Online Estimation of Secondary Path

in Active Noise Control System



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Thesis Acceptance Certificate

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Dedication

This thesis is dedicated to my beloved parents, teachers and siblings

Abstract

In recent years a lot of work has been done in the field of Active Noise Cancellation (ANC) for online estimation of the secondary path. Although vigorous research has been carried out but mostly for Gaussian noise hence in this thesis online estimation of secondary path has been done for the Impulsive Noise with different values of α to make the environment more impulsive. Previously most of the techniques presented were for stationary acoustic paths and in practical environment noise is non-stationary hence we carried out simulations for non-stationary environment. Most researchers have worked on Fx-LMS algorithm which is widely used in the field of ANC and then different variants of Fx-LMS have been proposed in past but in this paper we have proposed Filtered x Least Mean Absolute Third (FxLMAT) algorithm for online estimation of the secondary path and further improvements have been added to existing Fx-LMAT algorithm by suggesting two more algorithms i.e. Variable Step-Size Filtered x Absolute Third (VSSFxLMAT) and Variable Step-Size Filtered x Robust Normalized Absolute Third (VSSFxRNLMAT). Proposed Algorithms outperforms the existing algorithms.

Keywords: ANC, IN, MNR, Fx-LMS, LMAT, VSS-FxRNLMAT

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First I am thankful to ALMIGHTY ALLAH who gave me strength and wisdom to complete this work. Moreover, I am also thankful to my supervisor and committee members for their continuous guidance, support and encouragement throughout this research work. In the end I am grateful to my parents for their support and prayers.

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List of Abbreviations and Symbols

Abbreviations

ANC	Active Noise Control
IN	Impulsive Noise
LMS	Least Mean Square
LMAT	Least Mean Absolute Third
RLS	Recursive Least Square
Fx-LMS	Filtered x Least Mean Square
MNR	Mean Noise Reduction
OSPM	Online Secondary Path Modelling

CHAPTER 1

Introduction

1.1 Overview

Noise is an irritating and unwanted signal causing tremor in human beings, affecting them mentally and physically[1]-[2]. To overcome this problem a lot of interest in Active Noise Control (ANC) system has been observed, as an increasing number of applications are being embedded with this technology[3]-[8]. Basic technique behind the working of an ANC system is the principal of superposition; two waves 180 Degrees out of phase cancel out each other resulting in a new wave[9]-[13]. Same is the case in ANC system where an out-of-phase signal cancels out the noise such that the resultant signal is noise free. There are different algorithms which can be implemented in ANC, however the most famous among them is Filtered-x Least Mean Square (FxLMS) algorithm. FxLMS is an enhanced version of the LMS algorithm, where a reference signal x(n) is filtered through a secondary path's model before entering the adaptive filter block and thus it is named as FxLMS[14].

1.2 Noise Controlling Techniques

The well known techniques in Noise Control Systems are:

- i) Active Noise Control
- ii) Passive Noise Control

1.2.1 Active Noise Control

An additional anti-noise signal is used in ANC systems to cancel out the noise; it is commonly used in electronic machines, headphones and anti snoring devices. Active noise controlling techniques greatly rely on the electrical equipment as that is where all the algorithm of noise cancellation takes place.

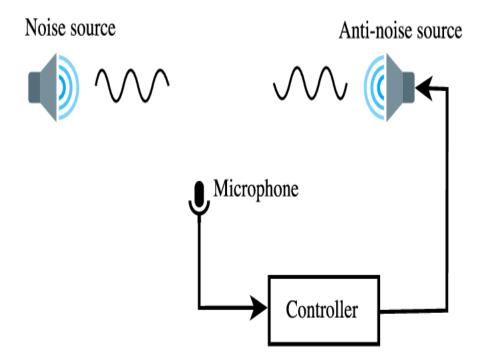


Figure 1.1: Active Noise Control System.

1.2.2 Passive Noise Control

In passive Noise Control Systems, noise is eliminated using passive elements such as noise absorbing tiles, rubber padding in headphones and earplugs, etc [15]. In simple words, it is the amount of vibration cancellation provided by the physical device, or how well the headset works as an earplug. Moreover, a unique design will provide good passive cancellation before electrical means of noise cancellation (i.e. ANC) are applied.



Figure 1.2: Passive Noise Control using Earplugs.

1.3 Motivation

In the past few years a lot of work has been done on Active Noise Cancellation and Online Estimation of the Secondary path. Most of the existing methods use Fx-LMS (Filtered x Least Mean Square) and its variants [16]. We wanted to come up with a solution where our method would not only work better than the existing techniques but would also bring forth fast convergence as well as robustness without increasing the computational complexity. Furthermore, in our daily lives, the noise we encounter is impulsive in nature and it was a dire need to address this issue. Hence, our proposed methods are designed to cater this problem as they eliminate the impulsive noise and provide improved online secondary path's estimation. The stability of the adaptive algorithms depends on the reference noise signal x(n) that is filtered through the estimated secondary path $\hat{s}(n)$ [17][18]. The work reported in this thesis is based on broadband feedforward ANC when impulsive noise (IN) is present.

1.4 Uses and Applications

Recently, ANC is used widely in noise cancelling headphones and other electronic devices where noise is an important issue to address. In noise cancelling headphones, an additional signal is used which is out of phase than the noise signal. Other applications where ANC is used nowadays are Snore ANC system, MRI ANC system and Infant Incubators. ANC is also used in industrial and heavy electrical equipment to cancel noise from generators, transformers, compressors, and fans.

1.5 Performance Analysis of ANC System

For any ANC system to be incorporated in practical applications, it must fulfil the following performance measures:

- 1. System should work for the biggest frequency band possible.
- 2. The ANC system should be able to withstand change in physical conditions like temperature, humidity, etc.
- 3. ANC system must be robust and simple.
- 4. The ANC system must be adaptive to endure time varying paramters such as change in noise, acoustic paths, etc.

1.6 Types of ANC Systems

ANC system has two main types: 1) Feedforward ANC system, where a reference noise signal is available before it goes past the secondary source 2) Feedback control ANC system, where the filter incorporates the reference signal without using an upstream reference input.

1.6.1 Broadband Feedforward ANC

In broadband feedforward ANC, the reference input is sensed by a microphone. The reference signal is handled by the ANC system to generate the control signal to drive

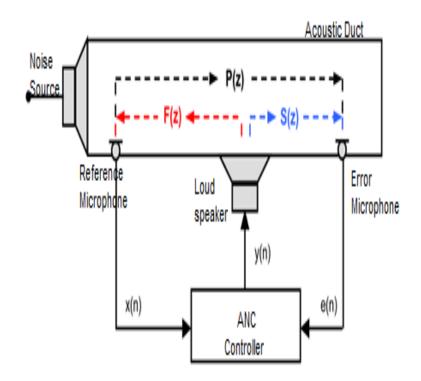


Figure 1.3: Broadband Feedforward ANC [14]

a loudspeaker. The ANC system's performance is evaluated by the error microphone. The controller's aim is to reduce the calculated acoustic noise. The basic functionality of a broadband feedforward ANC is such that there is an adaptive filter W(z) which estimates the unidentified primary path P(z). The P(z) comprises of the reply from the microphone acting as reference microphone to the error microphone, if our unknown plant is dynamic then our adaptive filter must be dynamic so that it can cater to the variations in our plant. The main purpose of the adaptive filter W(z) is to minimize the error e(n), i.e. e(n)=d(n) - y(n), which is the residual error.

1.6.2 Narrowband Feedforward ANC

In narrowband feedforward ANC a reference signal is generated internally through the reference sensor. This system is typically deployed where the primary is produced by rotating machines and is periodic in nature [17]. A sensor that is synchronised with the source of noise is used to generate an input signal containing the fundamental frequency of the noise source.

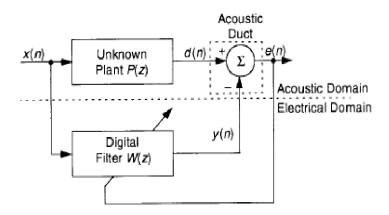


Figure 1.4: System Identification Viewpoint of Broadband ANC [15]

Quantity	Description
p(i)	Primary path
s(i)	Secondary path
$oldsymbol{\hat{S}}(i)$	OSPM filter
$oldsymbol{w}(i)$	ANC filter
$oldsymbol{x}(i)$	Input signal
$\boldsymbol{x'(i)} = \boldsymbol{s}(i) * \boldsymbol{x}(i)$	Input signal filtered
	through secondary path
$\hat{\boldsymbol{x}}' = \boldsymbol{\hat{s}}(i) * \boldsymbol{x}(i)$	Input signal filtered
	through secondary path estimate
$d(i) = \boldsymbol{p}(i) \ast \boldsymbol{x}(i)$	Disturbance signal
$y(i) = \boldsymbol{w}^{T}(i) * \boldsymbol{x}(i)$	Output of an ANC Filter
$y'(i) = \boldsymbol{s}(i) \ast \boldsymbol{x}(i)$	Canceling signal
$oldsymbol{v_m}(i)$	Noise Generated internally for OSPM filter
G(i)	Gain factor used for $v(i)$
$\boldsymbol{v}(i) = G(i) * \boldsymbol{v_m}(i)$	Noise with gain injected into the system
$\boldsymbol{\hat{v}'}(i) = \boldsymbol{\hat{s}}(i) * \boldsymbol{v_m}(i)$	Output of the OSPM filter
e(i) = d(i) - [y'(i) - vG'(i)]	Error Signal of ANC filter
$f(i) = e(i) - \hat{\boldsymbol{v}'}(i)$	Error Signal of OSPM filter

Table 1.1: Different perimeters for ANC systems with OSPM

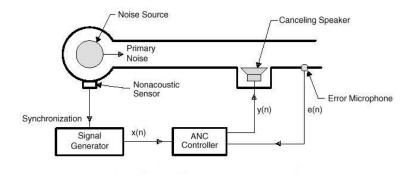


Figure 1.5: Narrowband Feedforward ANC.[17]

1.6.3 Feedback ANC

In feedback ANC, a microphone is deployed which acts as an error sensor to detect the unwanted noise. The erroneous signal is returned through a magnitude and phase amplifier such that it cancels out the error signal at sensor located near the microphone [18]. This configuration can be seen in Fig 1.6 and it only provides narrow attenuation over a brief range of frequencies for periodic or band-limited noise.

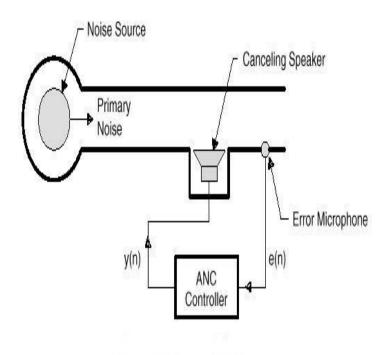


Figure 1.6: Feedback ANC System [11]

1.6.4 MultiChannel ANC

If there are many noise sources distributed in a broad location then it becomes impossible to diminish the undesirable noise with only single channel feedforward ANC, hence, a multichannel ANC is used as it has many reference microphones, secondary sources, and error microphones. A few of the uses are in control of exhaust turbulence in vehicles[16]-[19].

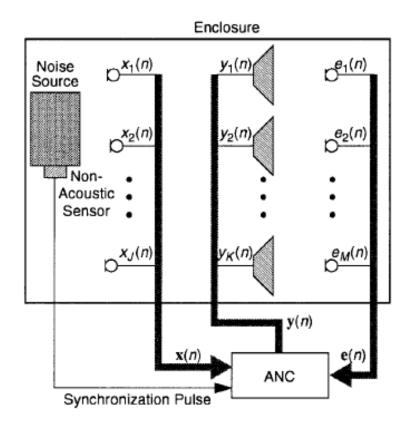


Figure 1.7: MultiChannel ANC System with J reference inputs, K secondary sources and M error sensors [19].

1.6.5 Hybrid ANC

In hybrid ANC systems, feedforward and feedback are deployed at the same time. The noise source is close to the sensor and hence the feedforward ANC gets the reference signal from the sensor. Error sensor measures the residual noise which is used to synthesize the feedback ANC filter and to update the variables of both feedforward and

CHAPTER 1: INTRODUCTION

feedback ANC filters [20].

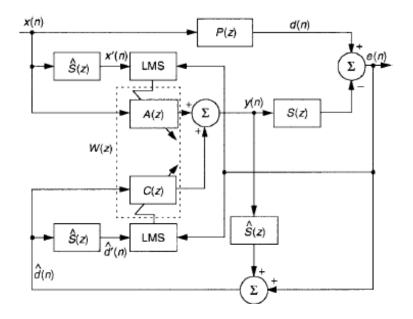


Figure 1.8: Hybrid ANC system[20].

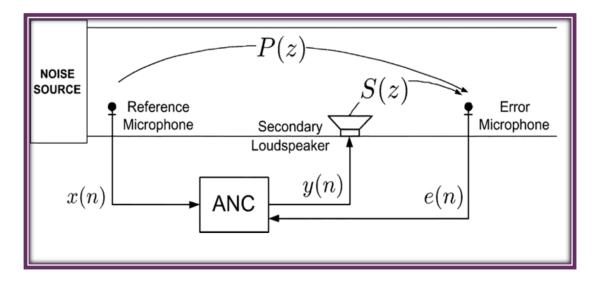


Figure 1.9: Basic ANC System [1].

1.7 Functioning of Basic ANC System

Fig 1.9 shows the block diagram of basic ANC system, where x(n) is the source of noise or our microphone used as a reference microphone. Primary path of noise to error microphone is P(z) and secondary path of noise while passing through ANC system to

secondary loud speaker is S(z). e(n) is the corrupted error signal and it is returned to the ANC system for enhanced performance. The ourput of the ANC system is denoted by y(n).

1.8 Adaptive Algorithms

An adaptive algorithm determines the process by which the filter is updated in relation to the exterior or required environment[20]-[28]. If w(k) is a vector of some length M and a time-varying finite impulse response (FIR) of the adaptive filter represents its components, a generalized equation to update its weight is given as,

$$\boldsymbol{w}(k+1) = \boldsymbol{w}(k) + \mu f[\boldsymbol{e}(k)\boldsymbol{x}(k)]^T, \qquad (1.8.1)$$

where, input reference signal is $\boldsymbol{x}(k)$, $\boldsymbol{w}(k)$ is the filter's weight at time k, $\boldsymbol{w}(k+1)$ is the updated weight at time k + 1 and μ controls the incremented size also known as step-size[29]-[32].

1.8.1 FxLMS Adaptive Algorithm

FxLMS is a widely used adaptive algorithm in ANC systems [33]. This algorithm functions by lowering the least mean square of the e(n) which is the error signal of the control filter w(n). The output of the ANC filter is y(n), p(n) and s(n) are the vectors representing primary and secondary paths, w(n) represents the ANC filter, and $\hat{s}(n)$ represents Online Secondary Path Modeling (OSPM) filter. The output of the ANC filter y(n) can be written as,

$$y(n) = \boldsymbol{w}^{T}(n) * \boldsymbol{x}(n)$$
(1.8.2)

The following equation is used to update the weights of the filter:

$$\boldsymbol{w}(n+1) = \boldsymbol{w}(n) + \mu_w e(n) \boldsymbol{x'}(n), \qquad (1.8.3)$$

where

$$e(n) = d(n) - y(n)$$
 (1.8.4)

$$d(n) = \boldsymbol{p}^{T}(n) * \boldsymbol{x}(n)$$
(1.8.5)

$$\boldsymbol{x}'(n) = \boldsymbol{s}^{T}(n) * \boldsymbol{x}(n)$$
(1.8.6)

and μ_w represents the step-size of the ANC filter.

1.8.2 FxLMAT

The LMAT is an adaptive algorithm which functions by reducing the third power of the mean of the absolute error, i.e., $|e(n)|^3$ [40]. The LMAT algorithm Weight update is given by:

$$\boldsymbol{w}(n+1) = \boldsymbol{w}(n) + \mu e^2(n) \operatorname{sign}[e(n)] \boldsymbol{x}(n), \qquad (1.8.7)$$

where, sign is the signum fuction, i.e., if $e(n) \ge 0$ then sign[e(n)] = 1, else sign[e(n)] = -1. μ is a small positive constant used as the step size parameter. Despite the fact that LMAT algorithm performs better than LMS for majority of the probability density functions (PDFs) of noise, its convergence depends on the strength of the reference signal $\mathbf{x}(n)$.

1.9 Online Secondary Path Modeling (OSPM)

ANC systems have become really favoured in recent times because of their enhanced functioning, lower cost and the invention of new applications embedded with this technology. The stableness of FxLMS algorithm depends on the reference signal $\boldsymbol{x}(n)$ filtering through the estimated secondary path $\hat{s}(n)$. Impoverished estimate, i.e., there is a difference of more than 90 degrees between the phase response of the original and the estimated secondary path coefficients. Although it is possible to estimate $\hat{s}(n)$ offline before the execution of the ANC system but in real time applications acoustic paths are time varying hence there is an obligation for OSPM.

1.9.1 Importance of OSPM

The basic adaptive LMS algorithm has degraded performance in the presence of secondary acoustic path, i.e., the channel between the controller and the error microphone. To cater for this problem, FxLMS was proposed. In [34]-[37], the reference signal is filtered through $\hat{s}(n)$ before being used in the ANC system. Secondary path can be estimated offline, however, ANC system comprises of digital to analog converters (DAC), analog to digital converters (ADC), etc. Change in any one of these components results in change of secondary path hence it creates the need to model the secondary path online.

1.10 Types of Noise

Noise is actually a type of signal that is undesirable and displeasing to our ears. Generally, noise is an irritating resonance that causes mild to severe discomfort.

1.10.1 Continuous Noise

It is the uninterrupted vibration caused, for example, by machinery that continues to run and emits undesirable noises. This type of noise generally comes from industrial area, ventilation systems or from engines.

1.10.2 Intermittent Noise

Intermittent noise is defined as noise that rapidly increases and decreases. This type of noise has a lot of fluctuation and is caused by factory machinery operating in cycles, a jet flying nearby or a noisy train passing by [38]-[40].

1.10.3 Low Frequency Noise

This type of noise is abundant in our daily lives. The frequency of this type of noise ranges from 10Hz-200Hz.

1.10.4 Impulsive Noise

This type of noise is most usually associated with industrial construction and explosions. IN is a short bursts of very high amplitude. IN has high power spectral density and it affects the communication signals badly [41]-[43]. The work reported in this thesis focuses on the active control of Impulsive Noise.

1.10.5 Symmetrical $-\alpha$ Stable Distribution

IN is modelled using Symmetrical - α Stable (S α S) Distribution , this distribution is given by the following characteristic expression,

$$\phi(t) = e^{-\gamma |t|^{\alpha}} \tag{1.10.1}$$

The (S α S) model is characterized by the help of the following parameters;

- 1. Characteristic component ' α ': α controls the shape of the distribution and its value ranges from 0< α <2. When α is closer to 0 the distribution becomes more impulsive, whereas, when α is closer to 2 the distribution becomes Gaussian.
- Skewness parameter 'β': With values between -1<β<1, β tells if the distribution is left skewed, i.e., β<0, centered at 0, i.e., β=0, or is right-skewed, i.e., β>0.
- 3. Scaling Parameter ' γ ': γ is the scaling parameter and its value is >0.
- 4. Location parameter ' δ ': δ is the parameter that depicts if the PDF lies on the x-axis or not.

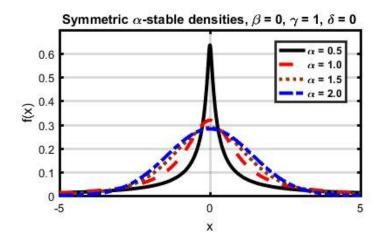


Figure 1.10: Symmetric- α Stable Distribution

1.11 Objectives

Following are the objectives of this research:

- 1. To propose an algorithm which helps in fast convergence of OSPM and reduces control filter error
- 2. To Improve performance of ANC system
- 3. To reduce the computational complexity

1.12 Area of Application

- 1. Headphones
- 2. Control of noise in air conditioning ducts
- 3. Control of noise in magnetic resonance imaging machines

1.13 Organization of Thesis

The thesis chapters are organized in five chapters. Chapter 1 provides brief introduction to ANC systems, their types, and applications. Chapter 2 is based on literature review,

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which tells about past techniques and their differences. Chapter 3 give details about our proposed algorithm and improvements. Chapter 4 covers simulations, results and complexity analysis of the proposed methods as compared ti past techniques. Chapter 5 tells about conclusion and future recommendations. CHAPTER 2

Literature Review

2.1 Background

Over the last few years, ANC has been a center of attention due to advancements in technology. The investigation of OSPM is required for boosting the effectiveness of ANC systems. FxLMS is one of the oldest and generally used adaptive algorithm in ANC systems. Different algorithms have been proposed in literature and all aim at improving the convergence while lowering down the computational and design complexities.

2.2 Different ANC Systems for OSPM

In the field of ANC systems authors in [44] were the pioneer in bringing in the concept of online secondary path estimation. In Eriksson's method [44], v(n) is used as a white noise which serves as a guidance signal and this technique consists of two filters. Eriksson's method was improved by Zhang [45] who proposed a third filter to lower the disturbance caused by the existing two filters, however, Zhangs method increased the design complexity. Due to increase in design complexity, this method was not very favored hence Akhter et al. proposed alternate method which required two filters and it improved the convergence as Variable Step Size LMS (VSS-LMS) algorithm was implemented in the modelling filter and modified FxLMS (MFxLMS) in the control filter [46]. Trade off of Akhter's method was that although it lowered the design complexity,

CHAPTER 2: LITERATURE REVIEW

the usage of MFxLMS algorithm increased the computational complexity. To solve this issue, Akhter suggested a modified version of [46] known as modified Akhter's method [47].This method uses FxLMS instead of MFxLMS algorithm.Drawback of Akhter's modified method is slow convergence due to use of FxLMS algorithm. To overcome slow convergence Carini proposed a self-tuning auxiliary noise power (ANP) scheduling alongside optimal VSS parameters, in which normalized FxLMS (Fx-NLMS) was used in both control and modelling filter [49]. Carini's method provided fast convergence but it increased the computational complexity. In [50] Ahmed suggested a new algorithm which used NFxLMS and NLMS for control and modeling filters, respectively. This algorithm also worked on two stage gain strategy. Pu et al. also introduced a technique in which modeling filter is updated using VSS-FxLMS algorithm along with simplified ANP scheduling strategy which decreased the computational complexity [48]. In [51] Yang proposed an enhanced algorithm in which the gain G(n) is determined only by the error function f(n) of the modeling filter. The convergence of all three filters depend on the step size of the modeling filters.

d(n)P(z)Noise Source v'(n)-v'(n)e(n)S(z)v(n) $\hat{v}(n)$ $\hat{S}(z)$ x(n)White Noise LMS Generat v(n)LMS **FxLMS** Algorithm

2.2.1 Eriksson's Method

Figure 2.1: Erikkson's Method for Feedforward ANC with OSPM [44].

Figure 2.1 shows Erikkson's method for OSPM in feedforward ANC. In Eriksson's method, v(n) is used as white noise which acts as a guidance signal. This technique

consist of two filters. One filter is used in controlling process and the other in modeling process. Control filter is implemented on Fx-LMS algorithm whereas modeling filter is updated using LMS algorithm. For Eriksson's ANC system, e(n), i.e., the error signal is calculated as:

$$e(n) = y'(n) * v'(n),$$
 (2.2.1)

where,

$$y'(n) = s(n) * y(n)$$
 (2.2.2)

$$v'(n) = s(n) * v(n)$$
 (2.2.3)

the white Gaussian noise is referred as v(n). f(n) is used as error signal for the modeling filter \hat{S} (z).

$$f(n) = [d(n) - y'(n) + v'(n)], \qquad (2.2.4)$$

which can be rephrased as

$$f(n) = e(n) - \hat{v'}(n).$$
 (2.2.5)

The variables for modelling filter are updated as

$$\hat{s}(n+1) = \hat{s}(n) + \beta * f(n) * v(n)$$
(2.2.6)

where, β is used as a step-size for modeling filter. The f(n) is used as error signal for the modeling filter \hat{S} (z).

coefficients for control filter are changed as

$$\boldsymbol{w}(n+1) = \boldsymbol{w}(n) + \alpha * \boldsymbol{e}(n) * \boldsymbol{x}(n), \qquad (2.2.7)$$

where the step-size for control filter is α . Erikkson's method was further improved by Zhang's.

2.2.2 Zhang's Method

The results of Erikkson's method are further enhanced by Zhang's et al [45], who proposed three filters instead of two filters. In Fig. 2.2, it can be seen that there is another

block of filter, h(n), which is included to reduce the distortion among the modeling and control filters. The error signal is then given as,

$$e(n) = d(n) - s(n) * y(n)$$
(2.2.8)

The new residual error signal, e'(n) can be defined as

$$e'(n) = e(n) - \hat{s}(n) * v(n).$$
(2.2.9)

Filter coefficients of the control and the modelling filter are given as:

$$w(n+1) = w(n) + \alpha * e'(n) * x'(n)$$
 (2.2.10)

$$\hat{s}(n+1) = \hat{s}(n) + \beta * v(n) * [g(n) - \hat{u}(n)]$$
(2.2.11)

The filter h(n) decreases the interference caused by noise v(n), update as:

$$h(n+1) = h(n) + \delta x(n) * e'(n).$$
(2.2.12)

Due to use of an additional filter in Zhang's method the design complexity of this

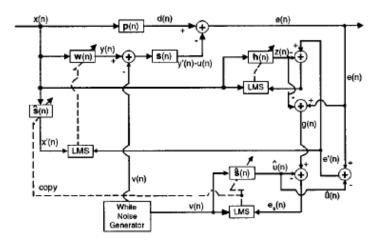


Figure 2.2: Zhang's Method for Feedforward ANC with OSPM [45].

this method was greatly increased and hence further improvements to this method were suggested.

2.2.3 Akhter's Method

Akhter proposed another method which required two filters and it improved the convergence[46]. VSS-LMS algorithm was used in the modelling filter and the control filter used modified FxLMS (MFxLMS), as shown in Fig. 2.3. In MFxLMS, a fixed block of filter is implemented to adjust the signal containing error for control filter hence increasing the upper bound of the step size. Due to implementation of the fixed filter, the convergence rate increases as compared to the previous methods. The step-size $\beta(n)$ for VSS-LMS adaptive algorithm is varied according to the ratio r(n), known as noise power scheduling, defined as

$$r(n) = \frac{\boldsymbol{P_f}(n)}{\boldsymbol{P_e}(n)}$$
(2.2.13)

 $P_f(n)$ and $P_e(n)$ are powers of the f(n) (modelling error signal) and the e(n) (residual error signal), respectively. The powers of error signals f(n) and e(n) are measured as

$$P_f(n) = \mu * P_f(n-1) + (1-\mu) * f^2(n)$$
(2.2.14)

$$P_e(n) = \mu * P_e(n-1) + (1-\mu) * e^2(n)$$
(2.2.15)

(where, μ is forgetting factor and its range is $(0.9 < \mu < 1)$. $\beta(n)$ is the step size that is estimated as

$$\beta(n) = r(n) * \beta_{min} + (1 - r(n)) * \beta_{max}$$
(2.2.16)

where β_{min} and β_{max} are minimum and maximum values of the step-size. Trade off of Akhter's method was that although it lowered the design complexity, the computational complexity was increased due to use of MFxLMS algorithm.

2.2.4 Akhter's Modified Method

To overcome increase in the computational complexity, Akhter suggested a modified version of his own method known as modified Akhter's method [47]. This method uses FxLMS instead of MFxLMS algorithm. Figure 2.4 shows block diagram of modified Akhter's method. In the previously proposed algorithms, gain G(n) of the random noise v(n) was fixed i.e. G(n) = 1 for all values of n. For a stable ANC system, G(n) should be 0 as the OSPM filter gets in steady state. If G(n) is kept fixed, v(n)

keeps on appearing in the residual noise e(n) resulting in overall degradation of the ANC process. ANP scheduling technique is implemented in this method which reduces v(n) in the residual error e(n) hence resulting in fast convergence. The gain G(n) is calculated as

$$G(n) = \sqrt{(1 - \rho(n))\sigma_{v_{min}}^2 + \rho(n)\sigma_{v_{max}}^2},$$
(2.2.17)

where, $\sigma_{v_{min}}^2$ and $\sigma_{v_{max}}^2$ are the minimum and maximum values of the variance of v(n), $\rho(n) = \frac{P_f(n)}{P_e(n)}$, and $P_e(n)$ and $P_f(n)$ are the powers of control and modelling error filters, respectively. Major drawback of the Akhter's modified method is gradual convergence due to use of the VSS-FxLMS adaptive algorithm.

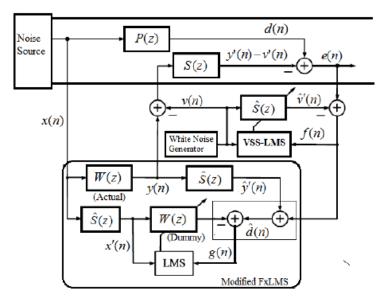


Figure 2.3: Akhter's Method for Feedforward ANC with OSPM [46].

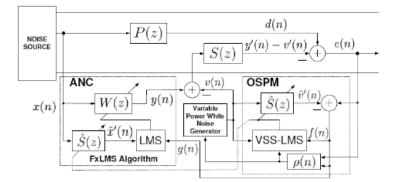


Figure 2.4: Akhter's Modified Method for Feedforward ANC with OSPM [47].

2.2.5 Carini's Method

To overcome slow convergence Carini proposed a self-tuning auxiliary noise power (ANP) scheduling alongside with optimal VSS parameters in which normalized FxLMS (Fx-NLMS) was used in both control and modelling filter [49]. In order to get the optimal VSS perimeters, ANC and OSPM filters are updated using the following Normalized LMS adaptations:

$$\boldsymbol{w}(n+1) = \boldsymbol{w}(n) + \mu_w(n) \frac{g(n)\hat{\boldsymbol{x}'}(n)}{\hat{\boldsymbol{x}'}^T(n)\hat{\boldsymbol{x}'}(n)}$$
(2.2.18)

$$\hat{\mathbf{s}}(n+1) = \hat{\mathbf{s}}(n) + \mu_s(n) \frac{f(n)v_1(n)}{\boldsymbol{v}^T(n)v(n)}$$
(2.2.19)

where $\mu_w(n)$ and $\mu_s(n)$ are variable step-sizes for ANC and OSPM filters respectively. To find the optimal value of μ_s , a delay coefficient technique is deployed. To implement the delay coefficient technique the memory length of OSPM filter is made M+D. $\hat{s}_0(n)$ and \hat{s}_1 are the vectors containing the first D and then the remaining M samples, respectively, and are updated as:

$$\hat{\boldsymbol{s}}_{\boldsymbol{0}}(n+1) = \hat{\boldsymbol{s}}_{\boldsymbol{0}}(n) + \mu_{\boldsymbol{s}}(n) \frac{f(n)\boldsymbol{v}_{\boldsymbol{0}}(n)}{\boldsymbol{v}^{T}(n)\boldsymbol{v}(n)}$$
(2.2.20)

$$\hat{s}_{1}(n+1) = \hat{s}_{1}(n) + \mu_{s}(n) \frac{f(n)v_{1}(n)}{v^{T}(n)v(n)}, \qquad (2.2.21)$$

where v_0 and v_1 are the vectors representing D and remaining M samples of v(n). Optimal value of μ_s is found by:

$$\mu_s(n) = \frac{\hat{\mathbf{N}}_s(n)}{\mathbf{P}_f(n)} \tag{2.2.22}$$

where using delay coefficient technique we obtain $\hat{N}(n)$. To find the optimal value for μ_w for each iteration we use the following equation:

$$\mu_w(n) = \frac{\mathbf{N}_w(n)}{\mathbf{P}_g(n)} \tag{2.2.23}$$

In this method self-tuning ANP scheduling methodology is implemented is used to keep the ratio R(n) constant and it is given by:

$$R(n) = \frac{E[((d(n) - y'(n))^2]}{E[(v'(n))^2]}$$
(2.2.24)

G(n), which is ANP scheduling gain, is calculated by:

$$G(n) = \sqrt{\frac{\boldsymbol{P}_{\boldsymbol{e}}(n)}{(R+1)\boldsymbol{P}_{\hat{\boldsymbol{s}}}(n)}}$$
(2.2.25)

In this method a new approach is used to schedule the noise power and furthermore normalized optimal step size parameters are used to model ANC and OSPM filters.

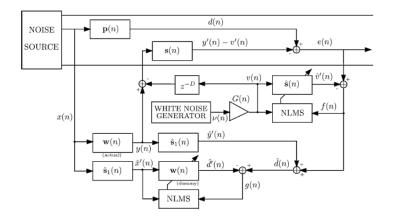


Figure 2.5: Carini's Method for Feedforward ANC with OSPM [49].

2.2.6 Ahmed's Method

Figure 2.6 shows Ahmed's method, this method uses a two-stage gain scheduling strategy for ANP [50]. In this method normalized step sizes are used. The gain sheduling in this proposed method is for two stages separately. The first stage, the ANC system is in a non steady state, i.e., $P_f > P_x$. Time varying G(n) is calculated :

$$G(n) = \sqrt{\frac{P_f(n-1)}{||\hat{s}(n)^2||}}$$
(2.2.26)

In the second stage, the ANC system is approaching steady state, i.e., $P_f \leq P_x$, then the time varying G(n) is given as:

$$G(n) = \sqrt{\frac{\boldsymbol{P}_{\boldsymbol{x}}(n)}{\boldsymbol{P}_{\boldsymbol{v}}(n)}} \quad \text{if} \quad \beta(n) > \frac{\boldsymbol{P}_{\boldsymbol{x}}(n)}{\boldsymbol{P}_{\boldsymbol{v}}(n)} \quad (2.2.27)$$

else

$$G(n) = \beta(n) \tag{2.2.28}$$

In this technique, normalized step sizes are used for the ANC and the OSPM filters. The weights of the ANC and the OSPM filters are updated as,

$$\boldsymbol{w}(n+1) = \boldsymbol{w}(n) + \mu_w(n)k(n)\boldsymbol{\hat{x}}'(n)$$
(2.2.29)

$$\hat{\boldsymbol{s}}(n+1) = \hat{\boldsymbol{s}}(n) + \mu_{\boldsymbol{s}}(n)f(n)\boldsymbol{v}(n)$$
 (2.2.30)

Ahmed's method has better convergence with less computational complexity, however, it performs computations for two separate stages. Therefore, this method is not suited for abrupt changes in acoustic paths.

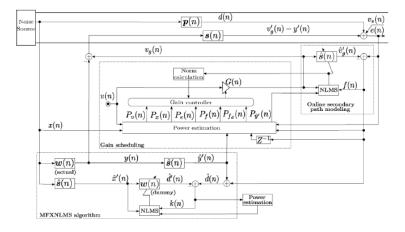


Figure 2.6: Ahmed's Method for Feedforward ANC with OSPM [50].

2.2.7 Pu's Method

Figure 2.7 shows block diagram of Pu's method. This method uses a new VSS FxLMS algorithm along with a new suggested ANP scheduling which only uses the parameters available [48]. In the ANP scheduling, the time varying gain G(n) is given as:

$$G(n+1) = \frac{\boldsymbol{P}_{\boldsymbol{x}}(n)\boldsymbol{P}_{\boldsymbol{f}}(n)}{\boldsymbol{P}_{\boldsymbol{e}}(n)}$$
(2.2.31)

The strategy suggested here is based on the OSPM system which is depicted in P_f and the reference signal power P_x . The ANC and OSPM filters are updated as:

$$\boldsymbol{w}(n+1) = \boldsymbol{w}(n) + \mu_w f(n) \boldsymbol{x'}(n) \tag{2.2.32}$$

$$\hat{\boldsymbol{s}}(n+1) = \hat{\boldsymbol{s}}(n) + \mu_{\boldsymbol{s}}(n)f(n)\boldsymbol{v}(n)$$
 (2.2.33)

where, μ_w is a controlling parameter for the ANC filter, and μ_s is formulated using a new VSSFxLMS technique defined as:

$$\mu_s(n) = \frac{\boldsymbol{P}_v(n)}{\boldsymbol{P}_e(n)} \tag{2.2.34}$$

Although Pu's algorithm has fast convergence with lower computational complexity, however, it cannot cater to abrupt changes in the acoustic paths.

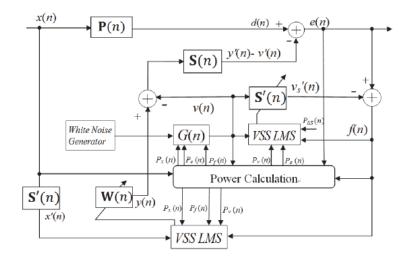


Figure 2.7: Pu's Method for Feedforward ANC with OSPM [48].

2.3 Comparison and Comments

It has been observed through simulations and analysis that Carini's method performs the best for ANC in the presence of IN for stationary acoustic paths, i.e., p(n) and s(n)whereas Ahmed's proposed method gives the accurate OSPM with fast convergence.

CHAPTER 3

Proposed Method

All the existing methods for OSPM in ANC used variants of FxLMS adaptive algorithm for control and modeling filters in the presence of Gaussian noise so there was a need to investigate an efficient method for IN. Our findings motivated us to switch to LMAT family as it provided more robustness in IN as compared to FxLMS.

3.1 Problem Statement

In ANC systems there is great difficulty in coming up with an algorithm that converges fast while keeping the computational complexity low. Most of the research work carried out in the ANC relies on the use of FxLMS algorithm which has slow convergence. Most of the research work is done in the presence of Gaussian noise and there is limited work done for IN. In order to cater for this problem we tested out FxLMAT algorithm in the presence of IN. We tested our proposed algorithms in low and high impulsive environments for ANC and OSPM and the results produced were promising surpassing the existing complex techniques. We proposed modification to FxLMAT algorithm called variable step size least mean absolute third (VSS-FxLMAT) where we optimised the step size and generated a new error signal that resulted in better ANC and OSPM. To further increase the convergence speed and improve OSPM while keeping ANC in mind we proposed yet another modification to FxLMAT called Variable step size filtered x robust normalized least mean absolute third (VSS-FXRNLMAT) algorithm. This al-

gorithm approached the convergence of recursive least square (RLS) family algorithms while providing the complexity of LMS algorithm. Our proposed technique worked in both stationary acoustic paths and Non-Stationary acoustic paths with different impulsive inputs. Our proposed techniques showed very promising results as compared to already developed ANC system algorithms while lowering down the complexity and improving the OSPM.

3.2 Proposed Fx-LMAT

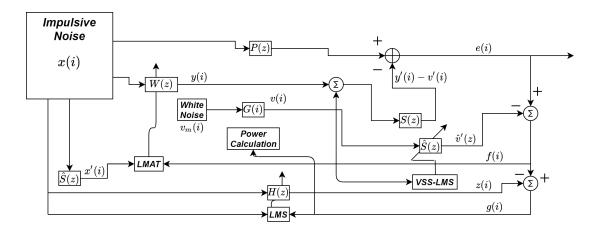


Figure 3.1: Block Diagram of Feedforward ANC with Fx-LMAT for OSPM

Figure 3.1 Shows the proposed technique with Fx-LMAT algorithm in the control filter. Control filter weight update equation for our proposed algorithm is given as

$$w(i+1) = w(i) + \mu_w * f(i)^2 * \operatorname{sign}(f(i)) * x'(i)$$
(3.2.1)

Modeling filter weight update equation is given as:

$$s(i+1) = s(i) + \mu_s(i) * g(i) * v(i)$$
(3.2.2)

where, μ_s is the step-size for our modeling filter, g(i) is the error signal which is used for the adaptation of modeling filter, and v(i) is the white noise injected in the modeling filter. e(i) is the error signal expressed as

$$e(i) = d(i) - [y'(i) - v'(i)]$$
(3.2.3)

The error signal f(i) which updates the coefficients of control filter w(i), is expressed as

$$f(i) = [d(i) - y'(i)] + [v'(i) - \hat{v}'(i)]$$
(3.2.4)

The error signal for the modelling filter $\hat{s}(i)$ is g(i) which is expressed as

$$g(i) = \varepsilon(i) + v(i) * [s(i) - \hat{s}(i-1)]$$
(3.2.5)

where $\varepsilon(i)$ is expressed as

$$\varepsilon(i) = d(i) - s(i) * y(i) - x(i) * h(i)$$
 (3.2.6)

The step size for the modelling filter is μ_s , expressed as

$$\mu_s(i) = 1/v^T v(i) + \delta_s(i)$$
(3.2.7)

v(i) is the white noise which is injected and δ_s is the parameter used for regularization often called regularization parameter

$$\hat{g}(i) = \varepsilon(i) + v(i) * [s(i) - \hat{s}(i)]$$
(3.2.8)

For the modeling filter $\hat{s}(i)$ to be updated, the component $\varepsilon(i)$ in the error signal g(i) is the disturbance and it has to be taken out speedily so that modelling process can be accelerated. By substituting Eq. 14 into Eq. 17 and replacing $\hat{s}(i-1)$, we get the following equation for δ_s

$$\delta_s(i) = \frac{\boldsymbol{P}_v(i)[\boldsymbol{P}_{\varepsilon}(i) + \sqrt{\boldsymbol{P}_g(i)\boldsymbol{P}_{\varepsilon}(i)}]}{\boldsymbol{P}_g(i) - \boldsymbol{P}_{\varepsilon}(i)}$$
(3.2.9)

The power P_v is expressed as

$$P_{v}(i) = \lambda P_{v}(i-1) + (1-\lambda)v^{2}(i)$$
(3.2.10)

The power $P_g(i)$ is expressed as

$$P_{g}(i) = \lambda P_{g}(i-1) + (1-\lambda)g^{2}(i)$$
(3.2.11)

 λ is another constant used as a forgetting factor, Value of λ ranges from .9 to 1. To make the modeling filter more stable, we multiply Eq. 19 with α , where α ranges from 0 to 1 Now the μ_s is expressed after multiplying with α as,

$$\mu_s(i) = \frac{\alpha}{\boldsymbol{v}^T(i)\boldsymbol{v}(i) + \delta_s(i)}$$
(3.2.12)

The third filter h(i) is updated as,

$$\boldsymbol{h}(i+1) = \boldsymbol{h}(i) + \mu_h g(i) \boldsymbol{x}(i) \tag{3.2.13}$$

where μ_h is the step size of the third filter and its main purpose is to minimize the interference hence it enables faster convergence of OSPM.

3.3 Proposed VSS-FxLMAT

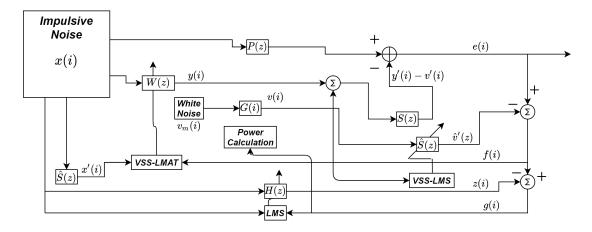


Figure 3.2: Block Diagram of Feedforward ANC with VSS FxLMAT for OSPM

Since the proposed Fx-LMAT performance in OSPM and in Control filter could be improved, we propose another algorithm, i.e., a Variable Step size Least Mean Absolute Third (VSS-LMAT). The block diagram of this technique is same as Fx-LMAT, however, there are changes in the control filter only which enabled for outclass performance. The block diagram of our proposed VSS-FxLMAT algorithm is shown in Figure 3.2. For VSS-LMAT we have suggested a new error signal which is expressed as

$$E_n(i) = [(\lambda E_n) + (1 - \lambda) * (|f(i)^2|)$$
(3.3.1)

Now we have a proposed a new step-size for the ANC filter μ_{wn} , expressed as

$$\mu_{wn} = \frac{\mu_w}{\delta_2 + |\hat{x}^2| + E_n(i)} \tag{3.3.2}$$

The control filter is updated as

$$w(i+1) = w(i) + \mu_{wn} * f(i)^2 * sign(f(i)) * x'(i)$$
(3.3.3)

Two new terms are introduced in this method which make the error signal low and that aids the process as the over all interference is also reduced. The new step size and the new error signal is used for the control filter which yields outclass results.

3.4 Proposed VSS-FxRNLMAT

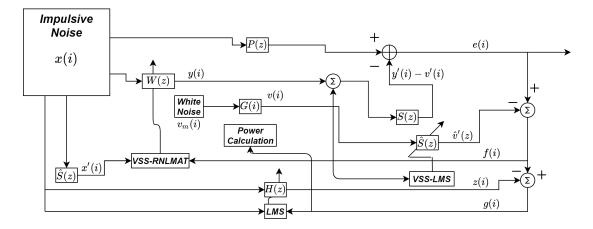


Figure 3.3: Block Diagram of Feedforward ANC with VSS-FxRNLMAT for OSPM

To achieve faster online estimation of the secondary path along with better results, we propose variable step size filtered x robust normalized least mean absolute third (VSS-FxRNLMAT) algorithm. This algorithm has far better results than most of the previous techniques. The block diagram of our proposed VSS-FxRNLMAT algorithm has been shown in Figure 3.3. We need to find a new error signal E_n like we did in previous technique, expressed as

$$E_n(i+1) = (\lambda E_n(i)) + ((1-\lambda)) * |f(i)^2|$$
(3.4.1)

Step size for the modeling of the control filter is then expressed as

$$\mu_{wn} = \frac{\mu_w}{||x'^2|| + \delta_2 + E_n(i+1)}$$
(3.4.2)

To further improve the control process we further propose a better step size defined μ_{w2} . μ_{w2} , which is our step size for the control filter, written as

$$\mu_{w2} = \frac{\mu_{wn}}{1 + \beta_2 * |f(i)^3|} \tag{3.4.3}$$

This yields outclass performance and the convergence is greatly improved. The weight update equation of the control filter then becomes

$$w(i+1) = w(i) + \mu_{w2} * f(i)^2 * sign(f(i)) * x'(i)$$
(3.4.4)

CHAPTER 4

Simulations, Results and Complexity Analysis

This chapter provides and discusses simulation results of various ANC methods, all of the simulations were carried out in MATLAB. A detailed analysis of all the existing and proposed methods for ANC in IN for OSPM is performed. The following algorithms are compared with our proposed methods:

- · Erikkson's method
- Akhter's method
- Carini's method
- Pu's method
- Yang's methof
- · Fareeha's method

In formulating our results, we tested the existing techniques and our proposed algorithms in IN and also in non stationary acoustic paths in the presence of IN. The performance matrices which we selected are Relative Modeling Error Δ S, Mean Noise Reduction (MNR) which tells us about the performance of Control Filter and vibration reduction parameter R.

$$MNR(i) = E(\frac{A_e(i)}{A_d(i)}) \tag{4.0.1}$$

Relative modeling Error ΔS is expressed as

$$\Delta S(i) = 10 \text{Log}_{10} \frac{||s(i) - \hat{s}(i)||^2}{||s(i)||}$$
(4.0.2)

Vibration Reduction R is expressed as

$$R(i) = -10\log(\frac{\sum e(i)^2}{\sum d(i)^2})$$
(4.0.3)

Finite impulse response (FIR) filters are used to mimic the primary p(n) and secondary s(n) acoustic paths. The impulse response of the primary acoustic path is set to a memory length of 48 while the secondary acoustic path is set to a memory length of 16. The tap vector length of ANC filter w(n) and OSPM filter $\hat{s}(n)$ is set to 32 and 16, respectively. In our simulations, two phases of ANC system have been taken into account. In the first phasem ANC filter has been kept inactive till 5000 iterations and only the OSPM filter is adapted to obtain the secondary path's estimate. In the second phase, after 5000 iterations, both ANC and OSPM filters are adapted simultaneously. The v(n) signal is white Gaussian noise with zero mean and 0.05 variance. A lot of simulations have been carried in order to get the optimal values of parameters for improving ANC system's OSPM performance. In our simulations we have considered the following four cases which are mentioned below:

- Case 1: Stationary acoustic paths with impulsive input, α =1.85
- Case 2: Stationary acoustic paths with impulsive input, α =1.65
- Case 3: Non-Stationary acoustic paths with impulsive input, α =1.65
- Case 4: Non-Stationary acoustic paths with impulsive input, α =1.85

4.1 Case 1: Stationary acoustic paths with impulsive input, α =1.85

In case 1, the input is IN with α =1.85 and the acoustic paths are kept stationary. Figure 4.2 shows Δ S for stationary acoustic paths when n=5000 as the ANC is in offline

Impulsive Noise <i>x(n)</i>					
Parameters	Symbol	Value			
Total Realizations	Avg	10			
Total Iterations	Ν	100,000			
Characteristic exponent	α	1.65, 1.85			
Scale Parameter	γ	1			
Location Parameter	С	0			
Skewness parameter	δ	0			

Table 4.1: Parameters for Impulsive Noise used in Simulations

Table 4.2: Values of Parameters for ANC System used in Simulations

ANC System with OSPM					
Parameters	Symbol	Value			
Primary path tap length	L	48			
Secondary path tap length	М	16			
ANC filter tap length	Lw	32			
OSPM filter tap length	М	16			

state before 5000 iterations and after that the proposed Fx-LMAT converges around 55000 iterations and reaches its steady state at 70000 iterations at -40dBs. Proposed VSS Fx-LMAT follows the same trend at -40dBs surpassing many proposed techniques. The simulation results show that our proposed VSS Fx-RNLMAT has the best convergence and steady state as it converges at n=2900 and shows robustness with no spikes and reaches -35dBs. Our proposed algorithm performs better than Yang's , Carini's, PU's, Akhter's and Erikkson's methods.

Figure 4.3 shows Mean noise Reduction after 5000 Iterations, our proposed Fx-LMAT

Methods	Value of Controlling Parameters			
	Case 1 for α =1.85	Case 2 for $\alpha = 1.65$	Case 3 for $\alpha = 1.65$	Case 4 for $\alpha = 1.85$
Erikson's Method	$\begin{array}{l} \mu_s \ = \ 1e-7, \\ \mu_w \ = \ 1e-7 \end{array}$	$\mu_s = 1e - 7,$ $\mu_w = 1e - 7$		
Akhtar's method	$\begin{array}{l} \mu_{w} = 1e-8, \\ \mu_{s_min} = 1e-6, \\ \mu_{s_max} = 1e-3, \\ \mathbf{e}_{v_{min}}^{2} = 1e-3, \\ \mathbf{e}_{v_{max}}^{2} = 1 \end{array}$	$\begin{array}{l} \mu_{W} = 1e-8, \\ \mu_{s_min} = 1e-6, \\ \mu_{s_max} = 1e-3, \\ e_{p_{min}}^{2} = 1e-3, \\ e_{p_{max}}^{2} = 1 \end{array}$		
Carini's Method	$\hat{\lambda} = 0.7$ $D = 8$ $\lambda = 0.99$ $R = 1$ $\mu_{s_min} = 1e - 4$	$\begin{aligned} \hat{\lambda} &= 0.7\\ D &= 8\\ \lambda &= 0.99\\ R &= 1\\ \mu_{s\ min} &= 1e-4 \end{aligned}$	$\begin{split} \hat{\lambda} &= 0.7\\ D &= 8\\ \lambda &= 0.99\\ R &= 1\\ \mu_{s_min} &= 1e-4 \end{split}$	$\begin{aligned} \hat{\lambda} &= 0.7\\ D &= 8\\ \lambda &= 0.99\\ R &= 1\\ \mu_{s_min} &= 1e-4 \end{aligned}$
Pu's Method	$\mu_w = 1e - 8$	-		
Yang's Method	$\mu_w = 1e - 8$ $\mu_h = 1e - 7$ $\alpha = 0.005$	$\mu_w = 1e - 8$ $\mu_h = 1e - 7$ $\alpha = 0.005$	$\mu_w = 1e - 8$ $\mu_h = 1e - 7$ $\alpha = 0.005$	$\mu_w = 1e - 8$ $\mu_h = 1e - 7$ $\alpha = 0.005$
Fareeha's Method	$\begin{array}{c} \delta_1 = 100,000 \\ \delta_2 = 1000 \\ \mu_h = 1e - 4 \\ \lambda = 0.99999 \end{array}$	$\begin{split} \delta_1 = 1000 \\ \delta_2 = 1000 \\ \mu_h = 1e - 4 \\ \lambda = 0.99999 \end{split}$	$\begin{split} \delta_1 = & 1000 \\ \delta_2 = & 1000 \\ \mu_h = & 1e - 4 \\ \lambda = & 0.99999 \end{split}$	$\delta_1 = 1000$ $\delta_2 = 1000$ $\mu_h = 1e - 4$ $\lambda = 0.999999$
<u>Proposed Fxlmat Method</u>	$\mu_w = 1e - 9$ $\mu_h = 1e - 6$ $\alpha = 0.001$	$\mu_w = 1e - 9$ $\mu_h = 1e - 6$ $\alpha = 0.001$	$\mu_w = 1e - 9$ $\mu_h = 1e - 6$ $\alpha = 0.03$	$\mu_w = 1e - 9$ $\mu_h = 1e - 6$ $\alpha = 0.03$
Proposed VSS FxLMAT Method	$\begin{array}{l} \mu_{wn} = 1e-3 \\ \mu_h = 1e-7 \\ \alpha = 0.001 \\ \delta_2 = .001 \end{array}$	$\begin{array}{l} \mu_{wn} = 1e-3 \\ \mu_h = 1e-7 \\ \alpha = 0.001 \\ \delta_2 = .001 \end{array}$	$\mu_{wn} = 1e - 3 \\ \mu_h = 1e - 7 \\ \alpha = 0.01 \\ \delta_2 = .001$	$\mu_{wn} = 1e - 3$ $\mu_h = 1e - 7$ $\alpha = 0.01$ $\delta_2 = .001$
Proposed VSS FxRNLMAT Method	$\mu_{w2} = 1e - 1 \\ \mu_h = 1e - 7 \\ \alpha = 0.002 \\ \delta_2 = .001 \\ \beta = .001$	$\begin{array}{c} \mu_{w2} = 1e-1 \\ \mu_h = 1e-7 \\ \alpha = 0.002 \\ \delta_2 = .001 \\ \beta = .001 \end{array}$	$\mu_{w2} = 1e - 2 \\ \mu_h = 1e - 7 \\ \alpha = 0.01 \\ \delta_2 = .001 \\ \beta = .001$	$\mu_{w2} = 1e - 2 \\ \mu_h = 1e - 7 \\ \alpha = 0.01 \\ \delta_2 = .001 \\ \beta = .001$

Table 4.3: Values of Controlling Parameters for Existing and Proposed Methods

Table 4.4: Complexity Analysis of Existing and Proposed Techniques

Methods	*,∕,√	+,-	Total	Example
Carini's method	7Lw+6M+4D+26	6Lw+6M+4D-1	13Lw+12M+8D+25	969
Pu's method	2Lw+3M+18			
		2Lw+3M+3	4Lw+4M+21	309
Yang's Method	2Lw+3M+2K+19	2Lw+3M+2K+5	4Lw+6M+4K+24	344
Fareeha's Method				
	3M2+7M+3Lw2+4Lw+2K+10	2M2+5M+2Lw2+2Lw+2K-2	5M2+12M+5Lw2+6Lw+4K+8	13320
Proposed FxLMAT				
	17+Lw+6M+3K	5+2L+6M+Lw+3K	22+2Lw+12M+2L+6K	422
Proposed VSS FxLMAT				
	20+L+Lw+6M+3K	8+3L+6M+Lw+3K	31+4L+2Lw+12M+6K	527
Proposed VSS FxRNLMAT				
	20+2L+Lw+6M+3K	9+4L+6M+Lw+3K	33+6L+2Lw+12M+6K	625

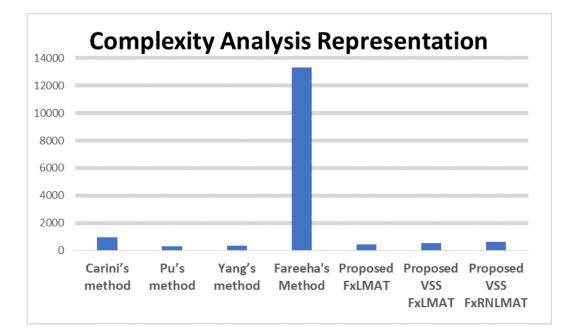


Figure 4.1: Complexity Analysis Representation

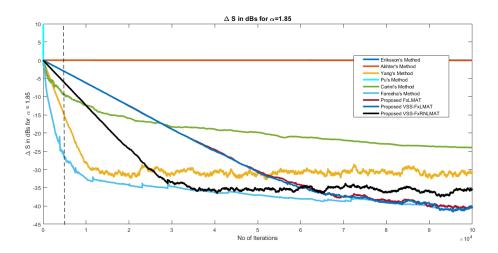


Figure 4.2: Relative Modelling Error for Case 1

shows good MNR and at 20000 iterations, it reaches 1.5 which surpasses Yangs. Carini's method diverges in impulsive environment and so does PU's method. Eriksson's and Akhter's method remains at 1. Proposed VSS-FxLMAT converges at 11000 iterations and reaches steady state at 20000 iterations and remains less than 0.5 showing amazing performance. Our proposed VSS-FxRNLMAT converges right at 5000 iterations as soon as the ANC is switched on and it reaches steady state at 6000 iterations and remains at 0.4, performing better than Fareeha's method and almost following the Carini's

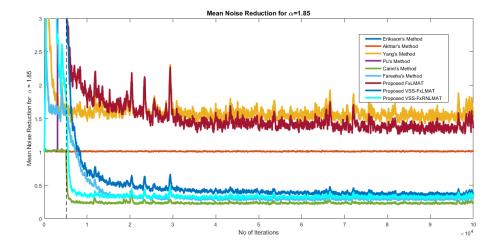


Figure 4.3: Mean Noise Reduction for Case 1

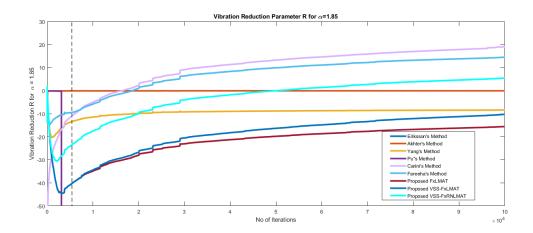


Figure 4.4: Vibration Reduction Parameter R for Case 1

method.

Figure 4.4 shows Vibration Reduction after 5000 Iterations when the ANC gets in online mode the convergence is good for all of the proposed Algorithms but VSS-FxRNLMAT out classes the rest of the algorithm and just falling under Fareeha's and Carini's method.

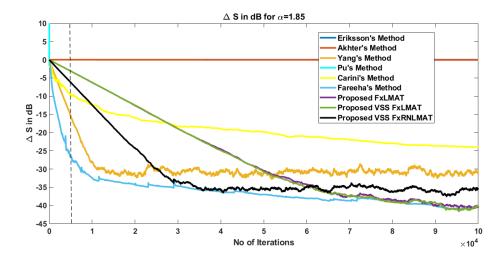


Figure 4.5: Relative Modelling Error for Case 2

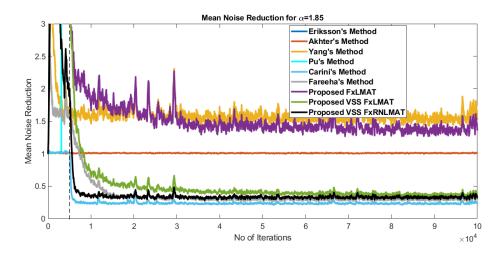


Figure 4.6: Mean Noise Reduction for Case 2

4.2 Case 2: Stationary acoustic paths with impulsive input, α =1.65

In case 2 the input is IN with α =1.65 and the acoustic paths are kept stationary. Figure 4.5 shows Δ S for stationary acoustic paths when n=5000 as the ANC is in offline state before 5000 iterations. Our proposed Fx-LMAT converges around 55000 iterations and reaches its steady state around 70000 iterations at -40dBs. Proposed VSS Fx-LMAT follows the same trend at -40dBs surpassing many proposed techniques. Figure 4.4 shows that our VSS Fx-RNLMAT has the best convergence and steady state as it converges at n=2900 and shows robustness with no spikes at -35dBs. Our proposed

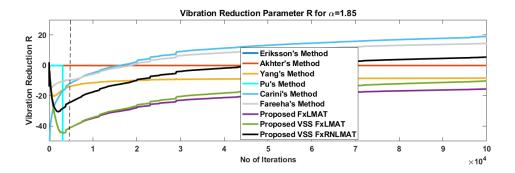


Figure 4.7: Vibration Reduction Parameter R for Case 2

Algos perform better than Yang's , Carini's, PU's, Akhter's and Erikkson's algorithm. Figure 4.6 shows mean noise reduction after 5000 Iterations. Our Proposed Fx-LMAT shows good MNR and at 20000 iterations, it reaches 1.5 which is better than Yangs. Carini's method diverges in impulsive environment and so does PU's method. Eriksson's and Akhter's method remains at 1. Proposed VSS-FxLMAT converges at 11000 Iterations and reaches steady state at 20000 iterations and remains less than 0.5 showing outstanding performance. The proposed VSS-FxRNLMAT converges right at 5000 iterations as soon as the ANC is switched on and it reaches steady state at 6000 iterations and remains at 0.4, thus, performing better than Fareeha's method and almost showing following the Carini's method.

Figure 4.7 Shows Vibration Reduction after 5000 iterations. When the ANC gets in online mode, the convergence is good for all of the proposed algorithms, however, the VSS-FxRNLMAT out classes the rest of the algorithms and just falling under Fareeha's and Carini's method.

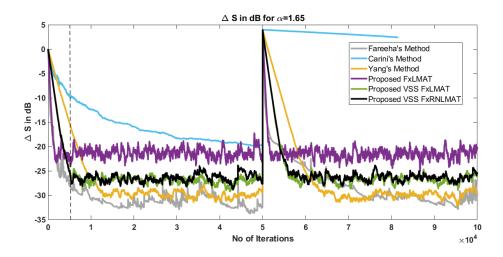


Figure 4.8: Relative Modelling Error for Case 3

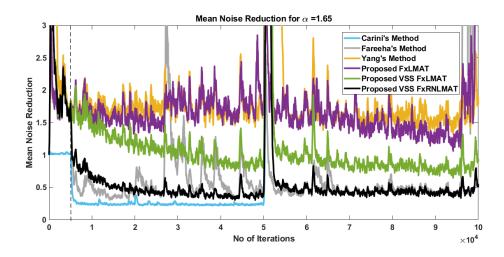


Figure 4.9: Mean Noise Reduction for Case 3

4.3 Case 3: Non-Stationary acoustic paths with impulsive input, α =1.65

In case 3, the acoustic paths are non stationary and the input noise is IN with α =1.65. Figure 4.8 confirms our findings that in varying acoustic paths with high impulsive environment the proposed Fx-LMAT algorithm shows robustness and stability as compared to all other algorithms. It converges right at 5000 iterations when the ANC is in online mode and continues to show rigidity and achieves steady state at 5000 iterations. At 50000 iterations when the acoustic paths are reversed the algorithm converges back showing stability at -22dBs. The proposed VSS-FxLMAT shows a similar robustness

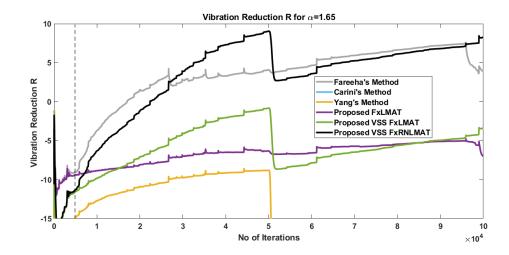


Figure 4.10: Vibration Reduction Parameter R for Case 3

and follows the same trend at -25dBs, whereas, our proposed VSS-FxRNLMAT shows fast convergence and stability as compared to both Yang's and Fareeha's methods which lack robustness. Moreover, at 50000 iterations when the acoustic paths are reversed our Algorithm reaches the stability and ends at -25dBs.

Figure 4.9 shows the Mean Noise Reduction Comparison of our proposed algorithms with the already existing techniques. In high impulsive non stationary environment, our proposed Fx-LMAT performs better than Yang's and shows stability and reaches MNR of 1.5 to improve the stability and robustness our proposed VSS-FxLMAT shows better convergence and reaches MNR of 0.8. VSS-FxRNLMAT has better convergence stability and performance than all other algorithms. It reaches the steady state at 20000 iterations and shows robustness as compared to Fareeha's method and falls back to stability even when the acoustic paths are reversed and reaches MNR of 0.4.

Figure 4.10 shows the vibration reduction parameter R. Yang's and Carini's methods don't show any results as these two algorithms don't converge for R, however, our proposed LMAT shows robustness and stability and reaches -6. VSS-LMAT shows better results but stability is affected by change in acoustic paths and it reaches -4. VSS-FxRNLMAT shows the best result although there is little deviation when the acoustic paths are reversed at 50000 iterations but the algorithm stabilizes and shows result of 7 surpassing Fareeha's method.

4.4 Case 4: Non-Stationary acoustic paths with impulsive input, α =1.85

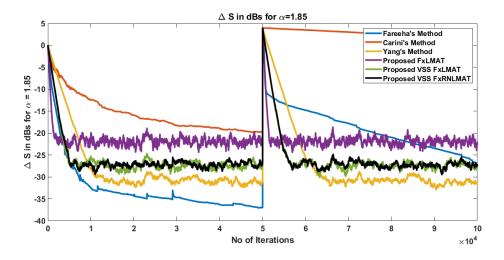


Figure 4.11: Relative Modelling Error for Case 4

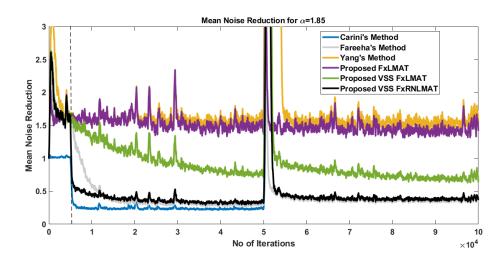


Figure 4.12: Mean Noise Reduction for Case 4

Figure 4.11 shows that After 5000 iterations the ANC gets in online mode and Fx-LMAT converges rapidly and reaches steady state right at instant and even when acoustic paths are reversed at 50000 Fx-LMAT converges back at -23dBs. Our proposed VSS-FxLMAT converges at 6000 iterations and reaches steady state and when the acoustic paths are reversed the proposed algorithm falls back and reaches steady state at 55000 iterations while being robust at -27dBs. Our proposed VSS-FxRNLMAT follows the

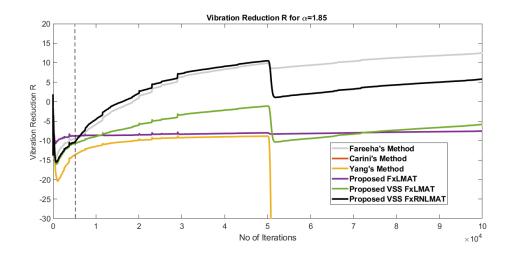


Figure 4.13: Vibration Reduction Parameter R for Case 4

same trend where as surpassing Fareeha's method as it takes a lot of time in converging back once the acoustic paths are reversed. Yang's method is slow at converging and after 50000 iterations it falls back slowly and shows lack of robustness.

Figure 4.12 shows that our proposed Fx-LMAT exhibits robustness and after 5000 iterations it remains robust. Moreover, there is hardly any fluctuation when the acoustic paths are reversed and it reaches an MNR value of 1.5 surpassing Yang's method. Our proposed VSS-FxLMAT algorithm show better results but the convergence is slow although it does not diverge even after 50000 iterations and reaches 0.8. VSS-FxRNLMAT out performs all the proposed algorithms as it converges at 10000 iterations and even when the acoustic paths are reversed it converges back with utmost robustness and reaches an MNR value of 0.4.

Figure 4.13 shows that our proposed Fx-LMAT shows robustness and does not get affected by change in acoustic paths and reaches -10. VSS-FxLMAT converges faster than Fx-LMAT but when the acoustic paths are reversed there is a big dip in the algorithm and it reaches -6. VSS-FxRNLMAT converges right at 5000 iterations and the algorithm handles the change in acoustic paths, finally reaching the value of 6. Carini's and Yang's methods diverge in non-stationary environment, only Fareeha's method is able to challenge VSS-FxRNLMAT.

4.5 Discussion

From our results and analysis it can be deduced that Carini's method provided the best noise control and vibration reduction for non stationary and stationary acoustic paths. Fareeha's method also achieved better noise control as compared to other existing methods. The proposed modifications in the FxLMAT algorithm proved fruitful as our proposed VSS-FxLMAT and VSS-FxRNLMAT provided excellent robustness against low and high IN. Our proposed algorithms were able to perform in both stationary and non stationary acoustic paths with fast convergence and excellent noise control.

CHAPTER 5

Conclusion and Future Recommendations

In this thesis we have reported research work based on Active Noise Control (ANC) of impulsive noise (IN) with online secondary path modelling (OSPM). In this research work we tested filtered x Least mean absolute third (FxLMAT) in IN and suggested two modifications to this adaptive algorithm. Our proposed modifications are variable step size filtered x least mean absolute third (VSS-FxLMAT) and variable step size filtered x robust normalized least mean absolute third (VSS-FxRNLMAT) algorthims. Simulation results showed that our proposed VSS-FxRNLMAT surpassed all current algorithms in noise control and OSPM while reducing the computational complexity. We also observed that VSS-FxRNLMAT approached recursive least square (RLS) adaptive filters family while providing the complexity of least mean square (LMS) adaptive algorithms family.

5.1 Future Recommendations

This research work focuses on active noise control systems that are feedforward in impulsive noise with online secondary path modelling. This research can be further extended to feedback and hybrid ANC systems. Modifications can be made to the proposed VSS-FxRNLMAT so that step size can be optimized further to aid the modeling

CHAPTER 5: CONCLUSION AND FUTURE RECOMMENDATIONS

process. The proposed methods can be further used for narrow-band or continuous noise inputs.

Furthermore, by making necessary changes to the control and modelling processes, computational complexity may be lowered even further.

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