Performance Analysis of Massive MIMO Systems in the Presence of Composite Shadowing Fading

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Approval

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To My Parents

Certificate of Originality

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Abstract

Massive MIMO (multiple-input multiple-output) has been identified as a key technology for next generation cellular systems. This thesis considers a multi-cellular system with large antenna arrays at the base station (BS) and single antenna user terminals (UTs), operating in a time division duplex (TDD) mode, under a composite fading-shadowing environment. In the uplink transmission, the pilot contamination occurs as the UTs transmit pilots to their respective BSs, and the serving BS estimates the channel state information using a minimum mean squared error estimation. This channel information is further used to design beamforming (BF) and regularized zero-forcing (RZF) precoders for downlink (DL) transmission. We analyze the ergodic rates for DL transmission using different precoding schemes and varying shadowing intensity. Next we analyze the outage probability of multi-user, multi-cellular system using uplink transmission and maximum ratio combining (MRC) receiver. It has been observed that shadowing does not average out as we increase the number of antennas as opposed to multi-path fading, and the severity of shadowing badly affects the performance of massive MIMO systems. Also probability of outage decreases as we decrease the severity of shadowing and vice versa.

List of Abbreviations

MIMO	Multiple input multiple output
SNR	Signal to noise ratio
SINR	Signal to interference plus noise ratio
MRC	Maximum ratio combining
RZF	Regularized zero forcing
MMSE	Minimum mean squared error
PDF	Probability density function
CDF	Cumulative distribution function

Contents

List of Figures

Chapter 1

Introduction

Urge to gain more performance from wireless systems leads us to use multiple antennas. Now it is a well known fact that increasing number of antennas can improve data rate without using extra bandwidth [10]. Multiple antennas benefits from spatial diversity and scattered data to improve performance of system.

1.1 What is MIMO System?

Multiple-input multiple-output (MIMO) system is antennas technology that uses multiple antennas at transmitting and receiving end. When MIMO communicates with multiple user terminals (UT) at the same time, it is called multi-user MIMO. MU-MIMO is used in wireless broadband standards like 4G LTE (Long Term Evolution)and LTE-Advanced. MIMO system is based on the concept that more the antennas are at base station more improvements in the performance of system could be made. This system uses diversity present in environment to improve the quality of communication [13]. MIMO systems also makes possible to send data into some specific direction. It improves the energy efficiency thus catering the ever increasing demand of data rate [17]. [1]

[1]Fig. 1.1 Source: "Multiple-Input-Multiple-Output (MIMO) Systems (Deke Guo National University of Defense Technology)"

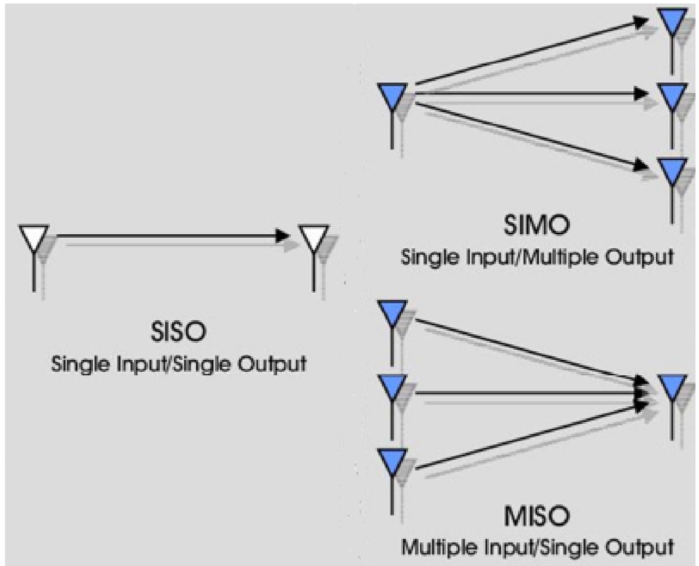

Figure 1.1: SISO, SIMO and MISO Systems

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In cellular system bandwidth and power are limited resources, these systems can increase the capacity without using much power and bandwidth. These improvements in performance of system could be classified into four categories i.e data rate, reliability, energy efficiency, interference mitigation.

There are two ways to improve efficiency of system either with spatial diversity or by spatial multiplexing. Spatial diversity is created when we transmit multiple copies of same data on different channel links using multiple antennas. Spatial multiplexing is a technique in which we send different channel links created by MIMO systems to transmit multiple data sets, this results in increases in the data rate of system.This technique does not guarantee the improvement in bit error rate (BER) of system. However, in spatial diversity due to the presence of multiple copies of data, it is possible to reduce BER efficiently thus improving the data rate. It is not possible to use both spatial diversity and spatial multiplexing at the same time. The spatial degrees of freedom (Number of antennas) can be used to provide diversity gain or

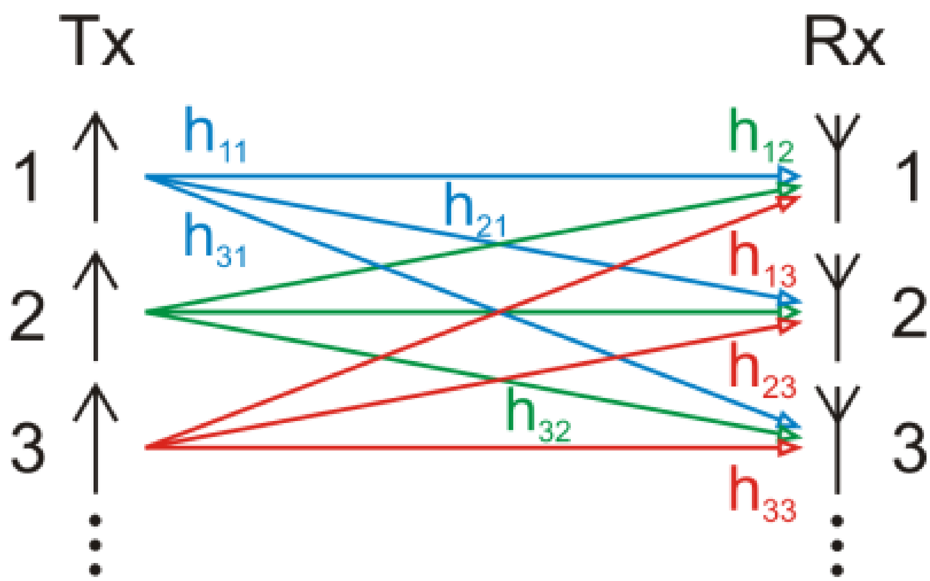

Figure 1.2: MIMO System

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Spatial multiplexing gain [15].

[2] These systems take advantage of multi path present in environment to enhance the reliability and capacity hence improving quality of service. These betterments are due to prsence of more than one antennas which insures multiplexing gains, array gains and interference cancellation. Multiple antennas provide more degrees of freedom to the propagation channel and improve the throughput and link reliability. Such systems exploit the phenomenon of multipath propagation, which is traditionally a drawback in wireless communications. The improvements in the performance offered by MIMO systems are due to diversity gain, spatial multiplexing gain, array gain and interference reduction.

[2]Fig. 1.2 Source: "https://en.wikipedia.org/wiki/MIMO"

1.2 Massive MIMO

The ever increasing demand of high data rates has initiated a spark in both academia and industry to work on the next generation of cellular systems, namely the fifth generation (5G) networks. 5G will be envisioned with a multitude of technologies such as device-to-device, millimeter wave communications, multi radio-access technologies (RATs) and heterogeneous topologies, and the very large-scale multiple-input multiple-output (MIMO) systems, also known as Massive MIMO. A massive MIMO system uses hundreds of antennas at the base station (BS) to simultaneously serve tens of users. It provides high throughput [11], [12], allowing to reap all the benefits of a conventional MIMO on a larger scale by using cheap antennas, thereby increasing data rates and improving the energy efficiency [17]. The large antenna arrays reduce the uplink (UL) and downlink (DL) transmit power through coherent combining, thus achieving higher data rates. It will provide the base of infrastructure of the future digital society by connecting Internet of people and Internet of things with clouds and other networks. Overall, Massive MIMO facilitates the development of future broadband services, which will efficiently utilize the energy and spectrum. Also it will be secure and robust [27].

Massive MIMO shown in Fig. 1.3 [3] operates in TDD mode. There are two reasons for operating in TDD mode. It needs knowledge of channel on uplink and downlink transmissions. In uplink communication, it is easy for BS to estimates the channel state information (CSI) using training sequence. First on downlink massive MIMO needs to send pilots to user terminal (UT) to estimate the CSI. These pilots need to be mutually orthogonal and due to the presence of hundreds of antennas it is difficult to create such large number of orthogonal pilots. Second, the number of

[3] Fig. 1.3 Source: "Joint Power Allocation and User Association Optimization for Massive MIMO Systems"

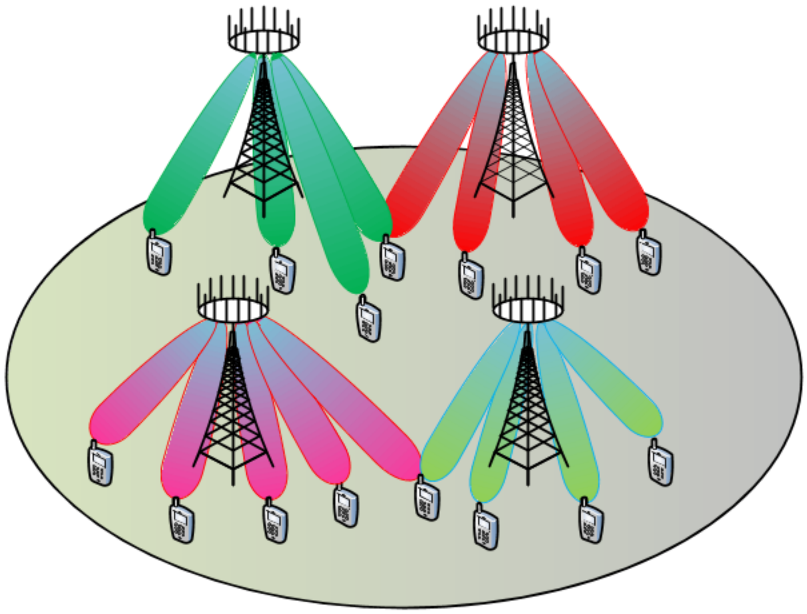

Figure 1.3: massive MIMO System

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channel responses that each terminal needs to estimate is also proportional to the number of base station antennas. Therefore we operate the massive MIMO in TDD system and assume that downlink channel is reciprocal of uplink channel.

It concludes that in a massive MIMO system, the user terminals (UTs) do not require any channel state information (CSI), however the BS acquires CSI and uses precoding schemes to nullify the effects of channel, thus reducing the overhead on UTs. For DL precoding, CSI is acquired through UL training [23], where the length of training sequence is dependent on the number of users and not on the number of antennas, thus additional antennas do not cause any overhead.

1.3 Why Massive MIMO?

Massive MIMO can increase the capacity of system by increasing the energy efficiency and efficient use of resources. This is based on the concept that with large number of antennas we can focus energy into some specific direction. This is done by coherent superposition of wavefronts. BS can add wave fronts constructively in some specific direction and destructively on other directions. This results in reduction of interference present in system.

It uses cheap single antennas that use low power and combine them in a way that increases the system efficiency. With massive MIMO, high power components at BS could be replaced by hundreds of low power cheap antennas. These antennas use energy in the range of mili-watts. It also reduces the need of amplifiers which are linear and accurate rather it urges to see the combined effects of these components. It is based on the law of large numbers which states that the effects of imperfect hardware are averaged out. This property makes the system much more robust as the system will continue to work despite the failure of one or two components. This leads us to operate massive MIMO using much less power than conventional systems. Hence we can power BS with wind or solar energy in remote areas where no electricity is available.

Massive MIMO can also reduce latency. Latency is caused by fading, it can cause the received signal strength to be very small at some times. When signal is sent through fading channel, it scatters and multi-paths are created. Sometimes these multi-paths interfere destructively which makes it difficult to reconstruct the signal. hence causing he latency. Massive MIMO reduces the effects of fading thus reducing the latency.

A huge advantage of massive MIMO is, it does not require rich scattering environment to operate. It provides excess degrees of freedom in the form of multiple antennas which could be used to create diversity hence improving the efficiency of system. Also in scattering environment, it can improve the diversity by sending multiple copies of same data.

1.4 Problem statement

Downlink data rate analysis of a multi-cellular massive MIMO system in the presence of composite shadowing-fading by applying beam-forming (BF) and regularized zero forcing (RZF) precoders with imperfect CSI on BS obtained by uplink training pilots and outage probability analysis with uplink transmission using maximum ratio combining (MRC) receiver with perfect CSI.

1.5 Thesis Contribution

In this research work, we consider a composite fading-shadowing channel, modeled by K-distribution, which is a combination of Rayleigh and gamma distributions [28], with imperfect CSI estimated by uplink training symbols. This work provides a simple model for composite fading-shadowing environment, unlike other models in which two RVs are multiplied to represent both fading and shadowing, which mostly ends in complicated mathematical forms. In [16] and [22], the authors restrict attention only to UL transmission. In this work, we consider both UL and DL aspects. We analyze the rates for DL transmission by assuming channel reciprocity, thus imperfect CSI will be utilized in designing DL precoders. In the UL, we estimate the CSI using minimum mean squared error (MMSE), whereas after obtaining the estimates of CSI, the BS precodes the downlink data using beam-forming (BF) and regularized zero forcing (RZF) precoders. Also, we perform outage probability analysis in the presence of K-fading using MF approach and a closed form expression of outage probability for composite shadowing/fading is derived. Simulation results provide proof of the derivation of outage probability. It also shows that as we decrease the severity of shadowing outage probability decreases and vice versa. We present DL ergodic rate analysis for both precoders by using simulations and show that severe shadowing in the environment limits the performance of the massive MIMO system under consideration.

1.6 Thesis Organization

The rest of the thesis is organized as follows: chapter 2 provides the literature review and highlights the relevant work on the composite shadowing fading environment. Chapter 3 describes the massive MIMO system model, in which we define the UL and DL data transmission methods. Chapter 4 describes channel estimation, downlink data rates using different linear precoders and provides the outage probability analysis. Chapter 5 provides the numerical results, discussions for different scenarios. Finally chapter 6 concludes the work with some future suggestions.

Chapter 2

Literature Review

The mobile communication systems have been enhancing day by day. Communication devices are now more powerful equipped with much improved processing units. These devices can process data at much higher rate hence demanding more data rates. The The European Mobile Observatory (EMO) pointed out that there has been a 92 percent growth in mobile broadband per year since 2006 [56]. For wireless communication these demands are increasing several times as more and more devices are connecting with wireless networks. To accommodate all these devices MIMO technology was introduced in cellular networks. MIMO systems was first introduced in 4G cellular networks. Basic idea of 4G network is that they provide high data rate to numbers of users at the same time [1]. 4G system uses advanced radio interference with orthogonal frequency-division multiplexing (OFDM), MIMO technologies. It can support data rates of up to $1Gps$ for low mobility and $100Mbps$ for high mobility [2].

Abrupt increase in number of wireless equipments concludes that 4G networks are not enough to fulfill the demand of consumers. Also cellular network uses high frequency waves to transmit data which means high energy is required at BS. Also high spectral efficiency, high mobility, seamless coverage are other are also the challenges that needs to be taken care of. To accommodate all these challenges, a new system needs to be design which is energy efficient, provides high data rate with seamless coverage. All these challenges lead us to work on 5G networks since 4G network has already reached to its theoretical limit. 5G uses large scale MIMO technology to provide energy efficiency, direct data to some specific direction and low latency.

Massive MIMO are being implemented in device o device (D2D) communication [3], [4], [5], [6]. In UL massive MIMO can nullify the effects of D2D-to-cellular interference [3]. It provides assistant in reducing pilot symbol's length by quickly acquiring CSI. It also helps in improving power use by using an iterative scheme [4].

Massive MIMO systrms are also being deployed in heterogeneous networks (HetNet) using different types of cells such as macro-, micro-, pico-, and femtocells. These cells enhance network capacity, coverage performance, and energy efficiency [7], [8], [9]. These cells share same frequency resource hence creating interference. Due to the co location of users in, we need to direct data into some specific direction creating less interference. Massive MIMO helps in reducing interference by directing data to particular user.

Our focus is on analysis of multi-cellular, multi-user massive MIMO system operating TDD mode. Due to the large number of degree of freedom provided by thousands of antennas, massive MIMO provides high data rate through linear precoding such as maximum ratio combining and Minimum Mean Square Error in uplink and Beamforming and Zero Forcing precoding in downlink.

In [11], [18] and [23], the rate analysis in the presence of pilot contamination and inter-cell interference over small-scale fading channel is performed, ignoring the effects of large-scale fading. For instant [11] considers multi-user, multi-cellular massive MIMO with infinite number of antennas. Due to the presence of multi-cells pilot contamination occurs as orthogonal pilots are used in different cells. This paper also shows that despite using infinite number of antennas inter-cellular interference still effects the performance of system. In [18], authors also study the effects of pilot contamination ant then they provide a solution to mitigate this problem. A compact analysis of massive MIMO in UL and DL scenarios is described in [23]. This paper considers multi-cellular, multi-user system with Rayleigh fading channels, Additive White Gaussian Noise (AWGN) and inter-cell interference. It estimates the CSI which is contaminated due to pilot reuse. Matched filter (MF) and minimum mean square error (MMSE) detectors are used in UL transmission to estimate the CSI and beam-forming (BF) and regularized zero-forcing (RZF) precoders are used to precode the DL data. They provide asymptotic approximations of achievable data rate using these detectors/precoders.

It has been observed that the small-scale fading is averaged out due to the presence of a large number of antennas, however, the largescale fading, known as shadowing, still remains a challenge for realizing a practical massive MIMO system. The combined effect of large and small-scale fading is generally modeled by Rayleigh-lognormal (RL) product distribution [32]. However, the rate analysis over composite fading-shadowing channel requires the probability density function (PDF) of the signalto-interference plus noise ratio (SINR). This PDF is not available in closed-form and hence approximation techniques are required, e.g., [19]-[21]. In [19], author analyzes the UL of multi-user, single cell system, which operates under a composite Rayleigh fading and lognormal shadowing channel. Three linear receiver known as MRC, ZF and MMSE were consider while assuming perfect CSI. They also perform outage analysis under composite fading/shadowing environment. For this purpose, authors approximate the PDF of SNR and SINR using a new log-normal RV and then compare the results of three receiver. Paper shows that small-scale fading and noise could be averaged out by using more antennas, but large-scale fading still remains a serious concern.

To study the effects of shadowing, [16] used the generalized-K fading channels for capacity analysis while [22] and [19] used the composite RL model for outage analysis. However, all of these works have analyzed the uplink transmissions only where [6] has assumed perfect CSI, while [3] has taken into consideration both cases, i.e., perfect and imperfect CSI.

Chapter 3

System Model

Consider a muti-cell, multi-user system given in Fig.3.1 with L cells, where each cell is equipped with M antennas. Every cell is serving K single user terminals (UT) uniformly distributed in each cell.In this system, we use time division duplex (TDD) mode and uplink pilot training to gather channel state information. We then rely on channel reciprocity to counter the effects of fading and shadowing. As described in literature review chapter, most of the previous work on massive-MIMO considers only small-scale fading ignoring the shadowing (Large-scale fading) effects. In our work, we take both small scale fading and large scale fading into count and analyze their effects. Our work considers composite fading-shadowing environment which caters both small-scale and large-scale fading. Channel coefficients are assumed to be independent and identically distributed (I.I.D) random variables and their envelop follows K-distribution. We consider uplink and downlink transmission models as follows. [1]

[1] Fig. 3.1 Source: "www.idc.lnt.de/en/forschung/massive-mimo-systems/"

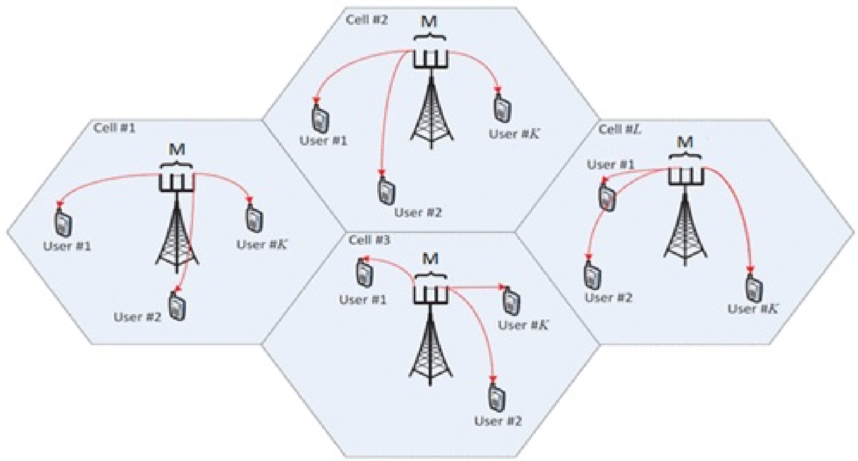

Figure 3.1: Massive-MIMO system with L cells, M antennas and K users

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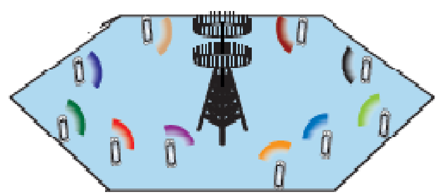

Figure 3.2: Uplink Transmission

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3.1 Uplink Transmission

The K single antenna UTs send their respective pilot sequence to the BS equipped with M antennas simultaneously serving all these UTs. Due to the reuse of pilots in other cells, pilot contamination occurs which causes imperfection in acquiring channel state information (CSI). Thus the received signal vector $\mathbf{y}_j^{ul} \, \epsilon \, \mathbb{C}^{M\times 1}$ at BS j is given by

$$\mathbf{y}_j^{ul} = \sqrt{\rho_{ul}} \sum_{l=1}^{L} \mathbf{H}_{jl}\mathbf{x}_l^{ul} + \mathbf{n}_j^{ul}, \tag{3.1}$$

2

where $\mathbf{H}_{jl} = [\mathbf{h}_{jl1}, \ldots, \mathbf{h}_{jlK}] \epsilon \mathbb{C}^{M\times K}$ is the channel matrix of K users from lth cell to jth cell, for any kth user and $\mathbf{h}_{jlk} \epsilon \mathbb{C}^{M\times 1}$ is the channel vector in cell l to base station j. The transmitted symbol matrix of K users and noise vector in the jth cell are $\mathbf{x}_l^{ul} = [x_{l1}^{ul}, \ldots, x_{lK}^{ul}]^T$ and $\mathbf{n}_j^{ul} \sim CN(0, \mathbf{I}_M)$, respectively and $\rho_{ul} > 0$ is the uplink transmit power. Each element of $\mathbf{h}_{jlk}$ is a product of i.i.d zero-mean complex Gaussian RV and square root of a gamma distributed RV given by

$$h = \sqrt{z}(g_R + jg_I) \quad ; \quad h \, \epsilon \, \mathbf{h}_{jlk} \tag{3.2}$$

where g_R, g_I are real and imaginary parts of a complex Gaussian RV and z denotes a gamma RV. It can be described as a complex Gaussian RV with a random variance following Gamma distribution. This implies that envelop of channel coefficients have Rayleigh distributed multi-path fading and Gamma distributed shadowing. Hence composite form of channel coefficients is Rayleigh-Gamma distributed. This composite form is actually defined as the K-distribution given by

$$f_{|h|}(|h|) = \frac{2}{\alpha\Gamma(\nu+1)} \left(\frac{|h|}{2\alpha}\right)^{\nu+1} K_\nu\left(\frac{|h|}{\alpha}\right), \tag{3.3}$$

where α indicates the scale parameter and ν denotes shape parameter. The shape parameter, ν, defines the severity of shadowing in our case. The intensity of shadowing increases for small values of ν and as $\nu \to \infty$, the effects of shadowing decreases and the distribution converge to Rayleigh fading distribution. In 3.3, $\Gamma(.)$ denotes the gamma function and $K_\nu(.)$ represents the modified Bessel function of second kind with order ν. Fig.3.4 shows K-distribution with different shaping parameters.

For the sake of simplicity, the envelope of the channel coefficients for the serving cell, i.e, $l = 1$ are assumed to follow K-distribution with shape parameter ν_o, whereas for $l = \{2, \ldots, L\}$, the fading envelope follows K-distribution with identical shape

[2] Fig. 3.2 source: "Massive MIMO: Ten Myths and One Critical Question"

parameter ν_I. Hence, we assume that the shadowing severity remains the same from all co-channel cells.

3.2 Downlink Transmission

After obtaining the CSI, respective data of all the users is transmitted simultaneously in available coherent time. Remember that, we need to send data in coherent time so that channel reciprocity could be taken into account. The received signal y_{jm}^{dl} at the mth user terminal in the jth cell is given as

$$y_{jm}^{dl} = \sqrt{\rho_{dl}} \sum_{l=1}^{L} \mathbf{h}_{ljm}^{H} \mathbf{s}_l + n_{jm}^{dl}, \tag{3.4}$$

where $\mathbf{s}_l \; \epsilon \; \mathbb{C}^{M\times 1}$ is the transmit vector of BS l, $n_{jm}^{dl} \sim CN(0,1)$ is additive white Gaussian noise (AWGN) and $\rho_{dl} > 0$ is transmit power of the jth BS. As mentioned earlier, we assume channel reciprocity, i.e., the downlink channel is the Hermitian transpose of uplink channel and the transmitted symbol vector is given by

$$\mathbf{s}_l = \sqrt{\varphi_l} \mathbf{W}_l \mathbf{x}_l^{dl}, \tag{3.5}$$

[3]

where $\mathbf{W}_l = [\mathbf{w}_{l1} \ldots, \mathbf{w}_{lk}] \; \epsilon \; \mathbb{C}^{N\times K}$ is a precoding matrix applied over the data symbols $\mathbf{x}_l = [x_{l1}^{dl}, \ldots, x_{lk}^{dl}] \; \epsilon \; \mathbb{C}^{K}$ for K UTs in cell l. The φ_l normalizes the average transmit power per UT of BS l, given by

$$\varphi_l = \frac{1}{\mathbb{E}\left[\frac{1}{k} tr(\mathbf{W}_l \mathbf{W}_l^H)\right]}, \tag{3.6}$$

[3]Fig.3.3Source: "www.commsys.isy.liu.se/en/research/projects/CENIIT-Radio-Resource-Management"

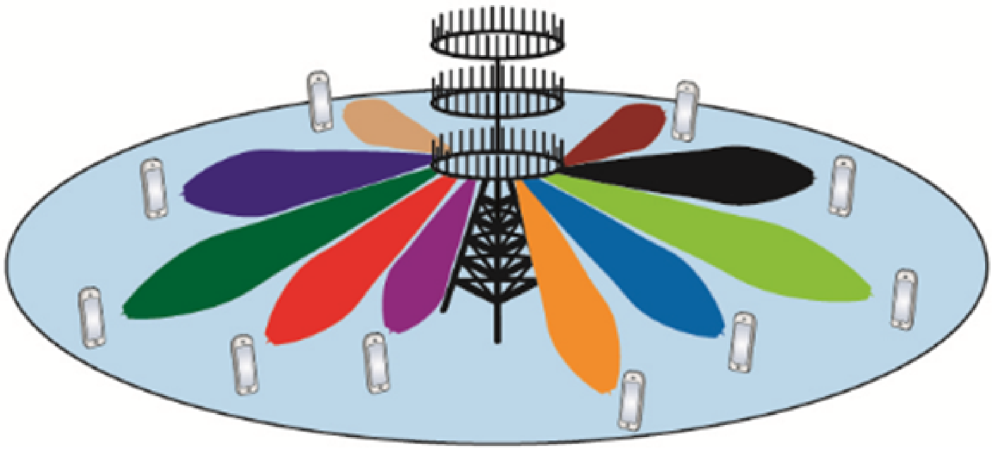

Figure 3.3: Downlink Transmission

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where $\mathbb{E}(.)$ denotes the expectation operator and $tr(.)$ defines the trace of matrix. The precoding vectors are designed such that effects of noise and interference are minimum.

3.3 K-Fading

The K-fading was first used in radar applications and for sea-echo cancellation [28], [39], [44]. It occurs when the resolution cell is much larger than the radiation wavelength and there are several number of scatters. In 5G cellular networks, usage of millimeter waves is being introduced. These waves have much smaller wavelengths, hence justifying the usage of K-fading in massive MIMO systems. Scattering experiments have shown that a class of distribution based on Bessel functions can be useful in non-Gaussian situations [45]. Therefor, we use K-distribution to represent composite shadowing fading. Traditionally Rayleigh-lognormal model is used for modeling shadowing fading, however it leads to complicated integral form. K-distribution is obtained when we replace log normal shadowing with Gamma shadowing. It is

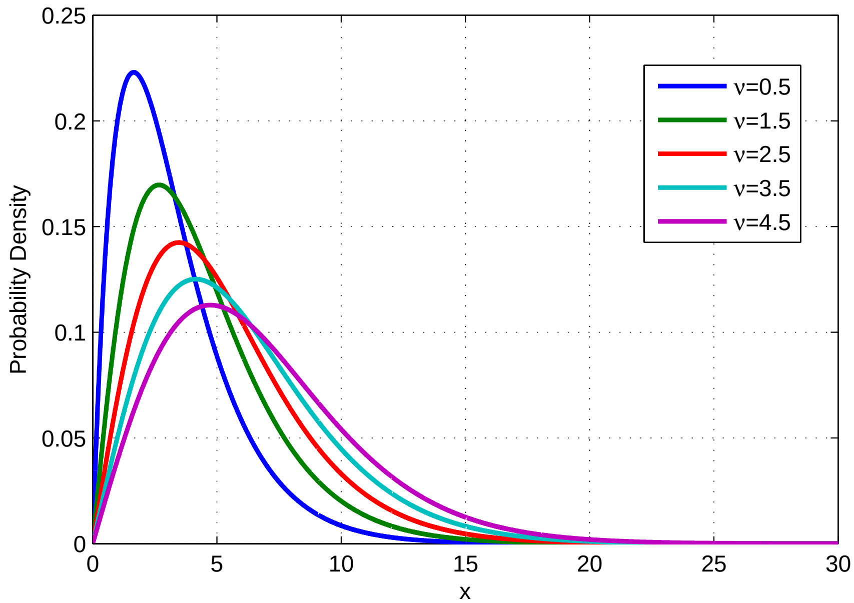

Figure 3.4: K-distribution with different shaping parameter

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given by

$$f_X(x) = \frac{2}{\alpha\Gamma(\nu+1)} \left(\frac{x}{2\alpha}\right)^{\nu+1} K_\nu \left(\frac{x}{\alpha}\right). \tag{3.7}$$

Its moments are defined by following equation

$$\mu_k = \mathbb{E}\left[x^k\right] = \frac{\Gamma(0.5k+1)\Gamma(\nu+1+0.5k)}{\Gamma(\nu+1)} (2\alpha)^k. \tag{3.8}$$

Moments are important as they will be used to find auto-correlation and cross-correlation of system. Fig. 3.4 shows K-distribution with different shadowing intensity parameters. Importance of ν could be described by calculating amount of fading (AF), which is given as [43]

$$AF = var[X^2]/(E[X^2])^2, \tag{3.9}$$

after using (3.8), final form of AF is given as

$$AF = \frac{\nu+3}{\nu+1}. \tag{3.10}$$

Chapter 4

Data Rate and Outage Probability Analysis

In this Chapter, we will be doing rate analysis over DL communication with imperfect CSI. Estimate of the CSI is obtained by using minimum mean square error (MMSE) estimator. After obtaining the CSI, we will use it to precode the DL data and cancel the effects of channel. Outage probability analysis over composite shadowing fading channel using UL communication is also provided in this chapter assuming perfect CSI obtained at BS. [1]

4.1 Channel Estimation

Each BS estimates $\hat{\mathbf{H}}_{jj}$ of its local channel $\mathbf{H}_{jj}$ by receiving orthogonal pilots from the UTs. Since other BSs are also estimating their local channels through an identical process by using same orthogonal pilots, the serving BS gets pilot contamination, which corrupts the estimation process. The jth BS estimates the channel vector $\mathbf{h}_{jjk}$

[1] Fig. 4.1 Source: "Large Scale MIMO in cellular Networks (Emil Bjornson)"

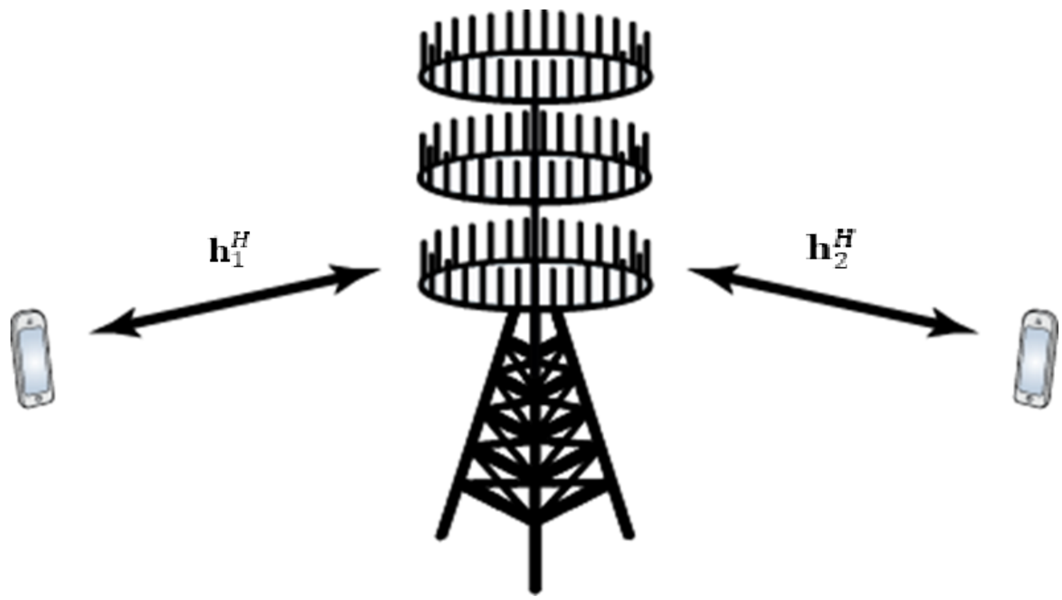

Figure 4.1: Uplink and Downlink channel

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for the kth user in jth cell, and the received pilot $\mathbf{y}_{jk}$ is given by[2]

$$\mathbf{y}_{jk} = \mathbf{h}_{jjk} + \sum_{l \neq j} \mathbf{h}_{jlk} + \frac{1}{\sqrt{\rho_{tr}}} \mathbf{n}_{jk}, \tag{4.1}$$

where $\mathbf{h}_{jlk}$ is interfering channel of kth user from cell l to cell j, and ρ_{tr} is the transmit power of UT k.

To estimate $\hat{\mathbf{h}}_{jjk}$ of $\mathbf{h}_{jjk}$, the classical minimum mean square error (MMSE) estimator is given as

$$\hat{\mathbf{h}}_{jjk} = \mathbf{R}_{jjk} \mathbf{Q}_{jk}^{-1} \mathbf{y}_{jk}^{tr}, \tag{4.2}$$

where $\mathbf{Q}_{jk} \epsilon \mathbb{C}^{M \times M}$ and $\mathbf{R}_{jjk} \epsilon \mathbb{C}^{M \times M}$ are the autocorrelation and cross correlation matrices, respectively. The autocorrelation matrix $\mathbf{Q}_{jk}$ is given by

$$\begin{aligned} \mathbf{Q}_{jk} &= \mathbb{E}\left[\mathbf{y}_{jk}\mathbf{y}_{jk}^H\right] \\ &= \mathbb{E}\left[\mathbf{h}_{jjk}\mathbf{h}_{jjk}^H\right] + \sum_{l \neq j}^{L} \mathbb{E}\left[\mathbf{h}_{jlk}\mathbf{h}_{jlk}^H\right] + \frac{1}{\sqrt{\rho_{tr}}} \mathbf{I}_M. \end{aligned} \tag{4.3}$$

[2]We assumed all-one vector for transmission, therefore, we omit the signal $\mathbf{x}_l$ from here onwards for ease of presentation.

From (4.3), we have $\mathbf{Q}_{jk} = \zeta \mathbf{I}_M$, where $\mathbf{I}_M$ is $M \times M$ identity matrix and ζ is given as

$$\zeta = \left((\nu_o + 1)(2\alpha)^2 + (L-1)(\nu_I + 1)(2\alpha)^2 + \frac{1}{\rho_{tr}} \right). \tag{4.4}$$

Similarly, the cross-correlation matrix $\mathbf{R}_{jjk}$ is given as

$$\mathbf{R}_{jjk} = \mathbb{E}\left[\mathbf{h}_{jjk}\mathbf{y}_{jk}^H\right]. \tag{4.5}$$

From (4.5), we have

$$\mathbf{R}_{jjk} = (\nu_o + 1)(2\alpha)^2 \mathbf{I}_M. \tag{4.6}$$

It can be noticed from (4.2) that the channel decoupling property can be used such that $\mathbf{h}_{jjk} = \hat{\mathbf{h}}_{jjk} + \tilde{\mathbf{h}}_{jjk}$, where $\tilde{\mathbf{h}}_{jjk}$ is the channel estimation error distributed as a complex normal RV with zero mean and covariance $\mathbf{Z}_{jk}$. Using (4.3) and (4.5), we have the following co-variance matrix of the channel estimation error

$$\begin{aligned} \mathbf{Z}_{jk}^{ul} &= \mathbb{E}\left[\tilde{\mathbf{h}}_{jjk}\tilde{\mathbf{h}}_{jjk}^H\right], \\ &= \mathbf{R}_{jjk} - \boldsymbol{\phi}_{jjk}, \end{aligned} \tag{4.7}$$

where we define

$$\boldsymbol{\phi}_{jjk} = \mathbf{R}_{jjk}\mathbf{Q}_{jk}^{-1}\mathbf{R}_{jjk}. \tag{4.8}$$

Simplifying (4.8) and putting the results in (4.7), we eventually obtain

$$\mathbf{Z}_{jk}^{ul} = \Theta \mathbf{I}_M, \tag{4.9}$$

where the Θ is given as

$$\Theta = (\nu_o + 1)(2\alpha)^2 \left(1 - \frac{(\nu_o + 1)(2\alpha)^2}{\zeta}\right). \tag{4.10}$$

4.2 Downlink rates with linear precoding

Since the channel estimation is computed only at the BS, hence the UTs does not contain knowledge of CSI. Therefore, this paper provide the ergodic achievable rates on the basis of techniques used in [18] and [23]. Here, UTs only have knowledge of the estimated value of the channel, which is effective during the channel coherence time. Hence, extra number of antennas always benefits when using channel reciprocity for DL transmission. This implies that as the number of antennas increases, the accuracy of estimated CSI by UTs improves. For this purpose, we decompose the signal y_{jm}^{dl} received at a UT m from the jth BS given as

$$\begin{aligned} y_{jm}^{dl} &= \sqrt{\rho_{dl}\varphi_j}\mathbb{E}[\mathbf{h}_{jjm}^H \mathbf{w}_{jm}]x_{jm}^{dl} \\ &+ \sqrt{\rho_{dl}\varphi_j}(\mathbf{h}_{jjm}^H \mathbf{w}_{jm} - \mathbb{E}[\mathbf{h}_{jjm}^H \mathbf{w}_{jm}])x_{jm}^{dl} \\ &+ \sum_{(l,k)\neq(j,m)} \sqrt{\rho_{dl}\varphi_j}\mathbf{h}_{ljm}^H \mathbf{w}_{lk} x_{lk}^{dl} + n_{jm}^{dl}, \end{aligned} \tag{4.11}$$

and assume UT optimistically learned the effective channel. Note that, we have taken downlink channel to be the reciprocal of the uplink channel. The ergodic achievable rate, R_{jm}^{dl}, of UT m in cell j is given as

$$R_{jm}^{dl} = log_2(1 + \gamma_{jm}^{dl}), \tag{4.12}$$

where the SINR γ_{jm}^{dl} is given as

$$\gamma_{jm}^{dl} = \frac{\varphi_j \left|\mathbb{E}[\mathbf{h}_{jjm}^H \mathbf{w}_{jm}]\right|^2}{\frac{1}{\rho_{dl}} + \varphi_j var[\mathbf{h}_{jjm}^H \mathbf{w}_{jm}] + \sum_{(l,k)\neq(j,m)} \varphi_l \mathbb{E}\left[\left|\mathbf{h}_{ljm}^H \mathbf{w}_{lk}\right|^2\right]}, \tag{4.13}$$

where var stands for the variance parameter. We consider two different linear precoders known as Eigen-Beamforming (BF) $\mathbf{W}_j^{BF}$ and regularized zero-forcing (RZF)

$\mathbf{W}_j^{RZF}$, which are defined as

$$\mathbf{W}_j^{BF} = \hat{\mathbf{H}}_{jj}, \tag{4.14}$$

$$\mathbf{W}_j^{RZF} = \left(\hat{\mathbf{H}}_{jj}\hat{\mathbf{H}}_{jj}^{H} + M\varepsilon_j^{dl}\mathbf{I}_M\right)^{-1}\hat{\mathbf{H}}_{jj}, \tag{4.15}$$

where $\varepsilon_j^{dl} > 0$ is a regularization parameter. We choose $\varepsilon_j^{dl} = \frac{1}{\rho_{dl}M}$, such that precoder could perfectly nullify the effect of noise present in the system.

4.3 Outage Probability Analysis

In this section, we will provide closed form expression for outage probability in UL communication using maximum ratio combining (MRC) receiver. First, we define the squared K distribution as it will be used to obtain the outage probability of system. Second, we will define the SIR of massive MIMO system using MRC receiver. Finally, we will derive the closed form expression of outage probability using SIR. Numerical proof of this closed form expression is provided in numerical results chapter.

4.3.1 Squred K Distribution

When the envelop of non-Gaussian channel follows the K-distribution then square of the envelop is modeled by squared K-distribution. It is derived from generalized K-distribution [40], which is given by following equation

$$f_X(x) = \frac{4m^{\frac{\beta+1}{2}}x^{\nu}}{\Gamma(m)\Gamma(\nu)\alpha^{\frac{\beta+1}{2}}}K_{\nu-m}\left[2\left(\frac{m}{\alpha}\right)^{1/2}x\right], \tag{4.16}$$

where $\beta = \nu + m - 1$. K-distribution is obtained by putting m=1, also when $m \to \infty$m and $\nu \to \infty$, generalized K-distribution approaches AWGN channel. By using eq. (4.16), PDF of square envelop is obtained as follows

$$f_Y(y) = 2\left(\frac{m}{\alpha}\right)^{\nu+1} y^{\frac{\nu-1}{2}} K_{\nu-m}\left[2\left(\frac{my}{\alpha}\right)^{1/2}\right], \tag{4.17}$$

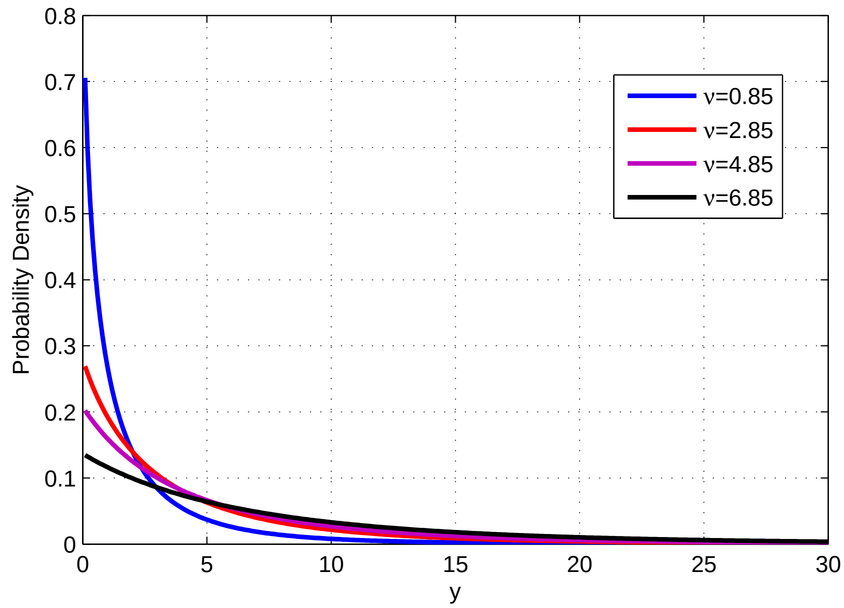

Figure 4.2: Squared K-distribution with different ν

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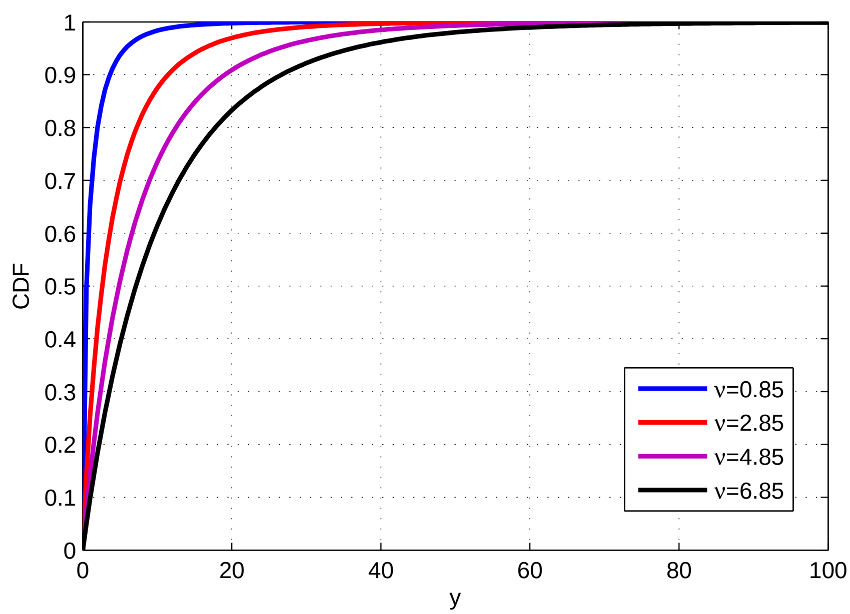

Figure 4.3: CDF with different ν

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where $Y = |X|^2$. By definition, we obtain squared K-distribution by putting $m = 1$ in eq. (4.17), which is given as

$$f_Y(y) = 2\left(\frac{1}{\alpha}\right)^{\nu+1} y^{\frac{\nu-1}{2}} K_{\nu-1}\left[2\left(\frac{y}{\alpha}\right)^{1/2}\right]. \tag{4.18}$$

Squared K-distribution with different shadowing intensity parameters ν is provided in Fig. 4.2. Also the CDF of squared K-distribution is given as

$$F_Y(y) = \pi csc(\pi(\nu-1))\left[\frac{\left(\frac{y}{\alpha}\right)_1 F_2\left(1; 2-\nu, 2; \frac{y}{\alpha}\right)}{\Gamma(\nu)\Gamma(2-\nu)\Gamma(2)} - \frac{\left(\frac{y}{\alpha}\right)_1^{\nu} F_2\left(\nu; \nu, 1+\nu; \frac{y}{\alpha}\right)}{\Gamma(\nu)\Gamma(1+\nu)}\right], \tag{4.19}$$

where ${}_pF_q(.)$ is the generalized hypergeometric function, and p and q are integers. CDF of square K-distribution is given in Fig. 4.3 with different shadowing intensity parameters.

4.3.2 Outage probability using MRC

In this section, we provide outage probability of massive MIMO system using signal to interference ratio (SIR). UL transmission is considered with perfect CSI known to BS. We choose maximum ratio combining (MRC) receiver, thus BS processes the signal by simply multiplying it with the conjugate-transpose of the channel. So receiver of kth user in jth cell is $\mathbf{r}_{jjk} = \mathbf{h}_{jjk}^H$. So SIR of kth user is given by

$$SIR_k = \frac{\rho_{tr}|\mathbf{r}_{jjk}\mathbf{h}_{jjk}|^2}{\rho_{tr}\sum\limits_{l=1,l\neq j}^{L}|\mathbf{r}_{jjk}\mathbf{h}_{jlk}|^2}, \tag{4.20}$$

where ρ_{tr} is transmitted power of UT. For better understanding and simplifying the analysis we take $M = 1$. In above equation, numerator is the power of desired signal and denominator is the sum of signal strengths from all the other interfering cells. So numerator is simply squared K RV and denominator is the sum of $L-1$ squared K RVs.

In order to obtain the outage probability, we simply calculate the CDF of ratio of two RVs. We write the SIR expression simply in the form of ratio of two RVs given as

$$SIR = z = \frac{u}{v}. \tag{4.21}$$

Here, we denote SIR with z for derivation purpose, where as u is squared K RV and v is sum of squared-K RVs. To derive the closed form expression of outage probability, we need to approximate v by another Gamma distributed RV. Assuming v as Gamma RV its PDF is given as

$$f_V(v) = \frac{\theta^{-\Xi}}{\Gamma(\Xi)} v^{\Xi-1} e^{-\frac{v}{\theta}}, \tag{4.22}$$

where $\Gamma(.)$ is Gamma function and Ξ and θ are shape and scale parameters respectively. To approximate sum of squared-K RVs with Gamma distribution, we use the moment matching method [36] and obtain the shape and scale parameter of Gamma distribution in the form of shape and scale parameters of squared-K RVs with some adjustment factor ϵ. Using moment matching method, we obtain Ξ and θ in the form of ν and α are as given below

$$\theta = (\vartheta - \epsilon)\alpha, \tag{4.23}$$

$$\Xi = \frac{L-1}{\vartheta - \epsilon}, \tag{4.24}$$

where ϑ is defined as

$$\vartheta = 1 + \frac{2}{\nu}. \tag{4.25}$$

To obtain the outage probability, we use the following equation

$$\mathcal{O} = \mathbb{P}\left\{SIR < \tau\right\} = \int_0^\infty \left[\int_0^{u/z} f_V(v)dv\right] f_U(u)du. \tag{4.26}$$

Here, the inner integral leads to the CDF of Gamma distribution which is given as

$$\int_0^{u/z} f_V(v)dv = \frac{\gamma(\Xi, \frac{u}{z\theta})}{\Gamma(\Xi)}, \tag{4.27}$$

where $\gamma(.)$ is incomplete gamma function. Using Eq. 4.27 and Eq. 4.18 in 4.26 we obtain the outage probability

$$\mathcal{O} = \int_0^\infty \frac{2}{\Gamma(\nu)} \left(\frac{1}{\alpha}\right)^{\frac{\nu+1}{2}} u^{\frac{\nu-1}{2}} K_{\nu-1}\left[2\sqrt{\frac{u}{\alpha}}\right] \frac{\gamma(\Xi, \frac{u}{\tau\theta})}{\Gamma(\Xi)} du \tag{4.28}$$

After integrating, we have

$$\mathcal{O} = \varrho\left[a\left[-1 +_1 F_1\left(\Xi, 1-\nu, \frac{\tau}{\alpha\theta}\right)\right] + b_1F_1\left(\Xi+\nu, 1+\nu, \frac{\tau}{\alpha\theta}\right)\right], \tag{4.29}$$

where ϱ, a and b are given as

$$\varrho = \frac{\alpha^{-\nu}}{\Gamma(\nu)\Gamma(\Xi)} \left(\frac{\theta}{\tau}\right)^{-\nu}, \tag{4.30}$$

$$a = \left(\frac{\alpha\theta}{\tau}\right)^{\nu} \Gamma(\Xi)\Gamma(\nu), \tag{4.31}$$

$$b = \Gamma(-\nu)\Gamma(\Xi+\nu), \tag{4.32}$$

such that $\Re(\nu) > 0$, $\Re(\Xi+\nu) > 0$, $\Re(\Xi) > -1$, $\Re(\frac{\theta}{\tau}) > 0$ and $\alpha > 0$.

Chapter 5

Numerical Results

In this chapter, we demonstrate the effects of changing the shadowing intensity on ergodic achievable rates of massive MIMO system under consideration. Various simulation results under different scenarios such as changing the number of BS antennas, shadowing intensity, and different DL precoding schemes have been obtained, where the uplink channel estimation is done using (4.2). We assume that ten users are uniformly distributed in a given cell. In all simulation results, the shadowing intensity for interfering cells is assumed unity, i.e., $\nu_I = 1$ implying a severe shadowing environment, the number of cells in a cluster is limited to $L = 7$, $\rho_{tr} = 10$dBm, $\rho_{dl} = 40$dBm and $\alpha = 0.15$, unless otherwise stated. Finally, we present the outage probability analysis of massive MIMO system with $M = 1$ using MF in uplink transmission. We assume the perfect CSI for this analysis. Analytical results acquired in Eq. (4.29) are supported by simulation.

Fig. 5.1 shows the ergodic rates of 10 users versus the number of antennas with BF and RZF precoders. We vary the shadowing intensity of serving cell by varying ν_o, where the shadowing intensity decreases as we increase the value of ν_o, which results in increasing the data rates as evident from the figure. It can also be noted that the RZF outperforms BF precoder, as it reduces interference more effectively.

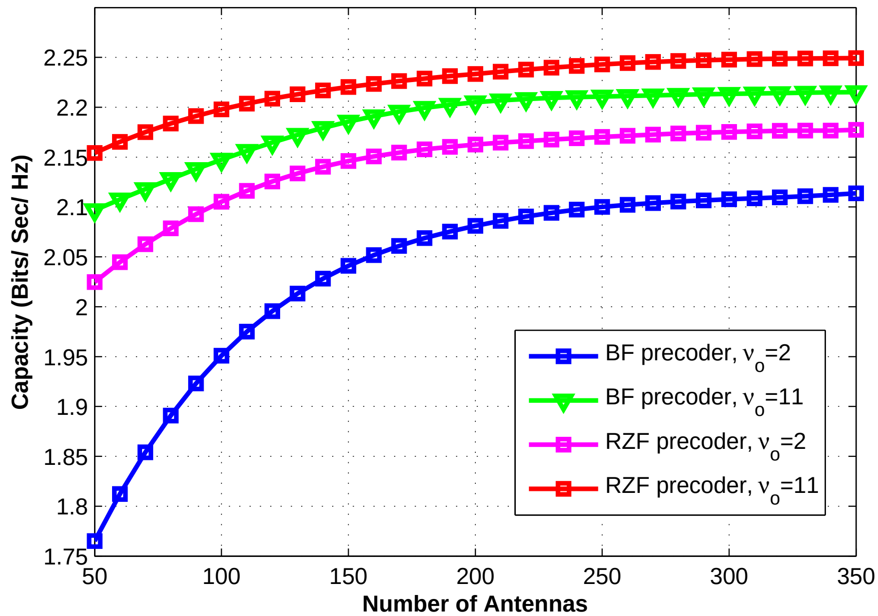

Figure 5.1: Downlink rates for different shadowing intensities with RZF, BF precoders, $L = 7$, $K = 10$

We also observe that an increase in the number of antennas does not increase the data rates linearly specially at low ν_o, rather a saturating trend in increase in data rate occurs as we further increase the number of antennas. This shows that shadowing does not average out as we increase the number of antennas in a massive MIMO system. However, it continues to affect the system even at a larger scale. We can observe from Fig. 5.1 that the data rate (with BF precoder and $\nu_o = 2$) increases 17% when we increase the number of antennas from 50 to 150, but it only increases by 3.4% as the number of antennas increase from 150 to 350. Thus the rates saturate as we increase the number of antennas in the presence of shadowing, as it limits the performance of massive MIMO system. Fig. 5.2 and Fig. 5.3 depict the ergodic rates of 10 users versus the number of antennas with RZF and BF precoders, and for different values of ν_o and L. It can be shown from the figures that as we increase the cell cluster size, data rates gradually decrease, which occurs due to an increase

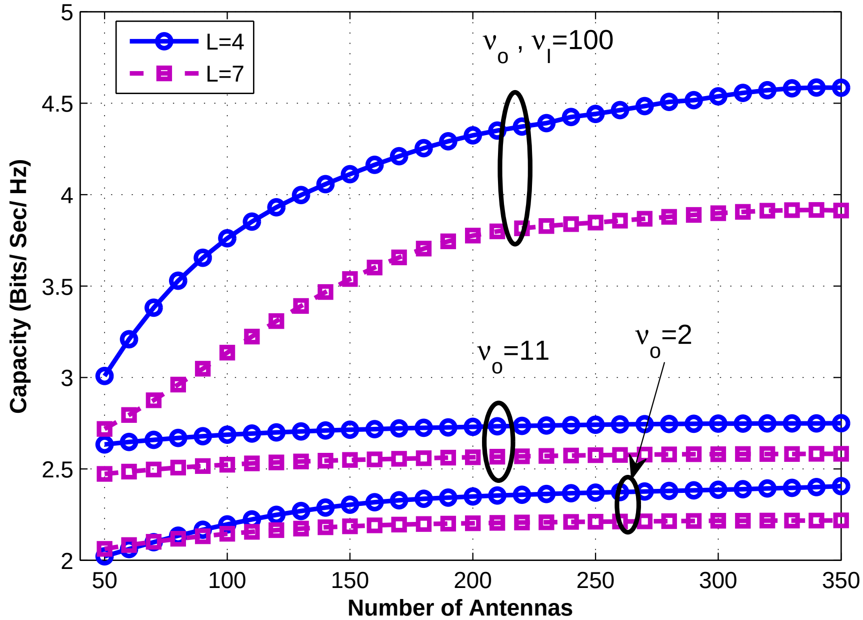

Figure 5.2: Downlink rates for different shadowing intensities with RZF precoder, $K = 10$

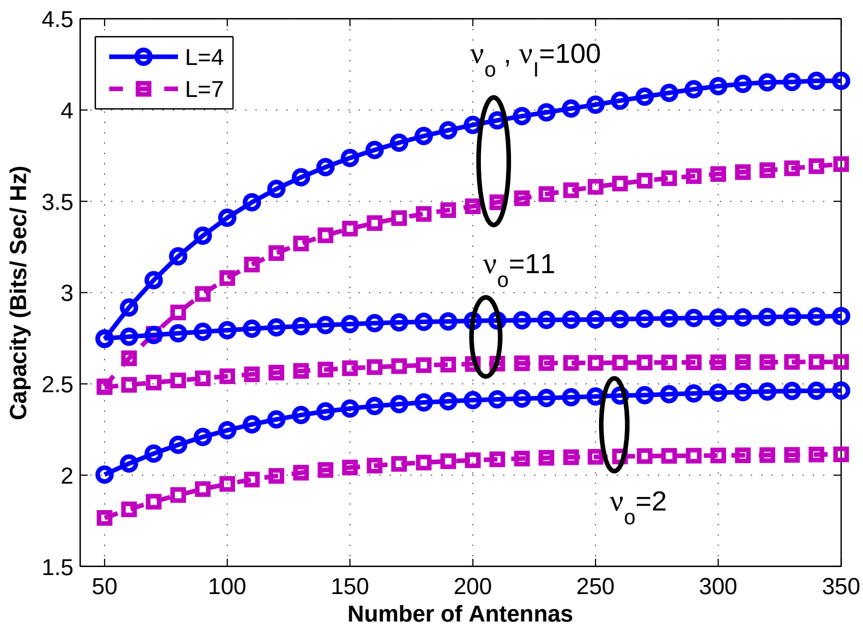

Figure 5.3: Downlink rates for different shadowing intensities with BF precoder, $K = 10$

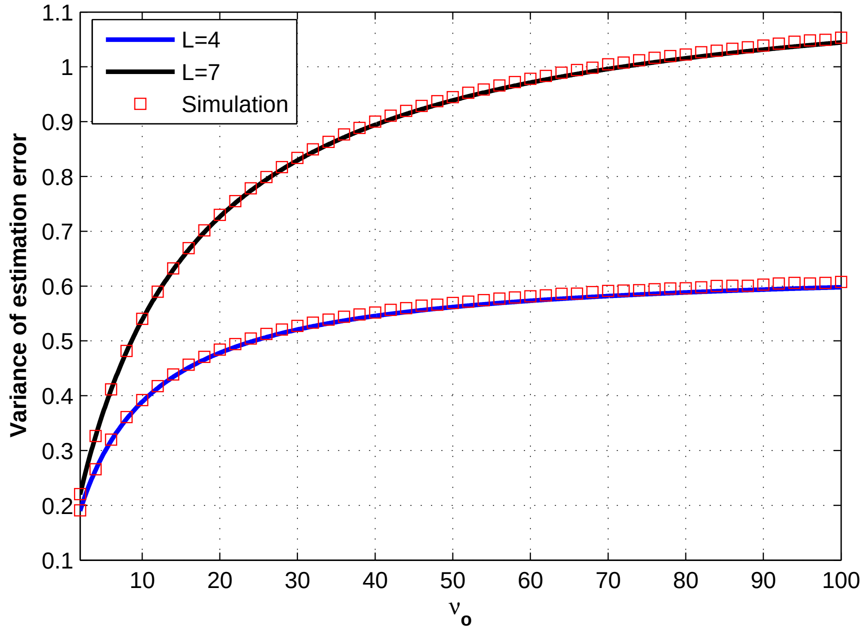

Figure 5.4: Estimation error variance for different shape parameters, $K = 10$, $M = 50$.

in the number of interfering cells. Also shadowing effect disappears as $\nu \rightarrow \infty$ and the only remaining effect is the Rayleigh fading. Therefore, we can see that at higher values of ν_o and ν_I, i.e., $\nu_o = 100$ and $\nu_I = 100$, which implies both serving cell and the interfering cells are only affected by Rayleigh fading, the performance of system is much better. We compare the rates for this scenario with our conventional scenarios, i.e., with parameter settings of $\nu_o = \{2, 11\}$ and $\nu_I = 1$. It can be shown that with Rayleigh fading only, the massive MIMO system performs better. The data rates dramatically increase as the number of antenna increases in the Rayleigh fading only scenario.

Fig. 5.4 shows the trend in the estimation error variance (Θ) for different cluster sizes, i.e, $L = \{4, 7\}$ as a function of ν_o. It can be observed that the theoretical values of Θ obtained via (4.10) are in agreement with the numerical simulations. Note that K-distribution becomes Rayleigh distribution as $\nu \rightarrow \infty$, hence as we increase ν_o, the value of channel estimation variance also saturates. The saturation is observed more prominently for $L = 4$ rather than $L = 7$ due to less inter-cell interference.

Fig. 5.5 shows the ergodic rates for 10 users as a function of the number of antennas and ν_o, with RZF precoder. The result quantifies the number of antennas required to achieve a specific data rate at a certain ν_o. For instance, to obtain data rate of 2.21, a massive MIMO requires 150 antennas for a moderately shadowed environment with $\nu_o = 9$. However, to maintain the same rate, the same system requires 250 BS antennas for a highly shadowed system with $\nu_o = 2$. Therefore, consideration of large-scale fading in designing of future massive MIMO systems is critical.

Similarly Fig. 5.6 represents the ergodic rates for BF precoder as a function of number of antennas and ν_o. This results shows that how many antennas are required for maintaining specific data rate with varying intensity of composite fading. It is evident from figure that to maintain a data rate of 2.15, 150 antennas are required for lightly shadowed system. On the other hand, it requires more than 350 antennas to maintain the required data rate for severe shadowing.

In Fig. 5.7, we present outage probability analysis of massive MIMO system. Here we take $M = 1, \alpha = 0.65, L = 7$ and interfering cells has identical severity of shadowing with $\nu_I = 0.85$. Note that we take severe shadowing for interference and scale parameter for all the system remains the same. Figure shows how outage probability varies as we vary the SINR threshold τ for different shadowing intensities. It is evident from figure, that as we decrease the shadowing intensity probability of outage also decreases. For example if we take $\tau = 1dBm$, then for highly shadowed system ($\nu = 0.85$), probability of outage is 0.95. As we decrease the shadowing by increasing $\nu = 3.85$, for similar threshold probability of outage reduces to 0.7.

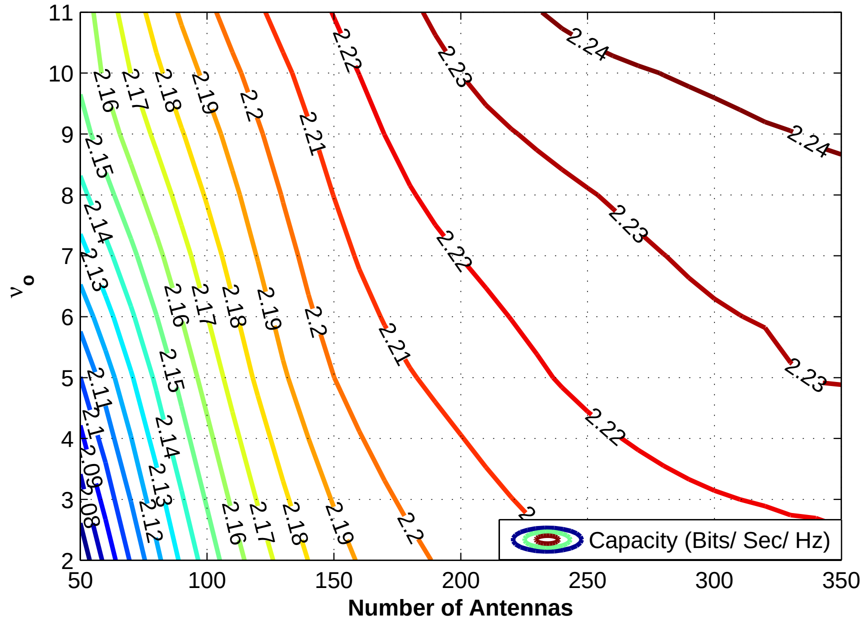

Figure 5.5: Downlink rates for RZF precoder as a function of number of antennas and ν_o, $K = 10$

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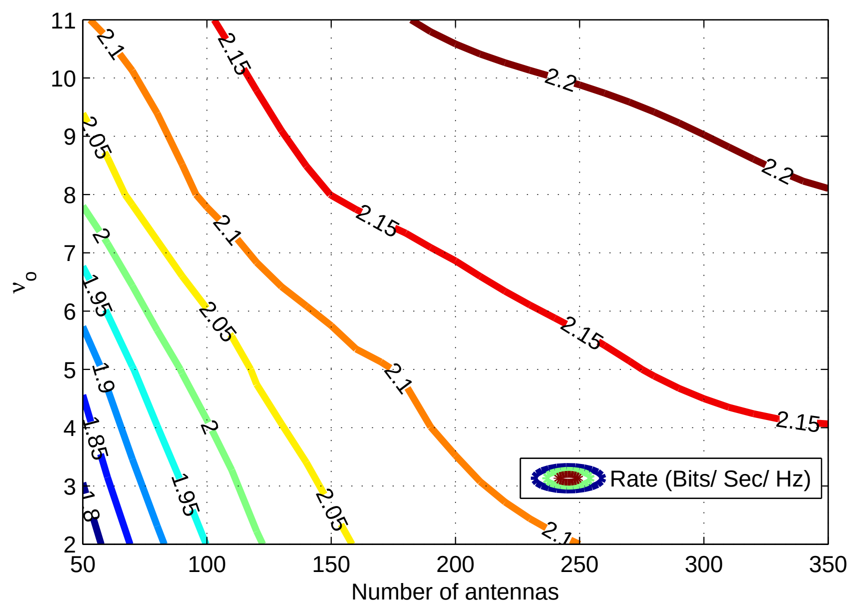

Figure 5.6: Downlink rates for BF precoder as a function of number of antennas and ν_o, $K = 10$

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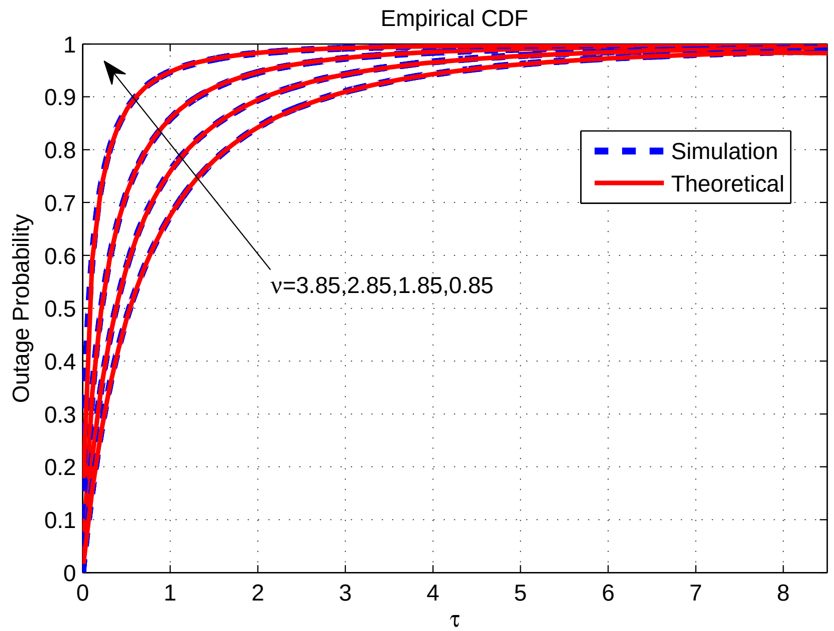

Figure 5.7: Outage probability with $\nu = 0.85, 1.85, 2.85, 3.85$ and different SIR threshold

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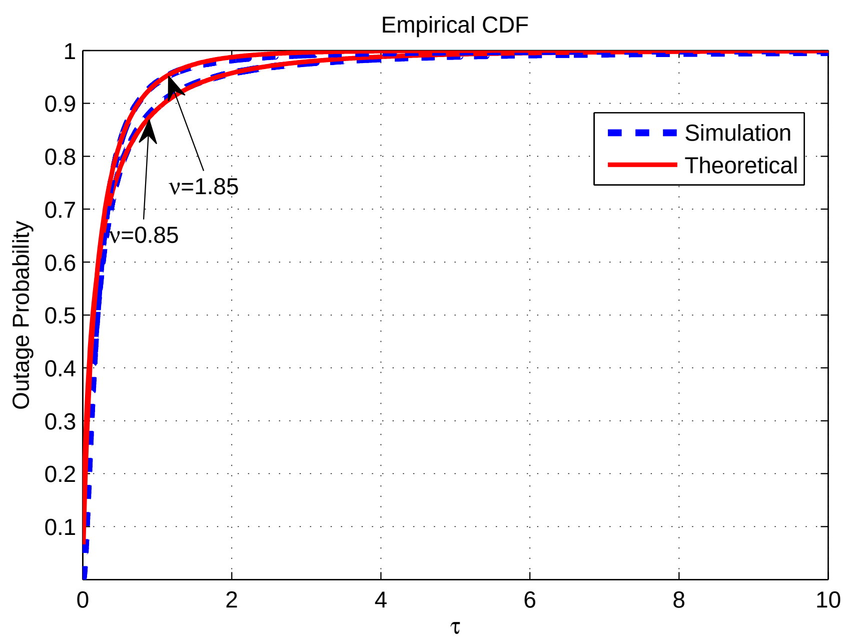

Figure 5.8: Outage probability with $\nu = 0.85, 1.85$

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Severe shadowing for both desired and interfering signal is taken into count in Fig. 5.8. Severe shadowing implies small ν has to be taken. For this purpose we have taken $\nu = 0.85, 1.85$ for not only interfering cells but also for desired cell. Results shows that UTs experience severe outage for even low threshold values of SIR.

Chapter 6

Conclusion and Future Work

In this chapter, we will conclude our work and some suggestions for future work are also provided.

6.1 Conclusion

This Thesis analyzed the effect of shadowing on the ergodic rates of a multi-cell, multi-user massive MIMO system and outage probability in the presence of K-fading. The uplink CSI for a K-fading environment has been estimated using MMSE estimation, whereas the ergodic rates of DL transmission for RZF and BF precoding schemes have also been provided. The results demonstrated that we estimate better channel as we decrease the severity of shadowing. It is also evident that RZF performs much better than BF precoder, however computational complexity of RZF is higher than BF precoder. Hence there needs to be a trade off between performance and energy saving. Results shows tha as we increase the number of antennas at BS performance of massive MIMO systems improves, however the shadowing effects cannot be completely averaged out even at a very large number of BS antennas. On the uplink transmission, outage probability analysis is done using MRC approach. The results shows that as we decrease the threshold of system outage probability also decreases.

6.2 Future Work

In future, we can extend outage probability analysis of system with large number of antennas. Motivated from this work, we aim to extend this work to asymptotic rate analysis over a composite fading channel under the conditions that number of antennas approach infinity, however a finite ratio between the number of antennas and users in a cell is maintained. One can check the performance analysis of this system with other channel estimators and precoders. We can also analyze the performance of massive MIMO in HetNet operating under composite shadowing fading channels. D2D massive MIMO could also be studied under K-fading environment operating indoor.

Appendix A – Communication Systems Symbols

$\mathbf{x}_l^{ul}$	Transmitted data vector from UT to BS in lth cell
$\mathbf{y}_j^{ul}$	Received data vector from UT to BS in jth cell
$\mathbf{H}_{jl}$	Complex channel vector of jth cell
$\mathbf{n}$	AWGN noise vector
$\mathbf{W}_l$	Precoding Vector of lth cell
$\mathbf{s}_l$	Precoded data vecotr of lth cell
$\mathbb{E}\left[\right]$	Expectation value
$\hat{\mathbf{h}}$	Estimated value of channel
γ_{jm}^{dl}	Downlink SINR of mth user in jth cell

Bibliography

[1] P. Gautam, S. Kaur, R. Kaur, S. Kaur and H. Kundra, "Review Paper on 4G Wireless Technology", *Int. Journal of Adv. in Sci. and Tech.*, Vol. 2, pp. 15-19, March 2014.

[2] C. Wang, F. Haider, X. Gao, X. You, Y. Yang, D. Yuan, H. M. Aggoune, H. Haas, S. Fletcher and E. Hepsaydir, "Cellular Architecture and Key Technologies for 5G Wireless Communication Networks", *IEEE Comm. Mag.*, vol. 52, pp 122-130, Feb. 2014.

[3] X. Liu, Y. Li, X. Li, L. Xiao, J. Wang, "Pilot Reuse and Interference-aided MMSE Detection for D2D Underlay Massive MIMO", *IEEE Trans. on Veh. Tech.*, July 2016.

[4] H. Xu, N. Huang, Z. Yang, J. Shi, B. Wu and M. Chen, "Pilot Allocation and Power Control in D2D Underlay Massive MIMO Systems", *IEEE Comm. Let.*, vol. 21, pp 112-115, Jan. 2017.

[5] M. Xie, X. Jia, M. Zhou and L. Yang, "Study on energy efficiency of D2D underlay massive MIMO networks with power beacons", *8th Int. Conf. on Wireless Comm. and Signal Proc. (WCSP)*, 13-15 Oct. 2016, Yangzhou, China.

[6] L. Cottatellucci, “D2D CSI feedback for D2D aided massive MIMO communications”, *IEEE Sensor Array and Multichannel Signal Proc. Workshop (SAM)*, 10-13 July 2016.

[7] T. E. Bogale L. B. Le, “Massive MIMO and mmWave for 5G Wireless HetNet: Potential Benefits and Challenges”, *IEEE Veh. Tech. Mag.*, Vol. 11, pp. 64-75, Feb. 2016.

[8] A. Adhikary, H. S. Dhillon and G. Caire, “Massive-MIMO Meets HetNet: Interference Coordination Through Spatial Blanking”, *IEEE J. on Sel. Areas in Comm.*, Vol. 33, pp. 1171-1186, June 2015.

[9] B. Xu, Y. Chen, M. Elkashlan T. Zhang and K. Wong, “User association in massive MIMO and mmWave enabled HetNets powered by renewable energy”, *IEEE Wireless Comm. and Net. Conf. (WCNC)*, 3-6 April 2016, Doha, Qatar.

[10] R. R. Muller, D. Guo and A. L. Moustakas, “Vector Precoding for Wireless MIMO Systems and its Replica Analysis”, *IEEE Journal on Sel. Areas in commun.*, Vol. 26, pp. 530-540, April 2008.

[11] T. L. Marzetta, “Noncooperative cellular wireless with unlimited numbers of base station antennas”, *IEEE Trans. Wireless Commun.*, vol. 9, pp. 3590-3600, Nov. 2010.

[12] F. Rusek, D. Persson, B. K. Lau, E. G. Larsson, T. L. Marzetta, O. Edfors, and F. Tufvesson, “Scaling up MIMO: opportunities and challenges with very large array”, *IEEE Signal Process. Mag.*, vol. 30, pp. 40-60, Jan. 2013.

[13] D. Gesbert , M. Shafi, D. Shiu, P. Smith and A. Naguib, "From theory to practice: An overview of space-time coded MIMO wireless systems", *IEEE J. Sel. Areas Commun.*, vol. 21, no. 3, pp. 281-302. 2003.

[14] M. T. Mushtaq, S. A. Hassan and D. N. K. Jayakody, "Ergodic Rate Analysis of Massive MIMO Systems in K-Fading Environment", *84th IEEE Vehicular Technology Conference (VTC-Fall)*, 1821 Sep 2016, Montral, Canada.

[15] A. Lozano and N. Jindal, "Transmit Diversity vs. Spatial Multiplexing in Modern MIMO Systems", *IEEE Trans. Wireless Commun.*, vol. 9, pp. 186-197, Jan. 2010.

[16] A. Yang, Z. He, C. Xing and Z. Fei, J. Kuang, "The role of large-scale fading in uplink massive MIMO systems", *IEEE Trans. Veh. Tech.*, vol. 65, pp. 477-483, Jan. 2016.

[17] H. Q. Ngo, E. G. Larsson and T. L. Marzetta, "Energy and spectral efficiency of very large multiuser MIMO systems", *IEEE Trans. Commun.*, vol. 61, pp. 1436-1449, Apr. 2013.

[18] J. Jose, A. Ashikhmin, T. L. Marzetta and S. Vishwanath, "Pilot contamination and precoding in multi-cell TDD systems", *IEEE Trans. Wireless Commun.*, vol. 10, pp. 2640-2651, June 2011.

[19] M. S. Zia and S. A. Hassan, "On the impacts of composite fading on large-scale multi-user MIMO systems", *IEEE Commun. Letters*, vol. 19, no.12, pp. 2286-2289, Dec. 2015.

[20] M. Bacha and S. A. Hassan, "Performance analysis of cooperative linear networks subject to composite shadowing fading", *IEEE Trans. Wireless Commun.*, vol. 12, pp. 5850-5858, Nov. 2013.

[21] A. Rahman and S. A. Hassan, "Analysis of composite fading in a single cell downlink cooperative heterogeneous networks", *81st IEEE Vehicular Technology Conference (VTC-Spring)*, Glasgow, United Kingdom, May 2015, pp. 1-5.

[22] M. S. Zia and S. A. Hassan, "Outage analysis of multi-user massive MIMO systems subject to composite fading", *81st IEEE Vehicular Technology Conference (VTC-Spring)*, Glasgow, United Kingdom, May 2015, pp. 1-5.

[23] J. Hoydis, S. Brink and M. Debbah, "Massive MIMO in the UL/DL of cellular networks: how many antennas do we need?", *IEEE J. Sel. Areas in Commun.*, vol. 31, pp. 160-171, Feb. 2013.

[24] P. S. Bithas and A. A. Rontogiannis, "Mobile communication systems in the presence of fading/shadowing, noise and interference", *IEEE Trans. Commun.*, vol. 63, pp. 724-737, Jan. 2015.

[25] A. Liu and V. Lau, "Hierarchical interference mitigation for massive MIMO cellular networks", *IEEE Trans. Signal Proces.*, vol. 62, pp. 4786-4797, July 2014.

[26] Z. Mulk and S. A. Hassan, On Achievable Rates in Massive MIMO-based Hexagonal Cellular System with Pilot Contamination,*81st IEEE Vehicular Technology Conference (VTC-Spring)*, May 2015, Glasgow, United Kingdom.

[27] E. G. Larsson, O. Edfors, F. Tufvesson and T. L. Marzetta“Massive MIMO for Next Generation Wireless Systems”, *IEEE Comm. Mag.*, vol. 52, pp. 186-195, Feb. 2014.

[28] A. Abdi and M. Kaveh, “K distribution: an appropriate substitute for Rayleigh-lognormal distribution in fading-shadowing wireless channels”, *Electron. Lett.*, vol.34, pp. 851-852, Apr.1998.

[29] I. Trigui and A. Laourine, S. Affes, and A. Stephenne, ”Performance analysis of mobile radio systems over composite fading/shadowing channels with co-located interference”, *IEEE Trans. Wireless Commun.*, vol. 8, pp. 3448-3453, July 2009.

[30] M. Johamm and W. Utschick, J.A. Nossek, “Linear transmit processing in MIMO communications systems”, *IEEE Trans. Signal Proces.*, vol. 53, pp. 2700-2712, Aug. 2005.

[31] O.N. Alrabadi and E. Tsakalaki, and H. Huang, G.F. Pedersen, “Beamforming via large and dense antenna arrays above a clutter”, *IEEE J. Sel. Areas Commun.*, vol. 31, pp. 314-325, Feb. 2013.

[32] G. L. Stuber, *Principles of Mobile Communication*, 3rd ed. Springer+Business Media, 2011.

[33] D. K. Nguyen and H. Ochi, “Two-way Cognitive DF Relaying in WSNs with Practical RF Energy Harvesting Relay Hardware”, *IEICE Trans. on Commun.*, vol.E99-B, no.3, pp.675-684 , Mar. 2016.

[34] M. Alfaqawi, J. Chebil, M. Habaebi and D. Datla, " Wireless Distributed Computing for Cyclostationary Feature Detection", *Elsevier Digital Communications and Networks journal*, vol. 2, no., pp. 47–56 1, Feb. 2016.

[35] R. Muller, D. Guo and A.L. Moustakas, "Vector precoding for wireless MIMO systems and its replica analysis", *IEEE Sel. Areas Commun.*, vol. 26, pp. 530-540, Apr. 2008.

[36] S. Al-Ahmadi and H. Yanikomeroglu, "On the approximation of the generalized-K distribution by a Gamma distribution for modeling composite fading Channels", *IEEE Trans. Wireless Commun.*, vol. 9, pp. 706-713, Feb. 2010.

[37] S. Wagner, R. Couillet, M. Debbah and D. T. M. Slock, "Large System Analysis of Linear Precoding in Correlated MISO Broadcast Channels Under Limited Feedback", *IEEE Trans. on Inf. Theory*, vol. 58, pp. 4509 - 4537, March 2012.

[38] A. H. Mehana and A. Nosratinia, "Diversity of MIMO Linear Precoding", *IEEE Trans. on Inf. Theory*, vol. 60, pp. 1019 - 1038, Nov. 2013.

[39] A. Abdi, H.A. Barger and M. Kaveh, "A Simple Alternative to the Lognormal Model of Shadow Fading in Terrestrial and Satellite Channels", *54th IEEE Vehicular Technology Conference (VTC-Spring)*, Atlantic City, New Jersey USA, Aug. 2001, pp. 2058-2062.

[40] P. S. Bithas, N. C. Sagias, P. T. Mathiopoulos, G. K. Karagiannidis and A. A. Rontogiannis, "On the Performance Analysis of Digital Communications over Generalized-K Fading Channels", *IEEE Commun. Letters*, vol. 10, no. 5, pp. 353-355, May 2006.

[41] A. Laourine, M. Alouini, S. Affes and A. Stephenne, "On the Capacity of Generalized-K Fading Channels", *IEEE Trans. Wireless Commun.*, vol. 7, pp. 2441-2445, July 2008.

[42] S. Al-Ahmadi and H. Yanikomeroglu, "On the Statistics of the Sum of Correlated Generalized-K RVs", *IEEE International Conference on Communications*, Cape Town, South Africa, May 2010, pp. 1-5.

[43] A. Abdi and M. Kaveh, "Comparison of DPSK and MSK Bit Error Rates for K and Rayleigh-Lognormal Fading Distributions", *IEEE Commun. Letters*, vol. 4, no. 4, pp. 122-124, April 2000.

[44] E. Jakeman and P. Pusey, "A model for non-Rayleigh sea echo", *IEEE Trans. on Antennas and Prop.*, vol. 24, pp. 806-814, Nov 1976.

[45] E. Jakeman and P. Pusey, "Significance of K Distributions in Scattering Experiments", *Phys. Rev. Lett.*, Vol. 40, Issue 9, Feb. 1978.

[46] P. S. Bithas, P. T. Mathiopoulos and S. A. Kotsopoulos, "Diversity Reception over Generalized-K (KG) Fading Channels", *IEEE Commun. Letters*, vol. 6, no. 12, pp. 4238-4243, Dec. 2007.

[47] C. Feng, Y. Jing and S. jin, "Interference and Outage Probability Analysis for Massive MIMO Downlink with MF Precoding", *IEEE Signal Proces. Letters*, vol. 23, pp. 366-370, March 2016.

[48] F. Fernandes, A. Ashikhmin and T. L. Marzetta, "Inter-Cell Interference in Noncooperative TDD Large Scale Antenna Systems", *IEEE Sel. Areas Commun.*, vol. 31, pp. 192-201, Feb. 2013.

[49] W. Fukuda, T. Abiko, T. Nishimura, T. Ohgane, Y. Ogawa, Y. Ohwatari and Y. Kishiyama, "Low-complexity Detection Based on Belief Propagation in a Massive MIMO System", *77th IEEE Vehicular Technology Conference (VTC-Spring)*, Dresden, Germany, June 2013, pp. 1-5.

[50] S. Zarei, W. Gerstacker, R. R. Mller and R. Schober, "Low-Complexity Linear Precoding for Downlink Large-Scale MIMO Systems", *24th IEEE International Symposium on Personal Indoor and Mobile Radio Communications (PIMRC)*, London, United Kingdom, Sep. 2013, pp. 1-5.

[51] C. Fragouli, N. Al-Dhahir and W. Turin, "Training-Based Channel Estimation for Multiple-Antenna Broadband Transmissions", *IEEE Trans. Wireless Commun.*, vol. 2, pp. 384-391, March 2003.

[52] M. Bigues and A. B. Gershman, "Training-Based MIMO Channel Estimation: A Study of Estimator Tradeoffs and Optimal Training Signals", *IEEE Trans. Signal Proces.*, vol. 54, pp. 884-893, March 2006.

[53] R. Zakhour and S. V. Hanly, "Base station cooperation on the downlink: Large system analysis", *IEEE Trans. on Inf. Theory*, vol. 58, pp. 2079-2106, April 2012.

[54] S. Atapattu, C. Tellambura and H. Jiang, "A Mixture Gamma Distribution to Model the SNR of Wireless Channels", *IEEE Trans. Wireless Commun.*, vol. 10, pp. 4193-4203, Dec. 2011.

[55] J. A. Anastasov, G. T. Djordjevic and M. C. Stefanovic, "Analytical Model for Outage Probability of Interference-Limited Systems over Extended Generalized-K Fading Channels", *IEEE Commun. Letters*, vol. 16, no.04, pp. 473-475, April 2012.

[56] Euro. Mobile Industry Observatory, GSMA, Nov. 2011.