DESIGN AND DEVELOPMENT OF MAGNETIC BRAKING SYSTEM

A Final Year Project Report

Presented to

SCHOOL OF MECHANICAL & MANUFACTURING ENGINEERING

Department of Mechanical Engineering

NUST

ISLAMABAD, PAKISTAN

In Partial Fulfillment
of the Requirements for the Degree of
Bachelors of Mechanical Engineering

by

Noman Mumtaz

Shahzaib

Talha Razzaq

June 2021

EXAMINATION COMMITTEE

We hereby recommend that the final year project report prepared under our supervision by:

NOMAN MUMTAZ	00000219524
SHAHZAIB	00000211356
TALHA RAZZAQ	00000235176

Titled: "Design And Development of Magnetic Braking System" be accepted in partial fulfillment of the requirements for the award of BE MECHANICAL ENGINEERING degree with grade ___

Supervisor: Dr. Jawad Aslam, Assistant Professor SMME, NUST	____________________ Dated:
Committee Member: Dr. Aamir Mubashar, Professor SMME, NUST	____________________ Dated:
Committee Member: Dr. Muhammad Safdar, Assistant Professor SMME, NUST	____________________ Dated:

______________________ ____________

(Head of Department) (Date)

COUNTERSIGNED

Dated: ________________ __________________________

(Dean / Principal)

ABSTRACT

This report deals with the design and simulation of Magnetic braking system for a passenger vehicle using computer aided engineering software. Braking torque calculations are done for the deceleration of a specific passenger car from highway speed. The torque requirement calculations and coil size calculations are done in Microsoft Excel. Then a magnetic braking system is designed for this required braking torque by making its 3D model and then doing its Torque Analysis in COMSOL Multiphysics software. The structural three-dimensional solid modelling of braking system is developed using the computer-aided drawing software SOLIDWORKS. The design is improved starting from simple basic design. The power and space constraints are considered while deciding final design of Magnetic Braking system. A common passenger vehicle Honda Civic 10th generation is considered because of the easy availability of its dimensional parameters. It is supposed that this vehicle decelerates from 120 kph to 60 kph in 4 seconds with a considerable load in it. Weight transfer is considered while braking. Thermal constraints are taken into account while designing the disc of braking system. The actual size of electromagnets is also implemented to ensure fitting. After torque analysis it can be said that the final design meets the required torque requirements. Then recommendations about prototyping and test bench are provided for future work.

ACKNOWLEDGMENTS

We would like to express our deepest and sincere gratitude to our respected supervisor, Dr. Jawad Aslam, for his enthusiasm, patience, helpful information, valuable guidance and practical advice that have helped us tremendously throughout our project. We would also like to express our gratitude to Dr. Aamir Mubashir for his guidance. We would also like to express our gratitude to our Final Year Project coordinators, Dr. Rehan Zahid and Dr. Niaz Bahadur Khan for their co-ordination regarding project timelines. And to some fellow students for their devotion and help during the project. We are forever humbled by the unwavering support provided to us by our parents throughout this time. Our parents have been the constant course of emotional and financial support. It is their faithful pride which enabled us to work with a sense of duty, the duty to return their trust with our untiring efforts and push our limits to achieve the goals which we set for ourselves. And we would like to thank NUST, for always providing us with all possible resources to help us pursue our dreams and take pride in serving the people of this dear land. We also place on record, a sense of gratitude to one and all, who directly or indirectly, have lent their hand in this venture.

ORIGINALITY REPORT

FYP Final report

ORIGINALITY REPORT

4%	3%	2%	2%
SIMILARITY INDEX	INTERNET SOURCES	PUBLICATIONS	STUDENT PAPERS

PRIMARY SOURCES

1	www.ijirst.org Internet Source	<1 %
2	en.wikipedia.org Internet Source	<1 %
3	Submitted to Higher Education Commission Pakistan Student Paper	<1 %
4	Submitted to Kingston College Student Paper	<1 %
5	www.variohm.com Internet Source	<1 %
6	idoc.pub Internet Source	<1 %
7	Submitted to University of New Orleans Student Paper	<1 %
8	www.coursehero.com Internet Source	<1 %
9	"Trends in Manufacturing and Engineering Management", Springer Science and Business	<1 %

Media LLC, 2021
Publication

10	www.slideshare.net Internet Source	<1 %
11	Submitted to International School of Kuala Lumpur Student Paper	<1 %
12	bboymike93.blogspot.com Internet Source	<1 %
13	Sergey I. Krivosheev, Sergey G. Magazinov, Yuri E. Adamyan, Dmitrii I. Alekseev, Maksim V. Manzuk. "Uniaxial High Strain Rate Tension with the Use of Magnetic Pulse Method", 2020 IEEE Conference of Russian Young Researchers in Electrical and Electronic Engineering (EIConRus), 2020 Publication	<1 %
14	Submitted to University of Strathclyde Student Paper	<1 %
15	A. Bezaguet, J. Casas-Cubillos, A. Cyvoct, P. Lebrun, R. Losserand-Madoux, M. Marquet, M. Schmidt. "A pulsed superconducting magnet for a static magnetic refrigerator operating between 1.8 K and 4.5 K", IEEE Transactions on Magnetics, 1994 Publication	<1 %

Submitted to Florida Institute of Technology

16	Student Paper	<1 %
17	Submitted to Manchester Metropolitan University Student Paper	<1 %
18	P. E. Allaire, E. H. Maslen, D. W. Lewis, R. D. Flack. "Magnetic Thrust Bearing Operation and Industrial Pump Application", Journal of Engineering for Gas Turbines and Power, 1997 Publication	<1 %
19	M. Talaat, N. H. Mostafa. "Use of finite element method for the numerical analysis of eddy current brake", 15th International Workshop on Research and Education in Mechatronics (REM), 2014 Publication	<1 %
20	K. Elis Norden. "Handbook of Electronic Weighing", Wiley, 1998 Publication	<1 %
21	docplayer.net Internet Source	<1 %
22	mafiadoc.com Internet Source	<1 %
23	www.meritnation.com Internet Source	<1 %
24	alevelhelp.blogspot.com Internet Source	

		<1 %
25	epdf.tips Internet Source	<1 %
26	Basic Electromagnetism, 1993. Publication	<1 %
27	Lezhi Ye, Desheng Li, Bingfeng Jiao. "Three-dimensional electromagnetic analysis and design of permanent magnet retarder", Frontiers of Mechanical Engineering in China, 2010 Publication	<1 %
28	eprints.nottingham.ac.uk Internet Source	<1 %
29	Shengqiang Yang, Wenhui Li. "Surface Finishing Theory and New Technology", Springer Science and Business Media LLC, 2018 Publication	<1 %
30	en.m.wikipedia.org Internet Source	<1 %
31	M.B. Badawi, W.A. Crosby, I.M. El Fahham, M.H. Alkomy. "Performance analysis of tilting pad journal bearing using COMSOL Multiphysics and Neural Networks", Alexandria Engineering Journal, 2020 Publication	<1 %

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

ABBREVIATIONS

ECB	Eddy Current Brake
COG	Center of Gravity
EM	Electromagnet
PM	Permanent Magnet
ABS	Anti-lock Braking System
EBD	Electronic Brake Distribution
ESC	Electronic Stability Controls
FE	Finite Element
PID	Proportional Integral Derivative
RPM	Revolution per minute
KPH	Kilometer per Hour
AC	Alternating Current
DC	Direct Current
SI	International System of units
IADS	International Alloy Designation System
mmf	Magnetomotive Force
CAD	Computer Aided Design
ODE	Ordinary Differential Equations
mef	Magnetic and Electric Field
SWG	Standard Wire Gauge

NOMENCLATURE

$W_{front\ dynamic}$	Load on front axle while braking (kg)
$W_{front\ static}$	Load on front axle while static (kg)
W_{total}	Vehicle Weight (kg)
E	Electric Field (V/m)
B	Magnetic Flux Density (T)
H	Magnetic Field Strength (A/m)
D	Electric Displacement Field (A/m^{2}s)
F	Lorentz Force on Charge (N)
q	Charge (C)
v	Velocity of charge (m/s)
T	Applied Torque (Nm)
L	Length of Shaft (m)
G	Shear Modulus of material (N/m^2)
J	Polar Moment of Inertia (m^4)
σ	Conductivity of material (S/m)
ρ	Density of material (kg/m^3)
c	Heat capacity (J/kg·K)
k	Thermal conductivity (W/m·K)
σ_t	Tensile strength (MPa)
Q	Heat Load in disc (W)
ΔT	Temperature Increase of disc (^{o}C)
V	Volume of the disc (m^3)
N	Number of Turns in one coil
I	Current through coils (A)
ω	Angular Velocity of disc (rad/s)
j	Current density (A/mm^2)
d	Wire Diameter (mm)

D_2	Outer Diameter of coil (mm)
D_1	Inner Diameter of coil (mm)
D_m	Mean diameter of coil (mm)
k_c	Packing Coefficient
l	Axial length of coil (mm)
L_w	Length of wire in one coil (mm)
ρ_w	Resistivity of Copper ($\Omega.mm^2/m$)
R	Resistance of Coil (Ω)
I_s	Setup Current (A)

CHAPTER 1: INTRODUCTION

1.1. Motivation:

Braking system is one of the main component of an automobile. Stopping or decelerating a vehicle in a safe, controllable and sustainable manner could be very critical for the safe operation of the automobiles and for the safety of the passengers in it. Majority of braking systems in the automobiles in the world work on the same principle. It is stated by Principle of Conservation of Energy. The kinetic Energy of moving object is converted into heat energy by friction. This friction is generated between two surfaces in which one is usually covered with a friction material. In maintenance, one of the parts is replaced regularly to keep the braking system operational in its best condition. The most common braking system that are being used in vehicles of all capacities around the world are either disc brakes and drum brakes. A considerable number of accidents in automobiles occur due to the failure of these friction brakes. Friction brakes of vehicles tend to have their effectiveness reduced when applied for a longer period of time or in emergency braking and thus leads to their failure most of the time. These failures mostly occur because of a condition called Brake Fade. In this condition of brake fade, the temperature of friction material and the brake rotor increases beyond safe limits and the braking system is unable to produce enough friction between the surfaces mostly because of change in properties of materials at elevated temperatures and thus the vehicle is unable to decelerate at the required rate and come to a stop.

These problems of friction brakes are inherent with them due to their design. Although these friction brakes have improved from time to time like newer and way more expensive Carbon ceramic brakes, still the problem of brake fade due to either high load usage or prolonged usage is there and can cause accident. Therefore, there is a need of an auxiliary braking system that will aid the conventional braking system and act as a secondary retarder in the vehicle. This can reduce the risk of accidents in automobiles. This auxiliary retarder should have better performance specially at high speed of vehicle where friction brakes perform worse and mostly fail and this retarder should not have the common problems of frictions brakes. In this way the service life of brakes can be increased and load on brakes can be reduced.

For this purpose Magnetic Brake System is proposed which is a type of contactless braking system that will work as a secondary retarder. This system has a very different working principle as compared to friction brakes. This concept of braking has many advantages over friction brakes and thus can be used as a secondary retarder.

1.2. Problem Statement:

A considerable number of automobile accidents happen due to the failure of friction braking systems. In emergency braking and it long term continuous load situation, these frictional braking systems experience phenomenon of brake fade which leads to their failure as the braking system becomes unable to stop the motion of the vehicle and prevent an accident. These braking systems also experience regular wear and degradation with time with their use and thus these systems require regular maintenance and sometimes replacement of some parts as they are used.

Friction brakes are also a cause of pollution in a way that when they wear, friction dust is released in atmosphere. If we consider number of vehicles running in the world every moment, this contributes considerably towards degradation of environment. Therefore to overcome these problems of friction brakes, a contactless braking system is proposed which will take load off the existing braking system of the car in certain high load situations and help the vehicle in braking safely from high speed. This system reduces the load from the friction brakes and has many other advantages. This Magnetic Braking system will act as a secondary retarder and work alongside the existing braking system. this is because of the property of Magnetic braking system to work best at higher speeds and then friction brakes will stop the motion from slow speed to a stand still.

1.3. Objectives:

The objective of this projects are as follow:

1. Required Braking Torque calculations for a common passenger vehicle decelerating from highway speed by using dimensional parameters of the vehicle and the required deceleration.
2. Design of 3D structural model of Magnetic Braking System by CAD Modelling and Torque Analysis using Software tools.
3. Parametric Analysis and improvement of design considering various constraints to achieve better performance.
4. Integration of the Magnetic Braking System in the existing electrical system of the vehicle.
5. Proposal of test bench design and parameters.

CHAPTER 2: LITERATURE REVIEW

2.1. Weight transfer:

When a vehicle is stationery, its weight is distributed on all of its wheels. The percentage of weight on each wheel depends on the design of the vehicle. When the vehicle is not moving or moving at a fixed speed, this percentage of weight distribution remains the same. But when the vehicle accelerates or decelerates, this percentage of load distribution changes. When the vehicle accelerates, its weight transfers towards the rear wheels, thus the normal force at the rear wheels increases which in turn increases the amount of friction force that can be generated at the rear wheels. When the vehicle decelerates, its weight transfers towards its front wheels, thus the normal force at its front wheels increases which in turn increases the amount of friction force that can be generated at the front wheels. This weight transfer changes the amount of braking torque that needs to be produced at the front wheels to decelerated the vehicle. Thus weight transfer will contribute towards the brake torque requirement of our braking system. This weight transfer depends upon the deceleration of the vehicle. This weight transfer also depends upon the static weight distribution and vehicle dimensions like wheelbase and the height of the center of gravity. Thus by knowing these parameters like deceleration and vehicle dimensions of our vehicle, the required braking torque will be calculated. The weight transfer on front wheels in braking is found by the given formula: [22]

$$W_{\text{front dynamic}} = W_{\text{front static}} + W_{\text{total}} * \left(\frac{\text{Height of center of gravity} * \text{decceleration}}{\text{Wheel base} * g}\right)$$

2.2. Braking systems:

The two most common types of brake that are implemented in common road vehicles nowadays are drum brakes and disc brakes. Both system have their advantages and disadvantages. A brief overview of these braking systems is presented below.

2.2.1. Drum Brakes:

The two most common types of brake that are implemented in common road vehicles nowadays are drum brakes and disc brakes. In drum brakes, the drum is connected to the rotating wheel of the vehicle. Two brake shoes are present inside the drum. The surfaces of these shoes are covered with friction materials. When the brakes are applied, these shoes press against the inner surface of the drum and create friction and thus braking torque is generated. This slows down the vehicle. the temperature of both brake shoes and brake drum increases as kinetic energy is dissipated in form of heat into the components of braking system. These drum brakes have their advantages and disadvantages. The advantages are that these systems are less expensive to make and maintain as compared to the disc brakes. These brakes have more corrosion resistance. They requires less input force in actuation and have larger contact area in brake shoe and brake drum. The disadvantages are that these brakes are difficult to cool because of their closed design. Also these brakes have more complexity as compared to disc brakes and there is a slight delay in their actuation. Some common designs of drum brakes are one leading one trailing shoe, duo-servo drum brakes, twin leading shoe drum brakes depending upon the actuation of brakes, the location of anchors and the movement of brake shoes.[23]

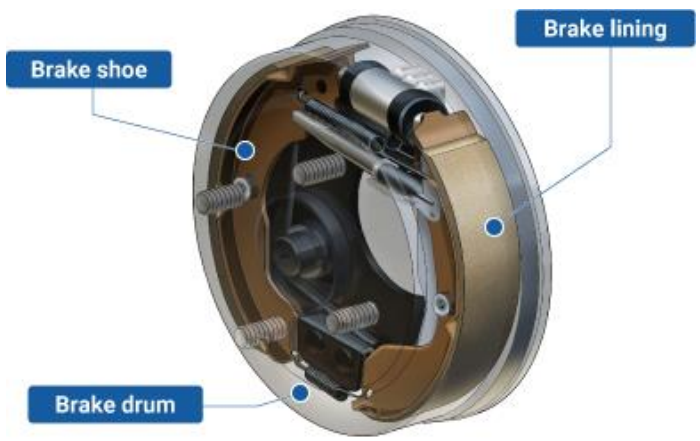


Figure 1: Drum Brakes.

2.2.2. Disc brake:

Disc brake is another widely used braking system. It uses a disc that rotates with the wheel and calipers that are stationery. These calipers have pistons in them that have friction brake pads on them. When the brakes are applied, the brake pads grip and clamp the outer surfaces of disc on both sides. The friction between brake pads and disc stops the disc and the energy is dissipated in the form of heat. These brakes also have their advantages and disadvantages. The advantages are that disc brakes have better heat dissipation because of their open design. These brakes have simpler design and less delay in actuation. Some disadvantages are that theses are expensive and prone to corrosion. Its design is superior to drum brakes.

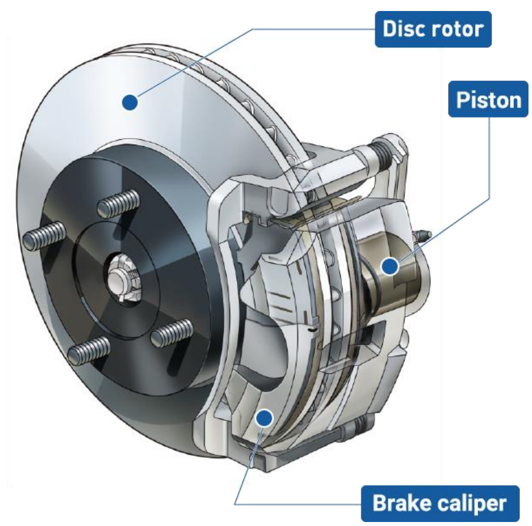


Figure 2: Disc Brakes.

In both systems, usually a hydraulic system having brake fluid is used to transfer the input motion of brake pedal to the motion of brake pads and drums by a series of hoses and tubes. Many other components like brake booster and master cylinder which contains the brake fluid are there. As these systems are actuated mechanically, there is usually some delay in their actuation. Also sometimes these fluid lines rupture due to excessive pressure which leads to the failure of braking system. sometimes there is leakage in this hydraulic system and due to loss of fluid, the system is unable to actuate brakes. [24]

2.3. Magnetic Braking:

There is a special type of braking system called as Magnetic Braking system that works on the principle of generation of eddy currents in conductors therefore sometimes also called as Eddy Current Braking (ECB) system. It is a modern braking system with several advantages over existing friction braking systems. This system has application in various equipment like in device that help climbers descend in a smooth and controlled way by providing retardation effect when the climber jumps of starts to fall. This device is called as Auto Belay device. The smooth retardation effect produced by the magnetic braking system is also used in some exercise equipment to simulate load when a shaft is rotated. This braking system is also used in some power tools with rotating metallic discs that need to be slowed down after operation. This system also work for linearly moving metallic bars instead of rotating discs.

2.3.1. Working Principle:

The Magnetic Braking system is a contactless braking system that works on the principle of eddy currents. Eddy currents are produced inside a conductor when magnetic flux changes through it. These currents produce magnetic fields. Some laws that govern this phenomenon are as follow:

Faraday's law of induction is a principle that states that when the magnetic flux density **B** is changed through time an electric field **E** is created. When a magnetic field changes along a conductor there will be a changing electric field that will cause electric currents to flow. These currents will produce their own magnetic field.

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Lenz's law explains the negative sign in the above equation. It states that the direction of induced field is such that it opposes the field that produced it. Thus there will be an interaction between these fields.

Ampère's law is the reverse of Faraday's law, it states that when a current is passed through a conductor, it produces a magnetic field **H** around it whose strength is proportional to the amount of current **J** passing through the conductor. **D** is the electric displacement field.

$$\nabla \times \mathbf{H} = \mathrm{J} + \frac{\partial \mathbf{D}}{\partial \mathrm{t}}$$

Lorentz' force law states the force acting upon a charged particle moving through a magnetic field. As current is the flow of charges, when a current carrying conductor is placed inside a magnetic field, it experiences a force. The direction is also given by Lorentz force law. The equation is as follow:

$$\mathbf{F} = \mathrm{q}\,(\mathbf{v} \times \mathbf{B})$$

Here **F** is the force on charge, q is value and polarity of charge, **v** is its velocity. This Lorentz force will produce the braking force and thus the braking torque for us.

2.3.2. Electromagnet:

As stated earlier that current flowing through a conductor produces a magnetic field around it. This effect is used to make electromagnet that produce magnetic field similar to permanent magnets but are more flexible as the magnetic field produced can be controlled by controlling the current through the conductor. [25]

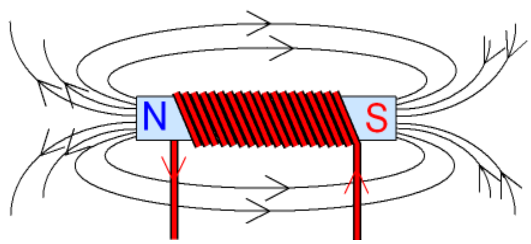


Figure 3: Electromagnet.

By winding conductor wires around to make a coil, we can get magnetic field similar to a bar permanent magnet. By winding these wires on a soft iron core, the field can be directed outside of the poles. A typical electromagnet is shown below:

2.3.3. Initial design:

Thus Lorentz force is the required force to produce braking torque in Eddy Current brakes. A magnetic field is produced by an electromagnet. A conductor passes through this produced magnetic field. When there is a relative motion between conductor and a magnetic field, eddy currents are produced in the conductor as stated by Faraday's law. These eddy currents produce their own magnetic field as stated by Ampere's law. This produced field is opposite to the applied magnetic field as stated by Lenz's law. Thus it opposes the applied Magnetic Field. In the region where conductor enters the applied magnetic field, as the magnetic flux increases in this region, the eddy currents produce a field such that it opposes the increasing flux. Thus retardation occurs and slows down the relative motion. In the region where conductor leaves the applied magnetic field, as the magnetic flux decreases in this region, the eddy currents produce a field such that it opposes the decreasing flux. Thus retardation occurs and slows down the relative motion. This effect is shown by the help of a diagram below:

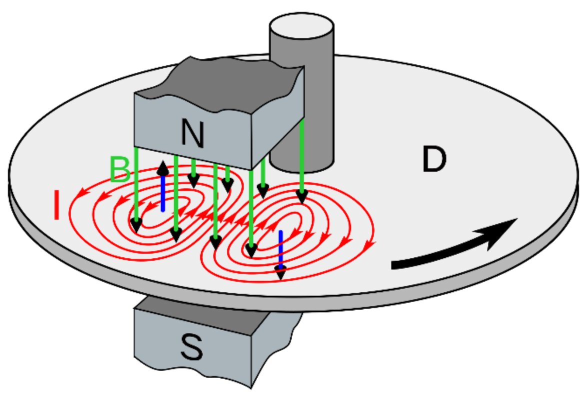


Figure 4: Eddy Current Brake.

2.3.4. Existing applications:

From literature review of existing research and applications of Magnetic Braking system, several aspects are found such that this system is being used in some heavy trucks in foreign markets alongside conventional friction brakes as it is mounted on the their drive shaft that is in between transmission and rear differential. This braking system do not suffer from common friction brake problems like fading due to excessive use, skidding due to wheel lockup in emergency braking, high maintenance requirement of friction brake components, low reliability of friction brakes, requirement of servo mechanisms for their engaging, breaking of components and higher weights. All of these properties are due to the contactless design of Magnetic Braking systems. Also as there are no moving parts, there is no chance of a mechanism failure. Also this device is easy to install in automobiles as it only requires integration with the electrical system of vehicle and not any mechanical or hydraulic system[2].

2.3.5. Advantages of Eddy Current Brakes:

Eddy Current Brakes have a large number of advantages as compared to commonly used friction brakes. There is no risk of brake fade or wrapping of brake disc due to stresses and excessive temperature rise. In these braking systems there is no need to change brake oils and thus there is no risk of oil leakage because this system works on electrical actuation and no mechanical actuation is involved. The actuation of these systems is electric thus it is instantaneous and also there is no delay as compared to hydraulic actuation as there is no fluid to be compressed [34]. This system helps to extend the life span of the regular brakes and keep the regular brakes cool for emergency situation. They have excellent heat dissipation efficiency because disc is free to rotate and whole surface is available for cooling and no contact is involved. Electromagnetic brake systems will reduce the overall maintenance cost because they are maintenance free. No part is to be changed at regular intervals as there is no wear and tear. The problem of brake fluid vaporization due to excessive temperature rise and hot outside temperature is eliminated. Also the freezing of brake fluid due to very cold climate is eliminated. These brakes produce no torque in case the relative motion between magnetic field and conductor stops thus there is no chance of wheel lockup therefore it is easier to integrate eddy current brakes with anti-lock braking system (ABS), Electronic Brake Distribution (EBD), traction control system, and electronic stability controls (ESC) of a car. Being independent system rather than connected hydraulic braking system, individual wheel braking control is easier. [3]. By controlling the current to the electromagnetic coils, the braking torque produced by the system can be controlled. Due to the above mentioned advantages, Eddy Current brakes are a compelling option as a modern braking system. Some papers have tried to implemented eddy current braking system in automobiles with different designs like using the already available friction disc of disc braking system also for eddy current brakes and passing magnetic field through it. But this is not suitable for thermal considerations and the choice for brake disc material and thickness will be limited by using the existing disc[4] Some designs has used a separate disc for eddy current brakes. The approach of a separate disc is found to be better for several reasons like heating produced in disc and parameter control over design of system as we can design our separate disc for ECB.[5][6][34].

2.3.6. Different configurations:

There are mainly two common configurations of eddy current brakes. One is horizontal conductor bar or rails moving through magnetic fields. These rails can be in horizontal or vertical direction. When magnets pass these rails, eddy currents develop in the conductor rails and apply force on downwards moving magnets thus slowing them down. Magnets can be either on moving part or on the stationery part of system [7][20][21].
Another configuration of Eddy current brakes that is the most common is having a conductor disc that rotates inside the poles of the magnet. This configuration produces braking torque on the disc and therefore on the shaft connected with the rotating disc. This configuration is mostly implemented to slow down rotating inertia like heavy disc and equipment with rotational inertia. Thus configuration is usually called as axial configuration [8][9][13][15]. Examples of these configurations are shown below.

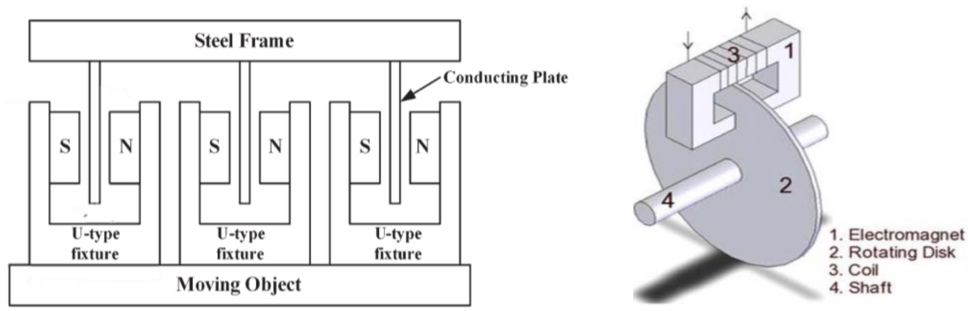


Figure 5: Horizontal and Axial configurations.

2.3.7. Critical velocity:

Eddy current brakes using non magnetic material for conductor disc have a property of having a critical velocity. When we observe braking torque produced by eddy current brakes, as the angular speed of disc rotating inside magnetic field increases at start in a linear fashion. But after some angular velocity, the linear fashion changes. As the angular speed of disc increases further, the braking torque increases but after a value of angular speed, the amount of braking torque produced by the system starts decreasing. This value of angular speed at which the value of torque is maximum is called as critical speed.[36]

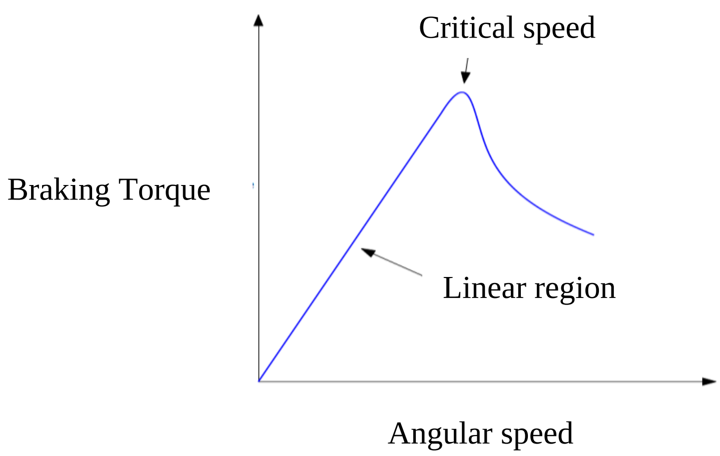


Figure 6: Critical Speed.

This behavior of eddy using non magnetic material for conductor disc is due to a phenomenon known as skin effect. Skin effect means that as current passes through a highly conductor material, due to the current passing, a magnetic field is generated inside the conductor. This magnetic field inside the conductor opposes motion of charges inside the conductor. Thus the charges prefer to move on the edges of conductor or can be said to be on the skin of conductor. As the speed increases, the eddy currents increase and this skin effect increases. Due to this skin effect, the braking torque decreases [18]. Thus due to this characteristic of eddy current brakes, we should operate them near the critical velocity where we will get the maximum braking torque.

2.3.8. Factors affecting braking torque:

The braking torque produced by Eddy Current depends on many factors. These factors include the geometrical parameters of system components like disc and parameters related to magnetic field source.

2.3.8.1. Magnetic field source:

For producing the required magnetic field, mainly two approaches are used, first is using permanent magnets to produce field. This configuration is easy to make because no designing of electromagnets is necessary. But this permanent magnet design will need some kind of physical shunting mechanism that will cut off the magnetic field when the braking is not required because the direction and intensity of field from the permanent magnets cannot be controlled and permanent magnets cannot be turned off like electromagnets.[10][20][21] But this approach contains mechanical movement of some kind of mechanism. Therefore permanent magnet design is not very ideal. Also we get poor control of magnetic field produced by permanent magnets. A better approach is to use electromagnets instead. Electromagnets provide excellent control and flexibility while designing Eddy Current Brakes[3]. In designing of electromagnets, the number of coil windings, the size of core and the current through electromagnet coils can be varied easily and the ideal value can be found to get desired magnetic field and thus the required braking torque. Also by increasing the number of coils the overall size of the system can be reduced. [6][11][15]. But this approach is more complex as it requires separate designing of electromagnets and we also need to take physical fitting of the coils into consideration as there will be space constraints.

2.3.8.2. Geometric and electromagnet parameters:

There are geometric factors that affect the amount of braking torque produced by eddy current brakes. These factors include the diameter of the conductor disc, the thickness of the disc, the value of air gap between the magnet pole and disc and the radius at which the magnet poles are located in front of the face of disc. Different parameters of electromagnet like the number of windings of coil and the current through them dictates braking torque. By decreasing the disc thickness, the torque increases [12] the same behavior can also be observed for decreasing the air gap, the torque increases [12][13][14], by increasing the number of windings in the electromagnet coil, the torque produced increases, also by increasing the current in these windings, the torque will increase [14]. Thus we can increase or decrease the performance of our system by varying these parameters.

2.3.8.3. Conductor material:

The material of conductor disc has effect on the amount of braking torque Eddy Current Brakes can produce. It is not only very important for the braking torque but also for the thermal performance of braking system. Usually non magnetic conducting materials are used because they are not attracted towards the magnetic field source. Copper and Aluminium are two most favorable materials [15] as they are non magnetic and also their electrical conductivity is considerable to allow production of eddy current. But these materials also suffer from skin effect problem. It can also be seen that by making conductor disc thin, the influence of skin effect can be reduced as thicker conductor have more skin effect [18]. Among these materials, Aluminium is the preferred one because it has some advantages over copper such that it has better performance at high angular velocity of discs. Also compared to copper, aluminium is lighter in weight, cheaper to purchase and work on and also more machinable.[8][16].

2.3.8.4. Thermal considerations:

As there is no friction heating in this system owing to its contactless design, the energy is dissipated by eddy currents in conductor. As the disc slows down due to magnetic field produced by eddy currents, these currents produce heating inside the conductor disc. Thus this puts a limit on geometry of system regarding thermal stability so that the temperature does not go out of control. The thermal behavior of practical models of Eddy Current Brakes has been investigated in some papers[6] and its effect on different parameters is also studied because when the temperature of disc increases, the heat is transferred to other nearby components by convection and radiation specially to the electromagnet coils. Thus as time passes, the performance of the system is affected. These effects are investigated from prototype testing in the above mentioned paper.

2.3.8.5. Mathematical models and controllers:

In order to calculate the torque produced by eddy current brakes mathematically, some mathematical models have been developed like by W.R. Smythe (1942), D. Schieber (1974), J.H. Wouterse (1991) [8][15][17][18]. These were some of the early researches on working and design of eddy current brakes. But the formulas developed by them are specific for some cases. This approach was used before the Finite Element Method technique. Now FE approach can be used to simulate eddy current braking and a comparison of theoretical results with experimentally obtained data from prototype development and testing is a better approach.[35] Different types of controllers like Sliding mode controller have been applied to control the system and to make it more responsive and robust so that it could cater for disturbances and act as a secondary retarder for the existing braking system [17]. Some approaches used a PID controller [19] for this purpose.

2.4. Car battery:

To power the electromagnet in Eddy current brakes, we need a current source. As we are implementing our magnetic braking system in a car, the power source we have in our car is its battery. Car battery provides current for different electrical systems of car and it charged by alternator that is powered by running engine in case of most cars that are powered by gasoline engine. Another important function of car battery is to provide current for starting motor also known as Crank Amperes. The capacities of car batteries are designated by its Amp-hour rating. For example, a 45 Amp-hour battery means it can theoretically provide 45 A current for one hour or 27.5 A for two hours and so on other combinations.[33] The battery already available in out chosen vehicle is 45 AH as shown below:

Figure 7: Car battery.

2.5. Torque Measurement:

In order to measure the torque being produced by a system, torque sensors are used. There is mainly two types of scenarios while measuring torque, stationary shafts and rotating shafts.

2.5.1. Torque Sensors:

First is the torque being applied on a static shaft which is not rotating. These sensors are called **Reaction torque sensors.** When the torque is applied on shaft on its axis, a deflection is produced in it because the material is in its elastic limit. With the information about this deflection, geometric and material properties of the shaft, the torque being applied can be calculated by using the formula below:

$$\theta = \frac{T * L}{J * G}$$

Here **θ** is the deflection, **T** is applied torque, **L** is length of shaft, **G** is the shear modulus of material, **J** is polar moment of inertia. Mostly strain gauges are used to measure the deflection. A number of strain gauges are used in order of a Wheatstone bridge. When strained, the strain gauge’s resistance changes. This change when in form of Wheatstone Bridge is converted in voltage change. This voltage signal is proportional to the deflection produced.

Figure 8: Reaction and Rotary torque sensors

The other type of torque sensors are **Rotary torque sensors** also known as dynamic torque sensors. The shaft keeps rotating in these sensors and the deflection is also measured with strain gauges that also rotate with the shaft. As the shaft is rotating, the continuous information of the produced deflection needs to be transferred to outside the rotating shaft.

This can be done by either slip rings contacts, or by inductive loop that uses magnetic field linkage and is also contactless. The most recent and contactless way is to use wireless communication between sensors on rotating shaft and static receiver outside. Wireless communication is the most efficient.

2.5.2. Load cells:

As the torque **T** is the product of an applied force $\mathbf{F_{applied}}$ and a moment arm **r**, there is another simpler way of measuring torque. It is by measuring force and then multiplying it with a moment arm. To measure load, usually load cells are used. Load cells also utilize the property of strain gauges. Load cells have a thinner section in their structure which shows deflection when load is applied, there are strain gauges on this thin section that record deflection. There are many types of load cells like Single point load cells, Bending Beam load cells, Compression load cells, Planar Beam load cells, Shear Beam load cells, Dual Shear Beam load cells, S-type load cells, Load pins and many more. Some are used for only tensile loads, some are only for compressions. Some designs like planner load cells are made keeping space constraints in mind. But for torque measurement usually Single point load cells are used with a stopper arm of known length. A single point load cell is shown below:

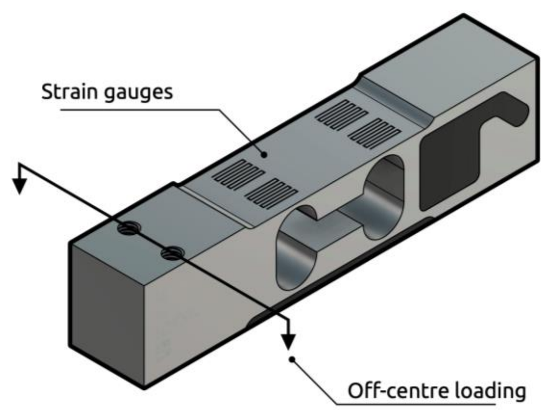


Figure 9: Single point load cell

CHAPTER 3: METHODOLOGY

3.1. Vehicle selection and parameters:

The vehicle selected for our calculation is 10^{th} generation Honda Civic available in our country. The reason for choosing this particular vehicle is that it is a common passenger vehicle in our country. Also the required data for this vehicle is easily available on manufacturer website. The required data of this car includes weight of the car and its static weight distribution. The required dimensional parameter include wheelbase and height of center of gravity of vehicle. Also the information regarding the wheel diameter, tire width and profile is needed to find wheel's outer diameter. These parameters are required alongside deceleration information to calculate required braking torque for our requirement.[26][27] The required parameters are shown in a table below:

Parameter	Value
Vehicle Weight (loaded)	1750 kg
Static Distribution Front Axle	60.6 %
Static Distribution Rear Axle	39.4 %
Center of Gravity Height	0.508 m
Wheelbase	2.7 m
Wheel Diameter	16 in

Table 1: Vehicle data

This data will be used to calculate torque at one front wheel for deceleration of the above mentioned car from 120 kph to 60 kph in 4 s. The data about tire and wheels being used in the specified vehicle is used to calculate vehicle

3.2. Torque and speed calculation equations:

Now after obtaining the required parameters of our vehicle, torque and front wheel rpm are calculated. First the deceleration in terms of g's is calculated and then dynamic load at front and rear axle is calculated. To simulate real life situation, data of a loaded vehicle is used. The formula is shown below:

$$W_{front\ dynamic}\ (kg) = W_{front\ static}\ (kg) + W_{total}\ (kg) * \left(\frac{\text{Height of COG (m)} * \text{decceleration (g's)}}{\text{Wheel base (m)}}\right)$$

$$W_{rear\ dynamic}\ (kg) = W_{rear\ static}\ (kg) - W_{total}\ (kg) * \left(\frac{\text{Height of COG (m)} * \text{decceleration (g's)}}{\text{Wheel base (m)}}\right)$$

After finding the dynamic load on the front and rear axle, the normal forces on front wheels are known. Now we need rpm of front tire at the specified velocities as this will also be the rpm of front drive shafts. The tire outer radius is also needed to find torque. The rpm of front tire is calculated by the following formula:

$$\text{Tire outer edge radius (m)} = \frac{\text{wheel diameter (m)}}{2} + \text{tire width (m)} * \frac{\text{tire profile(\%)}}{100}$$

$$\text{V (rpm)} = \frac{\text{V (kph)}}{3.6} * \frac{60}{2 * \pi * \text{tire outer edge radius (m)}}$$

After knowing front loads and tire outer edge radius, The value of braking torque required at one front wheel is calculated by using the following formula:

$$\text{T on one front wheel (Nm)} = \frac{\text{dynamic load one front axle (kg)}}{2} * \frac{\text{tire outer edge diameter (m)}}{2} * \text{deceleration (g's)} * 9.81$$

By using the above mentioned formulas, the required braking torque at one front wheel is calculated.

3.3. Torque requirement calculations:

These calculations are done with the help of Microsoft Excel software because of the easiness of automation of calculations. The above formulas for weight distribution, tire rpm and braking torque can be inserted in excel and linked with cell containing data. Microsoft Excel automatically calculates for you when the value of one or more parameters is changed. The calculations are as follow:

Vehicle Weight (empty)	Vehicle Weight (loaded)	static distribution front axle	static distribution rear axle	COG height	Wheel base	Tire Outer Edge Radius
kg	kg	%	%	m	m	m
1252	1750	60.60%	39.40%	0.508	2.7	0.32145

Static distribution front	Static distribution rear	Dynamic distribution front	Dynamic distribution rear
kg	kg	**kg**	**kg**
1060.5	689.5	**1200.35**	**549.65**

Initial velocity	Initial rpm	Initial angular velocity	Final Velocity	Final rpm	Final angular Velocity
kph	**rpm**	**rad/s**	kph	**rpm**	**rad/s**
120	**990.23**	**103.70**	60	**495.12**	**51.85**

Stopping time	Deceleration	Total Torque	Torque one front wheel	Torque one rear wheel
s	g	Nm	**Nm**	Nm
4	0.42473666	1171.95	**401.93**	184.05

Table 2: Required Torque

Thus our required braking torque for loaded car from 120 kph to 60 kph in 4 seconds is around **402 Nm** and the wheel slows down from **990 rpm to 495 rpm or 103.7 rad/s to 51.85 rad/s** in SI units.

3.4. Selection of parameters:

Keeping different constraints on our system, various parameters need to be decided. These parameters can also be called as control parameters. The decided values of these parameters are as follow:

3.4.1. Selection of disc diameter:

As we plan to mount our magnetic braking system inside the wheel of the selected car, the control parameter of the disc diameter and thus the maximum diameter of our system is the available space inside wheel. The wheels of 10th generation Honda Civic is **16 inches in diameter** and the size of existing friction brakes inside front wheel has **282mm rotor** inside it. Thus keeping this diameter in consideration, the **disc diameter is taken to be 300mm.**

Figure 10: 10th generation Honda Civic 16 in rim and tire.

3.4.2. Selection of disc material:

From literature review, two materials Copper and Aluminium are found to be the best materials for disc of Eddy Current Brakes as they are non magnetic and are also easily available materials. For our design, **Aluminium is chosen** over copper due to the following reasons[28][29]:

- It is lighter than copper
- It is cheaper than copper
- It is more machinable than copper
- It has less skin effect at the required velocities due to higher resistivity
- Its heat capacity is better than copper which means it can take more heat for less temperature rise.

In aluminium, we have many grades that are alloys of aluminium with different other materials like silicon, copper, magnesium, manganese, tin and zinc etc. These alloying materials when mixed with pure aluminium, change the properties of resulting alloys like its strength, its corrosion resistance and its machineability. Thus different alloys can be used for different specific applications. The percentage of these alloying materials also plays an important role in resulting properties. These alloys are designated by a naming scheme provided by The **International Alloy Designation System (IADS).**

These alloys are give a four digit number and an temper designation. the first digit indicates the major alloying elements, the second digit is usually 0. If not 0, then it indicates a variation of the alloy, and the third and fourth digits identify the specific alloy in the series. The temper designation follows designation number with a dash, a letter, and potentially a one to three digit number. This indicated treatments done on alloy. This also affects material properties.[30]

The alloys that are of our interest are **Al-2024, Al-6061** and **Al-7075.** Al-2024 is aluminium alloy with copper element. It is not very suitable because it have lower melting point and is also prone to corrosion which is very undesirable. Similarly Al-7075 has lower melting point and lower heat capacity. It has high strength but is expensive. These have different alloying elements thus different properties. Among these alloys, the **Al-6061 alloy is chosen** because of the following favorable properties:

- Best machinability of all aluminium alloys
- Corrosion resistance unlike Al-2024 alloy
- Highest melting point among these alloys
- Highest heat capacity among these alloys
- Most common and easily available Aluminium alloy.

Thus disc will be made of Al-6061 alloy which has the following properties[29][30][31]:

Parameter	**Symbol**	**Value**
Conductivity	σ	2.5×10^7 S/m
Density	ρ	2.700 kg/m^3
Heat capacity	c	897 J/kg·K
Melting point	$T_{melting}$	585^{o}C
Thermal conductivity	k	151–202 W/m·K
Tensile strength	σ_t	124–290 MPa
Alloy elements	magnesium (Mg) and silicon (Si)	

Table 3: Al-6061 properties.

3.4.3. Selection of disc thickness:

The braking torque produced by Eddy Current Brakes and the value of critical velocity depends upon the thickness of the disc, thus it is very important to select optimum thickness of disc. The constraint on the disc thickness is the maximum temperature rise. By using different disc thickness values and doing their Torque Analysis on COMSOL Multiphysics, we can get Total Heating in disc. From this total heating plot, we can get heat energy inside the disc for the deceleration of disc from 120kph to 60kph in 4 s. The heat produced in the conductor disc of our design due to eddy current is obtained from the following COMSOL Multiphysics plot:

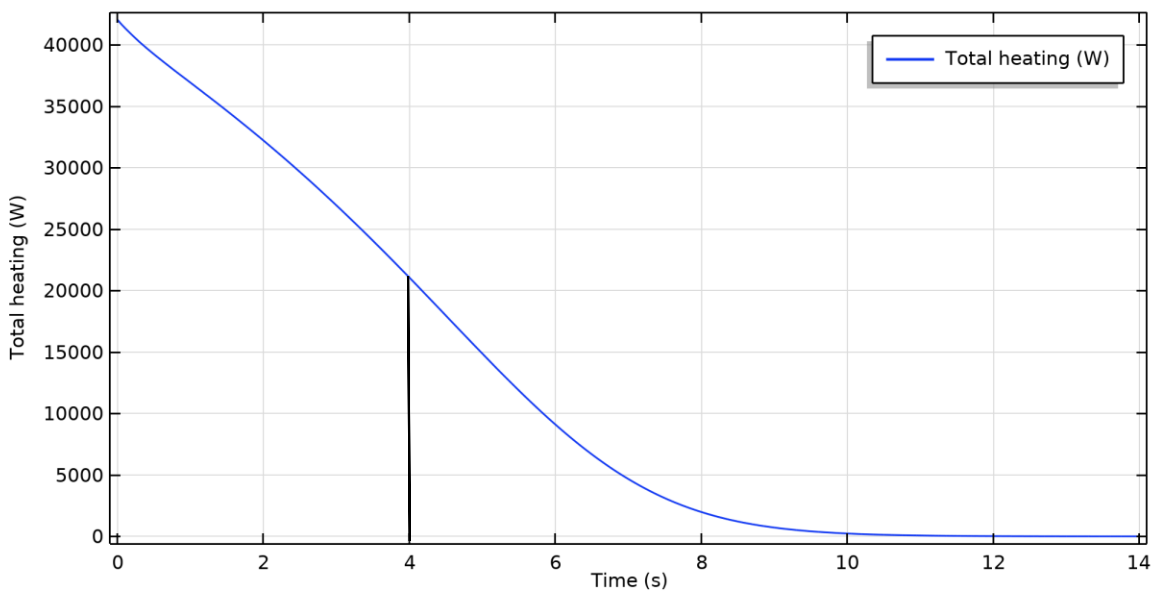


Figure 11: Heat produced curve

To make thing simple and to ensure that temperature does not increases out of maximum service temperature of our material, let us ignore convection cooling because of its complexity and dependence on surrounding environment of our system. By ignoring convection cooling, we will use the heating produced data from the above COMSOL plot. From above plot, we can find Heat Energy by noting down the Heat Load at 0 s to 4 s in the disc. We can note that the heat load decreases almost linearly. The value of Heat Load at the start and end of braking period is:

Time (s)	**rpm**	**Heat Load (W)**
0	990	42052
4	495	21058

Table 4: Value of Heat Load

The heat energy produced inside the disc during this time period will be equal to the area under the above plot in this time. The shape can be assumed to be a trapezoid. From formula of area of trapezoid.

$$Q = \frac{1}{2}(42052 + 21058) * 4 = 126220\ \mathrm{J}$$

Now the mass of disc is found from density and volume of disc. From the **heat capacity** value of Al-6061, the temperature increase is calculated by the following equation [31]:

$$Q = \rho * V * c * \Delta T$$

Material	Heating from COMSOL	Density of Aluminium	Outer diameter of disc	Inner diameter of disc	Thickness of disc
	J	kg/m^3	mm	mm	**mm**
Al-6061	126220	2700	300	64	**3**

C_p	Volume of disc	Mass of disc	Temperature increase
J/kg.K	m^3	**kg**	**°C**
897	0.00020240653	**0.546497635**	**257.48**

Table 5: Temperature Increase.

In real life application, the actual temperature increase of the disc due to this heating will be less than the above calculated value because of the convection cooling of disc by air flowing in the surroundings due to moving vehicle. Now the temperature increase comes out to be around 258^{o}C. Assuming an initial temperature of 30^{o}C, the final temperature without considering convection cooling of disc comes out to be around 288^{o}C. This resulting temperature is below the **maximum serviceable temperature of Al-6061 alloy which is around 350^{o}C**. [32] Thus the system is expected to be thermally stable for **disc thickness of 3 mm.**

3.4.4. Selection of electromagnet parameters:

The next component after the conductor disc is the electromagnet. This is the component that will produce the required magnetic field. Different parameters that determine the performance of electromagnet is number of turns of wire, current through the wire and the axial length of electromagnet. Keeping all the other parameters of Eddy Current Braking system design as constant, the value of braking torque depends upon electromagnet parameter that can be defined as **Magnetomotive Force (mmf)**. Magnetomotive force is the product of number of turns of wire and the current flowing through the wires. The equation is as follow:

$$\text{Magnetomotive Force (Amp} - \text{turns)} = \text{N (turns)} \times \text{I (A)}$$

Now if we increase the current, we can have less turns and thus smaller coils. But this will require more powerful current source. The control parameter here is the physical size of coils that is determined by number of turns. Thus we will go for the maximum size of coils that will fit and that will also reduce our current requirement. To find the required magnetomotive force, we will do a parametric analysis. After that be deciding maximum size of coil, the current required will be determined. But if we increase number of turns too much, this will increase our coil size thus our system will become heavier. Also by increasing the number of turns, the resistance of coil will increase and for our constant input voltage, the current through the coil will decrease. Thus a balance is to be found between these parametric values.

3.5. CAD Modeling :

After torque calculations we start the CAD Modeling of our magnetic braking system using SOLIDWORKS. This geometry consists of a solid brake disc of certain thickness, magnetic core made of solid magnetic material and a circular domain for coils on the magnetic core.

3.5.1. Initial model:

We started with a simple design of magnetic braking system which consisted of one permanent magnet and one disc. After that design with electromagnets is made because we are going to use electromagnets instead of permanent magnets. The simple initial design is shown below:

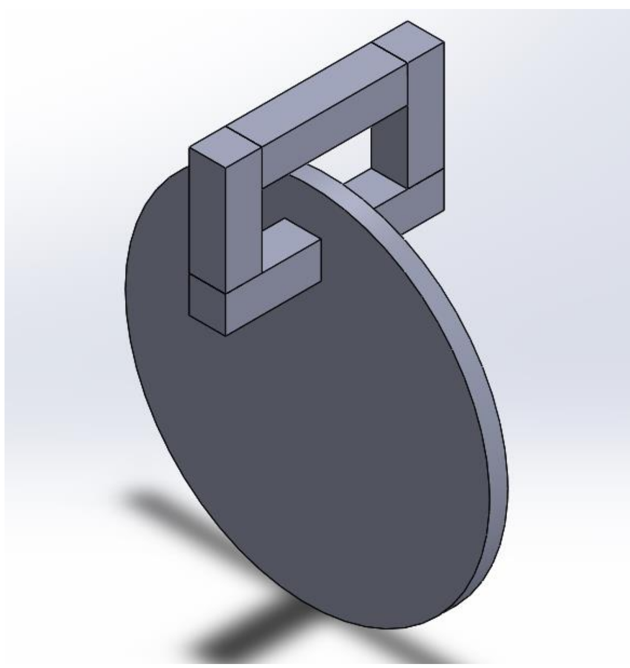


Figure 12: Starting model.

This model with permanent magnet was used to start Torque analysis later in simulation software. But this design produces very little output torque thus needs to be improved.

3.5.2. Improved designs:

After the torque analysis of starting design, the torque produced was found very low. Therefore new designs with improved performance and having multiple electromagnets were analyzed. Some of these designs are shown below:

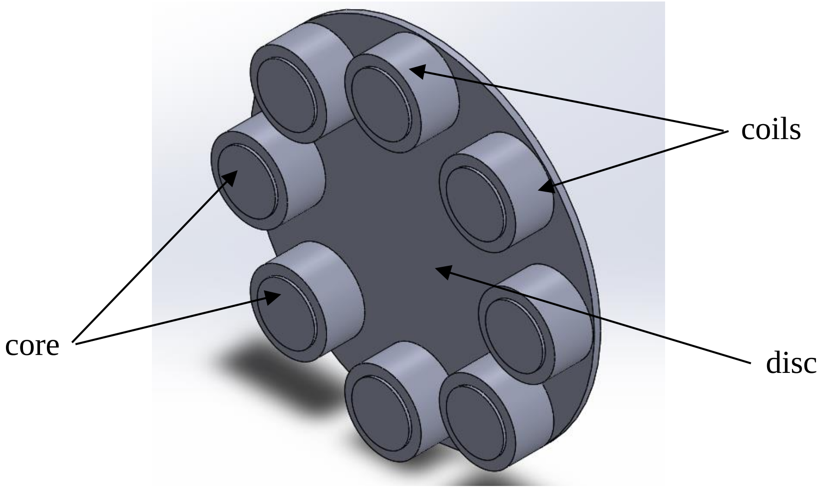


Figure 13: Model with electromagnets.

Another design with connected core is shown below:

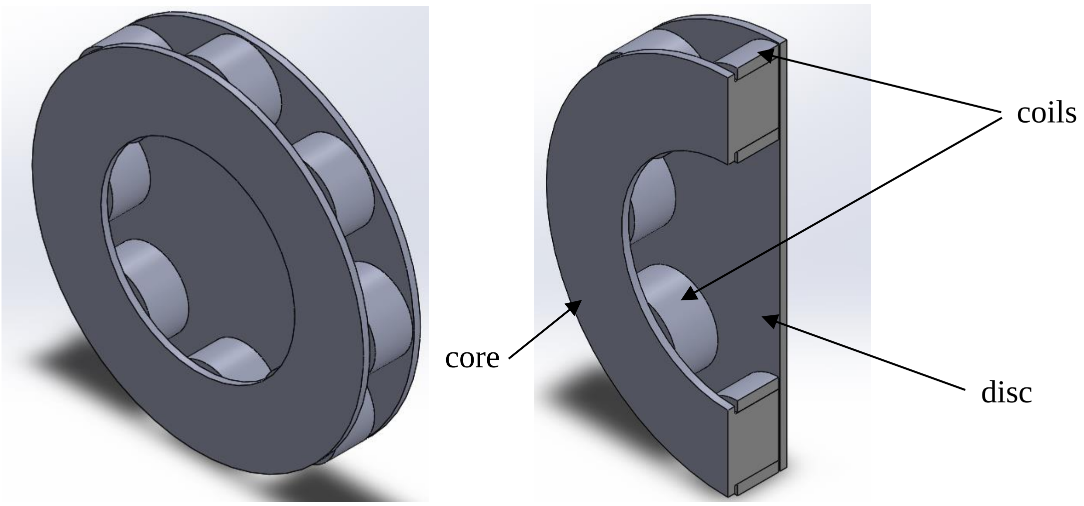


Figure 14: Model with connected core sectioned view.

3.5.3. Final design:

After the torque analysis of different designs with different geometries, the final design that was producing required braking torque and also satisfied all the constraints like thermal constraints and space constraints for our application was found. The components of our final design are as follow:

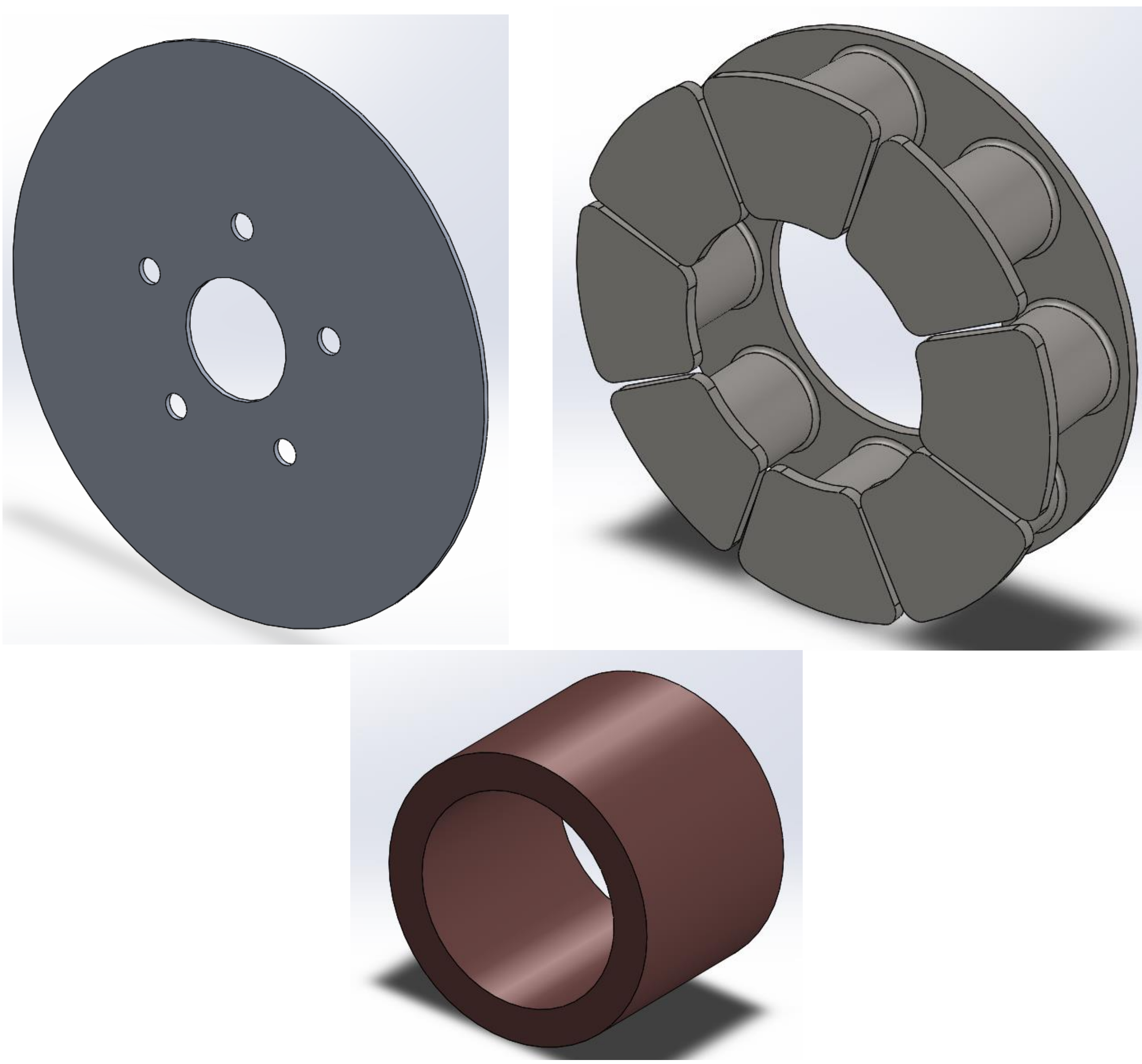


Figure 15: Components of final design: Disc, Core and Coil domains.

CAD model of final geometry containing disc, cores and coil domains that will be imported to COMSOL Multiphysics is shown below:

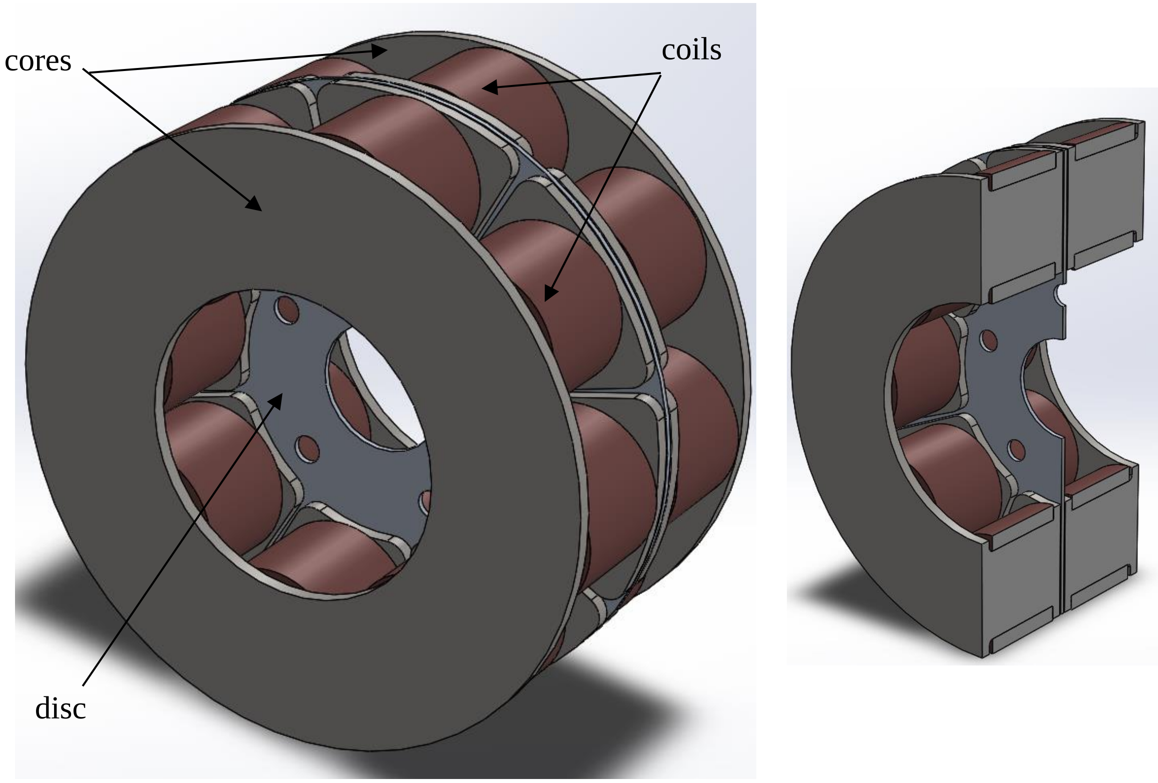


Figure 16: Final Model with sectioned view.

This final design as shown above consists of one solid disc with holes for shaft and flanges, 16 total coil domains around core poles and two sets of connected cores having 8 coils on one connected core with extended pole faces on either faces of disc. A sphere of a considerable diameter was also created around this geometry that will be assigned the air domain around our model. This sphere was made in COMSOL Multiphysics rather than SOLIDWORKS. This is necessary for analyzing the magnetic field produced by the electromagnets as the magnetic field will be in this air domain. Also the air gap between disc face and magnet pole faces comprises of this domain.

3.6. Torque Analysis in COMSOL Multiphysics:

This final design consisted of one solid disc with holes for shaft and flanges, 16 coil domains and two set of connected cores all inside an air domain. An introduction of COMSOL Multiphysics is given below

3.6.1. Introduction to COMSOL Multiphysics:

COMSOL Multiphysics is a finite element analysis, solver and Multiphysics simulation software. It has conventional physics-based user interface and a coupled systems of partial differential equations. Therefore, for the torque analysis of Magnetic Braking system, COMSOL Multiphysics software is used. The reason of using this software is that Eddy Current Brakes can be simulated easily in it by taking help from a simple built in example. Other parameters like torque, heating and deceleration can be calculated using integrations. A system of differential equations will be used to observe dynamics of system. COMSOL Multiphysics provides a user interface which is more user friendly than its counterparts and it also requires less computational power. Another important feature of COMSOL Multiphysics is the control available over meshing. Also it has built in unit conversion that automatically converts all values into SI units.

3.6.2. Lorentz force in AC/DC Module:

In the AC/DC module of COMSOL Multiphysics, Lorentz force that produces the braking force for our Eddy Current braking system can be found. The 3D model of braking system is solved using a stationary formulation for the electromagnetic part. Ordinary Differential Equations (ODE) are coupled with it for the calculations regarding rotational rigid body dynamics of brake disc. Magnetic vector potential A and electric scalar potential V formulation is used. Integrations are used to compute angular velocity, the torque produced by Lorentz force and the amount of heating occurring in disc.

3.6.3. Equations and Variables:

The angular velocity **ω,** Axial torque **T** and total heating **Q** is found by doing integration of following equations over the disc domain:

$$\omega = \omega * t - \int_{disc} \frac{T}{I}\, dt$$

$$T = \int_{disc} r \times (\text{Lorentz force})\ \ dV$$

$$Q = \int_{disc} (\text{Heating})\ dV$$

Here ω is the initial angular velocity of disc. Velocity is calculated in the node **Velocity (Lorentz terms)** in physics. **I is the moment of inertia** of the disc. This is also calculated in COMSOL Multiphysics by using a built in variable. **V is the volume** of disc. The software calculates it itself just like moment of inertia. These equations are added in the dorm of variables and software evaluates it and stores it in a table.

3.6.4. Integrations in COMSOL Multiphysics:

Axial torque **T** and total heating **Q** variables are put in **Variable section** in **Definition node.** This node can be found in **Model Builder** window. The following expressions are added using in built variable names:

Name	Expression	Unit	Description
axialTorque	-intdisc(x*mef.FLtzy-y*mef.FLtzx)	N·m	Axial torque
totalHeating	intdisc(mef.Qh)	W	Total heating

Figure 17: Torque and heating expressions.

COMSOL Multiphysics itself assigns SI units to the resulting variable.

3.6.5. Geometry:

As stated earlier, the geometry is made in SOLIDWORKS and then imported in COMSOL Multiphysics. A **sphere of 60cm diameter** is made around the imported geometry to act as air domain. The imported geometry is shown below:

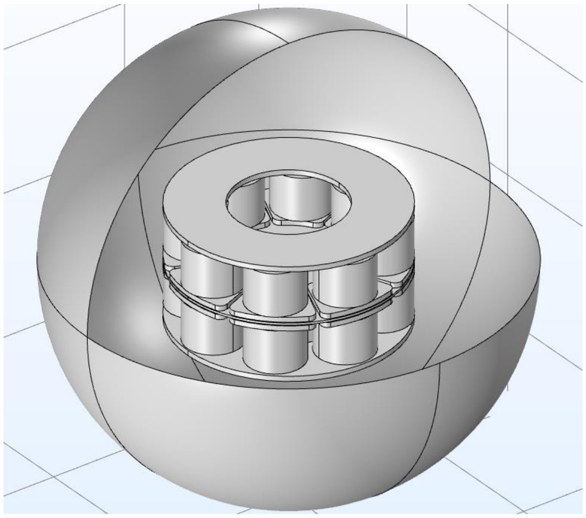


Figure 18 : Geometry in COMSOL Multiphysics.

3.6.6. Materials:

Air is assigned to surrounding domain. Aluminium 6061 is assigned to disc. Soft iron with permeability of 4000 to cores and copper is assigned to coil domains.

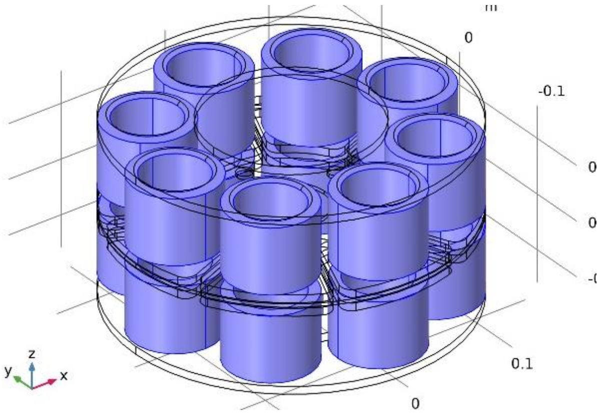


Figure 19: Coil domains.

3.6.7. Meshing:

In COMSOL Multiphysics, we have considerable control over meshing of different domains. Mesh should be coarser at places where we don not need more accurate results to save computational power and should be very fine in areas of interest to capture as much information as possible. The minimum sizes of elements in various domains are reduced as much as possible in order to make the solution converge because if the mesh is too coarse, the solution will not converge below minimum error level. The generated mesh is shown below:

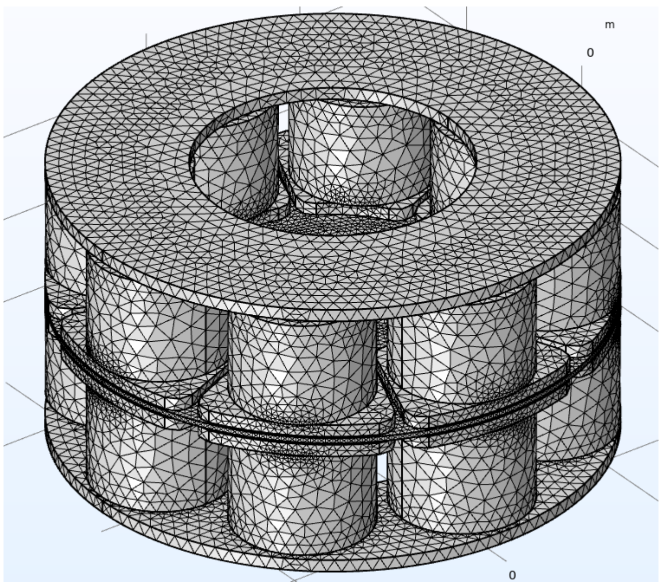


Figure 20: Mesh.

The generated mesh has the following statistics:

Parameter	Value
Total number of elements	513876
Total number of domains	20

Table 6: Mesh statistics.

3.6.8. Magnetic and Electric Field (mef) physics:

In Magnetic and Electric Field (mef) physics, Ampere's law and Current conservation is added for whole domains, 16 coil domains are added. The coils are given the follow directions to get a Halbach array type arrangement by giving alternate directions to coils to make a better magnetic field line network:

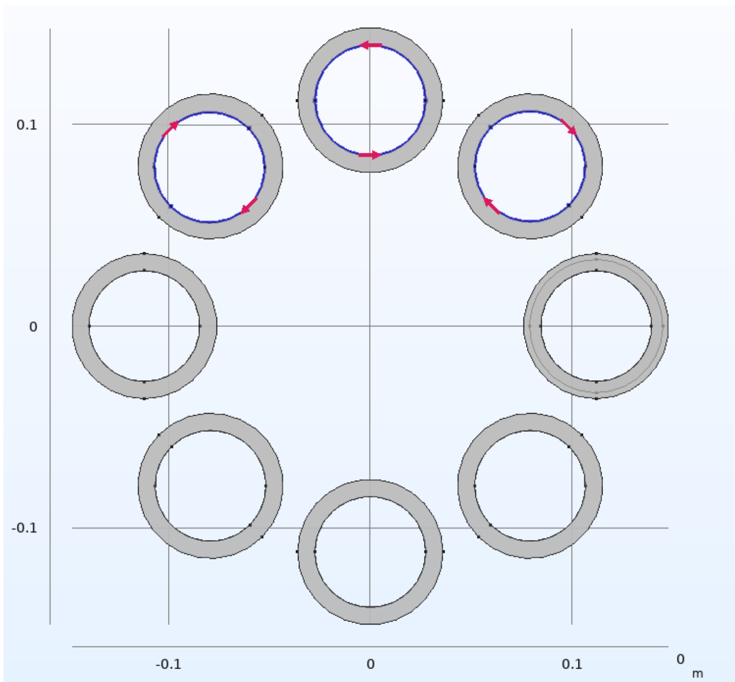


Figure 21: Coil directions.

3.6.9. Solver Studies :

Two studies are implemented for calculation. First one is stationery study which solves for Magnetic and Electric Field and the second study is a time dependent study which solves for both Magnetic and Electric Field and ODE's. The **time steps** for time dependent study are set as **range(0,0.05,14).** This means the time dependent study solves for 14 s and **time step of 0.05 s.** COMSOL Multiphysics will solve for intermediate steps for calculations but will only store data in form of a table for these specified time steps.

3.7. Coil calculations:

In COMSOL Multiphysics, we assign a domain to a coil and select homogenous multiturn conductor coil and provide number of turns and the current passing through the coil as an input. This does not take into account the actual space occupied by the coil. Therefore we need to calculate the actual size of coil separately. First of all, the diameter of wire is selected from the current needed to be passed through it. The equation is:

$$I = \frac{\pi}{4} * j * d^2$$

I is current in A**, j** is the current density in A/mm^2, **d** is wire diameter in mm. After finding the wire diameter, we find coil outer diameter by using the following formula:

$$D_2 = \frac{N * \pi * d^2}{2 * k_c * l} + D_1$$

In this equation **D_2** is coil outer diameter in mm, **N** is the number of turns, **d** is the wire diameter, **k_c** is a constant called as packing coefficient as it takes the space covered by voids in winding and the space covered by enamel coating on wires, **l** is the axial length of coil in mm and **D_1** is the inner diameter of coil in mm. After finding the outer diameter of coil, we can see whether it fits in our design. After that we can find length of wire **L_w** used in making one coil by the following formula:

$$L_w = \pi * \frac{D_2 - D_1}{2} * N$$

After that, the resistance of this wire of one coil can be calculated by:

$$R = \rho_w * \frac{4 * L_w}{\pi * d^2}$$

Here, **R** is the resistance of coil, **ρ_w** is the resistivity of copper. By finding resistance of coil and knowing voltage of battery, the current through the coil can be found. In this way calculation regarding size of coil and integration of setup with car battery can be done.

CHAPTER 4: RESULTS AND DISCUSSIONS

After doing required braking torque calculations in Microsoft Excel, deciding various control parameter of design, 3D CAD Modeling of simple design in SOLIDWORKS and doing its torque analysis in COMSOL Multiphysics, the output torque was noted. After that, the design of magnetic braking system was gradually improved by keeping constraints in mind by implementing more electromagnets, varying current and winding to find the required magnetomotive force. Also the geometry of pole is varied to get better path of magnetic field lines through the disc and thus better torque can be generated. Then the physical size of coils was calculated to confirm that it fits on either sides of disc and do not interfere with each other. The performance and dimensions of final design are summarized as follow.

4.1. Torque and speed requirements results:

After required torque and speed calculations by using Microsoft Excel, the following results can be summarized. Also the constraints and control parameters on our system can be recalled as:

Parameter	Value
Torque required	401.93 Nm
Initial speed of wheel	103.7 rad/s
Final speed of wheel	51.85 rad/s
Disc diameter	300 mm
Disc thickness	3 mm
Disc material	Al-6061

Table 7: Requirements and constraints.

4.2. Parametric analysis:

Parametric analysis was done in COMSOL Multiphysics to understand the variation of braking torque by changing various parameters of Eddy Current Brakes. The results of this parametric analysis helped in deciding number of electromagnets in our system and get an idea of how much Magnetomotive force is needed to produce required amount of torque. This analysis is shown below.

4.2.1. Design for analysis:

The following design is used for the parametric analysis of Eddy Current Brakes. This design has one Aluminium disc with 300mm diameter, 5mm thickness. The air gap is 1mm. The design has 16 coils with alternate windings and two sets of connected cores.

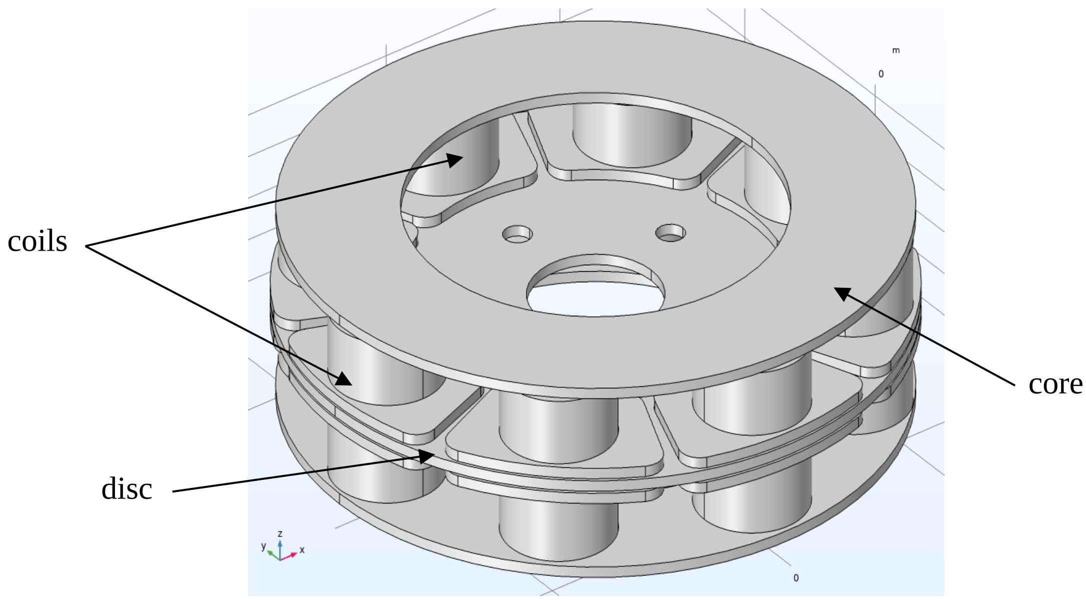


Figure 22: Design for Parametric Analysis.

The following parametric analysis are done to see the variation of Braking Torque:

- T vs number of windings
- T vs Current supplied
- T vs number of Electromagnets
- T vs Disc thickness

4.2.2. Torque vs number of windings:

The value of constant parameters for this analysis are as follow:

Parameter	Value
Current	8 A
Disc thickness	5 mm
Disc diameter	300 mm
Disc Material	Al

Table 8: Constant parameters for T vs windings.

The results are shown in form of a graph to see the variation.

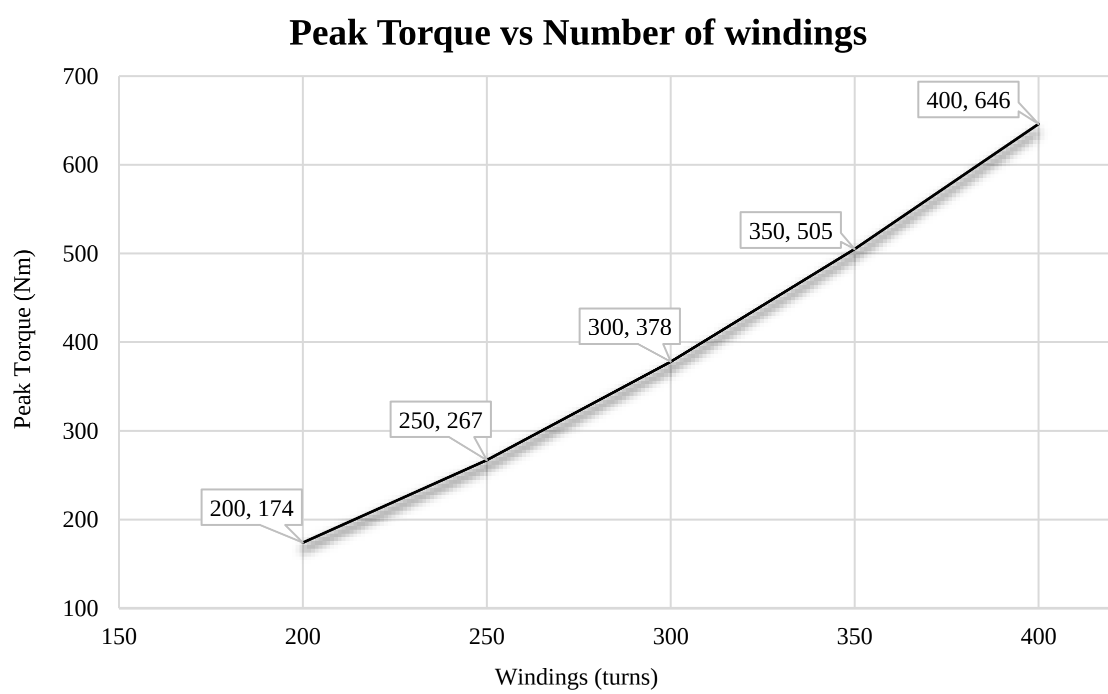


Figure 23: Torque vs Number of windings.

The trend of Braking torque variation can be seen from graph that **increasing the number of windings increases the Peak Torque value**. The **critical speed did not change**. The value of critical velocity remained around 35 rad/s.

4.2.3. Torque vs Current supplied:

The value of constant parameters for this analysis are as follow:

Parameter	Value
Windings	300
Disc thickness	5 mm
Disc diameter	300 mm
Disc Material	Al

Table 9: Constant parameters for T vs current.

The results are shown in form of a graph to see the variation.

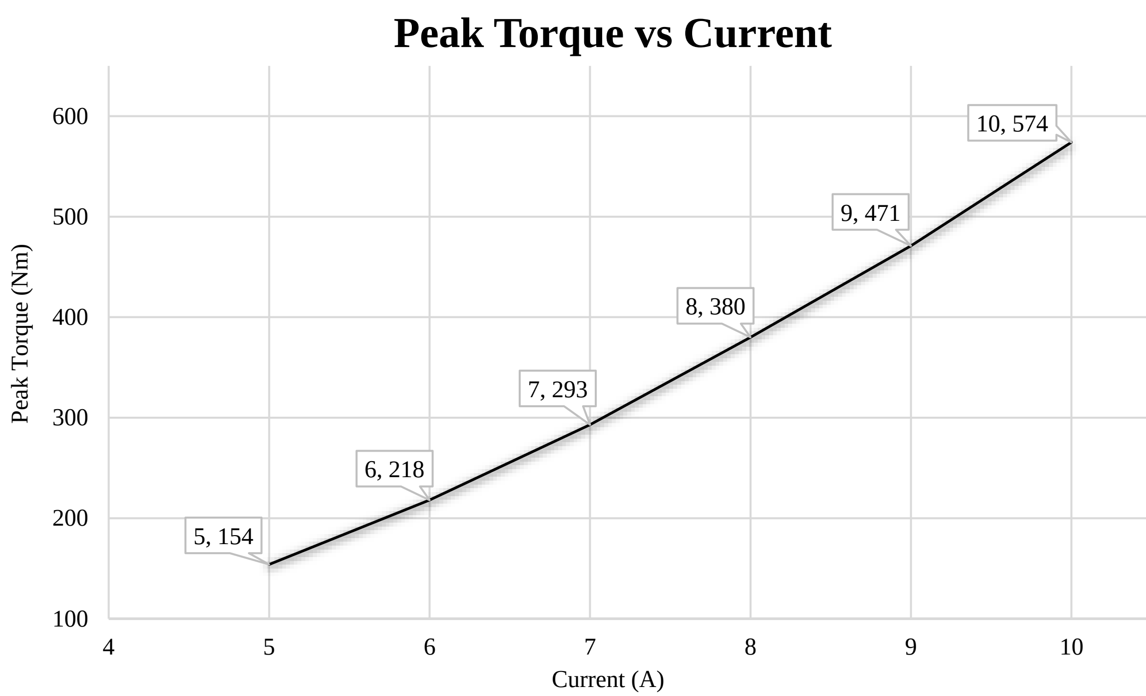


Figure 24: Torque vs current.

The trend of Braking torque variation can be seen from graph that **increasing the current in the windings of electromagnet increases the Peak Torque value**. The **critical speed did not change.** The value of critical velocity remained around 35 rad/s.

4.2.4. Torque vs number of Electromagnets:

The value of constant parameters for this analysis are as follow:

Parameter	Value
Windings	300
Current	8 A
Disc thickness	5 mm
Disc diameter	300 mm
Disc Material	Al

Table 10: Constant parameters for T vs number of Electromagnets.

The results are shown in form of a graph to see the variation.

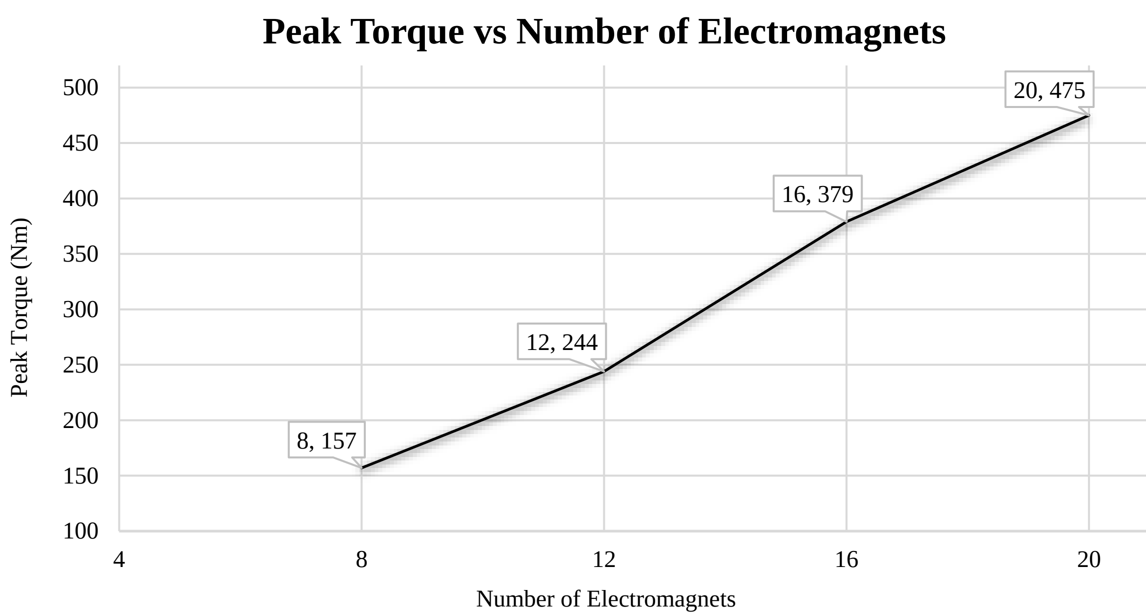


Figure 25: Torque vs number of Electromagnets.

The trend of Braking torque variation can be seen from graph that **increasing the number of Electromagnets increases the Peak Torque value**. The **critical speed did not change.** The value of critical velocity remained around 35 rad/s

4.2.5. Torque vs Disc thickness:

The value of constant parameters for this analysis are as follow:

Parameter	Value
Windings	300
Current	8 A
Disc diameter	300 mm
Disc Material	Al

Table 11: Constant parameters for T vs Disc thickness.

The results are shown in form of a graph to see the variation.

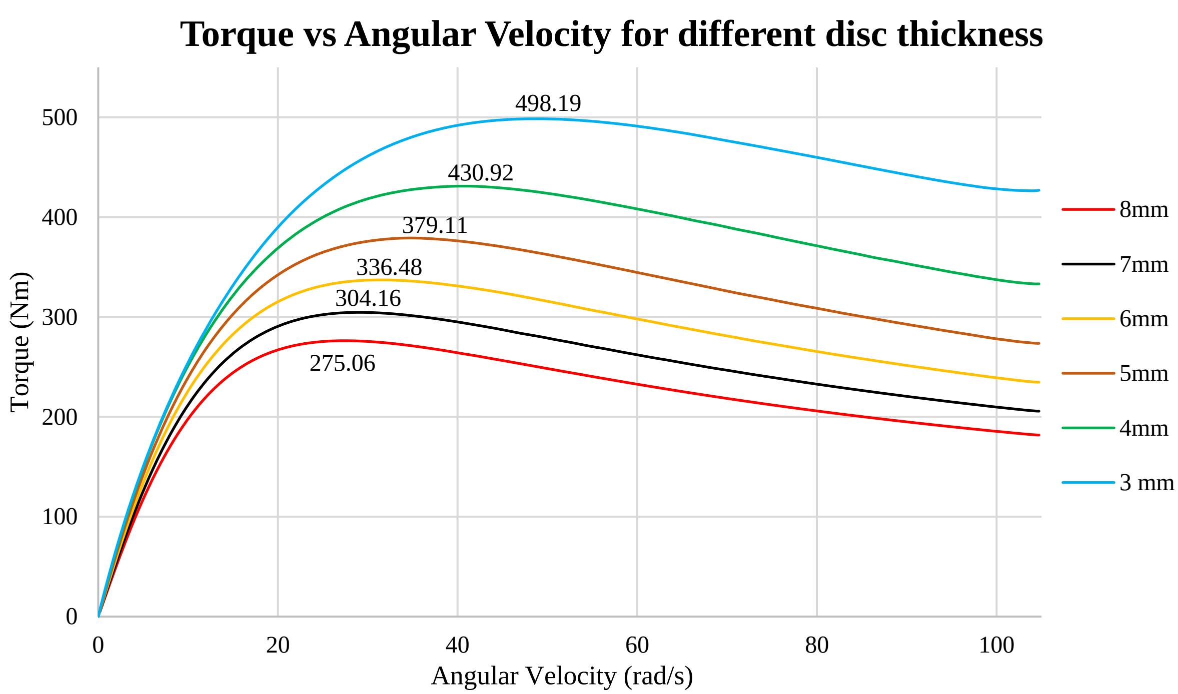


Figure 26: Torque vs Disc thickness.

The trend of Braking torque variation can be seen from graph that decreasing the disc thickness increases the Peak Torque value. The **critical speed changes.** The value of **critical speed decreases as the thickness of the disc increases.**

4.3. Final design dimensions :

The final design of our magnetic braking system that will give us the required braking torque is found. The dimensions of its various components are as follow. All the dimensions are in **mm**.

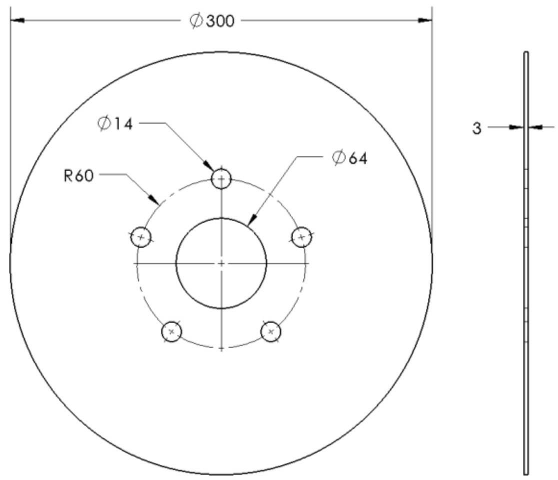


Figure 27: Disc dimensions.

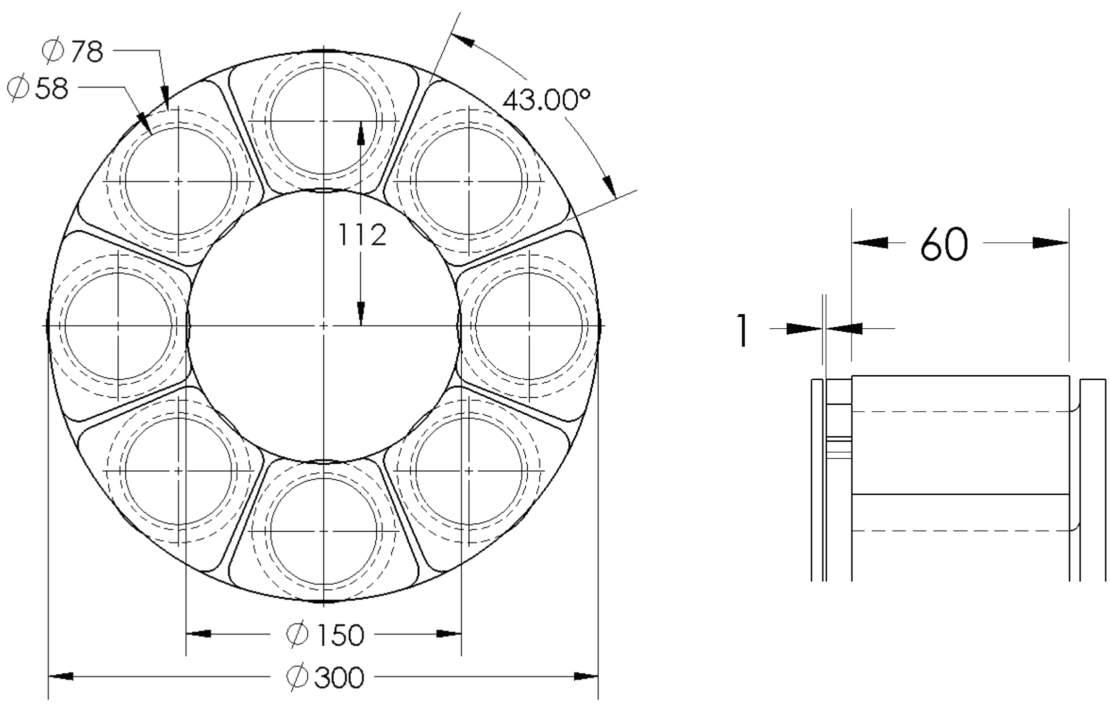


Figure 28: Electromagnet dimensions and air gap.

4.4. Output Torque results:

From COMSOL Multiphysics torque analysis, the plots of braking torque produced by the system, the deceleration of disc and heating produced inside the disc of our system are obtained. These plots are as follow:

4.4.1. Torque vs angular speed :

The torque vs angular speed graph of an Eddy Current Brake is considered as a characteristic curve to judge its performance. We can see how much torque can the system produce at a specific rotational speed of conductor disc. The Torque vs angular speed curve for our system as produced by COMSOL Multiphysics is as follow:

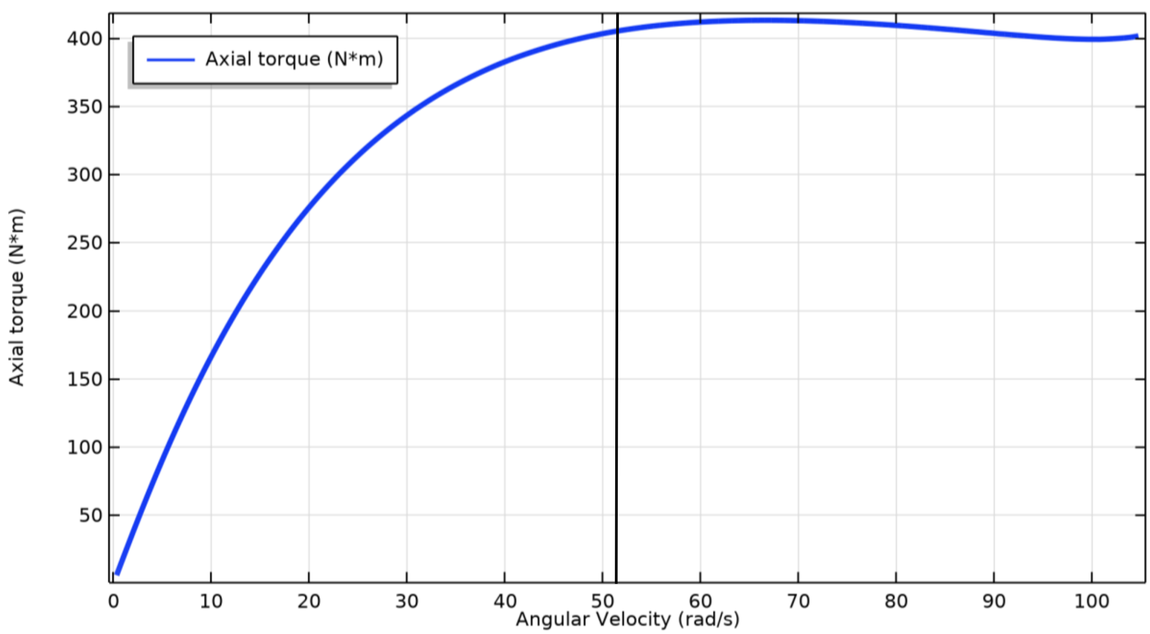


Figure 29: Torque curve of our system.

From this torque vs angular velocity curve of our eddy current braking system, we can see that in our required angular velocity range of **104 rad/s to 52 rad/s the torque produced by our system is above 400 Nm**. Thus in that velocity range, our system will produce the required deceleration.

4.4.2. Torque vs Time curve:

The deceleration of conductor disc can also be plotted in COMSOL Multiphysics. From this we can see that in first 4 s, the Torque is above 400 Nm.

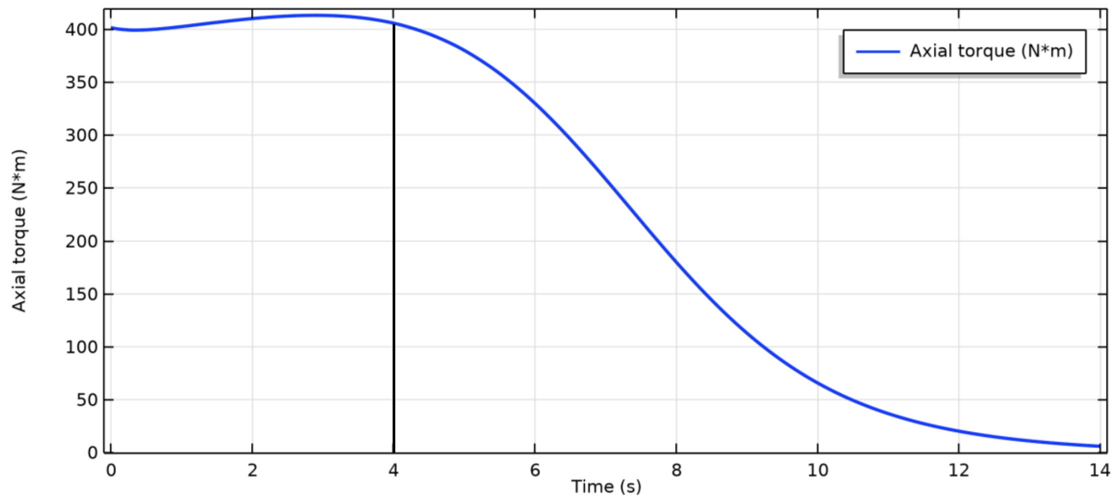


Figure 30: Torque vs Time curve

4.4.3. Deceleration plot:

The deceleration of conductor disc can also be plotted in COMSOL Multiphysics. From this we can see that disc slows from 104 rad/s to 52 rad/s in 4 s.

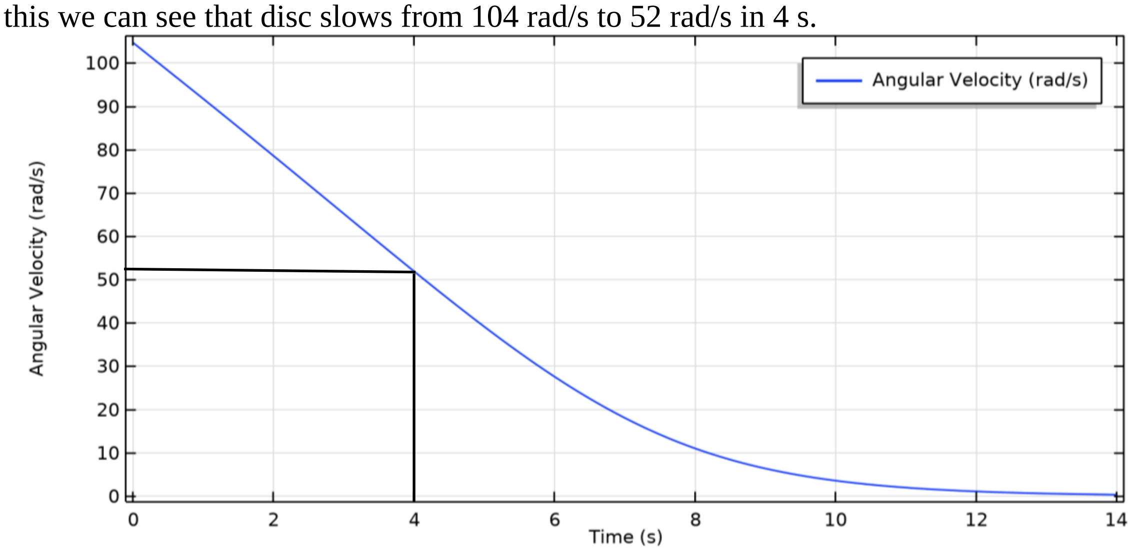


Figure 31: Deceleration curve of our system.

4.4.4. Heating produced plot:

The heating produced in the conductor disc can also be plotted in COMSOL Multiphysics. From this we calculated the temperature rise.

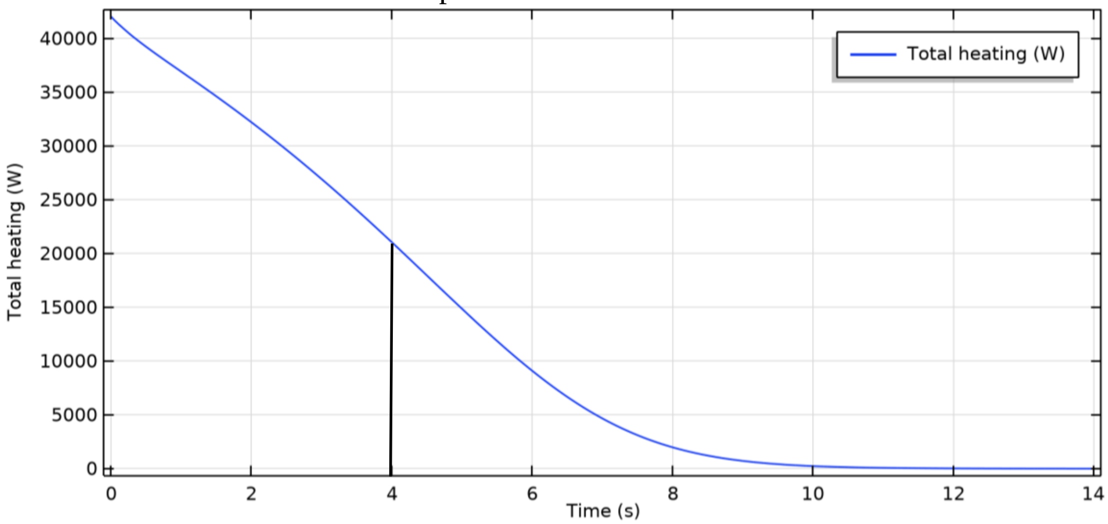


Figure 32: Heating curve of our system.

4.5. Magnetic Field Contours:

In order to keep a check on Magnetic saturation inside the two set of connected cores containing coils, the Magnetic Field contours are plotted at a surface on the cross section of core. This surface was added in COMSOL Multiphysics geometry node by first making a work plane and then drawing a circle on one of cores which is inside coil domains.

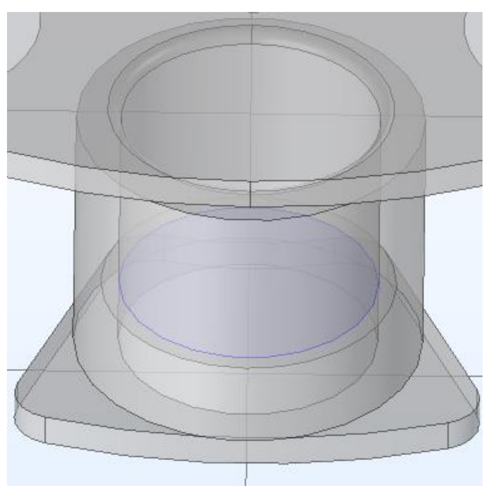


Figure 33: Core Cross-section Surface

The magnetic flux density in tesla (T) at start and end of braking period is shown below:

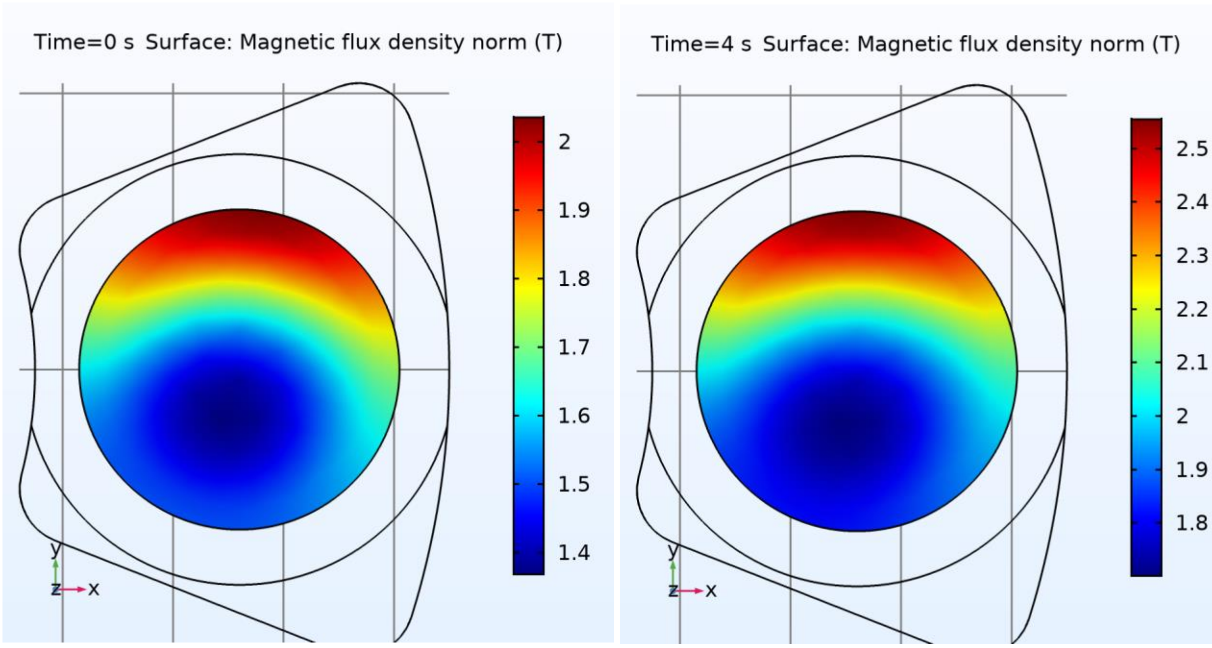


Figure 34: Magnetic Flux Density at start and end

If we observe from the above contours, the maximum value of the Magnetic Flux density (T) at start of braking period inside core is around **2 T** at one corner of core's cross section. This produced Magnetic Flux density **B** is giving us Braking Torque of around 400 Nm. This is the saturation limit for most high permeability iron alloys used in transformers [37]. Now as the braking continues, the eddy current density inside conductor disc shift more towards core mid and due to **fringing** effect at the sharp corners between cylindrical core and core shoe faces, there is very high value of B there. In real life, this will not occur but because of how the Finite Element Method works, by dividing domains in very small items and doing calculations at the resulting nodes. Now when it counters a sharp corner, there is an abrupt change in direction, due to this the calculations give a very high value of Magnetic flux density. Therefore we get this effect where the numerical value at sharp corner is very high. As it does not occur in real life, we will ignore this fringing effect. If we ignore this effect, there will be less value of B on the corner of core and it will be quite similar to the one at the beginning. Also as the Braking Torque at this point is a little above

400 Nm, in reality we will get around 400 Nm as this effect will not occur there. The Magnetic flux density at disc surface as well is shown below :

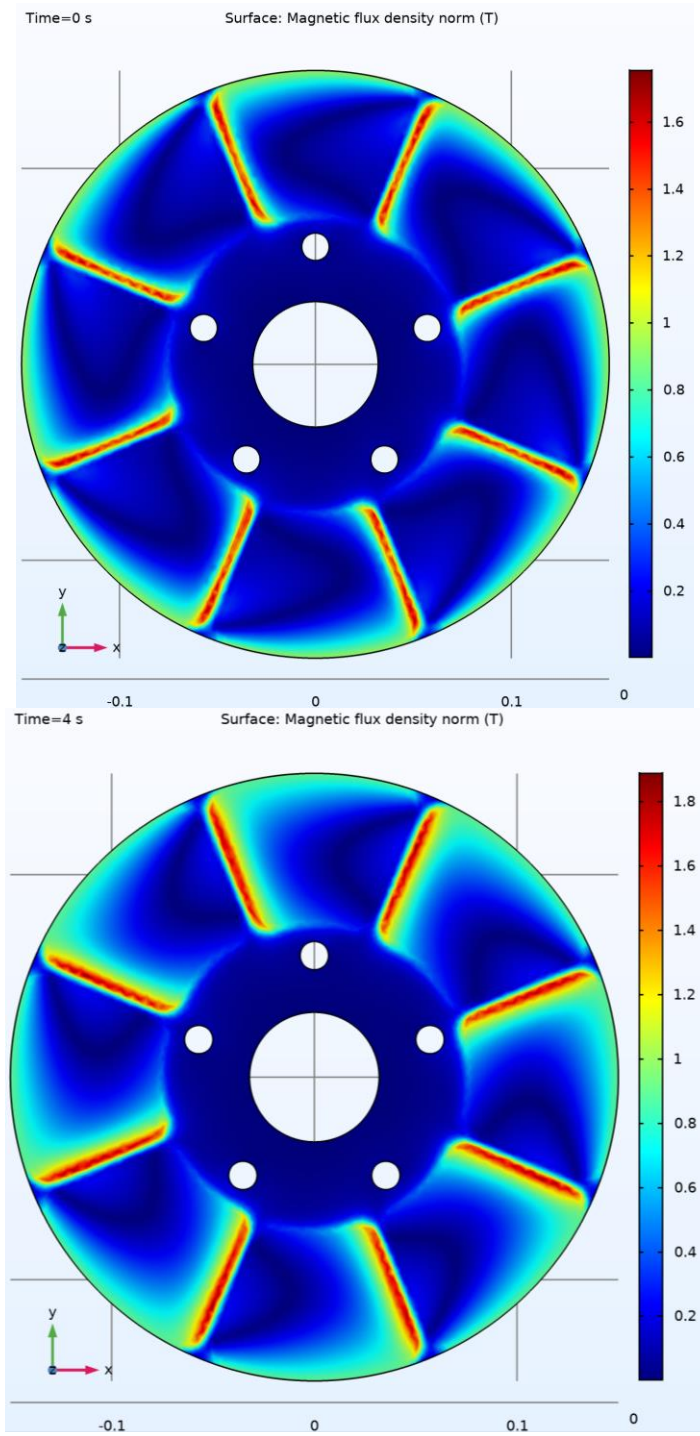


Figure 35: Magnetic Flux Density on disc surface at start and end

4.6. Coil calculations results:

From torque analysis, the **Magnetomotive Force (mmf)** for one coil that will produce this output torque comes out to be **1920 Amp-turns**. After designing Eddy Current braking system to meet the required deceleration for our system, the physical size of the electromagnet coils was calculated because as stated earlier, in COMSOL Multiphysics we only assign a domain to a coil and specify the number of turns and current through the wire. Thus the calculations of size were done with the following results.

The current density **j** for copper wires is around 3-8 A/mm^2. For coils of our extent is selected to be **6 A/mm^2**. From that for 6A current, are of wire comes out to be 1 mm^2. That gives wire of 1.13 mm. We select **wire diameter of 1.219 mm** which corresponds to wire gauge of **SWG 18.** Further calculations done using Microsoft Excel are as follow:

Parameter	Number of Turns	Wire diameter	Packing coefficient	Area of cross section of coil
unit	**N**	**d**	**k_c**	**A**
		mm		**mm^2**
Value	320	**1.219**	0.65	574.5580506

Coil inner diameter	Axial length of coils	Coil outer Diameter	Mean diameter of coil	Length of wire
D_1	**l**	**D_2**	**D_m**	**L_w**
mm	**mm**	**mm**	**mm**	**mm**
58	60	**77.15**	67.58	67935

Resistance of wire	Voltage of battery	Number of coils in series	Current through one coil	Total Coils in Setup	Total Setup Current
R	**V**		**I**		**I_s**
ohm	**volt**		**A**		**A**
0.9953847	12	2	**6.03**	16	**48.22**

Table 12: Coil results.

4.7. Coil connections:

For the integration of these coils with the car battery, the following arrangements of coil windings is proposed. This arrangement is proposed based on the calculated resistances of the coils. As the value of resistance of one coil comes out to be around 1 ohm, and as from above calculations, two coils in series will give a resistance of 2 ohm. When these coil set is connected to the battery, 6 A current will flow through the coils. An illustration of connections of 8 coils that are wrapped on one connected core on one side of disc will be like as shown in the picture below. The other 8 coils on the other side of disc will be in a similar way:

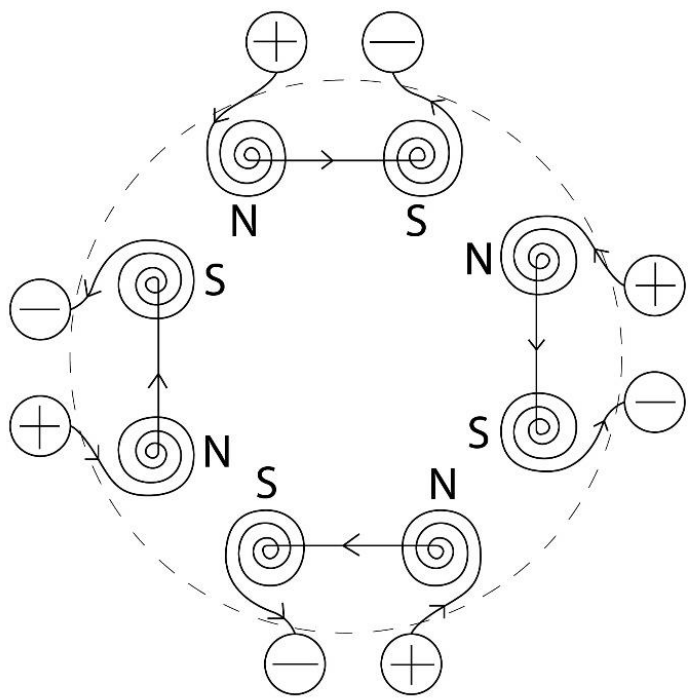


Figure 36: Coil Connections.

In COMSOL Multiphysics, the above feature of alternate coil directions in added by specifying coil directions in form of a circular path. In this way we can specify current flow direction through coil as it dictates the polarity of our coil. The directions of some adjacent coils is shown in Figure. 19.

4.8. Final design parameters:

After parametric analysis by COMSOL Multiphysics Torque Analysis, thermal considerations, coil size calculations, coil electrical calculations and defining coil connection, final design parameters are summarized in following table:

	Parameter	Value
System Parameters	Disc Diameter	300 mm
	Disc Material	Al-6061
	Number of Electromagnets	16
	Air gap	1 mm
	Setup Current	48.22 A
Coil parameters	Number of windings	320
	Resistance of one coil	0.9953847 Ω
	Number of coils in series	2
	Current through Coils	6 A
	Voltage of source	12 V
	Wire Diameter	1.219 mm
	Wire Gauge	SWG 18
	Wire Length of one Coil	67935 mm
	Coil Inner Diameter	58 mm
	Coil Outer Diameter	78 mm
	Coil Axial Length	60 mm

Table 13: Final design parameters

CHAPTER 5: CONCLUSION AND RECOMMENDATION

5.1. Analysis result conclusions:

As we saw from the results obtained from COMSOL Multiphysics Torque Analysis of our design, the proposed design is theoretically capable of producing the required torque in the required angular velocity range. But the real world performance of the system cannot be found out as no prototyping and testing of Magnetic Braking system is done and no comparison is done with any kind of benchmark.

5.2. Recommendations:

The following recommendations and future scope of work on Magnetic Braking system are proposed to work further in this field:

5.2.1. Thermal Analysis:

In our design, as mentioned in above chapters, we considered a rough overestimate of the temperature increase of conductor disc due to heating of eddy currents, but as we know that in real life application, there will be air blowing around the system and over the conductor disc. Also the heat produced in the disc will be more in front of magnetic core shoe faces and spread in other areas of disc by conduction and to other parts around it like magnetic core and coils by convection and radiation heat transfer. In order to simulate real world heat transfer from the brake disc and then study changes in behavior of these components due to temperature rise, we will need flow conditions around it. By knowing the speed and direction of wind traveling around our system and using the amount of heat produced data from COMSOL Multiphysics, we can do extended thermal analysis of our system and find the temperature increase and also the regions of disc where there is more temperature rise. Also the convection cooling and decreasing of temperature can be observed by this analysis. This can be done in any FEA software. By importing the same final geometry of our system in that software and providing the surrounding conditions in the form of boundary conditions, the heat transfer can be simulated. This can provide better understanding about the thermal behavior of system.

5.2.2. Prototyping:

Due to various circumstances and limitations, the prototyping of our Magnetic Braking System could not be done. Prototyping of the system will gives us several advantages most important being the comparison between theoretical results and experimentally obtained results. The required information about manufacturing of prototype like the dimensions of various parts and the their materials have been specified in the report that will be helpful in prototyping.

5.2.3. Test bench:

After prototyping, another recommendation is of making a test bench to gauge the real world performance of developed Magnetic Braking System prototype. This will help us compare our Torque Analysis results we got from COMSOL Multiphysics with actual output of our system and find out the error in results.

There can be **two type of test bench setups** for magnetic braking system prototype. **First** one will measure **braking torque produced by the braking system at a certain rpm** of conductor disc. This bench will consist of a driving motor with controller to control its rpm, coupling to connect conductor disc shaft with motor, then the magnetic braking system will be mounted on a separate shaft which will have a stopper arm and with this stopper arm, a single point load cell will be attached. The rpm of driving motor will be varied and the torque produced by braking system at that rpm will be measured by multiplying the load indicated by load cell with the length of stopper arm. No clutches and inertial flywheels will be needed in this setup. Other sensors like current and voltage meters for excitation coil can be implemented. A speed sensor to measure rpm of shaft can also be used. A non contact temperature sensor can also be used to observe the actual temperature rise in the disc. The required rating of motor is presented in next design, the recommended rating of single point load cell will depend upon length of arm attached with braking system. A schematic diagram of this test bench is shown as follow:

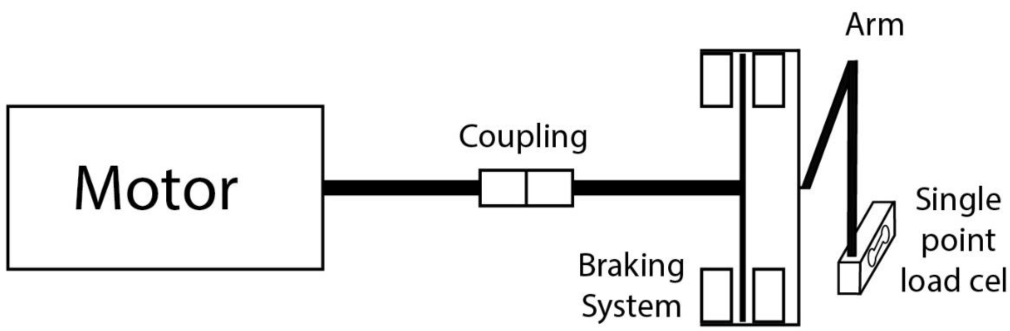


Figure 37: Test bench first recommendation

A **second type** of test bench will use inertial flywheels. It will consist of a driving motor, one or more inertia flywheels that can simulate the inertia of quarter car model, there will be clutches that will engage and disengage these inertia flywheels with the shaft of our Magnetic Braking system. And to measure the torque being produced, a rotary non contact sensor will be used on the shaft. This sensor will also provide the information about rpm of the shaft. Similar to previous test bench design, various other sensors like current and voltage meters for excitation coil, temperature sensors for disc and other components and digital data acquisition can be implemented. The required rating for motor is recommended with an approximate safety factor of around 1.5 times. The inertia value was calculated and can also be found from COMSOL Multiphysics. An approximate values of the test bench components rating is presented as follow:

Parameter	**Recommended Value**
Moment of inertia	31 kgm^2
Motor power	60 kW
Rotary Torque Sensor rating	500 Nm

Table 14: Test Bench Parameters

A schematic diagram of this test bench is shown as follow:

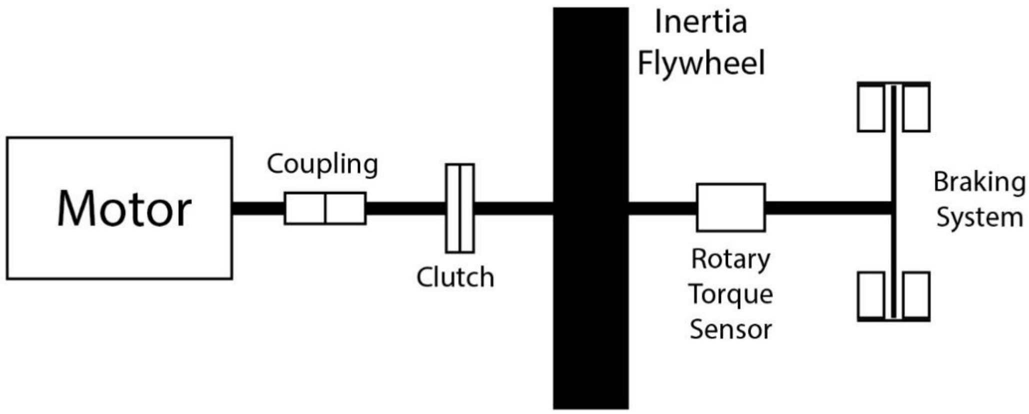


Figure 38: Test bench second recommendation

In this test bench, first the motor will be used o drive inertial flywheel to the required speed of 990 rpm. The clutch will remain engaged and the coils will not be excited during this time. After the required speed is achieved, the clutch will be disengaged and the current supply to the coils will be turned on. The braking torque will slow the inertial flywheel down and this will simulate the deceleration of one front wheel of the car according to the calculations. The deceleration time will be noted, the power consumed by Magnetic Braking system will be noted. The temperature increase will also be noted during the braking process. The inertia wheel can be one wheel or can also be multiple wheels on the same shaft to simulate multiple scenarios.

REFRENCES

[1] Sevvel P, Nirmal Kannan V, Mars Mukesh S (2014) “Innovative Electro Magnetic Braking System”, IJIRSET, Volume 3, Special Issue 2, April 2014, ISSN: 2319 – 8753.

[2] Senthil Kumar.L, Dinesh Kumar.V, Gokula Krishnan.P, Gokulan.M, Jagadesh.V, (2018), “Design and Fabrication of Automatic Eddy Current Braking”, IJCRT, Volume 6, Issue 2 April 2018. ISSN: 2320-2882.

[3] Smit Patel, Anand Patel, Meet Patel, Chetan Sanghani, (2015), “Development of the Electro-Magnetic Brake”, IJIRST, Volume 1 | Issue 12 | May 2015, ISSN (online): 2349-6010.

[4] Huang, S., Bao, J., Ge, S., Yin, Y., & Liu, T. (2020) “Design of a frictional–electromagnetic compound disk brake for automotives ” Proceedings of the IMechE, Part D: Journal of Automobile Engineering, 234(4), 1113 1122.

[5] Shi Jun, a, He Ren,b, Gu Xiaodan (2013) “Study and Design of Integrated System of Electromagnetic and Frictional Brake of Car”, Advanced Materials Research Vols. 779-780 (2013) pp 739-746. doi:10.4028/www.scientific.net/AMR.779-780.739

[6] C. Y. LIU, K. J. JIANG and Y. ZHANG (2011), “Design and use of an Eddy Current Retarder in an Automobile”, International Journal of Automotive Technology, Vol. 12, No. 4, pp. 611−616 (2011) app

[7] Ma, Der-Ming & Shiau, Jaw-Kuen. (2011). “The design of eddy-current magnet brakes.” Transactions of the Canadian Society for Mechanical Engineering. 35. 10.1139/tcsme-2011-0002.

[8] Oscar Rodrigues, Omkar Taskar, Shrutika Sawardekar, Henderson Clemente, Girish Dalvi (2016), “Design & Fabrication of Eddy Current Braking System”, IRJET, Volume: 03 Issue: 04, Apr-2016, e-ISSN: 2395 -0056.

[9] Pratyush Kumar Routh, Prem Sagar, Prince Kumar, Rajesh Kumar, Sarnendu Paul (2019), “Electromagnetic Braking System”, IRJET Volume: 06 Issue: 05 May 2019 e-ISSN:2395-0056

[10] Nyemba, Wilson R. & Pote, Chris & Chikuku, Tauyanashe & Mbohwa, Charles. (2017). “Conceptualization of a New Product Development Framework for Eddy Current Braking Systems for Heavy Vehicles in Zimbabwe”, (IEOM) Bristol, UK, July 24-25, 2017.

[11] Putra, Mufti R.A. Nizam, Muhammad; Tjahjana, Dominicus D.D.P Aziz, Muhammad; Prabowo, Aditya R. (2020) "Application of Multiple Unipolar Axial Eddy Current Brakes for Lightweight Electric Vehicle Braking" Appl. Sci. 10, no. 13: 4659. https://doi.org/10.3390/app10134659.

[12] Zhi-ding Ying, Xin-fu Xu, Jian-an Zhu, (2010). “Analysis of simulation design of the disc eddy current braking device” 2010 International Conference on Computer, Mechatronics, Control and Electronic Engineering (CMCE).

[13] MRA Putra, M. Nizam, DDDP. Tjahjana, HT Waloyo (2019) “The Effect of Air Gap on Braking Performance of Eddy Current Brakes on Electric Vehicle Braking System” (ICEVT) ,November 18-21, 2019, Bali, Indonesia.

[14] Zhijun Long, Changyou LI, Bin Huang, Weiwei Wen, (2009), “Simulation Research on Braking Torque of Eddy Current Retarder”, 2009 First International Workshop on Education Technology and Computer Science.

[15] Rhythm Dhoot, Sanket Gaikar, Nitish Kulkarni , Ojus Jain, (2016), “Design & Theoretical Study of Electromagnetic Braking System”, IOSR-JMCE, e-ISSN: 2278-1684, Volume 13, Issue 6 Ver. VI (Nov. - Dec. 2016), PP 87-96.

[16] V. Gyaneshwar, Bino Shaji, N. Ramanaryanan, S. Vishnu Chandar, D. Devika, K. Sathya Narayanan, (2020), “Material Selection and Optimisation of an Eddy Current Braking System with Regeneration”, Materials Today: Proceedings, Volume 24, Part 2, 2020, Pages 1608-1617, ISSN 2214-7853, https://doi.org/10.1016/j.matpr.2020.04.482.

[17] Ming Qian, Pushkin Kachroo, “Modeling and Control of Electromagnetic Brakes for Enhanced Braking Capabilities for Automated Highway Systems”.

[18] Waloyo, H. T., Ubaidillah, U., Tjahjana, D. D. D. P., Nizam, M., & Aziz, M. (2020). “A Novel Approach on the Unipolar Axial Type Eddy Current Brake Model Considering the Skin Effect.” Energies, 13(7), 1561. doi:10.3390/en13071561

[19] Tirta Nahari, Endra Joelianto, Suyatman (2012) “An Eddy Brakes Dynamometer Control System Design using State Space Based PID Controller”, 2012 IEEE Conference

on Control, Systems and Industrial Informatics (ICCSII) Bandung, Indonesia, September 23-26,2012

[20] J.D.Edwards, B.V.Jayawant, W.R.C.Dawson and D.T.Wright (1999) "Permanent-magnet linear eddy-current brake with a non-magnetic reaction plate", IEE Proceedings online no. 19990574, DOI 10.1049/ipepa:19990574

[21] Baoquan Kou , Yinxi Jin, Lu Zhang and He Zhang (2015) "Characteristic Analysis and Control of a Hybrid Excitation Linear Eddy Current Brake", Energies 2015, 8, 7441-7464; doi:10.3390/en8077441

[22] Weight transfer – Wikipedia . Retrieved from: https://en.wikipedia.org/wiki/Weight_transfer

[23] Drum Brakes – Wikipedia . Retrieved from: https://en.wikipedia.org/wiki/Drum_brake

[24] Disc Brakes – Wikipedia . Retrieved from: https://en.wikipedia.org/wiki/Disc_brake

[25] Electromagnet – Wikipedia . Retrieved from: https://en.wikipedia.org/wiki/Electromagnet

[26] Honda Civic Oriel 1.8 i-VTEC CVT Specifications. Retrieved from: https://www.pakwheels.com/new-cars/honda/civic/oriel-1-8-i-vtec-cvt/specifications/

[27] Vehicle Specifications 2020 Honda Civic Sedan. Retrieved from: https://owners.honda.com/vehicles/information/2020/Civic-Sedan/specs#mid^FC2E6LEW

[28] Copper. Retrieved from: https://en.wikipedia.org/wiki/Copper

[29] Aluminium. Retrieved from: https://en.wikipedia.org/wiki/Aluminium

[30] Aluminium Alloy. Retrieved from: https://en.wikipedia.org/wiki/Aluminium_alloy

[31] Specific heat capacity of materials. Retrieved from: https://theengineeringmindset.com/specific-heat-capacity-of-materials/

[32] Aluminum Workshop. Retrieved from: https://www.thefabricator.com/thewelder/article/aluminumwelding/aluminum-workshop-how-hot-is-too-hot-for-aluminum-

[33] Automotive battery. Retrieved from: https://en.wikipedia.org/wiki/Automotive_battery

[34] He Ren1, Gu XiaoDan, Shi Jun (2014) “Design of double-disc friction and electromagnetic hybrid brake system of passenger car”, Applied Mechanics and Materials Vol 610 (2014) pp 156-163; doi: 10.4028/www.scientific.net/AMM.610.156

[35] M. O. Gulbahce, D. A. Kocabas and I. Habir, (2012) "Finite elements analysis of a small power eddy current brake," Proceedings of 15th International Conference MECHATRONIKA, 2012, pp. 1-5.

[36] Cho, S., Lee, J., Kang, D., & Lee, H. (2017). Calculation of the Eddy Current Loss and Braking Characteristic Analysis of the Eddy Current Brake.

[37] Saturation (magnetic). Retrieved from: https://en.wikipedia.org/wiki/Saturation_%28magnetic%29#cite_note-4

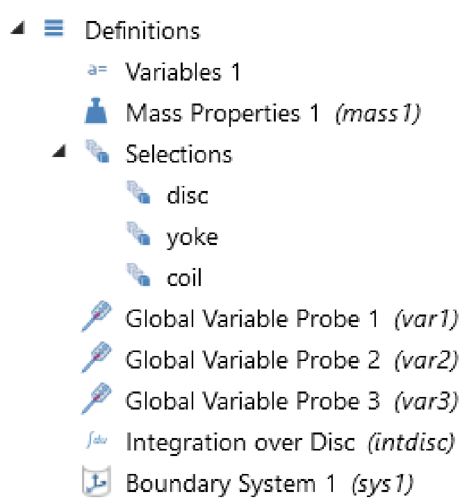
APPENDIX I: COMSOL SETTINGS

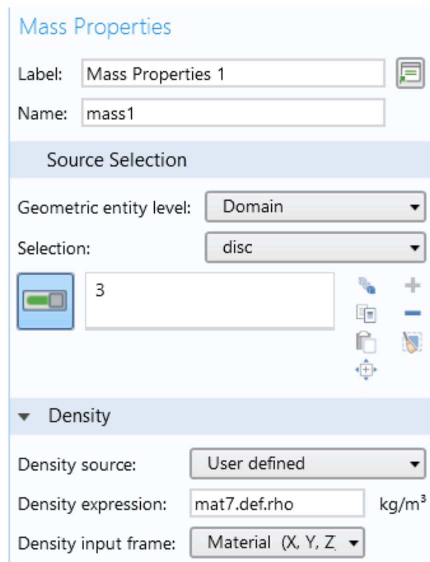
Various settings used in Torque Analysis in COMSOL Multiphysics:

COMSOL parameters:

ymur	4000	4000	Core relative permeability
dV0	990[rpm]	16.5 1/s	Disc initial angular frequency
turn	320	320	number of windings
current	6[A]	6 A	current

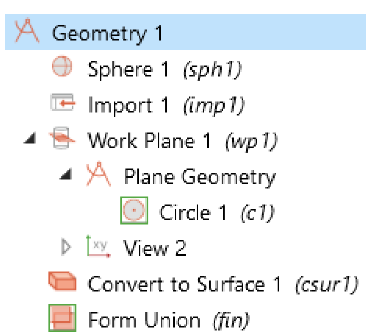
Definitions and mass properties:

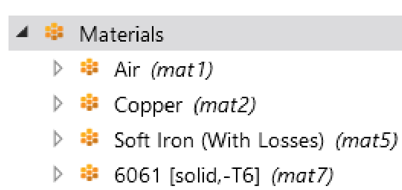




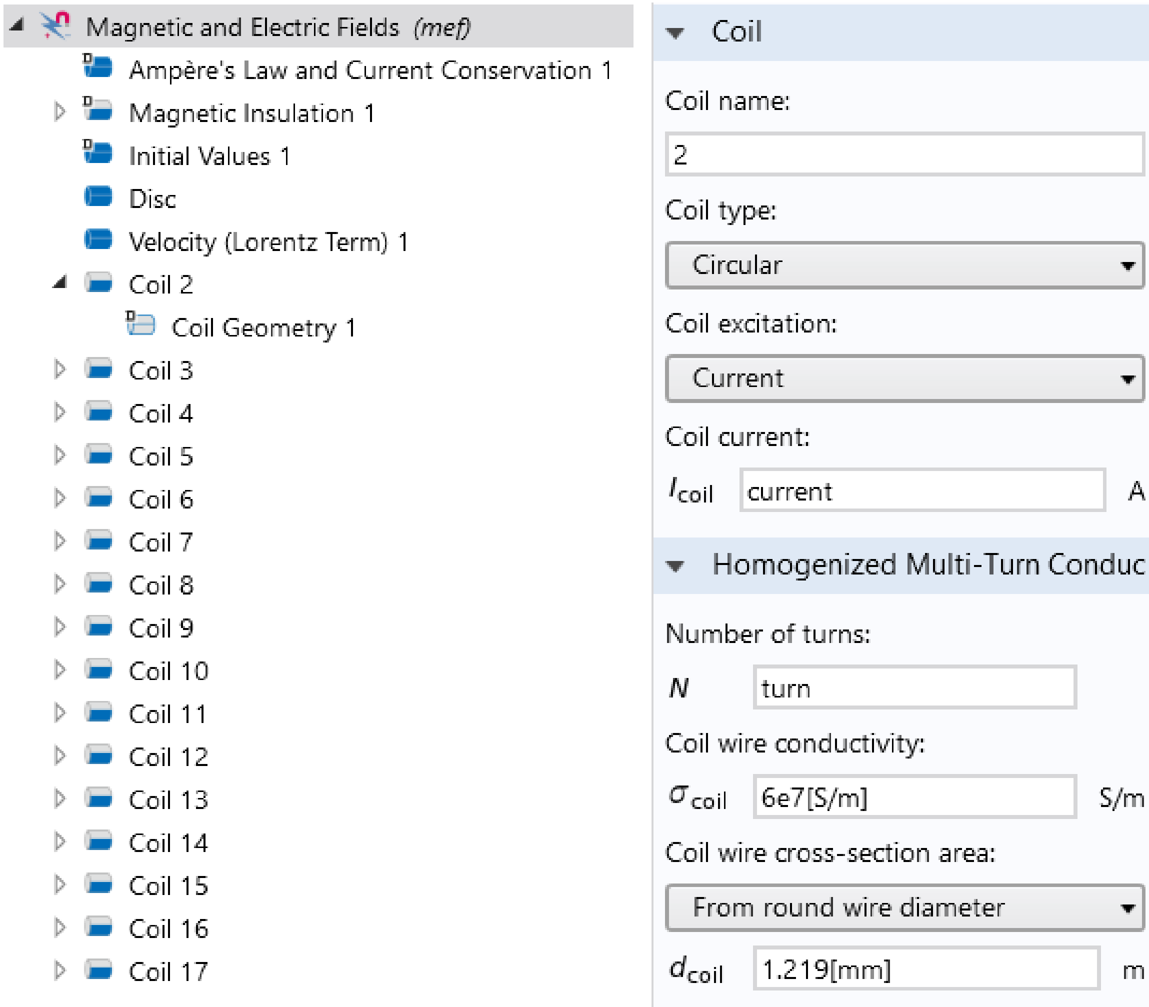
COMSOL Geometry and Materials:

Built in materials were used except the value of Magnetic permeability of soft irn was provided as shown in parameters:

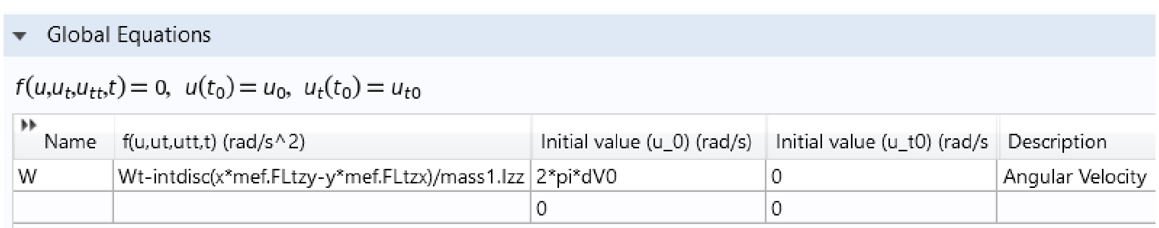




COMSOL Physics Settings and coil parameters:



ODE’s in COMSOL for angular velocity:



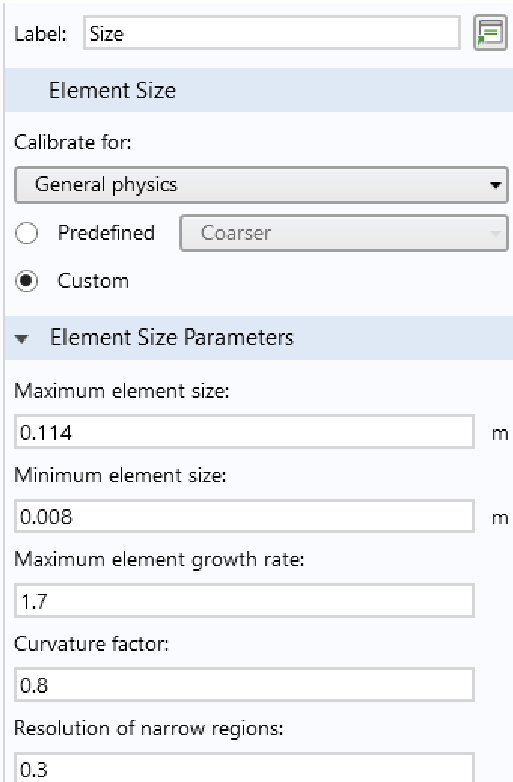
Global Equations

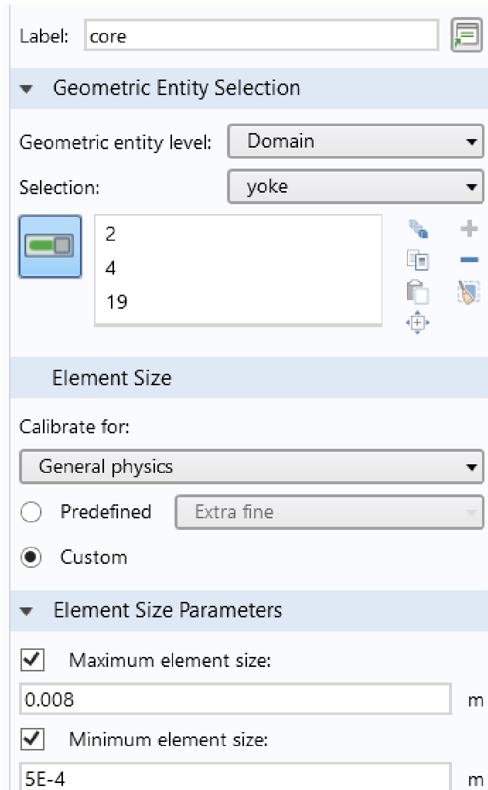
$f(u,u_t,u_{tt},t)=0,\ \ u(t_0)=u_0,\ \ u_t(t_0)=u_{t0}$

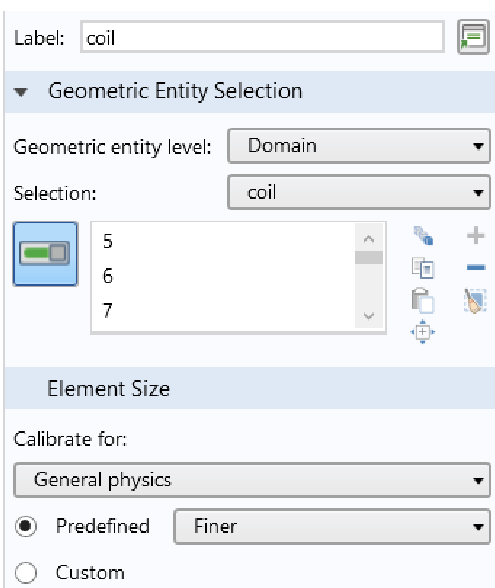
Name	f(u,ut,utt,t) (rad/s^2)	Initial value (u_0) (rad/s)	Initial value (u_t0) (rad/s	Description
W	Wt-intdisc(x*mef.FLtzy-y*mef.FLtzx)/mass1.Izz	2*pi*dV0	0	Angular Velocity
		0	0	

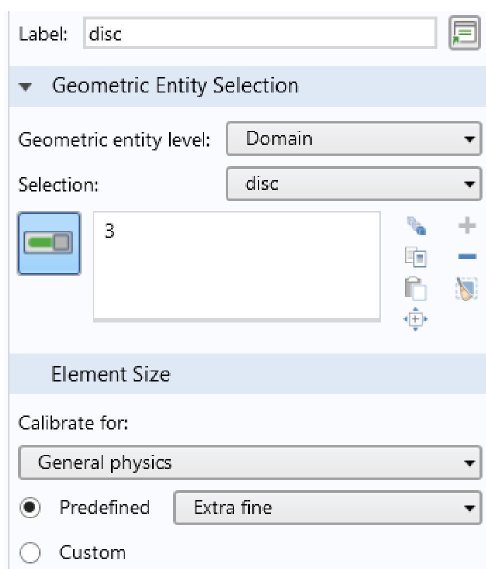
Mesh sizes:

Appropriate mesh size is selected to achieve convergence:









Study settings and solver configurations: