Quantum modification of dust shear Alfven waves in plasmas

Faryal Mughal
Regn.# 00000278314

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Supervised by
Dr. Muddasir Ali Shah

Department of Physics

School of Natural Sciences
National University of Sciences and Technology
H-12, Islamabad, Pakistan

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We hereby recommend that the dissertation prepared under our supervision by: Faryal Mughal, Regn No. 00000278314 Titled: **"Quantum modification of dust shear Alfven waves in plasmas"** accepted in partial fulfillment of the requirements for the award of **MS** degree.

Examination Committee Members

1. Name: DR. SHAHID IQBAL Signature: ____________

2. Name: DR. HUSSAIN GOHAR Signature: ____________

External Examiner: DR. AZHAR HUSSAIN Signature: ____________

Supervisor's Name: DR. MUDDASIR ALI SHAH Signature: ____________

Head of Department

1/02/2021
Date

COUNTERSINGED

Date: 01/02/2021

Dean/Principal

Dedicated to

My Father "Abdul Razaq Mirza", My Mother "Rehana Mirza", My Husband "Ahmad Haider Nawaz" and my siblings.

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All thanks and praises be to Allah, the lord of the Alamin(mankind, jinns and all that exists) the most beneficent the most merciful. The only owner of day of recompense. You (alone) we worship and you (alone) we ask for help. You have done for me through all my years I've been lost. I thank You with every breath I take. All praises for beloved Holy Prophet (P.B.U.H), succorer of humanity, gem of mankind, the unparalleled, the unrivaled, and the infallible. Special thanks to my supervisor **"Dr.Muddasir Ali Shah"**. You are a wonderful teacher who knows how to bring best in your students. I appreciate you for being my supervisor, you helped shape my career and showed me how to transform my mistakes into skills. Words can neither qualify nor quantify how helpful your guidance and advice has been. I am forever grateful for your support! I would like to express my thankfulness to my guidance and examination committee members Dr. Shahid Iqbal and Dr. Gohar for their guidance and support. A lot of thanks to my parents, my husband, my siblings "Sheheryar Mirza", "Sarosh Mughal". Especial thanks to "Shahrukh Mirza" for taking up all the troubles. The only person in this whole world with whom I can switch between saying 'I hate you' and 'I love you' without caring about anything is my younger brother. Thank you for being such an integral part of my life. I would like to says thanks to my friends especially Nayera Frooq, Zeeshan Rasheed, Waseem khan, Huzaifa Bilal, Muhammad Usman, Danish Ali Hamza, Saira Parveen, Mahpara and Sabeen Fatima for their support and love.

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ABSTRACT

Interest in quantum plasma has been developed due to its applications in ultra-small electronic devices, laser-plasma, and dense astrophysical plasmas. Vast research has been done in Quantum plasmas with high densities and low temperature but quantum effects can't be neglected in modest density, high-temperature plasmas which we normally consider as classical plasmas. Due to a high number density in quantum plasma electron plasma frequency become extremely high and it exceeds electron collision frequency. Because of these properties, new effects are generated in a plasma. Shear Alfven Waves (low frequency, long wavelength) in quantum dusty magnetoplasma have been studied using the quantum magnetohydrodynamic model (QMHD), which considers numbers of forces like Lorentz force, Bohm potential, Quantum force etc. A modified dispersion relation for Shear Alfven waves is formed while considering electrons and ions, magnetized and electrons are also considered quantized. Dust is considered to be magnetized in the first case and unmagnetized in the second case.

CONTENTS

1. INTRODUCTION

1.1 INTRODUCTION TO PLASMA

The word plasma is derived from a Greek word ‘plassein’ which means ‘to shape’ or ‘to mould’. It describes a system which macroscopically appears to be neutral but at microscopic level contains a large number of interacting electrons, ionized atoms and molecules. This term was firstly used by Tonks and Langmuir in 1929 to describe inner regions of glowing ionized gas produced by electric discharge. These particles interact with each other through long range Coulomb forces and show collective behaviour. Not all type of collection of neutral and charged particles qualify as plasma. There are certain conditions which must be satisfied in order to exhibit plasma behavior. We classify matter into four states i.e. solid, liquid, gas and plasma. This classification is based upon the strength of bonds which hold the particles together. These binding forces are stronger in solid as compared to liquids and in case of gases, they are almost negligible. For a substance to exist in any one of these states depends upon the thermal energy of its atoms and molecules. Equilibrium between thermal energy and the binding forces determines the state. When we supply a specific amount of heat phase transition occurs, as a result, thermal kinetic energy overcomes the binding potential energy. This specific amount of heat is known as latent heat. If we supply enough heat to a molecular gas it will be converted into an atomic gas due to the collision of particles that have thermal kinetic energy greater than the binding potential energy. Now at sufficiently elevated temperature, the kinetic energy of atoms will overcome the binding energy of outermost orbital electrons and as a result, an ionized gas (plasma) is produced. From the thermodynamic point of view, there is no phase transition involved as the process occur slowly with increasing temperature. The degree of ionization and electron temperature under the thermodynamic equilibrium condition are related by the Saha equation.

$$\frac{n_i}{n_n} = 2.4 * 10^{21}\frac{T^{\frac{3}{2}}}{n_i}e^{\frac{-U_i}{k_B T}} \tag{1.1}$$

Here n_i and n_n are densities of ionized and neutral atoms(number per m^3), T is the temperature of gas in kelvin and k_B is the Boltzmann's constant and U_i is the ionization energy. From this equation we can see ionization occurs at high temperature. So plasma usually exists at high temperature.

Plasma can be generated in a laboratory by using different methods and depending upon these methods plasma can have high or low densities and temperature, it can be steady or transient, stable or unstable etc.[1].

1.1.1 CRITERIA FOR PLASMA

We define plasma as a quasi-neutral gas exhibiting collective behavior. For an ionized gas to be called plasma there are certain conditions which must be satisfied.

1. COLLECTIVE BEHAVIOR:

Neutral molecules exert force on one another through collisions e.g. force generated by loudspeaker but in plasma the situation is entirely different, electrons and ions influence each other through long range Coulomb forces. Because of the presence of charged particles there might arise a local concentration of charges which generate electric field. Due to the motion of these charges electric currents are generated and hence magnetic fields are produced. These fields effect the motion of charges which are far away. If the system is disturbed from its equilibrium state, a local concentration of charges occurs and oppositely charged particles will rush towards the point of concentration in order to shield the charge. This shielding of charge reduces the Coulomb potential, which is called 'Debye potential', given by:

$$\phi = \phi_c e^{\frac{-x}{\lambda_D}}, \tag{1.2}$$

here ϕ_c is the Coulomb potential and λ_D is the Debye length i.e. the size of shield,

$$\Rightarrow \lambda_D = (\frac{\epsilon_o k_B T}{ne^2}). \tag{1.3}$$

In order to achieve shielding, there must be a large number of particles in the Debye sphere i.e. $N_D >>> 1$. as number density is defined as:

$$n = \frac{N_D}{\frac{4}{3}\pi\lambda_D^3}, \tag{1.4}$$

therefore

$$N_D = \frac{4}{3}\pi\lambda_D^3 n. \tag{1.5}$$

2. QUSAI-NEUTRALITY:

If the system has dimension 'L' and if it is much larger than λ_D then whenever a local concentration of charges will arise or external potentials are introduced into the system, these are shielded out in a distance short compared with 'L' and the rest of plasma will remain unaffected by external potentials or fields. Plasma is quasi-neutral. It means that $n_i \approx n_e \approx n$ where n is the plasma density. It must not be so neutral that all interesting electromagnetic forces disappear.

So for an ionized gas to qualify as a plasma, it has to be dense enough so that the condition

$$\lambda_D << L \tag{1.6}$$

is satisfied.

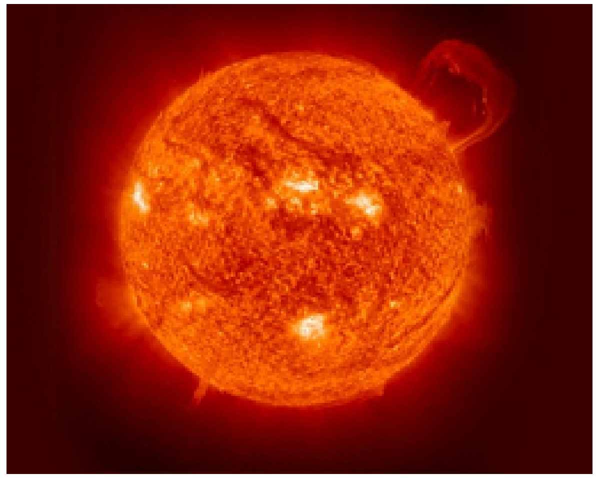
3. $\omega\tau > 1$:

Here ω is the plasma frequency and $\tau = \frac{1}{v}$ is the mean time between collisions with neutral atoms. For a gas to behave as a plasma the following condition must be satisfied

$$\omega\tau > 1 \tag{1.7}$$

1.1.2 OCCURANCE OF PLASMA IN NATURE

About 99% of matter in the universe is in a plasma state. It is found naturally in the ionosphere (layer of earth's atmosphere which lies about $75-1000$ km above the surface of the earth and is ionized due to solar and cosmic radiations), solar wind (it is a stream of highly energized charged particles which flow outward from the sun its speed is approximately $900km/s$ and the temperature is about1 million degree Celsius)[2], Van Allen radiation belts (it is a zone of charged particles most of which is originated by the solar wind. These particles are captured by the magnetic field of planets. It was first discovered by James Van Allen so are named as Van Allen radiation belts), interstellar hydrogen and Steller interiors etc.

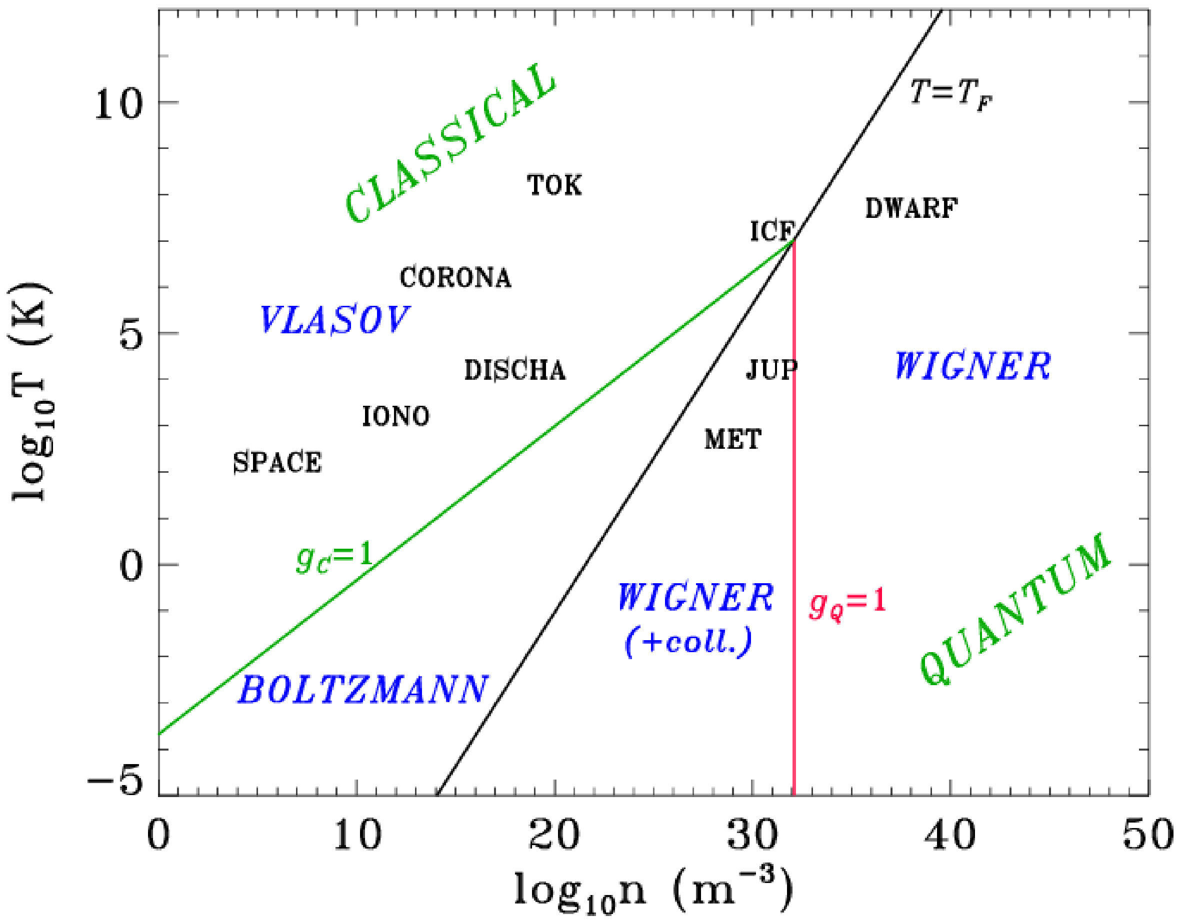


1.1.3 APPLICATION OF PLASMA

Plasma was firstly created in a laboratory in the 1920s when the research was being conducted on vacuum tubes that could carry large currents so there was a need to fill an ionized gas in the vacuum tube to carry large currents. Gas discharges are extensively used for lightening purposes, e.g. sodium lamps emit yellow light due to the arc burning in sodium vapours. Mercury vapour lamps emit radiations which are in the ultraviolet region. Neon lamps are used for decoration purposes. Xenon lamps are sources of good colour composition for stage shows. Nuclear fusion is one of the most important plasma application, in this process light nuclei fuse to form heavier one with the production of a large amount of energy. Plasma also plays a vital role in gas lasers and solid state lasers [3].

1.2 REGIMES OF PLASMA

Plasma is classified as classical plasma and quantum plasma. Plasma span a wide region of phase space in terms of temperature and density. Classical and quantum plasma can be classified based on parameters such as density, degeneracy parameter and length scale by using dimensional analysis and by defining dimensionless parameters, dimensionless parameters allow us to differentiate between different physical regimes characterized by a situation where one effect dominates over other.



Plasma-diagram-in-the-log-T-log-n-plane

1.2.1 CLASSICAL PLASMA

Considering a plasma with a number density 'n' and is mainly composed of particles with mass 'm', charge 'e' and interactions are governed by the Coulomb forces (with electric permittivity ϵ_o). With these parameters, we can define a quantity which is known as plasma frequency, mathematically we can express it as,

$$\omega_p = (\frac{e^2 n}{m\epsilon_o})^{\frac{1}{2}}. \tag{1.8}$$

It represents the oscillation frequency of electrons immersed in the background of positive ions. We usually consider ions to be motionless due to their large masses as compared to electrons. These oscillations are generated because when we deplete a region of plasma from electrons a net positive charge is created, Coulomb forces comes into action and pull back the electrons. These electrons due to their inertia not only replenish the positive region but also travel further away, hence again creating a positively charged region. This will cause undamped oscillations in the absence of collisions with the frequency known as ***plasma frequency***. It is independent of temperature.

Now if the plasma has finite temperature 'T', a random thermal motion will be generated, typical speed due to this motion is known as thermal velocity and is expressed as

$$v_T = [\frac{k_B T}{m}]^{\frac{1}{2}}. \tag{1.9}$$

By combining the above two relations we can define a typical length scale, the Debye length.

$$\lambda_D = (\frac{v_T}{\omega_p}) = \frac{(\frac{k_B T}{m})^{\frac{1}{2}}}{(\frac{e^2 n}{m\epsilon_o})^{\frac{1}{2}}} = (\frac{\epsilon_o k_B T}{ne^2})^{\frac{1}{2}} \tag{1.10}$$

It represents the phenomena of electrostatic screening when external potential is introduced [4].

We can explain the shielding effect in a non-degenerate plasma by introducing a test charge q_t, which is positive. When it is injected in an electron gas with number density n(r) and having fixed ionic background with number density n_0, electrons will rush towards the test charge due to Columbic forces. The electrons will be accumulated around the test charge and at equilibrium when the thermal energy will be equal to the potential energy, an external observer will observe an effective shielded charge instead of test charge q_t. This is what we call static screening or screening in plasma. It itself manifest the quasi-neutrality property. An excess charge is compensated due to an electric force. The Poisson's equation is given as:

$$\nabla^2 \phi = \frac{e}{\epsilon_o}(n(r) - n_0) - \frac{q_t}{\epsilon_o}\delta(r), \tag{1.11}$$

where e is the charge of electron, ϵ_o is vacuum permittivity and we have assumed that the charge is placed at the origin. Also, we have considered that the charge has enough mass that it can be considered to be at rest. The particles are in thermal equilibrium at temperature T and will follow Maxwell's -Boltzmann statistics, so we can write:

$$n(r) = n_o exp(\frac{e\phi}{k_B T}). \tag{1.12}$$

Maxwell's- Boltzmann statistics is appropriate when we are dealing with non-degenerate plasmas. Degeneracy parameter is given by: $\chi = \frac{T_F}{T} \ll 1$, here $T_F = \frac{E_F}{k_B}$ which is known as Fermi temperature and E_F is the fermi energy given by

$$E_F = \frac{\hbar^2}{2m}(3\pi^2 n_o)^{\frac{2}{3}}, \tag{1.13}$$

Were m is the mass of electrons. As electrons are fermions and have spin$\frac{1}{2}$, so all of them can't be accommodated in the ground state because of the Pauli Exclusion Principle which states that two or more identical fermions can't occupy the same quantum state simultaneously. So electrons will occupy the higher states until they occupy the highest energy state with energy E_F. We can linearize equation (1.11) by assuming that the scalar potential is zero before we inserted the test charge. So we can write equation (1.11) as

$$\nabla^2\phi = \frac{e^2 n_o \phi}{\epsilon_o k_B T} - \frac{q_t}{\epsilon_o}\delta(r), \tag{1.14}$$

here we have used the value of n(r) from equation (1.12) and expanded the exponential term. By applying proper boundary conditions, we get radially symmetric solution

$$\phi = \frac{q_t}{4\pi\epsilon_o r} e^{(\frac{-r}{\lambda_D})}. \tag{1.15}$$

This potential does not fall as (1/r) but it obeys potential which are Yukawa like i.e. $e^{(\frac{-r}{\lambda_D})}$ which decays quickly on a distance compareable to Debye length[5].

We can say that a classical plasma is collision-less when long range interactions dominate the short range interactions. This is due to the reason that potential energy of the particles which are separated by average interparticle distance becomes smaller than the average kinetic energy [4]. Now we will define a dimensionless parameter known as ***graininess parameter*** or ***coupling parameter*** (Classical). It is defined as ratio of interaction energy and average kinetic energy of the particles situated at an interparticle distance of $d = n^{\frac{-1}{3}}$. the expression for coupling parameter is given by

$$g_C = \frac{U_{int}}{K_{avg}} = (\frac{\frac{e^2}{\epsilon_o d}}{k_B T}). \tag{1.16}$$

Thermal effects dominate when the coupling parameter is small. This is known as the ***collision less regime***. When the coupling parameter is approximately equal to unity or greater, we cannot neglect the binary collisions and plasma is said to be strongly coupled or collisional plasma.

we can also express coupling parameter as inverse of number of particles contained in a volume of linear dimension λ_D

$$g_C = (\frac{1}{n\lambda_D^3})^{\frac{2}{3}} \tag{1.17}$$

When we are dealing with equilibrium classical plasma, we use Boltzmann distribution function, which is given by:

$$f(\alpha) = \frac{1}{e^{\beta(\epsilon - \mu_\alpha)}} \tag{1.18}$$

here $\beta = \frac{1}{k_B T}$, k_B is the Boltzmann constant and T is the equilibrium temperature. μ is the chemical potential and $\epsilon = \frac{p^2}{2m_\alpha}$ (α is used for different species which compose plasma for example electrons, ions etc) [6].

1.2.2 QUANTUM PLASMA

Plasma is a many body system containing a large number of particles and their motion are influenced by electromagnetic forces. The two basic parameters are temperature and density. According to the big bang theory, the temperature in the beginning was so high that atoms and molecules could not have existed. Corresponding the only thing existed was fully ionized gas which was in 100% plasma state, which was known as quark-gluon plasma.

With the passage of time when the temperature dropped the matter was able to exist in other states of matter. It is difficult to compute and verify the exact percentage of plasma existing in the universe but some people claims that 99% of the universe is in a plasma state [7]. Plasma is characterized by regimes of low temperature, high densities and high temperature, low densities which naturally present in interstellar and interplanetary media. In a laboratory, plasma has vide applications in discharge tubes, fusion reactions etc. Dynamics of plasma are governed by internal fields which are produced by particles and externally applied fields. Particles with low temperature and high densities also exhibit plasma behavior, particle densities may be up to $10^{24} cm^{-3}$ which naturally occur in the interiors of Jovian planets (Jupiter and Saturn), brown and white dwarfs. The outer crust of neutron stars is considered to be ultra-dense. With the particle densities of about $10^{36} cm^{-3}$.

In a laboratory, these conditions appear in a gas of free electrons in metals and semimetals. In compression techniques e.g. diamond anvils, launching shock waves into matter and laser and ion beams etc, the study of collective effects at such high densities become very complex. In such conditions we cannot ignore quantum effects. Also in astrophysical environments where the temperature is very high, we cannot neglect the quantum effects because of the restriction of the Pauli principle. Many quantum mechanical phenomena's such as pressure ionization, condensation, crystallization and tunneling of electrons becomes important. Different types of non- idealities and correlation arise which makes dense plasma more complex [9].

The particles composing plasma have an associated thermal de Broglie wavelength which measures quantum effects.

$$\lambda_B = (\frac{h}{mv_T}) \tag{1.19}$$

It represents the spatial extension of particle wave function due to quantum uncertainty. When we are dealing with classical plasma this de Broglie wavelength is so small that we can treat particles point-like. There is no quantum interference as there is no overlapping of wave functions involved, so we can say that quantum effects are prominent when de Broglie wavelength is approximately equal to or larger than interparticle distance $n^{\frac{-1}{3}}$:

$$n\lambda_B^3 \geq 1. \tag{1.20}$$

Now to understand the concept of shielding in degenerate plasma we again consider the same scenario of test charge being injected in an electron gas as we consider in case of a classical plasma. We consider the Maxwell-Boltzmann distribution function from where we derive equation (1.12) by considering the zeroth-order moment of Maxwell-Boltzmann distribution, given as:

$$f_{cl}(r,v) = n_0(\frac{m}{2\pi k_B T})^{\frac{3}{2}} exp[\frac{-1}{k_B T}(\frac{mv^2}{2} - e\phi)] \tag{1.21}$$

also we know that

$$n(r) = \int f_{cl}(r,v)dv. \tag{1.22}$$

Now we consider a uniform distribution of charges (electrons) with energy less than Fermi energy and no electron lies in energy level above Fermi level. Considering $(\frac{mv^2}{2} - e\phi) < E_F$. Eq. (1.21) reduces to

$$f_{cl}(r,v) = (\frac{3n_0}{4\pi v_F^3}), \tag{1.23}$$

otherwise $f_{cl}(r,v) = 0$, Here we have used $v_F = (\frac{2E_F}{m})^{\frac{1}{2}}$ which is the Fermi velocity. This distribution represents zero temperature Thomas-Fermi equilibrium. It also represents the equal occupation probability for energies which are less than Fermi energy and that no particle will lie in energy level greater than Fermi level. Till now we have not considered the temperature effects. Using equation (1.22) we can calculate n(r). $n(r) = \int f_{cl}(r,v)dv$

$$n(r) = \int (\frac{3n_0}{4\pi v_F^3})dv, \tag{1.24}$$

$$n(r) = n_0(1 + \frac{e\phi}{E_F})^{\frac{3}{2}}. \tag{1.25}$$

After using this result in Poisson's eq. (1.11), we get

$$\nabla^2\phi = \frac{3n_o e^2\phi}{2\epsilon_o E_F} - \frac{q_t}{\epsilon_o}\delta(r), \tag{1.26}$$

here we have replaced k_BT by E_F. Now we can define a shielding distance for degenerate plasma also known as Fermi length, defined as

$$\lambda_F = (\frac{2\epsilon_o E_F}{3n_o e^2})^{\frac{1}{2}}. \tag{1.27}$$

Unlike Debye length, Fermi length is non zero at zero temperature. This is because of the Pauli exclusion principle which does not allow particles to accumulate at the point where test charge is placed in this sense we can treat each particle in plasma (nondegenerate or degenerate) as a test charge with screening cloud around it. We have now the Yukawa interaction field instead of a Coulomb field.We can now define the degeneracy parameter

$$\chi = \frac{T_F}{T} = \frac{1}{2}(3\pi^2 n_o \lambda_B^3)^{\frac{2}{3}}. \tag{1.28}$$

When interparticle distance will be comparable to the Debye length, we can no longer use the Maxwell-Boltzmann distribution function, instead, we will use Fermi-Dirac distribution function. Such case arises when we are dealing with high densities of the order of $10^{24}cm^{-3}$ or higher.

We can summarize the fundamental scale for both the regimes as:

1. For both regimes, we define a time scale given by ω_p^{-1} where $\omega_p = (\frac{e^2 n}{m\epsilon_o})^{\frac{1}{2}}$.If there occurs a charge depletion electric forces will appear which restores the charge neutrality.

2. For both quantum and classical plasma we can define interaction energy as:

$$U_{int} = \frac{e^2}{\epsilon_o d} \tag{1.29}$$

where $d = n_o$

3. Kinetic energy for the non-degenerate case is k_BT while for the degenerate case, it is k_BT_F.

4. Coupling parameter for the nondegenerate case is given by:

$$g_C = \frac{U_{int}}{K_{avg}} = (\frac{\frac{e^2}{\epsilon_o d}}{k_BT}) = 2.1 \times 10^{-4} \times \frac{n_o\frac{1}{3}}{T}, \tag{1.30}$$

while in case of a degenerate plasma it is given by:

$$g_Q = \frac{U_{int}}{K_{avg}} = \frac{2me^2}{(3\pi^2)^{\frac{2}{3}}\epsilon_o h^2 n_o^{\frac{1}{3}}} = 5 \times 10^{10} \times n_o^{\frac{-1}{3}}.(1.31)$$

[5].

1.2.3 WHITE DWARF

Quantum plasmas are commonly found in the astrophysical environment such as stars, radio pulsars, magnetars etc. Usually, stars end up in three ways as:

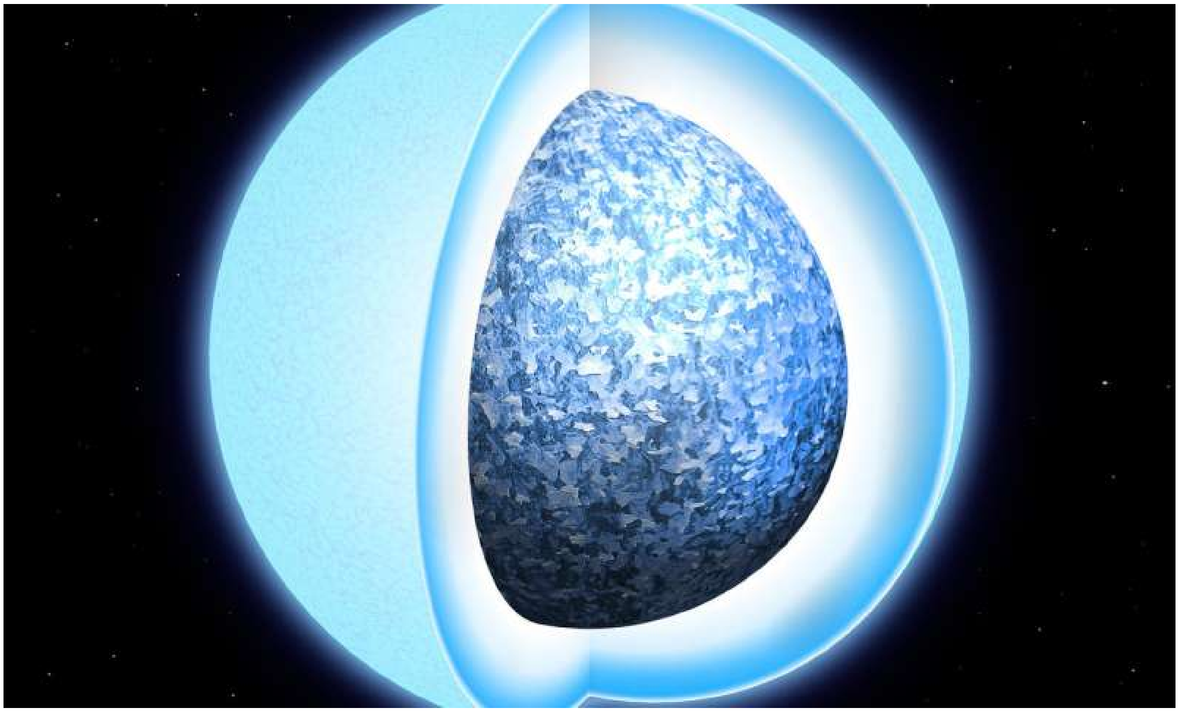
1. White dwarfs,

2. Neutron stars (their mass is equivalent to the mass of the sun but the radius is approximately 10 km, extremely dense),

3. Black holes (the end product of binary star system which is mostly present at the center of galaxies).

These compact objects have very high densities. White dwarfs have masses approximately equal to that of neutron stars but their sizes are approximately equal to the earth [10]. A white dwarf is a degenerate star, they are usually composed of carbon and oxygen. With time they cool down and are no longer visible and are then known as a cold black dwarf their number density is approximately equal to $10^{30}/cm^3$ and temperature range is usually 150K- 4K. The magnetic field can range from$10^3 to 10^9$ Gauss. So under these conditions plasma behavior can be studied by considering quantum effects and including terms like fermi pressure and Bohm potential in the Vlasov equation

In such environments, plasma can not only consist of electrons, ions and neutrals but there might exist another species called dust as explained in the next section.



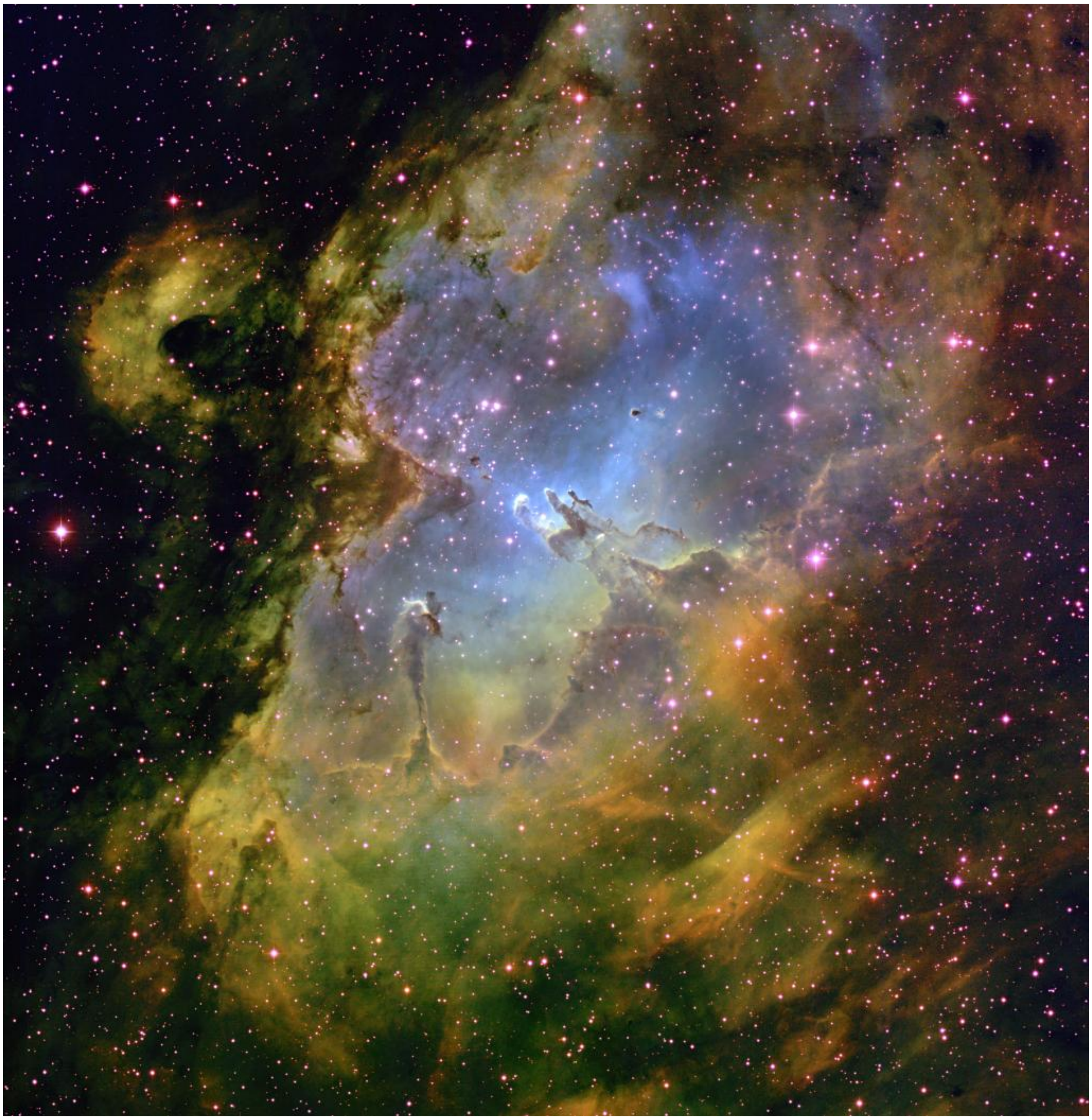
White dwarf

1.3 DUSTY PLASMA

Most of the plasma in our universe coexists with dust particulates. So we can say that dust is an omnipresent ingredient of plasma. They can have the size of a micron. Dust particles gain their charge from the surrounding environment. So depending upon their surrounding they can either be positively charged or negatively charged

[11].

The presence of such particles makes the dynamics of plasma more complex so we call such plasma as a “complex plasma” or “dusty plasma” or "dust in plasma" depending upon their sizes. Such plasmas have been observed in Orion nebula (Orion nebula a diffused nebulae and is one of the brightest nebulae present in our Milky Way. One can observe it with the naked eye it appears as a star in the sword of a hunter in Orion constellation), Noctilucent clouds (Noctilucent means visible at night, these are the night-shining clouds or we can say that it is cloud-like phenomena which occur at the upper atmosphere of earth. They are visible during astronomical twilight and are composed of ice crystals. The existence of such plasmas has been observed in the images of Eagle nebula and planetary rings which have been taken with the help of Hubble telescope [12].



Eagle nebula

1.3.1 CHARACTERISTICS OF DUSTY PLASMA

Dust can be of micron or sub-micron size, dusty plasma is a low temperature partially or fully ionized charged gas composed of electrons, ions, dust particles and neutrals. They can differ in size and shape unless they are man-made. Their sizes can range from Nano to millimeters and they can be metallic, conducting or can be simply ice particulates. They can be considered as point particles when observed from long distances. Now we can classify such plasmas as “Dusty plasma” or “Dust in plasma”. This classification depends upon their characteristic lengths. We can simply explain both the terms by considering that the radius of the dust particle is r_d and let the average intergrain distance be "a" also taking into account the plasma debye length λ_D.

When $r_d \ll \lambda_D < a$, the dust particles are isolated entities that are screened out and is called “dust in plasma” where local plasma inhomogeneity is taken into account. While in the other situation $r_d \ll a < \lambda_D$, dust grain will take part in collective behavior. This situation corresponds to “Dusty plasma”, where we will treat dust particles as massive charged particles similar to ions with a charge either positive or negative. The interaction between dust grains is screened out by electrons and ions which are present in the background.

When we are considering dust particles they not only modify the low- frequency waves (e.g. ion-acoustic waves, lower hybrid waves etc.) which are already present in the environment but also generates new low-frequency dust waves (e.g. dust acoustic and dust ion-acoustic waves etc.). Now for understanding the concept of dusty plasma properly we have to re-examine its characteristics, for example: neutrality, characteristic frequency, Debye shielding, coupling parameter etc.

1.NEUTRALITY: Dusty plasma is said to be neutral when there is no external disturbance as in electron-ion plasma. The net electric charge in the absence of external influences is therefore zero. So neutrality condition can be stated as:

$$q_i n_{io} = e n_{eo} - q_d n_{do}, \tag{1.32}$$

here n_{jo} is the unperturbed density with j=e,i,d. Also $q_i = Z_i e$ is ion charge, we have taken $Z_i = 1$ here and $q_d = Z_d e$ or $= -Z_d e$ where e is the charge on electron and Z_d is the number of charges residing on dust grain. Dust grain can acquire one thousand to several thousand of charge particles and $Z_d n_{do}$ is comparable to n_{io} even when we have $n_{do} \ll n_{io}$. In most of the situations in ambient dusty plasma (laboratory or space plasma), during the charging process, the electrons in the background stick themselves on the surface of dust grain, and as a result, their might arise the depletion of electrons number density. However complete depletion of electrons is not possible. This is because when $T_i \approx T_e$, minimum ratio of electron number density and ion number density turns out to be the square root of mass ratio of electrons and ions. Potential at grain surface approaches to zero, So for negatively charged dust grains we can write equation (1.32) as,

$$n_{io} \approx Z_d n_{do} \tag{1.33}$$

2.DEBYE SHIELDING: One of the most important characteristics of plasma physics is its ability to shield an individual charged particle or the surface at non-zero potential. It provides the distance (debye radius) over which other particles in a plasma feels the effects of an electric field of single particle or surface with non-zero potential. Now our main purpose is to explain this phenomenon when the dust is also present in plasma. Just as in electron-ion plasma, we consider that by inserting a charged ball in a dusty plasma, which is composed of electrons, ions and dust particles which are either negative or positively charged. Ball would attract particles of opposite charge i.e., it will attract electrons and dust (if negatively charged) if it is positively charged. Otherwise, it will attract ions and dust (if positively charged). Also, we are assuming that particles will not recombine at the surface of the ball [13].

In cold plasma shielding would be perfect and number of charges in the cloud will be equal to the number of charges in the ball. As a result of perfect shielding, no electric field will escape the cloud. Now if we have a finite temperature then the particles at the edge of the cloud might have enough thermal energy to escape the cloud. Cloud edge will then occur where the potential energy and thermal energy are equal. A finite electric potential will exist there and shielding would be incomplete. To calculate the approximate thickness of the cloud we assume that the potential $\phi_j(r)$ at the center $r = 0$ is ϕ_{jo}. We are also assuming that the dust-ion mass ratio $\frac{m_d}{m_i}$ is very large and the inertia of dust particles prevents them from moving significantly. So dust (negative) particles will form a uniform background. Here we are assuming that electrons and ions are in thermodynamic equilibrium and will follow Maxwell Boltzmann distribution. i.e.

$$n_e = n_{eo} exp(\frac{e\phi}{k_B T_e}), \tag{1.34}$$

and

$$n_i = n_{io} exp(\frac{e\phi}{k_B T_i}), \tag{1.35}$$

Now for our present case when the dust is also present we can modify Poisson's equation as;

$$\nabla^2\phi_j = 4\pi(en_e - en_i - q_d n_d), \tag{1.36}$$

here n_d is the number density of dust particles. As we have assumed that number density of dust particles in the cloud is the same as outside the cloud so we can write: $q_d n_d = q_d n_{do} = en_{eo} - en_{io}$. After putting eq. (1.34) and (1.35) in eq. (1.36) and also assuming that $\frac{e\phi}{k_B T_e} \ll 1$ and $\frac{e\phi}{k_B T_i} \ll 1$, the Poisson's equation become:

$$\nabla^2\phi_j = \frac{1}{\lambda_{De}^2} + \frac{1}{\lambda_{Di}^2}. \tag{1.37}$$

We can obtain dusty plasma Debye radius by assuming the solution of eq. (1.37) to be $\phi_j = \phi_{jo} exp(\frac{-r}{\lambda_D})$ so the Debye radius comes out to be:

$$\lambda_D = \frac{\lambda_{De}\lambda_{Di}}{\sqrt{\lambda_{De}^2 + \lambda_{Di}^2}}. \tag{1.38}$$

When we have negatively charged dust particles $n_{eo} \ll n_{io}$ and $T_e \geq T_i$ that means $\lambda_{De} \gg \lambda_{Di}$. So eq. (1.38) implies that $\lambda_D \simeq \lambda_{Di}$, it means that thickness of sheath is determined by the temperature and the number density of ions. If dust is positively charged then in that case it is determined by the density and the temperature of electrons.

3.CHARACTERISTIC FREQUENCY: When we disturb the plasma from its equlibrium, it results in the internal space-charge field, due to which there is a collective motion of particles that tends to restore neutrality. A natural frequency is associated with the particle motion known as plasma frequency ω_P. Now we assume cold unmagnetized dusty plasma, electrostatic oscillations are described by continuity equation:

$$\frac{\partial n_j}{\partial t} + \nabla.(n_j.v_j) = 0, \tag{1.39}$$

Momentum equation

$$\frac{\partial v_j}{\partial t} + (v_j.\nabla)(v_j) = \frac{-q_j}{m_j}(\nabla\phi), \tag{1.40}$$

And the Poisson's equation;

$$\nabla^2\phi = -4\pi\sum_j q_j n_j. \tag{1.41}$$

Just for simplicity, we have not considered pressure gradient forces and there exist no source or sink. We also assume that linear theory is valid i.e., amplitude of oscillation is so small that we can neglect higher-order terms. At equilibrium, all species are at rest and there exists no equilibrium space-charge field. Therefore, assuming that $n_j = n_{jo} + n_{j1}$ where $n_{j1} \ll n_{jo}$. Now linearizing eq. (1.39), (1.40) and (1.41) and combining the results to get :

$$\frac{\partial^2 \nabla^2 \phi}{\partial^2 t} + 4\pi \sum_j \frac{n_{oj} q_j^2}{m_j} \nabla^2 \phi = 0. \tag{1.42}$$

Now integrating equation (1.42) with the boundary condition that $\phi = 0$ at $r = 0$ and replacing partial derivative by complete derivative, the above equation will become:

$$\frac{d^2 \phi}{d^2 t} + \omega_P^2 \phi = 0, \tag{1.43}$$

where

$$\omega_P^2 = \sum_j \frac{4\pi n_{jo} q_j^2}{m_j} = \sum_j \omega_{Pj}^2 \tag{1.44}$$

This frequency will differ for each species, for example, an electron will oscillate around ions with electron plasma frequency given by $\omega_{Pe} = (\frac{4\pi n_{eo} e^2}{m_e})^{\frac{1}{2}}$, similarly ions will oscillate around dust particles with ion plasma frequency $\omega_{Pi} = (\frac{4\pi n_{io} e^2}{m_i})^{\frac{1}{2}}$ and lastly dust will oscillate around their equilibrium with dust plasma frequency$\omega_{Pd} = (\frac{4\pi n_{do} Z_d^2 e^2}{m_d})^{\frac{1}{2}}$.

There exist some other characteristic frequencies which are associated with the collisions of particles with neutrals i.e., electron-neutral, ion-neutral and dust-neutral collision frequency. The collisional frequency for the scattering of j species is given by

$$v_{jn} = n_n \sigma_j^n v_{Tj}, \tag{1.45}$$

here n_n is the number density of neutrals and scattering cross-section is represented by σ_j^n and v_{Tj} is the thermal speed given by $v_{Tj} = (\frac{k_B T_j}{m_j})^{\frac{1}{2}}$. These collisions damp the collective oscillations and gradually decrease their amplitude. The oscillations will slightly damp when $v_{Tj} < \omega_P$.

4.COULOMB COUPLING PARAMETER: The probability of the formation of dusty plasma crystal is determined by the Coulomb coupling parameter. To explain this effect we consider two dust particles with the same charge and are separated by a distance "a" after including the shielding effects. We can write dust Coulomb potential energy as $\epsilon_c = \frac{q_d^2 exp(\frac{-a}{\lambda D})}{a}$ and thermal energy is $k_B T_d$.

So now we can define the Coulomb coupling parameter (ratio of potential and thermal energy) as: $T_C = \frac{Z_d^2 e^2 exp(\frac{-a}{\lambda D})}{a k_B T_d}$. When $T_C \ll 1$, dusty plasma is weakly coupled and when $T_C \gg 1$ dusty plasma is strongly coupled [14].

As a dusty plasma is widely found in astrophysical environments and also in the laboratory, a tremendous amount of research has been conducted in the past few years on low-frequency electrostatic waves in the presence or absence of an external ambient field(static). Detailed work has already been done that how the massive and highly charged dust grains affect these waves. We apply external fields to control plasma properties. If we apply a static external field, the most significant low-frequency wave that is generated is dust lower hybrid wave(this mode is generated by the magnetized ions and unmagnetized dust). However, in the electromagnetic regime, a very small amount of research work has been done in a magnetized dusty plasma [15].

1.3.2 CHARGING OF DUST

One of the main properties of dust is its charge. For a finite system, the normal mode of particles is a pattern of motion where all particles oscillate with the same frequency and different normal modes have different frequencies associated with them. By taking the superposition of all such modes the dynamics of a system can be understood. For calculating the charge one has to solve eigenvalue problem and the final result for dust charge comes out to be:

$$Z_d = \sqrt{\frac{2\pi\epsilon_o m_d \omega_o^2 r_o^3}{e^2}}. \tag{1.46}$$

[16].

In case when dust grains are simply embedded in electron-ion (positive) plasma they will acquire a negative charge due to preferential attachment of electrons that are more mobile. Their negative charge will continue to increase unless they gain enough negative charge that they start repelling electrons and start attracting ions. We can also explain this process in terms of current. As ions are much more massive than electrons, so initially ion current Ii is much less than Ie as the dust starts accumulating negative charge this will cause a decrease in Ie until both Ii and Ie become equivalent. When ions and electrons have enough energies they can even pass through the dust and over there they might lose their energies partially or completely. Due to this energy, the electrons on dust particles might get excited and will escape from the surface this is known as secondary emission. If plasma contains highly energetic electrons then the effects of secondary emission are also taken into account. Dust can also acquire a positive charge due to the photoelectric effect that can occur in the presence of UV light. There are several phenomena through which dust can get charged for example thermionic emission, radiation emission, field emission, and impact ionization etc. [17][18].

1.4 EXISTANCE OF ELECTROMAGNETIC-HYDRODYNAMIC WAVES

Hannes Alfven discovered electromagnetic-hydrodynamic waves when he was working on sunspots (dark regions on the surface of sun that occur because of the reduced temperature which is the result of magnetic flux concentrations which inhibit convections).According to him, if we place a conducting fluid in a constant magnetic field (conducting fluid can either be plasma, liquid metals, saltwater etc) an emf is generated due to the motion of charges and as a result currents are produced. These currents result in mechanical forces and change the motion of the fluid and an electromagnetic-hydrodynamic wave is generated. This phenomenon is described by electrodynamic and hydrodynamic equations. The velocity of such waves comes out to be

$$V = (\frac{B_O^2}{\mu_O \rho})^{\frac{1}{2}}, \tag{1.47}$$

These waves have great importance in solar physics. As the sun has a conducting fluid and magnetic field so all the conditions for the existence of such waves are already present over there. For example, the particular region of the sun has a density of $0.005 g/cm^3$ and a magnetic field of 10^{15} Gauss, the velocity of such waves comes out to be $60 cm/s$. This is the velocity with which sunspots move toward the equator during the sunspot cycle [19].

1.4.1 MAGNETO HYDRODYNAMIC WAVES

Longitudinal sound waves are the most fundamental type of waves that exist in a nonconducting, compressible fluid. As these waves travel in the form of compressions and rarefactions so their motion is dependent upon pressure (P) and density ρ and obey adiabatic energy equation,

$$P\rho^{-\gamma} = CONSTANT, \tag{1.48}$$

here $\gamma = \frac{C_P}{C_V}$ i.e, ratio of specific heat at constant pressure and specific heat at constant volume. After differentiating equation (1.46), we get:

$$\nabla P = \frac{\gamma P}{\rho}(\nabla \rho) = V_s^2 \nabla \rho, \tag{1.49}$$

here,

$$V_s^2 = (\frac{\gamma P}{\rho})^{\frac{1}{2}} = (\frac{\gamma k_b T}{m})^{\frac{1}{2}} \tag{1.50}$$

represents the adiabatic sound speed [20].

1.4.2 ALFVEN WAVES

One of the most fundamental principles of magnetohydrodynamics is that electromagnetic force is generated due to the motion of conductive fluid in the presence of a magnetic field, which as a result produces electric currents. There arises the interaction between the magnetic field and the fluid because the currents generated will modify the initial magnetic field and will produce a mechanical force, which will modify the motion of a fluid [21]. These mechanical effects are equal to hydrostatic pressure given by $\frac{B_0^2}{2\mu_0}$ here B_0 is the magnetic field. Magnetic stresses are then equivalent to tension $\frac{B_0^2}{\mu_0}$ along field lines and hydrostatic pressure $\frac{B_0^2}{2\mu_0}$ which is isotropic in nature. The field lines will act as an elastic cords. Plasma particles will behave as they are tied to field lines and it seems that field lines are mass-loaded strings. By making an analogy with transverse vibration of the string we can say that the magnetic field will perform transverse vibrations when we disturb fluid from its equilibrium position. The speed of these vibrations is given by:

$$V_A = (\frac{TENSION}{DENSITY})^{\frac{1}{2}}, \tag{1.51}$$

or we can write it as

$$V_A = (\frac{B_O^2}{\mu_O \rho})^{\frac{1}{2}}. \tag{1.52}$$

which is called the Alfven speed.

1.4.3 EXISTANCE OF ALFVEN WAVES IN SOLAR CORONA

Alfven in 1942 proposed that there exist oscillations in a magnetized plasma. There are three modes of magneto-hydrodynamic waves out of which two modes which are slow and fast magnetoacoustic modes are compressible and are damped out. But the third one is transverse, incompressible oscillations that move in the direction of the magnetic field and magnetic tension will act as a restoring force. Scientists after proceeding the work of Alfven realized that these waves cause the transfer of energy from the turbulent photosphere to the solar corona. Alfven waves remain undetected (14 decades) for two reasons,

1. As these waves are incompressible so they are not visible as intensity fluctuations.

2. Velocity fluctuations require spectrograph, spectrograph can't measure the large field of view in a short time compared to wave periods.

Experimental evidence: Scientists obtained their results from a coronal multi-channel polari meter (CoMP) which consists of a filter that is tunable (in this case it is 0.13 nm filter band pass and is tuned to 3 wavelengths of FeXlll, coronal emission line at 1074.7nm) and a Polari meter (device to measure optical activity) measures properties of infrared coronal emission line over a large field over a short time. Polarization is expressed by strokes vector [I, Q, U, V] strokes parameters (I=INTENSITY, Q and U =NET LINEAR POLARIZATION STATES and V=NET CIRCULAR POLARIZATION) are entities which describe the polarization state of electromagnetic waves, these parameters are expressed in terms of a vector known as strokes vector. Strokes parameters at different wavelengths were measured and motion of images was observed using cross-correlation techniques (it's a measure of similarity of two series as a function of displacement of one relative to the other), these images were then translated to a common center. At last, these images were plotted in time with a grid spacing of 29 s, after inserting all the data the velocity image showed the quasi-periodic fluctuations. Fourier analysis showed a broad peak which was absent in the power spectrum of intensity fluctuation. From the obtained results scientists concluded that they observed Alfven waves because of three reasons which were,

1. The phase speed of observed waves is much larger than that of sound waves so they are not slow magnetoacoustic mode waves.

2. The spatiotemporal measurements showed that these waves travel along field lines so they are not fast magnetoacoustic mode waves either.

3. A source that causes the wave to occur on the solar surface would not produce coherent spatial structures that are present in the velocity data [22].

1.4.4 SHEAR ALFVEN WAVES

Alfven waves are omnipresent in space, they are found in earth's ionosphere, magnetosphere interstellar clouds,rings of planets, solar corona and solar wind. They transport electromagnetic energy to the magnetized plasma. In the laboratory, they are used for the heating of plasma in 'Tokamak' and other fusion devices. They are also used to communicate information regarding magnetic field structures and plasma currents. Alfven waves which (exist below cyclotron frequency) can be categorized as compressional Alfven waves as described in subsection (1.4.3), moves along the direction of the external magnetic field. The other one is shear Alfven wave that propagates at a certain angle with the externally applied field. They further give rise to kinetic Alfven waves which can be described as shear Alfven waves with finite Larmor radius effects. Due to the finite Larmor radius, these waves generate the longitudinal parallel electrostatic field. They can transfer energy to electrons and can even accelerate them in the direction of the field. But in the presence of dusty plasma, these waves are damped out because of the charge fluctuation effect involving dust dynamics [15].

2. MODELS IN QUANTUM PLASMA

2.1 INTRODUCTION

Systems we mostly deal with is many-body or particle system, if we have a system consisting of atoms or molecules which consists of a few hundred particles inside them i.e. few hundred electrons or nucleons. But when dealing with solids liquids or gases the number of particles reaches an enormous value. Plasma shows different behavior as compared to the other forms of matter and consists of free charges which give rise to current densities and electromagnetic fields through which particles interact. Mostly we have to deal with Coulomb interactions, which leads to screening effects, collective behavior and plasma oscillations. But when one deals with transport or thermodynamic properties then long-range character leads to special difficulties. In such scenarios, it is not suitable to use few particle approximations to explain screening effects. Complication arises when we are dealing with quantum plasma which is highly dense and has a low temperature. New effects are generated at a very short interval that are difficult to deal with. So to cope with such situations we require models based on previous knowledge.

2.2 Schrodinger- Poisson model:

In classical mechanics, identical particles are distinguishable, while dealing with identical particles in quantum mechanics there exists no mechanism to distinguish them. Also due to uncertainty principle, we can't exactly locate particle so determining the path of the particle becomes meaningless. If we can locate a particle at a certain position at some specific point its coordinates can not be specified at some other instant. We can say that in quantum mechanics, particles lose their individuality, so we use the probabilistic approach. Now there arises a question: How we will be able to describe the dynamics of a system which consists of N number of particles? To solve such a system, we generalize the dynamics of a single particle [8]. As we are considering dense plasma i.e. the density of particles is extremely high within a small region of volume, it means that if we need to describe quantum plasma we need solution of Schrodinger equation for N particles. But the problem is still not solved because the Schrodinger equation for N wave functions can't be solved [23]. If one has to solve the problem in Schrodinger picture, the only way is to neglect the two bodies and higher-order correlations (weak correlations), this assumption is valid as the value of the coupling parameter is very small as discussed in chapter 1. So now we can write the wave function of N particles as a product of wave functions of individual particles (one-particle wave function). Product wave function for fermions is not identical it means that the Schrodinger equation consists of a product of uncorrelated terms and each particle is treated individually. Here we have not considered the entanglement of states. So we need to know N independent Schrodinger equations for N wave functions [9].

$$\imath\hbar\frac{\partial\psi_i(x,t)}{\partial t} = \frac{-\hbar^2}{2m}\Delta\psi_i(x,t) + e\varphi(x,t)\psi_i(x,t), i = 1,2,3....N. \tag{2.1}$$

For the sake of simplicity, considering only one spatial dimension i.e. $\Delta = \frac{\partial^2}{\partial x^2}$, $\varphi(x,t)$ is the electrostatic potential which is given by the Poisson's equation:

$$\Delta\varphi(x,t) = 4\pi e(\sum_{i=1}^{N} p_i \mid \psi_i(x,t) \mid^2 - n_o), \tag{2.2}$$

here n_o is the ion density which is considered to be fixed in the background. It is a continuous function of position i.e $n_i(x)$. Each electron is in a well-defined pure N quantum state. The probability of occurring in any quantum state is given by Fermi-Dirac distribution function, given by:

$$p_i = [e^{\frac{(\epsilon-\epsilon_F)}{K_B T}+1}]^{-1}. \tag{2.3}$$

The sum of probabilities is always equal to one. Hartree firstly developed this model to study how the Coulomb potential of the nucleus is effected by the self-consistent effects of atomic electrons.

This model is numerically efficient and is simple as it contains two major terms i.e. quantum mechanical equation of motion and self-consistent long-range interactions. But here spin effects, dissipation, and relativistic corrections are neglected. But when we are considering more realistic situations these effects can no longer be neglected. We can say that this model is the quantum analog of the Vlasov- Poisson model. As in both the models most of the assumptions are same, for example, collisions are neglected in both, single-particle wave functions are considered in both the models and only electrostatic interactions are taken into account. It is useful to start with a simple model that covers the main aspects of quantum plasma and the study of macroscopic properties of a system under consideration becomes much easier.

2.3 Wigner- Poisson model:

In position space probability, density is equal to the square of the magnitude of the wave function. When the wave function is known it becomes easier to determine the distribution. Also in momentum space, the distribution is straight forward. But there was a need to have a function that represents probability distribution simultaneously in position and momenta space [24]. When we are dealing with classical statistical mechanics, the probability for coordinates and momenta are given by Gibb's Boltzmann formula according to which probability distribution tells us that the system will occur in a certain state as a function of the energy of the state and system's temperature. [25].

$$P_i \propto e^{\frac{-\epsilon_i}{k_B T}}, \tag{2.4}$$

$$P_i \propto e^{-\epsilon_i \beta}. \tag{2.5}$$

where,

ϵ_i =sum of kinetic energy and potential energy.

While dealing with quantum theory there exists no such relation for probability because we can't have a simultaneous probability for coordinate and momenta. Neumann formula in thermodynamics of quantum mechanical system shows that the mean value of a physical quantity (with normalizing constant only dependent upon temperature) is equal to the sum of diagonal elements of a matrix $Qe^{-\beta H}$.

where Q= matrix operator of any quantity and

H= Hamiltonian of the system

Under the transformation diagonal sum remain invariant so one can choose any operator representation or matrix for Q and H. Non- commutability of different parts of H must be taken into account. From above equation, it does not seem easy to make explicit calculations of the mean value. In 1932, Wigner proposed the concept of phase space representation of quantum mechanics by using the concept of quasi probabilities. He was trying to figure out the quantum correction in classical statistical mechanics. Now if the wave function is given, one may build up the relation, which is called probability- function for simultaneous values of coordinate and momenta. The expression for probability function is real but not positive everywhere. Correct values of probabilities for different values of coordinates are obtained when integrated with respect to “p” and also when integrated with respect to “x” for different values of “p”.

Wigner formalism has great importance since Schrodinger- Poisson system can be developed completely by making use of this formalism. For quantum mixture of states, each of which is characterized by occupation probability p_i, the Wigner distribution is given by:

$$W(x,v,t) = \frac{m}{2\pi\hbar}\sum_{i=1}^{N} p_i \int_{-\infty}^{\infty} d\lambda \psi_i^*(x+\frac{\lambda}{2},t) \times \psi_i(x-\frac{\lambda}{2},t) e^{\frac{\iota m v \lambda}{\hbar}}. \tag{2.6}$$

We can describe the evolution of the Wigner function under the action of electrostatic potential. Wigner function does not necessarily remain nonnegative in its evolution process. It can’t be interpreted as true probability density as we can do in a classical case instead it is real and normalizable to unity and it gives averages just as in classical statistical mechanics.

The assumptions and limitations which we considered in Schrodinger- Poisson model are also valid here but this model can work with both pure and mixed states.

2.4 *QUANTUM HYDRODYNAMIC MODEL*

The transport equations in a classical fluid model for plasma can be expressed in the form of conservation laws of energy and momentum. The Quantum hydrodynamic (fluid) model is a generalization of the classical model. Instead of dealing with the complexities of 2N Schrodinger- Poisson’s equation or phase space dynamics (Wigner- Poisson’s model) quantum hydrodynamic model provides a simpler way to investigate collective dynamics. Using standard definitions of the average of macroscopic quantities like pressure, density velocity, etc. We can drive standard QHD equations by making use of Schrodinger-Poisson’s equation and Wigner-Poisson’s equation.

2.4.1 *SCHRODINGER-POISSON APPROACH*

We can drive QHD model by making use of the Schrodinger-Poisson’s system in which macroscopic quantities such as average velocity and density are used. Continuity and momentum conservation of electron in the framework of the Vlasov model is explained by the Dawson multistream model of classical plasma. We can consider the Schrodinger-Poisson's equation to be the quantum analog of the Dawson multistream model. In Dawson model, N streams are considered which represents thin filaments (infinitely small) of plasma each of which has some velocity, number density, and probability [26] [27]. The same reasoning is considered in Schrodinger-Poisson's model by using Madelung representation to each stream. For pure state, wave function is represented by:

$$\psi_i = A_i(x,t)exp(\frac{\iota S_i(x,t)}{\hbar}), \tag{2.7}$$

here we have considered $S_i(x,t)$ to be the real phase and $A_i(x,t)$ is the real amplitude. We can now write density and velocity as:

$$n_i = \mid \psi_i \mid^2 = A_i^2; v_i = \frac{\partial x S_i}{m}. \tag{2.8}$$

After separating real and imaginary parts, we get reduced Schrodinger-Poisson's equation as:

$$\frac{\partial n_i}{\partial t} + \frac{\partial (n_i v_i)}{\partial x} = 0, \tag{2.9}$$

$$(\frac{\partial}{\partial t} + v_i \frac{\partial}{\partial x})v_i = \frac{e}{m}\frac{\partial \varphi}{\partial x} + \frac{\hbar^2}{2m}\frac{\partial}{\partial x}(\frac{\partial x^2 \sqrt{n}_i}{\sqrt{n}_i}). \tag{2.10}$$

Poisson's equation can now be written as:

$$\frac{\partial \varphi}{\partial x} = 4\pi e(\sum_{i=1}^{N} n_i - n_o). \tag{2.11}$$

Equation (2.9)-(2.11) constitute the quantum multistream model which can be reduced to the classical model when we consider the limit $\hbar \to 0$. Quantum mechanical effect is which is represents in the last term of eq. (2.8) interpreted as gradient of quantum Bohm potential. Global average density n(x,t)and global average velocity u(x,t)are defined as:

$$n(x,t) = \sum_{i=1}^{N} p_i n_i, \tag{2.12}$$

and

$$u(x,t) = \sum_{i=1}^{N} p_i \frac{n_i}{n} v_i \equiv < v_i > . \tag{2.13}$$

The probability of occupying state ψ_i can be obtained by multiplying eq. (2.9) and (2.10) and then summing over i=1. . . .N. Continuity and momentum equation for global average quantities n and u using Fermi-Dirac statistics are given by:

$$\frac{\partial n}{\partial t} + \frac{\partial (nu)}{\partial x} = 0. \tag{2.14}$$

$$(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x})u = \frac{e}{m}\frac{\partial \varphi}{\partial x} + \frac{\hbar^2}{2m}\frac{\partial}{\partial x}(\frac{\partial x^2 \sqrt{n}}{\sqrt{n}}) - \frac{1}{mn}\frac{\partial P}{\partial x}. \tag{2.15}$$

where,

$$P(x,t) = mn[\sum_{i=1}^{N} \frac{p_i n_i v_i^2}{n} - (\sum_{i=1}^{N} \frac{p_i n_i v_i}{n})^2]. \tag{2.16}$$

The last term on R.H.S of eq. (2.15) represents quantum statistical pressure which is due to the electrons which show fermionic nature when the temperature is low. Equation (2.14) and (2.15) are known as quantum hydrodynamic equations, which include quantum statistical and diffraction effects.

We can represent this model in another way by making use of effective wave function $\psi(x,t)$, which is based on global density n(x,t) and global velocity u(x,t) as,

$$\psi_i = \sqrt{n}(x,t)exp(\frac{\iota S_i(x,t)}{\hbar}). \tag{2.17}$$

here,

$$n = \mid \psi \mid^2 = A_i^2; u = \frac{\partial x S}{m}. \tag{2.18}$$

This results in non-linear Schrodinger equation of form;

$$\iota\hbar\frac{\partial \psi(x,t)}{\partial t} = \frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x,t) - e\varphi(x,t)\psi(x,t) + V_{eff}(n)\psi(x,t). \tag{2.19}$$

Here $V_{eff} = \int^n \frac{dn'}{n'}\frac{dP(n')}{dn'}$ is effective potential. For one dimensional case $\gamma = 3$ and $P_F = (\frac{mv_F^2}{3n_o^2})n^3$, where v_F is the fermi velocity. The relation for effective potential comes out to be $V_{eff} = (\frac{mv_F^2}{2n_o^2}) \mid \psi \mid^4$.

2.4.2 WIGNER-POISSON APPROACH

The distribution of particles in phase space at equilibrium is explained by the classical distribution function. We can derive the classical fluid model by taking moments of the Vlasov equation(or any suitable kinetic equation). Similarly, by taking moments of the Wigner equation we can derive the quantum hydrodynamic equations. Quantities such as particle density, pressure and average velocity are expressed by lower-order moments. We can define average density, velocity and pressure as:

$$n = \int W dv = \sum_{i=1}^{N} p_i \mid \psi_i \mid^2, \tag{2.20}$$

$$du = \frac{1}{n} \int W dv = \frac{\iota\hbar}{2mn} \sum_{i=1}^{N} p_i (\psi_i \frac{\partial \psi^*(x,t)}{\partial x} - \psi^*(x,t) \frac{\partial \psi^*(x,t)}{\partial x}), \tag{2.21}$$

and

$$P = m(\int W v^2 dv - nv^2). \tag{2.22}$$

By using Taylor expansion:

$$f(x \pm a) = f(x) \pm \frac{af(x)}{2x} + \frac{a^2}{2!} \frac{\partial^2 f(x)}{\partial x^2} + ... \tag{2.23}$$

Eq. (2.6) leads us to continuity equation and momentum equation:

$$\frac{\partial n}{\partial t} + \frac{\partial (nu)}{\partial x} = 0, \tag{2.24}$$

and

$$(\frac{\partial}{\partial t} + u \frac{\partial}{\partial x}) u = \frac{e}{m} \frac{\partial \varphi}{\partial x} + \frac{1}{mn} \frac{\partial P}{\partial x}. \tag{2.25}$$

Taking the derivative of ψ_i and ψ_i^*, we get

$$\frac{\partial \psi_i}{\partial x} = \frac{\partial A_i}{\partial x} exp(\frac{\iota S_i(x,t)}{\hbar}) + \frac{\iota}{\hbar} A_i \frac{\partial S_i}{\partial x} exp(\frac{\iota S_i(x,t)}{\hbar}), \tag{2.26}$$

$$\frac{\partial \psi_i^*}{\partial x} = \frac{\partial A_i}{\partial x} exp(-\frac{\iota S_i(x,t)}{\hbar}) - \frac{\iota}{\hbar} A_i \frac{\partial S_i}{\partial x} exp(-\frac{\iota S_i(x,t)}{\hbar}), \tag{2.27}$$

$$\frac{\partial^2\psi_i}{\partial x^2} = \frac{\partial^2 A_i}{\partial x^2}exp(\frac{\iota S_i(x,t)}{\hbar})+2\frac{\iota}{\hbar}\frac{\partial A_i}{\partial x}\frac{\partial S_i}{\partial x}exp(\frac{\iota S_i(x,t)}{\hbar})+\frac{\iota}{\hbar}A_i\frac{\partial^2 S_i}{\partial x^2}exp(\frac{\iota S_i(x,t)}{\hbar})-\frac{A_i}{\hbar^2}(\frac{\partial S_i}{\partial x})^2 exp(\frac{\iota S_i(x,t)}{\hbar}), \tag{2.28}$$

$$\frac{\partial^2\psi_i^*}{\partial x^2} = \frac{\partial^2 A_i}{\partial x^2}exp(-\frac{\iota S_i(x,t)}{\hbar})-2\frac{\iota}{\hbar}\frac{\partial A_i}{\partial x}\frac{\partial S_i}{\partial x}exp(-\frac{\iota S_i(x,t)}{\hbar})-\frac{\iota}{\hbar}A_i\frac{\partial^2 S_i}{\partial x^2}exp(-\frac{\iota S_i(x,t)}{\hbar})-\frac{A_i}{\hbar^2}(\frac{\partial S_i}{\partial x})^2 exp(-\frac{\iota S_i(x,t)}{\hbar} \tag{2.29}$$

Using equation (2.26-2.29) in equation (2.22) we get pressure terms.The classical pressure which do not include $\hbar$ is given as:

$$P_C = \frac{1}{2mn}\sum_{i,j} p_i p_j A_i^2 A_j^2[(\frac{\partial S_i}{\partial x})^2 - 2\frac{\partial S_i}{\partial x}\frac{\partial S_j}{\partial x} + (\frac{\partial S_j}{\partial x})^2], \tag{2.30}$$

$$P_C = mn[\sum_{i=1}^{N}\frac{p_i n_i v_i^2}{n} - (\sum_{i=1}^{N}\frac{p_i n_i v_i}{n})^2], \tag{2.31}$$

$$P_C = mn(< v_i^2 > - < v_i >^2), \tag{2.32}$$

Similarly, quantum pressure (terms which included $\hbar^2$) is given as:

$$P_Q = \frac{\hbar^2}{2m}\sum_i[(\frac{\partial A_i}{\partial x})^2 - A_i\frac{\partial^2 A_i}{\partial x^2}], \tag{2.33}$$

$$P_Q = \frac{\hbar^2}{2m}\sum_i P_i[(\frac{\partial\sqrt{n}_i}{\partial x})^2 - \sqrt{n}_i\frac{\partial^2\sqrt{n}_i}{\partial x^2}]. \tag{2.34}$$

So pressure term in equation (2.25) can be written as : $P = P_C + P_Q$. For a statistical system of pure states, amplitude of all states might be equal so we can write that $A_i(x) = A(x)$ which leads us to $n = A^2$. So equation (2.25) leads us to:

$$(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x})u = \frac{e}{m}\frac{\partial\varphi}{\partial x} - \frac{1}{mn}\frac{\partial P_C}{\partial x} + \frac{\hbar^2}{2m}\frac{\partial}{\partial x}(\frac{\partial x^2\sqrt{n}}{\sqrt{n}}). \tag{2.35}$$

Equation (2.24) and (2.35) gives us reduced quantum hydrodynamic approximation by using Wigner formalism. In the presence of magnetic field, fluid equations can be written as:

$$\frac{\partial n}{\partial t} + \nabla.(nu) = 0, \tag{2.36}$$

$$(\frac{\partial}{\partial t} + u.\nabla)u = \frac{q}{m}(E + u \times B) - \frac{1}{mn}\nabla P + \frac{\hbar^2}{2m}\nabla(\frac{\nabla\sqrt{n}}{\sqrt{n}}), \tag{2.37}$$

here $P = P_t + P_F$ where P_t =thermal pressure and P_F =Fermionic pressure. When we are considering plasma which is dense and has low temperature then P_F is sufficiently large as compared to P_t.

The quantum hydrodynamic approach gives better results for distances which are larger than Fermi screening length. With QHD some limitations are also associated e.g. it is not straight forward when we are treating system computationally because there arise some difficulties due to third-order derivatives of densities involved in Bohm potential term. Also, we are unable to study wave-particle interaction through this model. Furthermore, wavelength shorter then Fermi length can't be treated properly despite the fact that this model has many advantages due to its simplicity. As the kinetic model is too complicated to understand fundamental dynamics of quantum plasma so it is suitable for us to use QHD model.

3. QUANTUM MODIFICATION OF DUST SHEAR ALFVEN WAVE IN PLASMAS

In recent years, quantum plasma has become a topic of interest for many plasma researchers. It has wide applications in ultra-small electronic devices, e.g. metallic and semiconductor nanostructures, astrophysical systems which are highly dense and in laser-produced plasmas. The hot topic of research for scientists is the expansion of quantum electron gas in a vacuum and the dispersion properties of Landau damping [28]. Quantum effects significantly alter the dispersion properties and instabilities of the excited modes. Mostly we consider the case of unbound quantum plasma but in the laboratory when we are dealing with such plasmas we consider them to be spatially bound or spatially limited. We also consider the boundary conditions. Potential of plasma at the edge is zero. Because of finite cylindrical boundary effects dispersion properties (which can be affected by the boundary of the device and excited radial wave number) are discrete. So it is important to study boundary effects on quantum plasma [29].

Many scientists studied the quantum effects in low temperature and high-density range but even in a classical range where we are considering high temperature and modest-density we can't neglect the quantum effects [30].

In many schemes, we use magnetic fields for confining plasma in thermonuclear fusion reactions. These fields are sustained when the steady-state electric current is derived through the radio frequency fields. Generation of electric currents in low density has gained importance but very little importance has been given to the idea of steady-state currents in high densities. When we are considering moderately dense or dense medium, the plasma is Fermi degenerate plasma. It arises when a pallet of hydrogen is compressed to many times the solid densities for achieving inertial confinement fusion. These conditions are achieved by intense lasers or ion beams which will cause the intentional or incidental generation of steady currents [31]. While dealing with a quantum plasma, the electron plasma frequency is extremely high as the number density is extremely large so the electron plasma frequency exceeds the electron collision frequency. This will give rise to new effects in quantum plasmas. Collective interactions in quantum dense plasma are treated using different approaches. Here, we will be considering quantum magnetohydrodynamic model and several forces that will act on plasma species like Bohm potential and Lorentz force etc [30]. Also, dusty plasmas are rich in waves and instabilities. Dust particles are quite common in different environments such as interstellar media, comets etc as they are either positive or negatively charged depending upon the process like the bombardment of electrons and ions on the surface of dust particles from the background or by the process of photoemission, thermionic emission, secondary electron emission [32].

The wave function describes the individual system in the most complete possible form but it only provides us the probable results. This assumption can't be tested experimentally, the only way of testing this assumption is to find other interpretations of quantum theory in terms of present "HIDDEN" variables which can explain the precise behavior of an individual system. This assumption has been subjected to criticism, especially by Einstein who was of the view that there must exist a dynamical variable (as in classical physics) which can explain the actual behavior of an individual system and not just the probable behavior, but such variables are not present in the quantum theory. Many physicists felt that these objections raised by Einstein are not relevant, as the quantum theory with probability interpretation is in agreement with a wide range of experiments especially in the domain of distances which are larger than $10^{-13}cm$ and also because no other alternative interpretation was present at that time. In 1951 David Bohm in his paper "A SUGGESTED INTERPRETATION OF QUANTUM THEORY IN TERMS OF "HIDDEN" VARIABLES" gave an alternative interpretation. In this alternative interpretation each system is considered in a precise state, and changes with time are determined by equations which are analogous to the classical equation of motion, as long as general form of Schrodinger equation is obtained and physical results using alternative interpretation are same as those obtained by usual interpretation [33]. These effects are not prominent in the atomic domain. Present quantum theory is of crucial importance for dimensions of the order of $10^{-13}cm$ for which the usual interpretation is inadequate, an alternative interpretation of quantum theory was also given by de-Broglie in 1926 but it was given up by him because of the criticism made by Pauli and partly by himself [34]. The usual physical interpretation of the quantum theory is centered around uncertainty principle. Uncertainty principle is based upon two assumptions first that a wave function gives the most complete possible specification of the quantum state of the individual system and by using de Broglie relation $p = \hbar k$ (k being wave number) the uncertainty principle is readily deduced. This principle has limitation on the precision of momentum and position. For new physical interpretation of Schrodinger equation, we begin with the one particle Schrodinger equation and then later be generalized to the various number of particle[35]. The wave equation is:

$$\iota\hbar\frac{\partial\psi}{\partial t} = \frac{-\hbar^2}{2m}(\nabla^2\psi) + V(x)\psi. \tag{3.1}$$

here ψ is complex wave function given by:

$$\psi = Re^{(\frac{\iota S}{\hbar})}. \tag{3.2}$$

Now,

$$\nabla\psi = \nabla Re^{\frac{\iota S}{\hbar}} + \iota\frac{\nabla S}{\hbar}Re^{\frac{\iota S}{\hbar}}, \tag{3.3}$$

and

$$\frac{\partial\psi}{\partial t} = \frac{\partial R}{\partial t}e^{\frac{\iota S}{\hbar}} + \frac{\iota R}{\hbar}\frac{\partial S}{\partial t}e^{\frac{\iota S}{\hbar}}. \tag{3.4}$$

as,

$$\nabla^2\psi = \nabla.(\nabla\psi), \tag{3.5}$$

Using value of $(\nabla\psi)$ in the above equation, we get

$$\nabla^2\psi = \nabla.(\nabla Re^{\frac{\iota S}{\hbar}} + \iota\frac{\nabla S}{\hbar}Re^{\frac{\iota S}{\hbar}}). \tag{3.6}$$

After simplification, we can write

$$\nabla^2\psi = \nabla^2 Re^{\frac{\iota S}{\hbar}} + \nabla Re^{\frac{\iota S}{\hbar}}.\iota\frac{\nabla S}{\hbar} + \iota\frac{\nabla R\nabla S}{\hbar}e^{\frac{\iota S}{\hbar}} + \frac{\iota R}{\hbar}\nabla^2 Se^{\frac{\iota S}{\hbar}} + \frac{\iota R}{\hbar}\nabla Se^{\frac{\iota S}{\hbar}}.\frac{\iota}{\hbar}\nabla S, \tag{3.7}$$

$$\nabla^2\psi = \nabla^2 Re^{\frac{\iota S}{\hbar}} + \iota.\frac{\nabla R\nabla S}{\hbar}e^{\frac{\iota S}{\hbar}} + \iota\frac{\nabla R\nabla S}{\hbar}e^{\frac{\iota S}{\hbar}} + \frac{\iota R}{\hbar}\nabla^2 Se^{\frac{\iota S}{\hbar}} - \frac{R}{\hbar^2}(\nabla S)^2 e^{\frac{\iota S}{\hbar}}, \tag{3.8}$$

$$\nabla^2\psi = \nabla^2 Re^{\frac{\iota S}{\hbar}} + 2\iota.\frac{\nabla R\nabla S}{\hbar}e^{\frac{\iota S}{\hbar}} + \frac{\iota R}{\hbar}\nabla^2 Se^{\frac{\iota S}{\hbar}} - \frac{R}{\hbar^2}(\nabla S)^2 e^{\frac{\iota S}{\hbar}}. \tag{3.9}$$

using eq. (3.4) and (3.9) in eq. (3.1), we get

$$\iota\hbar[\frac{\partial R}{\partial t}e^{\frac{\iota S}{\hbar}} + \frac{\iota R}{\hbar}\frac{\partial S}{\partial t}e^{\frac{\iota S}{\hbar}}] = \frac{-\hbar^2}{2m}(\nabla^2 Re^{\frac{\iota S}{\hbar}} + 2\iota.\frac{\nabla R\nabla S}{\hbar}e^{\frac{\iota S}{\hbar}} + \frac{\iota R}{\hbar}\nabla^2 Se^{\frac{\iota S}{\hbar}} - \frac{R}{\hbar^2}(\nabla S)^2 e^{\frac{\iota S}{\hbar}}) + V(x)(Re^{\frac{\iota S}{\hbar}}), \tag{3.10}$$

$$\iota\hbar\frac{\partial R}{\partial t} - R\frac{\partial S}{\partial t} = \frac{-\hbar^2}{2m}\nabla^2 R - \frac{\iota\hbar}{m}(\nabla R\nabla S) - \frac{\iota\hbar}{2m}R\nabla^2 S + \frac{1}{2m}R(\nabla S)^2 + V(x)R. \tag{3.11}$$

or

$$\frac{\partial R}{\partial t} + \frac{\iota R}{\hbar}\frac{\partial S}{\partial t} = \frac{\iota\hbar}{2m}\nabla^2 R - \frac{1}{m}(\nabla R\nabla S) - \frac{1}{2m}R\nabla^2 S - \frac{\iota}{2m\hbar}R(\nabla S)^2 + \frac{\iota}{\hbar}V(x)R. \tag{3.12}$$

The real part of Eq.(3.4) can be written as:

$$\frac{\partial R}{\partial t} = \frac{-1}{2m}R\nabla^2 S + 2(\nabla R\nabla S). \tag{3.13}$$

and the imaginary part is

$$\frac{\iota R}{\hbar}\frac{\partial S}{\partial t} = \frac{\iota\hbar}{2m}\nabla^2 R - \frac{\iota}{2m\hbar}R(\nabla S)^2 - \frac{\iota}{\hbar}V(x)R. \tag{3.14}$$

$$\frac{\partial S}{\partial t} = \frac{\hbar^2}{2m}\frac{\nabla^2 R}{R} - \frac{1}{2m}(\nabla S)^2 - V(x), \tag{3.15}$$

or

$$\frac{\partial S}{\partial t} = -[\frac{1}{2m}(\nabla S)^2 + V(x) - \frac{\hbar^2}{2m}\frac{\nabla^2 R}{R}]. \tag{3.16}$$

We can write equation (3.13) as:

$$\frac{\partial R}{\partial t} = \frac{-1}{2m}R\nabla^2 S + 2(\nabla R\nabla S). \tag{3.17}$$

The above equation can also be written as;

$$2R(\frac{\partial R}{\partial t}) = \frac{-R^2}{m}\nabla^2 S - \frac{2R}{m}(\nabla R.\nabla S). \tag{3.18}$$

It is convenient to write $P(x) = R^2(x)$ or$R = P^{\frac{1}{2}}$, here $P(x)$ is the probability density.

$$\frac{\partial P}{\partial t} = \frac{-P}{m}\nabla^2 S - \nabla P.\frac{\nabla S}{m}. \tag{3.19}$$

$$\frac{\partial P}{\partial t} = -[\nabla P.\frac{\nabla S}{m} + \frac{P}{m}\nabla^2 S]. \tag{3.20}$$

$$\frac{\partial P}{\partial t} + [\nabla P.\frac{\nabla S}{m} + \frac{P}{m}\nabla^2 S] = 0. \tag{3.21}$$

$$\frac{\partial P}{\partial t} + \nabla.(P\frac{\nabla S}{m}) = 0. \tag{3.22}$$

And similarly, we can write the imaginary part in terms of P as;

$$\frac{\partial S}{\partial t} + \frac{1}{2m}(\nabla S)^2 + V(x) - \frac{\hbar^2}{4m}[\frac{\nabla^2 P}{P} - \frac{1}{2}\frac{(\nabla P)^2}{P^2}] = 0. \tag{3.23}$$

Now in the classical limit $\hbar \to 0$, so the function S(x) is a solution of Hamiltonian-Jacobi equation. If we are dealing with an ensemble in which particle trajectories are solutions of the equation of motion, then from an important theorem of mechanics we know that if all trajectories are normal to the given surface of constant "S", then they are normal to all surfaces of constant "S", and $\nabla\frac{S}{m}$ will be equivalent to V(x). For any particle passing the point "x" the equation can now be written as:

$$\frac{\partial P}{\partial t} + \nabla.(PV) = 0, \tag{3.24}$$

here P(x) is regarded as a probability density for particles in ensemble and PV can be regarded as a mean current of particles in an ensemble and the above equation gives us the conservation of probability. We assume that each particle is acted on not only by the classical potential V(x) but also a quantum mechanical potential. By comparison of Eq.(3.16) an Eq.(3.23) we get:

$$U(x) = -\frac{\hbar^2}{4m}[\frac{\nabla^2 P}{P} - \frac{1}{2}\frac{(\nabla P)^2}{P^2}] = -\frac{\hbar^2}{2m}\frac{\nabla^2 R}{R}. \tag{3.25}$$

Equation (2.23) is still regarded as the Hamilton-Jacobi equation, and $\frac{(\nabla S(x))}{m}$ is the velocity of particle and equation (3.22) describes the conservation of probability. It seems that we have an alternative interpretation of Schrodinger's equation. To develop this interpretation we associate with each electron precisely definable and continuously varying values of position and momentum. Solutions of modified Hamilton-Jacobi equation (3.16) provides us with an ensemble of all possible trajectories of these particles which can simply be obtained from S(x) (Hamilton-Jacobi function) by integrating velocity V(x). The equation for "S" tells us that particle motion is influenced by a force which is not entirely drivable from classical potential "V(x)" but it also has contributed from "quantum mechanical potential"

$$U(x) = -\frac{\hbar^2}{2m}\frac{\nabla^2 R}{R}. \tag{3.26}$$

here R(x) can be obtained in terms of S(x), R and S co-determine each other. It goes quite far when we make an analogy with electromagnetic fields obeys Maxwell's equation for both the cases, a complete specification of the field at a given instant for each point in space determines the value of the field for all times. If we know the field function we can calculate the force on a particle. So if we know the initial position and momentum of the particle, we can calculate the entire trajectory, equation of motion of particle acted by classical potential V(x) and quantum potential is given by,

$$m\frac{d^2x}{dt^2} = -\nabla[V(x) - \frac{\hbar^2}{2m}\frac{\nabla^2 R}{R}]. \tag{3.27}$$

Later in this thesis, we will replace R by $n^{\frac{1}{2}}$. In the last few years, the effect of dust particles on low-frequency electromagnetic waves has been observed. In this thesis, I will consider long-wavelength shear Alfven wave with arbitrary wave number using QMDH also we have here considered low-temperature quantum dusty magnetoplasma and a magnetic field is considered in the z-direction.

$$\frac{\partial V_j}{\partial t} = \frac{q_j}{m_j}\vec{E} + V_{ij} \times \omega_{cj}\vec{z} - \frac{\nabla P_{Fj}}{m_j n_{oj}} + \frac{\hbar^2}{4m_j^2 n_{oj}}\nabla(\nabla^2 n_j), \tag{3.28}$$

where

$$P_{Fj} = \frac{m_j V_{Fj}^2 n_j^3}{3n_{0j}^2}. \tag{3.29}$$

After simlification, we can write

$$(-\iota\omega)V_{1j} = \frac{q_j}{m_j}\vec{E} + V_{ij} \times \omega_{cj}\vec{z} - \frac{\iota\vec{k}V_{Fj}^2}{3}(\frac{n_{1j}}{n_{oj}})^3 + \frac{\hbar^2}{4m_j^2 n_{oj}}\iota\vec{k}(-k^2 n_{1j}). \tag{3.30}$$

From the equation of continuity,

$$\frac{\partial n_j}{\partial t} = \nabla.(n_j V_j). \tag{3.31}$$

$$\frac{\partial n_{0j}}{\partial t} + \frac{\partial n_{1j}}{\partial t} = \nabla.((n_{0j} + n_{1j}) + (V_{0j} + V_{1j})), \tag{3.32}$$

After linearization, we get

$$\frac{\partial n_{1j}}{\partial t} = \nabla.(n_{0j} V_{1j}), \tag{3.33}$$

By considering sinusoidal perturbation, the above eq. becomes

$$-\iota\omega n_{1j} + n_{0j}\nabla.V_{1j} = 0, \tag{3.34}$$

or

$$n_{1j} = \frac{n_{0j}}{\omega}(k).V_{1j}. \tag{3.35}$$

Now using the value of n_{1j} in eq. (1.7), we get

$$(-\iota\omega)V_{1j} = \frac{q_j}{m_j}\vec{E} + V_{ij} \times \omega_{cj}\vec{z} - \frac{\iota\vec{k}V_{Fj}^2}{3}\left(\frac{\frac{n_{0j}}{\omega}(k).V_{1j}}{n_{oj}}\right)^3 + \frac{\hbar^2}{4m_j^2 n_{oj}}\iota\vec{k}\left(-k^2\frac{n_{0j}}{\omega}(\vec{k}).V_{1j}\right), \tag{3.36}$$

or we can write

$$V_{1j} = \frac{\iota q_j}{m_j\omega}\vec{E} + \frac{\iota}{\omega}(V_{ij} \times \omega_{cj}\vec{z}) + \frac{V_{Fj}^2\vec{k}}{\omega^2}(\vec{k}.V_{1j})\left(1 + \frac{\hbar^2 k^2}{4m_j^2 V_{Fj}^2}\right). \tag{3.37}$$

Defining

$$\gamma_j = \frac{\hbar^2 k^2}{4m_j^2 V_{Fj}^2}, \tag{3.38}$$

Above equation can be written as:

$$V_{1j} = \frac{\iota q_j}{m_j\omega}\vec{E} + \frac{\iota}{\omega}(V_{ij} \times \omega_{cj}\vec{z}) + \frac{V_{Fj}^2\vec{k}}{\omega^2}(\vec{k}.V_{1j})(1 + \gamma_j). \tag{3.39}$$

Now solving the cross products and re-writing the above equation yields:

$$V_{1j} = \frac{\iota q_j}{m_j\omega}\vec{E} + \frac{\iota}{\omega}(V_{yj}\omega_{cj}\vec{i} - V_{xj}\omega_{cj}\vec{j}) + \frac{V_{Fj}^2\vec{k}}{\omega^2}(\vec{k}.V_{1j})(1 + \gamma_j). \tag{3.40}$$

By taking $V'_{Fj} = V_{Fj}(1 + \gamma_j)^{\frac{1}{2}}$,

we can write

$$V_{1j} = \frac{\iota q_j}{m_j\omega}\vec{E} + \frac{\iota}{\omega}(V_{yj}\omega_{cj}\vec{i} - V_{xj}\omega_{cj}\vec{j}) + \frac{\vec{k}}{\omega^2}(\vec{k}.V_{1j})V'^2_{Fj}. \tag{3.41}$$

Dropping the subscript 1 and the above equation in component form, we get

$$V_{xj}\vec{i} + V_{yj}\vec{j} + V_{zj}\vec{k} = \frac{\iota q_j}{m_j\omega}(\vec{E_x}\vec{i} + \vec{E_y}\vec{j} + \vec{E_z}\vec{k}) + \frac{\iota}{\omega}(V_{yj}\omega_{cj}\vec{i}) - \frac{\iota}{\omega}(V_{xj}\omega_{cj}\vec{j}) + \frac{(\vec{k}_x\vec{i} + \vec{k}_z\vec{k})}{\omega^2}(k_x.V_{xj} + k_z.V_{zj})V'^2_{Fj}. \tag{3.42}$$

x-component:

$$V_{xj} = \frac{\iota q_j}{m_j\omega}(\vec{E}_x) + \frac{\iota}{\omega}(V_{yj}\omega_{cj}) + \frac{V_{Fj}'^2}{\omega^2}(k_x^2 V_{xj}) + \frac{V_{Fj}'^2}{\omega^2}(k_x k_z.V_{zj}). \tag{3.43}$$

y-component:

$$V_{yj} = \frac{\iota q_j}{m_j\omega}(\vec{E}_y) - \frac{\iota}{\omega}(V_{xj}\omega_{cj}). \tag{3.44}$$

z-component:

$$V_{zj} = \frac{\iota q_j}{m_j\omega}(\vec{E}_z) + \frac{V_{Fj}'^2}{\omega^2}(k_x k_z V_{xj}) + \frac{V_{Fj}'^2}{\omega^2}(k_z^2.V_{zj}). \tag{3.45}$$

Now solving eq. (3.45), we can write

$$V_{zj}(1 - \frac{V_{Fj}'^2}{\omega^2}(k_z^2)) = \frac{\iota q_j}{m_j\omega}(\vec{E}_z) + \frac{V_{Fj}'^2}{\omega^2}(k_x k_z V_{xj}). \tag{3.46}$$

Now by defining $F = (1 - \frac{V_{Fj}'^2}{\omega^2}(k_z^2))$, we get

$$V_{zj} = \frac{1}{F}[\frac{\iota q_j}{m_j\omega}(\vec{E}_z) + \frac{V_{Fj}'^2}{\omega^2}(k_x k_z V_{xj})]. \tag{3.47}$$

Now to find V_{xj} we will use the value of V_{zj} from eq. (3.47) and value of V_{yj} from eq. (3.44) in eq. (3.43)

$$V_{xj} = \frac{\iota q_j}{m_j\omega}(\vec{E}_x) + \frac{\iota\omega_{cj}}{\omega}(\frac{\iota q_j}{m_j\omega}(\vec{E}_y) - \frac{\iota}{\omega}(V_{xj}\omega_{cj})) + \frac{V_{Fj}'^2}{\omega^2}(k_x^2 V_{xj}) + \frac{V_{Fj}'^2}{\omega^2}(k_x k_z.\frac{1}{(1 - \frac{V_{Fj}'^2}{\omega^2}(k_z^2))}[\frac{\iota q_j}{m_j\omega}(\vec{E}_z) + \frac{V_{Fj}'^2}{\omega^2}(k_x k_z V_{xj})]), \tag{3.48}$$

$$V_{xj} = \frac{\iota q_j}{m_j\omega}(\vec{E}_x) + \frac{\iota^2\omega_{cj}q_j}{m_j\omega^2}(\vec{E}_y) + \frac{\omega_{cj}^2}{\omega^2}(V_{xj}) + \frac{V_{Fj}'^2}{\omega^2}(k_x^2 V_{xj}) + \frac{\iota q_j V_{Fj}'^2 k_x k_z}{m_j\omega^3 F}(\vec{E}_z) + \frac{V_{Fj}'^4 k_x^2 k_z^2}{\omega^4(1 - \frac{V_{Fj}'^2}{\omega^2}(k_z^2))}V_{xj}, \tag{3.49}$$

$$V_{xj} - \frac{\omega_{cj}^2}{\omega^2}(V_{xj}) - \frac{V_{Fj}'^4 k_x^2 k_z^2}{\omega^2(\omega^2 - V_{Fj}'^2(k_z^2))}V_{xj} - \frac{V_{Fj}'^2}{\omega^2}(k_x^2 V_{xj}) = \frac{\iota q_j}{m_j\omega}(\vec{E}_x) + \frac{\iota^2\omega_{cj}q_j}{m_j\omega^2}(\vec{E}_y) + \frac{\iota q_j V_{Fj}'^2 k_x k_z}{m_j\omega^3 F}(\vec{E}_z), \tag{3.50}$$

$$V_{xj}[1-\frac{\omega_{cj}^2}{\omega^2}-\frac{V_{Fj}'^4k_x^2k_z^2}{\omega^2(\omega^2-V_{Fj}'^2(k_z^2))}-\frac{V_{Fj}'^2}{\omega^2}(k_x^2)]=$$

$$\frac{\iota q_j}{m_j\omega}[(\vec{E}_x)+\frac{\iota\omega_{cj}}{\omega}(\vec{E}_y)+\frac{V_{Fj}'^2k_xk_z}{\omega^2F}(\vec{E}_z)], \quad (3.51)$$

$$V_{xj}[\frac{\omega^2(\omega^2-V_{Fj}'^2(k_z^2))-V_{Fj}'^4k_x^2k_z^2-V_{Fj}'^2k_x^2(\omega^2-V_{Fj}'^2k_z^2)}{\omega^2(\omega^2-V_{Fj}'^2(k_z^2))}-\frac{\omega_{cj}^2}{\omega^2}]=$$

$$\frac{\iota q_j}{m_j\omega}[(\vec{E}_x)+\frac{\iota\omega_{cj}}{\omega}(\vec{E}_y)+\frac{V_{Fj}'^2k_xk_z}{\omega^2F}(\vec{E}_z)], \quad (3.52)$$

$$V_{xj}[\frac{\omega^4-\omega^2V_{Fj}'^2k_z^2-V_{Fj}'^4k_x^2k_z^2-\omega^2V_{Fj}'^2k_x^2+V_{Fj}'^4k_x^2k_z^2}{\omega^2(\omega^2-V_{Fj}'^2(k_z^2))}-\frac{\omega_{cj}^2}{\omega^2}]=$$

$$\frac{\iota q_j}{m_j\omega}[(\vec{E}_x)+\frac{\iota\omega_{cj}}{\omega}(\vec{E}_y)+\frac{V_{Fj}'^2k_xk_z}{\omega^2F}(\vec{E}_z)], \quad (3.53)$$

$$V_{xj}[\frac{\omega^4-\omega^2V_{Fj}'^2(k_x^2+k_z^2)}{\omega^2(\omega^2-V_{Fj}'^2(k_z^2))}-\frac{\omega_{cj}^2}{\omega^2}]=\frac{\iota q_j}{m_j\omega}[(\vec{E}_x)+\frac{\iota\omega_{cj}}{\omega}(\vec{E}_y)+\frac{V_{Fj}'^2k_xk_z}{\omega^2F}(\vec{E}_z)], \quad (3.54)$$

as we are taking

$$k_x=k\sin\theta. \quad (3.55)$$

and

$$k_z=k\cos\theta. \quad (3.56)$$

$$k_x^2+k_z^2=k^2. \quad (3.57)$$

$$V_{xj}[\frac{\omega^4-\omega^2V_{Fj}'^2(k^2)}{\omega^2(\omega^2-V_{Fj}'^2(k_z^2))}-\frac{\omega_{cj}^2}{\omega^2}]=\frac{\iota q_j}{m_j\omega}[(\vec{E}_x)+\frac{\iota\omega_{cj}}{\omega}(\vec{E}_y)+\frac{V_{Fj}'^2k_xk_z}{\omega^2F}(\vec{E}_z)] \quad (3.58)$$

$$V_{xj}[\frac{\omega^2[\omega^2-V_{Fj}'^2(k^2)]}{\omega^2(\omega^2-V_{Fj}'^2(k_z^2))}-\frac{\omega_{cj}^2}{\omega^2}]=\frac{\iota q_j}{m_j\omega}[(\vec{E}_x)+\frac{\iota\omega_{cj}}{\omega}(\vec{E}_y)+\frac{V_{Fj}'^2k_xk_z}{\omega^2F}(\vec{E}_z)] \quad (3.59)$$

Let

$$G = [\frac{\omega^2 - V_{Fj}'^2(k^2)}{(\omega^2 - V_{Fj}'^2(k_z^2))}], \tag{3.60}$$

then we can write

$$V_{xj}[G - \frac{\omega_{cj}^2}{\omega^2}] = \frac{\iota q_j}{m_j \omega}[(\vec{E}_x) + \frac{\iota \omega_{cj}}{\omega}(\vec{E}_y) + \frac{V_{Fj}'^2 k_x k_z}{\omega^2 F}(\vec{E}_z)] \tag{3.61}$$

$$V_{xj}[\frac{\omega^2 G - \omega_{cj}^2}{\omega^2}] = \frac{\iota q_j}{m_j \omega}[(\vec{E}_x) + \frac{\iota \omega_{cj}}{\omega}(\vec{E}_y) + \frac{V_{Fj}'^2 k_x k_z}{\omega^2 F}(\vec{E}_z)] \tag{3.62}$$

$$V_{xj} = \frac{\iota q_j}{m_j \omega}[\frac{\omega^2}{\omega^2 G - \omega_{cj}^2}(\vec{E}_x) + \frac{\iota \omega \omega_{cj}}{\omega^2 G - \omega_{cj}^2}(\vec{E}_y) + \frac{V_{Fj}'^2 k_x k_z}{(\omega^2 G - \omega_{cj}^2)F}(\vec{E}_z)] \tag{3.63}$$

Now calculating the value of V_{yj}. Eq.(3.44) is:

$V_{yj} = \frac{\iota q_j}{m_j \omega}(\vec{E}_y) - \frac{\iota}{\omega}(V_{xj}\omega_{cj})$

$$V_{yj} = \frac{\iota q_j}{m_j \omega}(\vec{E}_y) - \frac{\iota \omega_{cj}}{\omega}(\frac{\iota q_j}{m_j \omega}[\frac{\omega^2}{\omega^2 G - \omega_{cj}^2}(\vec{E}_x) + \frac{\iota \omega \omega_{cj}}{\omega^2 G - \omega_{cj}^2}(\vec{E}_y) + \frac{V_{Fj}'^2 k_x k_z}{(\omega^2 G - \omega_{cj}^2)F}(\vec{E}_z)]) \tag{3.64}$$

$$V_{yj} = \frac{\iota q_j}{m_j \omega}[\frac{-\iota \omega_{cj}\omega}{\omega^2 G - \omega_{cj}^2}(\vec{E}_x) + \vec{E}_y + \frac{\omega_{cj}^2}{\omega^2 G - \omega_{cj}^2}(\vec{E}_y) - \frac{\iota \omega_{cj} V_{Fj}'^2 k_x k_z}{\omega F(\omega^2 G - \omega_{cj}^2)}(\vec{E}_z)] \tag{3.65}$$

$$V_{yj} = \frac{\iota q_j}{m_j \omega}[\frac{-\iota \omega_{cj}\omega}{\omega^2 G - \omega_{cj}^2}(\vec{E}_x) + \frac{\omega^2 G}{\omega^2 G - \omega_{cj}^2}(\vec{E}_y) - \frac{\iota \omega_{cj} V_{Fj}'^2 k_x k_z}{\omega F(\omega^2 G - \omega_{cj}^2)}(\vec{E}_z)] \tag{3.66}$$

Now to find the value of V_{zj}, substituting the value of V_{xj} from eq.(3.63) in eq.(3.47)

$V_{zj} = \frac{1}{F}[\frac{\iota q_j}{m_j \omega}(\vec{E}_z) + \frac{V_{Fj}'^2}{\omega^2}(k_x k_z V_{xj})]$

$$V_{zj} = \frac{1}{F}\frac{\iota q_j}{m_j \omega}(\vec{E}_z) + \frac{V_{Fj}'^2}{F\omega^2}k_x k_z \frac{\iota q_j}{m_j \omega}[\frac{\omega^2}{\omega^2 G - \omega_{cj}^2}(\vec{E}_x) + \frac{\iota \omega \omega_{cj}}{\omega^2 G - \omega_{cj}^2}(\vec{E}_y) + \frac{V_{Fj}'^2 k_x k_z}{(\omega^2 G - \omega_{cj}^2)F}(\vec{E}_z)] \tag{3.67}$$

$$V_{zj} = \frac{\iota q_j}{F m_j \omega}[\frac{V_{Fj}'^2 k_x k_z}{(\omega^2 G - \omega_{cj}^2)}(\vec{E}_x) + \frac{\iota \omega_{cj} V_{Fj}'^2 k_x k_z}{\omega(\omega^2 G - \omega_{cj}^2)}(\vec{E}_y) + \vec{E}_z + \frac{V_{Fj}'^4 k_x^2 k_z^2}{\omega^2 F(\omega^2 G - \omega_{cj}^2)}(\vec{E}_z)] \tag{3.68}$$

$$V_{zj}=\frac{\iota q_j}{Fm_j\omega}[\frac{V_{Fj}'^2 k_x k_z}{(\omega^2G-\omega_{cj}^2)}(\vec{E}_x)+\frac{\iota\omega_{cj}V_{Fj}'^2 k_x k_z}{\omega(\omega^2G-\omega_{cj}^2)}(\vec{E}_y)+\vec{E}_z(1+\frac{V_{Fj}'^4 k_x^2 k_z^2}{\omega^2F(\omega^2G-\omega_{cj}^2)})] \tag{3.69}$$

Due to the electromagnetic shear Alfven waves, current density of plasma particles is given by:

$$\vec{J}=\sum_j q_j n_{oj}\vec{V}_j \tag{3.70}$$

By substituting values of $V_{xj}, V_{yj} and V_{zj}$, in above equation, we get

$$\begin{aligned}\vec{J}=\sum_j q_j n_{oj}[&\frac{\iota q_j}{m_j\omega}[\frac{\omega^2}{\omega^2G-\omega_{cj}^2}(\vec{E}_x)+\frac{\iota\omega\omega_{cj}}{\omega^2G-\omega_{cj}^2}(\vec{E}_y)+\frac{V_{Fj}'^2 k_x k_z}{(\omega^2G-\omega_{cj}^2)F}(\vec{E}_z)]+\\ &\frac{\iota q_j}{m_j\omega}[\frac{-\iota\omega_{cj}\omega}{\omega^2G-\omega_{cj}^2}(\vec{E}_x)+\frac{\omega^2G}{\omega^2G-\omega_{cj}^2}(\vec{E}_y)-\frac{\iota\omega_{cj}V_{Fj}'^2 k_x k_z}{\omega F(\omega^2G-\omega_{cj}^2)}(\vec{E}_z)]+\\ &\frac{\iota q_j}{Fm_j\omega}[\frac{V_{Fj}'^2 k_x k_z}{(\omega^2G-\omega_{cj}^2)}(\vec{E}_x)+\frac{\iota\omega_{cj}V_{Fj}'^2 k_x k_z}{\omega(\omega^2G-\omega_{cj}^2)}(\vec{E}_y)+\vec{E}_z(1+\frac{V_{Fj}'^4 k_x^2 k_z^2}{\omega^2F(\omega^2G-\omega_{cj}^2)})]]\end{aligned} \tag{3.71}$$

$$\vec{J}=\sum_j\frac{\iota q_j^2 n_{oj}}{m_j\omega}\begin{pmatrix}\frac{\omega^2}{\omega^2G-\omega_{cj}^2} & \frac{\iota\omega\omega_{cj}}{\omega^2G-\omega_{cj}^2} & \frac{V_{Fj}'^2 k_x k_z}{(\omega^2G-\omega_{cj}^2)F}\\ \frac{-\iota\omega_{cj}\omega}{\omega^2G-\omega_{cj}^2} & \frac{\omega^2G}{\omega^2G-\omega_{cj}^2} & -\frac{\iota\omega_{cj}V_{Fj}'^2 k_x k_z}{\omega F(\omega^2G-\omega_{cj}^2)}\\ \frac{V_{Fj}'^2 k_x k_z}{F(\omega^2G-\omega_{cj}^2)} & \frac{\iota\omega_{cj}V_{Fj}'^2 k_x k_z}{F\omega(\omega^2G-\omega_{cj}^2)} & \frac{1}{F}(1+\frac{V'4_{Fj}k_x^2 k_z^2}{\omega^2F(\omega^2G-\omega_{cj}^2)})\end{pmatrix}\begin{pmatrix}E_x\\ E_y\\ E_z\end{pmatrix} \tag{3.72}$$

$$\vec{J}=\sum_j\frac{\iota q_j^2 n_{oj}}{m_j\omega}\underline{\underline{\mathbf{K_j}}}.\vec{E}, \tag{3.73}$$

here;

$$\underline{\underline{\mathbf{K_j}}}=\begin{pmatrix}\frac{\omega^2}{\omega^2G-\omega_{cj}^2} & \frac{\iota\omega\omega_{cj}}{\omega^2G-\omega_{cj}^2} & \frac{V_{Fj}'^2 k_x k_z}{(\omega^2G-\omega_{cj}^2)F}\\ \frac{-\iota\omega_{cj}\omega}{\omega^2G-\omega_{cj}^2} & \frac{\omega^2G}{\omega^2G-\omega_{cj}^2} & -\frac{\iota\omega_{cj}V_{Fj}'^2 k_x k_z}{\omega F(\omega^2G-\omega_{cj}^2)}\\ \frac{V_{Fj}'^2 k_x k_z}{F(\omega^2G-\omega_{cj}^2)} & \frac{\iota\omega_{cj}V_{Fj}'^2 k_x k_z}{F\omega(\omega^2G-\omega_{cj}^2)} & \frac{1}{F}(1+\frac{V_{Fj}'^4 k_x^2 k_z^2}{\omega^2F(\omega^2G-\omega_{cj}^2)})\end{pmatrix} \tag{3.74}$$

$$\vec{J}=\underline{\underline{\sigma}}.\vec{E} \tag{3.75}$$

here σ is linear conductivity tensor and is given by:

$$\underline{\underline{\sigma}}=\sum_j\frac{\iota q_j^2 n_{oj}}{m_j\omega}\underline{\underline{\mathbf{K_j}}}. \tag{3.76}$$

Now relation between $\vec{E}$ and $\vec{B}$ is given by the Maxwell's equations:

$$\vec{\nabla}.\vec{E} = 4\pi\ \rho \tag{3.77}$$

$$\vec{\nabla}.\vec{B} = 0 \tag{3.78}$$

$$\vec{\nabla} \times \vec{E} = \frac{-1}{c}\frac{\partial \vec{B}}{\partial t}, \tag{3.79}$$

$$\vec{\nabla} \times \vec{B} = \frac{4\pi \vec{J}}{c} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t}. \tag{3.80}$$

Now taking curl of equation (3.79)

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \frac{-1}{c}\frac{\partial(\vec{\nabla} \times \vec{B})}{\partial t}, \tag{3.81}$$

$$\vec{\nabla}(\vec{\nabla}.\vec{E}) - \nabla^2 \vec{E} = \frac{-1}{c}\frac{\partial(\vec{\nabla} \times \vec{B})}{\partial t}. \tag{3.82}$$

Now using value of $(\vec{\nabla} \times \vec{B})$, we get

$$\vec{\nabla}(\vec{\nabla}.\vec{E}) - \nabla^2 \vec{E} = \frac{-1}{c}\frac{\partial(\frac{4\pi \vec{J}}{c} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t})}{\partial t}. \tag{3.83}$$

Applying sinusoidal perturbation, we get

$$\iota\vec{k}(\iota\vec{k}.\vec{E}) - (\iota\vec{k})^2\vec{E} = \frac{-1}{c}(-\iota\omega)(\frac{4\pi\vec{J}}{c} + \frac{1}{c}(-\iota\omega)\vec{E}), \tag{3.84}$$

$$-\vec{k}(\vec{k}.\vec{E}) + k^2\vec{E} = \frac{(\iota\omega)4\pi\vec{J}}{c^2} + \frac{(\omega^2)}{c^2}\vec{E}. \tag{3.85}$$

Now using value of $\vec{J} = \underline{\underline{\sigma}}.\vec{E}$

$$-\vec{k}(\vec{k}.\vec{E}) + k^2\vec{E} = \frac{(\iota\omega)4\pi(\underline{\underline{\text{œ}}}.\vec{E})}{c^2} + \frac{(\omega^2)}{c^2}\vec{E}, \tag{3.86}$$

or we can write it as,

$$-\vec{k}(\vec{k}.\vec{E}) + k^2\vec{E} = \frac{(\iota\omega)4\pi(\sum_j \frac{\iota q_j^2 n_{oj}}{m_j\omega}\underline{\underline{\mathbf{K_j}}}.\vec{E})}{c^2} + \frac{(\omega^2)}{c^2}\vec{E}, \tag{3.87}$$

$$-\vec{k}(\vec{k}.\vec{E}) + k^2\vec{E} = \frac{\iota 4\pi(\sum_j \frac{\iota q_j^2 n_{oj}}{m_j}\underline{\underline{\mathbf{K_j}}}.\vec{E})}{c^2} + \frac{(\omega^2)}{c^2}\vec{E}. \tag{3.88}$$

As plasma frequency is defined as: $\omega_{Pj} = \sqrt{\frac{4\pi n_{oj} q_j^2}{m_j}}$

Therefore,

$$-\vec{k}(\vec{k}.\vec{E}) + k^2\vec{E} = -\sum_j \frac{\omega_{Pj}^2}{c^2}\underline{\underline{\mathbf{K_j}}}.\vec{E} + \frac{\omega^2}{c^2}\vec{E}, \tag{3.89}$$

or

$$-\vec{k}(\vec{k}.\vec{E}) + k^2\vec{E} = (-\sum_j \frac{\omega_{Pj}^2}{\omega^2}\underline{\underline{\mathbf{K_j}}} + \underline{\underline{\mathbf{I}}}).\frac{\omega^2}{c^2}\vec{E}, \tag{3.90}$$

here I is a unit dyadic.

Now defining the quantitiy $\underline{\underline{\epsilon}} = \underline{\underline{\mathbf{I}}} - \sum_j \frac{\omega_{Pj}^2}{\omega^2}\underline{\underline{\mathbf{K_j}}}$, we can write equation (3.90) as:

$$-\vec{k}(\vec{k}.\vec{E}) + k^2\vec{E} = \underline{\underline{\epsilon}}.\frac{\omega^2}{c^2}\vec{E} \tag{3.91}$$

$$(-\vec{k}(\vec{k}) + k^2\underline{\underline{\mathbf{I}}} - \underline{\underline{\epsilon}}.\frac{\omega^2}{c^2}).\vec{E} = 0 \tag{3.92}$$

$Det \mid \underline{\underline{\mathbf{D}}} \mid .\vec{E} = 0$

$$Det \mid \underline{\underline{\mathbf{D}}} \mid = (k^2\underline{\underline{\mathbf{I}}} - \vec{k}(\vec{k}) - \frac{\omega^2}{c^2}\underline{\underline{\epsilon}}) \tag{3.93}$$

$$k^2\underline{\underline{\mathbf{I}}} - (\vec{k}_x + \vec{k}_z)(\vec{k}_x + \vec{k}_z) - \frac{\omega^2}{c^2}\begin{pmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz} \end{pmatrix} = 0 \tag{3.94}$$

$$\begin{pmatrix} k^2 & 0 & 0 \\ 0 & k^2 & 0 \\ 0 & 0 & k^2 \end{pmatrix} - \begin{pmatrix} k_xk_x & 0 & k_xk_z \\ 0 & 0 & 0 \\ k_zk_x & 0 & k_zk_z \end{pmatrix} - \begin{pmatrix} \frac{\omega^2}{c^2}\epsilon_{xx} & \frac{\omega^2}{c^2}\epsilon_{xy} & \frac{\omega^2}{c^2}\epsilon_{xz} \\ \frac{\omega^2}{c^2}\epsilon_{yx} & \frac{\omega^2}{c^2}\epsilon_{yy} & \frac{\omega^2}{c^2}\epsilon_{yz} \\ \frac{\omega^2}{c^2}\epsilon_{zx} & \frac{\omega^2}{c^2}\epsilon_{zy} & \frac{\omega^2}{c^2}\epsilon_{zz} \end{pmatrix} = 0 \tag{3.95}$$

$$\begin{pmatrix} k^2 - k_x^2 - \frac{\omega^2}{c^2}\epsilon_{xx} & -\frac{\omega^2}{c^2}\epsilon_{xy} & -k_x k_z - \frac{\omega^2}{c^2}\epsilon_{xz} \\ -\frac{\omega^2}{c^2}\epsilon_{yx} & k^2 - \frac{\omega^2}{c^2}\epsilon_{yy} & -\frac{\omega^2}{c^2}\epsilon_{yz} - \\ -k_z k_x - \frac{\omega^2}{c^2}\epsilon_{zx} & -\frac{\omega^2}{c^2}\epsilon_{zy} & k^2 - k_z^2 - \frac{\omega^2}{c^2}\epsilon_{zz} \end{pmatrix} \tag{3.96}$$

$$k^2 = k_x^2 + k_z^2 \tag{3.97}$$

and;

$$k_z^2 = k^2 - k_x^2 \tag{3.98}$$

$$k_x^2 = k^2 - k_z^2 \tag{3.99}$$

By substituting the above equations in matrix (3.96) we get:

$$\begin{pmatrix} k_z^2 - \frac{\omega^2}{c^2}\epsilon_{xx} & -\frac{\omega^2}{c^2}\epsilon_{xy} & -k_x k_z - \frac{\omega^2}{c^2}\epsilon_{xz} \\ -\frac{\omega^2}{c^2}\epsilon_{yx} & k^2 - \frac{\omega^2}{c^2}\epsilon_{yy} & -\frac{\omega^2}{c^2}\epsilon_{yz} - \\ -k_z k_x - \frac{\omega^2}{c^2}\epsilon_{zx} & -\frac{\omega^2}{c^2}\epsilon_{zy} & k_x^2 - \frac{\omega^2}{c^2}\epsilon_{zz} \end{pmatrix}. \tag{3.100}$$

As; $\underline{\underline{\epsilon}} = \underline{\underline{\mathbf{I}}} - \sum_j \frac{\omega_{Pj}^2}{\omega^2}\underline{\underline{\mathbf{K_j}}}$

$$\underline{\underline{\text{ffl}}} = \underline{\underline{\mathbf{I}}} - \sum_j \frac{\omega_{Pj}^2}{\omega^2} \begin{pmatrix} \frac{\omega^2}{\omega^2 G - \omega_{cj}^2} & \frac{\iota\omega\omega_{cj}}{\omega^2 G - \omega_{cj}^2} & \frac{V'^{2Fj} k_x k_z}{(\omega^2 G - \omega_{cj}^2)F} \\ \frac{-\iota\omega_{cj}\omega}{\omega^2 G - \omega_{cj}^2} & \frac{\omega^2 G}{\omega^2 G - \omega_{cj}^2} & -\frac{\iota\omega_{cj} V'^{2Fj} k_x k_z}{\omega F(\omega^2 G - \omega_{cj}^2)} \\ \frac{V'^{2Fj} k_x k_z}{F(\omega^2 G - \omega_{cj}^2)} & \frac{\iota\omega_{cj} V'^{2Fj} k_x k_z}{F\omega(\omega^2 G - \omega_{cj}^2)} & \frac{1}{F}(1 + \frac{V'^{4Fj} k_x^2 k_z^2}{\omega^2 F(\omega^2 G - \omega_{cj}^2)}) \end{pmatrix} \tag{3.101}$$

$$\begin{pmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} \sum_j \frac{\omega_{pj}^2}{\omega^2 G - \omega_{cj}^2} & \sum_j \frac{\iota\omega_{pj}^2\omega_{cj}}{\omega(\omega^2 G - \omega_{cj}^2)} & \sum_j \frac{V'^{2Fj}\omega_{pj}^2 k_x k_z}{(\omega^2 G - \omega_{cj}^2)F\omega^2} \\ \sum_j \frac{-\iota\omega_{pj}\omega_{cj}}{\omega(\omega^2 G - \omega_{cj}^2)} & \sum_j \frac{\omega_{pj}^2 G}{\omega^2 G - \omega_{cj}^2} & -\sum_j \frac{\iota\omega_{pj}^2\omega_{cj} V'^{2Fj} k_x k_z}{\omega^3 F(\omega^2 G - \omega_{cj}^2)} \\ \sum_j \frac{V'^{2Fj}\omega_{pj}^2 k_x k_z}{F\omega^2(\omega^2 G - \omega_{cj}^2)} & \sum_j \frac{\iota\omega_{pj}^2\omega_{cj} V'^{2Fj} k_x k_z}{F\omega^3(\omega^2 G - \omega_{cj}^2)} & \sum_j \frac{\omega_{pj}^2}{\omega^2 F}(1 + \frac{V'^{4Fj} k_x^2 k_z^2}{\omega^2 F(\omega^2 G - \omega_{cj}^2)}) \end{pmatrix} \tag{3.102}$$

.

$$\begin{pmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz} \end{pmatrix} = \begin{pmatrix} 1 - \sum_j \frac{\omega_{pj}^2}{\omega^2 G - \omega_{cj}^2} & -\sum_j \frac{\iota\omega_{pj}^2\omega_{cj}}{\omega(\omega^2 G - \omega_{cj}^2)} & -\sum_j \frac{V'^{2Fj}\omega_{pj}^2 k_x k_z}{(\omega^2 G - \omega_{cj}^2)F\omega^2} \\ \sum_j \frac{\iota\omega_{pj}\omega_{cj}}{\omega(\omega^2 G - \omega_{cj}^2)} & 1 - \sum_j \frac{\omega_{pj}^2 G}{\omega^2 G - \omega_{cj}^2} & \sum_j \frac{\iota\omega_{pj}^2\omega_{cj} V'^{2Fj} k_x k_z}{\omega^3 F(\omega^2 G - \omega_{cj}^2)} \\ -\sum_j \frac{V'^{2Fj}\omega_{pj}^2 k_x k_z}{F\omega^2(\omega^2 G - \omega_{cj}^2)} & -\sum_j \frac{\iota\omega_{pj}^2\omega_{cj} V'^{2Fj} k_x k_z}{F\omega^3(\omega^2 G - \omega_{cj}^2)} & 1 - \sum_j \frac{\omega_{pj}^2}{\omega^2 F}(1 + \frac{V'^{4Fj} k_x^2 k_z^2}{\omega^2 F(\omega^2 G - \omega_{cj}^2)}) \end{pmatrix}. \tag{3.103}$$

As three species are involved that are electrons, ions and dust so $\sum_j = \sum_e + \sum_i + \sum_d$.

The term G will only exist for "e" as they are the only fermi particles involved, so we can write

$$\epsilon_{xx} = 1 - \frac{\omega_{pe}^2}{\omega^2 G - \omega_{ce}^2} - \frac{\omega_{pi}^2}{\omega^2 - \omega_{ci}^2} - \frac{\omega_{pd}^2}{\omega^2 - \omega_{cd}^2}, \tag{3.104}$$

$$\epsilon_{xy} = -\frac{\iota\omega_{pe}^2\omega_{ce}}{\omega(\omega^2 G - \omega_{ce}^2)} - \frac{\iota\omega_{pi}^2\omega_{ci}}{\omega(\omega^2 - \omega_{ci}^2)} - \frac{\iota\omega_{pd}^2\omega_{cd}}{\omega(\omega^2 - \omega_{cd}^2)}, \tag{3.105}$$

$$\epsilon_{xz} = -\frac{V'^{2_{Fj}}\omega_{pe}^2 k_x k_z}{(\omega^2 G - \omega_{ce}^2)F\omega^2}, \tag{3.106}$$

$$\epsilon_{yx} = \frac{\iota\omega_{pe}^2\omega_{ce}}{\omega(\omega^2 G - \omega_{ce}^2)} + \frac{\iota\omega_{pi}^2\omega_{ci}}{\omega(\omega^2 - \omega_{ci}^2)} + \frac{\iota\omega_{pd}^2\omega_{cd}}{\omega(\omega^2 - \omega_{cd}^2)}, \tag{3.107}$$

$\epsilon_{yx} = -\epsilon_{xy}$

$$\epsilon_{yy} = 1 - \frac{\omega_{pe}^2 G}{\omega^2 G - \omega_{ce}^2} - \frac{\omega_{pi}^2}{\omega^2 - \omega_{ci}^2} - \frac{\omega_{pd}^2}{\omega^2 - \omega_{cd}^2}, \tag{3.108}$$

$$\epsilon_{yz} = \frac{\iota\omega_{pe}^2\omega_{ce}V'2_{Fe}k_x k_z}{\omega^3 F(\omega^2 G - \omega_{ce}^2)}, \tag{3.109}$$

$$\epsilon_{zx} = -\frac{V'2_{Fj}\omega_{pe}^2 k_x k_z}{F\omega^2(\omega^2 G - \omega_{ce}^2)}, \tag{3.110}$$

$\epsilon_{zx} = -\epsilon_{xz}$

$$\epsilon_{zy} = \frac{\iota\omega_{pe}^2\omega_{ce}V'^{2_{Fj}}k_x k_z}{F\omega^3(\omega^2 G - \omega_{ce}^2)}, \tag{3.111}$$

$\epsilon_{zy} = -\epsilon_{yz}$

and

$$\epsilon_{zz} = 1 - \frac{\omega_{pe}^2}{\omega^2 F}\left(1 + \frac{V'4_{Fe}k_x^2 k_z^2}{\omega^2 F(\omega^2 G - \omega_{ce}^2)}\right) - \frac{\omega_{pi}^2}{\omega^2} - \frac{\omega_{pd}^2}{\omega^2}. \tag{3.112}$$

In oblique shear Alfven waves both k and "E" are in same plane and therefore $E_y = 0$. Therefore low frequency long wavelength mode in β plasma, the non-diagonal components of dielectric tensor becomes negligibly small and therefore we can write,

$$Det \mid \underline{\underline{\mathbf{D}}} \mid= Det \begin{pmatrix} k_z^2 - \frac{\omega^2}{c^2}\epsilon_{xx} & -k_x k_z - \frac{\omega^2}{c^2}\epsilon_{xz} \\ -k_z k_x - \frac{\omega^2}{c^2}\epsilon_{zx} & k_x^2 - \frac{\omega^2}{c^2}\epsilon_{zz} \end{pmatrix} = 0 \tag{3.113}$$

or we can write it as;

$$\mid \begin{pmatrix} k_z^2 - \frac{\omega^2}{c^2}\epsilon_{xx} & -k_x k_z - \frac{\omega^2}{c^2}\epsilon_{xz} \\ -k_z k_x - \frac{\omega^2}{c^2}\epsilon_{zx} & k_x^2 - \frac{\omega^2}{c^2}\epsilon_{zz} \end{pmatrix} \mid= 0 \tag{3.114}$$

$$(k_z^2 - \frac{\omega^2}{c^2}\epsilon_{xx})(k_x^2 - \frac{\omega^2}{c^2}\epsilon_{zz}) - (-k_x k_z - \frac{\omega^2}{c^2}\epsilon_{xz})(-k_z k_x - \frac{\omega^2}{c^2}\epsilon_{zx}) = 0 \tag{3.115}$$

$$k_z^2 k_x^2 - \frac{\omega^2}{c^2}k_z^2\epsilon_{zz} - \frac{\omega^2}{c^2}k_x^2\epsilon_{xx} + \frac{\omega^4}{c^4}\epsilon_{xx}\epsilon_{zz} - k_z^2 k_x^2 - k_z k_x\frac{\omega^2}{c^2}\epsilon_{xz} - k_x k_z\frac{\omega^2}{c^2}\epsilon_{zx} - \frac{\omega^4}{c^4}\epsilon_{xz}\epsilon_{zx} = 0 \tag{3.116}$$

$$\frac{\omega^2}{c^2}[-k_z^2\epsilon_{zz} - k_x^2\epsilon_{xx} + \frac{\omega^2}{c^2}\epsilon_{xx}\epsilon_{zz} - \frac{\omega^2}{c^2}\epsilon_{xz}\epsilon_{zx} - k_x k_z\epsilon_{xz} - k_x k_z\epsilon_{zx}] = 0 \tag{3.117}$$

$$-k_z^2\epsilon_{zz} - k_x^2\epsilon_{xx} + \frac{\omega^2}{c^2}\epsilon_{xx}\epsilon_{zz} - \frac{\omega^2}{c^2}\epsilon_{xz}\epsilon_{zx} - k_x k_z\epsilon_{xz} - k_x k_z\epsilon_{zx} = 0 \tag{3.118}$$

as $\epsilon_{zx} = \epsilon_{xz}$

$$-k_z^2\epsilon_{zz} - k_x^2\epsilon_{xx} + \frac{\omega^2}{c^2}(\epsilon_{xx}\epsilon_{zz} - \epsilon_{xz}^2) - 2k_x k_z\epsilon_{xz} = 0 \tag{3.119}$$

$$\omega^2(\epsilon_{xx}\epsilon_{zz} - \epsilon_{xz}^2) - c^2k_z^2\epsilon_{zz} - c^2k_x^2\epsilon_{xx} - 2c^2k_x k_z\epsilon_{xz} = 0 \tag{3.120}$$

3.0.1 MAGNETIZED DUST

Firstly we will consider the case in which the inertia of electron is ignored as compared to ions and dust particles. Dust particles are considered magnetized as they are highly charged then the frequency ranges are considered as:

$$\omega^2 \ll \omega_{cd}^2, \tag{3.121}$$

$$\omega_{cd}^2 \ll \omega_{ci}^2, \tag{3.122}$$

$$\omega^2 \ll V'^{2_{Fj}} k_z^2, \tag{3.123}$$

and

$$\omega^2 \ll \omega_{ci}^2. \tag{3.124}$$

After using these conditions the simplified form of components of medium response function become:

$\epsilon_{xx} = 1 - \frac{\omega_{pe}^2}{\omega^2 G - \omega_{ce}^2} - \frac{\omega_{pi}^2}{\omega^2 - \omega_{ci}^2} - \frac{\omega_{pd}^2}{\omega^2 - \omega_{cd}^2}$ using condition $\omega^2 \ll \omega_{ci}^2$ and $\omega^2 \ll \omega_{cd}^2$

$$\epsilon_{xx} = 1 - \frac{\omega_{pe}^2}{\omega^2[\frac{\omega^2 - V'^{2_{Fe}}(k^2)}{(\omega^2 - V'^{2_{Fe}}(k_z^2))}] - \omega_{ce}^2} + \frac{\omega_{pi}^2}{\omega_{ci}^2} + \frac{\omega_{pd}^2}{\omega_{cd}^2} \tag{3.125}$$

as $k^2 = k_x^2 + k_z^2$ and $\omega^2 \ll V'^{2_{Fe}} k_z^2$ using these conditions in above the above relation yields:

$$\epsilon_{xx} = 1 - \frac{\omega_{pe}^2}{\omega^2[\frac{-V'^{2_{Fe}}(k_x^2) - V'^{2_{Fe}}(k_z^2)}{(-V'^{2_{Fe}}(k_z^2))}] - \omega_{ce}^2} + \frac{\omega_{pi}^2}{\omega_{ci}^2} + \frac{\omega_{pd}^2}{\omega_{cd}^2} \tag{3.126}$$

$$\epsilon_{xx} = 1 - \frac{\omega_{pe}^2}{[\frac{-\omega^2 V'2_{Fe}(k_x^2) - \omega^2 V'2_{Fe}(k_z^2)}{(-V'^{2_{Fe}}(k_z^2))}] - \omega_{ce}^2} + \frac{\omega_{pi}^2}{\omega_{ci}^2} + \frac{\omega_{pd}^2}{\omega_{cd}^2}. \tag{3.127}$$

As $k_z \gg k_x$, so we can neglect the term $\omega^2 V'^{2_{Fe}}(k_x^2)$.Also from $\omega^2 \ll \omega_{ci}^2$, we can infer that $\omega^2 \ll \omega_{ce}^2$. So above relation becomes:

$$\epsilon_{xx} = 1 + \frac{\omega_{pe}^2}{\omega_{ce}^2} + \frac{\omega_{pi}^2}{\omega_{ci}^2} + \frac{\omega_{pd}^2}{\omega_{cd}^2}. \tag{3.128}$$

Also;

$\epsilon_{xz} = -\frac{V'^{2_{Fe}} \omega_{pe}^2 k_x k_z}{F\omega^2(\omega^2 G - \omega_{ce}^2)}$

By using values of F and G in the above relation, we get

$$\epsilon_{xz} = -\frac{V'^{2_{Fe}} \omega_{pe}^2 k_x k_z}{(1 - \frac{V'^{2_{Fj}}}{\omega^2}(k_z^2))\omega^2(\omega^2[\frac{\omega^2 - V'^{2_{Fe}}(k^2)}{(\omega^2 - V'2_{Fe}(k_z^2))}] - \omega_{ce}^2)} \tag{3.129}$$

or

$$\epsilon_{xz} = -\frac{V'^{2}{}_{Fe}\omega_{pe}^2 k_x k_z}{\omega^2 - V'^{2}{}_{Fj}k_z^2)(\omega^2[\frac{\omega^2 - V'^{2}{}_{Fe}(k^2)}{(\omega^2 - V'^{2}{}_{Fe}(k_z^2))}] - \omega_{ce}^2)} \tag{3.130}$$

Using the privously mentioned conditions, we get

$$\epsilon_{xz} = -\frac{V'^{2}{}_{Fe}\omega_{pe}^2 k_x k_z}{-V'^{2}{}_{Fj}k_z^2([\frac{-\omega^2 V'^{2}{}_{Fe}(k_z^2) - \omega^2 V'^{2}{}_{Fe}(k_x^2)}{(-V'^{2}{}_{Fe}(k_z^2))}] - \omega_{ce}^2)}. \tag{3.131}$$

By using the same reasoning as above, we get

$$\epsilon_{xz} = -\frac{\omega_{pe}^2 k_x}{\omega_{ce}^2 k_z}. \tag{3.132}$$

Now zz-component of the tensoris given as $\epsilon_{zz} = 1 - \frac{\omega_{pe}^2}{\omega^2 F}(1 + \frac{V'^{4}{}_{Fe}k_x^2 k_z^2}{\omega^2 F(\omega^2 G - \omega_{ce}^2)}) - \frac{\omega_{pi}^2}{\omega^2} - \frac{\omega_{pd}^2}{\omega^2}$.

substituting the value of "F" and "G" in the above relation we get

$$\epsilon_{zz} = 1 - \frac{\omega_{pe}^2}{\omega^2(1 - \frac{V'^{2}{}_{Fj}}{\omega^2}(k_z^2))}(1 + \frac{V'^{4}{}_{Fe}k_x^2 k_z^2}{\omega^2(1 - \frac{V'^{2}{}_{Fe}}{\omega^2}(k_z^2))(\omega^2[\frac{\omega^2 - V'^{2}{}_{Fe}(k^2)}{(\omega^2 - V'^{2}{}_{Fe}(k_z^2))}] - \omega_{ce}^2)}) - \frac{\omega_{pi}^2}{\omega^2} - \frac{\omega_{pd}^2}{\omega^2} \tag{3.133}$$

After using the conditions, we finally get:

$$\epsilon_{zz} = 1 + \frac{\omega_{pe}^2}{V'^{2}{}_{Fe}(k_z^2)} + \frac{\omega_{pe}^2 k_x^2}{\omega_{ce}^2 k_z^2} - \frac{\omega_{pi}^2}{\omega^2} - \frac{\omega_{pd}^2}{\omega^2}. \tag{3.134}$$

Now by using values from equation (3.112), (3.116) and (3.118) in equation (3.103)

(3.103) is given by $\omega^2(\epsilon_{xx}\epsilon_{zz} - \epsilon_{xz}^2) - c^2k_z^2\epsilon_{zz} - c^2k_x^2\epsilon_{xx} - 2c^2k_xk_z\epsilon_{xz} = 0$ now using values will give us:

$$\begin{aligned}\omega^2((1 + \frac{\omega_{pe}^2}{\omega_{ce}^2} + \frac{\omega_{pi}^2}{\omega_{ci}^2} + \frac{\omega_{pd}^2}{\omega_{cd}^2})(1 + \frac{\omega_{pe}^2}{V'2_{Fe}(k_z^2)} + \frac{\omega_{pe}^2 k_x^2}{\omega_{ce}^2 k_z^2} - \frac{\omega_{pi}^2}{\omega^2} - \frac{\omega_{pd}^2}{\omega^2}) - (-\frac{\omega_{pe}^2 k_x}{\omega_{ce}^2 k_z})^2) - \\ c^2k_z^2(1 + \frac{\omega_{pe}^2}{V'2_{Fe}(k_z^2)} + \frac{\omega_{pe}^2 k_x^2}{\omega_{ce}^2 k_z^2} - \frac{\omega_{pi}^2}{\omega^2} - \frac{\omega_{pd}^2}{\omega^2}) - \\ c^2k_x^2(1 + \frac{\omega_{pe}^2}{\omega_{ce}^2} + \frac{\omega_{pi}^2}{\omega_{ci}^2} + \frac{\omega_{pd}^2}{\omega_{cd}^2}) - 2c^2k_xk_z(-\frac{\omega_{pe}^2 k_x}{\omega_{ce}^2 k_z}) = 0\end{aligned} \tag{3.135}$$

Now by taking $\frac{\omega_{pe}^2}{\omega_{ce}^2} = \frac{\rho_{Fe}^2}{\lambda_{DFe}^2}$ and $\frac{\omega_{pi}^2}{\omega_{ci}^2} + \frac{\omega_{pd}^2}{\omega_{cd}^2} = \frac{c^2}{V_{Ad}^2}$,

The above equation is modified as:

$$\omega^2((1+\frac{\rho_{Fe}^2}{\lambda_{DFe}^2}+\frac{c^2}{V_{Ad}^2})(1+\frac{1}{\lambda_{DFe}^2k_z^2}+\frac{\rho_{Fe}^2k_x^2}{\lambda_{DFe}^2k_z^2}-\frac{(\omega_{pi}^2+\omega_{pd}^2)}{\omega^2})-(\frac{\rho_{Fe}^4k_x^2}{\lambda_{DFe}^4k_z^2}))-$$
$$c^2k_z^2(1+\frac{1}{\lambda_{DFe}^2k_z^2}+\frac{\rho_{Fe}^2k_x^2}{\lambda_{DFe}^2k_z^2}-\frac{(\omega_{pi}^2+\omega_{pd}^2)}{\omega^2})-$$
$$c^2k_x^2(1+\frac{\rho_{Fe}^2}{\lambda_{DFe}^2}+\frac{c^2}{V_{Ad}^2})+2c^2k_xk_z\frac{\rho_{Fe}^2k_x^2}{\lambda_{DFe}^2k_z^2}=0. \quad (3.136)$$

After simplification we get:

$$\omega^2+\frac{\omega^2}{\lambda_{DFe}^2k_z^2}+\frac{\omega^2\rho_{Fe}^2k_x^2}{\lambda_{DFe}^2k_z^2}-(\omega_{pi}^2+\omega_{pd}^2)+\frac{\omega^2\rho_{Fe}^2}{\lambda_{DFe}^2}+\frac{\omega^2\rho_{Fe}^2}{\lambda_{DFe}^4k_z^2}$$
$$+\frac{\omega^2\rho_{Fe}^4k_x^2}{\lambda_{DFe}^4k_z^2}-\frac{\rho_{Fe}^2}{\lambda_{DFe}^2}(\omega_{pi}^2+\omega_{pd}^2)+\frac{\omega^2c^2}{V_{Ad}^2}+\frac{\omega^2c^2}{V_{Ad}^2\lambda_{DFe}^2k_z^2}+\frac{\omega^2c^2\rho_{Fe}^2k_x^2}{V_{Ad}^2\lambda_{DFe}^2k_z^2}$$
$$-\frac{c^2}{V_{Ad}^2}(\omega_{pi}^2+\omega_{pd}^2)-\frac{\omega^2\rho_{Fe}^4k_x^2}{\lambda_{DFe}^4k_z^2}-c^2k_z^2-\frac{c^2}{\lambda_{DFe}^2}-\frac{c^2\rho_{Fe}^2k_x^2}{\lambda_{DFe}^2}+\frac{c^2k_z^2}{\omega^2}(\omega_{pi}^2+\omega_{pd}^2)$$
$$-c^2k_x^2-\frac{c^2\rho_{Fe}^2k_x^2}{\lambda_{DFe}^2}-\frac{c^4k_x^2}{V_{Ad}^2}+\frac{2c^2\rho_{Fe}^2k_x^2}{\lambda_{DFe}^2}=0. \quad (3.137)$$

After multiplying the above equation with $\frac{\omega^2\lambda_{DFe}^2V_{Ad}^2k_z^2}{c^2}$ and neglecting some terms as $k_x \ll k_z$

The above equation reduces to:

$$\omega^4(1+\lambda_{DFe}^2k_z^2+\rho_{Fe}^2k_x^2-\frac{V_{Ad}^2}{c^2}(\rho_{Fe}^2k_x^2)\frac{\rho_{Fe}^2}{\lambda_{DFe}^2})+$$
$$\omega^2(-(V_{Ad}^2k_z^2)(1+\lambda_{DFe}^2k_z^2-\rho_{Fe}^2k_x^2)+\lambda_{DFe}^2k_z^2(\omega_{pi}^2+\omega_{pd}^2+c^2k_x^2))$$
$$+\lambda_{DFe}^2V_{Ad}^2k_z^4(\omega_{pi}^2+\omega_{pd}^2)=0. \quad (3.138)$$

For $\lambda_{DFe}^2k_z^2 \ll 1$, $\rho_{Fe}^2k_x^2 \ll 1$, $V_{Ad}^2 \ll c^2$, $\omega_{pd}^2 \ll \omega_{pi}^2$ the simplified form of coefficients "a", "b" and "c" can be written as :

$a \approx 1$

$b \approx -V_{Ad}^2k_z^2+\lambda_{DFe}^2k_z^2(\omega_{pi}^2+c^2k_x^2)$

$c \approx V_{Ad}^2\lambda_{DFe}^2k_z^4\omega_{pi}^2$

Equation (1.138) can be solved by using the quadratic formula and also implanting the above conditions, results will give quantum modified relation of dust shear Alfven waves. From quadratic formula,

$$\omega^2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{3.139}$$

$$\omega^2 = \frac{(V_{Ad}^2 k_z^2 + \lambda_{DFe}^2 k_z^2(\omega_{pi}^2 + c^2 k_x^2)) \pm \sqrt{(V_{Ad}^2 k_z^2 + \lambda_{DFe}^2 k_z^2(\omega_{pi}^2 + c^2 k_x^2))^2 - 4(1)(V_{Ad}^2 \lambda_{DFe}^2 k_z^4 \omega_{pi}^2)}}{2(1)} \tag{3.140}$$

$$\omega^2 = \frac{k_z^2(V_{Ad}^2 + \lambda_{DFe}^2(\omega_{pi}^2 + c^2 k_x^2)) \pm [V_{Ad}^4 k_z^4 + \lambda_{DFe}^4 k_z^4(\omega_{pi}^2 + c^2 k_x^2)^2 + 2V_{Ad}^2 \lambda_{DFe}^2 k_z^4(\omega_{pi}^2 + c^2 k_x^2) - 4V_{Ad}^2 \lambda_{DFe}^2 k_z^4 \omega \ldots}{2} \tag{3.141}$$

$$\omega^2 = \frac{k_z^2(V_{Ad}^2 + \lambda_{DFe}^2(\omega_{pi}^2 + c^2 k_x^2)) \pm [V_{Ad}^4 k_z^4 + \lambda_{DFe}^4 k_z^4(\omega_{pi}^4 + c^4 k_x^4 + 2\omega_{pi}^2 c^2 k_x^2) + 2V_{Ad}^2 \lambda_{DFe}^2 k_z^4 \omega_{pi}^2 + 2V_{Ad}^2 \lambda_{DFe} \ldots}{2}$$

$$\omega^2 = \frac{k_z^2(V_{Ad}^2 + \lambda_{DFe}^2(\omega_{pi}^2 + c^2 k_x^2)) \pm [V_{Ad}^4 k_z^4 + \lambda_{DFe}^4 k_z^4 \omega_{pi}^4 + \lambda_{DFe}^4 k_z^4 c^4 k_x^4 + 2\lambda_{DFe}^4 k_z^4 \omega_{pi}^2 c^2 k_x^2 + 2V_{Ad}^2 \lambda_{DFe}^2 k_z^4 \omega_{pi}^2 \ldots}{2}$$

After completing the square and cancelling the terms we get;

$$\omega^2 = \frac{k_z^2(V_{Ad}^2 + \lambda_{DFe}^2(\omega_{pi}^2 + c^2 k_x^2)) \pm [V_{Ad}^4 k_z^4 + \lambda_{DFe}^4 k_z^4(c^2 k_x^2 - \omega_{pi}^2)^2 + 2V_{Ad}^2 \lambda_{DFe}^2 k_z^4(c^2 k_x^2 - \omega_{pi}^2) + 4\omega_{pi}^2 c^2 k_x^2 \lambda_{D \ldots}^2}{2}$$

$$\omega^2 = \frac{k_z^2(V_{Ad}^2 + \lambda_{DFe}^2(\omega_{pi}^2 + c^2 k_x^2)) \pm [k_z^4(V_{Ad}^2 + \lambda_{DFe}^2(c^2 k_x^2 - \omega_{pi}^2))^2 + 4\omega_{pi}^2 c^2 k_x^2 \lambda_{DFe}^2 k_z^4]^{\frac{1}{2}}}{2}. \tag{3.145}$$

Now for the real frequency;

$$\omega^2 = \frac{k_z^2(V_{Ad}^2 + \lambda_{DFe}^2(\omega_{pi}^2 + c^2 k_x^2)) + [k_z^4(V_{Ad}^2 + \lambda_{DFe}^2(c^2 k_x^2 - \omega_{pi}^2))^2 + 4\omega_{pi}^2 c^2 k_x^2 \lambda_{DFe}^2 k_z^4]^{\frac{1}{2}}}{2} \tag{3.146}$$

as we are considering that $\lambda_{DFe}^2 \ll 1$ so the term in discriminant can be neglected and simplified dispersion relation can be obtained as:

$$\omega^2 = \frac{k_z^2(V_{Ad}^2 + \lambda_{DFe}^2(\omega_{pi}^2 + c^2k_x^2)) + [k_z^4(V_{Ad}^2 + \lambda_{DFe}^2(c^2k_x^2 - \omega_{pi}^2))^2]^{\frac{1}{2}}}{2} \quad (3.147)$$

$$\omega^2 = \frac{k_z^2(V_{Ad}^2 + \lambda_{DFe}^2(\omega_{pi}^2 + c^2k_x^2)) + [k_z^2(V_{Ad}^2 + \lambda_{DFe}^2(c^2k_x^2 - \omega_{pi}^2))]}{2} \quad (3.148)$$

$$\omega^2 = \frac{k_z^2V_{Ad}^2 + k_z^2\lambda_{DFe}^2\omega_{pi}^2 + k_z^2\lambda_{DFe}^2c^2k_x^2 + k_z^2V_{Ad}^2 + k_z^2\lambda_{DFe}^2c^2k_x^2 - k_z^2\lambda_{DFe}^2\omega_{pi}^2}{2} \quad (3.149)$$

After canceling the terms we get:

$$\omega^2 = \frac{2k_z^2V_{Ad}^2 + 2k_z^2\lambda_{DFe}^2c^2k_x^2}{2} \quad (3.150)$$

$$\omega^2 = k_z^2V_{Ad}^2 + k_z^2\lambda_{DFe}^2c^2k_x^2 \quad (3.151)$$

For electromagnetic waves, quantum effects depend upon the geometry and do not play any role for parallel propagation. So the standard dispersion relation can be written as:

$$\omega^2 = k_z^2V_{Ad}^2 \quad (3.152)$$

or

$$\omega = kV_{Ad} \quad (3.153)$$

For general K if we choose $\hbar = 0$ or $T_{Fe} = 0$ we get;

$$\gamma_j = \frac{\hbar^2k^2}{4m_j^2V_{Fj}^2} = 0 \quad (3.154)$$

Also as; $V'_{Fj} = V_{Fj}(1+\gamma_j)^{\frac{1}{2}}$

$$V'_{Fj} = V_{Fj}(1+0)^{\frac{1}{2}} \quad (3.155)$$

$$V'_{Fj} = V_{Fj} \tag{3.156}$$

or

$$V'_{Fe} = V_{Fe} \tag{3.157}$$

Also replacing λ_{DFe} by λ_{De} and ρ_{Fe} by ρ_e

now equation (3.135) can be written as:

$$\omega^2 = k_z^2 V_{Ad}^2 [1 + \frac{c^2 k_x^2 \lambda_{DFe}^2}{V_{Ad}^2}]. \tag{3.158}$$

By taking $\rho_{sd}^2 = (\frac{c^2}{V_{Ad}^2})\lambda_{DFe}^2$, our final result become,

$$\omega^2 = k_z^2 V_{Ad}^2 [1 + \rho_{sd}^2 k_x^2]. \tag{3.159}$$

3.0.2 UNMAGNETIZED DUST

Now we will choose the frequency range $\omega_{cd}^2 \ll \omega^2 \ll \omega_{ci}^2$ and $\omega^2 \ll V'2_{Fe} k_z^2$. Using these conditions components of response function comes out to be, as,

$\epsilon_{xz} = -\frac{V'2_{Fe}\omega_{pe}^2 k_x k_z}{F\omega^2(\omega^2 G - \omega_{ce}^2)}$.

Using the values of "F" and "G", we get

$$\epsilon_{xz} = -\frac{V'2_{Fe}\omega_{pe}^2 k_x k_z}{(1 - \frac{V'2_{Fj}}{\omega^2}(k_z^2))\omega^2(\omega^2[\frac{\omega^2 - V'2_{Fe}(k^2)}{(\omega^2 - V'2_{Fe}(k_z^2))}] - \omega_{ce}^2)} \tag{3.160}$$

or

$$\epsilon_{xz} = -\frac{V'2_{Fe}\omega_{pe}^2 k_x k_z}{\omega^2 - V'2_{Fj} k_z^2)(\omega^2[\frac{\omega^2 - V'2_{Fe}(k^2)}{(\omega^2 - V'2_{Fe}(k_z^2))}] - \omega_{ce}^2)} \tag{3.161}$$

After using conditions: $\omega^2 \ll V'2_{Fe} k_z^2$ and $k_x^2 \ll k_z^2$ we are left with,

$$\epsilon_{xz} = -\frac{V'2_{Fe}\omega_{pe}^2 k_x k_z}{-V'2_{Fj} k_z^2(\omega^2 - \omega_{ce}^2)}, \tag{3.162}$$

or

$$\epsilon_{xz} = -\frac{\omega_{pe}^2 k_x}{(\omega^2 - \omega_{ce}^2)k_z} \tag{3.163}$$

Using condition $\omega^2 \ll \omega_{ce}^2$ in the above expression, we get

$$\epsilon_{xz} = -\frac{\omega_{pe}^2 k_x}{(\omega_{ce}^2)k_z}. \tag{3.164}$$

Also, $\epsilon_{xx} = 1 - \frac{\omega_{pe}^2}{\omega^2 G - \omega_{ce}^2} - \frac{\omega_{pi}^2}{\omega^2 - \omega_{ci}^2} - \frac{\omega_{pd}^2}{\omega^2 - \omega_{cd}^2}$ using condition $\omega^2 \ll \omega_{ci}^2$ and $\omega_{cd}^2 \ll \omega^2$

$$\epsilon_{xx} = 1 - \frac{\omega_{pe}^2}{\omega^2[\frac{\omega^2 - V'2_{Fe}(k^2)}{(\omega^2 - V'2_{Fe}(k_z^2))}] - \omega_{ce}^2} + \frac{\omega_{pi}^2}{\omega_{ci}^2} + \frac{\omega_{pd}^2}{\omega^2}. \tag{3.165}$$

as $k^2 = k_x^2 + k_z^2$ and $\omega^2 \ll V'^{2_{Fe}} k_z^2$ the above relation becomes

$$\epsilon_{xx} = 1 - \frac{\omega_{pe}^2}{\omega^2[\frac{\omega^2 - V'^2{}_{Fe}(k_x^2) - V'^2{}_{Fe}(k_z^2)}{(-V'^2{}_{Fe}(k_z^2))}] - \omega_{ce}^2} + \frac{\omega_{pi}^2}{\omega_{ci}^2} + \frac{\omega_{pd}^2}{\omega^2} \tag{3.166}$$

$$\epsilon_{xx} = 1 - \frac{\omega_{pe}^2}{\omega^2[\frac{-V'^2{}_{Fe}(k_x^2) - V'^2{}_{Fe}(k_z^2)}{(-V'^2{}_{Fe}(k_z^2))}] - \omega_{ce}^2} + \frac{\omega_{pi}^2}{\omega_{ci}^2} + \frac{\omega_{pd}^2}{\omega^2} \tag{3.167}$$

$$\epsilon_{xx} = 1 - \frac{\omega_{pe}^2}{[\frac{-\omega^2 V'^2{}_{Fe}(k_x^2) - \omega^2 V'^2{}_{Fe}(k_z^2)}{(-V'^2{}_{Fe}(k_z^2))}] - \omega_{ce}^2} + \frac{\omega_{pi}^2}{\omega_{ci}^2} + \frac{\omega_{pd}^2}{\omega^2} \tag{3.168}$$

as $k_z \gg k_x$ so we can neglect the term $\omega^2 V'2_{Fe}(k_x^2)$. Also from $\omega^2 \ll \omega_{ci}^2$, we can infer that $\omega^2 \ll \omega_{ce}^2$ so the above relation now becomes:

$$\epsilon_{xx} = 1 + \frac{\omega_{pe}^2}{\omega_{ce}^2} + \frac{\omega_{pi}^2}{\omega_{ci}^2} + \frac{\omega_{pd}^2}{\omega^2}. \tag{3.169}$$

Now zz-component of the dielectric tensor is given as:
$\epsilon_{zz} = 1 - \frac{\omega_{pe}^2}{\omega^2 F}(1 + \frac{V'^{4_{Fe}} k_x^2 k_z^2}{\omega^2 F(\omega^2 G - \omega_{ce}^2)}) - \frac{\omega_{pi}^2}{\omega^2} - \frac{\omega_{pd}^2}{\omega^2}$,

$$\epsilon_{zz} = 1 - \frac{\omega_{pe}^2}{\omega^2(1 - \frac{V'^2{}_{Fe}}{\omega^2}(k_z^2))}(1 + \frac{V'^4{}_{Fe} k_x^2 k_z^2}{\omega^2(1 - \frac{V'^2{}_{Fe}}{\omega^2}(k_z^2))(\omega^2[\frac{\omega^2 - V'^2{}_{Fe}(k^2)}{(\omega^2 - V'^2{}_{Fe}(k_z^2))}] - \omega_{ce}^2)}) - \frac{\omega_{pi}^2}{\omega^2} - \frac{\omega_{pd}^2}{\omega^2} \tag{3.170}$$

After using conditions, we are left with :

$$\epsilon_{zz} = 1 + \frac{\omega_{pe}^2}{V'^2_{Fe}(k_z^2)} + \frac{\omega_{pe}^2 k_x^2}{\omega_{ce}^2 k_z^2} - \frac{\omega_{pi}^2}{\omega^2} - \frac{\omega_{pd}^2}{\omega^2}. \tag{3.171}$$

Now by using values from eq. (3.164), (3.169) and (3.171) in eq. (3.120), we get

$$\omega^2((1+\frac{\omega_{pe}^2}{\omega_{ce}^2}+\frac{\omega_{pi}^2}{\omega_{ci}^2}-\frac{\omega_{pd}^2}{\omega^2})(1+\frac{\omega_{pe}^2}{V'^{2}{}_{Fe}(k_z^2)}+\frac{\omega_{pe}^2k_x^2}{\omega_{ce}^2k_z^2}-\frac{\omega_{pi}^2}{\omega^2}-\frac{\omega_{pd}^2}{\omega^2})-(-\frac{\omega_{pe}^2k_x}{\omega_{ce}^2k_z})^2)-$$
$$c^2k_z^2(1+\frac{\omega_{pe}^2}{V'^{2}{}_{Fe}(k_z^2)}+\frac{\omega_{pe}^2k_x^2}{\omega_{ce}^2k_z^2}-\frac{\omega_{pi}^2}{\omega^2}-\frac{\omega_{pd}^2}{\omega^2})-$$
$$c^2k_x^2(1+\frac{\omega_{pe}^2}{\omega_{ce}^2}+\frac{\omega_{pi}^2}{\omega_{ci}^2}-\frac{\omega_{pd}^2}{\omega^2})-2c^2k_xk_z(-\frac{\omega_{pe}^2k_x}{\omega_{ce}^2k_z})=0. \quad (3.172)$$

Now taking $\frac{\omega_{pe}^2}{\omega_{ce}^2}=\frac{\rho_{Fe}^2}{\lambda_{DFe}^2}$ and $\frac{\omega_{pi}^2}{\omega_{ci}^2}+\frac{\omega_{pd}^2}{\omega_{cd}^2}=\frac{c^2}{V_{Ad}^2}$,

the above equation is modified as:

$$\omega^2((1+\frac{\rho_{Fe}^2}{\lambda_{DFe}^2}+\frac{c^2}{V_{Ad}^2}-\frac{\omega_{pd}^2}{\omega^2})(1+\frac{1}{\lambda_{DFe}^2k_z^2}+\frac{\rho_{Fe}^2k_x^2}{\lambda_{DFe}^2k_z^2}-\frac{(\omega_{pi}^2+\omega_{pd}^2)}{\omega^2})-(\frac{\rho_{Fe}^4k_x^2}{\lambda_{DFe}^4k_z^2}))-$$
$$c^2k_z^2(1+\frac{1}{\lambda_{DFe}^2k_z^2}+\frac{\rho_{Fe}^2k_x^2}{\lambda_{DFe}^2k_z^2}-\frac{(\omega_{pi}^2+\omega_{pd}^2)}{\omega^2})-$$
$$c^2k_x^2(1+\frac{\rho_{Fe}^2}{\lambda_{DFe}^2}+\frac{c^2}{V_{Ad}^2}-\frac{\omega_{pd}^2}{\omega^2})+2c^2k_xk_z\frac{\rho_{Fe}^2k_x^2}{\lambda_{DFe}^2k_z^2}=0 \quad (3.173)$$

After simplification we get:

$$\omega^2+\frac{\omega^2}{\lambda_{DFe}^2k_z^2}+\frac{\omega^2\rho_{Fe}^2k_x^2}{\lambda_{DFe}^2k_z^2}-(\omega_{pi}^2+\omega_{pd}^2)+\frac{\omega^2\rho_{Fe}^2}{\lambda_{DFe}^2}+\frac{\omega^2\rho_{Fe}^2}{\lambda_{DFe}^4k_z^2}$$
$$+\frac{\omega^2\rho_{Fe}^4k_x^2}{\lambda_{DFe}^4k_z^2}-\frac{\rho_{Fe}^2}{\lambda_{DFe}^2}(\omega_{pi}^2+\omega_{pd}^2)+\frac{\omega^2c^2}{V_A^2}+\frac{\omega^2c^2}{V_A^2\lambda_{DFe}^2k_z^2}+\frac{\omega^2c^2\rho_{Fe}^2k_x^2}{V_A^2\lambda_{DFe}^2k_z^2}$$
$$-\frac{c^2}{V_A^2}(\omega_{pi}^2+\omega_{pd}^2)-\omega_{pd}^2-\frac{\omega_{pd}^2}{\lambda_{DFe}^2k_z^2}-\frac{\omega_{pd}^2\rho_{Fe}^2k_x^2}{\lambda_{DFe}^2k_z^2}+\frac{\omega_{pd}^2}{\omega^2}(\omega_{pi}^2+\omega_{pd}^2)-$$
$$\frac{\omega^2\rho_{Fe}^4k_x^2}{\lambda_{DFe}^4k_z^2}-c^2k_z^2-\frac{c^2}{\lambda_{DFe}^2}-\frac{c^2\rho_{Fe}^2k_x^2}{\lambda_{DFe}^2}+\frac{c^2k_z^2}{\omega^2}(\omega_{pi}^2+\omega_{pd}^2)$$
$$-c^2k_x^2-\frac{c^2\rho_{Fe}^2k_x^2}{\lambda_{DFe}^2}-\frac{c^4k_x^2}{V_{Ad}^2}+\frac{c^2k_x^2\omega_{pd}^2}{\omega^2}+\frac{2c^2\rho_{Fe}^2k_x^2}{\lambda_{DFe}^2}=0. \quad (3.174)$$

After multiplying the above equation with $\frac{\omega^2\lambda_{DFe}^2V_{Ad}^2k_z^2}{c^2}$ and neglecting some terms as $k_x \ll k_z$ the above equation reduces to:

$$\omega^4((1+\frac{V_A^2}{c^2})1+\lambda_{DFe}^2k_z^2+\rho_{Fe}^2k_x^2-\frac{\omega_{pe}^2V_A^2}{\omega_{ce}^2c^2}(\rho_{Fe}^2k_x^2))+$$
$$\omega^2(-(V_A^2k_z^2)(1+\lambda_{DFe}^2k_z^2)+\omega_{dlh}^2(1+\lambda_{DFe}^2k_z^2+\rho_{Fe}^2k_x^2))$$
$$+c^2k_x^2\lambda_{DFe}^2k_z^2(1+\frac{V_A^2}{c^2}-\frac{V_A^2\rho_{Fe}^2}{c^2\lambda_{DFe}^2})+\lambda_{DFe}^2k_z^2\omega_{pi}^2(1+\frac{V_A^2}{c^2}+\frac{\omega_{pd}^2}{\omega_{pi}^2})+\lambda_{DFe}^2k_z^2\omega_{ci}^2\omega_{pd}^2+V_A^2k_z^2\omega_{pi}^2(1+\frac{\omega_{pd}^2}{\omega_{pi}^2}+$$
$$\frac{\omega_{pd}^2k_x^2}{\omega_{pi}^2k_z^2}+\frac{\omega_{pd}^4}{c^2k_z^2}\omega_{pi}^2)=0. \quad (3.175)$$

here,

$a=(1+\frac{V_A^2}{c^2})1+\lambda_{DFe}^2k_z^2+\rho_{Fe}^2k_x^2-\frac{\omega_{pe}^2V_A^2}{\omega_{ce}^2c^2}(\rho_{Fe}^2k_x^2)$,

$b=(-(V_A^2k_z^2)(1+\lambda_{DFe}^2k_z^2)+\omega_{dlh}^2(1+\lambda_{DFe}^2k_z^2+\rho_{Fe}^2k_x^2))$
$+c^2k_x^2\lambda_{DFe}^2k_z^2(1+\frac{V_A^2}{c^2}-\frac{V_A^2\rho_{Fe}^2}{c^2\lambda_{DFe}^2})+\lambda_{DFe}^2k_z^2\omega_{pi}^2(1+\frac{V_A^2}{c^2}+\frac{\omega_{pd}^2}{\omega_{pi}^2})$,

and

$c=\lambda_{DFe}^2k_z^2\omega_{ci}^2\omega_{pd}^2+V_A^2k_z^2\omega_{pi}^2(1+\frac{\omega_{pd}^2}{\omega_{pi}^2}+\frac{\omega_{pd}^2k_x^2}{\omega_{pi}^2k_z^2}+\frac{\omega_{pd}^4}{c^2k_z^2}\omega_{pi}^2)$.

For $\lambda_{DFe}^2k_z^2\ll 1$, $\rho_{Fe}^2k_x^2\ll 1$, $V_{Ad}^2\ll c^2$, $\omega_{cd}^2\ll\omega^2\ll\omega_{ci}^2$ the simplified form of coefficients "a", "b" and "c" can be written as :

$a\approx 1$,

$b\approx -[V_{Ad}^2k_z^2+\omega_{dlh}^2+c^2k_x^2\lambda_{DFe}^2k_z^2+\lambda_{DFe}^2k_z^2\omega_{pi}^2]$,

and

$c\approx\lambda_{DFe}^2k_z^2[\omega_{ci}^2\omega_{pd}^2+V_A^2k_z^2\omega_{pi}^2]$.

Equation (3.175) can be solved by using the quadratic formula and also implanting above conditions, results will give quantum modified relation of dust shear Alfven waves. From quadratic formula

$$\omega^2=\frac{-b\pm\sqrt{b^2-4ac}}{2a} \quad (3.176)$$

By substituting values of "a", "b" and "c",we get

$$\omega^2=\frac{(V_A^2k_z^2+\omega_{dlh}^2+\lambda_{DFe}^2k_z^2\omega_{pi}^2+\lambda_{DFe}^2k_z^2c^2k_x^2\pm\sqrt{(V_{Ad}^2k_z^2+\omega_{dlh}^2+\lambda_{DFe}^2k_z^2\omega_{pi}^2+\lambda_{DFe}^2k_z^2c^2k_x^2)^2-4(1)\lambda_{DFe}^2k_z^2\omega}}{2(1)} \quad (3.177)$$

$$\omega^2 = \frac{(V_{Ad}^2 k_z^2 + \omega_{dlh}^2 + \lambda_{DFe}^2 k_z^2 \omega_{pi}^2 + \lambda_{DFe}^2 k_z^2 c^2 k_x^2 \pm \sqrt{(V_A^4 k_z^4 + \omega_{dlh}^4 + \lambda_{DFe}^4 k_z^4 \omega_{pi}^4 + \lambda_{DFe}^4 k_z^4 c^4 k_x^4 + 2V_A^2 k_z^2 \omega_{dlh}^2 + 2}}{} \tag{3.178}$$

$$\omega^2 = \frac{(V_{Ad}^2 k_z^2 + \omega_{dlh}^2 + \lambda_{DFe}^2 k_z^2 \omega_{pi}^2 + \lambda_{DFe}^2 k_z^2 c^2 k_x^2 \pm \sqrt{(V_A^4 k_z^4 + \omega_{dlh}^4 + c^4 k_x^4 \lambda_{DFe}^4 k_z^4 + \lambda_{DFe}^4 k_z^4 \omega_{pi}^4 + 2V_A^2 k_z^2 \omega_{dlh}^2 + 2}}{} \tag{3.179}$$

After solving, we get,

$$\omega^2 = \frac{(V_A^2 k_z^2 + \omega_{dlh}^2 + \lambda_{DFe}^2 k_z^2 \omega_{pi}^2 + \lambda_{DFe}^2 k_z^2 c^2 k_x^2 \pm \sqrt{(V_A^2 k_z^2 + \omega_{dlh}^2 - \lambda_{DFe}^2 k_z^2 \omega_{pi}^2 + \lambda_{DFe}^2 k_z^2 c^2 k_x^2)^2 - 4c^2 k_x^2 \lambda_{DFe}^4 k_z^4}}{2(1)} \tag{3.180}$$

for real frequency and neglecting the last term as $\lambda_{DFe}^2 k_z^2 \ll 1$ we get,

$$\omega^2 = \frac{V_A^2 k_z^2 + \omega_{dlh}^2 + \lambda_{DFe}^2 k_z^2 \omega_{pi}^2 + \lambda_{DFe}^2 k_z^2 c^2 k_x^2 + V_A^2 k_z^2 + \omega_{dlh}^2 - \lambda_{DFe}^2 k_z^2 \omega_{pi}^2 + \lambda_{DFe}^2 k_z^2 c^2 k_x^2}{2(1)} \tag{3.181}$$

after cancelling the terms we get,

$$\omega^2 = \frac{2V_A^2 k_z^2 + 2\omega_{dlh}^2 + 2c^2 k_x^2 \lambda_{DFe}^2 k_z^2}{2} \tag{3.182}$$

$$\omega^2 = V_A^2 k_z^2 + \omega_{dlh}^2 + c^2 k_x^2 \lambda_{DFe}^2 k_z^2 \tag{3.183}$$

When quantum effect is eliminated by taking $k_x = 0$, then standard dispersion relation becomes,

$$\omega^2 = V_A^2 k_z^2 + \omega_{dlh}^2 \tag{3.184}$$

here $\omega_{dlh}^2 = \omega_{pd}^2 (\frac{\omega_{ci}^2}{\omega_{pi}^2})$ is the lower hybrid frequency.

For the dispersion relation of dust shear alfven wave in classical plasma, again considering eq.(1.83)

$$\omega^2 = V_A^2 k_z^2 + \omega_{dlh}^2 + c^2 k_x^2 \lambda_{DFe}^2 k_z^2 \tag{3.185}$$

$$\omega^2 = V_A^2 k_z^2 [1 + \frac{\omega_{dlh}^2}{V_A^2 k_z^2} + \frac{c^2 k_x^2 \lambda_{DFe}^2 k_z^2}{V_A^2 k_z^2}]. \tag{3.186}$$

After using the values of ω_{dlh}^2 and defining, we get $\rho_{sd}^2 = \frac{c^2}{V_A^2}(\lambda_{De}^2)$

$$\omega^2 = V_A^2 k_z^2 [1 + \rho_s^2 k_x^2 + \frac{\omega_{pd}^2}{c^2 k_z^2}]. \tag{3.187}$$

4. CONCLUSION

In this chapter, we will graphically represent dispersion relation of dust shear Alfven waves in quantum plasmas, We will plot ω vs k for eq. (3.151) and eq. (3.183). The typical parameter in interstellar and magnetospheric environments are $B_0 = 10^6 G$,$m_i = m_p$, $Z_d = \frac{(n_{oi} - n_{op})}{n_{op}}$.

In GRAPH 1 we have considered magnetized dust to be at $\theta = 5$. Also, $q_d = Z_{de} esu$, $n_{oe} = 10^{27} cm^{-3}$, $T_{Fe} = (3pi^2 n_o e)^{\frac{2}{3}} \frac{\hbar^2}{2m_e k_B}$,m_d$_{\overline{m_i = 10^9}}$, $n_{oi} = 1.001 \times 10^{27} cm^{-3}$, $n_{od} = 10^{-6} \times n_{oi} cm^{-3}$.

For magnetized dust $k = 0 - 3.25 cm^{-1}$ and for unmagnetized dust $k = 0 - 5.6 \times 10^4 cm^{-1}$, when there is no dust n_{oe}=n_{oi}, we will be using standard values of m_e and m_p, plank constant, Boltzmann constant, electron charge and ion charge in cgs system.

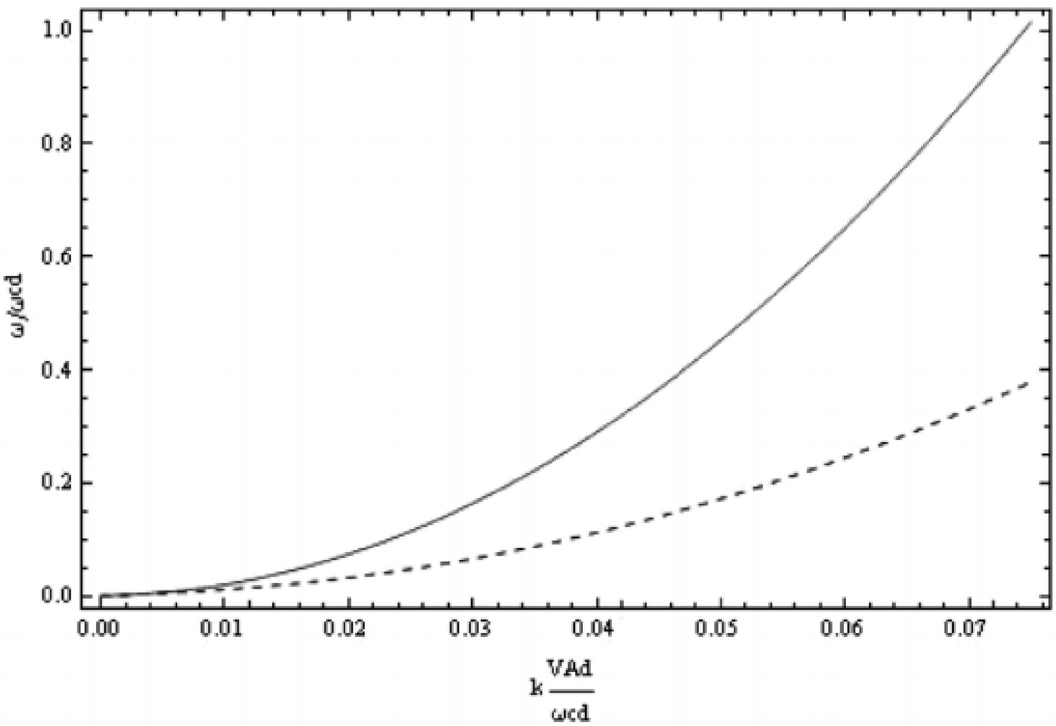


FIG. 1. ω versus k (case 1), for quantum (solid), nonquantum (dashed), and magnetized dust at $\theta = 5°$.

4.1 CASE 1

In $fig(1)$ the graph is plotted between ω and k for the case of magnetized dust. It shows the comparison of quantum and non-quantum dust shear Alfven waves. While considering the propagation of waves particles quantization has great importance. This fact is clarified from $fig(1)$. In dust shear Alfven wave in quantum plasma phase speed is much larger. Differences will increase for small wavelengths. The Alfven speed of ions is greater than the speed of dust particles due to the inertia, therefore phase speed of shear Alfven wave is smaller for dust and is larger for ion a quantum dusty magnetoplasma. Now an angle variation is made between the propagation vector and magnetic field. As we increase the angle, beyond some critical value, the wave goes to lower frequencies as shown in fig (2) and fig (3).

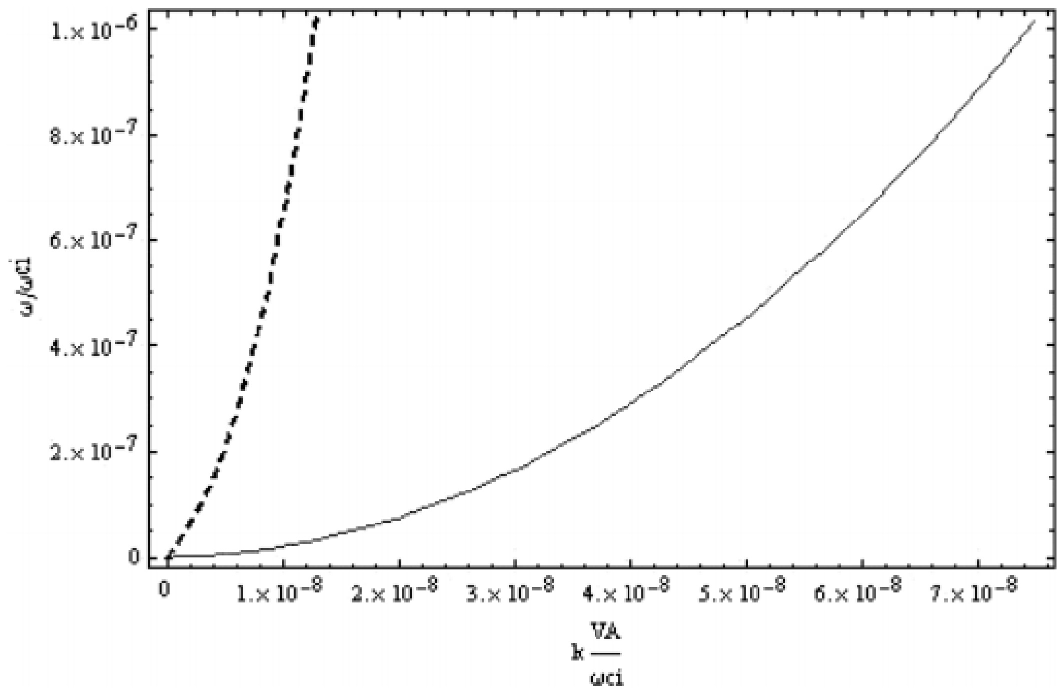


FIG. 2. ω versus k (case 1), the effect of dust (solid), and without dust (dashed) in a magnetized quantum plasmas at $\theta = 5^{\circ}$.

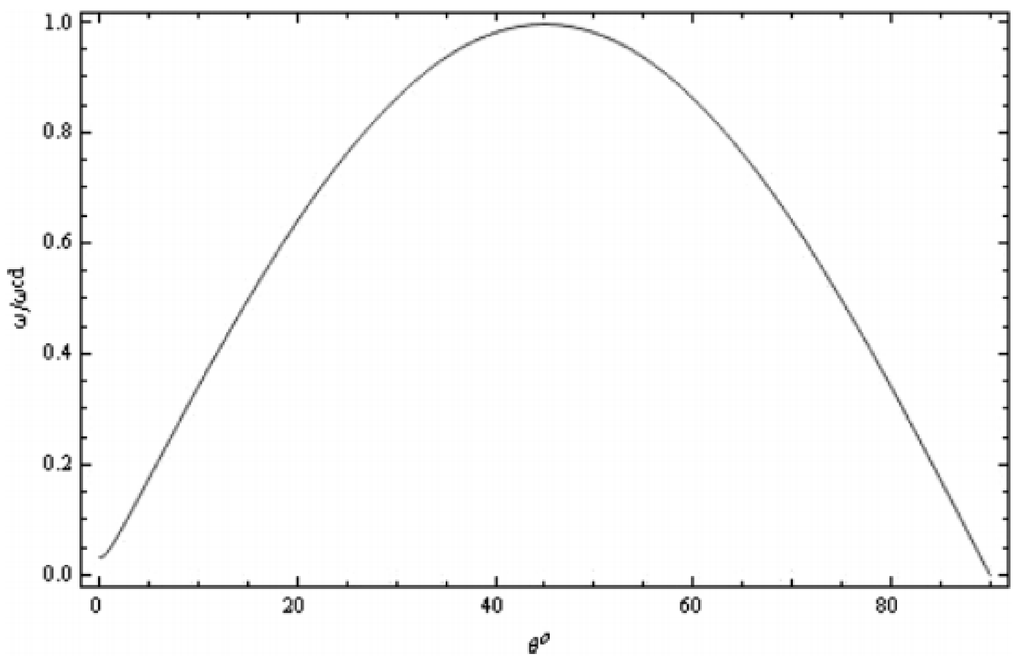


FIG. 3. ω versus k (case 1), variation of θ in a magnetized quantum dusty plasmas at $k = 1.36\,\mathrm{cm}^{-1}$.

4.2 CASE 2

In case of unmagnetized dust, Fig (4) elaborates the behavior of quantum modified shear Alfven waves when quantum effects are incorporated and in the presence of dust, frequency is enhanced at long wavelengths (smaller k). Similar response has been observed by quantized and no quantized medium.

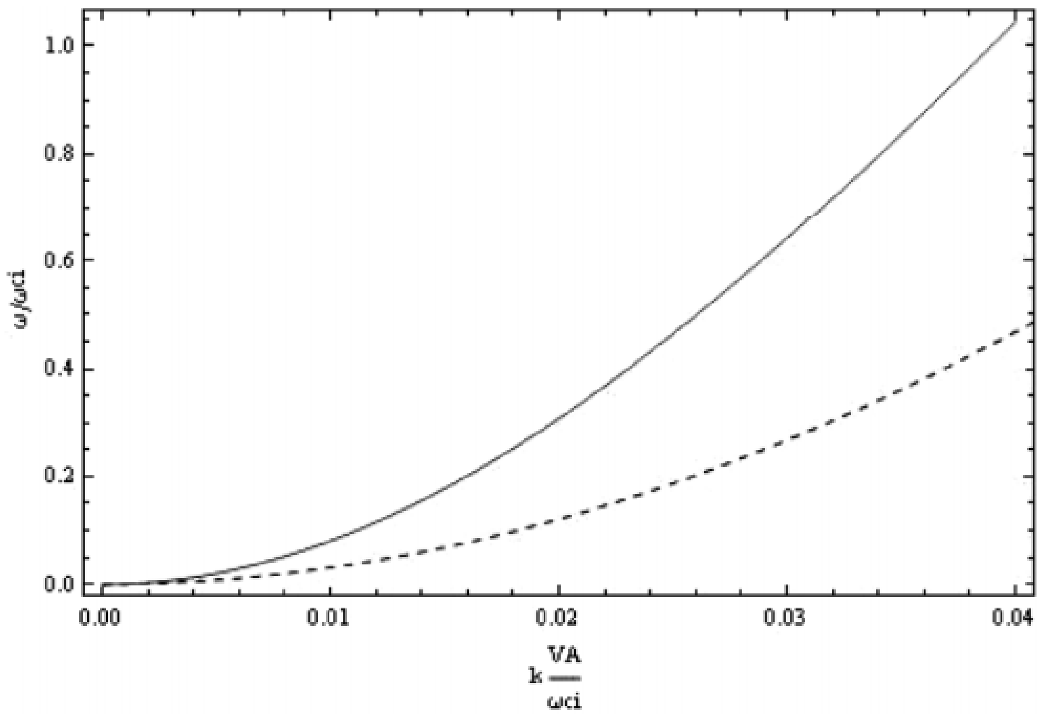


FIG. 4. ω versus k (case 2), for quantum (solid), nonquantum (dashed), and magnetized dust at $\theta = 5°$.

When small wavelengths are considered i.e. larger K value, quantum behaviour becomes significant. Fig (5) shows the effects of dust dynamics at large spatial scale lengths. Wave motion is also affected physically due to the presence of dust.

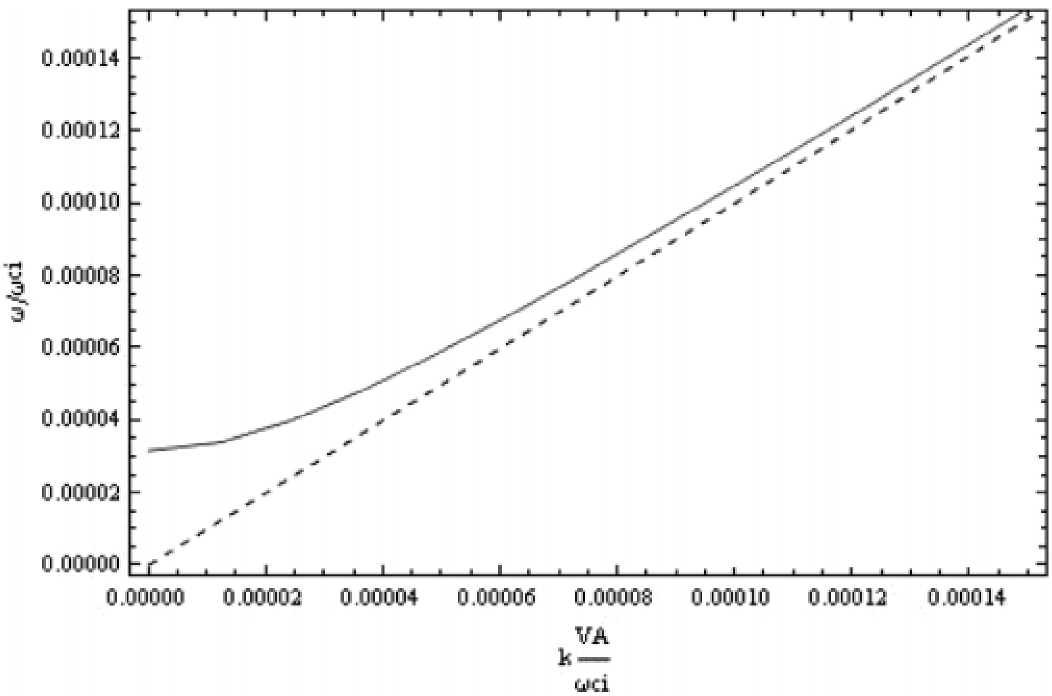


FIG. 5. ω versus k (case 2), with quantum effect, dust (solid) and without dust (dashed) at $\theta = 5°$.

At larger K i.e. small wavelengths dust effects are not visible and the wave does not observe the pressure of dust and therefore with or without dust medium will show the same behaviour.

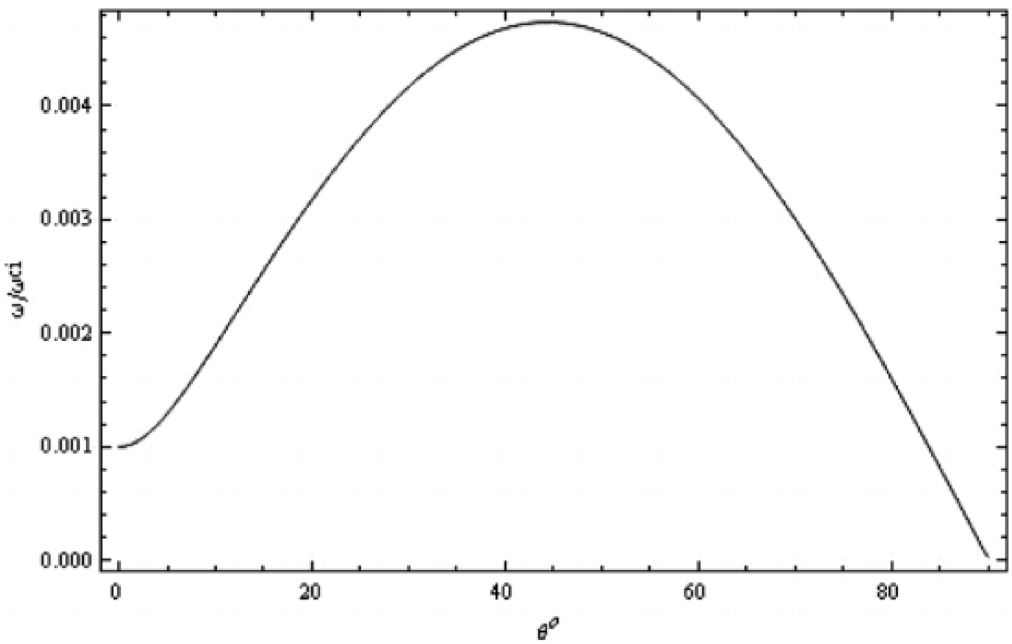


FIG. 6. ω versus k variation of θ in a magnetized quantum dusty plasmas at $k = 1.4 \times 10^3\,\text{cm}^{-1}$.

Fig(6) and fig(3) gives us the same description.

This dust modified shear Alfven waves is a natural mode of quantum dusty magnetoplasma which consists of electrons, ions, and dust in the presence of a static magnetic field.

As we have already considered that electrons are quantized and magnetized while ions are magnetized and dust particles are magnetized/unmagnetized in quantum plasmas.

As pure Alfven waves propagate parallel to a magnetic field and are not affected in quantized plasmas, but when low frequency shear Alfven waves propagate at small angles they get modified by quantum effects. In this thesis, we have investigated the electromagnetic shear Alfven waves in quantum dusty magnetoplasma. The new frequency involves the effects of magnetized, quantum electrons, magnetized ions, and dust particles, these conditions limit the propagation of electromagnetic waves in quantum dusty plasma environments in the presence of an ambient (static) magnetic field.

Parametric cascading of long-wavelength electromagnetic waves can produce electromagnetic noise in emission spectra from space plasma environments. Auroral Kilo-metric radiation and earthbound dust plasma clouds observed at earth polar summer meso may also be explained in terms of nonlinear interactions of large amplitudes low-frequency electromagnetic waves.

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