Some New Models of Compact Objects in Paraboloidal Spacetime

By

SAFI UR RAHMAN ZAFAR

A thesis
submitted in partial fulfillment of the
requirements for the degree of
Master of Science
in
Mathematics

Supervised by

PROF. AZAD AKHTER SIDDIQUI

Department of Mathematics,
School of Natural Sciences (SNS),
National University of Sciences and Technology (NUST),
Sector H-12, Islamabad, Pakistan.

2021

FORM TH-4

National University of Sciences & Technology

MS THESIS WORK

We hereby recommend that the dissertation prepared under our supervision by: SAFI UR RAHMAN ZAFAR, Regn No. 00000321365 Titled: "**Some New Models of Compact Objects in Paraboloidal Spacetime**" accepted in partial fulfillment of the requirements for the award of **MS** degree.

Examination Committee Members

1. Name: PROF. TOOBA FEROZE Signature:

2. Name: DR. M. ALI PARACHA Signature:

External Examiner: DR. ZAHID AHMAD Signature:

Supervisor's Name: PROF. AZAD A. SIDDIQUI Signature:

Head of Department

20/08/2021
Date

COUNTERSINGED

Date: 23/8/2021

Dean/Principal

Dedication

This thesis is dedicated to

my lovely parents
Abdur Rahman Nasir and **Namrah Mubarak**

and my supervisor
Prof. Azad A. Siddiqui

Acknowledgements

I am truly grateful to Allah, the omnipotent and the most merciful, Who blessed me with the strength to complete this thesis.

I express my heart-felt gratitude to my supervisor, **Prof. Azad Akhter Siddqui**, for his constant guidance, motivation and valuable suggestions. I am indebted to him.

I would like to thank my parents for their love and support. I also like to acknowledge my dear friend **Muhammad Furqan Nazeer** for his assistance.

SAFI UR RAHMAN ZAFAR

Abstract

Finding new exact or analytic solutions of the Einstein field equations is of great interest among researchers. In this thesis we have developed new classes of exact solutions of the Einstein-Maxwell field equations depicting interior of spherically symmetric compact objects. The solutions have been obtained for charged anisotropic fluid by assuming linear and quadratic equations of state. Solutions have been developed on background spacetime exhibiting paraboloidal geometry. Our solutions are well behaved and satisfy all physical conditions of a feasible star. Causality condition, energy conditions and region of stability are also discussed.

Contents

List of Figures

Chapter 1

Introduction

1.1 Background of Special and General Relativity

In late 1600's Isaac Newton gave his famous laws which revolutionized the modern science. His theory describes the motion of heavenly bodies and objects on earth. He considered universe as a flat and unbounded three dimensional space. He considered time as an absolute quantity which means time passes uniformly without regard to speed of object and observer. This theory worked very well for objects moving with very low speeds as compared to speed of light. According to this theory gravity manifests itself due to presence of matter.

In 1905 Einstein presented his theory of spacetime known as Special Theory of Relativity (SR). Spacetime is a manifold[1] endowed with a metric. SR is based on following postulates [1]:

1. Speed of light measured by inertial observers is constant in vacuum.

2. No specific inertial frame of reference is preferred.

Einstein gave the idea of time as a relative quantity as opposed to Newton's viewpoint. According to Einstein time measured by a moving observer is always less than the time measured by an observer at rest. He showed that mass and length are also relative

[1] A set of points with well understood connected properties [2].

quantities. In 1907 Hermann Minkowski gave geometrical formulation of Special Theory of Relativity. He expressed SR in terms of 4-dimensional manifold which is known as Minkowski spacetime. This theory is only valid in un-accelerated reference frames.

In 1915 Einstein introduced theory of General Relativity (GR). This theory deals with motion in accelerated reference frames and it tells us gravity manifests itself due to presence of energy and matter. He coined the idea of curved spacetime. In classical mechanics gravity is considered a force but in GR it is considered a geometric effect. It suggests that presence of massive objects in spacetime causes it to deviate from flatness and this curvature is the reason for gravity. In curved spacetime objects move along geodesics (nearly straight line path).

In the remaining part of Chapter 1 tensors and their algebra are discussed. Then compact objects, the Einstein field equations and its solutions are mentioned in this chapter. In Chapter 2, an exact solution is reviewed for spherically symmetric distribution of uncharged anisotropic fluid. Physical analysis of the solution is also discussed. In Chapter 3, a solution is developed for charged anisotropic fluid sphere. Linear and quadratic equations of state of the forms $p_r = \beta\rho - \delta$, and $p_r = \alpha\rho^2 - \gamma$ are assumed. This thesis is concluded with brief conclusion in Chapter 4.

1.2 Tensors

Tensors are generalization of vectors and scalars. Tensors are geometrical objects which remain unchanged under coordinate transformations. A tensor $\mathbf{T}$ of rank $m+n$ obeys following transformation law

$$T^{a_1',a_2',\dots,a_m'}{}_{b_1',b_2',\dots,b_n'}\left(x'\right) = \frac{\partial x^{a_1'}}{\partial x^{a_1}}\frac{\partial x^{a_2'}}{\partial x^{a_2}}\cdots\frac{\partial x^{a_m'}}{\partial x^{a_m}}\frac{\partial x^{b_1}}{\partial x^{b_1'}}\frac{\partial x^{b_2}}{\partial x^{b_2'}}\cdots\frac{\partial x^{b_n}}{\partial x^{b_n'}} T^{a_1,a_2,\dots,a_m}{}_{b_1,b_2,\dots,b_n}\left(x\right). \tag{1.1}$$

Here m represents contravariant indices and n represents covariant indices. Rank of a tensor is sum of its contravariant and covariant indices e.g. A scalar is a tensor of rank 0 and a vector is a tensor of rank 1. There are three types of a tensor.

1. A contravariant tensor is represented by upper indices. A rank two contravariant tensor can be written in terms of components and basis vectors as

$$\mathbf{T} = T^{ab}\overline{e}_a\overline{e}_b. \tag{1.2}$$

It obeys following transformation law

$$T^{a'b'}\left(x'\right) = \frac{\partial x^{a'}}{\partial x^a}\frac{\partial x^{b'}}{\partial x^b}T^{ab}\left(x\right). \tag{1.3}$$

2. A covariant tensor is denoted by lower indices. A rank two covariant tensor can be written in terms of components and basis vectors as

$$\mathbf{T} = T_{ab}\overline{e}^a\overline{e}^b. \tag{1.4}$$

It satisfies following transformation law

$$T_{a'b'}\left(x'\right) = \frac{\partial x^a}{\partial x^{a'}}\frac{\partial x^b}{\partial x^{b'}}T_{ab}\left(x\right). \tag{1.5}$$

3. Mixed tensor is combination of contravariant and covariant indices. It is denoted by both upper and lower indices. A mixed tensor of rank 2 can be written as

$$\mathbf{T} = T^a_b\overline{e}^a\overline{e}_b. \tag{1.6}$$

It obeys following transformation law

$$T^{a'}{}_{b'}\left(x'\right) = \frac{\partial x^{a'}}{\partial x^a}\frac{\partial x^b}{\partial x^{b'}}T^a{}_b\left(x\right). \tag{1.7}$$

1.2.1 Operations on Tensors

1. Addition and subtraction of two tensors of same type and rank gives another tensor of same type and rank. e.g. $S^{ij}{}_{kl}$ and $T^{ij}{}_{kl}$ are two tensors of same type and rank, then

$$S^{ij}{}_{kl} + T^{ij}{}_{kl} = W^{ij}{}_{kl}. \tag{1.8}$$

$W^{ij}{}_{kl}$ is also a tensor of same type and rank.

2. Product of two tensors gives another tensor whose rank is sum of the ranks of the given tensors. e.g. $S^{ij}{}_{kl}$ and $T^{m}{}_{np}$ are two tensors of different ranks, then

$$S^{ij}{}_{kl}T^{m}{}_{np} = W^{ijm}{}_{klnp}. \tag{1.9}$$

   Resultant $W^{ijm}{}_{klnp}$ is a tensor of rank 7.

3. In mixed tensors of rank greater than or equal to two, rank of a tensor can be reduced by setting a contravariant and covariant index equal to each other. Rank of the resultant tensor will be two less than that of original tensor. This process is known as contraction. e.g. $S^{ab}{}_{cd}$ is a tensor of rank 4 by setting $b = d$ we get $S^{a}{}_{c}$ which is of rank 2.

1.2.2 Metric Tensor and Kronecker Delta

To study the notion of distance on manifolds a useful quantity metric tensor is defined. It is a symmetric second rank tensor. Metric tensor is defined as

$$\mathbf{g} = g_{ab}\overline{e}^{a}\overline{e}^{b}, \tag{1.10}$$

g_{ab} are components of metric and $\overline{e}^{a}$, $\overline{e}^{b}$ are basis vectors. Other useful forms of metric tensor are

$$\mathbf{g} = g^{ab}\overline{e}_{a}\overline{e}_{b}, \quad \mathbf{g} = g^{a}{}_{b}\overline{e}_{a}\overline{e}^{b}. \tag{1.11}$$

Its components in terms of basis vectors are

$$g_{ab} = \mathbf{g}(\overline{e}_{a}\overline{e}_{b}) = \overline{e}_{a}.\overline{e}_{b} = \overline{e}_{b}.\overline{e}_{a} = g_{ba}. \tag{1.12}$$

Its properties are as follows

$$g^{ab}g_{ab} = 4 \quad \text{for} \quad a, b = 0, 1, 2, 3. \tag{1.13}$$

$$g^{ac}g_{bc} = \delta^{a}{}_{b}. \tag{1.14}$$

$\delta^{a}{}_{b}$ is known as Kronecker delta which is defined as

$$\delta^{a}{}_{b} = \begin{cases} 0 & \text{if} \quad a \neq b, \\ 1 & \text{if} \quad a = b. \end{cases} \tag{1.15}$$

Metric tensor is used to raise and lower indices of a tensor. A metric tensor is also used to define line element ds^2 as

$$ds^2 = g_{ba} dx^b dx^a. \tag{1.16}$$

This is known as the First Fundamental Form. It measures distance between two neighbouring points on a surface.

1.2.3 Christoffel and Levi-Civita Symbols

Christoffel symbols are named after Elvin Bruno Christoffel. Christoffel symbol of first kind is given as

$$\Gamma_{abc} = \frac{1}{2}\left[g_{ac,b} + g_{bc,a} - g_{ab,c}\right]. \tag{1.17}$$

Christoffel symbol of second kind is

$$\Gamma^a{}_{bc} = \frac{1}{2} g^{ad}\left[g_{cd,b} + g_{bd,c} - g_{bc,d}\right]. \tag{1.18}$$

Due to symmetry property of the metric tensor it is symmetric in lower indices.

$$\Gamma^a{}_{bc} = \Gamma^a{}_{cb}. \tag{1.19}$$

Levi-Civita symbol is defined as

$$\epsilon_{ijk} = \begin{cases} 0 & \text{if } \; i = j \text{ or } \; i = k \; \text{ or } \; j = k, \\ 1 & \text{for even permutation of indices}, \\ -1 & \text{for odd permutation of indices}. \end{cases} \tag{1.20}$$

1.2.4 Riemann, Ricci and Einstein Tensors

In GR curvature of spacetime is measured by a rank four tensor known as Riemann Curvature Tensor (RCT). It is named after Bernhard Riemann. It is mathematically written as

$$R^a{}_{bcd} = \Gamma^a{}_{bd,c} - \Gamma^a{}_{bc,d} + \Gamma^a{}_{cv}\Gamma^v{}_{bd} - \Gamma^a{}_{dv}\Gamma^v{}_{bc}. \tag{1.21}$$

An object is flat if all components of RCT vanish for the object. In n dimensions, it has total of n^4 components. Another form of RCT is as follows

$$R_{pbcd} = g_{pa} R^{a}_{bcd} = g_{pa} \left[\Gamma^{a}_{bd,c} - \Gamma^{a}_{bc,d} + \Gamma^{a}_{cv} \Gamma^{v}_{bd} - \Gamma^{a}_{dv} \Gamma^{v}_{bc} \right]. \tag{1.22}$$

Following symmetries are present in RCT

1. Presence of skew symmetry in first and last two indices

$$R_{pbcd} = -\, R_{bpcd}, \tag{1.23}$$

$$R_{pbcd} = -\, R_{pbdc}. \tag{1.24}$$

2. Presence of symmetry in interchanging first two and last two pair of indices

$$R_{pbcd} = R_{cdpb}. \tag{1.25}$$

3. If we fix the first index RCT is anti-symmetric in remaining three indices.

$$R_{p[bcd]} = R_{pbcd} + R_{pdbc} + R_{pcdb} = 0. \tag{1.26}$$

Due to these symmetries independent components of RCT reduce to $\frac{n^2\left(n^2-1\right)}{12}$. By taking $a = c$ in eq. (1.21) we get

$$R_{bd} = R^{a}{}_{bad} = \Gamma^{a}_{bd,a} - \Gamma^{a}_{ba,d} + \Gamma^{a}_{av} \Gamma^{v}_{bd} - \Gamma^{a}_{dv} \Gamma^{v}_{ba}. \tag{1.27}$$

This is known as the Ricci Tensor (RT) named after Gregorio Ricci. Rank of the RT is two. Alternatively it can be written as

$$R_{bd} = g^{st} R_{sbtd}. \tag{1.28}$$

Ricci tensor is symmetric in its indices

$$R_{bc} = R_{cb}. \tag{1.29}$$

Taking trace of the RT we get the Ricci Scalar (RS) which is denoted by R

$$R = g^{bc} R_{bc} = R^{b}{}_{b}. \tag{1.30}$$

Einstein introduced a tensor which describes the geometry of spacetime known as the Einstein tensor. It is mathematically written as

$$G_{ab} = R_{ab} - \frac{1}{2} g_{ab} R. \tag{1.31}$$

As metric tensor and the RT are symmetric this implies that the Einstein tensor is also symmetric,

$$G_{ab} = G_{ba}. \tag{1.32}$$

1.2.5 The Maxwell Tensor

It is also known as electromagnetic field tensor. The Maxwell tensor is anti-symmetric tensor of rank two. We can define it using 4-vector electromagnetic potential A as

$$F_{bc} = \frac{\partial A_c}{\partial x^b} - \frac{\partial A_b}{\partial x^c}. \tag{1.33}$$

Here 4-vector electromagnetic potential A is given as

$$A_\nu = \left(\frac{\phi}{c}, A_i \right). \tag{1.34}$$

Here ϕ is a scalar potential, $\nu = 0, 1, 2, 3$ and $i = 1, 2, 3$. Electric field $\mathbf{E}$ and magnetic field $\mathbf{B}$ can be expressed in the form of electromagnetic field tensor as

$$E_i = c F_{0i}, \tag{1.35}$$

$$B_i = -\frac{1}{2} \epsilon_{ist} F^{st}. \tag{1.36}$$

Here $i, s, t = 0, 1, 2, 3$. Components of the Maxwell tensor are

$$F_{bc} = \frac{1}{c} \begin{bmatrix} 0 & E_1 & E_2 & E_3 \\ -E_1 & 0 & cB_3 & -cB_2 \\ -E_2 & -cB_3 & 0 & cB_1 \\ -E_3 & cB_2 & -cB_1 & 0 \end{bmatrix}. \tag{1.37}$$

1.2.6 Stress Energy Tensor

It is also known as energy momentum tensor or matter tensor. It is the source of gravitational field. It tells us about distribution of matter at each point in spacetime. Components of T^{ab} are given as

$$T^{ab} = \begin{bmatrix} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{bmatrix}. \tag{1.38}$$

T^{00} represents the energy density. T^{0b} tells about flux of energy. T^{a0} show momentum density. T^{aa} represents isotropic pressure. T^{ab} where $a \neq b = 1, 2, 3$ represents shear stress. Some form of energy momentum tensor are given below

1. In **vacuum** as there is no source present so stress energy tensor vanishes i.e.

$$T^{ab} = 0. \tag{1.39}$$

2. **Dust** particles are non interacting matter particles. For dust, energy momentum tensor is [3]

$$T^{ab} = \rho_0 v^a v^b = \rho_0 \begin{bmatrix} v^0v^0 & v^0v^1 & v^0v^2 & v^0v^3 \\ v^1v^0 & v^1v^1 & v^1v^2 & v^1v^3 \\ v^2v^0 & v^2v^1 & v^2v^2 & v^2v^3 \\ v^3v^0 & v^3v^1 & v^3v^2 & v^3v^3 \end{bmatrix}. \tag{1.40}$$

ρ_0 is known as density and v^a is velocity 4-vector.

3. A perfect fluid is defined as one for which there are no forces present between the particles and no heat condition. Viscosity is also zero and there are no shear stresses. For perfect fluid energy momentum tensor takes the form

$$T^{bc} = (p + \rho_0)\, v^b v^c + p g^{bc}. \tag{1.41}$$

Here ρ_0 represents density, v is velocity and p represents pressure.

4. Anisotropic fluid is a fluid in which radial and tangential pressures are not same at each point. For anisotropic fluid distribution stress energy tensor takes the form [4]

$$T^a_b = \left[(\rho + p_t)\, v^a v_b - p_t g^a{}_b + (p_r - p_t)\, \chi^a \chi_b \right]. \tag{1.42}$$

Here $\rho,\ p_r,\ p_t$ are energy density, radial pressure and tangential pressure respectively. v^a and χ^a are 4-velocity and unit spacelike vector given as

$$v^a = \sqrt{\frac{1}{g_{00}}}\delta^a{}_0, \quad \chi^a = \sqrt{-\frac{1}{g_{11}}}\chi^a{}_1. \tag{1.43}$$

5. For electromagnetic field in absence of any source stress energy tensor is

$$E^{ab} = -\frac{1}{4\pi}\left[F^{ae}F^b{}_e - \frac{1}{4}g^{ab}F^{eg}F_{eg} \right]. \tag{1.44}$$

In the presence of a source it takes the form

$$T^{ab} = \rho_0 v^a v^b - F^{ae}F^b{}_e + \frac{1}{4}g^{ab}F^{eg}F_{eg}. \tag{1.45}$$

1.2.7 Covariant Derivative

Partial derivative of a tensor is not sufficient because it does not obey transformation law, therefore, we define a new type of derivative known as covariant derivative. Covariant derivative of a second rank tensor can be written as

$$S^a{}_{b;c} = S^a{}_{b,c} + \Gamma^a{}_{cd}S^d{}_b - \Gamma^d{}_{bc}S^a{}_d. \tag{1.46}$$

Here comma represents partial derivative and semi colon represents covariant derivative. It satisfies following properties

1. Linearity: Assume we have two tensors V and W then

$$\nabla (V + W) = \nabla V + \nabla W. \tag{1.47}$$

Here ∇ represents covariant derivative.

2. Product rule:

$$\nabla (V \otimes W) = (\nabla V) \otimes W + V \otimes (\nabla W), \tag{1.48}$$

where $\otimes$ represents tensor product.

3. Covariant derivative reduces to partial derivative for scalars

$$\nabla \psi = \partial_a \psi, \tag{1.49}$$

where ψ is a scalar.

1.3 The Einstein Field Equations(EFEs)

In 1915 Einstein gave a set of tensorial equations which describe curvature of spacetime as a consequence of presence of matter and energy. These equations are represented by

$$G_{ab} = R_{ab} - \frac{1}{2} g_{ab} = kT_{ab}, \tag{1.50}$$

where k is the coupling constant. We are adopting units where $k = 1$, the field equations takes the form

$$R_{ab} - \frac{1}{2} g_{ab} R = T_{ab}. \tag{1.51}$$

In 4-dimensions these field equations are set of ten non-linear partial differential equations (PDEs) involving twenty unknowns, namely 10 components of metric tensor and 10 components of stress energy tensor. Due to freedom of choice of coordinates (t, x^1, x^2, x^3) ten equations reduce to six equations. These equations relate geometry of spacetime to the presence of matter and energy.

Finding solutions of EFEs is quite difficult due to non linearity of PDEs. First solution of EFEs was given by Karl Schwarzchild in 1916. Other exact solutions of EFEs are also derived by imposing certain symmetry conditions which describe white dwarfs, neutron stars, black holes etc.

1.4 The Maxwell Equations

The Maxwell equations deal with the behaviour of electric field[2] and magnetic field[3] in space. There are two types of electric fields

1. Electrostatic field

2. Induced electric field.

Presence of electric charge produces electrostatic field around the source in space. Gauss's law [6] describes electrostatic field as

$$\overline{\nabla}.\overline{E} = \frac{\rho}{\epsilon_0}. \tag{1.52}$$

Here ρ is charge density and ϵ_0 is a constant known as permittivity of free space. Left hand side of this equation describes tendency of electric field to flow away from a specified position in space. $\overline{\nabla}$ is an operator known as nabla. Mathematically it is given as

$$\overline{\nabla} = \frac{\partial}{\partial x}\overline{e_1} + \frac{\partial}{\partial y}\overline{e_2} + \frac{\partial}{\partial z}\overline{e_3}. \tag{1.53}$$

If there is a positive charge (source) present in space then the electrostatic field produced around it tends to flow in opposite direction from the source. For negative charge direction of the electric field is inwards.

Induced electric field is produced by varying the magnetic field with respect to time. Magnetic fields are produced by moving electric charges. Magnetic fields have two poles i.e. North pole and South pole. These poles are not isolated (i.e always exists in pair) as opposed to electric field in which charges may or may not be isolated. Divergence of magnetic field at any point in space is zero and is known as Gauss's law for magnetic fields [6]. Mathematically, it is given as

$$\overline{\nabla}.\overline{B} = 0. \tag{1.54}$$

[2] An electric field $\overline{E}$ is the electrical force per unit charge exerted on a charged object [5].
[3] Magnetic field $\overline{B}$ is magnetic force experienced by a moving charge object [5].

Magnetic field lines form closed continuous loops and do not cross each other at any point in space.

A changing magnetic field induces an electric field which is known as Faraday's law [6]. Mathematical form of Faraday's law is

$$\overline{\nabla} \times \overline{E} + \frac{\partial \overline{B}}{\partial t} = 0. \tag{1.55}$$

First term represents curl of electric field and second term represents rate of change of magnetic field. Ampere Maxwell law [6] states that a magnetic field is produced by electric current and by time varying electric field.

$$\overline{\nabla} \times \overline{B} = \mu_0 \left(\overline{j} + \epsilon_0 \frac{\partial \overline{E}}{\partial t} \right), \tag{1.56}$$

where μ_0, ϵ_0 are permeability and permittivity of free space. First term on right side of equation represents current density and second term represents rate of change of electric field. These equations given by (1.52), (1.54), (1.55) and (1.56) are collectively known as Maxwell's equations.

Maxwell equations in tensorial form are

$$F^{bc}{}_{;c} = \mu_0 j^b, \tag{1.57}$$

$$F_{[bc,d]} = 0. \tag{1.58}$$

j^b is a 4-vector current density and $b, c, d = 0, 1, 2, 3$.

1.5 Compact Objects

Compact objects are endpoint states of stellar evolution [7]. Compact objects are classified into three categories

1. White dwarfs,

2. Neutron stars,

3. Black holes.

1.5.1 Formation of a Compact Object

Normal stars are made up of gasses held by gravity. A star is formed when a large amount of gas collapses on itself because of gravitational attraction [7]. More the gas contracts, faster the atoms of gas collide and as a result gas heats up. Eventually, a state is reached where gas atoms do not collide with each other instead they combine to form helium. Massive amount of heat is released as a result of this reaction. This heat increases pressure of gas until it is sufficient enough to balance the gravitational pull. At this point gas ceases to contract and star achieves a stable state.

With the passage of time, nuclear fuel resource of a star starts to diminish. When fuel is completely finished death of star happens. As a result gravitational attraction exceeds the pressure and gravitational collapse occurs and a compact object is formed.

In 1930 Subrahmanyan Chandrasekhar predicted that a star whose mass is less than or equal to 1.4 solar mass (known as Chandrasekhar limit) can settle down in final state known as White Dwarf. White dwarf uses pressure degenerate electrons to balance pull of gravity. It has radius of few thousand miles and its density is around $10^9\ kg/m^3$. White dwarfs are more dense than a normal star.

Another possible final state a star can attain is a Neutron Star. In this gravitational attraction is balanced by pressure of degenerate neutrons. Its mass is in the range of 1.5-2.9 solar mass (known as Tolman-Oppenheimer-Volkov limit). Neutron stars are smaller and more denser than white dwarfs.

Objects whose mass exceeds 3 solar masses cannot resist gravitational attraction and form a Black Hole. Its density is much higher than white dwarfs and neutron stars. Black hole is a region in space from which nothing (not even light) can escape. Due to gravitational collapse all matter shrinks to a single region of space with infinite density known as singularity. At singularity curvature is not finite. For non rotating black holes singularity is a point. On the other hand for rotating black holes it is in the form of a ring.

The surface around the black hole is known as event horizon also known as surface of black hole. The events inside an event horizon cannot affect an observer outside the event horizon. Although black holes are not visible but its presence can be detected by its gravitational pull on nearby objects. If a star is near to a black hole, matter starts falling into black hole. As it falls it gets extremely hot, an accretion disc is formed around black hole and radiations are emitted which are visible through telescopes.

1.6 Solutions of the Einstein Field Equations

Many solutions of Einstein field equations have been found by making use of certain ansatz. We will discuss only a few of them in this section.

1.6.1 The Schwarzchild Solution

In 1916 Karl Schwarzchild derived this solution for a point mass M. By taking following ansatz

1. Source is isolated.
2. Spacetime is static which means components of metric are independent of time coordinate. In other words we can says that components are symmetric with respect to time coordinate.
3. Spacetime is spherically symmetric.

Static and spherically symmetric line element is given as

$$ds^2 = -e^{2\nu(r)}dt^2 + e^{2\lambda(r)}dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right), \tag{1.59}$$

As there is vacuum present around the source so stress energy tensor vanishes and the Einstein field equations become

$$R_{ab} = 0. \tag{1.60}$$

By solving the field equations (1.60) with line element (1.59), in gravitational units, $G = c = 1$, we get

$$ds^2 = -\left(1 - \frac{2M}{r}\right)dt^2 + \left(1 - \frac{2M}{r}\right)^{-1}dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right). \tag{1.61}$$

This is known as the Schwarzchild black hole solution [8]. This solution has two singularities. First singularity is present at $r = 0$, here g_{00} becomes infinite. This is an essential singularity and it cannot be removed. Second singularity is present at $r = 2M$ (it is also known as the Schwarzchild radius). This is a coordinate or removable singularity which occurs due to faulty choice of coordinates. The Schwarzchild solution describes geometry of a spacetime outside a uncharged and non rotating compact object.

1.6.2 The Reissner-Nordstrom Solution

Taking following assumptions

1. Source is charged.
2. Spacetime is static and spherically symmetric.
3. Spacetime is asymptotically flat.

Stress energy tensor for this case is

$$T^i{}_j = \left(-E^2, -E^2, E^2, E^2\right). \tag{1.62}$$

By solving the Einstein-Maxwell field equations one gets

$$\begin{aligned} ds^2 = &\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-1} dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) - \\ &\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right) dt^2. \end{aligned} \tag{1.63}$$

Here M is mass and Q is total charge. This is also known as charged black hole solution or the Reissner-Nordstrom black hole solution [9]. At $r = 0$ we have an essential singularity. For other singularities we put

$$1 - \frac{2M}{r} + \frac{Q^2}{r^2} = 0, \tag{1.64}$$

$$r^2 - 2Mr + Q^2 = 0. \tag{1.65}$$

By solving this quadratic equation we get

$$r_+ = M + \sqrt{M^2 - Q^2}, \quad r_- = M - \sqrt{m^2 - Q^2}. \tag{1.66}$$

r_- and r_+ are coordinate singularities. It has two event horizons. r_- is called inner event horizon and r_+ is called outer event horizon.

1.6.3 The Kerr Solution

In 1963, Roy Kerr discovered a solution of rotating black holes. He generalized the Schwarzchild solution by adding spin into it. Following assumptions are used

1. Source of point mass M is rotating.
2. Source is surrounded by vacuum.
3. Spacetime is static and asymptotically flat.

The line element in the Boyer Lindquist form is given as

$$\begin{aligned} ds^2 = & \frac{\Delta}{\rho^2}\left(dt - asin^2\theta d\phi\right)^2 - \frac{sin^2\theta}{\rho^2}\left(\left(r^2 + a^2\right)d\phi - adt\right)^2 - \\ & \frac{\rho^2}{\Delta}dr^2 - \rho^2 d\theta^2, \end{aligned} \tag{1.67}$$

where

$$\Delta^2 = r^2 - 2Mr + a^2, \quad \rho^2 = r^2 + a^2cos^2\theta. \tag{1.68}$$

This is also known as the Kerr black hole solution [10]. It depends on mass and angular momentum $J = Ma$ here J is angular momentum. Event horizons are

$$r_+ = M + \sqrt{M^2 - a^2}, \quad r_- = M - \sqrt{M^2 - a^2}. \tag{1.69}$$

It describes geometry of a spacetime outside an uncharged and rotating compact object.

1.6.4 The Kerr-Neuman Solution

The Kerr-Neuman solution is generalization of the Reissner-Nordstrom solution [9]. Following ansatz are used

1. Source of point mass M and charge Q is rotating.
2. Source is surrounded by vacuum.
3. Spacetime is static and asymptotically flat.

The line element in the Boyer-Linquist coordinates is given as

$$\begin{aligned} ds^2 =& \frac{\Delta}{\rho^2}\left(dt - asin^2\theta d\phi\right)^2 - \frac{sin^2\theta}{\rho^2}\left(\left(r^2+a^2\right)d\phi - adt\right)^2 - \\ & \frac{\rho^2}{\Delta}dr^2 - \rho^2 d\theta^2, \end{aligned} \tag{1.70}$$

where

$$\Delta^2 = r^2 - 2Mr + a^2 + Q^2, \quad \rho^2 = r^2 + a^2cos^2\theta. \tag{1.71}$$

Inner and outer event horizons are

$$r_+ = M + \sqrt{M^2 - a^2 - Q^2}, \qquad r_- = M - \sqrt{M^2 - a^2 - Q^2}. \tag{1.72}$$

It describes geometry of a spacetime outside a charged and rotating compact object.

Chapter 2

Model of Uncharged Anisotropic Fluid Distribution

2.1 Introduction

Finding analytic solutions of the Einstein field equations is of great interest among astrophysicist and cosmologists. Some solutions of the field equations explain behaviour of compact objects like white dwarf, neutron star, pulsars etc. Generally, some equation of state is assumed to construct a physically viable model in the framework of general relativity.

Some spherically symmetric models assuming linear equation of state having isotropic pressure are developed in [11–15]. Some spherically symmetric models having non linear equation of state have been discussed in [16–20]. Ruderman [21] and Canuto [22] suggested that the stellar objects having density greater than $10^{15} gm/cm^3$ may have pressure anisotropy in the interior. Multiple factors such as presence of superfluid [23], strong electromagnetic field [24], viscosity [25] or phase transition [26] can be responsible for anisotropy. Liang and Bowers [27] discussed redshift and mass equilibrium of stellar objects composed of anisotropic matter distribution. Stability conditions for compact models are discussed by Pant and Fuloria [28]. Herrera [29] Maurya and Gupta [30] showed that the region is stable where radial speed of sound dominates over tangential speed of sound.

Paraboloidal metric was first used by Skea and Finch [31] to develop a stellar model and later on Jotania and Tikeraker [32] and Thomas and Pandya [33] developed some models taking linear equation of state. Thirukkanesh and Sharma and Das [35] extended the work for polytropic equation of state $p_r = k\rho^{1+\frac{1}{n}} - \beta$ for $n = 1, 2$, which is reviewed in the remaining part of this chapter.

2.2 Paraboloidal Spacetime Metric

A 3-paraboloid embedded in a 4-dimensional Euclidean space has equation (in Cartesian coordinates)

$$x^2 + y^2 + z^2 = 2Lw. \tag{2.1}$$

The section where w is constant are spheres, while sections where one of x, y, z is constant gives 2-paraboloids. Euclidean metric is given as

$$ds^2 = dw^2 + dx^2 + dy^2 + dz^2. \tag{2.2}$$

Using the transformation

$$w = \frac{r^2}{2L}, \quad x = r\sin\theta\cos\phi, \quad y = r\sin\theta\sin\phi, \quad z = r\cos\theta, \tag{2.3}$$

metric in eq (2.2) becomes

$$ds^2 = \left(1 + \frac{r^2}{L^2}\right) dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right). \tag{2.4}$$

In Schwarzchild coordinates (t, r, θ, ϕ) static and spherically symmetric metric is

$$ds^2 = -e^{2\nu(r)} + e^{2\lambda(r)}dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right). \tag{2.5}$$

$t =$ constant hypersurface[1] of the metric in eq (2.5) is

$$ds^2 = e^{2\lambda(r)}dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right). \tag{2.6}$$

Comparing eq (2.4) and (2.6), we get

$$e^{2\lambda(r)} = 1 + \frac{r^2}{L^2}, \tag{2.7}$$

where $e^{2\nu}$ and $e^{2\lambda}$ are metric potentials. The spacetime in eq (2.5) along with eq (2.7) has paraboloidal geometry characterized by parameter $L \neq 0$ which has units of length.

[1]Hypersurface is (n-1)-dimension submanifold of an n-dimesnion manifold [2].

2.3 The Einstein Field Equations

We are going to develop a model of static spherically symmetric anisotropic fluid distribution admitting a quadratic equation of state. The Einstein field equations given in eq (1.51) can also be written as

$$R^a{}_b - \frac{1}{2} R\delta^a{}_b = T^a{}_b. \tag{2.8}$$

The non zero components of the Christoffel symbols for the metric (2.6) are

$$\begin{aligned}
\Gamma^0_{10} = \Gamma^0_{01} =& \nu',\\
\Gamma^1_{00} =& e^{2\nu-2\lambda}\nu',\\
\Gamma^1_{11} =& \lambda',\\
\Gamma^1_{22} =& -re^{-2\lambda},\\
\Gamma^1_{33} =& -r\sin^2\theta e^{-2\lambda},\\
\Gamma^2_{21} =& \frac{1}{r},\\
\Gamma^2_{33} =& -\cos\theta\sin\theta,\\
\Gamma^3_{31} =& \frac{1}{r},\\
\Gamma^3_{32} =& \frac{\cos\theta}{\sin\theta}.
\end{aligned} \tag{2.9}$$

Inserting values of these symbols in eq (1.27), we obtain non zero components of Ricci tensor as

$$R^0{}_0 = e^{-2\lambda}\left(\nu'' - \nu'\lambda' + \nu'^2 + \frac{2\nu'}{r}\right), \tag{2.10}$$

$$R^1{}_1 = -e^{-2\lambda}\left(\nu'' - \nu'\lambda' + \nu'^2 - \frac{2\lambda'}{r}\right), \tag{2.11}$$

$$R^2{}_2 = \frac{1}{r^2} - \frac{e^{2\nu-2\lambda}}{r^2}\left(1 + r\nu' + r\lambda' - 2r\lambda'\right), \tag{2.12}$$

$$R^3{}_3 = \frac{1}{r^2}R_{22}. \tag{2.13}$$

By inserting these values in eq (1.30) we get the Ricci scalar as

$$R = \frac{2}{r^2} - 2e^{2\lambda}\left(\nu'' - \nu'\lambda' + \nu'^2 + \frac{2\nu'}{r} - \frac{2\lambda'}{r} + \frac{1}{r^2}\right). \tag{2.14}$$

For our case the stress energy tensor of anisotropic fluid is given as

$$T^i{}_j = (-\rho, p_r, p_t, p_t)\,. \tag{2.15}$$

Inserting values in eq (2.8) we obtain the Einstein field equations as

$$\begin{aligned} \frac{1}{r^2}\left[r\left(1-e^{-2\lambda}\right)\right]' &= \rho, && (2.16)\\ -\frac{1}{r^2}\left(1-e^{-2\lambda}\right)\frac{2\nu'}{r}e^{-2\lambda} &= p_r, && (2.17)\\ e^{-2\lambda}\left(v''+v'^2+\frac{v'}{r}-v'\lambda'-\frac{\lambda'}{r}\right) &= p_t. && (2.18)\end{aligned}$$

Here prime denotes differentiation with respect to r. Utilizing the following transformations [33]

$$xL^2 = r^2, \qquad y^2 = e^{2\nu(r)}, \qquad Z(x) = e^{-2\lambda(r)} = \frac{1}{1+x}. \tag{2.19}$$

in eqs (2.16)-(2.18) we get

$$\begin{aligned} \rho &= \frac{1}{L^2}\left[\frac{1}{1+x}+\frac{2}{(1+x)^2}\right], && (2.20)\\ p_r &= \frac{1}{L^2}\left[\frac{4}{1+x}\frac{\dot{y}}{y}-\frac{1}{1+x}\right], && (2.21)\\ p_t &= \frac{1}{L^2}\left[\frac{4x}{1+x}\frac{\ddot{y}}{y}+\left(\frac{4}{1+x}-\frac{2x}{(1+x)^2}\right)\frac{\dot{y}}{y}+\frac{1}{(1+x)^2}\right], && (2.22)\end{aligned}$$

$$=\frac{1}{L^2}\left[4xZ\left(\frac{d}{dx}\left(\frac{\dot{y}}{y}\right)+\left(\frac{\dot{y}}{y}\right)^2\right)+\left(4Z+2x\dot{Z}\right)\frac{\dot{y}}{y}+\dot{Z}\right].$$

Here $\frac{\ddot{y}}{y} = \left(\frac{d}{dx}\left(\frac{\dot{y}}{y}\right)+\left(\frac{\dot{y}}{y}\right)^2\right)$. The anisotropic factor denoted by Δ has the expression

$$\Delta = p_t - p_r = \frac{1}{L^2}\left[\frac{4x}{1+x}\ddot{y}-\frac{1}{(1+x)^2}\left(1+2x\frac{\dot{y}}{y}\right)+\frac{1}{1+x}\right]. \tag{2.23}$$

Notice that system of equations (2.20)-(2.22) involves four unknowns $\rho,\ p_r,\ p_t$ and y.

2.4 Solution of the Einstein Field Equations

We assume quadratic equation of state of the form

$$p_r = \alpha\rho^2 - \gamma. \tag{2.24}$$

Here $\alpha > 0$ and γ are constants. Using eqs (2.20) and (2.21) into eq (2.24) we get

$$\frac{\dot{y}}{y} = \frac{\alpha\,(3+x)^2}{4L^2\,(1+x)^3} + \frac{1}{4} - \gamma\frac{L^2(1+x)}{4}. \tag{2.25}$$

This is an ordinary differential equation with independent variable x and unknown y. Integrating eq (2.25) yields

$$\ln y = \frac{1}{8}\left(x(2-\gamma L^2\,(2+x)) - \frac{4\alpha\,(3+2x)}{L^2\,(1+x)^2}\right) + \ln d_1\,(1+x)^{\frac{\alpha}{4L^2}}. \tag{2.26}$$

Taking exponential on both sides of eq (2.26) we get

$$y = d_1\,(1+x)^{\frac{\alpha}{4L^2}}\, exp\left[\frac{1}{8}\left(x(2-\gamma L^2\,(2+x)) - \frac{4\alpha\,(3+2x)}{L^2\,(1+x)^2}\right)\right]. \tag{2.27}$$

d_1 is a constant of integration. Utilizing eq (2.27) in governing eqs (2.19)-(2.23) we obtain

$$e^{2\nu} = {d_1}^2\,(1+x)^{\frac{\alpha}{2L^2}}\, exp\left[\frac{1}{4}\left(x(2-\gamma L^2\,(2+x)) - \frac{4\alpha\,(3+2x)}{L^2\,(1+x)^2}\right)\right], \tag{2.28}$$

$$\rho = \frac{3+x}{L^2\,(1+x)^2}, \tag{2.29}$$

$$p_r = \frac{\alpha\,(3+x)^2}{L^4\,(1+x)^4} - \gamma, \tag{2.30}$$

$$\begin{aligned} p_t =& \frac{1}{4L^6\,(1+x)^7}\left[x\left((3+x)\left(L^4\,(1+x)^5 + \alpha^2\,(3+x)^3\right) + 2\alpha L^2\right.\right. \\ & \left.(1+x)^2\left(x^2+x-14\right)\right) - 2L^4\,(1+x)^4\left(L^2\,(1+x)^2\,(2+x) + \right. \\ & \left.\left.\alpha\,(3+x)^2\gamma + L^8\,(1+x)^8\gamma^2\right) + 4\alpha L^2\,(3+x)^2\,(1+x)^3\right] - \gamma. \end{aligned} \tag{2.31}$$

$$\begin{aligned} \Delta =& \frac{1}{4L^6\,(1+x)^7}\left[x\left((3+x)\left(L^4\,(1+x)^5 + \alpha^2\,(3+x)^3\right) + 2\alpha L^2\,(1+x)^2\right.\right. \\ & \left.\left(x^2+x-14\right)\right) - 2L^4\,(1+x)^4\left(L^2\,(1+x)^2\,(2+x)\right. \\ & \left.\left.+\alpha\,(3+x)^2\gamma + L^8\,(1+x)^8\gamma^2\right)\right]. \end{aligned} \tag{2.32}$$

Now reverse substituting, $x = \frac{r^2}{L^2}$, in governing eqs (2.27)-(2.32) yields

$$y = d_1 \left(1 + \frac{r^2}{L^2}\right)^{\frac{\alpha}{4L^2}} exp\left[\frac{1}{8}\left(\frac{r^2}{L^2}\left(2 - \gamma\left(2L^2 + r^2\right)\right) - \frac{4\alpha\left(3L^2 + 2r^2\right)}{\left(L^2 + r^2\right)^2}\right)\right]. \tag{2.33}$$

$$e^{2\lambda} = 1 + \frac{r^2}{L^2}, \tag{2.34}$$

$$e^{2\nu} = {d_1}^2 \left(1 + \frac{r^2}{L^2}\right)^{\frac{\alpha}{2L^2}} exp\left[\frac{1}{4}\left(\frac{r^2}{L^2}\left(2 - \gamma\left(2L^2 + r^2\right)\right) - \frac{4\alpha\left(3L^2 + 2r^2\right)}{\left(L^2 + r^2\right)^2}\right)\right], \tag{2.35}$$

$$\rho = \frac{3L^2 + r^2}{\left(L^2 + r^2\right)^2}, \tag{2.36}$$

$$p_r = \frac{\alpha(3L^2 + r^2)^2}{\left(L^2 + r^2\right)^4} - \gamma, \tag{2.37}$$

$$\begin{aligned} p_t = &\frac{L^8}{4\left(L^2 + r^2\right)^7}\left[\frac{r^2}{L^2}\left(\left(3 + \frac{r^2}{L^2}\right)\left(L^4\left(1 + \frac{r^2}{L^2}\right)^5 + \alpha^2\left(3 + \frac{r^2}{L^2}\right)^3\right)\right.\right. \\ &\left.\left. + 2\alpha L^2\left(1 + \frac{r^2}{L^2}\right)^2\left(\left(\frac{r^2}{L^2}\right)^2 + \frac{r^2}{L^2} - 14\right)\right) - 2L^4\left(1 + \frac{r^2}{L^2}\right)^4 \right. \\ &\left(L^2\left(1 + \frac{r^2}{L^2}\right)^2\left(2 + \frac{r^2}{L^2}\right) + \alpha\left(3 + \frac{r^2}{L^2}\right)^2 + L^8\left(1 + \frac{r^2}{L^2}\right)^8\gamma^2\right)\gamma \\ &\left. + 4\alpha L^2\left(3 + \frac{r^2}{L^2}\right)^2\left(1 + \frac{r^2}{L^2}\right)^3\right] - \gamma, \end{aligned} \tag{2.38}$$

$$\begin{aligned} \Delta = &\frac{L^8}{4\left(L^2 + r^2\right)^7}\left[\frac{r^2}{L^2}\left(\left(3 + \frac{r^2}{L^2}\right)\left(L^4\left(1 + \frac{r^2}{L^2}\right)^5 + \alpha^2\left(3 + \frac{r^2}{L^2}\right)^3\right)\right.\right. \\ &\left.\left. + 2\alpha L^2\left(1 + \frac{r^2}{L^2}\right)^2\left(\left(\frac{r^2}{L^2}\right)^2 + \frac{r^2}{L^2} - 14\right)\right) - 2L^4\left(1 + \frac{r^2}{L^2}\right)^4 \right. \\ &\left.\left(L^2\left(1 + \frac{r^2}{L^2}\right)^2\left(2 + \frac{r^2}{L^2}\right) + \alpha\left(3 + \frac{r^2}{L^2}\right)^2 + L^8\left(1 + \frac{r^2}{L^2}\right)^8\gamma^2\right)\gamma\right]. \end{aligned} \tag{2.39}$$

2.5 Exterior spacetime and Boundary conditions

Exterior spacetime of a static and spherically symmetric object is described by the Schwarzchild metric given in eq (1.61). Mass contained in sphere of radius r is given as

$$m(r) = \frac{1}{2}\int_0^r \omega^2 \left(\rho(\omega)\right) d\omega. \tag{2.40}$$

Inserting eq (2.20) into eq (2.40) we get

$$m(r = R) = M = \frac{R^3}{2(L^2 + R^2)}. \tag{2.41}$$

Rearranging (2.41) we can find model parameter as

$$L = \sqrt{\frac{R^3}{2M}\left(1 - \frac{2M}{R}\right)}. \tag{2.42}$$

At the boundary $r = R$ interior spacetime should match the Schwarzchild metric. Equating eq (1.61) with eq (2.5) we obtain

$$e^{2\nu(r=R)} = \left(1 - \frac{2M}{R}\right). \tag{2.43}$$

We can evaluate value of the integration constant, d_1, by substituting eqs (2.35) and (2.41) in eq (2.43) and simplifying as

$${d_1}^2 = \left(1 + \frac{R^2}{L^2}\right)^{-\left(1+\frac{\alpha}{2L^2}\right)} \exp\left[\frac{4\alpha\left(3L^2 + 2R^2\right)}{\left(L^2 + R^2\right)^2} - \frac{R^2}{L^2}\left(2 - \gamma\left(2L^2 + R^2\right)\right)\right]. \tag{2.44}$$

Radial pressure vanishes at the surface of the boundary i.e. $p_r|_{r=R} = 0$. Utilizing this boundary condition in eq (2.30) we get

$$\gamma = \frac{\alpha\left(3L^2 + R^2\right)^2}{\left(L^2 + R^2\right)^4}. \tag{2.45}$$

Substituting eq (2.42) into eq (2.45) we have

$$\gamma = \frac{4\alpha M^2\left(4M - 3R\right)^2}{R^8}. \tag{2.46}$$

2.6 Physical Analysis of the Solution

The following conditions must be satisfied by a feasible compact object

1. Metric potentials and matter variables should be well behaved and free from singularities.

2. Energy density should be positive and monotonically decreasing towards the boundary, i.e.

$$\rho(r) \geq 0, \quad \frac{d\rho}{dr} \leq 0. \tag{2.47}$$

3. Radial and tangential pressures should be positive and their gradients should be negative, i.e.

$$p_r \geq 0, \quad p_t \geq 0, \tag{2.48}$$
$$\frac{dp_r}{dr} \leq 0, \quad \frac{dp_t}{dr} \leq 0. \tag{2.49}$$

4. Mass to radius ratio should be less than or equal to $\frac{4}{9}$ (Buchdahl Limit [34]), i.e.

$$\frac{M}{R} \leq \frac{4}{9}. \tag{2.50}$$

5. Anisotropic factor should be zero at the centre, i.e.

$$p_r(r=0) = p_t(r=0) = 0. \tag{2.51}$$

6. The redshift should be well defined and positive at the centre and the boundary.

$$g_{11} \geq 0. \tag{2.52}$$

If a solution satisfies these conditions then it can be considered a potential model of a compact object.

Our model has following features:

(I) Metric potential, $e^{2\lambda}$, given in eq (2.34) is well behaved and no geometric singularites are present. Behaviour of $e^{2\lambda}$ is displayed in Fig. 2.1. Metric potential, $e^{2\nu}$, is also well defined. Behaviour of $e^{2\nu}$ is presented in Fig. 2.2.

(II) Energy density at centre of the compact object is

$$\rho(0) = \frac{3}{L^2}, \tag{2.53}$$

which is positive and from eq (2.36) it is clear that it remains positive and finite throughout the stellar interior. Graph in Fig. 2.3 shows its behaviour.

(III) Central radial and tangential pressures are

$$p_r(0) = p_t(0) = \frac{9\alpha}{L^4} - \gamma. \tag{2.54}$$

These are positive if $\frac{9\alpha}{L^4} > \gamma$. Radial and tangential pressures are finite throughout all the interior except the boundary where p_r vanishes. Behaviour of p_r and p_t are shown in Fig. 2.4.

(IV) As radial and tangential pressures are equal at the centre so anisotropy vanishes at the centre. p_t is lagging behind p_r upto some radial distance. After that p_t takes over. Anisotropy achieves its peak at the boundary. Behaviour of anisotropic factor is represented in Fig. 2.5.

(V) Surface Red shift (SRF) depends on metric component g_{11}. SRF at the origin and boundary is finite and positive.

$$\text{Redshift at origin} = Z(0) = 1, \tag{2.55}$$

$$\text{Redshift at boundary} = Z(R) = 1 + \frac{R^2}{L^2}. \tag{2.56}$$

(VI) Mass to radius ratio is 0.25697 which is less than $\frac{4}{9}$.

(VII) Taking derivative of eqs (2.36) with respect to r we get

$$\frac{d\rho}{dr} = -\frac{2r\left(5L^2 + r^2\right)}{\left(L^2 + r^2\right)^3}. \tag{2.57}$$

Gradient of the density is negative as r and L^2 are non negative which implies density is monotonically decreasing. This behaviour is represented in Figure 2.6.

(VIII) Taking derivative of eq (2.37) and eq (2.38) with respect to r we get

$$\frac{dp_r}{dr} = \frac{-4\alpha r\left(3L^2+r^2\right)\left(5L^2+r^2\right)}{\left(L^2+r^2\right)^5}, \tag{2.58}$$

$$\begin{aligned}\frac{dp_t}{dr} =& \frac{1}{2L^2\left(L^2+r^2\right)^8}\Big[r\Big(\alpha^2\left(3L^2+r^2\right)^3\left(3L^4-2r^4-13L^2r^2\right)- \\ & 2\alpha\left(L^2+r^2\right)^2\left(102L^8-54L^6r^2-9L^4r^4+8L^2r^6+r^8+3L^2\right. \\ & \left.\times\left(L^2-r^2\right)\left(L^2+r^2\right)^2\left(3L^2+r^2\right)\gamma\right)+\left(L^2+r^2\right)^5\left(3L^4\right. \\ & -L^2r^2-2\left(L^2+r^2\right)\left(2L^4+2L^2r^2+r^4\right)\gamma+\left(L^2+r^2\right)^3 \\ & \left.\times\left(L^2+2r^2\right)\gamma^2\right)\Big)\Big].\end{aligned} \tag{2.59}$$

Here gradient of p_r is negative as evident from its negative sign ($\alpha > 0$) and gradient of p_t is also negative which implies both pressures are monotonically decreasing. Graphical representation of gradients is given in Figure 2.7.

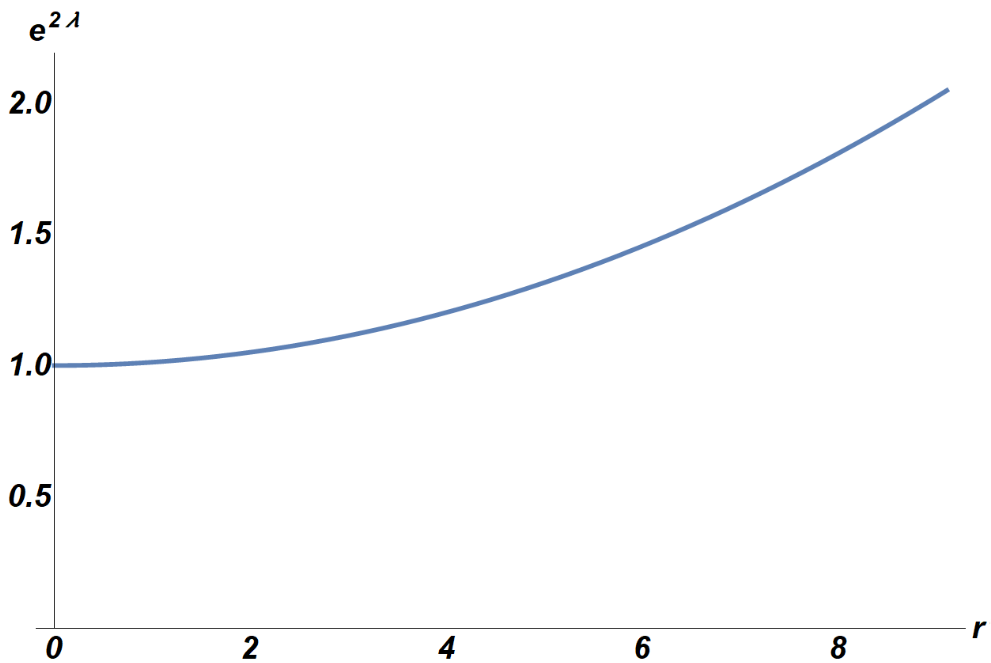

Figure 2.1: The metric potential, $e^{2\lambda}$, plotted against radial parameter r by selecting $R = 9.1$ and $L = 8.89$ [35].

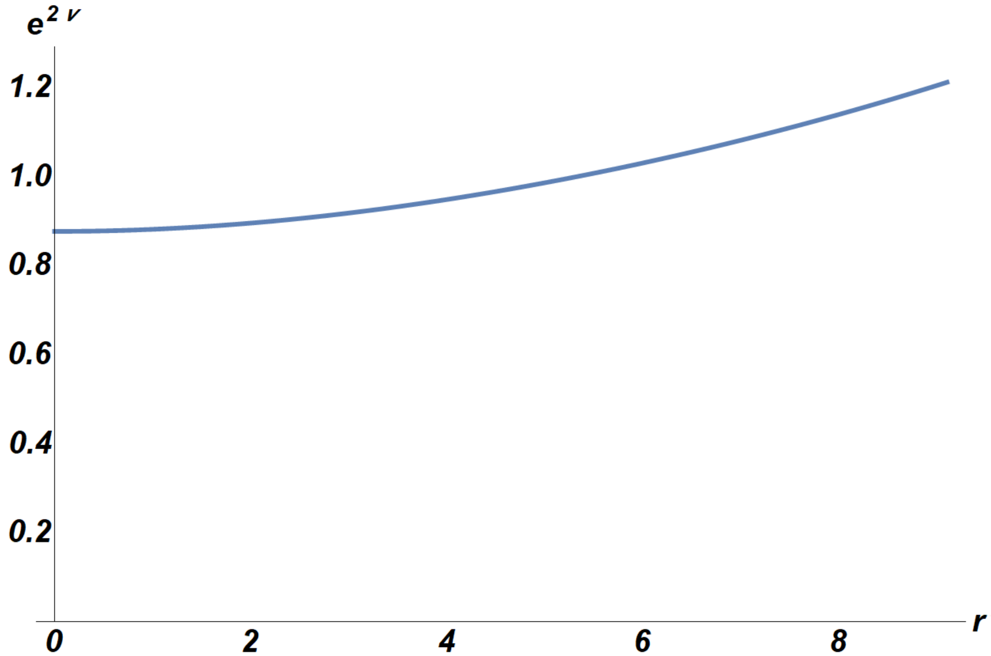

Figure 2.2: The metric potential, $e^{2\nu}$, plotted against radial parameter r by selecting $M = 1.58M_S$, $R = 9.1$, $L = 8.89$, $\alpha = 7$, $\delta = .00104522$ and $d_1 = 0.576662$ [35].

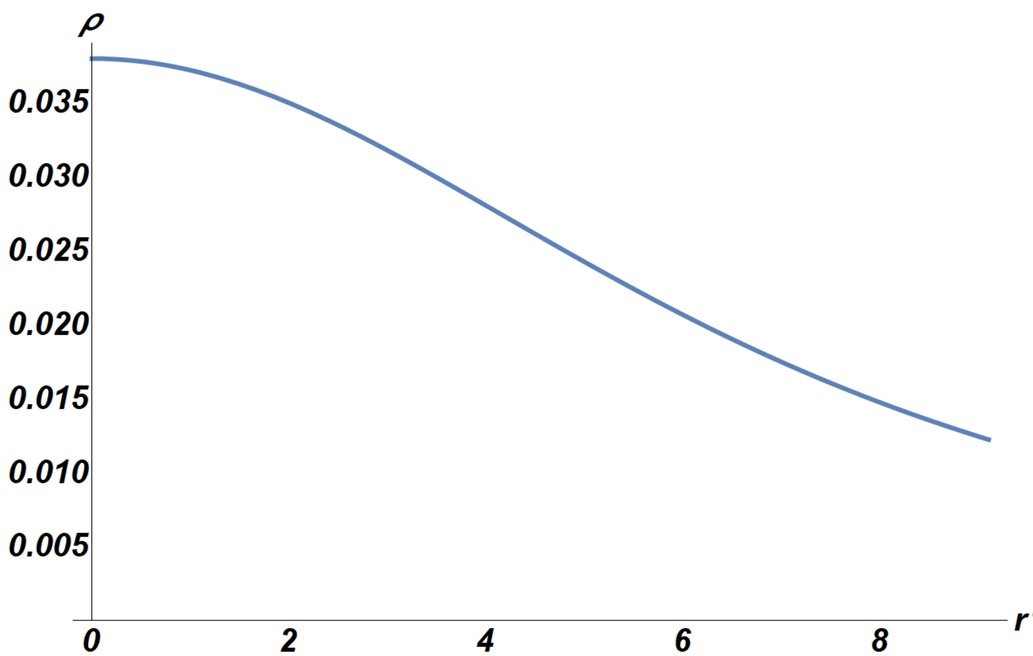

Figure 2.3: ρ plotted against r by selecting $M = 1.58M_S$, $R = 9.1$, $L = 8.89$, $\alpha = 7$, $\delta = .00104522$ and $d_1 = 0.576662$.

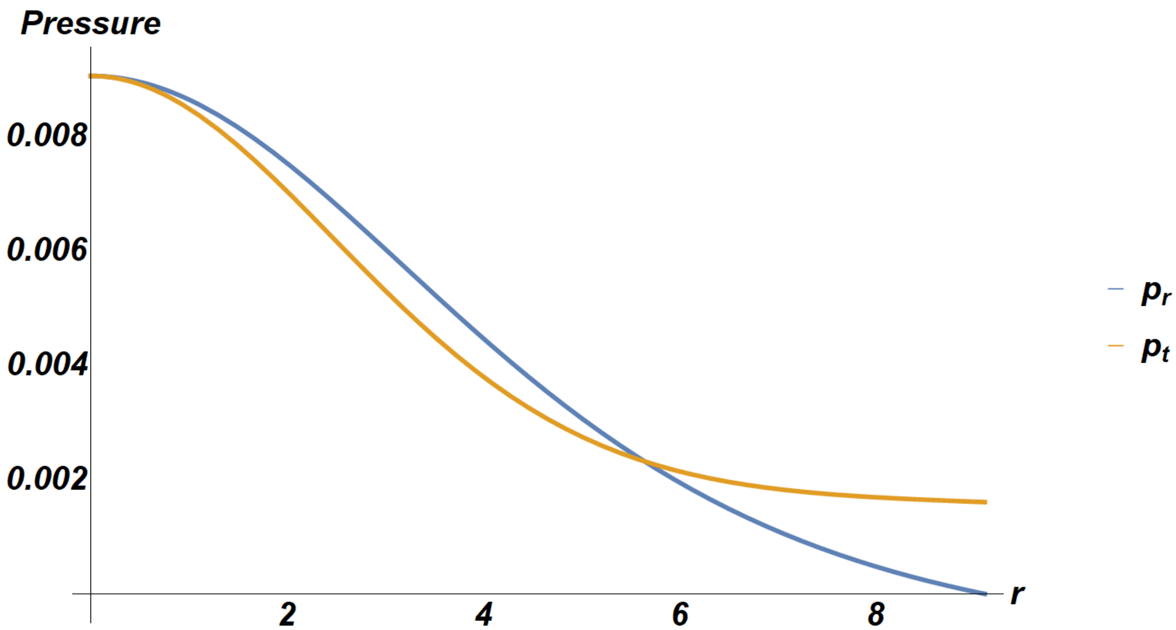

Figure 2.4: p_r and p_t plotted against r by choosing $M = 1.58M_S$, $R = 9.1$, $L = 8.89$, $\alpha = 7$, $\delta = .00104522$ and $d_1 = 0.576662$.

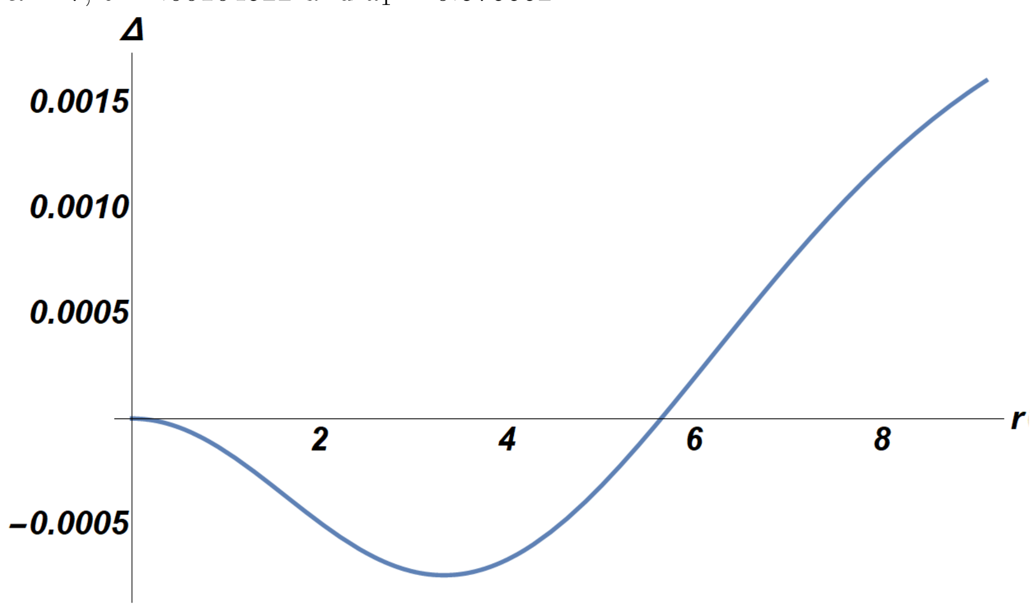

Figure 2.5: Difference of pressures are plotted against r by selecting $M = 1.58M_S$, $R = 9.1$, $L = 8.89$, $\alpha = 7$, $\delta = .00104522$ and $d_1 = 0.576662$.

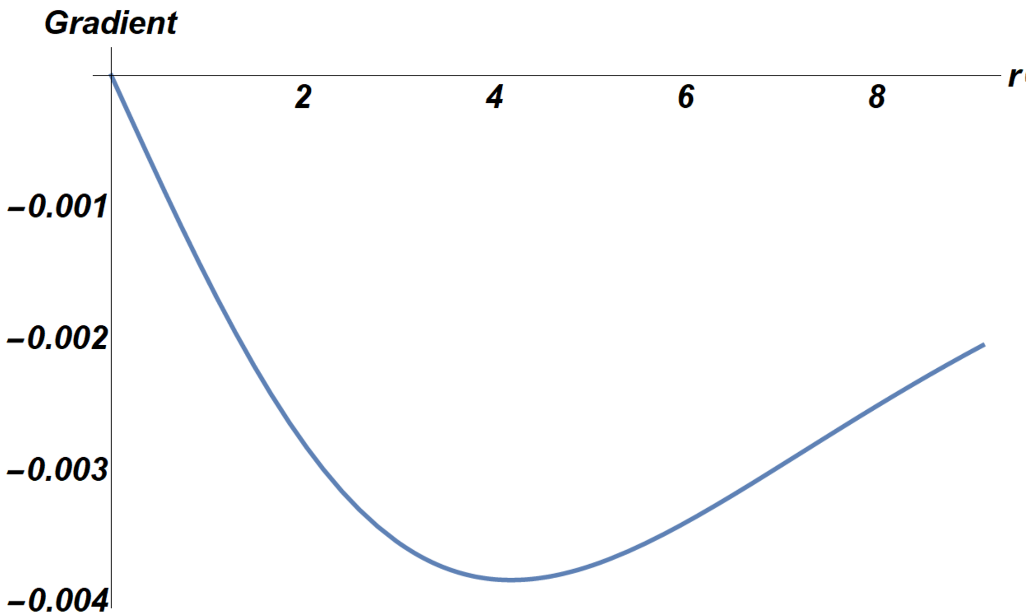

Figure 2.6: Gradient of density is plotted against r by selecting $M = 1.58M_S$, $R = 9.1$, $L = 8.89$, $\alpha = 7$, $\delta = .00104522$ and $d_1 = 0.576662$.

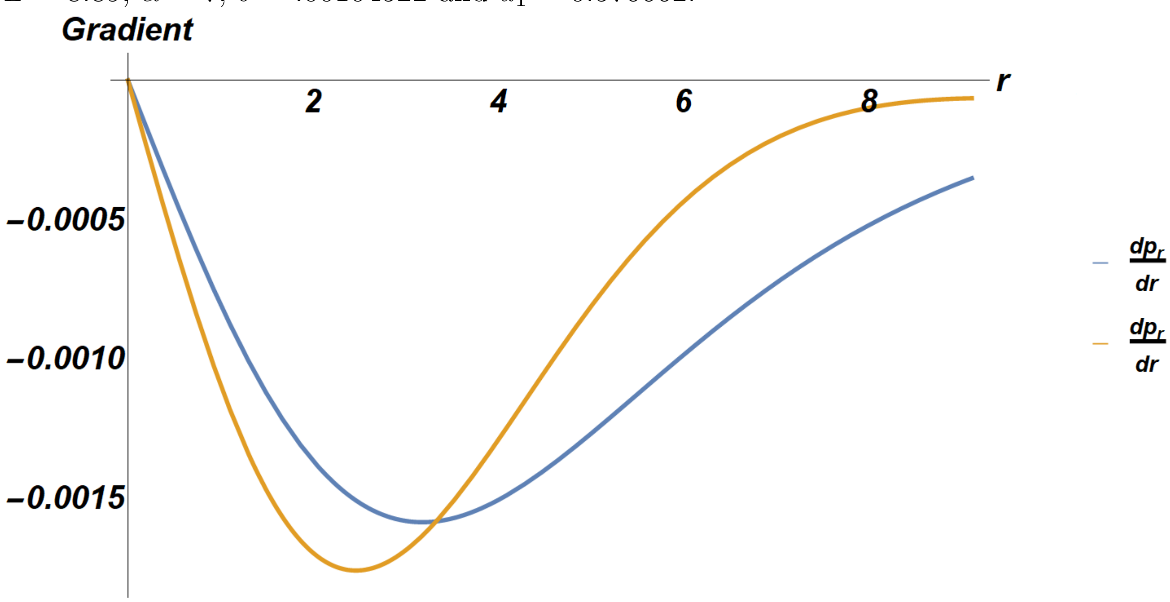

Figure 2.7: Gradient of pressures are plotted against r by selecting $M = 1.58M_S$, $R = 9.1$, $L = 8.89$, $\alpha = 7$, $\delta = .00104522$ and $d_1 = 0.576662$.

2.7 Stability Analysis

A model of compact object is considered good if it is stable. In this section we analyse two conditions for our models to be stable and determine stability region.

2.7.1 Energy Conditions

Matter content in a compact object must satisfy certain energy requirements are known as energy conditions. These conditions are in the form of inequalities and should be fulfilled within the stellar structure. Mathematical form of these conditions are:

1. Null Energy Condition $\rho \geq 0$,
2. Weak Energy Condition $\rho + p_r \geq 0$, $\rho + p_t \geq 0$,
3. Strong Energy Condition $\rho + p_r + 2p_t \geq 0$.

ρ is positive as discussed earlier so null energy condition is satisfied. As $\rho,\ p_r,\ p_t \geq 0$ so addition of any of these also gives a positive real number. Hence weak and strong energy conditions are also satisfied. Graph of energy conditions are presented in Figure 2.8.

2.7.2 Causality Condition

Speed of light is unity in relativistic units and speed of light is the maximum attainable speed, so for a physically stable model speed of sound along transverse and radial direction must be in the interval $[0, 1]$. Differentiating eq (2.24) with respect to ρ we get v_r which represents speed of sound in radial direction.

$$v_r^2 = \frac{dp_r}{d\rho} = 2\alpha\rho = \frac{2\alpha\left(3L^2 + r^2\right)}{\left(L^2 + r^2\right)^2}, \tag{2.60}$$

For speed of sound in transverse direction we make use of chain rule.

$$\begin{aligned} v_t^2 =& \frac{dp_t}{d\rho} = \frac{dp_t}{dr}\frac{dr}{d\rho} \\ =& -\frac{1}{4L^2\left(L^2+r^2\right)^5\left(5L^2+r^2\right)}\left[\alpha^2\left(3L^2+r^2\right)^3\left(3L^4-2r^4\right.\right. \\ & \left.-13L^2r^2\right)-2\alpha\left(L^2+r^2\right)^2\left(102L^8-54L^6r^2-9L^4r^4+8L^2r^6\right. \\ & \left.+r^8+3L^2\left(L^2-r^2\right)\left(L^2+r^2\right)^2\left(3L^2+r^2\right)\gamma\right)+\left(L^2+r^2\right)^5 \\ & \left(3L^4-L^2r^2-2\left(L^2+r^2\right)\left(2L^4+2L^2r^2+r^4\right)\gamma+\left(L^2+r^2\right)^3\right. \\ & \left.\left.\left(L^2+2r^2\right)\gamma^2\right)\right]. \end{aligned} \tag{2.61}$$

In Figure 2.9 graph is shown for certain values of parameters. It shows that the speed of sound lies between 0 and 1. Hence causality condition is satisfied.

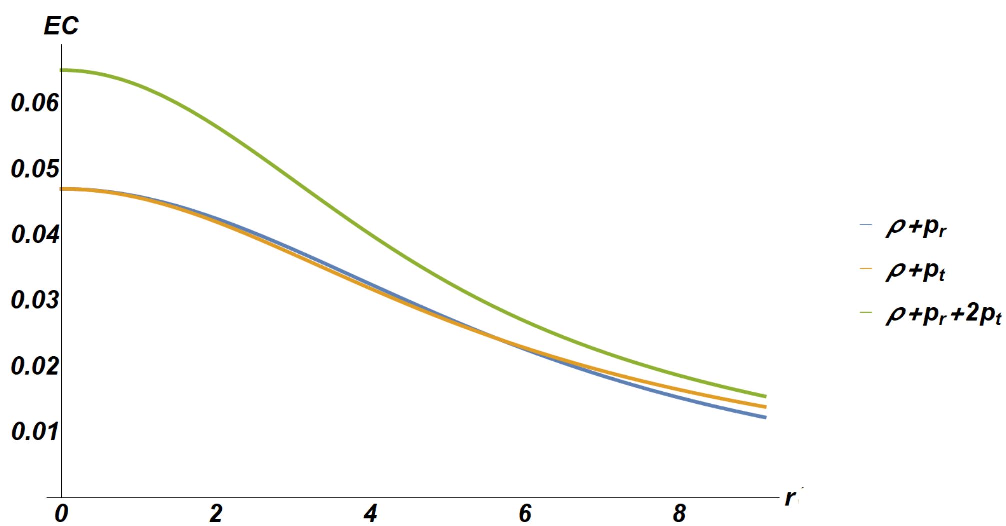

Figure 2.8: Energy conditions are plotted against r by choosing $M = 1.58M_S$, $R = 9.1$, $L = 8.89$, $\alpha = 7$, $\delta = .00104522$ and $d_1 = 0.576662$.

2.7.3 Region of Stability

The region where speed of sound in radial direction is greater than the speed of sound in transverse direction is known as stable region [29, 30]. Stability parameter Sp is shown in Figure 2.10. v_r is lagging behind v_t upto $r = 3.5$ km. After that v_r takes

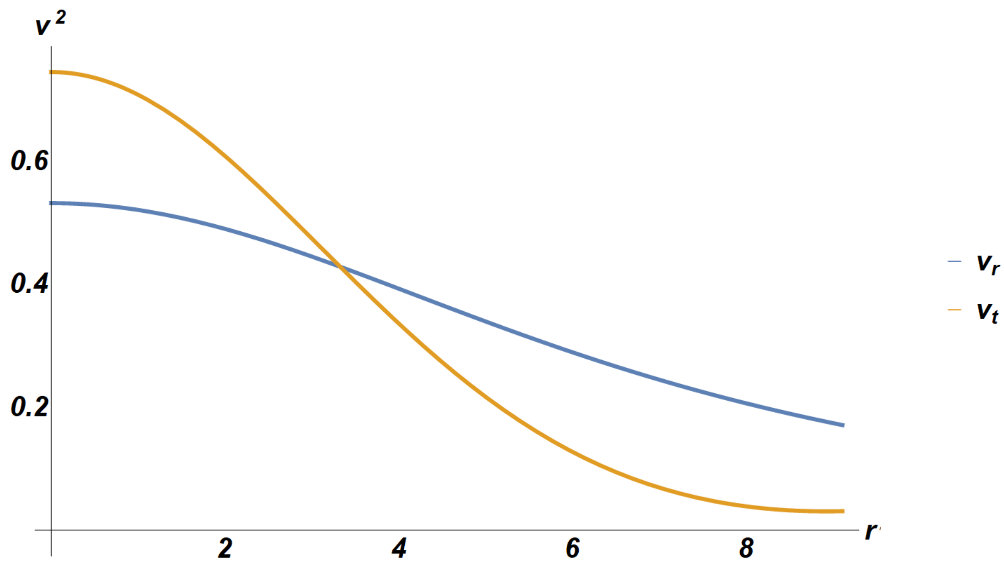

Figure 2.9: Speeds of sound are plotted against r for $M = 1.58M_S$, $R = 9.1$, $L = 8.89$, $\alpha = 7$, $\delta = .00104522$ and $d_1 = 0.576662$.

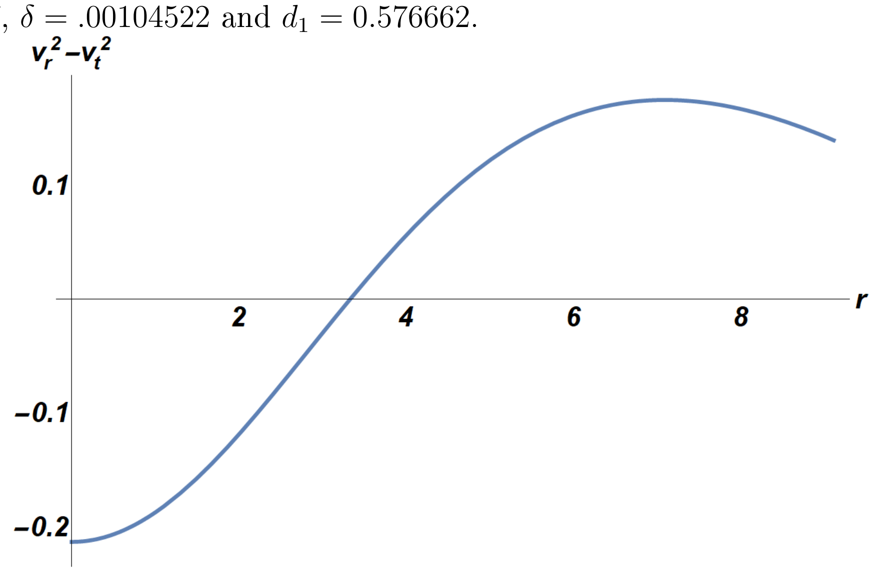

Figure 2.10: Behaviour of stability parameter against r for $M = 1.58M_S$, $R = 9.1$, $L = 8.89$, $\alpha = 7$, $\delta = .00104522$ and $d_1 = 0.576662$ is depicted.

over v_t, so we have the stable region. Stable region can be varied by choosing different values of parameters.

Chapter 3

Models of Charged Anisotropic Fluid Distribution

The interior spacetime metric is described by eq (2.5) and eq (2.7). The stress energy tensor of charged anisotropic fluid is obtained by adding eq (1.42) and eq (1.44) as

$$T^a_{\ b} = (\rho + p_t)\, v^a v_b + (p_r - p_t)\, \chi^a \chi_b - p_t g^a_{\ b} + E^a_{\ b}. \tag{3.1}$$

Here the electromagnetic field stress tensor is taken as

$$E^a_{\ b} = \begin{bmatrix} E^2 & 0 & 0 & 0 \\ 0 & -E^2 & 0 & 0 \\ 0 & 0 & E^2 & 0 \\ 0 & 0 & 0 & E^2 \end{bmatrix}. \tag{3.2}$$

Inserting values of v, $\chi$and $E^a_{\ b}$ from eqs (1.43)-(3.2), we get

$$T^a_{\ b} = \left(-\rho + E^2, p_r - E^2, p_t + E^2, p_t + E^2\right). \tag{3.3}$$

E is the electric field intensity. EMFEs for metric (2.5) with eq (2.7) and the stress energy tensor given in eq (3.3) are

$$\frac{1}{r^2}\left[r\left(1 - e^{-2\lambda}\right)\right]' = \rho + E^2, \tag{3.4}$$

$$\frac{-1}{r^2}\left(1 - e^{-2\lambda}\right)\frac{2\nu'}{r}e^{-2\lambda} = p_r - E^2, \tag{3.5}$$

$$e^{-2\lambda}\left(\nu'' + \nu'^2 + \frac{\nu'}{r} - \nu'\lambda' - \frac{\lambda'}{r}\right) = p_t + E^2. \tag{3.6}$$

$$\frac{4L^4}{r^2\left(L^2+r^2\right)}\left(\frac{r^2}{L^2}\dot{E}+E\right)^2 = \sigma^2, \tag{3.7}$$

where σ^2 represents the charge density and prime denotes differentiation with respect to r. By utilizing transformations proposed in eq (2.19) we get

$$\rho = \frac{1}{L^2}\left[\frac{1}{1+x}+\frac{2}{(1+x)^2}\right]-E^2, \tag{3.8}$$

$$p_r = \frac{1}{L^2}\left[\frac{4}{1+x}\frac{\dot{y}}{y}-\frac{1}{1+x}\right]+E^2, \tag{3.9}$$

$$p_t = \frac{1}{L^2}\left[\frac{4x}{1+x}\frac{\ddot{y}}{y}+\left(\frac{4}{1+x}-\frac{2x}{(1+x)^2}\right)\frac{\dot{y}}{y}+\frac{1}{(1+x)^2}\right]-E^2, \tag{3.10}$$

$$\sigma^2 = \frac{4}{x\left(1+x\right)}\left(x\dot{E}+E\right)^2. \tag{3.11}$$

Here dot denotes derivative with respect to x. The anisotropic factor denoted by Δ has the expression

$$\Delta = p_t - p_r = \frac{1}{L^2}\left[\frac{4x}{1+x}\ddot{y}-\frac{1}{(1+x)^2}\left(1+2x\frac{\dot{y}}{y}\right)+\frac{1}{1+x}\right]-2E^2. \tag{3.12}$$

Notice that the system of three eqs (3.8)-(3.10) involves five unknowns namely y, ρ, E, p_r and p_t. We assume electric field intensity of the form

$$E^2 = \frac{kx^2}{(1+x)^2}, \tag{3.13}$$

where k is a constant.

3.1 Model 1

We choose linear equation of state of the form

$$p_r = \beta\rho - \delta, \tag{3.14}$$

where $\beta > 0$ and δ are constants. Inserting values of ρ and p_r from eqs (3.8)-(3.9) into eq (3.14) we get

$$\frac{\dot{y}}{y} = \frac{\beta(3+x-kL^2x^2)}{4(1+x)}+\frac{1}{4}-\frac{L^2kx^2}{4(1+x)}-\delta\frac{L^2(1+x)}{4}. \tag{3.15}$$

Integrating eq (3.15) we get

$$\begin{aligned} y =& C_1 (1+x)^{-\frac{1}{4}\left(kL^2(1+\beta)-2\beta\right)} exp\left[\frac{1}{4}\left(x-\delta\left(L^2x+\frac{L^2x^2}{2}\right)-\frac{1}{2}(1+x)\left(kL^2\right.\right.\right. \\ & \left.\left.\left.(1+\beta)(x-3)-2\beta\right)\right)\right], \end{aligned} \tag{3.16}$$

where C_1 is a constant of integration. Using eq (3.16) in the governing equations (3.8)-(3.12) we finally obtain

$$\begin{aligned} e^{2\nu} =& C_1^2 (1+x)^{-\frac{1}{2}\left(kL^2(1+\beta)-2\beta\right)} exp\left[\frac{1}{2}\left(x-\delta\left(L^2x+\frac{L^2x^2}{2}\right)-\frac{1}{2}(1+x)\right.\right. \\ & \left.\left.\left(kL^2(1+\beta)(x-3)-2\beta\right)\right)\right], \end{aligned} \tag{3.17}$$

$$\rho = \frac{3+x}{L^2(1+x)^2} - \frac{kx^2}{(1+x)^2}, \tag{3.18}$$

$$\tag{3.19}$$

$$p_r = \frac{\beta(3+x-kL^2x^2)}{L^2(1+x)^2} - \delta, \tag{3.20}$$

$$\begin{aligned} p_t =& \frac{1}{4L^2(1+x)^3}\left[-4kL^2x^2(1+x) - 2L^2x^4\left(k(1+\beta)+\delta\right)(1+\beta-2L^2\delta)\right. \\ & -2x^2\left(-2-5\beta-3\beta^2+6kL^2(2+\beta)+11L^2\delta+7L^2\beta\delta-2L^4\delta^2\right) \\ & +x\left(3+9\beta^2-16L^2\delta+L^4\delta^2+\beta\left(8-6L^2\delta\right)\right)+L^4x^5\left(k(1+\beta)+\delta\right)^2 \\ & +x^3\left(1+2\beta+\beta^2-12L^2\delta-10L^2\beta\delta+6L^4\delta^2+2kL^2(1+\beta)\right. \\ & \left.\left.(4-3\beta+L^2\delta)\right)+12\beta-4L^2\delta\right], \end{aligned} \tag{3.21}$$

$$\sigma^2 = \frac{4k^2x^3}{(1+x)^7}, \tag{3.22}$$

$$\begin{aligned} \Delta =& \frac{1}{4L^2(1+x)^3}\left[-4kL^2x^2(1+x) - 2L^2x^4\left(k(1+\beta)+\delta\right)(1+\beta-2L^2\delta)\right. \\ & -2x^2\left(-2-5\beta-3\beta^2+6kL^2(2+\beta)+11L^2\delta+7L^2\beta\delta-2L^4\delta^2\right) \\ & +x\left(3+9\beta^2-16L^2\delta+L^4\delta^2+\beta\left(8-6L^2\delta\right)\right)+L^4x^5\left(k(1+\beta)+\delta\right)^2+\cdot \\ & x^3\left(1+2\beta+\beta^2-12L^2\delta-10L^2\beta\delta+6L^4\delta^2+2kL^2(4-3\beta+L^2\delta)\right. \\ & \left.\left.(1+\beta)\right)-4\beta(3+x-kL^2x^2)(1+x)+4L^2\delta(1+x)^3+12\beta-4L^2\delta\right]. \end{aligned} \tag{3.23}$$

Reverse substituting $x = \frac{r^2}{L^2}$ in eqs (3.17)-(3.23) to have

$$
\begin{aligned}
e^{2\nu} =& C_1^2 \left(1 + \frac{r^2}{L^2}\right)^{-\frac{1}{2}\left(kL^2(1+\beta)-2\beta\right)} exp\left[\frac{1}{2}\left(\frac{r^2}{L^2} - \delta\left(\frac{r^2}{L^2} + \frac{r^4}{2L^2}\right)\right.\right. \\
& \left.\left. -\frac{1}{2}\left(1 + \frac{r^2}{L^2}\right)\left(kL^2(1+\beta)\left(\frac{r^2}{L^2} - 3\right) - 2\beta\right)\right)\right], \qquad (3.24)\\
\rho =& \frac{3L^2 + r^2}{(L^2 + r^2)^2} - \frac{kr^4}{(L^2 + r^2)^2}, \qquad (3.25)\\
& \qquad (3.26)\\
p_r =& \frac{\beta(3L^2 + r^2 - kr^4)}{(L^2 + r^2)^2} - \delta, \qquad (3.27)\\
p_t =& \frac{L^4}{4(L^2 + r^2)^3}\left[-4k\frac{r^4}{L^2}\left(1 + \frac{r^2}{L^2}\right) - 2\frac{r^8}{L^6}\left(k(1+\beta) + \delta\right)(1 + \beta - 2L^2\delta)\right. \\
& -2\frac{r^4}{L^4}\left(-2 - 5\beta - 3\beta^2 + 6kL^2(2+\beta) + 11L^2\delta + 7L^2\beta\delta - 2L^4\delta^2\right) \\
& +\frac{r^2}{L^2}\left(3 + 9\beta^2 - 16L^2\delta + L^4\delta^2 + \beta\left(8 - 6L^2\delta\right)\right) + \frac{r^{10}}{L^6}\left(k(1+\beta) + \delta\right)^2 \qquad (3.28)\\
& +\frac{r^6}{L^6}\left(1 + 2\beta + \beta^2 - 12L^2\delta - 10L^2\beta\delta + 6L^4\delta^2 + 2kL^2(1+\beta)\right. \\
& \left.\left.(4 - 3\beta + L^2\delta)\right) + 12\beta - 4L^2\delta\right], \\
\sigma^2 =& \frac{4k^2L^8r^6}{(L^2 + r^2)^7}, \qquad (3.29)\\
\Delta =& \frac{L^4}{4(L^2 + r^2)^3}\left[-4k\frac{r^4}{L^2}(1 + x) - 2\frac{r^8}{L^6}\left(k(1+\beta) + \delta\right)(1 + \beta - 2L^2\delta)\right. \\
& -2\frac{r^4}{L^4}\left(-2 - 5\beta - 3\beta^2 + 6kL^2(2+\beta) + 11L^2\delta + 7L^2\beta\delta - 2L^4\delta^2\right) \\
& +\frac{r^2}{L^2}\left(3 + 9\beta^2 - 16L^2\delta + L^4\delta^2 + \beta\left(8 - 6L^2\delta\right)\right) + \frac{r^{10}}{L^6}\left(k(1+\beta) + \delta\right)^2 \quad . \qquad (3.30)\\
& +\frac{r^6}{L^6}\left(1 + 2\beta + \beta^2 - 12L^2\delta - 10L^2\beta\delta + 6L^4\delta^2 + 2kL^2(4 - 3\beta + L^2\delta)\right. \\
& \left.(1+\beta)) - 4\beta(3 + \frac{r^2}{L^2} - k\frac{r^4}{L^2})(1 + x) + 4L^2\delta(1 + \frac{r^2}{L^2})^3 + 12\beta - 4L^2\delta\right].
\end{aligned}
$$

3.1.1 Exterior Spacetime and Boundary Conditions

Exterior solution of a charged star is described by the Reissner-Nordstrom metric given in eq (1.63) where $Q = ER^2$ represents total charge and M is the total mass of the compact star within the boundary R. Mass contained in sphere of radius R is given as

$$M = m(R) = \int_0^R \omega^2 \left(\rho(\omega) + E^2\right) d\omega. \tag{3.31}$$

Substituting eq (3.8) and eq (3.13) into eq (3.31) we obtain

$$M = \frac{R^3}{2(L^2 + R^2)}. \tag{3.32}$$

From eq (3.32) we find the model parameter, L, as

$$L = \sqrt{\frac{R^3}{2M} - R^2}. \tag{3.33}$$

At $r = R$ interior spacetime should match the Reissner-Nordstrom solution, which requires

$$e^{2\nu}|_{r=R} = \left(1 - \frac{2M}{R} + \frac{Q^2}{R^2}\right). \tag{3.34}$$

We obtain constant of integration by substituting eqs (3.24) and (3.32) in eq (3.34) as

$$C_1^2 = \left(\left(1 + \frac{R^2}{L^2}\right)^{-1+\frac{1}{2}\left(kL^2(1+\beta)-2\beta\right)} exp\left[-\frac{1}{2}\left(-\delta\left(R^2 + \frac{R^4}{2L^2}\right) - \frac{1}{2}\left(1 + \frac{R^2}{L^2}\right)\right.\right.\right.$$
$$\left.\left.\left.\left(k\left(1+\beta\right)\left(R^2 - 3L^2\right) - 2\beta\right) + \frac{R^2}{L^2}\right)\right]\right)\left(1 + \frac{kR^4}{L^2(L^2+R^2)}\right). \tag{3.35}$$

Radial pressure vanishes at the boundary i.e. $p_r|_{r=R} = 0$. Utilizing this condition along with eq (3.33) we get

$$\delta = \frac{2\beta M}{R^4}(3R - 4M - 2kR^2M). \tag{3.36}$$

3.1.2 Physical Analysis of the Solution

Physical features of model are

- Metric potential, $e^{2\lambda}$, is well behaved and free from geometric singularities as evident from eq (2.34). Metric potential, $e^{2\nu}$, is also well defined and free from singularities and is shown in Figure 3.1 for certain values of the constants.

- Intensity of electric field is zero at the origin and for $k > 0$ increases towards the boundary as evident from eq (3.13). Graph is shown in Figure 3.2. Charge density is also finite, positive and monotonically increasing. Graph is displayed in Figure 3.3.

- Density is positive if

$$0 < k < \frac{3L^2 + R^2}{R^4}. \tag{3.37}$$

  Energy density at centre of the compact object is $\rho(0) = \frac{3}{L^2}$ which is positive and from eq (3.25) it is clear that it remains positive and finite throughout the interior and shown in Figure 3.4.

- Pressures at the centre are

$$p_r(0) = p_t(0) = \frac{3\beta}{L^2} - \delta. \tag{3.38}$$

  Pressures are positive if $\frac{3\beta}{L^2} > \delta$ and finite. Behaviour of pressures are displayed in Figure 3.5.

- Anisotropy vanishes at the centre. Anisotropic factor shows complete dominance of p_t over p_r. Figure 3.6 shows behaviour of Δ.

- Surface redshift at the origin and boundary is finite and positive.

$$\text{Redshift at origin} = Z(0) = 1, \tag{3.39}$$

$$\text{Redshift at boundary} = Z(R) = 1 + \frac{R^2}{L^2}. \tag{3.40}$$

- Mass to radius ratio is $\frac{M}{R} = 0.232 \leq \frac{4}{9}$.

- Gradient of the energy density is

$$\frac{d\rho}{dr} = -\frac{2\left(r^3 + L^2 r\left(5 + 2kr^2\right)\right)}{\left(L^2 + r^2\right)^3}. \tag{3.41}$$

Gradient is negative and it shows monotonically decreasing nature of density. Behaviour of gradient is presented in Figure 3.7.

- Gradient of pressures are

$$\frac{dp_r}{dr} = -\frac{2\beta\left(r^3 + L^2 r\left(5 + 2kr^2\right)\right)}{\left(L^2 + r^2\right)^3}, \tag{3.42}$$

$$\begin{aligned}\frac{dp_t}{dr} =& \frac{r}{2L^2\left(L^2+r^2\right)^4}\Big[L^{10}\delta^2 + 2r^8\left(k^2r^2\left(1+\beta\right)^2 - k\left(1+\beta\right)\left(1+\beta-2\delta r^2\right) + \right. \\ & \left.\delta\left(-1-\beta+\delta r^2\right)\right) + 2L^8\delta\left(-2-3\beta+3\delta r^2\right) - 4kL^6 r\left(L^2+r^2\right) \\ & + L^6\left(3+9\beta^2-12\delta r^2+14\delta^2r^4+6kr^2\left(1+\beta\right)\left(\delta r^2-4\right)-4\beta\left(7+4\delta r^2\right)\right) \\ & + L^2r^4\left(-1-3\beta^2+5k^2r^4\left(1+\beta\right)^2-8\delta r^2+9\delta^2r^4-4\beta\left(1+2\delta r^2\right)+\right. \\ & \left.2kr^2\left(1+\beta\right)\left(-4-4\beta+7\delta r^2\right)\right) + 2L^4r^2\left(1-3\beta^2-7\delta r^2+8\delta^2r^4+\right. \\ & \left.\beta\left(2-8\delta r^2\right)+kr^2\left(1+\beta\right)\left(8\delta r^2-9\beta-6\right)\right)\Big].\end{aligned} \tag{3.43}$$

Gradient of radial pressure is negative as evident from eq (3.42). Negative behaviour of gradient of the radial and tangential pressures are shown in Figure 3.15 for certain values of constants.

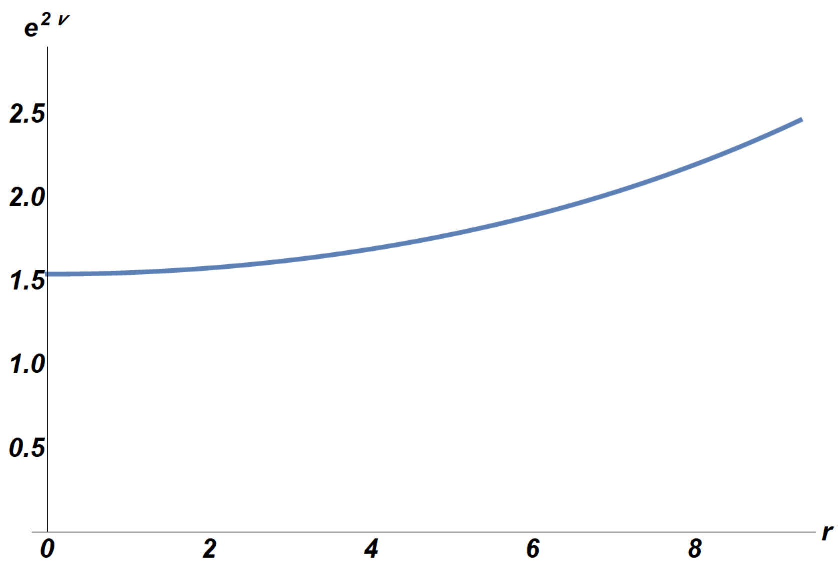

Figure 3.1: The metric potential, $e^{2\nu}$, plotted for $M = 1.46M_S$, $R = 9.3$, $L = 10$, $k = 0.00007$, $\beta = 0.1$, $\delta = .00111$ and $C_1 = 0.95340233$.

3.1.3 Stability Analysis

In this section we analyze conditions for our models to be stable and determine stability region.

Energy Conditions

All ECs are satisfied and graph is shown in Figure 3.9.

Causality Condition

Radial and transverse speeds of sound are:

$$v_r^2 = \beta, \tag{3.44}$$

$$\begin{aligned} v_t^2 = & -\frac{1}{4L^2\left(L^2+r^2\right)\left(r^2+L^2\left(5+2kr^2\right)\right)}\Big[L^{10}\delta^2+2r^8\left(k^2r^2\left(1+\beta\right)^2-4kL^6r\right. \\ & \left(L^2+r^2\right)-k\left(1+\beta\right)\left(1+\beta-2\delta r^2\right)+\delta\left(-1-\beta+\delta r^2\right)\Big)+L^6\Big(3+9\beta^2-12\delta \\ & r^2+14\delta^2r^4+6kr^2\left(1+\beta\right)\left(\delta r^2-4\right)-4\beta\left(7+4\delta r^2\right)\Big)+2L^8\delta\left(-2-3\beta+3\delta r^2\right) \\ & +L^2r^4\Big(-1-3\beta^2+5k^2r^4\left(1+\beta\right)^2-8\delta r^2+9\delta^2r^4-4\beta\left(1+2\delta r^2\right)+2kr^2 \\ & \left(1+\beta\right)\left(-4-4\beta+7\delta r^2\right)\Big)+2L^4r^2\Big(1-3\beta^2-7\delta r^2+8\delta^2r^4+\beta\left(2-8\delta r^2\right) \\ & +kr^2\left(1+\beta\right)\left(8\delta r^2-9\beta-6\right)\Big)\Big]. \end{aligned} \tag{3.45}$$

Speed of sound lies in [0,1] for $0 < \beta < 1$. Graph of speeds is shown in Figure 3.10.

Region of Stability

The stability parameter $Sp = v_r^2 - v_t^2 \geq 0$ that shows whole region is stable. The graph of Sp is shown in Figure 3.11.

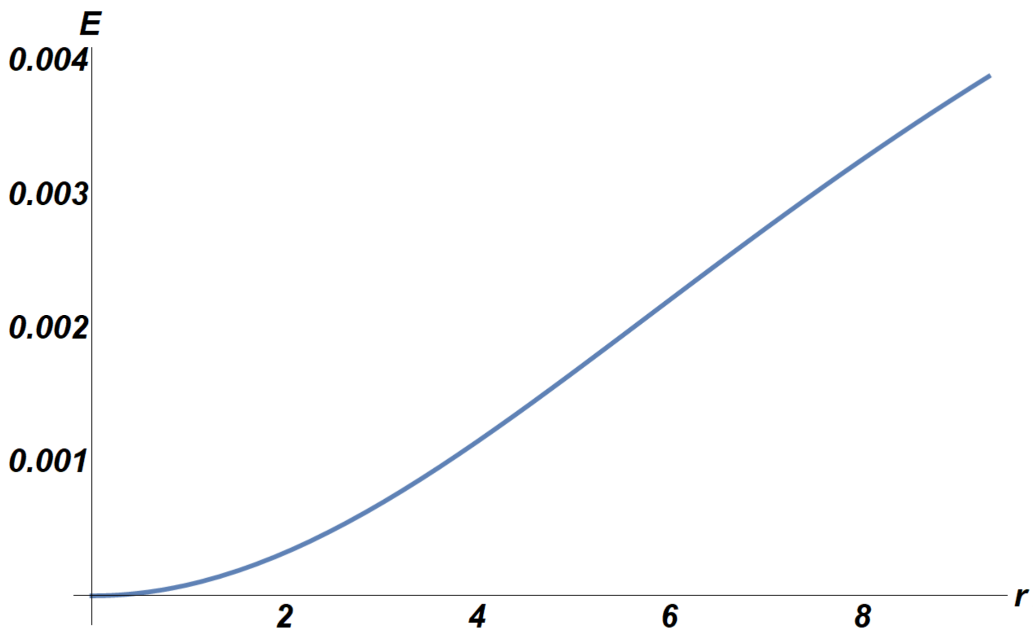

Figure 3.2: Variation of E with respect to r plotted for $k = 0.00007$ and $L = 10$.

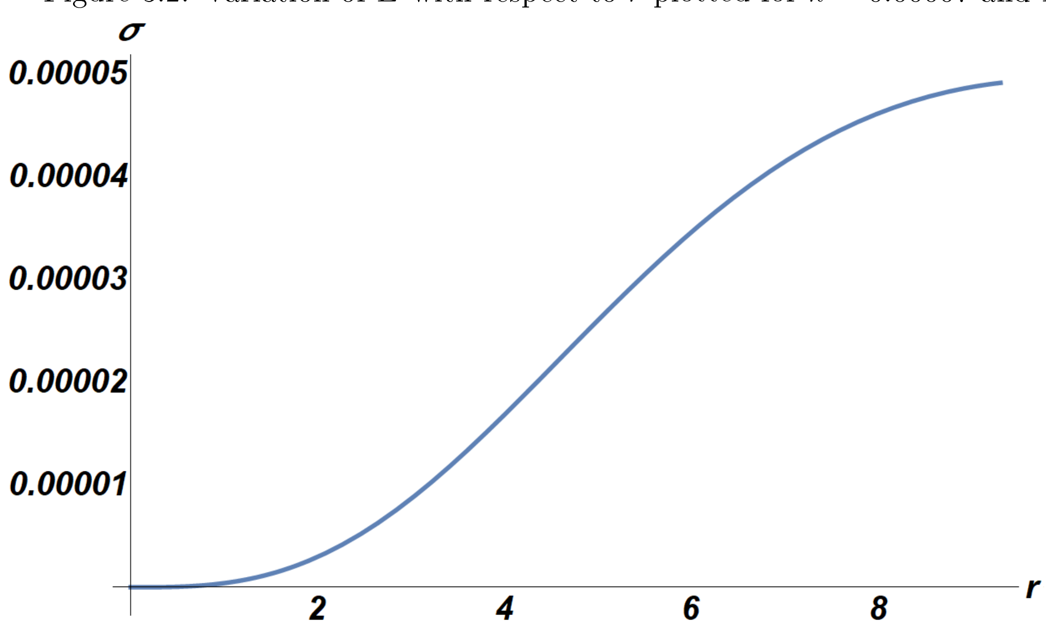

Figure 3.3: Variation of the charge density with respect to r represents its increasing behaviour.

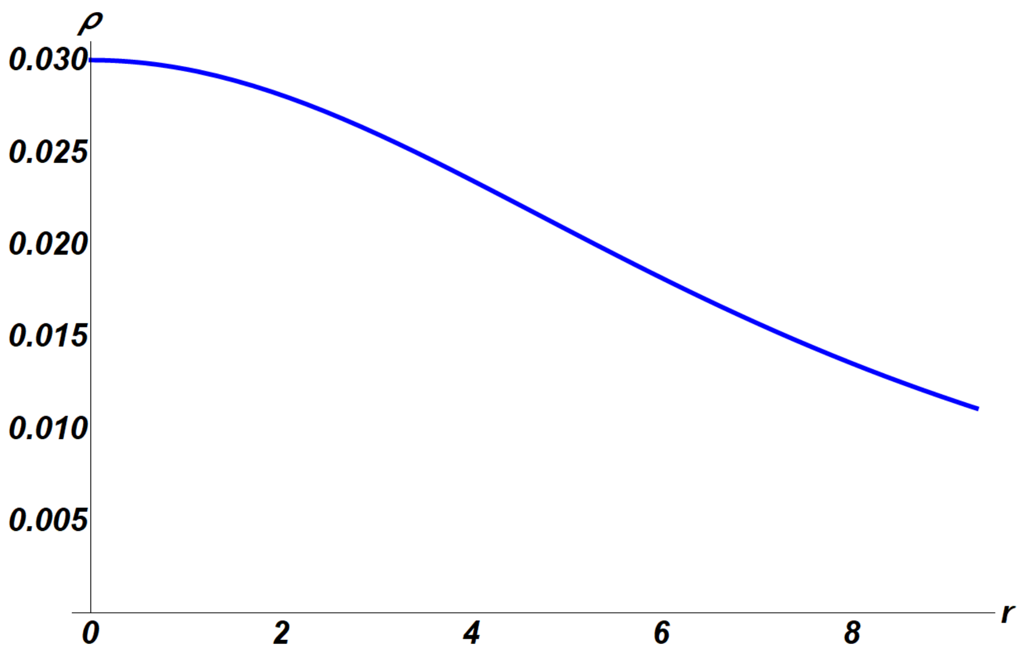

Figure 3.4: Variation of density with respect to r plotted for $M = 1.46M_S$, $R = 9.3$, $L = 10$, $k = 0.00007$, $\beta = 0.1$, $\delta = .00111$ and $C_1 = 0.95340233$.

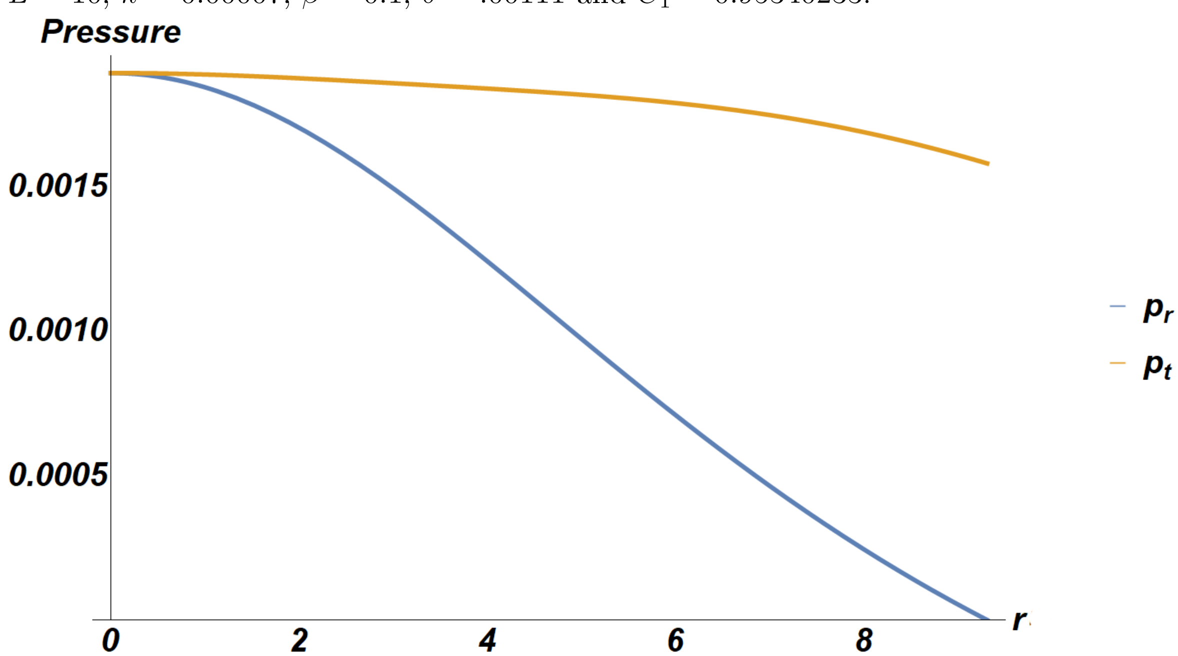

Figure 3.5: Variation of pressure with respect to r plotted for $M = 1.46M_S$, $R = 9.3$, $L = 10$, $k = 0.00007$, $\beta = 0.1$, $\delta = .00111$ and $C_1 = 0.95340233$.

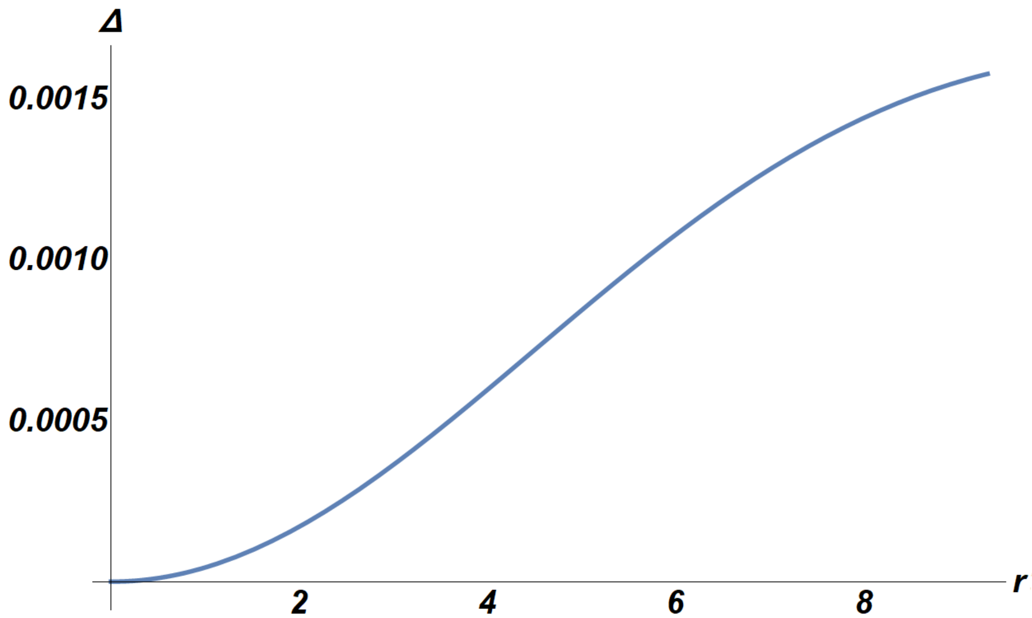

Figure 3.6: Anisotropic factor with respect to r plotted for $M = 1.46M_S$, $R = 9.3$, $L = 10$, $k = 0.00007$, $\beta = 0.1$, $\delta = .00111$ and $C_1 = 0.95340233$.

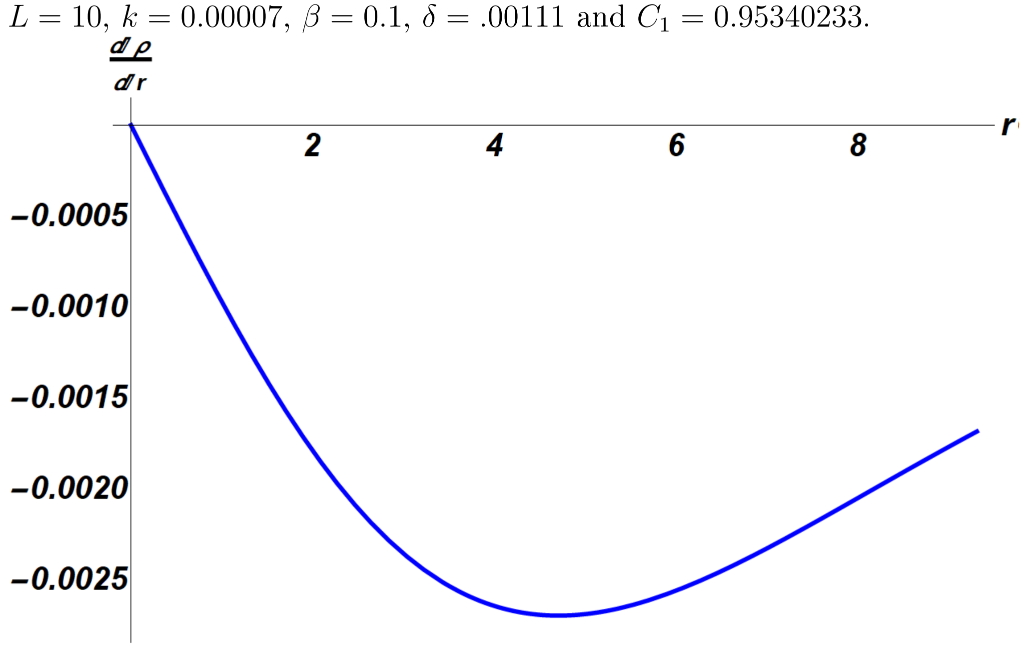

Figure 3.7: Graph of density gradient is plotted for $M = 1.46M_S$, $R = 9.3$, $L = 10$, $k = 0.00007$, $\beta = 0.1$, $\delta = .00111$ and $C_1 = 0.95340233$.

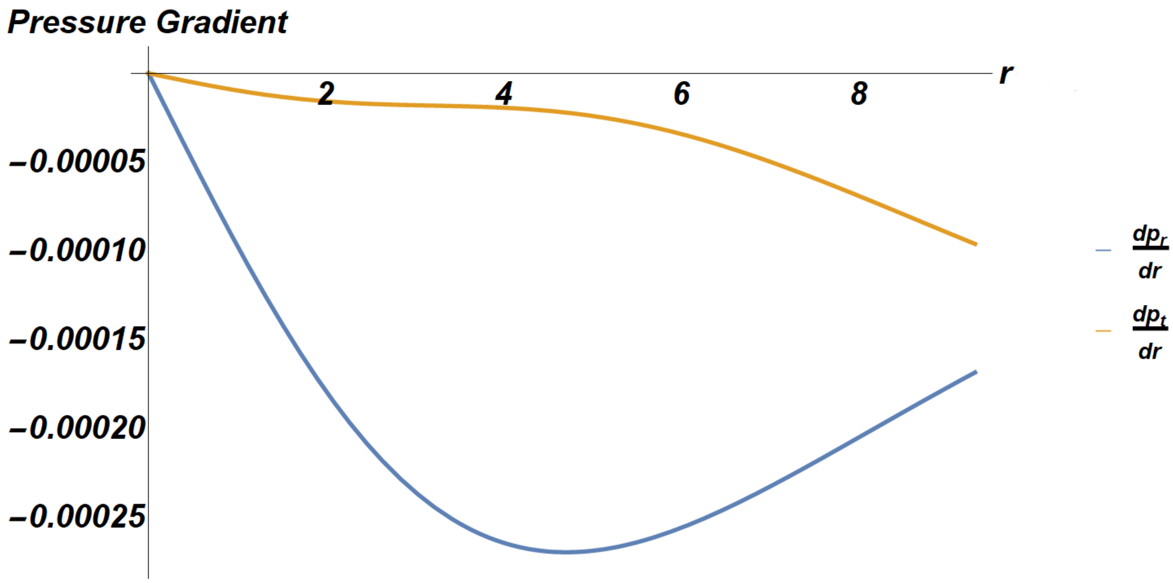

Figure 3.8: Variation of pressure gradients with respect to r depicts monotonically decreasing behaviour for $M = 1.46M_S$, $R = 9.3$, $L = 10$, $k = 0.00007$, $\beta = 0.1$, $\delta = .00111$ and $C_1 = 0.95340233$.

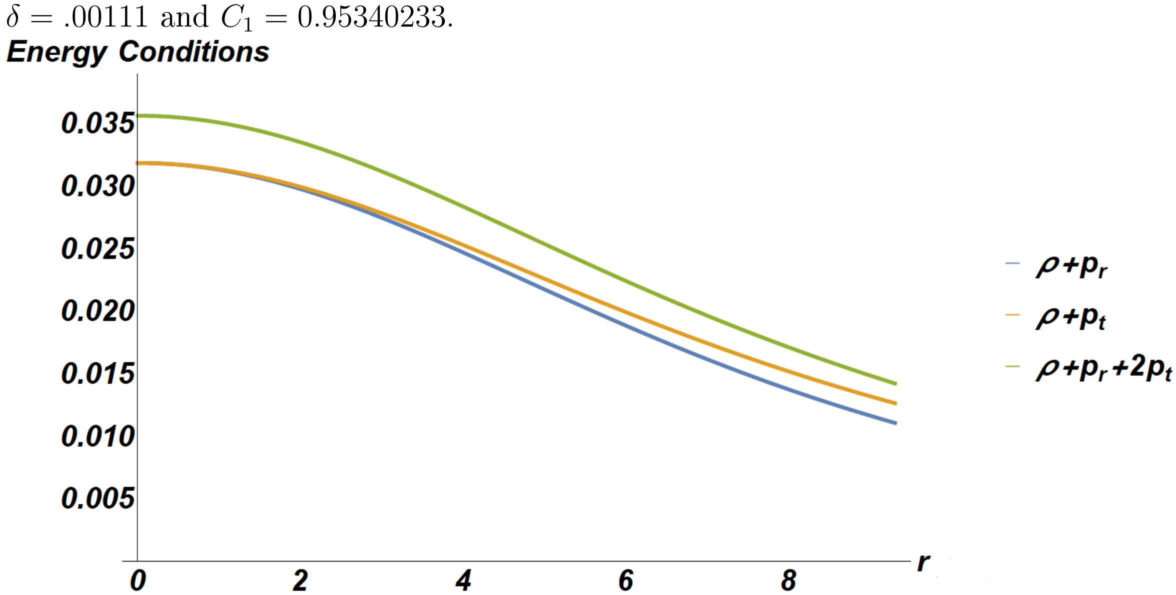

Figure 3.9: Energy conditions for linear EoS shows sum of pressures are positive.

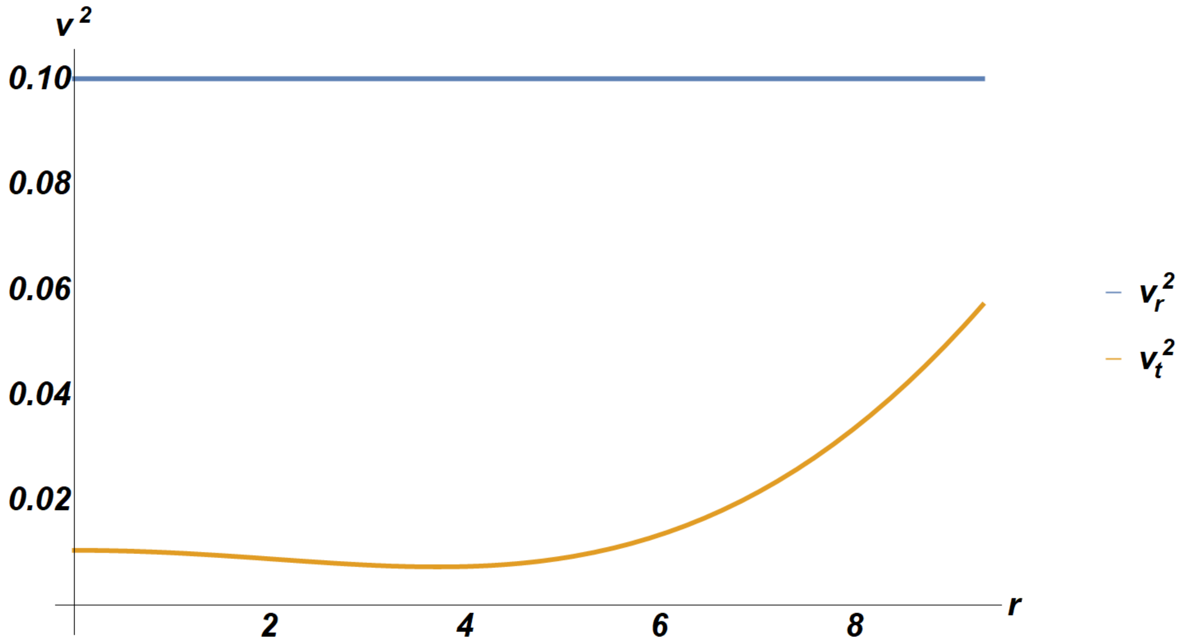

Figure 3.10: Radial speed of sound is constant and greater than transverse speed of sound.

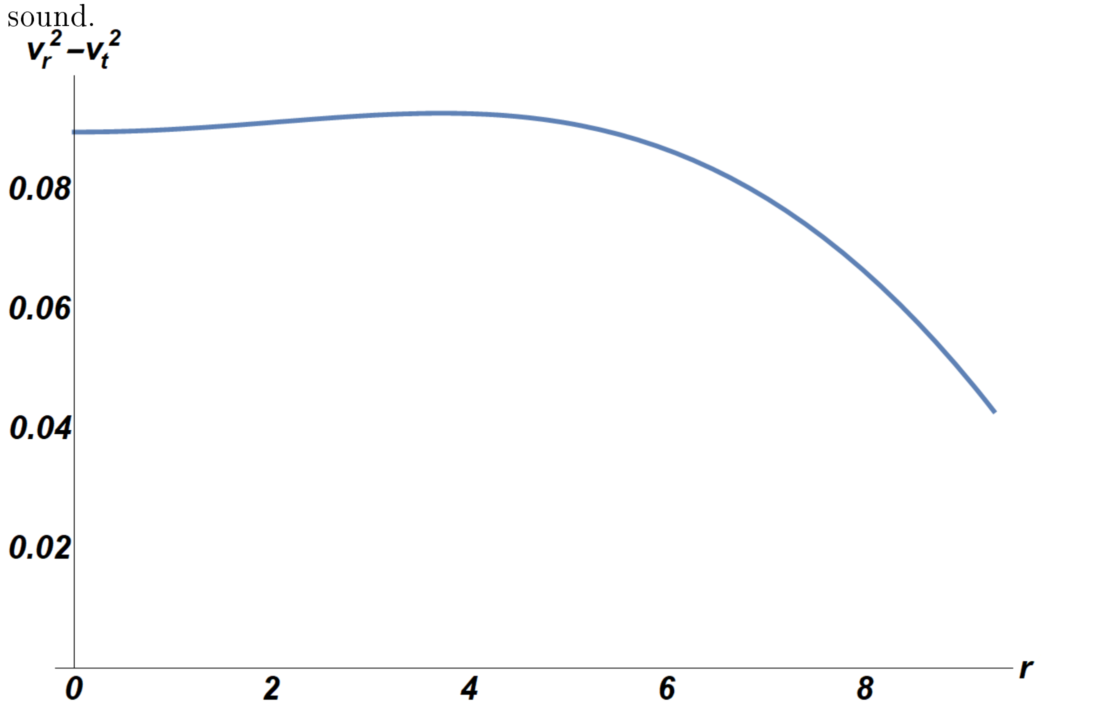

Figure 3.11: Difference of speeds plotted against r showing that the whole region is stable.

3.2 Model 2

We assume a quadratic equation of state of the form

$$p_r = \alpha\rho^2 - \gamma. \tag{3.46}$$

Here $\alpha > 0$ and γ are constants. Inserting values of ρ and p_r from eqs (3.8) and (3.9) into eq (3.46) we get

$$\frac{\dot{y}}{y} = \frac{\alpha\left(3 + x - kL^2x^2\right)}{4L^2\left(1+x\right)^3} + \frac{1}{4} - \frac{L^2kx^2}{4\left(1+x\right)^2} - \gamma\frac{L^2(1+x)}{4}. \tag{3.47}$$

Integrating eq (3.47) yields

$$\begin{aligned} y =& C_2\left(1+x\right)^{\frac{1}{4}\left(\frac{\left(1+6k^2L^4\right)\alpha}{L^2} - kL^2\right)} exp\left[\frac{1}{4}\left(2kL^2(1+x) - \frac{1}{2}kL^2\left(1+x\right)^2 + \right.\right. \\ & \alpha\left(\frac{1}{2}k^2L^2x^2 - k\left(2+3kL^2\right)x + \frac{2\left(-2-3kL^2+2k^2L^4\right)}{L^2\left(1+x\right)} \right. \\ & \left.\left.\left. - \frac{4kL^2 - k^2L^4 - 4}{2L^2(1+x)^2}\right) - \gamma\left(L^2x + \frac{1}{2}L^2x^2\right) + x\right)\right], \end{aligned} \tag{3.48}$$

where C_2 is a constant of integration. ρ, σ, E are same as in the previous model. Utilizing eq (3.48) in the governing equations (3.8)-(3.12) we get

$$\begin{aligned} e^{2\nu} =& C_2^2\left(1+x\right)^{\frac{1}{2}\left(\frac{\left(1+6k^2L^4\right)\alpha}{L^2} - kL^2\right)} exp\left[\frac{1}{2}\left(2kL^2(1+x) - \frac{1}{2}kL^2\left(1+x\right)^2 + \right.\right. \\ & \alpha\left(\frac{1}{2}k^2L^2x^2 - k\left(2+3kL^2\right)x + \frac{2\left(-2-3kL^2+2k^2L^4\right)}{L^2\left(1+x\right)} \right. \\ & \left.\left.\left. - \frac{4kL^2 - k^2L^4 - 4}{2L^2(1+x)^2}\right) - \gamma\left(L^2x + \frac{1}{2}L^2x^2\right) + x\right)\right], \end{aligned} \tag{3.49}$$

$$p_r = \frac{\alpha\left(3+x-kL^2x^2\right)^2}{L^4\left(1+x\right)^4} - \gamma, \tag{3.50}$$

$$\begin{aligned} p_t =& \frac{1}{4L^6\left(1+x\right)^7}\left[-4L^6\gamma\left(1+x\right)^6 - 4L^4\left(1+x\right)^5 - 2L^2\left(1+x\right)^2\left(2+x\right) + \right. \\ & 4kL^6x^2(1+x)^4 - 8kL^6x(1+x)^5\left(kL^4x^2\left(1+x\right)^2 - L^2\left(1+x\right)^3 \right. \\ & \left. -\alpha\left(3+x-kL^2x^2\right)^2 + L^4\gamma(1+x)^4\right) + \left(kL^4x^2(1+x)^2 - L^2(1+x)^3\right. \end{aligned} \tag{3.51}$$

$$-\left(3+x-kL^2x^2\right)^2+L^4\gamma(1+x)^3\Big)^2+\alpha\left(8L^2\left(1+x\right)^3\left(1-2kL^2x\right)\right.$$
$$\left.\left(3+x-kL^2x^2\right)-12L^2\left(1+x\right)^2\left(3+x-kL^2x^2\right)^2\right)-4kL^6x^2\left(1+x\right)^5\Big],$$

$$\begin{aligned}\Delta =&\frac{1}{4L^6\left(1+x\right)^7}\left[-4L^6\gamma\left(1+x\right)^6-4L^4\left(1+x\right)^5-2L^2\left(1+x\right)^2\left(2+x\right)\right.\\ &+4kL^6x^2(1+x)^4-8kL^6x(1+x)^5\left(kL^4x^2\left(1+x\right)^2-L^2\left(1+x\right)^3\right.\\ &\left.-\alpha\left(3+x-kL^2x^2\right)^2+L^4\gamma(1+x)^4\right)+\left(kL^4x^2(1+x)^2\right.\\ &\left.-L^2(1+x)^3-\left(3+x-kL^2x^2\right)^2+L^4\gamma(1+x)^3\right)^2+\alpha\left(8L^2\left(1+x\right)^3\right.\\ &\left.\times\left(1-2kL^2x\right)\left(3+x-kL^2x^2\right)-12L^2\left(1+x\right)^2\left(3+x-kL^2x^2\right)^2\right)\\ &\left.-4kL^6x^2\left(1+x\right)^5-4\alpha L^2\left(3+x-kL^2x^2\right)^2\left(1+x\right)^3\right]+\gamma.\end{aligned}\tag{3.52}$$

Reverse substituting $x=\frac{r^2}{L^2}$ in eqs (3.49)-(3.52) to get

$$\begin{aligned}e^{2\nu}=&C_2^2\left(1+\frac{r^2}{L^2}\right)^{\frac{1}{2}\left(\frac{\left(1+6k^2L^4\right)\alpha}{L^2}-kL^2\right)}exp\left[\frac{1}{2}\left(2k(L^2+r^2)-\frac{1}{2}kL^2\left(1+\frac{r^2}{L^2}\right)^2\right.\right.\\ &+\alpha\left(\frac{1}{2}k^2\frac{r^4}{L^2}-k\left(2+3kL^2\right)\frac{r^2}{L^2}+\frac{2\left(-2-3kL^2+2k^2L^4\right)}{\left(L^2+r^2\right)}\right.\\ &\left.\left.\left.-\frac{4kL^2-k^2L^4-4}{2(L^2+r^2)^2}\right)-\gamma\left(r^2+\frac{1}{2}\frac{r^4}{L^2}\right)+\frac{r^2}{L^2}\right)\right],\end{aligned}\tag{3.53}$$

$$p_r=\frac{\alpha\left(3L^2+r^2-kr^4\right)^2}{\left(L^2+r^2\right)^4}-\gamma,\tag{3.54}$$

$$\begin{aligned}p_t=&\frac{1}{4L^6\left(1+x\right)^7}\left[-4\frac{\gamma\left(L^2+r^2\right)^6}{L^6}-4\frac{\left(L^2+r^2\right)^5}{L^6}-2\frac{\left(L^2+r^2\right)^2\left(2L^2+r^2\right)}{L^4}+\right.\\ &4kL^2r^4(1+\frac{r^2}{L^2})^4-8kL^4r^2\left(1+\frac{r^2}{L^2}\right)^5\left(kr^4\left(1+\frac{r^2}{L^2}\right)^2-\right.\\ &\left.L^2\left(1+\frac{r^2}{L^2}\right)^3-\alpha\left(3+\frac{r^2}{L^2}-k\frac{r^4}{L^2}\right)^2+L^4\gamma(1+\frac{r^2}{L^2})^4\right)+\\ &\left(kr^4\left(1+\frac{r^2}{L^2}\right)^2-L^2(1+\frac{r^2}{L^2})^3-\left(3+\frac{r^2}{L^2}-k\frac{r^4}{L^2}\right)^2+L^4\gamma\right.\end{aligned}\tag{3.55}$$

$$\left(1+\frac{r^2}{L^2}\right)^3\Bigg)^2+\alpha\left(8L^2\left(1+\frac{r^2}{L^2}\right)^3\left(1-2kL^2\frac{r^2}{L^2}\right)\left(3+\frac{r^2}{L^2}-k\frac{r^4}{L^2}\right)\right.$$
$$\left.-12L^2\left(1+\frac{r^2}{L^2}\right)^2\left(3+\frac{r^2}{L^2}-k\frac{r^4}{L^2}\right)^2\right)-4kL^2r^2\left(1+\frac{r^2}{L^2}\right)^5\Bigg],$$

$$\begin{aligned}\Delta =&\frac{1}{4L^6\left(1+x\right)^7}\left[-4\frac{\gamma\left(L^2+r^2\right)^6}{L^6}-4\frac{\left(L^2+r^2\right)^5}{L^6}-2\frac{\left(L^2+r^2\right)^2\left(2L^2+r^2\right)}{L^4}+\right.\\
&4kL^2r^4(1+\frac{r^2}{L^2})^4-8kL^4r^2\left(1+\frac{r^2}{L^2}\right)^5\left(kr^4\left(1+\frac{r^2}{L^2}\right)^2-\right.\\
&\left.L^2\left(1+\frac{r^2}{L^2}\right)^3-\alpha\left(3+\frac{r^2}{L^2}-k\frac{r^4}{L^2}\right)^2+L^4\gamma(1+\frac{r^2}{L^2})^4\right)+\\
&\left(kr^4\left(1+\frac{r^2}{L^2}\right)^2-L^2(1+\frac{r^2}{L^2})^3-\left(3+\frac{r^2}{L^2}-k\frac{r^4}{L^2}\right)^2+L^4\gamma\right.\\
&\left.\left(1+\frac{r^2}{L^2}\right)^3\right)^2+\alpha\left(8L^2\left(1+\frac{r^2}{L^2}\right)^3\left(1-2kL^2\frac{r^2}{L^2}\right)\left(3+\frac{r^2}{L^2}-k\frac{r^4}{L^2}\right)\right.\\
&\left.-12L^2\left(1+\frac{r^2}{L^2}\right)^2\left(3+\frac{r^2}{L^2}-k\frac{r^4}{L^2}\right)^2\right)-4kL^2r^2\left(1+\frac{r^2}{L^2}\right)^5\\
&\left.-4\alpha L^2\left(3+\frac{r^2}{L^2}-k\frac{r^4}{L^2}\right)^2\left(1+\frac{r^2}{L^2}\right)^3\right]+\gamma.\end{aligned}\tag{3.56}$$

We evaluate constant of integration by substituting eq (3.49) in eq (3.34) as

$$\begin{aligned}C_2^2=&\left(\left(1+\frac{R^2}{L^2}\right)^{-\left(1+\frac{1}{2}\left(\frac{\left(1+6k^2L^4\right)\alpha}{L^2}-kL^2\right)\right)}exp\left[-\frac{1}{2}\left(2k\left(L^2+R^2\right)-\frac{1}{2}k\frac{\left(L^2+R^2\right)}{L^2}\right.\right.\right.\\
&+\frac{R^2}{L^2}-\gamma\left(R^2+\frac{R^4}{2L^2}\right)+\alpha\left(\frac{1}{2}k^2\frac{R^4}{L^2}-k\left(2+3kL^2\right)\frac{R^2}{L^2}-\frac{4kL^4-k^2L^6-4L^2}{2(L^2+R^2)^2}\right.\\
&\left.\left.\left.\left.+\frac{2\left(-2-3kL^2+2k^2L^4\right)}{\left(L^2+R^2\right)}\right)\right)\right]\right)\left(1+\frac{kR^4}{L^2(L^2+R^2)}\right).\end{aligned}\tag{3.57}$$

Utilizing boundary condition ($p_r|_{r=R}=0$) we evaluate

$$\gamma =\frac{4\alpha M^2}{R^8}(3R-4M-2kR^2M)^2.\tag{3.58}$$

3.2.1 Physical Analysis of the Solution

Physical features of our model are as follows:

- Metric potential, $e^{2\nu}$, is well defined and free from singularities. Its behaviour is shown in Fig. 3.12 for certain values of the constants.

- Pressures at the centre are

$$p_r(0) = p_t(0) = \frac{9\alpha}{L^4} - \gamma. \tag{3.59}$$

  Pressures are positive if $\frac{9\alpha}{L^4} \geq \gamma$. Pressures are equal at the centre and radial pressure vanishes at the boundary. Behaviour of pressures are displayed in Fig. 3.13.

- Behaviour of anisotropic factor is represented in Fig. 3.14. It shows p_r dominates over p_t upto $r = 7$ and then p_t takes over similar behaviour was reported by Thirukkanesh [35] and Varela [36].

- Gradient of pressures are negative for both cases and are given by eqs (3.60)-(3.61). Gradients is displayed in Fig. 3.15.

$$\frac{dp_r}{dr} = \frac{-4\alpha\left(3L^2 + r^2 - kr^4\right)\left(r^3 + L^2 r\left(5 + 2kr^2\right)\right)}{\left(L^2 + r^2\right)^5}, \tag{3.60}$$

$$\begin{aligned}\frac{dp_t}{dr} =& \frac{r}{2L^2\left(L^2 + r^2\right)^8}\Big[L^{18}\gamma^2 + 2L^{16}\gamma\left(5\gamma r^2 - 2\right) + L^{14}\Big(3 - 28\gamma r^2 - 18\alpha\gamma + \\ & 44\gamma^2 r^4 + 6kr^2\left(\gamma r^2 - 4\right)\Big) - 2L^4 r^6\Big(84\alpha^2 + 8\alpha r^2\left(1 - 9k\alpha\right) + 16r^8 \\ & \left(k^2\alpha - k - \gamma\right)\left(k + 2\gamma\right) + r^4\left(1 + 12k\alpha - 54k^2\alpha^2 - 18\alpha\gamma\right) + r^6 \\ & \left(28k - 79k^2\alpha + 42k^3\alpha^2 + 29\gamma - 62k\alpha\gamma\right)\Big) + L^2 r^8\Big(-31\alpha^2 + 4r^2\alpha \\ & \left(12k\alpha - 5\right) - 16r^6\left(2k\alpha - 1\right)\left(k^2\alpha - k - \gamma\right) + r^4\Big(2k\alpha - 1 + 6k^2 \\ & \alpha^2 + 6\alpha\gamma\Big) + r^8\left(-22k^3\alpha + 9k^4\alpha^2 + 30k\gamma + 17\gamma^2 + k^2\left(13 - 26\alpha\gamma\right)\right)\Big) \\ & + 2L^{12}\Big(-102\alpha + 4\gamma r^6\left(5k + 14\gamma\right) + r^2\left(7 - 72k\alpha - 30\alpha\gamma\right) + r^4\left(-43\gamma\right.\end{aligned} \tag{3.61}$$

$$+18k\left(\alpha\gamma-3\right))\right)+L^6r^4\left(128\alpha r^2-378\alpha^2+r^4\left(5+12k\alpha+270k^2\alpha^2+66\alpha\gamma\right)\right)$$
$$+4r^6\left(69k^2\alpha-31\gamma+32k\left(2\alpha\gamma-1\right)\right)-2r^8\left(7k^3\alpha-85k\gamma-70\gamma^2+19k^2\right.$$
$$\left.(2\alpha\gamma-1)\right)+2r^{10}\left(\alpha r^2-\alpha^2+k^4\alpha^2r^8-2k^3\alpha r^6\left(r^2+\alpha\right)-\gamma r^6+\gamma^2r^8+\right.$$
$$\left.k^2\left(3\alpha r^6+r^8\left(1-2\alpha\gamma\right)\right)+k\left(2\alpha^2r^2+2\gamma r^8+r^6\left(2\alpha\gamma-1\right)\right)\right)+L^{10}\left(-300\alpha r^2\right.$$
$$+81\alpha^2+r^4\left(25-222k\alpha-54\alpha\gamma\right)+r^8\left(114k\gamma+182\gamma^2+k^2\left(5-10\alpha\gamma\right)\right)$$
$$\left.+8r^6\left(10\alpha k^2-19\gamma+5k\left(1\alpha\gamma-5\right)\right)\right)+2L^8r^2\left(-3\alpha r^2\left(54k\alpha-5\right)\right.$$
$$-135\alpha^2+r^4\left(10-20k\alpha+12\alpha\gamma\right)+r^8\left(90k\gamma+98\gamma^2+k^2\left(11-22\alpha\gamma\right)\right)+$$
$$\left.\left.r^6\left(k\left(142\alpha\gamma-101\right)+120k^2\alpha-85\gamma\right)\right)-4kL^6r\left(L^2+r^2\right)^5\right].$$

- All ECs are satisfied. Fig. 3.9 shows graph of ECs.
- Radial and transverse speeds of sound are

$$v_r^2=\frac{dp_r}{d\rho}=\frac{2\alpha(L^2+r^2-4kr^4)}{(L^2+r^2)^2}, \tag{3.62}$$

$$\begin{aligned}v_t^2=&\frac{-1}{4L^2\left(r^2+L^2\left(5+2kr^2\right)\right)\left(L^2+r^2\right)^5}\Big[L^{18}\gamma^2-4kL^6r\left(L^2+r^2\right)^5\gamma\\&+2L^{16}\left(5\gamma r^2-2\right)-2L^4r^6\left(84\alpha^2+8\alpha r^2\left(1-9k\alpha\right)+16r^8\right.\\&\left(k^2\alpha-k-\gamma\right)\left(k+2\gamma\right)+r^4\left(1+12k\alpha-54k^2\alpha^2-18\alpha\gamma\right)+r^6\\&\left.\left(28k-79k^2\alpha+42k^3\alpha^2+29\gamma-62k\alpha\gamma\right)\right)+r^4\left(2k\alpha+L^2r^8\left(-31\alpha^2+\right.\right.\\&4r^2\alpha\left(12k\alpha-5\right)-16r^6\left(2k\alpha-1\right)\left(k^2\alpha-k-\gamma\right)-1+6k^2\alpha^2+6\alpha\gamma\Big)\\&\left.+r^8\left(-22k^3\alpha+9k^4\alpha^2+30k\gamma+17\gamma^2+k^2\left(13-26\alpha\gamma\right)\right)\right)+2L^{12}\left(-102\alpha\right.\\&\left.+4\gamma r^6\left(5k+14\gamma\right)+r^2\left(7-72k\alpha-30\alpha\gamma\right)+r^4\left(-43\gamma+18k\left(\alpha\gamma-3\right)\right)\right)\\&+L^6r^4\left(128\alpha r^2-378\alpha^2+r^4\left(5+12k\alpha+270k^2\alpha^2+66\alpha\gamma\right)\right)\\&+4r^6\left(69k^2\alpha-31\gamma+32k\left(2\alpha\gamma-1\right)\right)-2r^8\left(7k^3\alpha-85k\gamma\right.\\&\left.-70\gamma^2+19k^2\left(2\alpha\gamma-1\right)\right)+2r^{10}\left(\alpha r^2-\alpha^2+k^4\alpha^2+r^8-2k^3\alpha r^6\right.\\&\left(r^2+\alpha\right)-\gamma r^6+\gamma^2r^8+k^2\left(3\alpha r^6+r^8\left(1-2\alpha\gamma\right)\right)\\&\left.+k\left(2\alpha^2r^2+2\gamma r^8+r^6\left(2\alpha\gamma-1\right)\right)\right)+L^{10}\left(-300\alpha r^2+81\alpha^2\right.\end{aligned} \tag{3.63}$$

$$+r^4(25-222k\alpha-54\alpha\gamma)+r^8\left(114k\gamma+182\gamma^2+k^2(5-10\alpha\gamma)\right)$$
$$+8r^6\left(10\alpha k^2-19\gamma+5k(1\alpha\gamma-5)\right)\big)+2L^8r^2\left(-3\alpha r^2(54k\alpha-5)\right.$$
$$-135\alpha^2+r^4(10-20k\alpha+12\alpha\gamma)+r^8\left(90k\gamma+98\gamma^2+k^2(11-22\alpha\gamma)\right)+$$
$$r^6\left(k(142\alpha\gamma-101)+120k^2\alpha-85\gamma\right)\big)+L^{14}\left(3-28\gamma r^2-18\alpha\gamma+44\gamma^2r^4+\right.$$
$$\left.6kr^2\left(\gamma r^2-4\right)\right)\Big].$$

In Fig. 3.17 graph of speeds of sound are shown. Both speeds remain within the expected range of 0 and 1.

- In our case, v_r^2 is less than v_t^2 when r varies from 0 to 4. So stability region lies from $r=4$ to onwards for selected values of the parameters. Stability parameter is represented in Fig.3.18.

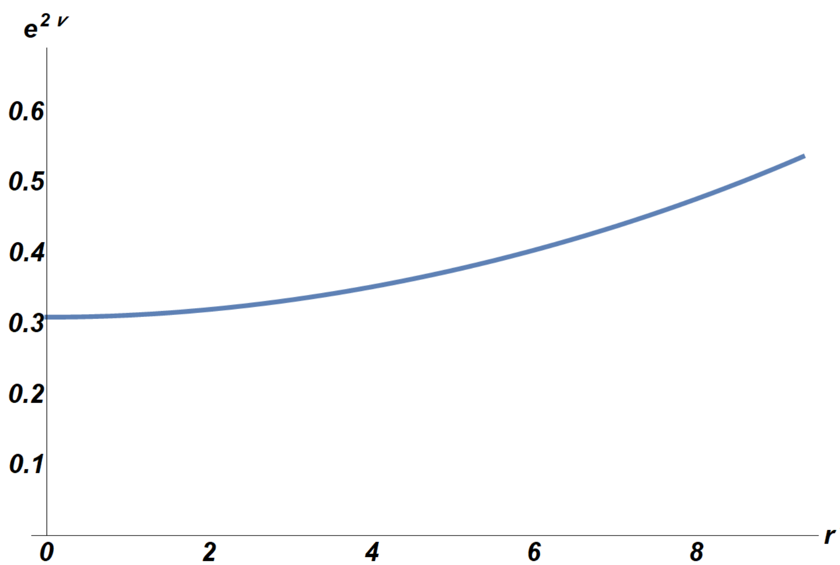

Figure 3.12: The metric potential, $e^{2\nu}$, plotted for $M=1.46M_S$, $R=9.3$, $L=10$, $k=0.00007$, $k=0.00007$, $\alpha=10$, $\gamma=.0012208$ and $C_2=0.64454092$.

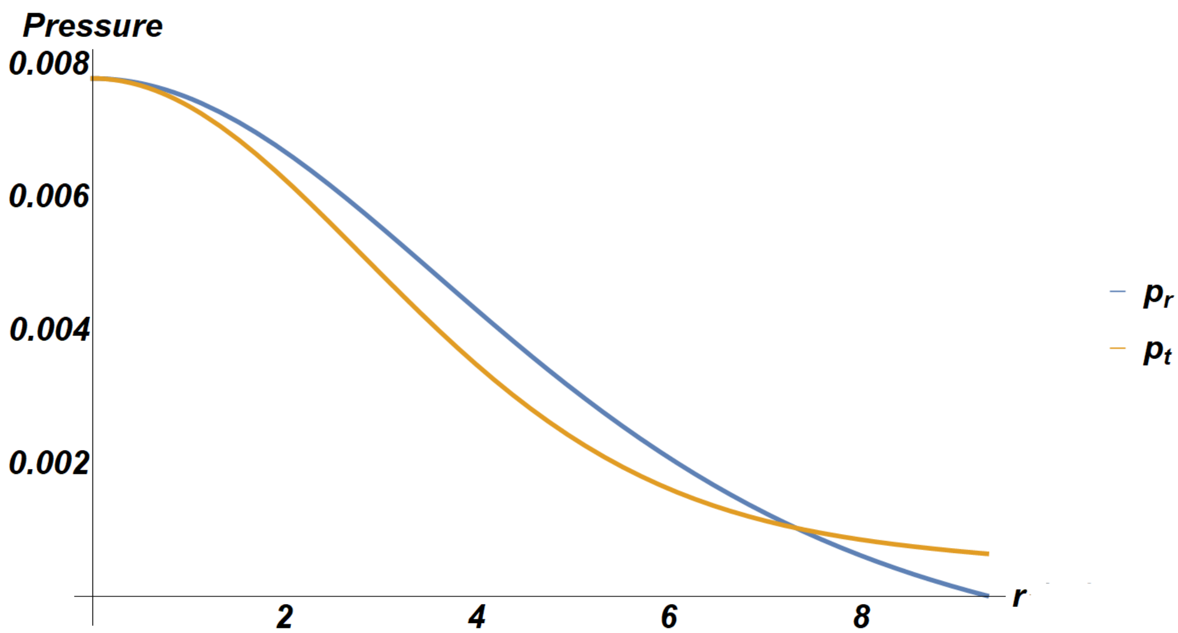

Figure 3.13: Variation of pressure with respect to r plotted for $k = 0.00007$, $\alpha = 10$, $\gamma = .0012208$ and $C_2 = 0.64454092$.

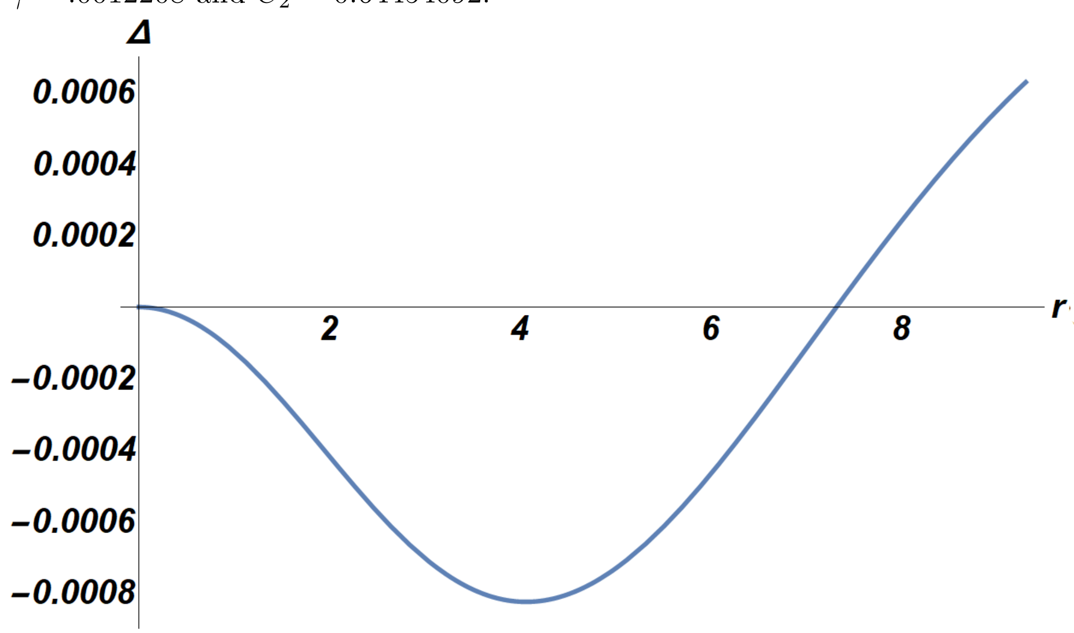

Figure 3.14: Anisotropic factor is plotted against $k = 0.00007$, $\alpha = 10$, $\gamma = .0012208$ and $C_2 = 0.64454092$. It shows dominance of p_r over p_t from $r = 0$ to $r = 7$.

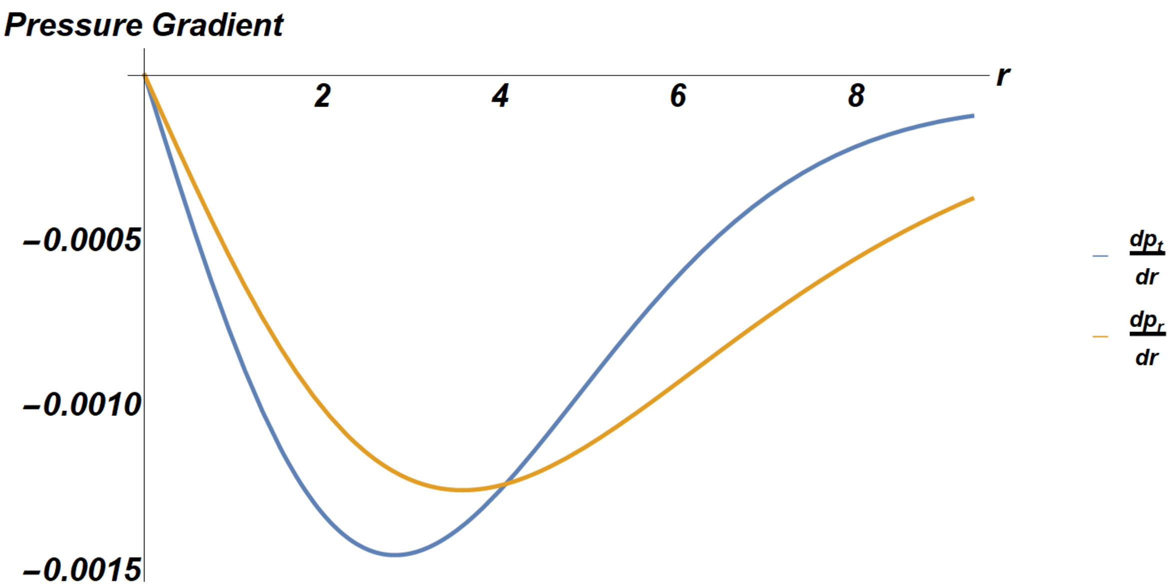

Figure 3.15: Pressure gradients are plotted for $k = 0.00007$, $\alpha = 10$, $\gamma = .0012208$ and $C_2 = 0.64454092$.

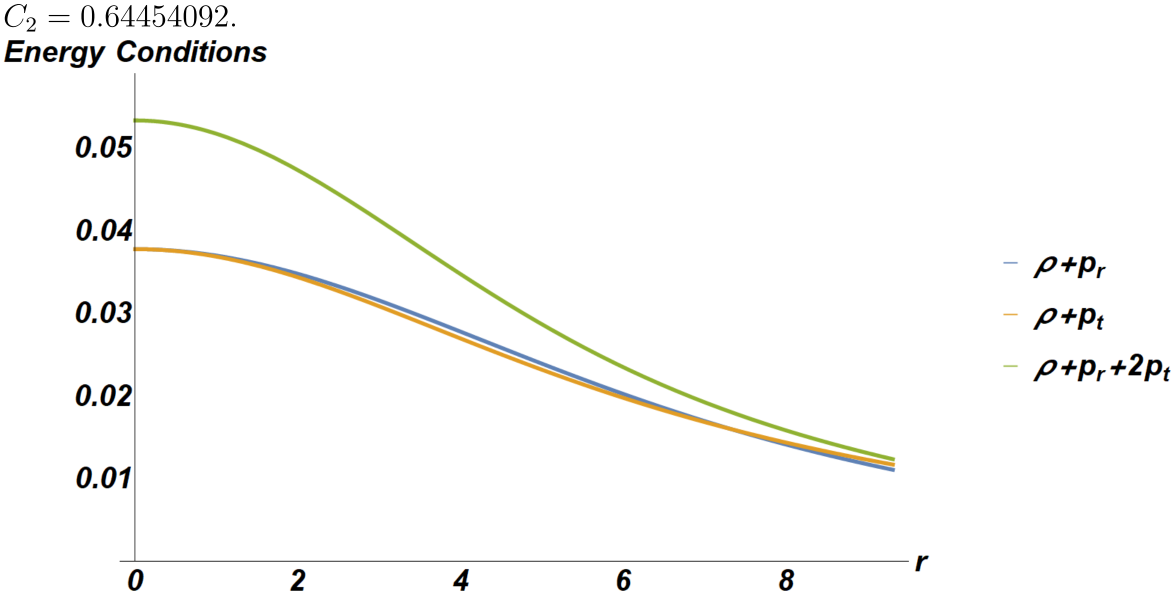

Figure 3.16: Energy conditions are plotted for $k = 0.00007$, $\alpha = 10$, $\gamma = .0012208$ and $C_2 = 0.64454092$.

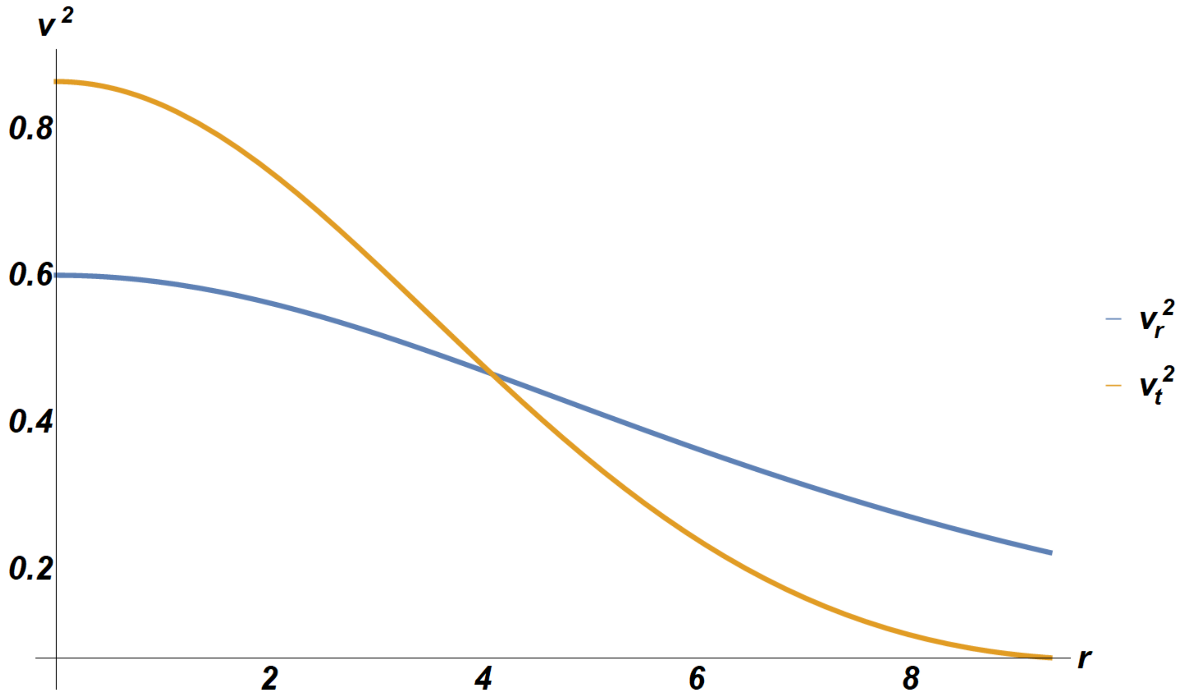

Figure 3.17: Speeds of sound are plotted for $k = 0.00007$, $\alpha = 10$, $\gamma = .0012208$ and $C_2 = 0.64454092$.

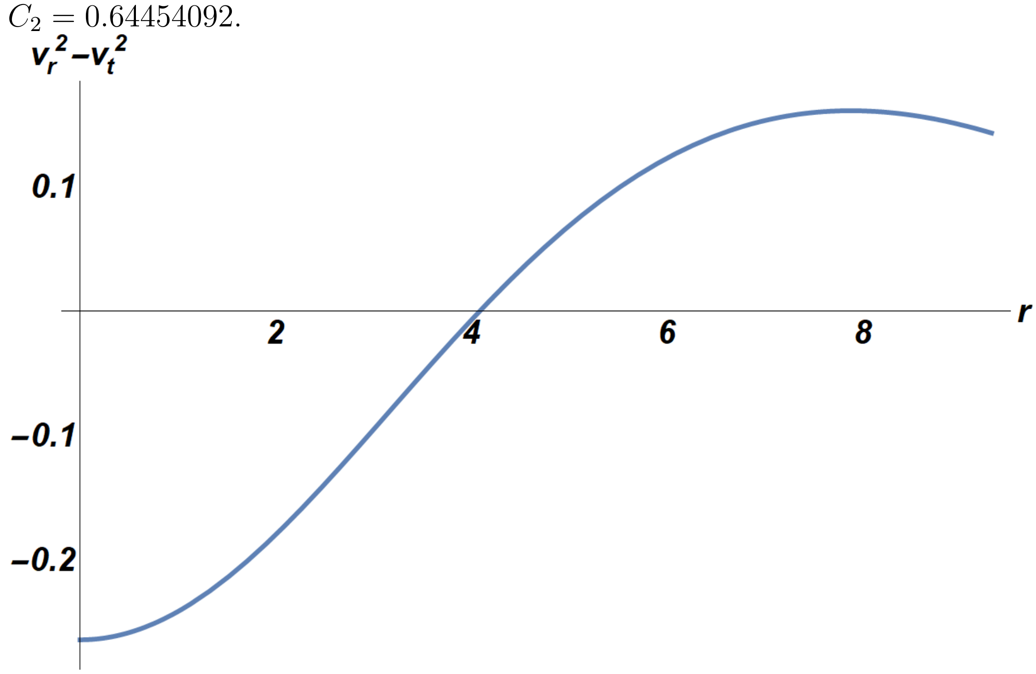

Figure 3.18: Stability parameter is plotted for $k = 0.00007$, $\alpha = 10$, $\gamma = .0012208$ and $C_2 = 0.64454092$. It represents stability region lies from $4 \le r \le 9.3$.

Chapter 4

Conclusion

In this thesis a class of new solutions of the Einstein-Maxwell field equations is obtained. To make the thesis self contained tensors and their algebra are reviewed in the first chapter. In the same chapter, the Einstein field equations and some well known solutions of these equations are also reviewed. This chapter is concluded with formation of compact objects.

In Chapter 2, we have reviewed the work of Thirukkanesh [35]. Solutions are obtained for spherical distribution of uncharged anisotropic fluid in paraboloidal spacetime. Physical analysis of the solution is also presented. Pressures and density are positive, finite and monotonically decreasing. Stability region shows partial stability for the selected values of the parameters.

In Chapter 3, we have generalized the work of Thirukkanesh [35] in the presence of charge. We have constructed models of charged compact objects with anisotropic pressure in paraboloidal spacetime admitting linear and quadratic equations of state. These models satisfy all the necessary conditions to be a physically feasible model. Density and pressures are positive and finite. For quadratic EoS case p_r is dominating over p_t for most of the interior. On the contrary p_t is exceeding p_r in linear case. Electric field intensity and charge density is showing increasing behaviour. Causality condition and Energy conditions are satisfied for both models. Stellar configuration with linear EoS is stable over the whole region while it is partially stable in quadratic EoS case for the selected values of the parameter.

Bibliography

[1] Albert Einstein. On the Electrodynamics of Moving Bodies.*Annals of Physics*, 322, (1905).

[2] Sean Carroll. *Spacetime and Geometry: An Introduction to General Relativity*. Cambridge University Press, (2019).

[3] Ray d'Inverno. *Introducing Einstein's Relativity*. Oxford University Press, (1998).

[4] S. Gedela, Neeraj Pant, R. P. Pant, and J. Upreti. Relativistic Anisotropic Model of Strange Star SAX J1808.4-3658 Admitting Quadratic Equation of State. *International Journal of Modern Physics A*, **34**:1950179, (2019).

[5] D. Fliesch. *A Students Guide to Maxwell Equations*. Cambridge University Press, (2008).

[6] David J. Griffiths. *Introduction to Electrodynamics*. Pearson Education Incorporation, (2013).

[7] M. Camenzind. *Compact Objects in Astrophysics*. Springer-Verlag Berlin Heidelberg, (2007).

[8] K. Schwarzchild. Uber das Gravitationsfeld Eines Masenpunktes nach der Einsteinschen Theorie. *Sitzber Preuss Akad Wiss*, 189, (2016).

[9] H. Reissner. Uber die Eigengravitation des Elektrischen Feldes nach der Ein- steinschen Theorie. *Annalen der Physik*, 50, (1916).

[10] R. P. Kerr. Gravitational Field of a Spinning Mass as an Example of Algebraically Special Metrics, *Physical Review Letters*. 11, (1963).

[11] S. Thirukkanesh. Strange star model with Tolmann IV type potential F.C. Ragel, *Astrophysics and Space Science*, **352**:743, (2014).

[12] G. Panotopoulos and Á. Rincón. Anisotropic Strange Quark Stars with a Non-Linear Equation-of-State. *The European Physical Journal Plus,* **134**:472, (2019).

[13] M. Malaver. Generalized Nonsingular Model for Compact Stars Electrically Charged. *World Science News*, **92**:327, (2018).

[14] R. P. Pant, S. Gedela, R. K. Bisht and N. Pant.*The European Physical Journal C,* **79**:602, (2019).

[15] M. Malaver. New Mathematical Models of Compact Stars with Charge Distributions. *International Journal of Systems Science and Applied Mathematics*, **2**:93, (2017).

[16] M. Malaver. Polytropic Stars with Tolman IV Type Potential. *American Association for Science and Technology,* **1**:309, (2015).

[17] T. Feroze and A. A. Siddiqui. Charged Anisotropic Matter with Quadratic Equation of State. *General Relativity and Gravitation*, **43**:25, (2011).

[18] T. Feroze and Hadia Tariq. Exact Solutions of the Einstein Equations with Polytropic Equations of State. *Canadian Journal of Physics*, **93**:637, (2015).

[19] F. T. Ortiz, M. Malaver, Á. Rincón and Y. G. Leyton. Relativistic Anisotropic Fluid Spheres Satisfying a Non-Linear Equation of State. *The European Physical Journal C*, **80**:371, (2020).

[20] F.C. Ragel and S. Thirukkanesh. General Relativistic Model for Mixed Fluid Sphere with Equation of State. *The European Physical Journal C*, **79**:306 (2019).

[21] R. Ruderman. Pulsars: Structure and Dynamics. *Annual Review of Astronomy and Astrophysics*, **10**:427, (1972).

[22] V. Canuto. Equation of State at Ultrahigh Densities. *Annual Review of Astronomy and Astrophysics*, **12**:167, (1974).

[23] R. Kippenhahn, A. Weigert, *Stellar Structure and Evolution* (Springer, Berlin, 1990)

[24] A. Pérez Martínez, H. Pérez Rojas and H.J. Mosquera Cuesta. *The European Physical Journal C,* **29**:111, (2003).

[25] B.V. Ivanov. The Importance of Anisotropy for Relativistic Fluids with Spherical Symmetry. *Internation Journal of Theoretical Physics,* **49**:1236, (2010).

[26] L. Herrera and L. Nez. Modeling hydrodynamic phase transitions in a radiating spherically symmetric distribution of matter. *The Astrophysical Journal,* **339**:339, (1989).

[27] R.L. Bowers and E.P.T. Liang. Anisotropic Spheres in General Relativity. *The Astrophysical Journal*, **188**:657, (1974).

[28] K. Pant, P. Fuloria. The Comparative Analysis of Dense Stellar Models Governed by Quadratic and Linear Equations of State. *New Astronomy*, **84**:101509, (2021).

[29] L.Herrera. Cracking of Self-Gravitating Compact Objects. *Physics Letters A,***165**:206, (1992).

[30] SK Maurya, MK Jasim, YK Gupta and TT Smitha. Modeling of Charged Anisotropic Compact Stars in General Relativity. *The European Physical Journal A,* **53**:141, (2017).

[31] M. R. Finch and J. E. F. Skea. J. E. F. A Realistic Stellar Model Based on an Ansatz of Duorah and Ray Class. *Classical and Quantum Gravity*, **6**:467, (1989).

[32] K. Jotania and R. Tikekar. Paraboloidal Space–Times and Relativistic Models of Strange Stars. *International Journal of Modern Physics D*, **15**:1175, (2006).

[33] V.O. Thomas and D.M. Pandya. Anisotropic Compacts Stars on Paraboloidal Spacetime with Linear Equation of State. *The European Physical Journal A*, **53**:120, (2017).

[34] H.A. Buchdahl. Regular general relativistic charged fluid spheres. *Acta Physica Polonica*, **10**:673, (1979).

[35] S. Thirukkanesh, R. Sharma and S. Das. Model of a Static Spherically Symmetric Anisotropic Fluid Distribution in Paraboloidal Spacetime Admitting a Polytropic Equation of State. *The European Physical Journal Plus*, **135**:620, (2020).

[36] V. Varela, F. Rahaman, S. Ray, K. Chakraborty and M. Kalam. Charged Anisotropic Matter with Linear or Nonlinear Equation of State. *Physical Review D,* **82**:044052, (2010).