Flow Dynamical Aspects of Black Holes in Rainbow Gravity

by

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Abstract

In this dissertation the accretion of the Schwarzschild black hole immersed in an electromagnetic field is discussed in the context of rainbow gravity. Discussion of the Hamilton Jacobi equations for astrophysical fluid is also given.

Then the cyclic and heteroclinic flow of the Schwarzschild black hole in rainbow gravity is discussed. Later, we find the Hamiltonian, sonic points, non-sonic critical points of the given system and discuss the isothermal test fluid with its solution. Further, conditions for critical accretion and polytropic solution for the Schwarzschild black hole are discussed in the framework of rainbow gravity.

Contents

Chapter 1

1.1 Introduction

On the basis of gravitational force, Isaac Newton compared the acceleration of different objects. He linked the acceleration of bodies on earth to the acceleration of bodies on moon. His comparison enabled him to wrap up the concept of gravitational force of attraction between different objects. He found out that the gravitational force of attraction shows a significant change when the distance between the two objects is varied. Further, he noticed distance is not only the variable quantity affecting the magnitude of gravitational force. He noticed that the gravitational force of attraction decreases when the square of the distance between the centers of two bodies is increased. The concept behind the mystery of the apple falling from tree under gravity must be contingent upon the mass of the apple. As acceleration of freely falling object is constant over small heights, thus the force F becomes entirely relied on the mass of the falling object/apple. On the basis of these ideas Newton discovered universal law of gravitation in which every body in the entire universe, attracts every other body with a force which changes directly with the product of their masses and that force changes inversely to the square of distance between their centers. The Newtonian gravity is treated as universal constant, i.e. same in all inertial frames [1].

In 1905, Einstein showed that if speed of light is treated as universal constant, then Lorentz transformations reduces to Galilean transformation, which also serve as a more generalized replacement. Further treating time on the same ground to the dimensions related to space, Einstein proposed special relativity (SR). This is grounded upon the two principles stated below [2]:

1. All inertial reference frames are physically same, i.e. in an inertial frame (a frame in which Newton's second law of motion holds), there is no physical dissension between two observers if they move relative to each other and "physical laws" appear to be same for both of them.

2. The speed of light remains the same in vacuum and observer independent.

Clearly, this theory is bounded to dealing with uniform and linear motion only. That is why it is called SR. Maxwell's theory is a remarkable theory of light, magnetism and electricity, which is merged into the framework of SR. Then one could think that the next step would be, to generalize Newton's theory of gravitation by making that compatible with Einstein's theory of SR as Maxwell did by generalizing Coulombs electrostatics and proposed new theory [2]. In contrast, Einstein selected completely different path for formulation of his theory, instead of generalized relativity. He proposed a mechanism, in which space and time are treated a continuum as spacetime. The idea behind this was that, mostly the forces of nature are represented by fields defined on spacetime fabric but in this theory of gravity known as general relativity (GR), gravity is the property of spacetime [1].

According to the equivalence principle, all bodies irrespective of their size and weight, experience same acceleration in the gravitational field. Now to develop further perspective on gravitation, one should consider how can we measure the electromagnetic filed in SR. First of all, set the "background observers", which are not in the range of electromagnetic field. These observers are called "inertial observers" and satisfy geodesic equation $u^a \partial_a u^b = 0$ [1]. Secondly, we place a charged test body in that electromagnetic field. Here the deviation of that test body from inertial motion will determine F_{ab} [1].

By the equivalence principle, we are unable to stay away a body or an observer from the force field. Thus, we do not have any straightforward, physical procedure for constructing mechanical approach parallel to that of electromagnetism. Any observer will move in an identical manner to the test body. So, we do not have an independent "background observer" to monitor the body under test. Therefore, we have no simple and direct method to assess the gravitational field in the present frames. If Einstein's theory of relativity were correct, one might device mechanical methods for independent observer by spacetime measurements, i.e. the (flat) spacetime metric can be measured using clocks and meter sticks, from which geodesic can be calculated. Also, the inertial observer can be thought of as driven by rocket engine. In this way equivalence principle can be viewed as an unexpected aspect of gravitational force field as was in Newtonian theory of gravity [2].

The concept of the GR emerges from our inability to construct independent "inertial observer" in SR. This can be sufficed by the hypothesis that " the spacetime is not flat". The world line of a free falling timelike observers is exactly same as the geodesic of the curved spacetime metric. So, this can be concluded that the motion of an object in gravitational field is equivalent to the geodesic of the curved spacetime. Hence gravity can be viewed as an aspect of spacetime structure instead of the force field. Einstein's theory of GR modifies the framework of SR by allowing the manifold to differ from $\mathbb{R}^4$ and allow the metric to be non flat. The physical quantities in GR can be described by tensor fields as were used in SR. In GR, perfect fluids are described in the terms of velocity 4-vector u^a (unit tangent (measured by g_{ab}) to its world line), density ρ, pressure P [1, 2].

1.1.1 Quantum Theory of Gravity

The Einstein's theory of GR put forth a revolutionary viewpoint on gravity and spacetime. However, in one important aspect, this theory is not comprehensive enough. It is well established that all physical fields must be described on a fundamental level by the principles of quantum theory. In quantum field theory, state of a system are represented by vectors in a Hilbert space and observable quantities are shown by self adjoint linear maps acting on Hilbert space [3]. Moreover, observable will not have definite value and one can only predicts possibilities for the outcomes of measurements. However, in GR the observable quantities, especially the spacetime metric always have a definite value. Thus, if the principles of quantum theory are apply to gravitational field, GR must correspond to quantum theory of gravity.

Classical description of matter are suitable for matter at macroscopic scale but becomes entirely inadequate on atomic and smaller scales. In this case, the scale at which the classical description breaks down is determined by the masses and velocity of the fundamental particles as well as the two fundamental constant of nature which enter the theory, namely Planck's constant h and the speed of light c. Similarly, in quantum theory of gravitation based on GR, one would expect that the fundamental scale at which the classical description become completely inadequate should be set by h, c and G. This is a unique combination of these constants which has the dimensions of length, namely the Planck's length $l_p = (Gh/c^3)^{\frac{1}{2}}$ [4, 5]. As might be expected, the Planck's length arises naturally in attempts to formulate a quantum theory of gravity. Thus, dimensional argument suggest that a classical description of space time structure should break down at scales of the order of the Planck length and below. Quantum mechanics and GR seems to describe all of observable reality and yet they cannot be simultaneously true. They must be united in a deeper yet undiscovered theory. In the start of 20^{th} century Einstein's relativity utterly change the means, how we anticipate space, time, mechanics and gravity. Then the revolution in quantum mechanics of the 20's and 30's upturned all of our thinking about the subatomic world [3].

These two philosophies have conflicts with each other in fundamental ways. On one hand as GR is the comprehensive theory of gravity, which describes the presence of matter and energy as to wrap the sheet of spacetime and the motion of objects is thereby changed. This result in the effect we perceive as gravity [4]. On the other hand, quantum mechanics talks about the subatomic world. It describes particle as waves of infinite possibility whose observed properties are indigenously uncertain. That development started with the Schroedinger equation, which tracks the probable waves through space and time. Here the space and time are treated as fundamentally separate variables like in Newtonian mechanics [5].

In order to build a comprehensive approach on gravitation, quantum mechanics must incorporate essential features of Einstein's relativity. This was initially addressed by Paul Dirac, who designed relativistic wave equation for the ultra fast electrons. Nowadays, modern quantum field theories fully incorporate the melding of space and time, yet inclusion of bending of spacetime is more complex. This causes issues some mild and flexible, other catastrophic. So, starting with the mild, we have paradox about black hole information. The black hole of pure GR swallow information in a way that can remove it completely from the universe. Especially when these black holes evaporate via Hawking radiation [3, 4]. A big conflict arises due to quantum theory which tells that quantum information should never be destroyed. This is known as information loss paradox, which is itself addressed by modern developments in quantum information theory. Following the work of Hawking, Bekenstein, Gerard and others, it becomes clear that an information engulfeds by the black hole can again become the part of the universe back through their Hawking radiation [3]. In a sense, both the source and the solution to the information loss paradox came from the discovery of Hawking radiation. Hawking was able to unite GR and quantum field theory but the union was approximate and incomplete [3].

In fact, it is very practicable to embed the curved geometry of GR into the theory of quantum field dealing with time and space but that approach completely fails when we have intense effects of gravity on the smaller scales of spacetime like the peculiarity of the black hole or at the instant of the Big Bang. For that, we need a generalized quantum theory of gravity, although the concepts of curved space on the smaller scales leads to catastrophic conflicts. We talk about these in two ways one after the another [3]. Let's start by thinking about what it means to define a location in a gravitational field with perfect precision. In order to measure a location in space say, the location of a particle you need to interact with it. One typically do that by striking a photon or other particle with the object. The more precisely you want to measure position the higher the energy of that interaction. That is why we use electron microscopes or X-rays or gamma rays to take images of extremely small things. So lets say we strike a particle with a beam from a particle accelerator to measure its location with extreme precision [3]. To measure more precisely we need more energy, which means we end in making even larger black hole. According to GR and Heisenberg it is impossible to measure a length smaller than the Planck length. As we know that uncertainty principle talks about the interdependence of uncertainty in position and momentum and large momentum also means large energy [4]. Keeping in view the Heisenberg uncertainty principle, we know that for a particle to be well localized in space, it's position wave function needs to be constructed from a broad momentum space wave functions, that include extremely high frequencies or extremely high momenta, i.e. the more certain is position, uncertainty in momentum increases. At Planck's length, momentum becomes highly uncertain and the

fluctuations are extremely high due to high energy probe at smaller scale [4].
Now by looking at the real conflict,the standard quantum theories treat the fabric of spacetime as the plate form on which all weird quantum stuff happens. Having understood the basic underlying features applying quantum principles to quantize most of the forces of nature is the standardize feature. For example, classical electromagnetism becomes quantum electrodynamics when you quantize the electron field and the electromagnetic field. To quantize gravity, spacetime itself must be quantized. That leaves no clean coordinate system on which to predicate our theory [3, 4].
In GR, the presence of mass or energy cause spacetime curvature, generating gravitational field causing gravity. Any energy must cause spacetime curvature. So, in quantum gravity theory of gravity, gravity is caused by excitation in our quantized space time. The energy of these excitation contribute to more spacetime curvature, represented as further excitation. In other words, gravity induces gravity. This type of self-interaction or self-energy is seen in other quantum field theories and is hard to deal with, even there. For example, in quantum electrodynamics (QED), the electron has a self-interaction due to its electric charge messing with the electric field of the surrounding. In QED, such situations are solved with something called perturbation theory [3, 5]. It's a scheme to calculate a complex interaction. The perturbation theory is more applicable throughout the quantum field theories of the standard model as it is more reliable and having little corrections even in the case, where the driving terms become infinite. In such a situation, the correction are constrained and brought back to reality through concrete physical measurements of some simple numbers in a process called renormalization [4].
None of them works when we try to quantize GR. When we deal with intense effects of gravity, the self energy corrections shatter to infinity. However, unlike other quantum field theories, there are no easy ways to measure we can do to re-normalize those rectifications. There is a dire need to have infinite measurements. Therefore, it can be said that a quantized spacetime of GR is non-renormalizable. The non-renormalizability of quantized GR is connected to the idea that precisely localized particles produces black holes [3]. Space and time cannot be dealt with the same way, i.e. below the Planck level and therefore the easiest economical way to quantizing gravity and spacetime must be inproper.

1.2 Metric Tensor and Energy Momentum Tensor

GR theory is described in terms of tensors. Before proceeding to Einstein field equation, we will define two basic tensors: energy momentum tensor $T_{\alpha\beta}$ that is the generalization of pressure (force) and the metric tensor $g_{\alpha\beta}$ that provides the information of coordinates of a spacetime [6].

1.2.1 Metric Tensor

We define the metric tensor by

$$g : \mathcal{D} \to \mathcal{D}^* \quad by \quad g_{\mu\nu}\mathcal{A}^\mu = \mathcal{A}_\nu,$$

with $\mathcal{D}$ and $\mathcal{D}^*$ are sets of derivation and dual derivation respectively and they form linear vector space over set of real numbers [6]. Its inverse is given by

$$g^{-1} : \mathcal{D}^* \to \mathcal{D} \quad by \quad g^{\mu\nu}\mathcal{A}_\mu = \mathcal{A}^\nu.$$

The metric and its inverse are symmetric tensors and can be used for raising and lowering indices. The mixed component of metric tensor is

$$g^{\mu\lambda}g_{\lambda\nu} = \delta^\mu_\nu,$$

here δ^μ_ν is known as the kronecker delta and defined as

$$\delta^\mu_\nu = 1 \quad for \quad \mu = \nu,$$

$$\delta^\mu_\nu = 0 \quad for \quad \mu \neq \nu.$$

1.2.2 Energy Momentum Tensor

GR deals with the gravitational field. This field rely on allocation of matter in space and its temporal evolution. So, we need a mathematical distribution of matter in spacetime. As SR relates mass and energy, the spacetime description must incorporate with the distribution of energy as well. The energy could either be carried by matter or stored in the field. In particular, it could be contained in stresses setup in a medium approximate as continuum. Before going on to the full relativistic description it is worthwhile to review the classical, 3-dimensional description of stresses. The generalization of the concept of pressure is stress, which is force per unit area. As force and area are vector quantities in 3-dimension. So, generally there is no meaning of dividing two vectors. The concept of pressure is applicable if the directions of the vector do not matter and only the magnitudes are relevant. A mean for which this requirement supports is called isotropic medium. For instance, the pressure of a gas, as deduced from the kinetic energy of gases, is the same in all directions. However, consider a helical spring stretched or squashed. The energy stored in it can be released by motion in one direction only not orthogonal to it. Such a medium is anisotropic. The generalization of anisotropic media is called stress. The stress is given by stress tensor [6].

$$\sigma^{\mu\nu} = \frac{dF^{\mu}}{dS_{\nu}} \qquad (\mu, \nu = 1, 2, 3). \tag{1.1}$$

Here dF^{μ} is the force which is acting on the area element dS_{ν}. The skew part of this tensor $\sigma^{[\mu,\nu]}$ gives the rotation. From this, we can define the vorticity vector by using the totally skew tensor in 3-dimensions,

$$\Omega_{\gamma} = \frac{1}{2} e_{\mu\nu\gamma} \sigma^{\mu\nu}. \tag{1.2}$$

Assuming an irrotational medium, so that $\sigma_{\gamma} = 0$, the stress tensor will be symmetric, $\sigma^{\mu\nu} = \sigma^{\nu\mu}$. It is symmetric part of the stress tensor that is the generalization of pressure. Thus, in the inertial frame in flat space $(Minkowskispacetime)$ the 4-dimensional tensor is

$$T^{\mu\nu} = \rho c^2 \delta_0^{\mu} \delta_0^{\nu} + \sigma^{ab} \delta_a^{\mu} \delta_b^{\nu}. \tag{1.3}$$

In GR, the spacetime does not remain flat if matter is present. Equation (2.3) can be generalized for any arbitrary frame and arbitrary manifold by

$$T^{\mu\nu} = \rho u^{\mu} u^{\nu} + \sigma^{ab} \delta_a^{\mu} \delta_b^{\nu}, \tag{1.4}$$

where u^μ is the velocity 4-vector. In the case that there are no stresses in an arbitrary frame $T^{\mu\nu}$ gives the total energy and the momentum of any portion of the fluid. Therefore, it is known as the energy momentum tensor [6].

1.3 Einstein Field Equations

Einstein field equations can be obtained from the action principle [1, 2].

$$S = S_M + S_G, \tag{1.5}$$

where S_M and S_G are components of the action because of matter and gravity (spacetime). The curvature of spacetime is sketched by Ricci tensor and metric tensor. S is defined as

$$S = \int \mathcal{L}\sqrt{-g}d^4x, \tag{1.6}$$

$$\mathcal{L} = \mathcal{L}_G + \mathcal{L}_M, \tag{1.7}$$

$$\mathcal{L}_G = \frac{1}{2}R. \tag{1.8}$$

By using (1.7) and (1.8) in (1.6) we get

$$S = \int_v (\frac{1}{2\kappa}R + \mathcal{L}_M)\sqrt{-g}d^4x,$$

where $\kappa = 8\pi$ and R is Ricci scalar.

$$S = \int_v \frac{1}{2\kappa}R\sqrt{-g}d^4x + \int_v \mathcal{L}_M\sqrt{-g}d^4x, \tag{1.9}$$

taking δS in (1.9) gives

$$\delta S = \frac{1}{2\kappa}\int_v \delta(R\sqrt{-g})d^4x + \int_v \delta(\mathcal{L}_M\sqrt{-g})d^4x,$$

now as $R = g^{\mu\nu}R_{\mu\nu}$ above equation becomes

$$\begin{aligned}
\delta S &= \tfrac{1}{2\kappa}\textstyle\int_v \delta(g^{\mu\nu}R_{\mu\nu}\sqrt{-g})d^4x + \int_v \delta(\mathcal{L}_M\sqrt{-g})d^4x, \\
\delta S &= \tfrac{1}{2\kappa}\textstyle\int_v \left[R_{\mu\nu}\delta(g^{\mu\nu}\sqrt{-g}) + g^{\mu\nu}\sqrt{-g}\delta R_{\mu\nu}\right]d^4x + \int_v \left[\delta(\mathcal{L}_M\sqrt{-g})\right. \\
&\quad \left. + \mathcal{L}_M\delta\sqrt{-g}\right]d^4x,
\end{aligned}$$

$$\delta S = \frac{1}{2\kappa}\int_v \left[R_{\mu\nu}g^{\mu\nu}\delta\sqrt{-g} + R_{\mu\nu}\sqrt{-g}\delta(g^{\mu\nu}) + \sqrt{-g}g^{\mu\nu}\delta R_{\mu\nu}\right]d^4x$$
$$+ \int_v \left[\delta\mathcal{L}_M\sqrt{-g} + \mathcal{L}_M\delta\sqrt{-g}\right]d^4x, \tag{1.10}$$

note that $\delta g = gg^{\mu\nu}\delta g_{\mu\nu} = -gg_{\mu\nu}\delta g^{\mu\nu}$

$$\delta\sqrt{-g} = -\frac{1}{2\sqrt{-g}}\delta g,$$
$$\delta\sqrt{-g} = -\frac{1}{2\sqrt{-g}}(-gg_{\mu\nu}\delta g^{\mu\nu}),$$
$$\delta\sqrt{-g} = -\frac{1}{2}\sqrt{-g}g_{\mu\nu}\delta g^{\mu\nu}, \tag{1.11}$$

now as right most part in (1.10) is δS_M, as follow

$$\delta S_M = \int_v [\delta(\mathcal{L}_M)\sqrt{-g} + \mathcal{L}_M\delta\sqrt{-g}]d^4x, \tag{1.12}$$

using (1.11) in (1.12) we have,

$$\delta S_M = \int_v (\delta(\mathcal{L}_M)\sqrt{-g})d^4x + \int_v \mathcal{L}_M[\frac{-1}{2}\sqrt{-g}g_{\mu\nu}\delta g^{\mu\nu}]d^4x, \tag{1.13}$$

let

$$\mathcal{L}_M g^{\mu\nu} \quad \Rightarrow \quad \delta\mathcal{L}_M = \frac{\partial\mathcal{L}_M}{\partial g^{\mu\nu}}\delta g^{\mu\nu},$$

by using above in (1.13), we have

$$\delta S_M = \int_v [\frac{\partial\mathcal{L}_M}{\partial g^{\mu\nu}} - \frac{1}{2}\mathcal{L}_M g_{\mu\nu}]\delta g^{\mu\nu}\sqrt{-g}d^4x, \tag{1.14}$$

$$\delta S_M = -\frac{1}{2}\int_v [-2\frac{\partial\mathcal{L}_M}{\partial g^{\mu\nu}} + \mathcal{L}_M g_{\mu\nu}]\delta g^{\mu\nu}\sqrt{-g}d^4x, \tag{1.15}$$

we define the energy momentum tensor $T^{\mu\nu}$ as

$$T_{\mu\nu} = -2\frac{\partial\mathcal{L}_M}{\partial g^{\mu\nu}} + \mathcal{L}_M g_{\mu\nu},$$

therefore (1.15) become,

$$\delta S_M = \int_v \sqrt{-g}T_{\mu\nu}\delta g^{\mu\nu}d^4x, \tag{1.16}$$

now, consider a local inertial frame, $\Gamma^{a}_{bc}(P) = 0 \quad \Rightarrow \Gamma^{a}_{bc,\rho}(P) \neq 0$, as

$$\begin{aligned} R_{\mu\nu} &= \Gamma^{\lambda}_{\mu\nu,\lambda} - \Gamma^{\lambda}_{\mu\lambda,\nu}, \\ \delta R_{\mu\nu} &= \delta(\Gamma^{\lambda}_{\mu\nu,\lambda}) - \delta(\Gamma^{\lambda}_{\mu\lambda,\nu}), \\ \delta R_{\mu\nu} &= \delta(\Gamma^{\lambda}_{\mu\nu})_{,\lambda} - \delta(\Gamma^{\lambda}_{\mu\lambda})_{,\nu}, \\ \delta R_{\mu\nu} &= \delta(\Gamma^{\lambda}_{\mu\nu})_{;\lambda} - \delta(\Gamma^{\lambda}_{\mu\lambda})_{;\nu}, \end{aligned} \tag{1.17}$$

$$g^{\mu\nu}\delta R_{\mu\nu} = g^{\mu\nu}(\delta\Gamma^{\lambda}_{\mu\nu})_{;\lambda} - g^{\mu\nu}(\delta\Gamma^{\lambda}_{\mu\lambda})_{;\nu},$$

by replacing λ by ν, above become,

$$g^{\mu\nu}\delta R_{\mu\nu} = (g^{\mu\nu}\delta\Gamma^{\lambda}_{\mu\nu} - g^{\mu\lambda}\delta\Gamma^{\nu}_{\mu\nu})_{;\lambda}.$$

We define a vector A^{λ} as, such that

$$A^{\lambda} = (g^{\mu\nu}\delta\Gamma^{\lambda}_{\mu\nu} - g^{\mu\lambda}\delta\Gamma^{\nu}_{\mu\nu}),$$

$$g^{\mu\nu}\delta R_{\mu\nu} = A^{\lambda}_{;\lambda}. \tag{1.18}$$

Now by multiplying by $\sqrt{-g}$ and by integrating over the volume element 4- dimension we get

$$\int_{v} g^{\mu\nu}\delta R_{\mu\nu}\sqrt{-g}d^4x = \int_{v} A^{\lambda}_{;\lambda}\sqrt{-g}d^4x.$$

By using Gauss-divergence theorem, where right side of above equation tends to zero, we have

$$\int_{v} A^{\lambda}_{;\lambda}\sqrt{-g}d^4x = \int_{\partial v} n_{\lambda}A^{\lambda}\sqrt{h}d^3y,$$

above become

$$\int_{v} g^{\mu\nu}\delta R_{\mu\nu}\sqrt{-g}d^4x = 0, \tag{1.19}$$

now

$$\begin{aligned} \delta(g^{\mu\nu}\sqrt{-g}) &= \sqrt{-g}\delta g^{\mu\nu} + g^{\mu\nu}\delta\sqrt{-g}, \\ \delta(g^{\mu\nu}\sqrt{-g}) &= \sqrt{-g}\delta g^{\mu\nu} + g^{\mu\nu}(-\frac{1}{2}\sqrt{-g}g_{\alpha\beta}\delta g_{\alpha\beta}). \end{aligned} \tag{1.20}$$

By using (1.16) , (1.19) and (1.20) in (1.10) we get,

$$\delta S = \frac{1}{2\kappa}\int_v \sqrt{-g}R_{\mu\nu}(\delta g^{\mu\nu} - \frac{1}{2}g^{\mu\nu}g_{\alpha\beta}\delta g^{\alpha\beta}) - \frac{1}{2}\int_v \sqrt{-g}T_{\mu\nu}\delta g^{\mu\nu}d^4x, \tag{1.21}$$

$$\delta S = \frac{1}{2\kappa}\int_v \sqrt{-g}(R_{\mu\nu}\delta g^{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}\delta g^{\mu\nu})d^4x - \frac{1}{2}\int_v \sqrt{-g}T_{\mu\nu}\delta g^{\mu\nu}d^4x,$$

as $\delta S = 0$, so

$$\delta S = \frac{1}{2\kappa}\int_v \sqrt{-g}[R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - kT_{\mu\nu}]\delta g^{\mu\nu}d^4x.$$

Above equation is valid for arbitrary $\delta g_{\mu\nu}$ and the volume element d^4x, the integral must be zero i.e.,

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} - \kappa T_{\mu\nu} + N_{\mu\nu} = 0,$$

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + N_{\mu\nu} = \kappa T_{\mu\nu}.$$

Here $N_{\mu\nu}$ is the constant of integration. We can write above as

$$G_{\mu\nu} = \kappa T_{\mu\nu},$$

where $G_{\mu\nu}$ is called Einstein tensor [2].

In Einstein's theory of relativity, equivalence of energy and mass that recommends every single kind of energy should behave as source of field. Wherever $G_{\mu\nu}$ tells spacetime how to bend and $T_{\mu\nu}$ tells mass how to flow. Here, $N_{\mu\nu}$ ought to have a universal value. For the conservation of energy momentum tensor, it is needed that the covariant derivative of $N_{\mu\nu}$ vanishes. So $N_{\mu\nu} \equiv \wedge g_{\mu\nu}$, wherever $\wedge$ is cosmological constant [2]. The Einstein Field Equations (EFEs) is written as

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \wedge g_{\mu\nu} = \kappa T_{\mu\nu}. \tag{1.22}$$

1.3.1 Schwarzschild Solution of Einstein Field Equations (EFEs)

The solution of EFEs that corresponds to exterior gravitational field of a static and spherically symmetric object (such as our sun and many other bodies) is proposed by Karl Schwarzschild only a few months later, when Einstein published his field equations [6]. The Schwarzschild solution remains the first and one of the most important known exact solution of EFEs. In

(t, r, θ, ϕ) coordinates, the metric of an arbitrary static, spherically symmetric spacetime takes the simple form with signature $(-, +, +, +)$.

$$ds^2 = -f(r)dt^2 + h(r)dr^2 + r^2(d\theta^2 + sin^2\theta d\phi^2), \tag{1.23}$$

where $f(r) = 1 - \frac{2M}{r}$ and $h(r) = f(r)^{-1}$ with M and r are mass and radial component respectively. The singularity occurs when $f(r) \to 0$ and is given by

$$1 - \frac{2M}{r} = 0 \quad \Rightarrow \quad r = 2M. \tag{1.24}$$

The metric (1.23) with $f(r) = 1 - \frac{2M}{r}$ has two singularities at $r = 2M$ and $r = 0$ namely co-ordinate and essential singularity respectively [1, 2]. The curvature invariants for given metric (1.23) having $f(r) = 1 - \frac{2M}{r}$ are:

$$I_1 = g^{\mu\nu}R_{\mu\nu} = 0,$$

$$I_2 = R^{\mu\nu}R_{\mu\nu} = 0,$$

$$I_3 = R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} = \frac{48M^2}{r^6}. \tag{1.25}$$

In general, if all curvature invariants of some given metric are finite then the metric has co-ordinate singularity and if any of these curvature invariant is infinite then the metric has essesntial singularity [6].

1.4 Hamilton-Jacobi Equations

A system that is characterize by the Hamiltonian is known as Hamiltonian system. If we use the generalized coordinates to define the state of the system, then the canonical form of the Hamiltonian can be derived from the Lagrangian, using the Legendre transformation [5]. We take a Lagrangian such that

$$\mathcal{L} = \mathcal{L}(t, \dot{q}_1, \cdots, \dot{q}_n, q_1, \cdots, q_n), \tag{1.26}$$

where $\dot{q}_i$, q_i and t are the generalized velocities, generalized coordinates and time coordinate respectively. Now, if we apply the Legendre transformation we yield new function $\mathcal{H}$ that depends on t and q_i and the derivatives of $\mathcal{L}$ with respect to $\dot{q}_i$, i.e.

$$\mathcal{H} = \mathcal{H}(t, q_i, \frac{\partial \mathcal{L}}{\partial \dot{q}}_i) \qquad (i = 1, 2, 3, ..., n),$$

$$\mathcal{H} = \mathcal{H}(t, q_i, p_i), \tag{1.27}$$

the generalized momentum conjugate to q_i is given by $p_i = \frac{\partial \mathcal{L}}{\partial \dot{q}_i}$. So by Legendre transform we have

$$\mathcal{H}(t, q_i, p_i) = \sum_{i=1}^{n} p_i \dot{q}_i - \mathcal{L}(t, q_i, \dot{q}_i). \tag{1.28}$$

Moreover, the Hamiltonian principle describes that the action integral of a system in a certain time period is stationary, i.e.

$$\delta I = \delta \int_{t_1}^{t_2} \mathcal{L} dt = 0. \tag{1.29}$$

In this formulation, the momentum and coordinates lie on the same point. By using (1.29) in (1.28) we get

$$\delta I = \delta I \int_{t_q}^{t_2} [\sum_{i=1}^{n} p_i \dot{q}_i - \mathcal{H}(t, q_i, \dot{q}_i)] dt = 0, \tag{1.30}$$

the square bracket has function of $t, p_i, \dot{p}_i, q_i$ and $\dot{q}_i$. Therefore,

$$\delta I = \delta \int_{t_q}^{t_2} f(t, p_i, \dot{p}_i, q_i, \dot{q}_i) dt = 0, \tag{1.31}$$

The Euler-Lagrange equations leads us to the equations,

$$\frac{d}{dt}(\frac{\partial f}{\partial \dot{q}_i}) - \frac{\partial f}{\partial \dot{q}_i} = 0, \tag{1.32}$$

$$\frac{d}{dt}(\frac{\partial f}{\partial \dot{p}_i}) - \frac{\partial f}{\partial \dot{p}_i} = 0. \tag{1.33}$$

$$\frac{\partial f}{\partial \dot{q}} = p, \qquad \frac{d}{dt}\frac{\partial f}{\partial \dot{q}} = \dot{p} \quad and \quad \frac{\partial f}{\partial q} = -\frac{\partial \mathcal{H}}{\partial q},$$

$$\frac{\partial f}{\partial \dot{p}} = q, \qquad \frac{d}{dt}\frac{\partial f}{\partial \dot{p}} = \dot{q} \quad and \quad \frac{\partial f}{\partial p} = \frac{\partial \mathcal{H}}{\partial p}. \tag{1.34}$$

Using (1.32) and (1.33) in above we get

$$\frac{\partial \mathcal{H}}{\partial q} = -\dot{p}, \tag{1.35}$$

$$\frac{\partial \mathcal{H}}{\partial p} = \dot{q}, \tag{1.36}$$

above are called as canonical forms of the Hamiltonian of the system [5].

1.5 Astrophysical Fluid and Flow

The shape of any material changes as it experiences a force. This may lead to a deformation. An unbounded deformation of the material, in such cases, is termed as flow. In other words, anything that can move is known as fluid. The fluid element is the landscape upon which the local variables, like density, temperature etc. can be defined [5].

The fluid can flow anywhere in space at the cosmic level is known as astrophysical fluids. Mathematical expressions providing the complete information of the flow are named as equation of state. These describe the link between thermodynamical properties and pressure of the systems. All thermodynamical properties can be calculated through these, such as the fluid moving at a fixed rate can be labeled as isothermal fluid. The equation of state is, $p = k\rho$, where ρ is the energy density, p is the pressure and k is a constant ranging between $0 < k \leq 1$ [5].

Chapter 2

Cyclic and Heteroclinic Flow of Schwarzschild Black Hole Immersed in an Electromagnetic Field in Rainbow Gravity

In this chapter, we review the cyclic and heteroclinic flow of Schwarzchild black hole immersed in an electromagnetic field that is formed by the coupling of electromagnetic waves with double polarization in the framework of rainbow gravity [7]. Here, we also discuss the general accretion of any static spherically symmetric metric. Further, we derive general formalism for accretion using the energy function of the dynamical system i.e. Hamiltonian, in the plane (r, v) where r and v are the position coordinate in radial direction and three dimensional velocity of fluid respectively. In addition, we investigate different cases and found that for some points, the speed of fluid is equal to the speed of sound and for some other points they are not equal. [8].

2.1 Rainbow Gravity

Several counter theories were proposed to develop quantum theory of gravity by unifying GR and quantum mechanics. String theory, M-theory, loop quantum theory, non-commutative geometry, rainbow gravity etc. are all the outcomes of this effort [9]. A fundamental issue arises in transition between GR at low energies and fundamental quantum description. At the same time, some important techniques are put forward to change the linear dispersion ratio $E^2 - p^2 = m^2$ with non- linear version. As relativistic phenomenon are not linear. It is likely that the linear version of the energy momentum relation is the first approximation of the actual non-linear ones [10].

$$E^2 = p^2 + m^2,$$

is modified to

$$E^2 = p^2 + m^2 + \lambda E^3 + \cdots ,$$

It forms the basis of quantum theory of gravity and it asserts to reconsider the Lorentz invariance relation and the structure of spacetime at high energy keeping Lorentz symmetry. Advances in this domain led to more generalized theories which indicate further minimum bounds to our probe of nature at high energy. It is likely that physics may change significantly over these thresholds on the study of theory of quantum gravity. Plancks length and Plancks energy are known to play vital role in quantum theory of gravity and are considered as universal constants, these can be think as the split between the classic and quantum structure [11]. This suggests that the Plancks length should remained invariant when measured in all inertial reference frames. However, this attribute will be associated with the Lorentz symmetry in SR. Plancks length found to violate the Lorentz transformation due to length [12]. Moreover, the absolute time or energy values also not fit in the description of Lorentz invariance and this gives an additional motivation to know the cause and solution of the Lorentz invariance. With all the suggestions for deepening our understanding of the type of nature of space by altering some well-formed concepts in physics, it is indeed very interesting to modify the Lorentz invariance by the proposals of the double special relativity recommendations (DSR) [13]. This adds the Planck length as a new required measure for the real version of Lorentz transformation of the spacetime. In DSR theory, the Lorentz invariance is valid only in classical gravity but remains no more feasible at the Planck scale. Although the properties of the inertial framework and the principle of equivalence are always preserved. The DSR theory in the framework of GR is called rainbow gravity. The rainbow gravity is characterized by the feature that the spacetime geometry rely upon the moving test particle's energy in the background, which implies that different observers having different energies will observe different classical geometries [14]. The metric description of spacetime must contain the parameter examining particle energy E, which forms an one parameter energy dependent metric family. Finally, it is worthwhile examining the effect of the modified dispersion ratio (corresponding to the quantum quantity) in a massive gravitational field like black hole [11, 13].

In DSR, when the ratio $E/E_p \rightarrow 0$, the speed of light goes to universal constant c. As a result energy momentum dispersion modifies to

$$E^2 f^2(l_p E) - \boldsymbol{p} \cdot \boldsymbol{p} g^2(l_p E) = m^2, \tag{2.1}$$

this can be obtained by the mapping of momentum space onto itself, i.e. $M : \mathcal{P} \rightarrow \mathcal{P}$, denoted by

$$M \cdot (E, \boldsymbol{p}_i) = (U_0, U_i) = (f(\frac{E}{E_p})E, g(\frac{E}{E_p})\boldsymbol{p}_i). \tag{2.2}$$

This concludes the non linear norm on momentum space, given as

$$|p|^2 = \eta^a b U_a(\boldsymbol{p}) U_b(\boldsymbol{p}). \tag{2.3}$$

The norm is preserved by real realization of the Lorentz group, given as

$$\tilde{L}_a^b = U^{-1} \cdot L_a^b \cdot U. \tag{2.4}$$

Where L_a^b are the usual operators.

In rainbow gravity, when effects of order $l_p E$ are taken under consideration, we do not have single classical spacetime geometry. We propose that when higher order of $l_p E$ are taken into account, classical spacetime geometry are presented by one parameter family of matrices, parametrized by the ratio E/E_p [12, 13]. Moreover, the energy of the particle moving in it depends on the geometry of spacetime. Therefore, there is no single spacetime is dual to momentum space. Instead, the matrice's family is dependent upon the energy of photon/particle [11, 14].

Moreover, argument E in spacetime metric $g_{ab}(E)$ does not represent the energy of the spacetime fabric. This shows the scale at which an inertial observer observes the trajectory of a particle or a system of particles, to determine the geometry of a spacetime [12]. In other words, when the gravity is weak or absent, the spacetime metric has energy dependence, i.e.

the geometry of particle of energy E is given by the set of energy dependent orthonoraml frames, i.e.

$$e_0 = f^{-1}(E/E_p)\tilde{e}_0, \qquad e_i = g^{-1}(E/E_p)\tilde{e}_i. \tag{2.5}$$

Here, tilde quantities represent the energy dependent frames. All these inertial frames have same metric but due to scaling, these inertial frames do not share all their geodesic. All these geodesic are energy dependent. This is equivalent to saying that energy momentum dispersion relation is modified. So, it remain no more quadratic. The modification of generally spherically symmetric exact and vaccum solution of EFEs called as modified Schwarzschild black Hole, described by the following metric [11].

$$ds^2_{Schw} = -\frac{(1-\frac{2GM}{\tilde{r}})}{f^2(E/E_{pl})}d\tilde{t}^2 + \frac{1}{(1-\frac{2GM}{\tilde{r}})g^2(E/E_{pl})}d\tilde{r}^2 + \frac{\tilde{r}^2}{g^2(E/E_{pl})}d\tilde{\Omega}^2, \tag{2.6}$$

with the condition

$$\lim_{E/E_p \to 0} f(E/E_p) = 1 \quad , \quad \lim_{E/E_p \to 0} g(E/E_p) = 1.$$

According to Amelino-Camelia, rainbow functions can be written as:

$$f(E/E_p) = 1 \quad , \quad g(E/E_p) = \sqrt{1-\eta\Big(\frac{E}{E_p}\Big)^2},$$

here η is dimensionless entity. By Heisenberg uncertainty principle that gives a relationship between momentum of Hawking particle P emitted from the blackhole and mass of blackhole as $P = \triangle P \sim \frac{1}{2GM}$ by regarding $\triangle x \sim \frac{1}{2GM}$ [12]. For simplicity we take $n = 2$ in modified dispersion relation. So that,

$$m^2 = E^2 - P^2 + \eta P^2\Big(\frac{E}{E_p}\Big)^2.$$

From above equation we can write

$$E^2 = (1 + 4G^2M^2m^2)/(\eta G + 4G^2M^2),$$

with

$$G = 1/{E_p}^2,$$

that for massless particle reduces to

$$E^2 = 1/(\eta G + 4G^2M^2).$$

Firstly, we discuss the equations for spherical accretion as well as the conservation laws for any static spherically symmetric, particularly Schwarzschild metric immersed in an electromagnetic field. Secondly, We have driven the pressure of ideal fluid for such spherically symmetric flow. Thirdly, we study the accretion development using Hamiltonian approach. At last, we study the polytropic fluid and verify answers that are strictly supersonic and solution with sonic flows and apply our results of Hamiltonian kinetic analysis to polytropic fluid [8].

2.2 General Equations of Accretion

We consider the modified static spherically symmetric metric of the form

$$ds^2 = -\frac{F(r)}{f^2(E/E_p)}d\tilde{t}^2 + \frac{1}{g^2(E/E_p)}[\frac{d\tilde{r}^2}{F(r)} + r^2 d\tilde{\Omega}^2], \tag{2.7}$$

where $d\tilde{\Omega} = d\tilde{\theta}^2 + sin^2\theta\tilde{\phi}^2$ and here the quantities $(\tilde{t}, \tilde{r}, \tilde{\Omega})$ are independent energy variables. The given metric also depends on the energy of a particle moving in it. The function $F(r)$ is given by [7]

$$F(r) = 1 - \frac{2M}{r} + \frac{Q_{eff}}{r^2}. \tag{2.8}$$

By taking effective charge as $Q_{eff} \equiv M^2(1 - a^2)$. As it is clear that when $a = 1$, i.e. $Q_{eff} = 0$, the metric (2.7) becomes the Schwarzschild black hole. Anyhow, when $a = 0$, i.e. $Q_{eff} = M^2$ corresponds to he Reissner-Nordstorm black hole. Where M denotes the mass-parameter and a is the interpolation parameter with $1 \geq a \geq 0$. On the other hand the metric function (2.8) can be written as:

$$F(r) = \frac{(r - r_e)(r - r_i)}{r^2}, \tag{2.9}$$

with $r_e = M(1 + a)$ and $r_i = M(1 - a)$ are two horizons named as the event and inner horizons respectively [15]. According to the law of particle conservation that states that particles neither created nor destroyed [8]. So, the divergence of particle flux is preserved, .i.e.

$$\nabla_\mu J^\mu = \nabla_\mu (n u^\mu) = 0, \tag{2.10}$$

where n is baryon number density of fluid and u^n is intrinsic four velocity of fluid. The stress energy momentum tensor for perfect fluid is given by

$$T^{\mu\nu} = (e+p)u^\mu u^\nu + p g^{\mu\nu}, \tag{2.11}$$

where ρ is energy density and p is pressure. Furthermore, we assume that the perfect fluid is moving in equatorial plane, as a result $u^\theta = 0$ and $u^\phi = 0$. For convenience, we use the notation $u^r = u$. Now, by normalization condition that is $u^\mu u_\mu = -1$. we get

$$g_{00}(u^0)^2 + g_{11}(u^1)^2 = -1,$$
$$u^t = \pm\sqrt{F(r)g_E^2 + u^2}\frac{f_E}{F(r)g_E}, \tag{2.12}$$

where $f_E = F^2(\frac{E}{E_p})$ and $g_E = G^2(\frac{E}{E_p})$ are energy dependent rainbow functions. The determinant of spacetime metric is given by

$$\sqrt{-g} = \sqrt{\frac{r^4 sin^2(\theta)}{f_E^2 g_E^6}}. \tag{2.13}$$

As the perfect fluid flows in equatorial plane so $\theta = \pi/2$. We obtain,

$$\sqrt{-g} = \frac{r^2}{f_E g_E^3}, \tag{2.14}$$

by using the law of conservation of particles, it yields

$$\nabla_\mu (n u^\mu)) = \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g} n u^\mu),$$
$$0 = \frac{f_E g_E^3}{r^2}(\partial_\mu(\frac{r^2}{f_E g_E^3}) n u^\mu), \tag{2.15}$$

by integrating we get

$$\frac{r^2 n u^2}{f_E g_E^3} = A_1. \tag{2.16}$$

Where A_1 is integration constant. Special enthalpy is defined as

$$h = \frac{e+P}{n}, \tag{2.17}$$

then

$$dh = \frac{n(de+dP)-(e+p)dn}{n^2},$$
$$dP = ndh - de + hdn. \tag{2.18}$$

By first law of thermodynamics [8] we have

$$de = hdn + nTds, \tag{2.19}$$

by using (2.19) in (2.20) we get

$$dp = ndh - nTds, \tag{2.20}$$

(2.19) and (2.20) are equations that describes the thermodynamics of simple fluids. Here T is the temperature and s is the entropy of per particle. According to theorem of relativistic hydrodynamics, the scalar $hu_\mu\xi^\mu$ is conserved along the trajectories of the fluid [8].

$$u^x\nabla_x(hu_\mu\xi^\mu) = 0,$$

where ξ^μ is killing vector. We consider special case, when $\xi^\mu = (1, 0, 0, 0)$. Now take

$$\nabla_\mu T^\mu_t = \nabla_\mu[(e+P)u^\mu u_t + P\delta^\mu_t],$$

$$\nabla_\mu T^\mu_t = \nabla_\mu[(hn)u^\mu u_t + P\delta^\mu_t],$$

$$\nabla_\mu T^\mu_t = \nabla_\mu[hnu^\mu u_t + P\delta^\mu_t],$$

$$\nabla_\mu T^\mu_t = hu_t\nabla_\mu(nu^\mu) + nu^\mu\nabla_\mu(hu_t) + \nabla_t(P).$$

By using (2.17), we get,

$$\nabla_\mu T^\mu_t = nu^\mu\nabla_\mu(hu_t) + \nabla_t(nh-e),$$

$$\nabla_\mu T^\mu_t = nu^\mu\nabla_\mu(hu_t),$$

therefore,

$$nu^\mu\nabla_\mu(hu_t) = 0,$$

$$nu^{\mu}[\partial_{\mu}(hu_t) - \Gamma^{\lambda}_{\mu t} hu_{\lambda}] = 0,$$

as $u = u(r)$ and $n \neq 0$. So

$$u^{\mu}[\partial_{\mu}(hu_t) - \Gamma^{\lambda}_{\mu t} hu_{\lambda}] = 0,$$

$$u^t[\partial_t(hu_t) + u^r\partial_r(hu^r) - h\Gamma^{\lambda}_{\mu t}u^{\mu}u_{\lambda}] = 0,$$

$$u^r\partial_r(hu_t) - h[u^t u_t \Gamma^t_{tt} + u^t u_r \Gamma^r_{tt} + u^r u_t \Gamma^t_{rt} + u^r u_r \Gamma^r_{rt}] = 0.$$

As all christoffel symbols vanishes, so

$$u^r\partial_r(hu_t) = 0,$$

as $u^r \neq 0$, so $\partial_r(hu_t) = 0$

$$\frac{d}{dr}(hu_t) = 0,$$

$$hu_t = A_2, \tag{2.21}$$

by using (2.12) in (2.21), we have

$$\frac{hf_E}{g_E}(\sqrt{F(r)g_E^2 + u^2}) = A_2. \tag{2.22}$$

Now for

$$u_{\nu}\nabla_{\mu}T^{\mu\nu} = u_{\nu}\nabla_{\mu}[nhu^{\mu}u^{\nu} + (nh - e)g^{\mu\nu}],$$

$$u_{\nu}\nabla_{\mu}T^{\mu\nu} = u_{\nu}[hu^{\nu}\nabla_{\mu}(nu^{\mu}) + (nu^{\mu})\nabla_{\mu}(hu^{\nu})] + (nh - e)g^{\mu\nu},$$

$$u_{\nu}\nabla_{\mu}T^{\mu\nu} = u^{\mu}\nabla_{\mu}(nh - e),$$

by special enthalpy

$$u_{\nu}\nabla_{\mu}T^{\mu\nu} = u^{\mu}\nabla_{\mu}(P). \tag{2.23}$$

by using (2.19) in (2.23), we get

$$u_{\nu}\nabla_{\mu}T^{\mu\nu} = -nTu^{\mu}\nabla_{\mu}(s) = 0,$$

as we are considering that special case in which the fluid has radial motion and stationary and no dependence on time. Moreover, it conserves the spherical symmetry of black hole.

As the flow is is-entropic, i.e. $(ds = 0)$. So (2.18) reduces to

$$dP = ndh, \tag{2.24}$$

and $de = hdn + nTds$ reduces to

$$de = hdn. \tag{2.25}$$

So the equation of state, that is $e = e(n, s)$ reduces to $e = e(n) = F(n)$. So, $F(n) = hn$. We will get

$$F'(n) = h. \tag{2.26}$$

From (2.24)

$$\frac{dP}{dn} . \frac{dn}{dh} = n,$$

$$P' = nF''(n), \tag{2.27}$$

by integrating by parts we get,

$$P = nF' - F. \tag{2.28}$$

Here F is Legendre transform of energy density. Above result is truly thermodynamic and it does not depend on the characteristics of flow. This is true for any isentropic flow. In locally inertial frame, three dimensional speed of sound $''c_s''$ is given by $c_s^2 = (\frac{\partial P}{\partial e})_s$ reduces to $c_s^2 = \frac{dP}{de}$

$$c_s^2 = \frac{dP}{de} = \frac{ndh}{hdn}, \tag{2.29}$$

$$\frac{dh}{h} = c_s^2 \frac{dn}{n},$$

as

$$c_s^2 = \frac{dP}{de} = \frac{ndh}{hdn} = \frac{n}{h} \cdot \frac{dh}{dn},$$

$$c_s^2 = \frac{n}{F'} \cdot F'',$$

$$c_s^2 = n(lnF'). \tag{2.30}$$

As

$$ds^2 = -\frac{F(r)}{f_E^2}dt^2 + \frac{dr^2}{F(r)g_E^2}, \tag{2.31}$$

or

$$ds^2 = -(\frac{\sqrt{F(r)}}{f_E}dt)^2 + (\frac{dr}{\sqrt{F(r)}g_E})^2. \tag{2.32}$$

As we have proper time and proper distance $d\tau_0 = \frac{\sqrt{F(r)}dt}{f_E}$ and $dl = \frac{dr}{\sqrt{F(r)}g_E}$ respectively. Corresponding to time and radial changes we have three dimensional speed v observed by locally static observer as

$$v = \frac{dl}{d\tau_0}, \tag{2.33}$$

$$v = \frac{dr}{dt} \cdot \frac{1}{\sqrt{F(r)}g_E} \cdot \frac{1}{\frac{\sqrt{F(r)}}{f_E}},$$

$$v = \frac{dr}{dt} \cdot \frac{f_E}{F(r)g_E}. \tag{2.34}$$

Now by squaring we get,

$$v^2 = \left(\frac{dr}{dt} \cdot \frac{f_E}{F(r)g_E}\right)^2,$$

$$v^2 = \left[\frac{f_E}{F(r)g_E} \cdot \frac{dr}{d\tau} \cdot \frac{d\tau}{dt}\right]^2,$$

$$v^2 = \frac{f_E^2}{g_E^2}\left(\frac{u}{u_t}\right)^2, \tag{2.35}$$

as

$$u_t = g_{tt}u^t,$$

by using (2.12)

$$u_t = -F(r)\left[\pm\sqrt{F(r)g_E^2 + u^2}\left(\frac{f_E}{F(r)g_E}\right)\right],$$

$$u_t = \mp\sqrt{F(r)g_E^2 + u^2}\left(\frac{f_E}{g_E}\right), \tag{2.36}$$

by using (2.12) in (2.35) we get

$$u^2 = \frac{v^2F(r)g_E^2}{1 - v^2}, \tag{2.37}$$

by (2.35) we can write

$$u_t^2 = \frac{f_E^2}{g_E^2}\left(\frac{u^2}{v^2}\right), \tag{2.38}$$

by using (2.37), we get

$$u_t^2 = \frac{f_E^2}{g_E^2}\left(\frac{F(r)g_E^2}{1-v^2}\right), \tag{2.39}$$

from (2.16) we can write $(r^2nu)^2 = A_1^2$. Now by using (2.39), (2.38) and (2.16) we can write

$$A_1^2 = \frac{r^4n^2v^2F(r)g_E^2}{1-v^2}. \tag{2.40}$$

Above equation represents the world lines of fluid element and that of a locally static observer.

2.3 Hamiltonian System

As in (2.16) and (2.21) we have derived two equations of motion. Either of these integral equations or their combination can be used as a Hamiltonian system for the fluid flow [16]. Now by taking left side of (2.21) as the Hamiltonian. By using (2.36) in (2.21) we have

$$\mathcal{H} = \frac{h^2f_E^2}{g_E^2}\left(F(r)g_E^2 + \frac{v^2F(r)g_E^2}{1-v^2}\right). \tag{2.41}$$

Using (2.40) we get

$$\mathcal{H} = \frac{h^2f_E^2}{g_E^2}\left(F(r)g_E^2 + \frac{A_1^2}{r^4n^2}\right). \tag{2.42}$$

Here n is a function of (r, v) and h is a function of the baryon number density n only so this implies $h(r, v)$. This applies to pressure p too. If we solve (2.41) without substitution, we get

$$\mathcal{H}(r,v) = \frac{h^2f_E^2}{g_E^2}\left(F(r)g_E^2 + \frac{v^2F(r)g_E^2}{1-v^2}\right),$$

$$\mathcal{H}(r,v) = h^2f_E^2\left(F(r) + \frac{F(r)v^2}{1-v^2}\right),$$

$$\mathcal{H}(r,v) = h^2f_E^2\left(\frac{F(r)}{1-v^2}\right). \tag{2.43}$$

2.4 Sonic Points

The parametric energy function of the dynamical energy system is given by (2.43). We use (2.43) to derive critical points of the dynamical system with $\mathcal{H}$ given by (2.43), with

$$\dot{r} = \mathcal{H}_{,v} \qquad \dot{v} = -\mathcal{H}_{,r}, \tag{2.44}$$

where $'' \cdot ''$ denotes the $\bar{t}$ derivative. Here radius is kept constant while taking partial derivative with respect to v in $\mathcal{H}_{,v}$ and speed (velocity) of sound is kept constant while performing partial derivative with respect to r in $\mathcal{H}_{,r}$. To find the critical points of (2.44) we put right hand side of that equation equal to zero. By evaluating right hand side we obtain,

$$\mathcal{H}(r,v) = f_E^2\left[\frac{h^2(r,v)F(r)}{1-v^2}\right].$$

Now by taking partial derivative with respect to v we get

$$\mathcal{H}_{,v} = f_E^2\left[\frac{(1-v^2)(2h.h_{,v}F(r)) - h^2F(r)(-2v)}{(1-v^2)^2}\right],$$

$$\mathcal{H}_{,v} = \frac{2h^2F(r)vf_E^2}{(1-v^2)^2}\left[1 + \frac{(1-v^2)lnh_{,v}}{v}\right], \tag{2.45}$$

and by taking partial derivative of (2.43) with respect to r, we have

$$\mathcal{H}_{,r} = \frac{f_E^2}{1-v^2}\left[2F(r)hh_{,r} + h^2F(r)_{,r}\right],$$

$$\mathcal{H}_{,r} = \frac{f_E^2h^2}{1-v^2}\left[F(r)_{,r} + \frac{2F(r)h_{,r}}{h}\right],$$

$$\mathcal{H}_{,r} = \frac{f_E^2h^2}{1-v^2}\left[F(r)_{,r} + 2(lnh)_{,r}\right]. \tag{2.46}$$

From (2.29) it's right most part yields,

$$(lnh)_{,v} = c_s^2(lnn)_{,v}, \tag{2.47}$$

$$(lnh)_{,r} = c_s^2(lnn)_{,r}. \tag{2.48}$$

Eq (2.40) gives

$$A_1 = \frac{r^2nv\sqrt{F(r)}g_E}{\sqrt{1-v^2}}.$$

If $''r''$ is kept constant, then

$$A_1 = \frac{nvg_E}{\sqrt{1-v^2}}. \tag{2.49}$$

Now take $A_1 = 1$, we get

$$\sqrt{1-v^2} = nvg_E.$$

By taking log of both sides and then taking partial derivative with respect to v we get,

$$ln\left(\frac{\sqrt{1-v^2}}{v}\right)_{,v} = (lnn)_{,v} + (lnG)_{,v},$$

$$\left(\frac{v}{\sqrt{1-v^2}}\right)\left[\frac{v \cdot \frac{-2v}{2\sqrt{1-v^2}} - (\sqrt{1-v^2})}{v^2}\right] = (lnn)_{,v},$$

$$(lnn)_{,v} = \frac{-1}{v\sqrt{1-v^2}}\left(\frac{1}{\sqrt{1-v^2}}\right).$$

From (2.47)

$$(lnh)_{,v} = \frac{-c_s^2}{v(1-v^2)}. \tag{2.50}$$

If $''v''$ is kept constant in (2.40) we get

$$A_1 = r^2 n\sqrt{F(r)}g_E. \tag{2.51}$$

If we take A_1 =1, then

$$1 = r^2 n\sqrt{F(r)}g_E,$$

$$ln\left(\frac{1}{r^2\sqrt{F(r)}}\right) = ln(ng_E).$$

Differentiating w.r.t r we get,

$$\left(ln\left(\frac{1}{r^2\sqrt{F(r)}}\right)\right)_{,r} = \Big(ln(ng_E)\Big)_{,r},$$

$$(lnn)_{,r} = -\left[\frac{r^2F(r)_{,r}}{2r^2F(r)} + \frac{4F(r)r}{2r^2F(r)}\right],$$

$$(lnn)_{,r} = -\left[\frac{1}{2}(lnF(r))_{,r} + \frac{2}{r}\right]. \tag{2.52}$$

Now from (2.48) we get

$$(lnh)_{,r} = -c_s^2 \left[\frac{4 + r(lnF(r))_{,r}}{2r} \right]. \tag{2.53}$$

Using (2.50) in (2.44) we get

$$\dot{r} = \mathcal{H}_{,v} = \frac{2h^2 F(r) v f_E^2}{(1-v^2)^2} \left[1 + \frac{(1-v^2)(\frac{-c_s^2}{v(1-v^2)})}{v} \right], \tag{2.54}$$

$$\dot{r} = \mathcal{H}_{,v} = \frac{2h^2 F(r) g_E^2}{v(1-v^2)^2} \left[v^2 - c_s^2 \right],$$

and

$$\dot{v} = -\mathcal{H}_{,r} = \frac{-f_E^2 h^2}{1-v^2} \left[F(r)_{,r} + 2F(r)(lnh)_{,r} \right],$$

$$\dot{v} = -\mathcal{H}_{,r} = -\frac{h^2 f_E^2}{1-v^2} \left[rF(r)_{,r} - 4c_s^2 F(r) - c_s^2 F(r) r (lnF(r))_{,r} \right],$$

$$\dot{v} = -\mathcal{H}_{,r} = -\frac{h^2 f_E^2}{1-v^2} \left[rF(r)_{,r}(1-c_s^2) - 4c_s^2 F(r) \right]. \tag{2.55}$$

At critical point, right hand side of above equations (2.54) and (2.55) vanishes if

$$v_c^2 = c_{sc}^2, \tag{2.56}$$

and

$$r_c F(r)_{c,r_c}(1-c_s^2) = 4F(r)_c c_{sc}^2. \tag{2.57}$$

This equation shows the speed of sound at the critical points c_{sc}^2 in the terms of r_c

$$u_c^2 = \frac{F(r)c_{sc}^2}{1-c_{sc}^2} = \frac{r_c F(r)_{c,r_c}}{4}. \tag{2.58}$$

Finally, we get

$$c_{sc}^2 = \frac{r_c F(r)_{c,r_c}}{4F(r)_c + r_c F(r)_{c,r_c}}. \tag{2.59}$$

Then we write constant A_1^2 as

$$C_1^2 = r_c^4 n_c^2 v_c^2 \frac{F(r)_c g_E^2}{1-v_c^2} = r_c^4 n_c^2 v_c^2 \frac{r_c F(r)_{c,r_c} g_E^2}{4v_c^2} = \frac{r_c^5 n_c^2 F(r)_{c,r_c} g_E^2}{4}, \tag{2.60}$$

and we obtain the following ratio:

$$\left(\frac{n}{n_c}\right)^2 = \frac{r_c^5 F(r)_{c,r_c}}{4} \, \frac{1-v^2}{r^4 F(r) v^2} \, \frac{(g_E)_c^2}{g_E^2}. \tag{2.61}$$

Here, we have two type of fluid motion approaching the horizon, one of them with speed v that vanishes and in the other the speed of fluid approaches the speed of light.

2.5 Isothermal Test Fluids

It is usually stated that for flow at a constant temperature, the sound speed of accretion remains fixed throughout the accretion process. It assures that the sound speed of accretion flow at any radii is always similar to the sound speed at the sonic point. As our system is adiabatic, so the flow of our fluid is isothermal. As equation of state of the form,

$$p = ke,$$

where $p = kF(n)$ with $G(n) = kF(n)$, where k is the state parameter constrained by $(0 < k \leq 1)$. Furthermore, the adiabatic sound speed is $c_s^2 = dp/de$. After we compare the adiabatic sound speed to the equation of state, it is found that $c_s^2 = k$.
The differential equation is obtained:

$$nF'(n) - F(n) = kF(n), \tag{2.62}$$

which is yielding

$$e = F = \frac{e_c}{n_c^{k+1}} \, n^{k+1}, \tag{2.63}$$

with constant e_c/n_c^{k+1}, chosen e_∞/n_∞^{k+1} or e_0/n_0^{k+1},

$$h = \frac{(k+1)e_c}{n_c^{k+1}} \, n^k = \frac{(k+1)e_c}{n_c} \left(\frac{n}{n_c}\right)^k, \tag{2.64}$$

and by taking square of above equation and taking

$$K = \left(\frac{r_c^5 F(r)_{c,r_c} (g_E)_c}{4}\right)^k \left(\frac{(k+1)e_c}{n_c}\right)^2 = \text{constant.}$$

We get

$$h^2 = K\Big(\frac{1-v^2}{v^2 r^4 F(r) g_E}\Big)^k. \tag{2.65}$$

The new Hamiltonian $\mathcal{H}$ and the dynamical system are

$$\mathcal{H}(r,v) = \frac{f_E^2 F(r)}{1-v^2}\Big(\frac{1-v^2}{v^2 r^4 F(r)}\Big)^k = \frac{f_E^2 F^{1-k}(r)}{g_E^k (1-v^2)^{1-k} v^{2k} r^{4k}}, \tag{2.66}$$

$$\dot{r} = \frac{2(v^2-c_s^2)F(r)g_E^2}{v(1-v^2)^2}\Big(\frac{1-v^2}{v^2 r^4 F(r) g_E}\Big)^k,$$

$$\dot{v} = -\frac{f_E^2}{r(1-v^2)}\Big(\frac{1-v^2}{v^2 r^4 F(r) g_E}\Big)^k \Big[rF(r)_{,r}(1-c_s^2) - 4F(r)c_s^2\Big].$$

With $p = ke$, we obtain

$$p \propto \Big(\frac{1-v^2}{v^2 r^4 F(r) g_E^2}\Big)^{\frac{k+1}{2}}, \tag{2.67}$$

the pressure diverges, as the curve approaches the horizon, as

$$p \sim (r-r_h)^{-\frac{k+1}{2k}}. \tag{2.68}$$

If r_h is a double root of $f = 0$, we obtain

$$p \sim (r-r_h)^{-\frac{k+1}{k}}.$$

A global flow solution is

$$v \simeq v_1 r^{-\alpha} + v_\infty \quad \text{as} \quad r \to \infty, \tag{2.69}$$

where ($\alpha > 0, v_1, |v_\infty| \leq 1$) are constants. Inserting this in the Hamiltonian:

$$\mathcal{H} \simeq \begin{cases} \text{(a):} \quad \frac{F(r)^{1-k}}{r^{4k}}, & (\text{if } 0 < |v_\infty| < 1); \\ \text{(b):} \quad \frac{F(r)^{1-k}}{r^{(4-2\alpha)k}}, & (\text{if } v_\infty = 0); \\ \text{(c):} \quad \frac{F(r)^{1-k}}{r^{(4+\alpha)k-\alpha}}, & (\text{if } |v_\infty| = 1). \end{cases} \tag{2.70}$$

Here, we will analyze the behave of perfect fluid by taking different values of $k = 1$, i.e. $k = 1$ for ultra-stiff fluid, $k = 1/2$ for relativistic fluid and $k = 1/3$ for radiation fluid. For our metric (2.59) becomes

$$k = \frac{-3(-1+a^2)M^2 - 5Mr_c + 2r_c^2}{2(-1+a^2)M^2 + 6Mr_c - 4r_c^2}. \tag{2.71}$$

2.5.1 Solution for Ultra-Stiff Fluid ($k = 1$)

The equation of state of ultra-stiff fluids are $p = ke$ with $k = 1$ [8]. The Hamiltonian becomes

$$\mathcal{H} = \frac{1}{v^2 r^4 g_E^2}, \tag{2.72}$$

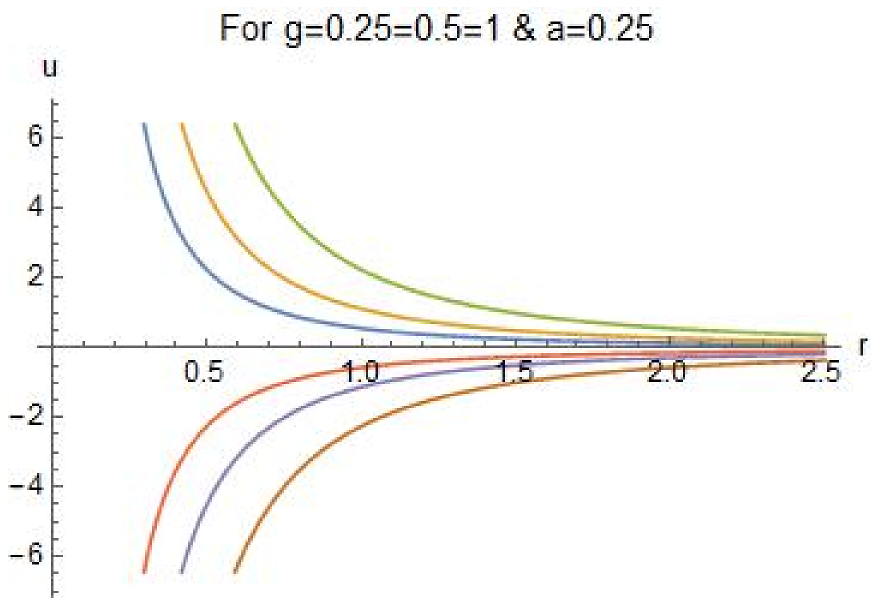


Fig:2.1. Solutions of $\mathcal{H}$ corresponding to k=1 when a=0.25 and 0< g $\leq$1

and one finds

$$v \sim 1/r^2 g_E, \tag{2.73}$$

$$\left(\frac{n}{n_c}\right)^2 = \frac{r_h^5 (G_E^2)_h F(r)_{,r}|_{r=r_h}}{4} \; \frac{\mathcal{H}_0 r^4 - 1}{r^4 F(r) g_E^2}, \tag{2.74}$$

for any solution curve $\mathcal{H}_0 > \mathcal{H}_{\min} = r_h^{-4}$, and

$$\left(\frac{n}{n_c}\right)^2 = \frac{r_c (g_E)_c^2 F(r)_{c,r_c}}{4} \; \frac{1 - v^2}{F(r)} = \frac{r_h (g_E^2)_h F(r)_{,r}|_{r=r_h}}{4} \; \frac{r^4 - r_h^4}{r^4 F(r) g_E^2}. \tag{2.75}$$

2.5.2 Solution for Ultra-Relativistic Fluid $(k = 1/2)$

These fluids have energy density greater than isotropic pressure [8]. Here, we put $p = \frac{e}{2}$ gives $k = 1/2$. The expression (2.71) reduces to

$$Z(r_c) = \frac{r^2}{2} - \frac{5}{2}Mr + \frac{3}{2}(-1 + a^2)M^2 = 0. \tag{2.76}$$

This polynomial has two positive roots as event horizon(r_h) and inner horizon(r_i) respectively,

$$r_h = \frac{1}{2}(5M + \sqrt{37M^2 - 12a^2M^2}), \quad r_i = \frac{1}{2}(5M - \sqrt{37M^2 - 12a^2M^2})$$

Furthermore, the Hamiltonian takes the form,

$$\mathcal{H} = \frac{\sqrt{f}}{r^2\sqrt{|v^2|}\sqrt{1 - v^2}\sqrt{g}}$$

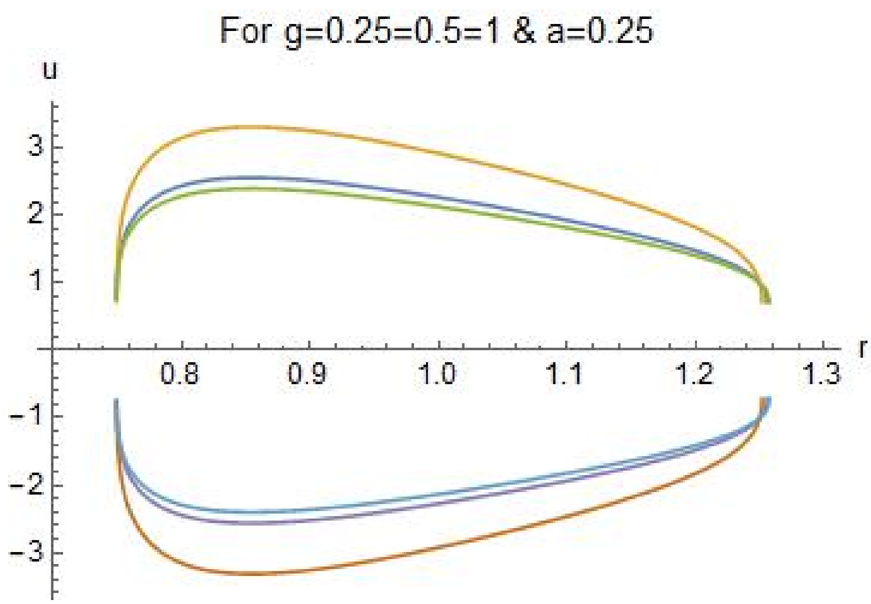


Fig:2.2. Solutions of $\mathcal{H}$ corresponding to k=1/2 when a=0.25 and $0 <$ g ≤ 1

2.5.3 Solution for Radiation Fluid ($k = 1/3$)

One of the most interesting case in astrophysics is radiation fluid which absorbs the radiation emitted by the black hole [8]. In this case, $k = 1/3$ reduces (2.71) to,

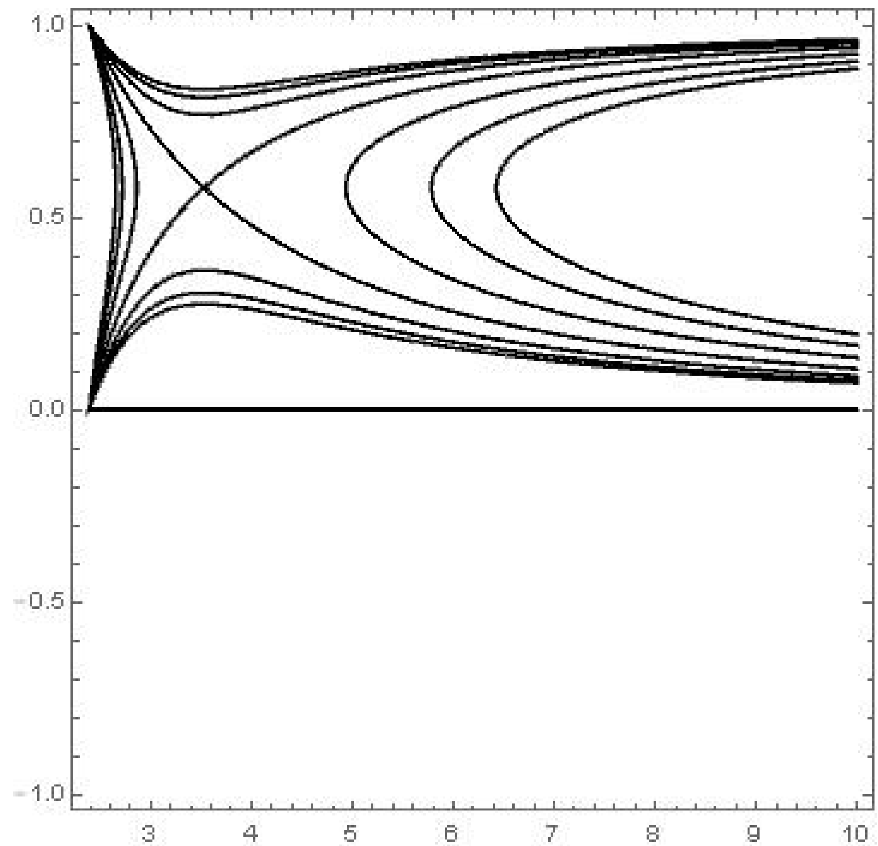
$$Q(r_c) = r^2 - 3Mr + 2(-1 + a^2)M^2 = 0, \tag{2.77}$$

gives two roots,

$$r_h = \frac{1}{2}(3M + \sqrt{17M^2 - 8a^2M^2}), \qquad r_i = \frac{1}{2}(3M - \sqrt{17M^2 - 8a^2M^2}).$$

The Hamiltonian (2.66) reduces to,

$$\mathcal{H} = \frac{f^{\frac{2}{3}}}{r^{\frac{4}{3}}|v|^{\frac{2}{3}}g^{\frac{1}{3}}(1-v^2)^{\frac{2}{3}}}$$



2.6 Polytropic Test Fluids

The equation of state is [8]:

$$p = An - \frac{B}{n^{\alpha}}, \tag{2.78}$$

where A and B are constants and $(0 < \alpha < 1)$. Inserting $p = G(n) = \mathcal{K}n^{\gamma}$ in the differential equation yields

$$nF' - F = \mathcal{K}n^{\gamma}.$$

The energy density $e = F$ is obtained as

$$e = F(n) = mn + \frac{\mathcal{K} n^\gamma}{\gamma - 1}. \tag{2.79}$$

It yields

$$h = m + \frac{\mathcal{K} \gamma n^{\gamma-1}}{\gamma - 1}. \tag{2.80}$$

The speed of sound

$$a^2 = \frac{(\gamma - 1)X}{m(\gamma - 1) + X} \qquad (X \equiv \mathcal{K} \gamma n^{\gamma-1}). \tag{2.81}$$

It is found as

$$h = m \, \frac{\gamma - 1}{\gamma - 1 - a^2}, \tag{2.82}$$

then we obtain

$$h = m\Big[1 + Y\Big(\frac{1 - v^2}{r^4 F(r) g_E^2 v^2}\Big)^{(\gamma-1)/2}\Big], \tag{2.83}$$

where

$$Y \equiv \frac{\mathcal{K} \gamma n_c^{\gamma-1}}{m(\gamma - 1)} \, \Big(\frac{r_c^5 (g_E^2)_c F(r)_{c,r_c}}{4}\Big)^{(\gamma-1)/2} = \text{ const.} \tag{2.84}$$

Then we evaluate the Hamiltonian by

$$\mathcal{H} = \frac{F(r)}{1 - v^2} \, \Big[1 + Y\Big(\frac{1 - v^2}{r^4 F(r) g_E^2 v^2}\Big)^{(\gamma-1)/2}\Big]^2. \tag{2.85}$$

If $Y < 0$,

$$1 + Y\Big(\frac{1 - v^2}{r^4 F(r) g_E^2 v^2}\Big)^{(\gamma-1)/2} \propto r^{-1} \quad \text{as} \quad r \to \infty. \tag{2.86}$$

Speed at spatial infinity (2.69)

$$v \simeq v_1 r^{-\alpha} + v_2 r^{-\delta} \text{ as } r \to \infty \qquad (\delta > \alpha > 0), \tag{2.87}$$

then from observing , we find $\alpha = 3$, $\delta \geq 4$, and

$$v_1^2 = (-3/\Lambda)(Y^2)^{1/(\gamma-1)}. \tag{2.88}$$

Chapter 3

Accretion of Schwarzschild Black Hole Immersed in an Electromagnetic Field in Rainbow Gravity

Around two decades back, astronomical observations suggested that the cosmos is experiencing a rushing extension. This finding is a revolt in cosmology with the reason accountable for this consequence labeled as dark energy. It has strange characteristic that yields repulsive gravitational effects and it voilates the null and weak energy conditions [17]. Astonishingly, explanations advise that around 70 percent of the energy of the cosmos fit in to the context of dark energy. Nevertheless, the properties of this energy are not well recognized and these days is the most interesting issue in theoretical physics. In previous few years, theorists are trying to solve this issue. Few proposals are presented like cosmological constant, phantom energy, quintessence, k-essence, dynamic scalar fields, and others. Generally, dark energy is shaped by correlating the parameters such as pressure, density and energy of an ideal fluid by that of a perfect fluid with the help of a barotropic equation $p = \omega\rho$, where ω is the state parameter. The gravitational accretion of mass is plentiful in astrophysics because it is an coherent mechanism to change gravitational energy into kinetic energy [18]. Basically, accretion is the procedure in which an enormous stellar object like a black hole or a dense compact object which can take particles from the surroundings of a fluid that primes to change the mass, size of accreting object. For example, the importance of accretion procedures are tangled with the presence of giant black holes at the middle of clusters. Though, black holes formation from the gravitational collapse is not the only way of their formation. Moreover, the specific space regions, where the conditions are favorable for the for black hole's formation . One way can be the union of numerous tiny black holes but it has very low probability [19]. The existence of giant black holes at the center of super massive ellipsoidal and circling galaxies advises that these black holes might have grew through accretion proceedings. Additional techniques of forming of super massive black holes such as union of planetary collapse of many stars or many tiny black holes in a very small realm resulting to union, seems nearly impossible [20, 21]. The most appealing picture of super massive and gigantic black holes is the deposit of matter or dust from neighboring areas for adequately longer times. The creation of mighty overextended and astrophysical jets from dynamic galaxies or small and compact substances designates the presence of huge quantities of hot dust nearby the space region of jet formation [22, 23]. Though accretion procedure not at all times increases the mass of the solid source, rarely the in falling substance is dissipated in the form of cosmic rays or jets. It is feasible that the accretion proceeding might not be fixed and the speed, an energy density of an in falling fluid alters with location and time. The compiling of matter and dust on these objects is a well-studied problem [24]. Anyhow, the pile up of matter of more bizarre kinds of energy-matter is not so usually probed including dark energy and ultra-stiff fluids. As the universe is dominated by dark energy and dark matter, it is more suitable to examine the accretion of different kinds of dark energy onto black holes [25, 26]. We have following metric

$$ds^2 = -F(r)dt^2 + \frac{1}{F(r)}dr^2 + r^2(d\theta^2 + sin^2\theta d\phi^2), \tag{3.1}$$

where

$$F(r) = 1 - \frac{2M}{r} + \frac{M^2(1-a^2)}{r^2}.$$

The matter is approximated as perfect fluid specified by energy momentum tensor

$$T_{\alpha\beta} = (\rho + P)u_\alpha u_\beta + P g_{\alpha\beta},$$

where ρ is proper energy density and P is proper pressure of fluid. The proper baryon number density is denoted by "n" and the baryon number flux,

$$J^\mu = nu^\mu.$$

The first law of thermodynamics,

$$dE + Pdv = Tds = 0.$$

If we take "N" as number of baryons, then

$$d(\frac{E}{N}) + Pd(\frac{V}{N}) = 0, \tag{3.2}$$

$$\frac{E}{N} = \frac{\frac{E}{V}}{\frac{N}{V}} = \frac{\rho}{n}, \tag{3.3}$$

$$\frac{V}{N} = \frac{\frac{V}{V}}{\frac{N}{V}} = \frac{1}{n}, \tag{3.4}$$

using (3.2) and (3.3) in (3.4) we get

$$d(\frac{\rho}{n}) + P(d\frac{1}{n}) = 0. \tag{3.5}$$

By taking differential of (3.5)

$$\frac{nd\rho - \rho dn}{n^2} + P(-\frac{dn}{n^2}) = 0,$$

$$nd\rho - \rho dn - Pdn = 0,$$

$$\frac{d\rho}{dn} = \frac{\rho + P}{n}. \tag{3.6}$$

The adiabatic sound speed of fluid is defined as

$$c_s^2 \equiv \frac{dP}{d\rho},$$

$$c_s^2 \equiv \frac{dP}{d\rho} \equiv \frac{dP}{dn} \cdot \frac{dn}{d\rho},$$

$$c_s^2 \equiv \frac{dP}{d\rho} \cdot \frac{1}{\frac{\rho+P}{n}} \Rightarrow c_s^2 = \frac{n}{\rho + P} \cdot \frac{dP}{dn}. \tag{3.7}$$

By conservation law of particle number, we have

$$\nabla_\mu j^\mu = (nu^\mu)_{;\mu}.$$

Above can be written as

$$\frac{1}{r^2}.\frac{d}{dr}\frac{r^2nu}{f_E g_E^3} = 0, \tag{3.8}$$

or

$$\frac{r^2nu}{f_E g_E^3} = C_1. \tag{3.9}$$

By integrating (3.8) over spatial volume and multiplying by the mass of each particle "m" [27]. We get

$$\dot{M} = 4\pi m \frac{r^2nu}{f_E g_E^3}, \tag{3.10}$$

where "$\dot{M}$" is constant of integration (Bondi mass accretion rate). Divergence of energy momentum tensor is given by

$$T^{\mu}_{\nu;\mu} = 0,$$

$$T^{\mu}_{\nu,\mu} + \Gamma^{\mu}_{\mu\alpha}T^{\alpha}_{\nu} - \Gamma^{\alpha}_{\mu\nu}T^{\mu}_{\alpha},$$

and

$$T^{\mu}_{\nu;\mu} = \frac{\sqrt{-g}}{\sqrt{-g}}T^{\mu}_{\nu,\mu} + \frac{1}{\sqrt{-g}}(\sqrt{-g})_{,\alpha}\, T^{\alpha}_{\nu} - \Gamma^{\alpha}_{\mu\nu}T^{\mu}_{\alpha},$$

$$T^{\mu}_{\nu;\mu} = \frac{1}{\sqrt{-g}}[\sqrt{-g}T^{\mu}_{\nu,\mu} + (\sqrt{-g})_{,\mu}\, T^{\alpha}_{\nu}] - \Gamma^{\alpha}_{\mu\nu}T^{\mu}_{\alpha},$$

$$T^{\mu}_{\nu;\mu} = \frac{1}{\sqrt{-g}}[\sqrt{-g}T^{\mu}_{\alpha}]_{,\alpha} - \Gamma^{\alpha}_{\nu\mu}T^{\mu}_{\alpha}.$$

Since

$$\Gamma^{\alpha}_{\nu\mu} = g^{\kappa\alpha}\Gamma_{\kappa\mu\nu}$$

and

$$\Gamma^{\alpha}_{\mu\nu}T^{\mu}_{\alpha} = \Gamma_{\kappa\nu\mu}g^{\kappa\alpha}T^{\mu}_{\alpha},$$

$$\Gamma^{\alpha}_{\mu\nu}T^{\mu}_{\alpha} = \Gamma_{\kappa\nu\mu}T^{\mu\kappa},$$

so

$$T^{\mu}_{\nu;\mu} = \frac{1}{\sqrt{-g}}[\sqrt{-g}T^{\mu}_{\nu,\mu} + (\sqrt{-g})_{,\mu}\, T^{\alpha}_{\nu}].$$

As we have $\sqrt{-g} = r^2$ and energy is conserved so put $\nu = 0$ in $T^{\mu}_{\nu;\mu} = 0$ and

$$T^{\mu}_{\nu} = (\rho + P)u^{\mu}_{\nu} + P\delta^{\mu}_{\nu},$$

$$T_0^\mu = (\rho + P)u_0^\mu + P\delta_0^\mu.$$

Now put "$\mu = 1$",

$$T_0^1 = (\rho + P)u_0^1,$$

$$T_0^1 = (\rho + P)\frac{\left[1 - \frac{2M}{r} + \frac{M^2(1-a^2)}{r^2}\right]\left[g_E^2\left(1 - \frac{2M}{r} + \frac{M^2(1-a^2)}{r^2}\right) + u^2\right]^{1/2}}{1 - \frac{2M}{r} + \frac{M^2(1-a^2)}{r^2}},$$

$$T_0^1 = (\rho + P)\left[g_E^2\left(1 - \frac{2M}{r} + \frac{M^2(1-a^2)}{r^2}\right) + u^2\right]^{1/2}. \tag{3.11}$$

We know

$$\nabla_\mu T_\nu^\mu = \frac{1}{\sqrt{-g}}(\sqrt{-g}T_\nu^\mu)_{,\mu},$$

$$\nabla_\mu T_\nu^\mu = \frac{1}{r^2}\left(\frac{r^2}{g_E^3}T_\nu^\mu\right)_{,\mu}.$$

Put $\nu = 0$,

$$\nabla_\mu T_0^\mu = \frac{1}{r^2}\left(\frac{r^2}{g_E^3}T_0^\mu\right)_{,\mu},$$

as $\mu = \mu(r)$, so

$$\frac{1}{r^2}\cdot\frac{d}{dr}\left(\frac{r^2}{g_E^3}T_0^1\right) = 0. \tag{3.12}$$

Put (1.14) in (1.15), we get

$$\frac{1}{r^2}\cdot\frac{d}{dr}\left(\frac{r^2 u}{g_E^3}(\rho + P)\left[g_E^2\left(1 - \frac{2M}{r} + \frac{M^2(1-a^2)}{r^2}\right) + u^2\right]^{1/2}\right) = 0. \tag{3.13}$$

Upon integration we get

$$\left(r^2(\rho + P)\frac{u}{g_E^3}\left[g_E^2\left(1 - \frac{2M}{r} + \frac{M^2(1-a^2)}{r^2}\right) + u^2\right]^{1/2}\right) = C_2. \tag{3.14}$$

The relativistic Euler equation is

$$(\rho + P)(\nabla_\mu u^\nu)u^\mu = -\left[g^{\mu\nu}\nabla_\mu P + u^\mu u^\nu \nabla_\mu P\right],$$

for momentum conservation put $\nu = 1$,

$$(\rho + P)(u^\mu u^1_{;\mu}) = -\left[g^{\mu 1}\nabla_\mu P + u^\mu u^1 \nabla_\mu P\right],$$

$$(\rho+P)\left(u^{\mu}\left[u^{1}_{,\mu}+\Gamma^{1}_{\mu\alpha}u^{\alpha}\right]\right)=-\left[g^{\mu 1}\nabla_{\mu}P+u^{\mu}u^{1}\nabla_{\mu}P\right],$$

$$(\rho+P)\left[u^{1}u^{1}_{,1}+\Gamma^{1}_{00}(u^{0})^{2}+\Gamma^{1}_{11}(u^{1})^{2}\right]=-\left[g^{11}\nabla_{1}P+(u^{1})^{2}\nabla_{1}P\right].$$

Now as these are the only no-vanishing components of christoffel symbol

$$\Gamma^{1}_{00}=\left[1-\frac{2M}{r}+\frac{M^{2}(1-a^{2})}{r^{2}}\right]\left[\frac{M}{r^{2}}-\frac{M^{2}(1-a^{2})}{r^{3}}\right],$$

and

$$\Gamma^{1}_{11}=-\frac{\left[\frac{M}{r^{2}}-\frac{M^{2}(1-a^{2})}{r^{3}}\right]}{1-\frac{2M}{r}+\frac{M^{2}(1-a^{2})}{r^{2}}},$$

inserting these in above equation we obtain,

$$u\frac{du}{dr}=-\frac{dp}{dr}\frac{\left[g_{E}^{2}\left(1-\frac{2M}{r}+\frac{M^{2}(1-a^{2})}{r^{2}}\right)+u^{2}\right]}{\rho+P}-g_{E}^{2}\left(\frac{M}{r^{2}}-\frac{M^{2}(1-a^{2})}{r^{3}}\right). \tag{3.15}$$

3.1 Conditions for Critical Accretion

By differentiating (3.9) w.r.t $''r''$, we get

$$(\frac{r^{2}nu}{g_{E}^{3}})_{,r}=(C_{1})_{,r},$$

$$2r(nu)+r^{2}(nu)_{,r}=0,$$

$$2r(nu)+r^{2}(nu^{'}+n^{'}u)_{,r}=0,$$

$$\frac{u^{'}}{u}+\frac{n^{'}}{n}=-\frac{2}{r}, \tag{3.16}$$

by (1.19)

$$u\frac{du}{dr}+\frac{dP}{dr}\frac{\left[g_{E}^{2}\left(1-\frac{2M}{r}+\frac{M^{2}(1-a^{2})}{r^{2}}\right)+u^{2}\right]}{\rho+P}+g_{E}^{2}\left(\frac{M}{r^{2}}-\frac{M^{2}(1-a^{2})}{r^{3}}\right)=0,$$

$$u\frac{du}{dr}+\frac{\frac{dP}{dn}\cdot\frac{dn}{dr}\left[g_{E}^{2}\left(1-\frac{2M}{r}+\frac{M^{2}(1-a^{2})}{r^{2}}\right)+u^{2}\right]}{\rho+P}+g_{E}^{2}\left(\frac{M}{r^{2}}-\frac{M^{2}(1-a^{2})}{r^{3}}\right)=0,$$

$$u\frac{du}{dr} + \frac{\frac{c_s^2(\rho+P)}{n} \cdot \frac{n\prime}{\rho+P}\left[g_E^2\left(1 - \frac{2M}{r} + \frac{M^2(1-a^2)}{r^2}\right) + u^2\right]}{\rho + P} + g_E^2\left(\frac{M}{r^2} - \frac{M^2(1-a^2)}{r^3}\right) = 0,$$

$$uu^{'} + \frac{c_s^2 n^{'}}{n}\left(g_E^2\left(1 - \frac{2M}{r} + \frac{M^2(1-a^2)}{r^2}\right) + u^2\right) = -g_E^2\left(\frac{M}{r^2} - \frac{M^2(1-a^2)}{r^3}\right). \quad (3.17)$$

Solving (3.16) and (3.17) simultaneously, we get

$$u^{'} = \frac{N_1}{N}, \quad (3.18)$$

$$n^{'} = -\frac{N_2}{N}, \quad (3.19)$$

where

$$N_1 = \frac{1}{n}\left[\frac{2c_s^2}{r}\left(g_E^2\left(1 - \frac{2M}{r} + \frac{M^2(1-a^2)}{r^2}\right) + u^2\right) - g_E^2\left(\frac{M}{r^2} - \frac{M^2(1-a^2)}{r^3}\right)\right], \quad (3.20)$$

$$N = \frac{u^2 - \left[g_E^2\left(1 - \frac{2M}{r} + \frac{M^2(1-a^2)}{r^2}\right) + u^2\right]c_s^2}{nu}, \quad (3.21)$$

$$N_2 = \frac{1}{u}\left[\frac{2u^2}{r} - g_E^2\left(\frac{M}{r^2} - \frac{M^2(1-a^2)}{r^3}\right)\right]. \quad (3.22)$$

For critical points

$$N_1 = \frac{1}{n_c}\left[\frac{2c_{sc}^2}{r_c}\left(g_E^2\left(1 - \frac{2M}{r_c} + \frac{M^2(1-a^2)}{r_c^2}\right) + u_c^2\right) - g_E^2\left(\frac{M}{r_c^2} - \frac{M^2(1-a^2)}{r_c^3}\right)\right], \quad (3.23)$$

$$N = \frac{u_c^2 - \left[g_E^2\left(1 - \frac{2M}{r_c} + \frac{M^2(1-a^2)}{r_c^2}\right) + u_c^2\right]c_{sc}^2}{n_c u_c}, \quad (3.24)$$

$$N_2 = \frac{1}{u_c}\left[\frac{2u_c^2}{r_c} - g_E^2\left(\frac{M}{r_c^2} - \frac{M^2(1-a^2)}{r_c^3}\right)\right]. \quad (3.25)$$

We put N_1, N_2 and N equal to zero. So, we can find radial velocity (u_c) and speed of sound (a_c) as follow:

$$u_c^2 = \frac{r_c g_E^2}{2}\left[\frac{M}{r_c^2} + \frac{M^2(1-a^2)}{r^3}\right],$$

i.e.

$$u_c^2 = \frac{g_E^2}{2}\left[\frac{M}{r_c} - \frac{M^2(1-a^2)}{r^2}\right], \quad (3.26)$$

and

$$c_{sc}^2 = \frac{u_c^2}{\left(1 - \frac{2M}{r_c} + \frac{M^2(1-a^2)}{r_c^2}\right)g_E^2 + u_c^2}. \tag{3.27}$$

Physically acceptable solutions of (3.26) and (3.27) exists if $''u_c^2 \geq 0''$ and $c_{sc}^2 \geq 0$, therefore

$$u_c^2 = \frac{g_E^2}{2}[\frac{M}{r_c} - \frac{M^2(1-a^2)}{r^2}] \geq 0.$$

$$c_{sc}^2 = \frac{u_c^2}{\left(1 - \frac{2M}{r_c} + \frac{M^2(1-a^2)}{r_c^2}\right)g_E^2 + u_c^2} \geq 0,$$

or

$$\frac{\frac{g_E^2}{2}[\frac{M}{r_c} - \frac{M^2(1-a^2)}{r_c^2}]}{\left(1 - \frac{2M}{r_c} + \frac{M^2(1-a^2)}{r_c^2}\right)g_E^2 + u_c^2} \geq 0,$$

therefore

$$c_{sc}^2 = \frac{\frac{g_E^2}{2}[\frac{M}{r_c} - \frac{M^2(1-a^2)}{r_c^2}]}{\left(1 - \frac{3M}{2r_c} + \frac{M^2(1-a^2)}{2r_c^2}\right)g_E^2} \geq 0,$$

where u_c^2 and c_{sc}^2 must satisfy following condition

$$(\frac{3M}{4} \pm \frac{1}{4}\sqrt{17M^2 - 8a^2}) < r_c < M(a^2 - 1). \tag{3.28}$$

Now to find the critical speed u_c and energy density ρ of fluid, we get these following equations from [17] labeled as equation 5, 9, 11 respectively,

$$\frac{uC(r)}{g_E^3}(\rho + p)\frac{A(r)}{B(r)}\sqrt{u^2 + g_E^2 B(r)} = A_1, \tag{3.29}$$

$$(\rho + p)\sqrt{u^2 + B(r)g_E^2}\sqrt{\frac{A(r)}{B(r)}}e^{-\int \frac{d\rho}{\rho + p(\rho)}} = \frac{-A_1}{A_0} = A_3, \tag{3.30}$$

$$\frac{1}{g_E^3}\frac{(\rho + p)}{\rho}\sqrt{\frac{A(r)}{B(r)}}\sqrt{u^2 + B(r)g_E^2} = \frac{A_1}{A_2} = A_4. \tag{3.31}$$

Solving (3.29), (3.30) and (3.31), we obtain,

$$u(r) = -\frac{g_E\sqrt{A_1^2 g_E^6 r^2 + A_0^2(M + aM - r)((-1+a)M + r)}}{A_0 r} \tag{3.32}$$

$$\rho(r) = \frac{A_0 A_2 g_E^2}{r\sqrt{A_1^2 g_E^6 r^2 + A_0^2(M + aM - r)((-1+a)M + r)}} \tag{3.33}$$

Here A_0, A_1 and A_2 are integration constants. We choose $A_0 = (1+\omega)$, $A_2 = 1$ and $A_4 = 0.01$. Using equation of state $p = \omega\rho$, which is used to model the perfect fluid with state parameter ω, with $\omega = -1$ corresponds to cosmological constant, $-1 < \omega < -1/3$ subjected to quintessence and $\omega < -1$ used for phantom models [17]. By substituting these in above we obtain,

$$u(r) = -\frac{g_E\sqrt{A_1^2 A_4^2 g_E^6 r^2 + (1+\omega)^2(M + aM - r)((-1+a)M + r)}}{(1+\omega) r} \tag{3.34}$$

$$\rho(r) = \frac{(1+\omega) A_2 g_E^2}{r\sqrt{A_1^2 A_4^2 g_E^6 r^2 + (1+\omega)^2(M + aM - r)((-1+a)M + r)}} \tag{3.35}$$

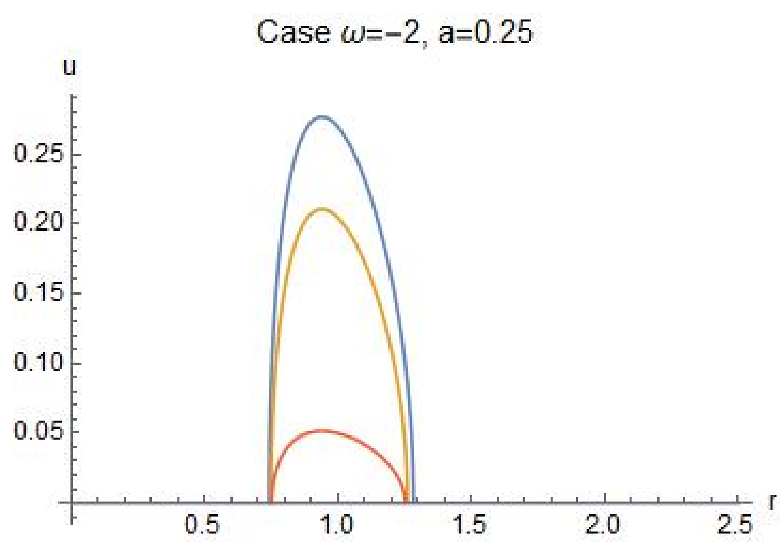


Fig:3.1. Speed of fluid versus radius for interpolation parameter a, where Red, Yellow and Blue graphs corresponds to $a = 0.25, 0.5, 0.8$ respectively with $\omega = -2$ and $g = 0.25$.

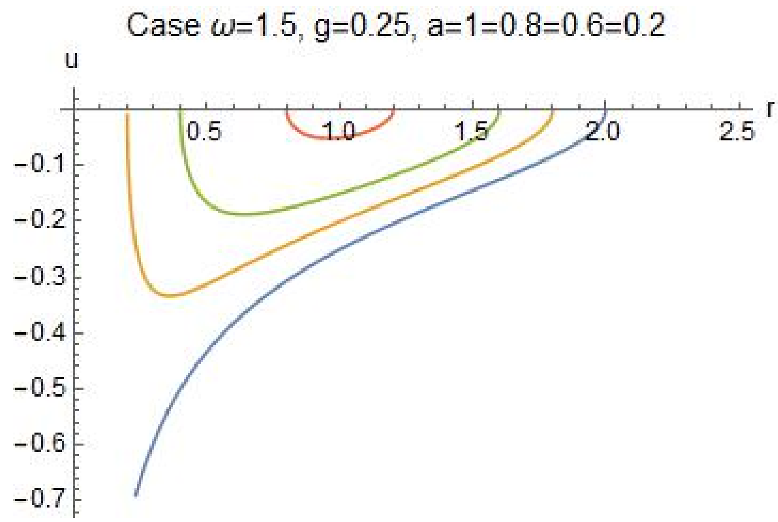


Fig:3.2. Speed of fluid versus radius for interpolation parameter a, where Red, Yellow and Blue graphs corresponds to $a = 0.2, 0.6, 0.8, 1$ respectively with $\omega = -1.5$ and $g = 0.25$.

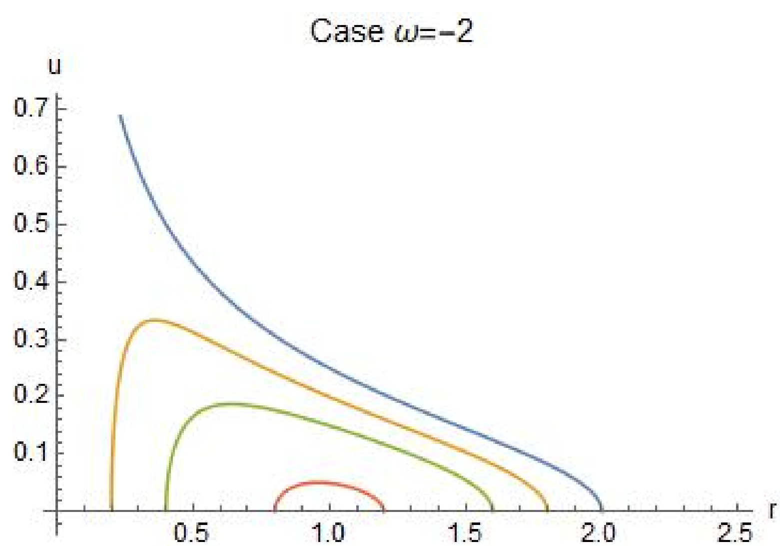


Fig:3.3. Speed of fluid versus radius for interpolation parameter a, where Red, Yellow and Blue graphs corresponds to $a = 0.25, 0.5, 0.8, 1$ respectively with $\omega = -2$ and $g = 0.5$.

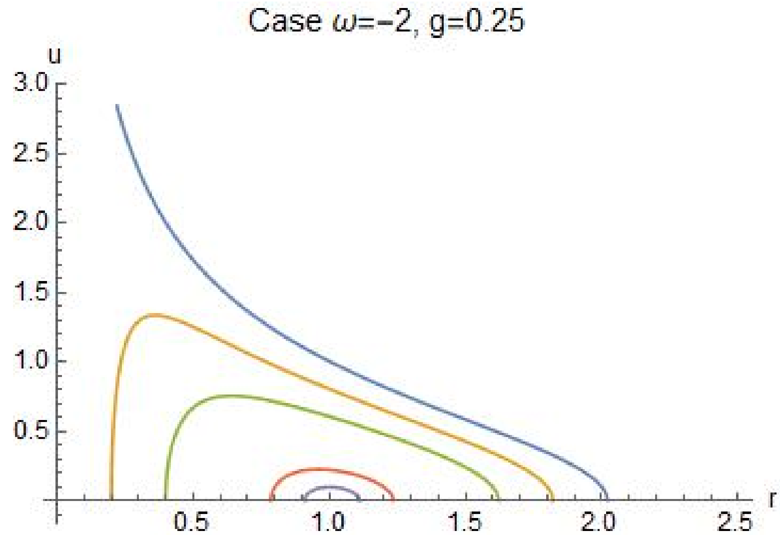


Fig:3.5. Speed of fluid versus radius for interpolation parameter a, where Red, Yellow and Blue graphs corresponds to $a = 0.25, 0.5, 0.8$ respectively with $\omega = -2$ and $g = 0.25$.

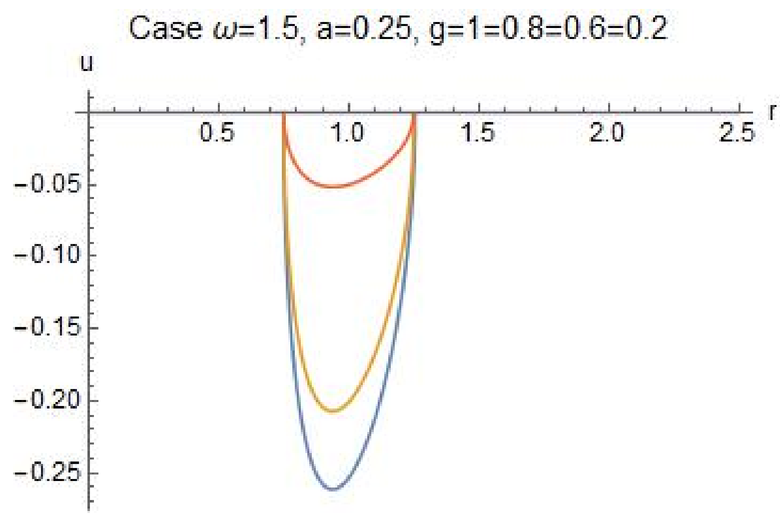


Fig:3.6. Speed of fluid versus radius for interpolation parameter a, where Red, Yellow and Blue graphs corresponds to $g = 0.25, 0.5, 0.8, 1$ respectively with $\omega = -2$ and $a = 0.25$.

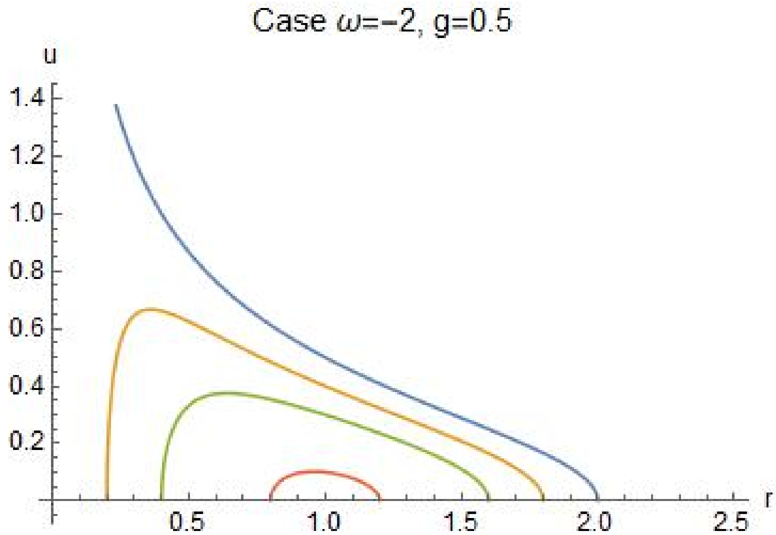


Fig:3.7. Speed of fluid versus radius for interpolation parameter a, where Red, Yellow and Blue graphs corresponds to $a = 0.25, 0.5, 0.8$ respectively with $\omega = -2$ and $g = 0.5$.

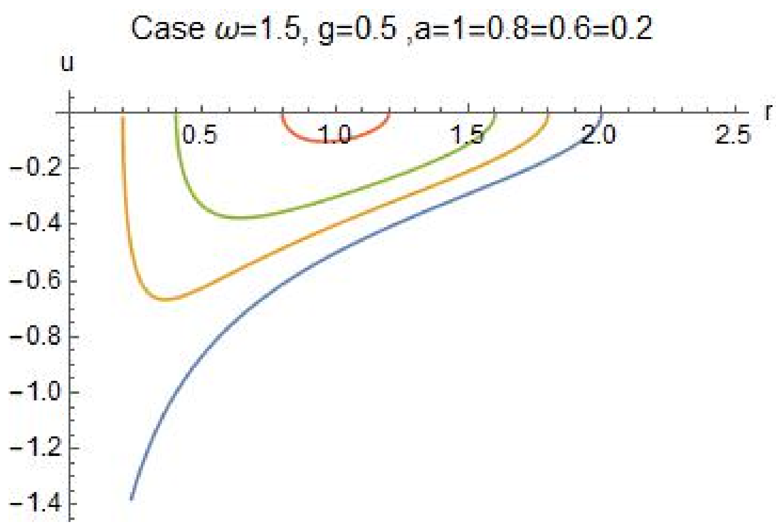


Fig:3.8. Speed of fluid versus radius for interpolation parameter a, where Red, Yellow and Blue graphs corresponds to $a = 0.25, 0.5, 0.8, 1$ respectively with $\omega = -1.5$ and $g = 0.5$.

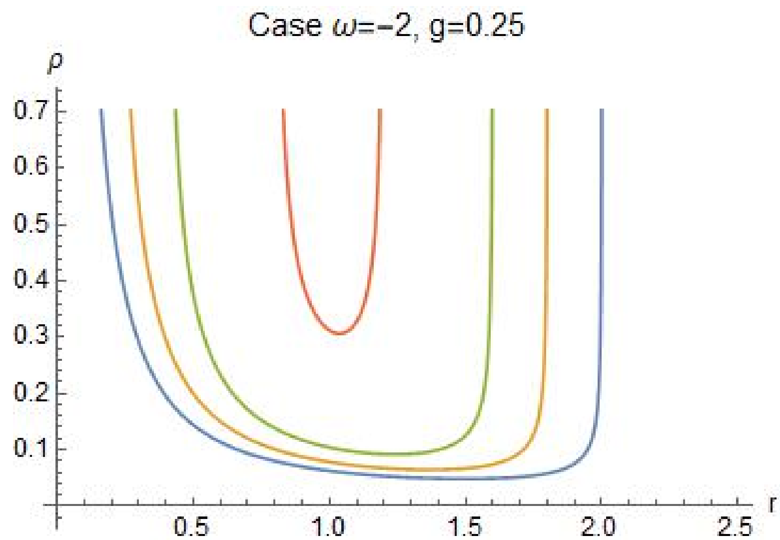


Fig:3.9. Density of fluid versus radius for interpolation parameter a, where Red, Yellow, Blue and Brown graphs corresponds to $a = 0.25, 0.5, 0.8, 1$ respectively with $\omega = -2$ and $g = 0.25$.

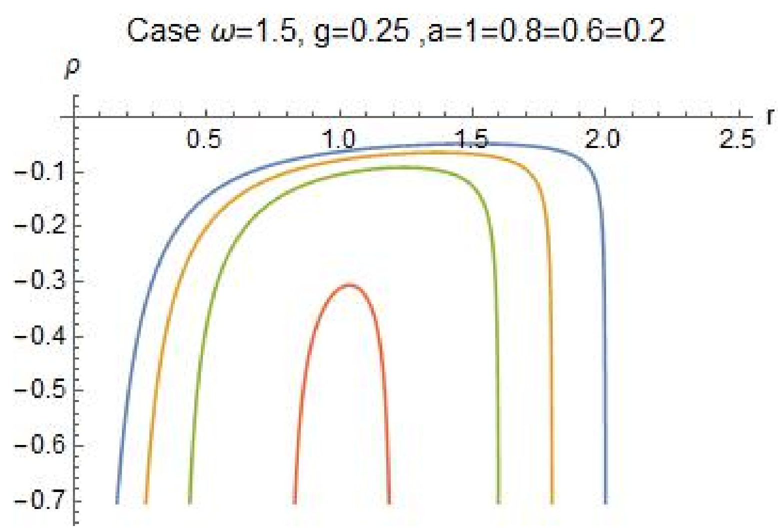


Fig:3.10. Density of fluid versus radius for interpolation parameter a, where Red, Yellow, Blue and brown graphs corresponds to $a = 0.25, 0.5, 0.8, 1$ respectively with $\omega = -2$ and $g = 0.25$.

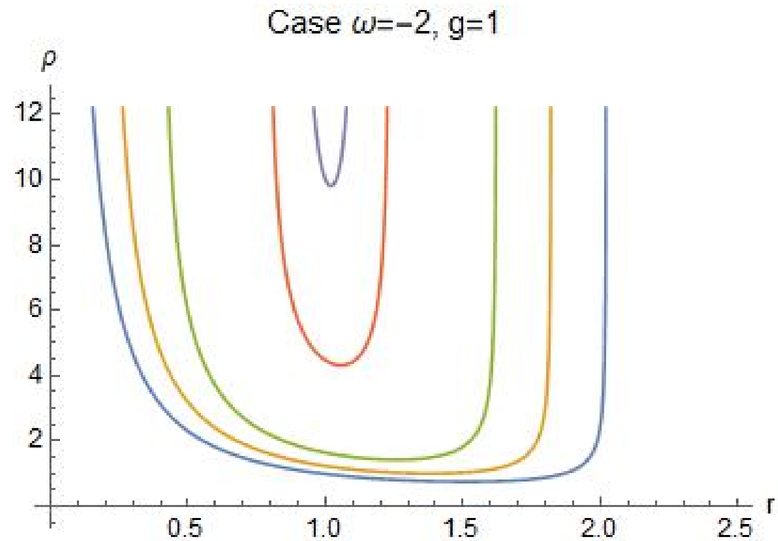


Fig:3.11. Density of fluid versus radius for interpolation parameter a, where Red, Yellow, Blue and brown graphs corresponds to $a = 0.25, 0.5, 0.8, 1$ respectively with $\omega = -2$ and $g = 1$.

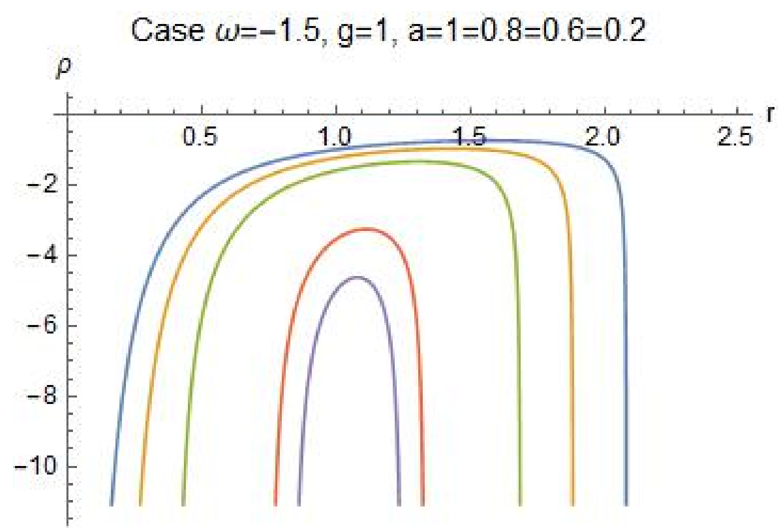


Fig:3.12. Density of fluid versus radius for interpolation parameter a, where Red, Yellow, Blue and Brown graphs corresponds to $a = 0.25, 0.6, 0.8, 1$ respectively with $\omega = -1.5$ and $g = 1$.

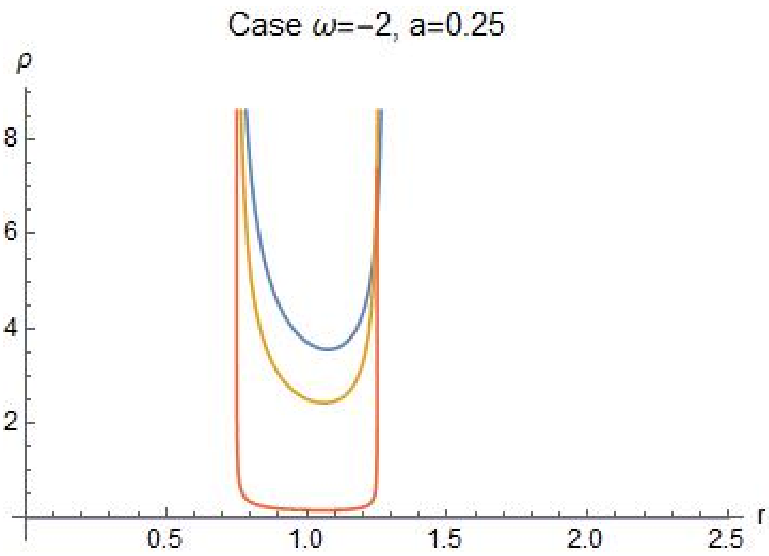


Fig:3.13. Density of fluid versus radius for interpolation parameter a, where Red, Yellow and Blue graphs corresponds to $g = 0.25, 0.5, 0.8$ respectively with $\omega = -2$ and $a = 0.25$.

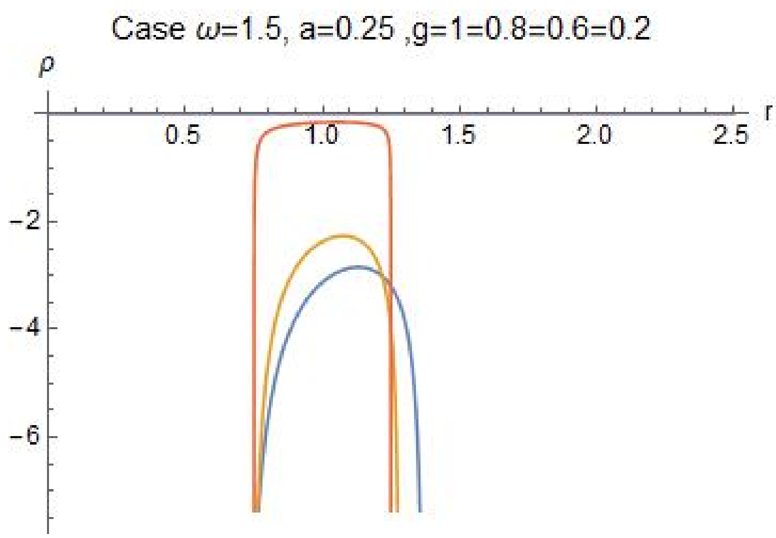


Fig:3.14. Speed of fluid versus radius for interpolation parameter a, where Red, Yellow and Blue graphs corresponds to $g = 0.25, 0.5, 0.8, 1$ respectively with $\omega = -1.5$ and $a = 0.25$.

3.2 Polytopic Solution

Consider polytropic state of equation [8]

$$P = kn^{\gamma}. \tag{3.36}$$

From (2.47) we have

$$d\frac{\rho}{n} + Pd(\frac{1}{n}) = 0,$$

$$\frac{nd\rho - \rho dn}{n^2} + P(-\frac{dn}{n^2}) = 0,$$

$$nd\rho - (\rho dn + P)dn = 0,$$

$$\frac{d\rho}{dn} = \frac{\rho + P}{n},$$

$$\frac{d\rho}{dn} = \frac{\rho + kn^{\gamma}}{n},$$

$$\frac{d\rho}{dn} - \frac{\rho}{n} = kn^{\gamma-1}.$$

Here the integrating factor is

$$I.F = e^{-\int \frac{1}{n}dn} \Rightarrow \frac{1}{n},$$

$$\int d(\rho n^{-1}) = \int kn^{\gamma-2}dr,$$

$$\rho = \frac{kn^{\gamma-1}}{\gamma - 1} + mn. \tag{3.37}$$

The adiabatic speed of sound is

$$c_s^2 = \frac{dP}{d\rho} \Rightarrow \frac{(kn^{\gamma})}{d\left(\frac{kn^{\gamma}}{\gamma-1} + mn\right)},$$

$$c_s^2\left[\frac{k}{\gamma - 1}(\gamma n^{\gamma-1}) + m\right] = k\gamma n^{\gamma-1},$$

$$k\gamma n^{\gamma-1} = \frac{c_s^2 m}{1 - \frac{c_s^2}{\gamma-1}}. \tag{3.38}$$

Bernoulli equation of the flow can be achieved by dividing (3.14) by (3.9) [27]. We get

$$\frac{ur^2(\rho+P)\left[\left(1-\frac{2M}{r}+\frac{M^2(1-a^2)}{r^2}\right)g_E^2+u^2\right]^{1/2}f_E}{r^2nu}=\frac{C_1}{C_2},$$

$$\frac{(\rho+P)^2}{n^2}\left[\left(1-\frac{2M}{r}+\frac{M^2(1-a^2)}{r^2}\right)g_E^2+u^2\right]=C_3,$$

when we take it from ρ to ρ_∞, we obtain

$$C=\frac{\rho_\infty+P_\infty}{n_\infty},$$

so that

$$\frac{(\rho+P)^2}{n^2}\left[\left(1-\frac{2M}{r}+\frac{M^2(1-a^2)}{r^2}\right)g_E^2+u^2\right]=\left(\frac{\rho_\infty+P_\infty}{n_\infty}\right)^2. \tag{3.39}$$

From (3.36) and (3.37), we can write

$$\frac{\rho+P}{n}=\frac{\frac{k}{\gamma-1}n^\gamma+mn+kn^\gamma}{n},$$

$$\frac{\rho+P}{n}=kn^{\gamma-1}\left(\frac{1}{\gamma-1}+1\right)+m,$$

$$\frac{\rho+P}{n}=\frac{c_s^2m}{1-\frac{c_s^2}{\gamma-1}}\left(\frac{1}{\gamma-1}+1\right)+m,$$

$$\frac{\rho+P}{n}=m\left(1+\frac{c_s^2}{\gamma-1-c_s^2}\right).$$

Take $m=1$ (as its constant of integration). So, the Bernoulli equation (3.39) transforms to

$$\left(1+\frac{c_s^2}{\gamma-1-c_s^2}\right)^2\left[\left(1-\frac{2M}{r}+\frac{M^2(1-a^2)}{r^2}\right)g_E^2+u^2\right]=\left(1+\frac{c_{s\infty}^2}{\gamma-1-c_{s\infty}^2}\right)^2. \tag{3.40}$$

Chapter 4

Conclusion

We investigated the effect of rainbow gravity theory proposed by J. Magueijo and L. Smolin [29] in accretion and flow process of spherically symmetric compact objects like Schwarzchild black hole using its metric in presence of electromagnetic field. Here, the Einstein equations and Schwarzchild metric in electromagnetic field are modified using the ratio E/E_p . We quantize the spacetime by using rainbow gravity functions $f(\frac{E}{E_p})$ and $g(\frac{E}{E_p})$ with $0 < f(\frac{E}{E_p}), g(\frac{E}{E_p}) < 1$. As we know that in rainbow gravity, the geometry depends upon the energy of the particle/test body that is under examination. So, we see that test body/particle with different energies show different geometries with same inertial frame of reference along with same equivalence principal.

In second chapter, we discussed the Hamiltonian for the given system and discuss the solution for ultra-stiff, ultra-relativistic and for radiation fluids. We found that ultra stiff fluid has un-physical solution if Hamiltonian is negative and if Hamiltonian is positive and $v > 0$, this implies that fluid will flow outward in the form of jets and $v < 0$ implies that the fluid will cause the accretion of black hole . Similarly, in solution for ultra-relativistic fluid, we found that fluid will only cause in increase in mass, no mass will flow out. Contrarily, in solution for radiation fluid, there is only supersonic and subsonic outward flow of fluid in form of jets there will be no accretion in this case.

In third chapter, instead of using Hamiltonian approach to study the speed of flow with energy density and discuss its several cases in presence of rainbow gravity. Here, we use equation of state and use different values of cosmological constant to study the speed of flow and energy density of perfect fluid. In this discussion, we took $-1 < \omega < -1/3$, $\omega \leq -1$ with different values of $0 < a < g \leq 1$ and discuss only those cases which posses physically possible solutions. Moreover, we conclude that it is conformal scaling of the original one and no-linear version leads to linear one when the ratio E/E_p tends to zero.

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