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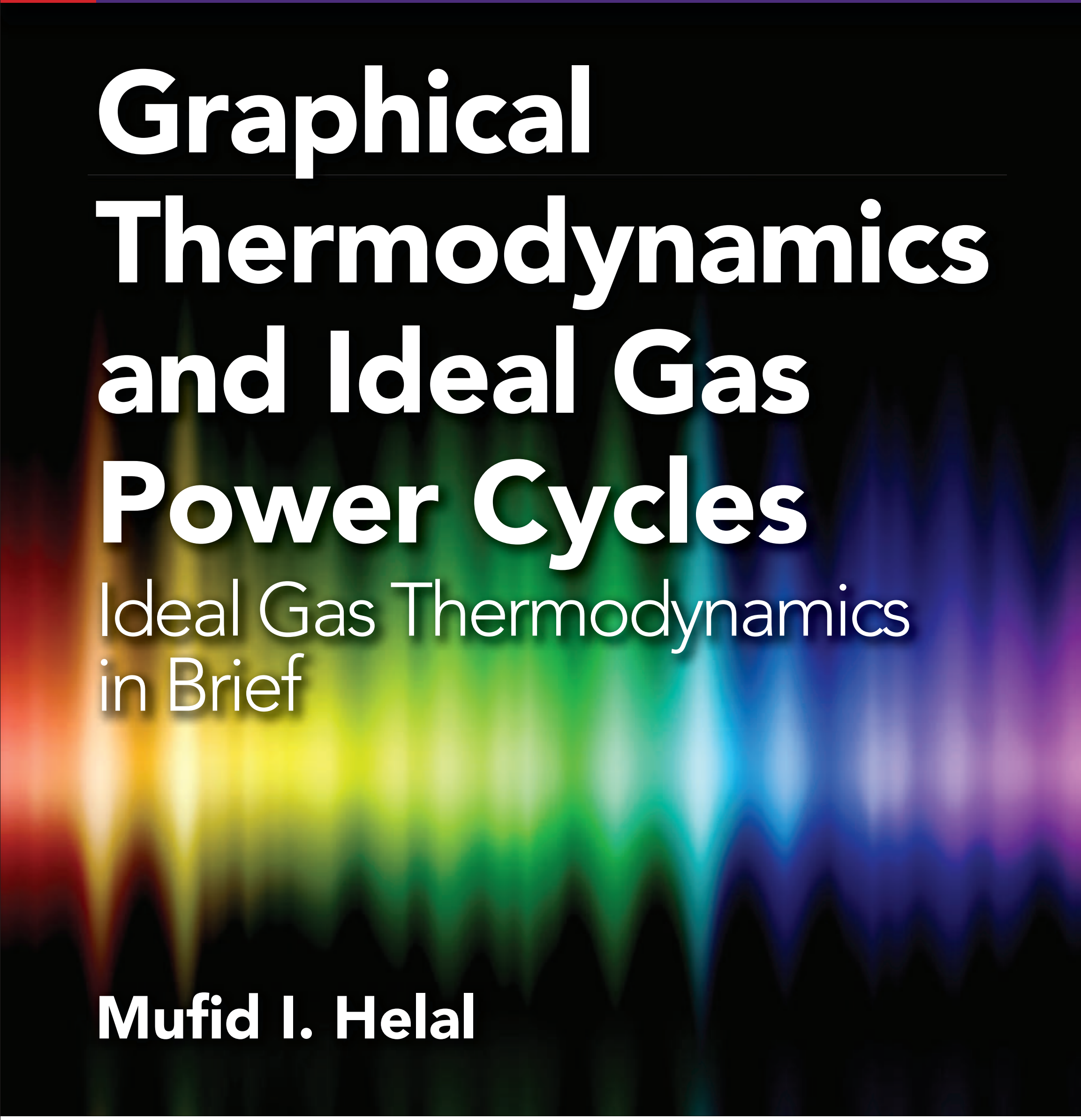

Graphical Thermodynamics and Ideal Gas Power Cycles

Ideal Gas Thermodynamics in Brief

Mufid I. Helal

MOMENTUM PRESS
ENGINEERING

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ABSTRACT

In this book, an almost new approach to modern thermodynamics has been applied. One or more useful qualitative discussion statements have been extracted from each equation. These and other important statements were numbered and their titles were situated in an index entitled "Helal and Others' statements, definitions and rules." This ensures very quick obtaining of the required (for discussing and solving problems) statements, rules, definitions, equations, and their theoretical base that much eases reader's qualitative discussions and calculations. Almost all ideal gas closed system thermodynamic topics are either discussed in depth or deeply abbreviated. The topics discussed in depth are either new original ones or valuable classical ones that increase reader's ability for better understanding but are overlooked or deeply abbreviated in modern thermodynamic books. In both cases, they are significantly improved. The main five new ideas that are discussed in depth in this book are: (1) The ideal gas polytropic process for $C_v = f(T)$ and its analysis (Chapter 6, Part I), (2) The theoretical realization of reversible gas state change processes (§ 5-10), (3) **Helal cycle** (§s 7-5-2), (4) **Helal graphical method** for comparing and discussing power cycles (§ 7-5-4), and (5) the imperfection in the classical proof of Carnot's efficiency (theorem) and its exclusion (Chapter 7, final section). The deeply abbreviated topics are rigorously discussed in depth in the majority of modern thermodynamic books. To dissipate any misunderstanding, the equations and statements that can be misunderstood are followed by explanatory sentences (see equation 1-38 and the paragraph following it).

KEYWORDS

absolutely reversible cycle; Carnot; Carnot's efficiency; closed thermodynamic system; cycle's ability for heat regeneration; the equivalent thermodynamic cycles; Ericsson and Dual cycles; Helal air standard cycle; Helal method; heat regeneration; ideal gas property tables; the imperfection in the classical proof of Carnot's efficiency (theorem) and its exclusion; a new, polytropic ideal gas state change process; polytropic state change process; the regeneratability condition; reversible cycle recognizing thermodynamic properties; second law of thermodynamics; Stirling cycle; the theoretical realization of reversible gas state change processes; thermal efficiency.

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PREFACE

This book *Graphical Thermodynamics and Ideal Gas Power Cycles* is based on author's book in Arabic "*Technical Thermodynamics*, Volume 1, Third Edition, Second Revision, Published in 2006 by Damascus University Press-Damascus University in Damascus-Syrian Arab Republic." We will refer to this book as the base book.

The base book discusses in depth many topics including the following five original topics in addition to a number of traditional topics that are useful for better understanding thermodynamics and that are rarely met in modern thermodynamics:

1. The analysis of the ideal gas polytropic process for $C_v = f(T)$: A new analysis making possible executing exact solutions of the polytropic process problems by using the already existing ideal gas property tables.
2. The theoretical realization of reversible gas state change processes: This topic consists of two parts: (1) the first part is **traditional**, but it was not successfully explained, and it does not exist in modern books. This part enables better understanding of concepts relating to equilibrium and reversibility. (2) The second part is new and original. It enables best understanding of concepts relating to equilibrium, reversibility, irreversibility, ability for heat regeneration, and so on.
3. **Helal graphical method** for comparing heat engine cycles. This method is more flexible and inclusive than the previous methods. It allows to examine qualitatively the influence of cycle's characteristics (r, r_p, and r_c) on the thermal efficiency of the dual cycle and enables additionally better understanding of T_s diagram.
4. **Helal cycle** (a new common case ideal gas ideal thermodynamic cycle): Five of its seven special case cycles have counterparts in reality. These are the Carnot, Otto, Brayton, Stirling, and Ericsson cycles.

5. Detecting and excluding the imperfection in the classical proof of Carnot's efficiency (theorem): The classical proof of the Carnot efficiency (theorem) is imperfect. It did not prove that the arbitrary reversible cycle used in the proof represents all ever possible engine cycles.

In the base book and especially in its new original topics, an almost new approach was applied. One or more statements, which are useful for qualitative discussions, were extracted from each equation. These statements were numbered as rules or definitions, and their titles were situated in a special index entitled "Helal definitions and rules." This ensured the quick finding of the required (for discussing and solving problems) rules, definitions, equations, and their theoretical base. The reduction in the required time and efforts to find the necessary tools for discussing and solving problems comforts the readers and eases their efforts. In this book:

1. The new approach that is partially applied in the base book is applied in the whole book. One or more statements, which are useful for qualitative discussions, were extracted from each equation. These and other important statements were numbered as rules or definitions, and their titles were situated in a special index entitled "Helal and Others' statements, definitions, and rules." This will ensure the very quick finding of the required (for discussing and solving problems) statements, rules, definitions, equations, and their theoretical base. The reduction in the required time and efforts to find the necessary tools for discussing and solving problems comforts the readers and eases their efforts. According to my experience, it will be much easier to the reader to use the rules instead of relations in qualitative discussions. Readers will benefit from the following: (1) Helal and others' statements, definitions, and rules, which will be included in the appendix, (2) the new topics, and (3) some traditional topics that are unusually explained.
2. Almost all ideal gas closed system thermodynamic topics are either discussed in depth or deeply abbreviated. (1) The topics discussed in depth are either new original ones that are based on the base book or valuable classical ones that increase reader's ability for better understanding but are overlooked or deeply abbreviated in modern thermodynamic books. And in both cases, they are significantly/gregariously improved. (2) The deeply abbreviated topics are significantly/gregariously discussed in depth in the majority of modern thermodynamic books. The following main new

ideas that were discussed in the base book are discussed here in depth: (1) the analysis of the ideal gas polytropic process for $C_v = f(T)$, (2) the theoretical realization of reversible gas state change processes, (3) Helal graphical method for comparing heat engines' cycles, (4) Helal cycle, and (5) detecting and excluding the imperfection in the classical proof of Carnot's efficiency (theorem).

3. The equation editor was used for writing equations and some other symbols. This forced the author to change some symbols by others. For example:

   - The short dash (-) does not exist in the equation editor and the trail to write (1 foot-pound force = 1 ft-lbf) gave $(1\ foot - pound\ \ force = 1\ ft - lbf)$. Therefore, the short dash that appeared as minus sign was replaced by the multiplying sign (·) and the former equation appeared as $(1\ foot \cdot pound\ \ force = 1\ ft \cdot lbf)$, which is much logical and never be wrongly understood. The ft·lbf is a perfect abbreviation for the foot-pound force because it shows (see Equation 1-25) that this unit is the product of multiplying one foot (ft) by one pound force (lbf). Thus, in this book (see Equations 1-19 and 1-25), the sign **(·)** is used as (1) a **multiplying sign** and (2) a **connection in the abbreviations of compound units' names**.
   - To uniform the abbreviations of all compound units throughout this book, the usage of (·) sign as mentioned above is also applied on SI units.

4. The following six signs are used as multiplication (×, *, ·) and division (÷, /, & –) signs:

   - Two signs (× and ·) are mostly used as multiplication signs. The first sign (×) is used only between a symbol and a digit "number" (see Equation 1-14), while the other one (·) is used between words or between symbols (see Equation 1-12). Another example is the Newton meter that is usually abbreviated as Nm. It is abbreviated here as N·m. The Newton meter is the product of multiplying one Newton by one meter (see the last side of Equation 1-25).
   - Three signs (÷, /, and –) can be used for division. The first two ones provide that each of the numerator and denominator must

be either simple or included between one or more types of brackets. No requirements are needed when using the third sign and when it is used in units no space or brackets (see Equations B and 1-37), but they usually occupy more place. (They are higher and shorter than the former ones, the thing which is absent in the former equations.)

5. To dissipate any misunderstanding, the equations and statements that can be misunderstood are followed by explanatory sentences (see Equation 1-38 and the paragraph following it).
6. If a multi-subfigures' figure is mainly used within a paragraph, we, at the beginning of the paragraph, remind that in this paragraph this figure is the main reference and then we refer to its subfigures directly.
7. It has been practiced in many references to differentiate between dimensions (or their numerical values) and their units by any type of brackets such as $T[R]=1.8\cdot T[K]$, separating by a space such as $T\ R=1.8\cdot T\ K$, or skipping the temperature symbol such as $R=1.8\cdot K$. We mostly use the separation by a space (see Equation 1-8).
8. The equations in this book are of three categories: (1) Equations to which you never need to refer to (they are unnumbered), (2) equations, which are locally (in a particular paragraph or example) used and are unexpected to be referred to in other parts of the book or outside it (their numbers do not denote to the paragraph or chapter numbers), and (3) equations to be referred to anytime. With ignoring the existence of the unnumbered and locally numbered equations, the equations of each chapter are sequentially numbered (the number of each equation of this category is composed of chapter's number followed by the sequential number of the equation within the chapter "e.g., the numbers of the equations of the first chapter of this book starting from the first numbered equations upto paragraph (1-2) are (A, B, B1, C, D and D) that belong to example (1-1) and its notes followed by equations (1-1 through 1-8, 1-8a, 1-8b, 1-8c, 1-9 through 1-17, 1-17a, 1-18 through 1-30, 1-30a, 1-31, 1-32, 1-33, (A, B, C, A', B', C', D), that belong to paragraph (1-1-11) and example (1-2). To refer to a locally numbered equation we need to denote each of its number and the number of the paragraph or example to which it belongs. For example, (see equation B of § 1-1-11).

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NOMENCLATURE

Symbol	Name	Unit
a	Acceleration	m/s^2
A	Area, the heat equivalent of work	m^2
A_r	The relative atomic mass (atomic weight)	–
B	Barometric or absolute pressure	Pa = N/m^2
C	Heat capacity, tabulated in Table 3-4 function	kJ/K
c, Mc or $\bar{c}$, c'	Specific heat, molar specific heat, volumetric specific heat	kJ/(kg K), kJ/(kmol K), kJ/(m^3 K)
c_p, Mc_p or $\bar{c}_p$, c'_p	Constant-pressure specific heats (mass, kilomolar and volumetric)	kJ/(kg K), kJ/(kmol K), kJ/(m^3 K)
c_v, Mc_v or $\bar{c}_v$, c'_v	Constant-volume specific heats (mass and kilomolar and volumetric)	kJ/(kg K), kJ/(kmol K), kJ/(m^3 K)
c_{v0} and c_{p0}	Zero-pressure (ideal-gas) constant-volume and constant-pressure specific heats	kJ/(kg K)
c_n	Polytropic-process specific heat	kJ/(kg K)
c_t	Constant-temperature specific heat	
C	The mean squared speed of the straight movement of the molecules	m/s^2
Da	Dalton—the unified atomic mass unit	
F	Force	N
G	Weight	N

Symbol	**Name**	**Unit**
g, g_n, g_k	Gravitational acceleration, normal gravitational acceleration, mass portion	m/s^2, m/s^2, ---
H, h, Mh, or $\overline{h}$	Enthalpy (total, specific, kilomolar/per kilomole)	kJ, kJ/kg, kJ/kmol
E, e, Me, or $\bar{e}$	Energy (total, specific, kilomolar/per kilomole)	J, J/(kg, J/\mol or J/m^3)
KE, ke, $\overline{ke}$	Kinetic energy (total, specific, kilomolar)	J, J/(kg, J/\mol or J/m^3)
K	The number of kilomoles of the total quantity, Kelvin	kmole^{-1}
k	Specific heat ratio (isentropic exponent) $k = (c_p / c_v)$; Boltzmann constant	No unit; N/m^2
M, m	The mass (molar, total)	**kg/kmol, kg**
M_r	The relative molar mass	-
M_u	The molar mass constant	**g/mol, kg/kmol**
m_m	The molecular mass	Da
N_A	Avogadro's number/constant	Mole^{-1}
n	Polytropic exponent; normal conditions	
p	Pressure (absolute pressure)	Pa = N/m^2
p_{abs}, p_g	Absolute pressure, gage pressure	N/m^2
p_k, p_{mix}, p_v	Partial pressure, mixture's pressure, VACUUM pressure	N/m^2
Q, q, (ǭ or $\overline{q}$)	Transferred heat (total, specific, kilomolar)	kJ, kJ/kg, and kJ/kmol
M_r (M.W. or F.W.)	The relative molar mass (molecular weight or formula weight)	
MR or $\overline{R}$ or Ru, R	Universal gas constant, gas constant	kJ/kg K, kJ/kmol K
r, r_c, r_k and r_p	The compression ratio, the compression ratio, the volumetric portion, and the pressure ratio	

Symbol	Name	Unit
S, s, Ms	Specific entropy (total, specific, kilomolar)	kJ/K, kJ/(kg K)
s_v, s_p, s°	Standard entropies (tabular entropy temperature functions)	kJ/(kg K)
T, t	absolute temperature, nonabsolute temperature	K, °C
U, u, Mu or $\bar{u}$	Internal energy (total, specific, kilomolar)	**J, J/kg**
V, v, Mv or $\bar{\upsilon}$	Volume (total, specific, and kilomolar)	m^3, m^3/kg, m^3/kmol
υ	Velocity	
W, w, Mw	Work, specific work	J, J/(kg, J/\mol or J/m^3)
W_b, w_b	Boundary work (total, specific)	J, J/(kg, J/\mol or J/m^3)
z_k	The molar portion	
δ	The inexact differential sign	
φ	Heat portion spent to change the internal energy of gas in the polytropic process	
γ	Specific weight	N/m^3
ρ	Density	kg/m^3
σ	The regeneration ratio (or the regenerator effectiveness “ε”) $\sigma = Q_R / Q_{R_{\max}} = \varepsilon$	
ω	Angular speed/**velocity**	1/s
θ, θ', $\bar{\theta}$	The lower heating value (specific per unit mass, volumetric per unit volume, kilomolar per kmole)	MJ/kg, MJ/m^3, MJ/kmole
θ_L^o, θ'^o_L, $\bar{\theta}_L^o$	The lower heating value “operative mass” (per unit mass, per unit volume, per kmole)	MJ/kg, MJ/m^3, MJ/kmole
η_t	Thermal efficiency	

SUBSCRIPTS

Avg—Average
Ct—Carnot cycle
ig—ideal gas
k—partial
mix—mixture
n—**normal**
rg—real gas
tot—total

ABBREVIATIONS

AR cycle—Absolutely reversible cycle
CP—The constant part
$\text{cycle}_{\text{DPPE}}$—Divided into pure processes exclusively
HTS—Heat transfer surface
ID or r—Intermolecular distance
IF—Intermolecular force
IPE—Intermolecular potential energy
$\text{DPPE}_{\text{cycle}}$—Divided into pure processes exclusively
KS—Known/fixed state
MC—Master/configuration curve
MKE—Molecular kinetic (thermal) energy
MR—mini heat reservoir
NPIGP—New-polytropic ideal gas state change process
NTP—Normal temperature and pressure also known as normal conditions
RC—Reversible thermodynamic cycle
RDCCs—Reversible differential Carnot cycles
SHC—The specific heat capacity
SNP—Same number points
STH—The specific transferred heat
TDTs—Temperature-dependent terms
TIT—Temperature-independent (constant) term
WF—Working fluid

CHAPTER 1

BASIC CONCEPTS AND DEFINITIONS

1.1 UNIT SYSTEMS

1.1.1 INTRODUCTION

In physics, we deal with so-called dimensions. According to Others' Definitions (ODs):

> **OD1-1:** *A dimension (physical characteristic), such as length, time, or mass, has certain measurement units that are related to each other in one way or another. The expression of a dimension must include its numerical and unit parts.* Therefore, we can say
>
> - this pencil is about 15 centimeters (cm) or 6 inches (in.) length, $L \approx 15\ cm \approx 6\ in.$, or
> - the length of this pencil in centimeters is about 15 and in inches about 6 $L_{cm} = L\,[cm] \approx 15 \quad \text{or} \quad L_{in.} = L\,[in.] \approx 6.$

The first (direct) expression is most common, but the second (indirect) expression is used especially in temperature conversions.

Some of dimensions' units (a unit for each dimension) are represented in the unit system and belong to the unit system, whereas others do not.

Note number 1-1 (Nt1-1): The expressions as used above are nonalgebraic ones because they include two equality signs each. They are equations with double equal signs. To ease referring to similar to them equations, the following terms are used in this book (1) the side of the equation for each of equation's expression that bordered by one or two equality sign(s),

(2) the multi equal-sign/side equation “abbreviation multi equation or simply equation” for the whole equation and (3) the kth side of the equation for the side N° k, where k is side’s sequential number from the left. The extreme sides keep additionally their traditional names (left and right sides). Thus, if the total number of equation’s sides is n, we can refer to its left side as equation’s first or left side and to its right side as equation’s right, last, or nth side.

Table 1.1. Standard prefixes in SI and technical units

Multiple	Prefix and its abbreviation	Multiple	Prefix and its abbreviation
10^{-1}	Deci (d)	10^{12}	Tera (T)
10^{-2}	Centi (c)	10^{9}	Giga (G)
10^{-3}	Milli (m)	10^{6}	Mega (M)
10^{-6}	Micro (μ)	10^{3}	Kilo (k)
10^{-9}	Nano (n)	10^{2}	Hecto (h)
10^{-12}	Pico (p)	10	Deka (da)

1.1.2 THE INTERNATIONAL SYSTEM OF UNITS

It is also known as metric SI (from *Le Système International d’ Unités*) and is the unit system that agreed to be applied all over the world. SI is the system that is used in this book. Its dimensions are classified into two groups: (1) the *primary (fundamental) dimensions* and (2) the *secondary dimensions*. It is most important to know the units of the primary dimensions because, as we shall see later, any secondary unit can be derived from the primary ones, but we cannot name and symbolize it. The current fundamental units of the metric SI are *meter* (m) for length (l), *kilogram* (kg) for mass (m), *second* (s) for time (t), *ampere* (A) for electric current, the *kelvin* (K) for temperature (T), *candela* (cd) for luminous intensity (amount of light), and *mole* (mol) for the amount of matter. ***The multiples and divisors of SI units form the major part of the units that do not belong to it (they are outside it or relative to it).***

The metric SI is simple and logical because the relationship between any of SI units and its standard multiples (or divisibles) is a decimal (see Table 1-1).

To simplify extracting notes and rules at the end of §1.1, the following unit conversions are explained in detail. Let us first solve and discuss the following example.

Example 1-1

Calculate the uniform velocity (in two different units "SI and another unit") of an object that displaced for 5 hectometers (hm) during 2 minutes. Determine also the conversion factor between these two units.

Solution

The uniform velocity (υ) equation is

$$\upsilon = (\Delta X)/t = \Delta X/t \tag{A}$$

Where ΔX is the displacement and t is the elapsed time.

1) Substituting the given values in Equation (A) and considering Table 1-1, we obtain

$$\begin{aligned} \upsilon &= \frac{\Delta \mathrm{X}}{\mathrm{t}} = \frac{5\ \mathrm{hm}}{2\ \mathrm{min}} = \frac{5 \cdot \mathrm{hm}}{2 \cdot \mathrm{min}} = \frac{5}{2}\frac{\mathrm{hm}}{\mathrm{min}} = 2.5\frac{\mathrm{hm}}{\mathrm{min}} \\ &= \frac{5 \cdot 100 \cdot \mathrm{m}}{2 \cdot 60 \cdot \mathrm{s}} = \frac{500 \cdot \mathrm{m}}{120 \cdot \mathrm{s}} \approx 4.1667\frac{\mathrm{m}}{\mathrm{s}} \end{aligned} \tag{B}$$

Or $$\upsilon = 2.5\ \mathrm{hm/min} \approx 4.1667\ \mathrm{m/s}. \tag{B1}$$

2) From the second and third sides of Equation (B1), we obtain the conversion factor between hm/min and m/s

$$1\ \ hm/min \approx 1.667\ m/s \tag{C}$$

The extracted from Example (1-1) notes (Nt) and rules (OR) or (HR) are stated below have been provided in the following.

Nt1-1A: The unit hm/min is not practiced and does not belong to any of existing unit systems.
Nt1-2: The unit m/s belongs to each of the metric SI and to the introduced later technical system of units.
Nt1-3: During calculations, we dealt with the given dimensions as if a multiplication sign (·) existed between (1) the numerical and unit parts of each dimension and (2) the base unit and its prefix (see Equation B). These signs always exist, but they are not visualized. If we denote dimension A by A and its numerical and unit parts by A_N and A_U, respectively, then we can write

$$\mathrm{A} = \mathrm{A}_{\mathrm{N}} \cdot \mathrm{A}_{\mathrm{U}} \tag{D}$$

Applying Equation (D) in Equation (A), we obtain

$$\upsilon_N \cdot \upsilon_U = \Delta X_N \cdot \Delta X_U / (t_N \cdot t_U) \tag{E}$$

This equation can be considered as the product of the following two functions

$$\upsilon_N = \Delta X_N / t_N \qquad \text{and} \qquad \upsilon_U = \Delta X_U / t_U \tag{1-1}$$

These functions/equations can be considered as the result of substituting each dimension, in Equation (A), by its (1) numerical part to produce the first equation and (2) unit part to produce the second equation and can be read as follows.

OR1-1: All equations, which are valid for calculating (not converting) a certain dimension (Y), are also valid for determining each of its unit (Y_U) and its numerical (Y_N) parts. Simply substitute each dimension's symbol (e.g., Y) by its unit symbol (e.g., Y_U) and you obtain, from the dimension's equation, the equation for determining the dimension's unit. Also, to obtain the equation for determining dimension's (Y) numerical part, substitute each dimension's symbol, in dimension's equation, by the symbol of the dimension's numerical part (e.g., If Equation A is dimension υ's determining equation, then the first part of Equation 1-1 can be used for determining dimension υ's numerical part υ_N and the second part of Equation 1-1 can be used for determining dimension υ's unit part υ_U.).

This leads to the following rules.

OR1-2: *To calculate a dimension, when one knows in advance its unit (e.g., the calculating equation is appropriate for the system of units) and when one is sure that the equation one applies is correct, one is requested to substitute each given dimension's symbol by its numerical value and it will not be wrong if the mentioned symbol is substituted by the whole dimension's value. In the case of not substituting the units from the beginning, the answer's unit must be added as soon as the answer's numerical calculation is fulfilled (see Equation D of Nt1-3).*

OR1-3: *To derive (calculate) the unit (A_U) of a dimension (A), (1) select an equation or more of those, which are appropriate for calculating this dimension, and (2) replace in any of the selected equations (the simplest one), each dimension's symbol by its unit (in this case dimension A's symbol is replaced by* A_U*). Such calculated* A_U *value is the derived dimension's (A) unit.*

OR1-4: *If the dimensions used in calculating a dimension are expressed in units that belong* (not relatives; see the bolded text in §1.1.2) *to a certain unit system* (SI or technical), *then the unit of the calculated dimension belongs to the same certain unit system.* Starting from the seventh side of Equation (B), the given dimensions (ΔX and t) were expressed in SI primary units (m and s) and the unit of calculated velocity (see the last sides of Equations B and B1) was expressed in the SI secondary unit (m/s).

OR1-5: Inducing a multiplication sign (· *or* ×) between the two (numerical and unit) parts of a dimension and also after the prefix of the unit symbol (see Equation B) is not practiced, but it is not wrong.

OR1-6: If two neighboring sub (simple) units of a compound unit are not interconnected by any algebraic sign (+, –, "÷ or /," *multiplication sign*), then it is allowed to interconnect these neighboring subunits with a multiplication sign (· *or* ×).

1.1.2.1 Deriving some secondary units from the primary ones

In this book, as we till now got acquainted with only the metric SI units, we shall derive some secondary SI units. This derivation (see OR1-1) requires applying any valid equation for calculating the dimension that is related to the required unit. Therefore, we shall use the simplest equations in these derivations:

1) To derive the acceleration unit, we chose the constant acceleration equation: $\mathrm{a} = (\Delta \upsilon)/\mathrm{t}$, where $\Delta \upsilon$ represents the velocity change and t the elapsed time. According to this equation and OR1-3, the SI acceleration unit is the result of dividing the SI velocity unit (m/s) by the SI time unit (s). Thus, the SI acceleration unit is $(m/s)/s = \dfrac{m}{s} \div s = \dfrac{m}{s} \times \dfrac{1}{s} = m/s^2$.
2) Similar to the acceleration unit, the SI force (F) unit can be found from the other SI units (the mass "m" unit "kg" and acceleration (a) unit "m/s^2") by applying Newton's second law

$$\mathrm{F} = \mathrm{m} \cdot \mathrm{a} \tag{1-2}$$

Thus,

$$\text{SI force unit} = \text{SI mass unit (kg)} \times \text{SI acceleration unit (m/s}^2) = \text{kg} \cdot \text{m/s}^2$$

This unit $(\text{kg}\cdot\text{m/s}^2)$ is named the newton (N) after Sir Isaac Newton (1642–1727).[1]

1.1.3 THE U.S. CUSTOMARY SYSTEM (ALSO KNOWN AS THE *ENGLISH SYSTEM*)

It is the second unit system that is used commonly in the United States today.

The developed from English units U.S. customary system was in use in the British Empire before American independence. Consequently, most U.S. units are virtually identical to the British imperial units. Several differences exist between the two systems.

> The majority of U.S. customary units were redefined in terms of meters and kilograms with the Mendenhall Order of 1893 and, in practice, many years before. These definitions were refined by the international yard and pound agreement of 1959. The United States primarily uses customary units in its commercial activities, whereas science, medicine, government, and many sectors of industry use metric units.[2]

In the ***English system***, the length is measured by yards (yd), the force by pound-force (lbf), and the time by seconds (s). The multiple of the yard is miles (mi), and its divisible are the feet (ft) and the inches (in.), where

$$\left.\begin{array}{c} 1\ \text{mi} = 5280\ \text{yd}, \qquad 1\ \text{in.} = 1'' = 25.4\ \text{mm}, \\ 1\ \text{yd} = 3\ \ \text{ft} = 36\ \text{in.} = 0.9144\ \text{meters} \end{array}\right\} \tag{1-3}$$

The divisible of the pound (lb) is the *ounce* (oz), where $1\ \text{lb} = 16\ \text{oz}$

> The English system has no apparent systematic numerical base, and various units in this system are related to each other arbitrarily (12 in. = 1 ft, 1 mile = 5,280 ft, 4 qt = 1gal, etc.), which makes it confusing and difficult to learn. The United States is the only industrialized country that has not yet fully converted to the metric system.[3]

1.1.4 THE TECHNICAL UNIT SYSTEM

Before 1960 (the date of producing the SI by the *General Conference of Weights and Measures*), ***the technical unit system*** was in common use. It was based on three fundamental quantities: *meters* (m) for length, *kilogram-force* (kgf) or *kiloponds* (kp, from Latin pounds meaning weight)[4] for force, and *seconds* (s) for time. It was prevalent in some countries, such as France, the former Soviet Union, and Germany. Similar to the metric SI, the relationship between any of the technical system units and its standard multiples (or divisibles) is a decimal. This eased and accelerated the transformation of countries that used the technical system to the metric system.

Although the technical system of units is already not in common use, we may encounter valuable old books that used this system and can be our favorite references it we can effectively use them. For such occasions, we need some brief knowledge about the conversion factors between the main units of the previously stated three unit systems.

1.1.5 FORCE AND MASS MAIN UNITS' CONVERSIONS

By definition

$$1\text{pound-mass} = 1\ \text{lbm} = 0.45359237\ \text{kg} \approx 0.454\ \text{kg} \tag{1-4}$$

$$1\ \text{pound-force} = 1\ \text{lbf} = 0.45359237\ \text{kgf} \approx 0.454\ \text{kgf} \tag{1-5}$$

$$\left.\begin{array}{ll} & 1\ \text{kgf} = 1\ \text{kp} = 9.80665\ \text{N} \approx 9.807\ \text{N} \\ \textbf{or} & 1\ \text{N} = 1\ \text{kgf}/9.80665 \approx 0.102\ \text{kgf} \end{array}\right\} \tag{1-6}$$

From these equations, we obtain the exact value of the conversion factor between the newton "N" and the kgf (CFNK),

$$\text{CFNK} = 9.80665\ \text{N / kgf} \tag{1-7}$$

On the other hand, the exact value of the nominal (or standard) gravitational acceleration of the earth (g_n), which is the gravitational acceleration at sea level and 45° latitude, is[5]

$$g_n = 9.80665\ \text{m/s}^2 \approx 32.174\ \ \text{ft/s}^2 \approx 9.807\ \text{m/s}^2 \tag{1-8}$$

Thus, the two different quantities CFNK and g_n have the same numerical value.

1.1.6 WEIGHT OF A BODY

The weight of a body (G) can be found from Newton's second law (see Equation 1-2)

$$G = m \cdot g \ \ N = (m \cdot g / 9.80665) \ \ kp \qquad (1\text{-}8a)$$

where g is the local gravitational acceleration[6] that depends on the geographical location and altitude, m the mass measured by weighbridges,[A] and 9.80665 N/kp the CFNK.

A. Weighbridges eliminate the effect of the local gravitational acceleration on the measurement result, since each of the standard weight and the object to be weighed are affected by the same local gravitational acceleration. This fits the concept of mass that is independent of the geographical location and altitude[7].

For approximate calculations of body's weight on earth and its surrounding atmosphere up to 10 km above sea level, and with a 0.6% tolerance, g in Equation (1-8a) can be replaced by g_n and Equation (1-8a) becomes

$$G \ \ kp \cong m \ \ kg \qquad (1\text{-}8b)$$

For English units, this equation becomes

$$G \ \ lbf \cong m \ \ lbm \qquad (1\text{-}8c)$$

1.1.7 PRESSURE UNITS

In all unit systems, the pressure (p) is a secondary dimension. As the pressure is defined as the force (F) acting normally on a unit of area (A) or $p = F/A$, the pressure unit is the result of dividing the force unit by the area unit, which is the squared length unit. The pressure unit in the metric SI is the newton per square meter (N/m^2) and is named the Pascal (Pa). As this unit is too small for practical use, its multiple (the bar, 1 bar = 10^5 Pa) is used instead. Thus,

$$1 \text{ bar} = 10^5 \ \ N/m^2 = 10^5 \ \ Pa = 10^5 \ \ N/(10^2 \ \ cm)^2 = 1 \ daN/cm^2 \qquad (1\text{-}9)$$

The pressure unit in the technical system is the kiloponds (or kgf) per square meter (kp/m^2) or (kgf/m^2). As this unit is too small for practical use, its multiple (the ***technical atmosphere* "at"**) is used instead. Thus,

$$1\ \text{at} = 10^4\ \text{kp/m}^2 = 10^4\ \text{kp}/(10^2\ \text{cm})^2 = 1\ \text{kp/cm}^2 \qquad (1\text{-}10)$$

The pressure unit in the English system is the pounds (lbf) per square inch [lbf/in.^{2} (see Nt1-6) or psi]. It is the pressure resulting from a force of 1 pound-force applied normally to an area of 1 square inch. Thus,

$$\left.\begin{aligned} 1\ \text{lbf} \approx 4.448\ \text{N}, \quad 1\,\text{psi} \approx 6895\ \text{N/m}^2, \\ 1\,\text{atm} \approx 14.696\ \text{psi} \approx 14.7\,\text{psi} \end{aligned}\right\} \qquad (1\text{-}11)$$

Today, pressure gauges are scaled in bars (bar), technical atmospheres (at), and pounds per square inch (psi).

The pressure (p) applied by a liquid column, with height (H) and density (ρ), is calculated by

$$p = H \cdot \rho \cdot g \qquad (1\text{-}12)$$

The liquid density is temperature dependent, $\rho = f(t)$, and the gravitational acceleration is geographical location dependent. Therefore, the fluid column height corresponding to a pressure is a function of the fluid type, temperature, and geographical location. This means that,

When the pressure is expressed by units (mmH_2O or cmH_2O or mmHg = Torr, etc.), it implies

$$g = g_n = 9.80665\ \text{m/s}^2, \quad \rho_{Hg} = \rho_{Hg(t=0^\circ C)} = 13{,}595.1\ \text{kg/m}^3$$

$$\text{And} \qquad \rho_{H_2O} = \rho_{H_2O(t=4^\circ C)} = 1000\ \text{kg/m}^3$$

1.1.8 OTHERS' DEFINITIONS

OD1-2: The standard atmosphere (***atm***) is an international reference pressure defined as 1,01,325 Pa. It is the mean atmospheric pressure at sea level and at 0°C. First, it was named as the physical atmosphere and later *the normal pressure.*

OD1-2A: The ***normal temperature and pressure (NTP)***, *also known as* ***normal conditions,*** *is the normal temperature (abbreviation NT and Symbol T_n)* $T_n = 273.15\ \text{K} = 0\ ^\circ\text{C}$, and *the normal pressure*, which is also known as *the standard atmosphere.*

$$\begin{aligned} p_n &= 1\ \text{atm} = 760\ \text{Torr} = 760\ \text{mm Hg}\ [0\ ^\circ\text{C}] \\ &= 10.33256\ \text{m}\ H_2O\ [4^\circ\text{C}] = 101{,}325\ (\text{kPa}) \end{aligned} \qquad (1\text{-}13)$$

Earlier the ***normal conditions*** represented the standard ambient temperature and pressure (SATP), but today, they are discontinued and we are supposed to avoid using them. However, many valuable traditional references are not renewed and are still in use. Therefore, we shall continue using these terms in parallel with the new terms.

The existing SATP temperatures today are $15^\circ\mathrm{C} = 268.15\,\mathrm{K} = 59^\circ\mathrm{F}$, $20^\circ\mathrm{C} = 293.15\,\mathrm{K} = 68^\circ\mathrm{F}$, and $25^\circ\mathrm{C} = 298.15\,\mathrm{K} = 77^\circ\mathrm{F}$. We shall refer to the last temperature as the new normal temperature (symbol T_{nn}). Thus, $\mathrm{T_{nn}} = 25^\circ\mathrm{C} = 298.15\,\mathrm{K} = 77^\circ\mathrm{F}$.

1.1.9 ENERGY UNITS

As we will see later (see the first law of thermodynamics), the thermal, mechanical, electrical, and other energies are different forms of energy. This means that work, mechanical energy, thermal energy, and other energies must be measured by the same units, and therefore, let us derive the units of work in the three unit systems using the equation of the work (*W*) done by a constant force (*F*) on a point that moves a displacement (*s*) in the direction of the force, W = F·s. According to OR1-1 and §1.1.2.1, the work unit = the force unit × the displacement (length) unit. This means that the work units in the three systems are (1) the newton-meter that was named later (joule) in the SI system, (2) the kilopond-meter "kp-m or kp·m" (the kilogram-force-meter "kgf-m or kgf·m") in the technical system, and (3) the foot-pound force (ft-lbf or ft·lbf) in the English system.

The joule is defined by the equation

$$1\ J = 1\ N \times 1\ m = 1\ N \cdot m = 1\ (kg \cdot m / s^2) \cdot m = 1\ kg \cdot m^2 / s^2 \qquad (1\text{-}14)$$

The units of heat energy used in the technical and English systems are renewed. The new (**international**) calorie (cal) is defined as

$$1\ \mathrm{cal} = 4.1868\ \mathrm{J} \qquad (1\text{-}15)$$

Today's British thermal unit (Btu) equals

$$1\ \mathrm{Btu} = 1.055056\ \mathrm{kJ} = 0.2522\ \mathrm{kcal} \qquad (1\text{-}16)$$

The numerical relationship between the calorie and the kilopond (the mechanical equivalent of heat "J") was found experimentally by Joule. It assigns the quantity of work in kp·m (kgf·m) that can be done at the expense of the disappearance of 1 kcal of heat, which can be calculated from Equations (1-6), (1-11), and (1-12)

$$\begin{aligned} 1 \text{ cal} &= 4.1868 \text{ J} = 4.1868 \text{ N}\cdot\text{m} = 4.1868\cdot(1 \text{ kgf} / 9.80665)\cdot\text{m} \\ &= 0.426935 \text{ kgf}\cdot\text{m} \end{aligned} \tag{1-17}$$

or

$$1 \ kcal = 426.935 \ kgf\cdot m \approx 427 \ kgf\cdot m \tag{1-17a}$$

Relation (1-17a) allows determining the value of the mechanical equivalent of heat *J* and its reverse value *A*

$$\left.\begin{aligned} \text{J} &= 426.935 \frac{\text{kp}\cdot\text{m}}{\text{kcal}} \approx 427 \frac{\text{kp}\cdot\text{m}}{\text{kcal}} \\ \text{and} \qquad \text{A} &= \frac{1}{426.953}\frac{\text{kcal}}{\text{kp}\cdot\text{m}} \approx \frac{1}{427}\frac{\text{kcal}}{\text{kp}\cdot\text{m}} \end{aligned}\right\} \tag{1-18}$$

One of English units of work is the foot-pound force (ft-lbf or ft·lbf):

$$1 \text{ ft}\cdot\text{lbf} = 1 \text{ ft}\times 1 \text{ lbf} = 0.001285 \text{ Btu} = 1.3558 \text{ J} \tag{1-19}$$

Other units of energy (work) can be extracted from power units. As power (*P*) is defined as the amount of energy generated per unit time, the power unit is energy-unit/time-unit; thus, the SI power unit (J/s) is named watt (W). This means that

$$1 \text{ W} = 1 \text{ J} / \text{s} \tag{1-20}$$

This unit and the power units of the English system (ft-lbf/s) and the technical system (kp·m/s) are small and their multiples.

- The kilowatt (kW):

$$1 \text{ kW} = 1\cdot(1000) \text{ W} = 1000 \text{ W} \tag{1-21}$$

- The horse power (hp):

$$1 \text{ hp} = 1\cdot(550) \text{ ft}\cdot\text{lbf} / \text{s} = 550 \text{ ft}\cdot\text{lbf} / \text{s} \tag{1-22}$$

- The French *chevaux* (ch) or the *metric horsepower* (hp“M”)[8]:

$$1 \text{ hp(M)} = 1 \text{ ch} = 1\cdot(75) \text{ kp}\cdot\text{m} / \text{s} = 75 \text{ kp}\cdot\text{m} / \text{s} \tag{1-23}$$

These units interrelate as follows.

$$1\ \text{hp}(\text{M}) = 1\ \text{ch} = 75\ \text{kp}\cdot\text{m/s} = 75\times 9.80665\,\text{N}\cdot\text{m/s}$$
$$= 735.49875\ \text{W} \approx 735.5\ \text{W} \tag{1-24}$$

$$1\ \text{hp} = 550\ \text{ft}\cdot\text{lbf/s} = 550\ \text{in.}\cdot\text{lbf/s} = 550\cdot 12\times\text{in.}\cdot\text{lbf/s}$$
$$= 6600\ \text{in.}\cdot\text{lbf/s} = 6600\cdot 0.0254\ \text{m}\times 0.4536\ \text{kp/s}$$
$$= 76.04\ \text{kp}\cdot\text{m/s} = 76.04/75\ \text{ch} = 1.0139\times 735.49875\ \text{W}$$
$$\cong 745.7\ \text{W} \cong 746\ \text{W} \cong 746\ \text{N}\cdot\text{m/s} \tag{1-25}$$

From Equations (1-24) and (1-25), we conclude that

$$\left.\begin{array}{ll} 1\ \text{ch} = 1\ \text{hp(M)} \cong 1\ \text{hp}; & 1\text{hp} = 1.0139\ \text{ch} \\ \text{and} \quad 1\ \text{ch} = (1/1.0139)\ \text{hp} = 0.9863\ \text{hp} & \end{array}\right\} \tag{1-26}$$

The small difference between the metric and the English horse powers causes nonrecognition between them.

As the work equals the product of power and time, the following energy units are in common use:

- The kilowatt-hour (kW-h or kW·h), which is the work produced by an engine of 1 kW power during 1 hour:

$$1\ \text{kW}\cdot\text{h} = 1\ [\text{kJ/s}]\times 3600\ \text{s} = 3600\ \text{kJ} \tag{1-27}$$

- The metric horsepower-hour (hp“M”-h), which is the work produced by an engine of 1 hp(M) power during 1 hour:

$$1\ \text{hp(M)}\cdot\text{h} = 1\ \text{ch}\cdot\text{h} = 75\ [\text{kp}\cdot\text{m/s}]\times 3600\ \text{s}$$
$$= 270000\ \text{kp.m} \tag{1-28}$$

- The horsepower-hour (*hp·h*), which is the work produced by an engine of 1 hp power during 1 hour:

$$1\ \text{hp}\cdot\text{h} = 0.7068\ [\text{Btu/s}]\times 3600\ \text{s} = 2544.48\ \text{Btu} \tag{1-29}$$

1.1.10 TEMPERATURE UNITS

The following definitions are necessary to understand this paragraph.

OD1-3: *The **absolute zero** is the lowest conceivable temperature.*

OD1-3A: *The temperature of the* ***ice point*** *is defined as the temperature of a mixture of ice and water that is in equilibrium with saturated air at a pressure of 1 atm (see OD1-2 and Equation 1-13).*
OD1-3B: *The temperature of the* ***steam point*** *is the temperature of water and steam, which are in equilibrium at a pressure of 1 atm.*
OD1-3C: *The* ***triple point*** *of water is the state in which the solid, liquid, and vapor phases of water exist together in equilibrium.* This point is assigned the value of 0.01°C. On the Celsius scale, the steam point is experimentally found to be 100.00°C.

Four temperature scales are still in common use today: (1) The ***Celsius scale*** *(unit degree Celsius "°C")*, (2) The ***Fahrenheit scale*** *(unit degree Fahrenheit "F")*, (3) The ***Kelvin scale*** *(unit* kelvin *"K")*, and (4) The ***Rankine scale*** *(unit rankine "R")*.

The ***Celsius scale*** was formerly called the *centigrade scale*, but it is now designated the Celsius scale[9] and its symbol includes the degree symbol since the letter C alone denotes Coulomb, the unit of electrical charge in the SI system of units.[10]

Table 1.2. Comparison of temperature scales

Temperature / **Reference point**	**Temperature symbol and its unit abbreviation**			
	$T\,[K]$	$T\,[^{\circ}C]$	$T\,[R]$	$T\,[^{\circ}F]$
Water boiling at 1 atm (see OD1-3B) or the steam point	373.15	100	671.67	212
Water triple point (see OD1-3C)	273.16	0.01		
Water freezing at 1 atm or the ice point (see OD1-3A)	273.15	0	491.67	32
The zero Fahrenheit (T_{0F})			459.67	0
The absolute zero (AZ)	0	−273.15	0	−459.67

OD1-3D: Temperature scales whose zeros coincide with the absolute zero ***are absolute temperature scales,*** and those whose zeros do not coincide with the absolute zero are ***relative temperature scales.*** Therefore (see Table 1-2), The *Kelvin and Rankine scales are* ***absolute temperature scales,*** *while* the *Celsius and Fahrenheit scales are* ***relative temperature scales.*** Today's temperature scales and former absolute ones (see OD1-3D) are symbolized as *T*, while the former symbol of relative temperature scales was *t*. Therefore, both symbols are used in this book, which includes many old tables and diagrams. This means that in addition to existing temperature relations used in other modern thermodynamic books, the following relations are used. °C

$$\left.\begin{aligned}&\Delta T\ K = \Delta T\ ^{\circ}C = (T_2 - T_1)\ ^{\circ}C = (t_2 - t_1)\ ^{\circ}C = \Delta t\ ^{\circ}C;\\ &\Delta T\ R = \Delta T\ F = (T_2 - T_1)\ F = (t_2 - t_1)\ F = \Delta t\ F;\\ &dT\ K = dT\ ^{\circ}C = dt\ ^{\circ}C \quad \text{And} \quad dT\ R = dT\ F = dt\ F\end{aligned}\right\} \qquad (1\text{-}30)$$

The values of some reference temperatures in several units are listed in Table 1-2.

Although the relations between the four scales are well known, they can be derived from Table 1-2 because of the *constancy of the magnitude of the degree of each scale that insures the linearity of these relations*.

Using the data of Table 1-2, we can obtain many relations for temperature conversions from one scale to another. The temperatures of these relations are expressed indirectly (see OD1-1), and we represent from them the following main ones:

1)

$$\left.\begin{aligned}&T\ [K] = T\ [^{\circ}C] + 273.15\\ \text{or} \qquad &T\ [^{\circ}C] = T\ [K] - 273.15\end{aligned}\right\} \qquad (1\text{-}30a)$$

This is read as *the temperature in kelvins equals the sum of the temperature in degrees Celsius and 273.15*. Thus, if $\underline{T = 50\ ^{\circ}C}$ then $T\ [^{\circ}C] = 50$ and $T\ [K] = 50 + 273.15 = 323.15$, **so** $\underline{T = 323.15\ K.}$

2) $T\ [R] = 1.8 \cdot T\ [K]$ or $T\ [K] = (5/9) \cdot T\ [R]$ (1-31)

For $\underline{T = 323.15\ K}: T\ [R] = 1.8 \cdot 323.15 = 581.67$ **or** $\underline{T = 581.67 R.}$

3)

$$\left.\begin{aligned}&T\ [R] = T\ [F] + 459.67\\ \text{or} \qquad &T\ [F] = T\ [R] - 459.67\end{aligned}\right\} \qquad (1\text{-}32)$$

For $T\ [R] = 581.67$:

$T\ [F] = T\ [R] - 459.67 = 581.67 - 459.67 = 122$ so $\underline{T = 122\ F.}$

Any set of three equations those obtained/selected from the three pairs of Equations (1-30A) through (1-32) each is enough to (1) execute any temperature conversion from any of the previously represented temperature scales to another and (2) obtain other calculating equations such as

$$\left.\begin{array}{ll} T\,^{\circ}C = (5/9)\cdot(T\ F-32), & T\ F = 1.8\times T\,^{\circ}C + 32, \\ 1\ K = 1\,^{\circ}C = 1\ \text{deg} \quad \text{and} & 1\ R{=}1F \end{array}\right\} \qquad (1\text{-}33)$$

The underlined equations in the last three items are some of the *direct expressions* of temperature or temperature difference (see OD1-1). They can be read as follows: *The temperature equals 50 degrees Celsius, 323.15 kelvins, 323.15 kelvins, 581.67 rankines, and 122 degrees Fahrenheit. The total result of the calculations of the previous three items is* $50\,^{\circ}C = 323.15\ K = 581.67\ R = 122\ F$

Nt1-4: The deg or degree symbol *in Equation* (1-33) *was used for the unit of the temperature difference between each of Kelvin and Celsius scales.*
Nt1-5: *No distance exists between the abbreviations of the prefix and the unit to which it belongs.*
Nt1-6: *The pound-force, pound-mass, pound-mole, and kilogram-force are fundamental units; therefore, their abbreviations are lbf, lbm, lbmole, and kgf, respectively.*
Nt1-7: The recent *abbreviation of* inch is (in.) not (in).
Nt1-8: Assuming that a multiplication sign between the numerical part of a dimension and its unit eases unit conversions. The reader can apply this point to understand the operations (procedures and processes) in Equations (1-17), (1-27), (1-28), and (1-29).
Nt1-9: One of the purposes of the former equations was showing the beginners in detail how to deal with unit converting. In practice, calculating dimensions is much simpler, especially when the answer is required in SI units (or in their standard multiples or divisibles).

Let us explain the methods of solving problems in practice through the following paragraph and example.

1.1.11 ABOUT DIMENSIONS' UNITS IN THE CALCULATING EQUATIONS

Today, two types of calculating equations are in common use (1) the free of conversion factors equations. They consist of only dimensions' symbols and no information about dimensions' units exist in these equations or in accompanying them captions/legends. And (2) Equations with conversion factor(s), those include dimensions' symbols and are equipped with information about dimensions' units, which exist either in the equations themselves or in accompanying them captions/legends. For example, The equation for calculating the shaft power (P) can be written as any of the following three equations:

$$\text{Power} = \text{torque} \times \text{angular speed} \qquad (\text{A})^{11}$$

$$\text{Power(hp)} = \frac{\text{torque}(\text{lbf} \cdot \text{ft}) \times \text{ angular speed(rpm)}}{5252} \qquad (\text{B})$$

$$\text{Horsepower} = \text{Torque} \times \text{RPM} / 5252 \qquad (\text{C})^{12}$$

Where torque in (lbf · ft)

These unusual for thermodynamics and many other branches of science equations can be rewritten in the usual form as:

$$\text{P} = \text{T} \times \omega \qquad (\text{A}')$$

Where T is the torque and ω is the angular velocity

$$\text{P}(\text{hp}) = \frac{\text{T}(\text{lbf} - \text{ft}) \times \text{n(rpm)}}{5252} \qquad (\text{B}')$$

Where T is the torque and n is the rotational speed. Or

$$\text{P} = \frac{\text{T} \times \text{n}}{5252} \text{ hp} \qquad (\text{C}')$$

Where T is the torque in lbf-ft and N is the rotational speed in rpm.

The free of conversion factors equations (see Equations A and A') are mostly used and can be used with all systems of units. Once the user has chosen the system of units to be used with a particular free of conversion factors equation, all its dimensions' units are considered belonging to this particular chosen system.

Equations B, C, B' and C' above represent the versions of writing the equation with a conversion factor. The user of such equations is obliged to use dimension's units as provided in each equation and its explanation (caption/legend).

Example 1-2

Using the ideal gas equation of state ($p \cdot V = m \cdot R \cdot T$), calculate the volume (V) of a mass (m = 3 kg) of air that exists at a pressure p=1 bar and temperature T = 350 K. The gas constant for air is R = 287 J/kg-K.

Solution

Let us firstly note that (1) this equation is free of conversion factors, (2) the units of its given dimensions are either metric SI ones (m = 3 kg, T = 350 K, and R = 287 J/kg-K), or multiples of the same metric SI units (p = 1 bar), and (3) the only un given dimension is the required one (V). This means that although the given equation can be used with any system of units, the less time consuming solution of this example will be when we chose using the metric SI system of units in solving this example. Therefore, we rewrite the given equation in the form $V = m \cdot R \cdot T / p$ that is appropriate for calculating the required volume, substitute in it the values of all given dimensions in metric SI units, and perform the calculations:

$$\begin{aligned} p \cdot V = m \cdot R \cdot T \Rightarrow V = \frac{m \cdot R \cdot T}{p} &= \frac{3 \cdot kg \cdot 287 \cdot J \cdot 350 \cdot K}{kg \cdot K \cdot 10^5 \cdot Pa} \\ &= \frac{3 \cdot 287 \cdot J \cdot 350}{10^5 \cdot Pa} = 3.0135 \frac{J}{Pa} \\ &= 3.0135 \frac{N \cdot m}{N \cdot m^{-2}} = 3.0135\ m^3 \end{aligned} \tag{D}$$

Since we know in advance that the unit of the calculated volume will be (m^3), we do not need to apply and deal with the unit parts of the dimensions in the Intermediate sides of equation D. Therefore, calculating air volume in this example will be limited in calculating the numerical part of air volume, and equation D become,

$$p \cdot V = m \cdot R \cdot T \Rightarrow V = m \cdot R \cdot T / p = 3 \cdot 287 \cdot 350 / 10^5 = 3.0135\ m^3 \tag{E}$$

1.2 CALCULATIONS AND DISCUSSIONS IN THERMODYNAMICS

All three ways of calculations and discussions (the graphical, the analytical, and the tabular) are used in thermodynamics. The analytical way gives high accuracy and is usually used everywhere where other ways cannot be used.

The graphical way is simple for understanding and discussions, but it is not that accurate; therefore, it is mostly used in discussions and in those circumstances where high accuracy is not required. However, using large-scale diagrams in calculations is acceptable for thermodynamic accuracy.

The tabular way is used (1) when a high accuracy is required, while the analytical solution is almost not available or time-consuming, and (2) for approximate calculations in some circumstances.

Before starting the analysis, let us introduce some important definitions and notes.

OD1-4: *A plane curve is a curve that lies in a single plane. A plane curve may be closed or open.*[13]

OR1-7: *Two neighboring points of a plane $y = f(x)$ curve cannot have the same two coordinates*. This means that very neighboring points (1 and 2) of a plane $y = f(x)$ curve can have *(1)* $x_2 – x_1 = \mathrm{dx} = 0$ *and* $y_2 – y_1 = \mathrm{dy} \neq 0$, *(2)* $x_2 – x_1 = \mathrm{dx} \neq 0$, *and* $y_2 – y_1 = \mathrm{dy} = 0$, *or 3)* $x_2 – x_1 = \mathrm{dx} \neq 0$ *and* $y_2 – y_1 = \mathrm{dy} \neq 0$. *For these three conditions, the distance between the two points (1 and 2) is* $L_{1\_2} = \sqrt{(dx)^2 + (dx)^2} \neq 0.$ If the two points are characterized by $\Delta x = \Delta y = 0$ *or* $dx = dy = 0$, then the exact distance between them is zero and they are congruent (not neighbors). Defining two very neighboring points (1 and 2) of a curve *by* $x_2 - x_1 = \mathrm{dx} = 0$ and $y_2 - y_1 = \mathrm{dy} \neq 0$ means that the slope of the curve at point 1 is $\pm\infty$.

OD1-4A: An ***equation*** has two sides (left and right) that are separated by an equal sign.

Helal Definition 1-1 (HD1-1): A ***dual-side equation*** (*abbreviation* equation) consists of two sides (left and right) that are separated by an equal sign.

HD1-2: A ***multiside equation*** consists of more than two sides that are separated by equal signs.

HD1-3: *A* ***single-equation curve*** *is the curve that can be expressed/defined by a single equation. A* ***multiequation curve*** *is the curve that cannot be expressed by a single equation.* It requires more than one equation (a set of equations) to be expressed. Refracted curves (see curve 1a–2a–3a–4a–5a in Figure 1.1) are invariably multiequation curves.

OD1-5: *A smooth curve is a continuous curve (no gaps or discontinuities) with no corners (no abrupt changes in the slope such as a point that you would get from the intersection of two lines).*[14] The continuous plane line (1–2–3–4, Figure 1.1a) is a smooth curve. It consists of three-plane smooth curves, which are (1) line 1–2 (a part of an ellipse that is expressed by $f_1(x)$), (2) straight line 2–3 that is expressed by $f_2(x)$, and (3) line 3–4 (a part of another ellipse that is expressed by $f_3(x)$). Line (2–3) is the common tangent of ellipses $f_1(x)$ *and* $f_3(x)$ at points 2 and 3. This means that to define an *n*-equation smooth curve, its two extreme points should be given in addition to its (n – 1) points, which interconnect its single-equation curves. Curve 1b–2b–3b–4b–5b in Figure 1.1 is also a multiequation smooth curve.

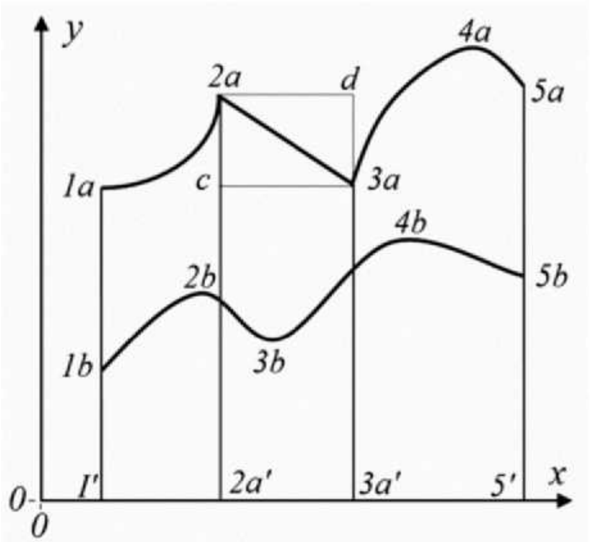

Figure 1.1. Multiequation curves

OD1-6: A plane *closed curve* is a curve with no endpoints and completely encloses an area.[15] In other words, it is the curve in which the start (*st*, 1 or I) and end (*ed*, 2 or II) points coincide.

OD1-7: A plane **simple closed curve** is a connected curve that does not cross itself and ends at the same point where it begins (e.g., circles, ellipses, and polygons). Note that despite the name "curve," a simple closed curve does not actually have to curve.[16] Curve (1'-b-2') in Figure 1.2 is a simple closed curve.

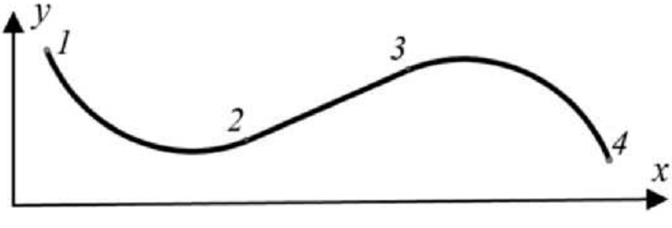

Figure 1.1a. A smooth triequation curve

OD1-8: The directional curve is the one that has a direction. It has a start (*st*, 1 or I) and an end (*ed*, 2 or II) point, and unless the curve is closed, the start and end points are enough to determine its direction. This means that (1) the direction of the closed curve should be defined through other means such as at least two additional points (3, 4, …) or through arrows, (2) calculations belonging to the whole closed curve must start from the curve's start point (if it does not exist, then the calculator adds it), passing successively through all additional curve's points (3, 4, …), and ending exactly at the curve's end point, which is the same start point. If the start and end points of a plane simple closed curve are not congruent, then the curve is more than plane simple closed curve (see the following new definition).

ND1-2: The *more than simple closed curve* (1"-c-2", Figure 1.2) is the curve that is composed of two parts: (1) the first part (1"-c-1") is a simple closed curve and (2) the second part (1"-2") is an additional open curve that coincides/superposes with the beginning of the first part. Therefore, the calculations result for the more than simple closed curve will be the algebraic sum of that for the closed curve and that for the additional curve. The only difference in the view between the simple closed curve and the more than simple closed curve is that the start and end points of the simple closed curve are congruent, whereas they are not for the *more than simple closed curve*.

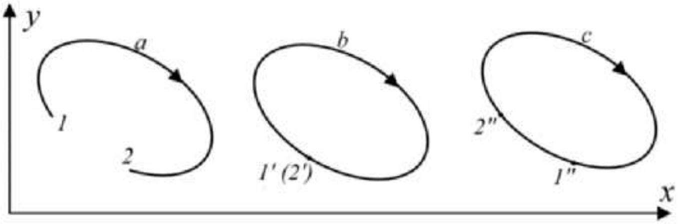

Figure 1.2. The open smooth curve (1-a-2), closed (1'-b-2'), and more than closed (1"-c-2") simple curves.

Nt1-10: The order of numbering a curve's points is mostly sequential because it allows writing multiterm equations in the abbreviated form (see Equation 1-64).

OD1-9: *A planimeter is a mechanical integrating instrument for measuring the area of an irregular plane figure, such as the area under a curve, by moving a point attached to an arm around the perimeter of the figure.*[17]

1.2.1 CALCULATING THE AREA UNDER A PLANE CURVE

Thermodynamic equations such as $q = \int_{t_1}^{t_2} C \cdot \mathrm{dt} = \int_{s_1}^{s_2} T \cdot \mathrm{ds}$, $w = \int_{t_1}^{t_2} p \cdot \mathrm{d}v$, and $W = \int_{x_1}^{x_2} F \cdot \mathrm{dx}$ are similar to

$$A=\int_{x_1}^{x_2} y\cdot \mathrm{dx} \tag{1-34}$$

This equation (see Figure 1.3) is used in calculating the *area ($A_{1\text{-}2\text{-}2'\text{-}1'\text{-}1}$) under curve (1–2) governed by equation* $y=f(x)$. For x-increasing or x-decreasing curves, this area is bordered by the curve (1–2) itself, its x-projection (1'–2'), x_1-constant, and x_2-constant lines. Rectangle (b–c–2'–1'–b) shares with curved leg right-angled trapezoid (1–2–2'–1'–1) each of the side/base/width (1'–2') and area, and has the average height:

$$y_{\text{avg}}=A_{1-2-2'-1'-1}/(x_2-x_1) \tag{1-35}$$

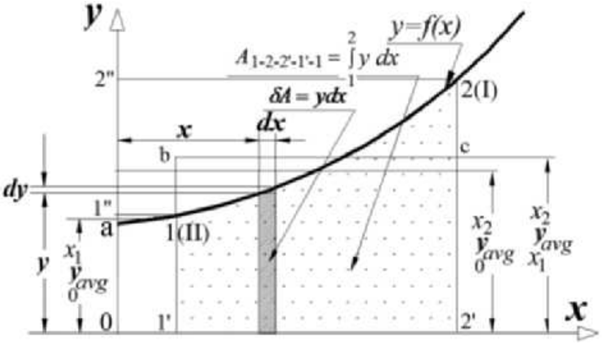

Figure 1.3. A schematic for calculating the area under a curve

In this equation, the symbol of the average height of the area under a curve is abbreviated to y_{avg}, and it could be abbreviated sometimes to y, because it is multiplied by ($x_2 - x_1$), which defines the x-limits of the area to which the average height belongs. In Figure 1.3, the average heights appear alone (they are not multiplied by Δt); therefore, their symbols must be denoted additionally by the x-limits of the area to which each average height belongs (the symbols can be $\overset{x_2}{\underset{x_1}{y_{\text{avg}}}}$ or $y_{\text{avg}}\Big|_{x_1}^{x_2}$ and so on).

Also, area (1–2–2'–1'–1) can be calculated as the difference between areas (a–2–2'–0–a) and (a–1–1'–0–a). This and Equations (1-34) and (1-35) lead to

$$\begin{aligned} \mathrm{A}_{\text{under }1-2}&=\mathrm{A}_{1-2-2'-1'-1}=\mathrm{A}_{a-2-2'-0-a}-\mathrm{A}_{a-1-1'-0-a}=\int_0^{x_2}\mathrm{y}\cdot\mathrm{dx}-\int_0^{x_1}\mathrm{y}\cdot\mathrm{dx}\\ &=\int_{x_1}^{x_2}\mathrm{y}\cdot\mathrm{dx}=\mathrm{y}_{\text{avg}}\cdot(\mathrm{x}_2-\mathrm{x}_1)=\mathrm{y}_{\text{avg}}\cdot(\mathrm{x}_2-0)-\mathrm{y}_{\text{avg}}\cdot(\mathrm{x}_1-0) \end{aligned} \tag{1-36}$$

Nt1-11: Function $y = f(x)$ is represented schematically in Figure 1.3. This means that we can use this representation in discussing and explaining several items, but we cannot use it in calculations. To use the graph in calculations, it should be created in scale and the user must take this scale into consideration.

Nt1-12: It is unnecessary for point *a* to be located on *y*-axis, where $x_a = 0$, but it is much important to keep its position on curve (1–2) or its extent throughout a full calculation.

Nt1-13: The three y_{avg} in Equation (1-36) are not the same. The first belongs to the $x_1\_x_2$ limits, the second to $x_a\_x_2$ limits, and the third to $x_a\_x_1$ limits.

Nt1-14: The expression $y \cdot dx$ (symbol δA) is the ***inexact differential*** of area *A* because unless the function $y = f(x)$ is defined (graphically, analytically, or tabular) integral, $A = \int_{x_1}^{x_2} y \cdot dx$ cannot be calculated. Symbolizing the expression $y \cdot dx$ by dA (instead of δA) is not rigorous (it does not remind the reader that the differential is inexact), but is not wrong, because it does not lead to any errors in the calculation.

Equation (1-34) assumes that (1) area A is sliced by an extremely large number of x-constant lines into extremely large number of right-angled trapezoidal slices of dx width, where the upper borders/legs of these trapezoids are differential straight lines, and (2) the original calculating equation for the differential of area "δA" (see Figure 1.3) is $dA = 0.5[(y + dy) + y] \cdot dx = y \cdot dx + 0.5(dy \cdot dx) = y \cdot dx$. In this equation, the second term ($dy \cdot dx / 2$) of the third side, which is the product of two differential terms, is much less than the first term $(y \cdot dx)$, and therefore it was ignored.

The equation for calculating the area under the straight line 2a_3a (the area of a no-differential right-angled trapezoidal slice cut by two *x*-constant lines x_{2a} and x_{3a} with upper straight-line border, 2a_3a, see Figure 1.1) is

$$\begin{aligned} A_{2a-3a-3a'-2a'-2a} &= A_{2a-d-3a'-2a'-2a} - A_{2a-d-3a-2a} \\ &= A_{c-3a-3a'-2a'-c} + A_{2a-3a-c-2a} \qquad (1\text{-}37) \end{aligned}$$

Here, it is not allowed to ignore any of the equal triangle areas $A_{2a-d-3a-2a}$ and $A_{2a-3a-c-2a}$ because each of them is not too small compared with the rectangle to/from which it is added/subtracted.

Nt1-15: (1) Line 1_2 in Figure 1.3 is ***monotony x-increasing*** because the *x*-projection (dx) of any of its differential parts is positive (dx > 0), while (2) line (I_II) that coincides with line (1_2) is ***monotony x-decreasing*** because the *x*-projection (dx) of any of its differential parts is negative (dx < 0). In addition, the special case directional $y = f(x)$ curve can be (3) a ***constant-x straight line*** $(\mathrm{dx} = 0)$ if all its points are described by $\mathrm{dx} = 0$, (4) a ***constant-y straight line*** $(\mathrm{dy} = 0)$ if all its points are described by $\mathrm{dy} = 0$, (5) a ***gradual x-increasing curve*** $(\mathrm{dx} \geq 0)$ if it consists of at least one *x*-increasing curve and one *x*-constant line or point and does not include any *x*-decreasing line, (6) a ***gradual x-decreasing curve*** $(\mathrm{dx} \leq 0)$ if it consists of at least one *x*-decreasing line and one *x*-constant line or point and does not include any x-increasing line, (7) an ***x-increasing curve*** $(\mathrm{dx} \geq 0)$ if it does not contain/include any differential curve that is described by $\mathrm{dx} < 0$, and (8) an ***x-decreasing curve*** $(\mathrm{dx} \leq 0)$ if it does not contain/include any differential curve that is described by $\mathrm{dx} > 0$.

It is clear from the previous paragraph that the *x*-increasing curve $(\mathrm{dx} \geq 0)$ is a common case for both the monotony and the gradual *x*-increasing curves, and the *x*-decreasing curve $(\mathrm{dx} \leq 0)$ is a common case for both the monotony and the gradual *x*-decreasing curves.

ND1-4: *The pure positive/negative area is the area, which does not include any negative/positive subareas.* Therefore:

For positive (*y*), the area under (1') the monotony *x*-increasing curve $(\mathrm{dx} > 0)$ is called ***absolutely pure positive area*** $(\delta A > 0)$, (2') the monotony x-decreasing curve $(\mathrm{dx} < 0)$ is called ***absolutely pure negative area*** $(\delta A < 0)$, (3') the constant-x straight line $(\mathrm{dx} = 0)$ equals zero because $\delta A = y \cdot 0 = 0$ and the definite integral of zero equals zero $(A = \int_1^2 0 = 0)$, (4') $y = 0$ straight line equals zero ($\delta A = y\mathrm{dx} = 0 \cdot \mathrm{dx} = 0 \Rightarrow A = 0$), (5') the gradual *x*-increasing curve $(\mathrm{dx} \geq 0)$ is ***pure positive area*** $(\delta A \geq 0)$, (6') the gradual x-decreasing curve $(\mathrm{dx} \leq 0)$ is ***pure negative area*** $(\delta A < 0)$, (7') the x-increasing curve $(\mathrm{dx} \geq 0)$ is ***pure positive area*** $(\delta A \geq 0)$, and (8') the *x*-decreasing curve $(\mathrm{dx} \leq 0)$ is ***pure negative area*** $(\delta A \leq 0)$.

For negative (*y*), the names of the previous curves must be inverted.

Taking into consideration items (3') and (4'), we conclude the following.

NR1-1: Areas under zero-*y* and constant-*x* straight lines are zero ones, and therefore, they do not affect areas' calculations under curve *y = f(x).*
NR1-2: *The sum of the absolutely pure positive/negative areas under a y = f(x) curve exactly equals the sum of the pure positive/negative areas under the same curve.* Therefore, we shall refer to both (the absolutely pure and pure) positive/negative areas as the pure positive/negative areas under a *y = f(x)* curve.

Let us introduce the following HDs.

HD1-4: *The* ***pure area*** *is the area under the* $y = f(x)$ *curve that is described by any of the following two conditions throughout: (1)* $(\delta A = y \cdot \mathrm{dx} \geq 0)$ *or (2)* $(\delta A = y \cdot \mathrm{dx} \leq 0)$. Its upper *boarder is the* ***pure area's curve****. The pure area is a right-angled trapezoid that is based (by its right-angled leg) on the x-axis. Its other leg (mostly curved and/or not-right-angled) is the pure area's curve.* Areas $A_{1\text{-}2\text{-}2'\text{-}1'\text{-}1}$ and $A_{1\text{-}1'\text{-}2'\text{-}2\text{-}1}$ in Figure 1.3 are pure positive and negative ones, respectively.
HD1-5: A $y = f(x)$ curve that is characterized by $y \cdot dx \geq 0$ is called the ***pure (homogenous) positive area's curve***. The condition $y \cdot dx \geq 0$ abbreviates the two pairs of conditions: (1) $(y \geq 0 \text{ and } dx \geq 0)$ and (2) $(y \leq 0 \text{ and } dx \leq 0)$ those can be expressed additionally as *the algebraic signs of "y" and "*dx*" are identical through the pure positive area's curve.* Also, the area under a *pure positive area's curve* is characterized by $\delta A = y \cdot \mathrm{dx} \geq 0$ and is called the *pure positive area.* Pure positive *area's* curves do not include any (differential/integral) curve's *slices,* characterized by $\delta A = y \cdot \mathrm{dx} \leq 0$ and/or $\delta \mathrm{A} = \mathrm{y} \cdot \mathrm{dx} < 0$. Pure positive areas do not include any negative (differential/integral) area's slices.
HD1-6: A curve that is characterized by $y \cdot dx \leq 0$ is called the *pure (homogenous) negative area's curve*. The *condition* $(y \cdot dx \leq 0)$ abbreviates the two pairs of conditions: (3) $(y \geq 0 \text{ and } dx \leq 0)$ and (4) $(y \leq 0 \text{ and } dx \geq 0)$. These two pairs of conditions can be expressed additionally as *the algebraic signs "*≥ 0 *or* ≤ 0*" of "y" and "*dx*" are, for the pure negative area's curve, opposite to each other*. Also, the area under a *pure negative area's curve* is characterized by $\delta A = y \cdot \mathrm{dx} \leq 0$ and is called the *pure negative area*. Pure negative area's curves do not include any positive (differential/integral) curve's slices, characterized by $\delta A = y \cdot \mathrm{dx} \geq 0$ and/or $\delta \mathrm{A} = \mathrm{y} \cdot \mathrm{dx} > 0$. Pure negative areas do not include any positive (differential/integral) area's slices.

Nt1-16: Thermodynamics is much interested in three kinds of areas under curves (the total, pure positive, and pure negative; see HD1-5 and HD1-6). Therefore, thermodynamic analysis requires dealing with these areas as algebraic values. This means that NR1-1 and NR1-2 are much useful in discussing and determining these three kinds of areas under curves. The three relations previously described ($dx > 0$, $dx < 0$, and $dx = 0$) are usually reduced (in thermodynamics) into two ***inequality–equality relations*** ($\delta A \geq 0$ and $\delta A \leq 0$), and we shall refer to the signs used in them as ***the inequality–equality signs***.

HD1-7: *For a particular plane curve y = f(x), Curve's characteristic points are the points those are vital/important for its full analysis. Such analysis includes* ***curve's creation*** *and all related to it calculations and discussions.*

The full set of Curve's characteristic points can be divided into the following sub-sets:

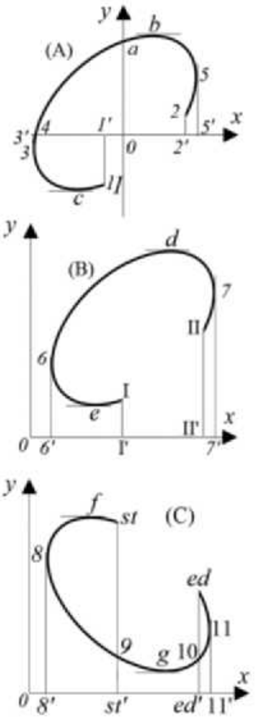

Figure 1.4. A schematic for determining the areas under curves

1) *points at which the* ***curve*** *starts, ends, or changes its mathematical expression (equation). These points can be additionally expressed/defined as "the points that limit the validity of the relations characterizing the curve and/or its parts".* For a continuous monoequation curve (see Figure 1.2 "elliptical curve 1-a-2"), these points are two (points 1 and 2), whereas for the triequation smooth curve 1-2-3-4 (see Figure 1.1a) these points are four (1, 2, 3, and 4) and for the miltiequation continuous refracted curve 1a-2a-3a-4a-5a (see Figure 1.1) these points are at least four (1a, 2a, 3a, and 5a) since we are not sure about the required number of equations to describe its two curves (1a-2a and 3a-4a-5a).
2) *Points those ease dealing-with (expressing, creating, calculating and discussing) the curve under study. These points are basic for the curve although they may not lie on it (e.g., circle's center does not lie on circular curve's line, but it with another circle's point allows creating the circle, whereas we need three circle's points to*

create the same circle. Also the simplest expression of a circle is that written for the case when its centre superposes with/on the origin of Cartesian coordinates).

3) *Points at which property sign inversions occur/may-occur*^{A} *on some properties of the curve such as curve's tangent*^{B} *and of the related to the curve properties such as the area under it.*

A. May-occur (instead of occur) eases the discussions for determining some characteristic points on the expense of increasing their number

B. The tangents of the curve at both sides of the refraction point are not the same slope.

The sign inversion of the differential $\delta A = ydx$ of the area (A) under the smooth curve $y = f(x)$ proceeds smoothly. Therefore, the necessary and insufficient condition for such inversion is $\delta A = ydx = 0$, which can be replaced by any of the conditions $y = 0, dx = 0$ and $y = dx = 0$. This means that: *"any point of Curve y = f(x) that comply with any of the conditions* $ydx = 0, y = 0, dx = 0$ *and* $y = dx = 0$ *can be Curve's sign inversion point/straight-line-segment if the algebraic signs of dA at both sides of this point/straight-line-segment are opposite (dA at one side is negative whereas at the other side is positive)".*

The above means that "the calculating equation for the area under a curve's $y = f(x)$ point/differential-part is $\delta A = ydx$. For the points at which curve's tangent is either x = const line (dx=0) or x-axis (y=0) $\delta A = ydx$ equals zero".

Curve's characteristic points at which the differential of the area under the curve $y = f(x)$ *equals zero* ($\delta A = ydx = 0$) *and* ***inverts*** *its sign* are (1) *curve's intersecting points with x*-axis (y = o), at which dx does not invert its sign (see Figures 1.4 "point 4" and 1-5 "points 10 and 12", where dx before, at, and after the mentioned points is positive. See also Figure 1.5 "points 15 and 17", where dx before, at, and after the mentioned points is negative). (2) Not lying-on/belonging-to *x*-axis (y = o), *curve's points,* at which dx inverts its sine (see Figure 1.4 "points 5, 6, 7, 8 and 11", at which dx=0 and inverts its algebraic sine whereas y ≠ o).

Curve's points at which the differential of the area under the curve $y = f(x)$ *equals zero* ($\delta A = ydx = 0$) *and* ***inverts*** *its sign* (see Figure 1.5 "points 1, 3, 5, and 7", at which dx=0 and inverts its algebraic sine whereas y = o) do not belong to *Curve's characteristic points. At* points 1, 3, 5, and 7, each of dx and y inverts its algebraic sign, therefore, the product ydx at each of the mentioned points $\delta A = ydx = 0$ does not invert its algebraic sine.

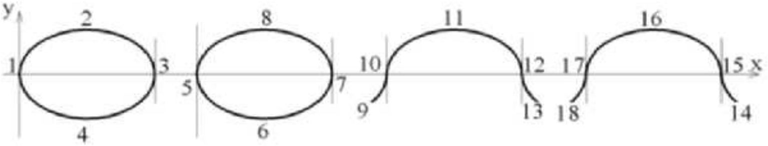

Figure 1.5. Curve's points 1, 3, 5, 7, 10, 12, 17, and 15 are specified by both (y = o) and (dx = o)

Nt1-17: If the complete curve (1–2, I–II, or *st* – *ed*, see Figure 1.4, where the numbers of the points are different from one subfigure A, B, and C to another) or its components suffer from heterogeneity in the sign of *y* and/or dx, then it is preferable to divide it (curve 1–2, I–II, or *st*–*ed*) into partial curves. The start and end points of the partial curves in the three illustrations in Figure 1.4 curves are (1) the intersection points of the curve with *x*-axis (point 4 $y_4 = 0$), (2) points 3, 5, 6, 7, 8, and 11, where dx = 0, and (3) the start and end points (1, 2, I, II, *st*, and *ed*) of the complete curve. Each of these partial curves belongs to one of the following four categories:

A. The first category curves comply with the conditions $(y \geq 0 \text{ and } dx \geq 0)$. These curves are located on and over *x*-axis $(y \geq 0)$ and are *x*-increasing ($dx \geq 0$, see Nt1-15, item 7). To this category belong partial curves (4–5), (6–7), (8–11), and their parts
B. The second category curves comply with the conditions $(y \leq 0 \text{ and } dx \leq 0)$. These curves are located on and under *x*-axis $(y \leq 0)$ and are *x*-decreasing ($dx \leq 0$, see Nt1-15, item 8). To this category belong partial curve (1–3) and its parts.
C. The third category curves comply with the conditions $(y \geq 0$ and dx $\leq 0)$. These curves are located on and over *x*-axis $(y \geq 0)$ and are *x*-decreasing $(dx \leq 0)$. To this category belong partial curves (5–2), (I–6), (7–II), (*st*–8), (11–*ed*), and their parts.
D. The fourth category curves comply with the conditions $(y \leq 0 \text{ and } dx \geq 0)$. These curves are located on and under *x*-axis $(y \leq 0)$ and are *x*-increasing $(dx \geq 0)$. To this category belong partial curve (3–4) and its parts.

The first and second categories' curves are *pure positive area's curves* (see HD1-5), whereas the third and fourth categories' curves are *pure negative area's curves* (see HD1-6).

Nt1-18: Equality signs in the "y" and "dx" relations mentioned previously mean that each of lines $y = 0$ and $x = \text{const}$ can belong to any category of curves.

Nt1-19: Curve categories A and C lie in the upper part of the diagram, while categories B and D lie in the lower part of the diagram. Categories A and C, which are characterized by $(y \geq 0)$, are the most common in thermodynamic calculations.

Nt1-20: Curve categories A and B are characterized by $(\delta A = y \cdot \mathrm{dx} \geq 0)$, which means that areas (1–3–3'–1'–1, 4–5–5'–4, 6–7–7'–6'–6, 8–9–10–11–11'–8'–8, 8–9–*st*–-8'–8, 9–10–*ed*'–*st*'–9, and 10–11–11'–*ed*'–10) under curves (1–3, 4–5, 6–7, 8–9–10–11, 8–9, 9–10, and 10–11) are *pure positive areas* (see HD1-5). On the other hand, curve categories C and D are characterized by $\delta A = y \cdot dx \leq 0$, which means that areas (3–4–3'–3, 5–2–2'–5'–5, I–6–6'–I'–I, 7–II–II'–7'–7, *st*–8–8'–*st*–-*st*, and 11–*ed*–*ed*'–11'–11) under curves (3–4, 5–2, I–6, 7–II, *st*–8, and 11–*ed*) are *pure negative areas* (see HD1-6). Actually, areas (1–3–3'–1'–1 and 3–4–3'–3) are over curves (1–3 and 3–4) not under them.

Nt1-21: Calculations of areas under a straight line segment are the simplest because its start and end points are enough to fully characterize the line.

Below some relations and rules for a straight line segment (st_ed), whose some arbitrarily chosen no-extreme points are sequentially numbered (2, 3,…, i+1,…, n–1 and n), where i is integer number that can be 1, n–1 or any integer number between them, whereas n–1 is the number of the parts of the straight line segment (st_ed) that partitioned by/with (n–2) arbitrarily chosen no-extreme points (such numbering makes st stands for point1 and n for ed):

$$\text{If } \mathrm{x_{ed}} - \mathrm{x_{st}} = 0 \Rightarrow \mathrm{x_{i+1}} - \mathrm{x_i} = 0,\ \mathrm{dx} = 0 \tag{1-38}$$

$$\text{If } \mathrm{x_{ed}} - \mathrm{x_{st}} > 0 \Rightarrow \mathrm{x_{i+1}} - \mathrm{x_i} > 0,\ \mathrm{dx} > 0 \text{ and vice versa} \tag{1-39}$$

$$\left.\begin{array}{l}\text{If } \mathrm{y_{ed}} > 0 \text{ and } \mathrm{y_{st}} > 0 \Rightarrow \mathrm{y_{i+1}} > 0,\ \mathrm{y_i} > 0 \ \&\ \mathrm{y} > 0 \\ \text{and vice versa}\end{array}\right\} \tag{1-40}$$

$$\left.\begin{array}{l}\text{If } \mathrm{y_{ed}} > 0 \;\&\; \mathrm{y_{st}} = 0 \;\Rightarrow \mathrm{y_{i+1}} > 0,\ \mathrm{y_i} \ge 0 \;\&\; \mathrm{y} \ge 0 \\ \text{and vice versa}\end{array}\right\} \tag{1-41}$$

$$\left.\begin{array}{l}\text{If } \mathrm{y_{ed}} > 0 \;\&\; \mathrm{y_{st}} < 0 \quad \text{or} \quad \mathrm{y_{ed}} < 0 \;\&\; \mathrm{y_{st}} > 0, \\ \text{then the straight line segment intersects } x-\text{axis.}\end{array}\right\} \tag{1-42}$$

OR1-8: *The slope of any (integral or differential) part of a straight line equals its own slope:*

$$\frac{\mathrm{y_{i+1}} - \mathrm{y_i}}{\mathrm{x_{i+1}} - \mathrm{x_i}} = \frac{\mathrm{y_{ed}} - \mathrm{y_{st}}}{\mathrm{x_{ed}} - \mathrm{x_{st}}} \tag{1-43}$$

which can be positive, negative, null, or $\pm\infty$ *. Here (ed) stands for the end point of the straight line and (st) for its start point. Also (i+1) stands for the end point of the straight line's part and (i) for its start point.*

OR1-9: *If the algebraic x-value of an imaginary point that moves along a straight line starting from its start point to its end one (1) increases, then the line is called x-increasing (*dx > *0) straight line and (2) decreases, then the line is called x-decreasing (*dx < *0).* This rule is also applicable for *y*-axis and leads to the following.

OR1-10: *For straight lines, Point with x = 0 cannot lie between two points with positive x-values or with negative ones, but it surely lies between a point with positive (x) and another with negative (x).* This rule is also applicable for y-axis and leads to the following.

OR1-11: *If* y_{st} *and* y_{ed} *are not zero and both are either positive or negative, then straight line segment (st_ed) does not intersect x-axis (* $\mathrm{y_{segment}} \neq 0$; *see Equation 1-40), but for opposite signs of* y_{st} *and* y_{ed}, *the straight line segment surely intersects x-axis (see Equation 1-42).* This rule is also applicable for *y*-axis.

As mentioned previously, calculations of areas under a linear partial line are the simplest. Let us have a look at straight line's illustration on *y_x* plane (or on the coordinates of start and end points of a straight line segment) and determine its characteristics: (1) is it *x*-increasing ($x_{ed} > x_{st}$), x-decreasing ($x_{ed} < x_{st}$), or constant-*x*? (2) Whether its dx_sign is the same as $(x_{ed} - x_{st})\_sign$, and (3) whether the straight line segment intersects *x*-axis (y_{st} and y_{ed} have different/opposite signs) or not, and if not, (4) does it coincide/superpose with *x*-axis (if both y_{st} and y_{ed} are equal to zero) or contacts it in one point (if only one of y_{st} and y_{ed}

equals zero). If you decide to continue, apply the following equation for pure straight line segments that may touch, but do not intersect x-axis

$$A_{\text{under (st\_ed) streight line}} = (y_{ed} + y_{st}) \cdot (x_{ed} - x_{st}) / 2 \qquad (1\text{-}44)$$

For lines those intersect *x*-axis, you first need to determine the abscissa ($x_{y=0}$) of the intersection point between the line under consideration and *x*-axis, and then apply the equation

$$A_{\text{under (st\_ed) streight line}} = \frac{y_{ed} \cdot (x_{ed} - x_{y=0})}{2} + \frac{y_{st} \cdot (x_{y=0} - x_{st})}{2} \qquad (1\text{-}45)$$

where items in baskets are the areas under the parts of the straight line that are located above and below the *x*-axis. These areas are pure ones. One of them is pure positive and the other is pure negative; therefore, when we need to deal with pure areas, this area must be divided into two positive and negative pure areas.

Nt1-22: We shall refer to the expression type $Z \geq 0$ as the pure positive Z-function and to $Z \leq 0$ as the pure negative Z-function, where Z is a variable or a function. Also we shall refer to the set of expressions type (1) $Z_1 \geq 0,\ Z_2 \geq 0,\ Z_3 \geq 0,$ etc. as the pure positive Zi-functions, (2) $Z_1 \leq 0,\ Z_2 \leq 0,\ Z_3 \leq 0,$ etc. as the pure negative Zi-functions, and (3) $Z_1 \geq 0,\ Z_2 \leq 0,\ Z_3 \geq 0,\ Z_4 \geq 0,\ Z_5 \leq 0,$ etc. as the pure Zi-functions, where index i is *a positive integer number and* Zi is a variable or a function.

Using HD1-5 and HD1-6, the relations (1-49 through 1-55), and the available data concerning the coordinates of the extreme points (*st* and *ed*) of a straight line, we obtain the following new and other authors' rules:

NR1-3: If the algebraic signs of $[y_{ed},\ y_{st},\ and\ (x_{ed} - x_{st})]$ -functions are identical, then the area under the straight line segment (*st_ed*) is *pure positive* (see HD1-5). Also, the area under the straight line segment (i_i+1) will be *pure positive* when $[y_{i+1},\ y_i,\ and\ (x_{i+1} - x_i)]$ have identical algebraic signs.

NR1-4: If the algebraic signs of (y_{ed} and y_{st}) are identical, while the algebraic sign of the difference $(x_{ed} - x_{dt})$ is opposite to their sign, then the area under the straight line segment (*st_ed*) is *pure negative* (see HD1-6). Also, the area under the straight line segment (*i_i*+1) will be *pure negative* when the algebraic signs of (y_{i+1} and y_i) are identical, while the sign of $(x_{i+1} - x_i)$ is opposite to their sign.

NR1-5: If the algebraic signs of (y_{ed} and y_{st}) are opposite to each other, then the area under the straight line segment (*st_ed*) is not pure. It consists of two parts (a pure positive part and a pure negative one; see HD1-5 and HD1-6). Also, the area under the straight line segment (*i_i*+1) will not be pure when the algebraic signs of (y_{i+1} and y_i) are opposite to each other.

NR1-6: If ($y_{ed} = y_{st} = 0$), then the area under the straight line segment (*st_ed*) is null (see Equation 1-44). Also, if ($y_{i+1} = y_i = 0$), then the area under the straight line segment (*i_i*+1) is null. As any curve can be divided into partial parts, which are defined by $(\delta A = y \cdot \mathrm{dx} \geq 0)$ and/or $(\delta A = y \cdot \mathrm{dx} \leq 0)$ each, then ***the area under the total curve (A*****tot*****) will be calculated using the equation***

$$A_{\mathrm{tot}} = \sum_{i=1}^{n} A_i \tag{1-46}$$

where A_i is the algebraic value of the area under curve's partial part number (*i*) and *n* the total number of curve's partial parts.

Let us determine the areas under curves (1–2, I–II, and *st–ed*, in Figure 1.4):

From Nt1-19, we obtain A_{under} $A_{\mathrm{under}_{6-7}} = A_{6-7-7'-6'-6} > 0$, $A_{\mathrm{under}_{I-6}} = A_{I-6-6'-I'-I} < 0$, and $A_{\mathrm{under}_{7-II}} = A_{7-II-II'-7'-7} < 0$. Therefore:

$$\left.\begin{aligned} \mathrm{A}_{\mathrm{under}_{6-7}} &= \mathrm{A}_{6-7-7'-6'-6} = \left|\mathrm{A}_{\mathrm{under}_{6-7}}\right| = \left|\mathrm{A}_{6-7-7'-6'-6}\right| \\ \mathrm{A}_{\mathrm{under}\ 7-\mathrm{II}} &= \mathrm{A}_{7-\mathrm{II}-\mathrm{II}'-7'-7} = -\left|\mathrm{A}_{\mathrm{under}\ 7-\mathrm{II}}\right| = -\left|\mathrm{A}_{7-\mathrm{II}-\mathrm{II}'-7'-7}\right| \\ \mathrm{A}_{\mathrm{under}_{\mathrm{I}-6}} &= \mathrm{A}_{\mathrm{I}-6-6'-\mathrm{I}'-\mathrm{I}} = -\left|\mathrm{A}_{\mathrm{under}_{\mathrm{I}-6}}\right| = -\left|\mathrm{A}_{\mathrm{I}-6-6'-\mathrm{I}'-\mathrm{I}}\right| \end{aligned}\right\} \tag{1-47}$$

It is obvious from Figure 1.4 that curve $(I-6-7-II)$ consists of the three curves $(I-6), (6-7)$, and $(7-II)$ exclusively; therefore, taking into consideration Equation (1-47), we obtain

$$\begin{aligned} \mathrm{A}_{\mathrm{under}_{I-6-7-\mathrm{II}}} &= \mathrm{A}_{\mathrm{under}_{\mathrm{I}-6}} + \mathrm{A}_{\mathrm{under}_{6-7}} + \mathrm{A}_{\mathrm{under}\ 7-\mathrm{II}} \\ &= -\left|\mathrm{A}_{\mathrm{I}-6-6'-\mathrm{I}'-\mathrm{I}}\right| + \left|\mathrm{A}_{6-7-7'-6'-6}\right| - \left|\mathrm{A}_{7-\mathrm{II}-\mathrm{II}'-7'-7}\right| \end{aligned} \tag{1-48}$$

It is clear from subfigure 1.4(B) that the absolute value of area $(6-7-7'-6'-6)$ is much bigger than the sum of the absolute values of the two areas $(I-6-6'-I'-I)$ and $(7-II-II'-7'-7)$, which leads to conclude that the right side of Equation (1-48) is positive and equals $\left|A_{I-6-7-II-II'-I'-I}\right|$, and therefore, Equation (1-48) becomes

$$\begin{aligned}
\mathrm{A}_{\mathrm{under}_{\mathrm{I-6-7-II}}} &= \left|\mathrm{A}_{\mathrm{I-6-7-II-II'-I'-I}}\right| = -\left|\mathrm{A}_{\mathrm{I-6-6'-I'-I}}\right| + \\
&\quad +\left|\mathrm{A}_{\mathrm{6-7-7'-6'-6}}\right| - \left|\mathrm{A}_{\mathrm{7-II-II'-7'-7}}\right| \\
&= \mathrm{A}_{\mathrm{I-6-7-II-II'-I'-I}} = \mathrm{A}_{\mathrm{6-7-7'-6'-6}} + \\
&\quad + \mathrm{A}_{\mathrm{I-6-6'-I'-I}} + \mathrm{A}_{\mathrm{7-II-II'-7'-7}} > 0 \qquad (1\text{-}49)
\end{aligned}$$

Similarly, we obtain for the other curves in subfigure 1.4(A) the following equations

$$\mathrm{A}_{\mathrm{under}_{1-3-4}} = \left|\mathrm{A}_{1-3-3'-1'-1}\right| - \left|\mathrm{A}_{3-4-3'-3}\right| = \mathrm{A}_{1-3-4-1'-1} > 0 \qquad (1\text{-}50)$$

$$\mathrm{A}_{\mathrm{under}_{4-a-5-2}} = \left|\mathrm{A}_{4-a-5-5'-4}\right| - \left|\mathrm{A}_{5-2-2'-5'-5}\right| = \mathrm{A}_{4-a-5-2-2'-4} > 0 \qquad (1\text{-}51)$$

$$\mathrm{A}_{\mathrm{under}_{1-2}} = \left|\mathrm{A}_{1-3-4-1'-1}\right| + \left|\mathrm{A}_{4-a-5-2-2'-4}\right| = \mathrm{A}_{1-3-4-a-5-2-2'-1'-1} > 0 \qquad (1\text{-}52)$$

And, for subfigure 1.4(C):

$$\begin{aligned}
\mathrm{A}_{\text{st-8-9-10-11-ed-ed'-st'-st}} &= (\mathrm{A}_{\text{st-8-8'-st'-st}} + \mathrm{A}_{\text{8-8-st'-8'-8}}) + \mathrm{A}_{\text{9-10-ed'-st'-9}} + \\
&\quad + (\mathrm{A}_{\text{10-11-11'-ed'-10}} + \mathrm{A}_{\text{11-ed-ed'-11'-11}}) \\
&= \mathrm{A}_{\text{st-8-9-st}} + \mathrm{A}_{\text{9-10-ed'-st'-9}} + \mathrm{A}_{\text{10-11-ed-10}} \\
&= -\left|\mathrm{A}_{\text{st-8-9-st}}\right| + \left|\mathrm{A}_{\text{9-10-ed'-st'-9}}\right| - \left|\mathrm{A}_{\text{10-11-ed-10}}\right| \qquad (1\text{-}53)
\end{aligned}$$

Nt1-22A: The first and third/fourth sides of Equation (1-53) insure that in the contrary with the areas under curves (1–2) and (I–II), the *area under curve* $(st-8-9-10-11-ed)$ *in the same Figure 1.4 does not consist of only one closed area, but it consists of a train of three closed areas (the positive* $A_{9-10-ed'-st'-9}$ *and the negative* $A_{st-8-9-st}$ *and* $A_{10-11-ed-10}$*), each two of which are (as train wagons) interconnected to each other by one point.* This is because the plane closed curve that surrounds this whole area is not simple (it intersects/crosses itself; see OD1-7).

Nt1-23: The start and end points of a closed curve coincide and accordingly their x-constant lines coincide, reducing the area between them to zero, and the area under the curve reduces to the area inside it.

OR1-12: *The area under a plane simple closed curve equals that surrounded by it area.*

NR1-7: *The area under a more than plane simple closed curve equals the sum of the area surrounded by the first closed part and that located under its second open part* (see ND1-2).

Nt1-24: Each of the previously discussed no-chain areas under curves (whether it complies with any of ($\delta A = y \cdot dx \leq 0$) or ($\delta A = y \cdot dx \geq 0$) conditions or not) and also each area that is a component (ring) of a chain area under a curve have a fixed closed directional perimeter, the direction of which is resulted from the direction of the curve that borders it and can be clockwise or anticlockwise. If an imaginary point moves along the perimeter of the area under the curve so that its motion along the bordering curve coincides with the curve's direction, then its circulation along the area's perimeter can be either clockwise or anticlockwise. All previously discussed positive areas under curves (whether they were partial or not) have perimeters with clockwise direction, and all previously discussed negative ones have perimeters with anticlockwise direction.

NR1-8: *The area under a curve or under a part of it is positive (not pure positive) if its perimeter has a clockwise direction, else it is a negative one.*

From Equations (1-49) and (1-53), we conclude the following.

NR1-9: *The algebraic sum of areas under neighboring curves is a new-closed area/new-chain-of-closed areas* $A_{st-ed-ed'-st'-st}$ *("st" stands for the curve's start point "1, I, or st" and "ed" stands for its end point "2, II, or ed") that is bordered by (1) the total curve (st–ed) that results from interconnecting these neighboring curves, (2) the x = constant line sector that connects the end point (ed) of the total curve with its x-projection (ed'), (3) the straight-line sector that interconnects the x-projections of the end and start points (ed' and st') of the total curve, and (4) the x = constant line sector that connects the start point (st) of the total curve with its x-projection (st').*

The way of calculating areas under curves (the graphical, analytical, and tabular) depends usually on the available data.

1.2.1.1 Graphical calculation of the area under y = f(x) curve

For this paragraph: 1) we shall number the projections of curves' points on the coordinate axes with/by their own numbers subscripted with/by the label/name of the coordinate axes (e.g., point $2_{x'}$ is the projection of point 2 on x'-axis), 2) our reference is Figure 1.6 that consists of five subfigures (I through V). We shall refer to any of these 5 subfigures within this paragraph directly without naming their figure's number

If the relation $y = f(x)$ is given graphically, then the priority will be given to the graphical calculation.

Let us discuss the calculation of the area under the unnumbered, smooth, and dotted curve that illustrated in Subfigure I It is clear from Subfigure I that this curve is x-increasing $(dx \geq 0)$ one that is located over x-axis $(y > 0)$, and accordingly (see HD1-5) it is a *pure positive curve,* so it does not suffer from heterogeneity in the sign of (y) and/or (dx), and the area under it is a *pure positive area*. Therefore, there is no need to divide it into partial curves before measuring it (see Nt1-16). The area under this curve and under any of its parts can be measured directly in a single-step measurement from Subfigure I using an existing planimeter (x-axis in Subfigure I is not displaced; it is congruent with y = 0 line). In the absence of instruments for measuring areas, and since the analytical

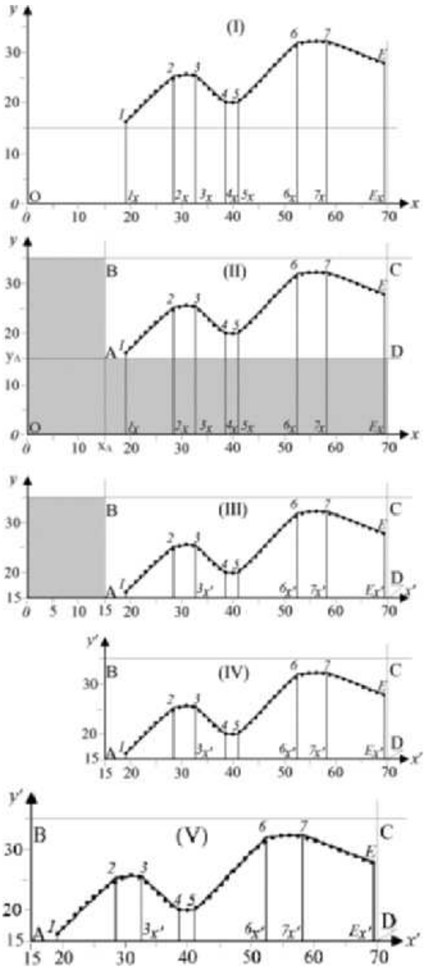

Figure 1.6. 1) Transforming the no compact graph into a compact one. 2) Approximating the unnumbered, smooth, and dotted curve by a numbered refracted line that consists of continuous straight line segments.

equation of the mentioned curve is unknown, the graphical calculation of the area under the dotted curve can be done as follows: 1) connect each two neighboring points with a straight line (this transforms the dotted line that consists of n points into a refracted one that consists of "n-1" straight line segments), 2) create from each line's point a vertical to x-axis line that ends on it (on x-axis). In so doing the area under the dotted line divides into (n-1) trapezoids (rectangles are special cases of trapezoids) and therefore (see Equation 1-44),

$$A_{\text{under dotted line}} = \sum_{i=1}^{n-1}\left[\frac{y_{i+1}+y_i}{2}\cdot(x_{i+1}-x_i)\right] = \sum_{i=1}^{n-1}\left(y_{\text{avg}i+1\ i}\cdot\Delta x_{i+1\ i}\right) \qquad (1\text{-}54)$$

In this equation, the point with number "i = 1" is point 1, which stands for the beginning of the replacing curve, and the point number "i = n + 1" is point E, which stands for the end of the replacing curve. The coordinates of line's points in this equation can be measured or read from line's graph.

The consumed time in the above described graphical calculation of the area under a dotted curve depends on the number of curve's points (n) that participate in calculations, the greater number of points (n) the longer consumed time and accordingly the greater exactness. Therefore, and since the dotted curve in Subfigure I consists of 45 points, it is impractical to calculate the area under it according to the above description. One of the practical ways is as follows: (1) Without any calculations we graphically approximate the plane dotted smooth line with/by a plane refracted line that consists of straight line segments to comply with the relation: $A_{URL} \approx A_{UDL}$ that transforms for subfigure I into: Area (1-2-3-4-5-6-7-E)$\approx A_{UDL}$. The indexes URL and UDL stand for under the refracted line and under the dotted line respectively. (2) We shall refer to the dotted line by *original curve* and to the refracted line by *replacing curve*. The start point of the original curve may not coincide with point 1, but it lies on x-constant line ($1\text{–}1_x$) or on its extension. Also, the end point of the original curve may not coincide with point E, but it lies on x-constant line ($E\text{–}E_x$) or on its extension. As can be seen from Subfigure I, (3) the original and the replacing curves are in good convergence, and (4) the replacing curve and the area under it are also *pure positive curve and area*, respectively. The decrease in the number (n) of the parts (straight-line segments) constituting the replacing curve eases the calculation of the area under it and simultaneously decreases the accuracy of the calculation. The calculation here will be executed according to Equation (1-54).

The following discussions require introducing the following definitions

> **HD1-7A:** We shall refer to: 1) the rectangle, whose vertices are the intersection points of the closest to plane graph's curve(s) gridlines; those may contact graph's curves without intersecting them as the ***main graph's frame***. Rectangle A-B-C-D-A in subfigures II through IV is ***graph's 1-6 main frame***, 2) the rectangular area that includes graph's curves (the main graph's frame) and the coordinate axes as the ***plane graph's range***. This means that the sides of each of the plane graph's range and the main graph's frame are parallel, whereas some of them (these sides) are parallel to the horizontal and the others to the vertical coordinate axes. 3) The graph, whose main and range frames are congruent as *the* ***compact graph***. A coordinate axis of the compact graph is either a crosser or a tangent to its main frame. The graphs in subfigures IV and V are compact; but that in subfigure V is a magnified copy of subfigure IV to occupy the same width as subfigure I.

Let us transform the no compact graph in subfigure (I) into a compact one (see Subfigure IV). To determine graph's 1-6 main frame we determine graph's gridlines that intersect curve 1-2-3-4-5-6-7-E. According to the labels of x and y axes these gridlines are y=20 [y-units], y=25 [y-units] and y=30 [y-units] that located between curve's 1-2-3-4-5-6-7-E no intersecting gridlines y=15 [y-units] and y=35 [y-units] in addition to Gridlines x=20 [x-units], x=25 [x-units], x=30 [x-units], x=35 [x-units], x=40 [x-units], x=45 [x-units], x=50 [x-units], x=55 [x-units], x=60 [x-units], and x=65 [x-units], that located between curve's 1-2-3-4-5-6-7-E no intersecting gridlines x=15 [x-units] and x=70 [x-units]. Thus the closest to graph's 1-6 curve 1-2-3-4-5-6-7-E gridlines those may contact the curves without intersecting them are grid lines y=35 [y-units], x=70 [x-units], y=15 [y-units] and x=15 [x-units] and therefore, the graph's 1-6 main frame is rectangle A-B-C-D-A.

To reach the compact graph (see subfigure IV), we cut-off/exclude /eliminate the graph's areas that are outside rectangle A-B-C-D-A. As seen from Subfigures II and IV the sum of these unnecessary areas is the highlighted area in Subfigure II. The existing mathematical methods allow eliminating this highlighted area without decreasing graph's calculating effectiveness. This can be done by: (A) cutting off the highlighted area under line y=15 [y-units] with deleting y-axis' part that borders it. Such cutting off results 1) disappearing each of the low part of y-axis and the gridlines in the cut off area, 2) displacing vertically x-axis with its notations to superpose on/with gridline y=15 [y-units] (see subfigures II and III) and 3) replacing each x in/on the displaced x-axis’ notations by x' (x-axis’ notations that confuse the view of subfigure III were deleted from it; because they can be restored any time from the label of the coordinate axis “e.g., x'” and curve’s points’ numbers “see the first lines of §1.2.1.1”) and (B) cutting off the highlighted area between the remaining of y-axis and line A-B that superposes on/with gridline x=15 [x-units] (see Subfigure IV). Such cutting off results 1) disappearing the gridlines in the cut off area, 2) displacing horizontally the remaining of y-axis with its notations to superpose on/with gridline x=15 [x-units] or line A-B (see Subfigures III and IV) and 3) replacing y in the displaced y-axis label by y'. The resulted compact graph in subfigure IV allows 1) reading the coordinates of any point of the represented curve 1-2-3-4-5-6-7-E and 2) calculating analytically and graphically the areas between the represented curve and each of the eliminated x-axis (Area 1-2-3-4-5-6-7-E-E_x-1_x-1) and the displaced x'-axis (Area 1-2-3-4-5-6-7-E-$E_{x'}$-$1_{x'}$-1). Point $1_{x'}$ exists in subfigure IV without labeling; it is the projection of point 1 on x'-axis, whose ordinate is (15 [y-units] or 15 [y'-units]).

In the following we shall prove that the compact graph in subfigure IV fully replaces the full graph in subfigure I. The coordinates of any point of the refracted line (1–2–3–4–5–6–7–E) or of its original dotted line can be easily read/obtained from any of the two subfigures, Therefore, calculating area (Area 1-2-3-4-5-6-7-E-E_x-1_x-1) that is the area between the refracted line (1–2–3–4–5–6–7–E) and the original x-axis can be done using the same equation (1-54) for both subfigures. The measurement of the mentioned area using an existing planimeter with taking in consideration graph’s scale does not need any other steps for subfigure I; but the measurement of area 1-2-3-4-5-6-7-E-E_x-1_x-1 using Subfigure IV and an existing planimeter cannot be done in the absence of the relationship between the mentioned area and other areas, those exist in Subfigure IV. From the comparison between Subfigures I and IV and taking in consideration the creation of figure 1.6 we find that x'-axis superposes with/on gridline y=y'=15 and therefore,

$$A_{1\text{-}2\text{-}3\text{-}4\text{-}5\text{-}6\text{-}7\text{-}E\text{-}Ex\text{-}1x\text{-}1} = A_{1\text{-}2\text{-}3\text{-}4\text{-}5\text{-}6\text{-}7\text{-}E\text{-}Ex'\text{-}1x'\text{-}1} + y_A\,(x_E - x_1)$$

$$= A_{1\text{-}2\text{-}3\text{-}4\text{-}5\text{-}6\text{-}7\text{-}E\text{-}Ex'\text{-}1x'\text{-}1} + y_A\,(x'_E - x'_1) \qquad (1\text{-}55)$$

All constituents of the right side of this equation can be obtained from Subfigure IV and therefore we can measure area 1-2-3-4-5-6-7-E-E_x-1_x-1 through: 1) Measuring the existing in Subfigure IV area 1-2-3-4-5-6-7-E-$E_{x'}$-$1_{x'}$-1 using an existing planimeter with taking in consideration graph’s scale. 2) Substituting in equation 1-55 the values of the measured area 1-2-3-4-5-6-7-E-E$_{x'}$-1$_{x'}$-1 in y'·x' units, y'$_A$ in y' units, x'$_E$ in x' units and x'$_1$ in x' units we obtain the measured value of area 1-2-3-4-5-6-7-E-E$_x$-1$_x$-1

Nt1-24A: No contradiction in equation (1-55). Ex is the label/name of the point that is the x-projection of point E of the refracted curve, whereas x$_E$ is the x-value of point E.
Nt1-24B: For closed curves the start and end points (1 and E) are congruent; therefore: $x_E - x_1 = x'_E - x'_1 = x''_E - x''_1 = 0$,
$y_{x''} \cdot (x''_E - x''_1) = y_{x'} \cdot (x'_E - x'_1) = y_x \cdot (x_E - x_1) = 0$, and
$A_{1-2-3-4-5-6-7-E-E'-1'-1} = A_{1-2-3-4-5-6-7-E-b-a-1} = A_{1-2-3-4-5-6-7-E-b'-a'-1}$,
which insures rules (OR1-12 and NR1-7).

Nt1-25: To increase the accuracy of the graphical calculation, the scale of the graphs and diagrams is usually increased. Often, this procedure is accompanied (associated) with transforming the no compact graph into a compact one.
Nt1-25A: Although the readings of the coordinates of any of diagram's point are the same, whether they were obtained using the original or the displaced coordinate axes, the distance from a point in the diagram to the axis changes with changing the displacement of the axis; therefore, with the exception of obtaining the coordinates of graph's points and/or calculating the areas of plane simple closed curves (see OR1-12), the displacement of the coordinate axes must be taken into consideration in graphical calculations.

Conclusion 1-1: Calculating the real area under function $y = f(x)$ that is represented in scale on a graph must be executed with taking into consideration the scale of the graph (see Nt1-11) and the displacement of the coordinate axes (see Nt1-25), see also (§1.2.1).

1.2.1.2 Analytical calculation of the area under y = f(x) curve

If the function $y = f(x)$ is given analytically, then the priority for calculating the area under this curve will be the analytical one because it is accurate and rarely difficult for execution (this depends on the complexity of the $y = f(x)$ function). To whatever extent the curve is complicated, the given data divide, if necessary, the multiequation curve into single-equation ones (parts) and almost completely defines each part by its equation and limits, but the calculator will have to determine the points of the inversion in the sign (≥ 0 or ≤ 0) of y or dx of a curve. Therefore, the analytical calculation of the area under each part is often simple. The area under the whole curve, which equals the algebraic sum of all partial areas under it, is also simple. To calculate each of the sum of all positive differential areas and the sum of all negative differential areas under the whole curve, we need to determine all points of the inversion in the sign of the expression $y{\cdot}\mathrm{dx}$.

> **Nt1-26:** According to Nt1-19 (see the underlined), the function y = f(x) is often positive in thermodynamic calculations; therefore, the determination of the area's sign is often reduced to the determination of dx_sign.

1.2.1.3 Tabular calculation of the area under y = f(x) curve

If the function $y = f(x)$ is given tabularly, then the priority for calculating the area under this curve will be the tabular one. The order of numbering each of the curve's points and the table's lines must be sequential. Therefore, the first line of the table (line no. 1) contains the two (x_1 and y_1) coordinates of the first (start) point of the curve, the second contains x_2 and y_2 of the second point, line (i) contains x_i and y_i of point no. (i), and so on. If the total number of the tabulated points is undefined, the last/end point of the curve can be numbered/symbolized as "E" and the point before it is (E – 1), … The corresponding line to E-point can be numbered as E-line and its coordinates' symbols can be x_E and y_E. If we imaginarily plot the points of the curve, which represents the tabular data, on a *y_x* plane, number each point by the table's line number that includes its coordinates, connect each neighboring two points by/with a straight line, and create x = constant lines from each point until x-axis, then we obtain a graph that is similar to the illustration in Figure 1.6. Obviously, the area under the plotted, on the basis of the given, tabular refracted curve can be calculated using Equation (1-54). This means that (1) the tabular calculation of the area under the given tabular curve does not require any graphical creations, it requires only applying Equation (1-54). If the partial lines (*i_i+1*) of the refracted tabular curve (*1–2–···–i–···–E*) are straight ones, then Nt1-22 and NR1-3 through NR1-6 can be applied to execute the calculations. There is no need here to determine the points at which dx inverts its sign as it keeps this sign throughout the straight-line sector. But the inversion in the sign (≥ 0 or ≤ 0) of y is still needed (we need to find the points with $y = 0$ that are situated between neighboring points with different y-signs). Each $y = 0$ point divides the straight-line sector (to which it belongs) into two *pure (positive and negative) area's curves* (straight lines) (see HD1-5 and HD1-6).

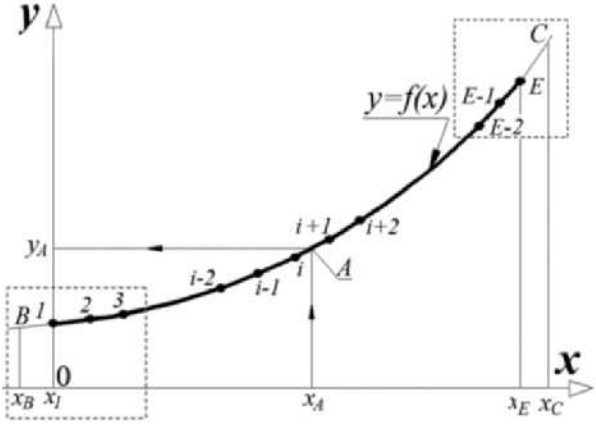

Figure 1.7. The graphical interpolation and extrapolation of the tabular data of a plane curve.

1.2.2 TABULAR DETERMINATION OF y-VALUE VERSUS A GIVEN x-VALUE

From §s 1.2.1.1 and 1.2.1.3 and Figure 1.7, whose symbolizing system is the same as that explained in §1.2.1.3, we conclude the following:

1) The procedure of graphical determination of the value of $y = f(x)$ versus any x-value within graph's x-interval $(x_1 - x_E)$ is also time-consuming and provide the same exactness value that depends on the graph's scale (see the determination of y_A in Figure 1.7).
2) Tabular determination of y-value versus any x-value that exists in the lines of the table is exact, very comfortable, and least time-consuming.

Nt1-27: It is clear from Figure 1.7 that a small part (i_i+1) of the curve (1_E), whose x-interval $(x_i\_x_{i+1})$ includes the given x_A-value is enough to determine the value of $y_A = f(x_A)$. *The creation of this small part (i_i + 1) for exact/almost-exact calculating* y_A*-value versus the given* x_A*-value requires (1) knowing the coordinates of points (i – 1 through i + 2) and (2) creating a curve (i – 1 through i + 2), to which we shall refer as* the interpolation *line, in a large scale.* Such partial graph creation warrantees obtaining y_A*-value's* exactness that is a little bit lower than the exactness of points (i and $i + 1$). This procedure is called ***nonlinear interpolation (or simply interpolation)***. Thus, to be exact, the nonlinear interpolation requires using at least three lines of the exact table of function $y = f(x)$, whose x-interval includes the given x-value (whose $x_{\max}$ and $x_{\min}$ comply with the condition $x_{\max} > x_{\text{given}} > x_{\min}$).

For the cases that allow considering linear relationship $y = f(x)$ between any two table's successive lines (e.g., i and $i + 1$), function's $y = f(x)$ curve (1_*E* in Figure 1.7) becomes a refracted line whose refraction points are defined by the table's lines and whose parts are the straight line segments (i_$i + 1$) that interconnect each two successive refraction points. We shall refer to this refracted line as the ***linear interpolation line***, and to the procedure applied to obtain y-value versus a given x-value that uses the linear interpolation line or a part of it as the ***linear interpolation***. In this case, the equation of any of the straight line segments (i_$i + 1$) is the well-known equation of the line passing through the two different points $i(x_i, y_i)$ and $i(x_{i+1}, y_{i+1})$, that is,

$$\frac{y - y_i}{y_{i+1} - y_i} = \frac{x - x_i}{x_{i+1} - x_i} \text{ from which } y = y_i + \left(y_{i+1} - y_i\right)\frac{x - x_i}{x_{i+1} - x_i} \qquad (1\text{-}56)$$

where x is the given x-value that lies between x_i and x_{i+1}.

This means that, ***when the table's data allow linear interpolation,*** it is sufficient, for tabular determination of y-value versus any x-value that included within the table's x-interval, to substitute table's successive lines (i and $i + 1$), whose x-interval includes the given x-value, with the given x-value in Equation (1-56) and calculate the required y-value.

Nt1-28: If the given x-value is outside the x-interval (x_1_x_E) of the table (this eliminates the possibility of using interpolation) and in the absence other possibilities for calculations, we find ourselves obliged to extend the tabular curve (1–E) so that the given x-value is inside the x-interval (x_B_x_C) of the extended curve "B–C" (see Figure 1.7) and determine the sought y-value through curve "B–C." This procedure is known as ***extrapolation*** and has two disadvantages: (1) there is no warrantee that the extension of the tabular curve (1 – E) is done correctly, and (2) it is not practical to create the whole curve "B–C" in a considerably large scale to execute one relatively exact calculation; therefore, the extrapolation that must be done only as a last resort must not require (1) large extension of the tabular curve and (2) must not involve all tabular lines. Thus, using at least three of the extreme lines of the table that adjoin the given x-value, we create the side part of curve

1_*E* that is closer to the point under consideration/to be found (B or C; see Figure 1.7) and somehow extend it so that it includes the point under consideration. In other words, we magnify the appropriate of the two dashed rectangles in Figure 1.7.

Note that the extrapolation can also be used as a last resort in the case of presenting the function $y = f(x)$ graphically.

The author's advice is creating the extensions (B_1 and/or E_C) as straight tangents to curves (1_2_3 and/or E-2_E-1_E), respectively.

Using exact and comprehensive/detailed y-value versus x-value tables with considering the linear relationship $y = f(x)$ between any two table's successive lines (e.g., i and $i + 1$) in calculating the areas under $y = f(x)$ curves gives almost exact results. Equation (1-54) is used here. We remind here that Equation (1-56) is used in calculating almost exactly the y-values versus given x-values from the same exact and comprehensive tables.

Using exact and abbreviated/not-detailed y-value versus x-value tables with Equations (1-54) and (1-56) or any of them in calculations gives approximate results. Almost exact results can be reached when using these tables to obtain the large scale graph of the nonlinear ***interpolation line*** $y = f(x)$ or its part that will be the base to execute the required calculations graphically as explained previously (see §**1.2.1.1** and the underlined text in Nt1-25 and Nt1-27).

1.2.3 DIFFERENCE BETWEEN TWO FUNCTIONS OF THE SAME VARIABLE

It is obvious that the sum/difference of two plane functions of the same variable [$y_2 = f_2(x)$ and $y_1 = f_1(x)$] equals/is a third plane function of the same variable [$y_3 = f_3(x)$] or

$$y_3 = y_2 \pm y_1 = f_3(x) \tag{1-56a}$$

Let us analyze Equation (1-56a) for the special case when each of the resulted function $y_3 = f_3(x)$ and any of the two added/subtracted functions [$y_2 = f_2(x)$ or $y_1 = f_1(x)$] are linear [e.g., $y_3 = f_3(x) = C_1 + C_2x$ and $y_1 = f_1(x) = A_1 + A_2x$]. Equation (1-56a) becomes (here $A_1, A_2\ldots, B_1, B_2\ldots$, and $C_1\ldots, C_2\ldots$ are the constants of functions y_1, y_2, and y_3, respectively):

$$\begin{aligned} y_2 &= y_3 \mp y_1 = (C_1 + C_2 \cdot x) \mp (A_1 + A_2 \cdot x) \\ &= (C_1 \mp A_1) + (C_2 \mp A_2) \cdot x = B_1 + B_2 \cdot x \end{aligned} \tag{1-57}$$

which is a linear equation.

A similar analysis on the inverse functions [$x_1 = f_1^{-1}(y)$, $x_2 = f_2^{-1}(y)$, and $x_3 = f_3^{-1}(y)$] of functions [$y_1 = f_1(x)$, $y_2 = f_2(x)$, and $y_3 = f_3(x)$] leads to

$$\begin{aligned} x_2 &= x_3 \mp x_1 = (C'_1 + C'_2 \cdot x) \mp (A'_1 + A'_2 \cdot x) \\ &= (C'_1 \mp A'_1) + (C'_2 \mp A'_2) \cdot x = B'_1 + B'_2 \cdot x \end{aligned} \tag{1-58}$$

where A'_1, A'_2…, B'_1, B'_2…, and C'_1…, C'_2… are the constants of inverse functions x_1, x_2, and x_3, respectively.

Equations (1-57) and (1-58) can be read as follows.

HR1-1: *If two of the three plane functions/inverse-functions of the same variable that constitute a subtracting/summing equation type* [$y_2 = y_3 \mp y_1$ or $x_2 = x_3 \mp x_1$] *are linear functions, then the third function/equation will invariably be a linear one that is represented in the y_x plane, when its second constant does not equal 0, by an inclined straight line, and when it equals 0, by function's/inverse-function's constant straight line.* Or:

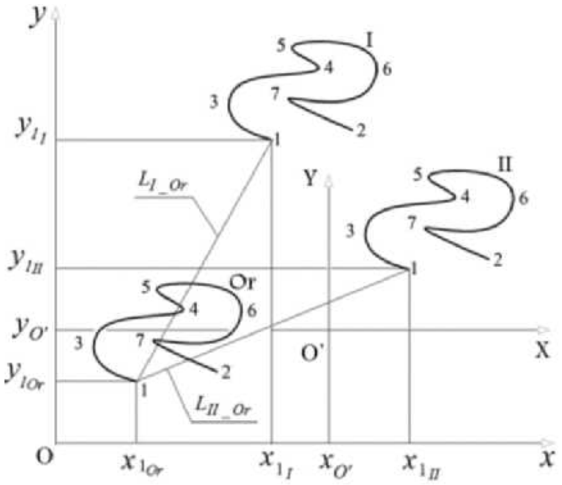

Figure 1.8. The twin curves

HR1-2: *In the y_x plane, the horizontal distance between two no-horizontal straight lines is (1) a linear function of x for different slope lines and (2) an x-independent (x = constant) for identical slopes (equal distanced in the horizontal direction) straight lines.*

ND1-3: If a curve *y = f(x)* that is illustrated in scale in plane Cartesian coordinates and denoted as (Or) (Original curve) is copied (with its numbered main points) and its copies are pasted without deforming or rotating on other places of the same graph (see Figure 1.8), then this original curve (Or) and its copies (I, II, …k…*n*) are the ***twin curves*****,** or ***the Same Size, Configuration/shape, and Orientation (SSCO)*** curves. We shall refer to a particular point (e.g., the start point 1) of a particular curve (e.g., curve II) as point (1_{II}) and to its coordinates in the Cartesian system, whose axes are (*x* & *y*), as $(x_{1_{II}}, y_{1_{II}})$. The creation of a ***twin*** (e.g., curve I) to/of this curve (II) can be imagined (see the above) as follows: we first copy curve (II) on site (the copy is now congruent-with/superposed-on its original curve "II") and starting from this initial position, the copy of curve (II) is rigidly transformed (it is straight displaced as a hard body "without rotating or deforming") into its position (I) on Figure 1.8). This means that the displacements of all points of the copy of curve (II) are the same in value and direction, and therefore *the distance* $(\mathrm{L}_{\mathrm{II\_I}})$ *between curve (II) and its copy/twin (I) is measured as the straight distance between any two Same-Number /**Counterpart** Points (*SNPs *or* CPs*) of the two twin curves,* or

$$\begin{aligned}\mathrm{L}_{\mathrm{II\_I}} &= \left(\mathrm{L}_{\mathrm{II\_I}}\right)_{\mathrm{SNP}} = \mathrm{L}_{1_{\mathrm{II}}\_1_{\mathrm{I}}} = \mathrm{L}_{2_{\mathrm{II}}\_2_{\mathrm{I}}} = \mathrm{L}_{3_{\mathrm{II}}\_3_{\mathrm{I}}} \\ &= \mathrm{L}_{4_{\mathrm{II}}\_4_{\mathrm{I}}} = \mathrm{L}_{5_{\mathrm{II}}\_5_{\mathrm{I}}} = \mathrm{L}_{6_{\mathrm{II}}\_6_{\mathrm{I}}} = \mathrm{L}_{7_{\mathrm{II}}\_7_{\mathrm{I}}}\end{aligned} \qquad \text{or}$$

$$\left.\begin{aligned}\Delta \mathrm{y}_{\mathrm{II\_I}} &= \left(\Delta \mathrm{y}_{\mathrm{II\_I}}\right)_{\mathrm{SNP}} = \left(\mathrm{y}_{\mathrm{II}} - \mathrm{y}_{\mathrm{I}}\right)_{\mathrm{SNP}} = \mathrm{y}_{1_{\mathrm{II}}} - \mathrm{y}_{1_{\mathrm{I}}} \\ &= \mathrm{y}_{2_{\mathrm{II}}} - \mathrm{y}_{2_{\mathrm{I}}} = \cdots = \mathrm{y}_{6_{\mathrm{II}}} - \mathrm{y}_{6_{\mathrm{I}}} = \mathrm{y}_{7_{\mathrm{II}}} - \mathrm{y}_{7_{\mathrm{I}}} \\ \Delta \mathrm{x}_{\mathrm{II\_I}} &= \left(\Delta \mathrm{x}_{\mathrm{II\_I}}\right)_{\mathrm{SNP}} = \left(\mathrm{x}_{\mathrm{II}} - \mathrm{x}_{\mathrm{I}}\right)_{\mathrm{SNP}} = \mathrm{x}_{1_{\mathrm{II}}} - \mathrm{x}_{1_{\mathrm{I}}} \\ &= \mathrm{x}_{2_{\mathrm{II}}} - \mathrm{x}_{2_{\mathrm{I}}} = \cdots = \mathrm{x}_{6_{\mathrm{II}}} - \mathrm{x}_{6_{\mathrm{I}}} = \mathrm{x}_{7_{\mathrm{II}}} - \mathrm{x}_{7_{\mathrm{I}}}\end{aligned}\right\} \qquad (1\text{-}59)$$

From which we obtain

$$\Delta \mathrm{y}_{\mathrm{II\_I}} = \mathrm{y}_{1_{\mathrm{II}}} - \mathrm{y}_{1_{\mathrm{I}}} \quad \text{and} \quad \Delta \mathrm{x}_{\mathrm{II\_I}} = \mathrm{x}_{1_{\mathrm{II}}} - \mathrm{x}_{1_{\mathrm{I}}} \qquad (1\text{-}60)$$

The creation of curve (II) in Figure 1.8 can be imagined as the result of the rigid transformation of the original curve (Or) with its system of coordinates vertically for $\Delta y_{II\_Or} = y_{1_{II}} - y_{1_{Or}}$ and horizontally for $\Delta x_{II\_Or} = x_{1_{II}} - x_{1_{Or}}$. After such transformation, curve (Or) superposes on/with curve (II). Denoting the displaced axes as Y and X and the origin of the displaced coordinates by O', we can say the following:

1) The equation of curve (II) in plane (Y_X) can be obtained from the equation of curve (Or) in plane (*y*_*x*) by replacement (Or by II, *y* by Y, and *x* by X). Thus, the equation of curve (II) in plane (*Y*_*X*) is

$$\mathrm{Y_{II}} = \mathrm{f}\left(\mathrm{X_{II}}\right) \tag{1-61}$$

2) The coordinates of point (1_{II}) in plane (*Y*_*X*) are

$$\mathrm{Y_{1_{II}}} = \mathrm{y_{1_{II}}} - \mathrm{y_{o'}} = \mathrm{y_{1_{Or}}} - 0 = \mathrm{y_{1_{Or}}} \quad \text{and} \quad \mathrm{X_{1_{II}}} = \mathrm{x_{1_{II}}} - \mathrm{x_{o'}} = \mathrm{x_{1_{Or}}} \tag{1-62}$$

To obtain the equation of curve (II) in plane (*y*_*x*) from Equation (1-61), we apply the mathematical method of translation of axes. For this purpose, we replace (Y_{II} and X_{II}) in Equation (1-61) by their equivalents from Figure 1.8, where $\mathrm{Y} = y - y_{o'}$ and $\mathrm{X} = \mathrm{x} - \mathrm{x_{o'}}$, and obtain

$$\mathrm{y_{II}} - \mathrm{y_{O'_{II}}} = \mathrm{f}\left(\mathrm{x_{II}} - \mathrm{x_{O'_{II}}}\right) \tag{1-63}$$

Taking into consideration Equations (1-60) and (1-62), we obtain

$$\begin{aligned} \mathrm{y_{II}} - \mathrm{y_{1_{II}}} + \mathrm{y_{1_{Or}}} &= \mathrm{f}\left(\mathrm{x_{II}} - \mathrm{x_{1_{II}}} + \mathrm{x_{1_{Or}}}\right) \\ &= \mathrm{y_{II}} - \Delta \mathrm{y_{II\_Or}} = \mathrm{f}\left(\mathrm{x_{II}} - \Delta \mathrm{x_{II\_Or}}\right) \end{aligned} \tag{1-64}$$

And, for copy (*k*) that is not illustrated in Figure 1.8

$$\begin{aligned} \mathrm{y_k} - \mathrm{y_{1_k}} + \mathrm{y_{1_{Or}}} &= \mathrm{f}\left(\mathrm{x_k} - \mathrm{x_{1_k}} + \mathrm{x_{1_{Or}}}\right) \\ &= \mathrm{y_k} - \Delta \mathrm{y_{k\_Or}} = \mathrm{f}\left(\mathrm{x_k} - \Delta \mathrm{x_{k\_Or}}\right) \end{aligned} \tag{1-64a}$$

If the original curve starts from the origin of coordinates (i.e., if $y_{1_{\mathrm{or}}} = 0$ and $x_{1_{\mathrm{or}}} = 0$), the values of $\Delta y_{II\_\mathrm{Or}}$ and $\Delta x_{II\_\mathrm{Or}}$ will be $y_{1_{II}}$ and $x_{1_{II}}$, respectively, and Equation (1-64a) becomes

$$\left(\mathrm{y_{II}} - \mathrm{y_{1_{II}}}\right) = \mathrm{f}\left(\mathrm{x_{II}} - \mathrm{x_{1_{II}}}\right) \tag{1-64b}$$

As y_{II} and x_{II} are the coordinates of the unfixed point of curve II (they are variables), while $y_{1_{II}}$ and $x_{1_{II}}$ are the coordinates of the start/initial fixed point 1_{II} (they are the constants of curve II), we can form an equation as

$$\left.\begin{array}{l} \text{For curve (II)}\ y - y_{1_{II}} = f\left(x - x_{1_{II}}\right) \qquad \textbf{or} \\ \left[y - y_{1_{II}} = f\left(x - x_{1_{II}}\right)\right]_{II} \quad \textbf{or} \quad y_{II} - y_{1_{II}} = f\left(x_{II} - x_{1_{II}}\right) \end{array}\right\} \tag{1-65}$$

Following the same procedure mentioned previously, we can form an equation for copy (*k*), regardless of the presence or absence of its illustration in Figure 1.8

$$\left.\begin{array}{l} \text{For curve (k):}\ y - y_{1_k} = f\left(x - x_{1_k}\right) \qquad \textbf{or} \\ \left[y - y_{1_k} = f\left(x - x_{1_k}\right)\right]_{k} \quad \textbf{or} \quad y_{k} - y_{1_k} = f\left(x_{k} - x_{1_k}\right) \end{array}\right\} \tag{1-66}$$

And for copy(I)

$$y - y_{1_I} = f\left(x - x_{1_I}\right) \tag{1-66a}$$

From Equation 1-59, we conclude the following.

Conclusion 1-2: *The general form equation of all twin curves that have the same SSCO in Cartesian coordinates is see the second of equations 1-66)*

$$y - y_{1_k} = f\left(x - x_{1_k}\right)$$

where k stands for twin curve no. k that can be any positive integer number and y_{1_k} and x_{1_k} *are the coordinates of the initial point of the curve under consideration (k).*

Nt1-29: From the beginning of ND1-3 till now, we used the symbol *f(...)*, where *(f)* is free from indexes, subscripts, superscripts, and primes to insist that, during this analysis, the structure of the function is kept the same. Here, three different structure functions are delivered to explain what we mean by the structure of a function:

- Functions $f\left(x-x_{1_k}\right)=A_1\left(x-x_{1_k}\right)+A_3\left(x-x_{1_k}\right)^3$ **and** $f(x)=A_1x+A_2x^3$ are the same-structure functions.
- Functions $f(x)=A_1\sin(x-B)-A_2\cos 2(x-B)$ **and** $f(x)=A_1\sin x-A_2\cos 2x$ are the same-structure functions.
- Functions $f\left(x-x_{1_k}\right)=A_1\left(x-x_{1_k}\right)+A_3e^{\left(x-x_{1_k}\right)}$ **and** $f(x)=A_1x+A_3e^x$ are the same-structure functions.

The difference between any two same-structure functions is limited in the variable's expression that may be a sum of the variable and a constant that are included inside parentheses such as [x, X_{II}, $(x_{II}-x_{O'_{II}})$, and so on]. Therefore, a set of same-structure functions can be expressed by one general-form equation. The terms "same-structure function and general-form equation" are widely used in Chapter 6 of this book.

Conclusion 1-3: *The constituents of each pair of a set of* ***twin curves*** *are equal distanced in the direction of any straight line that interconnects the counterpart points of its constituents. And, it happens that the counterpart (same code) points such as initial, final, and central points of more than two* ***twin curves*** *lie on the same straight line.*

In Chapter 6, we shall discuss two special cases of ***twin curves*** *when the constituents of each pair of curves are equal distanced in the vertical direction (for the first special case) or in the horizontal direction (for the second special case). For the first special case,* $\Delta X_{k\_Or}=\text{const}_{k_X}=0$, *Equation* (1-64a) becomes

$$y_k-\Delta y_{k\_Or}=f(x_k) \tag{1-67}$$

And, for *the second special case*, $\Delta Y_{k\_Or}=\text{const}_{k_Y}=0$, *Equation* (1-64a) becomes $y_k=f\left(x_k-\Delta x_{k\_Or}\right)$. This direct function equation is not appropriate for calculations and discussions; therefore, we use its inverse

$$f_1(y_k)=x_k-\Delta x_{k\_Or}=x_k-\text{const}_{k_x} \tag{1-68}$$

For the case when the original curve starts from the origin of coordinates, $y_{1_{Or}}=x_{1_{Or}}=0\Rightarrow\Delta y_{k\_Or}=y_{1_k}$ and $\Delta x_{k\_Or}=x_{1_k}$ (see the text between Equations (1-64a) and (1-65); therefore, Equation (1-67), for equal distanced in the vertical direction twin curves, becomes

$$y_k - y_{1_k} = f(x_k) \tag{1-69}$$

And, Equation (1-68), for equal distanced in the horizontal direction twin curves, becomes

$$f_1(y_k) = x_k - x_{1k} \tag{1-70}$$

HD1-8: In the common case, when the plane coordinate axes (y and x) divide their graph into 4 quadrants (I through IV), a set of twin *(SSCO)* curves that are expressed by equation $y - y_{1_k} = f\left(x - x_{1_k}\right)$ or its inverse $x - x_{1_k} = f_1\left(y - y_{1_k}\right)$ includes only one curve, whose equation is the simplest[A] among the equations of its twins and its characteristic point to which we shall refer as "main characteristic point" superposes on the *origin of coordinates.* We shall refer to this curve as "the *Master/Configuration curve (MC-curve) of the set of twin (SSCO) curves*". The *MC-curve* is governed/expressed by the *Master/Configuration equation (MC equation)* $[y = f(x)$ or its inverse $x = f_1(y)]$ that equals zero for $[x = 0 \ or \ y = 0]$.

A. The circle whose equation is the simplest is the one whose centre superposes on the origin of coordinates. Therefore, the graphs that do not occupy all four quadrants of the coordinate plane do not include any twin circle whose equation is the simplest.

As seen previously, *the condition "the original curve starts from the origin of coordinates" simplifies twin curves' equations, calculations, and accordingly creation.*

1.3 SUMMARY

A dimension (physical characteristic), such as temperature, pressure, and mass, is expressed by two interconnected measurements (parts), numerical and unit. And, it is denoted by a symbol (e.g., T for temperature, p for pressure, and m for mass). In the direct expression, the unit follows directly the numerical part, for example, $T = 65(\mathrm{F}) = 65F$, which is read as *the temperature is sixty-five degrees Fahrenheit.* But in the indirect expression, the two parts of the dimension are separated (in this case, the unit follows the dimension's symbol), for example, $T\,[F\,] = 65$, (F) which is read as *the temperature in degrees Fahrenheit is sixty-five*.

The main relations between temperature scales are as follows:

1) T[K] = T[°C] + 273.15, which is read as *the temperature in kelvins equals the sum of the temperature in degrees Celsius and 273.15*. Thus, if T = 50°C, then T = [°C] = 50 and T[K] = 50 + 273.15 = 323.15, so T = 323.15K.
2) $\text{T [R]} = 1.8 \cdot \text{T [K]}$. For $\text{T [K]} = 323.15$, $\text{T [R]} = 1.8 \cdot 323.15 = 581.67$, **or** $\text{T} = 581.67\ \text{R}$.
3) $\text{T [R]} = \text{T [F]} + 459.67$. For $\text{T [R]} = 581.67$, $\text{T [F]} = \text{T [R]} - 459.67 = 581.67 - 459.67 = 122$, so $\text{T} = 122\ \text{F}$.

A unit can be simple (its full name cannot be split into more than one unit name) or complex (its full name cannot be split into more than one unit name). The metric horsepower, hp(M), is a simple unit (the only unit name that can be extracted through splitting this full name is horsepower), whereas the Watt-hour is a complex unit (its full name can be split into Watt "a power unit" and hour "a time"). The unit of dimension D (see Equation 1-1) is a fraction in which each of the numerator and denominator is a complex unit consisting of two simple units, and it can be written as follows: $\left[\dfrac{\text{ch} \cdot \text{s}^2}{\text{K} \cdot \text{kgf}}\right], \dfrac{\text{ch} \cdot \text{s}^2}{\text{K} \cdot \text{kgf}}, \ (\text{ch} \cdot \text{s}^2)/(\text{K} \cdot \text{kgf})$, *[ch-$s^2$/kgf-K]*, *[ch s^2/kgf K]*, *ch-s^2/kgf-K*, and *ch s^2/kgf K*. In addition, we note that when the complex unit is inside brackets or parentheses, it can be written directly (without a space) after the number preceding it, otherwise a space is required.

CHAPTER ENDNOTES

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CHAPTER 2

THE WORKING FLUID AND ITS BASIC PROPERTIES

2.1 ENERGY AND ITS TRANSFORMATIONS

2.1.1 INTRODUCTION

Thermodynamics is defined as the science that studies energy (thermal, mechanical, electrical, chemical, etc.) transformations from one form to another, concentrating on the use of these transformations in the technique. The word thermodynamics stems from the Greek words *therme* (heat) and *dynamis* (power), which gives exact description to the early efforts to convert heat into power. At this time, thermodynamics deals with all aspects of energy—how it is generated and transformed, including power generation and refrigeration—and the relations between the properties of the substance.

2.1.2 TYPES OF ENERGY

Energy can be *transitional*, one can feel it through his senses, or stored (***latent*** energy), one cannot feel it unless it is converted into transitional one.

The brink of an abyss has mechanical potential (latent) energy that cannot be felt unless the rock falls down and its potential energy converts into mechanical kinetic (transitional) energy. The thermodynamic definition of the mechanical energy is "***mechanical*** energy is the energy that could lead to raising loads."

Also, the fuel has chemical latent energy that cannot be felt unless the fuel combusts and its chemical latent energy converts into heat (transitional) energy that can be seen in the form of a flame and felt due to temperature raise. In the other type of chemical reactions, the effect of the reaction is an electrical energy, which occurs in a form of electrical current passing through the car battery. From the above, we conclude that ***chemical energy*** resulted from chemical reaction's energy.

The third type of energy is the electrical energy that is defined as the energy made available by the flow of electric charge through a conductor.

In addition to the energy forms defined previously, we mention (1) the *electromagnetic energy*, which is the energy transmitted in the form of electromagnetic waves at the speed of light but with different wavelengths and (2) the *atomic energy*, which is stored inside atoms and appears only when the components of atoms of the material interact. The two main types of this energy are *Atomic Fission and Fusion energies.*

2.1.3 ENERGY TRANSFORMATION

Energy transforms (totally or partially) from one form to another. All no-thermal forms of energy transform fully and directly into thermal energy.

In a fuel cell, an isothermal chemical reaction occurs between the fuel and oxidizer, which results in direct generation of electrical energy. This reaction continues until the interruption of the fuel and oxidizer or any of them. In the battery, the chemical energy is converted directly into electrical energy. And, in the photovoltaic cell, the solar energy transforms directly into electrical energy. In the electric generator, the mechanical energy transforms directly to electrical energy, and in the electric motor, the electrical energy transforms directly into work.

The mechanical energy is obtained from the thermal energy by heat engines. What is a heat engine?

2.2 THE HEAT ENGINE

In this paragraph, we shall use some terms that need to be defined:

Others' Definition 2-1 (OD2-1): The **working fluid (WF)** is the fluid substance, in its gaseous or gaseous and liquid phases, that actuates cyclic devices (machines operating on a cycle such as heat engines, refrigerators, and heat pumps). The transformations of mechanical energy into thermal and thermal into mechanical are associated with changes in the state of the WF through which these transformations are performed. The WF is also known as working body.

> **OD2-1A:** The ***completely/fully/totally defined equations*** are the ones whose all constants are known directly or indirectly. These known constants can be grouped into (1) data to be given that change from one problem to another (temporary constituents) and (2) permanent constant constituents of the equation. If the permanent constant constituents exist in the equation in the form of symbols those are not defined in its legend (in the explanation that follows the equation), then the equation is a ***general form equation, else it is a calculating one***.
>
> **OD2-2:** The ***heat reservoir*** (also called ***reservoir***) is a *hypothetical* infinitely large body; whatever quantity of heat is exchanged with it, its temperature does not change, making it a constant temperature body. On the other hand, a ***mini reservoir*** (**MR**) may change its own temperature during exchanging heat with the WF.

The ***heat reservoir*** can (1) act as constant-temperature heat source (HS) that is also called *high-temperature heat reservoir or simply* ***high reservoir***, (2) act as constant-temperature heat sink (*low-temperature heat reservoir or simply* ***low reservoir***), and (3) exchange heat in both directions and we shall refer to it as *medial-temperature heat reservoir or simply* ***medial reservoir***. Therefore, <u>any heat transfer discussion between the WF and the heat reservoir is valid for all three above-listed heat exchanges.</u>

The ***mini heat reservoir*** (**MR**) can (1) act as constant-temperature HS*,* (2) act as constant-temperature heat sink, (3) act as nonconstant-temperature HS, (4) act as nonconstant-temperature heat sink, (5) be a simple compressible substance/system (see §2.4.9), and (6) exchange heat in both directions. Therefore, <u>any heat transfer discussion between the WF and the MR is valid for all six above-listed heat exchanges.</u>

In many places in this book, the direction of the WF's heat transfer during some processes is unknown in advance and we cannot predict the exact name of the other body that exchanges heat with the WF; therefore, we shall name it temporarily as the **MR** and introduce the following definition:

> **HD2-1:** The MR is the body that exchanges heat with the WF. If the heat machine is an engine, and the result of the exchange is MR's heat loss, then the MR is called HS, and if the result is MR's heat gain, then the MR is called heat sink/bath.

In the past, the mechanical energy was expressed as work and the work was expressed as raising a weight/load. And today:

> **OD2-3:** The engine is the machine that can continuously transfer energy into mechanical energy. In other words, the engine is the machine which produces mechanical energy from something else continuously. The "something else" can be either energy or energy source. The name of the engine is completed by the name of the "something else". Thus: (1) The wind engine is a machine that produces mechanical energy from the wind (wind kinetic energy). (2) The electric engine is a machine which produces mechanical energy from electricity (electric energy). (3) The heat engine is a machine which produces mechanical energy from heat (thermal energy). (4) The solar engine is a machine which produces mechanical energy from the solar energy. And (5) The combustion engine is a machine which produces mechanical energy from the fuel combustion.

The last three types of the listed in **OD2-3** engines (items 3 through 5) are heat engines, because they continuously transform heat into mechanical energy. But, the type described in item (3) is the general case, and the other two types are special cases of it.

Many different designs comply with the definition of heat engine stated in item (3), but they all have the following characteristics (see Figure 2.1):

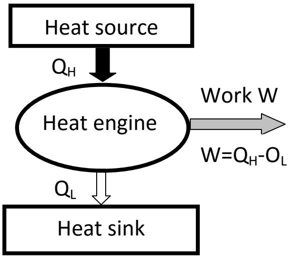

Figure 2.1. The working principle of a heat engine

1) They receive heat from an HS (burning fuels, solar energy, nuclear reactor, etc.) and convert a part of it into mechanical energy.
2) They reject the remaining waste heat, which is no longer valid for work production, to a heat sink (the atmosphere, lakes, oceans, etc.).
3) They operate on a cycle.

The existing heat engine's definition is:

OD2-3A: *The heat engine is the machine that operates cyclically (continuously), transforms a part of the heat transferred from an HS into work, and gets rid of (rejects) the remaining part, which is no longer valid for work production, to a heat sink.*

The burning fuels and solar rays are the often high-temperature reservoirs (HSs). That is why we divide HSs into three major categories: fuel combustion, natural power, and nuclear power. The environment (the atmospheric air, land, river, lake, sea, or ocean) is the often-met sink (the low-temperature heat reservoir).

2.3 THE PROCESS OF TRANSFORMING THERMAL ENERGY INTO MECHANICAL IN HEAT ENGINES

It is clear from the previous paragraph that heat engines are divided into two main types:

- The first consumes heat energy from noncombustion HSs (natural "solar power and geothermal energy" and nuclear power).
- The second consumes heat energy resulting from fuel combustion reaction. In turn, engines those consume the fuel combustion heat energy are divided into external combustion and internal combustion engines. Depending on how the heat is supplied to the WF, fuel combustion heat engines are classified into internal combustion engines (represented by automobile engines) and external combustion engines (represented by steam power plants). *In internal combustion engines (in the broad sense), fuel combusts inside the WF, whose initial components are fuel and oxidizer (mostly air)*. *In external combustion engines, fuel combusts outside the WF* (it combusts inside combustion chambers or furnaces), which allow only heat transfer between the combustion products and WF.

2.3.1 INTERNAL COMBUSTION ENGINES (IN THE BROAD SENSE)

The internal combustion engines (Figure 2.2) are divided into many types, including the following:

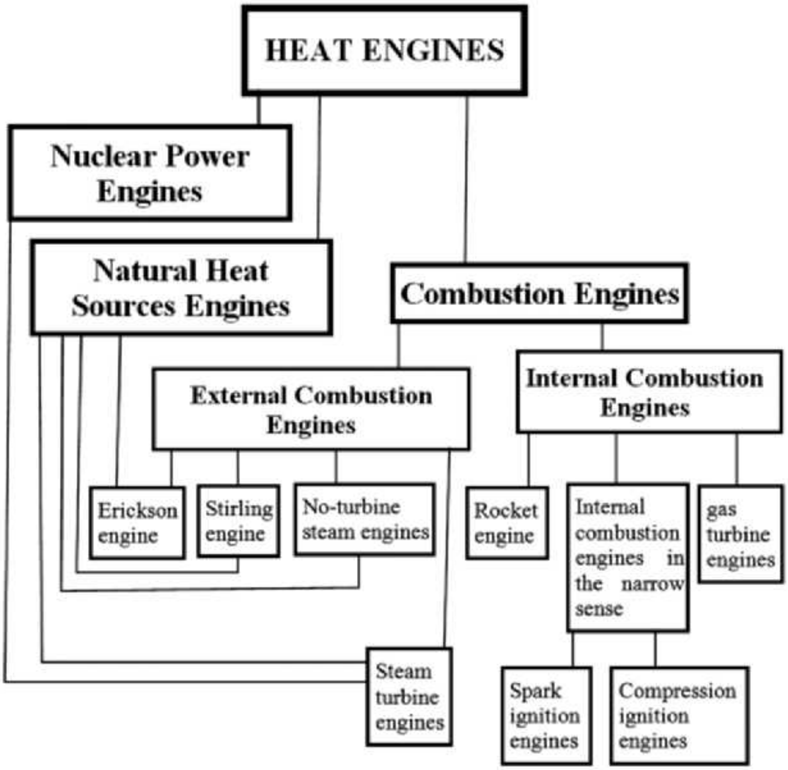

Figure 2.2. Types of heat engines

1) **Reciprocating (piston) engines:** *They are also called internal combustion engines (in the narrow sense) or simply internal combustion engines*. These engines operate using piston–cylinder arrangements (Figure 2.3). Somehow, compressed air and fuel (evaporated or sprayed) enter the combustion chamber (the small space trapped inside the cylinder when the piston in its top dead center [TDC] extreme position), where the ignition conditions are available (through ignition plug or fuel injector). The fuel combusts and the combustion products expand, pushing the piston strongly.

This leads to the generation of mechanical energy. Reciprocating engines are divided in terms of how the ignition is executed:

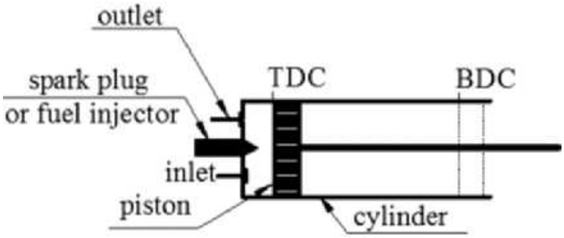

Figure 2.3. Nomenclature for the piston–cylinder arrangement of the internal combustion engine

a) Spark ignition engines: The charge (a mixture of air and fuel "gasoline" vapor) is sucked/drawn into the cylinder and compressed to the desired pressure in the combustion chamber, where the fuel is ignited by a high-voltage electric spark generated (by the ignition system) between the poles of spark plug installed in the top of the cylinder.

b) Compression ignition engines: Fresh air is drawn into the cylinder and compressed to the desired temperature in the combustion chamber, which exceeds the autoignition temperature of the fuel. The injection device starts injecting fuel (diesel or any other oil) into the combustion chamber and the combustion starts on contact as the fuel is injected and sprayed into this hot air.

2) **Rotary engines:** The most notable rotary engine is the *gas turbine engine,* the turbine of which is rotated by the high-pressure and -temperature combustion products, coming from a separate combustion chamber. In this chamber, fuel is sprayed and injected into the compressed air, coming from the rotary compressor, and combusts in it. According to the purpose of the gas turbine engine, the turbine produces either all mechanical energy that can be obtained from the combustion products or the part of it that is required by the compressor and the electric generator of the plane.

3) **Rocket engines:** In the simplest of this type of engines, there are no rotating parts, but there is a tank for the compressed fuel and another for the compressed oxidizer, which are connected to a combustion chamber through valves. The combustion products lunge through engine's exit at high speeds, causing the jet propulsion force or the rocket propulsion force.

2.3.2 EXTERNAL COMBUSTION ENGINES

Four types of external combustion engines are represented in Figure 2.2. Because of the aim of this book and its small size, we shall skip the steam engines and light in brief on the **Stirling and Ericsson engines**.

Both engines are reciprocating external combustion engines, which can use any ideal gas as WF.

To understand the principal *differences between the reciprocating external combustion engines and the internal combustion engines, imagine that the external combustion engine operates using piston–cylinder arrangement that differs from that in Figure 2.3 by the head of the cylinder. The head of the cylinder of the external combustion engine is equipped, instead of the instruments shown in Figure 2.3, with (1) an outside fire source that operates when the piston is around the TDC heating the fixed-mass WF without touching it (without changing its formula) and (2) another outside cooling source that operates when the piston is around the bottom* dead center *(BDC) cooling the fixed-mass WF without touching it (without changing its formula). This excludes the need to change the WF portion after each mechanical cycle, making the thermodynamic cycle of the explained external combustion engine closed.*

2.4 BASIC CONCEPTS AND DEFINITIONS

2.4.1 INTRODUCTION

OD2-4: *The **Macrostate** is the macroscopic thermodynamic state, and the **Microstate** is the microscopic thermodynamic state.*

OD2-5: ***Macrobodies** are large objects existing in nature, which we may see or not see; but we can have their measurements easily and simply. The kinetic energy of macrobodies is **mechanical energy**.*

OD2-6: ***Microbodies**: All macrobodies consist of minute objects called molecules, which in turn consist of atoms. These objects as well as clusters consisting of a small number of them are invisible minute objects, and cannot be measured using conventional methods. Therefore, they are called **microbodies.***

2.4.2 THE PURE SUBSTANCE AND ITS MOLECULES

OD2-7: A pure substance is any substance that has a fixed chemical composition throughout. For example, gold, water, oxygen, hydrogen, and carbon monoxide are pure substances. A pure substance (chemical element or compound) consists of a large number of homogeneous particles called molecules.

OD2-8: A molecule is the smallest particle of a pure substance that has all chemical properties of the pure substance. Therefore, the properties of a pure substance depend on the behavior of its molecules. A molecular pure substance is represented by its molecular formula; for example, hydrogen is represented by the formula H_2 and water by H_2O. Molecules differ in size, molecular mass, and structure. According to the number of atoms the molecule consists, molecules can be classified into monatomic, diatomic, triatomic, and multiatomic/polyatomic ones (prefixes mon = 1, di = 2, tri = 3, and multi = poly = more than 3). Many references use multi/poly for two and more. The molecule of a chemical compound can be diatomic, triatomic, or multiatomic/polyatomic.

OD2-9: A phase is identified as having a distinct molecular arrangement that is homogeneous throughout and separated from other phases by easily identifiable boundary surfaces (e.g., the two phases of H_2O in iced water).[1]

A substance can exist in three principal phases (solid, liquid, and gas), and it may have several phases within a principal phase. Each of these phases has its different molecular structure. Within an acceptable tolerance, the principal phases of a substance are recognized from their own characteristics, some of which are:

- A solid (1) retains its own shape and volume (2) is virtually incompressible; (3) diffusion within a solid occurs extremely slowly; and (4) does not flow.[2]
- A liquid (1) assumes the shape of the portion of the container it occupies, (2) does not expand to fill container, (3) is virtually incompressible; (4) diffusion within a liquid occurs slowly; and (5) flows readily.[2]
- A gas (1) assumes/takes both the volume and shape of its container (2) is compressible; (3) diffusion within a gas occurs rapidly; (4) flows readily;[2] and it (5) exerts the same pressure on all walls of the not high container. The first property can be formulated for gas mixtures as the following rule:

OR2-1: *if several gases unable to chemically interact were put in one container, they will form a homogeneous mixture as a result of the thermal movement (random motion).*

A substance can exist: (1) as a mono phase (solid, liquid, or gaseous) substance whose temperature and pressure are independent intensive properties. (2) as a mixture of two principal phases (solid and liquid, liquid and gas, and solid and gas) whose temperature and pressure are dependent intensive properties. In addition, each substance can exist as a mixture of its three principal phases (solid, liquid, and gaseous) in one fixed state whose temperature and pressure are fixed. The states of mater at which all three phases coexist are known as the *triple phase states,* and are represented in p_T diagram by one point that called the *triple point.* For water, the triple-point temperature and pressure are 0.01°C and 0.6117 kPa respectively.

When the matter undergoes a *melting/freezing* process, it consists of the liquid and solid phases. In the *boiling, vaporization/condensation* process, the matter consists of the liquid and gaseous phases, and in the *sublimation* process, the matter consists of the solid and gaseous phases.

OR2-2: *For a pure substance, the pressure and temperature of any of the states of the saturated liquid, vapor, and their mixture zone are interconnected by its liquid-vapor saturation equation. And therefore they are dependent to each other.*

In this book, we are interested in the gaseous phase exclusively.

OD2-10:[3] ***The mole*** *(abbreviation mol) is the SI unit* of *material quantity.* Its other name is the *gram-mole* (abbreviated gmol). It is the quantity of material consisting of as many specified elementary entities (atoms, molecules, electrons, ions, or other particles or specific groups) as there are atoms in 12 g of carbon-12. *The number of atoms in 12 g of carbon-12 is too close to* 6.022141 × 10^{23}*, which is known as Avogadro's number/constant for mole.* The authorized organization to adopt the value of Avogadro's constant (N_A) to be used throughout the world as the exact one is the Committee on Data for Science and Technology (CODATA). CODATA adopted in the years 2006, 2010 and2011 the following values of Avogadro constant (the first value is represented in both comprehensive and abbreviated forms, whereas the others in only the abbreviated form).

$$N_A = (6.022\,141\,79 \pm 0.00000030) \times 10^{23}\ \text{mol}^{-1}$$
$$= 6.022\,141\,79\,(30) \times 10^{23}\ \text{mol}^{-1};$$

$$N_A = 6.022\,141\,29\,(27) \times 10^{23}\ \text{mol}^{-1} \qquad \text{and}$$

$$N_A = 6.022\,140\,78\,(18) \times 10^{23}\ \text{mol}^{-1} \qquad \text{respectively.}^3$$

Therefore, we shall adopt $N_A = 6.022\ 141 \times 10^{23}\ \text{mol}^{-1}$ as the exact value of N_A throughout this book.

Also, the ***mole*** can be defined as the quantity of a pure substance/composition whose mass in grams is equal to its *relative molar mass (M_r).*

OD2-11: ***The kilomole*** *(kmol)* or the ***kilogram-mole*** (kgmol) equals 1,000 moles. It is also defined as (1) the quantity of a substance whose mass in kilograms is equal to its molecular weight and (2) the number of entities in 12 kg of ^{12}C.

OD2-12: The ***pound-mole*** (denoted ***lb-mol*** or ***lbmol***) is defined as the number of entities in 12 lbm of ^{12}C (carbon-12); one lb-mol equals 453.59237 moles.[4]

Also, the ***pound-mole*** can be defined as the quantity of a substance whose mass in pounds is equal to its molecular weight.

Nt2-1: *Since Avogadro's number mentioned above* $\left(N_A = 6.022\ 141 \times 10^{23}\ \text{mol}^{-1}\right)$ *was assigned for 1 mole and since the kmol and lbmol are the doubles of the mole, then we can refer to the number as the per mole Avogadro's number, and to the number of the molecules in one kmole that* equals $\left(1000 \times 6.022\ 141 \times 10^{23} = 6.022\ 141 \times 10^{26}\right)$ as *the per kmole Avogadro's number "*$N_A = 6.022\ 141 \times 10^{26}\ \text{kmol}^{-1}$*,"* and *to the number of the molecules in one lbmol that* equals $453.59237 \times 6.022\ 141 \times 10^{23} = 2731.597\ 208\ 664\ 17 \times 10^{23} \approx 2.732 \times 10^{26}$ as *the per lbmole Avogadro's number "*$N_A \approx 2.732 \times 10^{26}\ \text{lbmol}^{-1}$*."* Thus, we already have three *Avogadro's numbers* assigned for the three quantities *(1 mole, 1 kmol, and 1 lbmol), and we may need to speak about more Avogadro's numbers in the future. To differentiate one Avogadro's number from another, this number was treated as a dimension, whose today's units are mole^{-1}, kmole^{-1}, and lbmole^{-1}. This allows defining Avogadro's number as follows:*

$$N_A = 6.022\ 141 \times 10^{23}\ \text{mol}^{-1} = 6.022\ 141 \times 10^{26}\ \text{kmol}^{-1} \approx 2.732 \times 10^{26}\ \text{lbmol}^{-1} \tag{2-1}$$

Nt2-1A: In classical thermodynamics and accordingly in this book, where we practically do not deal with other elementary entities than molecules (the molecule of a mono-atomic substance consists of one atom), we usually understand 1 mol of water (H_2O) as 1 mol of water (H_2O) molecules exclusively. Therefore, we can define the mole here as the Avogadro's number/constant (6.022141× $\times 10^{23}$) of molecules and consider the units of Avogadro's number/constant as molecule/mol, molecule/kmol, and molecule/lbmol, instead of mol^{-1}, kmol^{-1}, and lbmol^{-1}, which are used in chemistry. Thus, we can rewrite Equation (2-1) as

$$\begin{aligned} N_A &= 6.022141 \cdot 10^{23} \ \frac{\text{molecule}}{\text{mol}} \\ &= 6.022\,141 \cdot 10^{26} \ \frac{\text{molecule}}{\text{kmol}} \approx 2.732 \cdot 10^{26} \ \frac{\text{molecule}}{\text{lbmol}} \end{aligned} \qquad (2\text{-}1a)$$

This procedure helps in some explanations since it clarifies the essence of Avogadro's number/constant that is a dimensional number whose numerical value depends on the chosen unit, but it will surely cause misunderstanding in the units of the gas and universal gas constants that have deep roots in thermodynamics, which are introduced below. Therefore, we shall not apply any changes that affect the shapes of the units of the gas and universal gas constants.

OD2-13: The chemistry definitions—of the atomic mass, molecular mass, molar mass, molecular weight, and formula weight—lead to the following conclusions that can be applied in classical thermodynamics:

1) The *atomic mass (m_a), molecular mass (m_m),* and *molar mass (M) are the masses of one atom, one molecule, and one mole of a pure substance/compound, respectively. Therefore, the molecular mass* equals the sum of the *atomic masses of the atoms those constitute* the molecule $(m_m = \sum_{\text{molecule}} m_a)$ and *the molar mass is calculated as* $(M = N_A \cdot m_m)$, where N_A is Avogadro's number for mole. Thus,

$$M = N_A \cdot m_m \qquad \text{or} \qquad m_m = M / N_A \qquad (2\text{-}1b)$$

2) The ***relative atomic mass (atomic weight)*** **symbol** (A_r) is the ***atomic mass*** related to 1/12 of the mass of a single carbon-12 atom (at rest). *It can be obtained from the standard atomic weights that are used in periodic tables and many standard references in ordinary terrestrial chemistry.*[5]

3) The *relative molar mass M_r* (the older terms are the *molecular weight* "MW or μ" and *formula weight* FW) is a dimensionless quantity (i.e., a pure number, without units) that equals, in SI system, to the molar mass in grams per mole divided by the *molar mass constant* (M_u) that is[6]

$$\mathrm{M_u} = 10^{-3}\ \text{kg/mol} = 1\ \text{g/mol}.^{6} \qquad (2\text{-}2)$$

Thus,

$$\mathrm{M_r} = \mathrm{M[g/mol]/M_u\,[g/mol]};\ \text{Abbreviation}\ M_r = M / M_u. \qquad (2\text{-}2a)$$

4) The numerical value of the ***molar mass in g/mol*** (or ***kg/kmol***) is equal to the numerical value of the ***molecular mass*** and the dimensionless values of the ***molecular weight*** and ***relative molar mass*** (M_r); therefore, we can accept that knowing M_r means knowing the values of all above-mentioned molecular properties.
5) The ***relative molar mass*** (M_r) of a compound can be computed by obtaining, from the compound's chemical formula, the kinds and numbers of atoms (nuclides) constituting the molecule and summing the relative atomic masses (see item 1 above) of all atoms constituting the compound's molecule. For example: $M_{r(\mathrm{H_2O})} = 2A_{r(\mathrm{H})} + A_{r(\mathrm{O})} \approx 2\times1+16 \approx 18.$

Nt2-2: *To keep the numerical value of the molar mass equal to the relative molar mass. Almost all thermodynamic references do not use the basic SI unit* (kg/mol) *in measuring the molar mass,* but use its part [(g/mol)= (kg/kmol)]. The practiced units here are the (kg/kmol) in the SI and (lb/lbmol) in the English system.
Nt2-2A: Let us obtain the equations for calculating *pure substance/compound quantities through the already introduced material in this book. Since the metric SI unit system is adopted in this book, these quantities can be measured by the numbers of molecules (symbol N), moles or kilomoles (symbol K), kilograms (symbol kg), and grams (symbol g).*

A quantity of a pure substance/compound consists of a number (N) of molecules or of (K) moles or of (m) *grams*, while 1 mole of the same substance consists of Avogadro's number (N_A = 6.022141× 10^{23} mol^{-1}) of molecules and its mass is the molar mass M “ $M\,[g\,/\,\mathrm{mol}]$ ” (see Equations 2-1 and 2-1b). Therefore, the mass of the molecule of a pure substance (m_m) can be calculated as

$$\mathrm{m_m} = \frac{\mathrm{m}}{\mathrm{N}} = \frac{\mathrm{M}}{\mathrm{N_A}} = \frac{10^{-23}\,\mathrm{M_r}}{6.022141}\ \mathrm{g} \qquad \text{(2-2b)}$$

or $$\mathrm{m} = \mathrm{N}\cdot\mathrm{m_m} = \frac{\mathrm{N}\cdot\mathrm{M}}{\mathrm{N_A}} = \mathrm{K}\cdot\mathrm{M} \qquad \text{(2-2c)}$$

where,

$$N[\,\text{molecules}] = K\,[\text{mol}]\cdot \mathrm{N_A}\,[\text{molecules/mol}] = K\cdot \mathrm{N_A}\,[\text{molecules}].$$

The units in this equation are SI ones exclusively.

Nt2-2B: *The* molar *mass of a compound is scientifically acceptable because the chemical formula of any of its molecules is the same as its/compound’s formula, while the atomic mass of a compound is meaningless because the different atoms that constitute compound’s molecule do not belong to the same chemical element. On the other hand, each of the atomic and molecular masses of a homogeneous mixture of gases is scientifically unacceptable/meaningless because the chemical formula of the molecules of a homogeneous mixture of gases is not the same for/of all its molecules.*

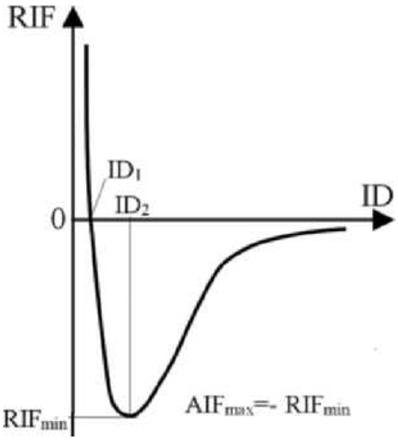

Figure 2.3A. A schematic for the relationship between the IF and the ID in a pure substance. The part of the curve that is located above the ID axis is characterized by RTF>0 belongs to the repulsive IF, whereas the other part that is characterized by RTF<0 belongs to the attractive IF and therefore, the minimum of the IF/RIF curve is the maximum of the AIF one.

2.4.3 INTERMOLECULAR FORCES:

OD2-14: *The force of attraction or repulsion acting between the molecules is known as* ***intermolecular force*** *(IF). The nature of intermolecular force is electromagnetic.*[7]

A pure substance experiences IFs. The IF in a pure substance can be attractive (cohesive) or repulsive. This force is a function of the intermolecular distance ID or r (see Figure 2.3A) and of the properties of the interacting molecules but not of temperature. The energy of attraction/repulsion force is an intermolecular potential energy. At a too small ID (ID_1), corresponding to a solid state that precedes melting and is far enough from it, the IF is zero. Starting from ID=ID_1 the decrease in (ID) monotony increases the repulsive force trying to return (ID) to its ID_1 value and prevent the molecules from touching or colliding. At ID=∞ the IF is zero and for $ID_1 < ID < \infty$ the IF is cohesive. It is an attractive intermolecular force (AIF). The maximum of the AIF is somewhere around the ID_2 corresponding to melting point. Starting from ID_2 (the ID corresponding to the maximum of the AIF), AIF decreases with increasing ID. If we imaginarily divide the solid phase region into strong repulsive, medial intermolecular and strong cohesive force subregions, then the strong cohesive force subregion will be neighboring to the melting point, and unless the molecule has enough kinetic energy, it will not have the opportunity to leave its relative position. Therefore, we can say that *the forces that bind the molecules to each other are strongest in solids and weakest in gases*.

2.4.4 THE IDEAL AND REAL GAS SUBPHASES

The theoretical study of substances in gaseous phase (which is the case that concerns us here) with taking in consideration the volumes of the molecules and their cohesive forces is extremely complicated, and it is too difficult to determine the laws of the behavior of gas molecules. This is because of the nature of the cohesive forces between the molecules.

On the other hand, the laws of gases derived on experimental basis are very complicated. That is why science, through the Kinetic Molecular Theory of gases (KMT), created the idea of the ***Ideal Gas***.

OD2-15: The ***Ideal Gas*** is a hypothetical (imaginary) gas consisting of zero-volume molecules, between which no cohesive forces act.

It is clear from the definition that ideal gases do not exist in nature. The analysis of the ideal gas is of great scientific value because

1) any gas existing in nature and is very far away from the liquid phase (its molecule's relative volume approaches zero ($MRV \to 0$) or which is the same when its volume per unit mass or per mole approaches infinity $v \to \infty$ or which is also the same when its pressure approaches zero $p \to 0)$ is ***ideal gas***, and
2) *the required accuracy in heat engineering calculations allows dealing with all gases used as WFs outside the scope of refrigerating and heat-pump machines, except water vapor, as ideal gases.* The latter can be found in two cases: (1) when the water vapor is a component in gaseous mixtures resulting from the combustion of fuel. In this case, water vapor is at high temperature and low pressure, and therefore, it can be considered as an ideal gas. And (2) when water vapor is used as a WF in steam engines or as a heat carrier in heat exchangers. In this case, the water vapor is close enough to the liquid phase and therefore it cannot be considered an ideal gas.

Properties No 1, 5, and 1a of gases (see OD2-9) result from the random translational motion of a huge amount of particles that do not give a certain privilege for a particular direction.

For a particular gas the intermolecular cohesive forces decrease with increasing temperature and/or decreasing pressure. Both the increase in temperature and decrease in pressure lead to increasing gas volume, and accordingly increasing the average ID and thus weakening of the cohesion forces between molecules, and simultaneously reducing Molecules' Relative Volume (MRV). The MRV is the ratio between gas molecules' volume, which is approximately constant, and the **gas volume** that is the space in which gas molecules move.

For gas states that are far enough from the liquid phase, both the intermolecular cohesive forces and MRV are so small that they can be neglected. In this case the characteristics of this gas will be consistent with the characteristics of the gas that we called **ideal gas**. This gives us a basis to (1) designate any gas in nature, for which the MRV and cohesive forces are small enough to be neglected an *ideal gas*, and (2) consider that the gas phase consists two subphases: (I) the ***ideal-gas subphase*** that is far away from the liquid phase and (II) the ***real gas subphase*** that lies between the liquid phase and ideal-gas subphase. The separating line between the two gaseous subphases cannot be fixed, because it is an accuracy function.

OD2-16: Real gases are the ones existing in nature, for which neglecting cohesive forces is not allowed.

2.4.5 THE THERMODYNAMIC SYSTEM (THE SYSTEM)

2.4.5.1 Introduction

OD2-17: A system is defined as a quantity of matter or a region in space chosen for study.[8]

OD2-17A: A thermodynamic system is the set of objects that interact with each other and with other bodies, which we call the surroundings. Depending on the objectives of the study we determine the objects belonging to the system, system's boundary and boundary's nature, since system's boundary can be real (like the walls of a pot) or imaginary surface, which surrounds the volume that includes the bodies of the system.[9]

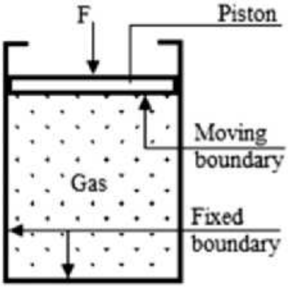

Figure 2.4. A schematic for a system (gas) and its boundary

OD2-18: The system **surroundings** is the mass or region outside the system.[10]

OD2-19: The system **boundary** is the real or imaginary surface that separates the system from its surroundings.[11] The system boundary can be fixed or moving and can have a changeable or unchangeable shape, but in all cases, it is zero mass (does not include any matter) and zero thickness (does not occupy any volume). For more clarity, the terms for the above three definitions are illustrated in Figure 2.4. Here, the inner surface of a piston–cylinder arrangement is the system boundary. It is real and has a changeable shape (the piston moves inside the cylinder). If the piston–cylinder arrangement is on a moving vehicle (mounted or transported), then the system is in motion; else it is almost *stationary*.

2.4.5.2 Types of thermodynamic systems

A number of bases are used for classifying the thermodynamic systems:

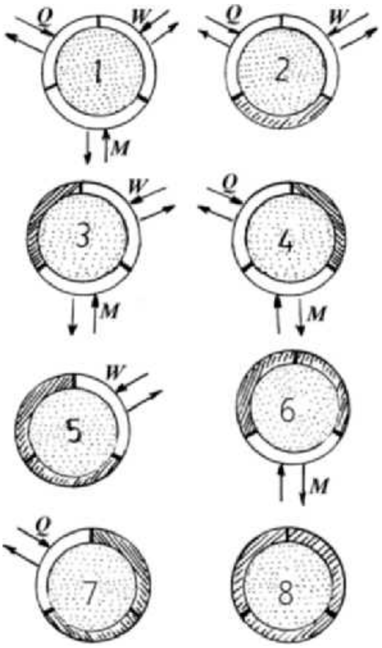

Figure 2.5. The classification on the basis of the properties of system boundaries

A) The classification on the basis of the properties of system boundaries (Figure 2.5 "main figure"), in terms of boundaries' ability to allow the transfer of thermal energy (heat), nonthermal energy (work), and matter (mass):

- The system that does not interact in any way with its surroundings is called an ***isolated system*** (illustration 8).
- The full name of the system other than the isolated system must include a notification that highlights mass interaction. The two main categories of the system are (1) the *open* system (also known as control volume system) that allows mass transfer through its boundary and (2) the *closed* system (also known as control mass system) that does not allow mass transfer through its boundary. If the name of the system does not include any additional notification, then heat and work transfer through system's boundary is allowed (see illustrations 1 and 2).
- The additional notification (adiabatic) means thermally isolated. Illustrations (3 and 5) are *adiabatic* open and closed systems, respectively.
- The no-work interaction open and closed systems are schematically shown in illustrations (4 and 7), respectively.
- Illustration (6) stands for the *no* ***energy*** *interaction open system*.

B) The classification on the basis of the structure of the components of the system and the stability of this structure. We can list here: (1) the physically homogeneous, (2) the physically heterogeneous, (3) the steady state, and (4) the unsteady systems.

In this book we shall deal with the ***closed stationary systems*** of ideal gases.

OD2-20: The ***closed stationary systems*** are closed systems whose velocity and elevation of their center of gravity (CG) remain constant during a process (e.g., a closed system inside a stationary closed vessel). Theoretically, the gas undergoing an expansion or compression process in a stationary piston–cylinder device is not a closed stationary system. This gas changes the position of its CG as the piston moves. A good example for exactly stationary closed system is the gas undergoing an expansion or compression process in the stationary double-piston–cylinder arrangement (see Figure 2.6). Here the system's CG will not move during pistons' movement and therefore the *system is* a *closed stationary one.*

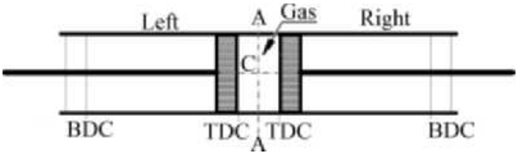

Figure 2.6. The CG of the system (Gas) superposes with the centre (C) of the double-piston-cylinder arrangement regardless of pistons' movement.

Nt2-3: Calculations show that for the gas undergoing an expansion or compression process in a piston–cylinder device that is mounted on a moving modern *overland* vehicle, the changes in kinetic and potential energies, compared with internal energy changes, are negligible. Therefore, the gas undergoing an expansion or compression process in a piston–cylinder device is classified as a ***closed stationary system.***

2.4.6 INTRODUCTION TO THE KINETIC-MOLECULAR THEORY

It was created in 1857 by Rudolf Clausius. Its other name is "The theory of moving molecules." It studies the behavior of ideal gases and shows the internal connection between the laws of these gases and the limits of their use. He invented the idea of the ideal gas and gave to its molecules the following properties:

1) Molecules are smooth spherical material (have a mass) points (do not have volume) and perfectly elastic (the total kinetic energy of collided molecules remains the same). This means that the sole energy of these molecules is the kinetic energy of their straight movement (the **translational** kinetic energy).

2) Gas molecules move at high speeds and straight paths and after each collision, whether with the vessel walls or with other molecules, follow new paths, and therefore the molecular pathways are broken lines composed of straight segments.
3) Molecules are not affected by cohesion forces.
4) The collision time is too small relative to molecule's passing time through its free path and can be neglected.
5) Since the molecules are perfectly elastic the kinetic energy losses due to collisions are nonexistent.

After that, the laws of mechanics were applied on the ideal-gas molecules, and the basic equation of the kinetic theory of gases was derived (see Equation 2-5). Then, Equation (2-10) that interconnects the absolute temperature with the average kinetic energy of ideal-gas molecules was derived. And after that, all ideal-gas laws were derived using Equations (2-5) and (2-10).

2.4.7 THE STATE OF A GAS

It is difficult (at least to the author) to give a general definition to the state of a substance (or a system) at a given time. Therefore, we will start from dealing with the ***equilibrium*** *state* (the special case of the state), which is usually expressed by using ***properties***.

OD2-21: A ***property*** is a macroscopic characteristic of a system such as mass, volume, energy, pressure, and temperature to which a numerical value can be assigned at a given time without knowing the previous behavior (*history*) of the system.[12] See also the beginning of §2.8.

OD2-22: An ***intensive property*** is independent of the mass; *the value of an* ***extensive property*** *varies directly with the mass*. Thus, if a quantity of matter in a given state is divided into two equal parts, each part will have the same value of intensive properties as the original and half the value of the extensive properties. Pressure, temperature and density are examples of intensive properties. Mass and total volume are examples of extensive properties. Extensive properties per unit mass such as specific volume are intensive properties.[13]

To specify a state, we need to measure and/or calculate all system properties throughout the entire system (each property can be measured in several macroscopic parts of the system, and "intensive properties that are not uniform throughout may require additional measurements and calculations"). This gives us a set of properties (a property map) that completely describes the condition, or the **state,** of the system. At a given state, all the properties of the macroscopic parts of a system have fixed values. If the value of even one property in one macroscopic system's part changes, the state will change to a different one.

> **HD2-1A:** The property map for a given state of a system is the map or table that includes full information about the values of its (system's) nonuniform intensive properties and their distribution throughout.

2.4.7.1 The definition of the state of a system

> **OD2-22A:** The state of a system is its condition as described by its properties (property map).
>
> **OD2-23:** *The* ***additive dimension*** *is the one that is directly proportional to/with the mass.* Therefore, if the *additive dimension is a state property, then it is an* extensive property, which means that the extensive properties are special cases of the additive dimensions. The ***additive*** dimensions such as volume "V," work "W," and transferred heat (TH) "*Q*" are usually symbolized by capital letters. Dividing these dimensions by their own quantities (in mass units, moles, or ***normal-volume*** "the volume that the gas would occupy if it existed at the normal conditions—see OD1-2A" units) transforms them into per mass unit, per mole, or per normal-volume unit specific dimensions that are abbreviated as mass, molar, or volumetric specific dimensions, respectively. The unit of each specific dimension includes/contains a quantity unit in its denominator.

Table 2.1. Obtaining equation (A) using cross-multiplication

	Mass	**Volume**	**Normal volume**	**No of kmoles**
Total	m	V	V_n	K
Per mole	Molar mass (M)	Molar volume Mv	Normal molar volume Mv_n	1
Per unit mass		v	v_n	

If the additive dimension has a name (e.g., "TH") and a symbol (*Q*) that is a capital letter, then its mass, molar, and volumetric specific dimensions will be denoted by (*q*), (Mq or $\overline{q}$), and (q'), respectively (*Mq* here is a combined symbol that consists of the prefix "M" for molar and "q" for the specific of the additive dimension "Q"). Thus, the basic part of any of the three specific dimensions of the additive dimension (*Q*) is (*q*) that is the small letter of (*Q*). When we need to speak in general about the specific additive dimension of (*Q*) that is called the specific transferred heat (STH), we can use its abbreviation STH even in equations.

Most of the following relationships (*A*) between the mass (*m*), molar mass (*M*), volume (*V*), normal volume (*V_n*), molar volume (Mv or $\overline{v}$), normal molar volume (Mv_n or $\overline{v}_n$), the number of kilo-moles (*K*) of the additive dimension (e.g., *Q*), and its specific dimensions (*q*), (Mq or $\overline{q}$), and (q') are obtained using Table 2.1 and the cross-multiplication:

$$\left.\begin{array}{l} K = \dfrac{m}{M} = \dfrac{V_n}{MV_n} = \dfrac{V_n}{\overline{v}_n} = \dfrac{V}{MV} = \dfrac{V}{\overline{v}}; \qquad q = \dfrac{Q}{m}; \qquad q' = \dfrac{Q}{V_n}; \\ \overline{q} = Mq = Q/K \ \textbf{and} \ Q = m \cdot q = V_n \cdot q' = K \cdot Mq = K \cdot \overline{q} \end{array}\right\} \quad (A)$$

The first of Equations (A), which is a six-sides multiequation, can be the origin of the equations (see Table 2.1): $Mv = M \cdot v$ and $Mv_n = M \cdot v_n$, while the last of Equations (A) is a four-side multiequation that is written for any quantity of the pure substance and can replace all three equations that precede it. Thus, we can replace Equations (A) by the following ones:

$$K = \frac{m}{M} = \frac{V_n}{\overline{v}_n} = \frac{V}{\overline{v}}; \quad \overline{v} = Mv = M \cdot v; \quad \text{and} \quad \overline{v}_n = Mv_n = M \cdot v_n \quad (2\text{-}3)$$

$$\left.\begin{array}{ll} STH = \dfrac{Q}{\text{Matter quantity}} \Rightarrow q = \dfrac{Q}{m}; \quad q' = \dfrac{Q}{V_n}; \quad \overline{q} = Mq = \dfrac{Q}{K} \\ \text{and} \qquad Q = m \times q = V_n \times q' = K \times Mq = K \times \overline{q} \end{array}\right\} \quad (2\text{-}3a)$$

Obviously the first of Equations (2-3a) is the abbreviation (the general form) of their last multiequation.

Nt2-3A: Equations (2-3a) are assigned for transferred heats calculations and they are able to be transformed into other equations that are assigned for calculating other additive dimensions. To obtain the equations for a new additive dimension, from Equations (2-3a), replace (1) (STH) with/by the abbreviation of the new specific additive dimension, (2) (Q) with/by the symbol of the new additive dimension, and (3) each (*q*) with/by the symbol of the specific dimension of the new additive one. For example, the obtained from equations (2-3a) equations for the *moving* ***boundary work*** *(symbol W_b) and its specific boundary work (abbreviated SBW and symbols w, w', and Mw)* will be Equation (2-3) as it is, in addition to the following Equation (2-3b):

$$\left.\begin{aligned}&\mathrm{SBW}=\frac{W}{\text{Matter quantity}}\Rightarrow w=\frac{w}{m};\quad w'=\frac{W}{V_n};\quad \overline{w}=Mw=\frac{w}{k}\\ &\text{and}\qquad W=m\cdot w=V_n\cdot w'=K\cdot Mw=K\cdot\overline{w}\end{aligned}\right\}\tag{2-3b}$$

Nt2-4: Symbolizing the molar volume and normal molar volume as Mv instead of the more compact modern symbol ($\overline{v}$) and Mv_n instead of ($\overline{v}_n$) is very successful since each of these symbols denotes additionally to its calculating equation (see the last two of Equation "2-3"). Therefore, we shall adopt this style of symbolizing throughout this book. For the moment, all molar specific dimensions will be symbolized so.

Nt2-4A: From the above we conclude that overlining a mass specific *dimension* transforms it into molar one/specific-*dimension*.

Nt2-5: When defining or speaking about the specific additive dimension (SAD) in common (regardless of the matter quantity kind), some books use/apply the symbol of the mass specific additive dimension (MSAD) in abbreviating the defining equation of the SAD. In these books the general form equation for the STH will look like $q = Q/(\text{Matter quantity})$ instead of the one adopted in this book (see equations 2-3a): $\mathrm{STH} = Q/(\text{Matter quantity})$.

HR2-1: *If the unit of a dimension contains a unit quantity in the denominator, then the dimension is a specific one, and the original dimension (the unit of which does not contain a unit quantity in the denominator) is an additive one.*

OD2-24: ***Two intensive properties*** are ***independent*** if one of them can change while the other one is held constant.[14] If two intensive properties are directly interconnected by a relationship, then fixing one of them fixes the other and this means that they are not independent. ***They are dependent properties***.

2.4.7.2 The equilibrium state

2.4.7.2.1 The definition of the equilibrium state

The simplest definition of the most common case equilibrium state is:

> **OD2-25:** *The equilibrium state is the state of a system that does not undergo any change.*

And the comprehensive definition of the same state is/can be:

> **OD2-26 (Abbreviated):** *The* ***equilibrium*** *state of a system is defined as the state where no system's macroscopic part tends to change its state*, and it is possible (when the system is affected by one or more of *velocity, elevation, surface tension, external force fields "electric, magnetic and gravitational" and/or* contacting with two or more different temperatures heat reservoirs) that the same parameter (intensive property) can have different values in different macroscopic parts of the system in its equilibrium state.
>
> **HD2-2:** The ***simple*** *equilibrium* ***(or uniform)*** *state of a system is defined as the state where no system's macroscopic part tends to change its state, and the values of its intensive properties are uniform throughout. Because of the state postulate (see OD2-30C), the* ***simple*** *equilibrium state of a system is also defined as the state where the values of any two of its independent intensive properties are uniform throughout.* The equations of state can be written and used only for the ***simple*** *equilibrium state of a system, or for its macroscopic part that is* ***simple*** *equilibrium*. The graphical representation of a thermodynamic state is legal only when the state is *simple equilibrium.* The graphical representation of nonuniform states is illegal or idiomatic/conventional. It can be used for some discussions and explanations but not for measurements and calculations (it is not permissible to apply the graph properties on no uniform states). Therefore:
>
> **Nt2-6:** *Classical thermodynamics' primary emphasis is on studying* ***simple*** *equilibrium states and constituted/created from only them so-called state change processes (see §2.4.10). In this book only these states and processes will be studied.*
>
> **HD2-3:** *Two states of a system that are represented on a property diagram by one point are the same/congruent only when their state*

property maps are congruent. A state property map includes (1) the borders between system's macroscopic parts in which state determining/defining intensive properties of these parts are uniform, (2) the values of the state determining/defining intensive properties in all these parts. *The* ***simplest state property map belongs to the simple equilibrium state and includes no more than the values of two independent intensive properties*** (*see* OD2-30C).

2.4.7.3 Some state properties

2.4.7.3.1 Density, specific volume, and specific weight

The **mass *specific volume*** (v) of a substance (abbreviation ***specific volume***) is defined as its volume per unit mass, while the ***density*** (ρ) of the same substance is its mass per unit volume.

Also, the specific weight (γ) of a substance is the weight per unit volume of this substance. Thus,

$$v = \mathrm{V} / \mathrm{m}, \rho = \mathrm{m} / \mathrm{V} \quad \text{and} \quad \gamma = \mathrm{G} / \mathrm{V} = \mathrm{m} \times \mathrm{g} / \mathrm{V} = \rho \times \mathrm{g} \qquad \text{(2-3c)}$$

From the first two equations, we conclude that density is the inverse of the specific volume and that

$$\rho \times \mathrm{v} = 1 \qquad \text{(2-4)}$$

Only two (the specific volume and density) of the three quantities defined above are intensive properties, because they as the pressure are functions of the substance state exclusively and accordingly uniform for the *simple equilibrium state*, but the specific weight is not an intensive property, because it changes with the change of geographical location ("g" is a function of geographical location).

2.4.7.3.2 The pressure

The KMT derived Equation (2-5), and named it the *basic equation of the KMT*:

$$\mathrm{p} = (2/3) \cdot \mathrm{n} \cdot (\mathrm{m}_{\mathrm{m}} \cdot \mathrm{c}^2 / 2) \qquad \text{(2-5)}$$

where, p is the gas pressure, m_m the mass of one molecule, and c the mean squared speed of the straight movement of the molecules. It is calculated from the equation: $c = (\sum_{i=1}^{n} c_i^2)/n$

where, n is the molecules' concentration or the number of molecules per unit volume:

$$n = \rho / m_m = (1/v)/m_m = 1/(v \cdot m_m) \qquad (2\text{-}6)$$

where, ρ and v are gas density and specific volume, respectively (see Equation 2-3b).

Equation (2-5) means that *the pressure equals two thirds of the translational (straight movement) kinetic energy of the molecules existing in a unit volume of the gas.*

Nt2-6A: In Equation (2-5) n and c^2 cannot be negative; whereas m_m is always positive integer and cannot be zero therefore, the calculated pressure through Equation (2-5) cannot be negative. That is why the calculated pressure using the *basic equation of the KMT* is measured starting from zero. Such pressure is called absolute pressure p_{abs}.

Fluid (gas or liquid) pressure *(p)* is measured by the force applied normally on a unit surface. If F is the force acting normally on the surface (A), then:

$$p = F/A \qquad (2\text{-}7)$$

Three principal kinds of devices are used in measuring pressure:

- The ***Barometer*** that measures the so-called ***absolute pressure*** (its reading is zero when the vessel contains no gas). It is denoted by p_{abs}. Thus, the ideal-gas absolute pressure is calculated by Equation (2-4).
- The ***vacuum meter*** that measures vacuum pressures p_v (the difference between the barometric "atmospheric" pressure B and the absolute pressure inside the vessel, which is below the barometric pressure). Thus,

$$B = p_{abs} + p_v = p + p_v \qquad (2\text{-}8)$$

- The manometer that measures gage pressures p_g (the difference between the absolute pressure inside the vessel, which is above the barometric pressure B, and the barometric pressure). Thus,

$$p_g = p_{abs} - B = p - B \tag{2-9}$$

From now on we shall denote to the absolute pressure by p. Therefore*, unless it is clearly/explicitly indicated to the dimension represented by p, it should be understood that p represents the absolute pressure.*

Equations (2-8) and (2-9) show that in case of a change in atmospheric pressure, while the gas state inside the vessel is still the same, the gage and vacuum pressures change, but the absolute pressure remains constant. Therefore, p_g and p_v do not describe the gas state, while the absolute pressure is an intensive property (a parameter) of state and it will be used in different thermodynamic relations.

2.4.7.3.3 The temperature

To evaluate the thermal movement, the KMT interconnected between the mean kinetic energy of molecules' straight motion and the absolute temperature concept proving the direct proportionality between these two values:

$$m_m \cdot c^2 / 2 = 3k \cdot T / 2 \tag{2-10}$$

where, *k* is the Boltzmann constant $k = 1.3806505 \cdot 10^{-23}$ J/K and T is the absolute temperature.

Nt2-6B: Equation (2-10) can be discussed similarly to the discussion of Equation (2-5) in Nt2-6. The result of such discussion will be "the calculated temperature through Equation (2-10) cannot be negative. That is why the calculated temperature using *the KMT's equation for* temperature is measured starting from the absolute zero". Such temperature is called absolute temperature (T_{abs} or T).

According to the KMT (see Equation 2-10) the ideal-gas absolute temperature approaches zero, when its molecules' straight velocity approaches zero. This zero-temperature state is out of the specter of ideal-gas states.

The concept of the absolute temperature (2-10) given by the KMT coincides with the usual concept of temperature defined in physics courses as the degree of hotness or coldness of a body. In other words, *the objects that have the same temperature in the usual concept of temperature have the same temperature according to the concept, which says that the temperature is a measure of the average kinetic energy of the straight movement of molecules.*

2.4.8 MODES OF WORK

This place is not appropriate to discuss the work interaction between a thermodynamic system and its surroundings. But some of the following definitions require, to be understood, a minimum knowledge about the modes (models or forms) of the energy transfer by work. This knowledge is abbreviated as follows:

Some of the mechanical work modes:
(1) The ***moving boundary work (W_b)*** is the work produced by a simple compressible system (gas) during its expansion or compression. Its calculating differential equation is $\delta W_b = p \cdot dV$, where p and V are the absolute pressure and volume of the system, respectively. It is the only work mode that is appropriate to be used with the content of this book, and we will deal with it in the following chapters. (2) The ***shaft work*** (here energy is transmitted with a rotating shaft). (3) The ***spring work***. (4) Work done on ***elastic solid bars***. (5) Work associated with the ***stretching of a liquid film***. (6) Work done to ***raise or to accelerate a body***. On the other hand, the ***electrical, magnetic,*** and ***electrical polarization work*** are nonmechanical work modes.[15]

2.4.9 THE SIMPLE COMPRESSIBLE SUBSTANCE AND THE SIMPLE COMPRESSIBLE SYSTEM

The theoretical study of the behavior of substances in gaseous state (which is the case that concerns us here) with taking in consideration the effects of velocity, elevation, surface tension, and external force fields (electric, magnetic, and gravitational) is too difficult. That is why science created the ideas of the simple substance and simple system, which simplify the study of a group of problems resulting, in terms of accuracy, acceptable for *heat engineering* answers.

OD2-27: A substance that can exchange energy through only work mode is called a ***simple substance***.[16]

OD2-28: A ***simple compressible substance*** is one in which the effects of the following are negligible: motion, fluid shear, surface tension, gravity, and magnetic and electrical fields.[17]

A more rigorous definition of a simple compressible substance is as follows.

> **OD2-29:** If the work mode through which a simple substance can exchange energy is boundary movement (or compression/expansion) then it is called a ***simple compressible substance***. In other words a simple compressible substance can exchange energy with its surroundings through $p \cdot dv$ work mode in addition to the energy exchange through heat interaction.[18]
>
> **OD2-29A:** ***Simple thermodynamic systems*** *are those whose states are defined in terms of only three coordinates: the uniform pressure p exerted by the system on the surroundings, the volume V, and the absolute temperature T.* It is assumed that the effects of surface tension or capillarity, external force fields (electric, magnetic and gravitational), and distortion of solid phases (shear) are absent.[19]
>
> **OD2-30:** The term ***simple system*** is applied when there is only *one* way the system energy can be significantly altered by work as the system undergoes quasiequilibrium processes.[20]
>
> **Nt2-7:** A comparison between definitions OD2-29A and OD2-29B leads to introduce HD2-2 and adopt the name *simple equilibrium state.*
>
> **OD2-30A:** A ***simple compressible system*** does not involve *kinetic or potential energy or energy from magnetic or electric fields*.[21]
>
> **OD2-30B:** A ***system*** is considered to be a ***simple compressible*** one in the absence of certain effects which are uncommon in many engineering applications. These are *electromagnetic and gravitational fields, surface tension, and motion*.[22]

The number of properties you need to describe the state of a system is spelled out in the state *postulate.*

> **OD2-30C:** ***The state postulate*** says: Two independent intensive properties are necessary to completely define the state of a simple compressible system. A simple compressible system does not involve kinetic or potential energy or energy from magnetic or electric fields. If the energy forms are involved in a process, then you need to specify properties related to those forms of energy in addition to the properties required by the postulate.[23]

From OD2-26, HD2-2, and OD2-30C we conclude the following.

HD2-4: *Thermodynamic equilibrium states can be simple or complicated with different degrees of complication. For a simple equilibrium state all intensive properties are uniform throughout and the number of independent properties you need to describe it is two.*
HD2-5: *If the deviations from uniformity of the intensive properties of a simple thermodynamic state are infinitesimal then the state is called a **simple quasi-equilibrium state**.*

2.4.10 THE STATE CHANGE PROCESSES OF A SYSTEM (GAS)

OD2-30D: If a system changes its state from one state to another passing through a number of intermediate states, then a ***state change process*** or simply ***process*** is performed.

OD2-31: The series of states through which a system passes during a process is called the **path** of the process.[24] It was agreed to give the code (1) for the beginning of the process and code (2) for the end of the process, and therefore the state change process of a system begins always from state (1) with parameters p_1, v_1, T_1 passing through states 3, 4, 5, 6, and 7, and always ends at state (2) with parameters p_2, v_2, T_2 (see Figure 2.7).

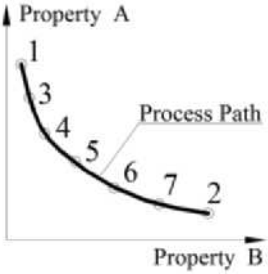

Figure 2.7. Numbering the distinguished points of a process' path

2.4.11 THE THERMODYNAMIC CYCLE

In the common case there are no conditions: (1) on the initial and final states of the state change process of a system and (2) on the process (does it cross itself or not), so it is possible for process's two states (initial and final) to be congruent. In this case, the process is called circular/closed/cyclic process or thermal/thermodynamic cycle.

2.4.12 THE EQUILIBRIUM PROCESS AND THE CONDITIONS TO REALIZE IT

2.4.12.1 The equilibrium process

OD2-32: ***The equilibrium*** *process is the state change process that consists of only equilibrium states* (see OD2-26); otherwise, the process is of no equilibrium. Such process can never exist because if its states are *equilibrium, how it can proceed? More exact expression is that the quasi-equilibrium process is the state change process that consists of only quasi-equilibrium states;* otherwise the process is of no *quasi-equilibrium*. Note that not all expressions in classical thermodynamics are exact ones.

OD2-33: *A* ***quasi-equilibrium process*** *is one, in which the departure from thermodynamic equilibrium is at most infinitesimal.* All states through which the system passes in a quasi-equilibrium process may be considered equilibrium states.[25] This definition is a good one and we shall adopt it.

Conclusion 2-1: *Since all the states through which the system passes in a quasi-equilibrium process may be considered equilibrium ones (see OD2-33), then the quasi-equilibrium process itself may be considered an equilibrium one. Therefore, we shall believe* in this book that *the quasi-equilibrium process and the equilibrium process are two names of the same process to which we shall refer to as the equilibrium process.*

Conclusion 2-2: *A state change process of a system (WF) is recognized equilibrium (we have just decided that quasi-equilibrium and equilibrium processes are two names for the same process) if it consists of only equilibrium and/or quasi-equilibrium states regardless of what happened to the other bodies that participated in this same process*.

Conclusion 2-3: Equilibrium processes are theoretical/hypothetical ones (they do not exist in reality). According to the above (see also HD2-2 *and* the § following it), each of them may consist of one or more of the kinds of equilibrium states. The equilibrium process that includes nonsimple equilibrium states cannot be represented by any of the equations of state or on property diagrams. Therefore, it is important to introduce the following definition:

HD2-6: The simple quasi-equilibrium/equilibrium process consists of only simple quasi-equilibrium/equilibrium states. This process can be legally represented by thermodynamic equations and/or on property diagrams.

2.4.12.2 The conditions for achieving an equilibrium (quasi-equilibrium) process

If the simple compressible substance/system undergoes a quasi-equilibrium process in a piston–cylinder arrangement, then at any piston's position and during piston's movement, system's state must be quasi-equilibrium and all its intensive properties must be almost uniform (the variation in any intensive property throughout the system must be infinitesimal, "the maximum difference between the values of any intensive property of any two macroscopic parts of the system must be infinitesimal/differential") throughout. Since two independent intensive properties are enough to completely define the state of a simple compressible system (see OD2-30C), then it will be enough to speak about two conditions those make the state change process of a simple compressible system quasi-equilibrium. The primary conditions to realize a quasi-equilibrium process can be that (1) the pressure difference between any two macroscopic parts of the system must not exceed dp and (2) the temperature difference between any two macroscopic parts of the system must not exceed dT.

To achieve the first condition the piston in the piston–cylinder arrangement must move extremely slowly, because its quick movement breaks down the equilibrium in the gas differently according to the direction and speed of piston's movement.

Also to achieve the second condition for *the gas that exchanges heat with an external (MR), the temperature of the heat exchange surface (the surface that separates between the gas and "MR") must vary from the temperature of the gas by* dT. Only in this case, the temperature difference between the hottest and coldest two macroscopic parts of the whole gas mass will be less than dT, and this is enough to consider that the temperature equilibrium in the gas is achieved. The temperature condition above indicates that the thermal equilibrium in the gas is inevitably achieved when (1) the external **MR** temperature differs from the gas temperature by dT, or (2) the **MR**, having significantly different temperature from that of the gas, is equipped with/by means (a layer of heat insulation) that allow making the temperature of the heat exchange surface that contacts the gas differ by dT from the gas temperature. The two traditionally well-known conditions for this special case equilibrium/quasi-equilibrium process are as follows.

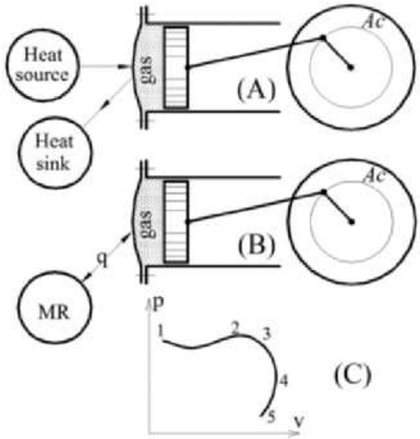

Figure 2.8. Executing an equilibrium/quasi-equilibrium dual-directional WF heat exchange process 1–2–3–4–5, using heat insulations and one MR. Partial processes 1–2–3 and 3–4–5 (in sub-figure C) are WF heat exporting and importing ones, respectively

> **OR2-3:** ***The pressure condition:*** *to achieve the equilibrium in pressure during a quasi-equilibrium process of a gas the speed of the piston must be infinitesimal.*

Although real processes taking place in reciprocating heat machines conduct/proceed at finite (not infinite) speeds and with friction, they do not much deviate from equilibrium processes, because the pressure changes in gases spread at high speed that equals to the speed of sound in the gas. This means that ***it is considered that the pressure condition is achieved in piston machines***.

> **OR2-4:** ***The temperature condition*** *for the equilibrium process:* *To achieve the equilibrium in temperature during a gas state change process, where the gas exchanges heat with an external (MR), the temperature of the heat exchange surface (the surface that separates between the gas and "MR") must vary from the temperature of the gas by* $\mathrm{d}T$. *Thus, the equilibrium in temperature can be achieved in two cases:*

(1) when the (MR_WF) temperature difference is significant and/but the MR is equipped with/by means (e.g., a layer of heat insulation) that allow making the temperature difference between *the gas and its heat exchange surface with the insulation infinitesimal and (2) when the (MR_WF) temperature difference is infinitesimal.*

If the WF undergoes a process that is fully/partially located in the liquid-vapor zone/region, the pressure (*p*) and temperature (*T*) will not be enough to completely define its saturated states ("*p* and *T*" here are dependent on each other's intensive properties "see OR2-1, the underlined in OD2-22 and OD2-24"). The conditions that are suitable for all gases will be better, which are as follows:

NR2-1: The mechanical equilibrium condition: *To achieve the mechanical quasi-equilibrium gas state change process, the mechanical power exchange (the speed of work exchange) between the WF and its surroundings must be infinitesimal.*
NR2-2: The thermal equilibrium condition: *To achieve the thermal quasi-equilibrium gas state change process the heat power exchange (the speed of heat exchange) between the WF and the MR must be infinitesimal.*

2.4.12.3 The minimum required number of MRs to achieve an equilibrium (quasi-equilibrium) WF state change process

According to the heat exchange direction between the MR and the WF, the minimum required number of MRs to achieve an equilibrium (quasi-equilibrium) WF state change process can be (1) one HS for the heat transfer from the MR to the WF (the sketch of the arrangement here is as in sub-Figure 2.8A but without heat sink), (2) one heat sink for the heat transfer from the WF to the MR (the sketch of the arrangement here is as in sub-Figure 2.8A but without HS), (3) one HS and one heat sink for the dual-directional heat transfer process (the sketch of the arrangement here is as in sub-Figure 2.8A). The HS serves process' parts in which the heat is imported to the WF; therefore, HS temperature must be higher than the highest temperature of process' parts those import heat to the WF. On the other hand, the heat sink serves process' parts in which the heat is exported from the WF; therefore, the temperature of the heat sink must be lower than the lowest temperature of process' parts those export heat from the WF.

Nt2-8: If the temperature interval that includes process' parts that import heat to the WF (1) do not interfere with that that includes process's parts that export heat from the WF and (2) has lower temperatures than that that includes exporting heat from the WF subprocesses, then the two MRs (the HS and the heat sink) can be replaced by one heat reservoir/MR of temperature lying between the two intervals (the sketch of the arrangement here is as in sub-Figure 2.8B).

Nt2-9: *Each MR, HS, heat sink, and heat reservoir mentioned in this paragraph is supposed to be equipped with/by means (e.g., a layer of heat insulation) that allow making the temperatures of the heat exchange surface with the gas and gas temperature almost equal* (see §2.4.3.2).

2.4.13 THE REVERSIBLE PROCESS AND THE CONDITIONS TO REALIZE IT

2.4.13.1 The definitions of the reversible process

OD2-34: *The* ***reversible process*** *happens so that it can—during reversing—pass through all states of the direct process/direction, and when the reverse process is completed, all elements that participated in it return to their initial states (the states they had before the start of the direct process).* This leads to that the reversible process and after its two-way going in the direct and reverse directions does not leave any trace/impact (change) on any of the elements that participated or did not participate in it. *It is said about the reversible processes that after their two-way going in the direct and reverse directions they do not leave any trace/impact (change) in the world,* and this saying can be a definition for the reversible process.

OD2-34A*: The* ***reversible process*** *for a thermodynamic system is the process that can be reversed after occurrence/happening and if it were completely reversed, then the compound process (the direct + the reversed one) do not leave any change/trace in/on both the system and the surroundings.*[26]

OD2-34B: *The* ***r*** *is the process that proceeds so that we can during reversing make it pass through all states of the direct direction, so that any changes do not occur at/on all the elements involved with it.*

> **OD2-34C:** *A* ***reversible process*** *is a process where the effects of following a thermodynamic path can be undone by exactly reversing the path.*[27]
>
> **OD2-34D:** *A* ***reversible process for a system*** *is defined as a process that, once having taken place, can be reversed and in so doing leave no change in either system or surroundings.*[28]
>
> **OD2-34E:** The reversible process is the one that can be reversed, and the system restored to its initial state, without leaving net effects in any of the systems involved.[29]

In addition to other definitions which speak about the system exclusively.

Apparently the above definitions do not lead to the same meaning for the reversible process, but logical discussion of all available definitions of the reversible process leads to the following comprehensive definition:

> **HD2-7:** *The* ***reversible process*** *is the one that if it, after being executed, was reversed until its start state, all elements (bodies) participating in it (1) pass through all the states passed during the direct process/proceeding, but in reverse order. And in doing so, (2) each of the participating elements exports all imported kinds of energy and imports all exported kinds of energy in the same doses of the direct process but in reverse order.* In other words, during each reverse of the differential/minute parts of the process each of the participating elements exports all imported, during this differential part, kinds of energy and imports all exported, during the same part, kinds of energy.

2.4.13.2 The practiced in thermodynamics conditions for achieving a reversible process

As it was stated above (see HD2-7) *achieving the reversibility of a WF state change process requires complying with a group of conditions that* secure *the exact reversing of the paths of all bodies participating in it.* Obtaining the conditions *that secure the exact reversing of each of the paths of the bodies participating in the reversible process requires a comprehensive analysis that cannot be included in the current size of this book. The main results of this analysis are*:

> **Nt2-10:** *WF's temperature uniformity is necessary for achieving the reversible process.*

Nt2-11: *WF's pressure and temperature uniformity is necessary for achiteving the reversible process.*
Nt2-12: *All states of the reversible process are quasi-equilibrium/ equilibrium.* Or:
HD2-8: *The reversible process is that special case of the quasi-equilibrium/equilibrium process, where, after reversing, all participants participating in it return back to their initial states along the same direct paths but in reverse order.*

Achieving temperature uniformity

According to the underlined one in (§2.9.3.2) the thermal equilibrium in the WF is inevitably achieved when:

OR2-5: ***The temperature condition for the (WF) temperature reversibility:*** *to achieve the possibility for the (WF) temperature reversibility during a reversible process of a gas,* the (MR_WF) temperature difference *must be infinitesimal.*
OR2-6: ***The temperature condition for a reversible process of a gas:*** *to achieve the reversibility in temperature during a gas state change process, where the gas exchanges heat with an external MR, the (MR_WF) temperature difference must be infinitesimal.*

A comparison between OR2-6 and OR2-4 leads to the following:

HR2-2: *The temperature condition for the equilibrium process allows (for some cases) the heat exchange between the MR and the WF (gas) at significant differences in temperatures, while the condition for the reversible process does not.*

2.4.13.3 Irreversible processes

OD2-35: Irreversible process is one that is not reversible.

2.4.13.3.1 Factors that make processes irreversible (in brief)

The factors that render/make a process irreversible are also called irreversibilities. Normally irreversible processes include one or more irreversibilities.

The following list comprises a few of the irreversibilities that are commonly encountered.

- Heat transfer through a finite temperature difference
- Unrestrained expansion of a gas or liquid to a lower pressure
- Spontaneous chemical reaction
- Spontaneous mixing of matter at different compositions or states
- Friction-sliding friction as well as friction in the flow of fluids
- Electric current flow through a resistance
- Magnetization or polarization with hysteresis
- Inelastic deformation.[30]

2.4.13.4 The internally reversible processes

> **OD2-36:** Irreversibilities present inside the system (the WF), such as the internal friction between different speed neighboring streams of the WF, are called *internal irreversibilities* and those present outside the system are *external irreversibilities*. The presence of any number of internal irreversibilities during a process is enough to render it *irreversible process*. And the absence of all internal and external irreversibilities, during a process, renders it *reversible process*. The absence of any internal irreversibility associated with the presence of any number of external irreversibilities, during a process, renders it an *internally reversible but externally irreversible* (or simply *internally reversible*) *process*. So
>
> **Nt2-13:** The internally reversible and the equilibrium/quasi-equilibrium processes are almost the same.

Thermodynamics studies primarily the equilibrium, internally reversible, and the reversible processes. These processes are the only ones that can be represented graphically and without hesitation for being composed of only equilibrium states while the representation of irreversible processes in the diagrams is mostly idiomatic/conventional (as we will see later) and it is not always allowed. The study of the irreversible process is executed in practice through two stages:

1) The irreversible process is idealized. It is replaced by a reversible or internally reversible one that shares with the irreversible

process (1) the initial and final states and (2) the thermal effect. After that the idealized (alternative reversible) process is studied thermodynamically.

2) Then the transition from the reversible process to the real irreversible one, through introducing experimental factors that take into consideration the deviation of real processes from the reversible ones.

All mathematical relationships that describe state change processes in thermodynamics belong to equilibrium processes, and accordingly they are valid for reversible processes.

NT2-14: A comparison between OR2-4 with OR2-5 shows that the second part of OR2-4 is absent in OR2-5 and the reversible process is invariably equilibrium/quasi-equilibrium.

2.5 IDEAL-GAS LAWS

2.5.1 INTRODUCTION

Subscripts (1 and 2) for $p, V,$ and T in the equations of this paragraph denote to state's number (1 and 2) of the same ideal gas.

The ideal-gas laws in parathions (Boyle–Mariette's $p_1 \cdot V_1 = p_2 \cdot V2$, Gay-Lussac's $V_1 / T_1 = V_2 / T_2$, the combined gas law "also known as the common Boyles Mariotte and Gay-Lussac's law" $p_1 \cdot V_1 / T_1 = p_2 \cdot V_2 / T_2$, and *the ideal-gas equation of state "also known as Clapeyron equation"* $p \cdot v = R \cdot T$) were first derived on an experimental basis using the real gases other than water vapor (such as air and hydrogen at low pressures) that existed at that time. It was found that the lower the densities of gases the better the accordance with calculation results by the mentioned laws. These ideal-gas laws were derived again and completed by the KMT. The derivation by the KMT started from substituting the (*n*) and ($m_m \cdot c^2 / 2$) expressions from Equations (2-6) and (2-10) in the KMT basic Equation (2-5). Thus (see Equations 2-2c and 2-3b),

$$\mathrm{p \cdot v = k \cdot T/m_m}, \quad \mathrm{m \cdot p \cdot v = k \cdot T \cdot N}, \quad \text{or} \quad \mathrm{p \cdot V = k \cdot T \cdot N} \qquad (2\text{-}11)$$

2.5.2 IDEAL-GAS EQUATION OF STATE (CLAPEYRON EQUATION) ABR

Applying Equation (2-1b) in Equation (2-11) we obtain

$$\mathrm{p \cdot v = k \cdot T \cdot N_A / M} \qquad \text{or} \qquad \mathrm{M \cdot p \cdot v/T = k \cdot N_A} \tag{2-11a}$$

The molecular mass (the mass of one molecule m_m) and Boltzmann constant (*k*) are dimensional values; therefore, unless we choose the unit system, we cannot substitute their numerical values. For the metric SI units: $\mathrm{N_A} = 6.022\ 141 \times 10^{23}\ \mathrm{mol^{-1}}$ (see Equation 2-1) and Boltzmann constant (see the notice that explains the constituents of Equation 2-10) $k = 1.3806505 \cdot 10^{-23}\ \mathrm{J/K}$ and therefore,

$$\begin{aligned}\frac{\mathrm{Mpv}}{\mathrm{T}} &= 8.314472383109145\frac{\mathrm{J}}{\mathrm{mol \cdot K}} = \mathrm{R_u} = \mathrm{MR} \\ &= \overline{\mathrm{R}} \approx 8.3145\frac{\mathrm{J}}{\mathrm{mol \cdot K}} = 8.3145\frac{\mathrm{kJ}}{\mathrm{kmol \cdot K}}\end{aligned} \tag{2-12}$$

The second side of Equation (2-12) is the same for all ideal gases. It is called the ***universal gas constant*** and denoted by *R_u*, *MR*, or $\overline{R}$ and equals additionally the sixth and seventh sides of the same equations. The R_u unit here belongs to the metric SI (it is one of its base units). Also, the exact numerical value of R_u that constitutes the second side is rounded up in the other sides to consist of only five digits.

Substituting SI units' values from Equations (1-19) and (1-31) and from (OD2-12) in Equation (2-12) we obtain

$$R_u = 8.314472383109145\frac{\mathrm{J}}{\mathrm{mol \cdot K}} = 1\ 545\ \frac{\mathrm{ft \cdot lbf}}{\mathrm{lbmol} \cdot R} \tag{2-13}$$

For a particular gas, the molar and relative molar masses are constants and their values can be calculated according to OD2-13 (see items 3 and 4). From Equations (2-2a) and (2-12) we obtain

$$\frac{\mathrm{p \cdot v}}{\mathrm{T}} = \frac{\mathrm{R_u}}{\mathrm{M}} = \frac{\overline{\mathrm{R}}}{\mathrm{M}} = \frac{8.3145\ [\mathrm{J/(mol \cdot K)}]}{10^{-3}\mathrm{M_r}\ [\mathrm{kg/mol}]} = \frac{8314.5}{\mathrm{M_r}}\frac{\mathrm{J}}{\mathrm{kg \cdot K}} = \frac{8.3145}{\mathrm{M_r}}\frac{\mathrm{kJ}}{\mathrm{kg \cdot K}} = \mathrm{R}$$

from which we obtain

$$\left.\begin{aligned}&\mathrm{R_u = M \cdot R = MR} \qquad\qquad \text{and} \\ &\mathrm{R} = \frac{\mathrm{R_u}}{\mathrm{M}} = \frac{8.3145}{\mathrm{M_r}}\frac{\mathrm{kJ}}{\mathrm{kg\ K}} = \frac{8314.5}{\mathrm{M_r}}\frac{\mathrm{J}}{\mathrm{kg\ K}}\end{aligned}\right\} \tag{2-13a}$$

and $$p \cdot v = R \cdot T \tag{2-14}$$

where, *R* is the gas constant. It is constant for each gas and is an exclusive function of the molar mass *M* (see Equation "2-2a").

As seen from the first part of Equation (2-13a), the third side (*MR*) can be used as the symbol of the universal gas constant (R_u), and in so doing the symbol will denote additionally its calculating equation $MR = M \cdot R$. Therefore, we shall adopt this symbol throughout this book.

In this book, we shall deal with only the numerical value, rounded up to only five digits, of $R_u = MR$.

Each of $R_u = MR$ and M_r is well known and published in many references and tables. Definition OD2-13 is helpful in obtaining (M_r).

Multiplying both sides of Equation (2-14) by the ideal-gas mass (*m*) we obtain:

$$p \cdot V = m \cdot R \cdot T \qquad \text{or} \qquad p \cdot V = m \cdot R_u \cdot T/M \tag{2-15}$$

The differential of Equation (2-14) is $p \cdot dv + v \cdot dp = R \cdot dT$. Dividing this differential/equation by Equation (2-14) we obtain $\frac{dp}{p} + \frac{dv}{v} = \frac{dT}{T}$. Thus, both equations

$$p \cdot dv + v \cdot dp = R \cdot dT \qquad \text{and} \qquad dp/p + dv/v = dT/T \tag{2-16}$$

are differentials of Equation (2-14): the first one is dimensional, while the second one is dimensionless.

Integrating the second part of Equations (2-16) from state (1) to state (2), we obtain

$$\ln\frac{p_2}{p_1} + \ln\frac{v_2}{v_1} = \ln\frac{T_2}{T_1} \tag{2-16a}$$

Equations (2-12) and (2-14) through (2-16) are known as Clapeyron equations, after the French engineer and physicist Benoît Paul Émile Clapeyron. They are also known as ideal-gas equations of state. The full name of each of them is Clapeyron equation for (mole or kmole), Clapeyron equation for unit mass (or simply Clapeyron equation), Clapeyron equation for a mass (*m*), and the differential Clapeyron equation, respectively.

2.5.3 AVOGADRO'S LAW

The following topics will be much abbreviated, with keeping the possibility to use them in solving problems.

2.5.3.1 Others' statements (OSs)

> ***OS2-1*:** *Equal volumes of different gases, at the same temperature and pressure, contain the same number of molecules. This is the statement o:* Avogadro's (law, hypothesis, theory or principle).

Writing the third of Equations (2-11) twice (once for the ideal gas No 1 that is defined by its pressure p_1, volume V_1, mass m_1, molar mass M_1, mass of one molecule m_{m_1}, temperature T_1, number of moles K_1, and number of molecules N_1. And once for the ideal gas No 2 defined by its pressure p_2, volume V_2, mass m_2, molar mass M_2, mass of one molecule m_{m_2}, temperature T_2, number of moles K_2, and number of molecules N_2) and dividing the equation for the first gas by the equation for the second gas we obtain:

$$\frac{p_1}{p_2}\cdot\frac{V_1}{V_2}\cdot\frac{N_2}{N_1}\cdot\frac{T_2}{T_1}=1 \tag{2-17}$$

Repeating the above procedure on the third of Equations (2-2c) we obtain:

$$\frac{m_1}{m_2}=\frac{N_1\cdot m_{m_1}}{N_2\cdot m_{m_2}}=\frac{N_1\cdot M_1}{N_2\cdot M_2}=\frac{K_1\cdot M_1}{K_2\cdot M_2}$$

from which we obtain that:

$$(1)\ \frac{N_1}{N_2}=\frac{K_1}{K_2},\ (2)\ \frac{N_1}{N_2}=\frac{K_1}{K_2}=1 \Rightarrow \frac{m_1}{m_2}=\frac{m_{m_1}}{m_{m_2}}=\frac{M_1}{M_2} \qquad \text{and}$$

$$(3)\ \frac{m_1}{m_2}=\frac{m_{m_1}}{m_{m_2}}=\frac{M_1}{M_2} \quad \Rightarrow \quad \frac{N_1}{N_2}=\frac{K_1}{K_2}=1\ .$$

The last two items can be combined as:

$$\frac{N_1}{N_2} = \frac{K_1}{K_2} = 1 \Leftrightarrow \frac{m_1}{m_2} = \frac{m_{m_1}}{m_{m_2}} = \frac{M_1}{M_2} \tag{2-17a}$$

Equations (2-17) and (2-17a) are the base for stating Avogadro's law, to which we suggest the following abbreviated statement "***If three relations of the following four groups (a relation of each group) a—*** $V_1 = V_2$***, b—*** $T_1 = T_2$***, c—*** $p_1 = p_2$, and ***d—(*** $N_1 = N_2$***,*** $m_1 \div M_1 = m_2 \div M_2$, or $K_1 = K_2$***) are realized, then the other relations of these groups are inevitably realized.***"

As stated, at the beginning of this paragraph, by Avogadro himself, Avogadro's law can be easily derived from the bolded abbreviated statement in italics for the case of realizing equations (**a, b,** and **c**). Avogadro's law can be introduced as the following abbreviation/equation:

$$\left.\begin{array}{ll} \text{If} & V_1 = V_2,\ T_1 = T_2 \text{ and } p_1 = p_2, \\ \text{then} & N_1 = N_2,\ \dfrac{m_1}{M_1} = \dfrac{m_2}{M_2} \text{ and } K_1 = K_2 \end{array}\right\} \tag{2-17b}$$

For the case of realizing relations (**b, c,** and one of **d-**relations) the statement can be introduced by one of the following abbreviations/equations:

$$\left.\begin{array}{ll} \text{If} & T_1 = T_2,\ p_1 = p_2 \text{ and } N_1 = N_2, \\ \text{then} & V_1 = V_2,\ \dfrac{m_1}{m_2} = \dfrac{M_1}{M_2} \text{ and } K_1 = K_2 \end{array}\right\} \tag{2-17c}$$

$$\left.\begin{array}{ll} \text{If} & T_1 = T_2,\ p_1 = p_2 \text{ and } \dfrac{m_1}{m_2} = \dfrac{M_1}{M_2}, \\ \text{then} & V_1 = V_2,\ N_1 = N_2 \text{ and } K_1 = K_2 \end{array}\right\} \tag{2-17d}$$

$$\left.\begin{array}{ll} \text{If} & T_1 = T_2,\ p_1 = p_2 \text{ and } K_1 = K_2, \\ \text{then} & V_1 = V_2,\ \dfrac{m_1}{m_2} = \dfrac{M_1}{M_2} \text{ and } N_1 = N_2 \end{array}\right\} \tag{2-17e}$$

Abbreviation/equation (2-17d) can be read as "***The masses of any two different ideal gases, at the same temperature and pressure, and whose***

ratio equals the ratio of their molar masses, occupy equal volumes. " This statement is the statement of the ***inverse Avogadro's law.***

Rewriting abbreviation/Equation (2-17e) for $K_1 = K2 = 1$ (m_1, m_2, V_1, and V_2 become M_1, M_2, Mv_1 and Mv_2, respectively) we obtain:

$$\left.\begin{aligned}&\text{If }\ K_1 = K_2 = 1,\ p_1 = p_2 \text{ and } T_1 = T_2,\ \text{ then}\\ &Mv_1 = Mv_2,\ \frac{m_1}{m_2} = \frac{M_1}{M_2} \text{ and } N_1 = N_2 = N_A\end{aligned}\right\} \tag{2-17f}$$

which can be read as "***the molar specific volume at the same pressure and temperature is the same for all ideal gases.***" Thus,

$$\left.\begin{aligned}&v = \frac{Mv}{M}; \qquad K = \frac{m}{M} = \frac{V_n}{Mv_n} = \frac{V}{Mv}; \qquad \rho = \frac{M}{Mv} \qquad \text{and}\\ &Mv_n = M \cdot v_n = 22.4146\ \frac{m^3}{kmol} \approx 22.4\ \frac{m^3}{kmol} = 0.0224\ \frac{m^3}{kmol}\end{aligned}\right\} \tag{2-18}$$

2.6 IDEAL-GAS MIXTURES

OD2-37: ***The homogeneous gas:*** *The gas whose molecules are of the same chemical formula.* Hydrogen is a homogeneous diatomic gas, because the chemical formula of any of its molecules is H_2, but air when considered composed of only oxygen and nitrogen is a diatomic non-homogeneous gas, because the chemical formula of some of its molecules is N_2 while the chemical formula for the rest of its molecules is O_2.
OD2-37A: ***The homogeneous mixture:*** *The gas whose molecules are not of the same chemical formula, but the molecules of each chemical formula are uniformly distributed throughout the mixture.* The atmospheric air is a homogeneous mixture.

Nonreacting ideal-gas mixtures are accepted as pure substances, because they are homogeneous (see the underlined in italics in §2.4.3). Thus, we are allowed to apply ideal-gas laws to them if we, when dealing with these mixtures, modify the international/chemical mole definition to be additionally appropriate for ideal-gas mixture calculations. This was traditionally done by introducing the following **OD2-38.**

OD2-38: ***the apparent molar mass of a gas mixture*** M_{mix} ***is the molar mass of an imaginary (does not exist in nature) pure gas, which has the same mass and number of molecules*** N_{mix} ***that the mixture has.***

Thus, we can use the same ideal-gas relationships in calculations of ideal-gas mixtures. From these relations,

mixture's quantity in kmol:

$$K_{mix} = K_1 + K_2 + K_3 + \cdots + K_k + \cdots + K_n \tag{2-19}$$

$$K_{mix} = \frac{m_{mix}}{M_{mix}} = \frac{V_{n_{mix}}}{(Mv)_{n_{mix}}} = \frac{V_{n_{mix}}}{\bar{v}_{n_{mix}}} = \frac{V_{mix}}{(Mv)_{mix}} = \frac{V_{mix}}{\bar{v}_{mix}} \tag{2-19a}$$

Mixture's specific volume:

$$v_{mix} = \frac{(Mv)_{mix}}{M_{mix}}; \qquad v_{mix_n} = \frac{(Mv)_{n_{mix}}}{M_{mix}} = \frac{22.4146}{M_{mix}} \tag{2-20}$$

Mixture's density:

$$\rho_{mix} = \frac{M_{mix}}{(Mv)_{mix}}; \qquad \rho_{mix_n} = \frac{M_{mix}}{(Mv)_{n_{mix}}} = \frac{M_{mix}}{22.4146} \tag{2-21}$$

where, K_k is the quantity of component (*k*) in one kmol. All other symbols in equations (2-19 through 2-21) last with the subscript/sub-subscript (mix) that denotes to the fact that they are mixture's dimensions. And when these symbols are freed from this subscript/sub-subscript they become familiar to the reader, because they were described previously in this book.

The unit of 22.4146 is m^3/kmol (or dm^3/mol), therefore, the unit of M_{mix} that shares with it the same third side of the second of equations (2-20 or 2-21) cannot be other than (mol or kmol) and accordingly the mentioned sides must be excluded from equations (2-20 and 2-21) when used for the calculations in English units.

Although OD2-38 successfully solved the problem of the equations to be used in dealing with ideal-gas mixtures, it includes some contradictions that can be abbreviated as "the mole, molar-mass M_{mix} and number of moles K_{mix} of an ideal-gas mixture are real (not apparent) values." For details see Additional Discussions in Volume II of this book.

2.6.1 THE LAWS OF IDEAL-GAS MIXTURES THAT CAN BE DERIVED ON THE BASIS OF THE KMT

These laws were first derived experimentally (exactly as the ideal-gas laws) and then were derived from the ideal-gas laws.

2.6.1.1 Dalton's law

It is also known as "Dalton's law of partial pressure" or "Dalton's law of additive pressure" and was derived experimentally and it states, "The pressure of a gas mixture is equal to the sum of the pressures each gas would exert if it existed alone at mixture's temperature and volume."[31] Thus,

$$\mathrm{p}_{\mathrm{mix}} = \mathrm{p}_1 + \mathrm{p}_2 + \cdots + \mathrm{p}_{\mathrm{k}} + \cdots + \mathrm{p}_{\mathrm{n}} = \sum_{\mathrm{k}=1}^{\mathrm{n}} \mathrm{p}_{\mathrm{k}} \tag{2-22}$$

where, p_k is the partial pressure of mixture's component *k* or the pressure that mixture's component *k* would exert, if it occupied the volume of the mixture alone and had mixture's temperature.

From Dalton's law we conclude that *each gas in the mixture occupies all mixture's volume, has mixture's temperature and is affected by its own (partial) pressure*.

2.6.1.2 Amagat's law

It is also known as "Amagat's law of partial volume" or "Amagat's law of additive volumes." It was derived experimentally and it states that "The volume of a gas mixture is equal to the sum of the volumes each gas would occupy if it existed alone at the mixture temperature and pressure." Thus,

$$V_{\mathrm{mix}} = V_1 + V_2 + \cdots + V_k + \cdots + V_n = \sum_{k=1}^{n} V_k \tag{2-23}$$

where, V_k is the partial volume of mixture's component *k*, or the volume that mixture's component *k* will occupy if it were alone at the pressure and temperature of the mixture. This means that *partial volumes measure quantities*.

2.6.2 GAS MIXTURE COMPOSITION

To calculate the characteristics of a gas mixture, one must know the composition of this mixture that can be given by specifying either the amounts of the components of the mixture (or the composition of the mixture), or the portions of each of the components in the mixture (or the relative composition of the mixture) that can be (1) mass (given by mass portions g_k), (2) volumetric (given by volumetric portions r_k), or (3) molar (given by molar portions z_k).

If a gas mixture is composed of (n) components having masses m_1, m_2, m_3, …, m_k, … and m_n, then the mass portion g_k of any component k is

$$\mathrm{g_k = m_k \div \sum_{i=1}^{n} m_k = m_k \div m_{mix}} \tag{2-24}$$

The sum of all mass portions:

$$\sum_{1}^{n} g_k = 1 \tag{2-25}$$

Giving the relative mass composition means giving g_k values of all gas mixture components. By multiplying each mass portion by 100, we obtain the percentage mass composition of the gas mixture.

The relative volumetric composition of the gas mixture can be given using partial volumes, because they (partial volumes) measure quantities (see the end of §2.6.1.2). If a gas mixture is composed of (n) components having partial volumes V_1, V_2, V_3, … , V_k, … and V_n, then the volumetric portion r_k of any component k is:

$$r_k = V_k \div \sum_{k=1}^{n} V_k = V_k \div V_{\mathrm{mix}} \tag{2-26}$$

Each V_k is taken at mixture's pressure p_{mix} and temperature T_{mix}. The right side of Equation (2-26) is obtained from the middle side using Amagat's law (2-23).

The sum of all volumetric portions:

$$\sum_{1}^{n} r_k = 1 \tag{2-27}$$

The relative molar/kilomolar composition of a gas mixture can also be used. In this case we need to introduce additional information that relates to the mole of the nonreacting ideal-gas mixture. Historically the following definition was introduced:

Ratio $K_k \div K_{\mathrm{mix}}$ is called molar portion and is denoted by z_k. Thus,

$$z_k = K_k \div K_{\mathrm{mix}} \tag{2-28}$$

Applying Equation (2-19) in (2-28) we obtain:

$$\sum_1^n z_k = 1 \tag{2-28a}$$

From Equations (2-18) through (2-19a) and (2-28a) we conclude that giving all portions of the components of a gas mixture (giving all gas mixture's components' portions), except one whose full name must be also given is enough to completely define its composition, because it is possible to calculate the only not given portion from the appropriate of the above-mentioned equation.

It is easy to prove that the molar and volumetric portions are numerically equal, and obtain the following equation:

$$r_k = V_k \div \sum_1^n V_k = K_k \div \sum_1^n K_k = z_k \tag{2-29}$$

This means that if a mixture's volumetric composition is known, then its molar composition is also known and vice versa.

If a mixture's mass composition is known ($\mathrm{g_1, g_2, g_3, \cdots, g_k, \cdots g_n}$ are known), then we can calculate all values of $\mathrm{r_1 = z_1}$, $\mathrm{r_2 = z_2}$, $\mathrm{r_3 = z_3, \cdots, r_k = z_k, \cdots r_n = z_n}$, which constitute the volumetric/molar composition, by using the equation:

$$\mathrm{r_k = z_k = \frac{g_k}{M_k} \div \sum_{k=1}^{n} \frac{g_k}{M_k}} \tag{2-30}$$

And if the mixture's volumetric/molar composition is known ($r_1 = z_1, r_2 = z_2,\ \mathrm{r_3 = z_3, \cdots, r_k = z_k, \cdots}$ and $\mathrm{r_n = z_n}$ are known), then we can calculate all the values of $\mathrm{g_1, g_2, g_3, \cdots, g_k, \cdots g_n}$, which constitute the mass composition, by using the equation:

$$\mathrm{g_k = M_k \cdot r_k / \sum_{k=1}^{n} (M_k \cdot r_k) = M_k \cdot z_k / \sum_{k=1}^{n} (M_k \cdot z_k)} \tag{2-31}$$

To calculate ideal-gas mixture's specific volume and/or density, the following equations are used:

$$v_{mix} = \sum_{k=1}^{n} \left({}^{g_k}\!/_{\rho_k} \right) \text{ and } \rho_{mix} = \sum_{k=1}^{n} (r_k \cdot \rho_k) = \sum_{k=1}^{n} (z_k \cdot \rho_k) \qquad (2\text{-}32)$$

To calculate gas mixture's **apparent/average** molar mass (see OD2-38 and Equations 2-19 and 2-19a), we use the following equations:

$$M_{mix} = \sum_{1}^{n} M_k . r_k \quad \text{and} \quad M_{mix} = 1 / \sum_{1}^{n} (g_k / M_k) \qquad (2\text{-}33)$$

$$\frac{V_{mix}}{K_{mix}} = \frac{V_k}{K_k} = (Mv)_{mix} = (Mv)_k = \overline{v}_{mix} = \overline{v}_k \qquad (2\text{-}34)$$

We remind that V_k here is the partial volume of gas (k) in the mixture this gas is found alone at mixture's pressure and temperature, which leads according to Avogadro's law to $(Mv)_{mix} = (Mv)_k$.

The dry atmospheric air that is close to earth surface has approximately constant composition that allows us to consider its apparent molecular weight μ_{air}=29.[32] Today's expression for the previous sentence is: the apparent relative molar mass of Air is $M_{r_{air}} = 29$ and its more accurate value is $M_{r_{air}} = 28.97$, which leads to the following atmospheric air characteristic constant $R_a = 8314.5/28.97 = 287.003797031411805315 8\ \text{J/(kg} \cdot \text{K)}$ and we shall adopt it in this book as,

$$R_a = 287 \ \text{J/(kg} \cdot \text{K)} \qquad (2\text{-}35)$$

$R_a = 287\,\text{J}/(\text{kg} \cdot \text{K})$ is well known and it can be used directly in thermodynamic calculations. To calculate partial pressures we can use the following equations:

$$p_k = p_{mix} \cdot g_k \cdot R_k / R_{mix} = p_{mix} \cdot g_k \cdot M_{mix} / M_k \ \ \textbf{and} \ \ p_k = r_k \cdot p_{mix} \qquad (2\text{-}36)$$

2.7 THE BOUNDARY WORK CALCULATION

The calculating differential equation of the moving boundary work (W) is (see §2.4.8)

$$\delta W = pdV \qquad \textbf{or} \qquad \delta w = pdv \qquad (2\text{-}37)$$

where, p, V, v, and w are the absolute pressure, volume, specific volume, and SBW of the system, respectively.

The work of 1 kg of the gas for the specific volume change from v_1 to v_2 equals to:

$$w = \int_{v_1}^{v_2} p \cdot dv \tag{2-38}$$

If the mass of the gas inside the cylinder is *M* (its volume will be denoted by *V*), then the result of a similar discussion would be:

$$W = \int_{V_1}^{V_2} p \cdot dV = \int_{v_1}^{v_2} p \cdot M \cdot dv = M \int_{v_1}^{v_2} p \cdot dv = M \cdot w \tag{2-39}$$

This means that the work of $M\,\text{kg}$ of the gas at the equality of all other conditions is greater than that of 1 kg by *M* times. In other words the work is an **additive dimension** (see OD2-23), and we can write for it relations of the type of Equation (2-3).

$$W = M \cdot w = V_n \cdot w' = K \cdot (\mu w) \qquad \text{and} \qquad (\mu w) = \mu \cdot w \tag{2-40}$$

In the common case the work of the gas may not consist of only the gas expansion work. In many cases the work of the gas includes other modes of work. Therefore, we will assume that the work (*w*) of the gas is composed of the expansion (boundary) work in addition to other works that are symbolized by w^*. Thus, the general formula for gas work will be:

$$w = \int_{v_1}^{v_2} p \cdot dv + W^* \tag{2-41}$$

It should be mentioned that relationship (2-41) for calculating the work of the gas in reversible process is valid to calculate gas work in the irreversible process, but there is a fundamental difference between the two cases:

In the first case this work is clear whether it was an expansion or compression work (work spent on compressing the gas), while in the second case (for the irreversible process) the integral $\int p \cdot dv$ expresses a total value that includes:

- The work that appeared or disappeared
- The friction work
- The work that resulted from absorbing the heat of friction or a part of it by the (WF).

In other words, the integral $\int p \cdot dv$ represents the WF work against the piston; here, we will meet problems on how to estimate the values of the intensive properties of nonequilibrium states.

2.8 RECOGNIZING THERMODYNAMIC PROPERTIES

A quantity is a property if its change in value between two states is independent of the process. It follows that if the value of a particular quantity depends on the details of the process, and not solely on the end states, that quantity cannot be a property.[33]

Integrating the differential of the volume (V) from state (1) to state (2) does not require any additional information. Therefore, the differential of the volume (V) is exact one, its symbol is (dV), and its integral from state (1) to state (2) is

$$\int_1^2 dV = V_2 - V_1 \tag{2-42}$$

Also, integrating the differential $(R \cdot dp)/p$ from state (1) to state (2) does not require any additional information, because R_0 is constant and

$$\int_1^2 (R \cdot dp)/p = R \cdot \int_1^2 dp/p = R \cdot \ln(p_2/p_1) = R(\ln p_2 - \ln p_1) \tag{2-42a}$$

Thus, volume (V) and pressure (p) are properties since the change in value of each of them between two states is independent of the process (since their differentials are exact ones). It is obvious that if the change of $R \cdot \ln p$ value between two states is independent of the process, when R = constant, means that the change of (p) value between the same two states is independent of the process too.

From the above we obtain the following.

<u>Nt2-15:</u> *The* mono variable *differentials are exact ones and their integrals are thermodynamic/state properties.*
<u>OR2-7:</u> *If the differential of a dimension is exact and does not include process' path functions, then the dimension is a thermodynamic/state property, else it is not.* With the exception of the intensive properties *density, pressure*, *and temperature,* this thermodynamic dimension can be either intensive property if its unit includes a quantity unit in its

denominator or extensive property if its unit does not include a quantity unit in its denominator.

The mostly used in thermodynamics dimensions are mono-variable dimensions/functions (functions of one other dimension) or dual-variable dimensions/functions (functions of two other dimensions). Therefore, Let us discuss:

- The differential of the mono-variable dimensions/functions $y = f(x)$ that equals the product $f'(x) \times dx$. In this differential, $f'(x)$ is either another function of *x* [e.g., $F(x)$] or constant, and in both cases we can refer to $f'(x)$ as $F(x)$ and to the product $f'(x) \times dx$ as $F(x) \times dx$. This product $F(x) \times dx$ is a sole function of (*x*), and the only requirement for its integration is to know the integration rules, which means that the mono-variable *differentials are always exact.*
- The dimensions' differentials are:

$$\delta q = c_0 dT, \quad \delta W_b = p \cdot dV \quad \textbf{and} \quad ds = c_{p0} \frac{dT}{T} - R\frac{dp}{p} \qquad \text{(A)}$$

See Equations 3-6, 2-37, and 3-25). These three *differentials are actually four, because the third differential is the sum of two differentials. (1) In the differential* $\delta q = c_0 \cdot dT$, c_0 by definition (see Equation 3-5) is process path function, which means (see the beginning of §2.8) that *q* is not a thermodynamic property because the *change in value of q between two states of the system will be dependent on the process. We did not try to evaluate whether the differential of q is exact or not, because this analysis through the expression* $\delta q = c_0 \cdot dT$ will not lead to any result. The *analysis of the expression* $\delta q = T \cdot ds$ (see the second of Equation 3-22) by applying Equation (3-25) leads to the differential $\delta q = T \cdot ds = (c_{p0} \cdot dT) - (R \cdot T \cdot dp) / p$, whose variables cannot be separated that assures the inexactness of *differential* $\delta q = T \cdot ds$. (2) The *differential* $\delta W_b = p \cdot dV$ is ***inexact differential***, because *p* by definition (see Equations (2-5), (2-7), (2-11), (2-11a), and OD2-30C that can be stated as "an intensive property such as pressure of a simple compressible system is a function of two other independent properties among which can be the specific volume) cannot be a sole function of *v* or *V*. This means that the *differential* $\delta W_b = p \cdot dV$ cannot be integrated without adding another thermodynamic condition that is process' path dependent and that makes *w* a nonthermodynamic property. Symbol δ *in the above equations* is used instead of *d* to indicate the inexact differentials, whose integrals from state (1) to state (2) are not $q_2 - q_1 = \int_1^2 c_0 \cdot dT$ and $W_2 - W_1 = \int_1^2 p \cdot dV$ but:

$$q_{1-2} = \int_1^2 c_0 \cdot dT \qquad \text{and} \qquad W_{1-2} = \int_1^2 p \cdot dV \qquad (2\text{-}43)$$

Contrary to the first two differentials (see Equations A above), the third one that is $ds=(c_{p0}\cdot dT/T)-R\cdot dp/p$ is an exact one (it can be directly integrated) since each of its two terms is a mono variable differential [$c_{p0}\cdot dT/T$for ideal gases is a mono temperature differential since $c_{p0} = f(T)$ and $R\cdot dp/p$ is a mono pressure differential]. The variables of a differential that is an algebraic sum of mono variable differentials are always separated.

The above means that ***s*** is an intensive property. It is fully analyzed in §3.1.2.3.1.

Thus, ***the TH per unit mass (q) and the boundary work (W_b) are not thermodynamic state properties*** since their value changes between states 1 and 2 depend on the process.

2.9 A BRIEF OVERVIEW OF THE PROPERTIES OF REAL GASES

The following is only a reminder to the reader that thermodynamics does not only consider ideal gases (as we do in this book), but also considers real gases.

Experiments, conducted on gases in a wide range of changes in pressure and temperature, proved that the properties of all gases deviate in one way or another from the laws that characterize the behavior of ideal gases. The ideal-gas properties (*p*, *v,* and *T*) are always governed by the simple and exact equation of state $pv = RT$ (with the only constant *R*), and therefore, the ideal-gas isotherms in *pv_p* diagram are always horizontal straight (zero slope) lines. While the exact real gas equation of state is very complicated (with many constants) and therefore the real gas isotherms in *pv_p* diagram are mostly non-zero-slope ones. A sample of the $pv = f(p)$ curves of the same particular real gas that experimented in wide intervals of temperatures and pressures (from 1 to 3,000) kgf/cm^2 (see Figure 2.9) showed that, in an interval of temperatures, (1) the slope of the real gas isotherm is negative in all its points (isotherm t_1), which means that this gas is more compressible than the ideal one; (2) the slope of the real gas isotherm is positive in all its points (isotherm t_3), which means that this gas is less compressible than the ideal one; (3) the real gas isotherm has a minimum (isotherm t_2), which means that this gas at relatively low pressures is more compressible than the ideal one and at relatively high pressures is less compressible; (4) in the interval between t_2 and t_3, there is a temperature (t_B) at which no deviation appears in a wide range of pressure changes, that is, the real gas at this temperature in the mentioned range of pressure changes behaves as an ideal gas. This temperature (t_B) is called Boyle's temperature (or Boyle's point).

CONCLUSIONS

1) Gas (ideal and real) state equations are of the type $f(p, v, \text{and } T) = 0$ (see Equations 2-14, 2-43, and 2-44). It is valid for pure gases and for nonreacting gas mixtures.
2) The quantity of a pure gas (a nonreacting gas mixture) can be measured in mass units, moles, and volume units (see Equations 2-19 and 2-19a), but the values of pressure and temperature should be added to the volume units (e.g., m^3 at normal conditions, or, m^3 at 1 kPa absolute and 288 K).

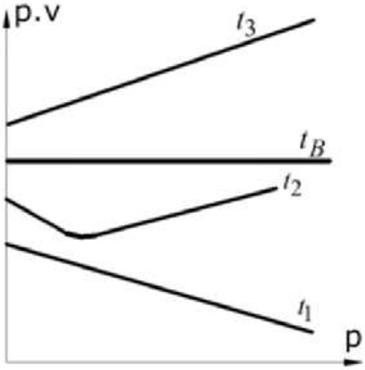

Figure 2.9. A schematic of four different temperatures isotherms of the same real gas in pv_p diagram. All isotherms of any ideal gas in pv_p diagram are straight horizontal lines.

2.10 SUMMARY

As seen from the table of contents, many topics are introduced in brief or comprehensively in this chapter. The equilibrium and reversibility are among the discussed in depth topics. And we shall introduce below some of the definitions and rules extracted from the discussions of these items in addition to those resulted from the comparison between the equilibrium and reversible processes

> **OD2-33:** ***A quasi-equilibrium process*** *is one, in which the departure from thermodynamic equilibrium is at most infinitesimal.* All states through which the system passes in a quasi-equilibrium process may be considered equilibrium states.

We shall believe that in this book the quasi-equilibrium process and the equilibrium process are two names for the same process that we shall refer to as the equilibrium process.

> **HD2-6:** The simple quasi-equilibrium/equilibrium process consists of only simple quasi-equilibrium/equilibrium states. This process can be legally represented by thermodynamic equations and/or by property diagrams.
> **OD2-34C** is one of the definitions of the reversible process: *The* ***reversible process*** *is the process that proceeds so that we can during reversing make it pass through all states of the direct direction; so that any changes do not occur at/on all the elements involved with it.*
> **OR2-3*****: The pressure condition*** *for both equilibrium and reversible processes**:** to achieve the equilibrium/reversibility in pressure during a quasi-equilibrium process of a gas the speed of the piston must be infinitesimal.*

The temperature condition for the equilibrium process: to achieve the equilibrium in temperature during a gas state change process, where the gas exchanges heat with an external *MR*, the temperature of the heat exchange surface (the surface that separates between the gas and “MR”) must vary from the temperature of the gas by $\mathrm{d}T$. Thus, the equilibrium in temperature can be achieved in two cases: (1) when the (*MR_WF*) temperature difference is significant and/but the *MR* is equipped with/by means (e.g., a layer of heat insulation) that allow making the temperature difference between the gas and its heat exchange surface with the insulation infinitesimal and (2) when the (*MR_WF*) temperature difference is infinitesimal. This second case constitutes the temperature condition of the reversible process.

The comparison between the equilibrium and reversible processes led to the following.

HD2-8: *The reversible process is that special case of the quasi-equilibrium/equilibrium process, where, after reversing, all participants participating in it return back to their initial states along the same direct paths but in reverse order.*

HR2-2: *The temperature condition for the equilibrium process allows (for some cases) the heat exchange between the MR and the WF (gas) at significant differences in temperatures, while the condition for the reversible process does not.*

CHAPTER ENDNOTES

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CHAPTER 3

THE FIRST LAW OF THERMODYNAMICS

3.1 HEAT TRANSFER CALCULATIONS DURING GAS STATE CHANGE PROCESSES

The heat transfer calculations during gas state change processes are the most common and important calculations in thermal engineering. Gas state changes can occur during physical processes (without any change in the molecular structure) or during chemical processes (with changes in the molecular structure).

We will discuss here, in some details, the first kind of calculations, which requires knowing the values of specific heats (SHs). Then, we will briefly introduce the second type.

3.1.1 THE SPECIFIC TRANSFERRED HEAT AND THE SPECIFIC HEAT

3.1.1.1 Basic definitions and relations

Others' Definitions (ODs): <u>OD3-1:</u> The specific transferred heat (STH) is the transferred heat (*Q*) to a unit quantity of matter (gas, liquid, or solid), and therefore (see Equations 2-3a),

$$\mathrm{STH} = \mathrm{Q} / \text{Matter quantity} \tag{3-1}$$

$$\left.\begin{array}{ll} \text{and} & \mathrm{q} = \mathrm{Q}/\mathrm{m}; \quad \mathrm{q}' = \mathrm{Q}/\mathrm{V_n}; \quad \overline{q} = \mathrm{Mq} = \mathrm{Q}/\mathrm{K}; \\ \text{or} & \mathrm{Q} = \mathrm{m}\times\mathrm{q} = \mathrm{V_n}\times\mathrm{q}' = \mathrm{K}\times\mathrm{Mq} = K\times\overline{q} \end{array}\right\} \tag{3-2}$$

OD3-2: The **heat capacity** (*C*) is the amount of heat (*Q*) to be transferred to the body (gas, liquid, or solid) during a particular process, to raise its temperature by one degree. Therefore, the heat capacity depends initially on the amount of the body, its kind (type), and how the heat transfer process proceeds, and its calculating equation is

$$C = Q/(T_2 - T_1) \tag{3-3}$$

OD3-2A: The specific heat capacity (SHC) or the SH (basic symbol *c*) is the share of the amount of heat (*Q*) to be transferred to a unit quantity of the body (gas, liquid, or solid) during a particular process, to raise its temperature by one degree. Thus, the SH depends initially on the kind of the body and how the heat transfer process proceeds, and its general form equation (see Equations 3-1 and 3-3) is

$$\mathrm{SHC} = \frac{C}{\text{Matter quantity}} = \frac{Q/(T_2 - T_1)}{\text{Matter quantity}} = \frac{\mathrm{STH}}{T_2 - T_1} \tag{3-4}$$

Let us introduce the following notes

Nt3-1: The most practiced SHs in thermodynamics are the constant volume (subscript *v*) and the constant pressure (subscript *p*) SHs. These SHs are usually positive and cannot equal zero or infinity. On the other hand, and according to the process undergone by the gas, its SH can be positive, negative, zero value, or infinite value. As we shall see later (see St6-1 and Table 6-1) the SH of (1) the constant-temperature (isothermal) process $c_t = \pm\infty$, (2) the isentropic (constant-entropy) process $c_s = 0$, and (3) several traditional polytropic processes *may have any value between* $+\infty$ and $-\infty$.

Nt3-1A: Because the symbols of ideal gas SHs are much repeated throughout this book and since all gases approach the behavior of the ideal gas when the pressure approaches zero (see **OD2-15**), we—as many other authors—will refer to the ideal gas SHs as the *zero-pressure ones,* symbolizing the constant-volume SH by c_{v0} and the constant-pressure one by c_{p0}, when we need/want to emphasize that the equation that includes SHs is assigned to ideal gases.

Based on the above definitions, the experimental data, and theoretical rules, (1) the SH (c_{rg}) of a particular real gas at a particular process depends on its temperature and pressure $c_{rg} = f(T, p)$, and with the presence of the equation of state, we can write $c_{rg} = f(T, v)$. And, the SH of a particular ideal gas and a particular process depends on temperature only $c_{ig} = f(T)$. Thus, we can write

$$\left.\begin{aligned} & c_{rg} = f(T, p, \text{gas kind, and process path}) \\ \text{and} \quad & c_0 = c_{ig} = f(T, \text{gas kind, and process path}) \end{aligned}\right\} \qquad (3\text{-}5)$$

Because (c) is temperature dependent, the specific heat (c) and heat capacity (C) calculated by Equations (3-3) and (3-4) are not the actual ones. They are the average ones. Any of the upper two symbols besides stands for “the average specific heat (c_{avg}) within temperature interval $T_1 \_ T_2$,” and any of the lower symbols stands for “the average heat capacity (C_{avg}) within temperature interval $T_1 \_ T_2$.” The availability of the temperature limits T_1 and T_2 of the average SHs and heat capacities in their equations or in the neighboring text allows using the abbreviated symbols $(c_{avg}$ and $C_{avg})$.

$$c_{avg}\Big|_{T_1}^{T_2} \quad \text{or} \quad \overset{T_2}{\underset{T_1}{c_{avg}}}$$

$$\overset{T_2}{\underset{T_1}{C_{avg}}} \quad \text{or} \quad C_{avg}\Big|_{T_1}^{T_2}$$

Thus, Equation (3-3) must be rewritten as

$$C_{avg} = Q / (T_2 - T_1) \qquad (3\text{-}5a)$$

And Equation (3-4) must be rewritten as

$$\left.\begin{aligned} & c_{avg} = C_{avg} / m = Q / (m \cdot \Delta T) = q / \Delta T \\ \text{or} \quad & Q = m \cdot q = m \cdot c_{avg} \cdot \Delta T = C_{avg} \cdot \Delta T \end{aligned}\right\} \qquad (3\text{-}4b)$$

To reach the ***actual SH,*** the temperature rise (ΔT) must be differential (dT). In this case, heat transfer will be differential too and since Q and q are not thermodynamic properties because they depend on the path of the process, their differentials are inexact (see § 2.8). The symbols of the differentials of Q and q are (δQ and δq) but not (dQ and dq). The ideal gas actual SH and transferred heat will be calculated from the following general relations:

$$\left.\begin{aligned} & c_0 = \delta Q / (m \cdot dT) = \delta q / dT \qquad \text{or} \\ & \delta Q = m \cdot \delta q = m \cdot c_0 \cdot dT \quad \text{or} \quad \delta q = c_0 \cdot dT \end{aligned}\right\} \qquad (3\text{-}6)$$

$$\textbf{or} \qquad Q = m \cdot q = \int_{T_1}^{T_2} m \cdot c_0 \cdot dT \qquad \textbf{or} \qquad q = \int_{T_1}^{T_2} c_0 \cdot dT \tag{3-7}$$

For the real gas, the ideal gas actual SH (c_0) in the Equations (3-6) and (3-7) is replaced by the real gas actual SH (*c*).

In Equation (3-4), as in any other equation, the units of all sides are the same, which means that both STH and SHC in the equation are either mass or molar or volumetric specific dimensions, and Equation (3-4) can be rewritten as the following group of calculating equations:

$$c = \frac{q}{T_2 - T_1}; \quad \bar{c} = Mc = \frac{Mq}{T_2 - T_1} \quad \text{and} \quad c' = \frac{q'}{T_2 - T_1} \tag{3-8}$$

Applying the group of equations (3-2), we obtain Group (3-9) of calculating equations:

$$\left.\begin{aligned} c = \frac{q}{T_2 - T_1} = \frac{Q}{m(T_2 - T_1)}; \quad Mc = \frac{Mq}{T_2 - T_1} = \frac{Q}{K(T_2 - T_1)}; \\ \text{and} \qquad\qquad\qquad c' = \frac{q'}{T_2 - T_1} = \frac{Q}{V_n (T_2 - T_1)} \end{aligned}\right\} \tag{3-9}$$

from which we obtain

$$Q / (T_2 - T_1) = m \cdot c = K \cdot Mc = V_n \cdot c' \tag{3-10}$$

<u>OD3-3:</u> The SH *is the amount of heat required per unit mass to raise the temperature by one degree.* It is considered here a homogeneous phase of a substance of constant composition. This phase may be a solid, a liquid, or a gas, but no change of phase will occur.[1]

Adding the last two sentences to each of OD3-2 and OD3-2A improves these definitions.

<u>Nt3-1B:</u> It is clear from the above that the heat capacity (*C*) is an additive dimension (OD2-23); therefore (see Nt2-3), its equations can be obtained from Equations (2-3) and (2-3a), and we can additionally write

$$\bar{c} = Mc = M \cdot c \tag{3-11}$$

For the case when allowed to ignore the SH temperature dependence in approximate calculations, (*c*) will still be a function of gas kind and process path so that we can write

$$\text{If } c \neq f(T) \Rightarrow c_0 = f(\text{gas kind and process path}) \qquad (3\text{-}12)$$

Nt3-2: Many authors define the SH as the heat transferred to a unit mass (see ODs 3-2A & 3-3).

Nt3-3: Although relations (3-5 and 3-12) are the most accurate expression of the reality of the SH, we usually deal with the expressions $c = f(T, p)$, $c = f(T)$, and $c = \text{constant}$, but *we must never forget the SH dependence on gas type and process path during calculations.* This means that the previous equations of this chapter and many other equations those include specific heat symbols and do not include information about process's kind are general form equations those can be written in three (the mass, the molar, and the volumetric) forms for each particular process. For example,

The forms of the first of Equation 3-8 for the constant pressure process are

$$c_p = \frac{q_p}{T_2 - T_1}; \qquad Mc_p = \frac{Mq_p}{T_2 - T_1} \quad \text{and} \quad c'_p = \frac{q'_p}{T_2 - T_1} \qquad (3\text{-}8a)$$

And the forms of the second of equations 3-7 for the constant volume process are

$$\left.\begin{aligned} q_v = \int_{T_1}^{T_2} c_{v0} \cdot dT; \qquad & Mq = \int_{T_1}^{T_2} Mc_{v0} \cdot dT \\ \text{And} \qquad & q' = \int_{T_1}^{T_2} c'_{v0} \cdot dT \end{aligned}\right\} \qquad (3\text{-}7a)$$

Nt3-4: In this book, we deal with only simple compressible systems.

3.1.2 THE CALORIC INTENSIVE PROPERTIES

In §2.4.7.3, we introduced three state intensive properties (specific volume, pressure, and temperature) called the thermal intensive properties, and the material delivered till now is enough to make the reader understand the following three new intensive properties (internal energy, enthalpy, and entropy) called the caloric intensive properties.

3.1.2.1 Internal energy

3.1.2.1.1 Introduction

The concept *internal energy* comes from the study of the forms of energy owned by the molecules/particles constituting a system. The molecules of any gas existing in nature/reality are different from that suggested by the kinetic-molecular theory (KMT) molecules, and accordingly, the properties of the molecules to be discussed in this paragraph do not comply with the properties of the KMT molecules stated in OD2-15 and §2.4.6. Thus, real molecules (1) may attract/repulse each other and (2) are not smooth spherical material points that are perfectly elastic. *They, according to their average concentration in the container, may attract/repulse each other and, according to their chemical formulas, have a volume, a mass that is not uniformly distributed throughout its volume, and have a shape.*

3.1.2.1.2 The molecular kinetic energies

Researches show that the molecules of real systems own different forms of kinetic and potential energies. During physical state change processes, only *the so-called molecular kinetic energies (MKEs)* and the cohesion forces' potential energies change. The other forms of molecules' potential energies such as the chemical and atomic energies (see §2.1.2) arise during the appropriate nonphysical change of each state, when changes occur in the structure of the molecule and/or atom.

Five forms of motions and accordingly five forms of *the so-called MKEs* may be owned by a molecule. These are as follows:

(A) The **translational** MKE (MKE_{trans}) is associated with the random motion of molecules. It fully exists in gases and slightly in liquids, and is absent in solids.[2]

(B) The **rotational** MKE (MKE_{rot}) does not exist in monoatomic substances nd exists in gases, liquids, and some solids, particularly those with a loose crystal structure, for example, solid N_2 and H_2 (nonpolar molecules).[2]

(C) The **vibrational** molecular energy (the energy of **intramolecular vibration**) is the kinetic energy associated with the relative motion of atoms that constitute the molecule. It exists in all three phases of diatomic and polyatomic substances.[2]

(D) The kinetic energy of **electrons** (the **electronic** MKE) is the energy of the electrons arising from their positions relative to the nuclei in the molecule.[2] This form is small compared with the other forms and exists in all three phases.

(E) The fifth form is the **lattice vibrational** molecular energy (the energy of the molecules of a solid crystal lattice). It exists only in the crystalic solid phase. Each of the lattice and intramolecular vibrations reaches its nonzero minimum when $T = 0$ K. This energy also increases with the increase of the crystal temperature (the higher the temperature of a crystal, the more intensively its atoms oscillate about their equilibrium position).[3]

Nt3-4A: (***can be skipped***) The MKE_{tr} and MKE_{rot} can be expressed in terms of molecule's mass (m_m), linear velocity (υ), moment of inertia (I), and angular velocity (ϖ) as $MKE_{tr} = m \cdot \upsilon^2/2$ and $MKE_{rot} = I \cdot \varpi^2/2$. This means that even when the molecule translates and rotates, MKE_{rot} will be zero for monoatomic substances because $I_{\text{monatomic}} = 0$ (because it is agreed that the mass of an atom is concentrated in its center of gravity, then the mass of a monatomic molecule is also concentrated in its atom's center of gravity having no extent, and accordingly, the monatomic molecule has no moment of inertia and no rotational energy, while diatomic, triatomic, and multiatomic molecules have extent and nonzero moment of inertia and accordingly they can have MKE_{rot}).

Nt3-4B: From the above, we conclude that for the gases existing in nature (1) *the so-called* MKE in diatomic and polyatomic gases consists of the translational, rotational, vibrational, and electronic forms (see items A through D) because no crystal lattices exist in gases, while (2) the MKE in monatomic gases consists of the MKE_{trans} and the ***electronic*** kinetic energies because they cannot have MKE_{rot} ($I_{\text{monatomic}} = 0$) (see item B, and Nt3-4A), vibrational molecular energy, or crystal lattices (see items C and E).

HD3-1: If we divide the five forms of *the so-called* MKE, introduced above, into two intermolecular and intramolecular groups, then *the so-called* translational, rotational, and lattice vibrational MKEs will constitute *the so-called* ***intermolecular MKEs***, while *the so-called* **vibrational** and the electronic MKEs will constitute *the so-called* ***intramolecular MKE***.

Nt3-4C: The intramolecular vibrational energy, as any oscillating movement's energy, is not purely a kinetic energy. Each vibrational energy between two interacting atoms consists, when it is constant by time, of two (kinetic and potential) equal sinusoidal but displaced by time/phase energies with zero minimums. Therefore, *the intramolecular vibrational energy of a substance/compound equals the maximum value of any of its kinetic or potential energy*. A similar discussion can be done for the lattice vibrational *molecular* energy that leads to *the lattice vibrational molecular energy of a substance/compound equaling the maximum value of any of its kinetic or potential energy*. This means that (1) *the intramolecular vibrational energy of a substance/compound is a function of the same variables of its intramolecular kinetic vibrational energy*, and (2) *the lattice vibrational molecular energy of a substance/compound is a function of the same variables of its lattice vibrational MKE.*

3.1.2.1.3 The constituents of the internal energy of a system

Since the *internal energy* of a system is the energy owned by its molecules, and since the forms of energy those owned by a molecule of a particular phase of the system are listed above, then the forms of *internal energy* of a system are the appropriate, of the following forms, to system's phase:

(I) The kinetic molecular forms of energy that are, according to the phase of the substance/compound, consist of some of the (1) translational, (2) rotational, (3) vibrational, (4) electronic, and (5) lattice vibrational kinetic energy forms.

(II) The potential molecular forms of energy that consist of (1) the cohesion forces' potential energy that changes with changing the distance between the molecules (i.e., with changing the volume occupied by the gas), and (2) the other potential energies that arise when changes occur in the structure of the molecule and atom (the chemical and atomic energies; see §2.1.2).

And, the sum of all appropriate to system's phase internal energy forms specified above (the values of the absent forms will be substituted by zero in this sum) of all molecules of the system is called the internal thermal energy or just the internal energy of the system.

We should mention here that in thermodynamics, we usually care about the internal energy changes but not about its total/absolute value; therefore, we usually neglect the constituents of the internal energy, which do not change throughout the processes under study. In particular, (a) for all *physical* state change processes we neglect the constituents of the internal energy, which do not change throughout these processes (see subitem 2 of item **II**, mentioned earlier) and consider that the potential internal energy for a system that undergoes only *physical* state change processes is caused by the intermolecular cohesion forces exclusively; (b) for physical real gas state change processes we additionally neglect the lattice vibrational kinetic energy that does not exist in gases and liquids (see item E of §3.1.2.1.2 and subitem 5 of item **I);** and (c) for physical ideal gas state change processes we additionally neglect the potential energy of cohesion forces (see subitem (1) of item ***II)***;

therefore, ***during physical state change processes, the internal energy:***

(A) For ideal gases and for the cases, when the real gas can be considered ideal one (when its specific volume is very large), we consider that the internal energy consists of:

1—the ***translational*** kinetic energy of the straight *random* motion of the molecules;
2—the ***rotational*** kinetic energy of the rotational motion of the molecules;
3—the ***intramolecular*** vibrational energy; and
4—the electronic MKE.

(B) The real gas internal energy consists of the ideal gas internal energy (see item F above) in addition to:

1—the potential energy of the intermolecular cohesive forces.

We remind here that (1) if the real gas, whether considered ideal or not, consists of only monatomic molecules, then each of the constituents of the internal energy that resulted from the rotational and intramolecular-vibrational motions of the molecules equals zero/does-not-exist, and (2) the mean electronic KME is always the smallest between the mean non-zero molecular energies in the sum for calculating the internal energy and can be ignored, because of its very small value relative to the other values in the sum.

Nt3-4D: Throughout the previous part of this §3.1.2.1.3 we were speaking about energy forms of molecules and internal energy and this does not mean that the same chemical formula molecules own the same molecular energy that is distributed according to the same law into the listed forms. The actual seen is totally different, and therefore, we need, when speaking about the macroscopic internal energy, to use terms like "the mean KME," "mean electronic KME," and so on.

It is clear from the above that the internal energy of a quantity of a substance/compound is directly proportional to the number of its molecules, or what is the same, the internal energy of a substance/compound is directly proportional to its mass/number-of-kilomoles. This means that the internal energy is an additive dimension. And therefore:

- The internal energy of the system is denoted/symbolized by "U."
- The specific "per unit mass" internal energy is denoted/symbolized by "u."
- The molar "rarely per mole and mostly per kilomole" internal energy is denoted/symbolized by "$Mu\, or\ \bar{u}$."

The above-mentioned three values are related to each other as follows (see Equations "2-3 through 2-3b" and the text between them):

$$\mathrm{U = m \cdot u = K \cdot (Mu)} = K \cdot \bar{u} \qquad \text{and} \qquad \bar{u} = (\mathrm{Mu}) = \mathrm{M \cdot u} \qquad (3\text{-}13)$$

3.1.2.1.4 Internal energy is a state property

Let us see to which of the intensive properties relates each of the listed above five elements of the internal energy (see items F and G above).

(H) The experimental and theoretical studies on substances in their solid, liquid, and gaseous phases show that all forms of the so-called "the mean microscopic kinetic energy" that are the mean "kinetic translational, kinetic rotational, intramolecular vibrational, kinetic electronic, and lattice vibrational" molecular energies (see items A through E above) are functions of temperature, increasing with its increment and decreasing with its decrease. The microscopic kinetic energy portion of the internal energy gives rise to the temperature of the system.[4] The above means that all forms of the internal energy of ideal and those considered ideal gases (see item F) are sole functions of temperature, and therefore, *the ideal gas internal energy is a function of its temperature.*

(I) The cohesive forces are functions of the average distance between the molecules, and this distance is determined by knowing the specific volume of the gas; therefore, the cohesion forces' potential energy in a real gas (see item G) is a function of its specific volume, and therefore, *the real gas internal energy is a function of its both specific volume and temperature.*

After taking into consideration what has been said about the intensive properties to which the internal energy relates we conclude:

- The internal energy of an ideal gas is a function of only temperature:

$$u = f(T); \quad \left(\frac{\partial u}{\partial v}\right)_T = \left(\frac{\partial u}{\partial p}\right)_T = 0; \quad \left(\frac{\partial u}{\partial T}\right)_v = \left(\frac{\partial u}{\partial T}\right)_p = \frac{du}{dT}$$

Or $$du_v = du_p = du = f(T) \tag{3-14}$$

This means that the internal energy of the ideal gas is not a new state property, but it carries the same meaning of the temperature. In other words, the internal energy and temperature are not independent state properties, but they depend on each other.

- The internal energy of the real gas is a function of both temperature and specific volume:

$$u_{rg} = f_1(T, v) \tag{3-15}$$

Because of the existence of the characteristic equation $F(p, v, T) = 0$, the real gas internal energy is a function of any two of the three intensive properties $(p, v, \text{and } T)$, that is, in addition to Equation (3-15), we can write the following equations:

$$u = f_2(p, v) \tag{3-15a}$$

$$u = f_3(T, p) \tag{3-15b}$$

Accordingly, the value of *u*, which can be determined for any gas state by knowing two of its intensive properties, can itself be an intensive property of the gas, that is, it jointly with any other intensive property $(p, v, \text{ or } T)$ can determine the gas state. The internal energy units are generally the same energy units (see §1.1.9). The often used units for measuring the specific (per mass) internal energy in thermodynamics are kJ/kg, kcal/kg, and Btu/lbf.

Unlike equation type $F(p,v,T)=0$, Equations (3-15), (3-15a), and (3-15b) are named the caloric equations of gas state.

3.1.2.2 Enthalpy

In thermal calculations, the expression $u+pv$ was often encountered; this expression has been given the name heat stock "enthalpy", and it has two symbols: today's symbol *h* and the old one *i*. Therefore, by definition

$$h=u+p\cdot v \tag{3-16}$$

Later it was discovered that the enthalpy is a property of state.

For ideal gases, where $u=f(T)$ and $p\cdot v=R\cdot T$, Equation 3-16 becomes:

$h=u+R\cdot T=f(T)+R\cdot T=f_7(T)$ from which we obtain:

$$dh=du+RdT \tag{3-17}$$

Relationship (3-17) clearly confirms that the ideal gas enthalpy is a function of only temperature. An additional expression for this relationship can be:

$$\left(\frac{\partial h}{\partial v}\right)_T=\left(\frac{\partial h}{\partial p}\right)_T=0 \quad \text{or} \quad \left(\frac{\partial h}{\partial T}\right)_V=\left(\frac{\partial h}{\partial T}\right)_P=\frac{dh}{dT}$$

$$\text{or} \qquad dh_v=dh_p=dh=f(T) \tag{3-18}$$

To prove that the enthalpy is a state property, we substitute *u* in the enthalpy definition Equation (3-16) from Equation (3-15a) and obtain $h=f_2(p,v)+pv$ or:

$$h=f_4(p,v) \tag{3-19}$$

Equation (3-19) is similar to Equation (3-15a) and on the basis of this similarity we can derive the following equations:

$$H=m\cdot h=K\cdot Mh=K\cdot\overline{h} \quad \text{and} \quad \overline{h}=Mh=M\cdot h \tag{3-20}$$

$$h=f_5(T,p) \quad \text{and} \quad h=f_6(T,v) \tag{3-19a}$$

Accordingly, the value of *h*, which can be determined for any gas state by knowing two of its intensive properties, can itself be an intensive property of the gas, that is, it jointly with any other intensive property $(p, v, \text{or } T)$ can determine the gas state, and thus, the ***enthalpy is a state property***.

3.1.2.3 Ideal gas entropy

3.1.2.3.1 Ideal gas entropy is a state property

In thermal calculations the expression $(\delta q / T)$ was often encountered. This expression was symbolized by ds. Later it was discovered that *s* is a state property and has been given the name entropy. Therefore, by definition

$$ds = \delta q / T \tag{3-21}$$

Applying Equation (3-6) we write:

$$ds = \delta q / T = c \cdot dT / T; \qquad \delta q = T \cdot ds = c \cdot dT \tag{3-22}$$

where $c = f(T, p)$.

From Equation (3-22) we conclude that the STH to the gas during the process between points 1 and 2 is (see Equation 3-6)

$$q = \int_1^2 T \cdot ds = \int_1^2 c \cdot dT \tag{3-23}$$

From Equation (3-21), we conclude the following.

> **<u>OR3-1:</u>** the STH is positive when both d*s* and d*T* have the same algebraic sign. And, when the algebraic signs of d*s* and d*T* are different, the SH is negative.

Let us first prove the existence of this state property for ideal gases. For a rigorous proof, we shall use the Equations (4-13) and (4-18), derived later. Dividing all sides of Equation (4-13) by *T* and applying the ideal gas equation of state (2-14), we obtain $\frac{\delta q}{T} = c_v \cdot \frac{dT}{T} + \frac{p}{T} \cdot dv = c_v \cdot \frac{dT}{T} + R \frac{dv}{v}$. Applying the entropy definition (3-21), we obtain

$$ds = c_v \cdot \frac{dT}{T} + R \frac{dv}{v} \tag{3-24}$$

Nt3-4E: Since the SHs (c_v and c_p) for ideal gases (see Nt3-1A) are only temperature dependent, the variables in this differential equation are separated; therefore, d*s is exact, and Equation (3-24) can be integrated directly, and the ideal gas entropy is a state property* (see OR2-7).

Nt3-4F: Since the ratio dT/T in the first of equations (3-22) is dimensionless, the units of the entropy (s) and the SH (c) are the same and the often used units of entropy are $\mathrm{kJ/(kg \cdot K)}$, $\mathrm{kcal/(kg \cdot deg)}$, and Btu/(lb.R). As it can be seen the unit of the ideal gas entropy (s) contains a unit mass in the denominator. According to "HR2-1" ***(s) is the specific ideal gas entropy***, while ***(S = ms) is the total ideal gas entropy*** and at the same time ***is additive dimension*** (see OD2-23).

Combining the results of Nt3-4E and Nt3-4F above and considering OD2-23, we conclude that "***The total ideal gas entropy (S) is an extensive property.***"

Therefore, we can write:

$$S = m \cdot s = K \cdot Ms = K \cdot \overline{s} \quad \text{and} \quad \overline{s} = Ms = M \cdot s \tag{3-24a}$$

Applying (Nt3-1A) and Equations (4-18) and (2-16) in Equation (2-24a), we obtain

$$ds = c_{v0}\frac{dT}{T} + R\frac{dv}{v} = c_{p0}\frac{dT}{T} - R\frac{dp}{p} = c_{v0}\frac{dp}{p} + c_{p0}\frac{dv}{v} \tag{3-25}$$

For $c_{v0} = f(T)$ and accordingly $c_{p0} = f(T)$, the variables in the second and third side of this equation are separated, while in the fourth side are not. Therefore, the fourth side is excluded from the exact entropy calculations. The special cases that change the look of the equations for the exact entropy calculations are:

$$\left.\begin{aligned} &\text{If } dv = 0 \Rightarrow v = \text{constant and } ds_v = c_{v0}\frac{dT}{T} \\ \text{and} \quad &\text{If } dp = 0 \Rightarrow p = \text{constant and } ds_p = c_{p0}\frac{dT}{T} \end{aligned}\right\} \tag{3-26}$$

Integrating the second differential equation from T_1 to T_2, we obtain —after taking in consideration that *the* ***ideal gas entropy*** s is ***a state property*** (see §2.8 "OR2-7") and accordingly d*s is exact* and $\int_1^2 ds = s_2 - s_1$ — *the calculating equation for the* ***ideal gas entropy change*** *in the* ***isobaric process 1_2****, which can be written as:*

$$\left.\begin{array}{l}\text{For } \ T_{ref*} > AZ = 0K = 0R: \\ \int_{T_1}^{T_2} ds_{p0} = s_{p0_2} - s_{p0_1} = \int_{T_1}^{T_2} c_{p0}\frac{dT}{T} = \int_{T_{ref*}}^{T_2} c_{p0}\frac{dT}{T} - \int_{T_{ref*}}^{T_1} c_{p0}\frac{dT}{T}\end{array}\right\} \qquad (3\text{-}27)$$

The 0 in the subscripts of this set of equations stands for ***ideal gas***—see Nt3-1A—which means that the mentioned 0 can be skipped if denoted somewhere else (in equation's explanatory texts) that this equation is written for ideal gases. The paragraph that precedes the conditioned equation (3-27) clearly denotes that this equation is written for ideal gases, therefore we can write equation (3-27) in any of the following two forms:

$$\left.\begin{array}{l}\text{For ideal gas and } T_{ref*} > AZ = 0K = 0R: \\ \int_{T_1}^{T_2} ds_p = s_{p_2} - s_{p_1} = \int_{T_1}^{T_2} c_p\frac{dT}{T} = \int_{T_{ref*}}^{T_2} c_p\frac{dT}{T} - \int_{T_{ref*}}^{T_1} c_p\frac{dT}{T}\end{array}\right\} \qquad (3\text{-}27\text{-}1)$$

$$\left.\begin{array}{l}\text{For } \ T_{ref*} > AZ = 0K = 0R: \\ \int_{T_1}^{T_2} ds_p = s_{p_2} - s_{p_1} = \int_{T_1}^{T_2} c_p\frac{dT}{T} = \int_{T_{ref*}}^{T_2} c_p\frac{dT}{T} - \int_{T_{ref*}}^{T_1} c_p\frac{dT}{T}\end{array}\right\} \qquad (3\text{-}27\text{-}2)$$

In equations (3-27 through 3-27-2): 1) s_{p0_2} (s_{p_2}) and s_{p0_1} (s_{p_1}) are the ideal gas entropies of the end and start states in the isobaric process 1-2 respectively. 2) T_{ref*} is an arbitrary reference temperature at which integral $\int_{T_1}^{T_2} c_p(dT/T)$ is separated into integrals $\int_{T_{ref*}}^{T_2} c_p(dT/T)$ and $\int_{T_{ref*}}^{T_1} c_p(dT/T)$. The value of T_{ref*} is essential in tabulating integral $\int_{T_{ref*}}^{T} c_p(dT/T)$ versus T, therefore, no table of integral $\int_{T_{ref*}}^{T} c_p(dT/T)$ versus T can be created without determining/fixing—in advance—the value of its T_{ref*} by table's author, who declares (or does not declare) its value.

Nt3-4G: Obviously/mathematically, the values of T_{ref*} in the two integrals resulted from the separation of integral $\int_{T_1}^{T_2} c_p(dT/T)$ into two, must be the same throughout integral's tabular calculation. This means that the tabular calculations through Equation (3-27-1) will be correct when the values of its two integrals $\int_{T_{ref*}}^{T_2} c_p(dT/T)$ and $\int_{T_{ref*}}^{T_1} c_p(dT/T)$ are obtained: 1) either from the same table of the so called Standard Entropy, or Thermodynamic relations, or Ideal Gas Properties, of a particular gas (Air for example), or 2) from two tables of standard entropies those assigned to the same ideal gas and have the same T_{ref}. Other options are risky for not experienced people.

Since the entropy value versus T_{ref} equals zero, the integrals with lower limit of T_{ref} will calculate *the relative, to table's principle of entropy, ideal gas entropies in the constant pressure process* instead of the ideal-gas entropy change in the constant pressure process. Therefore, any of the limits/temperatures of the integrals of Equations "3-27 through 3-27-2" ($T_{\text{ref*}}$, T_2 and/or T_1) cannot equal the absolute zero; because, the absolute zero limit, in such integrals (see the following Nt3-5A), makes its integral indeterminate (the AZ-value of any of T_2 and/or T_1 is enough to make the third and fourth sides of Equations "3-27 through 3-27-2" indeterminate, whereas $T_{\text{ref*}}$=0JK is enough to make the fourth side integral of Equations "3-27 through 3-27-2" indeterminate).

Nt3-4H: The limits T_1 and T_2 of integral $\int_{T_1}^{T_2} c_p(dT/T)$ that have definite values throughout a particular calculation (see equations 3-27 through 3-27-2), have other values throughout another calculation and so on. The two limits of such integral vary from one calculation to another and therefore, its limits are considered variables and its tables are of the double/dual entry type. Separating the definite integral $\int_{T_1}^{T_2} c_p(dT/T)$ into two definite integrals according to the mathematical rule

$$\int_{T_1}^{T_2} c_p \frac{dT}{T} = \int_{T_1}^{T_{\text{ref*}}} c_p \frac{dT}{T} + \int_{T_{\text{ref*}}}^{T_2} c_p \frac{dT}{T} = \int_{T_{\text{ref*}}}^{T_2} c_p \frac{dT}{T} - \int_{T_{\text{ref*}}}^{T_1} c_p \frac{dT}{T}$$

(See the last two sides of any of equations 3-27 through 3-27-2) considerably simplifies its tabular calculation; because, the resulted from this separation integrals $\int_{T_{\text{ref*}}}^{T_2} c_p(dT/T)$ and $\int_{T_{\text{ref*}}}^{T_1} c_p(dT/T)$ with their common case $\int_{T_{\text{ref*}}}^{T} c_{p0}(dT/T)$ are able for tabulation, versus T, in single-entry (temperature) ideal gas table since they have only one variable limit (T_2, T_1 or T) each. The table of integral $\int_{T_{\text{ref*}}}^{T} c_{p0}(dT/T)$ versus T is enough to obtain additionally the values of each of $\int_{T_{\text{ref*}}}^{T_2} c_p(dT/T)$ and/or $\int_{T_{\text{ref*}}}^{T_1} c_p(dT/T)$ versus (T_2 and/or T_1).

Mathematically, each of the mentioned three special cases of integral $\int_{T_1}^{T_2} c_{p0}(dT/T)$, equals zero for $T=T_{ref*}$ or:

$$\int_{T_{ref*}}^{T=T_{ref*}} c_p(dT/T) = \int_{T_{ref*}}^{T_2=T_{ref*}} c_p(dT/T) = \int_{T_{ref*}}^{T_1=T_{ref*}} c_p(dT/T) = 0 \qquad (3\text{-}27\text{-}3)$$

Symbolizing each of these integrals as $s_{p_{ref*}}$ leads to $s_{p_{ref*}} = 0$ and

$$\left.\begin{array}{lll} \int_{T_{ref*}}^{T} c_p \dfrac{dT}{T} = s_p - s_{p_{ref*}} = s_p, & & \text{which leads to} \\ \int_{T_{ref*}}^{T_2} c_p \dfrac{dT}{T} = s_{p_2} & \text{and} & \int_{T_{ref*}}^{T_1} c_p \dfrac{dT}{T} = s_{p_1} \end{array}\right\} \qquad (3\text{-}27a)$$

The chosen above arbitrary reference temperature T_{ref*} for tabular calculation of the ideal gas entropy change during the constant pressure process $s_{p_2} - s_{p_1}$ (see Equation 3-27-2) is actually choosing an ideal gas state to be the entropy principle for the table of integral $\int_{T_{ref*}}^{T} c_p(dT/T)$, because the other intensive property of the chosen state is the pressure of the constant pressure process. such procedure is usual since, in thermodynamics when studying physical state change processes, we are not interested in the values of the intensive caloric properties of state[A] but in their changes and therefore, we can, during a particular ideal gas entropy change determination/calculation, chose any ideal gas state to be the entropy principle.

A. The intensive caloric properties of state are the internal energy, enthalpy and entropy.[5]

In equations (3-27a): s_p, s_{p_2}, and s_{p_1} are *the* ***relative, to table's principle of entropy, ideal gas entropies in the constant pressure process*** ***(****Abbreviation* ***Table's ideal gas constant pressure entropies*** *or simply* ***Constant Pressure Entropies)*** for temperatures T, T_2, and T_1, respectively. The other symbols of (1) s_p are s_T^0 s°, or s_{p_T}; (2) s_{p_2} are $s_{T_2}^0$, s_2°, or $s_{p_{T_2}}$, and (3) s_{p_1} are $s_{T_1}^0$, s_1°, or $s_{p_{T_1}}$. Therefore (see Equations 3-27 through 3-27-2 and 3-27a):

$$\int_{T_1}^{T_2} c_{p0} \frac{dT}{T} = s_{p_2} - s_{p_1} = s_{T_2}^0 - s_{T_1}^0 = s_{T_2}^\circ - s_{T_1}^\circ \qquad (3\text{-}27b)$$

Where: $s_{p_2} - s_{p_1}$ or $(s_2 - s_1)_p$ or $(\Delta s_{1\_2})_{p0}$ is the exact ideal gas entropy change in the constant pressure process (1_2) that *totally lies within/inside the ideal gas zone.*

Some authors refer to: 1) the integrals type $\int_{T_{ref*}}^{T} c_{p0}(dT/T)$ *as the standard entropies*[6]. Whereas many other authors do not name integral $\int_{T_{ref*}}^{T} c_{p0}(dT/T)$, when speaking about it, but name one of its symbols (see Nt3-4G below).

A similar analysis for the first of differential Equations (3-26) will lead to similar results that concern the exact ideal gas entropy change in the constant volume process. Here also:

$$\left.\begin{aligned} &\int_{T_{ref*}}^{T} c_{v0}\frac{dT}{T} = s_v; \quad \int_{T_{ref*}}^{T_2} c_{v0}\frac{dT}{T} = s_{v2}; \quad \int_{T_{ref*}}^{T_1} c_{v0}\frac{dT}{T} = s_{v1} \\ &\text{and} \qquad \int_{T_1}^{T_2} c_{v0}\frac{dT}{T} = s_{v2} - s_{v1} = (s_2 - s_1)_v \end{aligned}\right\} \quad (3\text{-}27c)$$

where: s_v, s_{v2}, and s_{v1} are ***the relative, to table's principle of entropy, ideal gas entropies in the constant volume process*** *(abbreviation* ***Table's ideal gas constant volume entropies*** *or simply* ***Constant Volume Entropies****)* for temperatures T, T_2, and T_1, respectively. The determination of T_{ref} value is left to table's author, who chooses it and declares (or does not declare) his choice. Also $(s_2 - s_1)_v$ or $(\Delta s_{1\_2})_v$ is the exact ideal gas entropy change in the constant volume process (1_2) that *totally lies within/inside the ideal gas zone*.

From the first equations of (3-27a) and (3-27c) we obtain:

$$s_p - s_v = \int_{T_{ref*}}^{T} c_p (dT/T) - \int_{T_{ref*}}^{T} c_v (dT/T) = \int_{T_{ref*}}^{T} (c_p - c_v)\cdot(dT/T)$$

Applying Meyer Equation (see equation 4-18) on this equation we obtain:

$$s_v = s_p - \int_{T_{ref*}}^{T} R\frac{dT}{T} = s_p - R\ln\frac{T}{T_{ref*}} = s_p - R\ln T + RT_{ref*} \quad (3\text{-}27d)$$

$$\text{Or} \quad \ln T_{ref*} = \frac{s_v - s_p}{R} + \ln T \quad \text{or} \quad T_{ref*} = T\cdot\exp\frac{s_v - s_p}{R} \quad (3\text{-}27e)$$

Nt3-4I: We should mention here that different ideal gas property tables differently symbolize the constant pressure entropy s_p. Thus, s_p in (1) Borgnakke and Sonntag (2013)[6] is s_T^0, (2) Cengel and Boles (2006)[7] is s°, (3) Moran and Shapiro (2006)[8] is $s^\circ(T)$, (4) Rivkin (1973)[9] is s°, and (5) Vukalovich (1958)[10] is *s* (subscript *p* is skipped only inside the tables, but not in the descriptions). Also, these property tables have their own reference temperature T_{ref} that is mostly not declared. Only in Vukalovich (1958)[10], T_{ref} is declared as $T_{\text{ref}} = T_n = 273.15\ \text{K}$.

Nt3-5: Remember that T_{ref} in any table of thermodynamic properties of substances that includes the values of any s_p or $s\,s_p$ temperature functions can never be AZ.

The following is a deep abbreviation of the rest of the analysis of the ideal gas entropy:

$$(s_2 - s_1)_{\text{exact}} = s_{v_2} - s_{v_1} + R\ln\frac{v_2}{v_1} = s_{p_2} - s_{p_1} - R\ln\frac{p_2}{p_1} \tag{3-28}$$

$$(s_{p_2} - s_{p_1}) - (s_{v_2} - s_{v_1}) = R\ln\frac{p_2}{p_1} + R\ln\frac{v_2}{v_1} = R\ln\frac{T_2}{T_1} \tag{3-28a}$$

For $c_{v0} = \text{constant}$ and accordingly $c_{p0} = \text{constant}$,

$$\begin{aligned} s_2 - s_1 &\cong c_{v0}\ln\frac{T_2}{T_1} + R\ln\frac{v_2}{v_1} \cong c_{p0}\ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1} \\ &\cong c_{v0}\ln\frac{p_2}{p_1} + c_{p0}\ln\frac{v_2}{v_1} \end{aligned} \tag{3-29}$$

To increase the accuracy replace c_{v0} by $c_{v0_{\text{avg}}}$ and c_{p0} by $c_{p0_{\text{avg}}}$.

$$\left(\frac{\partial s}{\partial T}\right)_p = \frac{c_{p0}}{T} \quad \text{and} \quad \left(\frac{\partial s}{\partial T}\right)_v = \frac{c_{v0}}{T} \tag{3-30}$$

<u>Nt3-5A:</u> Usually, the ideal gas SH expression is an algebraic sum of a constant term (*CT*) and one or more temperature-dependent term(s) (TDTs) (see Equations 3-33 through 3-36 for c_{p0} or c_{v0} and Equation 6-6 for the SH of the new polytropic process c_{n0}), but some SH expressions consist of only constant term, whereas in some rare cases the SH expression consists of only temperature dependent terms; therefore, the integral:

$$\int_{T_{ref}}^{T} SH_0 \frac{dT}{T} = \int_{T_{ref}}^{T} (CT + TDTs)\frac{dT}{T} = CT \cdot \ln\frac{T}{T_{ref}} + \int_{T_{ref}}^{T} TDTs\frac{dT}{T}$$

$$= CT \cdot \ln\frac{T}{T_{ref}} + F_1(T) - F_1(T_{ref}) \qquad (3\text{-}31)$$

Which almost[B] invariably includes the term $\text{const} \times \ln\left(T / T_{ref}\right)$ that is undefined (equals either $+\infty$ or $-\infty$) for the case when one of its limits (T or T_{ref}) equals *AZ* = 0 K = 0 R, because $\ln 0 = -\infty$ and $\ln + \infty = +\infty$.

B. The last lines of items (1) through (3) of §3.1.6.1 clearly show that each of the ideal gas SH expressions that included in references [11] through [13] has a constant term and therefore, it is almost improbable/impossible to find a table for standard entropies, whose T_{ref} = AZ

Thus, the following equations are highly probale:

$$\left.\begin{array}{ll} \int_{T_{ref}=0\,K}^{T} SH_0 \dfrac{dT}{T} = \pm\infty; & \int_{T_{ref}=0\,K}^{T} c_{p0} \dfrac{dT}{T} = \pm\infty \\ \text{and} & \int_{T_{ref}=0\,K}^{T} c_{v0} \dfrac{dT}{T} = \pm\infty \end{array}\right\} \qquad (3\text{-}31A)$$

Therefore, (1) the condition $\underline{T_{ref} \neq 0\,K}$ is inevitable/necessary to tabulate the integrals $\int_{T_{ref}}^{T} c_{p0}(dT/T)$ and $\int_{T_{ref}}^{T} c_{v0}(dT/T)$. (2) In this book the star (*) in (*T*$_{ref*}$) is to remind that (*T*$_{ref}$) in temperature functions $\int_{T_{ref}}^{T} c_{p0}(dT/T)$ and $\int_{T_{ref}}^{T} c_{v0}(dT/T)$ cannot equal the AZ.

Nt3-5B: Temperature functions s_p, s_{p2}, and s_{p1} whose other symbols are $\left(s_T^0, s^\circ, \text{and/or } s_{pT}\right)$, $\left(s_{T_2}^0, s_2^\circ, \text{and/or } s_{pT_2}\right)$, and $\left(s_{T_1}^0, s_1^\circ, \text{and/or } s_{pT_1}\right)$, respectively, in addition to s_v, s_{v2}, s_{v1}, and $(s_p - s_v)$ are T_{ref*} functions, while any of the differences $\left(s_{p2} - s_{p1}\right)$ and $\left(s_{v2} - s_{v1}\right)$ whose both components are obtained from the same property table is not.

3.1.3 THE GRAPHICAL REPRESENTATION OF THE GAS STATE AND OF THE GAS STATE CHANGE PROCESSES)

The equation of state (the characteristic equation) is the equation of a space surface in the coordinate system (p, v, T), and the gas state corresponds with a certain point on this surface. This point is called the *figurative point*.

If we represent a number of gas states infinitesimally close to each other in the chosen coordinate system, then we obtain a representation of these states in the chosen coordinate system in the form of line 1–2 (Figure 3.1), and this line is applicable to the surface representing equation $F(p, v, T) = 0$. Line (1–2) that represents the gas state changes is called the **gas curve** or **process path**. Each point of this line represents an equilibrium state of the body (gas), and accordingly the graphical representation can exist only for thermodynamically equilibrium processes.

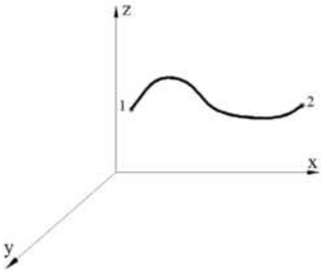

Figure 3.1. The equilibrium gas state change process the space coordinate system p_v_T

If necessary to represent the nonequilibrium processes on graphs, then that would be idiomatic. And then it will not be permissible to make use of the properties of the graphs.

> **HD3-1A:** A ***legally created*** state change process path in ***T_s, C_T, p_v, … diagram*** of a substance is the ***graphical representation in scale*** of this state change process, which must be in ***thermodynamical equilibrium***, in ***plane T_s, C_T, p_v,*** …respectively. Only *legally created process paths are appropriate/suitable for graphical calculations.*

The use of the space coordinate system is uncomfortable; therefore, usually, we use the plane coordinate system. This allows tracking the changes of two of the gas state properties, while the value of the third property for each of the points of the process can be calculated from the equation of state. The most comfortable of the plane coordinate systems is that which interconnects the pressure and specific volume.

Gas state change processes may/can run inside a piston–cylinder arrangement as illustrated in Figure 3.2, where the gas-specific volume change is proportional with piston displacement; therefore, we represent the specific volume on *x*-axis and the pressure at each piston position on *y*-axis, and thus we obtain the so-called *p_v* diagram (Figure 3.2).

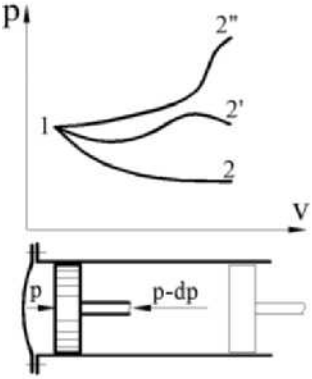

Figure 3.2. Expansion processes in p_v plane coordinates

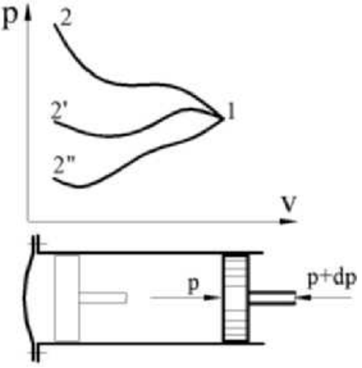

Figure 3.3. Compression processes in p_v plane coordinates

In Figure 3.2, three gas state change curves (1-2, 1-2', and 1-2") are illustrated. The gas state change according to this curve or that depends on how the heat is transferred to/from the body during the process. All these processes are expansion ones because the specific volume increases during the process, and we shall refer to these curves as the **gas expansion curves**.

In Figure 3.3, three gas state compression curves are illustrated (the specific volume decreases during the process), and we shall call them the **gas compression curves**.

> **Nt3-5c:** From now on, we shall additionally denote to the relative temperature (see OD1-3D and Equations (1-30)) by *t*.

3.1.4 GENERAL FORM EQUATIONS FOR CALCULATING THE BOUNDARY WORK AND TRANSFERRED HEAT

If we know the function $p = f(v)$ and any of the functions $c = f(t)$, $c = f(T)$, or $T = f(s)$ for a process, then we can calculate the boundary work (see §2.10) from Equation (2-38) and the transferred heat from Equation (3-23); for example, for the constant temperature process where $T = \text{constant} \neq f(s)$:

$$q_T = \int_{s_1}^{s_2} T \cdot ds = T \cdot \int_{s_1}^{s_2} ds = T \cdot (s_2 - s_1) \qquad (3\text{-}32)$$

Since the gas state change process in coordinates (*p_v*, *T_s*, *c_T,* or *c_t*) is expressed by the general form function $p = f(v)$, $T = f(s)$, $c = f(T)$, or $c = f(t)$ that is similar to the general form function $y = f(x)$, which expresses the plane line in coordinates *(y_x),* then the general discussion and calculation of each of the boundary work and the STH during physical state change processes can be executed using the appropriate of Equations (2-38) and/or (3-23), which are similar to Equation (1-44), that was comprehensively discussed in Chapter 1. Just replace in Equation (1-44) (*A*, *y*, and *x*) by, respectively, (*w*, *p*, and *v*), (*q*, *T*, and *s*), (*q*, *c*, and *T*), or (*q*, *c*, and *t*), and you will have a lot of material that belongs to the boundary work and/or STH analyses. For example, the STH $q = \int_1^2 T \cdot ds = \int_1^2 c \cdot dT$ is measured as the area under process path in the appropriate diagram T_s or c_T. Also, the specific boundary work $w = \int_1^2 p \cdot dv$ is measured as the area under process path in *p_v* diagram. All three diagrams must be created in scale. The graph scale and the displacement of the co-ordinate axes must be taken in consideration (see Nt1-12, Nt1-25, and §1.2.1). For cyclic processes, *the area under the curve equals the area inside it (see Nt1-23).*

3.1.5 SOME OF THE RULES, DEFINITIONS, AND NOTES, MAINLY USED IN THIS BOOK, THAT SIMPLIFY THE GRAPHICAL CALCULATIONS AND DISCUSSIONS

P_v and *T_s* diagrams are extensively used in studying and analyzing heat engines. We shall extract here the rules, definitions, and notes, mainly used in this book, from Chapter 1 by replacing the area (*A*) under the curve $y = f(x)$ by the (1) specific boundary work (*w*) done by the gas during process $p = f(v)$. (2) STH (*q*) to the gas during process $T = f(s)$, $c = f(T)$, or $c = f(t)$. In addition to replacing:

1) When discussing (*p_v*) diagram:
    - *y* by the absolute pressure *p* of the gas,
    - *x* by the specific volume *v* of the gas.
2) When discussing (*T_s*) diagram:
    - *y* by the absolute temperature *T* of the gas,
    - *x* by the specific entropy *s* of the gas.

3) When discussing *c_T* or *c_t* diagram:
   - *y* by the SH *c* of the gas,
   - *x* by the temperature *T* with *c_T* diagram or *t* with *c_t* one.

Thus, we obtain the following.

HD3-2: *The **stationary point** (graphical thermodynamics)* is *the point on a process path (curve) at which the tangent is either horizontal or vertical. If the tangent at the stationary point intersects the curve at the same stationary point, then the stationary point is an **inflection point** (a point of inflection), else, it is a **turning point**.*

According to the relative position between curve's part that includes a turning point and its tangent the turning point can be:

1) *Function's maximum (e.g., T-maximum in T_s diagram) if the Part of the Process Path Including It (PPPII) lies under its horizontal tangent.*
2) *Function's minimum (e.g., T-minimum) if PPPII lies above its horizontal tangent.*
3) *Variable's maximum (e.g., s-maximum) if PPPII lies rightward its vertical tangent.*
4) *Variable's minimum (e.g., s-minimum) if PPPII lies leftward its vertical tangent.*

If the process path includes some *vertical and/or horizontal straight parts, then these parts are stationary parts and each of them can be (1) function's flat maximum (e.g., p-flat-maximum in p_v graph), (2) function's flat minimum (e.g., p-flat-maximum), (3) variable's flat maximum (e.g.,_ v-flat maximum), (4) variable's flat minimum (e.g., v-flat-minimum), (5) function's flat inversion (e.g., C-flat-inversion in c_T or c_t diagram), and (6) variable's flat inversion (e.g., t-flat-inversion).*

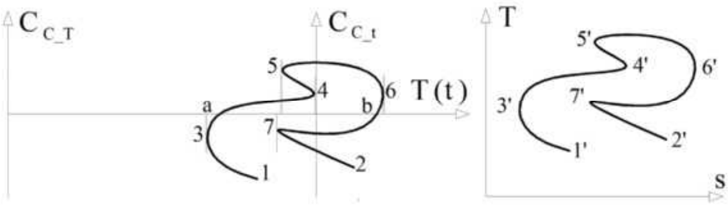

Figure 3.4. The two figures represented here do not belong to the same process

OD3-4: (Developed from Nt1-15): In the following paragraphs and through subfigures (I and II) of Figure 3.4 we shall discuss *T_s*, *c_T*, *c_t*, and *p_v* diagrams at the same time. Such group discussion requires, to be fulfilled, knowing in advance the algebraic sign of the ordinate of each of the coordinate systems under discussion. This means that we need to know in advance the algebraic signs of T, c, and p. T and p are always positive, because they are the absolute temperature and pressure respectively (see Nt2-6B and Nt2-6A), whereas the algebraic sign of c for gases depends on the process and can be positive or negative (Nt3-1 and equations 3-5).

Processes characterized by:

1) $\mathrm{ds} \geq 0$, $\mathrm{dT} = \mathrm{dt} \geq 0$, and $\mathrm{d}v \geq 0$ are called entropy-increasing, temperature-increasing and expansion (specific-volume-increasing) processes, respectively. They are abbreviated as s-increasing, (T or t)-increasing, and v-increasing processes, respectively. These processes consist of at least one $(\mathrm{ds} > 0)$, $(\mathrm{dT} = \mathrm{dt} > 0)$, or $(\mathrm{dv} > 0)$ process with some/without any differential/integral constant (*s*, *t*, or *v*) processes, but they do not contain (include) any differential/integral (*s*, *t*, or *v*)-decreasing processes.
2) $\mathrm{ds} \leq 0$, $\mathrm{dT} = \mathrm{dt} \leq 0$, and $\mathrm{dv} \leq 0$ are called entropy-decreasing, temperature-decreasing and compression (specific-volume-decreasing) processes, respectively. They are abbreviated as s-decreasing, (*T* or *t*)-decreasing, and *v*-decreasing (compression) processes, respectively. These processes consist of at least one $(ds < 0)$, $(dT = dt < 0)$, or $(dv < 0)$ process with some/without any differential/integral constant (*s*, *t*, or *v*) processes, but they do not contain (include) any differential/Integral (*s*, *t*, or *v*)-increasing processes.
3) $\mathrm{d}s = 0$ in *T_s* diagram, $\mathrm{d}T = \mathrm{d}t = 0$ in *c_T* (or *c_t*) diagram, or $\mathrm{dv} = 0$ in *p_v* diagram can be either the vertical straight parts of the process (variable flat maximums, minimums, or inversions "see HD3-2") or the process points at which the tangent of the process is parallel to the vertical axis (variable maximums, minimums, or inversions "see HD3-2").

Nt3-6: (Developed from Nt1-16): We shall discuss here/below a common case state change process that is supposed to be represented in the four different property diagrams (*T_s*, *c_T*, *c_t* and *p_v*) through Figure 3.4. Such representation results, when done in scale, in three different-shape paths: one for *p_v* diagram, another for *T_s* diagram, and a third one for both *c_T* and *c_t* diagrams since $T = t +$ constant (“see Equations 1-39 and 1-40”).

If, in *T_s*, *c_T*, *c_t*, or *p_v* diagram, the complete process or its components suffer from heterogeneity in the sign of the product *T·ds, c·dt (or c·dT), or p·dv,* then it is preferable to divide the process into main partial processes that comply with the purpose of the study and the available data.

For graphical transferred heat calculations any of *T_s, c_T, or c_t* diagrams can be used. If the available diagram *is* (1) *T_s,* then the process is divided according to the sign of the product *T*·d*s* and (2) *c_T* or *c_t*, then the process is divided according to the sign of any of the products *(c*·d*t or c*·d*T)* since d*t* = d*T* and accordingly *c*·d*t* = *c*·d*T.*

For graphical boundary work calculations *p_v* diagram must be available. The process here is divided according to the sign of the product *p*·d*v.*

The common/bordering states between the main partial processes are (see Figure 3.4) (1) the variable’s turning points (maximum “4, 6, 4' and 6''” and minimum “3, 5, 7, 3', 5' and 7''”), (2) points at which the original/whole/complete process completely intersects (not coincides or contacts with) the variable (mostly horizontal) axis (*s, t, or v*), see points (a and b), and (3) the initial and final points of the original/whole/complete process. This division is enough for calculating the main (pure positive, pure negative, and total) transferred heats from any of *T_s*, *c_T* or *c_t* diagrams and boundary works from *p_v* diagram. On the other hand, additional fourth item should be added to do detailed analyses and calculations, which is (4) all common points between mono-equation processes constituting the complete process, in addition to (5) any point that helps in executing the analysis. Each of the partial processes divided according to the above belongs to one of the following four categories:

- The first category processes comply with one of the conditions ($T \geq 0$ and $ds \geq 0$), ($c \geq 0$ and $dT = dt \geq 0$), or ($p \geq 0$ and $dv \geq 0$) each. Partial curves (a-4), (5-6), (3'-4'), (5'-6'), and (7'-2') belong to this category. These processes are located on and over the horizontal axis $(T \geq 0)$, $(c \geq 0)$, or $(p \geq 0)$ and are (*s*, *t*, or *v*)-increasing $(ds \geq 0)$, $(dT = dt \geq 0)$, or $(dv \geq 0)$ ones. Since $(\delta q = T \cdot ds = c \cdot dT) = c \cdot dt$ and $(\delta w = p \cdot dv)$, then $(\delta q \geq 0)$ or $(\delta w \geq 0)$ for this category.
- The second category processes comply with the conditions ($T \geq 0$ and $ds \leq 0$) ($c \geq 0$ and $dT = dt \leq 0$), or ($p \geq 0$ and $dv \leq 0$). Partial curves (4-5), (6-b), (1'-3'), (4'-5'), and (6'-7') belong to this

categroy. These processes are located on and over the horizontal axis $(T \geq 0)$, $(c \geq 0)$, or $(p \geq 0)$ and are, (*s, t or T, or v*)-decreasing $(\mathrm{ds} \leq 0)$, $(\mathrm{dT} = \mathrm{dt} \leq 0)$, or $(\mathrm{dv} \leq 0)$ ones. Since $(\delta \mathrm{q} = \mathrm{T} \cdot \mathrm{ds} = \mathrm{c} \cdot \mathrm{dT} = \mathrm{c} \cdot \mathrm{dt})$ and $(\delta \mathrm{w} = \mathrm{p} \cdot \mathrm{dv})$, then $(\delta \mathrm{q} \leq 0)$ or $(\delta \mathrm{w} \leq 0)$ for this category.

The following categories are assigned for processes, which are located under diagram's horizontal axis. Such processes do not exist in *T_s* and *p_v* diagrams because the absolute temperature and pressure cannot be negative. Therefore, the following covers only *c_t* or *c_T* diagram.

- The third category processes comply with the conditions $\mathrm{c} \leq 0$ and $\mathrm{dT} = \mathrm{dt} \leq 0$. Partial curves (1-3) and (b-7) belong to this category. These processes are located on and under the horizontal axis $(c \leq 0)$ and are *T*- or *t*-decreasing $(dT = dt \leq 0)$ ones. Since $\delta \mathrm{q} = \mathrm{c} \cdot \mathrm{dT} = \mathrm{c} \cdot \mathrm{dt}$, then $(\delta \mathrm{q} \geq 0)$ for this category.
- The fourth category processes comply with the conditions $c \leq 0$ and $\mathrm{d}T = \mathrm{d}t \geq 0$. Partial curves (3-a) and (7-2) belong to this category. These processes are located on and under the horizontal axis $(c \leq 0)$ and are *T*- or *t*-increasing $(\mathrm{dT} = \mathrm{dt} \geq 0)$ ones. Since $(\delta \mathrm{q} = \mathrm{c} \cdot \mathrm{dT} = \mathrm{c} \cdot \mathrm{dt})$, then $(\delta \mathrm{q} \leq 0)$ for this category.

Nt3-6A: The only difference between *c_T* and *c_t* diagrams for the same process/cycle is that its path is located at different distances from the vertical axes.

HD3-3: *The* ***monodirectional heat transfer*** $(\mathrm{Q} = \int_{t_1}^{t_2} \mathrm{m} \cdot \mathrm{c} \cdot \mathrm{dt} = \int_{T_1}^{T_2} \mathrm{m} \cdot \mathrm{c} \cdot \mathrm{dT} = \int_{s_1}^{s_2} \mathrm{T} \cdot \mathrm{dS})$ *to the system is that which complies with any of the following conditions throughout:* $(\delta \mathrm{q} = \mathrm{T} \cdot \mathrm{ds} \geq 0)$, $(\delta q = c \cdot dT \geq 0)$, $(\delta \mathrm{q} = c \cdot \mathrm{d}t \geq 0)$, $(\delta \mathrm{q} = \mathrm{T} \cdot \mathrm{ds} \leq 0)$, $(\delta \mathrm{q} = \mathrm{c} \cdot \mathrm{dT} \leq 0)$, *or* $(\delta \mathrm{q} = \mathrm{c} \cdot \mathrm{dt} \leq 0)$. *The state change process associated with* ***monodirectional heat transfer is called the monodirectional transferred heat process.*** *And, the* ***monodirectional moving boundary work*** *of the system* $(\mathrm{W} = \int_1^2 \mathrm{p} \cdot \mathrm{dV})$ *is that which complies with any of the following conditions throughout:* $(\delta \mathrm{W} = \mathrm{pdV} \geq 0)$ *or* $(\delta \mathrm{W} = \mathrm{pdV} \leq 0)$. *The state change process associated with* ***monodirectional moving boundary work is called the monodirectional boundary work process****.*

HD3-4: (Developed from HD1-2): *The **pure positive transferred heat** to the system* $(Q_{PP} = m \cdot \int_1^2 \delta q_{PP})$ *is that which complies with any of the following conditions throughout:* $(\delta q_{PP} = T \cdot \mathrm{d}s = c \cdot \mathrm{d}T = c \cdot d \geq 0)$. *The state change process associated with **pure positive transferred heat is called the pure positive transferred heat process.** And the **pure positive moving boundary work** of the system* $(\mathrm{W_{PP}} = \int_1^2 \delta\mathrm{W_{PP}} = \mathrm{m} \cdot \int_1^2 \delta\mathrm{w_{PP}})$ *is that which complies with the following condition throughout:* $(\delta\mathrm{W_{PP}} = \mathrm{pdV} = \mathrm{m} \cdot \mathrm{pdV} \geq 0)$. *The state change process associated with **pure positive moving boundary work is called the pure positive boundary work process**.*

HD3-5: (Developed from HD1-3): *The **pure negative transferred heat** to the system* $(Q_{PNe} = m \cdot \int_1^2 \delta q_{PNe})$ *is that which complies with any of the following conditions throughout:* $(\delta q_{PNe} = T \cdot \mathrm{d}s = c \cdot \mathrm{d}T = c \cdot d \leq 0)$. *The state change process associated with **pure negative transferred heat is called the pure negative transferred heat process.** And the **pure negative moving boundary work** of the system* $(W_{PNe} = \int_1^2 \delta W_{PNe} = m \cdot \int_1^2 \delta w_{PNe})$ *is that which complies with the following condition throughout:* $(\delta\mathrm{W_{PNe}} = \mathrm{pdV} \leq 0)$. *The state change process associated with **pure negative moving boundary work is called the pure negative boundary work process**.*

HD3-6: *The **pure neutral transferred heat** to the system* $(\mathrm{Q_{PNl}} = \mathrm{m} \cdot \int_1^2 \delta\mathrm{q_{PNl}})$ *is that which complies with any of the following conditions throughout:* $(\delta\mathrm{q_{PNl}} = \mathrm{T} \cdot \mathrm{ds} = \mathrm{c} \cdot \mathrm{dT} = \mathrm{c} \cdot \mathrm{dt} = 0)$. *The state change process associated with **pure neutral transferred heat is called the pure neutral transferred heat process.** Pure neutral heat transferred heat processes do not affect heat transfer calculations. Also, the **pure neutral moving boundary work** of the system* $(\mathrm{W_{PNl}} = \int_1^2 \delta\mathrm{W_{PNl}} = \mathrm{m} \cdot \int_1^2 \delta\mathrm{w_{PNl}})$ *is that which complies with the following condition throughout:* $(\delta\mathrm{W_{PNl}} = \mathrm{pdV} = 0)$. *The state change process associated with **pure neutral moving boundary work is called the pure neutral boundary work process**. Pure neutral moving boundary work processes do not affect moving boundary work calculations.*

HD3-7: (Developed from HD1-1): *In any of diagrams* (T_s), (c_T), (c_t), *or* (p_v), *the pure positive/negative transferred heat to the* *system or boundary work is represented by a right-angled trapezoid that is based (by its right-angled leg) on the horizontal axis. Its other leg (mostly not right-angled and may be not straight) is the partial process, which forms its upper border and is defined by one of the following relations:* $(\delta q = T \cdot ds = c \cdot dT = c \cdot dt \geq 0)$, $(\delta q = T \cdot ds = c \cdot dT = c \cdot dt \leq 0)$, $(\delta w = p \cdot dv \geq 0)$, *or* $(\delta w = p \cdot dv \leq 0)$. *The area under 1_2 in Figure 1.3 measures the pure positive transferred heat to the system.*

OR3-2: *The transferred heat to the system in T_s plane is pure positive during equilibrium entropy increasing* $(ds \geq 0)$ *processes and pure negative during equilibrium entropy decreasing* $(ds \leq 0)$ *ones. In other words, adding heat to the system (gas) during the equilibrium process is always accompanied with entropy increasing, and rejecting heat from it is accompanied with entropy decreasing. Also, the boundary work in p_v plane is pure positive during expansion* $(dv \geq 0)$ *processes and pure negative during compression* $(dv \leq 0)$ *ones.*

HD3-8: (Developed from *HD1-4): **Process'/cycle's characteristic points** in a property diagram (T_s, c_T, c_t, or p_v) are the points:*

1) *at which the **process** changes its mathematical expression (equation).* These points are mostly path's refraction points (points 6, 8, and 10 in Figure 3.5). The tangents of the path at both sides of the refraction point are not the same slope.
2) *which are its/path's intersection points with the horizontal axis.* These points do not exist in *T_s* and/or *p_v* diagrams because the absolute temperature and pressure cannot be negative and the whole process/cycle is always situated above the horizontal axis. They can exist only in *c_t or c_T* diagram. At each of these points c changes its sign, while $dT = dt$ does not, and therefore, $\delta q = c dT = c dt$ changes its sign too. Thus, *at each intersection point between process path in c_T (or c_t) diagram and the horizontal axis T (or t) the **process** changes the sign of its STH.*
3) *which are its/path's initial and final points. These points do not exist in cyclic processes (cycles), and*
4) *at which the tangents of the smooth parts of the path are either horizontal or vertical.* In this book, we shall refer to these points as **path's stationary *points***.

HD3-8A: ***Process'/cycle's characteristic isotherm*** *in a T_s diagram is the isotherm that passes through any of its characteristic points.*

> **HD3-9:** ***Process'/cycle's semicharacteristic points*** *in a T_s diagram are the intersection points between the process/cycle and its characteristic isotherms that are not process'/cycle's characteristic points.*

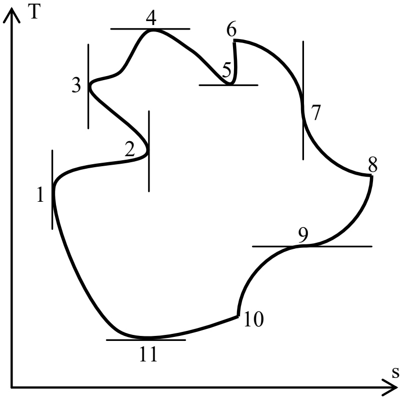

Figure 3.5. Some cycle's characteristic points in T_s diagram

> **HD3-9A:** ***Process'/cycle's maim part*** *in a T_s diagram is its part that extends/is located between its two neighboring characteristic isotherms.*
>
> **HD3-10:** ***Process'/cycle's main points*** *in a T_s diagram are both process'/cycle's characteristic points and its semicharacteristic points.*

Some distinguished points of a cycle

In Figure 3.5 some distinguished points of a cycle are illustrated in an imaginary *T-s* diagram. This figure will be our reference to this paragraph.

Path's stationary points *(1 through 5, 7, 9, and 11) can be either* ***function's stationary points*** *(4, 5, 9, and 11) that belong to horizontal tangents and known in Mathematics as* ***"stationary points"*** *or* ***variable's stationary points*** *(1, 2, 3, and 7) that belong to vertical tangents. The branches of the function's stationary points are (1)* ***function's inflection points*** *"9," (2)* ***function's turning points*** *"4, 5, and 11," (2a)* ***function's points of maximum*** *"4" (local and absolute), and (2b)* ***function's points of minimum*** *"5 and 11" (local "5" and absolute "11"). On the other hand, the branches of the variable's stationary points are (3)* ***variable's inflection points*** *"7," (4)* ***variable's turning points*** *"1, 2, and 3," (4a)* ***variable's points of maximum*** *"2" (local "2" and absolute), and (4b)* ***variable's points of minimum*** *"1 and 3" (local "3" and absolute "1").*

According to (OR3-2), the STH to the system throughout any of the ***subprocesses that*** *resulted from* ***dividing the process/cycle*** *in the property* *diagram (T_s, c_T, or c_t)* ***with/by all its characteristic points will be pure (positive, negative, or neutral), while the boundary work*** *throughout any of the* ***subprocesses that*** *resulted from* ***dividing it*** *in p_v* ***with/by all its characteristic points will also be pure.***

We shall refer to the ***lines of constant temperature, pressure, volume, and entropy that pass through characteristic points as characteristic isotherms, isobars, isochors, and isentrops, respectively***.

3.1.6 THE DETERMINATION OF THE SPECIFIC HEATS OF GASES

SH capacities were first determined experimentally and then theoretically. We shall discuss here only the ideal gas SHs.

3.1.6.1 The experimental determination of the specific heats of ideal gases

Table 3.1. The experimental function $Mc_{p\,=\,f(T)}$ for the carbon monoxide

T [K]	300	500	700	800	1,500	2,000
Mc_p [kJ/kmol deg]	29.13	29.76	31.10	32.44	34.99	35.96

After conducting tests on each gas for a particular process and obtaining the tabular experimental data such as that for the carbon monoxide (see Table 3.1), the data are treated analytically and/or graphically. The functions and/or curves extracted from the treatment represent the experimental relation $c = f(T)$ between the ideal gas SH (at constant pressure or at constant volume) and temperature.

The maximum required in thermodynamic calculation exactness for highly accurate SH ***calculations is usually reached, when using the equation that resulted from approximating, according to the above exactness, the experimental curve c = f(T) or c = f(t). The given data in such exact approximation is (1) the experimental function c = f(T) or c = f(t) in its tabular or graphical form and (2) the degree of the required exactness, while the required is the calculating lowest-degree polynomial function that ensures the highly accurate calculations of the SHs. This lowest-degree function includes four or five terms whose factors must be determined during this approximation.***

Three samples of equations, existing in western thermodynamic references, for highly accurate calculations of the ideal gas constant pressure SHs are as follows:

1)[11]

$$\left.\begin{array}{c} c_{p0} = C_0 + C_1\theta + C_2\theta^2 + C_3\theta^3 \text{ kJ/kg K} \\ \theta = T(\text{Kelvin})/1000 \end{array}\right\} \quad (3\text{-}33)$$

This is the general form equation for 29 calculating equations belonging to 29 different ideal gases. The constants for the steam (H_2O) are $C_0 = 1.79$, $C_1 = 0.107$, $C_2 = 0.586$, and $C_3 = -0.20$, and for air are $C_0 = 1.05$, $C_1 = -0.365$, $C_2 = 0.85$, and $C_3 = -0.39$.

∗This equation is approximate and is valid from 250 to 1,200 K.

Non of the 29 tabulated in this table SHs of different gases had zero constant term ($C_0 = 0$)

2)[12]

$$\left.\begin{array}{l} \overline{c}_p = \mathbf{Mc_p} = a + bT + cT^2 + dT^3 \qquad \text{or} \\ \overline{c}_p\ [\text{kJ}/(\text{kmol K})] = \mathbf{Mc_p} = a + bT + cT^2 + dT^3 \end{array}\right\} \quad (3\text{-}34)$$

where *T* is in K, and $\overline{c}_p$ in kJ/(kmol K). This is the general form equation for 28 calculating equations belonging to 28 different ideal gases. The constants for the oxygen (O_2) are $a = 25.48$, $b = 1.520\times10^{-2}$, $c = -7.155\times10^{-5}$, $d = 1.312\times10^{-9}$, maximum error 1.19%, and minimum error 0.28%. Also, the constants for air are $a = 28.11$, $b = 0.1967\times10^{-2}$, $c = 0.4802\times10^{-5}$, $d = -1.966\times10^{-9}$, maximum error 0.72%, and maximum error 0.33%. For both gases this equation is valid from 273 to 1,800 K.

Non of the 28 tabulated in this table SHs of gases had zero constant term (a = 0)

3)[13]

$$\frac{\overline{c}_p}{\overline{R}} = \frac{\mathbf{Mc_p}}{\mathbf{MR}} = \alpha + \beta T + \gamma T^2 + \delta T^3 + \varepsilon T^4 \quad (3\text{-}34a)$$

where *T* is in K, and equations are valid from 300 to 1,000 K. This is the general form equation for 12 calculating equations belonging to 12 different ideal gases. The constants for hydrogen (H_2) are $\alpha = 3.057$, $\beta = 2.677\cdot10^{-3}$, $\gamma = -5.810\cdot10^{-6}$, $\delta = 5.521\cdot10^{-9}$, and $\varepsilon = -1.812\cdot10^{-12}$, and for methane ($CH_4$) are $\alpha = 3.826$, $\beta = -3.979\cdot10^{-3}$, $\gamma = 24.558\cdot10^{-6}$, $\delta = -22.733\cdot10^{-9}$, and $\varepsilon = 6.963\cdot10^{-12}$.

Non of the 12 tabulated in this table SHs of gases had zero constant term ($\alpha = \text{o}$)

Nt3-7: The bolded sides in Equations (3-34) and (3-34a) are added by the author to clarify their original right sides according to the nomenclature of this book. These equations are not suitable for daily use; therefore, they are usually tabulated (see Table 3.1).

Nt3-7A: The values of the factors/constants of each of Equations (3-33) through (3-34A) are gas kind dependent (see items 1, 2, and 3); therefore, the ideal gas constant pressure SH and its terms, not the first, are temperature and gas kind dependent [$c_{p0} = f(T \text{ and gas kind})$].

Nt3-7B: Although the factors of Equations (3-33) through (3-34A) are mostly dimensional ones, only their numerical values are directly given in references. They are given either after the given calculating equation (see Equations 3-33 through 3-34A) or within it (see Table 3.2).

Nt3-7C: As seen, Equations (3-33) and (3-34) are polynomials of the third degree (their right sides include four terms each), while Equation (3-34A) is a polynomial of the fourth degree (its right side includes five terms).

3.1.6.2 The theoretical determination of the specific heats of ideal gases

The development of morphology helped in the theoretical determination of the SH. Important successes were achieved in this direction after announcing the quantum theory. Using this theory, the following ***Einstein's equation*** for determining the ideal gas molar SHs at constant volume was derived. This equation gives sufficient accuracy for the temperatures used in heat engineering and has the following three forms:

$$\left.\begin{aligned}
Mc_v &= \frac{3+n}{2}MR + \sum_{i=1}^{n_2} MR\cdot f\left(\frac{\theta}{T}\right)_i ;\\
Mc_v &= n_1\frac{\mu R}{2} + \sum_{i=1}^{n_2}\left\{MR\frac{(\theta/T)^2\cdot e^{(\theta/T)}}{\left[e^{(\theta/T)}-1\right]^2}\right\}_i ;\\
Mc_v &= \frac{3+n}{2}MR + MR\sum_{i=`1}^{n_2}\left[\left(\frac{hv}{kT}\right)^2\times\frac{e^{hv/kT}}{(e^{hv/kT}-1)^2}\right]_i\\
\text{and}\qquad Mc_v &= \frac{3+n}{2}MR + \sum_{i=`1}^{n_2}C_i
\end{aligned}\right\}\qquad (3\text{-}35)$$

where MR is the universal gas constant (see §2.5.2), *e* the basis of the natural logarithm, *T* the absolute temperature of the gas [K], and θ a function called the distinctive temperature:

$$\theta = h \cdot \nu / k = \omega hc / k \tag{3-35a}$$

> where *h* is the Planck's constant $h = 6.626\ 10^{-34}$ J sec, *k* the Boltzmann's constant $k = 1.38 \cdot 10^{-23}$ J/deg, ν the frequency of the intramolecular vibrations $\nu = \omega \cdot c$ [$\sec^{-1}$], c the speed of light in vacuum $c = 2{,}998.10^{10}$ cm/sec, ω the waveform number [cm^{-1}]. It is determined by spectroscopy, where the frequency is expressed by the waveform number (ω) that is interconnected with the ordinary frequency (ν).

Also *n* is the number of degrees of freedom of the rotational movement of the molecule:

$$\left.\begin{array}{ll}\text{For monatomic molecules, ------------------------------} & n = 0\\ \text{For diatomic and linear multiatomic molecules, --------} & n = 2\\ \text{For nonlinear molecules (triatomic and multiatomic),} & n = 3.\end{array}\right\} \tag{3-35b}$$

Also n_1 is the number of degrees of freedom of the translational and rotational movements of the molecule (see OD2-16). Since the number of degrees of freedom for the translational movement of the molecules equals 3, $n_1 = 3 + n$ and:

$$\left.\begin{array}{ll}\text{For monatomic molecules, -----------------------------} & n_1 = 3\\ \text{For diatomic and linear multiatomic molecules, ------} & n_1 = 5\\ \text{For nonlinear molecules (triatomic and multiatomic),} & n_1 = 6\end{array}\right\} \tag{3-35c}$$

Also n_2 *is the number of degrees of freedom of the intramolecular vibrations* $n_2 = 3m - (3 + n)$ and m is the number of atoms in the molecule:

For monatomic molecules $n_2 = 3 \cdot 1 - (3 + 0) = 0$
For diatomic molecules $n_2 = 3 \cdot 2 - (3 + 2) = 1$

For a particular substance, the spectroscopy determines the values of its waveform numbers (ω_i) and the value of n_2. For example, the result of the spectroscopy is $\omega_1 = \text{x cm}^{-1}$, $\omega_{2,3} = \omega_2 = \omega_3 = \text{y cm}^{-1}$, and $\omega_4 = \text{z cm}^{-1}$, which means that four frequencies are measured ($n_2 = 4$), of which ω_2 and ω_3 are equal. This leads to determining four distinctive temperatures θ_1, $\theta_2 = \theta_3$, and θ_4 (see Equation 3-35a) and to the sum in Equations (3-35) (see the title of Table 3.2) that equals $C_1 + C_2 + C_3 + C_4$ or $C_1 + 2C_2 + C_4$. For the result $\omega = x\ cm^{-1} \rightarrow n_2 = 1$ and $\omega_1 = x\ cm^{-1}$ *and the mentioned sum is C_1.*

The first term of the right side of Equation (3-35) calculates the amount of heat spent on increasing the translational and rotational kinetic energies of the molecules of 1 kmole of the gas, when it is heated at constant volume by $1^\circ C$, while the second term calculates the amount of heat spent on increasing the intramolecular vibrational kinetic energy of the same molecules during the same heating process.

The second term of the right side of Equation (3-35) is not suitable for daily use; therefore, we usually resort to one of the following two solutions:

I) Calculating for once the $C = \mu R \cdot f(\theta / T)$ values versus (θ / T), arranging the tables, publishing them, and using the tables in daily calculations (see Table 3.2). The $C = \mu R \cdot f(\theta / T)$ values are tabulated and listed in Table 3.2.
II) Replacing Equation (3-35) by a polynomial equivalent to it so that takes the form of relationship (3-33).

In the last paragraph the talk was about the molar SH at constant volume and this is enough to calculate all other ideal gas SHs.

Example 3-1:

Determine the constant volume SH c_v for the nitric oxide NO at temperature t = 1,600°C, taking into account/consideration the vibrational MKE (the kinetic energy of intramolecular vibration) and considering the vibrations harmonic. From the spectroscopy of the gas, it is known that the frequency of vibrations is $\omega = 1906\ \text{cm}^{-1}$.

Compare the obtained SH with/by its tabular value that equals to $c_v = 0,940\ \text{kJ}/(\text{kg.deg})$ ·

Solution:

For approximately calculating the molar SHs of gases in their ideal states, we can use the Einstein's Equation (3-35). For this purpose (see Equation 3-35a and its explanation/legend), we calculate the value of the only distinctive temperature (θ_1) :

$$\theta_1 = \omega_1 \frac{h \cdot c}{k} = 1906 \frac{6.62 \cdot 10^{-34}}{1.38 \cdot 10^{-23}} 2.998 \cdot 10^{10} = 2741\text{K}$$

Then we calculate θ_1 / T value: $\theta_1 / T = 2741/1873 = 1.464$.

According to this value, we find from Table 3.2 the value of function:

$$C_1 = MR \cdot f(\theta_1 / T) = 6.988 \text{ kJ}/(\text{kg.deg})$$

From this equation, the fourth of Equations (3-35), the second of Equations (3-35b), Equation (3-11), and taking in consideration that $n_2 = 1$ (only one ω=1906 cm^{-1} resulted from the spectroscopy), we calculate the required SH: $c_v = \frac{Mc_v}{M} = \frac{1}{30.00}\left(\frac{3+2}{2}.8.314 + 6.988\right) = 0.926\,\text{kJ}/(\text{kg.K})$

Mismatching with tabular values $\Delta c_v = 0.940 - 0.926 = 0.014 \frac{\text{kJ}}{\text{kg.K}}$ constitutes 1.5%. Its explanation is that Einstein's equation does not take into account the lack of compatibility between the vibrational, rotational, and electronic energies.

Nt3-7D: In this book, we shall encode the first five terms of the polynomial as *a*, *b*, *d*, *e*, and *g* (*c* and *f* are excluded to avoid the confusion with the symbols of the SH and function).

3.1.7 CALCULATING THE TRANSFERRED HEAT DURING PHYSICAL IDEAL GAS STATE CHANGE PROCESSES

To calculate the STH during a physical ideal gas state change process using any of the Equations of Group (3-7), we need to know the expression of the SH during that process. As seen in/from Equation (3-10), the transferred heat can be calculated through any of the three different forms of the SH (*c*, *Mc*, and *c*') that belong to the same ideal gas state change process under consideration. Also, as we shall see later (see Equations 3-56 and 3-57), ***this same transferred heat can be calculated through the internal energy and/or enthalpy changes (Δu and Δh)***, which means that the same calculations can be additionally done through any form of the constant pressure and/or constant volume SHs (c_p, Mc_p, c'_p, c_v, Mc_v, and c'_v). Thus, to calculate the ideal gas transferred heat the expression of any of the temperature functions *(u, h, c, Mc, c', c_p, Mc_p, c'_p, c_v, Mc_v, and c'_v) is needed*, and can be given analytically similar to Equations (3-34), graphically through a curve like curve (a-1-2) in Figure 1-3 or tabular through a table like in paragraph (3-1-6-1). Earlier (see §3.1.3) we highlighted on the graphical calculations; therefore, ***we will be interested here in the analytical and tabular calculations of the transferred heats at constant pressure and constant volume.***

Table 3.2. The values $C = MR \cdot \left(\theta / T\right)^2 \frac{e^{\theta/T}}{(e^{\theta/T} - 1)^2} \frac{\text{kmole}}{\text{kJ} \cdot \text{K}}$

$\frac{\theta}{T}$	C	$\frac{\theta}{T}$	C	$\frac{\theta}{T}$	C	$\frac{\theta}{T}$	C
0	8,314	1,10	7,528	2,30	5,451	4,80	1,604
0,10	8,302	1,15	7,461	2,40	5,259	5,00	1,419
0,15	8,294	1,20	7,390	2,50	5,066	5,20	1,256
0,20	8,286	1,25	7,314	2,60	4,873	5,40	1,097
0,25	8,273	1,30	7,239	2,70	4,685	5,60	0,971
0,30	8,265	1,35	7,164	2,80	4,497	5,80	0,854
0,35	8,235	1,40	7,084	2,90	4,312	6,00	0,745
0,40	8,206	1,45	7,004	3,00	4,128	6,40	0,569
0,45	8,173	1,50	6,946	3,10	3,948	6,80	0,431
0,50	8,143	1,55	6,837	3,20	3,772	7,20	0,322
0,55	8,114	1,60	6,749	3,30	3,601	7,60	0,239
0,60	8,072	1,65	6,665	3,40	3,433	8,00	0,179
0,65	8,030	1,70	6,573	3,50	3,270	8,40	0,134
0,70	7,988	1,75	6,485	3,60	3,115	8,80	0,096
0,75	7,938	1,80	6,393	3,70	2,960	9,20	0,071
0,80	7,888	1,85	6,301	3,80	2,814	9,60	0,050
0,85	7,834	1,90	6,209	3,90	2,667	10,00	0,038
0,90	7,779	1,95	6,117	4,00	2,529	11,00	0,017
0,95	7,720	2,00	6,025	4,20	2,269	12 00	0,0071
1,00	7,658	2,10	5,932	4,40	2,026	13,00	0,0029
1,05	7,595	2,20	5,644	4,60	1,804		

Nt3-8: Since we are mainly interested in ideal gas (see Equations 4-9a and 4-11a) $q_v = \Delta u = \int_{T_1}^{T_2} c_{v0} \cdot dT$ or $q_p = \Delta h = \int_{t_1}^{t_2} c_{p0}\, dT$, but not in ($\int_0^{t_2} c_v dt$, $\int_0^{t_1} c_v dt$, $\int_0^{T_2} c_v dT$, $\int_0^{T_1} c_v dT$, $\int_{T_{ref}}^{T_2} c_v dT$, $\int_{T_{ref}}^{T_1} c_v dT$, $\int_0^{t_2} c_p dt$, $\int_0^{t_1} c_p dt$, $\int_0^{T_2} c_p dT$, $\int_0^{T_1} c_p dT$, $\int_{T_{ref}}^{T_2} c_p dT$ or $\int_{T_{ref}}^{T_1} c_p dT$) total (absolute) values, then we mostly are not interested in the values of the constant lower limits of the last 12 integrals. These lower limits ($t = 0$, $T = 0$, or T_{ref}) are usually chosen by the authors of the tables of ideal gas properties of various substances, which usually include the values of a set of ideal gas properties against temperature, and among which are *u* and *h*. Here, T_{ref} can equal the AZ and the units of both upper and lower limits of each integral are the same.

Nt3-8A: Almost all published tables that use the nonlinear relation between the SH and temperature deal with the isobaric and isochoric processes (see the Nt3-1).

3.1.7.1 The pure analytical calculations of the transferred heat during physical ideal gas state change processes

The highest accuracy in thermodynamics is rarely required (it is required for some precise calculations but not for discussions and usual analyses and calculations). And it happens sometimes that a distinct accuracy is required, while the required calculating equations (see OD2-1A) for such calculation are not available. In such cases, we find ourselves obliged to execute approximate calculations/solutions that do not require unavailable data. That is why we shall discuss here different accuracy calculations.

The procedure for calculating the transferred heat during physical ideal gas state change processes *analytically* is very simple. Just choose the calculating equations, through the available data and references, to be used during the calculations, and in case of the absence of calculating errors, you shall sooner or later obtain the required results. Executing this simple procedure is not always easy since it depends on the structure of the calculating equations to be used. And this structure is required to be accuracy dependent.

Almost all published calculating equations for SHs are polynomial ones:

- Equations (3-33) through (3-34A) are samples of the fourth- or third-degree polynomial calculating equations that are used in exact calculations of the SHs.

- Equation $c'_{p\ \mathrm{avg}} = a + b \cdot t/2 + d \cdot t^2/3$ (see Table 3.3) is an evidence that the almost exact polynomial calculating equations of the second degree that are used in approximate calculations of the SHs exist in some published references.
- Also, the equations in Table 3.4 are evident that the polynomial calculating equations of the first degree that are used in approximate calculations of the SHs exist in some published references.

If (1) all required calculating equations for obtaining all exactness (exact, almost exact, linear approximate, and constant approximate), SHs, and STHs for all homogeneous materials were always available, and (2) all exactness analytical calculations were consuming the same time, we would always use the exact calculations. But since (1) the available references never include the required calculating equations for all homogeneous materials, and (2) the exact calculations are the most time consuming, while the constant approximate SH calculations are the less time consuming, we shall discuss below all possible analytical calculations.

To avoid misunderstanding or mistakes in such multipurpose/multivariant discussion we *first* obtain the general form equations of all calculating equations to be used in calculating the required dimensions, and *second* we perform these general form equations into calculating ones, and *only after that* we calculate the required dimensions using the obtained calculating equations.

Let us obtain the multivariant calculating equations for the STH (its symbol here will be *q*). To do this we, using the encoding of this book, start from rewriting the general form polynomials of the third and higher degrees that are available in references (see Equations 3-33 and 3-34A):

$$\mathrm{c_0 = a + bT + dT^2 + eT^3 + gT^4} \quad \text{and} \quad \mathrm{c_0 = a + bT + dT^2 + eT^3} \qquad (3\text{-}36)$$

where subscript (0) stands for ideal gas.

We shall refer to each of these equations as *the general form of the exact (highly accurate) equation for calculating the* SH.

The values of each of the factors/constants (*a*, *b*, *d*, *e,* and *g*) vary from one gas to another, from one process to another, from one unit quantity to another, from one temperature scale to another, and from one polynomial degree to another.

The lower exactness from the group of equations (3-36) are:

- ***The general form of the calculating equations of the almost exact (accurate) SHs:***

$$c_0 = a + bT + dT^2 \tag{3-36'}$$

- ***The general form of the calculating equations of the moderately accurate* SH*s:***

$$c_0 = a + bT \tag{3-36''}$$

- ***The general form of the calculating equations of the less accurate* SH*s:***

$$c_0 = a \tag{3-36'''}$$

Applying the first of Equations (3-36) in the second of Equations (3-7), we, after performing, obtain ***the general form equation of the calculating equations of the exact (***high accurate) ***STH***:

$$q = q\Big|_{T_1}^{T_2} = \int_{T_1}^{T_2} c_0 \cdot dT = \int_{T_1}^{T_2} \left(a + bT + dT^2 + eT^3 + gT^4\right) \cdot dT \qquad \text{or}$$

$$q = q\Big|_{T_1}^{T_2} = a\left(T_2 - T_1\right) + b\frac{T_2^2 - T_1^2}{2} + d\frac{T_2^3 - T_1^3}{3} + e\frac{T_2^4 - T_1^4}{4} + g\frac{T_2^5 - T_1^5}{5} \tag{3-37}$$

From this equation we obtain:

- ***The general form equation of the calculating equations of the almost exact (accurate) STHs:***

$$q = q\Big|_{T_1}^{T_2} = a \cdot \left(T_2 - T_1\right) + b \cdot \frac{T_2^2 - T_1^2}{2} + d \cdot \frac{T_2^3 - T_1^3}{3} \tag{3-37'}$$

- ***The general form equation of the moderately accurate calculating equations of the STHs:***

$$q = q\Big|_{T_1}^{T_2} = a\left(T_2 - T_1\right) + b \cdot (T_2^2 - T_1^2)/2 \tag{3-37''}$$

- ***The general form equation of the less accurate calculating equations of STHs:***

$$q = q\Big|_{T_1}^{T_2} = a \cdot \left(T_2 - T_1\right) \tag{3-37'''}$$

Based on any of Equations (3-36) through (3-37''') (e.g., the first of Equations 3-36), ***analytical calculations*** cannot be done in the absence of its calculating equation for the SH or for the STH of the material/ideal gas under consideration (e.g., ideal gas steam (IGS)). The only available, in this book, completely defined/calculating equation for the exact calculation of IGS SH is Equation (3-33), whose factors when it is rewritten for (*T*) instead of (*θ*) are (see the explanation that follows Equation 3-33): $C_0 = 1.79$, $C_1 = 0.107 \cdot 10^{-3}$, $C_2 = 0.586 \cdot 10^{-6}$, and $C_3 = -0.20 \cdot 10^{-9}$. Therefore, for the case, when this book is the only available reference, the exact equations for calculating the IGS constant pressure molar SH and the molar STH at constant pressure are:

$$\left.\begin{array}{l} \mathrm{Mc}_{\mathrm{p_{IGS}}}^{\mathrm{exact}}\ [\mathrm{kJ/(kmol \cdot K)}] = \mathrm{a + bT + dT^2 + eT^3} \qquad \text{and} \\ \mathrm{Mq}_{\mathrm{p_{IGS}}}^{\mathrm{exact}}\Big|_{\mathrm{T_1}}^{\mathrm{T_2}} \left[\dfrac{\mathrm{kJ}}{\mathrm{kmol}}\right] = \mathrm{a}\left(\mathrm{T_2 - T_1}\right) + \mathrm{b}\dfrac{\mathrm{T_2^2 - T_1^2}}{2} + \mathrm{d}\dfrac{\mathrm{T_2^3 - T_1^3}}{3} + \mathrm{e}\dfrac{\mathrm{T_2^4 - T_1^4}}{4} \\ \text{where (see the above)}: a = C_0 = 1.79, \qquad b = C_1 = 0.107 \cdot 10^{-3}, \\ \qquad d = C_2 = 0.586 \cdot 10^{-6}, \text{ and } e = C_3 = -0.20 \cdot 10^{-9}. \end{array}\right\} \quad (3\text{-}38)$$

The other form of Equations (3-38) is the two equations (3-38') and (3-38''):

$$\mathrm{Mc}_{\mathrm{p_{IGS}}}^{\mathrm{exact}}\ [\mathrm{kJ/(kmol \cdot K)}] = 1.79 + 107 \cdot 10^{-6} \cdot \mathrm{T} + \\ + 586 \cdot 10^{-9} \cdot \mathrm{T^2} - 2 \cdot 10^{-10} \cdot \mathrm{T^3} \qquad (3\text{-}38')$$

and

$$\mathrm{Mq}_{\mathrm{p_{IGS}}}^{\mathrm{exact}}\Big|_{\mathrm{T_1}}^{\mathrm{T_2}} \left[\frac{\mathrm{kJ}}{\mathrm{kmol}}\right] = 1.79\ \left(\mathrm{T_2 - T_1}\right) + 107 \cdot 10^{-6} \cdot \frac{\mathrm{T_2^2 - T_1^2}}{2} + \\ + 586 \cdot 10^{-9} \cdot \frac{\mathrm{T_2^3 - T_1^3}}{3} - 2 \cdot 10^{-10} \cdot \frac{\mathrm{T_2^4 - T_1^4}}{4} \qquad (3\text{-}38'')$$

If these exact equations do not suite some persons because of their large time consumption, especially when good calculating instrumentation is absent, then they (these persons)) can reduce the calculating time consumption on the expense of calculating exactness by moving to other calculating equations of less exactness if the new calculating equations to be used or their factors are directly or indirectly available in available references (e.g., the calculating equations of the ideal gas steam constant pressure/volume moderate accuracy SHs are indirectly available in this book as $c_{p_{avg}} = 1.833 + 0.0003111\ t$, $c_{v_{avg}} = 1.3716 + 0.0003111\ t$, … and can be obtained from Table 3.4. The first of them is $c_{p_{IGS}} [kJ/(kg \cdot^{\circ} C)] \approx 1.833 + 2 \cdot 0.0003111\ t$ and the numerical values/parts of its factors are $a = 1.833$ and $b = 2 \cdot 0.0003111$ "see Equations 1-33, 3-36", 3-44 and 3-45"). The just explained way for reducing the calculating time consumption on the expense of calculating exactness is the best between the other existing ways (see the following paragraph).

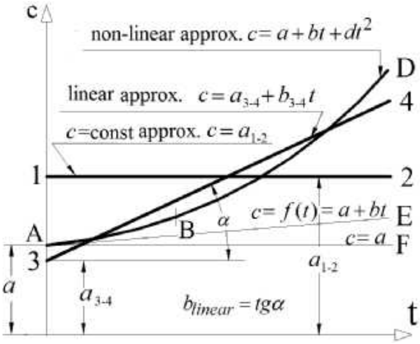

Figure 3.6. The experimental SH curve (A_B_C), its linear (3_4), and c=constant (line 1_2) approximations.

3.1.7.2 About the bad effect of abbreviating the calculating equations by cutting off their higher-degree terms

Simplifying the expression of the SH, by getting rid of its last term, causes losses in the accuracy of calculating each of the SH and the STH. These losses are usually minimized by optimizing the values of the factors/constants of the abbreviated equation/expression to reach its best approximation to the experimental curve. Applying such optimization on Equation (3-36") leads to the same exactness of the equations of Table 3.4 that is the best approximation to the experimental curve reached by a linear equation (see line 3–4 in Figure 3.6). And we shall refer to it as ***the best linear approximation to the experimental curve.*** Also, we shall refer to the other polynomials that best approximate the experimental curve as the ***best second-degree polynomial approximation to the experimental curve, best third-degree polynomial approximation to the experimental curve, best fourth-degree polynomial approximation to the experimental***

curve, and so on. As stated above, any of the ***best fourth- and higher-degree polynomial approximation*** *to the experimental curve* is considered an exact equation of the *experimental curve and the lowest (fourth)* ***degree is the less time consuming and the most appropriate for exact thermodynamic calculations.***

To illustrate the loss of accuracy caused by simplifying the expression of the SH we drive Figure 3.6. To simplify this discussion that is qualitative, suppose that the experimental curve is a quadratic one $[c_0 = a + bT + dT^2]$, not Equation (3-36); this means that almost all experimental points lie on curve (A–B–D) or near it.

The other four lines are:

- (3–4) that is considered the **best** linear approximation to the experimental curve,
- (1–2) is considered the **best** *c* = constant approximation to it,
- (A–E) that is defined by the equation ($c = a + bt$) and is the abbreviation of the mentioned quadratic equation after excluding its third term, and
- (A–F) that is defined by the equation ($c = a$) that is the abbreviation of the same quadratic equation after excluding all its second and third terms.

As is obvious from the illustration the best calculating accuracy is assumed to be reached with the quadratic equation. Concerning the linear approximation, line (3–4) is much closer to the experimental line than line (A–E) and for the c = constant approximation, line (1–2) is much closer to the experimental line than line (A–F). This means that ***the sole abbreviation of the expression of the SH, by only getting rid of its last term, causes losses in the accuracy of calculating each of the SH and the*** STH ***that exceed the minimum unavoidable ones;*** therefore, it is necessary to optimize/change the values of the factors/constants of the abbreviated equations to reach their best approximation to the experimental curve. And that is why the values of constant (*a*), which is the *ordinate (*the *c*-coordinate) of the intersection point of $c = f(t)$ line with *c*-axis, is not the same in the different (nonlinear (quadratic), linear, and temperature-independent) approximations/equations of the same SH (see points A, 3, and 1 in Figure 3.6). Also, constant (*b*), which is the *slope* of $c = f(t)$ line at its intersection point with *c*-axis, is not the same in the mentioned nonlinear and inclined linear types of approximations/equations.

3.1.7.3 Some additional analyses on Equations (3-36)

Several mathematical ways can be used to calculate analytically the STH expressed by Equations (3-7). These ways are expressed by the following multiequation:

$$\begin{aligned} q\Big|_{t_1}^{t_2} &= q\Big|_{T_1}^{T_2} = \int_{t_1}^{t_2} c\cdot dt = \int_0^{t_2} c\cdot dt - \int_0^{t_1} c\cdot dt = c_{avg}\cdot(t_2 - t_1) \\ &= c_{avg}\cdot(t_2 - 0) - c_{avg}\cdot(t_1 - 0) = \int_{T_1}^{T_2} c\cdot dT = \int_{T_{ref}}^{T_2} c\cdot dT - \int_{T_{ref}}^{T_1} c\cdot dT \\ &= c_{avg}\cdot(T_2 - T_1) = c_{avg}\cdot(T_2 - T_{ref}) - c_{avg}\cdot(T_1 - T_{ref}) \end{aligned} \qquad (3\text{-}38''')$$

from which we obtain:

$$\left.\begin{aligned} &c_{avg} = \frac{\int_{t_1}^{t_2} c\cdot dt}{\Delta t}; \quad c_{avg} = \frac{\int_0^{t} c\cdot dt}{t}; \quad c_{avg} = \frac{\int_{T_1}^{T_2} c\cdot dT}{\Delta T}; \quad c_{avg} = \frac{\int_{T_{ref}}^{T} c\cdot dT}{T - T_{ref}}; \\ &q\Big|_{T_1}^{T_2} = \int_0^{T_2} c\cdot dT - \int_0^{T_1} c\cdot dT\ ; \qquad q\Big|_{T_1}^{T_2} = c_{avg}\cdot(T_2 - 0) - c_{avg}\cdot(T_1 - 0); \\ &q\Big|_{t_1}^{t_2} = \int_0^{t_2} c\cdot dt - \int_0^{t_1} c\cdot dt \qquad \text{or} \qquad q\Big|_{t_1}^{t_2} = c_{avg}\cdot(t_2 - 0) - c_{avg}\cdot(t_1 - 0) \end{aligned}\right\} \quad (3\text{-}39)$$

In Equations (3-38''' and 3-39), any pair of temperature scales (the Rankine–Fahrenheit or the Kelvin–Celsius) can be used. Here no restrictions on T_{ref} that can equal the AZ (T_{ref} = the AZ was used in the last of Equations 3-39). Noting that the temperature scale used in the expressions of the SHs (see Equations 3-7) is rather of the duty of reference's creator than of table's user, which means that we as users must be ready for the calculations using any temperature scale. From now on we shall use the relative temperature scale but do not forget that you can transfer to the absolute scale whenever you want. Substituting the value of c_0 (index 0 stands for ideal gas) from the first of Equations (3-36) in the first of Equations (3-7) we obtain the equation $q = \int_{t_1}^{t_2} (a + bt + et^2)\cdot dt$ that performs after integrating into:

$$q\Big|_{t_1}^{t_2} = \int_{t_1}^{t_2} c\cdot dt = a(t_2 - t_1) + b\frac{t_2^2 - t_1^2}{2} + e\frac{t_2^3 - t_1^3}{3} = c_{avg}(t_2 - t_1) \qquad (3\text{-}40)$$

where $c_{avg} = q/(t_2 - t_1) = a + (b/2)(t_2 + t_1) + (e/3)(t_2^2 + t_1 t_2 + t_1^2)$

Noting that

$$q\Big|_{t_1}^{t_2} = q\Big|_0^{t_2} - q\Big|_0^{t_1} \qquad (3\text{-}41)$$

Table 3.3. The average volumetric SHs (the nonlinear relationship) at constant pressure $c'_{pavg} = a + \frac{b}{2} \cdot t + \frac{e}{3} \cdot t^2$ [2]

kcal/(m³ deg)		kJ/(m³ deg)				
H_2O	Air	O_2	N_2	CO_2	H_2O	Air
0.3569	0.3098	1.306	1.295	1.600	1.494	1.297
0.3595	0.3106	1.318	1.296	1.700	1.505	1.300
0.3636	0.3122	1.335	1.300	1.787	1.522	1.307
0.3684	0.3146	1.356	1.307	1.863	1.542	1.317
0.3739	0.3174	1.378	1.316	1.930	1.565	1.329
0.3797	0.3207	1.398	1.328	1.989	1.590	1.343
0.3857	0.3240	1.417	1.340	2.041	1.615	1.357
0.3920	0.3274	1.434	1.354	2.088	1.641	1.371
0.3984	0.3306	1.450	1.367	2.131	1.668	1.384
0.4050	0.3338	1.465	1.380	2.169	1.696	1.398
0.4115	0.3367	1.478	1.392	2.204	1.723	1.410
0.4180	0.3395	1.489	1.403	2.235	1.750	1.421
0.4244	0.3422	1.501	1.414	2.264	1.777	1.433
0.4306	0.3447	1.511	1.425	1.290	1.803	1.443
0.4366	0.3470	1.529	1.435	2.314	1.828	1.453
0.4425	0.3492	1.529	1.444	2.335	1.853	1.462

and that $q\big|_0^{t_2}$ and $c_{avg}\big|_0^{t_1}$ can be obtained from $q\big|_{t_1}^{t_2}$ and $c_{avg}\big|_{t_1}^{t_2}$ by replacing (t_1) by zero and that $q\big|_0^{t_1}$ and $c_{avg}\big|_0^{t_1}$ can be obtained from $q\big|_0^{t_2}$ and $c_{avg}\big|_0^{t_2}$ by replacing (t_2) by (t_1) and thus:

$$\left.\begin{aligned}
q\big|_0^{t_2} &= \int_0^{t_2} c \cdot dt = at_2 + \frac{b}{2} \cdot t_2^2 + \frac{e}{3} \cdot t_2^3 = c_{avg} \cdot (t_2 - 0); \\
c_{avg} &= q\big|_0^{t_2} / (t_2 - 0) = a + \frac{b}{2} \cdot t_2 + \frac{e}{3} \cdot t_2^2; \\
q\big|_0^{t_1} &= \int_0^{t_1} c \cdot dt = at_1 + \frac{b}{2} \cdot t_1^2 + \frac{e}{3} \cdot t_1^3 = c_{avg} \cdot t_1; \\
c_{avg} &= q\big|_0^{t_1} / (t_1 - 0) = a + \frac{b}{2} \cdot t_1 + \frac{e}{3} \cdot t_1^2 \\
q\big|_0^{t} &= \int_0^{t} c \cdot dt = at + \frac{b}{2} \cdot t^2 + \frac{e}{3} \cdot t^3 = c_{avg} \cdot t; \\
\text{And} \qquad c_{avg} &= q\big|_0^{t} / (t - 0) = a + \frac{b}{2} \cdot t + \frac{e}{3} \cdot t^2
\end{aligned}\right\} \qquad (3\text{-}42)$$

Nt3-9: The symbols of the average SHs (c_{avg}) in Equations (3-38''') through (3-42) are free from temperature limits because these limits (of dimension c or c_{avg}), in all these equations, are somehow defined (either by multiplying c_{avg} by ($t_2 - t_1$) or by the other terms of the equation).

Using any of multiequations (3-40) or (3-42) means that we are executing the almost exact analytical calculation of the STH (the exact calculation could be done if we used in the analysis of the transferred heat any of the more exact Equations (3-33) through (3-34). Such exact calculation will be more time-consuming). ***Multiequations (3-40) and (3-42) are of the same time consumption and exactness and are not suitable for daily use.*** From the above we conclude the following:

Nt3-10: Multiequation (3-40) is difficult for tabulating because each of $q\Big|_{t_1}^{t_2}$ and $c_{\text{avg}}\Big|_{t_1}^{t_2}$ is a function of the two variables t_1 and t_2. But multiequations (3-42) are easy for tabulating since (1) each of $q\Big|_0^{t_1}$ and $c_{\text{avg}}\Big|_0^{t_1}$ is a function of the only variable t_1 and (2) each of $q\Big|_0^{t_2}$ and $c_{\text{avg}}\Big|_0^{t_2}$ is a function of the only variable t_2. What we can meet in one table is either $q\Big|_0^{t}$ or $c_{\text{avg}}\Big|_0^{t}$ because we can obtain from the column of $q\Big|_0^{t}$ each of $q\Big|_0^{t_1}$ and $q\Big|_0^{t_2}$ and from the column of $c_{\text{avg}}\Big|_0^{t}$ we can obtain each of $c_{\text{avg}}\Big|_0^{t_1}$ and $c_{\text{avg}}\Big|_0^{t_2}$. The tables of $\mathrm{c}_{\text{avg}}\Big|_0^{\mathrm{t}} = \mathrm{f(t)}$ are organized/created on the basis of the first and third sides of the sixth multiequation 3-42 and are called the tables of the average SH within the temperature interval from $0^{\circ}\mathrm{C}$ up to "*t*" (the nonlinear function). Each table is allocated for one specific gas, and it may happen that one table is allocated for more than one gas (Tables 3.3 and 3.4 are allocated for five gases and for two unit systems each).

The availability of tabular function $c_{\text{avg}}\Big|_0^{t} = f(t)$ much eases the exact calculations of the STH (q), when the given data (t_1 and t_2) and the sixth of Equations (3-39) are used. But it complicates the exact calculations, when one of the given data is q because the substitution of the given data in the sixth of Equations (3-39) determines one of the expressions ($c_{avg}\Big|_0^{t_2} \cdot t_2$ or ($c_{avg}\Big|_0^{t_1} \cdot t_1$), which requires applying time-consuming approximating methods such as *the successive approximation method* (see OD3-4A) to determine the required temperature (t_2) through the sixth of Equations (3-39). This disadvantage of the tabular function $c_{\mathrm{avg}}\big|_0^t = f(t)$ does not exist in tabular function $\int_0^t \mathrm{cdt} = \mathrm{f(t)}$. If function $\int_0^t \mathrm{cdt} = \mathrm{f(t)}$ is tabulated and its tables are available, then the exact calculation will be totally comfortable. And the STH is calculated from the fifth of Equations (3-39).

> **OD3-4A:** The ***successive approximation method*** is summarized/defined as the calculating method that leads to the required, not absolutely correct answer by repeated attempts so that the input of each decided subsequent attempt is the output of the previous attempt. The output of each attempt can be a decision about (1) stopping the calculations due to reaching the required answer, or (2) the value of the input of the subsequent attempt. To start the calculations an expected answer to the problem can be adopted as the input to/of the first repeated attempt. Obtaining this first input is the duty of the solver and depends on his experience. The closer to the correct answer the adopted input to the first repeated attempt, the lesser the number of attempts to reach the acceptable answer.
>
> **Nt3-11:** It is not practical to publish specified tables for each thermodynamic function. The usual procedure is to publish many thermodynamic functions in one table. Some of these functions do not accept the AZ as a lower limit of integration (see the explanation that follows Equation 3-27 and Nt3-5A). At this point table's authors decide (1) whether they use one common reference temperature for all temperature functions or not and what is the value of each reference temperature, (2) to declare the values of the chosen reference temperatures or not. Therefore, ***the user of tables must use only one table throughout the problem calculation***. It is obvious that one of Equations (3-38) is used to fulfill the calculations.

3.1.7.4 The tabular calculations of the transferred heat during physical ideal gas state change processes

If a scientific institution obtains the exact calculating equation $\mathrm{q}\big|_0^{\mathrm{t}}\ \mathrm{f(t)}$ or $\mathrm{c}_{\mathrm{avg}}\big|_0^{\mathrm{t}} = \mathrm{f(t)}$ for a particular gas undergoing a particular state change process, creates its computer program to calculate the values of $\mathrm{q}\big|_0^{\mathrm{t}}$ or $\mathrm{c}_{\mathrm{avg}}\big|_0^{\mathrm{t}}$ versus distinct temperature values, organize the calculated results in a table, and publish it, then we will have the opportunity to obtain from this table the value of either $q\Big|_{t_1}^{t_2}$ or $c_{\text{avg}}\Big|_{t_1}^{t_2}$ and apply the obtained value in the appropriate of the equations:

$$\mathrm{q}\Big|_{\mathrm{t}_1}^{\mathrm{t}_2} = \mathrm{q}\Big|_0^{\mathrm{t}_2} - \mathrm{q}\Big|_0^{\mathrm{t}_1} \qquad \text{and} \qquad \mathrm{q}\Big|_{\mathrm{t}_1}^{\mathrm{t}_2} = \mathrm{c}_{\text{avg}}\Big|_0^{\mathrm{t}_2} \cdot \mathrm{t}_2 - \mathrm{c}_{\text{avg}}\Big|_0^{\mathrm{t}_1} \cdot \mathrm{t}_1 \qquad (3\text{-}43)$$

Calculations through published tabulated functions are easy and less time-consuming and are of the same accuracy of the used tables. The equations like (3-33 through 3-34) based on tables are exact ones and tabular calculations using these tables are practically of the same exactness.

The usual publications of scientific institutions include property tables for the widely used gases that depend on the constant pressure and/or constant volume SHs; therefore, one can find in these tables some of/all the functions/dimensions (c_p, c_v, u, h, s_p, s_v …). One can also find but rarely $\mathrm{c}_{\mathrm{v}_{\text{avg}}}\Big|_0^{\mathrm{t}}$ … $\mathrm{c}_{\mathrm{p}_{\text{avg}}}\Big|_0^{\mathrm{T}}$. As we shall see later $u = q_v\Big|_0^t$ and $h = q_p\Big|_0^t$ (see Equations 4-2 and 4-3) and their tables much ease the exact calculations of not only q_p and q_v, but also the exact calculations of the transferred heats during any gas state change process (see Equations 3-54 and 3-57).

3.1.7.5 The almost exact (highly accurate) calculation of the transferred heat

The above leads to:

(A) ***The exact (highly accurate) calculation of the transferred heat can only be done through exact calculating functions (equations, tables, and/or graphs)***. This means that both tables and graphs must be created using exact equations and exact creations (the distance between two neighboring represented points must be small and the scale of the ***graph be large).***

(B) ***The most practical/convenient way for the exact (highly accurate) calculation of the transferred heat during physical state change processes is the tabular one that can only be used in the presence of the exact tables of any of the functions (u, h,*** $c_{v_{\text{avg}}}\Big|_0^t$ ***…*** $c_{p_{\text{avg}}}\Big|_0^T$ ***). It is suitable for daily calculations and is the most exact and the less time consuming one.***

(C) ***The graphical way for the almost exact (accurate) calculation of the transferred heat during physical state change processes can only be used in the presence of the exact large-scale graphs of any of the functions (u, h,*** $c_{v_{\text{avg}}}\Big|_0^t$ ***…*** $c_{p_{\text{avg}}}\Big|_0^T$ ***…). It is less exact than the tabular way, almost the same time consumption and suitable for daily calculations.***

(D) ***The exact and quasi-exact pure analytical calculations of the transferred heat during a physical state change process that is executed by applying the exact and quasi-exact nonlinear relations between the SHs and temperature are slightly more exact than the tabular calculations that apply the same equations but, unless they were previously computer programmed, they are much more time consuming and accordingly they are not appropriate for daily calculations.***

3.1.7.6 The approximate calculations of the transferred heat during physical ideal gas state change processes

For the cases of (1) the available published tables or large-scale exact graphs that do not include calculated data for the SHs and STHs of the gas under consideration or (2) the absence of the published tables or large-scale exact graphs that include calculated data for the SHs and STHs of/for the gas under consideration, we find ourselves obliged to choose from/of the following calculations the approximate one that gives acceptable results within an acceptable time.

If the constants of Equations (3-33), (3-34), (3-34A), and (3-36) are known for the ideal gas under consideration, while its calculated data for the SHs and STHs are absent, then the simplest way to execute the calculations is to deal with the available one of Equations (3-36) (if both are available you can choose any one of them) that allows almost high, moderate, and/or low accuracy calculations:

3.1.7.6.1 The approximate calculation using the linear specific heats $c = a + bt$

Replacing *e* by zero in Equations (3-36), (3-40), and (3-42) we obtain:

$$c = a + bt \qquad (3\text{-}44)$$

$$\left.\begin{aligned} q = \int_{t_1}^{t_2} c \cdot dt = a(t_2 - t_1) + (b/2)(t_2^2 - t_1^2) = c_{avg} \cdot (t_2 - t_1); \\ c_{avg}\Big|_{t_1}^{t_2} = a + (b/2)(t_2 + t_1) \end{aligned}\right\} \quad (3\text{-}45)$$

$$\left.\begin{aligned} &q\Big|_0^{t_2} = \int_0^{t_2} c \cdot dt = at_2 + \frac{b}{2} \cdot t_2^2 = c_{avg} \cdot t_2 \quad \text{where} \quad c_{avg}\Big|_0^{t_2} = a + \frac{b}{2} \cdot t_2 \\ \text{and} \quad &q\Big|_0^{t_1} = \int_0^{t_1} c \cdot dt = at_1 + \frac{b}{2} \cdot t_1^2 = c_{avg} \cdot t_1 \quad \text{where} \quad c_{avg} = a + \frac{b}{2} \cdot t_1 \end{aligned}\right\} \quad (3\text{-}46)$$

It is clear from Equations (3-45) and (3-46) that assuming the function $C = f(t)$ linear simplifies the calculations of the transferred heat and makes them almost suitable for daily use. In addition, the values of $c_{avg}\Big|_0^t$ and $q\Big|_0^t$ are suitable for tabulation and from their tables (if available) the values of $c_{avg}\Big|_0^{t_2}$, $c_{avg}\Big|_0^{t_1}$, $c_{avg}\Big|_{t_1}^{t_2}$, $q\Big|_0^{t_2}$, and $q\Big|_0^{t_1}$ can be obtained. And such tables exist at least in soviet references. The table of SHs in a reference provides either the actual or the average SHs from the zero relative temperature scale until "*t*" (as in Tables 3.3 and 3.4).

It is clear that Equation (3-46) is simple, suitable for daily use, does not need to be tabulated, and all that required to calculate the STH is to know the values of the constants (*a* and *b*) of any of the different SHs (c, Mc, c', c_p, Mc_p, c'_p, c_v, Mc_v, and c'_v) for each gas. These values can either be given in their own tables or derived from the tables of SHs (the nonlinear relationship "see Table 3.3"). It is obvious that to fulfill the calculations, one of Equation (3-38) is used.

3.1.7.6.2 The approximate (less accurate) calculation using the constant specific heats

For c = constant = a, we obtain from the second of Equations (3-7):

$$c = a = \text{const} \Rightarrow q = \int_{t_1}^{t_2} \delta q = \int_{t_1}^{t_2} c \cdot dt = a \cdot (t_2 - t_1)$$

From which we obtain: $c_{avg}\Big|_{t_1}^{t_2} = \dfrac{q}{\Delta T} = \dfrac{a \cdot (t_2 - t_1)}{t_2 - t_1} = a$ or

$$c = a = \text{const} \Rightarrow c_{avg}\Big|_{t_1}^{t_2} = c = a = \text{const} \quad (3\text{-}46a)$$

Table 3.4. The average SHs for gases (from 0 to 1,500°C) and the linear relationship

Gas	The SH per unit mass kJ/(kg.K)	The volumetric SH kJ/(m^3 K)
A—Energy unit kilojoules (kJ)		
O_2	$c_{p_{avg}} = 0.9203 + 0.0001065\,t$ $c_{v_{avg}} = 0.6603 + 0.0001065\,t$	$c'_{p_{avg}} = 1.3138 + 0.0001577\,t$ $c'_{v_{avg}} = 0.9429 + 0.0001577\,t$
N_2	$c_{p_{avg}} = 1.024 + 0.00008855\,t$ $c_{v_{avg}} = 0.7272 + 0.00008855\,t$	$c'_{p_{avg}} = 1.2799 + 0.0001107\,t$ $c_{v_{avg}} = 0.9089 + 0.0001107\,t$
Dry air	$c_{p_{avg}} = 0.9965 + 0.00009299\,t$ $c_{v_{avg}} = 0.7088 + 0.00009299\,t$	$c'_{p_{avg}} = 1.2866 + 0.0001201\,t$ $c'_{v_{avg}} = 0.9157 + 0.0001201\,t$
H_2O	$c_{p_{avg}} = 1.833 + 0.0003111\,t$ $c_{v_{avg}} = 1.3716 + 0.0003111\,t$	$c'_{p_{avg}} = 1.4733 + 0.0002498\,t$ $c'_{p_{avg}} = 1.1024 + 0.0002498\,t$
CO_2	$c_{p_{avg}} = 0.8654 + 0.0002443\,t$ $c_{v_{avg}} = 0.6764 + 0.0002443\,t$	$c'_{p_{avg}} = 1.6990 + 0.0004798\,t$ $c'_{v_{avg}} = 1.3281 + 0.0004798\,t$
B—Energy unit kilocalorie (kcal)		
O_2	$c_{p_{avg}} = 0.2198 + 0.00002544\,t$ $c_{v_{avg}} = 0.1577 + 0.00002544\,t$	$c'_{p_{avg}} = 0.3138 + 0.00003766\,t$ $c'_{v_{avg}} = 0.2252 + 0.00003766\,t$
N_2	$c_{p_{avg}} = 0.2446 + 0.00002115\,t$ $c_{v_{avg}} = 0.1737 + 0.00002115\,t$	$c'_{p_{avg}} = 0.3057 + 0.00002643\,t$ $c'_{v_{avg}} = 0.2171 + 0.00002643\,t$
Dry air	$c_{p_{avg}} = 0.2378 + 0.00002221\,t$ $c_{v_{avg}} = 0.1693 + 0.00002221\,t$	$c'_{p_{avg}} = 0.3073 + 0.00002869\,t$ $c'_{v_{avg}} = 0.2187 + 0.00002869\,t$
H_2O	$c_{p_{avg}} = 0.4379 + 0.0000743\,t$ $c_{v_{avg}} = 0.3276 + 0.0000743\,t$	$c'_{p_{avg}} = 0.3519 + 0.00005967\,t$ $c'_{v_{avg}} = 0.2633 + 0.00005967\,t$
CO_2	$c_{p_{avg}} = 0.2067 + 0.00005836\,t$ $c_{v_{avg}} = 0.1616 + 0.00005836\,t$	$c'_{p_{avg}} = 0.4058 + 0.0001146\,t$ $c'_{v_{avg}} = 0.3172 + 0.0001146\,t$

The accuracy of the calculation here is the worst possible, and this is evident from Figure 3.6. (The points of line 1–2 are more distant from the accurate curved line A–B–D compared with those of line 1–2.)

The value of the SH can be obtained from the KMT of gases. And its equation can be derived from Einstein's Equation (3-35) by deleting the second term of its right side. Thus:

$$Mc_v = n_1 \cdot MR / 2 \tag{3-47}$$

Since the adoption of the SH as temperature independent gives inaccurate results, auditing the value of MR in Equation (3-47) is not feasible; the values of the universal gas constant are considered as:

$$MR \cong 8.3 \text{ kJ/(kmol·K)} \cong 2 \text{ kcal/(kmol·K)} \tag{3-48}$$

Thus, for inaccurate calculations $\frac{MR}{2} \cong 4.15 \frac{\text{kJ}}{\text{kmol·K}} \cong 1 \frac{\text{kcal}}{\text{kmol·K}}$ and Equation (3-47) becomes:

$$Mc_v \cong 4.15\ n_1 \text{ kJ/(kmol·K)} \cong n_1 \text{ kcal/(kmol·K)} \tag{3-49}$$

Applying Equations (3-48) and (3-49) in Meyer's equation $Mc_{pMc} = Mc_v + MR$ (see Equation 4-18a), we obtain:

$$Mc_p \cong (8.3 + 4.15\ n_1) \frac{\text{kJ}}{\text{kmol·K}} \cong (2 + n_1) \frac{\text{kcal}}{\text{kmol·K}} \tag{3-49a}$$

Equations (3-35c), (3-49), and (3-49a) were used for creating Table 3.5.

The deviations of the molar SHs, listed in Table 3.5, according to the KMT from the experimental values are acceptable for the monatomic and diatomic gases but they are significant and unacceptable for polyatomic gases. These large deviations are explained by the presence of considerable vibrations inside the molecules/particles. If we unelaborately take into account the values of intramolecular vibrations, we can use the approximate experimental values of the constant molar SHs listed in Table 3.6: (1) for some particular calculations and discussions, and (2) for the case when we find ourselves obliged to consider/assume the constancy of the SHs because of/due to the lack of data.

Table 3.5. Molar specific heats at constant volume and at constant pressure for gases (according to the ***KMT***)

Gas	kJ / (kmole · K)		kcal / (kmole · K)	
	Mc_p	Mc_v	Mc_p	Mc_v
Monatomic	20.9	12.6	5	3
Diatomic and linear polyatomic	29.3	20.9	7	5
Nonlinear polyatomic	33.5	25.1	8	6

Table 3.6. Molar specific heats at constant volume and at constant pressure for gases. (The ***experimental*** constant specific)

Gas	kcal / (kmole · K)		kJ / (kmole · K)	
	Mc_p	Mc_v	Mc_p	Mc_v
Monatomic	5	3	20.9	12.6
Diatomic and linear polyatomic	7	5	29.3	20.9
Nonlinear polyatomic	9	7	37.7	29.3

3.1.7.6.3 About the transferred heat and boundary work signs

In this book we will adopt the algebraic signs of the calculated by Equations (3-7) and (2-38) through (2-40) transferred heat to the system and produced boundary work by the system as the real signs of these dimensions. Therefore:

> **OR3-6:** *The transferred heat to the system (WF) is positive, while the transferred heat from the system is negative.*
> **OR3-7:** *The produced boundary work by the system (WF) is positive during system's expansion and negative during its compression.*

3.1.8 THE SPECIFIC HEAT OF A MIXTURE

To calculate the SH of a gas mixture, we usually use, from the following three equations, the one that appropriates the given data:

$$\left.\begin{aligned} c_{mix} &= \sum_{k=1}^{n}(c_k \times g_k), \quad (Mc)_{mix} = \sum_{k=1}^{n}\left(r_k \times (Mc)_k\right) \\ \text{and} &\qquad\qquad c'_{mix} = \sum_{k=1}^{n}(c'_k \times r_k) \end{aligned}\right\} \qquad (3\text{-}50)$$

If calculated SH is not the required one, we apply the suitable of the following equations to determine the required answer:

$$(Mc)_{mix} = M_{mix} \times c_{mix} = 22.41 \times c'_{mix} \qquad (3\text{-}51)$$

For example, if the given is the mass composition $g_1, g_2, g_3, \cdots g_k, \cdots g_n$ and the required is mixture's molar SH "$(Mc)_{mix}$," we first apply the first equation of (3-50) and determine (c_{mix}) then apply the equation $(Mc)_{mix} = M_{mix} \times c_{mix}$ and determine the required "$(Mc)_{mix}$".

3.2 ABOUT HEAT TRANSFER CALCULATIONS FOR CHEMICAL STATE CHANGE PROCESSES OF A GAS

What concerns us here is to calculate the amount of heat produced by the combustion of the fuel or the so-called calculation of the thermal effect of a chemical reaction.

Usually, fuel consists of a combination of combustible and incombustible substances. The interaction of each of combustible elements/compounds with Oxygen releases a considerable quantity of heat whereas the incombustible substances that are called impurities either do not interact with Oxygen or interact with Oxygen without releasing considerable quantities of heat.

The most important combustible elements are hydrogen H_2, carbon C, and sulfur S.

The equations of the complete combustion of hydrogen in air under constant pressure are:

$$H_2 + \frac{1}{2}O_2 = H_2O_{\text{Vapor}} + 58000 \text{ kcal/kmole}$$

$$H_2 + \frac{1}{2}O_2 = H_2O_{\text{Liquid}} + 68300 \text{ kcal/kmole}$$

Thus, the heat released by the complete combustion of 1 kmol of hydrogen is a function of the polarization state (solid, liquid, or gas) of the resulted water, if the formed/resulted water was in the liquid phase, then the released heat is 68,300 kcal, and if the resulted water was vapor, then 58,000 kcal of heat is released. The difference in the amount of the released heat is the required amount of heat to convert 1 kmol of water into water vapor.

The heat released by the complete combustion of a unit quantity of fuel at constant pressure is called ***the heating (caloric) value of fuel***. Since the amount of fuel can be measured by kmol, unit mass, or the quantity that occupies, in certain conditions, a unit volume, we distinguish between a ***molar heating value ($M\theta$),*** a ***mass heating value (θ),*** and a ***volumetric heating value (θ').*** The relationships that bind these values to each other are:

$$M\theta = M \times \theta = Mv_n \times \theta'$$

where Mv_n is the volume of one kilomole in the normal conditions.

Table 3.7: Some higher and lower heating values

Gas	Product	Heating value θ
C	CO_2	33.65 MJ/kg
H_2	H_2O Liquid	141.5 MJ/kg
H_2	H_2O Steam	121.42 MJ/kg
S	SO_2	9 MJ/kg
C	CO	About 10 MJ/kg

The heating value of fuel can be related to a unit amount of (1) pure fuel (superscript p), (2) dry fuel (superscript d) "fuel containing impurities and not containing moisture (superscript ") or (3) operative fuel (fuel as it enters to the burner (superscript o); it contains impurities and moisture).

As can be noticed, hydrogen has two heating values: higher (subscript H) “when the product of combustion is liquid water” and lower (subscript L) “when the result of combustion is water vapor.” As the product of combustion in thermal engineering problems is water vapor, not liquid water, we usually deal with the lower heating value of the fuel.

When the combustion of the carbon is complete the following equation realizes:

$$C + O_2 = CO_2 + 96000 \text{ kcal/kmole}$$

From the last equation we conclude that carbon has only one heating value that is used in combustion equations.

The combustion interaction of hydrocarbons proceeds according to the following equation:

$$C_nH_m + \left(n + \frac{m}{4}\right)O_2 = nCO_2 + \frac{m}{2}H_2O + \theta$$

And the combustion products contain H_2O; therefore, this kind of fuel has two (higher and lower) heating values, and in heat calculations the lower heating value θ_L is used.

It should be mentioned here that the heating value of the fuel is a function of combustion conditions (does the combustion take place at constant volume or at constant pressure and at which temperature it happens). But the calculations that we will execute does not require high accuracy, and we will use the heating value of fuel that is given in the text of the problem to be solved; if it was not given, we shall use Table 3.7.

The amount of heat released from the completely burned fuel is calculated from the relationship:

$$Q = m \times \theta_L^o = V_n \times \theta'^o_L = K \times M\theta_L^o$$

where *m* is the operative mass of the fuel, θ_L^o the lower heating value (operative mass) per unit mass, θ'^o_L the lower heating value (operative mass) per unit volume, $M\theta_L^o$ the lower heating value (operative mass) per kmole, and *K* the number of fuel kmoles.

The quantity of air required for combustion is calculated as follows:

The minimum required quantity of oxygen for the complete combustion of fuel $(m_{O_2})_{min}$ (the theoretical oxygen) is determined from the combustion equation, then the minimum required quantity of air for the complete combustion (the stoichiometric or theoretical air) is calculated as $(m_a)_{min} = \frac{(m_{O_2})_{min}}{0.232}$, and after that the required quantity of air for combustion is calculated as:

$m_a = \alpha \cdot (m_a)_{min}$, where α is the coefficient of the excess air.

3.3 THE ZEROTH LAW OF THERMODYNAMICS

The zeroth law of thermodynamics states that *if two thermodynamic systems are each in thermal equilibrium with a third, then all three are in thermal equilibrium with each other.*[14]

For long time before 1931, the content of this law was obvious, a well-known fact, and a familiar experience. This fact is not derived from other laws; therefore, it is a thermodynamic law and it was not adopted in science as a law in 1931 (i.e., after announcing the first and second laws of thermodynamics), when Ralph H. Fowler was the first to formulate and label it.[15] From the scientific point of view Fowler's law must precede the first law of thermodynamics and that is why Fowler's law was called the zeroth law of thermodynamics. The zeroth law of thermodynamics is the base of the method of measuring temperature.

3.4 THE CONSERVATION OF ENERGY PRINCIPLE. THE FIRST LAW OF THERMODYNAMICS

3.4.1 INTRODUCTION

The first steam machine was created at the end of the eighteenth century as a result of Newcomen, Simithson, Polzunove, and James Watt, and thus people could transform steam energy into mechanical. The invention of this machine, as well as observing the different phenomena such as the release of amounts of heat during drilling the barrels of the canons (Rumford), attracted the attention of researchers to thermal phenomena and to the relationship between heat and mechanical energy.

A number of experimental works was conducted that led to determining the equivalence in the mutual transformations of energy. As a result *the conservation of energy principle was widely approved* and it is now a fundamental law in Natural Sciences and has many formulations that share the statement: "*Energy can be neither created nor destroyed.*" One of the very long statements is: *Energy can be neither created nor destroyed. However, energy can change forms, and energy can flow from one place to another. The total energy of an isolated system does not change.*[16]

3.4.2 THE FIRST LAW FORMS

The most common case form of the first law of thermodynamics that deals with all kinds of energy is the formulation stated in the previous paragraph for the conservation of energy principle.

The first law of thermodynamics for thermal and mechanical phenomena can be formulated as follows: *In all cases, when the quantity of heat disappears, a certain amount of mechanical energy (in the form of work performed) appears, and on the contrary, when a work is executed (at the expense of spending an equal amount of mechanical energy) a certain amount of thermal energy appears.*[17] Therefore:

$$Q = W \tag{3-52}$$

Here, *Q* is the heat energy that disappeared and *W* is the resultant work at the expense of mechanical energy that appeared (Q and W here have the same energy units, see §1.1.9).

Recognizing (The verification of) the first law of thermodynamics in natural sciences led the absolute majority of inventors to stop trying to construct/create engines that can produce work (mechanical energy) from *nothing* (i.e., engines that can produce energy without consuming any kind of energy "such as energy resulted from fuel combustion"). This imaginary engine was given the name *Perpetuum-Mobile*[18] of the first kind.

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[18] Today the Italian *Perpetuum Mobile* means perpetual-motion machine. And the following formula for the first law of thermodynamics was introduced as:

It is impossible to construct a perpetual-motion machine of the first kind.

Some authors prefer using the original Italian name of the imaginary engine and the formula of the first law becomes as follows: *It is impossible to construct a* ***Perpetuum Mobile*** *of the first kind.*

3.5 THE ANALYTICAL EXPRESSION FOR THE FIRST LAW OF THERMODYNAMICS

The equation of the first law of thermodynamics, as it is in Equation (3-52), does not completely describe the energy balance during gas state change processes. Generally, these processes proceed with exchanging heat between the system (gas) and its surroundings; therefore, we should know on which independent phenomena of the process the heat was spent, and what is the heat amount spent on each phenomenon.

Let us consider the gas state change process through this point of view.

Consider 1 kg of gas is located inside the piston–cylinder arrangement illustrated in Figure 3.7. Let us give to this gas the infinitesimal quantity of heat (δq) and find what changes it does.

In the common case the gas state will infinitesimally change so that almost all independent properties of the system will infinitesimally change (only one independent property of the

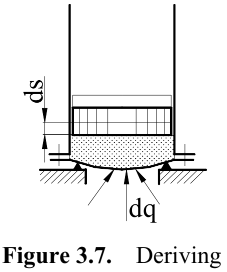

Figure 3.7. Deriving Equation (3-53)

system may not change). Therefore, we should expect infinitesimal changes in system's pressure (dp), temperature (dT), specific volume (dv), and internal energy (du). The specific volume change (dv) is caused by the piston displacement (ds) that caused the volume change (dV). The specific volume change (dv) causes the specific boundary work change (δw). Thus, because of adding the heat energy (δq) to the system, the system increased its internal energy by (du) and produced the boundary work (δw).

<u>Nt3-12:</u> At the time when the first and second laws of thermodynamics were recognized many scientists used the terms "raising a load" to express the work and even the mechanical energy in its broad meaning and recognized the internal energy as a thermal/heat one; therefore, the traditional derivation of the analytical expression of the first law of thermodynamics was as follows.

Since the internal energy of body is a thermal one, then the heat (Q) that lost/disappeared during our infinitesimal process equals the difference ($\delta q - du$) between the added heat (δq) and the internal energy increase/change (du), while the appeared work (W) equals δw. Thus, Equation (3-52) transforms into:

$$\delta q - du = \delta w \qquad \text{or} \qquad \delta q = du + \delta w \tag{3-53}$$

Because of the small size of this book we shall skip the more rigorous derivation for Equation (3-53).

<u>OD3-5:</u> No-heat energies are those kinds of energy, which can fully transform to other ones.

<u>Nt3-13:</u> w in Equation (3-53) carries the meaning of per unit mass mechanical energy, because we stipulated that and we can use this equation for all kinds of phenomena supposing that "W" denotes to all kinds of no-heat energies ("see OD3-5").

Integrating the differential Equation (3-53) we obtain the per unit mass first law:

$$q = \Delta u + w \tag{3-54}$$

where q is the specific transferred to the WF heat during the process, or as we call it most often the external heat, $\Delta u = u_2 - u_1$ the internal energy change during the process, and w the work produced by unit mass of the gas during the process.

Multiplying both sides by *m* we obtain:

$$Q = \Delta U + W \tag{3-54a}$$

Equations (3-53) and (3-54) are the most common case expressions for the first law, because the analytical expression of *w* is not substituted in them yet.

For those classified as simple closed stationary systems (see OD2-20 and Nt2-3), which are the main topic of this book, the only kind of the mechanical energy in its broad meaning (see Nt3-12) that exists is the boundary work. Its defining equation (see Equation 2-37) is:

$$\delta w = pdv \qquad \text{or} \qquad w_{1-2} = \int_1^2 p \cdot dv \tag{3-55}$$

Substituting this equation in Equation (3-53) we obtain the analytical expression for the first law of thermodynamics (for the simple closed stationary systems) in the following forms:

$$\delta q = du + pdv \qquad \text{and} \qquad q = \Delta u + \int_{v_1}^{v_2} pdv \tag{3-56}$$

It must be taken into consideration here that du is exact differential because u is a state function, while $dw = pdv$ and accordingly dq are inexact ones (see the forwarding to Equation 2-13, OR2-7 and the § that follows it).

We can obtain the equation of the first law of thermodynamics for the simple closed stationary systems in terms of enthalpy (*h*) from Equation (3-16).

Substituting u from Equation (3.17) in Equation (3-56) we obtain the first law of thermodynamics for those classified as simple closed stationary systems in terms of the enthalpy (*h*):

$$\left.\begin{array}{lll} \delta q = d(h - p \cdot v) + p \cdot dv & \text{or} & \delta q = dh - v \cdot dp \\ \text{and} & & q = \Delta h + \int_{p_1}^{p_2} vdp \end{array}\right\} \tag{3-57}$$

3.6 SUMMARY

(I) The following statement that is written for the SH (*c*):

"For real gases, the SH (c_{rg}) of a particular gas and a particular process depends on its temperature and pressure $c_{rg} = f(t, p)$

and with the presence of the equation of state we can write $c_{rg} = f(t,v)$. And for ideal gases, the SH of a particular gas and a particular process depends on temperature only $c_{ig} = c_0 = f(t)$.”

It can be rewritten for each of the internal energy (u) and the enthalpy $(h = u + p \cdot v)$ but not for the entropy ($\mathrm{ds} = \delta\mathrm{q}/\mathrm{T} = \mathrm{c} \cdot \mathrm{dT}/\mathrm{T}$), which is a function of two other independent intensive properties even for ideal gases.

(II) The following thermodynamic functions: “$\int_{T_{ref}}^{T} c_{v0} \cdot dT$ (symbol u), $\int_{T_{ref}}^{T} c_{p0} \cdot dT$ (symbol h), $\int_{T_{ref*}}^{T} c_{p0} \frac{dT}{T}$ (symbol s_p) and $\int_{T_{ref*}}^{T} c_{v0} \frac{dT}{T}$ (symbol s_v),” where T_{ref} is an arbitrary reference temperature that is constant and can equal the AZ, while T_{ref*} is an arbitrary reference temperature that is constant and cannot equal the AZ since $\int_{0K=0R}^{T} c_{p0} \frac{dT}{T} = \pm\infty$ and $\int_{0K=0R}^{T} c_{v0} \frac{dT}{T} = \pm\infty$ (see Nt3-5A)

Are able for tabulation versus T (or t) and they are tabulated for many substances. These tabulated functions allow the exact calculations of the functions ($u_2 = \int_{T_{ref}}^{T_2} c_{v0} \cdot dT$, $u_1 = \int_{T_{ref}}^{T_1} c_{v0} \cdot dT$, $h_2 = \int_{T_{ref}}^{T_2} c_{p0} \cdot dT$, $h_1 = \int_{T_{ref}}^{T_1} c_{p0} \cdot dT$, $s_{p_2} = \int_{T_{ref*}}^{T_2} c_{p0} \cdot \frac{dT}{T}$, $s_{p_1} = \int_{T_{ref*}}^{T_1} c_{p0} \cdot \frac{dT}{T}$, $s_{v_2} = \int_{T_{ref*}}^{T_2} c_{v0} \cdot \frac{dT}{T}$ and $s_{v_1} = \int_{T_{ref*}}^{T_1} c_{v0} \cdot \frac{dT}{T}$). In addition to:

- the specific internal energy change $\Delta u_{exact} = u_{2_{tab}} - u_{1_{tab}}$
- the specific enthalpy change $\Delta h_{exact} = h_{2_{tab}} - h_{1_{tab}}$
- the exact specific entropy change (see Equation 3-28):

$$\left(s_2 - s_1\right)_{exact} = s_{v_{2_{tab}}} - s_{v_{1_{tab}}} + R \ln \frac{v_2}{v_1} = s_{p_{2_{tab}}} - s_{p_{1_{tab}}} - R \ln \frac{p_2}{p_1} \qquad (3\text{-}58)$$

And for the approximate calculation, the SHs are considered constants (see Equation 3-29):

$$s_2 - s_1 \cong c_{v0} \ln\frac{T_2}{T_1} + R\ln\frac{v_2}{v_1} \cong c_{p0} \ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1}$$

$$\cong c_{v0} \ln\frac{p_2}{p_1} + c_{p0} \ln\frac{v_2}{v_1} \qquad (3\text{-}59)$$

The exact transferred heats' calculations can also be done through the tabulated thermodynamic functions. For this current level of knowledge the transferred heats in the isochoric and isobaric processes equations $q_v = u_2 - u_1$ and $q_p = h_2 - h_1$ (see Equations (4-9), (4-9a), and (4-11a) can be used.

In the absence of the tabulated thermodynamic functions and in the presence of SHs' exact equations (such as Equation 3-34), we can use the appropriate of these equations with Equation (3-7) for the exact calculations of the transferred heats. For the approximate calculations simplified equations can be used (see §s 3.1.7.6.1 and 3.1.7.6.2).

The equations of the first law of thermodynamics (see equations 3-56 and 3-57) are:

$$\left.\begin{aligned} \delta q &= du + pdv = dh - v\cdot dp \\ q &= \Delta u + \int_{v_1}^{v_2} pdv = \Delta h - \int_{p_1}^{p_2} vdp \end{aligned}\right\} \qquad (3\text{-}60)$$

and

make the exact and approximate transferred heats' calculations possible through the tabulated thermodynamic functions.

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CHAPTER 4

CALCULATIONS OF IDEAL GAS PHYSICAL STATE CHANGE PROCESSES IN CLOSED SYSTEMS (PART I)

4.1 INTRODUCTION

The following definitions are used throughout this book and some other ones:

OD4-1: The expansion process (in the broad sense) is defined by $v_2 > v_1$ and for $v_2 = v_1$ by $p_2 < p_1$. And, the compression process (in the broad sense) is defined by $v_2 < v_1$ and for $v_2 = v_1$ by $p_2 > p_1$.

4.2 THE SPECIAL CASES OF THE GAS STATE CHANGE PROCESSES AND THEIR REPRESENTATION ON PROPERTY DIAGRAMS

Starting from now, we shall deal with the special cases of the gas state change processes and their representation on property diagrams, and because some of the terms/shortened names are uncommon, we introduce them here.

OD4-2: The shortened names of paths of the special cases of the gas state change processes: (1) isochoric (constant volume) is isochore, (2) isobaric (constant pressure) is isobar, (3) isothermal (constant temperature) is isotherm, (4) adiabatic is adiabate, (5) isentropic (reversible adiabatic or isentropic) is isentrop, and (6) polytropic is polytrope.

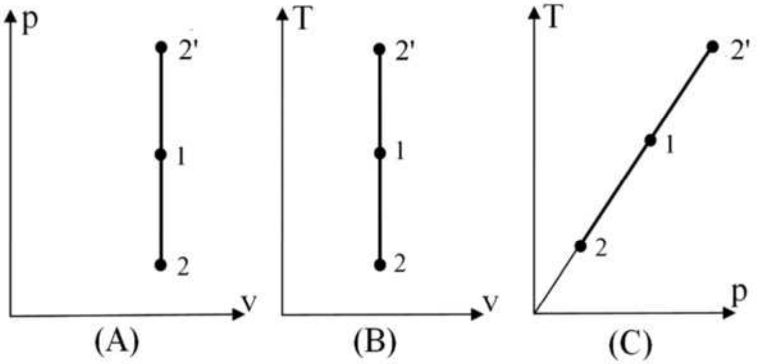


Figure 4.1. Representing the isochoric process on diagrams. Diagrams (A and B) are valid for all cases

In the common case of the gas (simple compressible system) state change, we can change two independent properties arbitrarily. In this case, the path of the process can have any shape in the plane coordinates. This path may be smooth and able for describing by a single simple equation, and may not be so. It will be possible to create the paths of the same process on other coordinate planes through the given path because we can (for a simple compressible system—see OD2-30C and HD2-4), knowing two independent intensive properties of each state, find all its remaining intensive properties through the relationships that are available in different forms (the relationships can have analytical, tabular, or graphical form).

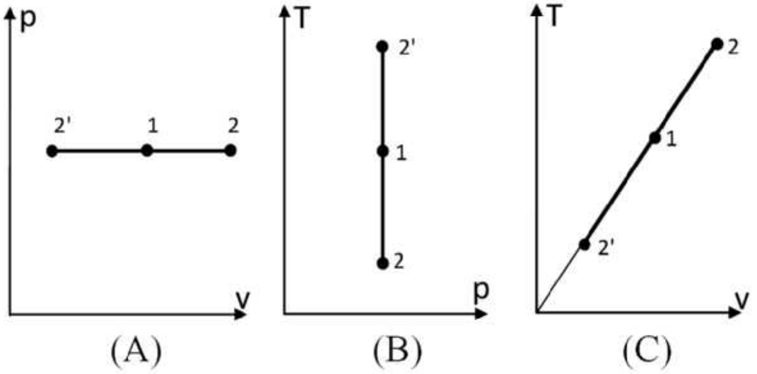


Figure 4.2. Representing the isobaric process on diagrams. Diagrams (A and B) are valid for all cases

Note that it is possible to find the relationship between the specific heat and temperature (the law of the process), but this can be difficult sometimes.

If we put some restrictions on some intensive properties or on other properties, we get special cases of the gas state change process. In Figures 4.1 through 4.4, several special case processes are illustrated on several property diagrams. In each diagram a combined/compound process (2–1–2'), governed by the same restriction, is illustrated. Points 2 and 2' represent the final states of the expansion and compression processes, respectively (see OD4-1).

Nt4-1: The constant property line/process is always perpendicular to the coordinate axis that is named by the same constant property name (see diagrams "A and B" in all four figures). All these diagrams are valid for all gases, but diagram (C) of each of Figures 4.1 through 4.4 is valid for only ideal gases:

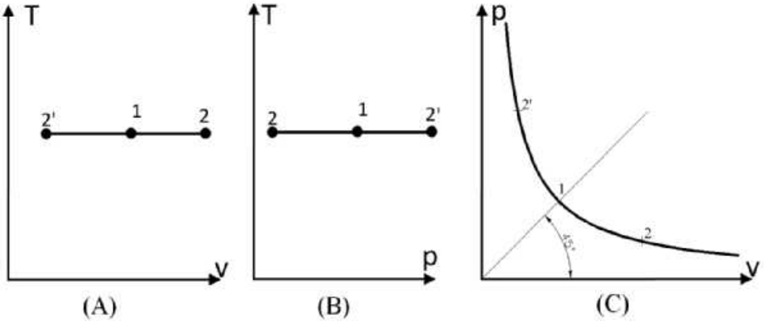


Figure 4.3. Representing the isothermal process on diagrams. Diagrams (A and B) are valid for all cases

1) If we fix the specific volume (i.e., we fix the piston) during the process, we get the isochoric (constant volume) gas state change process. Its main definition (law) is v = constant (see Figure "4.1"). In Figure 4.1C, the straight line 2'–1–2 represents the **ideal gas** isochoric process. It passes through the principle of coordinates, and its slope (see Equation 2-14) equals v/R.
2) If we fix the pressure (i.e., we apply on the piston a constant force "F_1" that tries to move the unfixed piston toward the head of the cylinder) during the process, we get the isobaric (constant pressure) gas state change process. Its main definition (law) is p = constant

(see Figure "4.2"). In Figure 4.2C, the straight line 2–1–2' represents the **ideal gas** isobaric process. It passes through the principle of coordinates, and its slope (see Equation 2-14) equals p/R.

3) If we fix the temperature during the process, we get the isothermal (constant temperature) gas state change process. Its main definition (law) is $T =$ constant (see Figure "4.3"). In Figure 4.3C, line 2'–1–2, which is a hyperbolic isosceles, represents the **ideal gas** isothermal process in *p_v* diagram. *Its equation in this diagram* (see the ideal gas equation of state "2-14") is

$$p \cdot v = RT = \text{constant}$$

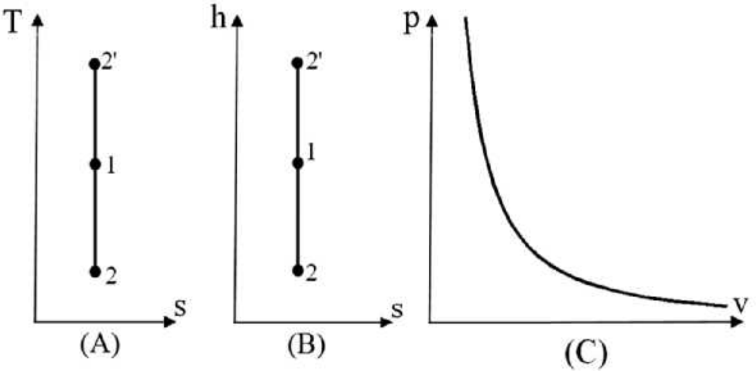


Figure 4.4. Representing the isentropic process on diagrams. Diagrams (A and B) are valid for all cases

4) If we prevent heat transfer between the working fluid (WF) and the surroundings (i.e., if we thermally isolate the cylinder and piston very well), we get the WF adiabatic state change process (at any moment of time no heat is exchanged with the "WF"). Thus, the definition of the adiabatic process is

$$\delta q = 0 \tag{4-1}$$

From the definitions of the entropy (see Equation 3-21)^{A} and the adiabatic process (see Equation 4-1), we obtain $\delta q = T\mathrm{d}s = 0$ and because $T \neq 0$, then,

$$\mathrm{d}s = 0 \text{ or } s = \text{constant} \qquad \text{or} \qquad s_2 - s_1 = 0 \tag{4-1a}$$

A. It was proved that the ideal gas specific entropy is an intensive property (see §3.1.2.3.1), and we shall prove later that the specific entropy is an intensive property for all gases.

Accordingly, in the equilibrium adiabatic process,^{B} as in other special case processes, there is a fixed (restricted) intensive property, which is the entropy. Based on Equations (4-1) and (4-1a), the equilibrium adiabatic process is called the **isentropic process**.

B. The process must be equilibrium; otherwise, the WF intensive properties cannot be determined. As we shall see later, the WF entropy increases during nonequilibrium adiabatic process.

Mathematically, $q = 0$ does not mean that $\delta q = 0$, because q is the algebraic sum of δq throughout the entire process (e.g., for a process "1–3–2"). Where, q = 0 we can write $q_{1-3-2} = \int_1^3 \delta q + \int_3^2 \delta q = q_{1-3} + q_{3-2} = 0$ from which $q_{1-3} = -q_{3-2}$. Therefore, $q = 0$ *cannot be another definition for the adiabatic process.*

In Figure 4.4C, line 2'–1–2 represents schematically the **ideal gas** isentropic process in *p_v* diagram.

4.3 THE SPECIAL CASES OF THE FIRST LAW OF THERMODYNAMICS FOR ANY GAS

Taking in consideration the title of this chapter, we shall discuss below the first law of thermodynamics for the simple closed stationary systems (see OD2-30 and OD2-20) exclusively. This means that Equations (3-56) and (3-57) are the ones to be discussed below.

4.3.1 THE FIRST LAW OF THERMODYNAMICS FOR THE ISOCHORIC PROCESS

To obtain the first law of thermodynamics for the isochoric process, we index all terms of the first of Equations (3-56) by *v* and substitute d*v* by zero:

$$\delta q_v = du_v + (pdv)_v = du_v \tag{4-2}$$

and after integrating from state (1) to state (2)

$$q_v = \Delta u_v = u_{2v} - u_{1v}$$

Applying the third of Equations (3-6) that must be written for the isochoric process and for any gas ($\delta q_v = c_v \mathrm{d}t$), we obtain

$$\delta q_v = du_v = c_v \cdot dt \qquad \text{or} \qquad q_v = \Delta u_v = \int_{t_1}^{t_2} c_v \, dt \qquad (4\text{-}3)$$

Equations (4-3) are valid for all gases (during their derivation we did not use any equation that belongs exclusively to the ideal gas), and *because* $v = \text{constant}$*, the fifth component of* Δu_v *(the change of the potential energy of the cohesive forces) equals zero, which means that* q_v *(or* Δu_v*) is a function of only* T_1 *and* T_2.

Equations (4-3) can be expressed by any of the following two rules:

OR4-1: *The internal energy change of any gas in the isochoric process equals the transferred heat to this gas during this process.*
OR4-2: *In the isochoric process, the transferred heat to any gas is spent on changing its internal energy.*

4.3.2 THE FIRST LAW OF THERMODYNAMICS FOR THE ISOBARIC PROCESS

To obtain the first law of thermodynamics for the isobaric process, we index all terms of the first of Equations (3-56) by *p* and substitute d*p* by zero:

$$\delta q_p = \mathrm{d}u_p + (p\mathrm{d}v)_p$$

and after integrating from state (1) to state (2)

$$q_p = \Delta u_p + [p(v_2 - v_1)]_p = [u_{2p} + (p \cdot v_2)_p] - [u_{1p} + (p \cdot v_1)_p]$$

Applying the third of Equations (3-6) that must be written for the isobaric process and for any gas ($\delta q_p = c_p dt$) and the enthalpy definition (3-16), we obtain

$$\delta q_p = dh_p = c_p\, dt \quad \textbf{or} \quad q_p = (h_2 - h_1)_p = \Delta h_p = \int_{t_1}^{t_2} c_p\, dt \qquad (4\text{-}4)$$

Equations (4-4) are valid for all gases (during their derivation we did not use any equation that belongs exclusively to the ideal gas).

OD4-3: *In isobaric processes and for any gas, the transferred heat to the gas (WF) can be defined as the enthalpy change between the final and initial states of the gas.*

4.3.3 THE FIRST LAW OF THERMODYNAMICS FOR THE ADIABATIC PROCESS

The condition of the adiabatic process is $\delta q = c dT = 0$ and because $dT = dt \neq 0$ then $c = 0$. Therefore, the first law of thermodynamics for the adiabatic process takes the form

$$0 = du + dw = du + p \cdot dv \qquad \text{or}$$

$$dw = -du \qquad \text{or} \qquad w = u_1 - u_2 \qquad (4\text{-}5)$$

OD4-4: *In the expansion adiabatic process of a gas, the work is produced on the expense of its internal energy* $(u_1 - u_2)$ *and vice versa,* that is, in the compression adiabatic process of a gas, the compression work applied by the surroundings on the gas is fully directed to increase the gas internal energy and is equal to its change $|w| = u_2 - u_1$.

Equations (4-5) are valid for real and ideal processes and for all real and ideal gases (during its derivation we did not use any equation that belongs exclusively to the ideal gas).

4.3.4 THE FIRST LAW OF THERMODYNAMICS FOR THE ISOTHERMAL PROCESS

None of the terms of the first law of thermodynamics is eliminated during the isothermal process; therefore, the first law for the isothermal process keeps its original form:

$$\delta q_t = du_t + dw_t = du_t + (pdv)_t \qquad \text{and} \qquad q_t = (u_2 - u_1)_t + w_t$$

These Equations are valid for all gases (during its derivation we did not use any equation that belongs exclusively to the ideal gas).

4.3.5 THE FIRST LAW OF THERMODYNAMICS FOR THE CLOSED PROCESS

As the WF in the closed process (see §2.9.2) returns at the end of the process to its initial state, then $u_2 = u_1$ or $\Delta u = 0$. Therefore, the first law of thermodynamics for the closed process (see Equation 3-54) is

$$\left.\begin{array}{llll} q = w & \text{or} \quad Q = W & \text{where,} & Q = \oint \delta Q; \\ W = \oint \delta W; & q = \oint \delta q & \text{and} & w = \oint \delta w \end{array}\right\} \qquad (4\text{-}6)$$

We shall refer to the reversible/internally-reversible cycle as the ideal cycle, the heat transferred to its WF as the useful heat (symbol $Q_\circ$ *or* $q_\circ$), and the work produced by its WF as the useful work (symbol $W_\circ$ *or* $w_\circ$). The other name of the useful heat is the net heat (symbol Q_{net} or q_{net}), and of the useful work is the network (symbol W_{net} or w_{net}). On the other hand, the path of the closed process is usually a multiequation curve (see HD1-3).

Taking in consideration the above stated we rewrite Equations (4-6) as follows:

$$\left.\begin{array}{ll} & W_{\circ alg} = Q_{\circ alg} = \oint \delta Q = \int\limits_{cycle} \delta Q = \sum\limits_{cycle} Q_{PPs} = \sum_{i=1}^{n} Q_i \\ \text{and} & w_{\circ alg} = q_{\circ alg} = \oint \delta q = \int\limits_{cycle} \delta q = \sum\limits_{cycle} q_{PPs} = \sum_{i=1}^{n} q_i \end{array}\right\} \qquad (4\text{-}7)$$

where PPs stands for partial processes, $Q_i = \int\limits_i \delta Q$, $q_i = \int\limits_i \delta q$, i (integer number) stands for the sequential number of the monoequation partial process of the cycle and n is the total number of these partial processes.

Equations (4-6) and (4-7) are valid for all gases (during its derivation we did not use any equation that belongs exclusively to the ideal gas).

From Equation (4-17) and (§3.1.4), we obtain the following:

OR4-5: In the cyclic process for any gas, (1) the useful work equals the useful heat and, accordingly, (2) in *p_v* and *T_s* diagrams, the areas inside the same legally created cyclic path and in the same energy units are equal.

Thus, q_o and w_o can be easily calculated as the area inside cycle's curve.

It is obvious that this cycle must run (must be executed) in a machine that excludes all irreversibilities. We will refer to such machine (engine) as ***ideal engine*** if it is also free from mechanical losses.

4.4 CALCULATING THE IDEAL GAS INTERNAL ENERGY AND ENTHALPY CHANGES

4.4.1 CALCULATING THE IDEAL GAS INTERNAL ENERGY CHANGE

As the internal energy of an ideal gas—in any process—is a function of only temperature (see Equations 3-14), Equations (4-3) take the following forms

$$\delta q_v = \mathrm{du}_v = c_{v0}\mathrm{dt} \quad \textbf{or} \quad q_v = \Delta u_v = \int_{t_1}^{t_2} c_{v0}\,\mathrm{dt} \tag{4-8}$$

Applying Equation (3-38''') on the second of Equations (4-8) we obtain:

$$\begin{aligned} q_v &= \Delta u_v = (u_2 - u_1)_v = q_v\Big|_{T_1}^{T_2} = \Delta u_v\Big|_{T_1}^{T_2} \\ &= \int_{T_1}^{T_2} c_{v0}\cdot dT = c_{v0_{avg}}\cdot(T_2 - T_1) \end{aligned} \tag{4-8a}$$

Calculating $q = \int_{t_1}^{t_2} C\,dt$ is discussed comprehensively in Chapter 3 (see §s 3.1.5 through 3.1.8); therefore, we shall extract from Equation (4-8) the following rule:

OR4-3: *Within any specified temperature interval and in any* ideal gas state *change process the internal energy change equals the transferred heat to it in the* $v = \text{constant}$ *process executed within the same specified temperature interval.* This means that it was unnecessary to indicate/denote the internal energy change in Equations (4-8) and (4-8a) by index (*v*). Thus,

$$\delta q_v = du = c_{v0} dt \qquad \text{or} \qquad q_v = \Delta u = \int_{t_1}^{t_2} c_{v0}\, dt \tag{4-9}$$

$$\begin{aligned} q_v &= \Delta u = u_2 - u_1 = q_v \Big|_{T_1}^{T_2} = \Delta u \Big|_{T_1}^{T_2} = \int_{T_1}^{T_2} c_{v0} \cdot dT \\ &= c_{v0_{avg}} \cdot (T_2 - T_1) \end{aligned} \tag{4-9a}$$

And for c_v = constant:

$$\begin{aligned} \text{If } c_{v0} = \text{constant} \Rightarrow q_v &= \Delta u = u_2 - u_1 = \int_{T_1}^{T_2} c_{v0} \cdot dT \\ &= c_{v0} \int_{T_1}^{T_2} dT = c_{v0} \cdot (T_2 - T_1) \end{aligned} \tag{4-9b}$$

Taking in consideration OR4-3 we obtain from Equations (4-8a), (3-13), and (3-46a):

$$\begin{aligned} \Delta U &= U_2 - U_1 = m \cdot (u_2 - u_1) = m \cdot \Delta u = K \cdot \Delta Mu \\ &= m \cdot c_{v0_{avg}} \cdot (T_2 - T_1) \approx m \cdot c_{v0} \cdot (T_2 - T_1) \end{aligned} \tag{4-10}$$

The specific heat in the right side of this Equation is assumed constant.

4.4.2 CALCULATING THE IDEAL GAS ENTHALPY CHANGE

As the enthalpy of an ideal gas—in any process—is a function of only temperature (see Equations 3-18), we apply them in Equations (4-4) and obtain

$$\left.\begin{aligned} &\delta q_p = dh_p = c_{p0}\, dT \qquad\qquad \text{or} \\ &q_p = (h_2 - h_1)_p = \Delta h_p = \int_{t_1}^{t_2} c_p\, dT = c_{p0_{avg}} \cdot (T_2 - T_1) \end{aligned}\right\} \tag{4-11}$$

Calculating $q = \int_{t_1}^{t_2} c\,\mathrm{dt}$ is discussed comprehensively in Chapter 3 (see §s 3.1.5 through 3.1.8); therefore, we shall extract from Equation (4-11) the following rule:

OR4-4: *Within any specified temperature interval and in any* ideal gas state *change process the enthalpy change equals the transferred heat to it in the* $p = \text{constant}$ *process executed within the same specified temperature interval.* This means that it was unnecessary to indicate/denote the enthalpy change in Equations (4-11) by index (*p*). Thus,

$$\left.\begin{aligned} &\delta q_p = dh = c_{p0}\,dT && \text{or} \\ &q_p = h_2 - h_1 = \Delta h = \int_{t_1}^{t_2} c_{p0}\,dT = c_{p0_{avg}} \cdot (T_2 - T_1) && \end{aligned}\right\} \tag{4-11a}$$

And for c_{v0} = constant, which leads to c_{po} = constant:

$$\begin{aligned} \text{If } c_{v0} = \text{constant} \Rightarrow q_p = \Delta h = h_2 - h_1 &= \int_{T_1}^{T_2} c_{p0} \cdot dT \\ &= c_{p0} \int_{T_1}^{T_2} dT = c_{po} \cdot (T_2 - T_1) \end{aligned} \tag{4-11b}$$

Taking in consideration OR4-4 we obtain from Equations (4-11), (3-20), and (3-46a):

$$\begin{aligned} \Delta H = H_2 - H_1 &= m \cdot (h_2 - h_1) = m \cdot \Delta h = K \cdot \Delta Mh \\ &= c_{p0_{avg}} \cdot (T_2 - T_1) \approx m \cdot c_{p0} \cdot (T_2 - T_1) \end{aligned}$$

The specific heat in the right side of this Equation is assumed constant.

Nt4-2: We should remind here that in thermodynamic calculations we are much interested in the changes of the internal energy and enthalpy, but not in their magnitudes. Therefore, we are not interested in the principle of the coordinates of the internal energy and enthalpy. However, we must always try to use the same property table in solving a thermodynamic problem or at least in finding a property change. Otherwise, one should exercise caution and do the necessary to avoid the error that would result from taking the initial and final values of the same property from two different tables. Note that the error could happen because of: (1) the lack of units' homogeneity between the two tables, (2) the differences in the reference temperature and pressure at which the value of the internal energy/enthalpy is chosen to be zero, or (3) both former reasons.

4.5 THE FIRST LAW OF THERMODYNAMICS FOR IDEAL GASES

Taking in consideration (OR4-3 and OR4-4) we obtain from Equations (4-9) and (4-11a) written for ideal gas:

$$\left.\begin{array}{ll} & du = c_{v0}dT \Rightarrow u_2 - u_1 = \int_1^2 c_{v0}dT \\ \text{and} & dh = c_{p0}\,dT \Rightarrow h_2 - h_1 = \int_1^2 c_{p0}dT \end{array}\right\} \quad (4\text{-}12)$$

From Equations (3-6), (3-56), and (4-8), we obtain the differential equation of the first law of thermodynamics for ideal gases:

$$\left.\begin{array}{lr} & \delta q = c_{v0}\cdot dT + p\cdot dv = c_0\cdot dT \\ \text{or} & (c_{v0} - c_0)\cdot dT + p\cdot dv = 0 \end{array}\right\} \quad (4\text{-}13)$$

where c_0 is a function of temperature, process path, and gas kind (see the second of Equations 3-5). Thus, the equation of the first law of thermodynamics for ideal gases is

$$q = \int_{t_1}^{t_2} c_{v0}\cdot dT + \int_{v_1}^{v_2} p\cdot dv = \int_{t_1}^{t_2} c_0\cdot dT$$

Also, from Equations (3-6), (3-57), and (4-11), we obtain the other form of the differential equation of the first law of thermodynamics for ideal gases:

$$\delta q = c_{p0}\cdot dT - v\cdot dp = c_0\cdot dT \quad (4\text{-}14)$$

Thus, the other form of the differential equation of the first law of thermodynamics for ideal gases:

$$q = \int_{t_1}^{t_2} c_{p0} \cdot dT - \int_1^2 v \cdot dp = \int_{t_1}^{t_2} c_0 \cdot dT$$

Let us try to integrate Equation (4-13) directly.

Substituting $\left(dT = \frac{p \cdot dv + v \cdot dp}{R} \right)$ from Equation (2-16) in Equation (4-13) we obtain $v \cdot dp \cdot \frac{c_{v0} - c_0}{R} + p \cdot dv \frac{c_{v0} - c_0 + R}{R} = 0$ that performs into $\frac{c_{v0} - c_0 + R}{c_{v0} - c_0} \cdot \frac{dv}{v} + \frac{dp}{p} = 0$ or:

$$n \cdot \frac{dv}{v} + \frac{dp}{p} = 0, \qquad \text{where} \qquad n = \frac{c_{v0} - c_0 + R}{c_{v0} - c_0} \tag{4-15}$$

Unless *n* = constant, the variables in this equation will not be separated, and the equation cannot be integrated. For *n* = constant the undefined integral of Equation (4-15) is $(n \cdot \ln v + \ln p = \text{const1} = \ln \text{const})$ or:

$$p \cdot v^n = \text{const} \qquad \text{where} \qquad n = \frac{c_{v0} - c_0 + R}{c_{v0} - c_0} = \text{const} \tag{4-16}$$

As we see Equation (4-16) is not for any of the discussed above special case state change processes of ideal gases, it is conditioned by $n = [(c_v - c + R)/(c_v - c + R)] = \text{constant}.$ The process governed by Equation (4-16) is called "the *polytropic process*." It is comprehensively discussed in Chapter 6.

4.5.1 THE FIRST LAW OF THERMODYNAMICS FOR IDEAL GAS ISOTHERMAL PROCESS

For ideal gas isothermal process (d*T* = 0), we obtain from Equations (2-37), (3-24), (4-12), (4-13), and (4-14):

$$\Delta u_T = \Delta h_T = 0 \qquad \text{or} \qquad \Delta u_t = \Delta h_t = 0$$

$$\delta q_T = p \cdot dv = -v \cdot dp = T \cdot ds = \delta w_T = c_T \cdot dT \qquad \text{or}$$

$$\delta q_T = \delta w_T = p \cdot dv = -v \cdot dp = T \cdot ds = \cancel{c_T \times dT} \tag{4-17}$$

$$q_T = w_T = \int_1^2 p \cdot dv = -\int_1^2 v \cdot dp = \int_1^2 T \cdot ds = \cancel{\int_1^2 c_T \times dT} \qquad (4\text{-}17a)$$

The right sides (product $c_T \cdot dT$ and its integral $\int_1^2 c_T \cdot dT$) of Equations (4-17 and 4-17a) were scarified/deleted because $c_T = \pm\infty$, dT = dt = 0 and accordingly $c_T \cdot dT = \pm\infty \times 0$ and its integral $\int_1^2 c_T \cdot dT = \int_1^2 \pm\infty \times 0$ are unknown, which means that they do not define any of δq_T or q_T.

From Equation (4-17) and (§3.1.4), we obtain the following:

> **OR4-6:** In the ideal gas isothermal process, (1) the WF produced work (w_T) equals the heat transferred to the WF (q_T) and, accordingly, (2) in the ideal gas isothermal expansion process w_T and q_T are positive and in the compression process w_T and q_T are negative, and (3) in *p_v* and *T_s* diagrams, the areas under the same legally created isotherm (till the not displaced horizontal axes) and in the same energy units are equal.

4.6 THE MEYER EQUATION

From Equations (3-17), (4-9), and (4-11a), we obtain $c_{p0}dt = c_{v0}dt + RdT$ and because $dt = dT$ (see the second and fourth of Equations "1-30"), we obtain after dividing by d*t* = d*T*:

$$c_{p0} = c_{v0} + R \qquad \text{or} \qquad c_{p0} - c_{v0} = R \qquad (4\text{-}18)$$

This means that c_{p0} and c_{v0} are the same type functions consisting of the same variable terms and differing only in the constant terms. The constant term of "c_p" is greater than that of c_v by R.

Multiplying all terms of Equation (4-18) by the molar mass (M) we obtain

$$Mc_p - Mc_v = MR = 8.314\frac{\text{kJ}}{\text{Kmole.deg}} = 1.986\frac{\text{kcal}}{\text{Kmole.deg}}$$

$$\approx 8.3\frac{\text{kJ}}{\text{Kmole.deg}} \approx 2\frac{\text{kcal}}{\text{Kmole.deg}} \qquad (4\text{-}18a)$$

Equations (4-18) and (4-18a) are known as ***Meyer equation.***

The isentropic exponent ($k = c_p/c_v$ "also known as the specific heat ratio") is used usually in thermodynamic calculations. For ideal gases (see Meyer Equation 4-18 or 4-18a):

$$k = \frac{c_{p0}}{c_{v0}} = \frac{c_{v0} + R}{c_{v0}} = 1 + \frac{R}{c_{v0}} = \frac{Mc_{p0}}{Mc_{v0}} \tag{4-19}$$

For $c_v = \text{constant}$, k values are (1) $k = 1.667$ for monatomic gases, (2) $k = 1.4$ for diatomic gases, and (3) $k = 1.29$ for tri- and polyatomic gases.

If Mc_v is a linear function of temperature, then according to Equation (4-18) k is a nonlinear function of temperature. In practical thermodynamic calculations equation:

$$k = k_o - \alpha \cdot t \tag{4-20}$$

is used. Up to $2000^\circ C$, (1) for air and diatomic gases: $k_o = 1.41$ and $\alpha = 55 \cdot 10^{-6}$ and (2) for the combustion products: $k_o = 1.37$ and $\alpha = 55 \cdot 10^{-6}$.

From the first and fourth sides of Equation (4-19) we obtain:

$$c_{v0} = R/(k-1) \tag{4-21}$$

Multiplying by k we obtain

$$c_{p0} = R \cdot k/(k-1) \tag{4-22}$$

4.7 SUMMARY

It behooves us in general to study more processes in a chapter entitled "calculations of ideal gas physical state change processes in closed systems" than the state change processes discussed here, and also in more details. But the small size of volume (I) of this book, coupled with the recently discovered valuable *new polytropic process*, which requires more discussion, has forced us to divide the traditional Chapter 4 into two parts (A) the current chapter that includes the first parts of almost all traditional processes as Part I and (B) Chapter 6 in volume 2 that comprehensively describes the new polytropic process and includes the modernized/improved second parts of part I processes as Part II.

As this chapter is very brief, the summary will be a list of the titles of chapter's paragraphs, which are (1) Introduction; (2) the special cases of the gas state change processes and their representation on property diagrams; (3) the special cases of the first law of thermodynamics for any gas (the first law for the isochoric, isobaric, adiabatic, isothermal, and closed processes); (4) calculating the ideal gas internal energy and enthalpy changes (calculating the ideal gas internal energy change and calculating the ideal gas enthalpy change); (5) the first law for ideal gases (the first law for ideal gas isothermal process); and (6) the Meyer equation.

CHAPTER 5

THE SECOND LAW OF THERMODYNAMICS

5.1 INTRODUCTION

The discovery of the conservation of energy principle (the law of the conservation and transformation of energy) put, theoretically, an end to the attempts for designing a machine that can, without consuming any energy from the surroundings, be a continuous source of energy. Such machine was named perpetuum mobile (perpetual-motion machine) of the first kind. This law is also called the first law of thermodynamics, and it is formulated as: ***It is impossible to construct a perpetuum mobile (perpetual-motion machine) of the first kind.***

The observation of heat machines has shown that the first law of thermodynamics cannot give a full explanation for the phenomena of thermal energy transformation into mechanical. It is understood from its first statement that *"if a quantity of heat disappears, a certain amount of mechanical energy appears and vice versa."* And this means that ***the first law concerns only in the consequences of the thermal–mechanical transformations and does not care about the conditions needed to achieve these transformations***. And therefore, it does not indicate the direction of energy transformation and does not specify the necessary conditions to ***achieve this transformation direction or that***. In other words, this law does not differentiate between, for example, the transformation of mechanical energy into thermal and the opposite transformation—the transformation of heat energy into mechanical. Although the numerical relations during the first and second transformations are governed by the first law, there is a fundamental difference between them. ***The transformation of mechanical energy, like electrical energy and all no-heat/thermal kinds of energy, into heat/thermal energy is easy and***

does not require taking into account any conditions. We can transform the entire available amount of mechanical and electric energy into thermal/heat energy: Mechanical energy, for example, transforms into heat by friction; electric energy transforms in heating devices, and on the contrary, as experience shows, ***the reverse conversion of heat energy to mechanical in Cyclic Operating heat Engine (COE) performs when certain conditions are taken into account.*** It is impossible in this engine to transform the entire thermal/heat energy available to us into mechanical energy, but a part of it remains untransformed into mechanical energy. The long experience in the use of heat engines and the studies of other heat phenomena led to the development of the terms of heat energy transformation into mechanical, and these conditions are the essence of the ***second law of thermodynamics***.

5.2 THE OBSERVED PHENOMENA ASSOCIATED WITH THE THERMAL–MECHANICAL TRANSFORMATIONS

5.2.1 PHENOMENA ASSOCIATED WITH THE TRANSFORMATION OF HEAT ENERGY INTO MECHANICAL ENERGY

Many natural phenomena and artificial (man-made) ones are associated with the transformation of heat energy into mechanical. As we are interested in this book in artificial phenomena, we will not cite here natural phenomena such as the phenomenon of winds.

The artificial phenomena exist in heat engines. The engine is **classically** defined as a machine that produces mechanical energy (work).

5.2.1.1 Types of heat engines in terms of their ability to operate continuously

In terms of the ability for continuous operating, engines are divided into two types:

- The first type is characterized by the ability to operate and produce mechanical energy continuously as long as it is not ordered or forced to stop, such as in emergency breakdowns. This type of engine operates on a cycle and is called *COE*.

- The second type is characterized by the inability to work and produce energy continuously. It works for a limited period, or to perform a limited stroke, and then cannot continue working even in the case of the availability of the energy source and the lack of breakdowns. This type of engine does not work cyclically and can be named the ***no-COE***.

5.2.1.2 The cyclic (periodical) operating engine

The imaginary reciprocating external combustion engine, explained at the end of §2.3, is a good example of the **COE**, in which the piston returns each time to its initial position, as well as the working fluid (WF) returns to its initial state**.** This naming must be understood in the sense that the machine operates according to frequently repeated process (cycles). The design of this machine may not be executed in the form of cylinder–piston arrangement. And then, the piston condition (the piston must return to its initial position…) can be replaced by another common condition (the continuous engine's operation), ***but the return of the WF to its initial state is a prerequisite requirement for whatever engine design.***

What is important in such a COE is the following: at the TDC, the WF takes the amount (q_1) of heat from the hot gases (the heat source or the hot reservoir) that resulted from fuel combustion and rejects the amount (q_2) of heat to the cooling water (the heat sink or the cold reservoir): The difference between these quantities $(q_1 - q_2 = q_o)$ will no longer exist in the form of heat and transforms into mechanical energy, at the expense of which the work (w_o) is done. According to the first law of thermodynamics, $w_o = q_o$. The heat taken from the WF at the BDC ***is invalid to continue transforming into mechanical energy because it was taken at the low temperature of the WF*** (at the surroundings temperature), and therefore, it cannot be used to obtain work and ***is considered missing***.

5.2.1.3 The no-COE

Let us discuss an example about the transformation of heat energy into mechanical in the heat engine that does not operate periodically.

A piston–cylinder arrangement is filled with a gas, and we can consider the gas as a source of heat energy. In this case, it is possible to transform a part of the internal thermal/heat energy of the gas that is located inside the arrangement into mechanical, and this can be done by allowing the gas expansion. This machine is a heat engine because it produces the mechanical energy and consumes the gas heat. However, its operation is limited by the end of piston's stroke, and hence it is not able to produce work constantly, and therefore it is a ***no-COE.***

Today, the label "heat engine" belongs only to COE, and the engine is defined as the machine that can continuously *produce mechanical energy (work)*. And more precisely:

OD5-1: The engine is the machine that continuously produces mechanical energy from something else.

5.2.2 PHENOMENA ASSOCIATED WITH THE TRANSFORMATION OF MECHANICAL ENERGY INTO THERMAL ENERGY

The observation results show that the transformation of mechanical energy, like electric energy and all kinds of no-heat/thermal kinds of energy, into heat (thermal energy) is easy and does not require taking into account any conditions. We can transform the entire amount of mechanical and electrical energy available to us into heat/thermal energy (for example, the mechanical energy entirely transforms into heat through friction, and electrical energy entirely transforms in heating devices).

5.2.3 PHENOMENA ASSOCIATED WITH THE HEAT TRANSFER WHEN TWO OBJECTS (HOT AND COLD) ARE CONTACTED

The observation results show that when two objects (hot and cold) come into contact, heat transfers from the hotter body to the cooler one, and only when the temperatures of the two bodies equalize (when the state of thermal equilibrium is accessed), the process of heat transfer stops.

5.3 THE SECOND LAW OF THERMODYNAMICS

There are several statements/formulations belonging to the second law of thermodynamics, resulted from observing several phenomena. Each statement/formulation of the law has resulted from one observed phenomenon, and therefore they may not appear to have much connection with each other, although they are equivalent.

We are primarily interested in the phenomena associated with either heat energy transformation into mechanical or heat transfer from one body to another. Therefore, we shall introduce the statements belonging to these phenomena after reminding that *in the past, the mechanical energy was expressed as work and before that, the work was expressed as raising a weight/load.*

A) The many years of experience shows that it is impossible to complete the process of transformation of heat energy into mechanical in the *COE* without the presence of heat reservoirs of at least two different temperatures. From this experience, in 1852, Max Planck, or maybe both William Thomson (Lord Kelvin) and Max Planck, gave a statement for the second law of thermodynamics that is differently entitled in different references, slightly different in text but with the same meaning in all references. This statement speaks about cyclic/continuously operating heat engines, and some of its looks are as follows:

   1) *Max Planck Description Statement:* ***"It is impossible to construct an engine that will work on a complete cycle and produce no effect except the raising of a weight and the cooling of a heat reservoir."***[1]
   2) *Planck's proposition*: ***"It is impossible to construct an engine which will work in a complete cycle, and produce no effect except the raising of a weight and cooling of a heat reservoir."***[2]
   3) The Kelvin–Planck statement: **"It is impossible to construct a device that will operate in a cycle and produce no effect other than the raising of a weight and the exchange of heat with a single reservoir."**[3]

   As seen from items 1 and 2 above, different references refer differently to the same statement to which we shall refer in this book as *Max Planck Statement.*

   Thus, the previous statements/formulations emphasize the impossibility of ***continuously obtaining mechanical energy*** (lifting/raising a load/weight) in case of the availability of only one heat source (single heat reservoir) that is cooled down to transform its entire cooling heat into work, although such transformation does not contradict any of the thermodynamic laws including the first law.

B) The second law of thermodynamics formulated by Planck shows the futility of the attempts to create an engine that operates constantly when only one heat reservoir is available. Such an engine is called perpetuum mobile (perpetual-motion machine) of the second type. In this case, the second law of thermodynamics is formulated as: ***It is impossible to construct a perpetuum mobile of the second kind.***[4]

The advantages of the perpetual-motion engine of the second type, if it were created, would be of very much importance. This kind of imaginary engines can absorb heat energy from sea water and transform it *completely* into mechanical energy. By cooling each m^3 of water for 1°C or kelvin, it will produce 4,186.8 kJ of mechanical energy.

C) The obtained results from the study of the possibility of transforming heat energy into mechanical in (1) a pot full of gas and located on a base, (2) the no-*COE* (see §5.2.1.2), and (3) a lot of examples like them have led William Thomson (Lord Kelvin) in 1851, or may be both William Thomson and Max Planck, to formulate the second law of thermodynamics that is differently entitled in different references, slightly different in text but has, in all references, the same meaning. This statement is interested in all heat energy transformation into mechanical and some of its looks are as follows:

4) William Thomson (Lord Kelvin) statement: **"*A process, whose full effect is limited in cooling a heat reservoir and in equivalent to this cooling, raise of a load,*** is ***impossible in nature.*"**[5]
5) Kelvin–Planck statement: "No ***process*** is possible whose **sole** result is the absorption of heat from a reservoir and the transform of this heat into work."[6]

In this book, we shall refer to this statement as *William Thomson (Lord Kelvin) statement:*

From this statement/formulation, we conclude that ***heat transformation into work can never be the only result (full effect) of any process in nature.***

From the two formulations of Planck and Thompson, we see that the second law of thermodynamics studies the same processes studied by the first law (i.e., studies the processes of transforming heat energy into mechanical energy). The first law of thermodynamics studies the results of the processes if they were realized, while the second law studies the conditions to be provided to achieve the required processes.

In contrast to the heat energy transformation process into mechanical, the mechanical energy transformation process into heat—the opposite of the first (for example, during friction)—can take place without being associated at the same time with any additional process in nature. Such processes that take place on their own, without being accompanied by other processes "alone," are called automatic/spontaneous processes (and sometimes they are called nonsubstitutable processes).

Thus, the process of mechanical energy transformation into heat through friction is a spontaneous process, but the reverse process or the process of heat energy transformation into mechanical is nonspontaneous.

The second law of thermodynamics that stresses on the need to follow certain conditions to be able to transform heat energy into mechanical (in the contrary to the reverse process, which proceeds, for example, during friction) describes qualitatively heat energy and how it differs from mechanical energy.

In addition to the studied process concerning the heat energy transformation into mechanical, in nature, there is a huge number of processes that take place easily ("alone," without the need for any additional conditions) in one direction and cannot be realized *alone* in the opposite direction. For example, the heat transfer process from the hot body to the cold one is of this kind. It proceeds (alone) regardless of our wishes, although it is impossible for the reverse heat transfer process from the cold body to the hot one to be done automatically/spontaneously. The existence of refrigerating machines convinces us that such a process (heat transfer from the cold body to the hot one "base process") can be carried out if it was associated in parallel with a mechanical energy transformation into heat process (additional process), that is, it is possible upon completion of a mechanical work.

D) The impossibility of the phenomenon described here without additional process is the foundation in formulating the following statement for/of the second law of thermodynamics: ***heat cannot transfer by itself (automatically/spontaneously) from one body to another one that has higher temperature than the first body.***

This statement was formulated by Clausius (1850). This statement must not be understood in only its narrow meaning, that is, heat does not transfer from a colder body to a warmer one by simple heat transfer, but by its broad meaning, and specifically: such heat transfer cannot be completed even by mechanisms if this heat transfer were the only result of the process.

The atomization and diffusion belong to the spontaneous processes.

E) As well as in the heat energy transformation into mechanical process, we can say, based on the data of daily observations, that *the spontaneous reversal of the described processes is impossible*. These daily observations have led to broader generalization to second law of thermodynamics by generalizing it to include all processes described here. And then, it can be formulated as follows: ***spontaneous/automatic processes are irreversible***.

These are the statements of the second law of thermodynamics in their experimental shapes, that is, those resulting from observation. It should be noted here that generalizing this law, on conditions other than those that took place during observation, is ineffective.

It must be emphasized here that the various statements of the second law of thermodynamics do not contradict with each other.

Finally, we must stress that ***the requirement/condition to achieve a process is not violating any of the first and second laws of thermodynamics***.

5.4 THE THERMODYNAMIC CYCLE

A thermodynamic cycle (abbreviation cycle) is a closed state-change process (see §2.4.11). The paths of all discussed in thermodynamics till now thermodynamic cycles are plane simple curves (see OD1-7).

Some properties of a reversible/internally reversible cycle are (1) **cycle's minimum temperature (T_{min}),** that, is the minimum temperature of its WF throughout the cycle; (2) **cycle's maximum temperature (T_{max}),** that is, the minimum temperature of its WF throughout the cycle; (3) **cycle's temperature interval,** that is, the temperature interval between cycle's minimum temperature (T_{min}) and its maximum temperature "T_{max}" (abbreviation T_{min}_T_{max} interval); and (4) **cycle*'s characteristic points*** (see HD3-8).

Nt5-1: If we refer to the limited by two neighboring isotherms cycle's area as ***cycle's mini-part***, then it is obvious for a common case cycle's configuration to meet some ***continuous mini-parts*** that consist of one elementary area each, whereas the other areas are discontinuous (they consist of more than one elementary area each).

HD5-1: *A cycle can be (1) simplest, if it consists of differential* ***continuous mini-parts*** exclusively (see all subfigures of Figure 5-12), (2) *simple, if it can be divided into three parts (so that its middle part consists of no more than all cycle's differential* ***continuous mini-parts***) or *into two parts (so that one of these parts consists of no more than all cycle's differential* ***continuous mini-parts*** ***"see Figure 5-2")****.* We shall refer to the cycle or to its part that consists of only continuous *differential* ***continuous mini-parts*** as the continuous cycle or continuous cycle's part. Also, we shall refer to the cycle that is no simple and no simplest as the *complicated cycle.*

Simplest cycle's intermediate parts in *T_s* diagram located between two neighboring characteristic isotherms (see HD3-8A and HD3-8) are trapezoids with curved and/or straight legs, while cycle's extreme parts can be either trapezoidal or triangular with curved and/or straight legs.

It is assumed in the following derivations that the thermodynamic cycle (see §3.1.3 and HD3-1A) is equilibrium/quasi-equilibrium (reversible or internally reversible).

5.4.1 THE DIRECT THERMODYNAMIC (POWER) CYCLE

The direct thermodynamic (power) cycle (symbol DC) is the cycle, which produces mechanical energy. *In p_v and T_s* ^{A} *diagrams (see Figure 5-1) this direct cycle proceeds clockwise* (cycles 1–3–2–5–1 and a–c–b–e–a are direct cycles). The specific (per unit mass) ^{B} gas (WF) expansion boundary work throughout the direct cycle $\left(w_{\exp DC}\right)$ is measured in *p_v* graph by the area under the expansion process curve 1–3–2, and it is positive $\left(w_{\exp DC} > 0\right)$ because $(dv > 0)$ as the specific transferred heat to the gas (the gained heat) throughout the cycle $\left(q_{1_{DC}}\right)$ is measured in *T_s* graph by the area under the entropy increasing process curve a–c–b, which is positive heat $\left(q_{1_{DC}} > 0\right)$ because $ds > 0$ (we remind here that the absolute pressure p and temperature T are always positive).

A. In §3.1.2.3.1 we already proved that the specific ideal gas entropy is an intensive property (see the bolded double underlined in Italics) and we justified the legality to deal with ideal gas *T_s* diagram. And we will prove later (see §5.8, Equation 5-56, and OR5-2) that the specific entropy is an intensive property for all gases.

B. q and w are specific dimensions, because s and v used in thieer calculations are specific ones.

Also, the specific gas compression boundary work throughout the same cycle $\left(w_{compDC}\right)$ is measured in *p_v* graph by the area under the compression process curve 2–5–1, and it, when calculated by gas work laws Equation (2-38), is negative work $\left(w_{compDC} < 0\right)$ because $dv < 0$, while the specific transferred heat from the gas (the rejected heat) throughout the cycle $\left(q_{2_{DC}}\right)$ is measured in *T_s* graph by the area under the entropy decreasing process curve b–d–a, and it, when calculated by transferred heat laws Equation (3-22), is negative heat because $ds < 0$.

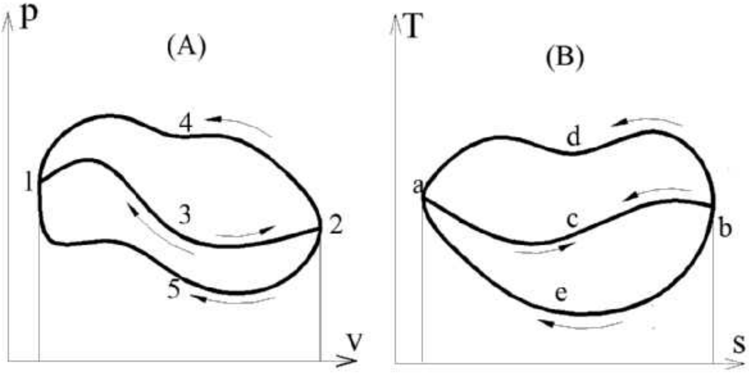


Figure 5.1. Direct and indirect thermodynamic cycles in p_v and T_s diagrams

Thus, the per unit mass gas work (the work done by a unit mass of the gas) throughout the cycle $w_o = \sum_{cycle} w$ is positive because the positive expansion work, as evident from subfigure (A), is greater than the absolute value of the negative compression work. Also, the transferred heat to a unit mass of the gas (the specific gained heat) throughout the cycle $q_o = \sum_{cycle} q$ is positive because the positive transferred heat to the gas during the cycle, as evident from subfigure (B), is greater than the absolute value of the negative transferred heat from the gas (the rejected heat) throughout the cycle. The above complies with OR1-12, and in graphical solutions and discussions, the area of the cycle in scale is cycle's (1) per unit mass produced work w_o in *p_v* graph and (2) per unit mass transformed into work heat q_o in *T_s* graph. Both are positive because the direct cycle precedes clockwise in both *T_s* and *p_v* graphs.

Note that $q_o = w_o$ as is clear from the relationship (4-7).

As the direct cycle produces work, it is an engine/a power cycle.

5.4.2 THE REVERSE THERMODYNAMIC CYCLE

The reverse thermodynamic cycle is the cycle, which produces heat energy. In *p_v* and *T_s* diagrams, this cycle proceeds anticlockwise (cycles 1–3–2–4–1 and a–c–b–d–a are reverse cycles). Throughout the reverse cycle 1–3–2–4–1 in *p_v* graph and a–c–b–d–a in *T_s* graph, (1) the gas expansion work is $w_{1-3-2} = A_{\text{under } 1-3-2} > 0$ because $dv > 0$, while the transferred heat to the gas is $q_{a-c-b} = A_{\text{under } a-c-b} > 0$ because $ds > 0$. (2) The gas compression work is $w_{2-4-1} = A_{\text{under } 2-4-1} < 0$ because $dv < 0$ (see Equation 2-38), while the transferred heat from the gas $q_{b-d-a} = A_{\text{under } b-d-a} < 0$ because $ds < 0$ (see equation 3-22).

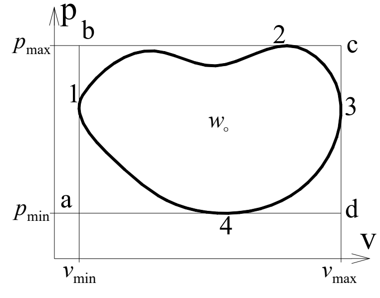


Figure 5.2. The volumetric specific work

Thus, the gas work (the work done by the gas) throughout the cycle $w_o = \sum_{\text{cycle}} w$ is negative because the positive expansion work, as evident from Figure 5-2, is less than the absolute value of the negative compression work. Also, the transferred heat to the gas (the gained heat) throughout the cycle $q_o = \sum_{\text{cycle}} q$ is negative because the positive transferred heat to the gas during the cycle, as evident from the same Figure 5-2, is less than the absolute value of the negative transferred heat from the gas (the rejected heat) throughout the cycle. The above complies with OR1-12, and in graphical solutions and discussions, the area of the cycle in scale is cycle's (1) produced work $w_\circ$ in *p_v* graph and (2) the transformed into work heat $q_\circ$ in *T_s* graph. Both are negative because the reverse cycle proceeds anticlockwise in both *T_s* and *p_v* graphs.

Note that $q_\circ = w_\circ$ as is clear from the relationship (4-7).

As the reverse cycle consumes work, it is not valid for engines. Also, in the reverse cycle (1) the rejected heat is greater than that absorbed and (2) the heat absorption proceeds at lower temperatures than those at which heat rejection proceeds. As we will see later, the heat effect of this cycle makes it suitable for use in **refrigerating and mechanical heating machines** (heat pumps).

5.4.3 NOTES ABOUT THERMODYNAMIC CYCLES AND CYCLIC OPERATING MACHINES

From the knowledge delivered through this book till now and the supposed scientific level of the readers, we can summarize, for cyclic operating machines, the following:

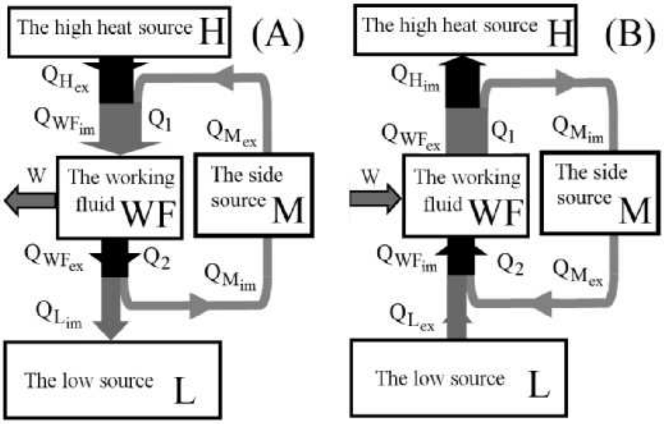


Figure 5.3. A schematic showing all kinds of bodies that can participate in executing a thermodynamic cycle.

A – a direct cycle B – a reverse cycle

<u>Nt5-1A:</u> The WF is one of the two energy (heat and work) exchanging bodies in each energy-exchanging process during the cycle. Therefore, when dealing with simple/complicated cyclic operating machines, we can mostly analyze the cyclic operating machines and their cycles though analyzing the state change process of the WF. Thus, in most cases:

- The analytical expressions (equations) of cycle's path in *T_s* and *p_v* planes are actually the equations of the state change processes of the WF of the machine.
- The representations of cycle's path in *T_s* and *p_v* diagrams are actually the paths of the state change processes of the WF of the machine.
- The calculating equations for work and heat in the machine/cycle are actually the same for the WF's state change processes.

We shall use the fact that the WF participates in all heat transfer processes of the cycle to shorten the names of cycle's heats:

Nt5-2: Today, the participants in executing a direct or reverse cycle are the WF and the other bodies that exchange heat with it during the cycle. They are (see Figure 5-3):

(1) Zero or more medium-temperature/side heat reservoir(s)—abbreviation side Mr(s). Each side MR exports heat to the WF during a part of the cycle (The symbol of the sum of these heats during a cycle is $Q_{M_{ex}}$), and imports heat from the same WF during another part of the same cycle (The symbol of the sum of these heats during a cycle is $Q_{M_{im}}$). Theoretically, it is unnecessary for the heat balance of each side MR to be zero, which leads to $Q_{M_{ex}} = Q_{M_{im}}$; but, according to former and today's definitions, for $Q_{M_{ex}} = Q_{M_{im}} \neq 0$ the cycle becomes regenerative (see §5.9.1.3) and for $Q_{M_{ex}} = Q_{M_{im}} = 0$ the side MRs will not participate in the cycle and the cycle becomes nonregenerative (see §5.9.1.4). As we shall see later the participation of at least one side MR (of/with any heat balance) in a cycle makes it regenerative (see §5.9.1.3).

(2) One or more high temperature heat reservoir(s)/MRs—symbol (H), and

(3) One or more low temperature heat reservoir(s)/MRs—symbol (L). For the last two MR's categories the direction of heat transfer depends on cycle's direction. Thus:

(A) In any ***direct*** cycle, the set of high MRs exports heat to the WF each time any of its MRs contacts the WF (The symbol of the sum of these heats during a cycle is $Q_{H_{ex}}$), whereas the set of low MRs imports heat from the WF each time any of its MRs contacts the WF (The symbol of the sum of these heats during a cycle is $Q_{H_{im}}$). And

(B) In any ***reverse*** cycle, the set of high MRs imports heat from the WF each time any of its MRs contacts the WF (The symbol of the sum of these heats during a cycle is $Q_{H_{im}}$), whereas the set of low MRs exports heat to the WF each time any of its MRs contacts the WF (The symbol of the sum of these heats during a cycle is $Q_{L_{ex}}$).

As, during a cycle, the sum of the imported to the WF heats equals the sum of the exported heats from all MRs that participate in executing this same cycle, whereas the sum of the exported from the WF heats equals the sum of the imported heats from all MRs that participate in executing this same cycle, we can write (see items 1, A and B above and Figure 5-3):

(C) In any ***direct*** cycle (see subfigure A), the WF imports the exported heats ($Q_{H_{ex}}$ and $Q_{M_{ex}}$) from the sets of high and side MRs and therefore the WF's imported heat during the cycle ($Q_{WF_{im}}$) equals $Q_{WF_{im}} = Q_{H_{ex}} + Q_{M_{ex}}$, whereas the same WF exports the imported heats ($Q_{L_{im}}$ and $Q_{M_{im}}$) to the sets of low and side MRs and therefore the WF's exported heat during the cycle ($Q_{WF_{ex}}$) is $Q_{WF_{ex}} = Q_{L_{im}} + Q_{M_{im}}$. Also, in any ***reverse*** cycle (see subfigure B), the WF exports the imported heats ($Q_{H_{im}}$ and $Q_{M_{im}}$) to the sets of high and side MRs and therefore the WF's exported heat during the cycle ($Q_{WF_{ex}}$) is $Q_{WF_{ex}} = Q_{H_{im}} + Q_{M_{im}}$, whereas the same WF imports the exported heats ($Q_{L_{ex}}$ and $Q_{M_{ex}}$) from the sets of low and side MRs and therefore the WF's imported heat during the cycle ($Q_{WF_{im}}$) is $Q_{WF_{im}} = Q_{L_{ex}} + Q_{M_{ex}}$.

Nt5-2A: The above symbols are introduced for the first time in this book to ease understanding the new derivations:

(1) To create the full names of the introduced symbols take into consideration that Q – stands for cycle's heat, H – stands for the set of high temperature MRs, L – stands for the set of low temperature MRs, WF – stands for the working fluid, im – stands for imported by and ex – stands for exported by. Thus, the full names of the new symbols of the introduced new cycle's heats (see Table 5-1) are: (1) $Q_{H_{ex}}$ – **Cycle's exported high MRs' heat,** (2) $Q_{L_{im}}$ – **Cycle's imported low MRs' heat,** (3 and 10) $Q_{WF_{im}}$ – **Cycle's imported WF's heat,** (4 and 9) $Q_{WF_{ex}}$ – **Cycle's exported WF's heat,** (5)

$Q_{net_{im}}$ – **Cycle's imported net heat,** (6) $W_{net_{ex}}$ – **Cycle's exported net work,** (7) $Q_{H_{im}}$ – **Cycle's imported high MRs' heat,** (8) $Q_{L_{ex}}$ – **Cycle's exported low MRs' heat,** (11) $Q_{net_{ex}}$ – **Cycle's exported net heat** and (12) $W_{net_{im}}$ – **Cycle's imported net work.**

The above are for the total amount of the WF in the machine. For a unit mass of the WF (see Equation 3-4): (1) $q_{H_{ex}}$ – **Specific cycle's exported high MRs' heat,** (2) $q_{L_{im}}$ – **Specific cycle's imported low MRs'**

Table 5-1: The current equivalents for the new symbols of the introduced new cycle's heats

Direct cycle				Reverse cycle		
No	**New symbol**	**Current symbol**		**No**	**New symbol**	**Current symbol**
1	$Q_{H_{ex}}$	Q_H		7	$Q_{H_{im}}$	Q_H
2	$Q_{L_{im}}$	Q_L		8	$Q_{L_{ex}}$	Q_L
3	$Q_{WF_{im}}$	Q_1		9	$Q_{WF_{ex}}$	Q_1
4	$Q_{WF_{ex}}$	Q_2		10	$Q_{WF_{im}}$	Q_2
5	$Q_{net_{im}}$	Q_{net}		11	$Q_{net_{ex}}$	Q_{net}
6	$W_{net_{ex}}$	W_{net}		12	$W_{net_{im}}$	W_{net}

Nt1: Formerly and currently discussed cycles do not include ones, whose $0 \neq Q_{M_{ex}} \neq Q_{M_{im}} \neq 0$. They include either regenerative cycles, whose $Q_{M_{ex}} = Q_{M_{im}} = Q_R \neq 0$ or nonregenerative cycles, whose $Q_{M_{ex}} = Q_{M_{im}} = Q_R = 0$, where Q_R is cycle's regenerated heat. Symbol Q_R that is adopted in this book for cycle's regenerated heat denotes also to the rejected heat Q_L in the direct cycle; therefore, *unless it is clearly/explicitly indicated to the dimension represented by* Q_R*, it should be understood that* Q_R *represents the regenerated heat during the cycle.*

Nt2: According to Or3-6, the algebraic values of the new cycle's heats that sub-subscripted by ex are positive and by im are negative.

Nt3: The above are the properties for the total amount of the WF in the machine. To obtain these properties per unit WF mass (the specific properties), we divide these properties by WF mass (e.g., $q_H = Q_H / M$)

heat, (3 and 10) $q_{WF_{im}}$ – **Specific cycle's imported WF's heat,** (4 and 9) $q_{WF_{ex}}$ – **Specific cycle's exported WF's heat,** (5) $q_{net_{im}}$ – **Specific cycle's imported net heat,** (6) $w_{net_{ex}}$ – **Specific cycle's exported net work,** (7) $q_{H_{im}}$ – **Specific cycle's imported high MRs' heat,** (8) $q_{L_{ex}}$ – **Specific cycle's exported low MRs' heat,** (11) $q_{net_{ex}}$ – **Specific cycle's exported net heat** and (12) $w_{net_{im}}$ – **Specific cycle's imported net work.**

> **Nt5-2A1:** In the traditional paragraphs below we continue using traditional and/or current symbols. The names of the current symbols can be (1) taken from other thermodynamic books (e.g., the names of Q_H can be the heat that costs or the added heat) or (2) extracted from the names of their counterpart new symbols by excluding the word (exported or imported) from them. For example, the name of Q_H can be **cycle's high MRs' heat.**
>
> It is obvious that cycle's heats calculation is possible only when the cycle is divided in *T_s*, *C_T,* or *C_t* diagram by/with its characteristic points into processes through which the transferred heats are pure (see HD3-8).
>
> **Nt5-2B:** Outside the effectiveness equations, where the symbols of cycles' heats (Q_H, Q_L, Q_1 and Q_2) represent their absolute values, a cycles' heat or work is an algebraic quantity whose sign depends on whether the WF imports or exports it. The imported by the WF cycles' heat is positive, whereas the exported by the WF cycles' heat is negative. Also the exported by the WF cycles' boundary work is positive, whereas the imported by the WF cycles' boundary work is negative. This means (see Nt5-2A) that Q_H, q_H, Q_1, q_1, Q_{net}, q_{net}, W_{net} and w_{net} are positive in direct cycles and negative in reverse cycles. Also Q_L, q_L, Q_2 and q_2 are negative in direct cycles and positive in reverse cycles. Concerning the introduced in Table 5-1 new symbols for cycle's heats see NT2 of Table 5-1.

5.4.4 EVALUATING THERMODYNAMIC CYCLES

The evaluation of any activity can be done through its effectiveness/efficiency (*E*), which is the ratio of its *valuable product* or *valuable output* (VO) to its *valuable consumption* or *valuable input* (VI).

$$\text{Effectiveness} = \text{VO/VI} \qquad (5\text{-}1)$$

Thus, what is the invaluable product?

> **HD5-1A:** *The invaluable product is the product that is available every time and anywhere*, while the valuable product is the product that is not invaluable. The surroundings, when it is used as a low reservoir, is a source for invaluable products. *All imported to the WF heats are valuable, and all exported to the surroundings heats are invaluable.*

The name of the effectiveness depends on the activity's category, kind, and the units used for the outputs and inputs. Therefore, Equation (5-1) is the common case of a lot of thermodynamic and nonthermodynamic effectiveness equations.

We shall discuss here the following special cases that are considered in thermodynamics. The inputs and outputs in these efficiencies are energy kinds.

The ideal direct thermodynamic cycles are the ideal cycles of heat engines, and the effectiveness of such cycles is called *thermal efficiency* (η_t). The ideal reverse thermodynamic cycles are the ideal cycles of two categories of machines: (1) the refrigerating (cooling) machines, the effectiveness of such cycles is called the cooling factor (β) or the coefficient of performance for a refrigerator "COP_R."[7] (2) The mechanical heating machines (the heat pumps), the effectiveness of such cycles is called the heating factor (β') or the coefficient of performance for a heat pump "COP_{HP}."[8]

All three mentioned devices are energy transformation devices and deal with mechanical energy that is always valuable and two kinds of heat energies (valuable and invaluable that is exported to a lower heat reservoir/MR "e.g., surroundings"); therefore, (1) the effectiveness of these devices can be denoted by energy transformation effectiveness (ETE), (2) the input of these devices from heat reservoirs/MRs can be denoted by valuable energy input (VEI), and (3) the output of these devices to others than the surroundings reservoirs can be denoted by valuable energy output (VEO). Thus, using Equation (5-1), we can write the following equation for the ETE:

$$\text{ETE} = \text{VEO}/\text{VEI} \tag{5-2}$$

Here the heat exported to heat sinks and/or the surroundings does not exist.

When talking about the electric generator, the electric power will be the VEO, while the mechanical energy is the VEI.

5.4.4.1 Evaluating engine cycles (the direct cycles)

It is necessary to introduce a measure (measures) to evaluate the engine cycles. There are quite a number of measures used in heat engineering for this purpose, including:

- The most important measure, from the thermodynamics point of view, is the *thermal efficiency* and we shall return to it soon.
- *The fullness factor of p_v diagram* (σ) is the ratio of the useful/network (w_o) in the given cycle to the maximum possible work (w_{max}) within cycle's pressure $(p_{max} - p_{min})$ and specific volume $(v_{max} - v_{min})$ intervals. Thus (see Figure 5-2)

$$\text{and} \qquad \left.\begin{aligned} w_{max} &= (p_{max} - p_{min}) \cdot (v_{max} - v_{min}) \\ \sigma &= \frac{w_o}{(p_{max} - p_{min}) \cdot (v_{max} - v_{min})} \end{aligned}\right\} \tag{5-2a}$$

The numerical value of this factor allows evaluating the validity of the cycle for practical use.

- *The volumetric specific work.* This factor is also for evaluating the validity of the cycle for practical use and it is calculated (see Figure 5-2) as the ratio of the useful/network (w_o) in the given cycle to the Wf's specific volume at the end of its expansion (v_{max}):

$$w_{ov} = w_o / v_{max} \tag{5-2b}$$

5.4.4.1.1 The thermal efficiency of the ideal direct cycle

The thermal efficiency is the measure that enables evaluating the economy of the process of heat energy transformation into mechanical in the ideal heat engine cycles (see §4.3.5). This measure is defined as follows:

> **HD5-2:** The thermal efficiency of an ideal heat engine is the ***ratio of the produced mechanical energy*** W_o ***to the amount of heat*** Q_H ***provided by (transferred from) the set of high-temperature heat reservoirs/MRs to the WF during an integer number of thermodynamic cycles.*** Note that the minimum number of high reservoirs/MRs participating during the cycle is one reservoir/MR. Thus:

$$\eta_t = W_o / Q_H \tag{5-3}$$

<u>Nt5-3:</u> The thermal efficiency is assigned to evaluate the economy of ideal engines/power cycles. To evaluate the economy of real engines/power cycles, other factors are used. One of these factors is expressed by the same right side of Equation (5-3), but carries another name.

During each thermodynamic cycle that achieved in a thermally insulated thermodynamic system that contains/includes the WF and the reservoirs/MRs (1) the set of high reservoirs/MRs loses Q_H of heat, (2) the set of low reservoirs/MRs gains Q_L of heat, (3) a work of W will appear, and (4) the WF returns to its initial state. Thus, the amount disappeared as heat is $Q_H - Q_L$ while that appeared as work is W.

Thus, according to the first law of thermodynamics:

$$W = Q = Q_H - Q_L \tag{5-4}$$

In paragraph (4-3-5) we discussed the first law for the closed process that can be understood as belonging to the WF and expressed for the ideal cycle as $q_o = w_o$ and $Q_o = W_o$ (see Equation (4-7)). There, we already named W_o as the useful work and named Q_o as the useful/net amount of heat (useful/net heat). As all heat transfer processes in the *COE* proceed through the WF's boundary (see Nt5-1A), Q_o is also the useful heat during one **COE** cycle, and q_o is also the specific useful heat during one **COE** cycle.

For the ideal **COE** cycle relationship (5-4) takes the shape:

$$Q_o = W_o = Q_H - Q_L \tag{5-4a}$$

From (5-3), (5-4a) and (3-4) we obtain:

$$\eta_t = \frac{Q_o}{Q_H} = \frac{Q_H - Q_L}{Q_H} = \frac{q_o}{q_H} = \frac{q_H - q_L}{q_H} \tag{5-4b}$$

To be able to calculate η_t, the given data must include: 1) either one of the following pairs of dimensions: (Q_H and Q_o), (Q_H and Q_L), (q_H and q_o), and (q_H and q_L) or 2) fully defined (mathematically or graphically) cycle's path with an information about the heat regeneration (see OD5-3 and OD5-4) in it. For this second case we additionally use some of the following calculations and definitions:

The fully defined cycle's path in T_s plane allows determining (see Nt5-2A and Table 5-1) cycle's WF's heats: (imported $Q_{WF_{im}}$ "Q_1", exported $Q_{WF_{ex}}$ "Q_2" and imported net $Q_{net_{im}}$ "Q_{net}") and/or specific cycle's WF's heats (see Equations 5-5 through 5-8 below that require dividing the cycle, in advance, into pure "positive, negative and neutral ***transferred heat"*** processes/subprocesses "see HD3-4 through HD3-6"). We shall refer to such divided cycles as the ***divided into pure processes exclusively (cycle*DPPE *or* DPPEcycle*)***. On the other hand the information about cycle's heat regeneration decides cycle's regenerated heat Q_R that allows for known $Q_{WF_{im}}$ and $Q_{WF_{ex}}$ determining Q_H and Q_L.

$$q_{1\,a\lg} = \sum_{cycle_{DPPE}} q_{pp} = \oint \delta q_{pp} > 0 \Rightarrow q_1 = \left|q_{1\,a\lg}\right| = q_{1\,a\lg} > 0 \qquad (5\text{-}5)$$

$$\left.\begin{aligned} & q_{2\,a\lg} = \sum_{cycle_{DPPE}} q_{PNe} = \oint \delta q_{PNe} < 0 \\ \text{and} \qquad & q_2 = \left|q_{2\,a\lg}\right| = -\sum_{cycle_{DPPE}} q_{PNe} > 0 \end{aligned}\right\} \qquad (5\text{-}6)$$

$$\begin{aligned} q_o &= \sum_{cycle} q_{\text{partial processes}} = \sum_{cycle_{DPPE}} (q_{PP} + q_{PNe} + 0) = \oint dq = q_1 - q_2 \\ &= q_1 + q_{2\,a\lg} = \sum_{cycle_{DPPE}} q_{PP} + \sum_{cycle_{DPPE}} q_{PNe} = \underline{\text{Cycle's area}} \end{aligned} \qquad (5\text{-}7)$$

$$\left.\begin{aligned} \text{Or} \qquad & q_o = q_1 + q_{2\,a\lg} = q_1 - q_2 \\ \text{and accordingly} \qquad & Q_o = Q_1 + Q_{2\,a\lg} = Q_1 - Q_2 \end{aligned}\right\} \qquad (5\text{-}8)$$

<u>OD5-3</u>: The ***heat regeneration*** is defined as the process that aims to increase the thermal efficiency of a mechanical energy–producing cycle (the direct or power cycle) by using a part of the heat Q_2, which is supposed to become a lost heat, and that is rejected by the WF during its *s*-decreasing process(es) through the entire cycle, in the WF heat-gaining (*s*-increasing) process(es).

<u>OD5-4</u>: The ***heat regenerator*** is the device through which the heat regeneration is achieved in the real heat engine.

The schemes in Figure 5-3 are the most common case ones. They represent the existed, existing and may ever exist schemes for direct and reverse thermodynamic cycles. For example, to obtain the scheme of/for the: 1) simple heat engine (the heat engine without heat regeneration) from the same subfigure A of Figure 5-3 we (see Figure 2-1) delete the right side branch ($Q_{M_{ex}}$, $Q_{M_{im}}$ and the side source M), widen Arrow $Q_{L_{im}}$

to the width of Arrow $Q_{WF_{ex}}$ (Q_2) and decrease the width of Arrow $Q_{WF_{im}}$ (Q_1) to the width of Arrow $Q_{H_{ex}}$ 2) heat engine with a heat regenerator from subfigure A of Figure 5-3 we change the names ($Q_{M_{ex}}$, $Q_{M_{im}}$ and the side source M) with (Q_R, Q_R and the heat regenerator R) respectively. And since the heat regenerator is not a heat source, neither a WF, it is usually excluded from the scheme (see Figure 5-3a). Figure (5-3a) will be our reference during the following explanation of the regenerative power cycle (the heat engine cycle with regeneration).

The heat Q_2 ($Q_{WF_{ex}}$) rejected from/by the WF branches out into two: the first branch $Q_{L_{im}}$ (Q_L) goes to the low heat reservoir/MR and the other (the regenerated heat) Q_R ($Q_{M_{im}}$ in the sub figure) goes through the regenerator (the side source M in the sub figure) to the WF to participate with the high heat reservoir/MR in supplying the WF with the heat required by it during its entropy-increasing process during the cycle.

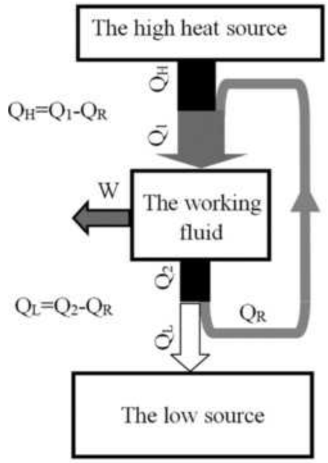


Figure 5.3a. A schematic showing all kinds of bodies that can participate in executing a thermodynamic cycle.

From Figure 5.3a we obtain:

$$\left.\begin{array}{l} Q_H = Q_1 - Q_R;\quad Q_L = Q_2 - Q_R;\quad Q_R = Q_1 - Q_H = Q_2 - Q_L \\ \text{and accordingly} \\ q_H = q_1 - q_R;\quad q_L = q_2 - q_R;\quad q_R = q_1 - q_H = q_2 - q_L \end{array}\right\} \quad (5\text{-}9)$$

From these equations and Equation (5-4a) we obtain

and accordingly
$$\left.\begin{aligned} Q_\circ &= W_\circ = Q_H - Q_L = Q_1 - Q_2 \\ q_\circ &= w_\circ = q_H - q_L = q_1 - q_2 \end{aligned}\right\} \quad (5\text{-}10)$$

And for the cycle without regeneration ($Q_R = 0$):

$$Q_H = Q_1, \quad Q_L = Q_2, \quad q_H = q_1 \quad \text{and} \quad q_L = q_2 \qquad (5\text{-}10a)$$

$$\begin{aligned} \eta_{t\,reg} &= \frac{Q_\circ}{Q_H} = \frac{W_\circ}{Q_H} = \frac{Q_H - Q_L}{Q_H} = \frac{Q_1 - Q_2}{Q_1 - Q_R} = \frac{Q_{1\,alg} + Q_{2\,alg}}{Q_{1\,alg} - Q_R} = 1 - \frac{Q_L}{Q_H} \\ &= \frac{q_\circ}{q_H} = \frac{w_\circ}{q_H} = \frac{q_H - q_L}{q_H} = \frac{q_1 - q_2}{q_1 - q_R} = \frac{q_{1\,alg} + q_{2\,alg}}{q_{1\,alg} - q_R} = 1 - \frac{q_L}{q_H} \end{aligned} \qquad (5\text{-}11)$$

This multiequation is an abbreviation of a set of equations that cosists of 12 equations those share with this multiequation the left sideand each of them share also with it one of its no left sides in addition to other equations whose both sides are two of the the no left sides of this same multiequation. Multiequation (5-11) is valid for both power cycles' (with and without) heat regeneration. Just substitute each of Q_R *(*q_R*) by zero,* Q_H *(*q_H*) by* Q_1 *(*q_1*),* Q_L *(*q_L*) by* Q_2 *(*q_2*), and* $W_\circ$ *(*$w_\circ$*) by* $Q_\circ$ *(*$q_\circ$*) to discover that Figure 5-3 transforms into Figure 2-1 and multiequation (5-11) transforms into:*

$$\begin{aligned} \eta_{t\,no-reg} &= \frac{Q_\circ}{Q_1} = \frac{W_\circ}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = \frac{Q_{1\,alg} + Q_{2\,alg}}{Q_{1\,alg}} = 1 - \frac{Q_2}{Q_1} \\ &= \frac{q_\circ}{q_1} = \frac{w_\circ}{q_1} = \frac{q_1 - q_2}{q_1} = \frac{q_{1\,alg} + q_{2\,alg}}{q_{1\,alg}} = 1 - \frac{q_2}{q_1} \end{aligned} \qquad (5\text{-}12)$$

For the case it was allowed to represent the nonregenerative cycle in *T_s* diagram (the cycle is reversible or internally reversible but externally irreversible whether it was regenerative or nonregenerative), all elements here can be obtained from *T_s* diagram of the WF, if the required accuracy allows.

Usually, the diagrams are used in the qualitative analyses of thermodynamic processes due to the outstanding ability of this method in the delivery of information to the receiver (reader of the research or listener's to a lecture).

Based on the above, the following can be concluded:

Nt5-4: The use of T_s diagram in studying the nonregenerative cycles (the reversible and internally reversible but externally irreversible cycles that are allowed processes to be represented graphically) is always possible, *but its use in studying the regenerative cycles is not always easy and is fraught with the dangers of falling into error and sometimes is not possible*; therefore, we emphasize that:

Nt5-5: *The use of the graphical methods for comparing thermodynamic cycles must be based on comparing nonregenerative cycles; if we want to do a graphical comparison between regenerative cycles* (or between a regenerative cycle and another nonregenerative), *we first have to replace the regenerative cycles by nonregenerative cycles that are completely equivalent to them (see HD5-3A) and then do the comparison using T_s diagram.*

From Multiequation (5-4b) we obtain $Q_L = Q_H(1-\eta_t)$

where $Q_H \times \eta_t$ is the heat transformed into mechanical energy during the cycle, while $Q_H(1-\eta_t)$ is the heat that did not transform into mechanical energy during the cycle, that is, the transferred/given heat to the low-temperature reservoir/MR. The greater the η_t the smaller the untransformed heat.

We remind here that we are speaking about the thermal efficiency of the ideal cycle; therefore, the values η_t, w_o, and q_o belong to the ideal cycle.

5.4.5 THE EQUIPOLLENT THERMODYNAMIC CYCLES

HD5-3: ***The equivalent/equipollent thermodynamic cycles*** *are those that share the* values of the (1) ***specific cycle's high MRs' heat***—symbol (q_H), (2) ***specific cycle's low MRs' heat***—symbol (q_L), (3) specific useful work $w_o = q_o = q_H - q_L = q_1 - q_2$, and (4) thermal efficiency η_t. And as these four characteristics/properties of the cycle are interconnected with each other through the two independent equations $\eta_t = q_o / q_H$ and $q_H - q_L = q_o$, we do not have to check all of them for being the same in all cycles under consideration to conclude/decide whether the equivalency between these cycles (under consideration) is-realized/exists or not. Cycles characterized by the same values of any two of the properties: (q_H, q_L, $w_o = q_o$, and/or η_t) are invariably equivalent ones.

HD5-3A: ***Completely equivalent cycles are the equivalent cycles that share the temperature interval T_{min}–T_{max} (see §5.4).*** To be completely equivalent, cycles must have/share the same values of the six cycle's properties $\underline{q_H, q_L, q_o, \eta_t,}$ T_{max}, and T_{min}. (To realize the completely equivalent cycles, each of the six cycle's properties $\underline{q_H, q_L, q_o, \eta_t,}$ T_{max}, and T_{min} must have the same value in all completely equivalent cycles.) And since $\underline{q_H, q_L, q_o, \text{ and } \eta_t}$ of the cycle are interconnected with each other through equations $\eta_t = q_o / q_H$ and $q_H - q_L = q_o$, we have to check, for being the same in all cycles under consideration, any two of the four (q_H, q_L, $w_o = q_o$, and/or η_t) in addition to T_{max} and T_{min}, and conclude/decide whether the complete equivalency between these cycles (under consideration) exists (is realized) or not. ***Cycles characterized by the same values of each of*** T_{max}, T_{min}, ***and any two of the properties (***q_H, q_L, $w_o = q_o$, ***and/or*** η_t***) are invariably completely equivalent ones.*** For the special case when all cycles under consideration are Carnot cycles (see the following paragraph), the thermal efficiency η_t is additionally interconnected with T_{max} and T_{min} through equation $\eta_t = (T_H - T_L)/T_H = (T_{max} - T_{min})/T_{max}$ that is independent from the former $\eta_t = q_o / q_H$ and $q_H - q_L = q_o$ equations. Therefore, ***Carnot cycles characterized by the same values of each of either*** T_{max}, T_{min} ***with any of*** q_H, q_L, and q_o, ***or any of*** T_{max} and T_{min} ***with any two of*** q_H, q_L, q_o, and η_t ***are invariably completely equivalent ones.***

5.5 THE CARNOT CYCLE

The French engineer and scientist Sadi Carnot created the ***simplest ideal cycle*** and calculated its thermal efficiency. ***The WF in this cycle is an ideal gas one and this eases the calculations***. But this, as we will see later, is not necessary for the studied cycle and does not affect its thermal efficiency.

The ***Carnot machine*** operates according to Carnot cycle or its reverse. It is a reciprocating ideal heat engine/refrigerator/heat pump with no irreversibilities (the piston that is perfectly thermally isolated moves smoothly and without friction inside the cylinder that is not equipped with inlet and outlet valves, while the cylinder that is perfectly thermally isolated during the isentropic processes performs to be an ideal thermal/heat selective conductor that transmits heat between the WF and each of the two, required to realize the cycle, heat reservoirs: the high "at constant temperature T_H" and low "at constant temperature T_L"). Thus, the cylinder and piston in the Carnot machine do not dissipate and not absorb any heat. Also, this arrangement does not allow any mass transfer between the WF inside the piston and its cylinder and the surroundings (no mass leakage is allowed).

In Figure 5-4 a *p_v* diagram for Carnot cycle is represented (subfigure A), and under it four states of ideal engine's piston–cylinder arrangement that carries out the cycle, which are:

- One of the states of the isothermal expansion process (1–2) is shown in subfigure (1).
- One of the states of the isentropic expansion process (2–3) is shown in subfigure (2).
- One of the states of the isothermal compression process (3–4) is shown in subfigure (3).
- One of the states of the isentropic compression process (4–1) is shown in subfigure (4).

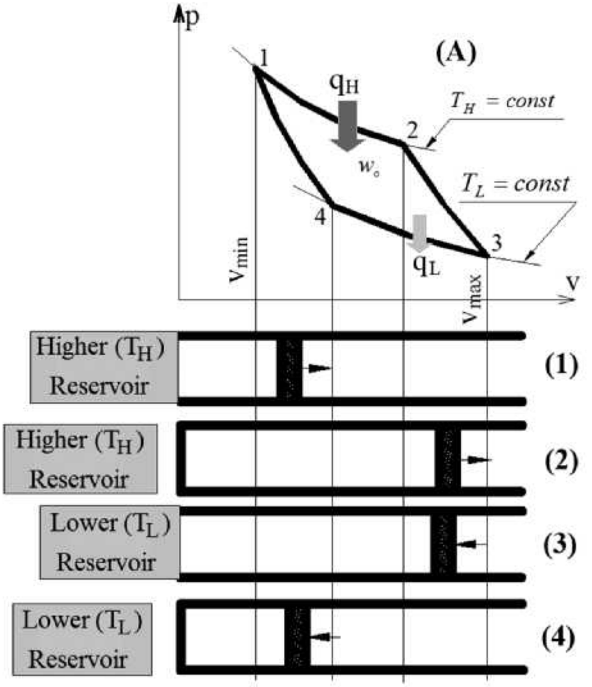


Figure 5.4. The Carnot cycle

The thick lines of the cylinder wall and the piston denote perfect isolation of both of them. When the WF comes into contact with the heat reservoir, the line/surface that separates the WF from the reservoir becomes thin denoting to the fact that the heat transfer between them is perfect.

Let 1 kg of the WF is enclosed in the piston–cylinder arrangement perfectly thermally isolated from the surroundings so that (with the exception of heat reservoirs) it is impossible for *the constant mass WF* to exchange heat with the surroundings and even with elements of the engine itself. ***In this ideal engine all processes proceed reversibly by our will***. *By our will* the heat reservoir, which we want, comes into perfect contact with the WF and an ideal heat transfer process is activated between the WF and the reservoir. And *by our will* the heat reservoir gets out of the contact with the WF and the WF becomes perfectly isolated.

The ***reversible*** Carnot cycle consists of two isotherms and two isentrops:

1) The isothermal expansion process that starts from WF's state 1 (v_1, T_H) and ends at state 2 (v_2, T_H) consumes the heat (see Equation 3-32 and OR4-6):

$$q_{1-2} = T_H \cdot (s_2 - s_1) > 0 \quad \textbf{and} \quad s_2 > s_1 \quad \textbf{because} \quad T_H > 0 \qquad (5\text{-}13)$$

That is specific transferred heat because we assumed that the amount of the WF inside the engine is 1 kg.

2) The adiabatic expansion process that starts from WF's state 2 (v_2, T_H) and ends at state 3 (v_3, T_L). This process is isentropic because it is reversible. Its equations (see Equations (4-1a) and (3-21)) are

$$ds = 0 \qquad \text{and} \qquad s_3 - s_2 = 0 \qquad \text{or} \qquad s_3 = s_2 \qquad (5\text{-}14)$$

$$q_{2-3} = \int_2^3 T \cdot ds = \int_2^3 T \cdot 0 = 0 \qquad (5\text{-}15)$$

3) The isothermal compression process that starts from WF's state 3 (v_3, T_L) and ends at state 4 (v_4, T_L) rejects the heat (see Equation 3-32 and "OR4-6"):

$$q_{3-4} = T_L \cdot (s_4 - s_3) < 0 \qquad (5\text{-}16)$$

4) The isentropic compression process that starts from WF's state 4 (v_4, T_L) and ends at state 1 (v_1, T_H). The equations of this process (see Equations (4-1a) and (3-21)) are

$$ds = 0; \qquad s_1 - s_4 = 0 \qquad (5\text{-}17)$$

$$\text{And} \quad q_{4-1} = \int_4^1 T \cdot ds = \int_4^1 T \cdot 0 = 0 \qquad (5\text{-}18)$$

From Equations (5-14) and (5-17) we obtain

$$s_2 - s_1 = s_3 - s_4 \tag{5-19}$$

We note here that the above heat exchange processes in the cycle are limited to/in two types: the WF gaining heat from the high reservoir in the isothermal process 1–2, and the WF rejecting heat to the low reservoir that is mostly the surroundings in the isothermal process 3–4. This means that the Carnot cycle is nonregenerative (the perfect proof is driven later “see the explanation of subfigure 5-12f”), and therefore $q_H = q_1$ and $q_L = q_2$ (see Equations 5-10a) and the equations suitable to calculate its thermal efficiency can be extracted from multiequation (5-11) after replacing each of Q_R (q_R) by zero or from multiequation (5-12). Thus, the following multiequation is suitable for calculating the thermal efficiency of the reversible Carnot cycle and other nonregenerative reversible cycles:

$$\begin{aligned}\eta_{t_{\text{no-reg}}} &= \frac{q_\circ}{q_H} = \frac{w_\circ}{q_H} = \frac{q_H - q_L}{q_H} = 1 - \frac{q_L}{q_H} = \frac{q_\circ}{q_1} \\ &= \frac{q_1 - q_2}{q_H} = \frac{q_1 - q_2}{q_1} = \frac{q_{1\text{alg}} + q_{2\text{alg}}}{q_{1\text{alg}}} = 1 - \frac{q_2}{q_1}\end{aligned} \tag{5-19a}$$

Here, q_2 is the specific rejected heat to the low reservoir that is, for heat engines, the surroundings. Therefore, q_2 is the absolute value of the amount of heat given to (actually taken of) the WF during its heat rejection process (3–4).

Now we can calculate $w_\circ$, q_1, and q_2 for the ideal gas Carnot cycle. The cycle consists of one pure positive heat transfer process (1–2), one pure negative heat transfer process (3–4), and two pure neutral heat transfer processes (2–3 and 4–1). From Equations (4-7), (5-7), and (5-13) through (5-18), we obtain

$$\begin{aligned}w_\circ = q_\circ &= \sum_{\text{cycle}} q_{\text{PPs}} = q_{1-2} + q_{2-3} + q_{3-4} + q_{4-1} \\ &= T_H \cdot (s_2 - s_1) + 0 + T_L \cdot (s_4 - s_3) + 0 \\ &= (T_H - T_L) \cdot (s_2 - s_1) = (T_H - T_L) \cdot \Delta s_{T_H} > 0\end{aligned} \tag{5-20}$$

From Equations (5-5), (5-6), (5-13), and (5-16), we obtain

$$q_1 = q_{1\text{alg}} = q_{1-2} = T_H \cdot (s_2 - s_1) = T_H \cdot \Delta s_{T_H} > 0 \tag{5-21}$$

$$\left.\begin{aligned} q_{2\,alg} = q_{3-4} &= T_L \cdot (s_4 - s_3) = T_L \cdot \Delta s_{T_L} < 0 \\ \text{and}\quad q_2 = \left|q_{2\,alg}\right| &= T_L \cdot (s_3 - s_4) = T_L \cdot \Delta s_{T_H} > 0 \end{aligned}\right\} \tag{5-22}$$

From Equations (5-19) through (5-22) we obtain

$$w_\circ = q_\circ = T_H(s_2 - s_1) + T_L(s_4 - s_3) = q_1 - q_2 = q_{1\,alg} + q_{2\,alg} > 0 \tag{5-23}$$

Applying Equations (5-21) and (5-22) in Equation (5-12) we obtain $\eta_{t\,Carnot} = 1 - \frac{q_2}{q_1} = 1 + \frac{q_{2\,alg}}{q_1} = 1 - \frac{T_L \cdot (s_3 - s_4)}{T_H \cdot (s_2 - s_1)}$ and applying Equation (5-19) in this equation we obtain:

$$\eta_{t\,Carnot} = 1 - \frac{q_2}{q_1} = 1 + \frac{q_{2\,alg}}{q_{1\,alg}} = 1 - \frac{q_L}{q_H} = 1 - \frac{T_L}{T_H} = 1 - \frac{T_2}{T_1} \tag{5-24}$$

Let us represent the Carnot cycle in *T_s* diagram (Figure 5-5). From T_1 and p_1 we obtain s_1 and then we create point 1 in Figure 5-5. Point 1 in the diagram represents the initial state of the ideal gas inside the cylinder. The isothermal expansion process that starts from point (1) is represented by line (1–2), and the specific gained heat (q_1) transferred from the high reservoir to the WF can be calculated by Equation (5-13) or measured by the area 1–2–5–6–1 (or area *a* + *b*) under line (1–2). Thus, $q_1 = T_H \cdot (s_2 - s_1) = a + b$.

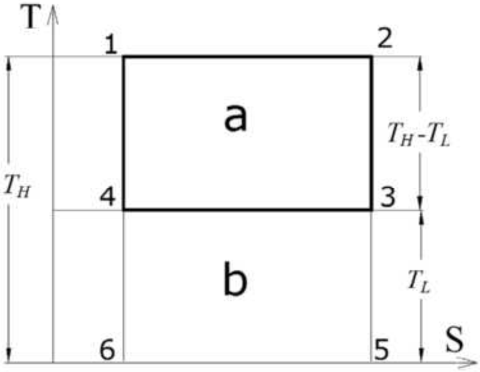


Figure 5.5. The Carnot cycle in T_s diagram

The following process (the isentropic expansion process) is represented by the *s* = constant line (2–3). The isothermal compression starts at point (3). It is entropy-decreasing process and is represented by line (3–4). During this process, the WF rejects to the low reservoir the specific rejected heat (q_2) that can be calculated by the equation $q_2 = |q_{3-4}| = q_{4-3} = T_L \cdot (s_3 - s_4) = b$ or measured by the area 4–3–5–6–4 (or area b) under line (4–3). Thus, $q_2 = T_L \cdot (s_3 - s_4) = b$.

Point (4) that represents the final state of the isothermal compression must be chosen so that the isentropic compression process passing from it passes through point (1), that is, so that the four processes form a closed process (a cycle). The difference between the areas ($a + b$) and (b), that is, area (a), measures the specific transformed heat into work. The thermal efficiency of the cycle, being nonregenerative, is calculated from Equation (5-12) and is equal to the ratio of area (a) by area ($a + b$). As the bases of these rectangles are equal, then their ratio is equal to the ratio of their heights or $\eta_t = \frac{a}{a+b} = \frac{\overline{1-4}}{\overline{1-6}} = \frac{T_H - T_L}{T_H}$. Thus, we came to the same Equation (5-24) obtained above.

5.6 THE REVERSE CARNOT CYCLE

The ***Carnot machine*** here (see §5.5) operates according to the reverse Carnot cycle. This machine performs the reversible reverse Carnot cycle and its only difference from the Carnot engine lies/is in the order of the four processes of the cycle, and in the results of such rearrangement (the functions of the heat reservoirs and direction of the cycle will be inverted/reflected).

We will explain this cycle very briefly in order to avoid repeating the explanation of the direct cycle.

In Figure 5-6 a *p_v* diagram for the reverse reversible Carnot cycle is represented (subfigure A), and under it four states of the piston–cylinder arrangement of the ideal Carnot machine that carries out the cycle.

Here also we assume that 1 kg of the WF is enclosed in the piston–cylinder arrangement.

The ***reversible reverse*** Carnot cycle consists of two isotherms and two isentrops:

1) The isentropic expansion process that starts from WF's state 1 (v_1, T_H) and ends at state 2 (v_2, T_L). The equations of this process (see Equations (4-1a) and (3-21) are

$$ds_{1-2} = 0 \quad \text{and} \quad s_2 - s_1 = 0 \quad \text{or} \quad s_2 = s_1 \quad \text{and} \quad T_2 < T_1 \qquad (5\text{-}25)$$

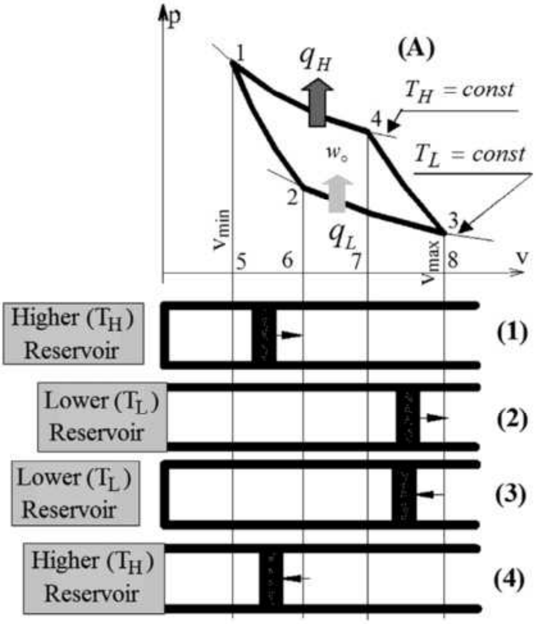


Figure 5.6. The reverse Carnot cycle

And the specific transferred heat to the WF is

$$q_{1-2} = \int_1^2 T \cdot ds = \int_1^2 T \cdot 0 = 0 \tag{5-26}$$

2) The isothermal expansion process that starts from WF's state 2 (v_2, T_L) and ends at state 3 (v_3, T_L) consumes the heat (see Equation 3-32 and OR4-6):

$$q_{2-3} = T_L \cdot (s_3 - s_2) > 0 \quad \text{and} \quad s_3 > s_2 \quad \text{because} \quad T_L > 0 \tag{5-27}$$

3) The isentropic compression process that starts from WF's state state 3 (v_3, T_L) and ends at state 4 (v_4, T_H). The equations of this process (see Equations (4-1a) and (3-21) are

$$ds_{3-4} = 0 \text{ and } s_4 - s_3 = 0 \text{ or } s_4 = s_3 \text{ and } T_3 < T_4 \tag{5-28}$$

And the specific transferred heat to the WF is

$$q_{3-4} = \int_3^4 T \cdot ds = \int_3^4 T \cdot 0 = 0 \tag{5-29}$$

From Equations (5-25 and 5-28) we obtain:

$$s_4 - s_1 = s_3 - s_2 \qquad \text{(5-29a)}$$

4) The isothermal compression process that starts from WF's state 4 (v_4, T_H) and ends at state 1 (v_1, T_H) rejects the heat (see Equation 3-32 and "OR4-6"):

$$q_{4-1} = T_H \cdot (s_1 - s_4) < 0 \quad \text{and} \quad s_4 > s_1, \quad \text{because} \quad T_H > 0 \qquad \text{(5-30)}$$

Let us create the reverse Carnot cycle that created in Figure (5-6) in T_s diagram (see Figure 5-7). From T_1 and p_1 we obtain s_1 and then we create point 1 in Figure 5-7. Point 2 lies under point 1 on the created from point 1 isentrop (see Equation 5-25) and point 3 lies to the right from point 2 on the created from point 2 isotherm (see Equation 5-27). Point 4 is the intersection point of the isentrop created from point 3 and the isotherm created from point 1.

It is clear from Figure (5-6) that the WF exchanges heat with only two bodies (reservoirs), which means (see items "1 through 4") that:

$$Q_1 = Q_H, \quad q_1 = q_H, \quad Q_2 = Q_L, \quad \text{and} \quad q_2 = q_L \qquad \text{(5-31)}$$

From relations (4-7, 5-7, 5-26, 5-27, 5-29 and 5-29a) we obtain the algebraic values of the imported to the WF specific useful work and heat $w_{\circ alg}$ & $q_{\circ alg}$:

$$w_{\circ alg} = q_{\circ alg} = \sum_{cycle} q_{PPs} = 0 + T_L \cdot (s_3 - s_2) + 0 + T_H \cdot (s_1 - s_4) \qquad \text{Or:}$$

$$w_{\circ alg} = q_{\circ alg} = \sum_{cycle} q_{PPs} = -(T_H - T_L) \cdot (s_3 - s_2) < 0 \qquad \text{(5-32)}$$

which means that the reverse Carnot machine consumes work (mechanical energy), and therefore, it is not an engine.

From relations (5-26, 5-27, 5-29 and 5-30) we obtain that the reverse Carnot cycle includes:

- Only one WF heat importing (gaining) process (2-3) and therefore (see Equation 5-27):

$$q_{2alg} = q_{Lalg} = q_{2-3} = T_L \cdot (s_3 - s_2) > 0 \qquad \text{(5-33)}$$

- Only one WF heat exporting (rejecting) process (4-1) and therefore (see Equation 5-30):

$$q_{1alg} = q_{Halg} = q_{4-1} = T_H \cdot (s_1 - s_4) < 0 \tag{5-34}$$

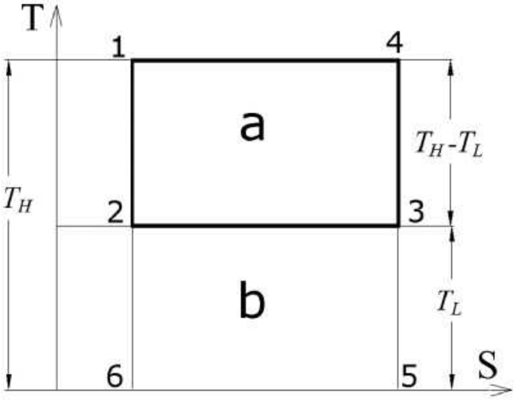


Figure 5.7. The reverse Carnot cycle in T_s diagram

Returning to Equation (5-2) the VEI for a reverse cycle is cycle's consumed $(W_\circ)$ that was additionally introduced above as cycle's imported net work $W_{net_{im}}$ (see Table 5-1 and Nt5-2A): The relation between cycle's imported net work symbols is:

$$W_{net_{im}} = W_\circ = \left|W_{\circ alg}\right| = m \cdot w_{net_{im}} = m \cdot w_\circ = -m \cdot w_{\circ alg} = m \cdot \left|w_{\circ alg}\right|$$

On the other hand, the VEO for the same reverse cycle can be either its imported high MRs' heat $(Q_{H_{im}})$ or its exported low MRs' heat $(Q_{L_{ex}})$, (see Table 5-1 and Nt5-2A).

The relation between the symbols of cycle's imported high MRs' heat is:

$$\begin{aligned} Q_{H_{im}} &= Q_1 = Q_H = \left|Q_{1alg}\right| = \left|Q_{Halg}\right| \\ &= m \cdot q_{H_{im}} = m \cdot q_1 = m \cdot q_H = m \cdot \left|q_{1alg}\right| = m \cdot \left|q_{Halg}\right| \end{aligned}$$

And the relation between the symbols of cycle's exported low MRs' heat is:

$$\begin{aligned} Q_{L_{ex}} &= Q_2 = Q_L = \left|Q_{2alg}\right| = \left|Q_{Lalg}\right| \\ &= m \cdot q_{L_{ex}} = m \cdot q_2 = m \cdot q_L = m \cdot \left|q_{2alg}\right| = m \cdot \left|q_{Lalg}\right| \end{aligned}$$

For the reverse Carnot cycle [see Equations (5-31), (5-33), and (5-34)], the VEO can be:

Either equation $$q_1 = q_H = \left|q_{1\,alg}\right| = \left|q_{H\,alg}\right| = T_H \cdot (s_4 - s_1) > 0 \tag{5-35}$$

Or equation $$q_2 = q_L = \left|q_{2\,alg}\right| = \left|q_{L\,alg}\right| = T_L \cdot (s_3 - s_2) > 0 \tag{5-35a}$$

From the above we conclude that *(1) in the Carnot engine, heat is imported to the WF from the high reservoir, transformed partially to mechanical energy that is exported to the mechanical energy consumer, and the nontransformed heat is rejected to the low reservoir that is usually the surroundings. (2) In the Carnot reverse machine, the mechanical energy is imported to the WF and transformed into heat that is added to the heat that is imported from the low heat reservoir/MR, and this sum of heats is transferred to the high reservoir. Here according to the purpose of the machine the surroundings can be either the high reservoir as in the refrigerating machines or the low reservoir as in the heat pumps.*

The purpose of the refrigerating machine is to cool their inside space to lower than the surroundings' temperature by transferring the heat Q_2 (q_2) from the cold space inside the refrigerator (the low heat reservoir/MR) to the *surroundings (the high reservoir).*

When refrigerator's motor is operating, its mechanical energy $W_{\circ}$ is imported to the WF, transformed into heat $Q_{\circ}$, and the sum $Q_{\circ} + Q_2 = Q_1$ is transferred/thrown to the surroundings. This can be rewritten for a unit mass of the WF as: $w_{\circ} = q_{\circ} = \left|w_{\circ\,alg}\right| = \left|q_{\circ\,alg}\right| = q_1 - q_2$

Thus, applying Equations (5-29a), (5-35), and (5-35a) in this equation we obtain:

$$\begin{aligned} w_{\circ} = q_{\circ} &= \left|w_{\circ\,alg}\right| = \left|q_{\circ\,alg}\right| = (T_H - T_L)\cdot(s_3 - s_2) \\ &= q_1 - q_2 = q_H - q_L > 0 \end{aligned} \tag{5-36}$$

According to the above and taking in consideration Equation (5-34), Equation (5-2) for refrigeration becomes:

$$\begin{aligned} \beta = \frac{Q_L}{\left|W_{\circ\,alg}\right|} &= \frac{Q_L}{Q_H - Q_L} = \frac{q_L}{\left|w_{\circ\,alg}\right|} = \frac{q_L}{q_H - q_L} \\ &= \frac{T_L}{T_H - T_L} = \frac{1}{(T_H / T_L) - 1} \end{aligned} \tag{5-37}$$

From Equation (5-37) we conclude that increasing T_L increases β, and for a particular value of T_L the efficiency of the process increases with decreasing T_H.

The purpose of the heat pump is to heat the inside space of the heated hall to higher than the surroundings' temperature by transferring to it the heat Q_1 (q_1) that is the sum of (1) the heat transferred from the atmosphere/surroundings (the low heat reservoir/MR) to the WF heat (Q_2), and (2) the useful heat ($Q_\circ$) transformed from the imported to the WF mechanical energy ($W_\circ$). Therefore, ***Q_1 is the product of the heat pump*** and accordingly ***is its VEO***.

To rewrite Equation (5-2) for heat pumps, we replace in this equation (1) the ETE by the heating factor (β'), (2) the VEO by the exported to the high reservoir/MR heat $q_1 = q_H$ ($Q_1 = Q_H$), and (3) the VEI by the imported mechanical energy $w_\circ = \left|w_{\circ alg}\right|$ or $W_\circ = \left|W_{\circ alg}\right|$.

According to the above and taking in consideration Equation (5-36), Equation (5-2) for refrigeration becomes:

$$\beta' = \frac{Q_H}{\left|W_{\circ alg}\right|} = \frac{Q_H}{Q_H - Q_L} = \frac{q_H}{\left|w_{\circ alg}\right|}$$

$$= \frac{q_H}{q_H - q_L} = \frac{T_H}{T_H - T_L} = \frac{1}{1-(T_L / T_H)} \tag{5-38}$$

Subtracting Equation (5-37) from Equation (5-38) we obtain:

$$\beta' - \beta = 1 \tag{5-39}$$

The achievement of ideal gas machines operating in accordance with the Carnot cycle or its reverse is impossible. Real heat engines, refrigerating machines, and heat pumps use other cycles. This is because of the impossibility to achieve ideal gas isothermal process in real machines.

Nt5-7: If we refer to the entropy change during the isothermal process at T_H by $\Delta s_{T_H} = (s_{final} - s_{initial})_{T_H}$ and to that at T_L by $\Delta s_{T_H} = (s_{final} - s_{initial})_{T_L}$ we obtain (see Figures 5-5 and 5-7):

From Equations (5-20) and (5-32):

$$w_{\circ alg} = q_{\circ alg} = (T_H - T_L) \cdot \Delta s_{T_H} \tag{5-40}$$

From Equations (5-21) and (5-34): $q_{1alg} = T_H \cdot \Delta s_{T_H}$ (5-41)

From Equations (5-22) and (5-33): $q_{2\,\mathrm{alg}} = T_L \cdot \Delta s_{T_L}$ (5-42)

As $T_H = T_{\max}$, $T_L = T_{\min}$, and $T_H - T_L$ are always positive, the signs of $w_{o\,\mathrm{alg}} = q_{o\,\mathrm{alg}}$ and $q_{1\,\mathrm{alg}}$ (see Equations "5-40, 5-41, and 5-42") are the same as that of Δs_{T_H} and the sign of $q_{2\,\mathrm{alg}}$ is the same as that of Δs_{T_L}, where (see Figures 5-5 and 5-7) Δs_{T_H} is positive for the direct Carnot cycle and negative for the indirect one, while Δs_{T_L} is negative for the direct Carnot cycle and positive for the indirect one.

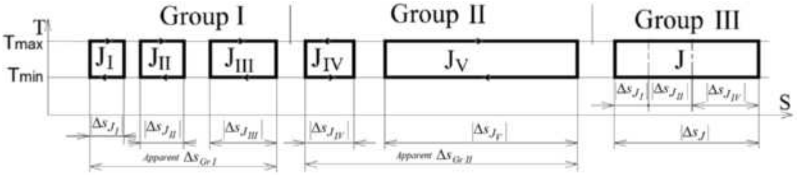


Figure 5.7A. Carnot cycles that share the same temperature extremes ($T_{\max}$ and $T_{\min}$). J_{IV} is the only reverse cycle

> **<u>HR5-1:</u>** The groups of Carnot (direct and/or reverse) cycles whose elements ***share the same maximum and minimum temperatures*** (whose elements operate over the same temperature interval) and share the same algebraic sum of areas (entropy changes) of their either upper or lower isothermal processes $\sum \Delta s_{T_{\max}}$ or $\sum \Delta s_{T_{\min}}$ are invariably completely equivalent groups. In Figure 5-7A, the six Carnot cycles J_I, J_{II}, J_{III}, J_{IV}, J_V, and J have/share the same temperature extremes ($T_{\max}$ and $T_{\min}$), while cycles J_I, J_{II}, and J_{III} constitute the first group (symbol gr_I), cycles J_{IV} and J_V constitute the second group (symbol gr_{II}), and cycle J constitutes the third group (symbol gr_{III}) of Carnot cycles. The three groups have/share the same algebraic sum of entropy changes of their upper isothermal processes (at $T_{\max}$), therefore, these three groups are completely equivalent ones.

In direct and reverse reversible/internally reversible Carnot cycles $T_H = T_{\max}$ and $T_L = T_{\min}$.

In area calculations of thermodynamic cycles we differentiate between the belonging to the process/cycle (*live/effective/actual*) areas and those occupied by cycle's parts on property diagram (***apparent***) ones.

5.7 INTRODUCTION TO CARNOT THEOREM (EXISTING FORMULATIONS OF CARNOT THEORY)

Authors agree with that Sadi Carnot performed a set of proofs and important conclusions relating to his thermodynamic cycle and which were crowned by the theory of S. Carnot that stated by himself as follows:

No engine can be more efficient than a Carnot engine

But authors do not agree on naming the mentioned conclusions. What is called by some authors ***Principle*** or ***Corollary*** some others call it ***Proposition*** or ***The Carnot efficiency***, as others consider it a basic part of the Carnot theory. Whatever the case is, and even if some of the proofs are incomplete, the Carnot proofs led to the following results:

1) The **First Carnot Principle (Carnot's first corollary)**
   The thermal efficiency of an irreversible power cycle is always less than the thermal efficiency of a reversible power cycle when each operates between the same two reservoirs.
2) *The efficiency of the Carnot reversible cycle does not depend on the nature of the WF, used for executing the cycle.*
3) The **Second Carnot Principle (Carnot's second corollary)**
   All reversible power cycles operating between the same two thermal reservoirs have the same thermal efficiencies.
4) The thermal efficiency of any reversible cycle, executed with more than two reservoirs, is less than the thermal efficiency of the reversible Carnot cycle, executed between the two temperature extremes.
5) Thermal efficiency of a Carnot reversible cycle, operating between two thermal reservoirs, is the highest efficiency that a reversible power cycle operating between two temperatures can have.

Statements 4 and 5 have the same proof, and carry the same meaning, but statement 5 is more inclusive than statement 4.

The Carnot theory/theorem has several statements resulting from combining the statements, which we have introduced above that is given by Carnot himself, and we will cite here the following statement:

Carnot's Theorem: All Carnot engines that operate between the same temperature reservoirs have the same efficiency, and no other heat engine operating between these same two temperatures can have a greater efficiency.

As we will see later (see §7.6), the proof of the fifth statement was imperfect and its new perfection did not affect Carnot's conclusions.

5.8 ENTROPY

Some details of discussing the effectivenesses of thermodynamic cycles are not classical (are unusual). These details were used for the first time in the base book and are used in this book. The energy terms of the effectiveness expressions ($Q_1, Q_2, Q_o, W_o, q_1, q_2, q_o, w_o$ and so on) that are usually positive are expressed through the absolute values ($|Q_{1alg}|$, $|Q_{2alg}|$, $|Q_{oalg}|$, $|W_{oalg}|$, $|q_{1alg}|$, $|q_{2alg}|$, $|q_{oalg}|$ and $|w_{oalg}|$) of their algebraic values (Q_{1alg}, Q_{2alg}, Q_{oalg}, W_{oalg}, q_{1alg}, q_{2alg}, q_{oalg} and w_{oalg}). This measure does not add any new idea although it, as the author hopes, improves readers understanding.

Using the (1) rules that resulted from the study of the two reversible Carnot cycles (direct and reverse) and (2) the second result that established to Carnot theory/theorem (see previous paragraph), it can be proved that for any substance there is a very important, in/for studying heat transfer processes, property, which is the entropy.

Previously we obtained the following relationship (5-24) that belongs to the ideal gas reversible Carnot cycle and, which, thanks to the second result of (§5.7) that will be proved in volume II of this book (see §7.7), became valid for the reversible Carnot cycle operating on/with any gas (WF): $\eta_{tCarnot} = 1 - \frac{q_2}{q_1} = 1 + \frac{q_{2alg}}{q_{1alg}} = 1 - \frac{T_2}{T_1}$. From this equation we obtain:

$$\frac{q_2}{q_1} = -\frac{q_{2alg}}{q_{1alg}} = \frac{T_2}{T_1} \tag{5-43}$$

Nt5-8: In direct thermodynamic cycles $q_{1alg} = q_1 > 0,\ q_{2alg} = -q_2 < 0$, and accordingly $(q_{2alg}/q_{1alg}) < 0$, while in reverse cycles $q_{1alg} = -q_1 < 0,\ q_{2alg} = q_2 > 0$, and accordingly $(q_{2alg}/q_{1alg}) < 0$. Therefore, in all thermodynamic cycles $(q_{2alg}/q_{1alg}) < 0$ (see Nt5-6 and §s 5.4.1 and 5.4.2).

From Equation (5-43) we obtain:

$$\left.\begin{array}{lcl} \frac{q_2}{T_2} = \frac{q_1}{T_1}, & & \frac{q_{2alg}}{T_2} = -\frac{q_{1alg}}{T_1} \\ \frac{q_1}{T_1} - \frac{q_2}{T_2} = 0 & \text{and} & \frac{q_{1alg}}{T_1} + \frac{q_{2alg}}{T_2} = 0 \end{array}\right\} \quad \text{or} \tag{5-44}$$

Here, q_{1alg} and q_{2alg} are the algebraic values of ***specific cycle's high WF heat*** and ***specific cycle's low WF heat,*** respectively. In ***direct cycles,*** q_{1alg}, its parts, and differentials are always positive, and q_{2alg}, its parts, and differentials are always negative because, when the cycle is divided into pure processes, q_{1alg} is the sum of all pure positive transferred to the WF, heats, and q_{2alg} is the sum of all pure negative transferred to the same WF heats. This means that ***for the direct Carnot cycle*** we can (1) identify q_{1alg} and q_{2alg} from the algebraic sign of the pure transferred heat q_{pure}, (2) express $\frac{q_{1alg}}{T_1}$ by $(q/T)_{s\uparrow}$, and $\frac{q_{2alg}}{T_2}$ by $(q/T)_{s\downarrow}$, (3) substitute these values in (5-43), and (4) obtain

$$\frac{q_{1alg}}{T_1}+\frac{q_{2alg}}{T_2}=\left(\frac{q}{T}\right)_{s\uparrow}+\left(\frac{q}{T}\right)_{s\downarrow}=\sum_{i=1}^{2}\frac{q_{ialg}}{T_i}=0$$

The discussion above was for the direct Carnot cycle. If we repeat it for the ***reverse Carnot cycle***, where $q_1=-q_{1alg}>0$, $q_2=q_{2alg}>0$, and accordingly $\frac{q_{1alg}}{T_1}=\left(\frac{q}{T}\right)_{s\downarrow}$ and $\frac{q_{2alg}}{T_2}=\left(\frac{q}{T}\right)_{s\uparrow}$, we obtain

$$\frac{q_{1alg}}{T_1}+\frac{q_{2alg}}{T_2}=\left(\frac{q}{T}\right)_{s\downarrow}+\left(\frac{q}{T}\right)_{s\uparrow}=\sum_{i=1}^{2}\frac{q_{ialg}}{T_i}=0$$

From the two equations above we obtain for the Carnot cycle and its reverse one:

$$\frac{q_{1alg}}{T_1}+\frac{q_{2alg}}{T_2}=\sum_{i=1}^{2}\frac{q_{ialg}}{T_i}=0 \qquad (5\text{-}45)$$

It is more convenient to execute (carry out) the analysis here as many other analyses using sine rules type (OR3-6) that is almost used throughout this book.

Equation (5-45) can be read/expressed as follows: for a reversible Carnot cycle, the algebraic sum of the fractions (resulted from dividing each of the transferred to the WF heats by the absolute temperature at which the heat transfer is done/carried out) equal to zero. Or:

HD5-3B: ***The relative heat is the product of dividing the transferred to the gas/WF heat by the absolute temperature at which the heat***

transfer is done (carried out). It is obvious that ***the relative heat is a characteristic of the isothermal process.***

<u>HR5-2:</u> ***The algebraic sum of all relative heats in the Carnot cycle is equal to zero.***

<u>Nt5-9:</u> Because the Carnot cycle, upon which we built our above conclusions, was reversible (this means that $T_H = T_1$ and $T_L = T_2$), and nonregenerative (Carnot cycle is naturally no regenerative "unable for regeneration"), and this means that $q_H = q_1$ and $q_L = q_2$ we have chosen to deal with the transferred to the WF heats, and with its temperatures instead of dealing with the values belonging to the two heat reservoirs.

Now consider a common case reversible cycle (1–2–3–4–1), which is represented in Figure 5-8A. In the contrary to the nonregenerative Carnot cycle, the reversible common case cycle can be regenerative, and therefore, it can be carried out with heat regeneration, and this means that $T_H = T_1$, $T_L = T_2$, $q_H \le q_1$, and $q_L \le q_2$. *It must be differentiated here between the heats belonging to the WF, and those belonging to heat* reservoirs/MRs*, the thing which is unnecessary for temperatures.*

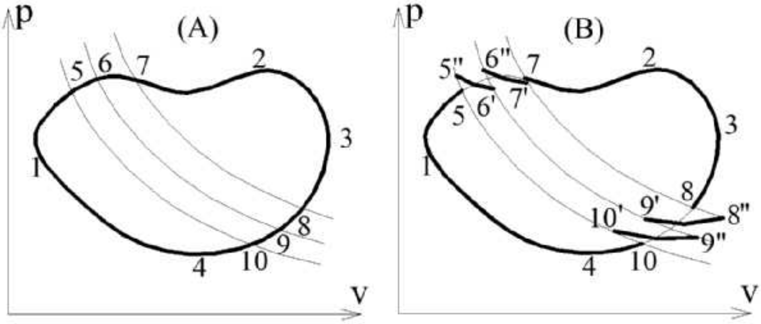


Figure 5.8. The set of partial cycles resulted from slicing a reversible cycle using reversible adiabates is equivalent to the original cycle. If the number of the slicing adiabates is huge, the slices become elementary Carnot cycles.

We must say the following about the way to achieve this cycle:

In the reversible Carnot cycle, heat was transferred to the WF at constant temperature, while in the common case cycle the heat is transferred to the WF at variable temperature. If we had in these conditions one highest heat reservoir/MR, with a temperature that equals the highest temperature of the WF, the heat would have been transferred in the remaining cases of transferring heat to the WF at finite (not infinitesimal) temperature differences, and such cycle is irreversible. To have a reversible cycle, the heat transfer between the heat reservoir/MR (heat accumulator) and the WF must proceed at infinitesimal temperature differences. Such heat transfer can be theoretically/imaginarily reached, when the WF temperature is variable, by using an infinite number of heat reservoirs/MRs and accumulators, which would have the same WF temperatures (or infinitesimally different from them) at all heat transfer stages of the cycle.[C] The same thing can be said about the low heat reservoirs/MRs: “In order to insure the reversibility of the WF heat rejection process, we can imagine that we have an ***unlimited number of low heat*** reservoirs/MRs that absorb heat from the WF, at infinitesimal differences between the WF temperatures and low reservoirs/MRs temperatures.”

C. As we will see later, the heat exchange between the WF and heat reservoirs/MRs at variable temperatures can be achieved in some reversible cycles by using those different from the infinite number of heat reservoirs/MR modes (such as the ideal heat regenerator).

Let us divide/slice our cycle to a number of partial cycles by the reversible adiabates 10-5, 9-6, 8-7, and so on. Then with the exception of the two extreme partial cycles, each partial cycle will consist of two reversible adiabates and two reversible heat exchange processes that may be isotherms or not. Thus, the original cycle (1–2–3–4–1) will consist of a set of partial reversible cycles (5–6–9–10–5), (6–7–8–9–6), and so on.

Let us first make sure that the obtained set of partial cycles (or slices) is equivalent to the original cycle (1–2–3–4–1).

To be able to replace the original cycle, the set of the obtained partial cycles must leave on each of the high set of heat reservoirs/MRs, the low set of heat reservoirs/MRs, and the recipients of mechanical energy, the same effects that the original cycle leaves on them. In other words, the following conditions must be met in the obtained set of partial cycles:

- The sum of all ***specific cycles’ high WF heats*** belonging to the resulted set of partial cycles equals to the original ***specific cycle’s high WF’s heat***:

$$q_{1\,a\,lg\,\text{original cycle}} = \sum_{\text{original cycle}} q_{1\,a\,lg\,\text{partial cycles}} \qquad (5\text{-}46)$$

- The sum of all ***specific cycles' low WF's heats*** belonging to the resulted set of partial cycles equals to the original ***specific cycle's low WF's heat***:

$$q_{2\,\mathrm{alg\,original\ cycle}} = \sum_{\mathrm{original\ cycle}} q_{2\,\mathrm{alg\,partial\ cycles}} \tag{5-47}$$

- The sum of all ***specific cycles' high MRs' heats*** belonging to the resulted set of partial cycles equals to the original ***specific cycle's high MRs' heat***:

$$w_{\circ\,\mathrm{alg\,original\ cycle}} = q_{\circ\,\mathrm{alg\,original\ cycle}} = \sum_{\mathrm{original\ cycle}} \left(q_{1\,\mathrm{alg}} + q_{2\,\mathrm{alg}}\right)_{\mathrm{partial\ cycles}} \tag{5-48}$$

In the partial cycles of the direct one, processes 5–6, 6–7, and so on are expansion ones, while processes 8–9, 9–10, and so on are compression ones. During cycle 5–6–9–10–5, the direction of adiabate 6–9 is from 6 to 9, while during cycle 6–7–8–9–6, the direction of the same adiabate will be from 9 to 6, that is, adiabate 6–9 will be common between the two partial cycle*s*, and throughout this adiabate the works in both partial cycle*s* are equal in absolute values and opposite in signs. As for the transferred heats, there is no transferred heat in both directions of the adiabatic process. Therefore, when adding the useful works in the neighboring partial cycles 5–6–9–10–5 and 6–7–8–9–6, the sum of the computed, for the common adiabate 6–9, expansion, and compression works will equal to zero. Thus, curves 6–9, as well as lines like it such as 7–8, 5–10, and so on, can fall out of the study for calculating the useful work and transferred heats, and in this case, it can be sufficient to deal with the heat exchange processes in the partial cycles, which are fully situated on the entire perimeter of the original cycle constituting the full curve of the original cycle. And when adding any kind of transferred heats in all studied partial cycles, we will obtain the value of this kind for the original cycle 1–2–3–4–1.

Let us convert the two partial cycles discussed above into two Carnot's cycles. For this purpose we will replace the heat exchange processes 5–6, 6–7, 8–9, and 9–10 in them by isothermal processes that pass from the middle of the original processes (see Figure 5-8B). We notice that the expansion adiabate 6'–9" is no longer fully coinciding with the compression adiabate 9'–6", and therefore, the adiabatic expansion work (in the Carnot cycle, located to the left of the adiabate) and the adiabatic compression work (in the Carnot cycle, located to the right of the adiabate) are not equal in absolute values. With decreasing the distance between neighboring adiabates , the heat exchange processes 5–6, 6–7, 8–9, and 9–10 approach more and more the alternative isothermal processes, and therefore the absolute values of the expansion work (in the Carnot cycle, located to the left of the adiabate) and the compression work (in the Carnot cycle, located to the right of the adiabate) approach each other more and more, and when the distance between the two neighboring adiabates limits to zero and the expansion adiabate 6'–9" coincides with the compression adiabate 9'–6", and therefore, the absolute values of the expansion work (in the Carnot cycle, located to the left of the isentrop) and the compression work (in the Carnot cycle, located to the right of the adiabate) equalize and the adiabatic processes of the neighboring partial Carnot cycles become congruent with the dividing adiabates of the original cycle (see Figure 5-8A). The above means that:

- The effect of the adiabates of the partial Carnot cycles that are congruent with the dividing adiabates of the original cycle on each of the elements of the original engine and the surroundings does not exist.
- The external borders (upper and lower) of the set of partial reversible Carnot cycles merge with the closed curve of the original cycle 1–2–3–4–1.

Thus, the processes of the partial Carnot cycles, which are congruent with the partition adiabates of the original cycle, could be dropped off the study for calculating the useful works and transferred heats. In this case, it can be sufficient to deal with the isothermal processes in the partial cycles, which are located entirely on the entire perimeter of the original cycle constituting the full curve of the original cycle. And when summing the useful works or any kind of transferred heats (useful or transferred to or from the WF) in all studied partial cycles we obtain the useful work (useful or transferred to or from the WF) in the original cycle (1–2–3–4–1).

The carried out analysis that was based on Figure 5-8 assumed that the original cycle (1–2–3–4–1) is a direct one. If we make points 2 and 4 exchange places, then the original cycle (1–2–3–4–1) and its parts (6"–9'–8"–7'–6") and (5"–10'–9"–6'–5") become reverse ones, reversible adiabates (8"–7', 9"–6') become compression ones, and 6"–9' and 5"–10' become expansion ones. Repeating the discussion for the reversed reversible cycle of that represented on Figure 5-8, we obtain the same Equations (5-45) through (5-48).

The above leads to the following conclusion: The set of the differential reversible Carnot cycles, resulting from the partition of a common case reversible thermodynamic cycle (RC) by an infinite number of reversible adiabates, is equivalent to the original cycle and leave the same effects left by the original cycle on each of the sets of high reservoirs/MRs, low reservoirs/MRs, heat accumulators, and the consumers/producers of mechanical energy; therefore and for the purpose of thermodynamic analysis, it is allowed to divide the original cycle by reversible adiabates into an infinite number of reversible differential Carnot cycles (RDCCs).

Now we can rewrite Equations (5-45) through (5-48) for a common case RC that is sliced by an infinite number of reversible adiabates into an infinite number of RDCCs.

For the sliced common case (direct or reverse) RC into RDCCs:

- The reversible original specific cycle's high WF's heat:

$$q_{1\,\mathrm{alg\,original\,RC}} = \int_{\mathrm{original\,RC}} \delta q_{1\,\mathrm{alg\,partial\,RDCCs}} \tag{5-49}$$

- The reversible original specific cycle's low WF's heat:

$$q_{2\,\mathrm{alg\,original\,RC}} = \int_{\mathrm{original\,RC}} \delta q_{2\,\mathrm{alg\,partial\,RDCCs}} \tag{5-50}$$

- The reversible original specific cycle's useful work:

$$w_{\circ\,\mathrm{alg\,original\,RC}} = q_{\circ\,\mathrm{alg\,original\,RC}} = \int_{\mathrm{original\,RC}} \left(\delta q_{1\,\mathrm{alg}} + \delta q_{2\,\mathrm{alg}}\right)_{\mathrm{partial\,RDCCs}} \tag{5-51}$$

For any RDCC (direct or reverse) sliced from the common case (direct or reverse) RC (see Equation 5-45):

$$\frac{\delta q_{1\,\mathrm{alg}}}{T_1} + \frac{\delta q_{2\,\mathrm{alg}}}{T_2} = \sum_{\mathrm{differential\,Carnot}} \frac{\delta q}{T} = 0 \tag{5-52}$$

T_1 and T_2 here are the highest and the lowest temperatures of the partial cycle, but not of the original one.

If we write for each differential Carnot cycle, sliced from the original reversible cycle, an equation that is similar to Equation (5-52), and create from all of them one equation so that its left side is the sum of the left sides of these differential equations and its right side is the sum of the right sides of them we obtain:

$$\oint_{RC}\left(\frac{\delta q_{1\,alg}}{T_1}+\frac{\delta q_{2\,alg}}{T_2}\right)=\oint_{RC}\frac{\delta q_{1\,alg}}{T_1}+\oint_{RC}\frac{\delta q_{2\,alg}}{T_2}+\oint_{RC}\frac{\delta q}{T}=0 \qquad (5\text{-}53)$$

which is the equation for the sliced, into RDCCs, common case (direct or reverse) RC.

We must note here that the two points (1 and 3) of the cycle in Figure 5-8A were chosen so that point (1) is located on the contact point (line) between the cycle and its left adiabatic tangent, while point (3) is located on the contact point (line) between the cycle and its right adiabatic tangent. By this choice, we enclosed all ***pure positive processes*** in the process (1–2–3), while all ***pure negative processes*** were enclosed in the process (3–4–1).

Choosing the location of points (1 and 3) according to the former paragraph allows to express the ratio $\frac{\delta q_{1\,alg}}{T_1}$ by one of the expressions $\left(\frac{\delta q}{T}\right)_{1-2-3}$ or $\left(\frac{\delta q}{T}\right)_{3-4-1}$ and the ratio $\frac{\delta q_{2\,alg}}{T_2}$ by the other expression, and thus we can express the sum of the two integrals $\oint_{RC}\frac{\delta q_{1\,alg}}{T_1}+\oint_{RC}\frac{\delta q_{2\,alg}}{T_2}$ by the sum of the two integrals $\int_{1-2-3}\left(\frac{\delta q}{T}\right)+\int_{3-4-1}\left(\frac{\delta q}{T}\right)$ that equals $\int_{1\text{-}2\text{-}3\text{-}4\text{-}1}\left(\frac{\delta q}{T}\right)=\oint_{RC}\frac{\delta q}{T}$.

Relations (5-53) can be expressed as follows:

<u>OR5-1</u>: ***For any reversible cycle the algebraic sum of all fractions (caused by dividing each of WF's transferred heats (differential and integral) by the absolute temperature at which this heat was transferred) equal to zero. Or:***

<u>HR5-3:</u> ***The algebraic sum of the relative heats in any reversible cycle equals zero.***

Let us symbolize $\oint_{RC}\frac{\delta q}{T}$ by Δs_{RC}, substitute it in relations (5-53) and obtain:

$$\Delta s_{RC} = 0 \tag{5-54}$$

which can be read as the entropy change (Δs) for a RC equals zero. Let us find out what does this mean.

In Figure 5-9 the reversible cycles (1–5–2–3–1 and 1–4–2–3–1) that share the reversible process (2–3–1) are represented.

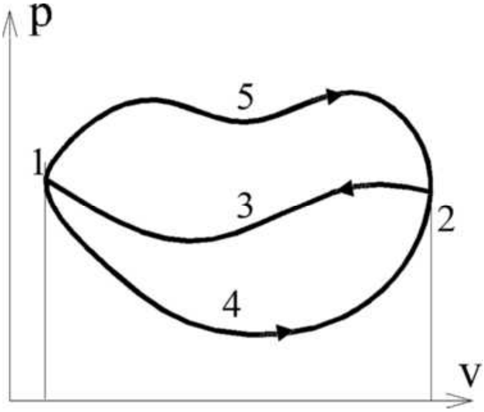


Figure 5.9. Two reversible cycles sharing the process 2-3-1

Applying Equation (5-54) on them we obtain:

$$\Delta s_{1-5-2-3-1} = \Delta s_{1-5-2} + \Delta s_{-2-3-1} = 0 \tag{5-55}$$

$$\Delta s_{1-4-2-3-1} = \Delta s_{1-4-2} + \Delta s_{-2-3-1} = 0 \tag{5-56}$$

from which we obtain $\Delta s_{1-5-2} = \Delta s_{1-4-2}$, which can be read as follows.

OR5-2: ***The s-change is initial and final states dependent, but not process path dependent***. This rule can be derived for all functions of state such as the internal energy and enthalpy for all gases.

According to the beginning of §2.8, ***s is a property of state for all gases,*** and since ***s for ideal gases*** was previously named (entropy), this name is used today for all gases.

Also, Equation (3-21) is now the definition of the entropy change in a differential reversible heat transfer process for any gas:

$$ds = \delta q / T \tag{5-57}$$

T, here, is the heat reservoir's/MR's *temperature. And because we assume that the temperature difference between the heat* reservoir/MR *and the WF is infinitesimal, T is also the WF's temperature.*

The entropy change in any reversible heat transfer process is calcul ated as:

$$\Delta s = s_2 - s_1 = \int_1^2 \frac{\delta q}{T} \qquad (5\text{-}58)$$

And the entropy change between any two common case states (1 and 2) is process path independent; therefore, *s*-value for a given state is determined by its state properties, without any need to know the path of the process through which the WF achieves this state and this means that *s* can be a property for this state.

As real gases are not of the interests of this book, we shall deliver in brief the following:

To calculate the value of the entropy of the WF at the state that is described, for example, by the pressure (*p*) and temperature (*T*), it must be agreed about the state for which s will be given the zero value, that is, which state will be considered a principle for measuring the entropy. As we, when studying physical state change processes in thermodynamics, are interested in entropy changes (not in *s* values), we do not care about which state will be chosen as a reference state to be the entropy principle. If the normal conditions (T_n and p_n) were chosen for the reference state then the entropy at *T* and *p* will be calculated using the equation:

$$s = \int_{p_n, T_n}^{p,T} \frac{\delta q}{T} \qquad (5\text{-}59)$$

For the reversible adiabatic process $dq = 0$ and $T \neq 0$, Equation (5-57) becomes $ds = \delta q / T = 0$ and its integral becomes $s = \text{constant}$. That is why the reversible adiabatic process that is defined by:

$$ds = 0 \quad \text{and/or} \quad s = \text{constant} \qquad (5\text{-}60)$$

is called the isentropic process.

For the isothermal process $T = \text{constant}$, *T*, in Equation (5-58), can be moved outside the integration signal, and then the result of integration will be:

$$q_T = T \cdot (s_2 - s_1) \qquad (5\text{-}61)$$

which is the same as Equation (3-32).

Equations (5-60) and (5-61) allow creating *T_s* diagrams of Carnot cycle and its reverse for all gases on the bases of their *p_v* diagrams (see Figures 5-4A and 5-6A). The result will be Figures 5-5 and 5-7.

Note that for real gases $w_t \neq q_t$ because $\left(\Delta u_t\right)_{\text{real gas}} \neq 0$.

The entropy is of particular importance when studying the cycles of internal combustion engines. Actually, if we create in *T_s* diagram a common case closed equilibrium (reversible) cycle (see Figure 5-10), then area 1–2–3–5–6–1 (area *a* + *b* or the area under curve 1–2–3 or $A_{1-2-3-5-6-1}$) measures the transferred, during the entropy increasing process 1–2–3, heat to the WF (q_1). Also, area 1–4–3–5–6–1 (area *b* or the area under curve 3–4–1 or $A_{1-4-3-5-6-1}$) measures the transferred, during the entropy decreasing process 3–4–1, heat from the WF (q_2). Thus, in cycle 1–2–3–4–1 the quantity of heat that is measured by area 1–2–3–4–1 (area *a* or $A_{1-2-3-4-1}$) disappears and will not exist as heat any more.

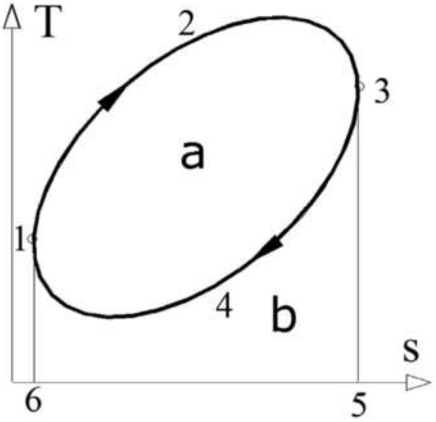


Figure 5.10. A common case closed equilibrium (reversible) cycle in T_s diagram.

It is clear from the above that encoding closed areas in diagrams by alphabetical letters located inside their boarders is the most comfortable method for graphical discussions. And we will adopt this method as a basic one for use in this book.

In paragraph (4-3-5), it was proved that the lost heat in the closed process transforms into useful work. From here we conclude that the area located inside the curve of the closed reversible process in *T_s* diagram measures the useful heat that is transformed into useful work.

According to the relationship (5-12): $\eta_{t\,\text{no-reg}} = \dfrac{q_1 - q_2}{q_1}$

As well as the meaning of areas in *T_s* diagram and from Figure 5-10:

$$\eta_{t\,no-reg} = \frac{A_{1-2-3-4-1}}{A_{6-1-2-3-5-6}} = \frac{a}{a+b} \tag{5-62}$$

<u>**OR5-3:**</u> *In T_s diagram, the ratio of the area, which measures the transformed heat into work during the cycle, to the area, which measures the transferred heat to the WF during the same cycle, is numerically equal to the thermal efficiency of the nonregenerative cycle.*

Thus, in addition to *p_v* diagram, *T_s* diagram as a result of the special features of the property entropy is an effective way to study the cycles of internal combustion engines. It is obvious from Figure 5-10 that:

OR5-4: Moving/displacing a thermodynamic cycle represented in *T_s* diagram horizontally without deforming or rotating does not affect its area, maximum temperature, minimum temperature, and the area under it. This leads to not affecting its added, rejected, and net heats, network, temperature interval, and thermal efficiency.

5.9 HEAT REGENERATION

From now on some differences can appear in the theoretical execution of some theoretical state change processes and cycles.

In OD5-3 we defined heat regeneration. To achieve heat regeneration, the following two conditions must be available:

- The cycle must be ***regeneratable*** (able for heat regeneration). Some configurations/forms of thermodynamic cycles do not allow heat regeneration, while others allow heat regeneration.
- The engine must be equipped with special devices called ***heat regenerator***.

Heat regeneration can be full $(Q_R = Q_{R_{max}})$, partial $(Q_{R_{max}} > Q_R > 0)$, or zero (without regeneration "$Q_R = o$"). In this last case the regeneration is absent and the cycle is ***nonregenerative cycle.*** A nonregenerative cycle can be ***regeneratable*** or ***nonregeneratable***.

The ratio $Q_R / Q_{R_{max}}$ is called the regeneration ratio and is symbolized by σ. It is also called the regenerator effectiveness and symbolized by ε.

Its value ranges between zero for the non***regenerative*** cycle and one for the ***regenerative*** cycle with full regeneration. The regeneration definitions about which we did not fully define yet are as follows:

5.9.1 BASIC CONCEPTS AND DEFINITIONS

5.9.1.1 The regeneratable cycle

The regeneratable cycle is the cycle having a shape/configuration that allows using heat regeneration. Depending on their shape/configuration, regeneratable cycles are classified into fully reversible, fully irreversible and partially reversible regeneratable ones. Also, depending on the availability of the heat regenerating means (e.g., regenerator) in the engine, the cycle can be regenerative or nonregenerative.

5.9.1.2 The fully reversible regeneratable cycle

5.9.1.2.1 The first-class regeneratable cycle

The first-class regeneratable cycle is the one that regenerates **the maximum possible for** its shape/configuration heat of regeneration, when executed reversibly. In this case the heat regeneration is also reversible.

5.9.1.2.2 The second-class regeneratable cycle

The second-class regeneratable cycle is the one that regenerates **only a part of the maximum possible for** its shape/configuration heat of regeneration, when executed reversibly. In this case the heat regeneration of the mentioned part is also reversible.

5.9.1.2.3 The third-class regeneratable cycle

The third-class regeneratable cycle is the one that its shape/configuration does not allow any heat regeneration, when executed reversibly. In this case the heat regeneration process is irreversible. Therefore, we can also define this cycle as the regeneratable one **with no ability for reversible heat regeneration.**

5.9.1.3 The regenerative cycle

The regenerative cycle is the regeneratable cycle that uses heat regeneration. The engine that operates on this cycle is provided with heat regenerating means (e.g., regenerator). This name is mostly associated with Rankine cycle.

5.9.1.4 The nonregenerative cycle

The nonregenerative cycle is the cycle without heat regeneration.

5.9.1.5 The nonregeneratable cycle

The nonregeneratable cycle is the cycle having a form that does not allow using regeneration. It is inevitably nonregenerative and we shall call it ***naturally nonregenerative***.

5.9.2 THE HEAT REGENERATOR

The heat regenerator is a heat exchanger used to increase the thermal efficiency of a power (engine) cycle by transferring heat—from the WF that is almost ready for heat rejection—to the WF requiring heat in another part of the cycle. Regenerators can be classified into intermittently operating regenerators (used actually in executing Stirling and Ericsson cycles) and continuously operating ones (used actually in Brayton regenerative cycle). Note that Stirling, Ericsson, and Brayton cycles will be introduced later.

Intermittently operating regenerators use intermediary objects (heat accumulators) to transfer heat from the exporting heat object (rejecting heat WF) to the importing heat one (gaining heat WF). These are less utilized than constantly operating regenerators.

The continuously/constantly operating regenerators are often counterflow heat exchanger type, in which the state change processes of the WFs participating in the regeneration process are isobaric ones.

The ideal heat regenerator is the regenerator, in which the heat transfer process from the rejecting heat WF to the wining heat WF is realized reversibly, that is, at an infinitesimal temperature difference (see Figure 5-11). The most commonly used example in explaining the real/ideal heat regenerator is the counterflow heat exchanger in which the state change processes of the WFs that participate in the heat regeneration are isobaric ones. This heat regenerator belongs to the continuously operating regenerators. It is easy to be realized and to analyze its function when it operates as an ideal heat regenerator. When calculating this type of regenerators, we assume that (1) The regenerator is perfectly thermally insulated from the outside, and that the heat is transferred only within the cross-section of the regenerator in the direction of the arrows, and (2) The cooling process of the heated WF and the heating process of the cooled WF are isobaric ones.

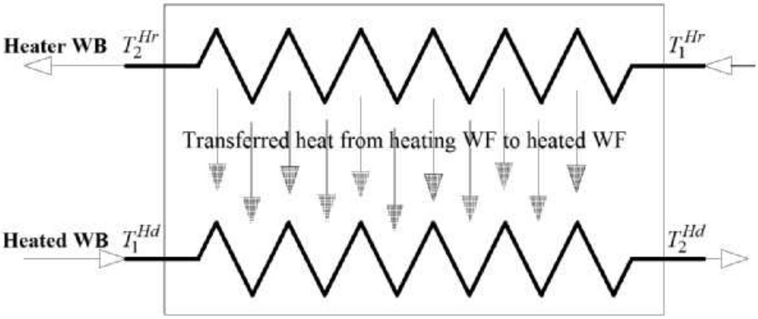


Figure 5.11. The ideal heat regenerator

In addition to the continuously operating regenerator of the counterflow heat exchanger type that is actually used in Brayton regenerative cycle, the intermittently operating regenerators that use intermediate bodies (heat accumulators) are used effectively in the implementation of some direct thermodynamic cycles, where the WF state change processes during regeneration are isochoric (Stirling cycle) or isobaric (Ericsson cycle). The WF heat exchange curves in any of the three mentioned cycles are equidistant in the horizontal direction of *T_s* diagram, and when you study these cycles you talk about the ideal regenerator.

As we will see later, ***the use of the ideal regenerator of the types described above to serve any part of cycle's temperature interval that is bordered, in T_s diagram, by two curves equidistant in the horizontal direction obviates the use of heat*** reservoirs/MRs in spite of the availability of the heat exchange processes.

In the real regenerator, which operates with a final (not infinitesimal) temperature difference, heat regeneration cannot be a reversible process. Thus, the regeneration process, in real engine, is irreversible and the regeneration ratio in it is inevitably less than one.

It is clear that achieving the reversible regeneration is easy for the case of equal specific heats of the two heat-exchanging WFs associated with the equality of their flows.

Nt5-10: ***Unlike power cycles, which are subject to the Planck statement, the parts of power cycles, which are subject to Thomson statement, can be achieved without the contribution from any heat reservoir/MR or with the contribution of only one heat reservoir/MR.*** This depends on the shape and quality/kind of cycle's part (depends on the participating processes in cycle's part and on the kind of regeneration).

5.9.3 ABOUT CYCLE'S ABILITY FOR HEAT REGENERATION

5.9.3.1 The regeneratability condition

From the definitions of heat regeneration and regeneratable cycle above (see OD5-3 and §5.9.1.1) we conclude that to be regeneratable, the cycle must have a configuration that theoretically allows, if the engine was equipped with a heat regeneration device, spontaneous transferring of an amount of heat (whatever small it was) from the rejecting heat WF to the gaining heat WF. As the spontaneous heat transfer goes from the higher-temperature body to the lower-temperature one, the maximum temperature of the rejecting heat WF (the *s*-decreasing process) $T_{s\downarrow_{max}}$ must be higher than the minimum temperature of the gaining heat WF (the *s*-increasing process) $T_{s\uparrow_{min}}$ by the differential d*T* at least. Thus, the ***regeneratability condition*** is as follows:

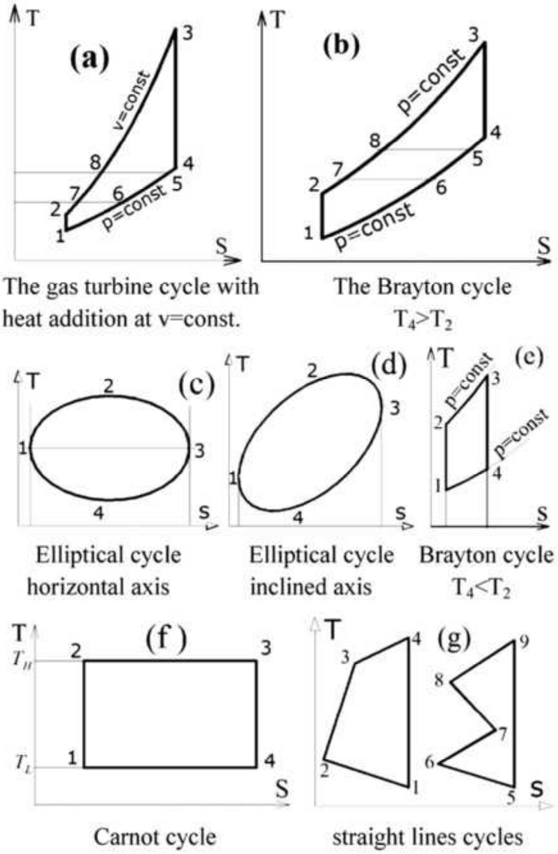
HR5-4: If the maximum temperature in the WF *s*-decreasing process(*s*) $T_{s\downarrow_{max}}$ during the thermodynamic cycle is higher than the minimum temperature in the WF *s*-increasing process(*s*) $T_{s\uparrow_{min}}$ during the same cycle; then the cycle is regeneratable, else it is nonregeneratable. Or:

$$\text{If } T_{s\downarrow_{max}} > T_{s\uparrow_{min}} \Rightarrow \text{Cycle regeneratable else Cycle nonregeneratable} \qquad (5\text{-}63)$$

5.9.3.2 Discussing the regeneratability of some direct thermodynamic cycles

In Figure 5-12 eight direct (engine) cycles (they proceed clockwise) are represented in seven *T_s* diagrams. Four of these cycles (diagrams "c, d, and g") are imaginary and except helping in explanations do not have any application:

(a) For the cycle in sub-figure (a), the heat rejection (s-decreasing) process is process (4–1) and the heat addition (s-increasing) process is (2–3), therefore, $T_{s\downarrow_{max}} = T_4$ and $T_{s\uparrow_{min}} = T_2$. As $T_4 > T_2$, then $T_{s\downarrow_{\max}} = T_4 > T_2 = T_{s\uparrow_{\min}}$, and according to the regeneratability condition (5-63), the illustrated in subfigure (a) gas turbine cycle with heat addition at constant volume is a regeneratable cycle.



The gas turbine cycle with heat addition at v=const.

The Brayton cycle $T_4>T_2$

Elliptical cycle horizontal axis

Elliptical cycle inclined axis

Brayton cycle $T_4<T_2$

Carnot cycle

straight lines cycles

Figure 5.12. Cycle's ability for heat regeneration

(b) For the cycle in sub-figure (b), the s-decreasing process is process (4–1) and the s-increasing process is process (2–3), therefore, $T_{s\downarrow_{\max}} = T_4$ and $T_{s\uparrow_{\min}} = T_2$. As $T_4 > T_2$, then $T_{s\downarrow_{\max}} > T_{s\uparrow_{\min}}$, and according to the regeneratability condition (5-63), the Brayton cycle illustrated in subfigure (b) is a regeneratable cycle.

(c) For the cycle in sub-figure (c), the s-decreasing process is process (3–4–1) and the s-increasing process is process (1–2–3), therefore, $T_{s\downarrow_{\max}} = T_3 = T_1$ and $T_{s\uparrow_{\min}} = T_3 = T_1$. This means that

$T_{s\uparrow_{\min}} = T_{s\downarrow_{\max}}$, and according to the regeneratability condition (5-63), the horizontal axis elliptical cycle illustrated in subfigure (c) is a nonregeneratable cycle.

(d) For the cycle in sub-figure (d), the s-decreasing process is process (3–4–1) and the s-increasing process is process (1–2–3), therefore, $T_{s\downarrow_{\max}} = T_3$ and $T_{s\uparrow_{\min}} = T_1$. As $T_3 > T_1$, then $T_{s\downarrow_{\max}} = T_3 > T_1 = T_{s\uparrow_{\min}}$, and according to the regeneratability condition (5-63), the inclined axis elliptical cycle illustrated in subfigure (d) is a regeneratable cycle.

(e) For the cycle in sub-figure (b), the s-decreasing process is process (4–1) and the s-increasing process is process (2–3), therefore, $T_{s\downarrow_{\max}} = T_4$ and $T_{s\uparrow_{\min}} = T_2$. As $T_4 < T_2$, then $T_{s\downarrow_{\max}} = T_4 < T_2 = T_{s\uparrow_{\min}}$, and according to the regeneratability condition (5-63), the Brayton cycle illustrated in subfigure (e) is a nonregeneratable cycle.

(f) For the cycle in sub-figure (f), the s-decreasing process is process (4–1) and the s-increasing process is process (2–3), therefore, $T_{s\downarrow_{\max}} = T_4 = T_1$ and $T_{s\uparrow_{\min}} = T_2 = T_3$. As $T_2 = T_3 > T_4 = T_1$, then $T_{s\downarrow_{\max}} < T_{s\uparrow_{\min}}$, and according to the regeneratability condition (5-63), the Carnot cycle illustrated in subfigure (f) is a nonregeneratable cycle.

(g) In cycle (1–2–3–4–1) of sub-figure (g), the *s*-decreasing process is process (1–2) and the s-increasing process is process (2–3–4), therefore, $T_{s\downarrow_{\max}} = T_2$ and $T_{s\uparrow_{\min}} = T_2$. This means that $T_{s\uparrow_{\min}} = T_{s\downarrow_{\max}}$, and according to the regeneratability condition (5-63), the straight lines' cycle (1–2–3–4–1) illustrated in subfigure (g) is a nonregeneratable cycle. Also, in cycle (5–6–7–8–9–5) of the same sub-figure (g), the *s*-decreasing processes are processes (5–6) and (7–8), while the s-increasing processes are processes (6–7), and (8–9), therefore, $T_{s\downarrow_{\max}} = T_8$ and $T_{s\uparrow_{\min}} = T_6$. As $T_8 > T_6$, then $T_{s\downarrow_{\max}} > T_{s\uparrow_{\min}}$, and according to the regeneratability condition (5-63) the straight lines cycle (5–6–7–8–9–5) illustrated in subfigure (g) is a regeneratable cycle.

Thus, cycles (1–2–3–4–1) in subfigures (c, e, f, and g) are ***nonregeneratable or naturally nonregenerative***. The other cycles in Figure 5-12 are ***regeneratable***.

Nt5-11: The four regeneratable cycles are not equivalent/equipollent in terms of the way to be followed to achieve them with full heat regeneration. As we shall see later:

- The one illustrated in subfigure (b) requires an ideal regenerator, a set of heat accumulators, or a set of heat reservoirs to serve the interval T_2–T_4 and achieve in this interval the ***full reversible regeneration***. This cycle is regeneratable for all authors (see §6.9.1).
- The one illustrated in subfigures (a) and (d) require a set of heat reservoirs each to serve the interval T_2–T_4 and achieve in this interval the ***full reversible regeneration***. The ideal regenerator is not appropriate for this job. Therefore, these can be executed as absolutely reversible (AR)—with ***full reversible regeneration***—from the point of view of the traditional model/method, and cannot be so from the other authors' point of view (see HD5-4 and §5.10.3).
- ***Cycle (5–6–7–8–9–5) in subfigure (g) is not reversibly regeneratable*** and a regenerator helps in irreversibly regenerating some heat.

Nt5-12: Not all cycles illustrated in Figure 5-12 are of thermodynamic interest. Only cycles a, b, and c are/were encountered in practice as ideal gas power cycles, while the Carnot cycle (f) and its reverse that are vital for the second law of thermodynamics is not used as ideal gas power and refrigeration cycles because of the impossibility of achieving/approaching ideal gas state change isothermal processes with acceptable productivity (energy/power exchange). The other cycles in Figure 5-12 are imaginary.

5.10 ABOUT THE THEORETICAL REALIZATION OF REVERSIBLE GAS STATE CHANGE PROCESSES: IN BRIEF (THE FULL ANALYSIS IN THE SECOND VOLUME)

5.10.1 INTRODUCTION

Although all authors adopt the definition of the reversible process (see §2.4.13.1 "OD2-34A through OD2-34E"), they are with respect to the theoretical achieving/realizing of reversible processes and cycles are divided into two groups: the first group adopts the standards of the traditional model (see §5.10.3), while the other speaks about a reversible cycle without heat regeneration and about another with heat regeneration. As we will see later, the results of reversibly executing the same cycle according to the standards of each of the two groups are not the same (there are a large number of mismatches). To avoid misunderstandings, it is preferable to rename the reversible cycles that are executed according to the traditional model standards.

The explanations given in the upcoming material/topics may give the impression that the author assumes that all users of his book own the technique that allows them, at any time and wherever they were, use it easily and successfully but the author is sure and understands that this explanation is always easy to be understood but not always easy to apply. Therefore, the reader after understanding the idea is free to apply it as is comfortable to him.

5.10.2 ABOUT THE IMAGINARY MODELS IN THERMODYNAMICS

As was stated earlier (see §s 2.4.4 and 2.4.6) some imaginary *models such as the ideal gas are successfully used* in thermodynamics, which means that using imagination in thermodynamics is not forbidden, but we need to be careful in using not approved imaginary *models*. And may be for this reason a group of authors do not adopt the standards of the traditional model. As we shall see in the following paragraph executing a nonisothermal heat transfer process according to the traditional model requires unlimited number of heat reservoirs, which can be understood for some people that this imagination is a higher-degree imagination that cannot be thermodynamically accepted, while the required two reservoirs for executing Carnot cycle is a lower-degree imagination that can be thermodynamically accepted. For us both imaginations require carefulness in using, and the results of each use should be carefully discussed before making conclusions about their thermodynamic validity (see §5.10.5)

5.10.3 THE TRADITIONAL/CLASSICAL MODEL/METHOD FOR REALIZING A REVERSIBLE PROCESS

During the explanation of the equilibrium and reversible processes above, we used a number of assumptions that form ***the traditional/classical method for realizing a reversible process***. Its most important items are as follows:

1) The ***heat reservoir*** is an infinitely large body; whatever the quantity of heat exchanged with it is, its temperature does not change (see OD2-2).
2) To achieve a reversible isothermal heat exchange process, a single heat reservoir that has the same temperature of the WF is needed.
3) To achieve a reversible nonisothermal heat exchange process, an almost infinite number of heat reservoirs is needed. Here, each nonisothermal nonisentropic process (cycle's) curve (e.g., curve 1–2 in subfigure A of Figure 5-13) is replaced by/for a similar to stairs refracted line consisting of differential/nondifferential isotherms and isentrops such as line 1–5–6–7–8–9–10–11–12–13–2 (see subfigure B of Figure 5-13), and so that the initial and final points of each stair lie on the original curve, while curve's nondifferential isotherms and/or isentrops remain unchanged (line 2–3–4 (subfigure A) remained unchanged (see subfigure B)), and thus all partial heat exchange processes become isothermal, and *each temperature level will be served by a single heat reservoir that has its same temperature*, and is able to exchange heat with the WF that has its temperature in both directions. A little different from the above, locating of the replacing refracted curve was discussed in the explanation of Figure 5-8. Both replacements will lead to the same original curve when $\Delta T \to dT \to 0$ or $\Delta s \to ds \to 0$.

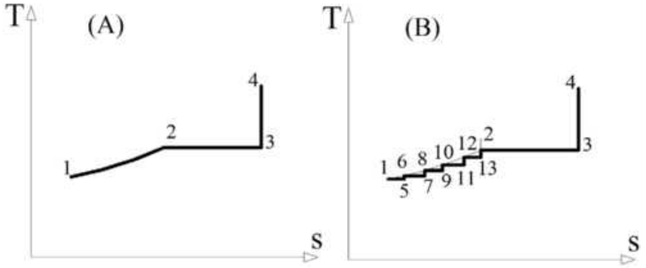


Figure 5.13. Replacing smooth process (cycle's) curve by/for a similar to stairs refracted line consisting of differential/nondifferential isotherms and isentrops.

Nt5-13: To avoid misunderstanding with the reversible cycle that is in use in modern thermodynamic books we shall call (refer to) the reversible cycles executed according to the standards of the traditional model as the ***AR cycles*** (see the beginning of §5.9.1) and define them as follows:

<u>HD5-4:</u> The AR cycle is the one (1) that, unless its configuration does not allow reversible heat regeneration, is always with full reversible heat regeneration,^{D} (2) that is equilibrium for all elements involved in it, (3) that is free from irreversibilities, and (4) that is carried out according to the traditional/classical method for realizing reversible processes, where the WF's heat exchange is carried out through contacting/touching it (its WF) sequentially, with the necessary number of heat reservoirs those/that reversibly exchange heat with the WF at its temperature and in the necessary directions. The number of the required reservoirs exactly equals the number of temperature levels secreted by the mathematically acceptable replacement, of the original cycle's line/path by/for a refracted line similar to stairs consisting of differential and nondifferential isotherms and isentrops. The AR cycle also complies with the definitions of the reversible process (see §2.4.13.1), and therefore, it is a special case of the reversible cycle according to the authors who do not adopt the standards of the traditional model.

D. In Figure 5-12, the configuration of cycles (c, e, and f) do not allow any heat regeneration, while the configuration of cycle 5–6–7–8–9–5 in subfigure g allows only irreversible heat regeneration (see the regeneratability condition 5-63 and §5.9.3.2).

The following discussion requires introducing some (abbreviated or not abbreviated) parts of the comprehensive discussion done in the second volume those are:

Nt5-14A: ***Cycle's temperature levels*** *are the unlimited-length isotherms (usually light-weight lines), whose temperatures are the temperatures of the heat reservoirs that serve the AR cycle, while* ***cycle's mini-part*** *is cycle's part (cycle's elementary Carnot cycles) limited by two neighboring cycle's temperature levels.*
Nt5-14B: Cycle's temperature mini-interval (abbreviation m-int) is the unlimited-width area of T_s graph that is limited by two neighboring cycle's temperature levels. The mini-interval and the mini-part of the same cycle that are limited/bordered by the same temperature levels share the same code.
Nt5-14C: Some mini-parts of the power AR cycles that have gaps and/or cavities are **discontinuous**. They consist of more than one elementary Carnot cycle and we shall refer to each elementary Carnot cycle of them as **mini-subpart** and to the whole group of elementary Carnot cycles that share the same temperature limits as mini-part.

The ***live/effective/actual*** *area of a group of elementary areas of an AR cycle is the sum of the absolute values of its elementary areas. Therefore,*

HD5-4A: *Regardless of the differences in numbers, areas, and entropy changes of their mini-subparts, two mini-parts of different* ***same direction*** *AR cycles in scale and in T_s diagram that share the same WF, the same mini-interval (the same upper and lower temperature levels), the same* ***live/effective/actual*** *area, and accordingly the same live/effective/actual width (Δs) are AR cycles'* ***completely equivalent mini-parts****.*

Completely equivalent mini-parts are special case of completely equivalent Carnot cycles (see HD5-3A).

HD5-5 ABR: Two neighbor AR cycle's mini-parts (lower "i" and upper "j") are briefly called *ij (dual) mini-part. An ij mini-part* can be (1) simple *if the t*wo (i and j) *mini-parts* of the *ij mini-part* are continuous ones and (2) complicated *if at least one of the t*wo (i and j) *mini-parts* of the *ij mini-part* is discontinuous one.

To avoid misunderstanding in the detailed discussions of *ij mini-parts* we introduce the following:

Symbol Tiij will stand for the border of mini-part (i) whose temperature is Tij.
Symbol Tjij will stand for the border of mini-part (j) whose temperature is Tij.

HD5-5A ABR: *The T_{ij} temperature level's contacting part is the continuous/****dis****continuous part(s) that contacts any of or both (i and/or j) mini-parts.* This line/part that is originally a light-weight one consists of (1) heavy-weight lines *that are, at the same time, heat transfer processes and* ***mono-side contacting lines*** *(each of them contacts its neighboring AR cycle's mini-parts from one side "upper or lower")* *and* (2) *medium-*weight lines *that are* ***dual-side contacting parts*** *(each of them contacts its neighboring AR cycle's mini-parts from both sides "upper and lower").*

In the following discussion we shall (1) use the the terms introduced in this volume, (2) use the terms introduced through the borrowed from the second volume notes Nt5-14A through Nt5-14C, and definitions HD5-4A through HD5-5A ABR, and (3) mainly refer to Figure 5-14, where the original cycle in subfigure (A) is divided into only six parts that simplifies the discussions on the expense of calculating accuracy. ***The replaced AR cycle*** *(the AR cycle, after replacing its original border line by a refracted one according to the classical model "see §5.10.3 and HD5-4") consists* *of a positive integer number of* ***reversible elementary Carnot cycles*** *that depends on the configuration of the original cycle and on the required accuracy of its calculations. This number is six (in Figure 5-14), can be one (in the AR Carnot cycle), and can be a very huge number for accurate calculations of common case cycles. The* ***elementary Carnot cycles,*** *their vertex points, the isotherms that limit them (****AR cycle's temperature levels****), and other elements of cycle's diagram (in subfigure 3-14B) are encoded according to the system that is comprehensively explained in the second volume of this book. The perimeter/border of the AR cycle in subfigure (5-14B) is represented as usual by heavy-weight lines and consists of only* differential and nondifferential isentrops and isotherms to which we shall refer as ***AR cycle's elementary isentrops and isotherms***. The AR cycle's elementary isotherms represent the heat exchange processes throughout the cycle, while the elementary isotherms that lay on a particular cycle's ***temperature level*** *(e.g., T_{de})* represent the heat exchange processes throughout this *temperature level. The AR cycle in Figure 5-14 consists of six mini-parts (a, b, c, d, e, and f) that are limited by the seven temperature levels ($T_{\min}$, T_{ab}, T_{bc}, T_{cd}, T_{de}, T_{ef}, and $T_{\max}$) of the seven heat reservoirs* $(R_{T_{i\,min}}, R_{T_{ab}}, R_{T_{bc}}, R_{T_{cd}}, R_{T_{de}}, R_{T_{ef}},$ *and* $R_{T_{i\,max}})$. *Each of these reservoirs* reversibly *exchanges* heat with the WF in the necessary directions, *when* WF's *temperature equals its temperature*. This means that for a particular reservoir such as $R_{T_{de}}$ we can write that *r*eservoir's $R_{T_{de}}$ heat balance $(R_{HB_{T_{de}}})$ and WF's heat balance $q_{\circ_{T_{de}}} = q_{net_{T_{de}}}$ throughout temperature level *T_{de}* are equal in absolute values and opposite in signs. Or

$$R_{HB_{T_{de}}} = -q_{\circ_{T_{de}}} = -q_{net_{T_{de}}} \tag{5-64}$$

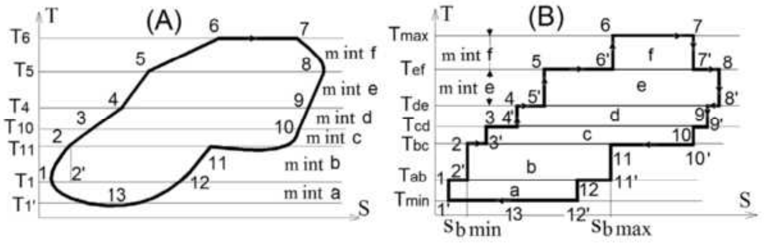


Figure 5.14. The heavy-weight lines are used for the perimeters of the original and replacing cycles and the medium-weight ones for the common borders between elementary Carnot cycles

And as reservoir $R_{T_{de}}$ does not exchange heat at others than temperature level T_{de}, $R_{HB_{T_{de}}}$ is also reservoir's $R_{T_{de}}$ heat balance throughout the *AR cycle in subfigure 5-14B.*

Thus, the continuous cycle's mini-part (j) in Figure 5-15 that included inside cycle's temperature mini-interval (j) consists of one mini-subpart, while the discontinuous mini-part (i) of the same cycle in the same Figure 5-15 consists of the three mini-sub-parts (i_1, i_2, and i_3) that constitute three parts (sub-parts) of cycle's temperature mini-interval (i).

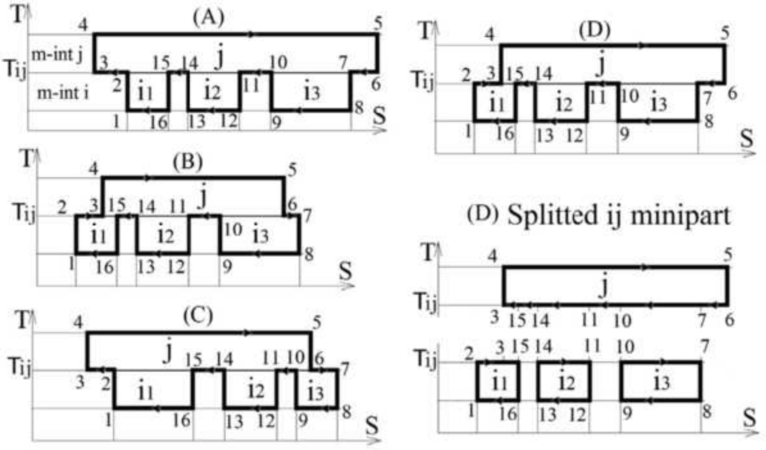


Figure 5.15. Some possible sketches (A–D) for understanding and deriving the heat balance $R_{HB_{T_{ij}}}$ of heat reservoir $R_{T_{ij}}$ that serves the complicated dual mini-part *ij* of a complicated AR cycle (see HD5-1 and HD5-5). The sketches are drawn for the case when the cycle consists of one dual mini-part, otherwise the upper (4–5) and lower (8–9, 12–13 and 16–1) heavy-weight lines must disappear.

It is clear from above that (1) cycle's temperature levels and mini-intervals are endless; they occupy the whole width of T_s diagram, while cycle's mini-parts and mini-subparts are not, and they occupy limited parts of temperature mini-intervals. (2) The set of light-weight temperature level's lines plays, as the set of gridlines, assisting in reading the ordinates of T_s graphs. These light-weight lines carry on some of their parts the heavy-weight lines that represent cycle's isothermal (heat transfer) elementary processes in addition to the medium-weight lines of the common borders between neighboring elementary Carnot cycles, where the heat transfer between the WF and the heat reservoir is absent.

The comprehensive analyses of the most ever complicated AR cycles that is done in the second part of this book is based on the four possible sketches of a part of the AR cycle that consists of two neighboring mini-parts (i and j) where i is the successive number of the lower mini-part and j = i + 1 is the successive number of the upper mini-part. Thus, we are supposed to expect that if the AR cycle consists of n mini-parts, i can be any number between 1 and n − 1.

This comprehensive analysis leads to the following equation for calculating reservoir's $(R_{T_{ij}})$ *heat balance* $(R_{HB_{T_{ij}}})$ *and WF's heat balance* $q_{\circ_{T_{ij}}} = q_{net_{T_{ij}}}$ *throughout temperature level T_{ij} that* are *equal in absolute values and opposite in signs. Or:*

$$R_{HB_{T_{ij}}} = -q_{\circ_{T_{ij}}} = -q_{net_{T_{ij}}} = -T_{ij}\sum_{T_{ijl}} \Delta s_{el} = q_{2_{T_{ij}}} - q_{1_{T_{ij}}} \tag{5-65}$$

where, in addition to that explained above, Δs_{el} is the entropy change of an elementary heat transfer process and $\sum_{T_{ijl}} \Delta s_{el}$ the algebraic sum of the entropy changes of all elementary heat transfer processes throughout the temperature level T_{ij}. For any of the *ij* mini-parts illustrated in Figure 5-15, this $\sum_{T_{ij}} \Delta s_{el}$ equals $\Delta s_{6-7} + \Delta s_{10-11} + \Delta s_{14-15} + \Delta s_{2-3}$.

Each of these four elementary heat transfer processes does not belong to the same mini-part in the four subfigures (e.g., Process 6–7 in subfigures "A and D" belongs to mini part "j," while it belongs to mini-part "i" in subfigures "B and C"). Therefore, we shall continue our discussion for a particular subfigure (e.g., subfigure "D").

In subfigure (D), state change processes 6–7, 10–11, and 14–15 belong to isotherm 6–3 that is the lower border of mini-part (*j*). Isotherm (6–3) consists of the six neighboring/subsequent direction isotherms (6–7, 7–10, 10–11, 11–14, 14–15, and 15–3), of which isotherms 7–10, 11–14, and 15–3 are not state change processes. While in the additional subfigure (D) that is imaginary one and where the *ij* mini-part is split into two noncontacted/individual (*i* and *j*) mini-parts, all six parts of the lower border of mini-part (*j*) are the same direction state change processes (6–7, 7–10, 10–11, 11–14, 14–15, and 15–3). This means that both subfigures (D and additional D) lead to the same calculating equation:

$$\Delta s_{T_{jij}} = s_3 - s_6 = \Delta s_{6-3} = \Delta s_{6-7} + \underline{\Delta s_{7-10}} + \Delta s_{10-11} + \\ + \underline{\Delta s_{11-14}} + \Delta s_{14-15} + \underline{\Delta s_{15-3}} \tag{5-66}$$

The underlined parts in this equation are the dimensions of the isothermal lines that represent neither *actual/elementary* state change processes nor the *live/effective/actual width (*Δ*s*) of cycle's mini-part. When we group these dimensions in the left side, Equation (5-66) transforms into:

$$\underline{\Delta s_{7-10}} + \underline{\Delta s_{11-14}} + \underline{\Delta s_{15-3}} = \Delta s_{T_{jij}} - \Delta s_{6-7} - \Delta s_{10-11} - \Delta s_{14-15} \qquad (5\text{-}67)$$

The value of these underlined dimensions can be obtained from the discontinuous upper border of mini-part *i* of subfigure (D) that consists of the three continuous sub-borders 2–3–15, 14–11, and 10–7, of which only part (2–3) is *actual/elementary* state change process (the other isothermal directional lines (3–15, 14–11, and 10–7) are not *actual/elementary* state change processes). Thus, the other calculating equation that can be obtained from both subfigures (D and additional D) is:

$$\begin{aligned}\Delta s_{T_{iij}} &= (s_3 - s_2) + (s_{15} - s_3) + (s_{11} - s_{14}) + (s_7 - s_{10}) \\ &= \Delta s_{2-3} + \underline{\Delta s_{3-15}} + \underline{\Delta s_{14-11}} + \underline{\Delta s_{10-7}}\end{aligned}$$

Or $$\underline{\Delta s_{3-15}} + \underline{\Delta s_{14-11}} + \underline{\Delta s_{10-7}} = \Delta s_{T_{iij}} - \Delta s_{2-3} \qquad (5\text{-}68)$$

As $\underline{\Delta s_{7-10}} = -\underline{\Delta s_{10-7}}$, $\underline{\Delta s_{11-14}} = -\underline{\Delta s_{14-11}}$ and $\underline{\Delta s_{15-3}} = \underline{-\Delta s_{3-15}}$, the left sides of Equations (5-67) and (5-68) are equal in absolute value and opposite in sign or $\underline{\Delta s_{7-10}} + \underline{\Delta s_{11-14}} + \underline{\Delta s_{15-3}} = -\left(\underline{\Delta s_{3-15}} + \underline{\Delta s_{14-11}} + \underline{\Delta s_{10-7}}\right)$; therefore, their right sides are also equal in absolute value and opposite in sign. Or $\Delta s_{T_{jij}} - \Delta s_{6-7} - \Delta s_{10-11} - \Delta s_{14-15} = -\left(\Delta s_{T_{iij}} - \Delta s_{2-3}\right)$ or:

$\Delta s_{T_{jij}} + \Delta s_{T_{iij}} = \Delta s_{2-3} + \Delta s_{6-7} + \Delta s_{10-11} + \Delta s_{14-15} = \sum_{T_{ijl}} \Delta s_{el}$ or (see the legend of Equation 5-65):

$$\sum_{T_{ijl}} \Delta s_{el} = \Delta s_{T_{jij}} + \Delta s_{T_{iij}} \qquad (5\text{-}69)$$

Applying this equation in Equation (5-65) we obtain:

$$\begin{aligned}R_{HB_{T_{ij}}} &= -q_{\varnothing_{T_{ij}}} = -q_{net_{T_{ij}}} \\ &= -T_{ij} \sum_{T_{ijl}} \Delta s_{el} = -T_{ij}\left(\Delta s_{T_{jij}} + \Delta s_{T_{iij}}\right) = q_{2_{T_{ij}}} - q_{1_{T_{ij}}} \qquad (5\text{-}70)\end{aligned}$$

is valid for any dual *ij* AR cycle's mini-part.

In this equation T_{ij} is always positive and in such calculations it cannot be zero, while $\Delta s_i = 0$ for $T_{ij} = T_{min}$ and $\Delta s_j = 0$ for $T_{ij} = T_{max}$. Therefore, $q_{net_{T_{ij}1}}$ according to the sum $\left(\Delta s_{T_{jij}} + \Delta s_{T_{iij}}\right)$ value can be positive, negative, or zero.

> **<u>Nt5-15:</u>** *It is obvious from the above that the traditional model is an imaginary one, and it cannot be used in thermodynamic actual calculations. But it is much helpful in understanding, explaining, and discussing complicated aspects, especially when (1) dealing with the comparisons between power cycles and (2) analyzing these cycles qualitatively.*

5.11 SUMMARY

All topics of this chapter are traditional; some of them are usually discussed topics under the title “The second law of thermodynamics” such as:

(1) The second law of thermodynamics, (2) the thermodynamic cycle (direct and reverse) and their evaluation, (3) the thermal efficiency of the ideal direct cycle, (4) the Carnot cycle, (5) the reverse Carnot cycle, (6) introduction to Carnot theorem (Existing formulations of Carnot theory), (7) the entropy is a property for all gases, and (8) the heat regenerator.

While some of the others are valuable traditional topics that are skipped from today’s thermodynamics such as (see Nt5-13) the traditional model for realizing a reversible process through which the discussion of (1) reversible and irreversible gas state change processes and (2) heat regeneration and the regeneratability of cycles.

Based on the following definition of/for the *heat reservoir:*

“The ***heat reservoir*** is an infinitely large body; whatever the quantity of heat exchanged with it is, its temperature does not change (see OD2-2)”

The AR cycle is defined as follows.

> **<u>HD5-4:</u>** The AR cycle is the one (1) that, unless its configuration does not allow reversible heat regeneration, is always with full reversible heat regeneration, (2) that is equilibrium for all elements involved in it, (3) that is free from irreversibilities, and (4) that is carried out according to the traditional/classical method for realizing reversible processes, where the WF’s heat exchange is carried out through contacting/touching it (its WF) sequentially, with the necessary number of heat reservoirs those/that reversibly exchange heat with the WF at its temperature

> and in the necessary directions. The number of the required reservoirs exactly equals the number of temperature levels secreted by the mathematically acceptable replacement, of the original cycle's line/path by/for a refracted line similar to stairs consisting of differential and nondifferential isotherms and isentrops. The AR cycle complies also with the definitions of the reversible process (see §2.4.13.1) and therefore it is a special case of the reversible cycle according to the authors who do not adopt the standards of the traditional model.

The AR cycle is discussed in detail in this and the following chapters because it is very helpful (1) in graphical comparisons between power cycles and (2) in qualitative discussions of power cycles. All basic discussions in this chapter show that we are able to create a nonregeneratable AR cycle that is completely equivalent to any ideal gas AR power cycle. This creation as was comprehensively stated (dealt individually with each cycle's mini-part that is located between two neighboring isotherms) is mostly time consuming and therefore *unless this creation is simplified, in the sixth chapter, it will not have any chance for practical application.*

Because of the limited size of volume (I) of this book, the following changes were done on this chapter:

A) The main paragraph "*5-10-3 the traditional/classical model/method for realizing a reversible process*" that consisted of an untitled introduction, two subparagraphs, and many sub-subparagraphs was briefed to become a paragraph with no branches that does not include all derivations of the original paragraph but includes all its results. It has the same original title. The discussions and derivations that were cut off this briefed paragraph "*5-10-3*"are moved to the second volume. They relate to:

- **HR5-7:** *The regenerated heat throughout any temperature level (T_{ij}) of an AR cycle equals the smaller between the WF rejected and gained heats. Therefore, temperature levels that do not include both s-decreasing and s-increasing elementary processes are nonregeneratable because q_R (the smaller between the WF rejected and gained heats in these levels) equals zero. Concerning the heat reservoir it loses heat when the WF gained heat is greater than its rejected heat, while it gains heat when the WF rejected heat is greater than its gained heat and it converts into an ideal heat accumulator when the WF rejected and gained heats are equal.*

- **HR5-8:** *All partial (elementary partial) cycles that resulted from partitioning (sequential partitioning) of the area of a direct cycle are direct ones and that resulted from partitioning the area of an indirect cycle are indirect ones*. For the elementary Carnot cycles of the AR cycle, this rule performs into ***"The elementary Carnot cycles of an AR cycle are the same direction as their original one."***
- **HR5-9:** *Simple AR cycles (see HD5-1), which consist of only simple fully contacted ij mini-parts, are always without reversible heat regeneration. The simple AR cycle is with reversible heat regeneration if at least one of its simple ij mini-parts is partially contacted or not contacted one.* In common*, AR cycles, which consist of only nonregenerative ij mini-parts, are always without reversible heat regeneration. The AR cycle is with reversible heat regeneration if at least one of its ij mini-parts is with heat regeneration.*

B) Paragraphs *"5-10-4 The absolute equipollency between thermodynamic cycles" and "5-10-5 About the maximum possible thermal efficiency of a power cycle"* were removed as they were moved to the second volume. And we drive here from them the following:

- **HD5-10:** The absolutely equivalent/equipollent cycles are the completely equivalent cycles, which share the same value of the ***full reversible heat regeneration***.
- **HR5-11:** The thermal efficiency of any power thermodynamic cycle that is executed with maximum ever heat regeneration between temperatures $T_{\max}$ and $T_{\min}$ equals the ratio $(T_{\max} - T_{\min}) / T_{\max}$.

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Dr. Moufid Hilal (Mufid Helal) was born in Syria in 1940. He completed his PhD in technology from Moscow Power Institute in 1972. From 1972 through 1977, he served as a lecturer at FMEE, University of Damascus, Syria. He then served as an assistant professor from 1977 to 1982 and as a professor from 1982 to 2012, all at the same institution, retiring from his service on December 12, 2012. He taught thermodynamics for undergraduate students and supervised graduation projects from 1972 to 2010, steam and gas turbines for the diploma students from 1998 to 2007, and advanced thermodynamics for master's students from 2007 to 2012, in addition to other undergraduate courses for 1 to 5 years each.

As a citizen of Syrian Arab Republic, his identity in Arabic is as follows: First name مفيد, family name هلال, and father's name إبراهيم. Upon obtaining his first international passport in 1958, his identity in English, in addition to his identity in Arabic, became thus: First name, Moufid; family name, Hilal; and father's name, Ibrahim. This identity in English continued being so in all his following passports except in the last (the current) one that was issued in October 28, 2009, wherein the identity in English became as follows: First name, Mufid; family name, Helal; and father's name, Ibrahim. Therefore, the publications of Helal M. I., Hilal M. I., and M. И. Хиляль (in Russian) all belong to the author of this book.

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AUTHOR OTHER PUBLICATIONS

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(Article provided by D.Sc Professor S. D. Ponomarev "Moscow Bawman Institute of Technology")

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- The theoretical realization of reversible gas state change processes,
- Helal cycle,
- Helal graphical method for comparing and discussing power cycles,
- The imperfection in the classical proof of Carnot's efficiency (theorem) and its exclusion.

The deeply abbreviated topics are rigorously discussed in depth in the majority of modern thermodynamic books. To dissipate any misunderstanding, the equations and statements that can be misunderstood are followed by explanatory sentences.

Mufid I. Helal (born 1940 in Syria) completed his high school in 1958 and obtained his PhD in technology from Moscow Power Institute in 1972. Since then he has held a number of academic positions at the Faculty of Mechanical and Electrical Engineering, Damascus University, Syria until his retirement in December 2012. His latest published book is *Technical Thermodynamics*, which was published in 2006.

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