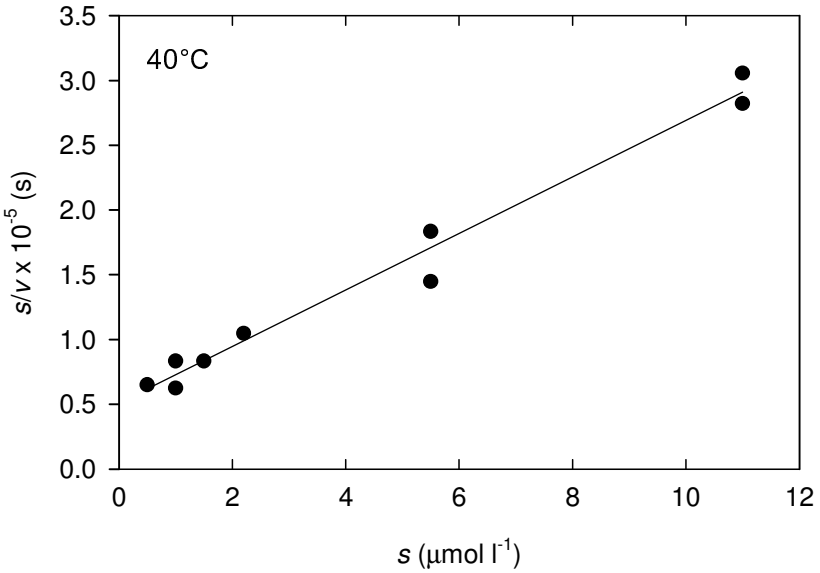
The kinetic parameters of the enzyme at the four different temperatures are evaluated by plotting *s*/*v* versus *s* as Langmuir plots (Section 12.4.4). The values are listed and plotted below.

	Temperature			
	10°C	20°C	30°C	40°C
s (μmol l^{-1})	*s*/*v* (s)	*s*/*v* (s)	*s*/*v* (s)	*s*/*v* (s)
0.5	1.67 × 10^5	9.09 × 10^4	1.19 × 10^5	6.49 × 10^4
1.0	1.96 × 10^5	1.09 × 10^5	1.05 × 10^5	8.33 × 10^4
1.0	2.38 × 10^5	1.03 × 10^5	1.12 × 10^5	6.25 × 10^4
1.5	2.46 × 10^5	1.36 × 10^5	1.15 × 10^5	8.33 × 10^4
2.2	3.10 × 10^5	1.38 × 10^5	2.24 × 10^5	1.05 × 10^5
5.5	5.00 × 10^5	3.67 × 10^5	2.89 × 10^5	1.45 × 10^5
5.5	5.61 × 10^5	2.89 × 10^5	2.12 × 10^5	1.83 × 10^5
11	9.17 × 10^5	5.00 × 10^5	3.79 × 10^5	2.82 × 10^5
11	1.16 × 10^6	5.24 × 10^5	4.40 × 10^5	3.06 × 10^5


10°C
s/v x 10-5 (s)
s (µmol l-1)
0
2
4
6
8
10
12
14


20°C
s/v x 10-5 (s)
s (µmol l-1)
0
1
2
3
4
5
6
7
8
10
12


30°C
s/v x 10-5 (s)
s (µmol l-1)
0
1
2
3
4
5
6
8
10
12



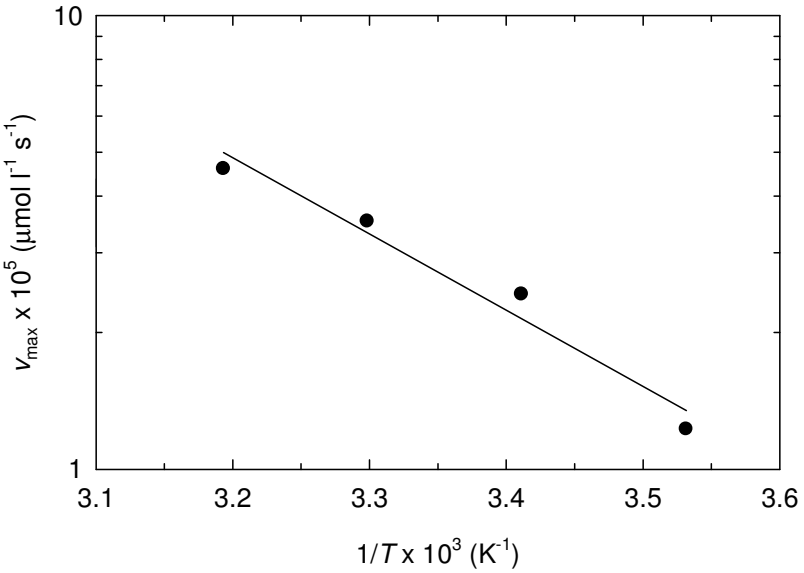
The equation for the straight line in the plot for 10°C is $s/v = 1.21 \times 10^5 + 8.17 \times 10^4 s$, where s has units of μmol l^{-1} and s/v has units of s. From Eq. (12.45), $1/v_{max} = 8.17 \times 10^4$ μmol^{-1} l s; therefore, $v_{max} = 1.22 \times 10^{-5}$ μmol l^{-1} s^{-1}. Also from Eq. (12.45), $K_m/v_{max} = 1.21 \times 10^5$ s. Multiplying this value by the result for v_{max} gives K_m = 1.48 μmol l^{-1}. When the plots for 20°C, 30°C and 40°C are analysed similarly, the following results are obtained.

Temperature	Langmuir equation	v_{max} (μmol l^{-1} s^{-1})	K_m (μmol l^{-1})
10°C	$s/v = 1.21 \times 10^5 + 8.17 \times 10^4 s$	1.22×10^{-5}	1.48
20°C	$s/v = 7.15 \times 10^4 + 4.12 \times 10^4 s$	2.43×10^{-5}	1.74
30°C	$s/v = 9.80 \times 10^4 + 2.85 \times 10^4 s$	3.51×10^{-5}	3.44
40°C	$s/v = 5.11 \times 10^4 + 2.18 \times 10^4 s$	4.59×10^{-5}	2.35

(b)

The activation energy is determined from the Arrhenius equation, Eq. (12.22), with v_{max} as the rate constant k. According to the Arrhenius equation, a plot of v_{max} versus $1/T$ on semi-logarithmic coordinates should give a straight line. T is converted to kelvin using Eq. (2.27). The parameter values are listed and plotted below.

T (°C)	T (K)	$1/T$ (K^{-1})	v_{max} (μmol l^{-1} s^{-1})
10	283.15	3.53×10^{-3}	1.22×10^{-5}
20	293.15	3.41×10^{-3}	2.43×10^{-5}
30	303.15	3.30×10^{-3}	3.51×10^{-5}
40	313.15	3.19×10^{-3}	4.59×10^{-5}



The equation for the straight line in the plot is $v_{max} = 11.40e^{-3864/T}$, where v_{max} has units of μmol l^{-1} s^{-1} and T has units of K. Therefore, from Eq. (12.22), E/R = 3864 K. From Table B.1 (Appendix B), R = 8.3144 J K^{-1} $gmol^{-1}$ = 8.3144 × 10^{-3} kJ K^{-1} $gmol^{-1}$; therefore, E = 3864 K × 8.3144 × 10^{-3} kJ K^{-1} $gmol^{-1}$ = 32.1 kJ $gmol^{-1}$.

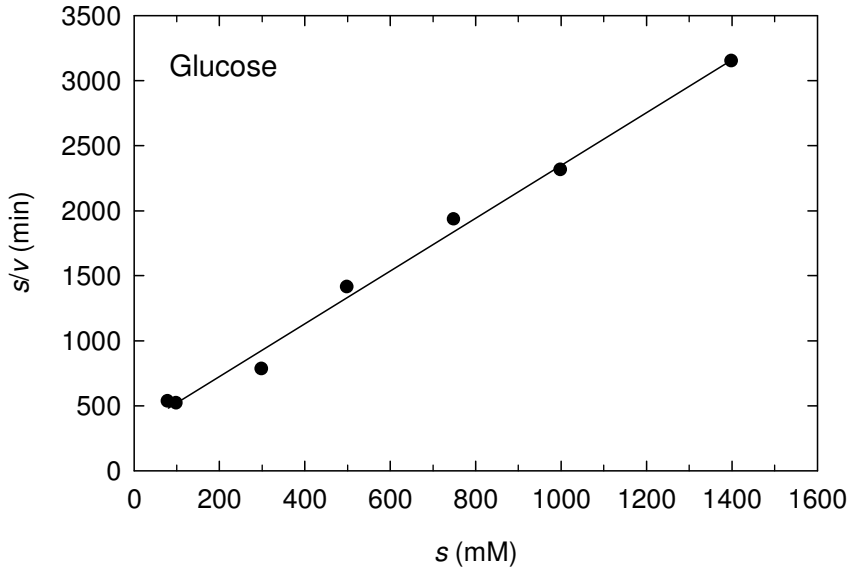
Answer: 32.1 kJ $gmol^{-1}$

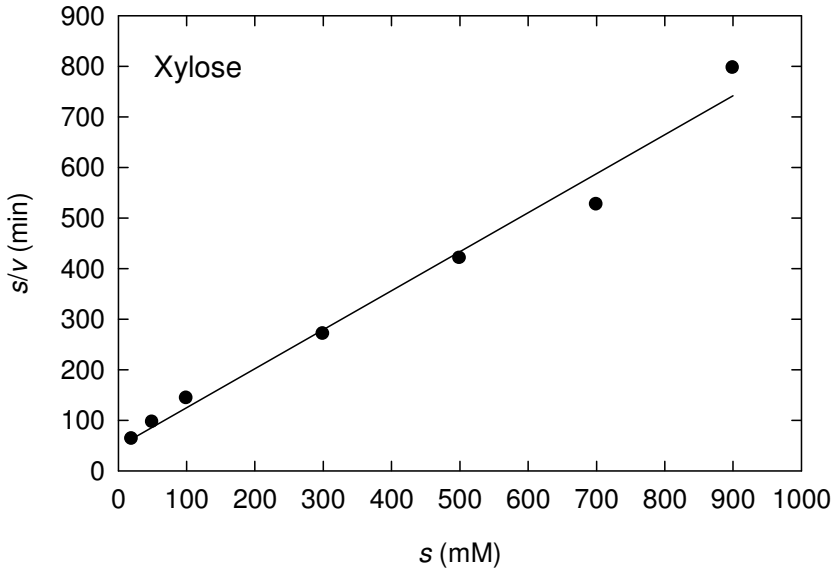
12.7 Enzyme substrate specificity

(a)

The kinetic parameters for the enzyme using glucose or xylose as substrate are evaluated by plotting s/v versus s as Langmuir plots (Section 12.4.4). The values are listed and plotted below.

Glucose			Xylose		
s (mM)	v (μmol ml^{-1} min^{-1})	s/v (min)	s (mM)	v (μmol ml^{-1} min^{-1})	s/v (min)
80	0.151	529.8	20	0.320	62.50
100	0.194	515.5	50	0.521	95.97
300	0.385	779.2	100	0.699	143.1
500	0.355	1408	300	1.11	270.3
750	0.389	1928	500	1.19	420.2
1000	0.433	2309	700	1.33	526.3
1400	0.445	3146	900	1.13	796.5





The equation for the straight line in the plot for glucose is $s/v = 318.7 + 2.03s$, where s has units of mM and s/v has units of min. From Eq. (12.45), $1/v_{max} = 2.03$ mM^{-1} min; therefore, $v_{max} = 0.493$ mM min^{-1}. Also from Eq. (12.45), $K_m/v_{max} = 318.7$ min. Multiplying this value by the result for v_{max}, $K_m = 157$ mM. From Section 12.3.3, the catalytic efficiency is defined as k_{cat}/K_m. From Eq. (12.39):

$$k_{cat} = \frac{v_{max}}{e_a}$$

Therefore:

$$\text{Catalytic efficiency} = \frac{v_{max}}{e_a K_m}$$

Substituting values for glucose as substrate with $e_a = 0.06$ mg ml^{-1}:

$$\text{Catalytic efficiency} = \frac{0.493 \text{ mM min}^{-1}}{0.06 \text{ mg ml}^{-1}\,(157 \text{ mM})} = 0.052 \text{ mg}^{-1} \text{ ml min}^{-1}$$

Similarly, the equation for the straight line in the plot for xylose is $s/v = 47.61 + 0.771s$, where s has units of mM and s/v has units of min. From Eq. (12.45), $1/v_{max} = 0.771$ mM^{-1} min; therefore, $v_{max} = 1.30$ mM min^{-1}. Also, $K_m/v_{max} = 47.61$ min; therefore, $K_m = 61.9$ mM. The catalytic efficiency is:

$$\text{Catalytic efficiency} = \frac{1.30 \text{ mM min}^{-1}}{0.06 \text{ mg ml}^{-1} (61.9 \text{ mM})} = 0.350 \text{ mg}^{-1} \text{ ml min}^{-1}$$

The catalytic efficiency of the enzyme with xylose as substrate is 0.350/0.052 = 6.7 times that with glucose as substrate.

Answer: With xylose as substrate, the catalytic efficiency of the enzyme is 6.7-fold greater than with glucose as substrate

(b)

From Section 12.3.3, catalytic efficiency is a measure of the substrate specificity of the enzyme. Therefore, this enzyme has a greater specificity for xylose as substrate than for glucose.

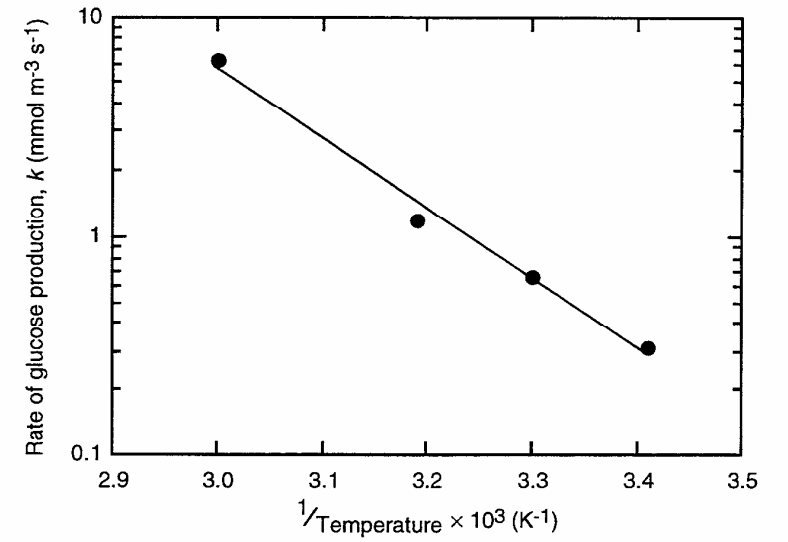
Answer: The enzyme has a greater specificity for xylose as substrate compared with glucose

12.8 Effect of temperature on the hydrolysis of starch

(a)

The activation energy is determined using the Arrhenius equation, Eq. (12.22), with k equal to the initial rate of glucose production. According to the Arrhenius equation, a plot of k versus $1/T$ on semi-logarithmic coordinates should give a straight line. The parameter values are listed and plotted below; T is converted to kelvin using Eq. (2.27).

T (°C)	T (K)	$1/T$ (K^{-1})	Rate, k (mmol m^{-3} s^{-1})
20	293.15	3.41×10^{-3}	0.31
30	303.15	3.30×10^{-3}	0.66
40	313.15	3.19×10^{-3}	1.20
60	333.15	3.00×10^{-3}	6.33



The equation for the straight line in the plot is $k = 1.87 \times 10^{10}\, e^{-7300/T}$, where k has units of mmol m^{-3} s^{-1} and T has units of K. Therefore, from Eq. (12.22), $E/R = 7300$ K. From Table B.1 (Appendix B), $R = 8.3144$ J K^{-1} gmol^{-1} $= 8.3144 \times 10^{-3}$ kJ K^{-1} gmol^{-1}; therefore, $E = 7300 \text{ K} \times 8.3144 \times 10^{-3}$ kJ K^{-1} gmol^{-1} $= 60.7$ kJ gmol^{-1}.

Answer: 60.7 kJ gmol^{-1}

(b)

Converting 55°C to kelvin using Eq. (2.27), T = (55 + 273.15) K = 328.15 K. Substituting this value into the equation for k obtained in **(a)**:

$$k = 1.87\times10^{10}\, e^{-7300/T} = 1.87\times10^{10}\, e^{-7300/328.15} = 4.08 \text{ mmol m}^{-3}\text{ s}^{-1}$$

Similarly, for T = 25°C = (25 + 273.15) K = 298.15 K:

$$k = 1.87\times10^{10}\, e^{-7300/T} = 1.87\times10^{10}\, e^{-7300/298.15} = 0.43 \text{ mmol m}^{-3}\text{ s}^{-1}$$

Therefore, the rate at 55°C is 4.08/0.43 = 9.5 times faster than at 25°C.

Answer: The reaction rate at 55°C is 4.08 mmol m^{-3} s^{-1} or 9.5 times faster than the rate of 0.43 mmol m^{-3} s^{-1} at 25°C

(c)

From Table B.1 (Appendix B), the value of R in the appropriate units is 1.9872 cal K^{-1} gmol^{-1}. Using the equation provided, at 55°C = 328.15 K:

$$k_\text{d} = 2.25\times10^{27}\, e^{-41{,}630/(1.9872\times328.15)} = 0.42 \text{ h}^{-1}$$

Therefore, from Eq. (12.73), the half-life of the enzyme at 55°C is:

$$t_\text{h} = \frac{\ln 2}{0.42 \text{ h}^{-1}} = 1.65 \text{ h}$$

At 25°C = 298.15 K:

$$k_\text{d} = 2.25\times10^{27}\, e^{-41{,}630/(1.9872\times298.15)} = 6.87\times10^{-4} \text{ h}^{-1}$$

and the enzyme half-life is:

$$t_\text{h} = \frac{\ln 2}{6.87\times10^{-4} \text{ h}^{-1}} = 1009 \text{ h}$$

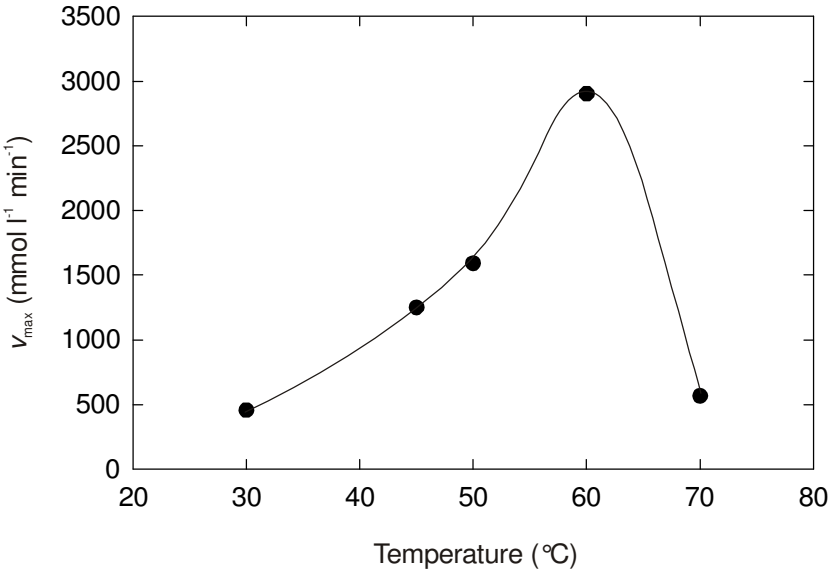
Although, from **(b)**, the reaction rate is 9.5 times faster at 55°C than at 25°C, the rate of deactivation is 0.42/(6.87 × 10^{-4}) = 611 times greater. Therefore, unless there are other considerations, 25°C would probably be the more practical temperature for processing operations.

Answer: The enzyme half-life at 25°C is 1009 h or 611 times longer than the half-life of 1.65 h at 55°C. The more practical operating temperature is probably 25°C.

12.9 Optimum temperature for enzymatic hydrolysis of cellulose

(a)

A plot of the raw data, v_max versus temperature, is shown below.



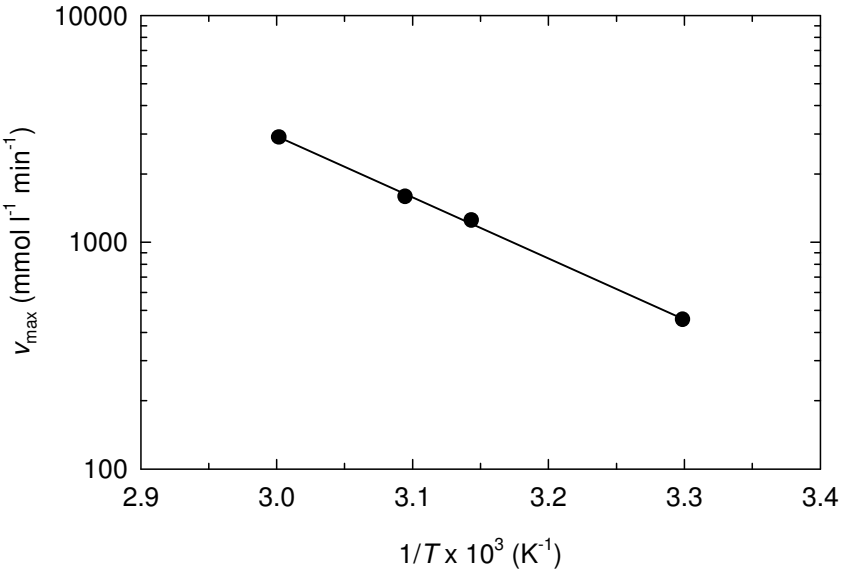
The activity of the enzyme is maximum at around 60°C.

Answer: Around 60°C

(b)

Thermal activation of the enzyme occurs between 30°C and 60°C; the result at 70°C reflects thermal destruction or inactivation of the enzyme (Section 12.3.4). The Arrhenius parameters are determined by fitting the data between 30°C and 60°C to Eq. (12.22) with v_{max} as the rate constant k. According to the Arrhenius equation, a plot of v_{max} versus $1/T$ on semi-logarithmic coordinates should give a straight line. T is converted to kelvin using Eq. (2.27). The parameter values are listed and plotted below.

T (°C)	T (K)	$1/T$ (K^{-1})	v_{max} (mmol l^{-1} min^{-1})
30	303.15	3.30×10^{-3}	456
45	318.15	3.14×10^{-3}	1250
50	323.15	3.09×10^{-3}	1590
60	333.15	3.00×10^{-3}	2900



The equation for the straight line in the plot is $v_{max} = 3.52 \times 10^{11}\ e^{-6201/T}$, where v_{max} has units of mmol l^{-1} min^{-1} and T has units of K. Therefore, from Eq. (12.22), E/R = 6201 K. From Table B.1 (Appendix B), R = 8.3144 J K^{-1} $gmol^{-1}$ = 8.3144×10^{-3} kJ K^{-1} $gmol^{-1}$. Therefore, E = 6201 K × 8.3144×10^{-3} kJ K^{-1} $gmol^{-1}$ = 51.6 kJ $gmol^{-1}$. Also from Eq. (12.22), $A = 3.52 \times 10^{11}$ mmol l^{-1} min^{-1}.

Answer: The activation energy E is 51.6 kJ $gmol^{-1}$; the Arrhenius constant A is 3.52×10^{11} mmol l^{-1} min^{-1}

(c)

From Eq. (2.27), 55°C = (55 + 273.15) K = 328.15 K. Applying the equation derived in **(b)** to evaluate v_{max}:

$$v_{max} = 3.52 \times 10^{11}\, e^{-6201/328.15} = 2186$$

Therefore, v_{max} = 2186 mmol l^{-1} min^{-1}.

Answer: 2190 mmol l^{-1} min^{-1}

12.10 Enzyme reaction and deactivation

K_m = 5 mM = 5×10^{-3} gmol l^{-1} = 5 gmol m^{-3}. The concentration of fat is reduced from 45 gmol m^{-3} to 0.2 × 45 gmol m^{-3} = 9.0 gmol m^{-3}. The reaction starts with $s >> K_m$ and ends with $s > K_m$. We will assume that s can be considered >> K_m for the entire duration of the reaction so that $v \approx v_{max}$ (Section 12.3.3) and an analytical solution can be obtained. Combining Eqs (12.41) and (12.72) and substituting $v = -ds/dt$ gives:

$$\frac{-ds}{dt} = v_{max0}\, e^{-k_d t}$$

In this equation, s and t are the only variables. Separating variables and integrating:

$$\int -ds = \int v_{max0}\, e^{-k_d t}\, dt$$

Applying Eq. (E.24) from Appendix E and the inverse of Eq. (E.17) for integration:

$$-s = \frac{-v_{max0}}{k_d} e^{-k_d t} + K$$

where K is the combined constant of integration. The initial condition is: at $t = 0$, $s = s_0$. Therefore:

$$-s_0 = \frac{-v_{max0}}{k_d} + K$$

$$K = \frac{v_{max0}}{k_d} - s_0$$

Substituting this expression for K into the equation for $-s$ gives:

$$-s = \frac{-v_{max0}}{k_d} e^{-k_d t} + \frac{v_{max0}}{k_d} - s_0$$

or

$$s = s_0 - \frac{v_{max0}}{k_d}(1 - e^{-k_d t})$$

At the beginning of the reaction, s_0 = 45 gmol m^{-3} and v_{max0} = 0.07 mmol l^{-1} s^{-1}. Converting the units of v_{max0}:

$$v_{max0} = 0.07 \text{ mmol } l^{-1} s^{-1} \cdot \left|\frac{1 \text{ gmol}}{1000 \text{ mmol}}\right| \cdot \left|\frac{1000\, l}{1\, m^3}\right| \cdot \left|\frac{60 \text{ s}}{1 \text{ min}}\right| = 4.2 \text{ gmol } m^{-3} \text{ min}^{-1}$$

From Eq. (12.73):

$$k_d = \frac{\ln 2}{t_h} = \frac{\ln 2}{8\ \text{min}} = 0.087\ \text{min}^{-1}$$

Substituting parameter values into the equation for s, when 80% of the fat is hydrolysed so that s = 9.0 gmol m^{-3}:

$$9.0\ \text{gmol m}^{-3} = 45\ \text{gmol m}^{-3} - \frac{4.2\ \text{gmol m}^{-3}\ \text{min}^{-1}}{0.087\ \text{min}^{-1}}(1 - e^{-0.087t})$$

where t has units of min. Grouping terms and solving for t:

$$12.276 = 48.276\, e^{-0.087t}$$

$$e^{-0.087t} = 0.254$$

$$-0.087t = -1.369$$

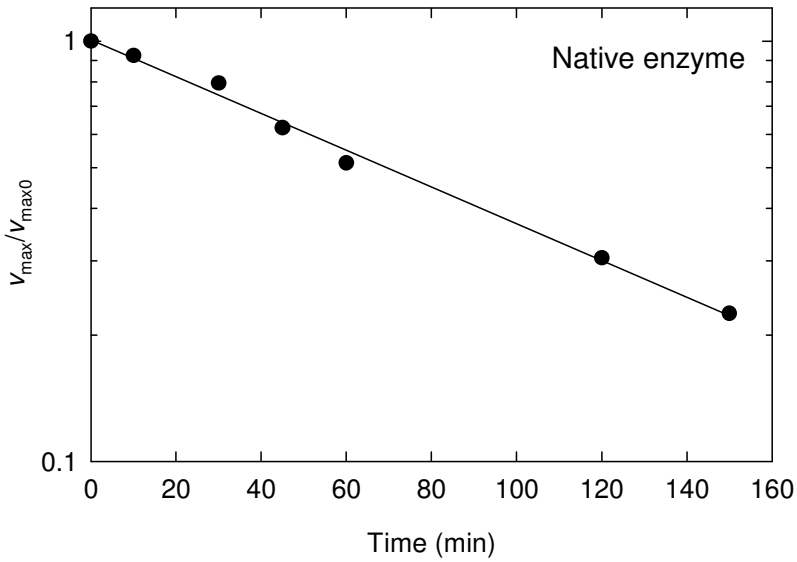
$$t = 15.7$$

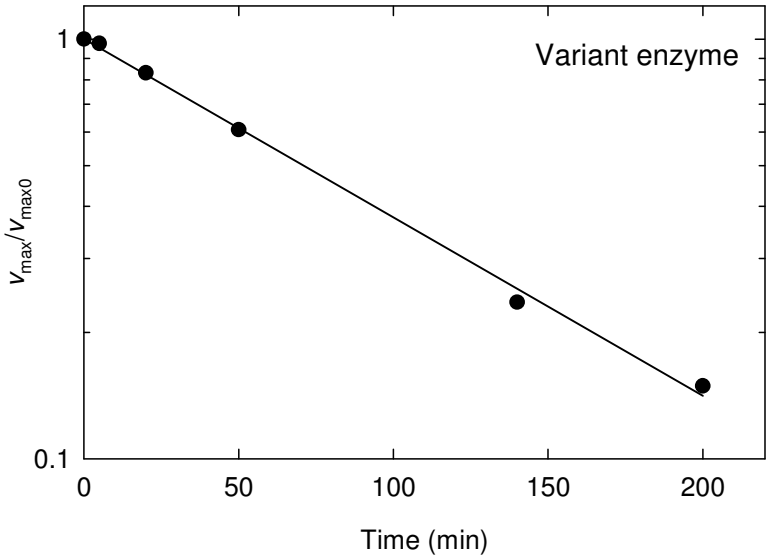
Answer: 16 min, assuming that $v \approx v_{max}$ for the entire reaction

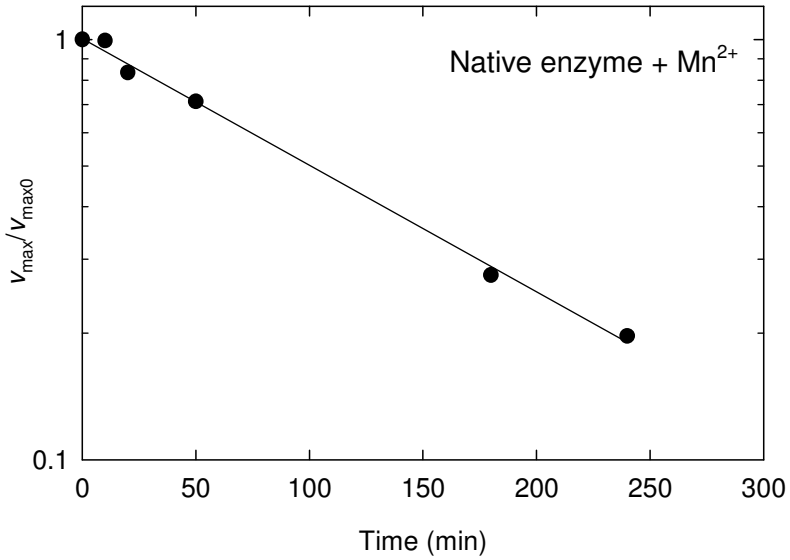
12.11 Effect of amino acid composition and metal binding on enzyme stability

(a)

Under conditions of excess substrate, the activity of the reaction = v_{max} and the ratio of activity to initial activity = v_{max}/v_{max0}. The equation for first-order enzyme deactivation is Eq. (12.72). From this equation, if the data for each of the reaction conditions follow first-order kinetics, semi-log plots of v_{max}/v_{max0} versus time should produce straight lines. These plots are shown below.







As straight-line plots are obtained for each of the conditions tested, we can say that the enzyme follows first-order deactivation kinetics.

Answer: Yes

(b)

The equations for each of the straight lines in the plots in **(a)** are listed below. The values of the deactivation rate constant k_d in Eq. (12.72) and the enzyme half-life t_h calculated using Eq. (12.73) are also shown.

Reaction condition	Equation	k_d (min^{-1})	t_h (min)
Native enzyme	$v_{max}/v_{max0} = 1.007\ e^{-0.0101t}$	0.0101	68.6
Variant enzyme	$v_{max}/v_{max0} = 1.002\ e^{-0.0098t}$	0.0098	70.7
Native enzyme + Mn^{2+}	$v_{max}/v_{max0} = 1.004\ e^{-0.0069t}$	0.0069	100.5

Answer: The half-life of the variant enzyme is 71 min, compared with 69 min for the native enzyme and 100 min for the native enzyme with Mn^{2+}

(c)

Relative to the native enzyme, the half-life of the variant enzyme is increased by only:

$$\frac{(70.7-68.6)\text{ min}}{68.6\text{ min}}\times 100\% = 3.1\%$$

In contrast, adding Mn^{2+} to the reaction with native enzyme increases the half-life by:

$$\frac{(100.5-68.6)\text{ min}}{68.6\text{ min}}\times 100\% = 47\%$$

Therefore, adding Mn^{2+} to the reaction mixture was a more effective strategy for increasing the stability of this enzyme than developing the variant enzyme using site-directed mutagenesis.

Answer: Adding Mn^{2+} was more effective than site-directed mutagenesis

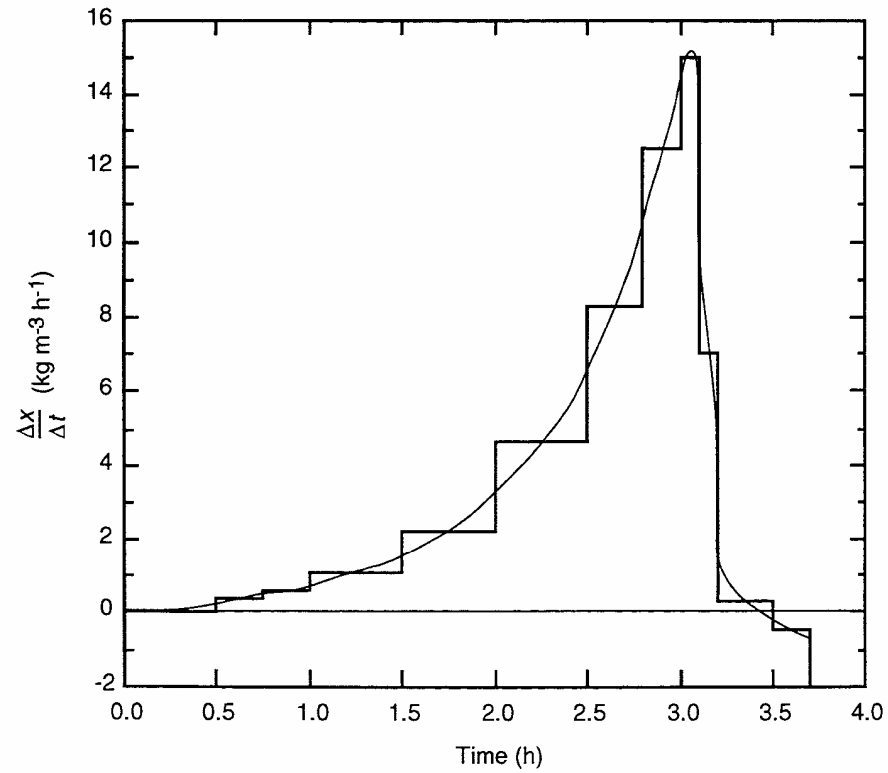
12.12 Growth parameters for recombinant *E. coli*

(a)

To evaluate μ as a function of time, the rate of cell growth must be determined at different times throughout the culture period. The average rate–equal area construction is used to determine growth rates from the concentration data. The data and calculations are tabulated below.

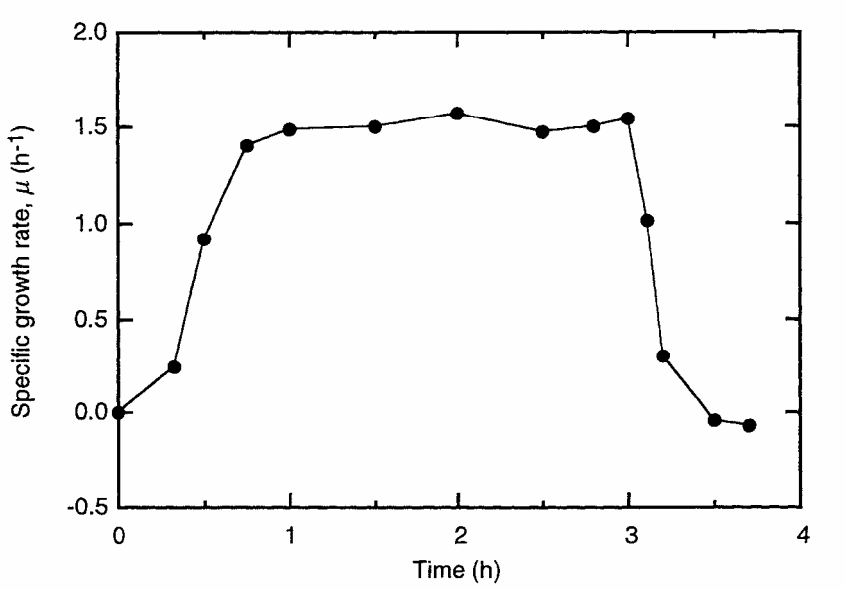
Time, t (h)	x (kg m^{-3})	Δx (kg m^{-3})	Δt (h)	$\Delta x/\Delta t$ (kg m^{-3} h^{-1})
0.0	0.20			
		0.01	0.33	0.03
0.33	0.21			
		0.01	0.17	0.06
0.5	0.22			
		0.10	0.25	0.40
0.75	0.32			
		0.15	0.25	0.60
1.0	0.47			
		0.53	0.50	1.06
1.5	1.00			
		1.10	0.50	2.20
2.0	2.10			
		2.32	0.50	4.64
2.5	4.42			
		2.48	0.30	8.27
2.8	6.9			
		2.50	0.20	12.50
3.0	9.4			
		1.50	0.10	15.00
3.1	10.9			
		0.70	0.10	7.00
3.2	11.6			
		0.10	0.30	0.33
3.5	11.7			
		–0.10	0.20	–0.50
3.7	11.6			

The values of $\Delta x/\Delta t$ are plotted as a function of time according to the method described in Section 12.2.1.



Values of dx/dt are read from the average rate–equal area curve. The specific growth rate μ is determined by dividing r_X by the corresponding cell concentration. The results are listed and plotted below.

Time, t (h)	$r_X = dx/dt$ (kg m^{-3} h^{-1})	$\mu = 1/x\ dx/dt$ (h^{-1})
0.0	0	0
0.33	0.05	0.24
0.5	0.20	0.91
0.75	0.45	1.41
1.0	0.70	1.49
1.5	1.5	1.50
2.0	3.3	1.57
2.5	6.5	1.47
2.8	10.4	1.51
3.0	14.5	1.54
3.1	11.0	1.01
3.2	3.5	0.30
3.5	–0.3	–0.03
3.7	–0.8	–0.07



(b)

As expected for most batch cell cultures, growth occurs at around the maximum specific growth rate for most of the culture period. Taking the average of the values of μ between times 0.75 h and 3 h, $\mu_{max} = 1.50 \pm 0.02\ h^{-1}$, where 0.02 is the standard error.

Answer: 1.50 h^{-1}

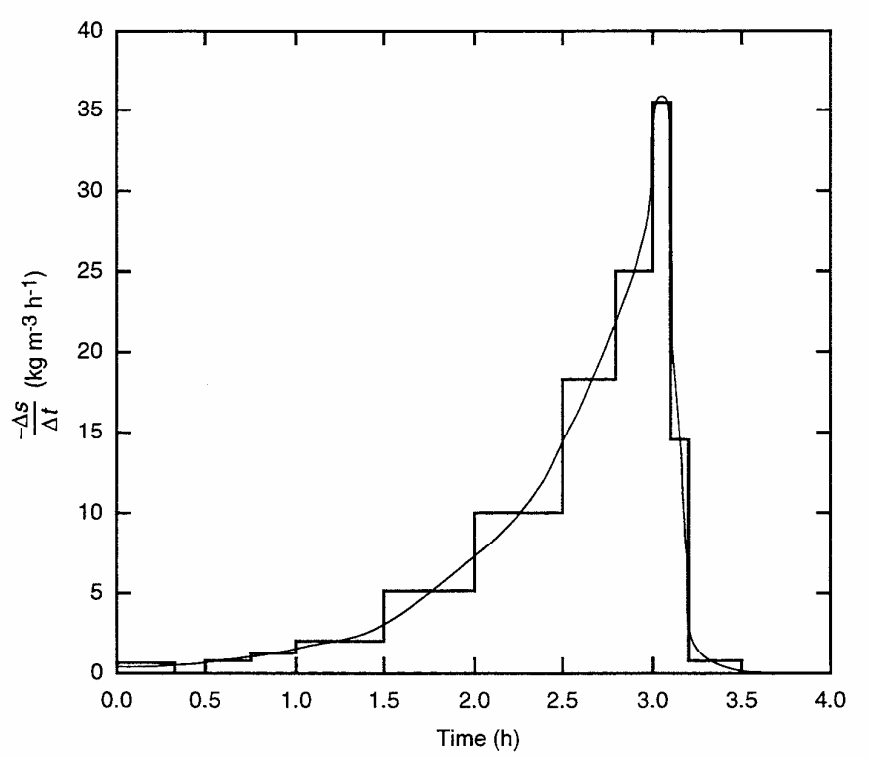
(c)

From Eq. (12.108), the observed biomass yield Y'_{XS} at any point in time is equal to the ratio of the observed growth and substrate consumption rates. The average rate–equal area construction is used to determine the substrate consumption rates $r_S = -ds/dt$ from the substrate concentration data.

Time, t (h)	s (kg m^{-3})	Δs (kg m^{-3})	Δt (h)	$-\Delta s/\Delta t$ (kg m^{-3} h^{-1})
0.0	25.0	–0.2	0.33	0.61
0.33	24.8	0.0	0.17	0.0
0.5	24.8	–0.2	0.25	0.80
0.75	24.6	–0.3	0.25	1.20
1.0	24.3	–1.0	0.50	2.00
1.5	23.3	–2.6	0.50	5.20
2.0	20.7	–5.0	0.50	10.0
2.5	15.7	–5.5	0.30	18.3
2.8	10.2	–5.0	0.20	25.0
3.0	5.2	–3.55	0.10	35.5

3.1	1.65	–1.45	0.10	14.5
3.2	0.2	–0.2	0.30	0.67
3.5	0.0	0.0	0.20	0.0
3.7	0.0			

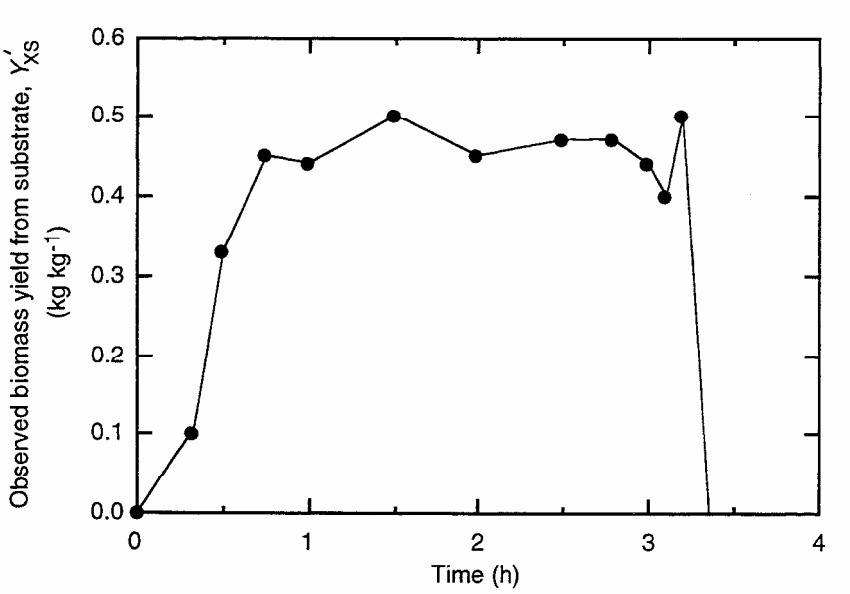
The values of $-\Delta s/\Delta t$ are plotted as a function of time according to the method described in Section 12.2.1.



Results for $-ds/dt$ read from the average rate–equal area curve are listed below. The instantaneous biomass yield coefficients calculated using Eq. (12.108) and the values of r_X from **(a)** are listed and plotted below.

Time, t (h)	$r_S = -ds/dt$ (kg m^{-3} h^{-1})	$Y'_{XS} = r_X/r_S$ (kg kg^{-1})
0.0	0.4	0
0.33	0.5	0.10
0.5	0.6	0.33
0.75	1.0	0.45
1.0	1.6	0.44
1.5	3.0	0.50
2.0	7.4	0.45
2.5	13.9	0.47
2.8	22.2	0.47
3.0	33.0	0.44
3.1	27.5	0.40

3.2	7.0	0.50
3.5	0.4	–0.75
3.7	0	–



During the exponential growth phase between 0.75 h and 3 h when $\mu = \mu_{max}$, Y'_{XS} is approximately constant with an average value of 0.46 ± 0.01 kg kg^{-1}, where 0.01 is the standard error.

Answer: 0.46 kg kg^{-1}; Y'_{XS} is approximately constant during exponential growth

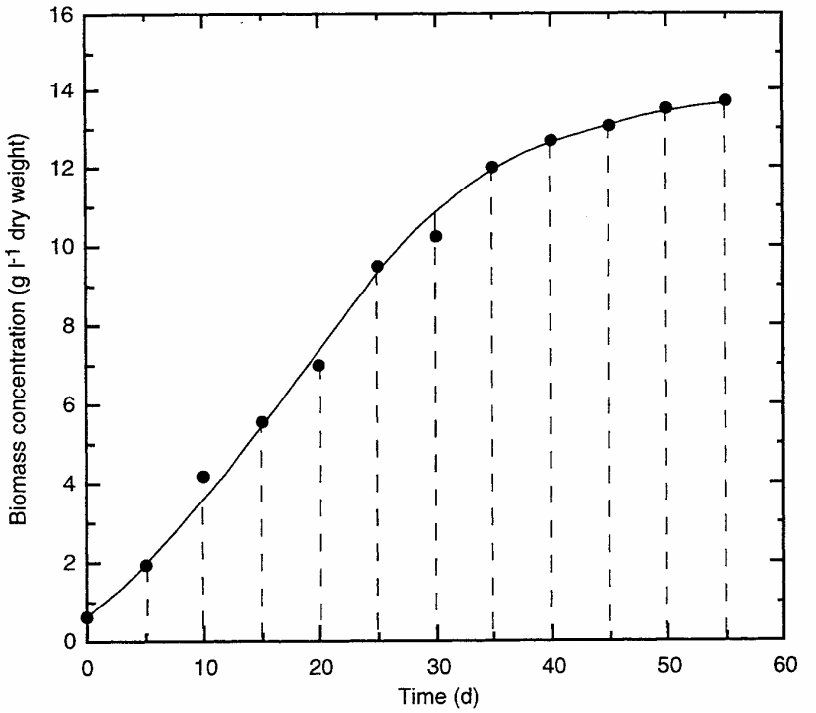
12.13 Growth parameters for hairy roots

(a)

To evaluate μ as a function of time, the rate of root growth must be determined at different times throughout the culture period. The mid-point slope method is used to determine growth rates from the biomass concentration data. The growth data are listed and plotted below according to the method described in Section 12.2.2. Values of $[(x)_{t+\varepsilon} - (x)_{t-\varepsilon}]$ are read from the graph.

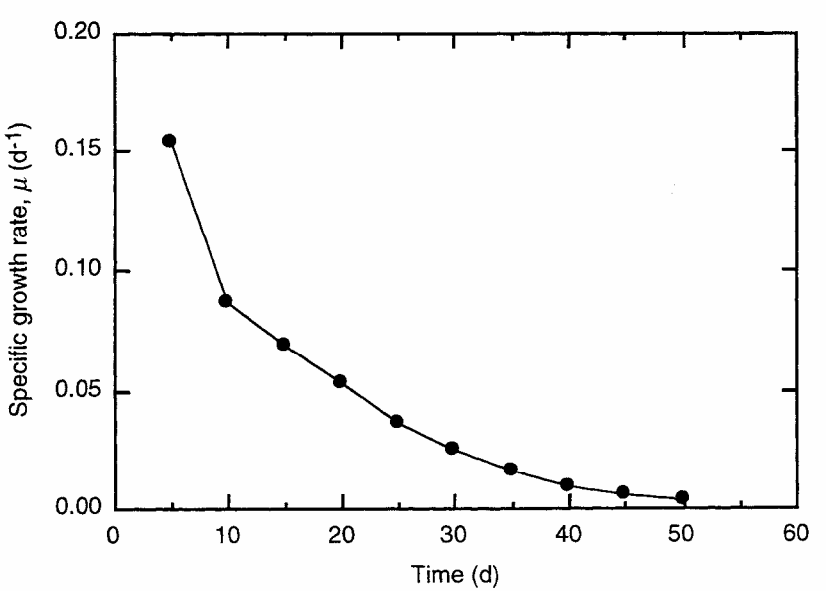
Time, t (days)	Biomass concentration, x (g l^{-1})	ε	$[(x)_{t+\varepsilon} - (x)_{t-\varepsilon}]$ g l^{-1}
0	0.64	5	–
5	1.95	5	3.0
10	4.21	5	3.6
15	5.54	5	3.8
20	6.98	5	3.7
25	9.50	5	3.4
30	10.3	5	2.6
35	12.0	5	1.9
40	12.7	5	1.2
45	13.1	5	0.8

50	13.5	5	0.6
55	13.7	5	–



The growth rate dx/dt is determined using the central-difference formula, Eq. (12.24). Values of the specific growth rate μ are determined by dividing r_X by the corresponding biomass concentration. The results are listed and plotted below.

Time, t (days)	$r_X = \mathrm{d}x/\mathrm{d}t$ (g l^{-1} day^{-1})	$\mu = 1/x\ \mathrm{d}x/\mathrm{d}t$ (day^{-1})
0	–	–
5	0.30	0.154
10	0.36	0.086
15	0.38	0.069
20	0.37	0.053
25	0.34	0.036
30	0.26	0.025
35	0.19	0.016
40	0.12	0.009
45	0.08	0.006
50	0.06	0.004
55	–	–

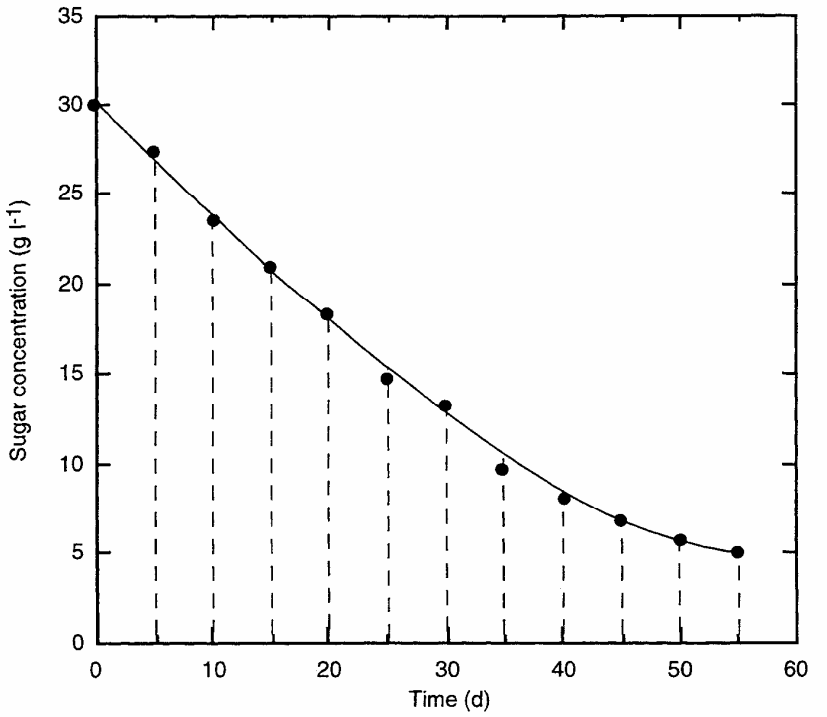


The results indicate that the specific growth rate of the roots declines continuously throughout the culture period.

Answer: Near the beginning of the culture

(b)

The mid-point slope method is used to determine the rate of substrate uptake as a function of time. The sugar concentration data are plotted below according to the method described in Section 12.2.2.

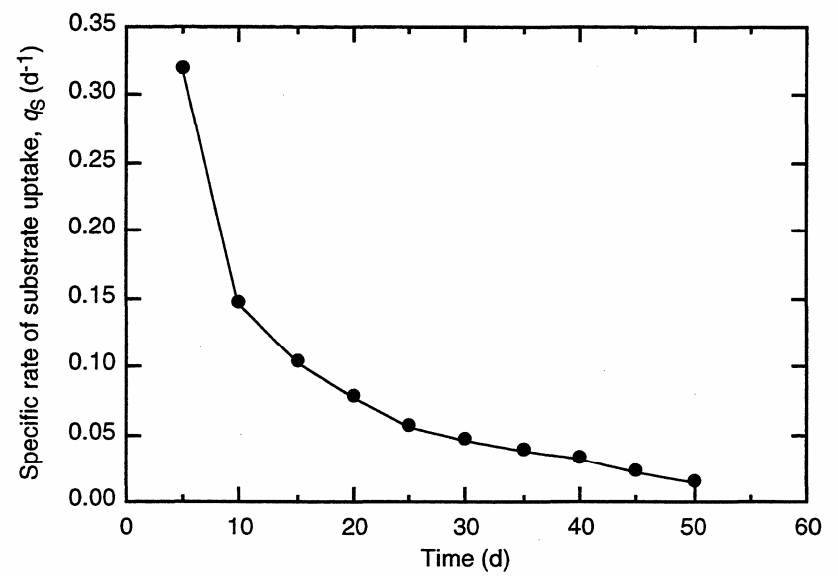


Values of $[(s)_{t+\varepsilon} - (s)_{t-\varepsilon}]$ read from the graph are listed in the table below.

Time, t (days)	Sugar concentration, s (g l^{-1})	ε	$[(s)_{t+\varepsilon} - (s)_{t-\varepsilon}]$ g l^{-1}
0	30.0	5	–
5	27.4	5	–6.2
10	23.6	5	–6.1
15	21.0	5	–5.7
20	18.4	5	–5.4
25	14.8	5	–5.2
30	13.3	5	–4.6
35	9.7	5	–4.4
40	8.0	5	–3.9
45	6.8	5	–2.7
50	5.7	5	–1.9
55	5.1	5	–

The rate of substrate uptake $-\mathrm{d}s/\mathrm{d}t$ is determined using the central-difference formula, Eq. (12.24). Values of the specific rate of substrate uptake q_S are determined by dividing r_S by the corresponding biomass concentration. The results are listed and plotted below.

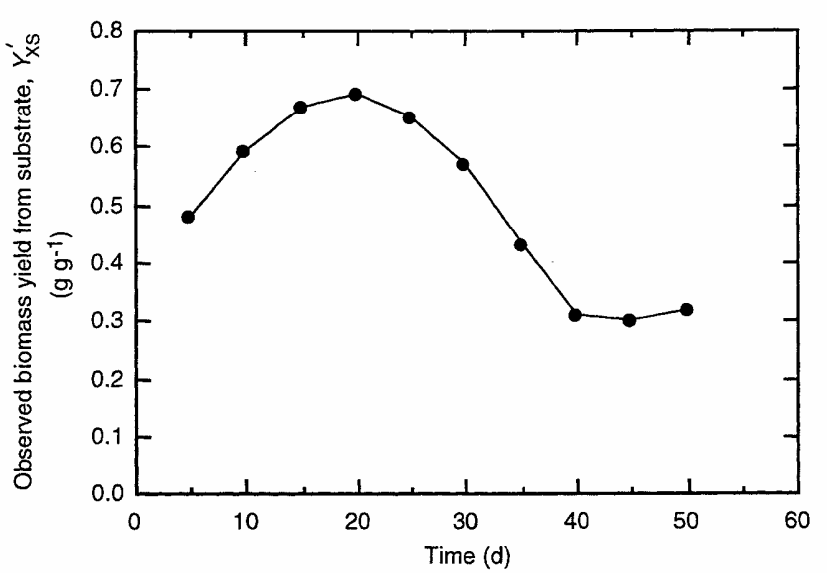
Time, t (days)	$r_S = -\mathrm{d}s/\mathrm{d}t$ (g l^{-1} day^{-1})	$q_S = -1/x\ \mathrm{d}s/\mathrm{d}t$ (day^{-1})
0	–	–
5	0.62	0.318
10	0.61	0.145
15	0.57	0.103
20	0.54	0.077
25	0.52	0.055
30	0.46	0.045
35	0.44	0.037
40	0.39	0.031
45	0.27	0.021
50	0.19	0.014
55	–	–



(c)

The instantaneous biomass yield coefficient is calculated using Eq. (12.108) and the values of r_X and r_S from **(a)** and **(b)**. The results are listed and plotted below.

Time, t (days)	$Y'_{XS} = r_X/r_S$ (g g^{-1})
0	–
5	0.48
10	0.59
15	0.67
20	0.69
25	0.65
30	0.57
35	0.43
40	0.31
45	0.30
50	0.32
55	–

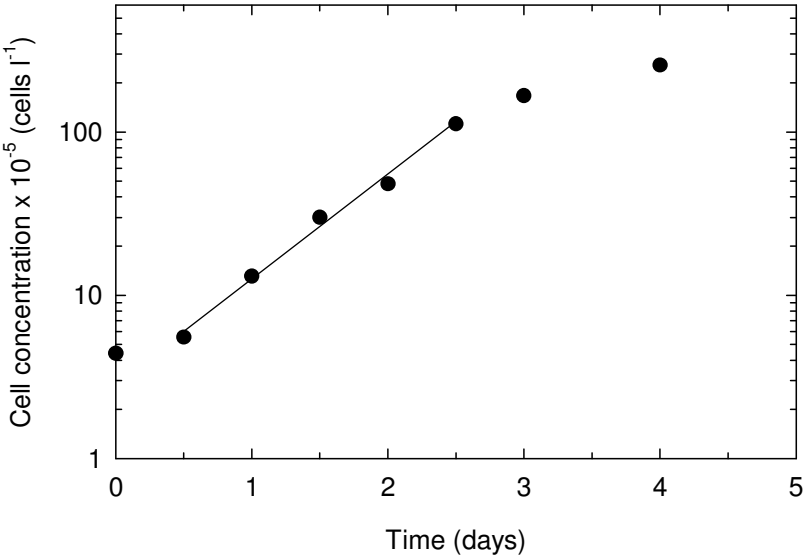


Answer: Y'_{XS} is not constant; it varies during the culture period within the range 0.30–0.69 g g^{-1}

12.14 Kinetics of diatom growth and silicate uptake

(a)

If the culture exhibits exponential growth, according to Eq. (12.85), a plot of cell concentration versus time on semi-logarithmic coordinates should give a straight line. This plot is shown below.

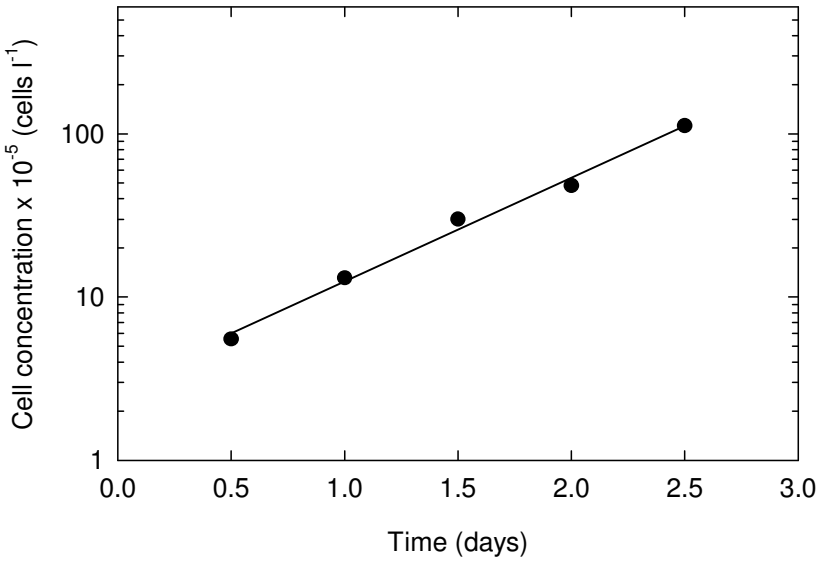


The cell concentration data can be fitted with a straight line for only part of the culture period: exponential growth occurs between about 0.5 and 2.5 days. There is a lag phase before 0.5 days; after 2.5 days, the rate of cell growth declines.

Answer: Yes, between about 0.5 and 2.5 days of the culture period

(b)

During exponential growth, μ is constant and equal to μ_{max} (Section 12.8.3). From Eq. (12.85), the value of μ can be obtained from a plot of cell concentration versus time on semi-logarithmic coordinates, where only those data corresponding to the exponential phase are analysed. This plot is shown below.



The equation to the straight line in the plot is $x = 2.89 \times 10^5 \, e^{1.464t}$, where x is cell concentration in units of cells l^{-1} and t is time in days. From Eq. (12.85), $\mu = 1.464$ day^{-1}; therefore, $\mu_{max} = 1.464$ day^{-1}.

Answer: 1.46 day^{-1}

(c)

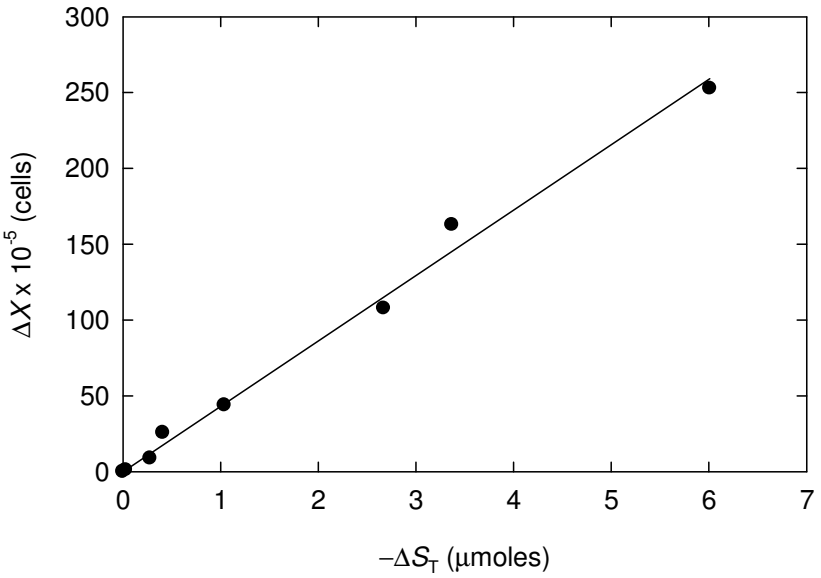
As shown in the plot in **(a)**, exponential growth does not begin until about 0.5 days. The culture has an initial lag phase before this time.

Answer: Yes

(d), (e)

The observed biomass yield from substrate is calculated using Eq. (12.78). As the measured data allow many values of ΔX and ΔS_T to be determined, we can use a plot of ΔX versus ΔS_T to estimate the yield. Using a basis of 1 litre and calculating ΔX and ΔS_T as the difference between the measured values and the initial value of X or S, the results are listed and plotted below.

Time (days)	ΔX (cells)	$-\Delta S_T$ (μmoles)
0	0.00	0.00
0.5	1.12×10^5	0.03
1.0	8.69×10^5	0.28
1.5	2.56×10^6	0.41
2.0	4.38×10^6	1.04
2.5	1.08×10^7	2.67
3.0	1.63×10^7	3.37
4.0	2.53×10^7	6.01



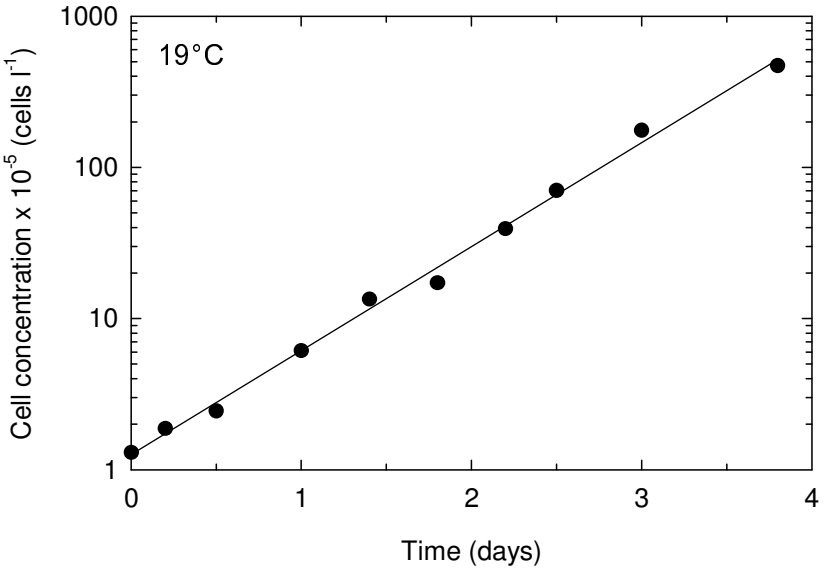
The data in the plot can be fitted with a straight line passing through the origin. The fit is reasonably good, indicating that the observed yield of biomass from substrate Y'_{XS} is constant throughout the culture period. The equation to the straight line is $-\Delta X/\Delta S_T = 4.31 \times 10^6$ cells μmole^{-1}; therefore, from Eq. (12.78), $Y'_{XS} = 4.31 \times 10^6$ cells μmole^{-1}.

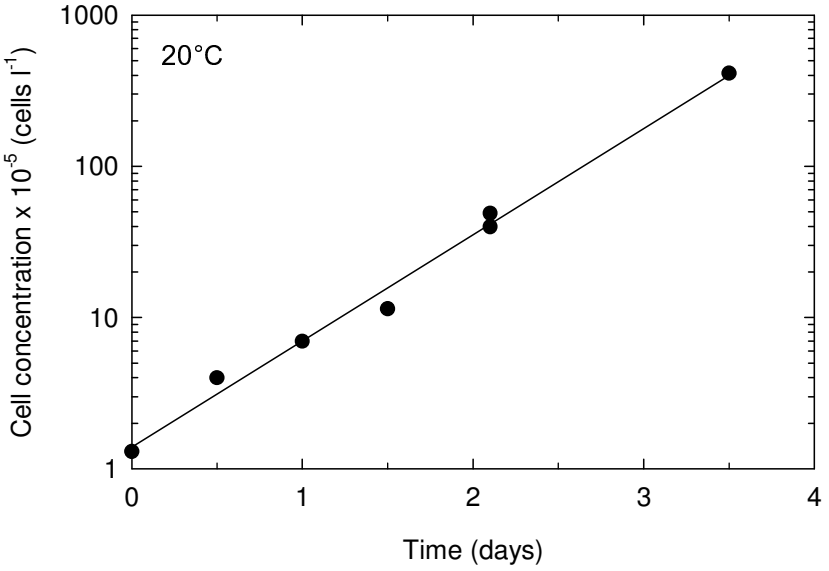
Answer: 4.31×10^6 cells μmole^{-1}; yes

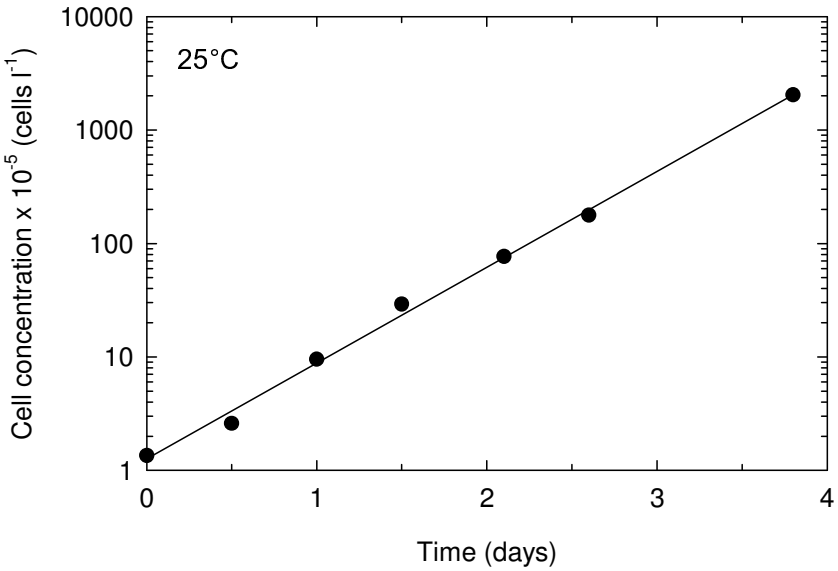
12.15 Algal batch growth kinetics

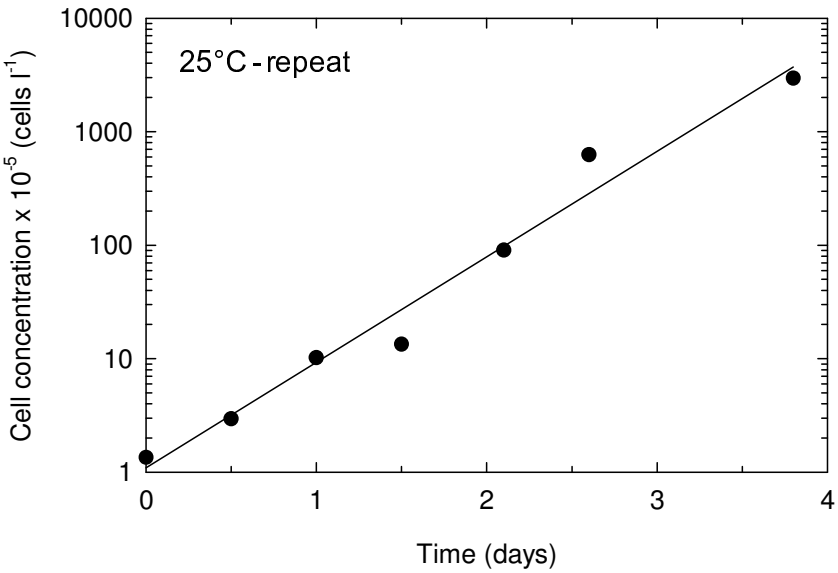
(a)

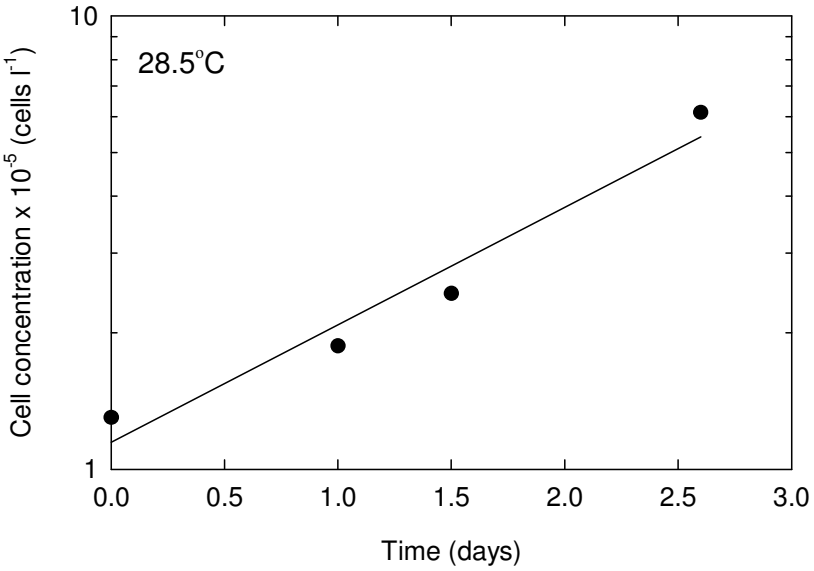
If the cultures exhibit exponential growth, according to Eq. (12.85), a plot of cell concentration versus time on semi-logarithmic coordinates should give a straight line. Separate semi-log plots for the data measured at different temperatures are shown below.




20°C
Cell concentration x 10-5 (cells l-1)
1000
100
10
1
0
1
2
3
4
Time (days)


25°C
Cell concentration x 10-5 (cells l-1)
10000
1000
100
10
1
0
1
2
3
4
Time (days)


25°C - repeat
Cell concentration x 10-5 (cells l-1)
10000
1000
100
10
1
0
1
2
3
4
Time (days)



The cell concentration data in each plot can be fitted using a straight line. Therefore, growth is exponential at each of the temperatures tested.

Answer: Yes

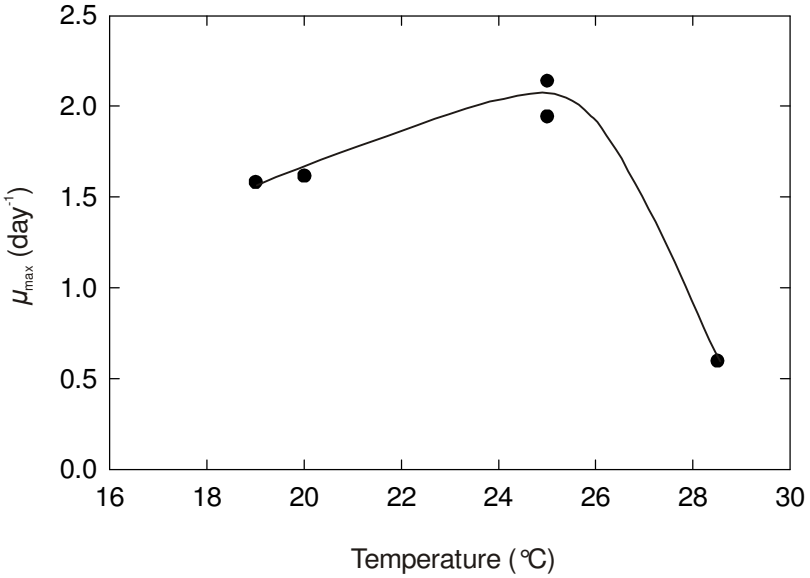
(b)

The equations for each of the straight lines in the plots in **(a)** are listed below, where x is cell concentration in units of cells l^{-1} and t is time in days. During exponential growth, μ is constant and equal to μ_{max} (Section 12.8.3); therefore, the value of μ_{max} is obtained directly from the equations. The cell doubling time t_d is calculated using Eq. (12.89).

Temperature, T	Equation	μ_{max} (day^{-1})	t_d (days)
19°C	$x = 1.261 \times 10^5\, e^{1.582t}$	1.58	0.44
20°C	$x = 1.393 \times 10^5\, e^{1.616t}$	1.62	0.43
25°C	$x = 1.265 \times 10^5\, e^{1.944t}$	1.94	0.36
25°C – repeat	$x = 1.092 \times 10^5\, e^{2.141t}$	2.14	0.32
28.5°C	$x = 1.146 \times 10^5\, e^{0.597t}$	0.60	1.16

(c)

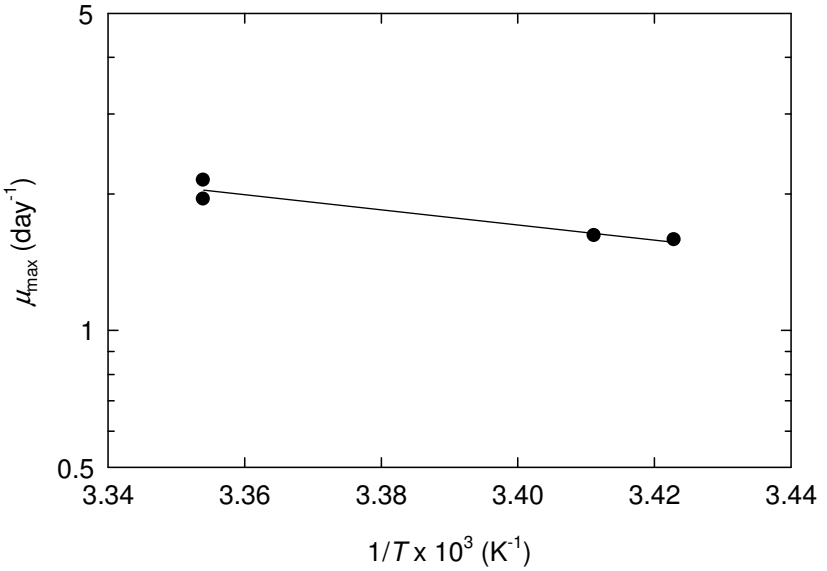
A plot of μ_{max} versus temperature is shown below.



Thermal activation of the algae occurs between 19°C and 25°C; the result at 28.5°C reflects thermal destruction or inactivation of the cells (Sections 12.3.4 and 12.12).

During thermal activation, the relationship between μ_{max} and temperature is expected to follow the Arrhenius equation, Eq. (12.22), with μ_{max} as the rate constant k. Therefore, a plot of μ_{max} versus $1/T$ on semi-logarithmic coordinates for the data between 19°C and 25°C should give a straight line. T is converted to kelvin using Eq. (2.27). The parameter values are listed and plotted below.

T (°C)	T (K)	$1/T$ (K^{-1})	μ_{max} (day^{-1})
19	292.15	3.42×10^{-3}	1.58
20	293.15	3.41×10^{-3}	1.62
25	298.15	3.35×10^{-3}	1.94
25 – repeat	298.15	3.35×10^{-3}	2.14



The equation for the straight line in the Arrhenius plot is $\mu_{max} = 7.637 \times 10^5 \, e^{-3827/T}$, where μ_{max} has units of day^{-1} and T has units of K.

Answer: $\mu_{max} = 7.637 \times 10^5 \, e^{-3827/T}$, where μ_{max} has units of day^{-1} and T has units of K

(d)

Converting 22°C to kelvin using Eq. (2.27), T = (22 + 273.15) K = 295.15 K. Substituting this value into the equation derived in **(c)**:

$$\mu_{\max} = 7.637 \times 10^5 e^{-3827/295.15} = 1.79$$

Therefore, $\mu_{\max}$ = 1.79 day^{-1}.

Answer: 1.79 day^{-1}

(e)

$x_0 = 2 \times 10^5$ cells l^{-1} and t = 2.25 days. From the result in **(d)** at 22°C, $\mu = \mu_{\max}$ = 1.79 day^{-1}. Applying Eq. (12.85) to calculate the cell concentration at the end of the growth period:

$$x = 2 \times 10^5 \text{ cells l}^{-1}\, e^{1.79 \text{ day}^{-1}(2.25 \text{ days})} = 1.12 \times 10^7 \text{ cells l}^{-1}$$

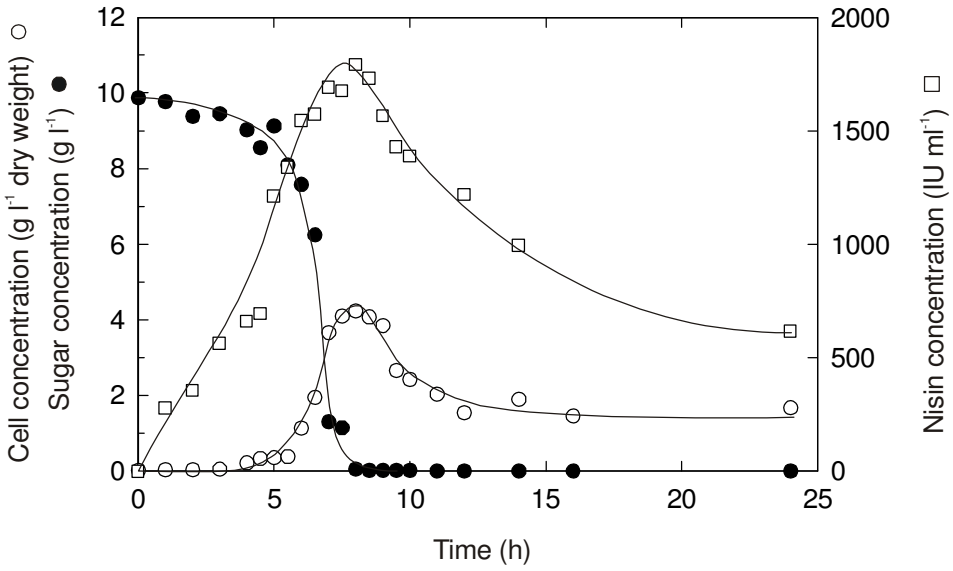
V = 1.6 m^3; therefore, from Table A.2 (Appendix A), $V = 1.6 \times 10^3$ l. The number of algal cells in the bioreactor at the end of the growth period is $xV = 1.12 \times 10^7$ cells l^{-1} × 1.6×10^3 l = 1.79×10^{10} cells. The number of cells added initially to the system at inoculation is $x_0V = 2 \times 10^5$ cells l^{-1} × 1.6×10^3 l = 3.20×10^8 cells. Therefore, the amount of cells produced during the culture is ($1.79 \times 10^{10} - 3.20 \times 10^8$) cells = 1.76×10^{10} cells.

Answer: 1.76×10^{10} cells

12.16 Kinetics of batch cell culture with nisin production

(a)

All of the data are plotted together below.



(i)

The cell and nisin concentrations follow similar time-courses, peaking at 7–8 h and then declining. From this, we can say that nisin production is growth-associated (Section 12.10.1).

Answer: Nisin production is growth-associated

(ii)

The data do not show evidence of premature cessation of growth, as growth does not stop before the sugar concentration reaches zero.

Answer: No; growth ceases when the sugar is depleted, not before

(iii)

Nisin concentrations decline after about 7–8 h. Therefore, 24 h is not an appropriate batch culture duration for harvesting maximum nisin levels.

Answer: No; nisin concentrations decline after about 7–8 h

(iv)

From Section 12.7.1, the overall biomass yield from substrate is calculated using the differences between the initial and final masses of cells and substrate. Applying Eq. (12.78) and a basis of 1 litre:

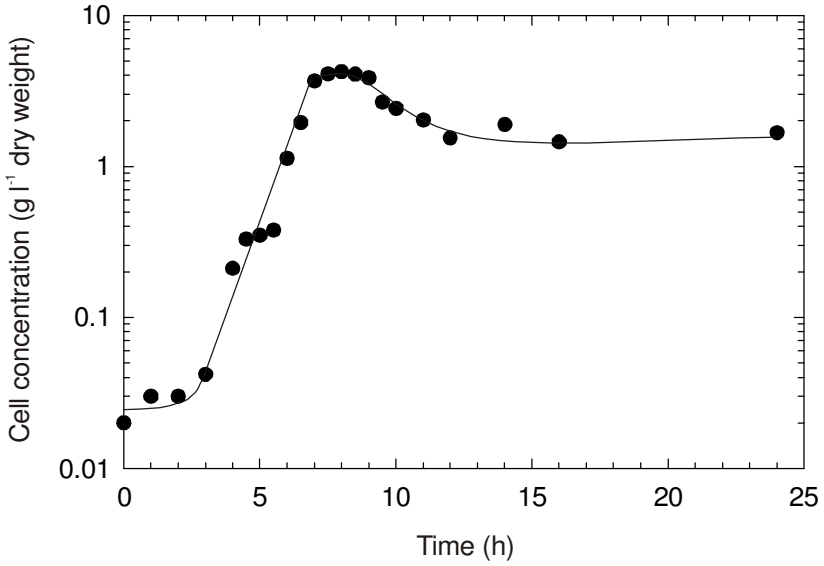
$$Y'_{\mathrm{xs}} = \frac{-(1.67 - 0.02)\ \mathrm{g}}{(0 - 9.87)\ \mathrm{g}} = 0.17\ \mathrm{g\ g^{-1}}$$

Because the cell concentration is reduced substantially during the death phase, the overall yield does not reflect the relationship between sugar uptake and biomass production during the culture.

Answer: 0.17 g g^{-1}; this result is not meaningful because the culture exhibits a significant death phase

(b)

The cell concentration data are plotted on semi-logarithmic coordinates below.



(i)

There is a lag phase at the beginning of the culture of duration 2–3 h.

Answer: Yes, 2–3 h

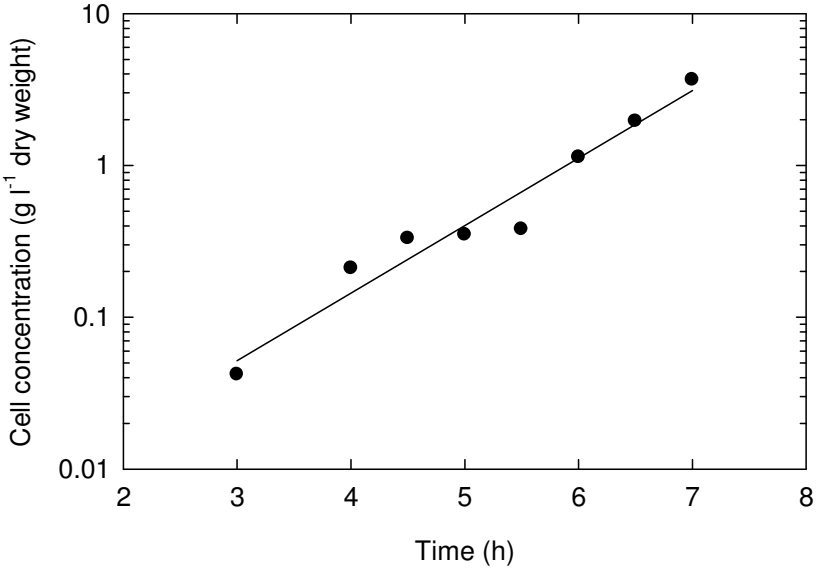
(ii)

In the semi-log plot, the growth data fall on a straight line between about 3 h and 7 h. Therefore, μ is constant and exponential growth occurs during this period (Section 12.8.1).

Answer: Yes, between about 3 h and 7 h

(iii)

Eq. (12.85) applies during exponential growth. According to this equation, μ is obtained by fitting a straight line to the data for cell concentration during the exponential phase plotted on semi-logarithmic coordinates. This plot is shown below.



The equation to the straight line in the plot is $x = 2.39 \times 10^{-3}\ e^{1.0243t}$, where x is cell concentration in units of g l^{-1} dry weight and t is time in h.

Answer: $x = 2.39 \times 10^{-3}\ e^{1.0243t}$, where x is cell concentration in units of g l^{-1} dry weight and t is time in h

(iv)

From the equation derived in **(b) (iii)**, $\mu = 1.0243\ h^{-1}$. During exponential growth, μ is constant and equal to μ_{max} (Section 12.8.3); therefore, $\mu_{max} = 1.0243\ h^{-1}$.

Answer: 1.02 h^{-1}

(v)

Applying Eq. (12.89):

$$t_d = \frac{\ln 2}{1.0243\ h^{-1}} = 0.68\ h$$

This doubling time applies only during the period of exponential growth when μ is constant, i.e. between 3 and 7 h after inoculation.

Answer: 0.68 h; this applies only during the exponential growth period, or between 3 and 7 h after the start of the culture

(vi)

From the semi-log plot in **(b)** of all the cell concentration data, after growth there is a relatively short stationary phase between about 7.5 h and 8.5 h before the cells enter the death phase.

Answer: Yes; stationary phase occurs between about 7.5 h and 8.5 h

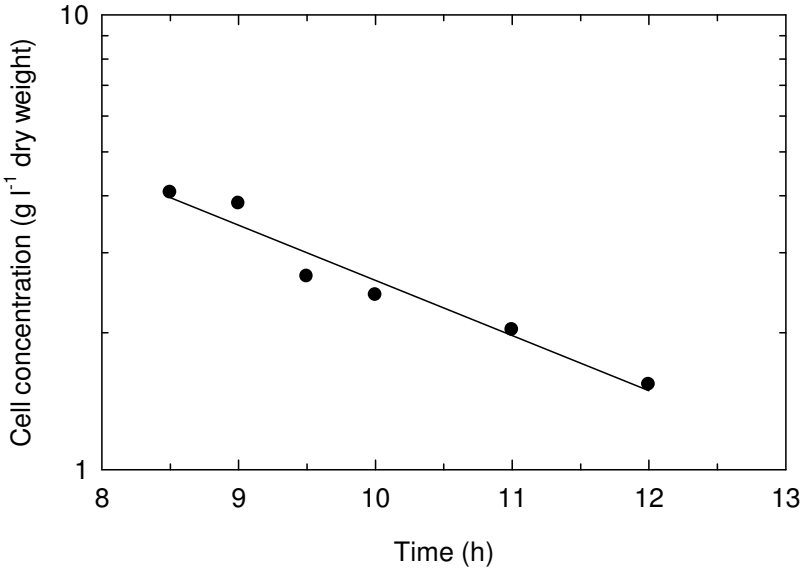
(vii)

As explained in Section 12.8.3, the transition from growth to stationary phase is often abrupt so that the decline phase is difficult to identify. However, from the semi-log plot in **(b)** of all the cell concentration data, a very short decline phase can be observed between about 7 and 7.5 h.

Answer: Between about 7 and 7.5 h

(viii)

From the semi-log plot in **(b)** of all the cell concentration data, the death phase occurs between about 8.5 h and 12 h. During this period, the reduction in cell concentration follows first-order kinetics, as indicated by the straight line through the data. Cell concentrations for the death phase only are plotted below using semi-logarithmic coordinates.



The equation to the straight line in the plot is $x = 42.5\ e^{-0.279t}$, where x is cell concentration in units of g l^{-1} dry weight and t is time in h. Therefore, from Eq. (12.118) for first-order cell death kinetics, the specific death constant $k_d = 0.279\ h^{-1}$.

Answer: 0.28 h^{-1}

(ix)

Applying Eq. (12.78) using a basis of 1 litre and the data for cell and substrate concentrations at 0 and 7 h:

$$Y'_{XS} = \frac{-(3.66 - 0.02)\ g}{(1.30 - 9.87)\ g} = 0.42\ g\ g^{-1}$$

This value is substantially higher than the result calculated in **(a) (iv)**. Because the culture exhibits a significant cell death phase, the final cell concentration used in the calculation of **(a) (iv)** does not reflect the extent of culture growth supported by the substrate. Determining the biomass yield using data from the end of the exponential growth phase gives a better estimate of this parameter.

Answer: 0.42 g g^{-1}; because the culture exhibits a death phase, this result provides a better estimate of the biomass yield from substrate than the overall yield

(x)

An equation analogous to Eq. (12.78) for the observed product yield from substrate is:

$$Y'_{PS} = \frac{-\Delta P}{\Delta S_T}$$

where ΔP is the mass of product formed and $-\Delta S_T$ is the total or observed mass of substrate consumed. Using a basis of 1 litre and the data for nisin and substrate concentrations at 0 and 7 h:

$$Y'_{PS} = \frac{-(1693-0)\times 1000 \text{ IU}}{(1.30-9.87) \text{ g}} = 1.975\times 10^5 \text{ IU g}^{-1}$$

Converting units:

$$Y'_{PS} = 1.975\times 10^5 \text{ IU g}^{-1} . \left| \frac{1 \text{ g}}{40\times 10^6 \text{ IU}} \right| = 4.94\times 10^{-3} \text{ g g}^{-1}$$

Answer: 4.94×10^{-3} g g^{-1}

(xi)

An equation analogous to Eq. (12.78) for the observed product yield from biomass is:

$$Y'_{PX} = \frac{\Delta P}{\Delta X}$$

where ΔP is the mass of product formed and ΔX is the biomass formed. Using a basis of 1 litre and the data for nisin and cell concentrations at 0 and 7 h:

$$Y'_{PX} = \frac{(1693-0)\times 1000 \text{ IU}}{(3.66-0.02) \text{ g}} = 4.651\times 10^5 \text{ IU g}^{-1}$$

Converting units:

$$Y'_{PX} = 4.651\times 10^5 \text{ IU g}^{-1} . \left| \frac{1 \text{ g}}{40\times 10^6 \text{ IU}} \right| . \left| \frac{1000 \text{ mg}}{1 \text{ g}} \right| = 11.6 \text{ mg g}^{-1}$$

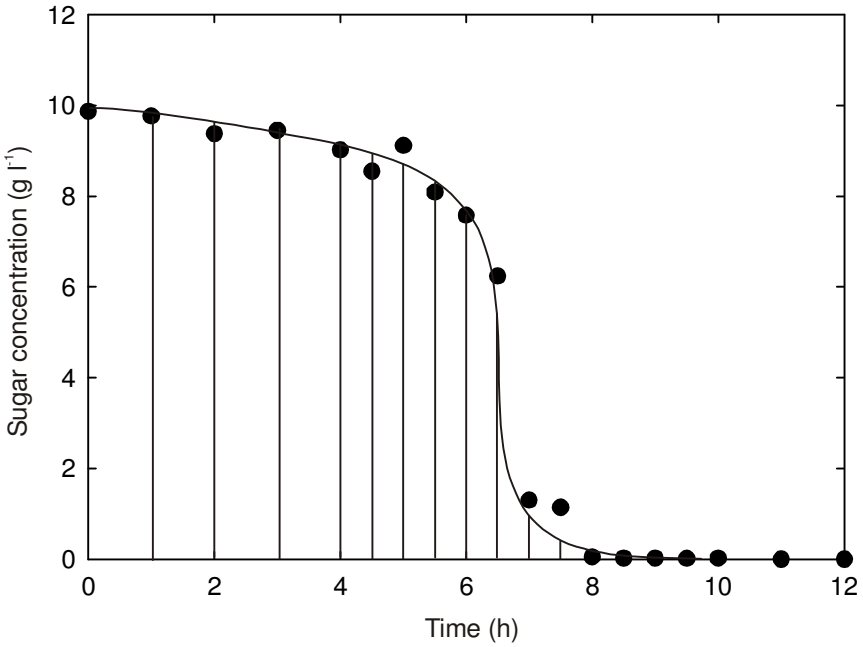
Answer: 11.6 mg g^{-1} dry weight

(c)

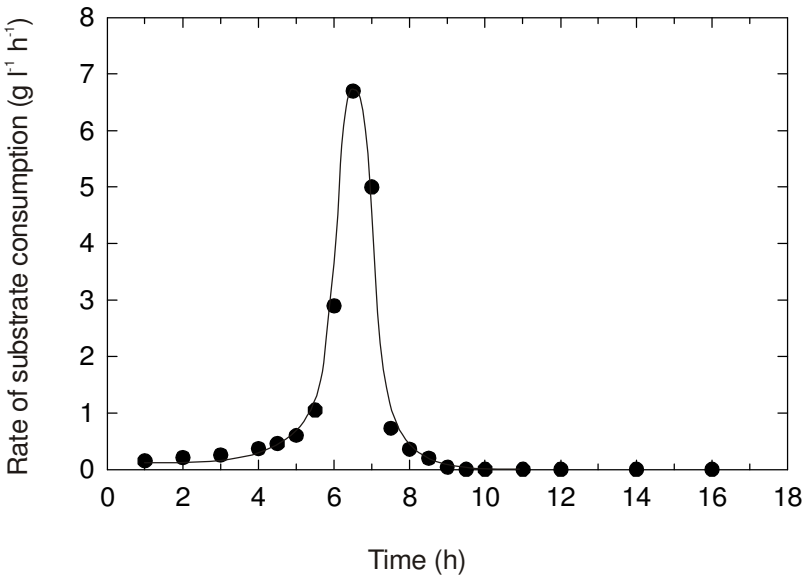
The mid-point slope method is used to determine the rate of substrate consumption at different times during the culture period from the sugar concentration data. The data are listed and plotted below according to the method described in Section 12.2.2. Values of $[(s)_{t+\varepsilon} - (s)_{t-\varepsilon}]$ are obtained from the smoothed curve on the graph. ds/dt is determined using the central-difference formula, Eq. (12.24). The rate of substrate consumption $r_S = -\text{d}s/\text{d}t$.

Time, t (h)	Sugar concentration, s (g l^{-1})	ε	$[(s)_{t+\varepsilon} - (s)_{t-\varepsilon}]$ g l^{-1}	ds/dt (g l^{-1} h^{-1})	r_S (g l^{-1} h^{-1})
0.0	9.87	1	–	–	–
1.0	9.77	1	–0.292	–0.146	0.146
2.0	9.38	1	–0.411	–0.206	0.206
3.0	9.45	1	–0.504	–0.252	0.252
4.0	9.02	1	–0.729	–0.365	0.365
4.5	8.55	0.5	–0.451	–0.451	0.451
5.0	9.12	0.5	–0.597	–0.597	0.597
5.5	8.09	0.5	–1.048	–1.048	1.048
6.0	7.58	0.5	–2.891	–2.891	2.891
6.5	6.24	0.5	–6.696	–6.696	6.696
7.0	1.30	0.5	–4.999	–4.999	4.999

7.5	1.14	0.5	–0.729	–0.729	0.729
8.0	0.05	0.5	–0.358	–0.358	0.358
8.5	0.02	0.5	–0.199	–0.199	0.199
9.0	0.02	0.5	–0.040	–0.040	0.040
9.5	0.015	0.5	0	0	0
10.0	0.018	0.5	0	0	0
11.0	0	1	0	0	0
12.0	0	1	0	0	0
14.0	0	2	0	0	0
16.0	0	2	0	0	0
24.0	0	8	–	–	–



The results for r_S are plotted below as a function of time.



(i)

The maximum rate of substrate consumption is about 6.7 g l^{-1} h^{-1}.

Answer: 6.7 g l^{-1} h^{-1}

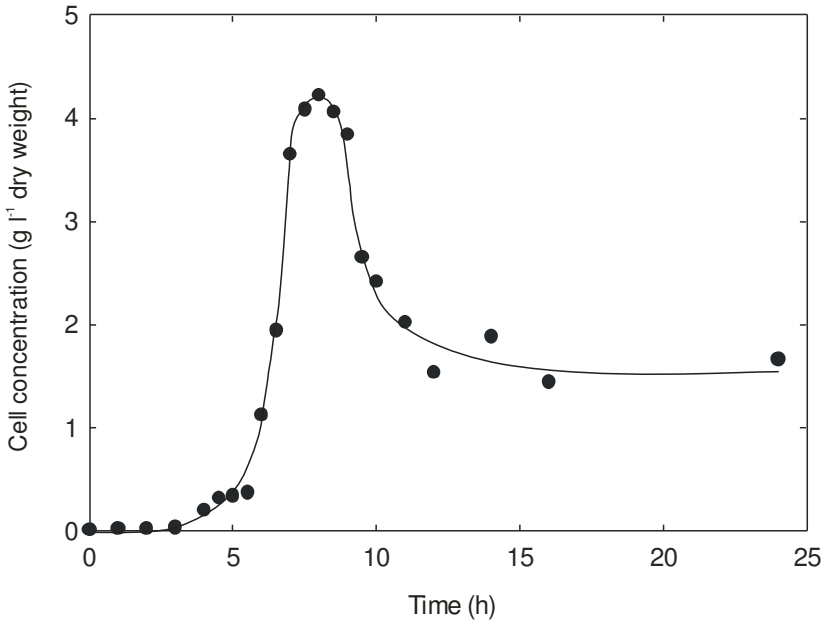
(ii)

The maximum rate of substrate consumption occurs at around 6.5 h.

Answer: 6.5 h

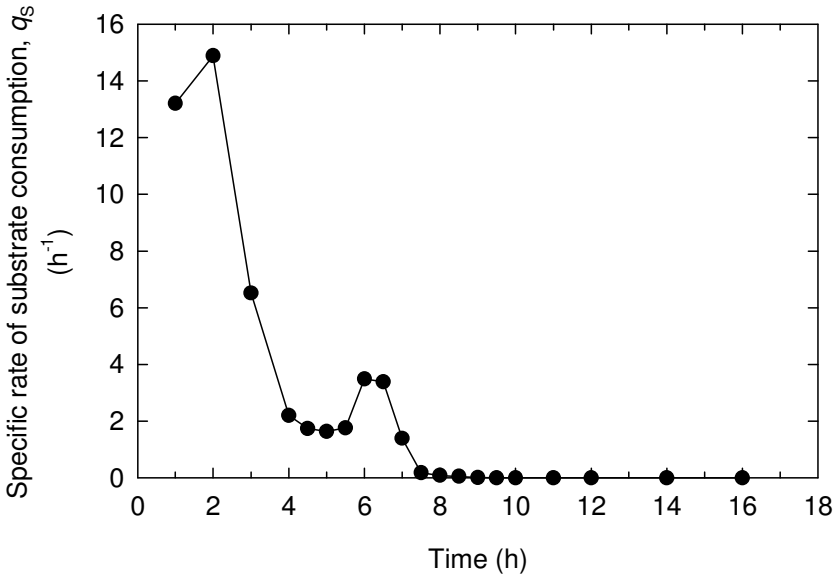
(d)

From Eq. (12.102), the specific rate of substrate consumption q_S is equal to r_S/x. This parameter could be evaluated as a function of time by dividing the values of r_S determined in **(c)** by the corresponding measured values of x. However, a better method is to use the x values obtained after smoothing the measured cell concentration data; this reduces the effects of any roughness in the measurements. A graph of the smoothed x data is shown below.



The values of x read from the smoothed curve and the calculated results for q_S are listed and plotted below.

Time, t (h)	Smoothed cell concentration, x (g l^{-1} dry weight)	r_S (g l^{-1} h^{-1})	$q_S = r_S/x$ (h^{-1})
0.0	0.000	–	–
1.0	0.011	0.146	13.2
2.0	0.014	0.206	14.9
3.0	0.039	0.252	6.51
4.0	0.166	0.365	2.20
4.5	0.260	0.451	1.74
5.0	0.365	0.597	1.64
5.5	0.597	1.048	1.76
6.0	0.829	2.891	3.49
6.5	1.978	6.696	3.39
7.0	3.591	4.999	1.39
7.5	4.122	0.729	0.177
8.0	4.199	0.358	0.085
8.5	4.127	0.199	0.048
9.0	3.481	0.040	0.011
9.5	2.707	0	0.00
10.0	2.271	0	0.00
11.0	1.989	0	0.00
12.0	1.823	0	0.00
14.0	1.646	0	0.00
16.0	1.575	0	0.00
24.0	1.547	–	–



(i)

The maximum specific rate of substrate consumption is about 15 h^{-1} and occurs during the lag phase at 2 h. However, the calculation at 2 h involves dividing a relatively small value of $[(s)_{t+\varepsilon} - (s)_{t-\varepsilon}]$ by a very small value of x. As it is often difficult to measure cell concentrations accurately at early times during batch culture when the cell density is very low, this reduces the confidence we can have in q_S values determined at low t.

Answer: 15 h^{-1} at 2 h; this result is subject to error as a result of dividing a small difference in sugar concentration by a low and likely inaccurate value of cell concentration

(ii)

From **(b) (ii)**, exponential growth occurs between about 3 h and 7 h. The results for q_S show variation in this region between q_S = 1.4 h^{-1} and q_S = 6.5 h^{-1}, reflecting the scatter in the original data and the amplification of that scatter inherent in slope calculations. However, the results for 3–7 h are distinct from those at earlier and later times during the culture, suggesting that q_S may be roughly constant during the growth period. Taking the average of the values between 3 h and 7 h gives $q_S \approx 2.8$ h^{-1} during the growth phase.

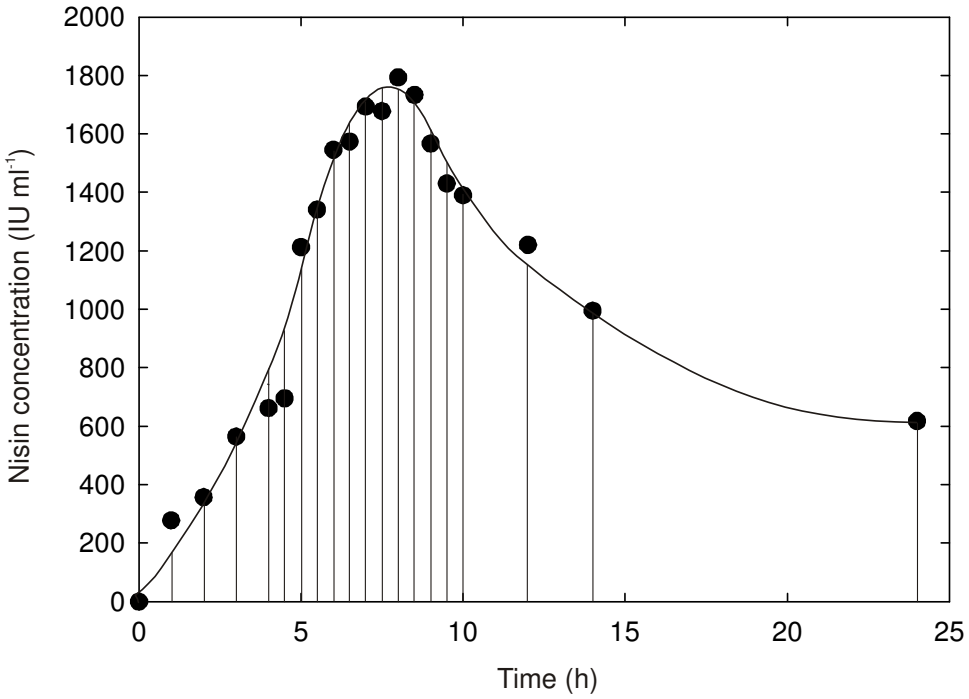
Answer: Approximately 2.8 h^{-1}

(e)

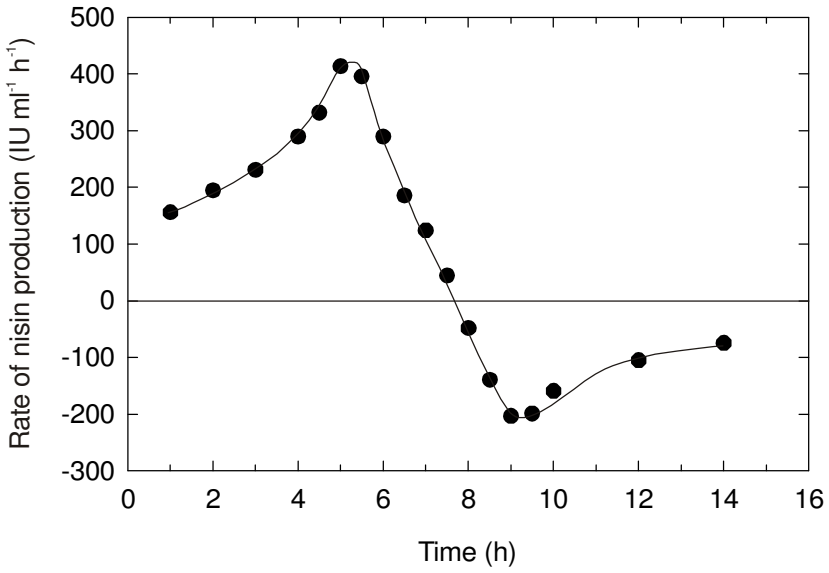
The mid-point slope method is used to determine the rate of nisin production as a function of time. The data are listed and plotted below according to the method described in Section 12.2.2. Values of $[(p)_{t+\varepsilon} - (p)_{t-\varepsilon}]$ are obtained from the smoothed curve on the graph. dp/dt is determined using the central-difference formula, Eq. (12.24). The rate of product formation r_P = dp/dt.

Time, t (h)	Nisin concentration, p (IU ml^{-1})	ε	$[(p)_{t+\varepsilon} - (p)_{t-\varepsilon}]$ IU ml^{-1}	dp/dt = r_P (IU ml^{-1} h^{-1})
0.0	0	1	–	–
1.0	278	1	311.6	155.8
2.0	357	1	389.0	194.5
3.0	564	1	461.9	230.9
4.0	662	1	579.0	289.5

4.5	695	0.5	331.5	331.5
5.0	1213	0.5	413.3	413.3
5.5	1341	0.5	395.6	395.6
6.0	1546	0.5	289.5	289.5
6.5	1574	0.5	185.6	185.6
7.0	1693	0.5	123.8	123.8
7.5	1678	0.5	44.2	44.2
8.0	1793	0.5	–48.6	–48.6
8.5	1733	0.5	–139.2	–139.2
9.0	1567	0.5	–203.3	–203.3
9.5	1430	0.5	–198.9	–198.9
10.0	1390	0.5	–159.1	–159.1
11.0	–		–	–
12.0	1220	2	–419.9	–105.0
14.0	995	2	–298.3	–74.6
16.0	–		–	–
24.0	617	8	–	–



The results for r_P are plotted below as a function of time.



(i)

From the graph, the maximum nisin productivity or nisin production rate (Section 12.1.3) is about 415 IU ml^{-1} h^{-1}. Converting units gives:

$$415 \text{ IU ml}^{-1} \text{ h}^{-1} . \left| \frac{1000 \text{ ml}}{1 \text{ l}} \right| . \left| \frac{1 \text{ g}}{40 \times 10^6 \text{ IU}} \right| . \left| \frac{1000 \text{ mg}}{1 \text{ g}} \right| = 10.4 \text{ mg l}^{-1} \text{ h}^{-1}$$

Answer: 10.4 mg l^{-1} h^{-1}

(ii)

From the graph, the maximum productivity occurs at about 5.3 h.

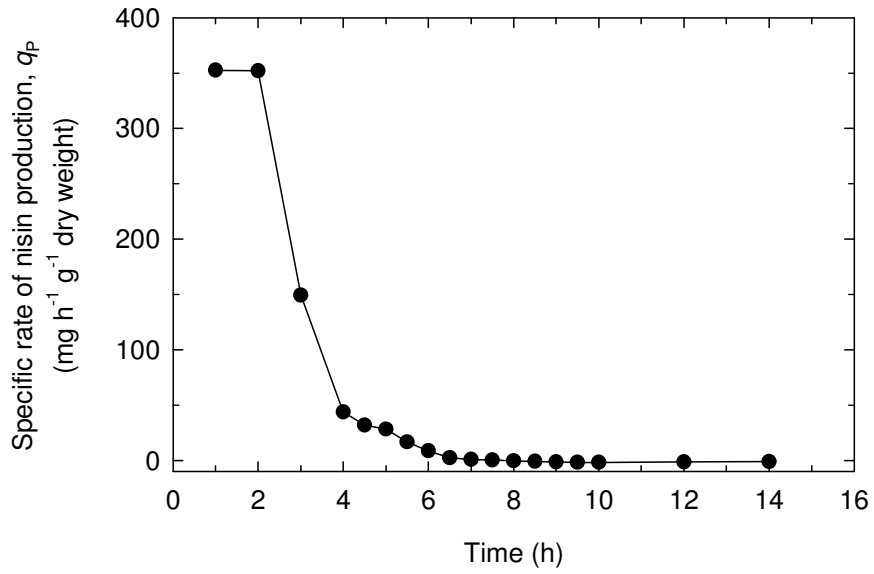
Answer: 5.3 h

(f)

From Eq. (12.98), the specific rate of nisin production q_P is equal to r_P/x. This parameter is evaluated using the results for r_P from **(e)** converted to units of mg l^{-1} h^{-1} and the smoothed data for x from **(d)**. The results are listed and plotted below.

Time, t (h)	Smoothed cell concentration, x (g l^{-1} dry weight)	r_P (IU ml^{-1} h^{-1})	r_P (mg l^{-1} h^{-1})	$q_P = r_P/x$ (mg h^{-1} g^{-1} dry weight)
0.0	0.000	–	–	–
1.0	0.011	155.8	3.895	353
2.0	0.014	194.5	4.862	352
3.0	0.039	230.9	5.773	149
4.0	0.166	289.5	7.238	43.7
4.5	0.260	331.5	8.287	31.9
5.0	0.365	413.3	10.331	28.3
5.5	0.597	395.6	9.890	16.6
6.0	0.829	289.5	7.238	8.73

6.5	1.978	185.6	4.641	2.35
7.0	3.591	123.8	3.094	0.862
7.5	4.122	44.2	1.105	0.268
8.0	4.199	–48.6	–1.215	–0.289
8.5	4.127	–139.2	–3.481	–0.843
9.0	3.481	–203.3	–5.083	–1.46
9.5	2.707	–198.9	–4.972	–1.84
10.0	2.271	–159.1	–3.978	–1.75
11.0	1.989	–	–	–
12.0	1.823	–105.0	–2.624	–1.44
14.0	1.646	–74.6	–1.865	–1.13
16.0	1.575	–	–	–
24.0	1.547	–	–	–



(i)

From the graph, the maximum specific rate of nisin production is about 350 mg h^{-1} g^{-1} dry weight.

Answer: 350 mg h^{-1} g^{-1} dry weight

(ii)

From the graph, the maximum q_P occurs during the lag phase at 1–2 h after inoculation.

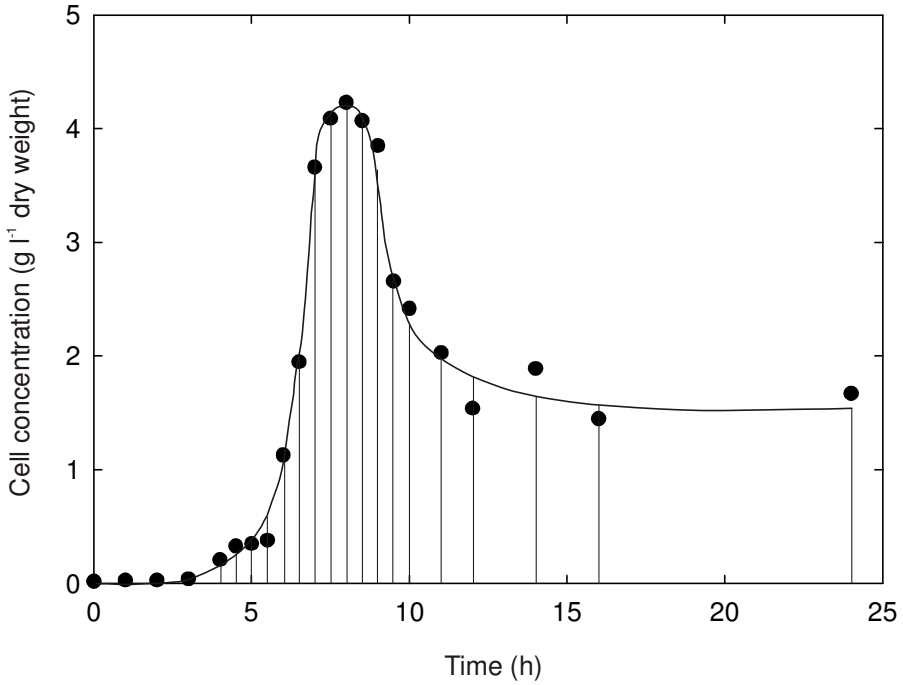
Answer: 1–2 h

(g)

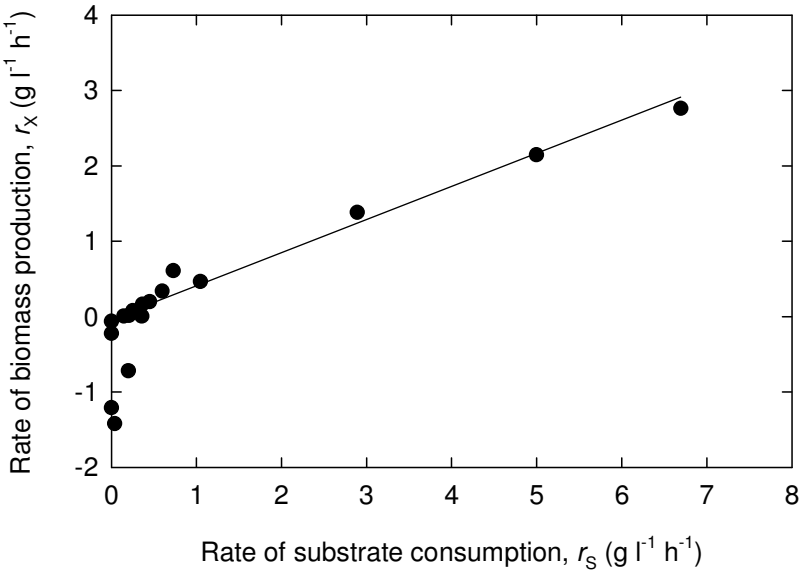
The mid-point slope method is used to determine the rate of biomass production as a function of time from the cell concentration data. The data are listed and plotted below according to the method described in Section 12.2.2. Values of $[(x)_{t+\varepsilon} - (x)_{t-\varepsilon}]$ are obtained from the smoothed curve on the graph. dx/dt is

determined using the central-difference formula, Eq. (12.24). The rate of biomass production $r_X = dx/dt$. The corresponding results for r_S from **(c)** are also listed.

Time, t (h)	Cell concentration, x (g l^{-1})	ε	$[(x)_{t+\varepsilon} - (x)_{t-\varepsilon}]$ g l^{-1}	$dx/dt = r_X$ (g l^{-1} h^{-1})	r_S (g l^{-1} h^{-1})
0.0	0.02	1	–	–	–
1.0	0.03	1	0.014	0.007	0.146
2.0	0.03	1	0.028	0.014	0.206
3.0	0.042	1	0.152	0.076	0.252
4.0	0.21	1	0.326	0.163	0.365
4.5	0.33	0.5	0.199	0.199	0.451
5.0	0.35	0.5	0.337	0.337	0.597
5.5	0.38	0.5	0.464	0.464	1.048
6.0	1.13	0.5	1.381	1.381	2.891
6.5	1.95	0.5	2.762	2.762	6.696
7.0	3.66	0.5	2.144	2.144	4.999
7.5	4.09	0.5	0.608	0.608	0.729
8.0	4.23	0.5	0.006	0.006	0.358
8.5	4.07	0.5	–0.718	–0.718	0.199
9.0	3.85	0.5	–1.420	–1.420	0.040
9.5	2.66	0.5	–1.210	–1.210	0
10.0	2.42	0.5	–	–	0
11.0	2.03	1	–0.448	–0.224	0
12.0	1.54	1	–	–	0
14.0	1.89	2	–0.249	–0.062	0
16.0	1.45	2	–	–	0
24.0	1.67	8	–	–	–



r_X is plotted versus r_S below.



(i)

From Eq. (12.108), the observed yield of biomass from substrate Y'_{XS} is equal to the ratio r_X/r_S. From the graph, for positive values of r_X during the growth phase, the points for r_X and r_S fall on a straight line passing through the origin, indicating that Y'_{XS} is constant during growth.

Answer: Yes

(ii)

The slope of the straight line passing through the origin for positive values of r_X is 0.43. Therefore, Y'_{XS} = 0.43 g g^{-1}.

This result deviates substantially from the overall yield of 0.17 g g^{-1} determined in **(a) (iv)**. The reason for this difference is the same as that outlined in **(b) (ix)** above: using cell concentration data after

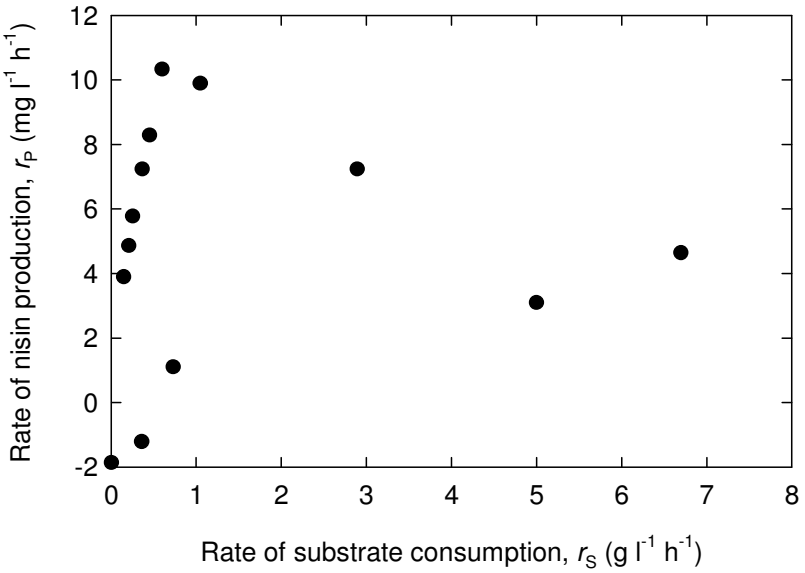
the end of the growth phase to evaluate Y'_{XS} does not reflect the extent of culture growth supported by the substrate.

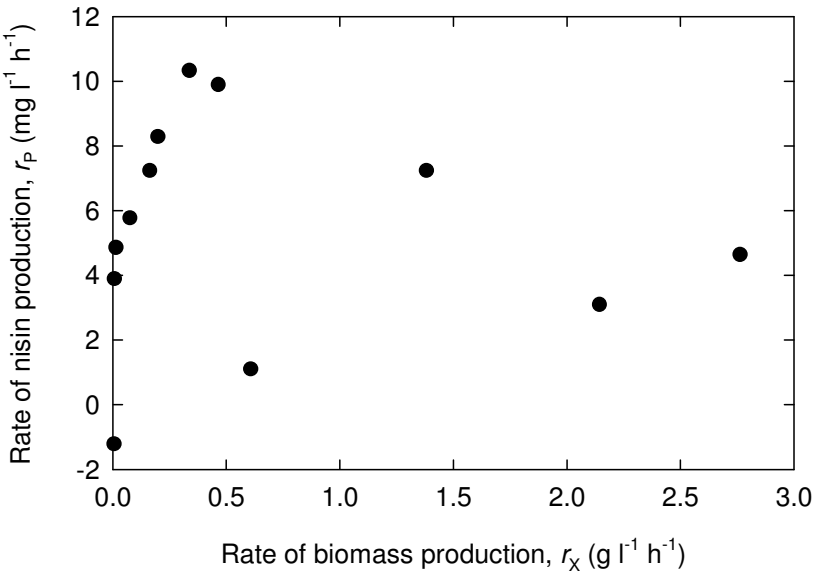
The result for Y'_{XS} determined here as the ratio r_X/r_S during growth is very close to that of 0.42 g g^{-1} determined in **(b) (ix)** based on differences in biomass and substrate concentrations between 0 and 7 h. The graphical method used here involving calculation of r_X and r_S is a much more lengthy and complex procedure compared with that employed in **(b) (ix)**. However, an advantage is that it takes into account all of the data during the growth phase and does not rely on calculation of differences between single datum points that may carry substantial error.

Answer: Y'_{XS} = 0.43 g g^{-1}; this result is similar to that found in **(b)** but deviates significantly from the overall yield determined in **(a)**

(h)

Using the results for r_X from **(g),** r_S from **(c)**, and r_P in units of mg l^{-1} h^{-1} from **(f)**, r_P versus r_S, and r_P versus r_X, are plotted below.





From Eqs (12.110) and (12.109), respectively, the observed yield of nisin from substrate Y'_{PS} is equal to the ratio r_P/r_S, and the observed yield of nisin from biomass Y'_{PX} is equal to r_P/r_X. In both of the above plots, the

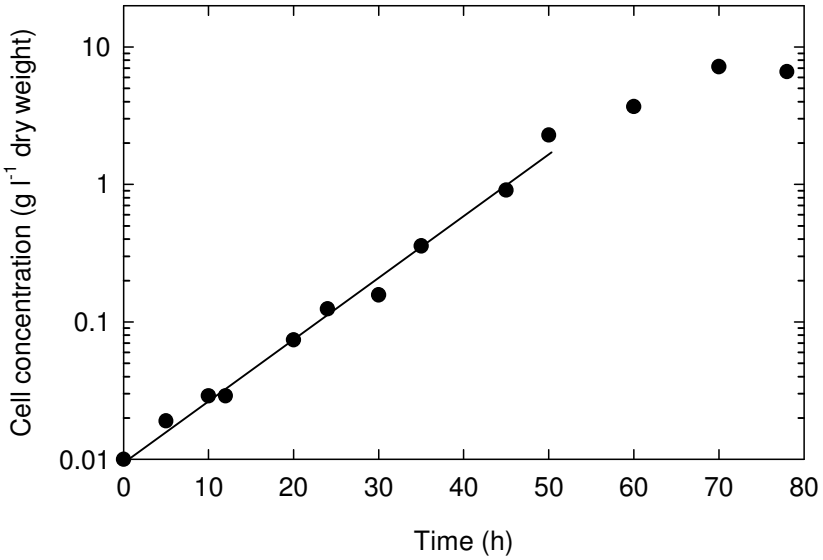
points for positive values of r_P are scattered and do not exhibit a straight-line relationship with either r_S or r_X during the growth period. Therefore, we can conclude that neither Y'_{PS} nor Y'_{PX} is constant during batch culture.

Answer: No

12.17 Yeast culture and astaxanthin production

(a)

The cell concentration data are plotted on semi-logarithmic coordinates below.



As the growth data fall on a straight line in the plot starting at time zero, the culture does not exhibit a lag phase.

Answer: No

(b)

The cell concentration data produce a straight line on the semi-log plot for much of the culture period. This is an indication of exponential or first-order growth kinetics (Section 12.8.1).

Answer: Yes

(c)

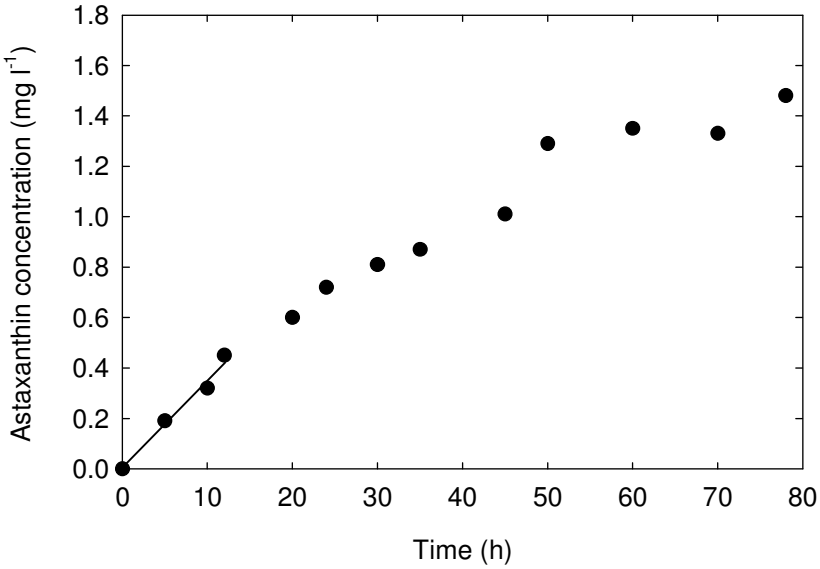
In the semi-log plot, the growth data fall on a straight line between 0 h and 50 h; the points after 50 h appear to be part of the decline and stationary phases. The period 0–50 h can therefore be considered as the exponential growth phase. The equation to the straight line on the semi-log plot between 0 h and 50 h is $x = 9.65 \times 10^{-3}\ e^{0.104t}$, where x is cell concentration in units of g l^{-1} dry weight and t is time in h. Therefore, from Eq. (12.85), $\mu = 0.104\ h^{-1}$. As the specific growth rate μ is constant and equal to μ_{max} during exponential growth (Sections 12.8.1 and 12.8.3), $\mu_{max} = 0.104\ h^{-1}$. The culture doubling time during exponential growth is evaluated using Eq. (12.89):

$$t_d = \frac{\ln 2}{0.104\ h^{-1}} = 6.7\ h$$

Answer: $\mu_{max} = 0.104\ h^{-1}$; $t_d = 6.7$ h; valid between 0 h and 50 h. *Different interpretations of the time period for exponential growth are possible: for example, the straight line in the semi-log plot may be considered to extend to 60 h or 70 h. The results for μ_{max} and t_d will then be slightly different.*

(d)

A plot of astaxanthin concentration versus time is shown below.



The initial rate of astaxanthin synthesis is the slope of a line through the data at time zero. As the first four points appear to fall on a straight line, these may be used to evaluate the initial rate. The use of four points rather than only the first two improves the accuracy of the analysis; the slope evaluated using only the data at $t = 0$ and $t = 5$ h is more subject to the errors in these individual measurements.

The equation for the straight line in the plot is $p = 0.0354t + 0.0011$, where p is astaxanthin concentration in mg l^{-1} and t is time in h. The slope is 0.0354; therefore, the initial rate of astaxanthin synthesis is 0.0354 mg l^{-1} h^{-1}.

Answer: 0.035 mg l^{-1} h^{-1}

(e)

From Eq. (12.98), the initial specific rate of astaxanthin production is equal to the initial rate of astaxanthin production divided by the initial cell concentration. Using the result from **(d)** and the cell concentration at time zero, the initial specific rate of astaxanthin production is (0.0354 mg l^{-1} h^{-1})/(0.01 g l^{-1}) = 3.54 mg g^{-1} h^{-1}. This calculation involves dividing by a very small value of x. As it is often difficult to measure cell concentrations accurately at early times during batch culture when the cell density is very low, this reduces the confidence we can have in the answer.

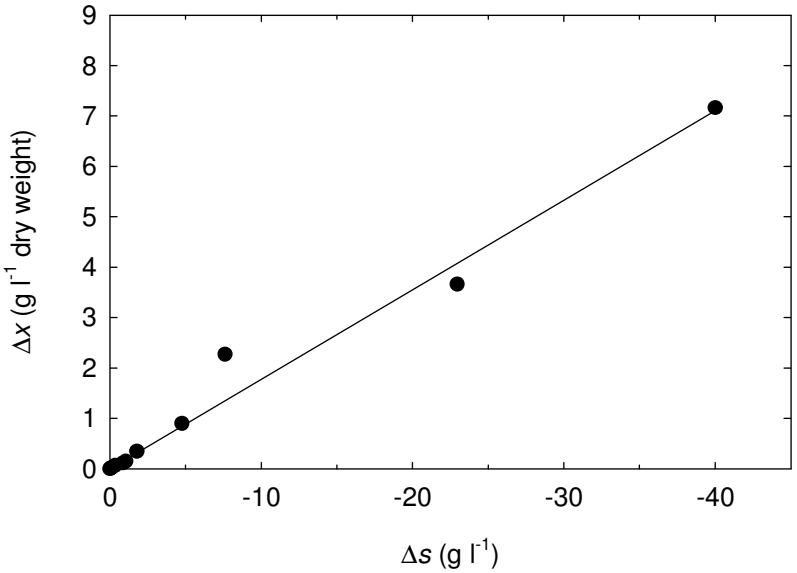
Answer: 3.54 mg g^{-1} h^{-1}; this result is subject to error as a result of dividing by a very low and likely inaccurate value of cell concentration

(f)

Sugar is depleted at 70 h and, as indicated by the graph in **(a)**, cells are produced between 0 h and 70 h. Δx and Δs are determined as differences between the measured data during this period and the initial cell and substrate concentrations, respectively. The results are listed and plotted below.

Time, t (h)	Δx (g l^{-1} dry weight)	Δs (g l^{-1})
0	0	0
5	0.009	–0.04
10	0.019	–0.10
12	0.019	–0.09

20	0.064	–0.35
24	0.114	–0.86
30	0.147	–1.05
35	0.346	–1.78
45	0.896	–4.76
50	2.27	–7.60
60	3.66	–22.95
70	7.16	–40



The observed biomass yield from substrate Y'_{XS} is defined in Eq. (12.78). For a system of constant volume, Y'_{XS} may also be determined as the ratio of concentration differences, $-\Delta x/\Delta s$, where Δs is the total or observed change in substrate concentration. From the graph, as the data for Δx versus Δs can be fitted using a straight line passing through the origin, the ratio $\Delta x/\Delta s$, and therefore Y'_{XS}, can be considered constant throughout the growth period. The equation to the straight line is $\Delta x/\Delta s = -0.178$ g g^{-1}; therefore, $Y'_{XS} = 0.178$ g g^{-1}.

Answer: 0.18 g g^{-1}; Y'_{XS} is constant during the growth phase

(g)

An equation analogous to Eq. (12.78) for the product yield from substrate is:

$$Y'_{PS} = \frac{-\Delta P}{\Delta S_T}$$

where ΔP is the mass of product formed and $-\Delta S_T$ is the total or observed mass of substrate consumed. For a system of constant volume, Y'_{PS} may also be determined as the ratio of concentration differences:

$$Y'_{PS} = \frac{-\Delta p}{\Delta s}$$

where Δp is the increase in product concentration and $-\Delta s$ is the total or observed decrease in substrate concentration. From Section 12.7.1, overall yields are determined using differences between the initial and final states. Therefore, the overall product yield from glucose is:

$$Y'_{PS} = \frac{-(1.48 - 0)\ \text{mg l}^{-1}}{(0 - 40)\ \text{g l}^{-1}} = 0.037\ \text{mg g}^{-1}$$

Answer: 0.037 mg g^{-1}

(h)

An equation analogous to Eq. (12.78) for the product yield from biomass is:

$$Y'_{PX} = \frac{\Delta P}{\Delta X}$$

where ΔP is the mass of product formed and ΔX is the biomass formed. In terms of concentration differences:

$$Y'_{PX} = \frac{\Delta p}{\Delta x}$$

where Δp is the increase in product concentration and Δx is the increase in cell concentration. From Section 12.7.1, overall yields are determined using differences between the initial and final states. Therefore, the overall product yield from biomass is:

$$Y'_{PX} = \frac{(1.48 - 0)\ \text{mg l}^{-1}}{(6.59 - 0.01)\ \text{g l}^{-1}} = 0.2249\ \text{mg g}^{-1}$$

Answer: 0.22 mg g^{-1}

12.18 Ethanol fermentation by yeast and bacteria

(a)

From Table C.8 (Appendix C), the molecular formulae for glucose and ethanol are $C_6H_{12}O_6$ and C_2H_6O, respectively. The reaction equation for fermentation of glucose to ethanol without cell growth is:

$$C_6H_{12}O_6 \rightarrow 2C_2H_6O + 2CO_2$$

From the stoichiometry, the maximum theoretical yield of ethanol from glucose is 2 gmol $gmol^{-1}$. From Table C.1 (Appendix C) the molecular weights are: glucose = 180.2; ethanol = 46.1. Therefore:

$$\text{Maximum theoretical yield} = \frac{2\ \text{gmol ethanol}}{1\ \text{gmol glucose}} = \frac{2\ \text{gmol ethanol} \,.\, \left|\frac{46.1\ \text{g}}{1\ \text{gmol}}\right|}{1\ \text{gmol glucose} \,.\, \left|\frac{180.2\ \text{g}}{1\ \text{gmol}}\right|} = 0.51\ \text{g g}^{-1}$$

Answer: 0.51 g g^{-1}

(b)

In the absence of growth, μ in Eq. (12.114) is zero and the equation reduces to:

$$Y'_{PS} = \frac{m_P}{m_S}$$

Therefore, for Y'_{PS} equal to the maximum theoretical yield:

$$m_P = 0.51 m_S$$

For *S. cerevisiae*, m_S = 0.18 kg kg^{-1} h^{-1}; for *Z. mobilis*, m_S = 2.2 kg kg^{-1} h^{-1}. Applying these parameter values in the above equation gives m_P = 0.092 h^{-1} for *S. cerevisiae* and m_P = 1.12 h^{-1} for *Z. mobilis*.

Answer: 0.092 h^{-1} for *S. cerevisiae*; 1.12 h^{-1} for *Z. mobilis*

(c)

During batch culture at typical glucose concentrations, $\mu = \mu_{max}$ for most of the culture period (Section 12.8.3). Therefore, Eq. (12.114) can be written as:

$$Y'_{PS} = \frac{Y_{PX}\mu_{max} + m_P}{\left(\frac{\mu_{max}}{Y_{XS}} + m_S\right)}$$

Substituting parameter values for *S. cerevisiae*, including the result for m_P from **(b)**:

$$Y'_{PS} = \frac{3.9\ \text{kg kg}^{-1}\,(0.4\ \text{h}^{-1}) + 0.092\ \text{h}^{-1}}{\left(\frac{0.4\ \text{h}^{-1}}{0.11\ \text{kg kg}^{-1}} + 0.18\ \text{kg kg}^{-1}\ \text{h}^{-1}\right)} = 0.43\ \text{kg kg}^{-1}$$

Similarly, for *Z. mobilis*:

$$Y'_{PS} = \frac{7.7\ \text{kg kg}^{-1}\,(0.3\ \text{h}^{-1}) + 1.12\ \text{h}^{-1}}{\left(\frac{0.3\ \text{h}^{-1}}{0.06\ \text{kg kg}^{-1}} + 2.2\ \text{kg kg}^{-1}\ \text{h}^{-1}\right)} = 0.48\ \text{kg kg}^{-1}$$

Answer: 0.43 kg kg^{-1} for *S. cerevisiae*; 0.48 kg kg^{-1} for *Z. mobilis*

(d)

$$\text{Efficiency} = \frac{Y'_{PS}}{\text{maximum theoretical product yield}}$$

Using the results from **(a)** and **(c)**, for *S. cerevisiae*:

$$\text{Efficiency} = \frac{0.43\ \text{kg kg}^{-1}}{0.51\ \text{g g}^{-1}} = 0.84$$

For *Z. mobilis*:

$$\text{Efficiency} = \frac{0.48\ \text{kg kg}^{-1}}{0.51\ \text{g g}^{-1}} = 0.94$$

Answer: 0.84 for *S. cerevisiae*; 0.94 for *Z. mobilis*

(e)

From Eq. (12.101) with $\mu = \mu_{max}$:

$$q_P = Y_{PX}\mu_{max} + m_P$$

Applying parameter values for *S. cerevisiae*:

$$q_P = 3.9\ \text{kg kg}^{-1}\,(0.4\ \text{h}^{-1}) + 0.092\ \text{h}^{-1} = 1.65\ \text{h}^{-1}$$

For *Z. mobilis*:

$$q_P = 7.7\ \text{kg kg}^{-1}\,(0.3\ \text{h}^{-1}) + 1.12\ \text{h}^{-1} = 3.43\ \text{h}^{-1}$$

Answer: 1.65 h^{-1} for *S. cerevisiae*; 3.43 h^{-1} for *Z. mobilis*

(f)

In Eq. (12.101), the growth-associated term is $Y_{PX}\mu$ and the non-growth-associated term is m_P. Therefore, for $\mu = \mu_{max}$, the proportion of ethanol production from growth for *S. cerevisiae* is:

$$\frac{Y_{PX}\mu_{max}}{Y_{PX}\mu_{max} + m_P} = \frac{3.9\ \text{kg kg}^{-1}(0.4\ \text{h}^{-1})}{3.9\ \text{kg kg}^{-1}(0.4\ \text{h}^{-1}) + 0.092\ \text{h}^{-1}} = 0.94$$

so that the proportion from non-growth metabolism is 0.06. For *Z. mobilis*:

$$\frac{Y_{PX}\mu_{max}}{Y_{PX}\mu_{max} + m_P} = \frac{7.7\ \text{kg kg}^{-1}(0.3\ \text{h}^{-1})}{7.7\ \text{kg kg}^{-1}(0.3\ \text{h}^{-1}) + 1.12\ \text{h}^{-1}} = 0.67$$

and the proportion from non-growth metabolism is 0.33. Non-growth-associated ethanol production is more substantial for *Z. mobilis*.

Answer: For *S. cerevisiae*, 0.94 of the ethanol production is growth-associated and 0.06 is non-growth-associated. For *Z. mobilis*, 0.67 is growth-associated and 0.33 is non-growth-associated. *Z. mobilis* produces a more substantial proportion of its ethanol in non-growth-associated metabolism.

(g)

From Section 12.1.3, volumetric productivity is equal to specific productivity multiplied by cell concentration. From **(e)**, as the specific ethanol productivity q_P for *Z. mobilis* is 3.43/1.65 = 2.1 times that of *S. cerevisiae*, to achieve the same volumetric productivity, the concentration of yeast must be 2.1 times that of bacteria.

Answer: 2.1 times the concentration of bacteria

(h)

From Section 12.1.3, total productivity is equal to specific productivity multiplied by cell concentration and fermenter volume. At zero growth, μ in Eq. (12.101) is zero and $q_P = m_P$. Therefore, from **(b)**, the specific productivity q_P for *S. cerevisiae* is 0.092 h^{-1} and q_P for *Z. mobilis* is 1.12 h^{-1}. As the value for *Z. mobilis* is 1.12/0.092 = 12.2 times that of *S. cerevisiae*, to achieve the same total productivity at the same cell concentration, the fermenter volume for the yeast culture must be 12.2 times that for the bacteria.

Answer: 12.2 times the volume for the bacterial culture

(i)

From Eq. (12.112) with $\mu = \mu_{max}$*:*

$$\frac{1}{Y'_{XS}} = \frac{1}{Y_{XS}} + \frac{m_S}{\mu_{max}}$$

Applying parameter values for *S. cerevisiae:*

$$\frac{1}{Y'_{XS}} = \frac{1}{0.11\ \text{kg kg}^{-1}} + \frac{0.18\ \text{kg kg}^{-1}\ \text{h}^{-1}}{0.4\ \text{h}^{-1}} = 9.54\ \text{kg kg}^{-1}$$

Therefore:

$$Y'_{XS} = 0.105\ \text{kg kg}^{-1}$$

For *Z. mobilis*:

$$\frac{1}{Y'_{XS}} = \frac{1}{0.06\ \text{kg kg}^{-1}} + \frac{2.2\ \text{kg kg}^{-1}\ \text{h}^{-1}}{0.3\ \text{h}^{-1}} = 24.0\ \text{kg kg}^{-1}$$

$$Y'_{XS} = 0.042 \text{ kg kg}^{-1}$$

Answer: 0.105 kg kg^{-1} for *S. cerevisiae*; 0.042 kg kg^{-1} for *Z. mobilis*. As *Z. mobilis* produces less biomass per mass of substrate consumed and per mass of ethanol produced than *S. cerevisiae*, biomass disposal is less of a problem with the bacteria.

(j)

Answer: The ethanol yield from substrate is 12% higher using *Z. mobilis* than *S. cerevisiae*, the specific productivity is 2.1 times higher so that a smaller and cheaper fermentation vessel is required to achieve the same rate of ethanol production, and *Z. mobilis* produces less than half the amount of biomass generated by *S. cerevisiae* per mass of glucose consumed. All of these factors mean that *Z. mobilis* performs better than *S. cerevisiae* for ethanol production.

However, other aspects of the cultures and the ethanol production industry also need to be considered. *Z. mobilis* requires a higher pH (5.0) for growth than *S. cerevisiae* (3.5–4.0), and is therefore more susceptible to contamination. *Z. mobilis* also does not grow well on molasses, a common substrate material for fermentations, because of its high salt content. The biomass produced in ethanol fermentation by yeast is often sold for use in animal feeds, whereas application of bacteria for this purpose is not as well accepted in the industry. These are some of the reasons why *Z. mobilis* has not been widely adopted for industrial ethanol production, despite its superior ethanol production characteristics.

12.19 Plasmid loss during culture maintenance

From Eq. (12.96):

$$\alpha = \frac{0.033 \text{ h}^{-1}}{0.025 \text{ h}^{-1}} = 1.32$$

The number of generations of plasmid-containing cells over the 28-day period is, from Eq. (12.97):

$$n = \frac{(0.025 \text{ h}^{-1})\ 28 \text{ days} \left| \frac{24 \text{ h}}{1 \text{ day}} \right|}{\ln 2} = 24.2$$

If the fraction of cells containing plasmid is $F = 0.66$, from Eq. (12.95):

$$0.66 = \frac{1 - 1.32 - p}{1 - 1.32 - (2^{24.2(1.32+p-1)})p} = \frac{0.32 + p}{0.32 + (2^{24.2(0.32+p)})p} = \frac{0.32 + p}{0.32 + 214.38\, p\, (2^{24.2\,p})}$$

Multiplying through by the denominator:

$$0.211 + 141.49\, p\, (2^{24.2\,p}) = 0.32 + p$$

$$141.49\, p\, (2^{24.2\,p}) - 0.109 - p = 0$$

This equation can be solved for p by trial and error. Values of the left-hand-side of the equation for various p are listed below, starting with $p = 0.001$ as the first guess.

p	$141.49\, p\, (2^{24.2p}) - 0.109 - p$
0.001	0.0339
0.002	0.1816
0.0007	–0.0095
0.00076	–0.0008
0.00077	0.0006
0.000766	0.000017

As the value of the expression when $p = 0.000766$ is sufficiently close to zero, the solution can be taken as $p = 0.000766$.

Answer: 0.000766

12.20 Medium sterilisation

(a), (b) and (c)

From Section 12.15, the specific death constant k_d is evaluated using Eq. (12.74) with $A = 10^{36.2}$ s^{-1} and $E_d = 283$ kJ gmol^{-1}. From Table B.1 (Appendix B), $R = 8.3144$ J K^{-1} gmol^{-1} = 8.3144×10^{-3} kJ K^{-1} gmol^{-1}. Converting the temperatures to kelvin using Eq. (2.27), 80°C = (80 + 273.15) K = 353.15 K, 121°C = (121 + 273.15) K = 394.15 K, and 140°C = (140 + 273.15) K = 413.15 K. Therefore, at 80°C:

$$k_d = (10^{36.2}\ \text{s}^{-1})\, e^{-283\ \text{kJ gmol}^{-1}/(8.3144\times10^{-3}\ \text{kJ K}^{-1}\ \text{gmol}^{-1}\times 353.15\ \text{K})} = 2.20\times10^{-6}\ \text{s}^{-1}$$

At 121°C:

$$k_d = (10^{36.2}\ \text{s}^{-1})\, e^{-283\ \text{kJ gmol}^{-1}/(8.3144\times10^{-3}\ \text{kJ K}^{-1}\ \text{gmol}^{-1}\times 394.15\ \text{K})} = 0.0497\ \text{s}^{-1}$$

At 141°C:

$$k_d = (10^{36.2}\ \text{s}^{-1})\, e^{-283\ \text{kJ gmol}^{-1}/(8.3144\times10^{-3}\ \text{kJ K}^{-1}\ \text{gmol}^{-1}\times 413.15\ \text{K})} = 2.63\ \text{s}^{-1}$$

The relationship between the number of viable cells and time is given by Eq. (12.118). Converting the units of the initial concentration of contaminants x_0:

$$x_0 = 10^8\ \text{cells l}^{-1} . \left|\frac{10^3\ \text{l}}{1\ \text{m}^3}\right| = 10^{11}\ \text{cells m}^{-3}$$

Therefore, for sterilisation of 1 m^3 of medium, $N_0 = 10^{11}$. After sterilisation, $N = 10^{-3}$. Substituting values into Eq. (12.118) gives:

$$10^{-3} = 10^{11} e^{-k_d t}$$

$$10^{-14} = e^{-k_d t}$$

Solving this equation by taking the logarithm of both sides and applying Eq. (E.3) from Appendix E:

$$-32.24 = -k_d t$$

or

$$t = \frac{32.24}{k_d}$$

Therefore, at 80°C:

$$t = \frac{32.24}{2.20\times10^{-6}\ \text{s}^{-1} . \left|\frac{3600\ \text{s}}{1\ \text{h}}\right|} = 4070\ \text{h}$$

At 121°C:

$$t = \frac{32.24}{0.0497\ \text{s}^{-1} . \left|\frac{60\ \text{s}}{1\ \text{min}}\right|} = 10.8\ \text{min}$$

At 140°C:

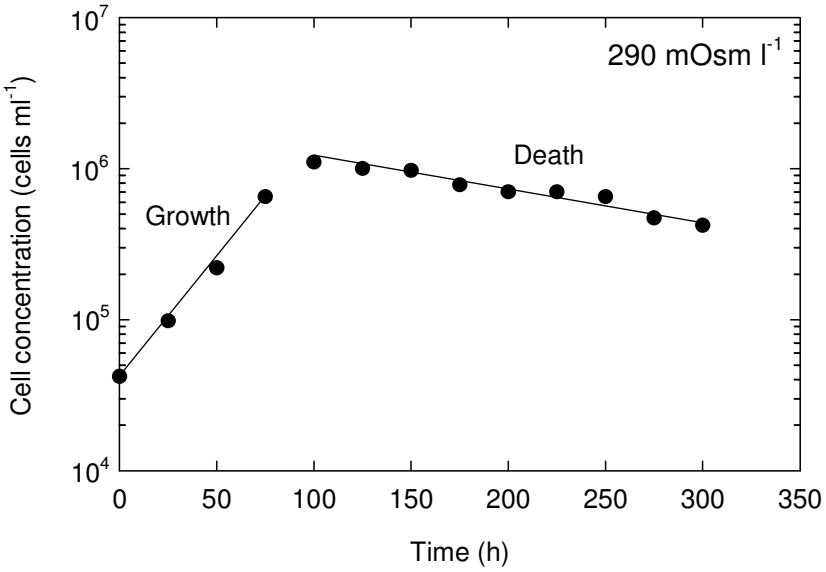
$$t = \frac{32.24}{2.63\ \text{s}^{-1}} = 12.3\ \text{s}$$

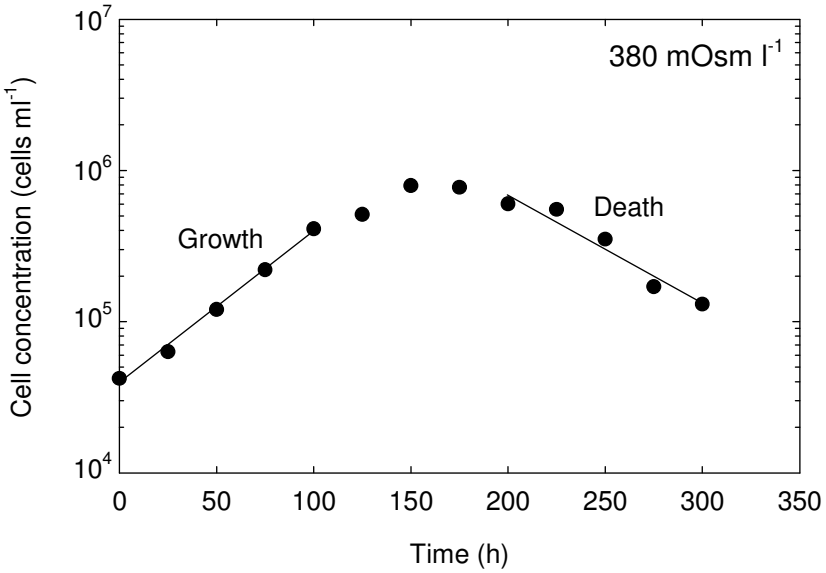
Answer: 4070 h at 80°C; 10.8 min at 121°C; 12.3 s at 140°C

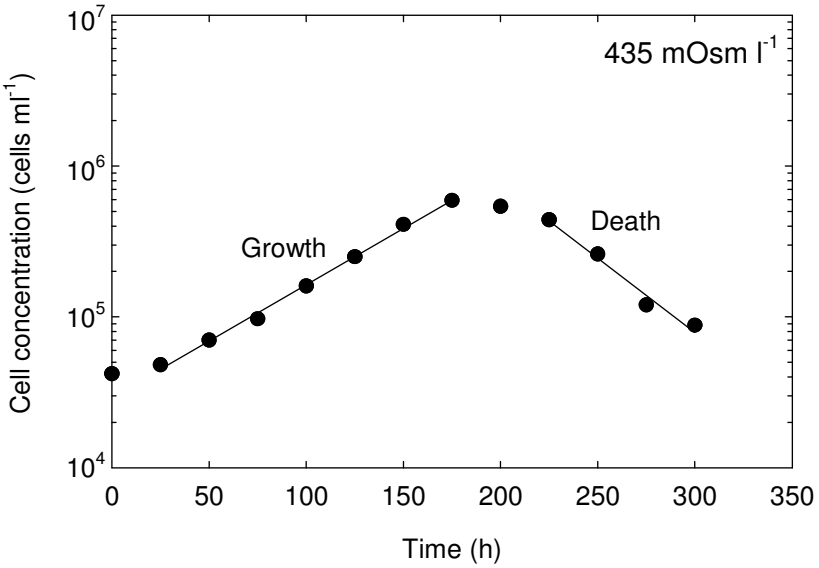
12.21 Effect of medium osmolarity on growth and death of hybridoma cells

(a)

The cell concentration data are plotted using semi-logarithmic coordinates for each of the media tested.







As shown in the graphs, there is no lag phase at 290 mOsm l^{-1} and probably no lag phase at 380 mOsm l^{-1}. At 435 mOsm l^{-1}, a lag phase of about 20 h can be observed.

Answer: Yes; there is a lag phase at 435 mOsm l^{-1} but not at the lower medium osmolarities

(b)

According to Eq. (12.85), during exponential growth, a plot of cell concentration versus time on semi-logarithmic coordinates gives a straight line with slope equal to the specific growth rate, μ. For each medium osmolarity, the data for cell growth plotted in **(a)** can be fitted using a straight line during only part of the culture period.

At 290 mOsm l^{-1}, exponential growth occurs between 0 and 75 h. The equation to the straight line on the graph during this period is $x = 4.02 \times 10^4 \, e^{0.0361t}$, where x is cell concentration in units of cells ml^{-1} and t is time in h. Therefore, from Eq. (12.85), $\mu = 0.0361$ h^{-1}. As μ is constant and equal to μ_{max} during exponential growth (Sections 12.8.1 and 12.8.3), $\mu_{max} = 0.0361$ h^{-1}.

At 380 mOsm l^{-1}, exponential growth occurs between 0 and 100 h. The equation to the straight line during this period is $x = 3.86 \times 10^4 \, e^{0.0232t}$, where x is cell concentration in units of cells ml^{-1} and t is time in h. Therefore, $\mu = 0.0232$ $h^{-1} = \mu_{max}$.

At 435 mOsm l^{-1}, exponential growth occurs between 25 and 175 h. The equation to the straight line during this period is $x = 2.94 \times 10^4 \, e^{0.0172t}$, where x is cell concentration in units of cells ml^{-1} and t is time in h. Therefore, $\mu = 0.0172$ $h^{-1} = \mu_{max}$.

Answer: μ_{max} decreases with increasing medium osmolarity; $\mu_{max} = 0.036$ h^{-1}, 0.023 h^{-1} and 0.017 h^{-1} at 290, 380 and 435 mOsm l^{-1}, respectively

(c)

From the data provided, at 290 mOsm l^{-1} the maximum cell density achieved is 1.1×10^6 cells ml^{-1} at 100 h. At 380 mOsm l^{-1} the maximum cell density is 7.9×10^5 cells ml^{-1} at 150 h. At 435 mOsm l^{-1} the maximum cell density is 5.9×10^5 cells ml^{-1} at 175 h. Therefore, the maximum cell density is reduced and the time taken to achieve maximum cell density is increased with increasing medium osmolarity.

Answer: With increasing medium osmolarity, the maximum cell density is reduced while the time taken to achieve maximum cell density is increased

(d)

According to Eq. (12.118), during cell death with first-order kinetics, a plot of viable cell number versus time on semi-logarithmic coordinates gives a straight line with negative slope. At constant volume, a semi-log plot of viable cell concentration versus time can also be used to evaluate k_d. For each medium osmolarity, the data for viable cell concentration plotted in **(a)** can be fitted using a straight line for cell death during the later part of the culture period.

At 290 mOsm l^{-1}, first-order cell death occurs between 100 and 300 h. The equation to the straight line on the graph during this period is $x = 1.84 \times 10^6 \, e^{-4.68 \times 10^{-3} t}$, where x is viable cell concentration in units of cells ml^{-1} and t is time in h. Therefore, by analogy with Eq. (12.118), $k_d = 4.68 \times 10^{-3}$ h^{-1}.

At 380 mOsm l^{-1}, first-order cell death occurs between 200 and 300 h. The equation to the straight line during this period is $x = 2.09 \times 10^7 \, e^{-1.69 \times 10^{-2} t}$. Therefore, $k_d = 1.69 \times 10^{-2}$ h^{-1}.

At 435 mOsm l^{-1}, first-order cell death occurs between 225 and 300 h. The equation to the straight line during this period is $x = 6.68 \times 10^7 \, e^{-2.24 \times 10^{-2} t}$. Therefore, $k_d = 2.24 \times 10^{-2}$ h^{-1}.

Answer: k_d increases with increasing medium osmolarity; $k_d = 4.68 \times 10^{-3}$ h^{-1}, 1.69×10^{-2} h^{-1} and 2.24×10^{-2} h^{-1} at 290, 380 and 435 mOsm l^{-1}, respectively

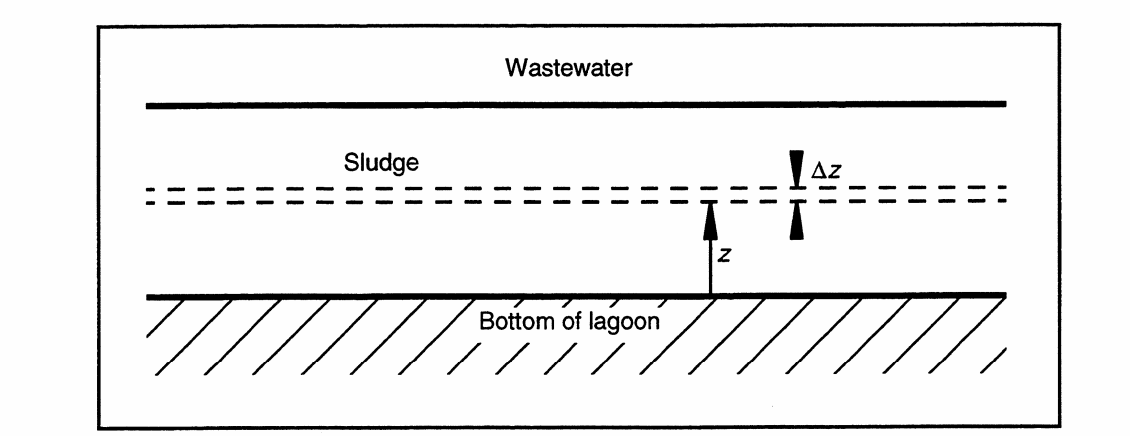
Chapter 13

Heterogeneous Reactions

13.1 Diffusion and reaction in a waste treatment lagoon

(a)

A substrate shell mass balance is performed around a thin slice of sludge of area A and thickness Δz located distance z from the bottom of the lagoon, as shown in the diagram below.



Substrate diffuses into the shell across the upper boundary at $z + \Delta z$, and diffuses out across the lower boundary at z. Substrate consumption within the shell follows first-order kinetics. Following the procedure outlined in Section 13.3.1, from Fick's law, the rate of substrate input by diffusion is:

$$\left(\mathscr{D}_{\text{Se}} A \frac{\mathrm{d}s}{\mathrm{d}z} \right)\Bigg|_{z+\Delta z}$$

where $\mathscr{D}_{\text{Se}}$ is the effective diffusivity of substrate in the sludge. Similarly, the rate of substrate output by diffusion is:

$$\left(\mathscr{D}_{\text{Se}} A \frac{\mathrm{d}s}{\mathrm{d}z} \right)\Bigg|_{z}$$

The rate of substrate consumption in the shell is $k_1 s A \Delta z$. Substituting these expressions into the mass balance equation, Eq. (4.1), with the rate of substrate generation = 0, at steady state:

$$\left(\mathscr{D}_{\text{Se}} A \frac{\mathrm{d}s}{\mathrm{d}z} \right)\Bigg|_{z+\Delta z} - \left(\mathscr{D}_{\text{Se}} A \frac{\mathrm{d}s}{\mathrm{d}z} \right)\Bigg|_{z} - k_1 s A \Delta z = 0$$

Assuming that the substrate diffusivity and sludge area do not vary with distance z:

$$\mathscr{D}_{\text{Se}} A \left(\frac{\mathrm{d}s}{\mathrm{d}z}\Big|_{z+\Delta z} - \frac{\mathrm{d}s}{\mathrm{d}z}\Big|_{z} \right) - k_1 s A \Delta z = 0$$

Cancelling A and dividing through by Δz gives:

$$\mathscr{D}_{\text{Se}} \frac{\dfrac{\mathrm{d}s}{\mathrm{d}z}\Big|_{z+\Delta z} - \dfrac{\mathrm{d}s}{\mathrm{d}z}\Big|_{z}}{\Delta z} - k_1 s = 0$$

Taking the limit as $\Delta z \to 0$ and invoking the definition of the derivative as in Eq. (E.13) in Appendix E:

$$\mathscr{D}_{\mathrm{Se}} \frac{\mathrm{d}}{\mathrm{d}z}\left(\frac{\mathrm{d}s}{\mathrm{d}z}\right) - k_1 s = 0$$

or

$$\mathscr{D}_{\mathrm{Se}} \frac{\mathrm{d}^2 s}{\mathrm{d}z^2} - k_1 s = 0$$

Answer: $\mathscr{D}_{\mathrm{Se}} \dfrac{\mathrm{d}^2 s}{\mathrm{d}z^2} - k_1 s = 0$

(b)

At $z = L$, $s = s_\mathrm{b}$. At $z = 0$, we assume that the substrate concentration profile reaches a minimum so that $\mathrm{d}s/\mathrm{d}z = 0$.

Answer: At $z = L$, $s = s_\mathrm{b}$; at $z = 0$, $\mathrm{d}s/\mathrm{d}z = 0$.

(c)

(i)

If $s = Ne^{pz}$, using differentiation rule Eq. (E.17) in Appendix E:

$$\frac{\mathrm{d}s}{\mathrm{d}z} = pNe^{pz}$$

and

$$\frac{\mathrm{d}^2 s}{\mathrm{d}z^2} = p^2 Ne^{pz}$$

Substituting the expressions for s and $\mathrm{d}^2s/\mathrm{d}z^2$ into the differential equation in **(a)**:

$$\mathscr{D}_{\mathrm{Se}} p^2 Ne^{pz} - k_1 Ne^{pz} = 0$$

Dividing through by Ne^{pz} and solving for p:

$$p^2 = \frac{k_1}{\mathscr{D}_{\mathrm{Se}}}$$

$$p = \sqrt{k_1 / \mathscr{D}_{\mathrm{Se}}} \qquad \text{or} \qquad p = -\sqrt{k_1 / \mathscr{D}_{\mathrm{Se}}}$$

Answer: $p = \pm\sqrt{k_1 / \mathscr{D}_{\mathrm{Se}}}$

(ii)

If $s = Ne^{pz} + Me^{-pz}$, substituting the values for p from **(i)** above:

$$s = Ne^{z\sqrt{k_1/\mathscr{D}_{\mathrm{Se}}}} + Me^{-z\sqrt{k_1/\mathscr{D}_{\mathrm{Se}}}}$$

Differentiating this equation using differentiation rule Eq. (E.17) in Appendix E:

$$\frac{\mathrm{d}s}{\mathrm{d}z} = N\sqrt{k_1 / \mathscr{D}_{\mathrm{Se}}}\, e^{z\sqrt{k_1/\mathscr{D}_{\mathrm{Se}}}} - M\sqrt{k_1 / \mathscr{D}_{\mathrm{Se}}}\, e^{-z\sqrt{k_1/\mathscr{D}_{\mathrm{Se}}}}$$

Applying the boundary condition at $z = 0$:

$$0 = N\sqrt{k_1 / \mathscr{D}_{\mathrm{Se}}} - M\sqrt{k_1 / \mathscr{D}_{\mathrm{Se}}}$$

Cancelling the square root terms gives:

$$N = M$$

Answer: $N = M$

(iii)

Substituting N for M in the equation for s in **(ii)**:

$$s = N\left(e^{z\sqrt{k_1/\mathscr{D}_{Se}}} + e^{-z\sqrt{k_1/\mathscr{D}_{Se}}}\right)$$

Applying the boundary condition at $z = L$:

$$s_b = N\left(e^{L\sqrt{k_1/\mathscr{D}_{Se}}} + e^{-L\sqrt{k_1/\mathscr{D}_{Se}}}\right)$$

Solving for N:

$$N = \frac{s_b}{e^{L\sqrt{k_1/\mathscr{D}_{Se}}} + e^{-L\sqrt{k_1/\mathscr{D}_{Se}}}}$$

Substituting this result into the equation for s above:

$$s = s_b \frac{e^{z\sqrt{k_1/\mathscr{D}_{Se}}} + e^{-z\sqrt{k_1/\mathscr{D}_{Se}}}}{e^{L\sqrt{k_1/\mathscr{D}_{Se}}} + e^{-L\sqrt{k_1/\mathscr{D}_{Se}}}}$$

Answer: $s = s_b \dfrac{e^{z\sqrt{k_1/\mathscr{D}_{Se}}} + e^{-z\sqrt{k_1/\mathscr{D}_{Se}}}}{e^{L\sqrt{k_1/\mathscr{D}_{Se}}} + e^{-L\sqrt{k_1/\mathscr{D}_{Se}}}}$

(iv)

From the definition of cosh x, the numerator in the equation for s in **(iii)** is equal to 2 cosh $(z\sqrt{k_1/\mathscr{D}_{Se}})$. Similarly, the denominator is equal to 2 cosh $(L\sqrt{k_1/\mathscr{D}_{Se}})$. Therefore:

$$s = s_b \frac{2\cosh\left(z\sqrt{k_1/\mathscr{D}_{Se}}\right)}{2\cosh\left(L\sqrt{k_1/\mathscr{D}_{Se}}\right)}$$

or

$$\frac{s}{s_b} = \frac{\cosh\left(z\sqrt{k_1/\mathscr{D}_{Se}}\right)}{\cosh\left(L\sqrt{k_1/\mathscr{D}_{Se}}\right)}$$

Answer: QED

(d)

Taking the derivative of s using the equation for s in **(c) (iv)**:

$$\frac{ds}{dz} = s_b\sqrt{k_1/\mathscr{D}_{Se}}\,\frac{\sinh\left(z\sqrt{k_1/\mathscr{D}_{Se}}\right)}{\cosh\left(L\sqrt{k_1/\mathscr{D}_{Se}}\right)}$$

Evaluating the derivative at $z = L$:

$$\left.\frac{ds}{dz}\right|_{z=L} = s_b\sqrt{k_1/\mathscr{D}_{Se}}\,\frac{\sinh\left(L\sqrt{k_1/\mathscr{D}_{Se}}\right)}{\cosh\left(L\sqrt{k_1/\mathscr{D}_{Se}}\right)}$$

Substituting this result into the expression for $r_{A,obs}$:

$$r_{\mathrm{A,obs}} = \mathscr{D}_{\mathrm{Se}} A s_{\mathrm{b}} \sqrt{k_1 / \mathscr{D}_{\mathrm{Se}}} \frac{\sinh\left(L\sqrt{k_1 / \mathscr{D}_{\mathrm{Se}}}\right)}{\cosh\left(L\sqrt{k_1 / \mathscr{D}_{\mathrm{Se}}}\right)}$$

Answer: $r_{\mathrm{A,obs}} = \mathscr{D}_{\mathrm{Se}} A s_{\mathrm{b}} \sqrt{k_1 / \mathscr{D}_{\mathrm{Se}}} \frac{\sinh\left(L\sqrt{k_1 / \mathscr{D}_{\mathrm{Se}}}\right)}{\cosh\left(L\sqrt{k_1 / \mathscr{D}_{\mathrm{Se}}}\right)}$

(e)

The internal effectiveness factor is defined in Eq. (13.26). For first-order reaction kinetics at substrate concentration s_{b} everywhere in the sludge, the rate of reaction within the entire sludge volume is:

$$r^*_{\mathrm{As}} = k_1 s_{\mathrm{b}} A L$$

Substituting this and the expression for $r_{\mathrm{A,obs}}$ from **(d)** into Eq. (13.26) for first-order kinetics:

$$\eta_{\mathrm{i1}} = \frac{\mathscr{D}_{\mathrm{Se}} A s_{\mathrm{b}} \sqrt{k_1 / \mathscr{D}_{\mathrm{Se}}} \dfrac{\sinh\left(L\sqrt{k_1 / \mathscr{D}_{\mathrm{Se}}}\right)}{\cosh\left(L\sqrt{k_1 / \mathscr{D}_{\mathrm{Se}}}\right)}}{k_1 s_{\mathrm{b}} A L}$$

Cancelling and grouping terms and applying the definition of tanh x gives:

$$\eta_{\mathrm{i1}} = \frac{\tanh\left(L\sqrt{k_1 / \mathscr{D}_{\mathrm{Se}}}\right)}{L\sqrt{k_1 / \mathscr{D}_{\mathrm{Se}}}}$$

Therefore, applying the definition of ϕ_1:

$$\eta_{\mathrm{i1}} = \frac{\tanh \phi_1}{\phi_1}$$

Answer: QED

(f)

From the definition of tanh x:

$$\tanh \phi_1 = \frac{e^{\phi_1} - e^{-\phi_1}}{e^{\phi_1} + e^{-\phi_1}}$$

For L = 2 cm, the values of ϕ_1, tanh ϕ_1 and η_{i1} evaluated for the three sets of conditions are listed below.

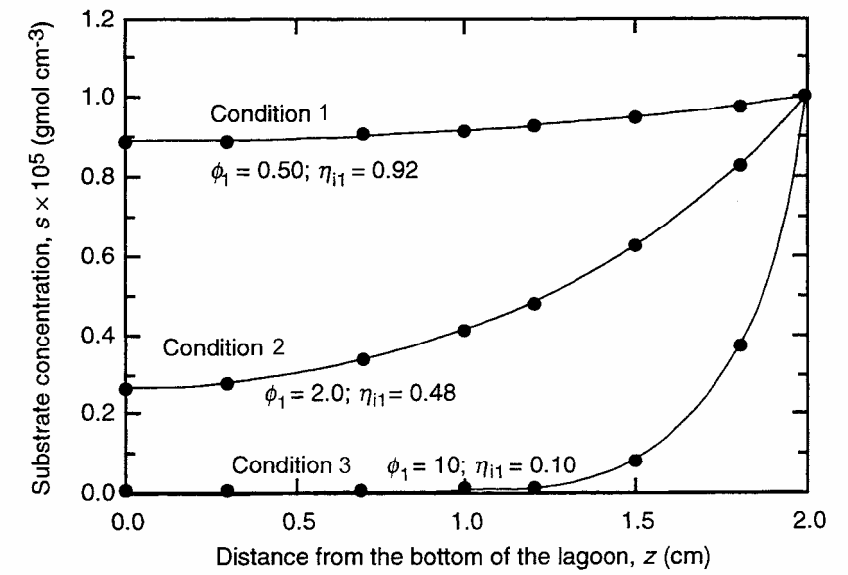
Condition	ϕ_1	tanh ϕ_1	η_{i1}
1	0.50	0.46	0.92
2	2.0	0.96	0.48
3	10	1.0	0.10

The substrate concentration profiles are calculated using the equation for s as a function of z from **(c) (iii)** and applying the definition of ϕ_1:

$$s = s_{\mathrm{b}} \frac{e^{z\sqrt{k_1/\mathscr{D}_{\mathrm{Se}}}} + e^{-z\sqrt{k_1/\mathscr{D}_{\mathrm{Se}}}}}{e^{\phi_1} + e^{-\phi_1}}$$

For $s_{\mathrm{b}} = 10^{-5}$ gmol cm^{-3}, the results for s at various values of z are listed and plotted below.

Distance, z (cm)	Substrate concentration, s (gmol cm^{-3})		
	Condition 1	Condition 2	Condition 3
0.0	8.87×10^{-6}	2.66×10^{-6}	9.08×10^{-10}
0.3	8.89×10^{-6}	2.78×10^{-6}	2.14×10^{-9}
0.7	9.00×10^{-6}	3.34×10^{-6}	1.50×10^{-8}
1.0	9.15×10^{-6}	4.10×10^{-6}	6.74×10^{-8}
1.2	9.27×10^{-6}	4.81×10^{-6}	1.83×10^{-7}
1.5	9.50×10^{-6}	6.25×10^{-6}	8.21×10^{-7}
1.8	9.78×10^{-6}	8.26×10^{-6}	3.68×10^{-6}
2.0	1.00×10^{-5}	1.00×10^{-5}	1.00×10^{-5}



Answer: As ϕ_1 increases, η_{i1} decreases, the concentration profile becomes steeper, and the minimum substrate concentration in the sludge is reduced.

13.2 Oxygen concentration profile in an immobilised enzyme catalyst

(a)

From Section 12.3.3, as C_{As} is (0.5 mM)/(0.015 mM) = 33 times the value of K_m, as a first approximation we can consider the kinetics to be effectively zero-order with $k_0 = v_{max}$. Converting the units of k_0 to a per volume of gel basis:

$$k_0 = v_{max} = 0.12 \text{ mol s}^{-1} \text{ kg}^{-1} (0.012 \text{ kg m}^{-3}) . \left| \frac{32 \text{ g}}{1 \text{ mol}} \right| . \left| \frac{1 \text{ kg}}{1000 \text{ g}} \right| = 4.61 \times 10^{-5} \text{ kg s}^{-1} \text{ m}^{-3}$$

Converting C_{As} to units of kg m^{-3}:

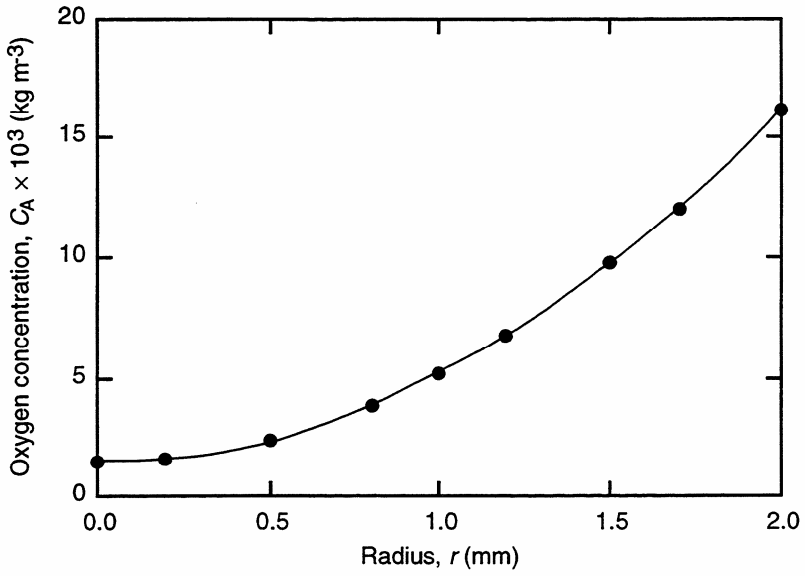
$$C_{As} = 0.5 \text{ mM} = 0.5 \times 10^{-3} \text{ gmol l}^{-1} . \left| \frac{1000 \text{ l}}{1 \text{ m}^3} \right| . \left| \frac{32 \text{ g}}{1 \text{ gmol}} \right| . \left| \frac{1 \text{ kg}}{1000 \text{ g}} \right| = 0.016 \text{ kg m}^{-3}$$

For zero-order reaction, the equation used to determine the substrate concentration inside the beads depends on whether C_A remains > 0 throughout the particle. The maximum particle radius for which this occurs can be calculated using Eq. (13.17):

$$R_{max} = \sqrt{\frac{6\,(2.1\times10^{-9}\ \text{m}^2\ \text{s}^{-1})\ 0.016\ \text{kg m}^{-3}}{4.61\times10^{-5}\ \text{kg s}^{-1}\ \text{m}^{-3}}} = 0.0021\ \text{m} = 2.1\ \text{mm}$$

Therefore, the maximum particle diameter for $C_A > 0$ everywhere is 2 × 2.1 mm = 4.2 mm. Because the immobilised enzyme beads are smaller than this, $C_A > 0$ and the oxygen concentration profile can be calculated using the equation for zero-order reaction and spherical geometry in Table 13.1. Values for C_A as a function of r are listed and plotted below.

Radius, r (m)	Oxygen concentration, C_A (kg m^{-3})
2.0×10^{-3}	1.60×10^{-2}
1.7×10^{-3}	1.19×10^{-2}
1.5×10^{-3}	9.60×10^{-3}
1.2×10^{-3}	6.63×10^{-3}
1.0×10^{-3}	5.02×10^{-3}
0.8×10^{-3}	3.71×10^{-3}
0.5×10^{-3}	2.28×10^{-3}
0.2×10^{-3}	1.51×10^{-3}
0.0	1.37×10^{-3}



As the minimum value of C_A at the centre of the bead is still about 2.9 times K_m, the assumption of zero-order kinetics is reasonable.

(b)

As $C_A > 0$ everywhere within the bead, the entire bead volume is active.

Answer: 1.0

(c)

For zero-order reaction, the maximum conversion rate occurs when the oxygen concentration is greater than zero everywhere in the particle. The largest bead size for this to occur was calculated in **(a)** as 4.2 mm.

Answer: 4.2 mm

13.3 Effect of oxygen transfer on recombinant cells

(a)

Converting the units of k_0 from moles to mass:

$$k_0 = 10^{-3}\ \text{mol s}^{-1}\ \text{m}^{-3} . \left|\frac{32\ \text{g}}{1\ \text{mol}}\right| . \left|\frac{1\ \text{kg}}{1000\ \text{g}}\right| = 3.2\times10^{-5}\ \text{kg s}^{-1}\ \text{m}^{-3}$$

The maximum particle radius for which oxygen concentration inside the beads remains greater than zero is calculated using Eq. (13.17):

$$R_{\max} = \sqrt{\frac{6\,(1.4\times10^{-9}\ \text{m}^2\ \text{s}^{-1})\,8\times10^{-3}\ \text{kg m}^{-3}}{3.2\times10^{-5}\ \text{kg s}^{-1}\ \text{m}^{-3}}} = 1.45\times10^{-3}\ \text{m} = 1.45\ \text{mm}$$

Therefore, the maximum particle diameter for aerobic conditions is 2×1.45 mm = 2.9 mm.

Answer: 2.9 mm

(b)

The particle radius is (1.45 mm)/2 = 0.725 mm = 7.25×10^{-4} m. As this radius is less than $R_{\max}$ determined in **(a)**, oxygen is present everywhere in the particle. Therefore, for zero-order kinetics, $\eta_i = 1$. From Eq. (13.26) with $\eta_i = 1$:

$$r_{A,obs} = r^*_{As}$$

where $r^*_{As} = k_0 = 3.2 \times 10^{-5}$ kg s^{-1} m^{-3}. Substituting parameter values into the equation for the observable Thiele modulus Φ in Table 13.4 for spherical geometry:

$$\Phi = \left(\frac{7.25\times10^{-4}\ \text{m}}{3}\right)^2 \frac{3.2\times10^{-5}\ \text{kg s}^{-1}\ \text{m}^{-3}}{1.4\times10^{-9}\ \text{m}^2\ \text{s}^{-1}\,(8\times10^{-3}\ \text{kg m}^{-3})} = 0.167$$

Using this value, the minimum intraparticle oxygen concentration can be calculated from the equation in Table 13.5 for spherical geometry and $\Phi < 0.667$:

$$C_{A,\min} = 8\times10^{-3}\ \text{kg m}^{-3}\left(1-\frac{3}{2}(0.167)\right) = 6.0\times10^{-3}\ \text{kg m}^{-3}$$

Answer: 6.0×10^{-3} kg m^{-3}

(c)

If the cell density is reduced by a factor of 5, k_0 is reduced to 1/5 its original value. Therefore, $k_0 = 1/5 \times (3.2 \times 10^{-5}$ kg s^{-1} m$^{-3}) = 6.4 \times 10^{-6}$ kg s^{-1} m^{-3}. From Eq. (13.17):

$$R_{\max} = \sqrt{\frac{6\,(1.4\times10^{-9}\ \text{m}^2\ \text{s}^{-1})\,8\times10^{-3}\ \text{kg m}^{-3}}{6.4\times10^{-6}\ \text{kg s}^{-1}\ \text{m}^{-3}}} = 3.24\times10^{-3}\ \text{m} = 3.24\ \text{mm}$$

Therefore, the maximum particle diameter for aerobic conditions is 2×3.24 mm = 6.5 mm.

Answer: 6.5 mm diameter

13.4 Ammonia oxidation by immobilised cells

(a)

R = 1.5 mm = 1.5 × 10^{-3} m. Calculating the observable modulus Ω for spherical geometry from Table 13.6:

$$\Omega = \frac{1.5\times10^{-3}\ \text{m}}{3}\frac{2.2\times10^{-5}\ \text{kg s}^{-1}\ \text{m}^{-3}}{6\times10^{-5}\ \text{m s}^{-1}\ (6\times10^{-3}\ \text{kg m}^{-3})} = 0.031$$

From Eqs (13.43) and (13.44):

$$C_{\text{As}} = C_{\text{Ab}}(1-\Omega) = C_{\text{Ab}}(1-0.031) = 0.97C_{\text{Ab}}$$

As $C_{\text{As}} \approx C_{\text{Ab}}$, external mass transfer effects are insignificant.

Answer: Insignificant; the surface oxygen concentration is only 3% lower than in the bulk medium.

(b)

From Section 13.5, for zero-order oxygen uptake kinetics, $C_{\text{As}} > 0$ means that $\eta_{\text{e0}} = 1$. The internal effectiveness factor η_{i0} can be determined from Figure 13.14 as a function of the observable Thiele modulus Φ. Evaluating Φ from the equation in Table 13.4 for spherical geometry using the result for C_{As} from **(a)**:

$$\Phi = \left(\frac{1.5\times10^{-3}\ \text{m}}{3}\right)^2 \frac{2.2\times10^{-5}\ \text{kg s}^{-1}\ \text{m}^{-3}}{1.9\times10^{-9}\ \text{m}^2\ \text{s}^{-1}\ (0.97\times6\times10^{-3}\ \text{kg m}^{-3})} = 0.50$$

From Figure 13.14, at $\Phi = 0.50$, $\eta_{\text{i0}} = 1$. Applying Eq. (13.46), $\eta_{\text{T}} = \eta_{\text{i0}}\eta_{\text{e0}} = 1 \times 1 = 1$.

Answer: 1

(c)

Using the results from **(a)** and **(b)**, the minimum intraparticle oxygen concentration is calculated from the equation in Table 13.5 for spherical geometry and $\Phi < 0.667$:

$$C_{\text{A,min}} = 0.97\times6\times10^{-3}\ \text{kg m}^{-3}\left(1-\frac{3}{2}(0.50)\right) = 1.5\times10^{-3}\ \text{kg m}^{-3}$$

This oxygen concentration is greater than the critical level.

Answer: Yes

13.5 Microcarrier culture and external mass transfer

D_{p} = 120 μm = 120 × 10^{-6} m. The external mass transfer coefficient k_{S} is determined using the equations in Section 13.6.1 for free-moving spheres. The Grashof number is calculated from Eq. (13.51) with g = 9.8 m s^{-2} (Section 2.3) and the viscosity unit conversion factor 1 N s m^{-2} = 1 kg m^{-1} s^{-1} from Table A.9 (Appendix A):

$$Gr = \frac{9.8\ \text{m s}^{-2}\ (120\times10^{-6}\ \text{m})^3\ (10^3\ \text{kg m}^{-3})\ (1.2\times10^3 - 10^3)\ \text{kg m}^{-3}}{\left(10^{-3}\ \text{N s m}^{-2}.\left|\frac{1\ \text{kg m}^{-1}\ \text{s}^{-1}}{1\ \text{N s m}^{-2}}\right|\right)^2} = 3.39$$

Therefore, from Eq. (13.52):

$$Re_p = \frac{3.39}{18} = 0.188$$

The Schmidt number from Eq. (13.49) is:

$$Sc = \frac{10^{-3}\ \text{N s m}^{-2} . \left| \frac{1\ \text{kg m}^{-1}\ \text{s}^{-1}}{1\ \text{N s m}^{-2}} \right|}{10^3\ \text{kg m}^{-3}\ (2.3\times10^{-9}\ \text{m}^2\ \text{s}^{-1})} = 435$$

Therefore, $Re_pSc = 0.188 \times 435 = 81.8$. As this value is less than 10^4, the Sherwood number can be evaluated using Eq. (13.55):

$$Sh = \sqrt{4 + 1.21(81.8)^{0.67}} = 5.21$$

From the definition of the Sherwood number in Eq. (13.50):

$$k_S = \frac{Sh\,\mathscr{D}_{AL}}{D_p} = \frac{5.21\,(2.3\times10^{-9}\ \text{m}^2\ \text{s}^{-1})}{120\times10^{-6}\ \text{m}} = 9.99\times10^{-5}\ \text{m s}^{-1}$$

Using this value of k_S to determine Ω from the equation in Table 13.6 for spherical geometry:

$$\Omega = \frac{\left(\frac{120\times10^{-6}\ \text{m}}{2}\right)}{3} \frac{0.015\ \text{mol s}^{-1}\ \text{m}^{-3}}{9.99\times10^{-5}\ \text{m s}^{-1}\ (0.2\ \text{mol m}^{-3})} = 0.015$$

From Eqs (13.43) and (13.44):

$$C_{As} = C_{Ab}(1-\Omega) = C_{Ab}(1-0.015) = 0.985\,C_{Ab}$$

External mass transfer effects are insignificant as $C_{As} \approx C_{Ab}$. Because respiration is zero-order and the cells are present only on the surface of the beads, $C_{As} > 0$ is all that is required to ensure maximum reaction rate.

Answer: No effect

13.6 Immobilised enzyme reaction kinetics

(a)

$R = 0.8$ mm $= 0.8 \times 10^{-3}$ m. As external boundary layers have been eliminated, $C_{As} = C_{Ab} = 0.85$ kg m^{-3} and $\eta_e = 1$. The value of β as defined in Section 13.4.3 is:

$$\beta = \frac{K_m}{C_{As}} = \frac{3.5\ \text{kg m}^{-3}}{0.85\ \text{kg m}^{-3}} = 4.12$$

From Figures 13.13–13.15, this value of β means that the reaction kinetics can be considered effectively first-order. Evaluating the observable Thiele modulus Φ from the equation in Table 13.4 for spherical geometry:

$$\Phi = \left(\frac{0.8\times10^{-3}\ \text{m}}{3}\right)^2 \frac{1.25\times10^{-3}\ \text{kg s}^{-1}\ \text{m}^{-3}}{1.3\times10^{-11}\ \text{m}^2\ \text{s}^{-1}\ (0.85\ \text{kg m}^{-3})} = 8.0$$

From Figure 13.14, at $\Phi = 8.0$, $\eta_{i1} = 0.12$. Therefore, from Eq. (13.46), $\eta_T = \eta_i\eta_e = 0.12 \times 1 = 0.12$.

Answer: 0.12

(b)

From the definition of the internal effectiveness factor, Eq. (13.26):

$$r_{As}^{*} = \frac{r_{A,obs}}{\eta_i} = \frac{1.25\times10^{-3}\ \text{kg s}^{-1}\ \text{m}^{-3}}{0.12} = 0.0104\ \text{kg s}^{-1}\ \text{m}^{-3}$$

For first-order kinetics, $r_{As}^{*} = k_1 C_{As}$; therefore:

$$k_1 = \frac{r_{As}^{*}}{C_{As}} = \frac{0.0104\ \text{kg s}^{-1}\ \text{m}^{-3}}{0.85\ \text{kg m}^{-3}} = 0.0122\ \text{s}^{-1}$$

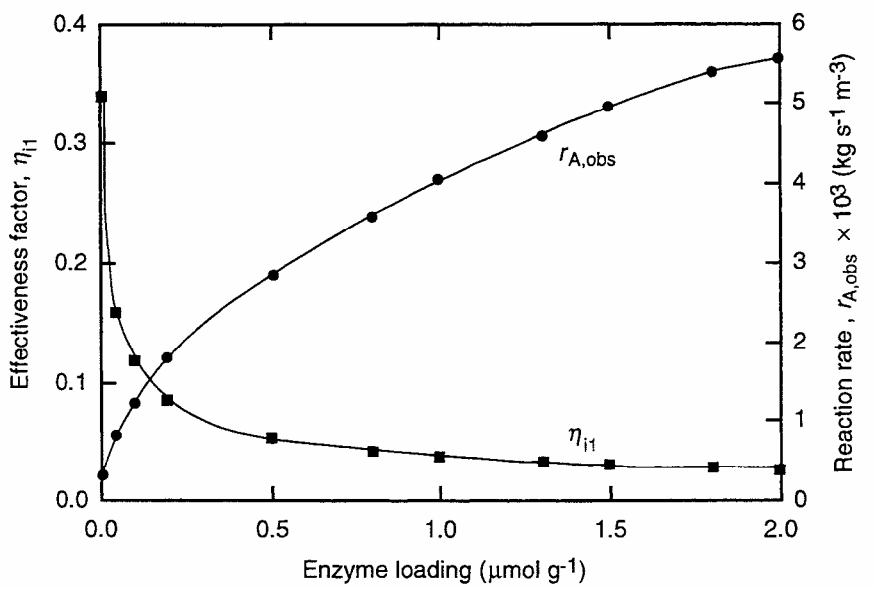
Answer: 0.0122 s^{-1}

(c)

The value of 0.0122 s^{-1} for k_1 corresponds to an enzyme loading of 0.1 μmol g^{-1}. The generalised Thiele modulus ϕ_1 can be evaluated as a function of enzyme loading using the equation for first-order reaction and spherical geometry from Table 13.2, with k_1 directly proportional to the enzyme loading. For $\phi_1 < 10$, the internal effectiveness factor η_{i1} is determined using the equation in Table 13.3 and the definition of coth x; for $\phi_1 > 10$, from Eq. (13.30), $\eta_{i1} \approx 1/\phi_1$. For each value of k_1, $r_{As}^{*} = k_1 C_{As}$ and $r_{A,obs}$ can be determined from these results and the definition of the internal effectiveness factor in Eq. (13.26). Calculated values of these parameters for several different enzyme loadings are listed below.

Enzyme loading (μmol g^{-1})	k_1 (s^{-1})	ϕ_1	η_{i1}	r_{As}^{*} (kg s^{-1} m^{-3})	$r_{A,obs}$ (kg s^{-1} m^{-3})
0.01	0.0012	2.6	0.34	0.0010	3.40×10^{-4}
0.05	0.0061	5.8	0.16	0.0052	8.32×10^{-4}
0.10	0.0122	8.2	0.12	0.0104	1.25×10^{-3}
0.20	0.0244	11.6	0.086	0.021	1.81×10^{-3}
0.50	0.0610	18.3	0.055	0.052	2.86×10^{-3}
0.80	0.0976	23.1	0.043	0.083	3.57×10^{-3}
1.0	0.122	25.8	0.039	0.104	4.06×10^{-3}
1.3	0.159	29.5	0.034	0.135	4.59×10^{-3}
1.5	0.183	31.6	0.032	0.156	4.99×10^{-3}
1.8	0.220	34.7	0.029	0.187	5.42×10^{-3}
2.0	0.244	36.5	0.027	0.207	5.59×10^{-3}

The results for η_{i1} and $r_{A,obs}$ are plotted below as a function of enzyme loading.



Answer: As the enzyme loading is increased from 0.01 μmol g^{-1}, the effectiveness factor drops significantly. Although the reaction rate continues to rise with increasing enzyme loading, at loadings above about 0.5 μmol g^{-1}, less than 5% of the potential activity of the enzyme is being utilised. Further increases in enzyme loading therefore represent an effective waste of more than 95% of that enzyme.

13.7 Mass transfer effects in plant cell culture

(a)

D_p = 1.5 mm = 1.5 × 10^{-3} m. The particle Reynolds number is evaluated using Eq. (13.48) with ρ_L = density of water = 1 g cm^{-3} (Section 2.4.1) = 10^3 kg m^{-3} and μ_L = viscosity of water = 10^{-3} kg m^{-1} s^{-1} (Eq. 7.8):

$$Re_p = \frac{1.5\times10^{-3}\text{ m }(0.83\times10^{-2}\text{ m s}^{-1})\,(10^3\text{ kg m}^{-3})}{10^{-3}\text{ kg m}^{-1}\text{ s}^{-1}} = 12.5$$

As this value is within the range $10 < Re_p < 10^4$, the external mass transfer coefficient can be determined using Eq. (13.57) for spherical particles in a packed bed. Converting the diffusivity units to m^2 s^{-1}:

$$\mathscr{D}_{Ae} = 9\times10^{-6}\text{ cm}^2\text{ s}^{-1}.\left|\frac{1\text{ m}}{100\text{ cm}}\right|^2 = 9\times10^{-10}\text{ m}^2\text{ s}^{-1}$$

$\mathscr{D}_{AL}$ = 2 × $\mathscr{D}_{Ae}$ = 2 × 9 × 10^{-10} m^2 s^{-1} = 1.8 × 10^{-9} m^2 s^{-1}. The Schmidt number from Eq. (13.49) is:

$$Sc = \frac{10^{-3}\text{ kg m}^{-1}\text{ s}^{-1}}{10^3\text{ kg m}^{-3}\,(1.8\times10^{-9}\text{ m}^2\text{ s}^{-1})} = 556$$

From Eq. (13.57), the Sherwood number is evaluated as:

$$Sh = 0.95\,(12.5)^{0.5}\,(556)^{0.33} = 27.0$$

From the definition of the Sherwood number in Eq. (13.50):

$$k_S = \frac{Sh\,\mathscr{D}_{AL}}{D_p} = \frac{27.0\,(1.8\times10^{-9}\text{ m}^2\text{ s}^{-1})}{1.5\times10^{-3}\text{ m}} = 3.24\times10^{-5}\text{ m s}^{-1}$$

This value of k_S can be used to determine the observable modulus for external mass transfer Ω from the equation in Table 13.6 for spherical geometry. As the specific gravity of the wet cells is 1, from Section 2.4.2, 1 g of wet cells occupies a volume of 1 cm^3 and $r_{A,obs}$ = 0.28 mg cm^{-3} h^{-1}. Converting the units of $r_{A,obs}$ to kg s^{-1} m^{-3}:

$$r_{A,obs} = 0.28\text{ mg cm}^{-3}\text{ h}^{-1}.\left|\frac{1\text{ kg}}{10^6\text{ mg}}\right|.\left|\frac{1\text{ h}}{3600\text{ s}}\right|.\left|\frac{100\text{ cm}}{1\text{ m}}\right|^3 = 7.78\times10^{-5}\text{ kg s}^{-1}\text{ m}^{-3}$$

Substituting this and the other parameter values into the equation for Ω:

$$\Omega = \frac{\left(\frac{1.5\times10^{-3}\text{ m}}{2}\right)}{3}\frac{7.78\times10^{-5}\text{ kg s}^{-1}\text{ m}^{-3}}{3.24\times10^{-5}\text{ m s}^{-1}\,(8\text{ mg l}^{-1}).\left|\frac{1\text{ kg}}{10^6\text{ mg}}\right|.\left|\frac{1000\text{ l}}{1\text{ m}^3}\right|} = 0.075$$

From Eqs (13.43) and (13.44):

$$C_{As} = C_{Ab}(1-\Omega) = C_{Ab}(1-0.075) = 0.925\,C_{Ab}$$

As C_{As} is close to C_{Ab}, external mass transfer effects are present but small.

Answer: The effect is small; the surface oxygen concentration is 0.925 times that in the bulk medium

(b)

Evaluating the observable Thiele modulus Φ from the equation in Table 13.4 for spherical geometry using the result for C_{As} from **(a)**:

$$\Phi = \left(\frac{\left(\frac{1.5\times10^{-3}\ \text{m}}{2} \right)}{3} \right)^2 \frac{7.78\times10^{-5}\ \text{kg s}^{-1}\ \text{m}^{-3}}{9\times10^{-10}\ \text{m}^2\ \text{s}^{-1} (0.925\times8\ \text{mg l}^{-1}) \cdot \left| \frac{1\ \text{kg}}{10^6\ \text{mg}} \right| \cdot \left| \frac{1000\ \text{l}}{1\ \text{m}^3} \right|} = 0.73$$

From Figure 13.14 for zero-order reaction, at $\Phi = 0.73$, η_{i0} is very close to 1.0. Therefore, internal mass transfer effects can be considered negligible.

Answer: Negligible

(c)

$C_{As} = 0.925 \times 8\ \text{mg l}^{-1} = 7.4\ \text{mg l}^{-1}$. As η_{i0} is very close to 1.0, it is likely that oxygen is exhausted just close to the centre of the clumps. Therefore, taking $r_{A,obs}$ to be virtually equal to the intrinsic zero-order rate constant k_0, the maximum particle radius for the oxygen concentration to remain greater than zero throughout the clump is evaluated using Eq. (13.17):

$$R_{max} = \sqrt{\frac{6\ (9\times10^{-10}\ \text{m}^2\ \text{s}^{-1})\ 7.4\ \text{mg l}^{-1} \cdot \left| \frac{1\ \text{kg}}{10^6\ \text{mg}} \right| \cdot \left| \frac{1000\ \text{l}}{1\ \text{m}^3} \right|}{7.78\times10^{-5}\ \text{kg s}^{-1}\ \text{m}^{-3}}} = 7.2\times10^{-4}\ \text{m} = 0.72\ \text{mm}$$

Therefore, the maximum particle diameter for oxygen through to the centre of the clump is 2×0.72 mm = 1.4 mm, which is only slightly less than the plant cell clump diameter of 1.5 mm.

Answer: The oxygen concentration falls from 7.4 mg l^{-1} at the external surface to zero just near the centre of the clumps.

13.8 Respiration in mycelial pellets

(a)

$R = 2.5\ \text{mm} = 2.5 \times 10^{-3}$ m. The presence of external boundary layers can be checked by calculating the observable modulus for external mass transfer, Ω. From Table 13.6 for spherical geometry:

$$\Omega = \frac{2.5\times10^{-3}\ \text{m}}{3} \frac{8.7\times10^{-5}\ \text{kg s}^{-1}\ \text{m}^{-3}}{3.8\times10^{-5}\ \text{m s}^{-1}\ (8\times10^{-3}\ \text{kg m}^{-3})} = 0.24$$

From Eqs (13.43) and (13.44):

$$C_{As} = C_{Ab}(1-\Omega) = C_{Ab}(1-0.24) = 0.76\,C_{Ab} = 0.76\ (8\times10^{-3}\ \text{kg m}^{-3}) = 6.1\times10^{-3}\ \text{kg m}^{-3}$$

As C_{As} is significantly less than C_{Ab}, external mass transfer effects are present.

Answer: Yes

(b)

As oxygen uptake is considered a zero-order reaction, for $C_{As} > 0$, $\eta_{e0} = 1$ (Section 13.5).

Answer: 1

(c)

In the absence of internal and external mass transfer resistances, the reaction rate is r^*_{Ab} for $C_A = C_{Ab}$ throughout the pellets. As r^*_{Ab} is related to $r_{A,obs}$ by Eq. (13.45), r^*_{Ab} can be determined if we know the value

of η_T corresponding to $r_{A,obs}$ in the presence of mass transfer limitations. Evaluating the observable Thiele modulus Φ using the equation in Table 13.4 for spherical geometry and the result for C_{As} from **(a)**:

$$\Phi = \left(\frac{2.5\times10^{-3}\text{ m}}{3}\right)^2 \frac{8.7\times10^{-5}\text{ kg s}^{-1}\text{ m}^{-3}}{1.75\times10^{-9}\text{ m}^2\text{ s}^{-1}(6.1\times10^{-3}\text{ kg m}^{-3})} = 5.66$$

From Figure 13.14 for zero-order reaction, at $\Phi = 5.66$, $\eta_{i0} = 0.30$. Therefore, from Eq. (13.46), as $\eta_{e0} = 1$ from **(b)**, $\eta_T = \eta_{i0}\eta_{e0} = 0.30 \times 1 = 0.30$. Using Eq. (13.45):

$$r_{Ab}^* = \frac{r_{A,obs}}{\eta_T} = \frac{8.7\times10^{-5}\text{ kg s}^{-1}\text{ m}^{-3}}{0.30} = 2.9\times10^{-4}\text{ kg s}^{-1}\text{ m}^{-3}$$

Answer: 2.9×10^{-4} kg s^{-1} m^{-3}, or more than three times the rate actually observed

(d)

If external mass transfer effects were eliminated, $C_{As} = C_{Ab} = 8 \times 10^{-3}$ kg m^{-3} and the observed reaction rate would be greater than 8.7×10^{-5} kg s^{-1} m^{-3}. Under these conditions, an expression for the observable Thiele modulus Φ from the equation in Table 13.4 for spherical geometry is:

$$\Phi = \left(\frac{2.5\times10^{-3}\text{ m}}{3}\right)^2 \frac{r_{A,obs}}{1.75\times10^{-9}\text{ m}^2\text{ s}^{-1}(8\times10^{-3}\text{ kg m}^{-3})} = 4.96\times10^4\, r_{A,obs} \quad (1)$$

where $r_{A,obs}$ has units of kg s^{-1} m^{-3}. Because the reaction is zero-order, $r_{As}^* = r_{Ab}^*$; therefore, from the result for r_{Ab}^* in **(c)**, $r_{As}^* = 2.9 \times 10^{-4}$ kg s^{-1} m^{-3}. From Eq. (13.26):

$$\eta_{i0} = \frac{r_{A,obs}}{2.9\times10^{-4}\text{ kg s}^{-1}\text{ m}^{-3}} \quad (2)$$

Φ and η_{i0} are related by the curve in Figure 13.14 for spheres and zero-order reaction. The value of $r_{A,obs}$ can be determined by trial-and-error using Figure 13.14 and the equations derived above. As a first guess, take $r_{A,obs} = 2.0 \times 10^{-4}$ kg s^{-1} m^{-3}. Depending on the difference between the values of η_{i0} obtained from the figure and from (2), $r_{A,obs}$ is adjusted as shown in the table below.

$r_{A,obs}$ (kg s^{-1} m^{-3})	η_{i0} – from (2)	Φ – from (1)	η_{i0} – from Figure 13.14
2.0×10^{-4}	0.69	9.92	0.19
1.0×10^{-4}	0.34	4.96	0.34

As the values for η_{i0} in the last row are as close as practical, $r_{A,obs} = 1.0 \times 10^{-4}$ kg s^{-1} m^{-3}. Therefore, compared with the reaction rate of 8.7×10^{-5} kg s^{-1} m^{-3} observed in the presence of both internal and external mass transfer resistances, eliminating the external boundary layers increases the reaction rate by about 15%.

Answer: 1.0×10^{-4} kg s^{-1} m^{-3}

13.9 Effect of mass transfer on glyphosate removal

(a)

From the experimental data, the observed rate of reaction increases with liquid recirculation rate until it reaches a maximum of 8.4×10^{-5} kg s^{-1} m^{-3} at a flow rate of 35 cm^3 s^{-1}. As explained in Section 13.8.2, external mass transfer effects are eliminated under these conditions.

Answer: 35 cm^3 s^{-1}. The reaction rate becomes independent of liquid velocity when external mass transfer effects are eliminated.

(b)

$R = 3.5$ mm $= 3.5 \times 10^{-3}$ m. $C_{Ab} = 60$ mg l^{-1} $= 60$ g m^{-3} $= 60 \times 10^{-3}$ kg m^{-3}. From **(a)**, at an operating flow rate of 40 cm^3 s^{-1}, $r_{A,obs} = 8.4 \times 10^{-5}$ kg s^{-1} m^{-3} and external mass transfer effects are eliminated. Therefore, at this flow rate, $\eta_{e1} = 1$. For first-order reactions, from Table 13.7, $\eta_{e1} = 1$ means that $C_{Ab} = C_{As} = 60 \times 10^{-3}$ kg m^{-3}. Evaluating the observable Thiele modulus Φ using the equation in Table 13.4 for spherical geometry:

$$\Phi = \left(\frac{3.5\times10^{-3}\ \text{m}}{3} \right)^2 \frac{8.4\times10^{-5}\ \text{kg s}^{-1}\ \text{m}^{-3}}{0.9\times10^{-9}\ \text{m}^2\ \text{s}^{-1}\ (60\times10^{-3}\ \text{kg m}^{-3})} = 2.12$$

From Figure 13.14 for first-order reaction, at $\Phi = 2.12$, $\eta_{i1} = 0.31$. Therefore, from Eq. (13.46), $\eta_T = \eta_{i1}\eta_{e1} = 0.31 \times 1 = 0.31$. Using Eq. (13.45):

$$r_{Ab}^{*} = \frac{r_{A,obs}}{\eta_T} = \frac{8.4\times10^{-5}\ \text{kg s}^{-1}\ \text{m}^{-3}}{0.31} = 2.71\times10^{-4}\ \text{kg s}^{-1}\ \text{m}^{-3}$$

Answer: 2.7×10^{-4} kg s^{-1} m^{-3}

13.10 Mass transfer effects in tissue-engineered cartilage

(a)

$b = 3$ mm $= 3 \times 10^{-3}$ m. $C_{Ab} = 7 \times 10^{-3}$ kg m^{-3}. $\mathscr{D}_{Ae} = 1.0 \times 10^{-9}$ m^2 s^{-1}; $k_S = 2 \times 10^{-5}$ m s^{-1}. $r_{A,obs} = 6.6 \times 10^{-6}$ kg s^{-1} m^{-3}. Calculating the observable modulus Ω for flat-plate geometry from Table 13.6:

$$\Omega = (3\times10^{-3}\ \text{m}) \frac{6.6\times10^{-6}\ \text{kg s}^{-1}\ \text{m}^{-3}}{2\times10^{-5}\ \text{m s}^{-1}\ (7\times10^{-3}\ \text{kg m}^{-3})} = 0.141$$

From Eqs (13.43) and (13.44):

$$C_{As} = C_{Ab}(1-\Omega) = 7\times10^{-3}\ \text{kg m}^{-3}\ (1-0.141) = 6.01\times10^{-3}\ \text{kg m}^{-3}$$

Even though $C_{As} < C_{Ab}$, because the kinetics of oxygen uptake are zero-order, for $C_{As} > 0$, $\eta_{e0} = 1$ (Section 13.5). To assess whether internal mass transfer limitations are present, the internal effectiveness factor η_{i0} is determined from Figure 13.15 as a function of the observable Thiele modulus Φ. Evaluating Φ from the equation in Table 13.4 for flat-plate geometry:

$$\Phi = (3\times10^{-3}\ \text{m})^2 \frac{6.6\times10^{-6}\ \text{kg s}^{-1}\ \text{m}^{-3}}{1.0\times10^{-9}\ \text{m}^2\ \text{s}^{-1}\ (6.01\times10^{-3}\ \text{kg m}^{-3})} = 9.88$$

From Figure 13.15 for zero-order kinetics, at $\Phi = 9.88$, $\eta_{i0} = 0.2$. This result indicates that the culture is affected by significant mass transfer limitations.

Answer: Yes

(b)

From Eq. (13.46) and the effectiveness factor results in **(a)**, $\eta_T = \eta_{i0}\eta_{e0} = 0.2 \times 1 = 0.2$. From Eq. (13.45):

$$r_{Ab}^{*} = \frac{r_{A,obs}}{\eta_T} = \frac{6.6\times10^{-6}\ \text{kg s}^{-1}\ \text{m}^{-3}}{0.2} = 3.30\times10^{-5}\ \text{kg s}^{-1}\ \text{m}^{-3}$$

Answer: 3.3×10^{-5} kg s^{-1} m^{-3}

(c)

From **(a)**, $\Phi = 9.88$. For flat-plate geometry and $\Phi \geq 2$, the minimum oxygen concentration within the cartilage tissue is given in Table 13.5 as $C_{A,min} = 0$.

Answer: Zero

(d)

If external boundary layers are eliminated by increasing the flow rate of medium over the tissue, $\eta_{e0} = 1$ and $C_{As} = C_{Ab} = 7 \times 10^{-3}$ kg m^{-3}. If the tissue thickness is also reduced so that $b = 2$ mm $= 2 \times 10^{-3}$ m, this will affect the observable rate of reaction $r_{A,obs}$, the observable Thiele modulus Φ, and the internal effectiveness factor η_{i0}. From Eq. (13.46), $\eta_T = \eta_{i0}\eta_{e0} = \eta_{i0} \times 1 = \eta_{i0}$. Using this result in Eq. (13.45):

$$r_{A,obs} = r^*_{Ab}\eta_{i0} \tag{1}$$

r^*_{Ab} is not affected by mass transfer conditions and remains constant if the cell density is unchanged. Substituting (1) and the result for r^*_{Ab} from **(b)** into the equation for Φ in Table 13.4 for flat-plate geometry:

$$\Phi = (2\times10^{-3}\text{ m})^2 \frac{(3.30\times10^{-5}\text{ kg s}^{-1}\text{ m}^{-3})\eta_{i0}}{1.0\times10^{-9}\text{ m}^2\text{ s}^{-1}(7\times10^{-3}\text{ kg m}^{-3})} = 18.86\eta_{i0}$$

Therefore:

$$\eta_{i0} = \frac{\Phi}{18.86} \tag{2}$$

$r_{A,obs}$ can be found by solving iteratively for Φ and η_{i0}. A value for Φ is estimated, η_{i0} is determined using (2), and the result is compared with the value of η_{i0} obtained from Figure 13.15 for zero-order kinetics. As mass transfer is improved by reducing the thickness of the tissue, we can expect that Φ will be less than the value of 9.88 determined in **(a)**. Results for several Φ are shown in the table below.

Estimated Φ	η_{i0} calculated using (2)	η_{i0} from Figure 13.15
7.0	0.37	0.30
5.5	0.29	0.36
6.0	0.32	0.32

We can take $\Phi = 6.0$ and $\eta_{i0} = 0.32$ as the solution. Applying this result for η_{i0} and the value of r^*_{Ab} from **(b)** in (1):

$$r_{A,obs} = (3.30\times10^{-5}\text{ kg s}^{-1}\text{ m}^{-3})\,0.32 = 1.06\times10^{-5}\text{ kg s}^{-1}\text{ m}^{-3}$$

Answer: 1.1×10^{-5} kg s^{-1} m^{-3}

(e)

When external boundary layers are removed, $C_{As} = C_{Ab} = 7 \times 10^{-3}$ kg m^{-3}. The maximum thickness of cartilage without oxygen depletion, b_{max}, is evaluated using the equation for zero-order reaction and flat-plate geometry in Table 13.1. By analogy with the derivation of Eq. (13.17), if we set $C_A = z = 0$ in this equation, oxygen is depleted just at the inner edge of the tissue. Substituting parameter values with the intrinsic zero-order rate constant k_0 equal to r^*_{Ab} from **(b)**:

$$0 = 7\times10^{-3}\text{ kg m}^{-3} + \frac{3.30\times10^{-5}\text{ kg s}^{-1}\text{ m}^{-3}}{2\,(1.0\times10^{-9}\text{ m}^2\text{ s}^{-1})}(-b^2_{max})$$

Solving for b_{max}:

$$b_{\max} = \sqrt{\frac{2\,(1.0\times10^{-9}\ \text{m}^2\ \text{s}^{-1})\ 7\times10^{-3}\ \text{kg m}^{-3}}{3.30\times10^{-5}\ \text{kg s}^{-1}\ \text{m}^{-3}}} = 6.51\times10^{-4}\ \text{m} = 0.65\ \text{mm}$$

Answer: 0.65 mm

13.11 Oxygen uptake by immobilised bacteria

R = (3 mm)/2 = 1.5 mm = 1.5 × 10^{-3} m. $\mathscr{D}_{Ae}$ = 1.9 × 10^{-9} m^2 s^{-1}. $r_{A,obs}$ = 2.2 × 10^{-5} kg s^{-1} m^{-3}. C_{Ab} = 6 × 10^{-3} kg m^{-3}. k_S = 6 × 10^{-5} m s^{-1}.

(a)

Calculating the observable modulus Ω for spherical geometry from Table 13.6:

$$\Omega = \frac{1.5\times10^{-3}\ \text{m}}{3}\,\frac{2.2\times10^{-5}\ \text{kg s}^{-1}\ \text{m}^{-3}}{6\times10^{-5}\ \text{m s}^{-1}\,(6\times10^{-3}\ \text{kg m}^{-3})} = 3.06\times10^{-2}$$

From Eqs (13.43) and (13.44):

$$C_{As} = C_{Ab}\,(1-\Omega) = C_{Ab}\,(1-3.06\times10^{-2}) = 0.97\,C_{Ab}$$

External mass transfer effects are insignificant as $C_{As} \approx C_{Ab}$.

Answer: Negligible

(b)

Evaluating the observable Thiele modulus Φ using the equation in Table 13.4 for spherical geometry and the result for C_{As} from **(a)**:

$$\Phi = \left(\frac{1.5\times10^{-3}\ \text{m}}{3}\right)^2 \frac{2.2\times10^{-5}\ \text{kg s}^{-1}\ \text{m}^{-3}}{1.9\times10^{-9}\ \text{m}^2\ \text{s}^{-1}\,(0.97\times6\times10^{-3}\ \text{kg m}^{-3})} = 0.497$$

From Figure 13.14 for zero-order reaction, at Φ = 0.497, η_{i0} = 1. Because oxygen uptake is zero-order and $C_{As} > 0$, from Section 13.5, η_{e0} = 1. Applying Eq. (13.46), $\eta_T = \eta_{i0}\eta_{e0} = 1 \times 1 = 1$.

Answer: $\eta_i = \eta_e = \eta_T = 1$

(c)

The minimum oxygen concentration within the beads is given by the equation in Table 13.5 for zero-order reaction, spherical geometry and $\Phi < 0.667$:

$$C_{A,\min} = 0.97\times6\times10^{-3}\ \text{kg m}^{-3}\left(1-\frac{3}{2}(0.497)\right) = 1.48\times10^{-3}\ \text{kg m}^{-3}$$

As this $C_{A,min}$ is greater than 4 × 10^{-4} kg m^{-3}, the condition is satisfied.

Answer: Yes

13.12 Three-dimensional culture of mesenchymal stem cells

k_S = 3.6 × 10^{-5} m s^{-1}. C_{Ab} = 8 × 10^{-3} kg m^{-3}. Converting the units of $\mathscr{D}_{Ae}$:

$$\mathscr{D}_{Ae} = 9\times10^{-6}\ \text{cm}^2\ \text{s}^{-1}\,.\left|\frac{1\ \text{m}}{100\ \text{cm}}\right|^2 = 9\times10^{-10}\ \text{m}^2\ \text{s}^{-1}$$

(a), (b)

R = (3.5 mm)/2 = 1.75 mm = 1.75 × 10^{-3} m. $r_{A,obs}$ = 7.6 × 10^{-5} kg s^{-1} m^{-3}. Calculating the observable modulus Ω for spherical geometry from Table 13.6:

$$\Omega = \frac{1.75\times10^{-3}\ \text{m}}{3}\frac{7.6\times10^{-5}\ \text{kg s}^{-1}\ \text{m}^{-3}}{3.6\times10^{-5}\ \text{m s}^{-1}\ (8\times10^{-3}\ \text{kg m}^{-3})} = 0.154$$

From Eqs (13.43) and (13.44):

$$C_{As} = C_{Ab}(1-\Omega) = 8\times10^{-3}\ \text{kg m}^{-3}\ (1-0.154) = 6.77\times10^{-3}\ \text{kg m}^{-3}$$

As C_{As} is less than C_{Ab}, external mass transfer effects are present. However, because oxygen consumption is zero-order and $C_{As} > 0$, from Section 13.5, η_{e0} = 1. We cannot tell if the reduction in C_A that occurs within the external boundary layer affects the oxygen uptake rate until we determine the observable Thiele modulus Φ for internal mass transfer.

Evaluating Φ using the equation in Table 13.4 for spherical geometry:

$$\Phi = \left(\frac{1.75\times10^{-3}\ \text{m}}{3}\right)^2 \frac{7.6\times10^{-5}\ \text{kg s}^{-1}\ \text{m}^{-3}}{9\times10^{-10}\ \text{m}^2\ \text{s}^{-1}\ (6.77\times10^{-3}\ \text{kg m}^{-3})} = 4.24$$

From Figure 13.14 for zero-order reaction, at Φ = 4.24, η_{i0} = 0.38. As $\eta_{i0} < 1$, we can say immediately that the oxygen uptake rate is affected by internal mass transfer effects. For zero-order kinetics, this result implies that C_A falls to zero within the beads (Table 13.5). As the reduction in oxygen concentration that occurs in the external boundary layer reduces C_{As} and increases the extent of oxygen depletion in the particle, we can conclude that the oxygen uptake rate in this system is affected by external mass transfer even though η_{e0} = 1 (Section 13.5).

Answer: The rate of oxygen uptake is affected by both internal and external mass transfer effects

(c)

From Table 13.5 for zero-order reaction, spherical geometry and $\Phi \geq 0.667$, $C_{A,min}$ = 0. Therefore, cells at the centre of the beads are not supplied with oxygen.

Answer: No

(d)

The maximum oxygen uptake rate in the absence of mass transfer effects is r^*_{Ab}. Applying the above results for η_{i0} and η_{e0} in Eq. (13.46), $\eta_T = \eta_{i0}\eta_{e0}$ = 0.38 × 1 = 0.38. Calculating r^*_{Ab} from Eq. (13.45):

$$r^*_{Ab} = \frac{r_{A,obs}}{\eta_T} = \frac{7.6\times10^{-5}\ \text{kg s}^{-1}\ \text{m}^{-3}}{0.38} = 2.0\times10^{-4}\ \text{kg s}^{-1}\ \text{m}^{-3}$$

Answer: 2.0 × 10^{-4} kg s^{-1} m^{-3}

(e)

When external boundary layers are removed, $C_{As} = C_{Ab}$ = 8 × 10^{-3} kg m^{-3}. The maximum particle radius for $C_A > 0$ everywhere in the beads is calculated using Eq. (13.17) with the intrinsic zero-order rate constant $k_0 = r^*_{Ab}$ from **(d)**:

$$R_{max} = \sqrt{\frac{6\ (9\times10^{-10}\ \text{m}^2\ \text{s}^{-1})\ 8\times10^{-3}\ \text{kg m}^{-3}}{2.0\times10^{-4}\ \text{kg s}^{-1}\ \text{m}^{-3}}} = 4.65\times10^{-4}\ \text{m} = 0.465\ \text{mm}$$

Therefore, the maximum bead diameter for $C_A > 0$ everywhere is 2 × 0.465 mm = 0.93 mm.

Answer: 0.93 mm

13.13 Diffusion and reaction of glucose and oxygen

Let subscript G represent glucose and subscript O represent oxygen. $\mathscr{D}_{Ge} = 0.42 \times 10^{-9}$ m^2 s^{-1}; $\mathscr{D}_{Oe} = 1.8 \times 10^{-9}$ m^2 s^{-1}. If glucose mass transfer limitations were to influence the reaction rate to the same extent as oxygen mass transfer effects, η_{TG} would be equal to η_{TO}. In the absence of external boundary layers, $\eta_{eG} = \eta_{eO} = 1$; from Eq. (13.46), this means that η_{iG} would be equal to η_{iO}. As oxygen uptake is strongly limited by internal mass transfer, from Eq. (13.40) for zero-order kinetics, we can say that $\eta_{iO} = 2/\Phi_O$. If glucose uptake were also strongly affected by glucose mass transfer, from Eq. (13.39) for first-order kinetics, $\eta_{iG} = 1/\Phi_G$. Equating η_{iG} and η_{iO} gives:

$$\frac{1}{\Phi_G} = \frac{2}{\Phi_O}$$

or

$$\frac{\Phi_G}{\Phi_O} = 0.5 \tag{1}$$

The observable Thiele modulus Φ is related to the observed reaction rate $r_{A,obs}$ for each substrate by the equation for spherical geometry in Table 13.4. Because the particle radius is the same for both substrates:

$$\frac{\Phi_G}{\Phi_O} = \frac{\left(\frac{r_{G,obs}}{\mathscr{D}_{Ge} C_{Gs}}\right)}{\left(\frac{r_{O,obs}}{\mathscr{D}_{Oe} C_{Os}}\right)} = \frac{r_{G,obs}}{r_{O,obs}}\left(\frac{\mathscr{D}_{Oe} C_{Os}}{\mathscr{D}_{Ge} C_{Gs}}\right) \tag{2}$$

From the reaction stoichiometry, on a molar basis, $r_{O,obs} = 2.7 r_{G,obs}$. Converting this relationship to a mass basis using the molecular weight of oxygen = 32 and the molecular weight of glucose =180 (Table C.1, Appendix C):

$$\frac{r_{O,obs}}{32} = \frac{2.7 r_{G,obs}}{180}$$

or

$$\frac{r_{G,obs}}{r_{O,obs}} = 2.08 \tag{3}$$

In the absence of external boundary layers, $C_{Gs} = C_{Gb}$ and $C_{Os} = C_{Ob} = 8 \times 10^{-3}$ kg m^{-3}. Substituting (1) and (3) into (2) and applying parameter values gives:

$$0.5 = 2.08\left(\frac{(1.8\times10^{-9}\ \text{m}^2\ \text{s}^{-1})\ 8\times10^{-3}\ \text{kg m}^{-3}}{(0.42\times10^{-9}\ \text{m}^2\ \text{s}^{-1})\, C_{Gb}}\right)$$

where C_{Gb} has units of kg m^{-3}. Calculating and rearranging gives:

$$0.5 = \frac{0.071}{C_{Gb}}$$

$$C_{Gb} = 0.14$$

Therefore, $C_{Gb} = 0.14$ kg m^{-3} = 0.14 g l^{-1}. This result demonstrates that, for glucose mass transfer to limit the reaction to the same extent as oxygen mass transfer, the external glucose concentration must be very low relative to the level of 20 g l^{-1} supplied to the culture. Such low glucose concentrations would occur for only a very short period of time at the end of batch growth. The implication of this result is that aerobic cultures are much less likely to be affected by glucose mass transfer limitations compared with oxygen.

Answer: 0.14 g l^{-1}; glucose mass transfer is much less likely than oxygen mass transfer to limit reaction rates in immobilised aerobic cell cultures

13.14 Uptake of growth factor by neural stem cell spheroids

D_p = 500 μm = 500 × 10^{-6} m. ρ_p = 1.15 g cm^{-3} = 1150 kg m^{-3}. ρ_L = 1 g cm^{-3} = 1000 kg m^{-3}. μ_L = 1 cP; from Table A.9 (Appendix A), $\mu_L = 10^{-3}$ kg m^{-1} s^{-1}. Converting the units of $\mathscr{D}_{AL}$:

$$\mathscr{D}_{AL} = 3.3\times10^{-6}\ \text{cm}^2\ \text{s}^{-1} . \left|\frac{1\ \text{m}}{100\ \text{cm}}\right|^2 = 3.3\times10^{-10}\ \text{m}^2\ \text{s}^{-1}$$

$\mathscr{D}_{Ae}$ = 5 × 10^{-11} m^2 s^{-1}. In 1 ml of nutrient medium there are 10 ng of growth factor. This is consumed by 10 mg of spheroids in 15 h. From the definition of density in Section 2.4.1, the volume of 10 mg of spheroids is:

$$\text{Volume of spheroids in 10 mg} = \frac{10\ \text{mg} . \left|\frac{1\ \text{g}}{1000\ \text{mg}}\right| . \left|\frac{1\ \text{kg}}{1000\ \text{g}}\right|}{1150\ \text{kg m}^{-3}} = 8.696\times10^{-9}\ \text{m}^3$$

Therefore, as 1 ng = 10^{-9} g (Table 2.4), $r_{A,obs}$ can be evaluated as:

$$r_{A,obs} = \frac{10\ \text{ng}}{15\ \text{h}\ (8.696\times10^{-9}\ \text{m}^3)} . \left|\frac{10^{-9}\ \text{g}}{1\ \text{ng}}\right| . \left|\frac{1\ \text{kg}}{1000\ \text{g}}\right| . \left|\frac{1\ \text{h}}{3600\ \text{s}}\right| = 2.13\times10^{-8}\ \text{kg s}^{-1}\ \text{m}^{-3}$$

Converting the units for C_{Ab} gives:

$$C_{Ab} = 10\ \text{ng ml}^{-1} . \left|\frac{10^{-9}\ \text{g}}{1\ \text{ng}}\right| . \left|\frac{1\ \text{kg}}{1000\ \text{g}}\right| . \left|\frac{1000\ \text{ml}}{1\ \text{l}}\right| . \left|\frac{1000\ \text{l}}{1\ \text{m}^3}\right| = 10^{-5}\ \text{kg m}^{-3}$$

(a)

The external mass transfer coefficient k_S is determined using the equations in Section 13.6.1 for free-moving spheres. The Grashof number is calculated from Eq. (13.51) with g = 9.8 m s^{-2} (Section 2.3):

$$Gr = \frac{9.8\ \text{m s}^{-2}\ (500\times10^{-6}\ \text{m})^3\ (1000\ \text{kg m}^{-3})\ (1150-1000)\ \text{kg m}^{-3}}{(10^{-3}\ \text{kg m}^{-1}\ \text{s}^{-1})^2} = 183.8$$

Therefore, from Eq. (13.53):

$$Re_p = 0.153\ (183.8)^{0.71} = 6.20$$

As $Re_p < 10^3$, we can use Eq. (13.56) to determine the Sherwood number, *Sh*. Evaluating *Sc* from Eq. (13.49):

$$Sc = \frac{10^{-3}\ \text{kg m}^{-1}\ \text{s}^{-1}}{1000\ \text{kg m}^{-3}\ (3.3\times10^{-10}\ \text{m}^2\ \text{s}^{-1})} = 3030$$

and applying Eq. (13.56):

$$Sh = 2 + 0.6\ (6.20)^{0.5}\ (3030)^{0.33} = 23.05$$

From the definition of the Sherwood number in Eq. (13.50):

$$k_S = \frac{Sh\ \mathscr{D}_{AL}}{D_p} = \frac{23.05\ (3.3\times10^{-10}\ \text{m}^2\ \text{s}^{-1})}{500\times10^{-6}\ \text{m}} = 1.52\times10^{-5}\ \text{m s}^{-1}$$

This value of k_S is used to determine Ω from the equation in Table 13.6 for spherical geometry:

$$\Omega = \frac{\left(\frac{500\times10^{-6}\text{ m}}{2}\right)}{3}\frac{2.13\times10^{-8}\text{ kg s}^{-1}\text{ m}^{-3}}{1.52\times10^{-5}\text{ m s}^{-1}(10^{-5}\text{ kg m}^{-3})} = 0.012$$

From Eqs (13.43) and (13.44):

$$C_{As} = C_{Ab}(1-\Omega) = 10^{-5}\text{ kg m}^{-3}(1-0.012) = 9.88\times10^{-6}\text{ kg m}^{-3}$$

For first-order kinetics, from Table 13.7:

$$\eta_{e1} = \frac{9.88\times10^{-6}\text{ kg m}^{-3}}{10^{-5}\text{ kg m}^{-3}} = 0.988$$

As $C_{As} \approx C_{Ab}$ and $\eta_{e1} \approx 1$, external mass transfer effects are not significant.

Answer: At the beginning of the culture when the concentration of growth factor in the medium is 10 ng ml^{-1}, external mass transfer has a negligible effect on the rate of growth factor uptake

(b)

Evaluating the observable Thiele modulus Φ using the equation in Table 13.4 for spherical geometry and the result for C_{As} from **(a)**:

$$\Phi = \left(\frac{\left(\frac{500\times10^{-6}\text{ m}}{2}\right)}{3}\right)^2 \frac{2.13\times10^{-8}\text{ kg s}^{-1}\text{ m}^{-3}}{5\times10^{-11}\text{ m}^2\text{ s}^{-1}(9.88\times10^{-6}\text{ kg m}^{-3})} = 0.299$$

From Figure 13.14 for first-order reaction, at Φ = 0.299, η_{i1} = 0.81. As $\eta_{i1} < 1$, the rate of reaction is affected by internal mass transfer limitations.

Answer: Yes

(c)

From Eq. (13.46), $\eta_T = \eta_{i1}\eta_{e1}$ = 0.81 × 0.988 = 0.80. From Eq. (13.45), the rate of reaction in the absence of all mass transfer resistances, r^*_{Ab}, is increased relative to $r_{A,obs}$ by a factor:

$$\frac{r^*_{Ab}}{r_{A,obs}} = \frac{1}{\eta_T} = \frac{1}{0.80} = 1.25$$

Answer: By a factor of 1.25

13.15 Maximum reaction rate for immobilised enzyme

R = (100 μm)/2 = 50 μm = 50 × 10^{-6} m. As external boundary layer effects are eliminated, η_e = 1 and C_{As} = C_{Ab} = 2.68 mM = 2.68 × 10^{-3} mol l^{-1} = 2.68 mol m^{-3}. Converting the units of $\mathscr{D}_{AL}$:

$$\mathscr{D}_{AL} = 4\times10^{-6}\text{ cm}^2\text{ s}^{-1}.\left|\frac{1\text{ m}}{100\text{ cm}}\right|^2 = 4\times10^{-10}\text{ m}^2\text{ s}^{-1}$$

$\mathscr{D}_{Ae} = 0.45 \times \mathscr{D}_{AL} = 0.45 \times (4 \times 10^{-10}\text{ m}^2\text{ s}^{-1}) = 1.8 \times 10^{-10}\text{ m}^2\text{ s}^{-1}$. Converting the units of $r_{A,obs}$:

$$r_{A,obs} = 125\text{ μmol min}^{-1}\text{ cm}^{-3}.\left|\frac{1\text{ mol}}{10^6\text{ μmol}}\right|.\left|\frac{1\text{ min}}{60\text{ s}}\right|.\left|\frac{100\text{ cm}}{1\text{ m}}\right|^3 = 2.08\text{ mol s}^{-1}\text{ m}^{-3}$$

(a)

The observable Thiele modulus Φ is evaluated using the equation in Table 13.4 for spherical geometry:

$$\Phi = \left(\frac{50\times10^{-6}\ \text{m}}{3} \right)^2 \frac{2.08\ \text{mol s}^{-1}\ \text{m}^{-3}}{1.8\times10^{-10}\ \text{m}^2\ \text{s}^{-1}\ (2.68\ \text{mol m}^{-3})} = 1.20$$

Figure 13.14 can be used to determine the internal effectiveness factor corresponding to this value of Φ for spherical geometry. However, if we do not know the K_m value for immobilised penicillin-G amidase, β is also unknown. Nevertheless, we can determine the upper and lower bounds of η_{im} by determining the results for zero- and first-order kinetics (Section 13.4.4). From Figure 13.14, for Φ = 1.20, η_{i0} = 0.80 and η_{i1} = 0.45. Therefore, eliminating internal mass transfer limitations can be expected to increase the rate of enzyme reaction by a factor of between 1/0.80 and 1/0.45, or between 1.25 and 2.2.

Answer: By a factor of between 1.25 and 2.2

(b)

Let K_m for immobilised penicillin-G amidase = 13 μM = 13 × 10^{-6} mol l^{-1} = 13 × 10^{-3} mol m^{-3}. Therefore, from the definition of β in Section 13.4.3:

$$\beta = \frac{K_m}{C_{As}} = \frac{13\times10^{-3}\ \text{mol m}^{-3}}{2.68\ \text{mol m}^{-3}} = 4.85\times10^{-3}$$

For interpolation of the effectiveness factor plots, this value of $\beta \approx 0$ and the enzyme can be considered to follow zero-order kinetics. Using Eq. (13.46) and η_{i0} from **(a)**, $\eta_T = \eta_{i0}\eta_e = 0.80 \times 1 = 0.80$. Calculating r^*_{Ab} from Eq. (13.45):

$$r^*_{Ab} = \frac{r_{A,obs}}{\eta_T} = \frac{2.08\ \text{mol s}^{-1}\ \text{m}^{-3}}{0.80} = 2.60\ \text{mol s}^{-1}\ \text{m}^{-3}$$

Answer: 2.60 mol s^{-1} m^{-3}

Chapter 14

Reactor Engineering

14.1 Economics of batch enzyme conversion

For 75% conversion, $s_f = 0.25 s_0$. The batch reaction time for enzyme processes is evaluated using Eq. (14.10):

$$t_b = \frac{1.5 \text{ g l}^{-1}}{0.9 \text{ g l}^{-1} \text{ h}^{-1}} \ln \frac{3 \text{ g l}^{-1}}{0.25 \times 3 \text{ g l}^{-1}} + \frac{3 \text{ g l}^{-1} - 0.25 \times 3 \text{ g l}^{-1}}{0.9 \text{ g l}^{-1} \text{ h}^{-1}} = 4.81 \text{ h}$$

The reactor operating cost is therefore:

$$\text{Reactor operating cost} = 4.81 \text{ h} \left| \frac{1 \text{ day}}{24 \text{ h}} \right| (\$4800 \text{ day}^{-1}) = \$962$$

Using the equation provided, for 75% substrate conversion, the cost of downstream processing per kg of product is:

$$C = 155 - 0.33 (75) = \$130.25 \text{ kg}^{-1}$$

The mass of product formed is determined from the mass of substrate consumed, which is equal to the change in substrate concentration multiplied by the volume of the reactor V:

$$\text{Mass of substrate consumed} = (s_0 - s_f) V = (3 - 0.25 \times 3) \text{ g l}^{-1} (1600 \text{ l}) = 3600 \text{ g}$$

As 1.2 g of product are formed per g of substrate consumed:

$$\text{Mass of product formed} = 1.2 \times 3600 \text{ g} = 4320 \text{ g} = 4.32 \text{ kg}$$

Therefore:

$$\text{Downstream processing cost} = \$130.25 \text{ kg}^{-1} (4.32 \text{ kg}) = \$563$$

The revenue from sale of the product is:

$$\text{Revenue} = \$750 \text{ kg}^{-1} (4.32 \text{ kg}) = \$3240$$

Therefore, the cost benefit at 75% substrate conversion is:

$$\begin{aligned} \text{Cost benefit} &= \text{revenue} - \text{reactor operating cost} - \text{downstream processing cost} \\ &= \$3240 - \$962 - \$563 \\ &= \$1715 \end{aligned}$$

Carrying out the calculations for 90% substrate conversion, the batch reaction time for $s_f = 0.10 s_0$ is:

$$t_b = \frac{1.5 \text{ g l}^{-1}}{0.9 \text{ g l}^{-1} \text{ h}^{-1}} \ln \frac{3 \text{ g l}^{-1}}{0.10 \times 3 \text{ g l}^{-1}} + \frac{3 \text{ g l}^{-1} - 0.10 \times 3 \text{ g l}^{-1}}{0.9 \text{ g l}^{-1} \text{ h}^{-1}} = 6.84 \text{ h}$$

At 90% conversion, the reactor operating cost is increased due to the longer reaction time:

$$\text{Reactor operating cost} = 6.84 \text{ h} \left| \frac{1 \text{ day}}{24 \text{ h}} \right| (\$4800 \text{ day}^{-1}) = \$1368$$

The cost of downstream processing per kg of product is:

$$C = 155 - 0.33 (90) = \$125.30 \text{ kg}^{-1}$$

The mass of substrate consumed is:

$$\text{Mass of substrate consumed} = (s_0 - s_f)V = (3 - 0.10\times 3)\ \text{g l}^{-1}\ (1600\ \text{l}) = 4320\ \text{g}$$

and the mass of product formed is:

$$\text{Mass of product formed} = 1.2\times 4320\ \text{g} = 5184\ \text{g} = 5.18\ \text{kg}$$

Therefore, the downstream processing cost is:

$$\text{Downstream processing cost} = \$125.30\ \text{kg}^{-1}\ (5.18\ \text{kg}) = \$649$$

The sales revenue is:

$$\text{Revenue} = \$750\ \text{kg}^{-1}\ (5.18\ \text{kg}) = \$3885$$

Therefore, the cost benefit at 90% substrate conversion is:

$$\begin{aligned}\text{Cost benefit} &= \text{revenue} - \text{reactor operating cost} - \text{downstream processing cost}\\ &= \$3885 - \$1368 - \$649\\ &= \$1868\end{aligned}$$

The gain per batch from increasing the conversion from 75% to 90% is therefore \$1868 – \$1715 = \$153.

Answer: There is a gain of \$153 per batch, representing a 9% increase relative to the cost benefit at 75% substrate conversion

14.2 Batch production of aspartic acid using cell-bound enzyme

(a)

The initial concentration of substrate s_0 = 15% (w/v) = 15 g per 100 ml = 150 g l^{-1}. The final substrate concentration $s_f = 0.15 s_0 = 0.15 \times 150$ g l^{-1} = 22.5 g l^{-1}. Calculating the deactivation rate constant at 32°C using Eq. (12.73):

$$k_d = \frac{\ln 2}{t_h} = \frac{\ln 2}{10.5\ \text{days}} \cdot \left|\frac{1\ \text{day}}{24\ \text{h}}\right| = 2.75\times 10^{-3}\ \text{h}^{-1}$$

For enzyme subject to deactivation, the batch reaction time at 32°C is evaluated using Eq. (14.13):

$$t_b = \frac{-1}{2.75\times 10^{-3}\ \text{h}^{-1}} \ln\left[1 - 2.75\times 10^{-3}\ \text{h}^{-1}\left(\frac{4.0\ \text{g l}^{-1}}{5.9\ \text{g l}^{-1}\ \text{h}^{-1}} \ln\frac{150\ \text{g l}^{-1}}{22.5\ \text{g l}^{-1}} + \frac{(150 - 22.5)\ \text{g l}^{-1}}{5.9\ \text{g l}^{-1}\ \text{h}^{-1}}\right)\right] = 23.6\ \text{h}$$

At 37°C, the deactivation rate constant is:

$$k_d = \frac{\ln 2}{t_h} = \frac{\ln 2}{2.3\ \text{days}} \cdot \left|\frac{1\ \text{day}}{24\ \text{h}}\right| = 1.26\times 10^{-2}\ \text{h}^{-1}$$

and the batch reaction time is:

$$t_b = \frac{-1}{1.26\times 10^{-2}\ \text{h}^{-1}} \ln\left[1 - 1.26\times 10^{-2}\ \text{h}^{-1}\left(\frac{4.0\ \text{g l}^{-1}}{8.5\ \text{g l}^{-1}\ \text{h}^{-1}} \ln\frac{150\ \text{g l}^{-1}}{22.5\ \text{g l}^{-1}} + \frac{(150 - 22.5)\ \text{g l}^{-1}}{8.5\ \text{g l}^{-1}\ \text{h}^{-1}}\right)\right] = 17.7\ \text{h}$$

As the batch reaction time is lower at 37°C than at 32°C, 37°C is the recommended operating temperature.

Answer: 37°C

(b)

From Eq. (14.33), the total batch reaction time at 37°C is:

$$t_T = 17.7\ \text{h} + 28\ \text{h} = 45.7\ \text{h}$$

Therefore, in one year or 365 days, the number of batches carried out is:

$$\text{Number of batches} = \frac{365 \text{ days} \,.\, \left| \frac{24 \text{ h}}{1 \text{ day}} \right|}{45.7 \text{ h per batch}} = 192$$

In each batch, the mass of ammonium fumarate converted is 0.85×150 g l^{-1} = 127.5 g l^{-1} multiplied by the reactor volume V. Therefore, the mass of substrates converted is $127.5V$ g = $0.1275V$ kg, where V has units of litres. From the reaction stoichiometry, as the molecular weights of ammonium fumarate and aspartic acid are approximately equal, the mass of aspartic acid produced is also $0.1275V$ kg. After one year or 192 batches, the mass of aspartic acid produced is $0.1275V \times 192 = 24.5V$ kg. Using the conversion factor 1 tonne = 10^3 kg (Table A.3, Appendix A), the target level of aspartic acid production each year is 5000×10^3 kg = 5×10^6 kg. To reach this target level:

$$24.5V = 5 \times 10^6$$

$$V = 2.04 \times 10^5$$

Therefore, $V = 2.04 \times 10^5$ l = 204 m^3.

Answer: 204 m^3

14.3 Prediction of batch culture time

(a)

The initial cell concentration x_0 = 12 g/100 l = 0.12 g l^{-1}. Assume that stationary phase is reached when s_f = 0. The batch culture time is determined using Eq. (14.27):

$$t_b = \frac{1}{0.9 \text{ h}^{-1}} \ln\left[1 + \frac{0.575 \text{ g g}^{-1}}{0.12 \text{ g l}^{-1}} (10 \text{ g l}^{-1} - 0)\right] = 4.3 \text{ h}$$

Answer: 4.3 h

(b)

If only 70% of the substrate is consumed, $s_f = 0.3 s_0 = 0.3 \times 10$ g l^{-1} = 3 g l^{-1}. From Eq. (14.27):

$$t_b = \frac{1}{0.9 \text{ h}^{-1}} \ln\left[1 + \frac{0.575 \text{ g g}^{-1}}{0.12 \text{ g l}^{-1}} (10 \text{ g l}^{-1} - 3 \text{ g l}^{-1})\right] = 3.9 \text{ h}$$

The biomass density at this time is calculated using Eq. (14.19):

$$x = x_0 e^{\mu_{max} t_b} = 0.12 \text{ g l}^{-1} \, e^{(0.9 \text{ h}^{-1} \times 3.9 \text{ h})} = 4.0 \text{ g l}^{-1}$$

Answer: 4.0 g l^{-1}

14.4 Fed-batch scheduling

(a)

The initial substrate concentration s_0 = 3% (w/v) = 3 g per 100 ml = 30 g l^{-1}. The batch culture time required to achieve $s_f = 0$ is determined using Eq. (14.27):

$$t_b = \frac{1}{0.18 \text{ day}^{-1}} \ln\left[1 + \frac{0.5 \text{ g g}^{-1}}{1.5 \text{ g l}^{-1}} (30 \text{ g l}^{-1} - 0)\right] = 13.3 \text{ days}$$

The biomass density at this time is calculated using Eq. (14.19):

$$x = x_0 e^{\mu_{\max} t_b} = 1.5 \text{ g l}^{-1}\, e^{(0.18 \text{ day}^{-1} \times 13.3 \text{ days})} = 16.4 \text{ g l}^{-1}$$

Answer: The batch culture time is 13.3 days; the final biomass concentration is 16.4 g l^{-1}

(b)

The mass of cells at the start of fed-batch operation is equal to the final batch cell concentration multiplied by the initial medium volume:

$$X_0 = xV = 16.4 \text{ g l}^{-1}\,(100 \text{ l}) = 1640 \text{ g}$$

The final mass of cells after 40 days of fed-batch culture is determined using Eq. (14.50):

$$X = 1640 \text{ g} + 0.5 \text{ g g}^{-1}\,(30 \text{ g l}^{-1})\,(4 \text{ l day}^{-1})\,40 \text{ days} = 4040 \text{ g} = 4.04 \text{ kg}$$

Answer: 4.04 kg

(c)

The mass of cells produced in each reactor run is equal to the final biomass minus the biomass used for inoculation:

$$\text{Biomass produced per run} = 4040 \text{ g} - 1.5 \text{ g l}^{-1}\,(100 \text{ l}) = 3890 \text{ g} = 3.89 \text{ kg}$$

By analogy with Eq. (14.33), the total reaction time is:

$$t_T = t_b + t_{fb} + t_{dn}$$

where t_b is the batch reaction time and t_{fb} is the fed-batch operation time. Substituting parameter values using the result for t_b from **(a)**:

$$t_T = 13.3 \text{ days} + 40 \text{ days} + 1 \text{ day} = 54.3 \text{ days}$$

In one year, the number of runs carried out is:

$$\text{Number of runs} = \frac{275 \text{ days}}{54.3 \text{ days per run}} = 5.06$$

The total biomass produced annually is equal to the biomass produced per run multiplied by the number of runs per year:

$$\text{Biomass produced per year} = 3.89 \text{ kg} \times 5.06 = 19.7 \text{ kg}$$

Answer: 19.7 kg

14.5 Fed-batch production of cheese starter culture

(a)

An expression for the liquid volume as a function of time during fed-batch reactor operation can be derived from an unsteady-state total mass balance, as shown in the solution to Problem 6.8(a) in Chapter 6. Using this expression:

$$V_0 = V - Ft = 40 \text{ m}^3 - 4 \text{ m}^3 \text{ h}^{-1}\,(6 \text{ h}) = 16 \text{ m}^3$$

Answer: 16 m^3

(b)

From the definition of the dilution rate in Eq. (14.39), after 6 h of fed-batch operation when $V = 40$ m^3:

$$D = \frac{4\ \text{m}^3\ \text{h}^{-1}}{40\ \text{m}^3} = 0.10\ \text{h}^{-1}$$

Substituting this value into Eq. (14.45) for the substrate concentration at quasi-steady state:

$$s \approx \frac{0.10\ \text{h}^{-1}\,(0.15\ \text{kg m}^{-3})}{0.35\ \text{h}^{-1} - 0.10\ \text{h}^{-1}} = 0.060\ \text{kg m}^{-3}$$

Answer: 0.060 kg m^{-3}

(c)

When product formation is coupled with energy metabolism, the equation for r_S does not include a separate term for product synthesis (Sections 12.11.2 and 14.5.1, Cell Culture subsection) so that Eq. (14.43) becomes:

$$\frac{\text{d}s}{\text{d}t} = D(s_\text{i} - s) - \left(\frac{\mu}{Y_\text{XS}} + m_\text{S}\right)x$$

At quasi-steady state, ds/dt ≈ 0, s << s_i and $\mu \approx D$. Therefore, the equation reduces to:

$$0 = Ds_\text{i} - \left(\frac{D}{Y_\text{XS}} + m_\text{S}\right)x$$

Solving for x:

$$x = \frac{Ds_\text{i}}{\dfrac{D}{Y_\text{XS}} + m_\text{S}} = \frac{0.10\ \text{h}^{-1}\,(80\ \text{kg m}^{-3})}{\dfrac{0.10\ \text{h}^{-1}}{0.23\ \text{kg kg}^{-1}} + 0.135\ \text{kg kg}^{-1}\ \text{h}^{-1}} = 14.0\ \text{kg m}^{-3}$$

Answer: 14.0 kg m^{-3}

(d)

After 6 h of fed-batch operation, the mass of cells is:

$$X = xV = 14.0\ \text{kg m}^{-3}\,(40\ \text{m}^3) = 560\ \text{kg}$$

At the start of fed-batch operation when the liquid volume is 16 m^3, if operation is at quasi-steady state, the cell concentration ≈ 14.0 kg m^{-3} and:

$$X = xV = 14.0\ \text{kg m}^{-3}\,(16\ \text{m}^3) = 224\ \text{kg}$$

Therefore, the mass of cells produced during 6 h of fed-batch operation is (560 kg – 224 kg) = 336 kg.

Answer: 336 kg

14.6 Continuous enzyme conversion in a fixed bed reactor

Convert the parameter values to units of kg, m, s. K_m = 0.54 g l^{-1} = 0.54 kg m^{-3}; s_i = 0.42 g l^{-1} = 0.42 kg m^{-3}. During reactor operation, s = 0.02 g l^{-1} = 0.02 kg m^{-3}. R = 1 mm = 10^{-3} m. The active enzyme concentration per unit volume of catalyst e_a is:

$$e_\text{a} = \frac{10^{-4}\ \text{g}}{250\ \text{cm}^3} \cdot \left|\frac{1\ \text{kg}}{1000\ \text{g}}\right| \cdot \left|\frac{100\ \text{cm}}{1\ \text{m}}\right|^3 = 4\times10^{-4}\ \text{kg m}^{-3}$$

The effective diffusivity of urea in the gel is:

$$\mathscr{D}_{Ae} = 7\times10^{-6}\ \text{cm}^2\ \text{s}^{-1}.\left|\frac{1\ \text{m}}{100\ \text{cm}}\right|^2 = 7\times10^{-10}\ \text{m}^2\ \text{s}^{-1}$$

From Table C.1 (Appendix C), the molecular weight of urea is 60.1 and the molecular weight of NH_4^+ is 18.0. Therefore, from the stoichiometry, reaction of 60.1 g of urea produces 2 × 18.0 = 36.0 g of NH_4^+. Expressing the turnover number k_{cat} (Section 12.3.3) in terms of urea:

$$k_{cat} = 11{,}000\ \text{g NH}_4^+\ \text{(g enzyme)}^{-1}\ \text{s}^{-1}.\left(\frac{60.1\ \text{g urea}}{36.0\ \text{g NH}_4^+}\right) = 1.84\times10^4\ \text{g urea (g enzyme)}^{-1}\ \text{s}^{-1}$$

$$= 1.84\times10^4\ \text{kg kg}^{-1}\ \text{s}^{-1}$$

From Eq. (12.39), v_{max} expressed on a per volume of gel basis is:

$$v_{max} = 1.84\times10^4\ \text{kg kg}^{-1}\ \text{s}^{-1}\ (4\times10^{-4}\ \text{kg m}^{-3}) = 7.36\ \text{kg m}^{-3}\ \text{s}^{-1}$$

As there are 250 cm^3 of gel per litre of liquid in the reactor, v_{max} expressed on a per volume of liquid basis is:

$$v_{max} = 7.36\ \text{kg m}^{-3}\ \text{s}^{-1}\left(\frac{250\ \text{cm}^3}{1\,l}\right).\left|\frac{1\ \text{m}}{100\ \text{cm}}\right|^3.\left|\frac{1000\,l}{1\ \text{m}^3}\right| = 1.84\ \text{kg m}^{-3}\ \text{s}^{-1}$$

The rate of reaction can be determined after evaluating the effectiveness factor in the absence of external boundary layers. From the definition of β in Section 13.4.3 with $C_{As} = s$:

$$\beta = \frac{K_m}{s} = \frac{0.54\ \text{kg m}^{-3}}{0.02\ \text{kg m}^{-3}} = 27$$

From Figure 13.13, for this value of β the reaction kinetics can be considered first-order. Based on Eq. (12.42), the effective first-order rate constant k_1 is:

$$k_1 = \frac{v_{max}}{K_m} = \frac{1.84\ \text{kg m}^{-3}\ \text{s}^{-1}}{0.54\ \text{kg m}^{-3}} = 3.41\ \text{s}^{-1}$$

Calculating the generalised Thiele modulus from the equation in Table 13.2 for first-order kinetics and spherical geometry:

$$\phi_1 = \frac{10^{-3}\ \text{m}}{3}\sqrt{\frac{3.41\ \text{s}^{-1}}{7\times10^{-10}\ \text{m}^2\ \text{s}^{-1}}} = 23.3$$

As $\phi_1 > 10$, from Eq. (13.30):

$$\eta_{i1} \approx \frac{1}{23.3} = 0.043$$

From Eq. (13.46), as $\eta_{e1} = 1$ in the absence of external boundary layer effects, $\eta_T = 0.043$. The flow rate of urea solution into and out of the reactor can be determined by evaluating the dilution rate D in the mass balance equation, Eq. (14.54):

$$D = \frac{\eta_T v_{max} s}{(K_m + s)(s_i - s)} = \frac{0.043\ (1.84\ \text{kg m}^{-3}\ \text{s}^{-1})\ 0.02\ \text{kg m}^{-3}}{(0.54 + 0.02)\ \text{kg m}^{-3}\ (0.42 - 0.02)\ \text{kg m}^{-3}} = 7.06\times10^{-3}\ \text{s}^{-1}$$

From the definition of the dilution rate, Eq. (14.39):

$$F = DV = 7.06\times10^{-3}\ \text{s}^{-1}\ (11) = 7.06\times10^{-3}\,l\,\text{s}^{-1}$$

In 30 min, the volume of urea solution treated is:

$$\text{Volume treated} = 7.06\times10^{-3}\,\text{l s}^{-1}\,(30\ \text{min})\,.\left|\frac{60\ \text{s}}{1\ \text{min}}\right| = 12.7\,\text{l}$$

Answer: 12.7 litres

14.7 Batch and continuous biomass production

$s_0 = s_i = 4\%$ (w/v) = 4 g per 100 ml = 40 g l^{-1} = 40 kg m^3. $s_f = s = 0.02 \times 40$ kg m^{-3} = 0.8 kg m^{-3}. For the batch reactor, $x_0 = 0.01\%$ (w/v) = 0.01 g per 100 ml = 0.1 g l^{-1} = 0.1 kg m^3.

The batch culture time is determined from Eq. (14.27):

$$t_b = \frac{1}{0.44\ \text{h}^{-1}}\ln\left[1+\frac{0.41\ \text{g g}^{-1}}{0.1\ \text{kg m}^{-3}}(40-0.8)\ \text{kg m}^{-3}\right] = 11.6\ \text{h}$$

The biomass density at this time is obtained from Eq. (14.19):

$$x_f = x_0 e^{\mu_{max} t_b} = 0.1\ \text{kg m}^{-3}\ e^{(0.44\ \text{h}^{-1}\times 11.6\ \text{h})} = 16.5\ \text{kg m}^{-3}$$

Calculating the mass of cells produced per batch:

$$X = (x_f - x_0)V = (16.5-0.1)\ \text{kg m}^{-3}\ (1000\ \text{m}^3) = 1.64\times10^4\ \text{kg}$$

If the downtime between batches t_{dn} is 20 h, from Eq. (14.33):

$$t_T = 11.6\ \text{h} + 20\ \text{h} = 31.6\ \text{h}$$

Therefore, in one year or 365 days, the number of batches carried out is:

$$\text{Number of batches} = \frac{365\ \text{days}\,.\left|\frac{24\ \text{h}}{1\ \text{day}}\right|}{31.6\ \text{h per batch}} = 277$$

The total annual biomass production from batch culture is therefore 1.64×10^4 kg $\times$ 277 = 4.54×10^6 kg.

For continuous reactor operation, the steady-state cell concentration is given by Eq. (14.62):

$$x = (40-0.8)\ \text{kg m}^{-3}\ (0.41\ \text{g g}^{-1}) = 16.1\ \text{kg m}^{-3}$$

The dilution rate D corresponding to s = 0.8 kg m^{-3} is determined using Eq. (14.89):

$$D = \frac{0.44\ \text{h}^{-1}\ (0.8\ \text{kg m}^{-3})}{0.7\ \text{mg l}^{-1}\,.\left|\frac{1000\,\text{l}}{1\ \text{m}^3}\right|.\left|\frac{1\ \text{kg}}{10^6\ \text{mg}}\right| + 0.8\ \text{kg m}^{-3}} = 0.44\ \text{h}^{-1}$$

From the definition of the dilution rate, Eq. (14.39):

$$F = DV = 0.44\ \text{h}^{-1}\ (1000\ \text{m}^3) = 440\ \text{m}^3\ \text{h}^{-1}$$

The rate of biomass production Fx = 440 m^3 h^{-1} $\times$ 16.1 kg m^{-3} = 7084 kg h^{-1}. The number of days per year available for continuous reactor operation is (365 – 25) days = 340 days; this corresponds to 340 days $\times$ 24 h day^{-1} = 8160 h. Therefore, the total biomass produced per year is 7084 kg h^{-1} $\times$ 8160 h = 5.78×10^7 kg. This production level is $(5.78 \times 10^7)/(4.54 \times 10^6)$ = 12.7 times the amount produced using batch culture.

Answer: The annual biomass production using continuous operation is 5.78×10^7 kg, which is 12.7 times the production of 4.54×10^6 kg from batch culture

14.8 Bioreactor design for immobilised enzymes

$s_0 = s_i = 10\%$ (w/v) = 10 g per 100 ml = 100 g l^{-1} = 100 kg m^{-3}. $s_f = s = 0.01 \times 100$ kg m^{-3} = 1 kg m^{-3}. Based on the unsteady-state mass balance equation for first-order reaction derived in Example 6.1 (Chapter 6), the equation for the rate of change of substrate concentration in a batch reactor is:

$$\frac{\mathrm{d}(Vs)}{\mathrm{d}t} = -k_1 s V$$

where V is the reaction volume and k_1 is the reaction rate constant. As V can be considered constant in a batch reactor, this term can be taken outside of the differential and cancelled from both sides of the equation:

$$\frac{\mathrm{d}s}{\mathrm{d}t} = -k_1 s$$

The differential equation contains only two variables, s and t. Separating variables and integrating:

$$\frac{\mathrm{d}s}{s} = -k_1 \mathrm{d}t$$

$$\int \frac{\mathrm{d}s}{s} = \int -k_1 \mathrm{d}t$$

Using integration rules (E.27) and (E.24) from Appendix E and combining the constants of integration:

$$\ln s = -k_1 t + K$$

The initial condition is: at $t = 0$, $s = s_0$. Therefore, from the equation, $\ln s_0 = K$. Substituting this expression for K gives:

$$\ln s = -k_1 t + \ln s_0$$

$$\ln \frac{s}{s_0} = -k_1 t$$

$$t = \frac{-\ln \dfrac{s}{s_0}}{k_1}$$

The batch culture time t_b is the time required for the substrate concentration to reach s_f:

$$t_b = \frac{-\ln \dfrac{s_f}{s_0}}{k_1} = \frac{-\ln \left(\dfrac{1 \text{ kg m}^{-3}}{100 \text{ kg m}^{-3}} \right)}{0.8 \times 10^{-4} \text{ s}^{-1} . \left| \dfrac{3600 \text{ s}}{1 \text{ h}} \right|} = 16.0 \text{ h}$$

If the downtime between batches t_{dn} is 20 h, from Eq. (14.33):

$$t_T = 16.0 \text{ h} + 20 \text{ h} = 36 \text{ h}$$

Therefore, in one year or 365 days, the number of batches carried out is:

$$\text{Number of batches} = \frac{365 \text{ days} . \left| \dfrac{24 \text{ h}}{1 \text{ day}} \right|}{36 \text{ h per batch}} = 243$$

To treat 400 tonnes of penicillin-G annually, using the unit conversion factor 1 tonne = 10^3 kg (Table A.3, Appendix A):

$$\text{Mass of penicillin G treated per batch} = \frac{400 \text{ tonne}}{\text{number of batches per year}}$$

$$= \frac{400 \text{ tonne} \,.\, \left| \frac{10^3 \text{ kg}}{1 \text{ tonne}} \right|}{243}$$

$$= 1.65 \times 10^3 \text{ kg}$$

As the concentration of penicillin-G added to the reactor is 100 kg m^{-3}:

$$\text{Reactor volume} = \frac{1.65 \times 10^3 \text{ kg}}{100 \text{ kg m}^{-3}} = 16.5 \text{ m}^3$$

The batch reactor volume required is 16.5 m^3.

For a CSTR operated under steady-state conditions, $F_i = F_o = F$, V is constant, and ds/dt = 0. Therefore, the mass balance equation for first-order reaction derived in Example 6.1 (Chapter 6) becomes:

$$0 = Fs_i - Fs - k_1 sV$$

$$0 = \frac{F}{V}(s_i - s) - k_1 s$$

Solving for F/V:

$$\frac{F}{V} = \frac{k_1 s}{s_i - s} = \frac{0.8 \times 10^{-4} \text{ s}^{-1} (1 \text{ kg m}^{-3})}{(100 - 1) \text{ kg m}^{-3}} = 8.08 \times 10^{-7} \text{ s}^{-1}$$

The mass flow rate of penicillin-G into the CSTR is 400 tonnes per year. As the concentration of substrate in the feed stream s_i = 100 kg m^{-3}, the volumetric flow rate of the feed stream F is:

$$F = \frac{400 \text{ tonne year}^{-1} \,.\, \left| \frac{10^3 \text{ kg}}{1 \text{ tonne}} \right| \,.\, \left| \frac{1 \text{ year}}{365 \text{ days}} \right| \,.\, \left| \frac{1 \text{ day}}{24 \text{ h}} \right| \,.\, \left| \frac{1 \text{ h}}{3600 \text{ s}} \right|}{100 \text{ kg m}^{-3}} = 1.27 \times 10^{-4} \text{ m}^3 \text{ s}^{-1}$$

Applying this with the above result for F/V:

$$V = \frac{F}{\left(\frac{F}{V}\right)} = \frac{1.27 \times 10^{-4} \text{ m}^3 \text{ s}^{-1}}{8.08 \times 10^{-7} \text{ s}^{-1}} = 157 \text{ m}^3$$

The CSTR reactor volume required is 157 m^3.

For the PFTR, if the density of enzyme beads is four times greater than in the other reactors, $k_1 = 4 \times 0.8 \times 10^{-4}$ s^{-1} = 3.2×10^{-4} s^{-1}. By analogy with Eq. (14.83), for first-order kinetics, the differential equation for the change in substrate concentration with position in the reactor is:

$$u \frac{\mathrm{d}s}{\mathrm{d}z} = -k_1 s$$

The differential equation contains only two variables, s and z. Separating variables and integrating:

$$\frac{\mathrm{d}s}{s} = \frac{-k_1}{u} \mathrm{d}z$$

$$\int \frac{\mathrm{d}s}{s} = \int \frac{-k_1}{u}\mathrm{d}z$$

Using integration rules (E.27) and (E.24) from Appendix E and combining the constants of integration:

$$\ln s = \frac{-k_1}{u} z + K$$

The boundary condition is: at $z = 0$, $s = s_i$. Therefore, from the equation, $\ln s_i = K$. Substituting this expression for K gives:

$$\ln s = \frac{-k_1}{u} z + \ln s_i$$

$$\ln \frac{s}{s_i} = \frac{-k_1}{u} z$$

At the end of the PFTR, $z = L$ and $s = s_f$. Therefore:

$$\ln \frac{s_f}{s_i} = \frac{-k_1}{u} L$$

Applying the definition of the reactor residence time τ from Eq. (14.85):

$$\ln \frac{s_f}{s_i} = -k_1 \tau$$

Rearranging and solving for τ:

$$\tau = \frac{-\ln \frac{s_f}{s_i}}{k_1} = \frac{-\ln\left(\frac{1\ \mathrm{kg\ m^{-3}}}{100\ \mathrm{kg\ m^{-3}}}\right)}{3.2\times10^{-4}\ \mathrm{s^{-1}} . \left|\frac{3600\ \mathrm{s}}{1\ \mathrm{h}}\right|} = 4.0\ \mathrm{h}$$

Note that this is 1/4 the value obtained for the batch reaction time t_b, as expected from the analogous kinetic characteristics of batch and PFTR reactors and the 4 × higher value of k_1 in the PFTR. As calculated for the CSTR, $F = 1.27 \times 10^{-4}\ \mathrm{m^3\ s^{-1}}$. Therefore, from the definition of τ in Eq. (14.51):

$$V = \tau F = 4.0\ \mathrm{h} . \left|\frac{3600\ \mathrm{s}}{1\ \mathrm{h}}\right| (1.27\times10^{-4}\ \mathrm{m^3\ s^{-1}}) = 1.83\ \mathrm{m^3}$$

The PFTR reactor volume required is 1.83 $\mathrm{m^3}$.

Answer: The smallest reactor volume is 1.83 $\mathrm{m^3}$ for the PFTR

14.9 Chemostat culture with protozoa

In this problem, bacteria are the limiting substrate. The doubling time for the protozoa is minimum when $\mu = \mu_{max}$. Therefore, from Eq. (12.89):

$$\mu_{max} = \frac{\ln 2}{t_d} = \frac{\ln 2}{6.5\ \mathrm{h}} = 0.107\ \mathrm{h^{-1}}$$

$Y_{XS} = 0.33\ \mathrm{g\ g^{-1}}$; $K_S = 12\ \mathrm{mg\ l^{-1}} = 12 \times 10^{-3}\ \mathrm{g\ l^{-1}}$; $s_i = 10\ \mathrm{g\ l^{-1}}$. We assume that no products are formed and protozoan cell death and maintenance requirements are negligible.

(a)

The maximum practical dilution rate is the critical dilution rate, D_{crit}. This is determined using Eq. (14.66):

$$D_{\text{crit}} = \frac{0.107\ \text{h}^{-1}\,(10\ \text{g l}^{-1})}{(12\times10^{-3}+10)\ \text{g l}^{-1}} = 0.107\ \text{h}^{-1}$$

Answer: 0.107 h^{-1}

(b)

D = (0.107 h^{-1})/2 = 0.0535 h^{-1}. Applying Eq. (14.63):

$$x = \left(10\ \text{g l}^{-1} - \frac{0.0535\ \text{h}^{-1}\,(12\times10^{-3})\ \text{g l}^{-1}}{(0.107-0.0535)\ \text{h}^{-1}}\right)0.33\ \text{g g}^{-1} = 3.30\ \text{g l}^{-1}$$

Answer: 3.3 g l^{-1}

(c)

D = 3/4 × 0.107 h^{-1} = 0.080 h^{-1}. Applying Eq. (14.58):

$$s = \frac{0.080\ \text{h}^{-1}\,(12\times10^{-3}\ \text{g l}^{-1})}{(0.107-0.080)\ \text{h}^{-1}} = 0.036\ \text{g l}^{-1}$$

Answer: 0.036 g l^{-1}

(d)

D = (0.107 h^{-1})/3 = 0.036 h^{-1}. Applying Eq. (14.69):

$$Q_{\text{X}} = 0.036\ \text{h}^{-1}\left(10\ \text{g l}^{-1} - \frac{0.036\ \text{h}^{-1}\,(12\times10^{-3})\ \text{g l}^{-1}}{(0.107-0.036)\ \text{h}^{-1}}\right)0.33\ \text{g g}^{-1} = 0.119\ \text{g l}^{-1}\ \text{h}^{-1}$$

Answer: 0.12 g l^{-1} h^{-1}

14.10 Two-stage chemostat for secondary metabolite production

(a)

s_{i} = 10 g l^{-1} = 10 kg m^{-3}. The dilution rate, which is the same for both reactors, is calculated using Eq. (14.39):

$$D = \frac{50\ \text{l h}^{-1}.\left|\frac{1\ \text{m}^3}{1000\ \text{l}}\right|}{0.5\ \text{m}^3} = 0.10\ \text{h}^{-1}$$

The cell and substrate concentrations entering the second reactor are the same as those leaving the first reactor. The substrate concentration leaving the first reactor is determined using Eq. (14.58):

$$s = \frac{0.10\ \text{h}^{-1}\,(1.0\ \text{kg m}^{-3})}{0.12\ \text{h}^{-1} - 0.10\ \text{h}^{-1}} = 5.0\ \text{kg m}^{-3}$$

As maintenance requirements are significant but there is no product formation in the first reactor, the cell concentration leaving the first vessel is calculated using Eq. (14.61):

$$x = \frac{0.10\ \text{h}^{-1}\,(10-5.0)\ \text{kg m}^{-3}}{\left(\frac{0.10\ \text{h}^{-1}}{0.5\ \text{kg kg}^{-1}}\right) + 0.025\ \text{kg kg}^{-1}\ \text{h}^{-1}} = 2.2\ \text{kg m}^{-3}$$

Answer: The cell concentration is 2.2 kg m^{-3}; the substrate concentration is 5.0 kg m^{-3}

(b)

As growth is negligible in the second reactor, $\mu = 0$ and $x = x_i = 2.2$ kg m^{-3}. The substrate concentration is determined by rearranging Eq. (14.59) and solving for s:

$$s = s_i - \left(\frac{q_P}{Y_{PS}} + m_S \right) x \frac{V}{F}$$

Substituting parameter values with $s_i = 5.0$ kg m^{-3}:

$$s = 5.0 \text{ kg m}^{-3} - \left(\frac{0.16 \text{ kg kg}^{-1} \text{ h}^{-1}}{0.85 \text{ kg kg}^{-1}} + 0.025 \text{ kg kg}^{-1} \text{ h}^{-1} \right) 2.2 \text{ kg m}^{-3} \left(\frac{0.5 \text{ m}^3}{50 \text{ l h}^{-1} . \left| \frac{1 \text{ m}^3}{1000 \text{ l}} \right|} \right) = 0.31 \text{ kg m}^{-3}$$

For the two reactors together:

$$\text{Overall substrate conversion} = \frac{(s_i - s)}{s_i} \times 100\% = \frac{(10 - 0.31) \text{ kg m}^{-3}}{10 \text{ kg m}^{-3}} \times 100\% = 97\%$$

Answer: 97%

(c)

As no product is formed in the first reactor, $p_i = 0$ for the second reactor. The product concentration is determined by rearranging Eq. (14.64) and solving for p:

$$p = \frac{q_P x V}{F} = \frac{0.16 \text{ kg kg}^{-1} \text{ h}^{-1} (2.2 \text{ kg m}^{-3}) \, 0.5 \text{ m}^3}{50 \text{ l h}^{-1} . \left| \frac{1 \text{ m}^3}{1000 \text{ l}} \right|} = 3.5 \text{ m}^{-3}$$

Answer: 3.5 kg m^{-3}

14.11 Growth of algae in a continuous bioreactor with cell recycle

$V = 1$ l; $s_i = 0.1$ mg l^{-1}; $K_S = 0.5$ µg l^{-1} = 0.5×10^{-3} mg l^{-1} ; $Y_{XS} = 18$ mg mg^{-1}. Converting the units for μ_{max}:

$$\mu_{max} = 0.5 \text{ day}^{-1} . \left| \frac{1 \text{ day}}{24 \text{ h}} \right| = 0.0208 \text{ h}^{-1}$$

(a)

$\tau = 60$ h. Applying Eq. (14.51):

$$D = \frac{1}{\tau} = \frac{1}{60 \text{ h}} = 0.0167 \text{ h}^{-1}$$

and

$$F = \frac{V}{\tau} = \frac{1 \text{ l}}{60 \text{ h}} = 0.0167 \text{ l h}^{-1}$$

The steady-state substrate concentration is determined using Eq. (14.58):

$$s = \frac{0.0167 \text{ h}^{-1} (0.5 \times 10^{-3} \text{ mg l}^{-1})}{(0.0208 - 0.0167) \text{ h}^{-1}} = 0.0020 \text{ mg l}^{-1}$$

Therefore, the percentage conversion of substrate is:

$$\text{Conversion} = \frac{s_\text{i} - s}{s_\text{i}} \times 100\% = \frac{(0.1 - 0.0020)\ \text{mg l}^{-1}}{0.1\ \text{mg l}^{-1}} \times 100\% = 98\%$$

Answer: 98%

(b)

Assuming that maintenance effects are negligible, the biomass productivity Q_X is obtained by combining Eqs (14.67) and (14.62):

$$Q_\text{X} = D(s_\text{i} - s)Y_\text{XS} = 0.0167\ \text{h}^{-1}\,(0.1 - 0.0020)\ \text{mg l}^{-1}\,(18\ \text{mg mg}^{-1}) = 0.02946\ \text{mg l}^{-1}\ \text{h}^{-1}$$

Answer: 0.029 mg l^{-1} h^{-1}

(c)

From Eqs (14.77) and (14.78):

$$\alpha = \frac{1}{4} = 0.25$$

$$\beta = \frac{3}{1} = 3$$

The equation provided is used to determine the dilution rate D required to achieve the substrate concentration s determined in **(a)**. Substituting values gives:

$$0.0020 = \frac{D(0.5 \times 10^{-3})\,(1 + 0.25 - 0.25 \times 3)}{0.0208 - D(1 + 0.25 - 0.25 \times 3)}$$

where D has units of h^{-1}. Rearranging and calculating terms:

$$4.16 \times 10^{-5} - 0.0010D = 2.50 \times 10^{-4} D$$

$$4.16 \times 10^{-5} = 1.25 \times 10^{-3} D$$

$$D = 0.033$$

Therefore, D = 0.033 h^{-1}. This is the value of the dilution rate based on the fresh feed flow rate. As the fresh feed flow rate with recycle is the same as that without recycle, from **(a)**, F = 0.0167 l h^{-1}. From the definition of the dilution rate in Eq. (14.39):

$$V = \frac{F}{D} = \frac{0.0167\ \text{l h}^{-1}}{0.033\ \text{h}^{-1}} = 0.506\ \text{l}$$

The reactor volume with cell recycle is (0.506 l)/(1 l) × 100% = 50.6% of that required without recycle, representing a reduction of 49.4%.

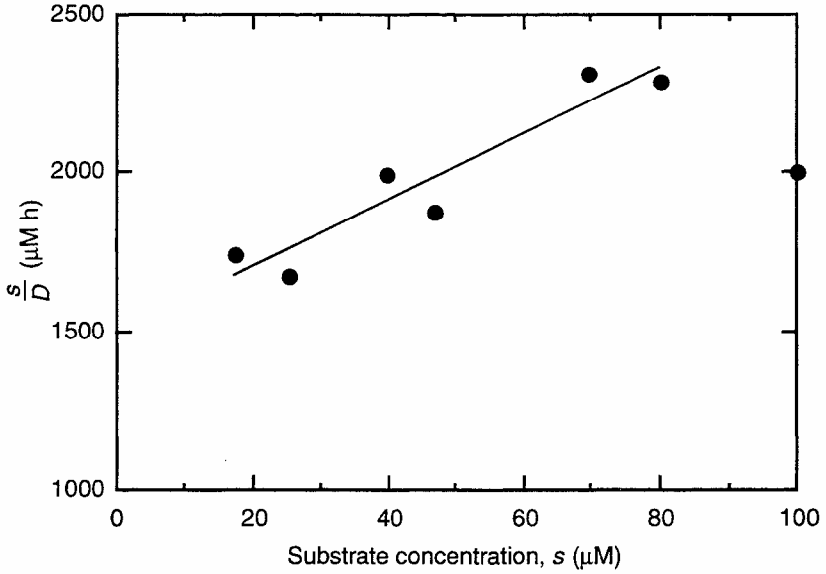
Answer: 49%

14.12 Kinetic analysis of bioremediating bacteria using a chemostat

(a)

From Eq. (14.92), μ_max and K_S can be determined from the slope and intercept of a plot of s/D versus s. Values of D are evaluated from the experimental flow rates using the definition of dilution rate in Eq. (14.39) and V = 1 l = 1000 ml. The measured substrate concentration at 50 ml h^{-1} indicates that washout occurs at this flow rate; therefore, this result is not included in the kinetic analysis. The data are listed and plotted below.

Flow rate, F (ml h^{-1})	Dilution rate, D (h^{-1})	Substrate concentration, s (μM)	s/D (μM h)
10	0.010	17.4	1740
15	0.015	25.1	1673
20	0.020	39.8	1990
25	0.025	46.8	1872
30	0.030	69.4	2313
35	0.035	80.1	2289
50	0.050	100	2000



The slope of the straight line in the plot is 10.48 h; the intercept is 1493 μM h. From Eq. (14.92), the slope = $1/\mu_{max}$; therefore, μ_{max} = 1/(10.48 h) = 0.095 h^{-1}. The intercept = K_S/μ_{max}; therefore K_S = 1493 μM h × 0.095 h^{-1} = 142 μM.

Answer: μ_{max} = 0.095 h^{-1}; K_S = 142 μM

(b)

The maximum practical operating flow rate corresponds to the critical dilution rate D_{crit} determined using Eq. (14.66):

$$D_{crit} = \frac{0.095\ h^{-1}(100\ \mu M)}{142\ \mu M + 100\ \mu M} = 0.039\ h^{-1}$$

Calculating the flow rate using Eq. (14.39) with V = 1000 ml:

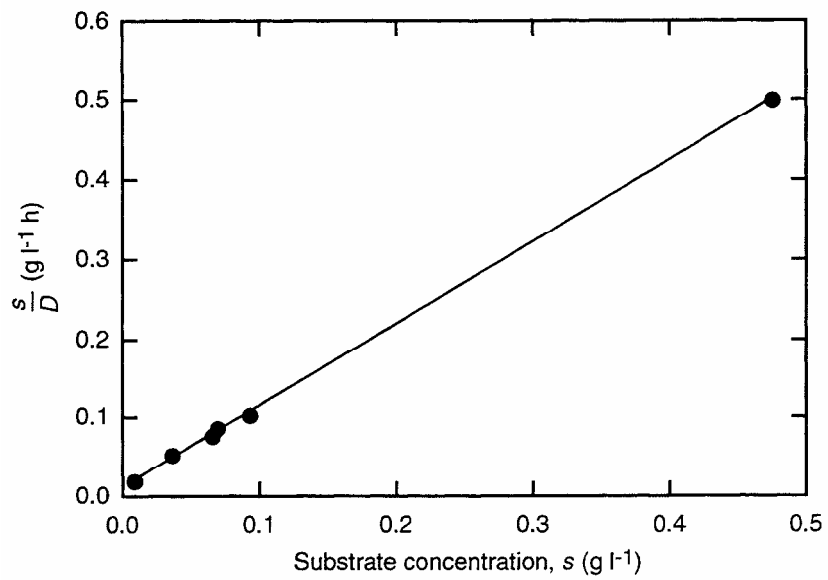
$$F = DV = 0.039\ h^{-1}(1000\ ml) = 39\ ml\ h^{-1}$$

Answer: 39 ml h^{-1}

14.13 Kinetic and yield parameters of an auxotrophic mutant

From Eq. (14.92), μ_{max} and K_S can be determined from the slope and intercept of a plot of s/D versus s. Values of D are evaluated from the experimental flow rates using the definition of dilution rate in Eq. (14.39) and $V = 2$ l. The relevant data are listed and plotted below.

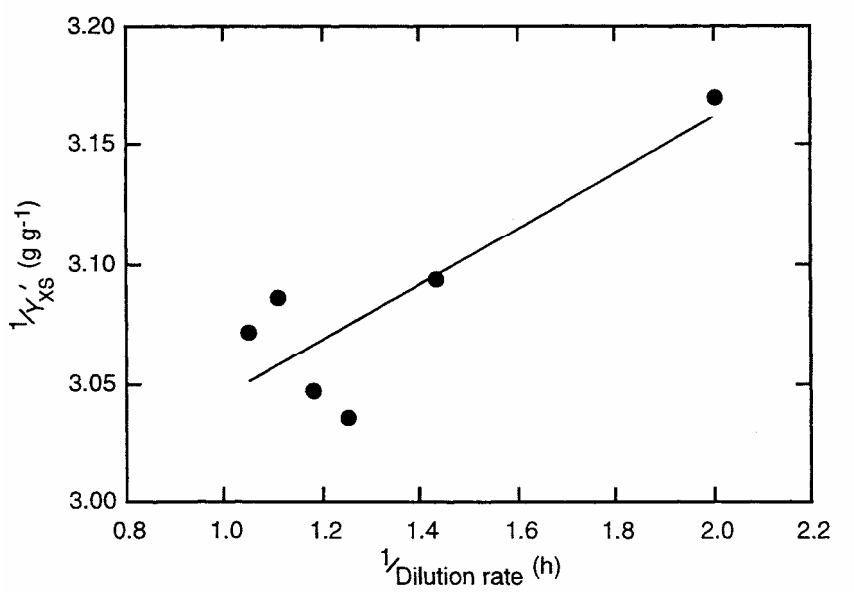
Flow rate, F (l h^{-1})	Dilution rate, D (h^{-1})	Substrate concentration, s (g l^{-1})	s/D (g l^{-1} h)
1.0	0.50	0.010	0.020
1.4	0.70	0.038	0.054
1.6	0.80	0.071	0.089
1.7	0.85	0.066	0.078
1.8	0.90	0.095	0.106
1.9	0.95	0.477	0.502



The slope of the straight line in the plot is 1.027 h; the intercept is 0.0119 g l^{-1} h. From Eq. (14.92), the slope = $1/\mu_{max}$; therefore, $\mu_{max} = 1/(1.027\ h) = 0.97\ h^{-1}$. The intercept = K_S/μ_{max}; therefore $K_S = 0.0119$ g l^{-1} h × 0.97 h^{-1} = 0.012 g l^{-1}.

For threonine production directly coupled with energy metabolism, from Eq. (14.93), Y_{XS} and m_S can be determined from the slope and intercept of a plot of $1/Y'_{XS}$ versus $1/D$, where Y'_{XS} is calculated using Eq. (14.94) with $s_i = 10$ g l^{-1}. The relevant data are listed and plotted below.

Flow rate, F (1 h^{-1})	Dilution rate, D (h^{-1})	$1/D$ (h)	Y'_{XS} (g g^{-1})	$1/Y'_{XS}$ (g g^{-1})
1.0	0.50	2.00	0.315	3.175
1.4	0.70	1.43	0.323	3.096
1.6	0.80	1.25	0.329	3.040
1.7	0.85	1.18	0.328	3.049
1.8	0.90	1.11	0.324	3.086
1.9	0.95	1.05	0.326	3.067



The slope of the straight line in the plot is 0.12 g g^{-1} h^{-1}; the intercept is 2.93 g g^{-1}. From Eq. (14.93), the slope = m_S; therefore m_S = 0.12 g g^{-1} h^{-1}. The intercept = $1/Y_{XS}$; therefore, Y_{XS} = 1/(2.93 g g^{-1}) = 0.34 g g^{-1}.

Answer: μ_{max} = 0.97 h^{-1}; K_S = 0.012 g l^{-1}; m_S = 0.12 g g^{-1} h^{-1}; Y_{XS} = 0.34 g g^{-1}

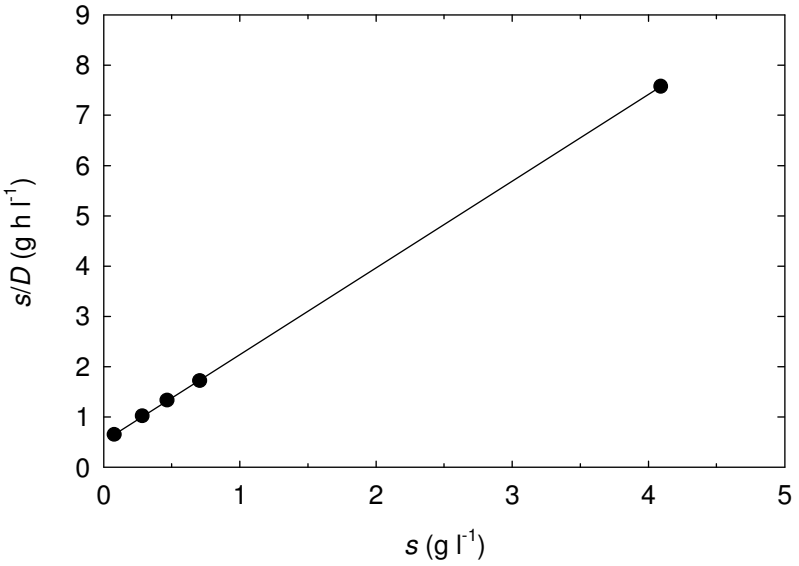
14.14 Chemostat culture for metabolic engineering

V = 400 ml; s_i = 12 g l^{-1}; p_i = 0.

(a)

For a system with sterile feed and negligible cell death, μ_{max} and K_S are determined from Eq. (14.92) by plotting s/D versus s. The dilution rate corresponding to each flow rate is calculated using Eq. (14.39). The results are listed and plotted below.

F (ml h^{-1})	D (h^{-1})	s (g l^{-1})	s/D (g h l^{-1})
48	0.12	0.078	0.650
112	0.28	0.285	1.018
140	0.35	0.466	1.331
164	0.41	0.706	1.722
216	0.54	4.09	7.574



The equation to the straight line in the plot is $s/D = 1.725s + 0.519$. Therefore, from Eq. (14.92), $1/\mu_{max} = 1.725$ h, so that $\mu_{max} = 0.580$ h^{-1}. Also from Eq. (14.92), $K_S/\mu_{max} = 0.519$ g h l^{-1}. Multiplying this value by μ_{max} gives $K_S = 0.519$ g h l^{-1} × 0.580 h^{-1} = 0.301 g l^{-1}.

Answer: $\mu_{max} = 0.58$ h^{-1}; $K_S = 0.30$ g l^{-1}

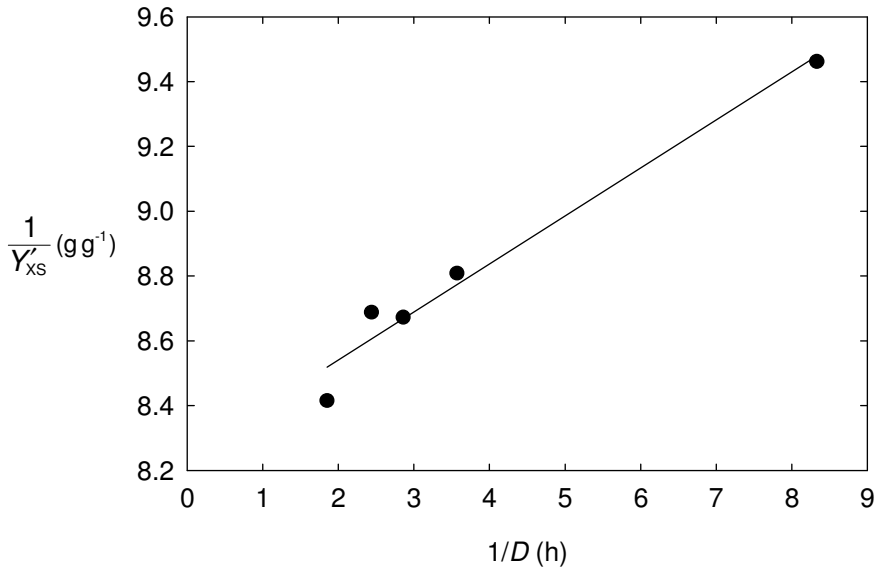
(b)

When product formation is directly coupled with energy metabolism, m_S and Y_{XS} can be determined from Eq. (14.93) by plotting $1/Y'_{XS}$ versus $1/D$. If the feed stream to the bioreactor is sterile, the observed biomass yield from substrate Y'_{XS} is given by Eq. (14.94); therefore, an expression for $1/Y'_{XS}$ is:

$$\frac{1}{Y'_{XS}} = \frac{s_i - s}{x}$$

The results are listed and plotted below.

F (ml h^{-1})	D (h^{-1})	$1/D$ (h)	$1/Y'_{XS}$ (g g^{-1})
48	0.12	8.33	9.462
112	0.28	3.57	8.808
140	0.35	2.86	8.672
164	0.41	2.44	8.688
216	0.54	1.85	8.415



The equation to the straight line in the plot is $1/Y'_{XS} = 0.148/D + 8.244$. Therefore, from Eq. (14.93), $m_S = 0.148$ h^{-1} and $1/Y_{XS} = 8.244$ g g^{-1}, giving $Y_{XS} = 0.121$ g g^{-1}.

Answer: $m_S = 0.15$ h^{-1}; $Y_{XS} = 0.12$ g g^{-1}

(c)

When product formation is directly coupled with energy metabolism, Eq. (12.113) applies. In a chemostat culture with $\mu = D$, Eq. (12.113) becomes:

$$Y'_{PX} = Y_{PX} + \frac{m_P}{D} \tag{1}$$

Therefore, if Y_{PX} and m_P are constant, a plot of Y'_{PX} versus $1/D$ should give a straight line with slope m_P and intercept Y_{PX}. When the feed stream to the bioreactor is sterile, the observed product yield from biomass Y'_{PX} is determined using an equation analogous to Eq. (14.94):

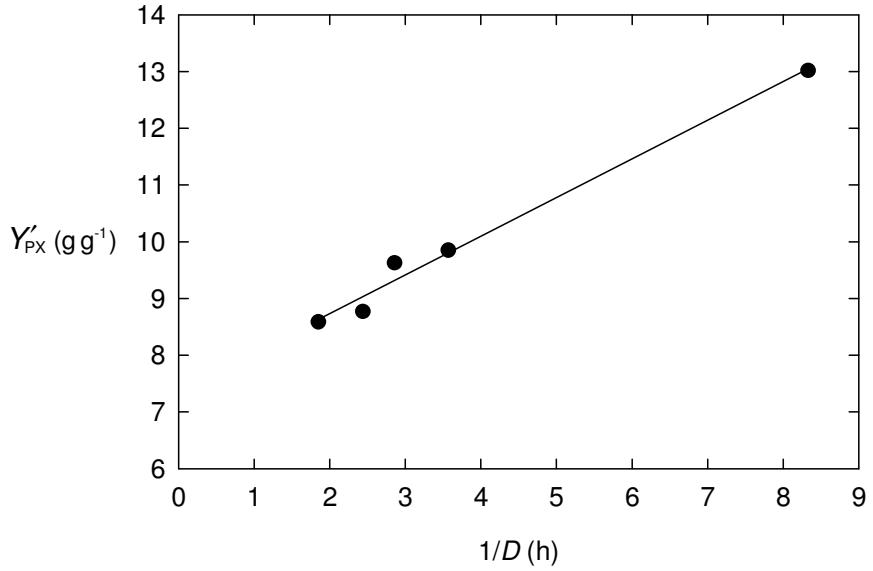
$$Y'_{PX} = \frac{p - p_i}{x}$$

As $p_i = 0$, this becomes:

$$Y'_{PX} = \frac{p}{x}$$

The results are listed and plotted below.

F (ml h^{-1})	D (h^{-1})	$1/D$ (h)	Y'_{PX} (g g^{-1})
48	0.12	8.33	13.016
112	0.28	3.57	9.850
140	0.35	2.86	9.624
164	0.41	2.44	8.769
216	0.54	1.85	8.585



The equation to the straight line in the plot is $Y'_{PX} = 0.682/D + 7.371$. Therefore, from (1), $m_P = 0.682\ h^{-1}$ and $Y_{PX} = 7.371\ g\ g^{-1}$.

Answer: $m_P = 0.68\ h^{-1}$; $Y_{PX} = 7.37\ g\ g^{-1}$

(d)

From the definition of yield in Section 12.7.1, it can be deduced that $Y_{PS} = Y_{PX}Y_{XS}$. Therefore, applying the results for Y_{PX} and Y_{XS} from **(b)** and **(c)**:

$$Y_{PS} = 7.371\ g\ g^{-1}\ (0.121\ g\ g^{-1}) = 0.892\ g\ g^{-1}$$

Answer: 0.89 g g^{-1}

(e)

The maximum practical dilution rate is the critical dilution rate D_{crit}. D_{crit} evaluated using Eq. (14.66) is valid for cultures with maintenance requirements; this can be demonstrated by combining Eqs (14.58) and (14.61) for cells with maintenance requirements and product formation linked with energy metabolism, substituting $x = 0$ for the washout condition, and solving for D. Applying Eq. (14.66) with the results for μ_{max} and K_S from **(a)** gives:

$$D_{crit} = \frac{0.580\ h^{-1}\ (12\ g\ l^{-1})}{(0.301+12)\ g\ l^{-1}} = 0.566\ h^{-1}$$

From the definition of the dilution rate in Eq. (14.39), the flow rate corresponding to D_{crit} is:

$$F = D_{crit}V = 0.566\ h^{-1}\ (400\ ml) = 226.4\ ml\ h^{-1}$$

Answer: 226 ml h^{-1}

(f)

The dilution rate for maximum biomass productivity is D_{opt} (Section 14.5.4, Cell Culture subsection). An expression for D_{opt} for cell cultures with negligible maintenance requirements is Eq. (14.70). Applying this equation using the results for μ_{max} and K_S from **(a)** gives:

$$D_{opt} = 0.580\ h^{-1}\left(1 - \sqrt{\frac{0.301\ g\ l^{-1}}{(0.301+12)\ g\ l^{-1}}}\right) = 0.489\ h^{-1}$$

Because the *Lactobacillus* culture has a non-zero maintenance coefficient as determined in **(b)**, strictly speaking, Eq. (14.70) is not valid for these cells and the above calculation cannot be considered correct. Although maintenance is expected to have only a small effect on the answer, an alternative expression for D_{opt} that applies to cultures with maintenance requirements can be derived. Combining Eqs (14.67) and (14.61), which is valid when maintenance requirements are present and product formation is coupled with energy metabolism, gives:

$$Q_X = \frac{D^2(s_i - s)}{\frac{D}{Y_{XS}} + m_S}$$

Substituting Eq. (14.58) for s:

$$Q_X = \frac{D^2\left(s_i - \frac{DK_S}{\mu_{max} - D}\right)}{\frac{D}{Y_{XS}} + m_S}$$

Applying parameter values for s_i, K_S, μ_{max}, Y_{XS} and m_S:

$$Q_X = \frac{D^2\left(12 - \frac{0.301D}{0.580 - D}\right)}{\frac{D}{0.121} + 0.148}$$

where Q_X has units of g l^{-1} h^{-1} and D has units h^{-1}. Combining terms gives:

$$Q_X = \frac{12D^2 - \frac{0.301D^3}{0.580 - D}}{\frac{D}{0.121} + 0.148}$$

To find the value of D that maximises Q_X, the derivative dQ_X/dD is determined and set equal to zero; the resulting equation is then solved for D. Differentiating the above equation for Q_X with respect to D by applying the differentiation rules in Section E.2 (Appendix E):

$$\frac{dQ_X}{dD} = \frac{24D - \frac{0.903D^2}{0.580 - D} - \frac{0.301D^3}{(0.580 - D)^2}}{\frac{D}{0.121} + 0.148} - \frac{12D^2 - \frac{0.301D^3}{0.580 - D}\left(\frac{1}{0.121}\right)}{\left(\frac{D}{0.121} + 0.148\right)^2}$$

Setting $dQ_X/dD = 0$, multiplying both sides of the equation by $\left(\frac{D}{0.121} + 0.148\right)^2$ and combining terms gives:

$$0 = \left(24D - \frac{0.903D^2}{0.580 - D} - \frac{0.301D^3}{(0.580 - D)^2}\right)\left(\frac{D}{0.121} + 0.148\right) - 99.2D^2 - \frac{2.49D^3}{0.580 - D}$$

Dividing through by D:

$$0 = \left(24 - \frac{0.903D}{0.580 - D} - \frac{0.301D^2}{(0.580 - D)^2}\right)\left(\frac{D}{0.121} + 0.148\right) - 99.2D - \frac{2.49D^2}{0.580 - D}$$

This equation can be solved by trial-and-error. As a first guess, take D = 0.49 h^{-1} as found above by applying Eq. (14.70). The value of the right-hand side of the equation is then compared with the value of

the left-hand side, i.e. zero. Depending on the deviation from zero, D is adjusted, as shown in the table below.

D (h^{-1})	Right-hand side of the equation
0.49	–12.597
0.45	18.255
0.48	–0.967
0.478	0.932
0.479	–0.00268

As the value of the right-hand side of the equation is as close as practicable to zero when $D = 0.479$, we can say that D_{opt} for this culture is 0.479 h^{-1}. Calculating the corresponding flow rate using Eq. (14.39):

$$F = D_{opt}V = 0.479\ \text{h}^{-1}\,(400\ \text{ml}) = 191.6\ \text{ml h}^{-1}$$

Answer: 192 ml h^{-1}

(g)

The steady-state substrate concentration is calculated using Eq. (14.58). For $D = 0.479\ h^{-1}$:

$$s = \frac{0.479\ \text{h}^{-1}(0.301\ \text{g l}^{-1})}{(0.580 - 0.479)\ \text{h}^{-1}} = 1.43\ \text{g l}^{-1}$$

The rate of substrate consumption is given by an equation analogous to Eqs (14.67) and (14.68):

$$Q_S = D(s_i - s)$$

Substituting parameter values:

$$Q_S = 0.479\ \text{h}^{-1}(12 - 1.43)\ \text{g l}^{-1} = 5.06\ \text{g l}^{-1}\ \text{h}^{-1}$$

For cultures with maintenance requirements and product formation linked with energy metabolism, the steady-state biomass concentration is given by Eq. (14.61). Substituting parameter values including the result for s:

$$x = \frac{0.479\ \text{h}^{-1}(12 - 1.43)\ \text{g l}^{-1}}{\left(\dfrac{0.479\ \text{h}^{-1}}{0.121\ \text{g g}^{-1}}\right) + 0.148\ \text{h}^{-1}} = 1.23\ \text{g l}^{-1}$$

The rate of biomass production is given by Eq. (14.67):

$$Q_X = 0.479\ \text{h}^{-1}\,(1.23\ \text{g l}^{-1}) = 0.589\ \text{g l}^{-1}\ \text{h}^{-1}$$

The steady-state lactic acid concentration is given by Eq. (14.65). For cultures with maintenance requirements and product formation linked with energy metabolism, q_P is given by Eq. (12.101). Combining these equations and substituting $\mu = D$ and $p_i = 0$ for chemostat operation gives:

$$p = \frac{(Y_{PX}D + m_P)\,x}{D}$$

Substituting parameter values including the result for x:

$$p = \frac{\left(7.371\ \text{g g}^{-1}\,(0.479\ \text{h}^{-1}) + 0.682\ \text{h}^{-1}\right)1.23\ \text{g l}^{-1}}{0.479\ \text{h}^{-1}} = 10.82\ \text{g l}^{-1}$$

The rate of lactic acid production is given by Eq. (14.68):

$$Q_P = 0.479\ \text{h}^{-1}\,(10.82\ \text{g l}^{-1}) = 5.18\ \text{g l}^{-1}\ \text{h}^{-1}$$

Answer: $Q_X = 0.59$ g l^{-1} h^{-1}; $Q_P = 5.2$ g l^{-1} h^{-1}; $Q_S = 5.1$ g l^{-1} h^{-1}

14.15 Effect of axial dispersion on continuous sterilisation

From the definition of dilution rate in Eq. (14.39), the volumetric flow rate of medium $F = DV = 0.1\ \text{h}^{-1} \times 15\ \text{m}^3 = 1.5\ \text{m}^3\ \text{h}^{-1}$. The linear velocity u in the holding section of the steriliser is determined by dividing F by the pipe cross-sectional area $A = \pi r^2$, where r is the pipe radius. For $r = 6$ cm = 0.06 m:

$$u = \frac{F}{A} = \frac{1.5\ \text{m}^3\ \text{h}^{-1}}{\pi\,(0.06\ \text{m})^2} = 132.6\ \text{m h}^{-1}$$

The value of the specific death constant is evaluated using Eq. (12.74) with $R = 8.3144$ J gmol^{-1} K^{-1} (Appendix B), $E_d = 288.5$ kJ gmol^{-1} $= 2.885 \times 10^5$ J gmol^{-1}, $A = 7.5 \times 10^{39}$ h^{-1}, and the temperature converted from °C to kelvin using Eq. (2.27):

$$k_d = 7.5\times10^{39}\ \text{h}^{-1}\, e^{-2.885\times10^5\ \text{J gmol}^{-1}/[(8.3144\ \text{J gmol}^{-1}\ \text{K}^{-1})\,(130\,+\,273.15)\ \text{K}]} = 313.1\ \text{h}^{-1}$$

Within a period of 3 months ≈ 90 days, the number of cells N_1 entering the steriliser is equal to the volumetric flow rate of the medium F multiplied by cell concentration × time:

$$N_1 = 1.5\ \text{m}^3\ \text{h}^{-1}\left(10^5\ \text{ml}^{-1}\,.\left|\frac{10^6\ \text{ml}}{1\ \text{m}^3}\right|\right) 90\ \text{days}\,.\left|\frac{24\ \text{h}}{1\ \text{day}}\right| = 3.24\times10^{14}$$

Within the same 3-month period, the acceptable number of cells remaining at the end of the sterilisation treatment is $N_2 = 1$. Therefore:

$$\frac{N_2}{N_1} = \frac{1}{3.24\times10^{14}} = 3.09\times10^{-15}$$

(a)

For perfect plug flow with no axial dispersion, the sterilisation time can be determined using Eq. (14.97):

$$t_{hd} = \frac{\ln\left(\dfrac{3.24\times10^{14}}{1}\right)}{313.1\ \text{h}^{-1}} = 0.107\ \text{h}$$

To allow the medium to remain for this period of time in the holding section of the steriliser pipe, the length of pipe required is equal to the linear velocity of the medium u multiplied by t_{hd}:

$$L = u\,t_{hd} = 132.6\ \text{m h}^{-1}\,(0.107\ \text{h}) = 14.2\ \text{m}$$

Answer: 14.2 m

(b)

Calculating the Reynolds number for pipe flow using Eq. (7.1) with pipe diameter $D = 12$ cm = 0.12 m:

$$Re = \frac{0.12\ \text{m}\,(132.6\ \text{m h}^{-1})\,1000\ \text{kg m}^{-3}}{4\ \text{kg m}^{-1}\ \text{h}^{-1}} = 3.98\times10^3$$

The value of $\mathscr{D}_z/uD$ corresponding to this Re is found from Figure 14.41. Using the experimental curve as this gives a higher $\mathscr{D}_z$ than the theoretical curve and thus a more conservative design, $\mathscr{D}_z/uD \approx 1.5$. Therefore:

$$\mathscr{D}_z = 1.5uD = 1.5\,(132.6\ \text{m h}^{-1})\,0.12\ \text{m} = 23.9\ \text{m}^2\ \text{h}^{-1}$$

From Eq. (14.101), an expression for the Peclet number *Pe* is:

$$Pe = \frac{(132.6\ \text{m h}^{-1})\ L}{23.9\ \text{m}^2\ \text{h}^{-1}} = 5.5L \tag{1}$$

where *L* has units of m. Similarly, an expression for the Damköhler number *Da* from Eq. (14.102) is:

$$Da = \frac{(313.1\ \text{h}^{-1})\ L}{132.6\ \text{m h}^{-1}} = 2.36L \tag{2}$$

The design problem can be solved from this point using trial-and-error methods and Figure 14.42. As a first guess, try $L = 20$ m. The values for *Pe* and *Da* are evaluated using (1) and (2) and the corresponding value for N_2/N_1 read from Figure 14.42. Depending on how this value compares with the target N_2/N_1 of 3.09×10^{-15}, the value of *L* is adjusted until the results for N_2/N_1 coincide. The calculations are shown in the table below.

L (m)	*Pe*	*Da*	N_2/N_1 (from Figure 14.42)
20	110	47	4×10^{-16}
18	99	42	1×10^{-14}
19	105	45	1.5×10^{-15}

Given the resolution of Figure 14.42, the last value of N_2/N_1 is as close as practicable to 3.09×10^{-15}. Therefore, the required length of pipe in the holding section is about 19 m, or 34% longer than that determined for ideal plug flow in **(a)**.

Answer: About 19 m

(c)

For $L = 14.2$ m, from equations (1) and (2) developed in **(b)**, $Pe = 78$ and $Da = 34$. From Figure 14.42, N_2/N_1 is about 5×10^{-12}; therefore, $N_1/N_2 = 2 \times 10^{11}$. For $N_2 = 1$, $N_1 = 2 \times 10^{11}$, i.e. one contaminant enters the fermenter for every 2×10^{11} that enter the steriliser. For $F = 1.5\ \text{m}^3\ \text{h}^{-1}$ and an input contaminant concentration of $10^5\ \text{ml}^{-1}$, the time required for 2×10^{11} contaminants to enter the steriliser is:

$$\text{Time} = \frac{2 \times 10^{11}}{1.5\ \text{m}^3\ \text{h}^{-1}\left(10^5\ \text{ml}^{-1} \cdot \left|\frac{10^6\ \text{ml}}{1\ \text{m}^3}\right|\right) \cdot \left|\frac{1\ \text{h}}{60\ \text{min}}\right|} = 80\ \text{min}$$

Therefore, contaminants enter the fermenter at a rate of one every 80 min.

Answer: One contaminant enters the fermenter every 80 min

14.16 Contamination frequency after continuous sterilisation

The linear velocity *u* in the steriliser pipe is determined by dividing the volumetric flow rate *F* by the pipe cross-sectional area $A = \pi r^2$, where *r* is the pipe radius. For $r = (8\ \text{cm})/2 = 4\ \text{cm} = 0.04$ m:

$$u = \frac{F}{A} = \frac{0.9\ \text{m}^3\ \text{h}^{-1}}{\pi\ (0.04\ \text{m})^2} = 179.0\ \text{m h}^{-1}$$

The Damköhler number *Da* is determined using Eq. (14.102):

$$Da = \frac{340\ \text{h}^{-1}\ (21\ \text{m})}{179.0\ \text{m h}^{-1}} = 39.9$$

$\mu = 0.9$ cP; therefore, from Table A.9 (Appendix A), $\mu = 0.9 \times 10^{-3}\ \text{kg m}^{-1}\ \text{s}^{-1}$. Calculating the Reynolds number for pipe flow using Eq. (7.1) with pipe diameter $D = 8\ \text{cm} = 0.08$ m:

$$Re = \frac{0.08\ \text{m}\,(179.0\ \text{m h}^{-1}).\left|\frac{1\ \text{h}}{3600\ \text{s}}\right|(1000\ \text{kg m}^{-3})}{0.9\times10^{-3}\ \text{kg m}^{-1}\,\text{s}^{-1}} = 4.42\times10^{3}$$

The value of $\mathscr{D}_z/uD$ corresponding to this *Re* is found from Figure 14.41. Using the experimental curve as this gives a higher $\mathscr{D}_z$ than the theoretical curve and thus a more conservative design, $\mathscr{D}_z/uD \approx 1.1$. Therefore:

$$\mathscr{D}_z = 1.1uD = 1.1\,(179.0\ \text{m h}^{-1})\ 0.08\ \text{m} = 15.75\ \text{m}^2\ \text{h}^{-1}$$

From Eq. (14.101), the Peclet number *Pe* is:

$$Pe = \frac{179.0\ \text{m h}^{-1}\,(21\ \text{m})}{15.75\ \text{m}^2\ \text{h}^{-1}} = 238.7$$

The value of N_2/N_1 corresponding to the calculated values of *Da* and *Pe* is obtained from Figure 14.42 as ~4×10^{-16}; therefore, $N_1/N_2 = 2.5 \times 10^{15}$. For $N_2 = 1$, $N_1 = 2.5 \times 10^{15}$, i.e. one contaminant enters the fermenters for every 2.5×10^{15} that enter the steriliser. For $F = 0.9\ \text{m}^3\ \text{h}^{-1}$ and an input contaminant concentration of $6.5 \times 10^5\ \text{ml}^{-1}$, the time required for 2.5×10^{15} contaminants to enter the steriliser is:

$$\text{Time} = \frac{2.5\times10^{15}}{0.9\ \text{m}^3\ \text{h}^{-1}\left(6.5\times10^{5}\ \text{ml}^{-1}.\left|\frac{10^{6}\ \text{ml}}{1\ \text{m}^3}\right|\right).\left|\frac{24\ \text{h}}{1\ \text{day}}\right|} = 178\ \text{days}$$

Therefore, contaminants enter the fermenters at a rate of one every 178 days.

Answer: Once every 178 days. *Note that this result depends strongly on interpolation of Figure 14.42. Small deviations in Pe and/or Da lead to large changes in the value of N_2/N_1.*