Modelling and Simulation of Gas Distribution Networks using Proper Orthogonal Decomposition

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Spring 2019-MS-CS&E 00000281793

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Declaration

I, *Shanzay khan* declare that the work provided in this thesis, titled "Modeling and Simulation of Gas Distribution Networks Using Proper Orthogonal Decomposition," is my own and was created as a result of my own original research.

I confirm that:

1. This work is completed entirely while pursuing a Master of Computational Science and Engineering degree at NUST.
2. This thesis has been explicitly indicated where any part of it has previously been submitted for a degree or other qualification at NUST or another institution.
3. When I consult other people's published work, I always make sure to credit them.
4. I always give the source when I quote from other people's work. This thesis is completely my own work, with the exception of such references.
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6. Where the thesis is based on work I did in collaboration with others, I've made it clear who did what and how much I contributed personally.

Shanzay Khan,
MS-CS&E-RCMS 00000281793

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This thesis is dedicated to my dearest parents, my siblings and my husband

Acknowledgments

I am grateful to Allah Subhana-Watala, my Creator, for guiding me through this task at every step and for every new concept You implanted in my head to help me improve it. Without Your invaluable assistance and direction, I would have been unable to do anything. It was Your desire that whomever assisted me over the duration of my thesis, whether it was my parents or any other individual, thus none be deserving of honour except You.

I am eternally grateful to my loving parents, who nurtured me from the time I was unable to walk and have continued to support me in every aspect of my life.

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Abbreviations

MOR	Model Order Reduction
POD	Proper Orthogonal Decomposition
ROM	Reduced Order Model
PDE	Partial Differential Equation
ODE	Ordinary Differential Equation
DAE	Differential Algebraic Equation
FVM	Finite Volume Method
FDM	Finite Difference Method
FOM	Full Order Model
LTI	Linear Time Invariant
CFD	Computational Fluid Dynamics
SVD	Singular Value Decomposition
VLSI	Very Large Scale Integration
DG	Directed Graph
DAC	Directed Acylic Graph
KLD	Karhunen-Loeve Decomposition

PCA Principal Component Analysis

SLP Sequential Linear Programming

Abstract

Gas distribution networks are systems with large pipelines, storage units, compressors, and many other devices such as regulators and valves. These networks cover broad geographical area and their analysis require large human resources, equipment and time which can lead to human/measurement device errors. To avoid these errors, one possibility is to analyse the network through modeling and simulation. This will require a mathematical model of the complete network and a computationally efficient simulation of the large scale complex network. In this thesis we explore fast simulation techniques for such complex models using model order reduction. The concept of model order reduction is to approximate large-scale dynamical systems effectively and efficiently into much smaller dimensions and produce nearly the same input/output characteristics. We observe the applicability of proper orthogonal decomposition (POD) based model order reduction on gas distribution network models as they are well used for nonlinear systems in the literature. A comparison between the original and the reduced models is made in terms of computational time and accuracy using different gas networks. Numerical analysis show that reduced order model is highly accurate, stable and takes lesser time to simulate as compared to the original model.

Keywords: *Gas Distribution Network, Differential Equations, Model Order Reduction, Proper Orthogonal Decomposition*

Chapter 1

Introduction

This chapter starts with the introduction of modelling and simulation of gas distribution networks. Further the importance of model order reduction (MOR) in the field of modelling and simulation is discussed. The problem statement, motivation and objectives are discussed afterwards. In the end of this chapter, thesis overview is given.

1.1 Gas Network

According to BP statistical review published in 2016, Pakistan is currently contributing to 1.2% of the total global production of natural gas. Due to incompetent distribution of natural gas resources, Pakistan has been experiencing a huge shortage of gas since 2004 [2]. Pakistan's economy has been considerably affected due to the over consumption and under production of natural gas. Natural gas used as an energy source can be transferred from suppliers to consumers using pipelines for long or short distances. The intricacy and size of these gas networks will therefore differ considerably. Perhaps the most frequently asked question about these transportation issues is whether the supply meets the demands of the consumer. The natural gas distribution grid comprises of pipelines, city gas stations, compressors and storage facilities. compressor station is the most important part of the gas network as it supplies the energy needed to keep the gas flowing at the specified flow rate and pressure. There are three types of onshore and offshore pipelines [1]:

1. Gathering pipelines

2. Transportation pipelines

3. Distribution pipelines

Natural gas distribution pipelines are mainly used to transfer gas from plant to consumers. Inter and Intrastate distribution pipelines are used to deliver the natural gas generated from gas reserves, either offshore or onshore, to commercial and residential, manufacturing and utility companies through gathering systems. Pipeline networks are widely used for the transportation of coal, gasoline, water and other chemicals due to their reliability and lower financial costs as measured against other things of transportation that undermine the economic stability. The unexpected increase in population has resulted in the growth of cities and towns, providing more connections to the already vast and complicated pipeline networks. Distribution networks have smaller diameter pipes and they run on low to medium pressure, whereas transmission networks operate on medium to high pressure and lack compressors and reduction valves. Figure 1.1 shows different types of pipelines in a gas network. In order to model a gas distribution network some parameters are considered to be more significant like pressure of a gas in a pipeline and mass flow of the gas. Modelling of these parameters results into a large number of differential and algebraic equations which are computationally expensive to solve. The detailed explanation of mathematical model of gas distribution network is explained in chapter 3.

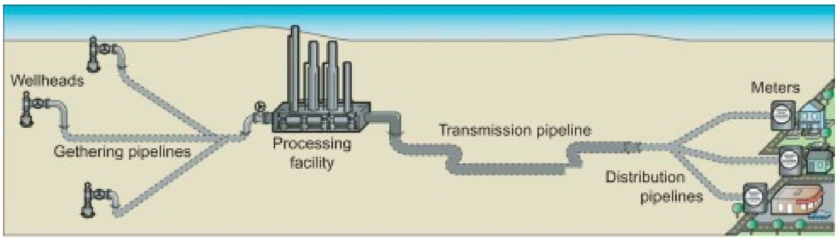


Figure 1.1: Pipeline structure of Gas network [1]

1.2 Model Order Reduction

Gas distribution networks are often modelled by a set of partial differential algebraic equations and the associated constraints are modelled by algebraic equations. The complete network model therefore involve partial differential algebraic equations (pDAE) and their simulation is known to be computationally complex. Model order reduction (MOR) can provide a remedy to this problem by constructing a reduced order model (ROM) that is computationally cheap to simulate and its behaviour is similar to the original large scale model. MOR (see [3] for details) was developed originally in the domain of system analysis and control theory, which focuses mainly on the characteristics of dynamical systems in order to decrease their complexity while preserving as much of their input/output characteristics as possible.

To explain the concept of MOR, let us consider a dynamical system which can be modelled in the form of non-linear differential algebraic equations.

$$\begin{aligned} E\dot{x}(t) &= f(x(t), u(t)) + Bu(t), \\ y(t) &= Cx(t) + Du(t), \end{aligned} \tag{1.2.1}$$

where f is non linear function f: $\mathbb{R}^n \rightarrow \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$ is the input of the system, $x(t) \in \mathbb{R}^n$ represent states of the system and $y(t) \in \mathbb{R}^p$ is the output of the system. $C \in \mathbb{R}^{p\times n}$, $B \in \mathbb{R}^{n\times m}$ and $D \in \mathbb{R}^{p\times m}$ are the state matrices of the system. The matrix $E \in \mathbb{R}^{n\times n}$ is singular and the rows that are null corresponds to the algebraic part of the model.

Now, using MOR technique, an approximation of the model in (1.2.1) can be computed which can be written as:

$$\begin{aligned} \hat{E}\dot{\hat{x}}(t) &= \hat{f}(\hat{x}(t), u(t)) + \hat{B}u(t), \\ \hat{y}(t) &= \hat{C}\hat{x}(t) + \hat{D}u(t), \end{aligned} \tag{1.2.2}$$

where $\hat{f}$ is a non linear function $\hat{f}$: $\mathbb{R}^r \rightarrow \mathbb{R}^r$, $\hat{x}(t) \in \mathbb{R}^r$ denotes state and $\hat{y}(t) \in \mathbb{R}^p$ represents the output of the approximated model of the system. $\hat{E} \in \mathbb{R}^{r\times r}$, $\hat{B} \in \mathbb{R}^{r\times m}$, $\hat{C} \in \mathbb{R}^{p\times r}$ and $\hat{D} \in \mathbb{R}^{p\times m}$ are reduced state matrices as given in (1.2.2). The number of states of the reduced model is much less as compared to that of the original model (i.e, $r << n$) and the output of both reduced and original system is almost the same with minimal approximation error ($\hat{y}(t) \approx y(t)$). This model is called reduced order model (ROM) and can be used for simulation purposes with less computational power as compared to the original model.

Some general characteristics of a reduced order model are as follows:

- The output of reduced order model must be almost equal to the output of original model with minimal approximation error.

- Structure of the system should be maintained.
- The reduced model should retain the properties of the original model such as stability and passivity.
- The reduced order model should take less computational time and should be computationally efficient.

1.3 Problem Statement

As discussed earlier, models representing mass flow and pressure of gas distribution network at different points are large and complex, which makes the simulation of such systems a challenging problem. In order to obtain fast simulation of such large scale complex networks, model order reduction technique is commonly used. The problem is to explore the applicability of an efficient MOR technique for the non-linear models as in (1.2.1) representing a gas distribution network.

1.4 Motivation/Applications

Natural gas is generally considered the most widely used energy sources. For years demand of natural gas has been rising. Gas flow in a pipeline network is considered to be effected by a number of factors like inconsistencies in meter measurements, theft, underground losses, human and mechanical errors. Modelling and simulation provides a fast and efficient means of predicting network parameters. Mathematical modelling of gas network enables us to check the flow rate and pressure of gas at every node which help in identification of losses in the network. The same model can be extended to other complex networks such as power networks and water distribution networks.

Model order reduction is used to minimise the computational cost and save time while simulating multiple complicated dynamic systems. Initially the model reduction theory is applied to control systems. It has been extended to several other fields like conservative systems, semiconductors, VLSI systems.

1.5 Objectives

Following objectives have been fixed for my research work:

- Mathematical representation of gas distribution networks in the form of PDE's and algebraic equations. Also, discretization of PDE's to ODE's and representation of complete network model in state space form.
- Simulation of the non-linear state space model using MATLAB solvers.
- Implementation of model order reduction technique on non-linear state space model to obtain reduced state space model.
- Fast simulation using ROM with MATLAB solvers and comparison to the simulation results of original model.
- Validate the model by comparing the data of simulation results with actual measurements of gas distribution network.

1.6 Thesis Overview

This thesis is organized into five chapter, the current chapter provides a brief introduction into the problem statement, background and applications of choosing a framework for efficient modelling of a gas distribution network. Chapter 2 focuses on literature review, it discusses the research techniques applied by researchers in the field of model order reduction. Chapter 3 deals with the methodology used to design the framework of the pipeline network including pipe dynamics, topology and approach to model order reduction. In chapter 4, different types of networks are used for simulation and the results are observed. In chapter 5, we discuss the observations derived from the analyses of the simulation results and future works.

Chapter 2

Literature Review

In this chapter, first the modeling and simulation of gas networks is discussed following with the overview of various MOR tehniques used by researchers. Although our main focus in this research is on the non linear problem but for better understanding of MOR, classical method used for the reduction of LTI systems are discussed.

2.1 Network modeling and simulation

There are certain rules which need to be taken care of while modelling any real world phenomena, such as model should comply with universal physical laws and should include components of physical systems. Consider the case of gas distribution network which when modelled should have constraints due to pipes, junctions and other passive equipments. Such models can be used to estimate the behaviour of the network at different nodes and pipes. One of the prominent research work done in [4], in which the Euler equations were used to simulate the transient behaviour of the network for different topologies. The work of Saeid Mokhatab and William A. Poe in [5] considers integrated Bernoulli's equation to model the flow equations, with the assumption that the flow is isothermal, meaning that the temperature is constant throughout the pipe and the pipe has zero height. The equation also includes an efficiency factor E, which is unity for dry gas flow and decreased from unity when the pipeline's efficiency is reduced due to corrosion. Gonzales in [6] models the unidirectional gas flow using 1D isothermal equation which uses the law conservation of mass and momentum along with elevation of pipes considered as well. In [7] a gas flow model was developed with isothermal flow, constant elevation and the Darcy-Weisbach equation to simulate friction along the pipe using the continuity equation (mass and momentum conservation). In [8] the system is modelled using Euler equations, which are a collection of PDEs that represent the equations of continuity, motion and energy. In [9], the general flow equations and principal of conservation of mass are used to model the gas dynamics.

2.1.1 Graph theory

Graph theory deals with the visualization of networks which consist of points and lines. The knowledge of graph theory traces back to early 18^th century in which the work of Leonhard Euler proved to be the cornerstone of the graph theory. To assess the flow and pressure of any pipe network such as gas network, graph theory comes in handy. It uses the properties of the given network to create a simplified structure of the network. The structure of the network is a graph comprised of nodes and edges where the former can be demand, supply or interconnected points and the later represents the pipes of the network. The relation of nodes connections with edges can be represented in the form of matrix which is known as incidence matrix. In [9], graph theory is applied where nodes and edges denotes the pressure and mass flow of a pipe, respectively. Similar work has been carried out in [10] where graph theory is used for steady state analysis of the pipe network. In the same way, graph theory can be used in PDE models as well, see [11].

A graph can be explained as a non-empty set of vertices or nodes V and edges $E \subseteq x, y$ where V represents as set of entities $x, y \in V | x \neq y$ and edges E represents the relationship between the pair of things. A graph with no edges is said to be empty, while a graph with no vertices is considered to be a null graph. Directed and un-directed graphs are the two types of graphs that are studied in graph theory.

Un-directed Graphs: Edges of an un-directed graph are a set of unordered bidirectional vertices such as $x, y \Leftrightarrow y, x$ where $x, y \in V$.

1. Simple Graph: It is a graph with no self-loops, many edges and no two edges connecting the same pair of nodes. It is also known as strict graph.

2. Planner Graph: Type of undirected graph in which no two edges intersect each other and is divided into regions called faces. The graph can also be expressed using Euler formula that is: $v - e + f = 2$ where v,f and e indicates vertices, faces

and edges of the graph respectively.

3. Complete Graph: Every vertex is next to each other. The number of edges in a graph of n vertices K_n is indicated as:

$$E_m = \frac{n(n-1)}{2} \tag{2.1.1}$$

4. Acyclic /Tree Graph: It is a connected graph that doesnot contain cycles, which indicates a full circuit. A relationaship between vertices and edges can be represented as $m = n - 1$ when G is linked.

Directed Graphs: Directed graph also known as digraph consists of V vertices and ordered pair of edges E, represented as $G_d = (V, E)$ where $x, y \in V$. The arcs incident out of a vertex point y defines the out-degree $d^+(y)$, whereas the arcs incident into y defines the in-degree $d^-(y)$. Both classes of the graphs (directed and undirected) are similar apart from the direction between the vertices.

Directed Acyclic graph (DAG): It is a graph with a sequential ordering of vertices. DAG's are commonly used to model data for systems or processes. In contrast to its counterpart, where the efficiency decreases, mathematical procedures may readily be applied to DAG's for determining longest path, shortest path, number of junction nodes and so forth.

An incidence matrix is used to describe the finite DAG's, and it represents the relationship connection between an ordered pair of vertices.

$$A_x y = \begin{cases} 1 & \text{y incident to vertex x} \\ 0 & \text{y not incident to vertex x} \\ -1 & \text{y not incident from vertex x} \end{cases} \tag{2.1.2}$$

Cyclic and Non cyclic Topology

Pipeline systems may also be divided into three different categories based on their configurations.

1. Gun-barrel topology: Consists of single pipe with one source node and one demand node.

2. Non-cyclic topology: Tree/fork scheme consists of a branching pipeline having single inflow element and multiple outflow elements.

3. Cyclic topology: Looped scheme is made up of diamond shaped network in which long pipes share two junctions. There are multiple inflow and outflow components at these junctions.

2.1.2 Dynamic flow model

When physical world phenomena is modelled, it gets the shape of PDEs which are set of mathematical equations having partial differential with respect to one or more independent variables. To find out the solutions of PDEs, it is first converted to ODEs using discretization. There are several techniques of discretization explained in [12, 13]. In [14], to solve the exhaust gas model of a diesel powered generator authors used the finite difference method to explain the heat transfer. The work of Peng Wang and Bo Yu in [15] models gas pipe network using equation of momentum, energy and continuity by applying adaptive implicit finite difference scheme. Various schemes of finite volume method (FVM) is explored in [16] for discretization of high pressure gas distribution network which suggests that FVM is efficient for discretization of 1D isothermal equations. [17], compares two discretization techniques for natural convection flow in a square cavity in which it is observed that FVM performs better than FDM. [18], performs an analysis of finite volume and finite difference schemes for a convection flow in a square cavity, concluding that the results produced by Finite volume method are more stable and closer to benchmark solutions, and show higher accuracy for refined mesh sizes, whereas FDM produces severe oscillations when compared to FVM, resulting in a loss of credibility. The literature suggests that Finite volume techniques work better than other techniques while discretizing the 1D isothermal Euler equation. Therefore, we carry discretization of gas network using FVM in this thesis.

2.2 MOR techniques for linear systems

Model order reduction is basically a technique for reducing the computing cost of solving a PDE problem numerous times for various parameters. In numerical analysis the system under observation has an abundant equations and variables, reduction is applied to reduce the computational time and complexity of the model keeping system's input and output relation as close to original as possible [19]. MOR techniques may be categorized into two main groups which are Krylov based methods and singular value decomposition based methods. The advantages of some of the methods are discussed in [20]. In [21], a genetic algorithm based MOR method was proposed for multi time scale discrete systems. Using this technique certain dynamics that have little impact on the system are eliminated.[22], provided a comprehensive paper based on various techniques for computing the reduced model for commercial applications, as they provide a software application *Model Order Reduction for Gas and Energy Networks* abbreviated as **morgen**, that provides independent analysis for modules such as solvers, MOR techniques and others to implement best solution to gas networks. The empirical structured subspace technique for model reduction offers the least number of errors, DMD Galerkin reductor was an effiecient reductor, and the first-order IMEX solver performed better for solving ODE's, according to the results.

2.2.1 MOR formulation

Let us consider a case of LTI system which is given as:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t), \\ y(t) &= Cx(t) + Du(t). \end{aligned} \tag{2.2.1}$$

where $A \in \mathbb{R}^{n \times n}$, $C \in \mathbb{R}^{p \times n}$, $B \in \mathbb{R}^{n \times m}$ and $D \in \mathbb{R}^{p \times m}$. In this case, the ROM will be represented as:

$$\begin{aligned} \dot{\hat{x}}(t) &= \hat{A}\hat{x}(t) + \hat{B}u(t), \\ \hat{y}(t) &= \hat{C}\hat{x}(t) + \hat{D}u(t). \end{aligned} \tag{2.2.2}$$

where $\hat{A} \in \mathbb{R}^{r \times r}$, $\hat{B} \in \mathbb{R}^{r \times m}$, $\hat{C} \in \mathbb{R}^{p \times r}$ and $D \in \mathbb{R}^{p \times m}$. The number of states of ROM is much less than that of the full order model (i.e. $r << n$). The output of reduced model is approximation of the output of the original model ($\hat{y}(t) \approx y(t)$).

In linear model order reduction there exists a number of algorithms to obtain a reduced system of the form given in equation (2.2.1) that also guarantees error estimates. Textbooks on linear model order reduction cover the approaches stated above in detail [19, 23]

2.2.2 Balanced Truncation

In balanced truncation method the transformation matrix T is computed by decomposing the controllability and observability gramians, see [24–26]. The basic observation of this approach is that only a system's greatest singular values are important.The square root approach [27], which is based on Cholesky factorization of observability and controllability gramians, is an effective way of implementing the balanced truncation method. Using SVD, V and W also known as basis matrices are built. Consider a dynamical system with state space representation given Equation (2.2.1).Assuming the dynamical system is stable, the associated Lyapunov equations for the gramians are given as:

$$AP + PA^T + BB^T = 0, \tag{2.2.3}$$

$$A^TQ + QA + C^TC = 0, . \tag{2.2.4}$$

where P is controllability gramian and Q is observability gramian of the system. We need to calculate the transformation matrix T which balances the model i..e. $P = Q = \Sigma = diag(\sigma_i)$. The transformed balanced realization along with transformed gramians are given as:

$$A' = T^{-1}AT, B' = T^{-1}B, C' = CT, D' = D, P' = T^{-1}PT^{-*}, Q' = T^*QT.$$

The transformation matrix can be found using Cholesky factorization of gramians and then applying SVD [19]. We partition the balanced realization as:

$$A' = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix},$$

$$B' = \begin{pmatrix} B_1 \\ B_2 \end{pmatrix},$$

$$C' = \begin{pmatrix} C_1 & C_2 \end{pmatrix}.$$

The ROM is represented as:

$$\dot{\tilde{x}}(t) = A_{11}\tilde{x}(t) + B_1u(t),$$

$$y(t) = C_1x(t) + Du(t).$$

The balance truncation technique works good for model having order of few thousands due to the fact that solving the Lyapunov equations require $O(n^3)$ operations with n the size of original model. The solution of Lyapunov equation for complex systems gets very expensive, computationally. Another drawback of the balanced truncation method is that it does not guarantee the preservation of passivity.

2.2.3 Interpolation Based Model Order Reduction

A LTI system can be treated as a problem of rational interpolation in which a transfer function $H(s)$ of a system is considered as n degree rational function. $H_r(s)$ represents the approximation of original transfer function with respect to H_2 norm and is known as transfer function of ROM. $H_r(s)$ is obtained by Petrov-Galerkin projection in which basis matrices V and W are constructed as discussed in [28]. The interpolation based model reduction gives good approximation of model but the error in output of original and reduced model depends on the selection of interpolation points and tangential directions. Apart from this, interpolation based model order reduction is possible only if there exists original model of a physical system.

In case of linear systems (2.2.1), there are various techniques in the literature to compute reduced-order models (ROMs), [3, 29]. Among these methods, projection-based moment-matching methods [30, 31] are well used. Using projection matrices $V \in \mathbb{R}^{nxr}$ and $W \in \mathbb{R}^{nxr}$, we approximate $x(t) \approx Vx_r(t)$ such that the Petrov-Galerkin orthogonality condition holds:

$$W^T(EV\dot{x}_r(t) - (AVx_r(t) + Bu(t))) = 0, \tag{2.2.5}$$

$$\hat{y}(t) = CVx_r(t). \tag{2.2.6}$$

The projection is called one-sided projection, if $W = V$. Otherwise it is known as two sided which gives ROM of the form:

$$E_r = W^T EV, \quad A_r = W^T AV, \quad B_r = W^T B, \quad C_r = CV. \tag{2.2.7}$$

In case of linear systems, a suitable choice of V and W, implicitly ensure moment matching, where moments are the coefficients of the Taylor series expansion of the transfer function at some predefined shift frequencies. Thus for projection-based moment matching, the choice of V and W depends on the transfer function of the system. In addition, all the existing moment-matching/interpolation approaches [32–34] are based on the simplification that the interpolation points is the same for each frequency variable.

2.3 Non-linear MOR

As explained in section 1.2, non linear model order reduction is widely used in many domains involving large scale CAD design and simulation of nonlinear dynamical systems. The methods for non linear MOR are mostly the extension of MOR techniques for linear systems, see [29]. But the most common and efficient technique of non linear MOR is proper orthogonal decomposition. Detailed explanation of POD is given below.

2.3.1 Interpolation based nonlinear MOR

Let us consider a nonlinear (Quadratic bilinear) single-input single-output (SISO) system which can be written as:

$$\begin{aligned} E\dot{x}(t) &= Ax(t) + Nx(t)u(t) + Qx(t) \otimes x(t) + Bu(t), \\ y(t) &= Cx(t), \end{aligned} \tag{2.3.1}$$

where $A, E, N \in \mathbb{R}^n, BandC^T \in \mathbb{R}^{nxn^2}.u(t) \in \mathbb{R}$ is the input, $y(t) \in \mathbb{R}$ is the output of the system and $x(t) \in \mathbb{R}$ is the state vector. Using projection matrices $V \in \mathbb{R}^{nxr}$ and $W \in \mathbb{R}^{nxr}$, we approximate $x(t) \approx Vx_r(t)$ such that the Petrov-Galerkin orthogonality condition holds:

$$\begin{aligned} W^T(EV\dot{x}_r(t) - (AVx_r(t) + NVx_r(t)u(t) + QVx_r(t) \otimes Vx_r(t) + Bu(t))) = 0, \\ \hat{y}(t) = CVx_r(t). \end{aligned} \tag{2.3.2}$$

However, nonlinear systems have no universal input-output representation though for some classes of nonlinear systems, including the QBDAE system, it is possible to generalise the transfer function concept by utilising the Volterra theory [35], where the input-output relationship is described by a set of high-order transfer functions. This makes the concept of moment-matching slightly complex in the nonlinear case, since the structure of the basis matrices V and W now depends on multiple high-order transfer functions.

2.3.2 Proper Orthogonal Decomposition

Another method of MOR which is based on SVD is proper orthogonal decomposition (POD). This method is more common in field of Computational Fluid Dynamics (CFD).

The POD is generally used to evaluate effective bases for complex systems. The main advantage of POD is that it can also be used for non-linear ODE's described in equation (1.2.2).

In this method, the inputs which consist of essential behaviour of the system are given to a certain model which builds outputs. These outputs are called 'snapshots' which consist of column vector [36]. Snapshot based reduction involves computation of a collection of representative samples of the solution in advance. The solution of reduced model is then expressed as a combination of linearly dependent vectors of all these snapshots. Galerkin projection is used to observe respective coefficients, based on a weak version of governing equations. Therefore, the reduced model preserves both spatial structure and underlying physics of typical solutions. The textbooks provide brief introductions to snapshot-based model order reduction [37, 38].

The first time POD was used is for the reduction of dynamical systems in 1990's [39]. The POD, also called Karhunen-Loeve decomposition (KLD), was initially developed in context of underlying continuous second-order processes and proposed independently among numerous scientists. The POD is identical as principal component analysis (PCA) when reduced to a very finite dimensional scenario and terminated after a few terms [40]. See references [41–44], for a thorough analysis of the KLD, POD, and PCA equivalency, as well as their link to the singular value decomposition (SVD). The technique was hardly used until 1950's, because computing the POD modes required a massive amount of computations. Significant developments occurred with the introduction of supercomputers. The POD is currently utilized in variety of fields. For example, several researchers employed POD to recover coherent structures from turbulence. In [45], the authors suggested the application of POD to detect number of signals used in multi channel time series. KLD is used to capture the modes of a chemical reaction in a diffusion process and determine its dynamic behaviour [46].dynamic characterization [47–49], active control [50], finite element model updating [51, 52], aeroelastic problems [53, 54], modal analysis [55], model order reduction [56–58], multibody systems [59] and stochastic structural dynamics [60–62] are just a few of the applications for the POD in structural dynamics.

Consider Y is a matrix which contains snapshots of the output and belongs to $\mathbb{R}^{m \times n}$, then there exist

$$U = (u_1, u_2, ..., u_m) \in \mathbb{R}^{m \times m}, \quad V = (v_1, v_2, ..., v_n) \in \mathbb{R}^{n \times n}. \tag{2.3.3}$$

Using singular value decomposition (SVD)

$$Y = U \Sigma V^* \quad or \quad U^* Y V = \begin{pmatrix} D & 0 \\ 0 & 0 \end{pmatrix} := \Sigma \in \mathbb{R}^{m \times n}. \tag{2.3.4}$$

The Y can be written as:

$$Y = (y_1, y_2, ..., y_n) = U^d D (V^d)^T = \sum_{i=1}^{d} < u_i, y_j >_{R^m} u_i. \tag{2.3.5}$$

The columns of snapshots matrix can be written in the form of linearly independent columns of U^d. Model order reduction using POD generates ROMs with good approximation of the output. POD has the advantage of being able to solve nonlinear PDEs.

Chapter 3

Methodology

In this chapter, modelling of gas distribution networks using 1D Euler equations is discussed and then the discretization of partial differential equations is explained. Furthermore the reduction of gas distribution networks using proper orthogonal decomposition is explained.

3.1 Modelling of Gas Distribution Network

We observe that most of the work carried out in the modelling and simulation of gas distribution network uses Euler equation for the dynamics of gas through a pipe [63].

3.1.1 Euler Equation

The Euler equation is mainly comprised of conservation principles that come together to generate a set of partial differential equations that describes the system's gas or fluid dynamics. In case of gas dynamics, Euler equations consists of law of mass conservation, momentum conservation and state equations as explained in above section. In this work, [6, 11, 64, 65], some assumptions are made to keep the model simple. Pipes are considered to be underground which means the temperature does not change significantly. Therefore, energy equation has been ignored here [8, 66–70].

To construct a model of complex network, Let us look into the flow of gas in a single pipe with cross-sectional area a, diameter d, and length L. Then, 1D isothermal Euler equation can be expressed as:

$$\frac{\partial}{\partial t}\rho = -\frac{\partial}{\partial x}\varphi, \tag{3.1.1}$$

$$\frac{\partial}{\partial t}\varphi = -\frac{\partial}{\partial x}p - \frac{\partial}{\partial x}(\rho v^2) - g\rho\frac{\partial}{\partial x}h - \frac{\lambda(\varphi)}{2d}\rho v|v|, \tag{3.1.2}$$

$$p = \gamma(T)z(p,T)\rho. \tag{3.1.3}$$

The spatial domain of the given equations is $[0, L]$. Variables used are defined in Table: 3.1.

Notation	Discription	Unit
p	Pressure	$\frac{N}{m^2}$
g	Gravitational constant	$\frac{m}{s^2}$
h	Pipe Elevation	m
q	Mass flow	$\frac{kg}{s}$
ρ	Density	$\frac{Kg}{m^3}$
λ	Friction factor	unitless
v	Velocity	$\frac{m}{s}$
φ	Flow rate per area	$\frac{kg}{m^2 s}$
L	Length	m
γ	Gas state	$-$
z	Compressibility Factor	$-$
T	Temperature	K
a	Cross sectional area	m^2
d	Diameter	m

Table 3.1: List of Symbols

Equation 3.1.1 represents the law of conservation of mass while equation 3.1.2 denotes the conservation of momentum. Equation 3.1.3 relates the relationship of pressure with density.

Notice that the flow rate φ can be expressed as $\varphi = \rho v$ with v being the velocity and ρ density of gas through the pipe. The notations of certain variables used in [17] are followed here. We consider the isothermal case where temperature remains constant throughout the network. For isothermal process, $\gamma(T) = \gamma(T_o)$ and $z(p,T) = z_o(p)$. The inertia term can be neglected due to small value which is explained in [6]. Since the variable of interest is mass flow rate, we use $q = a\varphi$ and substitute into (3.1.1) - (3.1.3), we get

$$\frac{\partial}{\partial t}\frac{p}{z_0(p)} = -\frac{\gamma_0}{a}\frac{\partial}{\partial x}q, \tag{3.1.4}$$

$$\frac{\partial}{\partial t}q = -a\frac{\partial}{\partial x}p - \frac{\lambda(q)\gamma_0}{2da}z_0(p)\frac{q|q|}{p}. \tag{3.1.5}$$

The nonlinearity in the above equation is due to friction term in (3.1.5). We rewrite the isothermal incompressible Euler equation with $z_0(p) = 1$:

$$\frac{\partial}{\partial t}p + \frac{c}{a}\frac{\partial}{\partial x}q = 0, \tag{3.1.6}$$

$$\frac{\partial}{\partial t}q + a\frac{\partial}{\partial x}p + \frac{c\lambda}{2da}\frac{q|q|}{p} = 0. \tag{3.1.7}$$

For our simplification we introduce $c = \gamma_0$ in the above equations (3.1.6 and 3.1.7).

3.1.2 Discretization of Gas Distribution Network

There are different types of discretization techniques used for the conversion of PDE's to ODE's. Among them the most common and effective technique of discretization of 1D Euler equation is finite volume method (FVM). Discretization of equations (3.1.6 and 3.1.7) is explained in detail in [17].

The boundary conditions are taken as the initial pressure at the start of the pipe and initial mass flow at the end of the pipe such that:

$$\begin{cases} p = p_s, & \text{at } x = 0, \\ q = q_d, & \text{at } x = L. \end{cases} \tag{3.1.8}$$

Applying FVM gives the model in matrix form as:

$$\underbrace{\begin{bmatrix} M_p & \\ & M_q \end{bmatrix}}_{\mathcal{M}} \begin{bmatrix} \partial_t p \\ \partial_t q \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & K_{pq} \\ K_{qp} & 0 \end{bmatrix}}_{\mathcal{K}} \begin{bmatrix} p \\ q \end{bmatrix} + \overbrace{\underbrace{\begin{bmatrix} B_q \\ 0 \end{bmatrix}}_{\mathcal{B}_q}}^{\text{right BC}} q_d + \overbrace{\underbrace{\begin{bmatrix} 0 \\ B_p \end{bmatrix}}_{\mathcal{B}_p}}^{\text{left BC}} p_s + \begin{bmatrix} 0 \\ g(p_s, p, q) \end{bmatrix}, \quad (3.1.9)$$

where B_p and B_q are the input vectors with q_d input demand mass flux and p_s the input supply pressure.

The discretized analogue of pressure and mass flow to be computed can be represented as:

$$p = \begin{bmatrix} p_2 & p_3 & \cdots & p_n \end{bmatrix}^T, \quad q = \begin{bmatrix} q_1 & q_2 & \cdots & q_{n-1} \end{bmatrix}^T.$$

Finite Volume method proved to be efficient than finite difference method (FDM) in terms of accuracy as discussed in section 2.1.2.

$$M_p = \begin{bmatrix} \frac{h_1+h_2}{2} & & & & \\ & \frac{h_2+h_3}{2} & & & \\ & & \ddots & & \\ & & & \frac{h_{n-2}+h_{n-1}}{2} & \\ & & & \frac{h_{n-1}}{8} & \frac{3h_{n-1}}{8} \end{bmatrix}, M_q = \begin{bmatrix} \frac{3h_1}{8} & \frac{h_1}{8} & & & \\ & \frac{h_1+h_2}{2} & & & \\ & & \frac{h_2+h_3}{2} & & \\ & & \ddots & & \\ & & & & \frac{h_{n-2}+h_{n-1}}{2} \end{bmatrix},$$

$$K_{pq} = -\frac{c}{2} \begin{bmatrix} -\frac{1}{a_1} & \frac{1}{a_1} - \frac{1}{a_2} & \frac{1}{a_2} & & & \\ & -\frac{1}{a_2} & \frac{1}{a_2} - \frac{1}{a_3} & \frac{1}{a_3} & & \\ & & \ddots & \ddots & \ddots & \\ & & & -\frac{1}{a_{n-3}} & \frac{1}{a_{n-3}} - \frac{1}{a_{n-2}} & \frac{1}{a_{n-2}} \\ & & & & -\frac{1}{a_{n-2}} & \frac{1}{a_{n-2}} - \frac{1}{a_{n-1}} \\ & & & & & -\frac{1}{a_{n-1}} \end{bmatrix},$$

$$K_{qp} = -\frac{1}{2} \begin{bmatrix} a_1 & & & & & \\ a_1 - a_2 & a_2 & & & & \\ -a_2 & a_2 - a_3 & a_3 & & & \\ & -a_3 & a_3 - a_4 & a_4 & & \\ & & \ddots & \ddots & \ddots & \\ & & & -a_{n-2} & a_{n-2} - a_{n-1} & a_{n-1} \end{bmatrix},$$

$$B_q = -\frac{c}{2}\begin{bmatrix} 0 \\ \vdots \\ 0 \\ \frac{1}{a_{n-1}} \\ \frac{1}{a_{n-1}} \end{bmatrix}, \quad B_p = \frac{1}{2}\begin{bmatrix} a_1 \\ a_1 \\ 0 \\ \vdots \\ 0 \end{bmatrix},$$

$$g(p_s,p,q) = -\frac{c}{4}\begin{bmatrix} \frac{h_1\lambda_1}{a_1d_1}\frac{q_1|q_1|}{p_s} \\ (\frac{h_1\lambda_1}{a_1d_1}+\frac{h_2\lambda_2}{a_2d_2})\frac{q_2|q_2|}{p_2} \\ (\frac{h_2\lambda_2}{a_2d_2}+\frac{h_3\lambda_3}{a_3d_3})\frac{q_3|q_3|}{p_3} \\ \vdots \\ (\frac{h_{n-2}\lambda_{n-2}}{a_{n-2}d_{n-2}}+\frac{h_{n-1}\lambda_{n-1}}{a_{n-1}d_{n-1}})\frac{q_{n-1}|q_{n-1}|}{p_{n-1}} \end{bmatrix}. \quad (3.1.10)$$

M_p and M_q are mass matrices. K_{pq} and K_{qp} are upper-triangular matrix and lower triangular matrix respectively. $g(p_s, p, q)$ is the non-linear term of the network, the non linearity occurs because of the quadratic term. The matrices of the pipe model increases exponentially as the number of pipes and nodes increase according to the distribution system. This significantly increases the computational time and therefore requires reduction techniques to decrease the model complexity while keeping the relation between the input and output relatively similar. The next section will briefly explain model reduction method to solve non linear model of gas distribution system.

3.1.3 Algebraic constraints

There exists certain constraints in the pipeline system concerning the connection of edges such as mass flow constraint and pressure constraint [71]. According to the mass conservation, the mass in the system must remain constant over time, this adds a constraint for the massflow in the model, and is given by:

$$\sum_{i\in I_k} q_d^{(i)} = \sum_{i\in O_k} q_1^{(i)} \quad for\ every\ node\ k, \quad (3.1.11)$$

where I_k is the set of edges incoming to the node k and O_k are the set edges outgoing of node k. Mass flow nodal condition in 3.1.11 states that inflow at the junction k should be equal to the outflow at the same junction k.

The second type of nodal condition describes the pressure equality among pipes connected to the same junction node, and is given by:

$$p_{n^{(i)}}^{(i)} = p_s^{(j)}, \ if \ the \ node \ connects \ incoming \ pipe \ i \ and \ outgoing \ pipe \ j. \tag{3.1.12}$$

The pressure nodal condition states that pressure at the outflow should be equal to the pressure at the inflow pipes that connects to the same junction node and ensures that there is only one pressure value at each node.

3.1.4 State space form

The state space model of equation (3.1.9) can be written as:

$$M\partial_t x = Kx + Bu(t) + f(x, u(t)). \tag{3.1.13}$$

Where M (mass matrix) is a singular matrix only when at least one junction of the network is included. f is known as the non linear function. Mathematical model in (3.1.13) consists of large number of non linear DAE's. The size of these DAE's depends on the length of the pipes of the network. Also, the size of the network increases when small discretization points are taken in order to increase the accuracy of the model. To solve the full order model of the network, we uses MATLAB solvers like ODE15s and implicit euler method. But as the size of the network increases it becomes computationally expensive to solve the FOM. To overcome this difficulty, model order reduction plays an important role as discussed in the next section.

3.2 Reduction of Gas Distribution Network using POD

In this section POD will be applied to the pipeline model. Proper orthogonal decomposition as explained in section 2.2 is a snapshot based model reduction technique referring to [72]. For the complete understanding of POD, one should understand the concept of singular value decomposition (SVD). From equation (2.3.5), linear combinations of the columns of U^d gives us the columns of Y. Proof of this is as follows:

$$\begin{aligned} Y &= U^d D (V^d)^T, \\ y_j &= \sum_{i=1}^{d} u_i^d (D(V^d)^T)_{ji}, \\ y_j &= \sum_{i=1}^{d} (D(V^d)^T)_{ji} u_i^d. \end{aligned} \tag{3.2.1}$$

As we know $(u^d)^T u^d = I$, the equation 3.2.1 becomes

$$\begin{aligned} y_j &= \sum_{i=1}^{d} ((u^d)^T (u^d)(D(V^d)^T)_{ji} u_i^d \\ &= \sum_{i=1}^{d} ((U^d)^T Y)_{ji} U_i^d. \end{aligned} \tag{3.2.2}$$

Orthogonal u^d having d dimensional basis is called the POD basis. The POD basis will give the best rank r approximation to the column of y.

As explained earlier, POD is a technique used to reduce the complexity of the network without changing the input/output characteristics of the network. For this, the algorithm used for the reduction of the state space model in equation (3.1.13) is explained below:

1. For the specific value of t solve the non-linear system (3.1.13) to get snapshots of the original solution.

$$X = (x_1, \ldots, x_{t_N}) \tag{3.2.3}$$

2. From the SVD of X, get the POD vectors of rank l.

$$X = \tilde{U}\Sigma\tilde{V}^T, \quad V = [\tilde{u}_1, \tilde{u}_2, \ldots, \tilde{u}_l] \tag{3.2.4}$$

3. Use V to get the reduced order model of the system.

$$x \approx V x_r$$

$$\begin{aligned} MV\dot{x_r} &= KVx_r + F(Vx_r, u(t)) + Bu(t) + r(t) \\ r(t) &= MV\dot{x_r}(t) - KVx_r - F(Vx_r, u(t)) - Bu(t) \end{aligned} \tag{3.2.5}$$

$$V^T r = 0$$

$$\begin{aligned} V^T MV\dot{x_r}(t) - V^T KVx_r - V^T f(Vx_r, u(t)) - V^T Bu(t) = 0 \\ V^T MV\dot{x_r}(t) = V^T KVx_r + V^T f(Vx_r, u(t)) + V^T Bu(t) \end{aligned} \tag{3.2.6}$$

Chapter 4

Results and Discussions

In Chapter 3, we have shown the modeling and simulation of gas distribution networks and the use of POD based model order reduction for its computationally efficient simulation. In this chapter, we will discuss the results of our implementation on different network structures from the literature and an actual network of gas distribution. The results include a comparison of the original and the reduced system responses, error plots of the reduced system relative to the original system and computational time calculations.

4.1 Fork Network

In this section, the results for three different fork network topologies are simulated with varying inputs of the network. The pipe number refers to the pipes connected between the supply, junction and demand nodes. The cross-sectional area of pipe is considered to be of the same value as that of the main pipe. Gamma is considered as specific heat capacity of natural gas, and the compressibility factor shows deviation of gas from ideal conditions. Pressure and mass-flux columns show the value at the specified node.

4.1.1 Fork Network 1

The first network is a simple fork network consisting of three pipes shown in figure 4.1. Node 1 is considered as the supply node. Node 3 and 4 are called as demand nodes. Supply pressure given to the system as input is 50 *bar* and demand mass flow as 30 and

40 kg/s. The mesh size for pipeline is taken to be 10 meters, where an auxiliary node is introduced into the pipeline after every 10 meters, and the pipe length is 1000 meters.

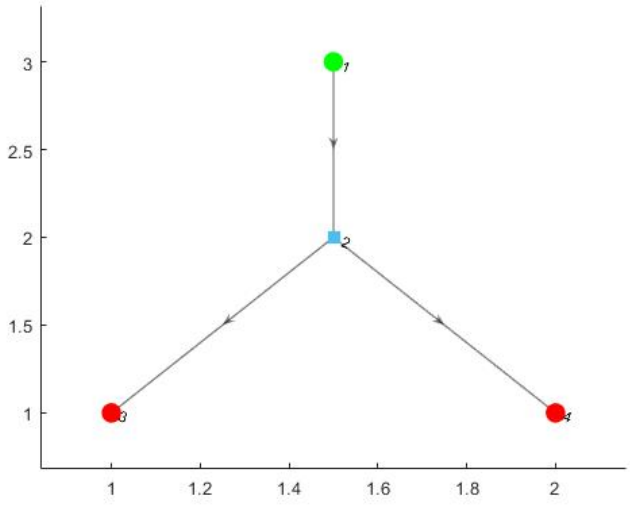


Figure 4.1: Simple Gas Fork network 1

The parameters of the network across every node is given in table 4.1.

supply pressure= 50 bar Demand mass flow= 30 kg and 40 kg							
Pipe Number	**Cross-sectional area** (m^2)	**pipe Length** (m)	**Node**	**Gas state** (γ)	**Compressibility Factor**	**Pressure** (bar)	**Mass flow** (kg/s)
1	0.785	1000	1	1.467	1	50	70
2	0.785	1000	3	1.467	1	50	30
3	0.785	1000	4	1.467	1	50	40

Table 4.1: Fork Network 1

Figure 4.2 and 4.3 show the mass flow and pressure at demand nodes 3 and 4, respectively. The findings are consistent with the system model's constraints, indicating that the pressure is maintained throughout the network and that the gas is transported correctly to its destination.

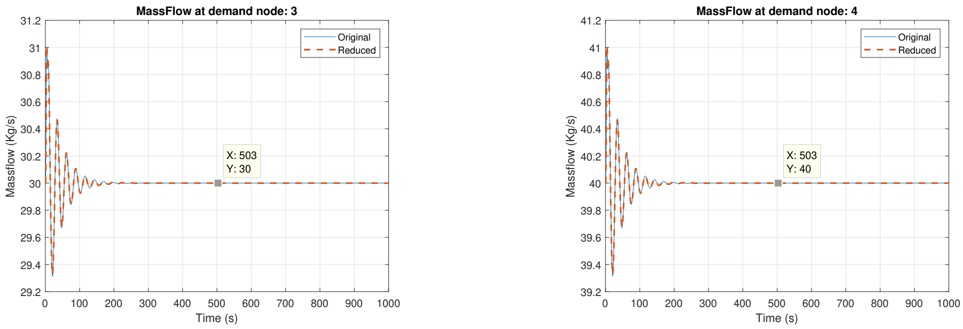


Figure 4.2: Fork Network 1 flow rates at Demand Nodes

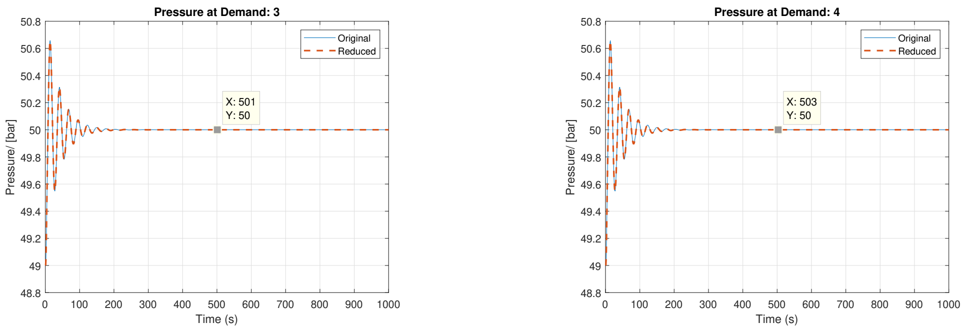


Figure 4.3: Fork Network 1 Pressure at Demand Nodes

Figure 4.4 shows the values of mass flow at supply node, which is equal to 70 kg/s. Value of mass flow rate at supply node is equal to sum of individual mass flow of demand nodes which in this case are 30 kg/s and 40 kg/s.

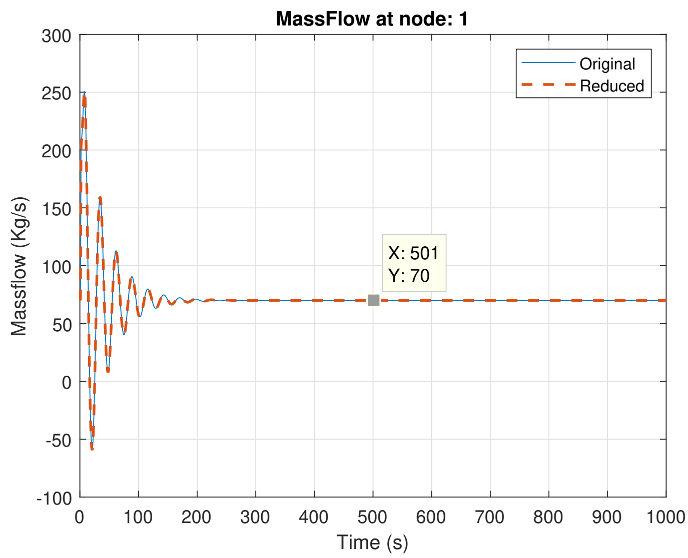


Figure 4.4: Fork Network 1 flow rates at Supply Node

The size of original model is $n = 601$ while the size of reduced model is $r = 8$. Figure 4.5 shows relative error between the results of FOM and ROM. Simulation time of the original system with h = 10m is **47.5675 sec** and simulation time of the reduced system is **18.0488 sec**. It has been found that ROM produces stable output results closely matching that of original model.

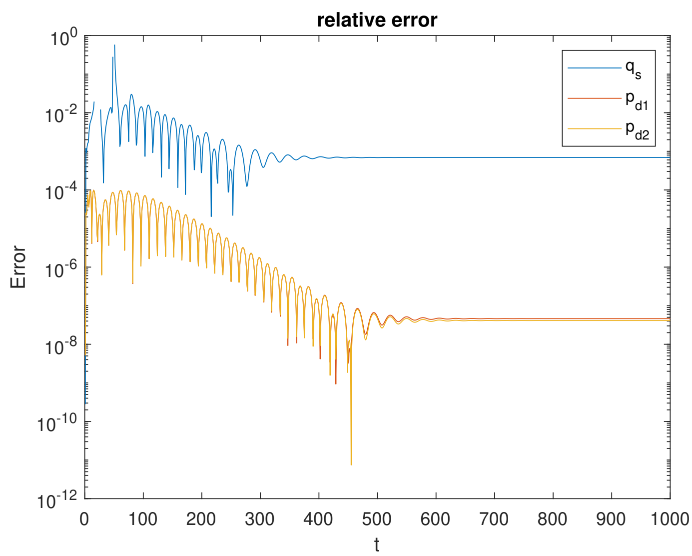


Figure 4.5: Fork Network 1 Relative Error

4.1.2 Fork Network 2

The fork network 2 is shown in figure 4.6 displays a total of 7 main pipes. The first pipe contain nodes 1, 2 and 3, second pipe 3, 4 and 5, third pipe 3, 6 and 7, fourth has 3, 8 and 9, fifth 9, 10 and 11, sixth has 9, 12 and 13 and seventh pipe has 9, 14 and 15 nodes. The input pressure is given to be 50 bar and demand mass flow is given as 30, 40, 50, 60, 70 kg/s to nodes 5, 7, 11, 13 and 15 respectively.

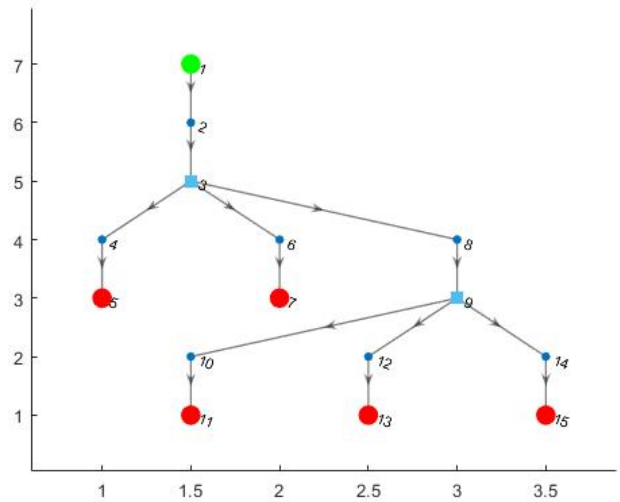


Figure 4.6: Gas Fork network 2

The table 4.2 shows the parameters of the simulated network. For junction 3 the pressure is stabilized at 50 bar and mass flow at junction is 250 kg/s because the total value of mass flow required at input junction is must be equal to the sum of mass flow of individual demand node. Similarly, at junction 9, the value pressure is taken to be 50 bar and mass-flow rate is equal to 180 kg/s, because it has three demand nodes(50, 60, 70 kg/s) and the total mass flow of these nodes is equal to 180 kg/s.

supply pressure= 50 bar Demand mass flow= 30, 40 ,50,60 and 70 kg/s							
Pipe Number	**Cross-sectional area** (m^2)	**pipe Length** (m)	**Node**	**Gas state** (γ)	**Compressibility Factor**	**Pressure** (bar)	**Mass flow** (kg/s)
1	0.785	1000	1	1.467	1	50	250
1	0.785	1000	3	1.467	1	50	250
2	0.785	1000	5	1.467	1	50	30
3	0.785	1000	7	1.467	1	50	40
4	0.785	1000	9	1.467	1	50	180
5	0.785	1000	11	1.467	1	50	50
6	0.785	1000	13	1.467	1	50	60
7	0.785	1000	15	1.467	1	50	70

Table 4.2: Fork Network 2

Figure 4.7 and figure 4.8 shows mass flow rate and pressure values at demand nodes. Figure 4.9 shows the mass flow rate at junction nodes 3 and 9 respectively. Figure 4.10 shows the pressure and mass flow rate at the node 1.

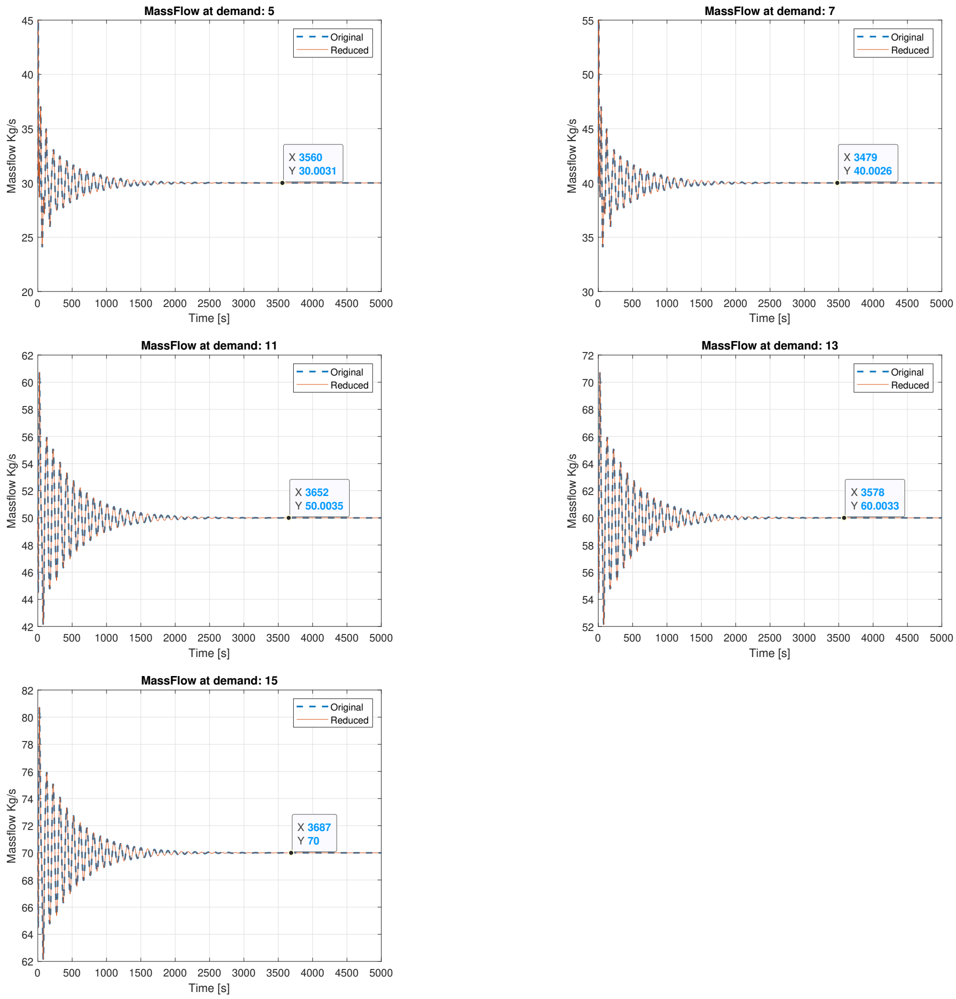


Figure 4.7: Fork Network 2 flow rates at Demand Nodes

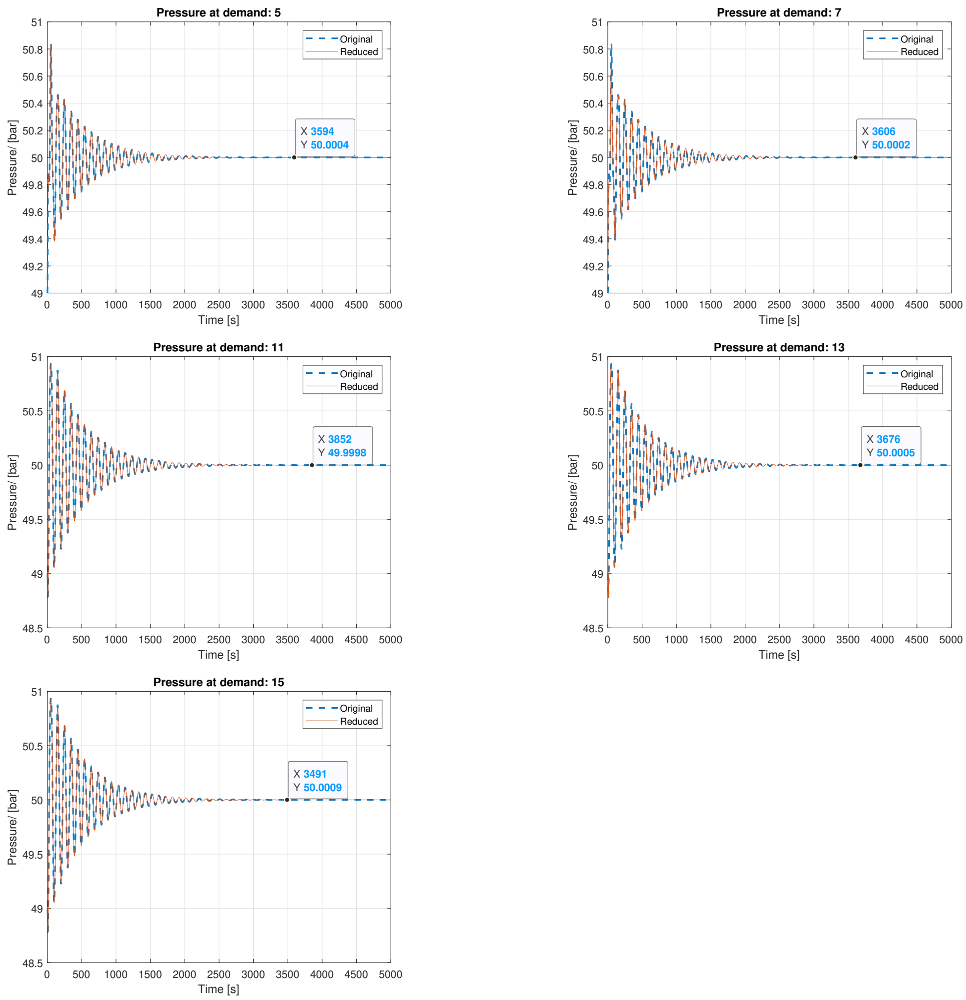


Figure 4.8: Fork Network 2 Pressure at Demand Nodes

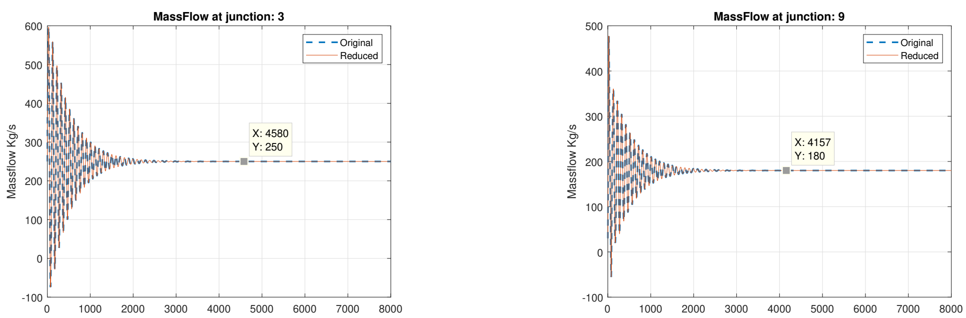


Figure 4.9: Fork Network 2 flow rates at junction Nodes

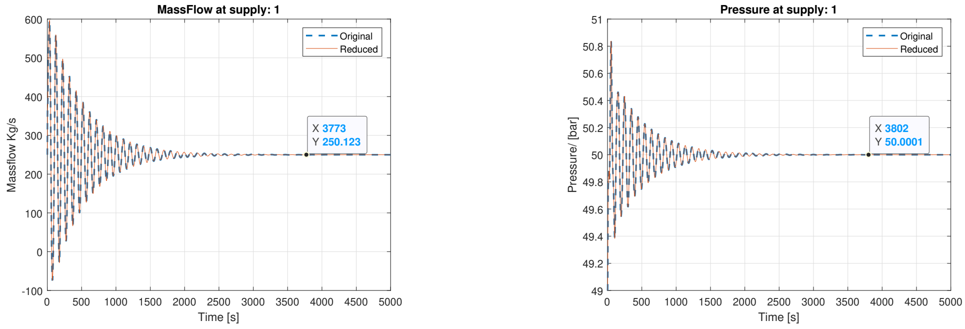


Figure 4.10: Mass Flow rate and Pressure at supply Node

For reduction of the network, mesh size of the pipeline is taken to be 10 *meters* where an auxiliary node is introduced after every 10 m, the total length of the pipeline is 1000 *meters*. The network is reduced to 20 states from the original 1402 states. The figure 4.11, shows minimal relative error between the outputs of FOM and ROM . Total simulation time of original system is **3.71 min** and the reduced time is **50.4 sec** only. It shows that POD is very effective technique because it reduces the computational time of the network without changing the output of the system.

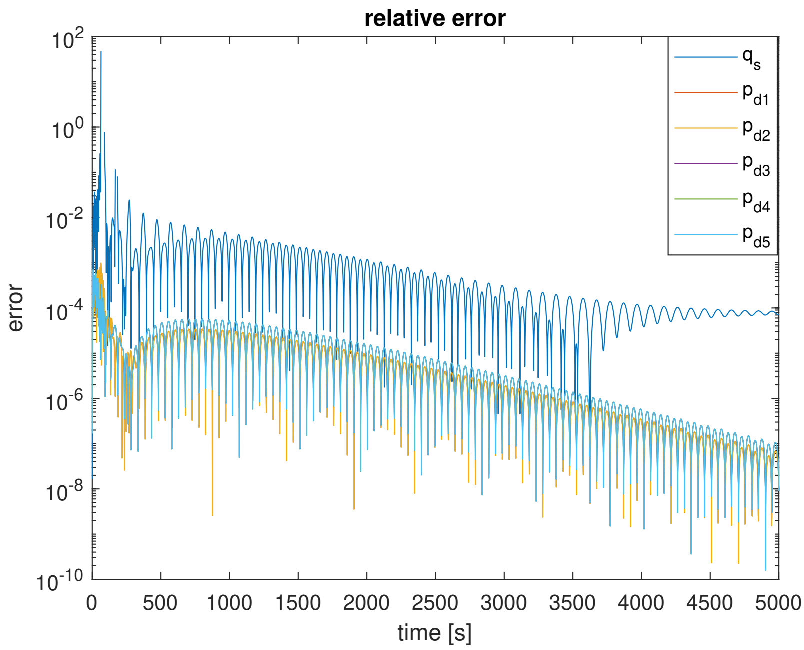


Figure 4.11: Fork Network 2 Relative error

4.2 Cyclic Network

4.2.1 Simple Cyclic Network

The cyclic network contains one source node, one demand node and two junction nodes. Total main pipes in the network is 4. Demand mass-flow of 30 kg/s and source pressure of 50 bar is given as the input to the system.

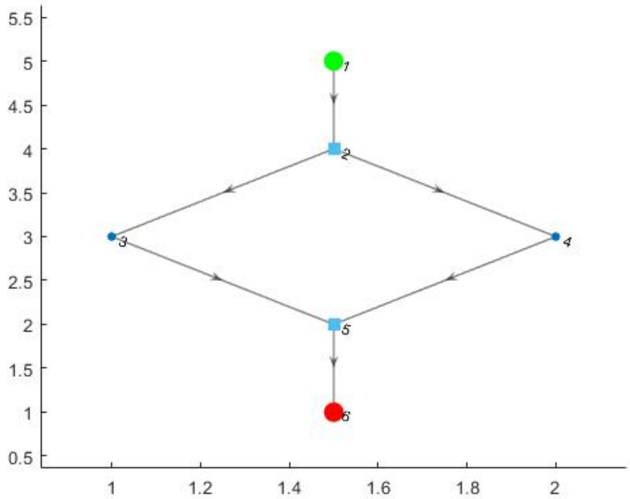


Figure 4.12: Simple Gas Cyclic Network

Table 4.3 shows the the values of mass flow and pressure at each demand, supply and junction nodes of network.

supply pressure= 50 bar Demand mass flow= 30 kg/s							
Pipe Number	**Cross-sectional area** (m^2)	**pipe Length** (m)	**Node**	**Gas state** (γ)	**Compressibility Factor**	**Pressure** (bar)	**Mass flow** (kg/s)
1	0.785	1000	1	1.467	1	50	30
6	0.785	1000	3	1.467	1	50	30

Table 4.3: Cyclic Network

For the reduction of the network, mesh size h is taken as 10m where the total length of the pipeline is 1000 m. The size matrix A is 803x803 due to which the computational time to solve the model is **13.6 sec**. To save the computational time we have applied POD which reduces the size of matrix A to 10x10 and reduced time is **5.3 sec** only. The relative error between the output of original and reduced system is shown in figure 4.13, from which it is concluded that POD is a very efficient technique for reduction because it reduces 61% of the computational time of the network by reducing the states to more than half of the original states.

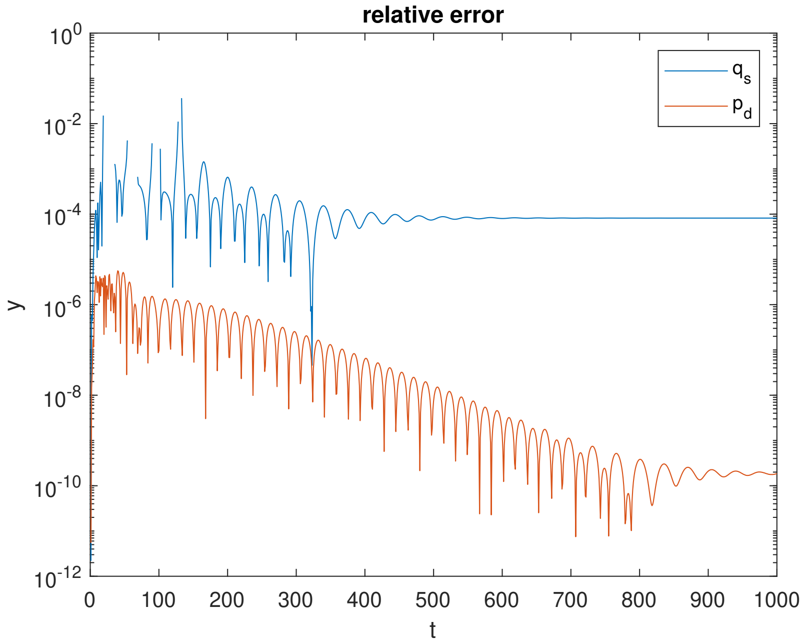


Figure 4.13: Cyclic Network Relative error

Figure 4.14 shows mass flow and pressure values at each demand node and figure 4.15 shows the values of mass flow and pressure at supply node.

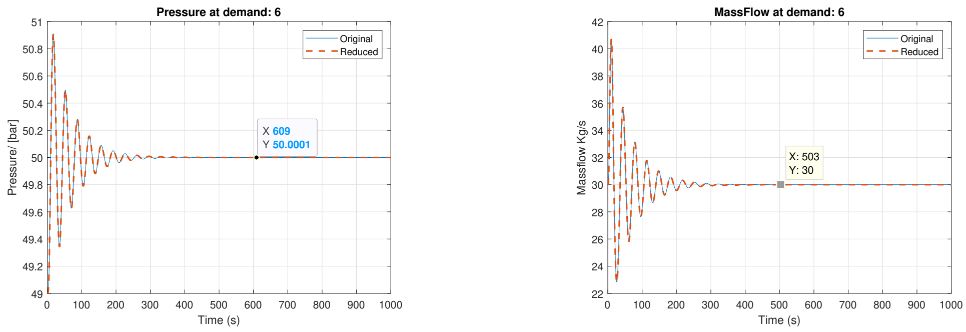


Figure 4.14: Mass Flow and Pressure at Demand Nodes

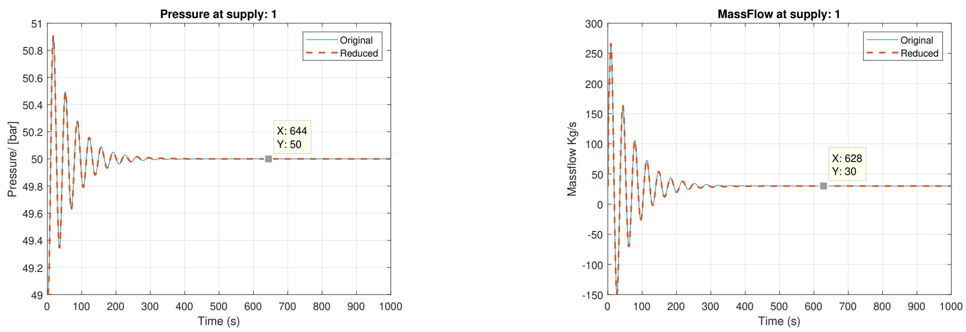


Figure 4.15: Mass Flow and Pressure at Supply node Nodes

4.2.2 Cyclic Network with variable parameters

We consider the same cyclic network in figure 4.12 but with different length, diameter and cross sectional area of the pipe. Input parameters of the network is defined in table 4.4.

Description	Symbol	Value
Pressure at supply node	p_{s1}	100 bar
Flow rate at demand node	q_{d1}	40 kg/s
Gamma	γ	1.46745319
Compressibility Factor	c	1
Mesh size	h	10

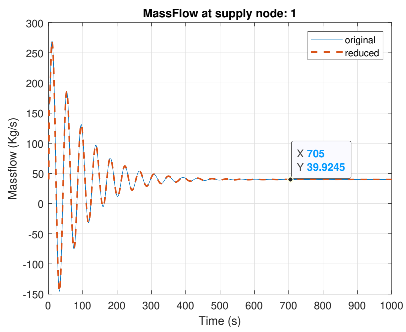
Table 4.4: Given Data of Variable Cyclic Network

Next, we take length and area of each pipe as shown in Table 4.5

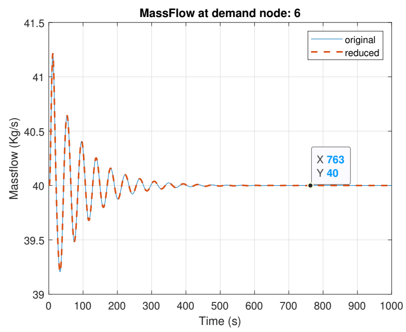
Pipe	**Length** (m)	**Area** (m^2)
Pipe12	1000	0.635
Pipe23	800	0.585
Pipe24	750	0.197
Pipe35	350	0.443
Pipe45	300	0.202
Pipe56	1200	0.987

Table 4.5: Length and Cross-Sectional Area of Variable Cyclic Network

We choose simulation time $t = 1000$ *seconds* for the above input parameters. Figure 4.16a and 4.16b show the mass flow at demand and supply node.



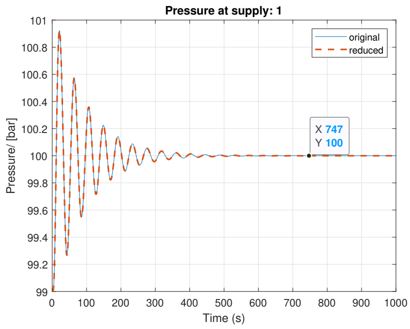
(a) Mass Flow at Supply Node



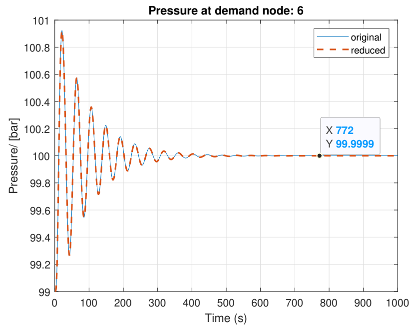
(b) Mass Flow at Demand Node

Figure 4.16: Cyclic Network 2 Mass Flow

Next, we look into pressure of the network at input and output node generated from both, original and reduced model. The pressure with respect to time can be seen in Figure 4.17a and 4.17b



(a) Pressure at Supply Node



(b) Pressure at Demand Node

Figure 4.17: Cyclic Network 2 Pressure

The size matrix A is 883x883 due to which the computational time to solve the model is **15.6 sec**. To save the computational time we have applied POD which reduces the size of matrix A to 10x10 and reduced time is **4.6 sec** only. The relative error between the output of original and reduced system is shown in figure 4.18, from which it is concluded that POD is a very efficient technique for reduction because it reduces 73% of the computational time of the network by reducing the states to more than half of the original states.

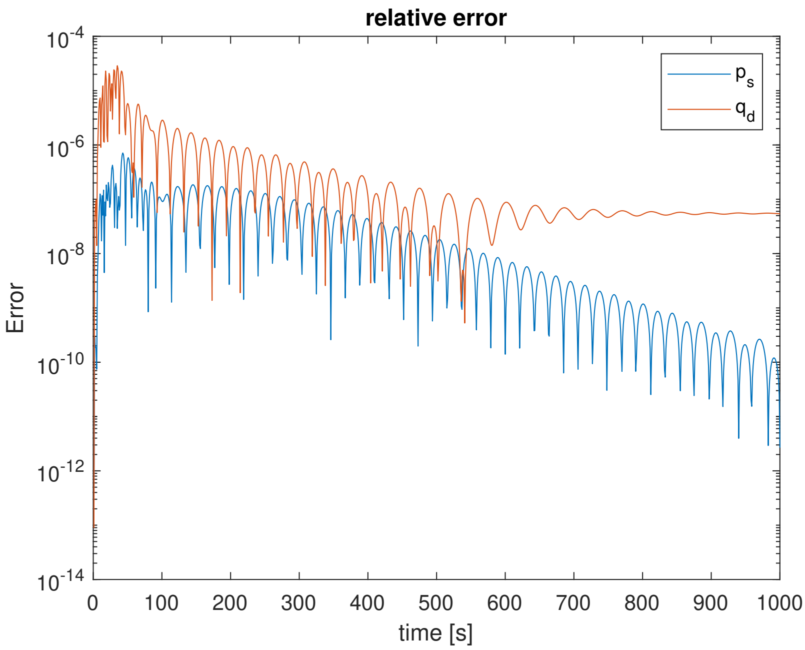


Figure 4.18: Cyclic Network 2 Relative Error

4.3 Medium Network

The specification of the network in fig 4.19 are shown in table 4.6. The initial system has one source node, which represents the natural gas supply through the city gates and then divides and expands into 30 demands nodes, which represents the customers. The mesh size is taken as 50 m, which means that an auxiliary node is being added after every 50 m in the pipeline network. The network consists of total 59 nodes.

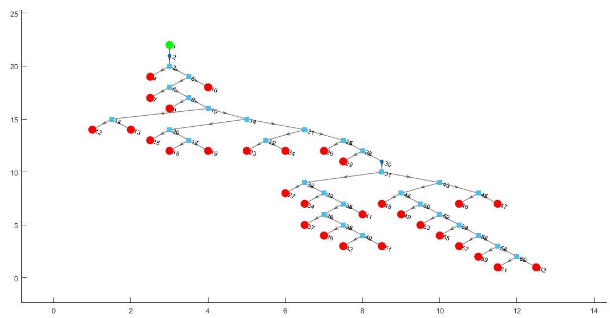


Figure 4.19: Large Gas Network

Table 4.6: Parameters used in the gas flow equation of the network

Pipe Number	**From Node**	**To Node**	**Node Type**	**Pipe length (m)**	**Pipe Friction Factor**	**Pipe Diameter (m)**	**Gas state (γ)**
1	1	3	supply	1000	0.021	0.9997	1.46745319
2	3	4	demand	920	0.021	0.9997	1.46745319
3	3	5	junction	940	0.021	0.9997	1.46745319
4	5	6	junction	950	0.021	0.9997	1.46745319
5	5	16	demand	980	0.021	0.9997	1.46745319
6	6	7	demand	1020	0.021	0.9997	1.46745319
7	6	8	junction	1050	0.021	0.9997	1.46745319
8	8	9	demand	1030	0.021	0.9997	1.46745319
9	8	10	junction	920	0.021	0.9997	1.46745319
10	10	11	junction	940	0.021	0.9997	1.46745319
11	10	14	junction	950	0.021	0.9997	1.46745319
12	11	12	demand	980	0.021	0.9997	1.46745319
13	11	13	demand	1020	0.021	0.9997	1.46745319

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Table 4.6 – *Continued from previous page*

Pipe Number	From Node	To Node	Node Type	Pipe length (m)	Pipe Friction Factor	Pipe Diameter (m)	Gas state (γ)
14	14	20	junction	1050	0.021	0.9997	1.46745319
15	14	21	junction	970	0.021	0.9997	1.46745319
16	17	18	demand	920	0.021	0.9997	1.46745319
17	17	19	demand	940	0.021	0.9997	1.46745319
18	20	15	demand	950	0.021	0.9997	1.46745319
19	20	17	junction	980	0.021	0.9997	1.46745319
20	21	22	junction	1020	0.021	0.9997	1.46745319
21	21	25	junction	910	0.021	0.9997	1.46745319
22	22	23	demand	1000	0.021	0.9997	1.46745319
23	22	24	demand	920	0.021	0.9997	1.46745319
24	25	26	demand	940	0.021	0.9997	1.46745319
25	25	28	junction	950	0.021	0.9997	1.46745319
26	28	29	demand	980	0.021	0.9997	1.46745319
27	28	31	junction	1020	0.021	0.9997	1.46745319
28	31	32	junction	1050	0.021	0.9997	1.46745319
29	31	43	junction	1000	0.021	0.9997	1.46745319
30	32	27	demand	920	0.021	0.9997	1.46745319
31	32	33	junction	940	0.021	0.9997	1.46745319
32	33	34	demand	950	0.021	0.9997	1.46745319
33	33	35	junction	980	0.021	0.9997	1.46745319
34	35	36	junction	1020	0.021	0.9997	1.46745319
35	35	41	demand	1050	0.021	0.9997	1.46745319
36	36	37	demand	1000	0.021	0.9997	1.46745319
37	36	38	junction	920	0.021	0.9997	1.46745319
38	38	39	demand	940	0.021	0.9997	1.46745319
39	38	40	junction	950	0.021	0.9997	1.46745319
40	40	42	demand	980	0.021	0.9997	1.46745319
41	40	51	demand	1020	0.021	0.9997	1.46745319
42	43	44	junction	1050	0.021	0.9997	1.46745319
43	43	45	junction	1000	0.021	0.9997	1.46745319
44	44	48	demand	920	0.021	0.9997	1.46745319
45	44	50	junction	940	0.021	0.9997	1.46745319
46	45	46	demand	950	0.021	0.9997	1.46745319

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Table 4.6 – *Continued from previous page*

Pipe Number	**From Node**	**To Node**	**Node Type**	**Pipe length** (m)	**Pipe Friction Factor**	**Pipe Diameter** (m)	**Gas state** (γ)
47	45	47	demand	980	0.021	0.9997	1.46745319
48	50	49	demand	1020	0.021	0.9997	1.46745319
49	50	52	junction	1050	0.021	0.9997	1.46745319
50	52	53	demand	1000	0.021	0.9997	1.46745319
51	52	54	junction	920	0.021	0.9997	1.46745319
52	54	55	demand	940	0.021	0.9997	1.46745319
53	54	56	junction	950	0.021	0.9997	1.46745319
54	56	57	demand	980	0.021	0.9997	1.46745319
55	56	58	junction	1020	0.021	0.9997	1.46745319
56	58	59	demand	1050	0.021	0.9997	1.46745319
57	58	60	junction	980	0.021	0.9997	1.46745319
58	60	61	demand	1020	0.021	0.9997	1.46745319
59	60	62	demand	1050	0.021	0.9997	1.46745319

Figure 4.20 shows the value of mass flow at different demand nodes 12, 16, 61 and 62.

Figure 4.21 shows the values of pressure at different demand nodes 12, 16, 61 and 62.

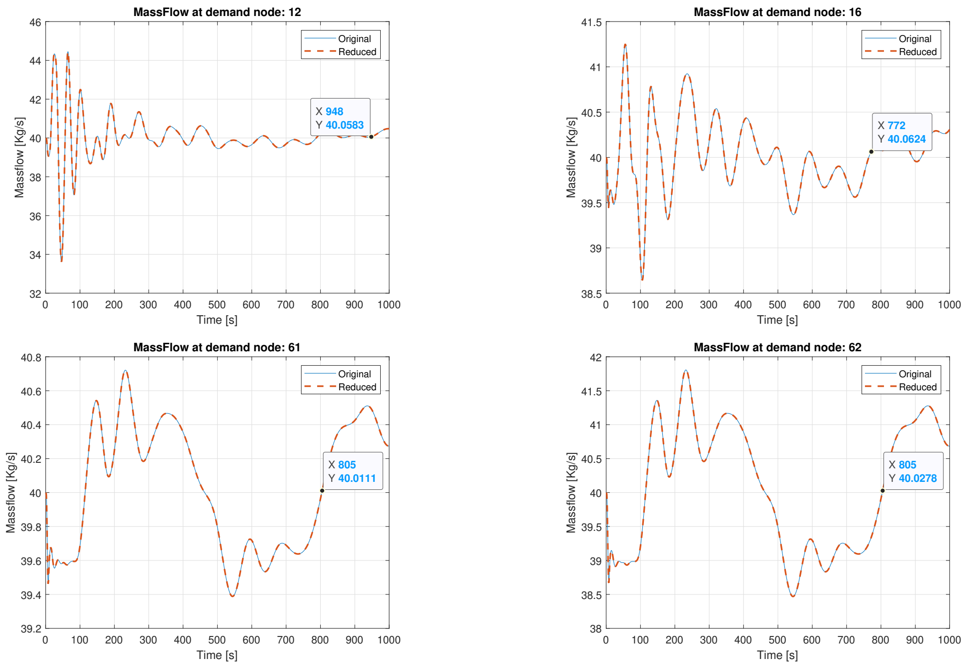

Figure 4.20: Large Network flow rate at demand nodes

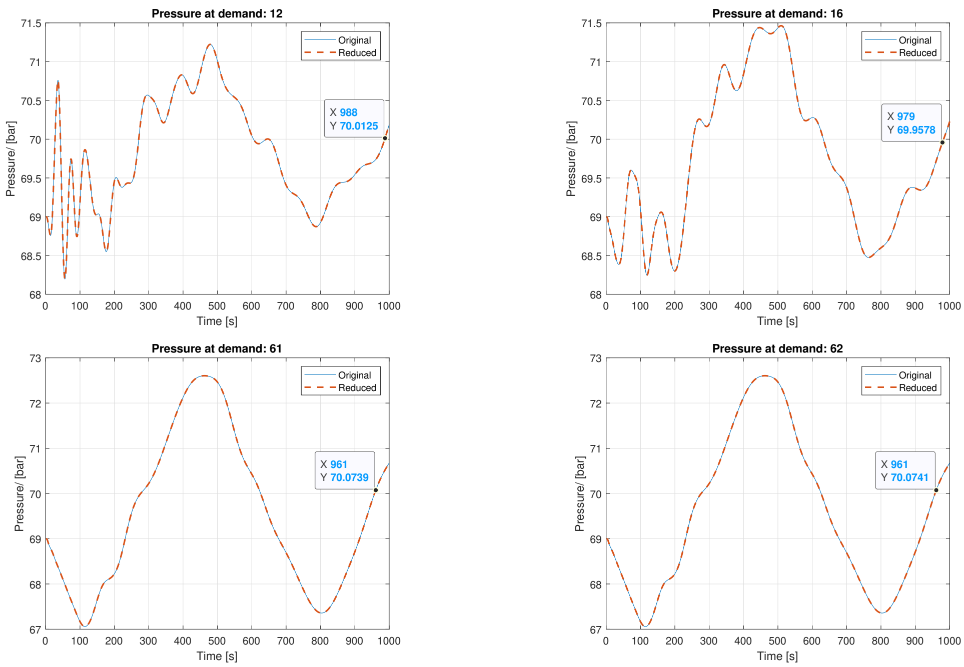

Figure 4.21: Large Network pressure at demand nodes

For reduction, total 1245 states of the network is reduced to 100 states only. The computational time of solve the original system is **18 min**, while POD only takes **31 sec** to solve the reduced system having the same output as that of the original system. The relative error of the reduced and original system is shown in the figure 4.22. The output values of pressure and mass flow are shown in table 4.7.

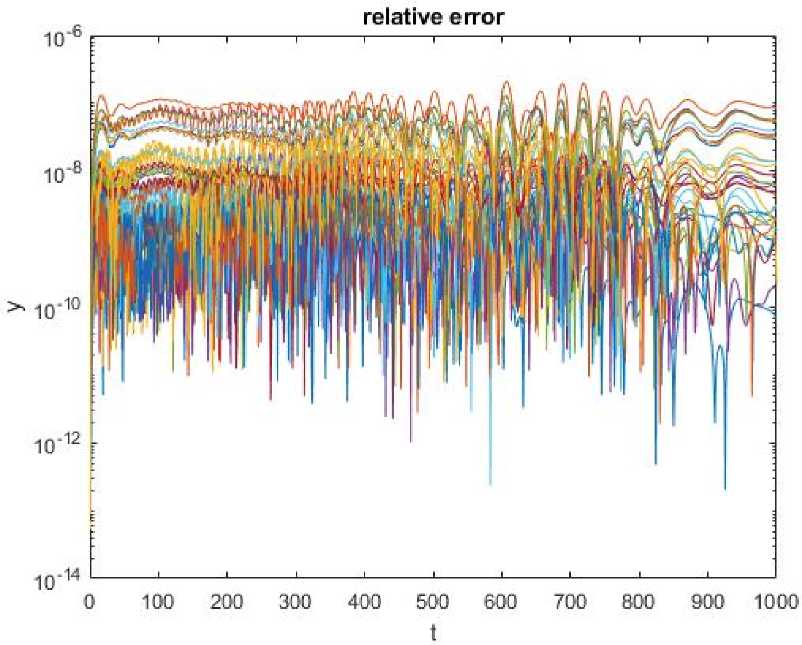

Figure 4.22: Relative Error of original and reduced system

Table 4.7: Output values of Reduced Network

Pipe Number	**From Node**	**To Node**	**Node Type**	**Pressure from MOR** *(bar)*	**Mass flow from MOR**
1	1	3	supply	70	1200
2	3	4	demand	69.02969135	43.21400107
5	5	16	demand	68.99925637	39.91950396
6	6	7	demand	68.99910686	39.90332008
8	8	9	demand	68.9991061	39.90323829
12	11	12	demand	68.9991061	39.90370692
13	11	13	demand	68.9991061	39.90370692
16	17	18	demand	68.9991061	39.90370923
17	17	19	demand	68.9991061	39.90370923

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Table 4.7 – *Continued from previous page*

Pipe Number	From Node	To Node	Node Type	Pressure from MOR (*bar*)	Mass flow from MOR
18	20	15	demand	68.9991061	39.90324018
22	22	23	demand	68.9991061	39.90370692
23	22	24	demand	68.9991061	39.90370692
24	25	26	demand	68.9991061	39.90323789
26	28	29	demand	68.9991061	39.90323788
30	32	27	demand	68.9991061	39.90323789
32	33	34	demand	68.9991061	39.90324019
35	35	41	demand	68.9991061	39.9032402
36	36	37	demand	68.9991061	39.90324021
38	38	39	demand	68.9991061	39.90324251
40	40	42	demand	68.9991061	39.90370924
41	40	51	demand	68.9991061	39.90370924
44	44	48	demand	68.9991061	39.9032379
46	45	46	demand	68.9991061	39.90370692
47	45	47	demand	68.9991061	39.90370692
48	50	49	demand	68.9991061	39.90324019
50	52	53	demand	68.9991061	39.9032402
52	54	55	demand	68.9991061	39.9032402
54	56	57	demand	68.9991061	39.90324021
56	58	59	demand	68.9991061	39.90324251
58	60	61	demand	68.9991061	39.90370924
59	60	62	demand	68.9991061	39.90370924

4.4 Actual Network

In my thesis I have taken the gas network of NUST H-12, Islamabad. The network was given to us by Project Management Office (PMO) NUST. They have provided us the gas bill of the month of December 2020 of NUST hostels. For the better visualization of the network ArcGIS pro is used to plot NUST gas network. The network shows hostels of the NUST as the demand nodes and the main supply of Gas is given to the network through main pipe shown through different colour in the figure 4.23 below :

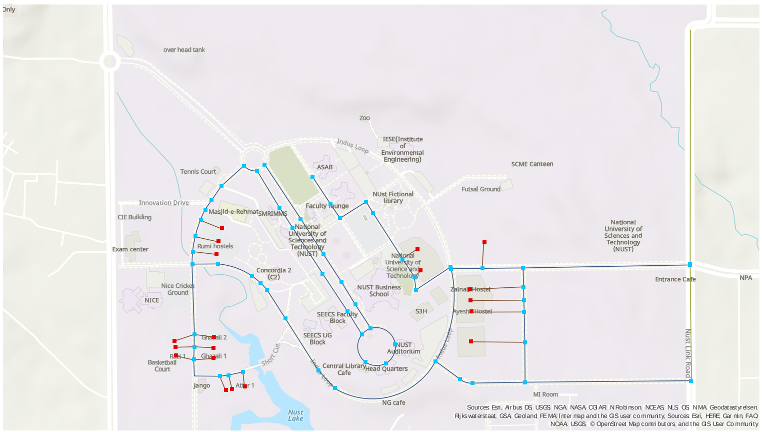


Figure 4.23: NUST Hostels Gas Network

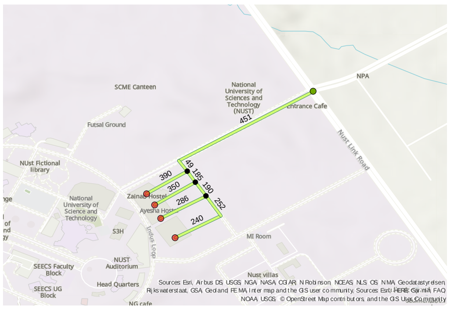
For our own convenience we have taken only four hostels under consideration Zainab Hostel, Zainab hostel kitchen, Ayesha hostel and Khadija hostel. Mass flow rate and pressure of these hostels are calculated using the Hm3 values given in the gas bill. The values of mass flow per sec and pressure is shown in table 4.8 below:

Summary of gas bills December 2020						
Location	**Cubic hectometer** (HM3)	**Metric Million British Thermal Unit** (MMBTU)	**Cubic meter** m^3	**Density** kg/m^3	**Mass flow** kg/sec	**Pressure** Psi
Zainab Hostel	8.167	29.968	816.7	0.8	5.85	15.54263234
Zainab hostel kitchen	7.746	28.425	774.6	0.8	5.55	15.54263234
Ayesha hostel	18.413	67.564	1841.3	0.8	13.20	15.54263234
Khadija hostel	20.785	76.269	2078.5	0.8	14.90	15.54263234

Table 4.8: Calculation of Mass Flow rate at each demand node

The directed graph of the network along with the GIS map of NUST network is shown in 4.24 . The network consists of 7 main pipes. Node 1 is considered as the supply node and nodes 3,5,7 and 8 are considered as demand nodes. Names of these nodes are

Zainab Hostel, Zainab Hostel Kitchen, Ayesha hostel and Khadija hostel, respectively.



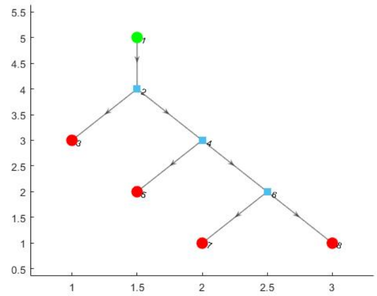


Figure 4.24: NUST Network

For the simulation of the network pressure at the supply node is given as 16 psi. The details of the network such as length of the pipes and boundary conditions of the network is shown in the table 4.9 and 4.10. The modelling of the network is done as isothermal system, temperature of the system is taken as $288.7K$.

Pipe Number	**From Node**	**To Node**	**Node Type**	**Pipe length** (m)	**Pipe Friction Factor**	**Pipe Diameter** (m)	**Gas state** (γ)
1	1	2	Supply	500	0.021	0.9997	1.46745319
2	2	3	Demand	390	0.0217	0.9997	1.46745319
3	2	4	Junction	185	0.021	0.9997	1.46745319
4	4	5	Demand	350	0.021	0.9997	1.46745319
5	4	6	Junction	190	0.021	0.9997	1.46745319
6	6	7	Demand	286	0.021	0.9997	1.46745319
7	6	8	Demand	492	0.021	0.9997	1.46745319

Table 4.9: Parameters used in the gas flow equation of the network

Pipe Number	From Node	To Node	Node Type	Pressure (psi)	Mass flow (kg/s)
1	1	2	Supply	16	–
2	2	3	Demand	–	5.854
4	4	5	Demand	–	5.553
6	6	7	Demand	–	13.199
7	6	8	Demand	–	14.9

Table 4.10: Boundary Conditions of pressure and Mass flow

The size of the matrix A of the original network is 485x485 which means that to solve the mathematical model of the network 485 differential equations are used for calculation of the output of the network. To reduce the complexity of the network model order reduction is applied due to which network equations are reduced from 485 to 10 only keeping the input and output characteristics of the system unchanged. Figure 4.25 shows the output of the original and reduced system's mass flow at supply and demand nodes. Mass flow rate at supply node is equal to 39.5 kg/s because the value of mass flow rate required at the input junction is equal to sum of individual mass flow at each demand node.

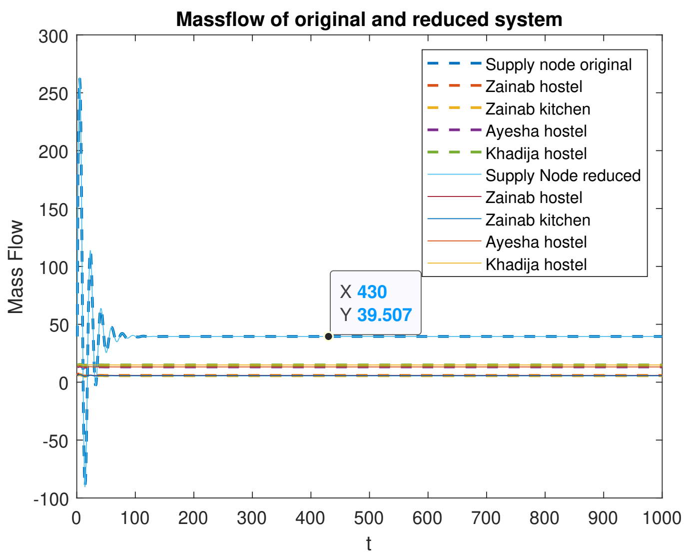


Figure 4.25: Mass Flow of Original and Reduced system

Pipe Number	From Node	To Node	Node Type	Actual values of pressure (psi)	Actual Values of Mass flow (kg/s)	Pressure from MOR (psi)	Mass flow from MOR (kg/s)	Relative Error of Pressure %
1	1	2	Supply	16	39.506	15.8690119	39.49391961	0.8
2	2	3	Demand	15.54263	5.854	15.26303588	5.85362129	1.8
4	4	5	Demand	15.54263	5.553	15.12794447	5.552847394	2.7
6	6	7	Demand	15.54263	13.199	15.05589642	13.19908503	3.1
7	6	8	Demand	15.54263	14.899	15.03827624	14.90000286	3.2

Table 4.11: Pressure and mass flow details of the network from actual and MOR values

The comparison of actual and reduced values of the network is shown in table 4.11. From the relative error of the pressure values it is observed that the estimated value of pressure decreases from demand node 3 to 7. The increase in the relative error is because of the different length of pipes. If the length of pipe is longer than the error is high. From this table we can also find the abnormalities of the network such that if we assume that for a specific pipe in the network if the value of relative error is higher than as calculated in the table then we can say that there is occurrence of some losses in that pipeline.

Chapter 5

Conclusions

The thesis was aimed towards developing a much efficient reduced order model for simulating the dynamics of non-linear gas distribution pipeline network displaying the actual and predicted values of mass flow at pressure at demand nodes. The dynamics of the gas distribution pipeline network is modelled using 1D Euler equations and further finite volume discretization is used for converting the PDE's into ODE's to represent the network in the form of state space model. The network generated is computationally expensive because of the large number of ODE's generating after the discretization of the network. To overcome this problem model reduction is used for reducing the model keeping input/output characteristics of reduce and original models almost similar. For the model reduction Proper orthogonal decomposition was implemented on the non-linear state space model of the network. Results were simulated for various fork and cyclic networks for the reduced and original systems. From the results we have seen that POD provides efficient results even when the model is reduced to more than half of its states. Results also show the reduction in time and also maintain adequate approximation of input and output relation between original and reduced systems. Also, a small portion of an actual gas network of NUST is also modelled using 1D Euler equations and the reduced results are validated with the actual value of pressure and mass flow. However, the only drawback of POD is that it does not reduce the non linear function, as a result, even if the overall dimension of the POD model is smaller, the non linear term's complexity remains the same.

5.1 Future Works

The current system was modelled based on the nonlinear term due to the gravity term of 1D Euler equation for the simplification of the overall model design. In order to extend our current system the assumptions made in chapter 3 would be neglected and one should observe the system behaviour by introducing the gravity and inertial term and also its affects over the network model. For reduction Discrete Empirical interpolation method should be applied to gas distribution pipeline networks for the reduction of nonlinear term as well. The network modelled is primarily focused on the gas distribution networks and can be extended to transportation networks such as power networks or water distribution networks.

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