Heavy to Light Meson Transition Form Factors and Applications in the framework of Standard Model

by

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Dedicated to my wife,

who supported me each step of the way.

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It is surreal to say that venture ends finally.

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Abstract

For the rare B meson decays, $B_d \to K^* l^+ l^-$ and $B_s \to \phi l^+ l^-$, the long distance effects are studied. The form factors which give a description of meson transition amplitudes in the effective Hamiltonian approach, are investigated by means of Wards Identities. These form factors are then compared with the other approaches in the literature , like Light Cone Sum Rules (LCSR) approach and Lattice QCD (LQCD) approach. Moreover, the $B_d \to K^* l^+ l^-$ and $B_s \to \phi l^+ l^-$ branching ratios are computed in the framework of Standard Model and are compared with the experimental results and the other approaches, LCSR and LQCD. The differential branching fractions are given as a function of the squared-momentum transferred.

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Chapter 1

Introduction

The semi-leptonic decays are of great interest since last two decades. These decays provide rigorous tests for Standard Model and also provide physics beyond the Standard Model. At tree level these decays are forbidden in Standard Model and occur only at loop level via GIM-mechanism. This mechanism was first presented by Glashow, Iliopulos, and Maiani [1]. It is easier to observe the exclusive rare B decays experimentally than inclusive rare B decays [2, 3] and converse is true for theoretical point of view i.e. theoretically the exclusive decays are more difficult to observe due to uncertainties in calculations of transition form-factors [4]. These form-factors are calculated in literature by using different approaches like Wards Identities (WI) [5], Light Cone Sum Rules (LCSR) [6], Lattice QCD (LQCD) [7], Dyson-Schwinger Equation (DSE) approaches amongst many others. These hadronic transition form-factors are the constituents of physical observables, such as branching ratios, forward-backward asymmetry, helicity fraction etc.

Rare decays, especially the $B_d \to K^* l^+ l^-$ and $B_s \to \phi l^+ l^-$ are interesting as they are the subject of intense experimental inspection by the LHCb Collaboration [8]. The experimental figures of branching ratios for these B decays are [9]:

$$\begin{aligned} Br(B_d \to K^* l^+ l^-) &= (1.06 \pm 0.09) \times 10^{-6}, \\ Br(B_s \to \phi l^+ l^-) &= (7.6 \pm 1.5) \times 10^{-7}. \end{aligned} \tag{1.1}$$

In our work we investigated the transition form-factors using Wards Identities (WI). We computed their values t $q^2 = 0$ and then extrapolated in Vector meson Dominance Model for all q^2 values for the said decays. Then we compared these form-factors with the other form-factors, already existing in the literature, such as Lattice QCD (LQCD) and Light Cone Sum Rules (LCSR) approaches. We give a comparison of these form-factors for the processes $B_d \to K^* l^+ l^-$ and $B_s \to \phi l^+ l^-$. By using these transition form-factors we calculate the branching ratio for the said processes in the framework of Standard Model and compare them with experimental results and other approaches.

1.1 Standard Model Highlights

The Standard Model (SM) of fundamental particles has been successful since its beginning in 1960 [10–12] and early 1970 [13, 14]. One of the beautiful features of SM is that it predicted the weak neutral current which was experimentally observed in the "Gargamelle Neutrino Experiment" in 1973 [15]] and was the first success of the SM. The experiment was run to look the processes of the form $\nu_\mu/\overline{\nu_\mu} + N \to \nu_\mu/\overline{\nu_\mu} + hadrons$(neutral current) and $\nu_\mu/\overline{\nu_\mu} + N \to \mu^- \mu^+ + hadrons$ i.e. charged current. Together with the data taken from processes and other similar experiments in the 1970 [16, 17], the Standard Model predicted the masses of the mediating Z and $W^\pm$ vector bosons. The W^- and Z-bosons were first directly produced at CERN (an European Organization for Nuclear Research) in 1983 [18,19]. The calculated mass well matched with the Standard Model predictions. The LEP (Large Electro-Positron collider) measured more precise Z-mass, a couple of years later. These experiments also probed the theory at the level of one-loop and found good agreements in many observables. Another success of SM is the prediction of top quark, which was experimentally tested at Fermilab at CDF (Collider Detector at Fermilab) in 1995 [20].

The Standard Model has some limitations which hinder its status as fundamental theory. Some of these limitations are as follows:

- Why the electroweak unification scale is so small (hierarchy problem)
- Gravity is not included in Standard Model

- What is the origin of mass-patterns among the fermions
- Why are there only three generations of quarks and leptons
- Neutrinos are massless in SM while experiments have shown that the neutrinos have mass

Despite all these deficiencies, the Standard Model is still a very successful model. Theoretical calculations within the SM and precised measurements of observables have been continued to show good agreements with each other [21–23].

In the following section we give an overview of the Standard Model. We put our focus on the particles, their parameters and interactions such as masses and coupling constants.

1.1.1 Overview of SM

The Standard Model (SM) is a gauge field theory established on the gauge group G_{SM}:

$$G_{SM} = SU(3)_C \otimes SU(2)_L \otimes U(1)_Y.$$

The groups $SU(3)_C$ and $G_{EW} = SU(2)_L \otimes U(1)_Y$ corresponds the strong interaction and electroweak interaction, respectively. The strong interaction acts only on $color - charged$ particles (qurks and gluons) while leaves the other particles of SM untouched. The subscript C in $SU(3)_C$ refers to $color - charge$. The group $SU(2)_L$ acts only on the left-handed fields. The left-handed particles (when the directions of spin and motion of the particle are opposite) participate in weak interactions where as the right-handed ones (when the direction of spin is the same as the direction of motion of the particle) do not (that is why we put the subscript L). The group $U(1)_Y$ acts on the particles with weakhypercharge and Y refers to the weak hypercharge. The generator for electric charge is the combination of I_3 (third isospin component) from $SU(2)$ and the hypercharge Y (corresponding to the gauge symmetry $U(1)$). They are combined with each other in well known

Gell-Mann-Nishijima formula:

$$Q = I_3 + \frac{Y}{2}.$$

The quark fields have a color-index. Each quark flavor q (where $q = u, d, s, c, t, b$) have three different types or colors. In order to make a tight notation, we collect these three types in a vector as follows;

$$q = (q^1\ q^2\ q^3)^T \tag{1.2}$$

$$\tag{1.3}$$

where $1, 2, 3$ is the color index. and $q = u, d, s, c, t, b$ is the flavor of quark.

The Weyl fermions are the fundamental constituents of Dirac-spinor ψ. The Dirac-spinor in chiral basis can be written as;

$$\psi = \begin{pmatrix} \phi \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \chi \end{pmatrix}$$

where the ϕ and χ are the Weyl spinors. These are two component spinors.

By introducing the two chiral projectors L and R as;

$$\begin{aligned} L &= \frac{1}{2}(1 - \gamma_5) \\ R &= \frac{1}{2}(1 + \gamma_5) \\ . \end{aligned} \tag{1.4}$$

	Symbol	Generations	$SU(3)_C$ Rep	$SU(2)_L$Rep	Y
Quarks	$q_L^{i,\alpha}$	$\begin{pmatrix} u^\alpha \\ d^\alpha \end{pmatrix}_L \begin{pmatrix} t^\alpha \\ b^\alpha \end{pmatrix}_L \begin{pmatrix} c^\alpha \\ s^\alpha \end{pmatrix}_L$	3	2	1/6
	$q_R^{i,\alpha}$	$u_R^\alpha \quad t_R^\alpha \quad c_R^\alpha$	3	1	2/3
	$d_R^{i,\alpha}$	$d_R^\alpha \quad b_R^\alpha \quad s_R^\alpha$	3	1	-1/3
Leptons	l_L^i	$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$	1	2	-1/2
	l_R^i	$e_R \quad \mu_R \quad \tau_R$	1	1	-1

Table 1.1: Fermion sector (quarks and leptons) of the SM and its transformation properties. The subscript L(R) represents the left/right handed fields. The superscript α represents the color index.

We define left handed and right handed fields as;

$$\begin{aligned} \psi_L &= L\psi \leftrightarrow \phi \\ \psi_R &= R\psi \leftrightarrow \chi \end{aligned} \tag{1.5}$$

There are 12 gauge bosons, 45 Weyl fermions and 1 Higgs doublet in the Standard Model. There are 18 independent parameters in SM, given as;

- Three lepton masses: m_l ; l= e, μ, τ ;
- Six fermion masses: m_i ; i= u, d, c, s, t, b;
- One gauge boson mass: m_Z (where $m_W^{\pm}$ is connected with m_Z via coupling);
- Three couplings: g, g', g_s;
- One Higgs boson mass: m_H;
- Four CKM matrix parameters.

	Gauge boson	Symbol	$SU(3)_C$ Rep	$SU(2)_L$Rep	Y
Electromagnetic interactions	photon	γ	1	1	
Weak interactions	W-boson , Z-boson	$W^{\pm}$, Z	1	3	
Strong interactions	gluon	g	8	1	0
	Higgs	ϕ	1	2	1/2

Table 1.2: Boson sector (Higgs boson, guage fields) of the SM and its transformation properties.

1.2 The Gauge Group of Electroweak Interactions

In the following section we discuss the gauge groups $SU(2)_L$, $U(1)_Y$ and $SU(2)_L \otimes U(1)_Y$. We study the local invariance of these groups and see how gauging the group $SU(2)_L \otimes U(1)_Y$ gives massless W, Z bosons and the photon A_μ.

1.2.1 Gauge group $SU(2)_L$

To begin with, we consider electron and its neutrino at first and then we will generalize. The Dirac field operator can be written in left/right handed and components. For electron as a Dirac field operator we write;

$$e(x) = e_L(x) + e_R(x), \tag{1.6}$$

where

$$e_L(x) = \frac{1}{2}(1 - \gamma_5)e(x), \tag{1.7}$$

$$e_R(x) = \frac{1}{2}(1 + \gamma_5)e(x),$$

where e_L is left-handed while e_R is right-handed chiral state. These are two orthogonal subspaces and are projection operators P_L and P_R.

The matrix γ_5 is defined as $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$. In the chiral basis γ_5 is written as,

$$\gamma^5 = \begin{pmatrix} -I_2 & 0 \\ 0 & I_2 \end{pmatrix}.$$

As the Dirac field representation is not an irreducible representation. It actually splits into two irreducible representations. Their behaviour is similar under rotation while behave differently under boost. We can split the representation into two irreducible representations out of which one is called left-handed (e_L) while the other one is known as right-handed (e_R). The e_L and e_R are doublet and singlet respectively under $SU(2)_L$.

The Dirac mass term for fermions is of the type $-m_f\bar{\psi}\psi$, but such terms are not allowed in the Lagrangian as are not invariant under $SU(2)_L$.

$$\begin{aligned} -m_f\bar{\psi}\psi &= -m_f(\bar{e}_L + \bar{e}_R)(e_L + e_R), \\ &= -m_f\left(\bar{e}_L e_R + \bar{e}_R e_L\right). \quad (\text{since } \bar{e}_L e_L = \bar{e}_R e_R = 0) \end{aligned} \tag{1.8}$$

Since e_L is left-handed doublet (vector) and e_R right-handed singlet (scalar) so both behave differently under transformation which results the mass term transformation not to be a scalar. This type of term is not invariant in the Lagrangian. So we can not add this type of term in the Lagrangian by hand

We have so far seen the left-handed neutrino in the experiments. The Lagrange density for the three fields ν_{eL}, e_L) and e_R is,

$$\begin{aligned} \mathcal{L}_0(x) &= \bar{\ell}_L(x) i\gamma^\mu \partial_\mu \ell_L(x) + \bar{e}_R(x) i\gamma^\mu \partial_\mu e_R(x), \\ &= \begin{pmatrix} \bar{\nu}_{eL}(x) & \bar{e}_L(x) \end{pmatrix} (i\gamma^\mu\partial_\mu) \begin{pmatrix} \nu_{eL}(x) \\ e_L(x) \end{pmatrix} + \bar{e}_R(x) i\gamma^\mu \partial_\mu e_R(x). \end{aligned} \tag{1.9}$$

The Lagrange density (1.9) is invariant under arbitrary or global $SU(2)$ rotation or transformations but is not invariant under local $SU(2)$ transformations,

$$\begin{pmatrix} \nu_{eL}(x) \\ e_L(x) \end{pmatrix} \to U(x) \begin{pmatrix} \nu_{eL}(x) \\ e_L(x) \end{pmatrix},$$

where $U(x) \in SU(2)$. This Lagrangian is made invariant by replacing ∂_μ with covariant derivative

D_μ. This introduces three vector fields, one for each generator of $SU(2)$. The covariant derivative for $SU(2)$ is,

$$D^\mu = \partial^\mu + ig\frac{W_a^\mu(x)\tau^a}{2}, \tag{1.10}$$

where W_a^μ $(a = 1, 2, 3)$ are the three vector fields introduced for the sake of invariance and τ^a are Pauli spin matrices and g is the gauge coupling constant.

We define,

$$W_\mu(x) = \frac{W_\mu^a(x)\tau_a}{2}.$$

which is a 2×2 hermitian matrix with zero trace.

We define the matrix of field strengths as,

$$\begin{aligned} W_{\mu\nu}(x) &= \partial_\mu W_\nu(x) - \partial_\nu W_\mu(x) + ig[W_\mu(x), W_\nu(x)], \\ &= \frac{W_{\mu\nu}^a(x)\tau_a}{2}. \end{aligned} \tag{1.11}$$

Using $[\tau_a, \tau_b] = \frac{i}{2}\epsilon_{abc}\tau^c$, where ϵ_{abc} are the structure constants for $SU(2)$, we get,

$$W_{\mu\nu}^a(x) = \partial_\mu W_\nu^a(x) - \partial_\nu W_\mu^a(x) - g\epsilon_{abc}W_\mu^b(x)W_\nu^c(x).$$

Hence the Lagrangian density for the neutrino, W-fields and the electron is,

$$\mathcal{L} = \frac{1}{2}Tr(W_{\mu\nu})(W^{\mu\nu}) + \begin{pmatrix} \bar{\nu}_{eL} & \bar{e}_L \end{pmatrix} i\gamma^\mu(D_\mu) \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} + \bar{e}_R i\gamma^\mu\partial_\mu e_R.$$

$$\mathcal{L} = \frac{1}{2}Tr(W_{\mu\nu})(W^{\mu\nu}) + \begin{pmatrix} \bar{\nu}_{eL} & \bar{e}_L \end{pmatrix} i\gamma^\mu \left(\partial_\mu + ig\frac{W_\mu^a\tau_a}{2}\right) \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} + \bar{e}_R i\gamma^\mu\partial_\mu e_R. \tag{1.12}$$

This Lagrange density (1.12) is invariant under local $SU(2)$ transformations. Or in other words$\mathcal{L} \rightarrow \mathcal{L}$ for

$$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \rightarrow U(x) \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix},$$

$$e_R \rightarrow e_R,$$

$$W_\mu \to U(x) W_\mu U^\dagger(x) - \frac{i}{g} U(x) \partial_\mu U^\dagger(x), \tag{1.13}$$

where $U(x) \in SU(2)_L$ is a local gauge transformation. The gauge group W_μ (that we have introduced) is the weak isospin group and the fields ν_{eL} and e_L form a weak doublet; whereas e_R is singlet under $SU(2)_L$.

The process to gauge the global $SU(2)_L$ symmetry introduces not only vector fields, but also an interaction. The structure of the interaction can be read from Eq. (1.12) as,

$$\begin{aligned}
\mathcal{L}_{e\nu W} &= -g \begin{pmatrix} \bar{\nu}_{eL} & \bar{e}_L \end{pmatrix} \gamma^\mu \frac{W_\mu^a \tau_a}{2} \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}, \\
&= -g \begin{pmatrix} \bar{\nu}_{eL} & \bar{e}_L \end{pmatrix} \gamma^\mu \frac{1}{2} \left(\begin{pmatrix} 0 & W_\mu^1 \\ W_\mu^1 & 0 \end{pmatrix} + \begin{pmatrix} 0 & -iW_\mu^2 \\ iW_\mu^2 & 0 \end{pmatrix} \right. \\
&\quad \left. + \begin{pmatrix} W_\mu^3 & 0 \\ 0 & -W_\mu^3 \end{pmatrix} \right) \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}.
\end{aligned} \tag{1.14}$$

We define electric charge basis as,

$$W_\mu^\pm = \frac{1}{\sqrt{2}} \left(W_\mu^1 \mp i W_\mu^2 \right), \tag{1.15}$$

$$W_\mu^+ = (W_\mu^-)^\dagger,$$

The field $W_\mu^- W_\mu^+$ has the effect of annihilating $W^-(W^+)$ particles and creating $W^+(W^-)$ particles. So $\mathcal{L}_{e\nu W}$ in new basis is ,

$$\begin{aligned}
\mathcal{L}_{e\nu W} &= -g \begin{pmatrix} \bar{\nu}_{eL} & \bar{e}_L \end{pmatrix} \gamma^\mu \frac{1}{2} \begin{pmatrix} W_\mu^3 & \sqrt{2} W_\mu^+ \\ \sqrt{2} W_\mu^- & -W_\mu^3 \end{pmatrix} \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}, \\
&= -\frac{g}{2} \left(W_\mu^3 (\bar{\nu}_{eL} \gamma^\mu \nu_{eL} - \bar{e}_L \gamma^\mu e_L) \right. \\
&\quad \left. + \sqrt{2} W_\mu^+ \bar{\nu}_{eL} \gamma^\mu e_L + \sqrt{2} W_\mu^- \bar{e}_L \gamma^\mu \nu_{eL} \right).
\end{aligned} \tag{1.16}$$

The coupling (given in Eq.(1.16))describes the neutrino transformation into an electron with absorption of W^- particle. It also decribes that W_μ^3-boson couples with left-handed electron (e_L) and to the left-handed neutrino (ν_{eL}), but not with e_R, showing that W_μ^3 can not be identified as photon field. The photon couples to the left/right handed electron and not to the neutrino.

1.2.2 Gauge group $U(1)_Y$

Now, let see the Lagrange density $\mathcal{L}_0$ (1.9) under $U(1)$ transformations. Again, $\mathcal{L}_0$ is invariant under global $U(1)$ transformation (where θ, θ' are the constant phases for right-handed and left-handed parts respectively),

$$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \to e^{i\theta'} \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix},$$

$$e_R \to e^{i\theta} e_R. \tag{1.17}$$

Gauging these two $U(1)$ groups yield to massless gauge bosons. Which would lead to two photon like bosons in theory (that is a contradiction to the experiment). Gauging special combination of the $U(1)$ transformations of the form,

$$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \to e^{+iy_L\chi} \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix},$$

$$e_R \to e^{+iy_R\chi} e_R. \tag{1.18}$$

The operators generating the above mentioned group (y_L and y_R) would be referred to as weak hypercharge Y. Where hypercharge y_L is given to the fields ν_{eL} and e_L while y_R to e_R, then the transformation of $U(1)$ hypercharge group is,

$$\begin{pmatrix} \nu_{eL} \\ e_L \\ e_R \end{pmatrix} \to e^{i\chi Y} \begin{pmatrix} \nu_{eL} \\ e_L \\ e_R \end{pmatrix}, \tag{1.19}$$

with

$$Y = \begin{pmatrix} y_L & 0 & 0 \\ 0 & y_L & 0 \\ 0 & 0 & y_R \end{pmatrix}. \tag{1.20}$$

Again introducing the real vector field B_μ and gauge coupling constant g' (for gauge group $U(1)$), the field strength tensor is,

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu.$$

1.2.3 $SU(2)_L \otimes U(1)_Y$

The Lagrange density for $SU(2)_L \otimes U(1)_Y$ is given by,

$$\mathcal{L} = -\frac{1}{2}Tr(W_{\mu\nu})(W^{\mu\nu}) - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \bar{\psi}i\gamma^\mu D_\mu\psi, \tag{1.21}$$

where covariant derivative for the $SU(2)_L \otimes U(1)_Y$ gauge group is,

$$D_\mu = \partial_\mu + igW_\mu^a T_a + \frac{i}{2}g' B_\mu Y,$$

with

$$T_a = \begin{pmatrix} \frac{1}{2}\tau_a & 0_{2\times1} \\ 0_{1\times2} & 0_{1\times1} \end{pmatrix}.$$

where the matrix for hypercharge Y is the same as it is given in Eq. (1.20).
The Lie algebra of $SU(2)_L \otimes U(1)_Y$ is,

$$[T_a, T_b] = i\epsilon_{abc}T^c,$$

$$[T_a, Y] = 0.$$

The interaction term $\mathcal{L}_{int}$, in Eq. (1.21) are:

$$\begin{aligned}
\mathcal{L}_{int} &= -\bar{\psi}\gamma^\mu(gW^a_\mu T_a + g'B_\mu Y)\psi, \\
&= -\frac{g}{\sqrt{2}}(W^+_\mu \bar{\nu}_{eL}\gamma^\mu e_L + W^-_\mu \bar{e}_L\gamma^\mu \nu_{eL}) \\
&\quad -\frac{1}{2}\left(gW^3_\mu + 2y_L g'B_\mu\right)\bar{\nu}_{eL}\gamma^\mu\nu_{eL} \\
&\quad +\frac{1}{2}\left(gW^3_\mu - 2y_L g'B_\mu\right)\bar{e}_L\gamma^\mu e_L - y_R g'B_\mu \bar{e}_R\gamma^\mu e_R, \qquad (1.22)
\end{aligned}$$

where

$$\psi = \begin{pmatrix} \nu_{eL} \\ e_L \\ e_R \end{pmatrix}$$

As y_L and y_R are constants, these constants can be chosen freely because we already have another free parameter g'. Let us conventionally choose $Y_L = -\frac{1}{2}$.

The W^3_μ and B_μ both are electrically neutral and massless so far, which means both are on an equal footing. Their linear combinations can form an equivalent basis. We choose these two orthogonal linear combinations,

$$Z_\mu = \frac{1}{\sqrt{g^2+g'^2}}\left(gW^3_\mu - g'B_\mu\right). \qquad (1.23)$$

The gauge field orthogonal to Z_μ is,

$$A_\mu = \frac{1}{\sqrt{g^2+g'^2}}\left(g'W^3_\mu + gB_\mu\right). \qquad (1.24)$$

Definig the weak mixing angle θ_w,

$$\cos\theta_w = \frac{g'}{\sqrt{g^2+g'^2}}. \qquad (1.25)$$

and

$$\sin\theta_w = \frac{g}{\sqrt{g^2+g'^2}}.$$

Eqs. (1.23) and (1.24) can be re-expressed as,

$$\begin{aligned} Z_\mu &= \cos\theta_w W_\mu^3 - \sin\theta_w B_\mu, \qquad (1.26)\\ A_\mu &= \sin\theta_w W_\mu^3 + \cos\theta_w B_\mu. \end{aligned}$$

The interaction term in Eq.(1.22) becomes,

$$\begin{aligned} \mathcal{L}_{int} &= -\frac{g}{\sqrt{2}}\left(W_\mu^+\bar{\nu}_{eL}\gamma^\mu e_L + W_\mu^-\bar{e}_L\gamma^\mu\nu_{eL}\right) \qquad (1.27)\\ &-\sqrt{g^2+g'^2}Z_\mu\left(\frac{1}{2}\bar{\nu}_{eL}\gamma^\mu\nu_{eL} - \frac{1}{2}\bar{e}_L\gamma^\mu e_L - \sin^2\theta_w\left(-\bar{e}_L\gamma^\mu e_L + y_R\bar{e}_R\gamma^\mu e_R\right)\right)\\ &-\frac{gg'}{\sqrt{g^2+g'^2}}A_\mu\left(-\bar{e}_L\gamma^\mu e_L + y_R\bar{e}_R\gamma^\mu e_R\right). \end{aligned}$$

Now it has become obvious that one of the combination is coupling with the electrons and not with the neutrinos so we can identify that as a photon (the last term in above expression). If we choose

$$y_R = -1, \quad \frac{gg'}{\sqrt{g^2+g'^2}} = e$$

then

$$e = \sqrt{g^2+g'^2}\sin\theta_w\cos\theta_w$$

So the Lagrange density $\mathcal{L}_{int}$ is written in more compact form as follows:

$$\begin{aligned} \mathcal{L}_{int} &= -e[A_\mu J^\mu_{em} + \frac{1}{\sqrt{2}\sin\theta_w}(W_\mu^+\bar{\nu}_{eL}\gamma^\mu e_L + W_\mu^-\bar{e}_L\gamma^\mu\nu_{eL}) \qquad (1.28)\\ &+\frac{1}{\sin\theta_w\cos\theta_w}Z_\mu J^\mu_{NC}] \end{aligned}$$

where

$$J^\mu_{em} = -\bar{e}_L\gamma^\mu e_L - \bar{e}_R\gamma^\mu e_R = -\bar{e}\gamma^\mu e \qquad (1.29)$$

$$J^\mu_{NC} = \frac{1}{2}\bar{\nu}_{eL}\gamma^\mu\nu_{eL} - \frac{1}{2}\bar{e}_L\gamma^\mu e_L - \sin^2\theta_w J^\mu_{em} \qquad (1.30)$$

The J^{μ}_{em} and J^{μ}_{NC} are electromagnetic and neutral currents which couples with photon and Z-boson respectively.

Note that from the above discussion that gauging the group $SU(2)_L \otimes U(1)_Y$ gives massless photon field $A_\mu W$ and Z bosons. The linear combinations of W^3_μ and B_μ give the neutral Z boson and the photon field A_μ. The value of weak mixing angle θ_w (free parameter in the theory) is determined from experiments.

To make the above theory realistic, we have to assign masses to the bosons $W^{\pm}$ and Z and to the electrons. A possible solution was presented by Salam (1968) and Weinberg (1967) which is the spontaneous symmetry breaking (SSB).

Let us see the effect of SSB in $SU(2)_L \otimes U(1)_Y$.

1.3 Higgs field and spontaneous symmetry breaking

Symmetry is spontaneously broken when the Lagrangian is invariant but not at ground state. The mechanism that Weinberg (1967) and Salam (1968) presented suggested that along with the fermions and the vector fields, we require an additional scalar field (the Higgs field). The simplest idea is to introduce a complex scalar fields ϕ such that ϕ_1 and ϕ_2 are complex scalar fields. In order to get the similar type of Lagrangian (as in the previous section) in which we already have an $SU(2)_L$ doublet. The simplest way is to add another doublet (Higgs doublet ϕ) . So that $doublet \otimes doublet$ gives us $3 \oplus 1$ theory. The introduced complex scalar field ϕ such that ϕ_1 and ϕ_2 being the complex scalar parts is,

$$\phi(x) = \begin{pmatrix} \phi_1(x) \\ \phi_2(x) \end{pmatrix}.$$

This introduced field is a Lorentz scalar $SU(2)_L$ doublet. Looking for a Lagrange density $\mathcal{L}_\phi$ which is invariant under local $SU(2)_L$ transformations.

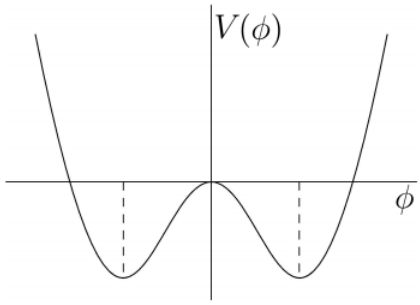


Figure 1.1: $V(\phi)$ Effective potential for $(\mu)^2 \ < \ 0$, shows local minima.

We consider a scalar potential that is amenable to spontaneous symmetry breaking. The Lagrange density would look like,

$$\mathcal{L}_\phi = (\partial_\mu \phi^\dagger)(\partial_\mu \phi) - V(\phi), \quad V(\phi) = \kappa(\phi^\dagger \phi) + \lambda(\phi^\dagger \phi)^2. \tag{1.31}$$

with the conditions,

$$\kappa = -\mu^2 < 0 \ , \qquad \lambda > 0.$$

The $V(\phi)$ is, illustrated in figure(1.1),

$$V(\phi) = \frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4.$$

To find the extrema we derivate it twice and set equal to zero. Doing so we get,

$$-\mu^2\phi + \lambda\phi^3 = 0 \tag{1.32}$$

which means $\phi = 0$ and $\phi = \pm\phi_0$ where $\phi_0 = \sqrt{\frac{\mu^2}{\lambda}}$ corresponds to minima of the potential. The field configuration $\phi = \left(0, \frac{1}{\sqrt{2}}\phi_0\right)^T$ is non-invariant under local $SU(2)_L$ transformation $U(x)$ (with $U(x) \in SU(2)$). The ground state of the gauge group $SU(2)_L$ has been broken spontaneously. Choosing ,

$$\langle 0|\phi(x)|0\rangle = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}\phi_0 \end{pmatrix}. \tag{1.33}$$

As the symmetry of the system has been broken spontaneously, which means that the field has been shifted. The new shifted field ϕ' is

$$\phi'(x) = \phi(x) - \langle 0|\phi(x)|0\rangle. \tag{1.34}$$

whose vacuum expectation value is zero i.e.

$$\langle 0|\phi(x')|0\rangle = 0. \tag{1.35}$$

Now minima is at $x = 0$.

We now will allow the Higgs field to interact with fermions and the gauge bosons. Thus adding this term in the Lagrange density, the Yukawa interaction is given by,

$$\begin{aligned}\mathcal{L}_{Yuk} &= -c_e \bar{e}_R \phi^\dagger \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} + h.c. \\ &= -c_e \left(\phi_1^\dagger \bar{e}_R \nu_{eL} + \phi_2^\dagger \bar{e}_R e_L\right) + h.c.\end{aligned} \tag{1.36}$$

where c_e is the Yukawa coupling constant here. We assign a suitable hypercharge Y_H to the Higgs field

$$y_H = y_L - y_R = \frac{1}{2}. \tag{1.37}$$

Let us write the full $SU(2)_L \otimes U(1)$ invariant Lagrangian. For the Higgs field the covariant derivative is,

$$\partial_\mu \phi \to D_\mu \phi = (\partial_\mu + igW_\mu^a \frac{\tau_a}{2} + ig' B_\mu y_H)\phi. \tag{1.38}$$

The total Lagrange density is,

$$\begin{aligned}\mathcal{L} &= -\frac{1}{2} Tr(W_{\mu\nu})(W^{\mu\nu}) - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \begin{pmatrix} \bar{\nu}_{eL} & \bar{e}_L \end{pmatrix} \left(i\gamma^\mu D_\mu\right) \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \\ &\quad + \bar{e}_R i\gamma^\mu D_\mu e_R - c_e \bar{e}_R \phi^\dagger \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} - h.c + (D_\mu \phi)^\dagger (D^\mu \phi) - V(\phi).\end{aligned} \tag{1.39}$$

In above Lagrange density, $3rd$ and $4th$ terms are $SU(2)$ doublet and $U(1)$ singlet respectively, while $5th$ and $6th$ terms are the yukawa terms (note that only yukawa term contains coupled right-handed and left-handed fermi fields) and the last terms are for the Higgs field (with ϕ as Higgs doublet).

Under $SU(2)_L \otimes U(1)_Y$ group of gauge transformations the Lagrange density (1.39) is invariant. $SU(2)$ gauge transformations are,

$$W_\mu \to U(x) W_\mu U^\dagger(x) - \frac{i}{g} U(x) \partial_\mu U^\dagger(x),$$

$$B_\mu \to B_\mu,$$

$$e_R \to e_R,$$

$$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \to U(x) \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}, \tag{1.40}$$

$$\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \to U(x) \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix},$$

where $U(x) \in SU(2)_L$, given by,

$$U(x) = e^{i(\frac{\tau_a}{2} \varphi^a(x))}, \tag{1.41}$$

where τ_a being 2×2 pauli spin matricles with ($a = 1, 2, 3$) and $\varphi^a(x)$ is an arbitrary function of x. $U(1)$ gauge transformations are given as,

$$W_\mu \to W_\mu,$$

$$B_\mu \to B_\mu - \frac{1}{g'} \partial_\mu \chi(x),$$

$$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \to e^{iy_L \chi(x)} \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}, \tag{1.42}$$

$$e_R \to e^{iy_R \chi(x)} e_R,$$

$$\phi(x) \rightarrow e^{iy_H\chi(x)}\phi(x),$$

with $\chi(x)$ being an arbitrary real function of x for $U(1)$ transformation group.
We can have rotation of the Higgs field (ϕ) in any direction in isospin space by means of gauge transformation.

$$U(x)\phi(x) = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}\rho(x) \end{pmatrix}, \tag{1.43}$$

Now the vacuum expectation value (vev) of the Higgs field is given by minimizing the potential, which is,

$$\langle 0|\rho(x)|0\rangle = \rho_0,$$

$$\langle 0|\phi(x)|0\rangle = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}\rho_0 \end{pmatrix}. \tag{1.44}$$

This vacuum expectation value in non-invariant under the full $SU(2)_L \otimes U(1)_Y$ group. Only $U(1)$ subgroup which is generated by $T_3 + Y$ leaves it invariant:

$$e^{i\chi(x)(\frac{\tau_3}{2}+y_H)} \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}\rho_0 \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}\rho_0 \end{pmatrix}. \tag{1.45}$$

1.3.1 Boson Masses

The next step of spontaneous symmetry breaking is to shift the field $\rho(x)$ by vev. Chosing the shifted field ($\rho'(x)$) such that,

$$\rho'(x) = \rho(x) - \rho_0(x). \tag{1.46}$$

Before using this shifted field to get the Higgs mass from Eq. (1.39), let's apply vev to the term $(D_\mu\phi)^\dagger(D^\mu\phi)$ (without considering ∂_μ part of the covariant derivative) given in Eq. (1.39),

$$
\begin{aligned}
(D_\mu\phi^\dagger)(D^\mu\phi) &= \langle\phi^\dagger\rangle_0(-ig\frac{W^{\mu a}\tau_a}{2} - ig'B^\mu Y_H)(ig\frac{W^a_\mu\tau_a}{2} + ig'B_\mu Y_H)\langle\phi\rangle_0, \\
&= \begin{pmatrix} 0 & \frac{1}{\sqrt{2}}\rho_0 \end{pmatrix} \begin{pmatrix} \frac{2gg'A_\mu + (g^2 - g'^2)Z_\mu}{2\sqrt{g^2+g'^2}} & \frac{g}{\sqrt{2}}W^+_\mu \\ \frac{g}{\sqrt{2}}W^-_\mu & -\frac{\sqrt{g^2+g'^2}}{2}Z_\mu \end{pmatrix} \\
&\quad \begin{pmatrix} \frac{2gg'A^\mu + (g^2 - g'^2)Z^\mu}{2\sqrt{g^2+g'^2}} & \frac{g}{\sqrt{2}}W^{+\mu} \\ \frac{g}{\sqrt{2}}W^{-\mu} & -\frac{\sqrt{g^2+g'^2}}{2}Z^\mu \end{pmatrix} \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}\rho_0 \end{pmatrix}, \\
&= \frac{g^2\rho_0^2}{4}W^-_\mu W^{\mu+} + \frac{(g^2+g'^2)\rho_0^2}{8}Z_\mu Z^\mu. \qquad (1.47)
\end{aligned}
$$

It is clear from the above expression that $W^\pm$ and Z boson have become massive, while the Photon A_μ is massless. From Eq. (1.47) boson masses are,

$$
\begin{aligned}
m_W &= \frac{1}{2}g\rho_0, \qquad (1.48) \\
m_Z &= \frac{1}{2}\rho_0\sqrt{g^2+g'^2},
\end{aligned}
$$

which shows that vev of the Higgs field is directly propotional to the masses of W and Z boson .Here ρ_0 is the vev of Higgs Field.

1.3.2 Fermion masses

Now let us go for the fermion masses. The Yukawa coupling in the Lagrange density (1.39) is,

$$
\begin{aligned}
-c_e\left(\bar{e}_R\langle\phi^\dagger\rangle_0\begin{pmatrix}\nu_{eL} \\ e_L\end{pmatrix} + \begin{pmatrix}\bar{\nu}_{eL} & \bar{e}_L\end{pmatrix}\langle\phi\rangle_0 e_R\right) &= -c_e\frac{1}{\sqrt{2}}\rho_0\left(\bar{e}_R e_L + \bar{e}_L e_R\right), \\
&= -c_e\frac{\rho_0}{\sqrt{2}}\bar{e}e. \qquad (1.49)
\end{aligned}
$$

The fermion (electron here) has acquired standard mass terms with,

$$m_e = \frac{1}{\sqrt{2}} c_e \rho_0. \tag{1.50}$$

Now the total Lagrange density in terms of new shifted field (1.34) is,

$$\begin{aligned}
\mathcal{L} \;=\; & -\frac{1}{2}Tr(W_{\mu\nu})(W^{\mu\nu}) - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \bar{\nu}_{eL} i\gamma^{\mu}\partial_{\mu}\nu_{eL} + \bar{e}i\gamma^{\mu}\partial_{\mu}e \\
& + m_W^2 W_{\mu}^- W^{\mu +}\left(1+\frac{\rho'}{\rho_0}\right)^2 + \frac{1}{2}m_Z^2 Z_{\mu}Z^{\mu}\left(1+\frac{\rho'}{\rho_0}\right)^2 - m_e\bar{e}e\left(1+\frac{\rho'}{\rho_0}\right) \\
& + \frac{1}{2}\partial_{\mu}\rho'\partial^{\mu}\rho' - \frac{1}{2}m_{\rho'}^2\rho'^2\left(1+\frac{\rho'}{\rho_0}+\frac{1}{4}\left(\frac{\rho'}{\rho_0}\right)^2\right) + \mathcal{L}',
\end{aligned} \tag{1.51}$$

where $\mathcal{L}'$ represents the coupling between the fermions and bosons. The term $m_{\rho'}$ (Higgs mass, as the field ρ' belongs to Higgs particle) is,

$$m_{\rho'} = \sqrt{2\lambda}\langle\phi\rangle_0. \tag{1.52}$$

According to Eq.(1.51), we can say that the Higgs mechanism has generated the masses for all the fermions and weak bosons ($W^{\pm}$, Z). The gauge symmetries ($SU(2)_L \otimes U(1)_Y$) are broken spontaneously, while the electromegnatic symmetry $U(1)_{EM}$ are unbroken. We can say that the symmetry group of the SM has broken down to a lower symmetry group of $SU(3)_C \otimes U(1)_{EM}$. The theory has given us the masses of the particles in the SM which include three massive vector bosons ($W^{\pm}$, Z)), a massive fermion (electron) and a massive, spin zero, neutral boson with mass $m_{\rho'}$ (the Higgs particle). While vector boson (photon with field A_{μ}) remains massless. We have also got a left-handed fermion with zero mass (neutrino). This can be extended for other fermion families.

1.3.3 GIM Mechanism and CKM matrix

Glashow, Iliopoulos and Maiani proposed a mechanism (called GIM mechanism) in 1970 [1]. This mechanism tells us that there are no transitions that would change flavour but not charge. At tree level flavour changing neutral currents (FCNC) do not occur. In weak interactions the flavor quantum nunmbers are not conserved and also weak interaction eigenstatesd', s', b' are different from mass eigen states. They are connected with each other via linear combinitions as,

$$\begin{aligned} d' &= V_{ud}\, d + V_{us}\, s + V_{ub}\, b \\ s' &= V_{cd}\, d + V_{cs}\, s + V_{cb}\, b \\ b' &= V_{td}\, d + V_{ts}\, s + V_{tb}\, b \end{aligned} \tag{1.53}$$

We have

$$\bar{d}'\gamma_\mu(1-\gamma_5)d' = [V^*_{ud}\,\bar{d} + V^*_{us}\,\bar{s} + V^*_{ub}\,\bar{b}]\gamma_\mu(1-\gamma_5)[V_{ud}\,d + V_{us}\,s + V_{ub}\,b] \tag{1.54}$$

The CKM matrix operate on downtype quarks by definition.

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}, \tag{1.55}$$

where V_{CKM} is CKM matric (or Cabibbo-Kobayashi-Maskawa Matrix) and

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}, \tag{1.56}$$

The CKM matrix has nine real parameters. Howerver only four (a phase and three angles) of them are physical and other five parameters are eliminated of by suitable transformation which leave the remaining terms in the lagrangian invariant.

The standard parametrization was proposed [24] in which $\theta_{12}, \theta_{13}, \theta_{23}$ are Euler angles and δ is the phase.

$$V_{CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}s_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \qquad (1.57)$$

with $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$. This parametrization has an benefit that the rotation angles are defined and labled in a way which related the mixing of two generations and one of the angles vanishes, the mixing between these two generations vanishes as well.

Chapter 2

The Theoretical Framework for Exclusive B-Meson Decays

In this Chapter we present the theoretical framework for exclusive B-meson decays, especially $B_d \to K^* l^+ l^-$ and $B_s \to \phi l^+ l^-$. In the first section we give an overview of effective Hamiltonian which is the main ingredient of this thesis. This effective Hamiltonian is going to be used to compute the amplitude. Finally we write the decay distribution of these decays in terms of helicity basis.

2.1 Effective Hamiltonian

The b-quark decay normally has two parts. They are treated at different energy scales. We elaborate this with the example of transition $b \to c\, d\, \overline{u}$. In figure (2.1), on left, we see that the dominant SM Feynman diagram corresponds to this decay process. The amplitude of the process is

$$A = (\frac{-ig}{\sqrt{2}})^2\, V_{cb}\, V_{ud}^*\, (\overline{d}\gamma_\mu L\, u) \frac{g^{\mu\nu} - \frac{k^\mu k^\nu}{m_W^2}}{i(k^2 - m_W^2)} (\overline{c}\gamma_\nu L\, b), \tag{2.1}$$

where k is the $4-$momentum. The involved two scales are m_W (≈ 80 Gev) and k, which is of the order of the mass of decaying b quark, m_b (≈ 4.8 Gev). The ratio of the two scales is small so we can write amplitude in k as:

$$A = \frac{g^2}{2i} V_{cb} V_{ud}^{*} [\frac{1}{m_W^2} \underbrace{(\bar{d}\gamma_\mu L\, u)(\bar{c}\gamma^\mu L\, b)} + \frac{1}{m_W^4}(dim\, 8\, operators) + \,...], \tag{2.2}$$

where $(\bar{d}\gamma_\mu L\, u)(\bar{c}\gamma\mu L\, b)$ is dimension 6 operator.

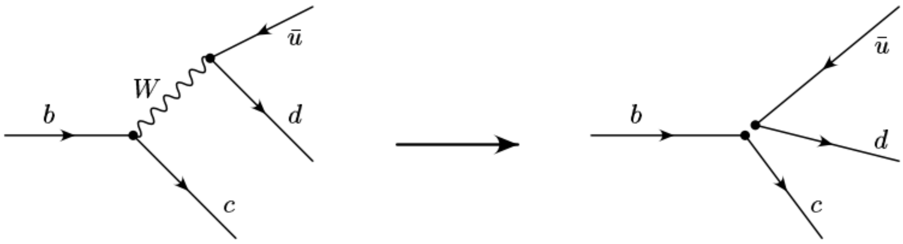


Figure 2.1: The decay process $b \rightarrow c\; d\; \overline{u}$ from a high-energy, on left, and low-energy, on right, point of view.

The effective Hamiltonian of this process is:

$$\begin{aligned} H_{eff} &= -\frac{4G_F}{\sqrt{2}} V_{cb} V_{ud}^{*} C\, (\bar{d}\gamma_\mu L\, u)(\bar{c}\gamma^\mu L\, b) \\ G_F &= \frac{g^2}{4\sqrt{2}m_W^2}, \end{aligned} \tag{2.3}$$

where G_F is Fermi constantV_{cb} V_{ud}^{*} are Cabibo-Kobayashia and Maskawa matrix elements, and C represents the Wilson coefficients at energy scale μ [4, 25, 26].

When considering QCD corrections the H_{eff} takes the form:

$$\begin{aligned} H_{eff} &= -\frac{4G_F}{\sqrt{2}}\, V_{cb}\, V_{ud}^*\, [C_1(\mu)\, O_1 + C_2(\mu)\, O_2], \\ O_1 &= (\bar{d}_\alpha \gamma_\mu L\, u_\beta)(\bar{c}_\beta \gamma^\mu L\, b_\alpha), \\ O_2 &= (\bar{d}_\alpha \gamma_\mu L\, u_\alpha)(\bar{c}_\beta \gamma^\mu L\, b_\beta), \end{aligned} \tag{2.4}$$

with α and β being the color indices. The Wilson coefficients give short distance effects (above μ) while the matrix elements of the operators deals with the long distance effects (below μ).
The method described above is called operator product expansion (OPE) which was first put forward by Wilson in 1969. OPE is important for weak decays. The important feature of this method is the factorization of short- and long distance contributions, as discussed above.

As we have already mentioned, this thesis focusses on the two exclusive decay processes $B_d \to K^* l^+ l^-$ and $B_s \to \phi l^+ l^-$. We now discuss the relevant part of the effective Hamiltonian for these processes. At quark level these processes are described by the transition $b \to s l^+ l^-$, which is described in SM as:

$$H_{eff} = -\frac{G_F}{\sqrt{2}} \lambda_t \sum_{i=1}^{10} C_i(\mu) O_i(\mu) \tag{2.5}$$

Whit G_F as Fermi constant, $\lambda_t \equiv V_{tb} V_{ts}^*$, $C_i(\mu)$ are the Wilson coefficients(short-distance contributions) and the $O_i(\mu)$ are the operators(long distance contribution) and μ is the effective limit. These above operators can explicitly written as [27];

Current-Current Operators

$$\begin{aligned} O_1 &= (\bar{s}_i u_j)_{V-A} (\bar{u} d_i)_{V-A} \\ O_2 &= (\bar{s} u)_{V-A} (\bar{u} d)_{V-A} \end{aligned} \tag{2.6}$$

QCD-Penguins Operators

$$\begin{aligned} O_3 &= (\overline{s}d)_{V-A}\sum_q(\overline{q}q)_{V-A} \\ O_4 &= (\overline{s_i}d_j)_{V-A}\sum_q(\overline{q_j}q_i)_{V-A} \\ O_5 &= (\overline{s}d)_{V-A}\sum_q(\overline{q}q)_{V+A} \\ O_6 &= (\overline{s_i}d_j)_{V-A}\sum_q(\overline{q_j}q_i)_{V+A} \end{aligned} \tag{2.7}$$

Electroweak-Penguins Operators

$$\begin{aligned} O_7 &= \frac{3}{2}(\overline{s}d)_{V-A}\sum_q e_q(\overline{q}q)_{V+A} \\ O_8 &= \frac{3}{2}(\overline{s_i}d_j)_{V-A}\sum_q e_q(\overline{q_j}q_i)_{V+A} \\ O_9 &= \frac{3}{2}(\overline{s}d)_{V-A}\sum_q e_q(\overline{q}q)_{V+A} \\ O_{10} &= \frac{3}{2}(\overline{s_i}d_j)_{V-A}\sum_q e_q(\overline{q_j}q_i)_{V-A} \end{aligned} \tag{2.8}$$

Magnetic-Penguins Operators

$$\begin{aligned} O_{7\gamma} &= \frac{e}{8\pi^2}\, m_b\, \overline{s_i}\, \sigma^{\mu\nu}\, (1+\gamma_5)\, b_i\, F^{\mu\nu} \\ O_{8G} &= \frac{g}{8\pi^2}\, m_b\, \overline{s_i}\, \sigma^{\mu\nu}\, (1+\gamma_5)\, T^a_{ij}\, b_j\, G^a_{\mu\nu} \end{aligned} \tag{2.9}$$

Here $F^{\mu\nu}$ and $G_{\mu\nu}$ are photon and gluon field strength tensors, T_{ij} are the generators of the $SU(3)$ group; i,j denotes the color indices.

Semi-Leptonic Operators

$$\begin{aligned} Q_{7V} &= (\bar{s}d)_{(V-A)}\,(\bar{e}e)_V \\ Q_{7A} &= (\bar{s}d)_{(V-A)}\,(\bar{e}e)_A \\ Q_{9V} &= (\bar{b}s)_{(V-A)}\,(\bar{e}e)_V \\ Q_{10A} &= (\bar{b}s)_{(V-A)}\,(\bar{e}e)_A \\ Q(\bar{\nu}\nu) &= (\bar{s}d)_{(V-A)}\,(\bar{\nu}\nu)_{(V-A)} \\ Q(\bar{\mu}\mu) &= (\bar{s}d)_{(V-A)}\,(\bar{\mu}\mu)_{(V-A)} \end{aligned} \tag{2.10}$$

where $V \equiv \gamma^{\mu}$ and $A \equiv \gamma^{\mu}\gamma^{5}$.

In terms of the above Hamiltonian in equation (2.5) , for $B \to V\ l^{+}\ l^{-}(\ ;\ V = K^{*}$ or $\phi)$ the amplitude is :

$$\begin{aligned} M_{SM}(B \to V\ l^{+}\ l^{-}) &= -\frac{G_F\alpha}{2\sqrt{2}\pi}V_{tb}V_{ts}^{*}[\ C_9^{eff}\langle V|\bar{s}\gamma^{\mu}(1-\gamma5)b|B\rangle\ \bar{l}\gamma^{\mu}l \\ &\quad +C_{10}\langle V|\bar{s}\gamma^{\mu}(1-\gamma_5)b|B)\rangle\ \bar{l}\gamma^{\mu}\gamma5l \\ &\quad -\frac{2m_B}{q^2}C_7^{eff}\langle V|\bar{s}\iota\sigma^{\mu\nu}q_{\nu}(1+\gamma_5)b|B\rangle\ \bar{l}\gamma^{\mu}l\ ]. \end{aligned} \tag{2.11}$$

The Wilson coefficient C_{10} is not normalized under QCD corrections and is independent of energy scale i.e. not function of μ. The effective Wilson coefficient $C_9^{eff}(\mu)$ is:

$$C_9^{eff}(\mu) = C_9(\mu) + Y_{SD}(z,s') + Y_{LD}(z,s'), \tag{2.12}$$

with $z = m_c/m_b$ and $s' = q^2/m_b^2$. The $Y_{SD}(z,s')$ and $Y_{LD}(z,s')$ describe the short distance and long distance contribution respectively. The mathematical expressions for $Y_{SD}(z,s')$ and $Y_{LD}(z,s')$ are:

$$
\begin{aligned}
Y_{SD}(z,s') &= h(z,s')(3C_1(\mu)+C_2(\mu)+3C_3(\mu)+C_4(\mu)+3C_5(\mu)+C_6(\mu)) \\
&\quad -\frac{1}{2}h(1,s')(4C_3(\mu)+4C_4(\mu)+3C_5(\mu)+C_6(\mu)) \\
&\quad -\frac{1}{2}h(0,s')(C_3(\mu)+3C_4(\mu))+\frac{2}{9}(3C_3(\mu)+C_4(\mu)+3C_5(\mu)+C_6(\mu)),
\end{aligned}
\tag{2.13}
$$

and

$$
Y_{LD}(z,s') = \frac{3\pi}{\alpha^2}C^{(0)}\sum_{V_i=\psi_i} k_i \frac{\Gamma(V_i \to l^+l^-)}{m_{V_i}^2 - s'm_b^2 - im_{V_i}\Gamma_{V_i}}, \tag{2.14}
$$

with $C^{(0)} = 3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6$.

The Wilson coefficient C_7^{eff} is:

$$
\begin{aligned}
C_7^{eff}(\mu) &= C_7(\mu) + C_{b\to s\gamma}(\mu), \\
C_{b\to\gamma}(\mu) &= i\alpha_s[\frac{2}{9}\eta^{14/23}(G_1(x_t) - 0.1687) - 0.03C_2(\mu)], \\
G_1(x_t) &= \frac{x(x^2-5x-2)}{8(x-1)^3} + \frac{3x^2\ln^2(x)}{4(x-1)^4},
\end{aligned}
\tag{2.15}
$$

with $\eta = \alpha_s(m_W)/\alpha_s(\mu)$, $x = m_t^2/m_W^2$.

The matrix elements, $\langle V|O_i|B\rangle$, in amplitude can be calculated by many ways. For Wards Identities the martix elements and form factors are related as, [5];

$$\langle V(k,\varepsilon)|V^\mu|B(p)\rangle = \frac{2i\epsilon^{\mu\nu\alpha\beta}}{m_B+m_V}\varepsilon^*_\nu k_\alpha p_\beta V(q^2) \tag{2.16}$$

$$\begin{aligned}\langle V(k,\varepsilon)|A^\mu|B(p)\rangle &= (m_B+m_V)\varepsilon^{*\mu}A_1(q^2) - \frac{\varepsilon^*.q}{m_B+m_V}(k+p)^\mu A_2(q^2) - \\ &\quad -2m_V\frac{\varepsilon^*.q}{m_B+m_V}\frac{\varepsilon^*.q}{q^2}[A_3(q^2)-A_0(q^2)]\end{aligned} \tag{2.17}$$

Where $V^\mu = \overline{u}\gamma^\mu b$, $A^\mu = \overline{u}\gamma^\mu\gamma^5 b$, and

$$A_3(q^2) = \frac{m_B+m_V}{2m_V}A_1(q^2) - \frac{m_B-m_V}{2m_V}A_2(q^2) \tag{2.18}$$

where $A_3(0) = A_0(0)$

$$\langle V(k,\varepsilon)|\overline{u}i\sigma^{\mu\nu}q_\nu b|B(p)\rangle = -i\varepsilon^{\mu\nu\alpha\beta}\varepsilon^*_\nu k_\alpha p_\beta F_1(q^2) \tag{2.19}$$

$$\begin{aligned}\langle V(k,\varepsilon)|\overline{u}i\sigma^{\mu\nu}\gamma^5 q_\nu b|B(p)\rangle &= [(m_B^2-m_V^2)\varepsilon^{*\mu} - (\varepsilon^*.q)(k+p)^\mu]F_2(q^2) + \\ &\quad +(\varepsilon^*.q)[q^\mu - \frac{q^2}{m_B^2-m_V^2}(k+p)^\mu]F_3(q^2)\end{aligned} \tag{2.20}$$

with $F_1(0) = 2F_2(0)$.

Putting these matrix elements in equation (2.11) and separating the $\bar{l}\gamma^\mu l$ and $\bar{l}\gamma^\mu\gamma^5 l$ terms we get,

$$M(b\to s\, l^+\, l^-) = \frac{G_f.\alpha}{2\sqrt{2}\pi}V_{tb}V^*_{ts}[(T_1^\mu)\bar{l}\gamma^\mu l + (T_2^\mu)\bar{l}\gamma^\mu\gamma^5 l] \tag{2.21}$$

(2.22)

$$T_i^\mu = \epsilon^*_\nu T_i^{\mu\nu} \quad ;\ i=1,2$$

$$T_1^{\mu\nu} = 2iX_1\epsilon^{\mu\nu\alpha\beta}k_\alpha p_\beta + X_2 q^\mu q^\nu - X_3 g^{\mu\nu} + X_4(p+k)^\mu q^\nu \tag{2.23}$$

$$T_2^{\mu\nu} = 2iX_5\epsilon^{\mu\nu\alpha\beta}k_\alpha p_\beta + X_6 q^\mu q^\nu - X_7 g^{\mu\nu} + X_8(p+k)^\mu q^\nu \tag{2.24}$$

Where

$$\begin{aligned}
X_1 &= C_9^{eff}\frac{V(q^2)}{m_B+m_V}+C_7^{eff}\frac{m_B}{q^2}F_1(q^2) \qquad (2.25)\\
X_2 &= \frac{C_9^{eff}}{q^2}[(-2m_V)A_0(q^2)+(m_B+m_V)A_1(q^2)-(m_B-m_V)A_2(q^2)]\\
&\quad -C_7^{eff}\frac{2m_B}{q^2}F_3(q^2)\\
X_3 &= C_9^{eff}(m_B+m_V)A_1(q^2)+\frac{2m_B}{q^2}C_7^{eff}(m_B^2-m_V^2)F_2(q^2)\\
X_4 &= C_9^{eff}\frac{A_2(q^2)}{m_B+m_V}+\frac{2m_B}{q^2}C_7^{eff}[F_2(q^2)+\frac{q^2}{m_B^2+m_V^2}F_3(q^2)]\\
X_5 &= C_{10}\frac{V(q^2)}{m_B+m_V}\\
X_6 &= \frac{C_{10}}{q^2}[-2m_VA_0(q^2)+(m_V+m_B)A_1(q^2)-(m_B-m_V)A_2(q^2)]\\
X_7 &= C_{10}(m_V+m_B)A_1(q^2)\\
X_8 &= \frac{C_{10}}{m_V+m_B}A_1(q^2)
\end{aligned}$$

Now as $|M|^2 = M^\dagger M$ so,

$$\begin{aligned}
|M|^2 &= (\frac{G_f.\alpha.\lambda_t}{2\sqrt{2}\pi})^2[T_1^\mu T_1^{\dagger\nu}(\bar{l}\gamma_\mu l)(\bar{l}\gamma_\nu l)^\dagger + T_1^\mu T_2^{\dagger\nu}(\bar{l}\gamma_\mu l)(\bar{l}\gamma_\nu\gamma_5 l)^\dagger \qquad (2.26)\\
&\quad +T_2^\mu T_1^{\dagger\nu}(\bar{l}\gamma_\mu\gamma_5 l)(\bar{l}\gamma_\nu l)^\dagger + T_2^\mu T_2^{\dagger\nu}(\bar{l}\gamma_\mu\gamma_5 l)(\bar{l}\gamma_\nu\gamma_5 l)^\dagger]
\end{aligned}$$

(2.27)

$$\begin{aligned}
|M|^2 &= (\frac{G_f.\alpha.\lambda_t}{2\sqrt{2}\pi})^2[H_{11}^{\mu\nu}(\bar{l}\gamma_\mu l)(\bar{l}\gamma_\nu l)^\dagger + H_{12}^{\mu\nu}(\bar{l}\gamma_\mu l)(\bar{l}\gamma_\nu\gamma_5 l)^\dagger\\
&\quad +H_{21}^{\mu\nu}(\bar{l}\gamma_\mu\gamma_5 l)(\bar{l}\gamma_\nu l)^\dagger + H_{22}^{\mu\nu}(\bar{l}\gamma_\mu\gamma_5 l)(\bar{l}\gamma_\nu\gamma_5 l)^\dagger],
\end{aligned}$$

where $\lambda_t = V_{tb}V_{ts}^*$ and $H_{ij}^{\mu\nu} = T_i^\mu T_j^{\nu\dagger}$ [28].

Now as,

$$
\begin{aligned}
(\bar{l}\gamma_\mu l)(\bar{l}\gamma_\nu l)^\dagger &= tr[\gamma^\mu(\not{p_1}^\mu - m_l)\gamma_\nu(\not{p_2}^\mu + m_l)] \qquad (2.28)\\
(\bar{l}\gamma_\mu\gamma_5 l)(\bar{l}\gamma_\nu\gamma_5 l)^\dagger &= tr[\gamma^\mu\gamma_5(\not{p_1}^\mu - m_l)\gamma_\nu\gamma_5(\not{p_2}^\mu + m_l)]\\
(\bar{l}\gamma_\mu l)(\bar{l}\gamma_\nu\gamma_5 l)^\dagger &= -tr[\gamma^\mu(\not{p_1}^\mu - m_l)\gamma_\nu\gamma_5(\not{p_2}^\mu + m_l)]\\
(\bar{l}\gamma_\mu\gamma_5 l)(\bar{l}\gamma_\nu l)^\dagger &= -tr[\gamma^\mu\gamma_5(\not{p_1}^\mu - m_l)\gamma_\nu(\not{p_2}^\mu + m_l)]
\end{aligned}
$$

so therefore,

$$
\begin{aligned}
\sum_{pol}|M|^2 &= [H_{11}^{\mu\nu}.tr[\gamma^\mu(\not{p_1}^\mu - m_l)\gamma_\nu(\not{p_2}^\mu + m_l) \qquad (2.29)\\
&+H_{22}^{\mu\nu}.tr[\gamma^\mu\gamma_5(\not{p_1}^\mu - m_l)\gamma_\nu\gamma_5(\not{p_1}^\mu + m_l)\\
&-H_{12}^{\mu\nu}.tr[\gamma^\mu(\not{p_1}^\mu - m_l)\gamma_\nu\gamma_5(\not{p_2}^\mu + m_l)\\
&-H_{21}^{\mu\nu}.tr[\gamma^\mu\gamma_5(\not{p_1}^\mu - m_l)\gamma_\nu(\not{p_2}^\mu + m_l)]
\end{aligned}
$$

(2.30)

$$
\begin{aligned}
&= [H_{11}^{\mu\nu}.4(-g_{\mu\nu}(m_l^2 + p_1.p_1) + p_1^\mu p_2^\nu + p_2^\mu p_1^\nu)\\
&+H_{22}^{\mu\nu}4(g_{\mu\nu}(m_l^2 - p_1.p_1) + p_1^\mu p_2^\nu + p_2^\mu p_1^\nu)\\
&+H_{12}^{\mu\nu}.4(i\varepsilon_{\mu\nu\alpha\beta}p_1^\alpha p_2^\beta) + H_{21}^{\mu\nu}.4(i\varepsilon_{\mu\nu\alpha\beta}p_1^\alpha p_2^\beta)]
\end{aligned}
$$

(2.31)

$$
\begin{aligned}
\sum_{pol}|M|^2 &= 4[H_{11}^{\mu\nu}(-L_{\mu\nu}^{(2)}(m_l^2 + \frac{q^2 - 2m_l^2}{2}) + L_{\mu\nu}^{(1)})\\
&+H_{22}^{\mu\nu}(L_{\mu\nu}^{(2)}(m_l^2 - \frac{q^2 - 2m_l^2}{2}) + L_{\mu\nu}^{(1)})\\
&+(H_{12}^{\mu\nu} + H_{21}^{\mu\nu})L_{\mu\nu}^{(3)}]\\
&= 4[L_{\mu\nu}^{(1)}(H_{11}^{\mu\nu} + H_{22}^{\mu\nu}) - \frac{1}{2}L_{\mu\nu}^{(2)}(q^2 H_{11}^{\mu\nu} + (q^2 - 4m_l^2)H_{22}^{\mu\nu}) \qquad (2.32)\\
&+L_{\mu\nu}^{(3)}(H_{12}^{\mu\nu} + H_{21}^{\mu\nu})].
\end{aligned}
$$

We have defined hadron and lepton tensors as [28];

$$
\begin{aligned}
L^{(1)}_{\mu\nu} &= p_{1\mu}p_{2\nu} + p_{2\mu}p_{1\nu}, \quad (2.33)\\
L^{(2)}_{\mu\nu} &= g_{\mu\nu}\\
L^{(3)}_{\mu\nu} &= i\varepsilon_{\mu\nu\alpha\beta}p_1^{\alpha}p_2^{\beta},\\
H^{\mu\nu}_{ij} &= T^{\mu}_{i}T^{\nu\dagger}_{j}
\end{aligned}
$$

These tensors can be solved, as given in the following sections.

2.1.1 Hadronic part

The hadronic tensor in terms of helicity basis $\varepsilon^{\dagger\mu}(m)$ as,

$$
\begin{aligned}
H^{(i)}_{m} &= \varepsilon^{\dagger\mu}(m)T^{(i)}_{\mu} \quad (2.34)\\
H^{(i)}_{m} &= \varepsilon^{\dagger\mu}(m)\epsilon^{\dagger\nu}(n)T^{(i)}_{\mu\nu}
\end{aligned}
$$

Where $T^{(i)}_{\mu} = \epsilon^{\dagger\nu}(n)T^{(i)}_{\mu\nu}$; ϵ^{ν} is the polarization vector of the V meson ($V = K^{*},\ \phi$) ; $m, n = 0, \pm, t$, are the longitudinal, transverse and time components ; and $i = 1, 2$.
The helicity components of polarization vector reads as;

$$
\begin{aligned}
\epsilon^{\mu}(\pm) &= \frac{1}{\sqrt{2}}(0, \pm 1, -i, 0) \quad (2.35)\\
\epsilon^{\mu}(0) &= \frac{1}{m_V}(|k|, 0, 0, -E_V)
\end{aligned}
$$

(2.36)

and in the B meson rest frame i.e

$$\begin{aligned}
\mathrm{p}^{\mu} &= (m_B, 0, 0, 0) && (2.37)\\
\mathrm{k}^{\mu} &= (E_k, 0, 0, |k|)\\
\mathrm{q}^{\mu} &= (q_0, 0, 0, -|k|)
\end{aligned}$$

the polarization vectors reads as

$$\begin{aligned}
\varepsilon^{\mu}(t) &= \frac{1}{\sqrt{q^2}}(q_0, 0, 0, |k|) && (2.38)\\
\varepsilon^{\mu}(\pm) &= \frac{1}{\sqrt{2}}(0, \mp 1, -i, 0)\\
\varepsilon^{\mu}(0) &= \frac{1}{\sqrt{q^2}}(|k|, 0, 0, q_0)
\end{aligned}$$

where $|K| = \frac{\lambda^{\frac{1}{2}}}{2m_B}$; $\lambda = \sqrt{m_B^4 + m_V^4 + q^4 - 2(m_B^2 m_V^2 + m_V^2 q^2 + m_B^2 q^2)}$ and $E_V = \frac{m_B^2 + m_V^2 - q^2}{2m_B}$ so using equation (2.34) we have,

$$\begin{aligned}
H_0^{(1)} &= \frac{1}{m_V\sqrt{q^2}}[2q_0|k|^2(q_0 - E_V)X_2 + (|k|^2 + q_0 E_V)X_3 && (2.39)\\
&\quad +|k|^2(q_0(m_B + 2E_V) - q_0^2 - E_V(m_B + E_V))X_4]\\
H_0^{(2)} &= \frac{1}{m_V\sqrt{q^2}}[2q_0|k|^2(q_0 - E_V)X_6 + (|k|^2 + q_0 E_V)X_7 && (2.40)\\
&\quad +|k|^2(q_0(m_B + 2E_V) - q_0^2 - E_V(m_B + E_V))X_8]\\
H_+^{(1)} &= -i|k|m_B X_1 + X_3 && (2.41)\\
H_+^{(2)} &= -i|k|m_B X_5 + X_7 && (2.42)\\
H_-^{(1)} &= i|k|m_B X_1 + X_3 && (2.43)\\
H_-^{(2)} &= i|k|m_B X_5 + X_7 && (2.44)
\end{aligned}$$

These are the components of hadronic tensor, The subscripts $\pm, 0$ denotes the transverse and longitudinal helicity components, respectively.

2.1.2 Leptonic part

For the leptonic tensors $L_{\mu\nu}^{(k)}$ (in $\bar{l}l$-CM frame we can write,

$$\begin{aligned} q^{\mu} &= (\sqrt{q^2},\ \overrightarrow{0}) \\ p_1^{\mu} &= (E_l,\ |p_1|\sin\theta,\ 0,\ |p_1|\cos\theta) \\ p_2^{\mu} &= (E_l,\ -|p_1|\sin\theta,\ 0,\ -|p_1|\cos\theta) \end{aligned} \tag{2.45}$$

with $E_l = \sqrt{q^2}/2$ and $|p_1| = \sqrt{q^2 - 4m_l^2}/2$ and the polarization vectors in $\bar{l}l-$CM frame are;

$$\begin{aligned} \epsilon^{\mu}(\pm) &= \frac{1}{\sqrt{2}}(0, \mp 1, -i, 0) \\ \epsilon^{\mu}(0) &= (0,\ 0,\ 0,\ 1) \\ \epsilon^{\mu}(t) &= (1,\ 0,\ 0,\ 0) \end{aligned} \tag{2.46}$$

Hence by using the this information of polarization of vectors and lepton kinematics we have calculated the following lepton tensor components;

$$L^{(1)}_{00} = -2|p_1|^2\cos^2\theta \tag{2.47}$$

$$L^{(2)}_{00} = -1 \tag{2.48}$$

$$L^{(3)}_{00} = 0 \tag{2.49}$$

$$L^{(1)}_{++} = E_l^2 - |p_1|^2\sin^2\theta \tag{2.50}$$

$$L^{(2)}_{++} = -1 \tag{2.51}$$

$$L^{(3)}_{++} = -2E_l|p_1|\cos\theta \tag{2.52}$$

$$L^{(1)}_{--} = E_l^2 \tag{2.53}$$

$$L^{(2)}_{--} = -1 \tag{2.54}$$

$$L^{(3)}_{--} = 2E_l|p_1|\cos\theta \tag{2.55}$$

We have ignored the time component for both leptonic and hadronic tensors. Now by using these leptonic tensor components the hadronic tensor components and the matrix elements, we can write our amplitude M of equation (2.11). Now we are in a position to write our decay distribution.

2.2 Differential decay rate

We can write differential decay rate in terms of helicity amplitude, which is:

$$\begin{aligned}\frac{d^2\Gamma}{dq^2 d\cos\theta} &= \frac{G_F^2}{(2\pi)^3}(\frac{\alpha|\lambda_t|}{2\pi})^2\frac{|k|\beta_l}{8m_l^2}\times\frac{1}{2}[L^{(1)}_{\mu\nu}.(H^{\mu\nu}_{11}+H^{\mu\nu}_{22}) \\ &\quad -\frac{1}{2}L^{(2)}_{\mu\nu}.(q^2H^{\mu\nu}_{11}+(q^2-4m_l^2)H^{\mu\nu}_{22})+L^{(3)}_{\mu\nu}.(H^{\mu\nu}_{12}+H^{\mu\nu}_{21})],\end{aligned} \tag{2.56}$$

$|k|$ is the momentum of vector meson, given in the rest frame of B meson and $\beta_l = \sqrt{1-4m_l^2/q^2}$. After integration over $\cos\theta$ and putting the values of the leptonic and hadronic tensor components (calculated in above section) $L^{(k)}(m,n)$ and $H^{ij}(m,n)$, respectively, we get;

$$\frac{d\Gamma(B \rightarrow Vl^+l^-)}{dq^2} = \frac{G_F^2}{(2\pi)^3}(\frac{\alpha|\lambda_t|}{2\pi})^2\frac{\lambda^{1/2}q^2}{48M_B^3}\beta_l[H^1H^{1\dagger}(1+4m_l^2/q^2) + H^2H^{2\dagger}(1-4m_l^2/q^2)], \tag{2.57}$$

where m_l is the lepton mass and

$$H^iH^{i\dagger} \equiv H_+^iH_+^{i\dagger} + H_-^iH_-^{i\dagger} + H_0^iH_0^{i\dagger}. \tag{2.58}$$

We will use decay rate $\frac{d\Gamma}{dq^2}$ to calculate the branching fractions for the decays $B_d \rightarrow K^*l^+l^-$ and $B_s \rightarrow \phi l^+l^-$ in next chapter.

Chapter 3

$B \to V$ Exclusive transition form-factors and applications

This chapter deals with the transition form factors and its applications for $B \to V\mu^+\mu^-$ ($V = K^*, \phi$) decays and its applications. More precisely we discuss the calculations of form factors of the said decays using experimental constraints, relate them through Wards Identities and extrapolate these form factors over whole physical region within the general vector meson dominance model. These form factors are then compared with other approaches existing in literature such as LCSR and LQCD.

3.1 Form factors and Wards Identities

The transition form-factors are the major hadronic uncertainties in the decays. There are many approaches to calculate these form-factors. We have used form-factors from Wards Identities (W.I.) [5], Lattice QCD (LQCD) [7] and Light Cone Sum Rules (LCSR) [6] approaches.
In the case of $B \to V\mu^+\mu^-$ ($V = K^*, \phi$) decays, there are seven form-factors [4, 5], which are given in equation (2.16)-(2.20). These form-factors can be related with each other using Wards

Identities [5, 29] as given below,

$$\langle V(k,\varepsilon)|\bar{s}\sigma^{\mu\nu}q_\nu b|B(p)\rangle = -(m_b+m_s)\langle V(k,\varepsilon)|\bar{s}\gamma^\mu b|B(p)\rangle \tag{3.1}$$
$$\langle V(k,\varepsilon)|\bar{s}\sigma^{\mu\nu}\gamma^5 q_\nu b|B(p)\rangle = (m_b-m_s)\langle V(k,\varepsilon)|\bar{s}\gamma^\mu\gamma_5 b|B(p)\rangle$$

Using matrix elements in Wards Identity equations we can relate the transition form-factors as :

$$F_1(q^2) = \frac{m_b+m_s}{M_B+M_V}V(q^2) \tag{3.2}$$
$$F_2(q^2) = \frac{m_b-m_s}{M_B-M_V}A_1(q^2) \tag{3.3}$$
$$F_3(q^2) = -\frac{2M_V}{q^2}V(q^2)(m_b-m_s)[A_3(q^2)-A_0(q^2)] \tag{3.4}$$

These form-factors $F_1(q^2), F_2(q^2), F_3(q^2)$ are model independent.The form-factors $V(q^2)$, $A_1(q^2)$, $A_2(q^2)$, $F_1(q^2)$, $F_2(q^2)$, $F_3(q^2)$ can be written as;

$$F_1(q^2) = g_+(q^2) - q^2h(q^2) \tag{3.5}$$
$$F_2(q^2) = g_+(q^2) + \frac{q^2}{M_B^2-M_V^2}g_-(q^2)$$
$$F_3(q^2) = -g_-(q^2) - (M_B^2-M_V^2)h(q^2), \tag{3.6}$$

and

$$V(q^2) = \frac{M_B+M_V}{m_b+m_s}[g_+(q^2) - q^2h_1(q^2)] \tag{3.7}$$
$$A_1(q^2) = \frac{M_B-M_V}{m_b-m_s}[g_+(q^2) + \frac{q^2}{M_B^2-M_V^2}g_-(q^2)]$$
$$A_2(q^2) = \frac{M_B-M_V}{m_b-m_s}[g_+(q^2) - q^2h(q^2)] - \frac{2M_V}{M_B-M_V}A_0(q^2)$$

The $V(q^2), A_1(q^2), F_1(q^2), F_2(q^2)$ can be parameterized at $q^2 = 0$ by $g_+(0)$ whereas $A_0(q^2)$ and $A_2(q^2)$ is expressed in terms of $g_+(0)$ and $A_0(0)$.

The decay rate in terms of $g_+(0)$ is written as [30]:

$$\Gamma(B \to V\gamma) = \frac{G_F^2 \alpha_{em}}{32\pi^4}|V_{tb}V_{ts}^*|^2 m_b^2 M_B^3 (1 - \frac{M_V^2}{M_B^2})^3 |C_7^{eff}|^2 |g_+(0)|^2 \tag{3.8}$$

The branching ratios of $B_d \to K^*\gamma$ and $B_s \to \phi\gamma$ have following experimental values [30].

$$\begin{aligned} Br(B_d \to K^*\gamma) &= (4.33 \pm 0.15) \times 10^{-5} \\ Br(B_s \to \phi\gamma) &= (3.6 \pm 0.4) \times 10^{-5} \end{aligned} \tag{3.9}$$

.

Using these branching ratios the extracted values of $g_+(0)$ are [4]:

$$\begin{aligned} g_+(0)(B_d \to K^*) &= 0.365^{+0.025}_{-0.025}, \\ g_+(0)(B_s \to \phi) &= 0.335^{+0.020}_{-0.020}. \end{aligned} \tag{3.10}$$

By using these $g_+(0)$ we can find the value of $A_0(0)$ [5] as;

$$A_0(0) = (\frac{1 - M_V^2/M_B^2}{1 + M_V^2/M_B^2} + \frac{M_B}{M_V}) g_+(0). \tag{3.11}$$

The form-factors from Wards Identities do not work for the whole region of q^2. So in that case we have to apply some parameterizations between $q^2 = 0$ and near the pole. Such a parameterization is given below [5]:

$$F(q^2) = \frac{F(0)}{(1 - q^2/M'^2)(1 - q^2/M^2)} \tag{3.12}$$

where M is $M_{B^*}(1^-)$ or $M_{B_A^*}(1^+)$ and M' is the radial excitation of M. This parameterization incorporates the correction to the single-pole-dominance and also helps to determine the coupling constant of B^* or B_A^* [31].

By using the above parameterization we can write the transition form-factors as [5]:

$$
\begin{aligned}
V(q^2) &= \frac{V(0)}{(1-q^2/M_B^2)(1-q^2/M_B'^2)}, \qquad (3.13)\\
A_1(q^2) &= \frac{A_1(0)}{(1-q^2/M_{B_A^*}^2)(1-q^2/M_{B_A^*}'^2)}(1-\frac{q^2}{M_B^2-M_V^2}),\\
A_2(q^2) &= \frac{\tilde{A}_2(0)}{(1-q^2/M_{B_A^*}^2)(1-q^2/M_{B_A^*}'^2)} - \frac{2M_V}{M_B-M_V}\frac{A_0(0)}{(1-q^2/M_B^2)(1-q^2/M_B'^2)},
\end{aligned}
$$

where $\tilde{A}_2(0)$ is defined as

$$
\tilde{A}_2(0) = \frac{M_B-M_V}{m_b-m_s}g_+(0). \qquad (3.14)
$$

$$
\qquad (3.15)
$$

3.1.1 Analysis for $B \to V\mu^+\mu^-$

A comparison of form-factors for $B_d \to K^*\mu^+\mu^-$ is presented in figure 3.1. The numerical results are given in table 3.1.

	V	A_1	A_2	$\frac{V(q^2_{max})}{V(0)}$	$\frac{A_1(q^2_{max})}{A_1(0)}$	$\frac{A_2(q^2_{max})}{A_2(0)}$
Ward Identities	0.457(2.692)	0.343(0.585)	0.343(0.977)	5.890	1.705	2.848
Light Cone Sum Rules	0.377(1.84)	0.295(0.593)	0.283(0.425)	4.880	2.010	1.502
Lattice QCD	0.330(1.910)	0.310(0.630)	0.240(0.420)	5.788	2.032	1.750

Table 3.1: Different form-factors for $B_d \to K^*\mu^+\mu^-$ at $q^2 = 0$ and $q^2 = q^2_{max}$ in different approaches. The first valve is for $q^2 = 0$ and the value in parenthesis is for $q^2 = q^2_{max}$. Also ratio of form-factors at $q^2 = 0$ and $q^2 = q^2_{max}$ is presented in last columns.

In $B_d \to K^*\mu^+\mu^-$ the form factor $V(q^2)$ curves are parallel to each other at low and high q^2 for LCSR and LQCD approaches . For Wards Identities, below $10Gev^2$ the trend is same as LCSR and LQCD but above $10Gev^2$ the curve increases sharply. In case of A_1 trends are same for all

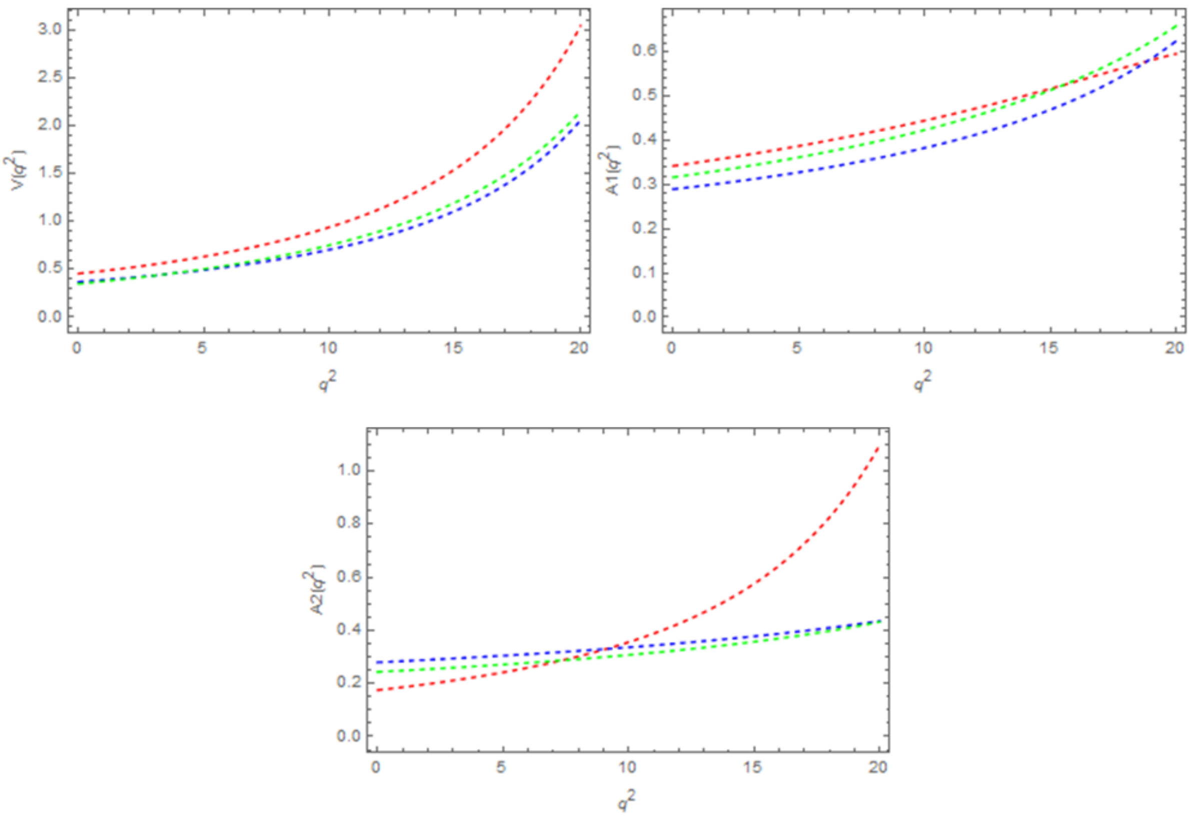


Figure 3.1: Comparison of form-factors for $B_d \rightarrow K^*\mu^+\mu^-$. Red line-for Wards Identities (W.I.), blue line- for LCSR and green line-for LQCD results.

three aproaches. For A_2 LCSR parallels the LQCD while below $10Gev^2$ curve of WI, LCSR and LQCD behave alike and above $10Gev^2$ WI curve increases with larger slope. These differences for different approaches are more clearly given numerically in Table3.1.

	V	A_1	A_2	$\frac{V(q^2_{max})}{V(0)}$	$\frac{A_1(q^2_{max})}{A_1(0)}$	$\frac{A_2(q^2_{max})}{A_2(0)}$
Ward Identities	0.428(2.155)	0.306(0.498)	0.295(0.504)	5.035	1.627	4.624
Light Cone Sum Rules	0.412(1.770)	0.320(0.622)	0.271(0.364)	4.296	1.944	1.343
Lattice QCD	0.280(1.714)	0.301(0.602)	0.250(0.371)	6.121	2.011	1.480

Table 3.2: Different form-factors for $B \rightarrow \phi\mu^+\mu^-$ at $q^2 = 0$ and $q^2 = q^2_{max}$. The first valve is for $q^2 = 0$ and the value in parenthesis is for $q^2 = q^2_{max}$.

For $B \rightarrow \phi\mu^+\mu^-$ form-factors are compared in figure3.2 and the numerical data is presented in

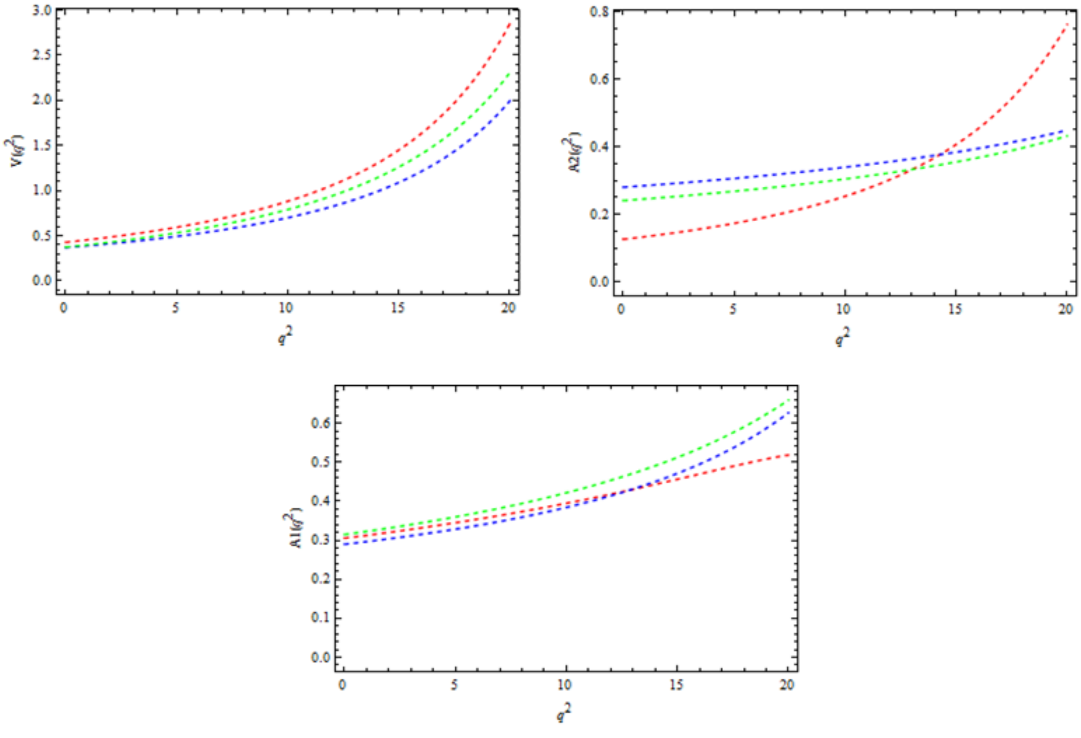


Figure 3.2: A comparison of form-factors for $B \rightarrow \phi\mu^+\mu^-$. Where the Red line-for Wards Identities (W.I.), blue line- for LCSR and green line for LQCD results.

table3.2.In case of $V(q^2)$ the trend of curves for WI, LCSR and LQCD approaches is same below $10Gev^2$ while above $10Gev^2$ this trend differed slightly. For $A_1(q^2)$, below $15Gev^2$, the curve of WI form-factors have same trend as of LCSR and LQCD form-factors and above $15Gev^2$ trend of WI have a slight decrease in slope. In case of$A_2(q^2)$ below$15Gev^2$ all three approaches have same increasing trend while above $15Gev^2$ WI differes slightly with more positive slope. The numerical values of form-factors are given in detail in table3.2

3.2 Application of transition form-factors

In the following section we present and discuss the branching ratio. We will use form-factors from different approaches (WI, LCSR, LQCD) to calculate this observable.

3.2.1 Analysis for Branching Fractions (Br)

Branching ratio is the ratio of decay width of a particular mode of decay to the full decay. For B meson decaying into a vector meson V and a lepton pair, we can write;

$$Br = \frac{\Gamma(B \rightarrow V\,\mu^+\mu^-)}{\Gamma_{tot}}.$$

The differential branching fraction is,

$$\frac{dBr}{dq^2} = \frac{1}{\Gamma_{tot}}\frac{d\Gamma(B \rightarrow Vl^+l^-)}{dq^2},$$

where $V = K^* or\ \phi$ meson and value of $\frac{d\Gamma}{dq^2}$ is given in equation 2.57. We have used Wards Identities (W.I.), Light Cone Sum Rules (LCSR) and Lattice QCD (LQCD) form-factors to calculate branching fractions. A comparison is given in 3.3. The numerical results for $B \rightarrow V\mu^+\mu^-$ is given in Table3.3.

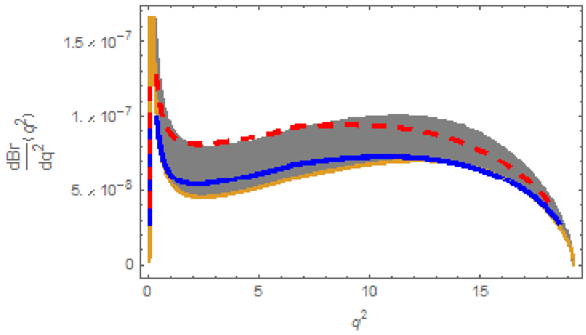
In case of $B_d \rightarrow K^*\mu^+\mu^-$ the branching ratios obtained with WI, LCSR and LQCD form-factors are close to the experimental value. The calculated numerical results for the said decays agree with PDG averages. As we have no error estimates in case of LCSR and LQCD so we can not comment in that cases. In case of Lattice QCD branching ratios are comparatively larger. This is due to reason that LQCD does not produce more rising form-factors and hence may not explain VMD behavior.

Similarly in case of $B_s \rightarrow \phi\mu^+\mu^-$ decay, values of LCSR and LQCD are larger. In case of Wards Identities as we have separate values of $g_+(0)$ for both decays but with a small difference. So the functional form of the $V(q^2), A_1(q^2), A_2(q^2)$ form-factors for both the decays is nearly same. As

	$Br(B_d \to K^*\mu^+\mu^-)$	$Br(B_s \to \phi\mu^+\mu^-)$
Ward Identities	$(1.38 \pm 0.20) \times 10^{-6}$	$(1.24 \pm 0.14) \times 10^{-6}$
Light Cone Sum Rules	1.22×10^{-6}	1.62×10^{-6}
Lattice QCD	1.89×10^{-6}	1.71×10^{-6}
PDG [9]	$(1.06 \pm 0.09) \times 10^{-6}$	$(7.6 \pm 1.5) \times 10^{-6}$

Table 3.3: Numarical values of branching ratios for semileptonic decays $B \to V\mu^+\mu^-$ in different form factor approaches and PDG

the difference is minor so we do not expect too much different branching ratios.



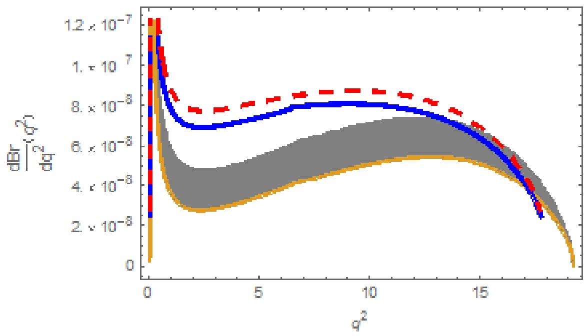


Figure 3.3: Differential branching fraction for $B \to K^*\mu^+\mu^-$ (on left panel) and $B \to \phi\mu^+\mu^-$ (on right panel). The branching fractions for WI, LCSR and LQCD form-factors are plotted as gray, blue, and red curves, respectively.

Chapter 4

Conclusion

In this dissertation we analyze the transition form factors for $B_d \to K^* l^+ l^-$ and $B_s \to \phi l^+ l^-$ by analyzing Ward Identities to determine their value at $q^2 = 0$ and then extrapolated them in a general VDM to large q^2 values. Furthermore, these form factors are used to calculate the physical observable such as branching fraction for the decays $B_d \to K^* l^+ l^-$ and $B_s \to \phi l^+ l^-$ and compared them to those obtained with form factors of corresponding LQCD, LCSR and experimental values of the branching ratios for the said decay.

We started our thesis with the introduction of Standard Model from group theoretical point of view and discussed its limitations. We gave highlights of this model and gave some calculations of the gauge group of electroweak interactions. Then we described briefly the Spontaneous Symmetry Breaking (SSB) and the GIM-mechanism and CKM-matrics.

In chapter two we gave theoretical framework of exclusive B-meson decays. We gave some details of effective Hamiltonian, its operators and Wilson coefficients. We wrote amplitude of these decays in the helicity basis and then used it in the differential decay distribution of these processes.

In third chapter we discussed form factors and Wards Identities briefly, and compared its results with the other approaches, like Light Cone Sum Rules and Lattice QCD. The form factors for $B_d \to K^* l^+ l^-$ are discussed below and above the $10Gev^2$. Similarly for $B_s \to \phi l^+ l^-$ comparison is made for below the $15Gev^2$ and above the $15Gev^2$. Finally the branching fraction of both $B_d \to K^* l^+ l^-$ and $B_s \to \phi l^+ l^-$ is calculated from these form factors and comparison for both decays is given for Light Cone Sum Rules, Lattice QCD and Wards Identities. For $B_d \to K^* l^+ l^-$ the trend of curves is more closer to each other for all approaches as compared with the $B_s \to \phi l^+ l^-$.

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