Charged Anisotropic Compact Object with Karmarkar Condition

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A thesis
submitted in partial fulfillment of the
requirements for the degree of
Master of Science
in
Mathematics

Supervised by

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2021

FORM TH-4

National University of Sciences & Technology

MS THESIS WORK

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Dedication

This thesis is dedicated to

my best teacher
Late Majid Ajaz

my lovely parents
Muhammad Nazeer and **Najma bibi**

and my supervisor
Prof. Azad A. Siddiqui

Acknowledgements

All praise is for **Allah**, Lord of all worlds, the Most Compassionate, Most Merciful, Master of the Day of Judgment, Who has provided me ability of thinking, capability to observe this universe, strength and knowledge to write up this thesis.

I am very thankful of my supervisor **Prof. Azad A. Siddiqui**, who helped me, guided me, encouraged me, and motivated me with his patience and, more importantly, I am always impressed by his very logical way of talking.

I am in debt of my dear teacher **Late Majid Ajaz**, who was principal of **Scholars Model School, Chichawatni**. May Allah reward him with all that is good. He was the person, who taught me the importance of education and motivated me to get knowledge to choose right path in every aspect of life.

I am very grateful to my parents **Muhammad Nazeer** and **Najma bibi**, who always motivated and helped me in all possible ways. Without their support I could not go through the journey of my MS. May Allah have mercy on them.

I am very thankful to all my teachers for their kind guidance and affection, especially **Prof. Tooba Feroze**. And finally, I want to express my appreciation to all family members and a close group of wonderful friends for their help, good wishes and encouragement.

Furqan Nazeer

Abstract

In this thesis, a new generalized solution by using the Karmarkar condition for charged anisotropic matter distribution is presented. Under the embedding Class-I space-times, we find out the possibility of constructing an electromagnetic model where physical parameters have a purely electromagnetic origin. This specific solution of charged anisotropic relativistic compact objects is used to model the internal composition so that it satisfies compulsory physical conditions. The metric potentials, density and pressure have no singularities and satisfy the required conditions inside the anisotropic compact object. The anisotropic factor is zero at the center and increases afterwards.

Contents

List of Figures

Chapter 1

Introduction

1.1 Historical Background

Many philosophers and scientists throughout history have held different views on the concepts of space and time. Throughout Plato's era, time was linked to cosmic regularity (the movement of the Sun and Moon), but Aristotle disagreed with this framework, believing that space is proportional to its substance content, and time is proportional to the succession of events. At the sphere of the moon, Aristotle separated the world into two different portions. Everything was made up of the four elements air, fire, water, and earth below this sphere where all types of motion were conceivable, but only the fifth element ether was present above it where the circular movement was possible only. However, the movements above were classified into two categories by him: natural motion and enforced motion [1]. These misapprehensions continued until the mid-seventeenth century when substantial work by Newton and Galileo uncovered them. Galileo was the first to make a logical connection between past hazy notions with subsequent discoveries in 1632. His interpretation of relativity was devoid of any relation to the mechanics of natural motion. Galileo moved beyond the Principle of Inertia, which had fixed a simple equivalence of state between rest and uniform motion, and he defined a complete equivalence of physical laws concerning all inertial references. Aristotle's "natural motion" was similarly reinforced by the gravitational force for Newton. He added different concepts like including laws of motion and gravitation. Moreover in Newtonian mechanics, the notions of space and time are considered to be entirely separable and time is also assumed to be an absolute quantity capable of precise description irrespective of the reference frame. Later on, Leibniz came and contradicted Newton's idea of absolute space, he was of the view that laws of gravitation might be universalized. To begin with, he said that Galileo had demonstrated that there is no such thing as absolute velocity, and hence no such thing as absolute space; from which it is formed. Second, Leibniz criticized Newton's depiction of absolute space as a physical substance because it lacked causal powers and independent existence. Space, according to Leibniz, is just a mental concept [2, 3].

1.1.1 Role of Electrodynamics

Maxwell discovered the equations combining electricity, magnetism, and light moving like a wave into a single frame termed electromagnetism in 1864 [4]. These are set of coupled partial differential equations given as

$$\nabla \cdot \mathbf{E} = \rho, \tag{1.1}$$

$$\nabla \times \mathbf{B} - \partial_t \mathbf{E} = \mathbf{j}, \tag{1.2}$$

$$\nabla \cdot \mathbf{B} = 0, \tag{1.3}$$

$$\nabla \times \mathbf{E} - \partial_t \mathbf{B} = 0, \tag{1.4}$$

where ρ is the electric charge density, $\mathbf{j}$ is the current density, $\mathbf{E}$ is the electric field and $\mathbf{B}$ is the magnetic field. It was found that Galilei's laws do not apply to Maxwell equations or the processes they govern. Moreover, taking the curl of equations $(\nabla \times \mathbf{E})$ and $(\nabla \times \mathbf{B})$ one derives the equation for electromagnetic waves with a propagation speed $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$ where μ_0 is the permeability of vacuum and ϵ_0 is the permittivity of a vacuum. One of Maxwell's most surprising implications was the relationship between the constants μ_0, ϵ_0 and the speed of light. Based on the success of Maxwell's theory and the inconsistency of Newtonian mechanics many questions of incompatibility were raised [5]; what exactly is a 'vacuum'? Is it a set of rules that must be followed at all times? How can we develop an electromagnetic theory if we reject the existence of an absolute frame of rest? In frames moving with respect to each other, how do Maxwell's equations appear? Is it necessary to alter the value of c?

1.1.2 The Mach's Principle

By thoroughly studying the classic bucket experiment, in 1872 Ernst Mach came to certain findings, claiming that thought experiments like the bucket argument are problematic. His results are said to be one of the factors that influenced the creation of Einstein's hypothesis. Newton imagined a bucket of water hung on a rope and the rope coils up as one turns the bucket clockwise. Now on relinquishing control the bucket starts spinning counter-clockwise, slowly at first, then more quickly. According to newton the water's surface will gradually retreat from the center and climb up the edge of the vessel, acquiring a concave shape. The bucket and the water spin together for a time. The bucket eventually slows down and its rotation reverses; the water slows down as well, finally smoothing out again. The only way Newton could explain the bucket experiment was to claim that the water was whirling in absolute space. Mach objected to this, he believed it was permissible in a hypothetically empty universe because the matter content in the real world did not support such existence. Absolute space and absolute motion, he maintained, should not be utilized in scientific contexts since they are useless philosophical ideas. Mach came to think that any explanation of motion and inertia, including the motion of water in a spinning bucket, could be understood exclusively in terms of the rest of the universe's matter. He summarized the results as follows [6]

1. The geometry of spacetime is determined by the distribution of matter.
2. If there is no matter then there is no geometry.
3. A body in an otherwise empty universe should posses no inertial properties.

1.1.3 Michelson Morley Experiment

Physicists were used to thinking that waves needed a medium to travel, the possibility of an ether in which electromagnetic waves might travel was proposed. The earth should then travel through this ether, and its absolute velocity through the ether should be detectable.

In 1881, Michelson and Morley designed an experiment in this regard and performed on apparatus known as Michelson interferometer. They reasoned that if the speed of light was constant with regard to the hypothetical ether through which Earth was traveling then its motion might be detected by comparing the speed of light in the direction of Earth's motion along with the speed of light at right angles to Earth's motion. This famous experiment yields a negative result, which refutes the ether theory. Failure of this lead to certain possibilities [6]:

1. The ether is attached rigidly to the earth.
2. It was assumed that rigid bodies contract and clocks slow down when moving through the ether.
3. There is no ether.

1.2 Theory of Relativity

There was no proper explanation for all these problems until a proposal led by Einstein in 1905 was worked out. It severely discredited the ether theories and got rid of all reference frames for space and time. Einstein's view was that space and time are not absolute quantities but they depend on the motion of the observers. He believed that the universe can be visualized as a 4-dimensional continuum with 3 spatial and 1 time coordinates. Einstein's thinking revolved around electrodynamics. He figured it out that Maxwell's condition of electromagnetism required a Special Theory of Relativity [7].

Postulates of Special Theory of Relativity

The two postulates of his theory are:

1. All inertial observers are equivalent.
2. The velocity of light is the same in all inertial systems.

The most significant consequences of this theory, which had been experimentally tested and validated are:

1. **Time Dilation:** The slowing of the passage of time observed by moving objects relative to an observer.

2. **Length Contraction:** Observers perceive the length of a moving item to be shorter than it would be if it were still.

Minkowski later expressed events occurring in the cosmos in a four-dimensional perspective in 1907. In Minkowski's theory, a spacetime event was simply regarded as a point. The Lorentz transformations became the focal point of this new theory of relativity, much as the Galilean transformations were in the classical era. These are essentially linear coordinate transformations that relate two frames traveling at a constant speed to each other. The Minkowski line element is invariant under Lorentzian transformations in flat spacetimes. It is the square of the infinitesimal gap between two events (ct, x) and (ct + cdt, x + dx) that are separated infinitesimally and represented as

$$ds^2 = -c^2dt^2 + dx^2 + dy^2 + dz^2. \tag{1.5}$$

The fact that the Maxwell equations are invariant under these transformations is one of the reasons for their relevance in the special theory of relativity. But this was not the end, after presenting a special theory that described the uniform linear motion of an object restricted to the inertial frame of reference, Einstein started working for the general case which involves the arbitrary motion of an object. Einstein took ten years to include acceleration and generalize the special, or restricted theory from uniform linear motion to the arbitrary motion. This new theory of space, time and gravitation is known as the General Theory of Relativity (GR), published in 1915 [8]. It is a geometric theory which proposes that the spacetime is not a flat structure, but it can be distorted by the presence of massive objects producing curvature in the spacetime. GR replaces the Newtonian gravitational force with the curvature of spacetime. This theory is based on the following premises:

1.2.1 Equivalence Principle

In November 1907, Einstein had a thought that he subsequently described as "the happiest idea of his life". He recognized there had to be a relationship between inertia and gravitation by envisioning a guy standing in an elevator freely falling under gravity. He came to the conclusion that tests carried out in a uniformly accelerated frame of reference are compatible with experiments carried out in a non-accelerated frame of reference placed in a gravitational field. To put it another way, being stationary in a gravitational field and speeding upward are both the same thing. The principle states as [5]:

"*The laws of physics are the same in uniform static gravitational field and in the accelerated frame of reference.*"

1.2.2 Principle of General Covariance

The equivalence of all inertial observers is a premise of special relativity, but in order to deal with gravity, general relativity seeks to incorporate non-inertial observers. All observers, whether inertial or not, should be capable of discovering the rules of physics, according to Einstein. As a logical conclusion to Einstein's principle of special relativity, he offered the following [6]

"All observers are equivalent"

There is a canonical or preferred coordinate system in special relativity when the metric is flat and the connection is integrable; specifically, Minkowski coordinates, but there is none in a curved spacetime that is a manifold with a non-flat metric. This is merely another assertion that a global inertial observer does not exist. As a result, the principle of general covariance acquires its ultimate form as [6]

"In a gravitational field, the universal principles of physics are to be represented in tensorial form"

1.3 Tensors

Physical laws must be independent of any particular coordinates used in describing them mathematically, if they are to be valid. A study of the consequences of this requirement leads to tensors. Tensors are algebraic objects which remain unchanged under coordinate transformations.

Tensors are fundamentally the generalization of vectors. If we have a finite set of vectors spaces $\{V_1, V_2, ..., V_n\}$ over a common field F, one can form tensor product as $V_1 \otimes V_2 ... \otimes V_n$, which is termed a tensor. If there are i copies of V and j copies of $V^\star$ ($V^\star$ is dual space of V) in our tensor product, the tensor is said to be of type (i, j). The space of tensors of type (i, j) is expressed as

$$T^i_j = V_1 \otimes V_2 \otimes ... \otimes V_i \otimes V^\star_{i+1} \otimes V^\star_{i+2} \otimes ... \otimes V^\star_{i+j}. \tag{1.6}$$

A scalar is a zero rank tensor, a vector is a 1st rank tensor and matrix is 2nd rank tensor. Addition and subtraction are only possible for tensors of same rank and gives tensor of same rank. Product of different rank tensors gives tensor whose rank is sum of the ranks of the given tensors.

1.3.1 The Metric Tensor

In differential geometry, metric tensor is a type of function which takes tangent vectors u and v at a point of a surface (or higher dimensional differentiable manifold) as input and produces a scalar

$$g(\mathbf{u}, \mathbf{v}) = \mathbf{u}.\mathbf{v}, \tag{1.7}$$

$$g(\mathbf{u}, \mathbf{v}) = u^i w^j \mathbf{e}_i.\mathbf{e}_j. \tag{1.8}$$

In terms of basis vectors

$$g_{ij}(\mathbf{e}_i, \mathbf{e}_j) = \mathbf{e}_i.\mathbf{e}_j. \tag{1.9}$$

As dot product is commutative so metric tensor is symmetric tensor i.e $g_{ij} = g_{ji}$ and g^{ij} is inverse of g_{ij}.

Metric tensor is also used to find first fundamental form as

$$ds^2 = g_{ij} dx^i dx^j, \tag{1.10}$$

also known as the line element or metric.

1.3.2 The Curvature Tensor

The Riemann curvature tensor can be defined with the help of the Christoffel symbols Γ^i_{jk}.

$$R^i_{jkl} = (\Gamma^i_{jl})_{,k} - (\Gamma^i_{jk})_{,l} + \Gamma^i_{ke}\Gamma^e_{lj} - \Gamma^i_{le}\Gamma^e_{kj}, \tag{1.11}$$

where Christoffel symbols are expressed in terms of metric tensor as

$$\Gamma^i_{jk} = \frac{1}{2} g^{il} \left(g_{kl,j} + g_{jl,k} - g_{jk,l} \right). \tag{1.12}$$

R^i_{jkl} can be transformed into covariant tensor by using the transformation

$$R_{ijkl} = g_{im} R^m_{jkl}, \tag{1.13}$$

and satisfies the Bianchi identity of first and second kind given as

$$R_{i[jkl]} = R_{ijkl} + R_{iljk} + R_{iklj} = 0, \tag{1.14}$$

$$R^i_{m[jk;l]} = R^i_{mjk;l} + R^i_{mlj;k} + R^i_{mkl;j} = 0. \tag{1.15}$$

Here ' ;' is covariant derivative that is the generalization of ∂_a for a curved spacetime. For example the components of covariant derivative of a tensor of rank $(1, 1)$ are defined as

$$A^i_{j;k} = A^i_{j,k} + \Gamma^i_{jl} A^l_k - \Gamma^l_{kj} A^i_l. \tag{1.16}$$

By contracting first and third indices of the Riemann curvature tensor one can construct the Ricci curvature tensor, R_{ij}, and transformation of that gives the Ricci scalar, R, respectively as

$$R_{ij} = R^k_{ikj}, \tag{1.17}$$

$$R = g^{ij} R_{ij}. \tag{1.18}$$

An interesting feature of Ricci scalar is that it determines along with other invariants the nature of a singularity, as it is invariant under coordinate transformations. There are two types of singularities, that are coordinate (arises due to coordinates and are removable) and essential (occurs due to problem in geometry and are non removable) singularities. By examining the invariant quantities given below in eqs. (1.19) to (1.22), one can check the nature of a singularity. If the curvature invariants are finite then there is a coordinate singularity otherwise essential singularity.

$$R_1 = R, \tag{1.19}$$

$$R_2 = R^{ij}_{kl} R^{kl}_{ij}, \tag{1.20}$$

$$R_3 = R^{ij}_{kl} R^{kl}_{mn} R^{mn}_{ij}, \tag{1.21}$$

$$R_4 = R^{ij}_{kl} R^{kl}_{mn} R^{mn}_{op} R^{op}_{ij}. \tag{1.22}$$

1.3.3 The Einstein Tensor

The Einstein tensor can be derived from second kind of the Bianchi identity by putting $i = j$,

$$R^i_{mik;l} + R^i_{mli;k} + R^i_{mkl;i} = 0. \tag{1.23}$$

Substituting $R^i{}_{mik} = R_{mk}$ and $R^i{}_{mli} = R^i{}_{mil} = -R_{ml}$, we have

$$R_{mk;l} - R_{ml;k} + R^i_{mkl;i} = 0, \tag{1.24}$$

multiply by g^{mn} to get

$$R^n_{k;l} + R^n_{l;k} + R^{in}_{lk;i} = 0, \tag{1.25}$$

with some contractions one obtains

$$\left(R^i_k - \frac{1}{2}\delta^i_k R \right)_{;i} = 0, \tag{1.26}$$

or

$$G^i_{k;i} = 0, \tag{1.27}$$

where

$$G^i_k = R^k_i - \frac{1}{2}\delta^i_k R, \tag{1.28}$$

or

$$G_{ij} = R_{ij} - \frac{1}{2}Rg_{ij}, \tag{1.29}$$

is called the Einstein Tensor.

1.3.4 The Maxwell Tensor

Maxwell tensor is also known as elector magnetic field tensor, for its construction, we define the four vector potential, A_i, and skew symmetric tensor, F_{ij}, as

$$A_i = (-\phi, \mathbf{A}), \tag{1.30}$$

$$F_{ij} = \partial_i A_j - \partial_j A_i. \tag{1.31}$$

Here, ϕ is the scalar potential and $\mathbf{A}$ is vector potential. Electric and magnetic fields in terms of ϕ and $\mathbf{A}$ become

$$\mathbf{E} = -\boldsymbol{\nabla}\phi - \partial_t \mathbf{A}, \tag{1.32}$$

$$\mathbf{B} = \boldsymbol{\nabla} \times \mathbf{A}. \tag{1.33}$$

By these equations, we get $F_{0j} = E_j$ and $F_{ij} = \epsilon_{ijk}B^k$ where ϵ_{ijk} is the Levi-Civita tensor defined as

$$\epsilon_{ijk} = \begin{cases} +1 & \text{if } ijk \text{ is an even permutaion} \\ -1 & \text{if } ijk \text{ is an odd permutaion} \\ 0 & \text{otherwise.} \end{cases} \tag{1.34}$$

Therefore the electromagnetic field tensor is given as

$$F_{ij} = \begin{bmatrix} 0 & E_1 & E_2 & E_3 \\ -E_1 & 0 & B^3 & -B^2 \\ -E_2 & -B^3 & 0 & B^1 \\ -E_3 & B^2 & -B^1 & 0 \end{bmatrix} \tag{1.35}$$

1.3.5 The Energy Momentum Tensor

Energy momentum tensor (T^{ab}) has very important role in the Einstein field equations. The energy momentum tensor (EMT) of "dust" can be characterized by density ρ and four velocity, v^i, of fluid in some coordinates, x^i, as

$$T^{ij} = \rho v^i v^j. \tag{1.36}$$

The physical explanation of the components are given as

1. T^{00} represents energy density ρ.
2. T^{i0} is called the momentum density.
3. T^{0i} is called the energy flux.
4. T^{ij} with $i = j \neq 0$, gives force per unit area called pressure.

EMT related to "perfect fluid" can be defined by adding scalar pressure, p, along with energy density, ρ, and flow vector, v^i, as

$$T^{ij} = (\rho_0 + p)v^i v^j - pg^{ij}. \tag{1.37}$$

Notice that the perfect fluid becomes dust if $p \to 0$.
For anisotropic fluid distribution EMT takes the form

$$T^i_j = \left[(\rho + p_t)\, v^i v_j - p_t g^i_j + (p_r - p_t)\, \chi^i \chi_j\right], \tag{1.38}$$

where χ^i is unit space vector in radial direction.
The Maxwell EMT is given as

$$T^i_j = \frac{1}{4\pi}\left(-F^{ik}F_{jk} + \frac{1}{4}g^i_j F^{kl}F_{kl}\right). \tag{1.39}$$

Chapter 2

The Einstein-Maxwell Field Equations

The field equations given by Einstein in 1915, in the form of a tensor equation relate the spacetime curvature with energy, momentum and stress within that spacetime. The electromagnetic fields are related to the distribution of charges and currents via Maxwell's equations. The relationship between the metric tensor and the Einstein tensor allows the Einstein field equations to be written as a set of non-linear partial differential equations. The solutions of the Einstein field equations are the components of the metric tensor.

2.1 Derivation of the Einstein Field Equations

Beginning the derivation of field equations by defining the general form of Einstein-Hilbert action as

$$S = \int L\sqrt{-g}\, d^4x. \tag{2.1}$$

The Lagrangian L for gravitational source and matter is taken as $L = L_G + L_m$ where $L_G = \frac{1}{2k}R$ and $k = \frac{8\pi G}{c^4}$. Throughout this discussion the speed of light c and gravitational constant G are taken to be 1, so action becomes

$$S = \int \frac{1}{2k}R\sqrt{-g}\, d^4x + \int L_m\sqrt{-g}\, d^4x. \tag{2.2}$$

Since $\delta S = 0$, using this and eq (1.18) in (2.2) gives

$$\delta S = \frac{1}{2k}\int \left(R_{ij}g^{ij}\delta\sqrt{-g} + R_{ij}\sqrt{-g}\delta g^{ij} + \sqrt{-g}g^{ij}(\delta R_{ij})\right) d^4x$$
$$+ \int \left(L_m\delta\sqrt{-g} + \sqrt{-g}\delta(L_m)\right) d^4x = 0. \tag{2.3}$$

Now considering geodesic coordinates where $\Gamma^i_{jk} = 0$ at an arbitrary point P. This then reduces Riemann tensor as

$$R^k_{ilj} = \Gamma^k_{ij,l} - \Gamma^k_{il,j}. \tag{2.4}$$

or

$$\delta R^k_{ilj} = \delta\Gamma^k_{ij,l} - \delta\Gamma^k_{il,j}. \tag{2.5}$$

Since in geodesic coordinates the partial derivative is equivalent to covariant derivative and commutes with variation. So famous Palatini equation is obtained given as

$$\delta R^k_{ilj} = \delta\Gamma^k_{ij;l} - \delta\Gamma^k_{il;j}. \tag{2.6}$$

Contraction of k and l gives

$$\delta R_{ij} = \delta\Gamma^k_{ij;k} - \delta\Gamma^k_{ik;j}, \tag{2.7}$$

or

$$g^{ij}\delta R_{ij} = g^{ij}\delta\Gamma^k_{ij;k} - g^{ij}\delta\Gamma^k_{ik;j}, \tag{2.8}$$
$$= g^{ij}\delta\Gamma^k_{ij;k} - g^{ik}\delta\Gamma^j_{ij;k}, \tag{2.9}$$
$$= \left(g^{ij}\delta\Gamma^k_{ij} - g^{ik}\delta\Gamma^j_{ij}\right)_{;k}, \tag{2.10}$$

or

$$\int_v g^{ij}\delta R_{ij}\sqrt{-g}\, d^4x = \int_v A^k{}_{;k}\sqrt{-g}\, d^4x, \tag{2.11}$$

where

$$A^k = g^{ij}\delta\Gamma^k_{ij} - g^{ij}\delta\Gamma^j_{ij}. \tag{2.12}$$

Using divergence theorem eq (2.11) yields

$$\int_v g^{ij}\delta R_{ij}\sqrt{-g}\, d^4x = 0. \tag{2.13}$$

By eq (2.13) and substitution of the identity $\delta\sqrt{-g} = \frac{-1}{2}\sqrt{-g}g_{ij}\delta g^{ij}$, eq (2.3) becomes

$$\begin{aligned}\frac{1}{2k}\int_v \left(R_{ij}g^{ij}\left(\frac{-1}{2}\sqrt{-g}\, g_{ij}\delta g^{ij}\right) + R_{ij}\sqrt{-g}\,\delta g^{ij}\right) d^4x \\ + \int_v \left(L_m\left(\frac{-1}{2}\sqrt{-g}\, g_{ij}\delta g^{ij}\right) + \sqrt{-g}\,\delta(L_m)\right) d^4x = 0.\end{aligned} \tag{2.14}$$

As $L_m = L_m(g^{ij})$ this implies $\delta L_m = \frac{\partial L_m}{\partial g^{ij}}\delta g^{ij}$, thus eq (2.14) becomes

$$\begin{aligned}\frac{1}{2k}\int_v \left(\frac{-1}{2}Rg_{ij}\delta g^{ij} + R_{ij}\delta g^{ij}\right)\sqrt{-g}\, d^4x \\ -\frac{1}{2}\int_v \left(-2\frac{\partial L_m}{\partial g^{ij}} + L_m g_{ij}\right)\delta g^{ij}\sqrt{-g}\, d^4x = 0.\end{aligned} \tag{2.15}$$

Energy momentum tensor in terms of Lagrangian is defined as

$$T_{ij} = -2\frac{\partial L_m}{\partial g^{ij}} + L_m g^{ij}. \tag{2.16}$$

Thus eq (2.15) takes the form

$$\frac{1}{2k}\int_v \left(R_{ij} - \frac{1}{2}g_{ij}R - kT_{ij}\right)\delta g^{ij}\sqrt{-g}\, d^4x = 0, \tag{2.17}$$

which implies

$$R_{ij} - \frac{1}{2}g_{ij}R = kT_{ij}. \tag{2.18}$$

Eq (2.18) are the Einstein filed equations(EFEs) on the left we have curvature that determines the presence of gravitational source and on the right energy momentum tensor that represents the matter contents.

2.2 The Maxwell Equations

Now the main goal is to observe electromagnetism in the context of relativity. For this eqs. (1.1) to (1.4) need to be transformed into tensor notation by the electromagnetic field tensor given by eq (1.35) and the four vector J^i defined as $J^i = (\rho, \mathbf{J})$. Thus the set of traditional Maxwell equations in tensor form are reduced to two equations as follows

$$\partial_j F^{ij} = J^i, \tag{2.19}$$

$$\partial_{[j} F_{ik]} = 0. \tag{2.20}$$

Source eq (2.19) is valid in Minkowski space (inertial coordinates), one can express it in coordinate invariant (arbitrary coordinates) way as

$$\nabla_j F^{ij} = J^i, \tag{2.21}$$

where as the internal eq (2.20) subject to continuity equation is satisfied automatically as there is no change in $\nabla_{[j} F_{ik]} = \partial_{[j} F_{ik]} = 0$. If these covariant tensor equations are valid in one coordinate system then they are valid for all.

2.3 Exact Solutions of the Field Equations

Consider the most general static spherically symmetric metric as

$$ds^2 = e^{\nu(r)} c^2 dt^2 - e^{\lambda(r)} dr^2 - r^2(d\theta^2 + sin^2\theta d\phi^2). \tag{2.22}$$

So the metric tensor and its inverse are

$$\begin{aligned} g_{ij} &= \left(e^{\nu(r)}, -e^{\lambda(r)}, -r^2, -r^2 sin^2\theta\right), \\ g^{ij} &= \left(e^{-\nu(r)}, -e^{-\lambda(r)}, -\frac{1}{r^2}, -\frac{1}{r^2 sin^2\theta}\right). \end{aligned} \tag{2.23}$$

The independent, non zero components of the Christoffel symbols are

$$\begin{aligned} &\Gamma^1_{00} = \nu' e^{(\nu-\lambda)}/2, && \Gamma^1_{22} = -re^{-\lambda}, && \Gamma^1_{11} = \lambda'/2, \\ &\Gamma^2_{21} = \Gamma^3_{31} = \frac{1}{r}, && \Gamma^1_{33} = -re^{-\lambda} sin^2\theta, && \Gamma^2_{33} = -sin\theta cos\theta, \\ &\Gamma^3_{32} = cot\theta, && \Gamma^0_{01} = \nu'/2. \end{aligned} \tag{2.24}$$

So the non-vanishing components of the Ricci curvature tensor are

$$R_{00} = e^{\nu-\lambda}\left(\frac{\nu''}{2} + \frac{\nu'}{4}(\nu' - \lambda') + \frac{\nu'}{r}\right), \tag{2.25}$$

$$R_{11} = -\frac{\nu''}{2} + \frac{1}{4}\nu'(\lambda' - \nu') + \frac{\lambda'}{r}, \tag{2.26}$$

$$R_{22} = 1 - e^{-\lambda} + \frac{1}{2}re^{-\lambda}(\lambda' - \nu'), \tag{2.27}$$

$$R_{33} = R_{22}sin^2\theta, \tag{2.28}$$

and the Ricci scalar is

$$R = e^{-\lambda}\left(\nu'' + \frac{\nu'}{2}(\nu' - \lambda') + \frac{2}{r}(\nu' - \lambda') + \frac{2}{r^2}\right) - \frac{2}{r^2}. \tag{2.29}$$

2.3.1 The Schwarzschild Solution

The first exact solution of the Einstein field equations is the Schwarzschild solution. Schwarzchild found the metric that represents static, spherically symmetric gravitational field in the empty space. By considering vacuum, the Einstein field equations become

$$R_{ij} = 0. \tag{2.30}$$

So by eqs. (2.25) to (2.27), the EFEs becomes

$$\frac{\nu''}{2} + \frac{\nu'}{4}(\nu' - \lambda') + \frac{\nu'}{r} = 0, \tag{2.31}$$

$$-\frac{\nu''}{2} + \frac{1}{4}\nu'(\lambda' - \nu') + \frac{\lambda'}{r} = 0, \tag{2.32}$$

$$1 - e^{-\lambda} + \frac{1}{2}re^{-\lambda}(\lambda' - \nu') = 0. \tag{2.33}$$

Simplifying eqs. (2.31) and (2.32), we get

$$\nu = -\lambda, \tag{2.34}$$

substituting it in eq (2.33), we have

$$(re^{-\lambda})' = 1. \tag{2.35}$$

This implies

$$e^{\nu} = e^{-\lambda} = \left(1 + \frac{\alpha}{r}\right), \tag{2.36}$$

where $\alpha = \frac{-2Gm}{c^2}$. Thus metric in eq (2.22) takes the form

$$ds^2 = \left(1 - \frac{2Gm}{c^2 r}\right) dt^2 - \left(1 - \frac{2Gm}{c^2 r}\right)^{-1} dr^2 - r^2(d\theta^2 + sin^2\theta d\phi^2), \tag{2.37}$$

which is the Schwarzschild metric. There exist two singularities in the metric, at $r = 0$ and $r = 2Gm/c^2$, which are essential and coordinate singularities respectively, the later called the event horizon. In gravitational units, $G = c = 1$, the Schwarzchild metric takes the form

$$ds^2 = \left(1 - \frac{2m}{r}\right) dt^2 - \left(1 - \frac{2m}{r}\right)^{-1} dr^2 - r^2(d\theta^2 + sin^2\theta d\phi^2). \tag{2.38}$$

2.3.2 The Reissner-Nordstrom Solution

The analogue of the Schwarzschild solution having charged point mass was found by Reissner [9] in 1916 and by Nordstrom [10] in 1918 independently, hence the solution is known as Reissner-Nordstrom solution. Therefore, adding charge into the previous assumptions the Einstein-Maxwell field equation become

$$R_{ij} = 8\pi T_{ij}. \tag{2.39}$$

Here due to spherical symmetry and static point charge the components of electrostatic field are in the radial direction i.e. $E = E(r)$ with magnetic field equal to zero. So the Maxwell tensor F_{ij} is

$$F_{ij} = \begin{bmatrix} 0 & -E(r) & 0 & 0 \\ E(r) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{2.40}$$

Components of the Maxwell tensor, eq (1.39) become

$$T_{ij} = \frac{1}{8\pi}(E^2, E^2, -E^2, -E^2). \tag{2.41}$$

So by eqs. (2.31) to (2.33), the EMFEs (2.39) become

$$e^{\nu-\lambda}\left(\frac{\nu''}{2}+\frac{\nu'}{4}(\nu'-\lambda')+\frac{\nu'}{r}\right)=E^2, \tag{2.42}$$

$$-\frac{\nu''}{2}+\frac{1}{4}\nu'(\lambda'-\nu')+\frac{\lambda'}{r}=E^2, \tag{2.43}$$

$$1-e^{-\lambda}+\frac{1}{2}re^{-\lambda}(\lambda'-\nu')=-E^2. \tag{2.44}$$

The Maxwell equations $\nabla_j F^{ij}=F^{ij}_{;j}=0$ reduce to

$$(e^{-(\nu+\lambda)/2}r^2E)'=0, \tag{2.45}$$

which leads to

$$E(r)=\frac{Qe^{(\nu+\lambda)/2}}{r^2}. \tag{2.46}$$

By solving eqs. (2.42) to (2.44) with (2.46), we get

$$\nu=-\lambda, \tag{2.47}$$

$$(re^{-\lambda})'=1-\frac{Q^2}{r^2}. \tag{2.48}$$

By integration, we get

$$e^{\nu}=e^{-\lambda}=\left(1+\frac{constant}{r}+\frac{Q^2}{r^2}\right), \tag{2.49}$$

for $Q=0$ the solution reduces to the Schwarzschild solution which implies $constant=-2m$, so,

$$e^{\nu}=e^{-\lambda}=\left(1-\frac{2m}{r}+\frac{Q^2}{r^2}\right). \tag{2.50}$$

Therefore, the Reissner-Nordstrom metric is

$$ds^2=\left(1-\frac{2m}{r}+\frac{Q^2}{r^2}\right)dt^2-\left(1-\frac{2m}{r}+\frac{Q^2}{r^2}\right)^{-1}dr^2-r^2(d\theta^2+sin^2\theta d\phi^2). \tag{2.51}$$

Here again at $r=0$, we have essential singularity. For finding other singularities take

$$1-\frac{2m}{r}+\frac{Q^2}{r^2}=0, \tag{2.52}$$

or

$$r_{\pm}=m\pm\sqrt{m^2-Q^2}. \tag{2.53}$$

Here $r=r_{\pm}$ are coordinate singularities, r_+ and r_- are called outer and inner horizons respectively.

2.3.3 The Kerr Solution

In 1963, Roy Kerr discovered a solution of rotating black holes. He generalized the Schwarzchild solution by assuming a rotating point mass in vacuum with static space time. The line element in Boyer Lindquist form is

$$ds^2 = \frac{\Delta}{\rho^2}\left(dt - asin^2\theta d\phi\right)^2 - \frac{sin^2\theta}{\rho^2}\left(\left(r^2+a^2\right)d\phi - adt\right)^2 - \frac{\rho^2}{\Delta}dr^2 - \rho^2 d\theta^2, \quad (2.54)$$

where

$$\Delta^2 = r^2 - 2Mr + a^2, \quad \rho^2 = r^2 + a^2cos^2\theta. \quad (2.55)$$

This is the Kerr black hole solution. The inner and outer event horizons are

$$r_+ = M + \sqrt{M^2 - a^2}, \quad r_- = M - \sqrt{M^2 - a^2}. \quad (2.56)$$

2.3.4 The Kerr–Newman Solution

The Kerr–Newman solution is a generalization of the Reissner-Nordstrom solution by assuming a charged rotating point mass in vacuum with static space time. The line element in Boyer-Linquist coordinates is given as

$$ds^2 = \frac{\Delta}{\rho^2}\left(dt - asin^2\theta d\phi\right)^2 - \frac{sin^2\theta}{\rho^2}\left(\left(r^2+a^2\right)d\phi - adt\right)^2 - \frac{\rho^2}{\Delta}dr^2 - \rho^2 d\theta^2, \quad (2.57)$$

where

$$\Delta^2 = r^2 - 2Mr + a^2 + Q^2, \quad \rho^2 = r^2 + a^2cos^2\theta. \quad (2.58)$$

This is the Kerr–Newman black hole solution. The inner and outer event horizons are

$$r_+ = M + \sqrt{M^2 - a^2 - Q^2}, \quad r_- = M - \sqrt{M^2 - a^2 - Q^2}. \quad (2.59)$$

Chapter 3

Models of Compact Objects with Karmarkar Condition

Laplace introduced theoretical concept of the existence of a massive object from whose gravitational field no particle even light could not escape, but it did not receive much attention due to some strange properties. Astronomers believe that stars are formed from massive clouds of dust and gases in space. Gravity pulls dust and gases together to form a prostar. As gasses come together, they get hot. Finally the star remains in this state for several thousands of years, until the nuclear fusion process ignites crushing hydrogen atoms into helium then carbon until the formation of iron. Fusion process that creates iron dose not generate any energy. Due to nuclear fusion process if the gravitational force pressing inwards is greater than the outwards push of internal pressure, the core collapses under the dominant gravitational field, at this point compact objects are formed.
The study of compact objects can be categorized as

1. **White Dwarfs**: If the size (volume) of the star is comparable to earth and mass is comparable to sun then star will turn into a white dwarf. It is one of the densest forms of matter present with this density. Just three white dwarfs were discovered until 1926, they are held up against gravity not by heat but by electrons repelling each other. Chandrasekhar later developed the degenerate electron equation of state in 1930, taking into account special relativistic results.

The amount of maximum mass they can hold was to be $1.4M_{\odot}$, where $M_{\odot}$ is the solar mass [11].

2. **Neutron Stars**: When Chandrasekhar limit is reached massive stars will eventually have smaller radius and at this time protons combine with electrons forming neutrons and neutrinos, where neutrinos fly away resulting in neutron stars. Supernova remnants, solitary objects, and binary systems may all include neutron stars. Stars with radius about 10km and the maximum mass estimated to be is $3M_{\odot}$, massive stars could not resist the gravitational pull and continue to collapse, and Neutron star is formed. They were discovered as radio pulsars at the end of the 1960s and as X-ray stars at the start of the 1970s, also planets have been discovered in one neutron star [11].

3. **Black Holes**: As fluid in blackholes are the densest material so in the massive stars ($M > 3M_{\odot}$) entire mass of core collapses into a black hole. These are so dense objects with extremely strong gravity that even light cannot escape through. The most important feature in a black hole is event horizon, defined as "*a hypersurface separating those spacetime points that are connected to infinity by a timelike path from those that are not*" [12]. If something crosses this it falls into the black hole singularity (it is infinitely small and dense where laws of physics do not apply). The outside observer dose not get effected by events happening inside an event horizon. Depending on the mass distribution black holes can be defined as

   • Stellar black holes: These black holes are smaller in size and to grow in size they consume gases and dust present around them. These are the most common black holes, according to scientists millions of them can be present only in the Milky Way galaxy, and the mass range lies between $10^{\frac{1}{2}}$ to $10^{2}M_{\odot}$.
   • Intermediate black holes: Presence of this medium size black hole is still debatable astronomers believe that these are formed by collision of cluster of stars in a chain reaction. Usually there mass ranges between 10^{3} to $10^{5}M_{\odot}$.
   • Supermassive black holes: These black holes are located at the heart of each

galaxy and are formed by the merger of hundred thousands of stellar and intermediate black holes. The known supermassive black hole is $S50014+81$ which has 40 billions times mass of sun and its diameter is 236.7 billion km. The range of such black holes is 10^6 to $10^9 M_\odot$.

The internal composition of compact objects is unknown. In order to study and understand thermodynamical and gravitational behavior of compact objects, one generally develops their possible analytical models. These models are solutions of the Einstein field equations (EFEs)/Einstein-Maxwell field equations (EMFEs) for uncharged/charged objects respectivily. Many solutions of the EFEs/EMFEs for compact objects have been obtained by several authors taking different assumptions on the parameters involved. There are some necessary conditions for acceptable model of compact objects.

3.1 Admissible Conditions for Compact Objects

For any compact object to be physically acceptable the following conditions should be fulfilled:

1. There should be no singularity in metric potentials within the radius of the object.
2. The electric field and anisotropic factor should be increasing as moving towards the boundary and must be zero at the center.
3. ρ, p_r and p_t must be positive, monotonically decreasing with radial coordinate and finite inside the compact object.
4. The pressures p_r and p_t must be equal at the center of the compact object and at the boundary of the compact object p_r must be zero.
5. The trace of the energy-momentum tensor must be positive and decreasing.
6. The value of adiabatic-index must be larger than 4/3.
7. Energy, causality and hydrostatic equilibrium conditions should be satisfied.

3.2 The Karmarkar Condition for Class-I Space-time

Eddington [13] discussed that four-dimensional curved space-times can be embedded in higher-dimensional flat space-times. Randall and Sundram [14], Anchordoqui and Bergila [15] also discussed in details that the m-dimensional manifold, V_m, can be embedded in Pseudo-Euclidean space of dimension atleast n, where $n = m(m+1)/2$. The minimum extra dimensions $(n-m)$ is called the embedding class of V_m.

The metric given in eq (2.22) can represent Class-I space-time if it satisfies the Karmarkar condition given as

$$R_{2323}R_{1010} = R_{1212}R_{3030} + R_{1220}R_{1330}. \tag{3.1}$$

The non-zero components of Riemann curvature tensor for the metric (2.22) are

$$R_{1010} = -e^{\nu}\left(\frac{\nu''}{2} + \frac{\nu'^2}{4} - \frac{\lambda'\nu'}{4}\right), \tag{3.2}$$

$$R_{2323} = -e^{-\lambda}r^2 sin^2\theta(e^{\lambda} - 1), \tag{3.3}$$

$$R_{3030} = R_{2020}sin^2\theta = -\frac{1}{2}r sin^2\theta \nu' e^{\nu-\lambda}, \tag{3.4}$$

$$R_{1313} = R_{1212}sin^2\theta = \frac{1}{2}r\lambda' sin^2\theta. \tag{3.5}$$

So the Karmarkar condition for metric (2.22) can be written as

$$\frac{\nu''}{\nu'} = \frac{1}{2}\left[\frac{\lambda' e^{\lambda}}{e^{\lambda} - 1} - \nu'\right]. \tag{3.6}$$

Integrating eq (3.6), we obtain ν in terms of λ as

$$e^{\nu} = \left(B\int\sqrt{e^{\lambda} - 1}\,dr + A\right)^2, \tag{3.7}$$

where A and B are constants of integration.

3.3 Review of Some Solutions of the Field Equations with Karmarkar Condition

The Friedman-Lemaitre-Robertson-Walker metric and the Schwarzschild metric are of Class-I [16], whereas the Kerr spacetimes is of Class-V [17]. We are considering the metric of embedding Class-I spacetimes. Several authors have already obtained embedding Class-I solutions and used them as models of compact stars, some models are presented by Kumar and Gupta [18], Maurya et al [19] and K.N. Singh [20]. In the following subsections we will be discussing some known solutions presented the D. M. Pandya and V. O. Thomas [21], they presented an exact solution of the Einstein field equations that is static, spherically symmetric on paraboloidal spacetime with uncharged anisotropic distribution by using the Karmarkar Condition. And, Piyali Bhar, Ksh. Newton Singh and Farook Rahaman [22] analyzed solutions of the Einstein-Maxwell field equations using Karmarkar condition in a spherically symmetric space time with charged anisotropic distribution.

3.3.1 Models Of Compact Stars on Paraboloidal Spacetime Satisfying Karmarkar Condition

Consider the spacetime metric given in eq (2.22) with

$$e^{\lambda} = 1 + \frac{r^2}{a^2}, \tag{3.8}$$

where a is a geometric parameter, this shows the paraboloidal spacetimes. The components of energy momentum tensor (1.38) for the distribution in the presence of anisotropy takes the form

$$T_{ij} = (\rho, -p_r, -p_t, -p_t). \tag{3.9}$$

Thus the Einstein field equations (2.18) become

$$\rho = \frac{1}{8\pi}\left[\left(\lambda' - \frac{1}{r}\right)\frac{e^{-\lambda}}{r} + \frac{1}{r^2}\right], \tag{3.10}$$

$$p_r = \frac{1}{8\pi}\left[\left(\nu' + \frac{1}{r}\right)\frac{e^{-\lambda}}{r} - \frac{1}{r^2}\right], \tag{3.11}$$

$$p_t = \frac{1}{8\pi}\left[\left(\nu'' + \frac{\nu' - \lambda'}{r} + \frac{\nu'}{2}(\nu' - \lambda')\right)\frac{e^{-\lambda}}{2}\right]. \tag{3.12}$$

From eqs (3.7) and (3.8), we get

$$e^{\nu} = \left(A + B\frac{r^2}{a^2}\right)^2. \tag{3.13}$$

So eqs (3.10) to (3.12) and the anistropic factor $\Delta (= p_t - p_r)$ become

$$\rho = \frac{1}{8\pi}\left[\frac{3 + \frac{r^2}{a^2}}{a^2\left(1 + \frac{r^2}{a^2}\right)^2}\right], \tag{3.14}$$

$$p_r = \frac{1}{8\pi}\left[\frac{B\left(4 - \frac{r^2}{a^2}\right) - A}{a^2\left(A + B\frac{r^2}{a^2}\right)\left(1 + \frac{r^2}{a^2}\right)}\right], \tag{3.15}$$

$$p_t = \frac{1}{8\pi}\left[\frac{B\left(4 + \frac{r^2}{a^2}\right) - A}{a^2\left(A + B\frac{r^2}{a^2}\right)\left(1 + \frac{r^2}{a^2}\right)^2}\right], \tag{3.16}$$

$$\Delta = \frac{1}{8\pi}\left[\frac{\frac{r^2}{a^2}\left[A - B\left(2 - \frac{r^2}{a^2}\right)\right]}{a^2\left(A + B\frac{r^2}{a^2}\right)\left(1 + \frac{r^2}{a^2}\right)^2}\right]. \tag{3.17}$$

The spacetime metric (2.22) becomes

$$ds^2 = \left(A + B\frac{r^2}{a^2}\right)^2 dt^2 - \left(1 + \frac{r^2}{a^2}\right) dr^2 - r^2(d\theta^2 + sin^2\theta d\phi^2), \tag{3.18}$$

which should match the Schwarzschild metric (2.38) at the boundry $r = R$, to gives

$$e^{\nu_b} = \left(1 - \frac{2M}{R}\right) = \left(A + B\frac{R^2}{a^2}\right)^2, \tag{3.19}$$

$$e^{-\lambda_b} = \left(1 - \frac{2M}{R}\right) = (1 + \frac{R^2}{a^2})^{-1}. \tag{3.20}$$

Using the boundary condition $p_r(R) = 0$, we get

$$-A + B\left(4 - \frac{R^2}{a^2}\right) = 0. \tag{3.21}$$

From eqs (3.19) to (3.21), we obtain

$$a = R\sqrt{\frac{R}{2M} - 1}, \tag{3.22}$$

$$A = \frac{4 - \frac{R^2}{a^2}}{4\sqrt{1 + \frac{R^2}{a^2}}}, \tag{3.23}$$

$$B = \frac{1}{4\sqrt{1 + \frac{R^2}{a^2}}}. \tag{3.24}$$

By using eqs (3.23) and (3.24), we rewrite eqs (3.7) to (3.8) as

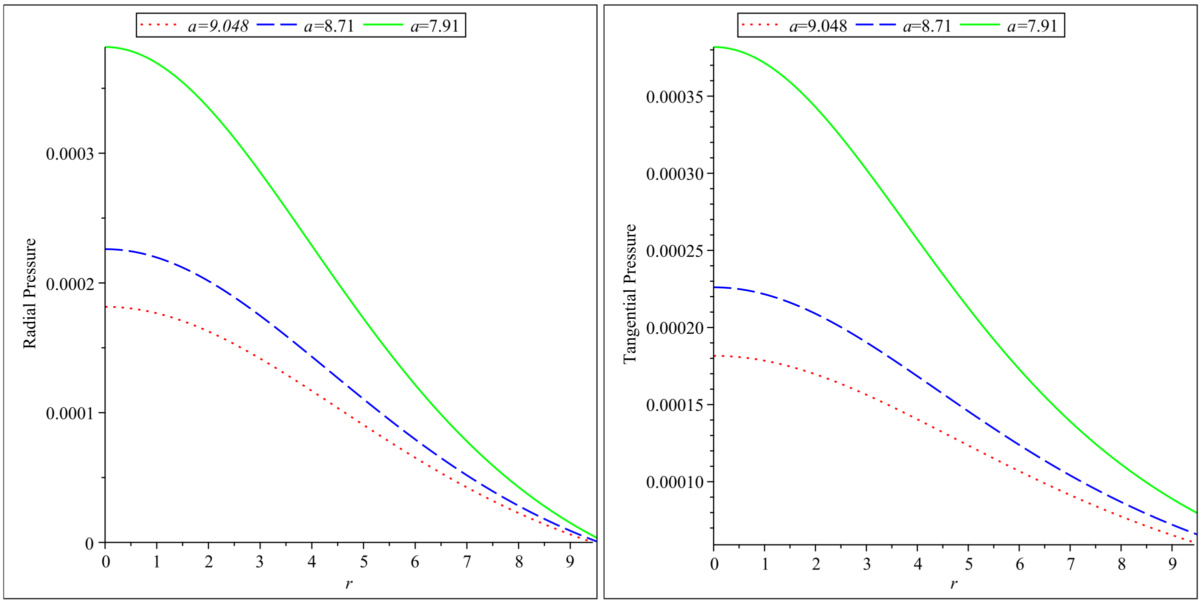
$$\rho = \frac{1}{8\pi}\left[\frac{3 + \frac{r^2}{a^2}}{a^2\left(1 + \frac{r^2}{a^2}\right)^2}\right], \tag{3.25}$$

$$p_r = \frac{1}{8\pi}\left[\frac{\frac{R^2}{a^2} - \frac{r^2}{a^2}}{a^2\left(4 + \frac{r^2}{a^2} - \frac{R^2}{a^2}\right)\left(1 + \frac{r^2}{a^2}\right)}\right], \tag{3.26}$$

$$p_t = \frac{1}{8\pi}\left[\frac{\frac{R^2}{a^2} + \frac{r^2}{a^2}}{a^2\left(4 + \frac{r^2}{a^2} - \frac{R^2}{a^2}\right)\left(1 + \frac{r^2}{a^2}\right)^2}\right], \tag{3.27}$$

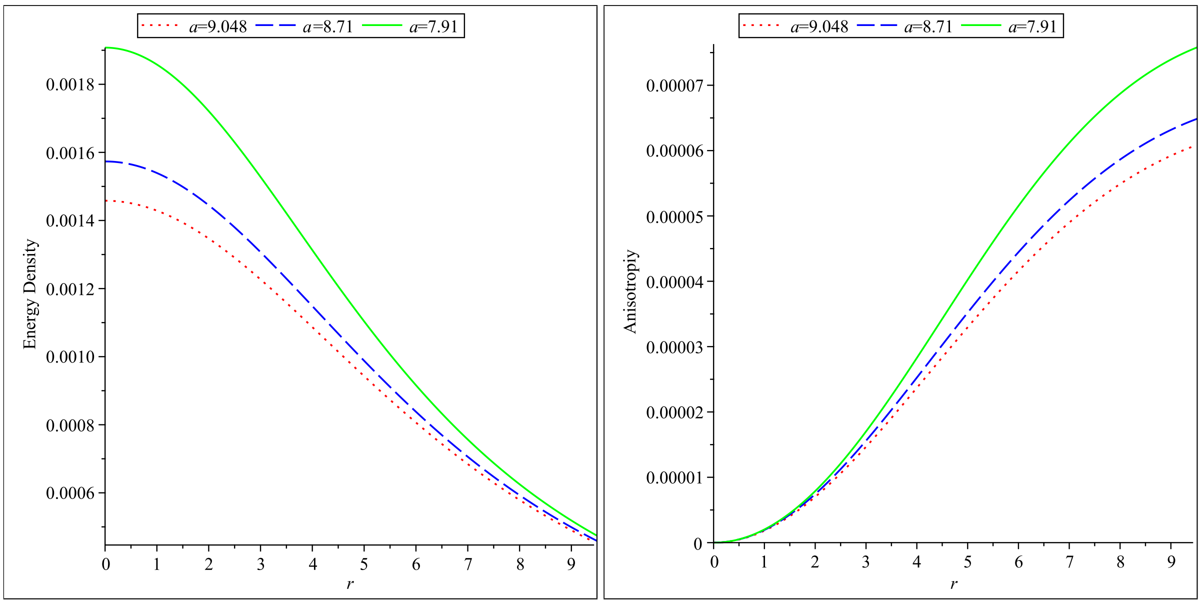
$$\Delta = \frac{1}{8\pi}\left[\frac{\frac{r^2}{a^2}\left(2 + \frac{r^2}{a^2} - \frac{R^2}{a^2}\right)}{a^2\left(4 + \frac{r^2}{a^2} - \frac{R^2}{a^2}\right)\left(1 + \frac{r^2}{a^2}\right)^2}\right]. \tag{3.28}$$

To examine the validity of the model with observational data, we have considered compact stars PSR J1903+327, Vela X-1 and PSR J1614-2230, whose mass and size are known. By using this observational data, the value of the geometric parameter a is found. The variation of radial pressure, p_r, and tangential pressure, p_t, are shown in Fig 3.1, both p_r and p_t decrease radially outward. Fig 3.2 shows the graph of density and anisotropy, density decreases with increase in radius and anisotropy is zero at the centre and positive otherwise.



(a) p_r (b) p_t

Figure 3.1: Graphs of pressures are plotted for compact stars PSR J1903+327, Vela X-1 and PSR J1614-2230.



(a) ρ (b) Δ

Figure 3.2: Graphs of energy density and anisotropy show that both are well defined and non-negative.

Discussion

In [21], authors have studied validity of the model developed by using the Karmakar condition in paraboloidal spacetime for compact stars Her X-1, LMC X-4, EXO 1785-248, PSR J1903+327, Vela X-1 and PSR J1614-2230. The model satisfies the admissible conditions for compact objects. It has been concluded by calculations that the energy density, radial and tangential pressures are within the limit, positive at the center and monotonically decreasing. However, the physical parameters anisotropy and compactness factors are increasing as moving toward boundary. Furthermore, the model satisfies all the energy conditions, which are necessary. The stability factor and adiabatic index fall within the limits.

3.3.2 A Charged Anisotropic Well-behaved Adler-Finch-Skea Solution Satisfying the Karmarkar Condition

Consider the spacetime metric given in eq (2.22) with

$$e^{\nu} = A(1+Br^2)^2, \tag{3.29}$$

where A and B are constants. By assuming that the matter within the star is charged and anisotropic in nature, the components of energy momentum tensor eqs. (1.38) and (1.39) take the form

$$T_{ij} = (\rho + E^2, -p_r + E^2, -p_t - E^2, -p_t - E^2). \tag{3.30}$$

Thus the Einstein field equations (2.18) become

$$8\pi\rho + E^2 = \left(\lambda' - \frac{1}{r}\right)\frac{e^{-\lambda}}{r} + \frac{1}{r^2}, \tag{3.31}$$

$$8\pi p_r - E^2 = \left(\nu' + \frac{1}{r}\right)\frac{e^{-\lambda}}{r} - \frac{1}{r^2}, \tag{3.32}$$

$$8\pi p_t + E^2 = \left(\nu'' + \frac{\nu' - \lambda'}{r} + \frac{\nu'}{2}(\nu' - \lambda')\right)\frac{e^{-\lambda}}{2}. \tag{3.33}$$

From eqs (3.6) and (3.29), we get

$$e^{\lambda} = 1 + 16AB^2Cr^2, \tag{3.34}$$

where C is a constant. Assume the electric field intensity for model as

$$E^2 = \frac{KBr^2}{1+Br^2}, \tag{3.35}$$

where K is positive constant.

Using eqs (3.29), (3.35) and (3.34), in eqs (3.31) and (3.33), we have

$$\rho = \frac{1}{8\pi}\left[\frac{16AB^2C(3+16AB^2Cr^2)}{(1+16AB^2Cr^2)^2} - \frac{KBr^2}{1+Br^2}\right], \tag{3.36}$$

$$p_r = \frac{1}{8\pi}\left[\frac{4B+KBr^2-16AB^2C\{1+Br^2(1-Kr^2)\}}{(1+Br^2)(1+16AB^2Cr^2)}\right], \tag{3.37}$$

$$p_t = \frac{1}{8\pi}\left[\frac{4B-KBr^2-256A^2B^5C^2Kr^6-16AB^2C\{1-Br^2(1-2Kr^2)\}}{(1+Br^2)(1+16AB^2Cr^2)^2}\right], \tag{3.38}$$

and the anistropic factor $\Delta(=p_t-p_r)$ becomes

$$\Delta = \frac{1}{8\pi}\left[\frac{2Br^2[-K-16AB^2C\{1+2Kr^2-8ABC(1+Br^2-2KBr^4)\}]}{(1+Br^2)(1+16AB^2Cr^2)^2}\right]. \tag{3.39}$$

The spacetime metric (2.22) becomes

$$ds^2 = A(1+Br^2)^2dt^2 - (1+16AB^2Cr^2)dr^2 - r^2(d\theta^2+sin^2\theta d\phi^2), \tag{3.40}$$

which should match the Reissner-Nordstrom metric (2.51) at the boundry, $r=R$, to give

$$e^{\nu_b} = \left(1-\frac{2M}{R}+\frac{Q^2}{R^2}\right) = A(1+BR^2)^2, \tag{3.41}$$

$$e^{-\lambda_b} = \left(1-\frac{2M}{R}+\frac{Q^2}{R^2}\right) = (1+16AB^2CR^2)^{-1}. \tag{3.42}$$

Now by using the boundary condition $p_r(R)=0$ and $Q(=Er^2)$ with (3.41) and (3.42), we get

$$B = \frac{-R^2\sqrt{K^2R^6 + 4KR^4 - 16MR + 4R^2 + 16M^2} - KR^5 + 6MR^2 - 2R^3}{2(3KR^7 - 5MR^4 + 2R^5)}, \tag{3.43}$$

$$A = \frac{BKR^5 - 2BMR^2 + BR^3 + R - 2M}{R(BR^2 + 1)^2}, \tag{3.44}$$

$$C = \frac{KR^2 + 4}{16AB(1 - BKR^4 + BR^2)}. \tag{3.45}$$

The metric potentials are regular inside the compact object. Notice that $e^{\lambda}(r = 0) = 1$ and $e^{\nu}(r = 0) = A$, a positive constant. Graph of metric potentials are plotted in Fig 3.3 (left). The metric potentials are monotonically increasing. The density, radial and transverse pressures are positive and monotonically decreasing. The radial pressure vanishes at the boundary of the compact object $p_r(R) = 0$. The graphs of density, radial and tangential pressures are shown in Fig 3.3 (right) and 3.4 (left) respectively. The graph of the anisotropic factor is shown in Fig 3.4 (right). The anisotropic factor vanishes at the center of the compact object, moreover, the anisotropic factor is negative from center till $r = 5.47$km and positive for $r > 5.47$km upto boundry.

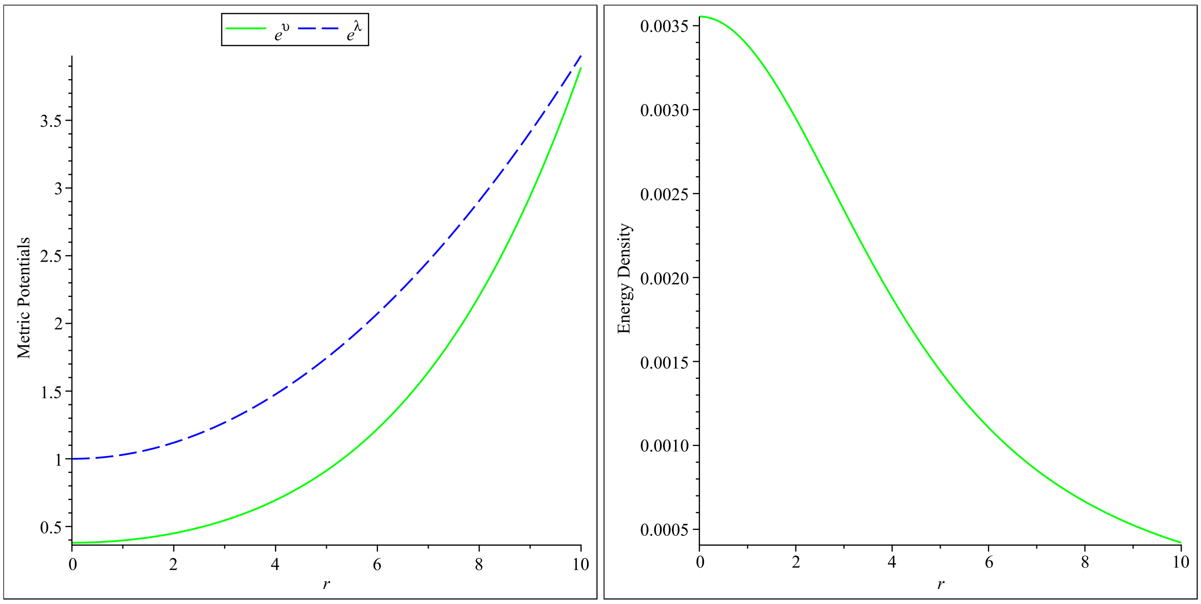


Figure 3.3: The graphs of metric potentials plotted by taking $A = 0.38$, $B = 0.022$, $C = 10.12$ and $K = 0.001$. Variation of density is plotted by taking the same values of the constant mentioned earlier.

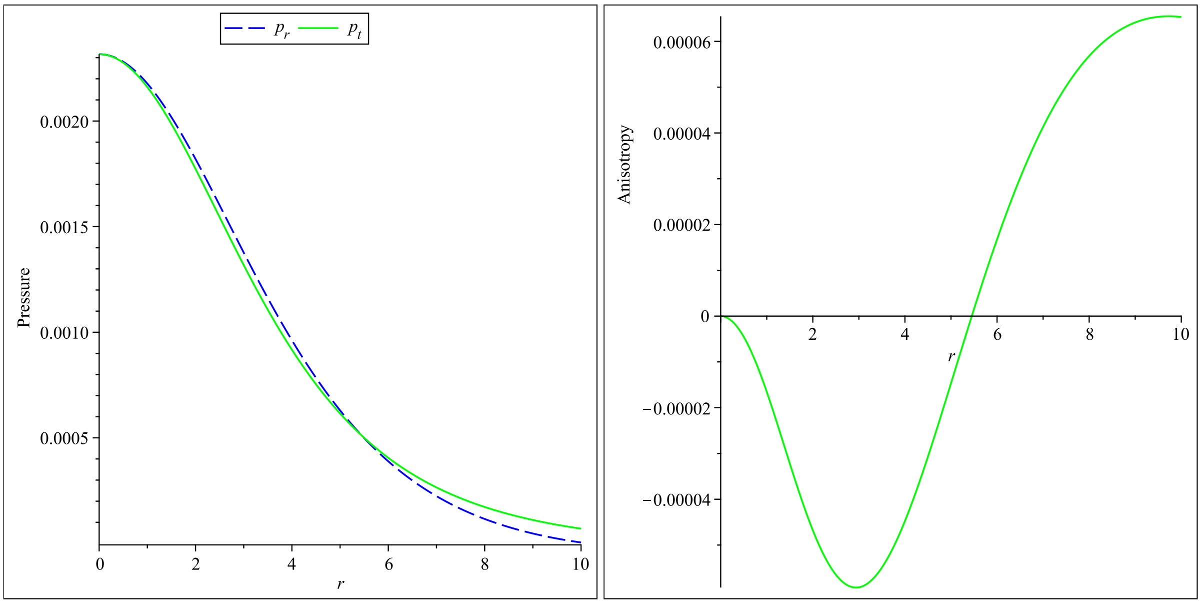


Figure 3.4: Variation of pressures and anisotropy are plotted by taking the same values of the constant mentioned in Fig 3.3.

Discussion

In [22], authors have observed that the physical parameters energy density, radial and tangential pressures, pressure-density ratio, radial and tangential speeds of sound are positive at the center, within the limit and monotonically decreasing outwards. However, the metric potentials, anisotropy, red shift, electric field intensity and adiabatic index are increasing outwards which is necessary for a physically viable configuration. The decreasing behaviour of pressures and density is also evident by their negative gradients. The solution also represents a static and equilibrium configuration as the forces acting on the compact object is counter-balancing each other.

Chapter 4

Charge Anisotropic Solution on Embedding Class-l Space-time

In this chapter, we find new charge anisotropic solutions of the Einstein-Maxwell field equations by using the karmarkar condition. The components of energy momentum tensor given by eqs (1.38) and (1.39), in the presence of electromagnetic field and anisotropic pressure, take the form

$$T_{ab} = diag = \left(-\rho - E^2, p_r - E^2, p_t + E^2, p_t + E^2\right). \tag{4.1}$$

For this configuration, the EMFE's (2.18) take the form

$$\rho = \frac{1}{8\pi}\left[\left(\lambda' - \frac{1}{r}\right)\frac{e^{-\lambda}}{r} + \frac{1}{r^2} - E^2\right], \tag{4.2}$$

$$p_r = \frac{1}{8\pi}\left[\left(\nu' + \frac{1}{r}\right)\frac{e^{-\lambda}}{r} - \frac{1}{r^2} + E^2\right], \tag{4.3}$$

$$p_t = \frac{1}{8\pi}\left[\left(\nu'' + \frac{\nu' - \lambda'}{r} + \frac{\nu'}{2}(\nu' - \lambda')\right)\frac{e^{-\lambda}}{2} - E^2\right], \tag{4.4}$$

$$\sigma = \frac{1}{4\pi r^2} e^{\frac{-\lambda}{2}} (r^2 E)', \tag{4.5}$$

where prime ($'$) represents differentiation with respect to r and σ is the charge density. For charge compact object the effective mass can be expressed as

$$M = \int_0^R \left(4\pi\rho + \frac{E^2}{2}\right) r^2 dr. \tag{4.6}$$

Consider the space-time metric given in eq (2.22) with metric potential for the interior of anisotropic configuration as

$$e^{\lambda} = 1 + ar^2. \tag{4.7}$$

Using eq (3.7) in eq (4.7) we get

$$e^{\nu} = \left(A + \frac{B\sqrt{a}r^2}{2}\right)^2. \tag{4.8}$$

Motivated by Pant and Fuloria [23], we assume electric field intensity for our model as

$$E^2 = \frac{ka^3r^6}{(1+ar^2)^4}, \tag{4.9}$$

where k and a are positive constants. Now by using eqs. (4.2) to (4.4), we obtain the expressions for ρ, p_r, and p_t as

$$\rho = \frac{1}{8\pi}\left[\frac{3a + a^2r^2}{(1+ar^2)^2} - \frac{ka^3r^6}{(1+ar^2)^4}\right], \tag{4.10}$$

$$p_r = \frac{1}{8\pi}\left[\frac{4B\sqrt{a} - Ba\sqrt{a}r^2 - 2Aa}{(1+ar^2)(B\sqrt{a}r^2 + 2A)} + \frac{ka^3r^6}{(1+ar^2)^4}\right], \tag{4.11}$$

$$p_t = \frac{1}{8\pi}\left[\frac{B^2a^2r^4 + (2ABa\sqrt{a} + 4B^2a - 2ABa)r^2 + 8AB\sqrt{a} - 4A^2a}{(1+ar^2)^2(B\sqrt{a}r^2 + 2A)^2} - \frac{ka^3r^6}{(1+ar^2)^4}\right]. \tag{4.12}$$

4.1 Boundary Conditions

The Reissner-Nordstrom metric (2.51) is the most general spherically symmetric and static metric for anisotropic charged matter. Our solution must match the Reissner-Nordstrom solution at the boundry, which requires

$$e^{\nu_b} = \left(1 - \frac{2M}{R} + \frac{Q^2}{R^2}\right) = \left(A + \frac{B\sqrt{a}R^2}{2}\right)^2, \tag{4.13}$$

$$e^{-\lambda_b} = \left(1 - \frac{2M}{R} + \frac{Q^2}{R^2}\right) = (1 + aR^2)^{-1}. \tag{4.14}$$

Now by using the boundary condition $p_r(R) = 0$ and $Q(= Er^2)$ with eqs (4.13) and (4.14), we obtain

$$B = \frac{1}{2\sqrt{a(1+aR^2)}}\left[a - \frac{ka^3R^6}{(1+aR^2)^3}\right], \tag{4.15}$$

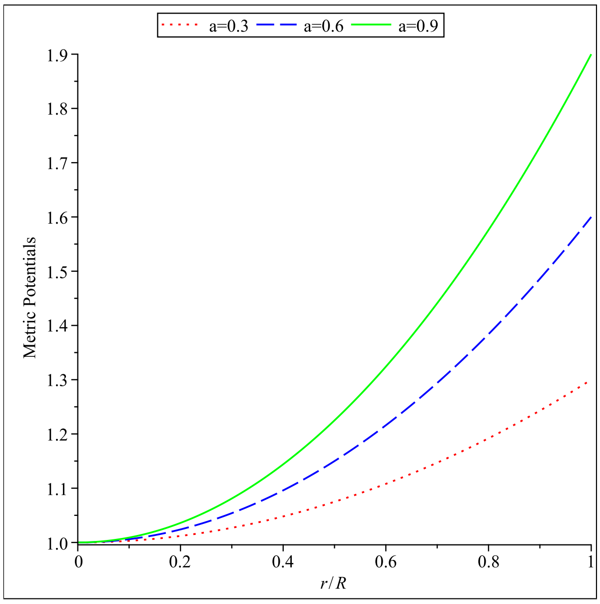
$$A = \left(\frac{1}{\sqrt{1+aR^2}}\right) - \frac{\sqrt{a}R^2}{2}B, \tag{4.16}$$

$$M = \frac{R}{2} - \frac{R}{2(1+aR^2)} + \frac{ka^3R^9}{2(aR^2+1)^4}. \tag{4.17}$$

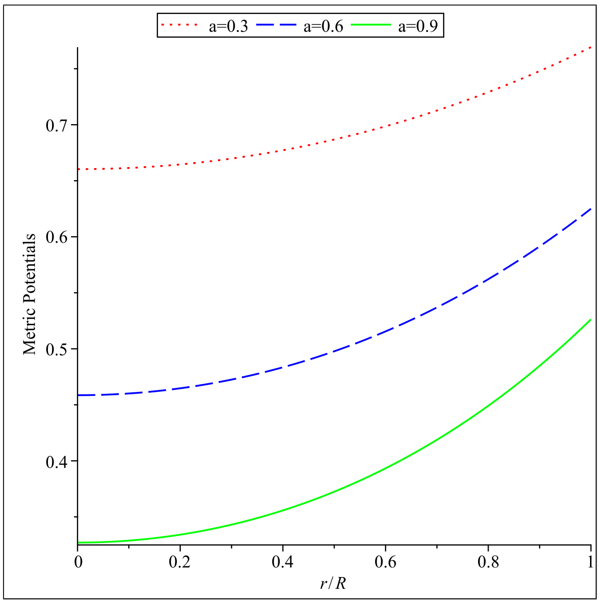
4.2 Physical Conditions

4.2.1 Metric potential and Electric Field Intensity

The metric potentials are free from singularities and both e^λ and e^ν are monotonically increasing moreover $e^\lambda = 1$ at the center. The behavior of metric potentials for different values of a is shown in Figure 4.1.



(a) e^λ



(b) e^ν

Figure 4.1: The metric potentials are plotted for $a = 0.3$, 0.6, and 0.9.

The electric field intensity is zero at the center and increases as moving towards the boundary. Figure 4.2 shows the behavior of electric field intensity for $k = 0.5$.

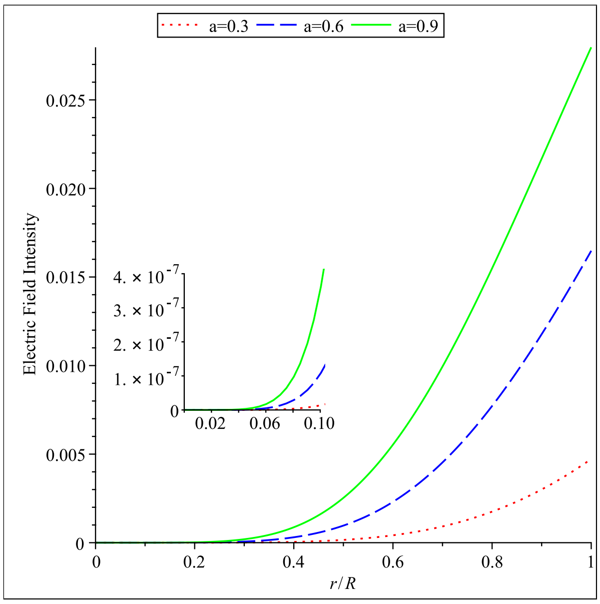


Figure 4.2: The electric field intensity is plotted for $a = 0.3$, 0.6, and 0.9 by taking $k = 0.5$.

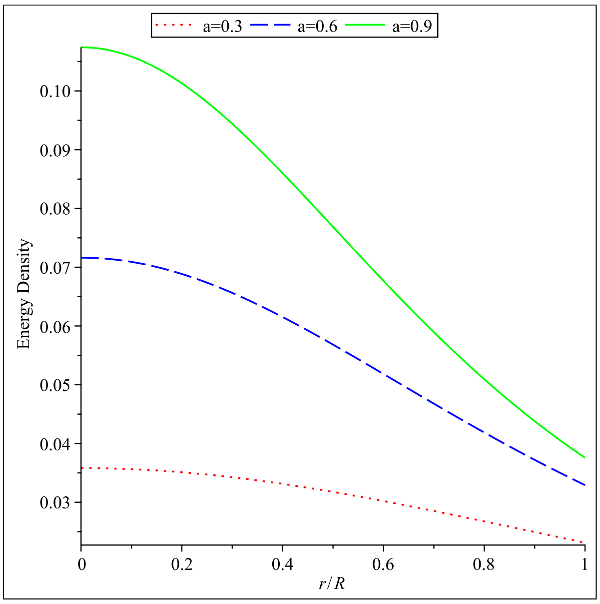


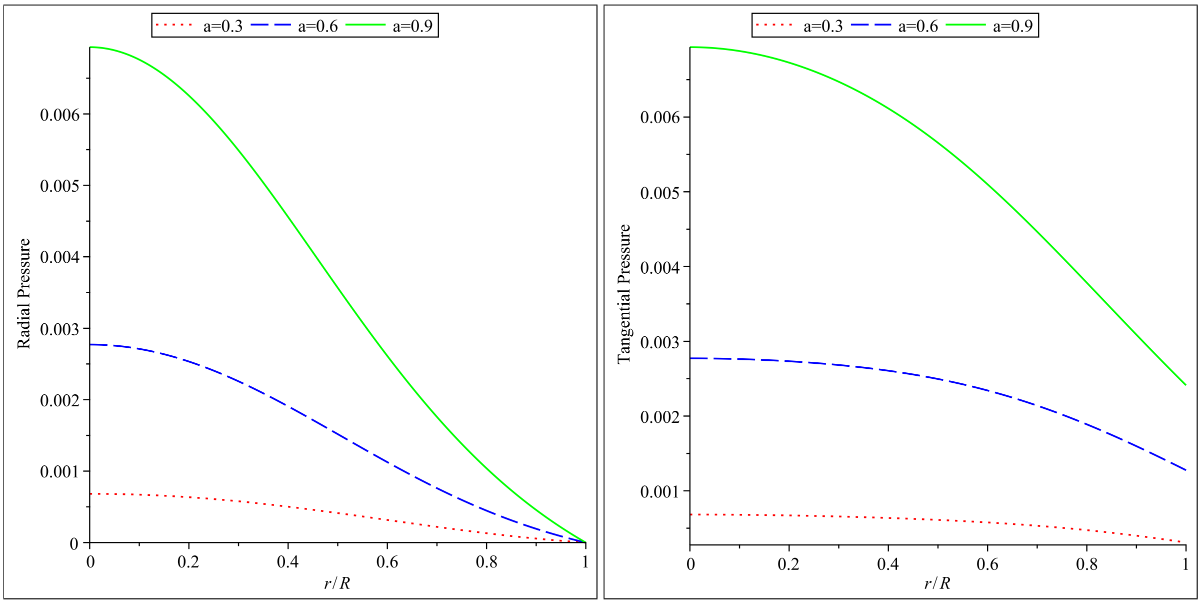
Figure 4.3: The density ρ is plotted for $a = 0.3$, 0.6, and 0.9 by taking $k = 0.5$. It is well defined and positive.

4.2.2 Density and Pressures

The density and pressures have no singularity. The values of density and pressures at $r = 0$ are obtained as

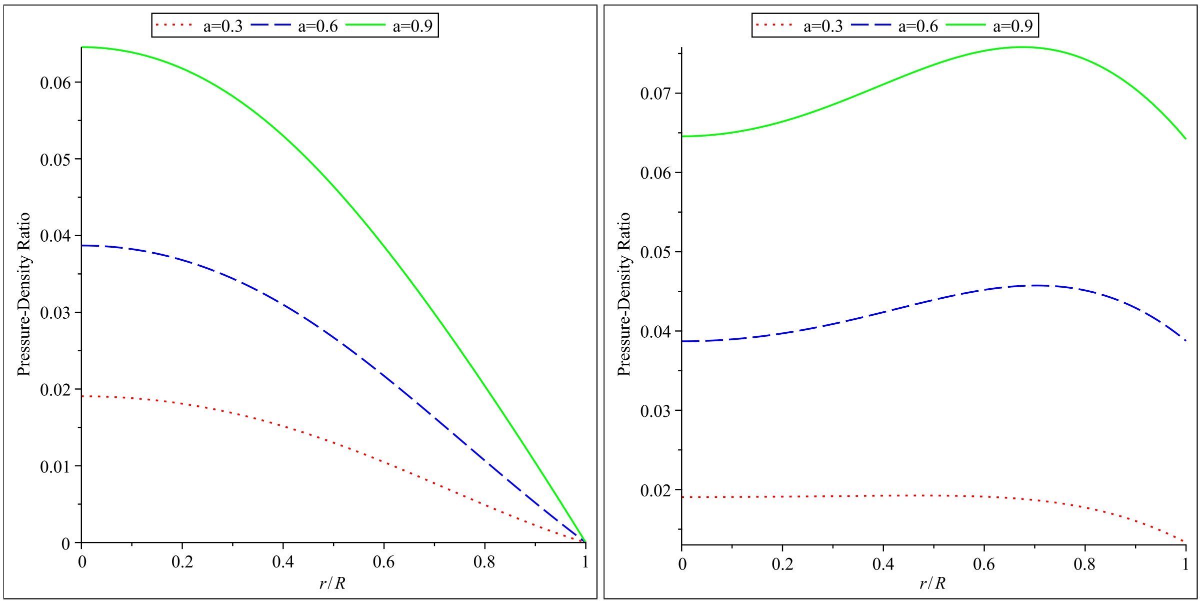
$$\rho(r=0) = \frac{3a}{8\pi} > 0, \qquad p_r(r=0) = p_t(r=0) = \frac{4B\sqrt{a} - 2Aa}{16\pi A} > 0. \tag{4.18}$$

Notice that radial and tangential pressures are equal and positive at $r = 0$. Figures 4.3 and 4.4 represent the behaviour of energy density and pressures respectively, which are monotonically decreasing, moreover $p_r(r = R) = 0$. The Zeldovich's [24] criteria that the pressure-density ratio must be less than 1 within the compact object is satisfied. Figure 4.5 shows the pressure-density ratios.



(a) p_r (b) p_t

Figure 4.4: The radial and tangential pressures are plotted for $a = 0.3$, 0.6, and 0.9 by taking $k = 0.5$. Both are well defined and non-negative.



(a) p_r/ρ (b) p_t/ρ

Figure 4.5: The pressures-density ratios are plotted, which are less than 1.

The gradients of density, radial and tangential pressures are given by eqs (4.19), (4.20) and (4.21) respectively. These gradients are decreasing for suitable values of the parameters involved. Figure 4.6 show graphs of the gradients for different values of a.

$$\frac{d\rho}{dr} = \frac{1}{8\pi(1+ar^2)^2}\left[2a^2r - \frac{4ar(a^2r^2+3a)}{(ar^2+1)} - \frac{6ka^3r^5}{(ar^2+1)^2} + \frac{8ka^4r^7}{(ar^2+1)^3}\right], \tag{4.19}$$

$$\begin{aligned}\frac{dp_r}{dr} =& -\frac{1}{8\pi}\left[\frac{8ka^4r^7}{(1+ar^2)^5} + \frac{2Ba^{3/2}r}{(1+ar^2)(B\sqrt{a}r^2+2A)} + \frac{2ar(4B\sqrt{a}-Ba^{3/2}r^2-2Aa)}{(1+ar^2)^2(B\sqrt{a}r^2+2A)}\right]\\ &-\frac{1}{8\pi}\left[\frac{2B\sqrt{a}r(4B\sqrt{a}-Ba^{3/2}r^2-2Aa)}{(1+ar^2)(B\sqrt{a}r^2+2A)^2} - \frac{6ka^3r^5}{(1+ar^2)^4}\right],\end{aligned} \tag{4.20}$$

$$\begin{aligned}\frac{dp_t}{dr} =& -\frac{1}{8\pi}\left[\frac{4ar(B^2a^2r^4+r^2(2ABa^{3/2}+4B^2a-2ABa)+8AB\sqrt{a}-4A^2a)}{(1+ar^2)^3(B\sqrt{a}r^2+2A)^2} + \frac{6ka^3r^5}{(ar^2+1)^4}\right]\\ &-\frac{1}{8\pi}\left[\frac{4B\sqrt{a}r(B^2a^2r^4+r^2(2ABa^{3/2}+4B^2a-2ABa)+8AB\sqrt{a}-4A^2a)}{(ar^2+1)^2(B\sqrt{a}r^2+2A)^3} - \frac{8ka^4r^7}{(1+ar^2)^5}\right]\\ &+\frac{1}{8\pi}\left[\frac{4B^2a^2r^3+2r(2ABa^{3/2}+4B^2a-2ABa))}{(1+ar^2)^2(B\sqrt{a}r^2+2A)^3}\right].\end{aligned} \tag{4.21}$$

4.2.3 Trace of the Energy-Momentum Tensor

The Bondi's [25] condition for anisotripic fluid sphere states that the trace of energy-momentum tensor must be positive for compact objects to be acceptable. For our model the condition, $\rho - p_r - 2p_t > 0$, is satisfied and graph is shown in Figure 4.7.

4.2.4 Anisotropic Factor

Anisotropy is defined as the difference between tangential and radial pressure. For the compact object it is necessary that the anisotropic factor must be zero at the center and positive otherwise. The anisotropic factor is obtained as

$$\begin{aligned}\Delta = p_t - p_r =& \frac{1}{8\pi}\left[\frac{B^2a^2r^4+(2ABa\sqrt{a}+4B^2a-2ABa)r^2+8AB\sqrt{a}-4A^2a}{(1+ar^2)^2(B\sqrt{a}r^2+2A)^2}\right]\\ &-\frac{1}{8\pi}\left[\frac{4B\sqrt{a}-Ba\sqrt{a}r^2-2Aa}{(1+ar^2)(B\sqrt{a}r^2+2A)} + \frac{2ka^3r^6}{(1+ar^2)^4}\right].\end{aligned} \tag{4.22}$$

The anisotropic factor satisfies the required condition and graph shown in Figure 4.8.

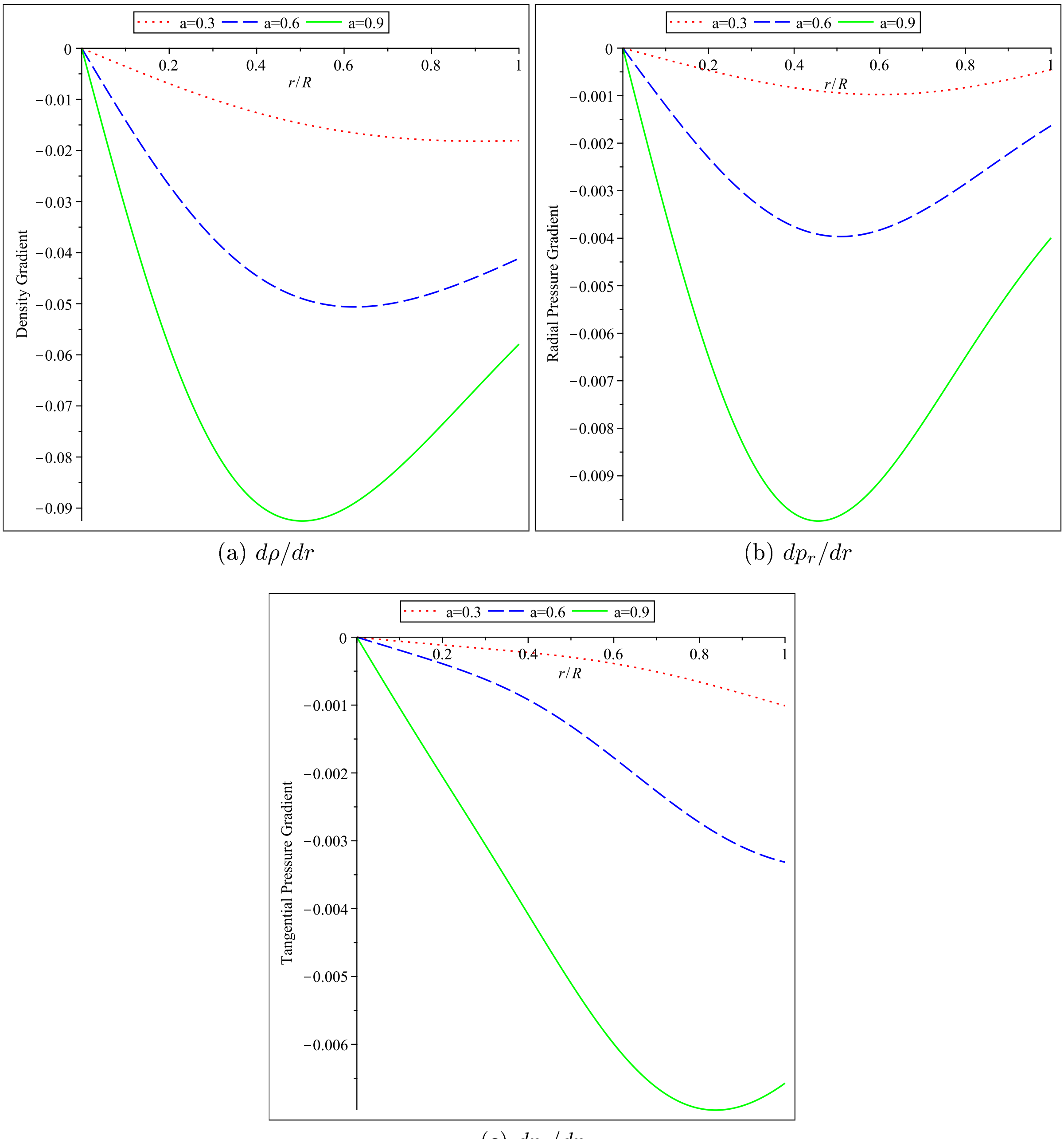


Figure 4.6: The gradients of density and pressures are plotted, which have non-positive values.

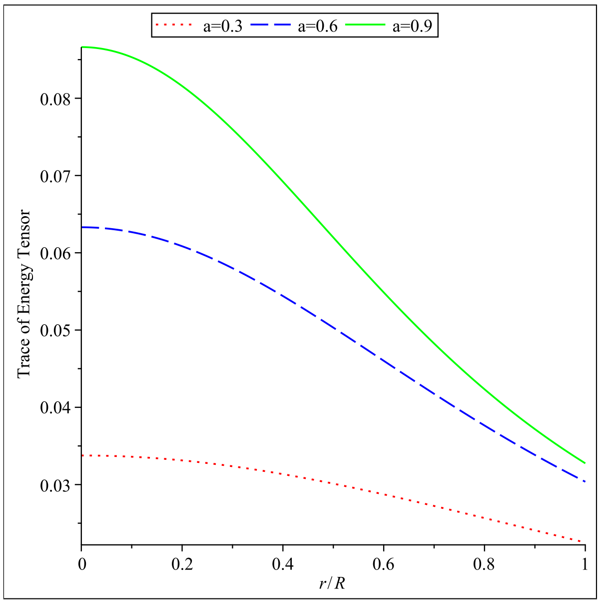


Figure 4.7: The trace of energy momentum tensor is plotted, which is positive and decreasing.

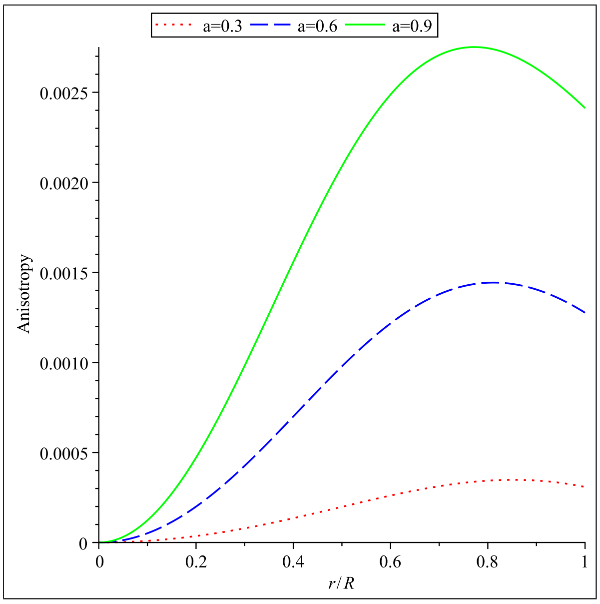


Figure 4.8: The anisotropic factor is plotted for $a = 0.3$, 0.6, and 0.9 by taking $k = 0.5$.

4.2.5 Mass-Radius Relation

The Buchdal [26] states that the condition of mass-radius ratio, $M/R < 4/9$, for a compact object must be satisfied. For our model the mass-radius ratio is, $M/R = 0.4 < 4/9$. The metric potential e^{λ} in view of eq (4.14) is

$$e^{-\lambda} = 1 - \frac{2m}{r} + \frac{Q^2}{r^2} = 1 - \frac{2m}{r} + E^2r^2 = (1 + ar^2)^{-1}, \tag{4.23}$$

so the mass function from eq (4.23) is obtained as

$$m = \frac{r}{2}[1 + E^2r^2 - (1 + ar^2)^{-1}]. \tag{4.24}$$

The mass function is positive and increasing. The variation of mass is shown in Figure 4.9. The compactness-factor must be positive, increasing and less than 8/9 for compact objects.

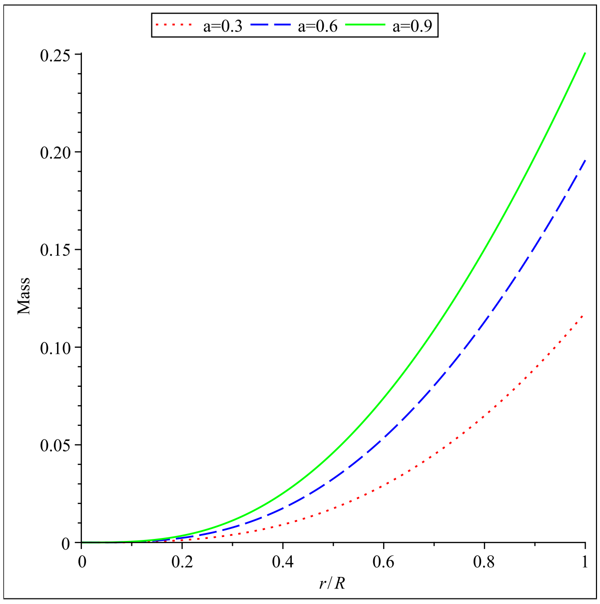


Figure 4.9: Variation of mass function.

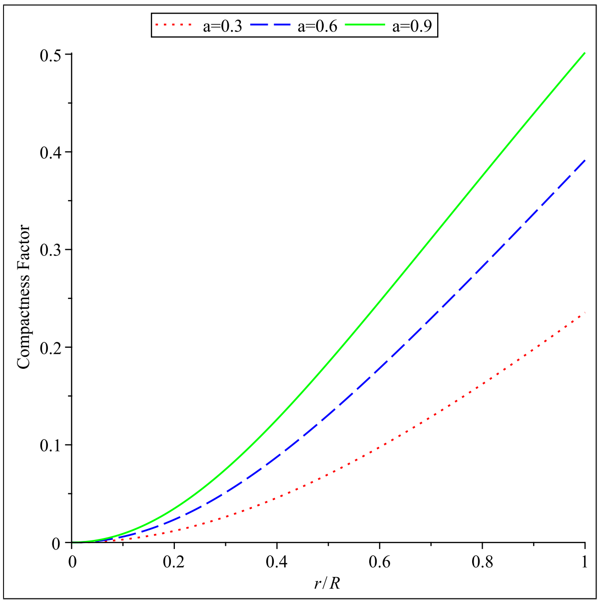


Figure 4.10: Graph of compactness factor.

For our model the compactness factor is given as

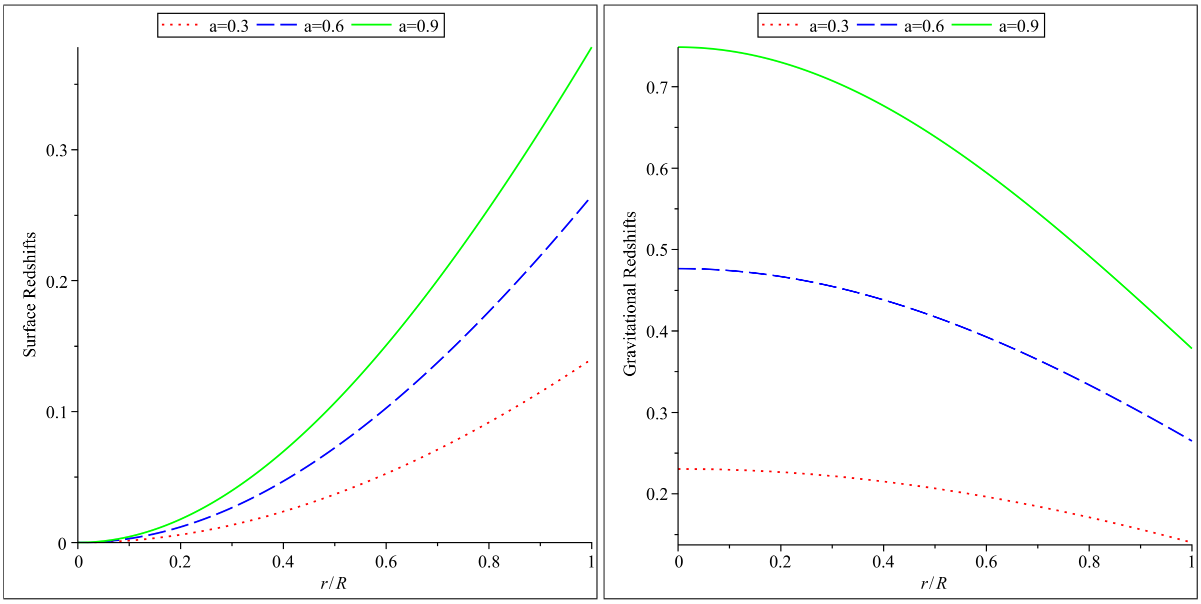
$$u = \frac{2m}{r} = 1 + E^2 r^2 - (1 + ar^2)^{-1}, \tag{4.25}$$

which satisfied the required condition and its graph is shown in Figure 4.10. The surface and gravitational redshifts are expressed as

$$z_s = e^{\lambda/2} - 1, \tag{4.26}$$

$$z = e^{-\nu/2} - 1. \tag{4.27}$$

Surface redshift, z_s, and gravitational redshift, z, have increasing and decreasing behaviour respectively, shown in Figure 4.11 .



(a) z_s (b) z

Figure 4.11: Graphs of Surface and gravitational redshifts.

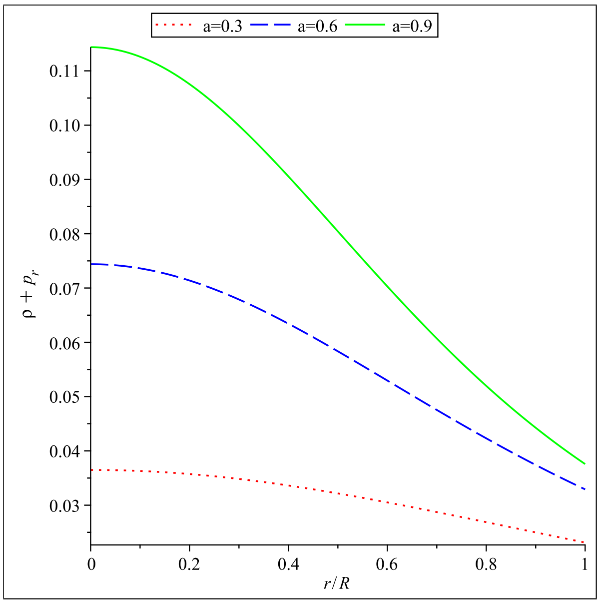
4.3 Stability Conditions

4.3.1 Energy Conditions

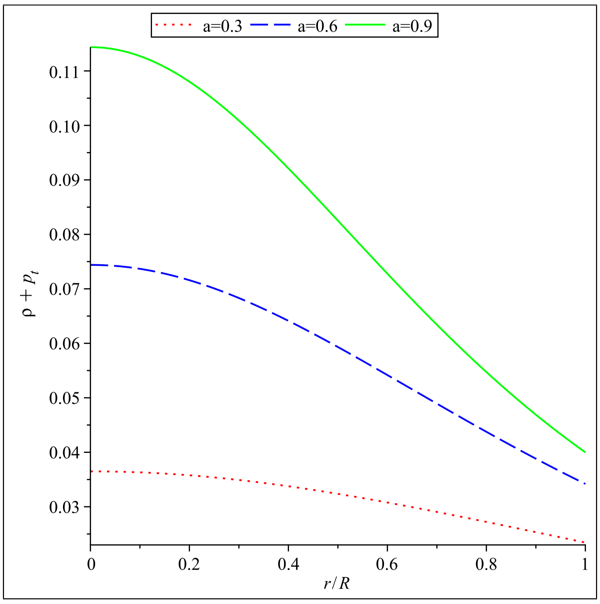
A physically reasonable compact object should satisfy the energy conditions:

1. WEC: $p_r + \rho \geq 0$, $p_t + \rho \geq 0$,
2. SEC: $2p_t + p_r + \rho \geq 0$,
3. NEC: $\rho \geq 0$.

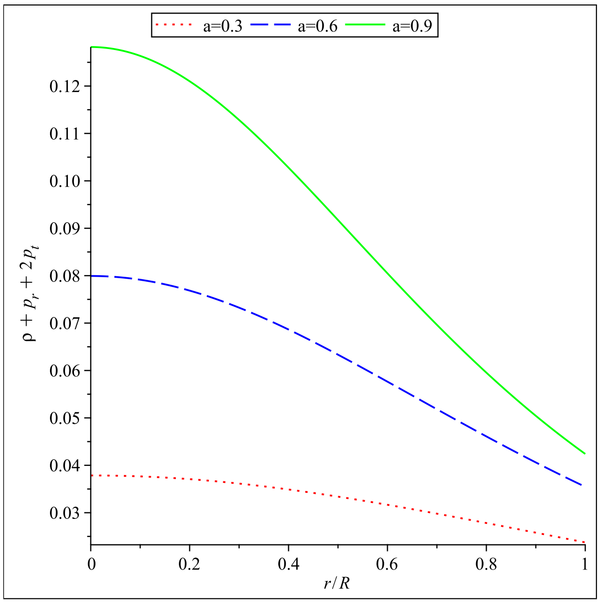
All energy conditions for our model are satisfied and graphs are shown in Figure 4.12.



(a) $\rho + p_r$



(b) $\rho + p_t$



(c) $\rho + p_r + 2p_t$

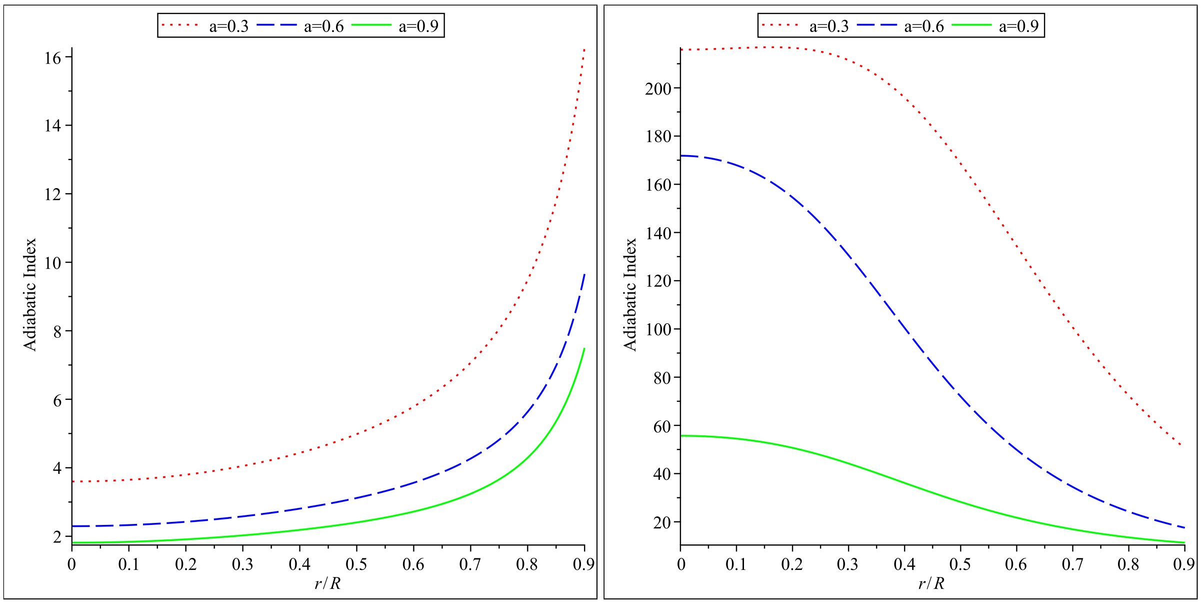
Figure 4.12: Graphs of the energy conditions.

4.3.2 Adiabatic Index

The adiabatic index, Γ_α, defined by the Heintzmann and Hillebrandt [27] is

$$\Gamma_\alpha = \frac{\rho + p_\alpha}{p_\alpha}\frac{dp_\alpha}{d\rho}. \tag{4.28}$$

The model is physically acceptable if the value of Γ_α is greater than $4/3$. For our model this condition is satisfied and graphs of adiabatic index are shown in Figure 4.13.



(a) Γr (b) Γt

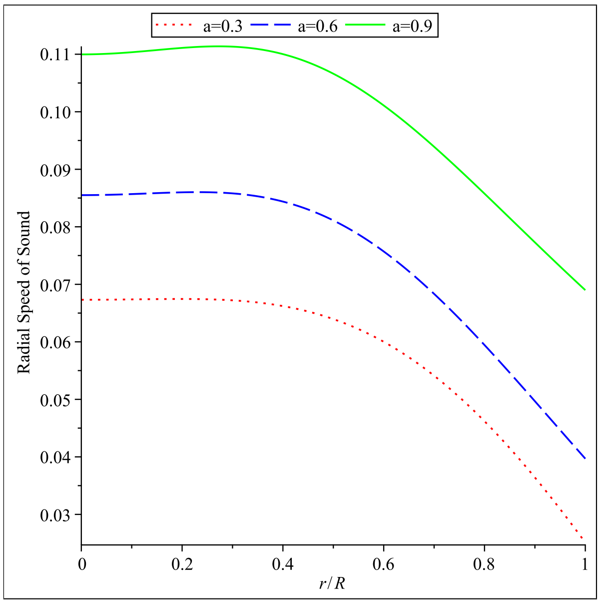
Figure 4.13: Graph of the Adiabatic Index

4.3.3 Casuality Condition

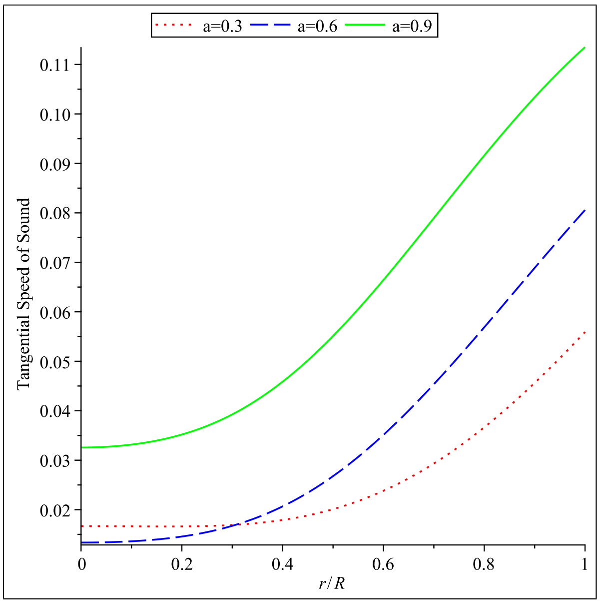
The speed of sound, v_α^2, in anisotropic fluid distribution is defined as

$$v_\alpha^2 = \frac{dp_\alpha}{d\rho} = \left(\frac{dp_\alpha/dr}{d\rho/dr}\right). \tag{4.29}$$

The speed of sound in radial and tangential direction inside the compact object must be less than 1, i.e. $0 < v_r^2,\ v_t^2 < 1$. The graph of radial and tangential speed of sound are shown in Figure 4.14, that shows the speeds lie between 0 and 1, obeying the causality condition. Moreover, for our model the condition $0 \le |v_t^2 - v_r^2| \le 1$ is also satisfied that is shown in Figure 4.15.



(a) ${v_r}^2$



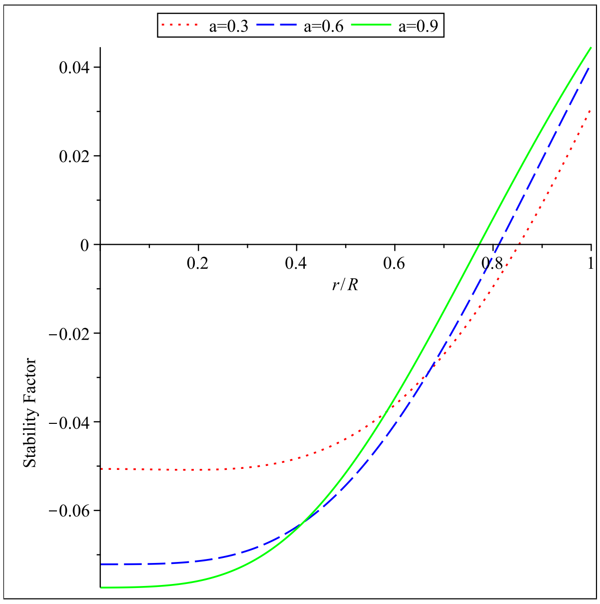
(b) ${v_t}^2$

Figure 4.14: Plots of the radial and tangential speeds of sound.

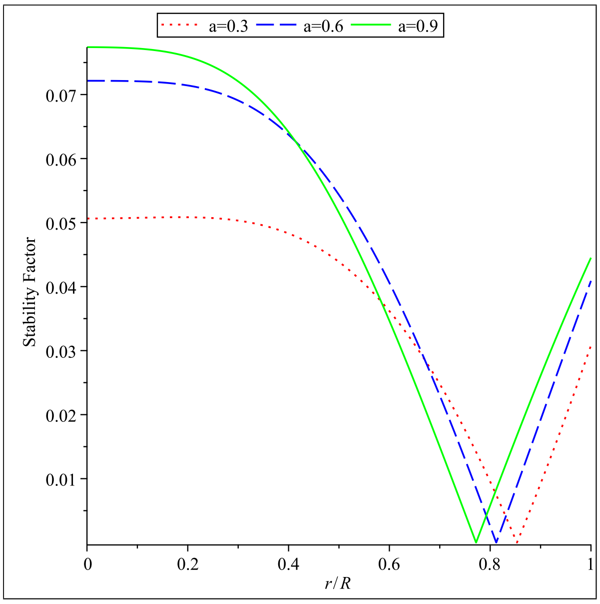
Herrara [28] and Abrue [29] proposed the concept of "Cracking" by which for anisotropic matter distribution, we can find the potentially stable and unstable regions. The compact object is stable if and only if the tangential speed of sound is less than the radial speed of sound.

$$0 \leq |v_t^2 - v_r^2| \leq 1 = \begin{cases} -1 \leq v_t^2 - v_r^2 \leq 0, & \text{Potentially Stable} \\ 0 \leq v_t^2 - v_r^2 \leq +1. & \text{Potentially Unstable} \end{cases}$$

The graphical behavior is shown in Figure 4.15. We clearly observe that the stability parameter is stable for large region and unstable for some region. We can choose such values of parameter a for which almost the complete region will be potentially stable.



(a) ${v_t}^2 - {v_r}^2$



(b) $|{v_t}^2 - {v_r}^2|$

Figure 4.15: Stability factor is plotted for a=0.3, 0.6, and 0.9.

4.3.4 Equilibrium State Under Various Forces

The stellar configuration is in equilibrium state if TOV eq (4.30) which is given by Tolman-Oppenheimer-Volkoff [30, 31] is satisfied

$$\frac{2}{r}(p_t - p_r) - \frac{dp_r}{dr} + \sigma E e^{\lambda/2} - \frac{(\rho + p_r)\nu'}{2} = 0. \tag{4.30}$$

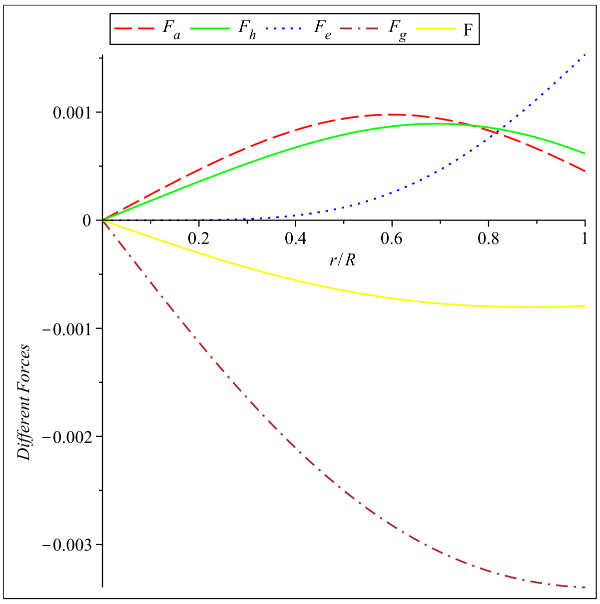
Eq (4.30) can be written as

$$F_a + F_h + F_g + F_e = 0, \tag{4.31}$$

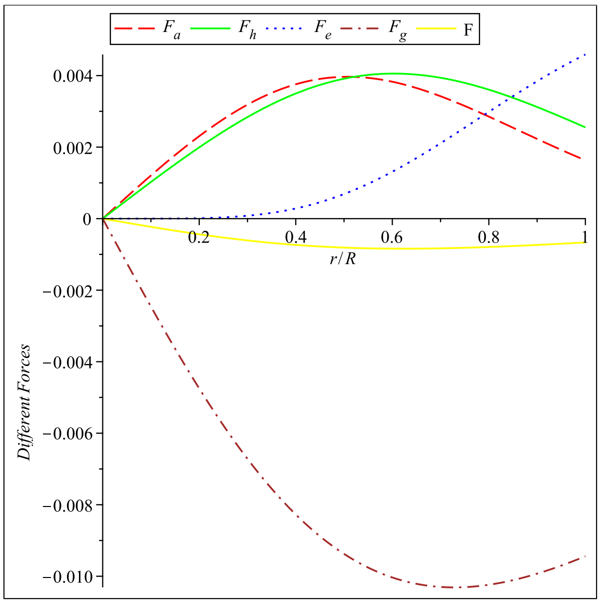
where F_a, F_h, F_e and F_g are anisotropic, hydrostaic, electric and gravitational forces respectively, given as

$$F_a = \frac{2}{r}(p_t - p_r), \qquad F_h = -\frac{dp_r}{dr}, \qquad F_e = \sigma E e^{\lambda/2}, \qquad F_g = -\frac{(\rho + p_r)\nu'}{2}.$$

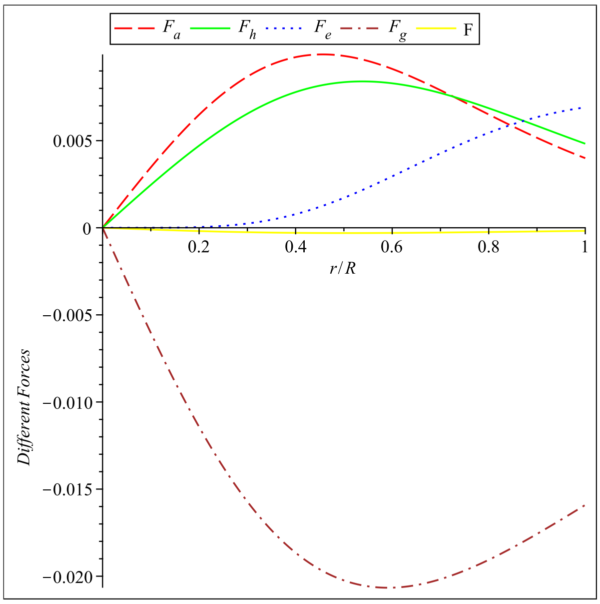
We may conclude that configuration of our compact object is in static equilibrium because the above four forces counterbalancing each other.



(a) $a = 0.3$



(b) $a = 0.6$



(c) $a = 0.9$

Figure 4.16: Variations of different forces with r/R, where F shows the resultant force.

Chapter 5

Conclusion

In this thesis, at first the historical background of space-time is discussed from where the theory started and how it transformed with time until it took the shape given by the genius mind of Einstein, then a brief discussion on different tensors is given in Chapter 1. In Chapter 2, the Einstein-Maxwell field equations and exact solutions of field equations are discussed in detail, including the most important ones the Schwarzschild solution for point mass and the Reissner-Nordstrom for the charged point mass presented in 1916 and 1918 respectively.
Some models for compact objects with Karmarkar condition are reviewed in Chapter 3. In Chapter 4, we have suggested a new model for anisotropic compact charged object taking the Karmarkar condition. We have checked all physical and stability conditions for our model, it is found that for suitable values of the arbitrary constants, our model satisfies all the physical and stability conditions. Therefore, it can be considered as a new analytical model of compact objects.

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