PREDICTION OF SHEAR STRENGTH IN HIGHER STRENGTH CONCRETE BEAMS

By

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A thesis submitted in partial fulfillment of
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DEDICATED

TO

MY FAMILY, TEACHERS

AND

WELL WISHERS

ACKNOWLEDGEMENT

I am grateful to Almighty Allah (the most merciful) whose blessing gave me the strength to complete this research work. I express my gratitude and sincere thanks to my advisor Brigadier (Retired) Dr Khaiq-ur-Rashid Kayani for his advice, untiring guidance and supervision throughout the research.

I extend my gratitude and special thanks to engineer Mr. Ahsan Ali, who provided full support and corporation for collection of database of beams failing in shear.

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I appreciate full support provided to me by my mother, sister, wife and children, this research work would not have been completed without their prayers and whole hearted encouragement.

ABSTRACT

A methodical investigation of the shear strength of high strength concrete beams was conducted. A database of 245 reinforced concrete beams failing in shear was collected, having 84 beams without web reinforcement and 161 beams with web reinforcements. A detailed literature review was carried out to find out the factors effecting shear in reinforced concrete beams followed by earmarking the important factors influencing shear. Graphical tool of ms-excel being was used to investigate the effect of different variables as well as for the development of formula was facilitated by this tool.

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LIST OF ABBREVIATIONS/NOTATIONS

a	Shear span
A_s	Longitudinal steel area
A_v	Area of stirrups
a/d	Shear span to depth ratio
b	Beam section width
b_w	Beam section web width
C	Compressive force in concrete compression zone
d	Beam section depth
f_c'	Concrete compressive strength
f_y	Yield strength of longitudinal steel
f_{vy}	Yield strength of stirrups steel
h	Beam section height
HSC	High strength concrete
in	Inch
jd	Flexural lever arm
M	Moment
M_n	Nominal moment
NSC	Normal strength concrete
psi	Pounds per square inch
RC	Reinforced concrete
s	Stirrups spacing
T	Tensile force in the reinforcement
V	Shear force

V_c	Shear force provided by concrete
V_s	Shear force provided by steel stirrups
v	Shear stress
v_c	Shear stress provided by concrete
v_s	Shear stress provided by steel stirrups
v_{cr}	Cracking shear strength
v_u	Ultimate shear strength
V_n	Nominal shear strength
ρ	Longitudinal reinforcement ratio
ρ_v	Shear reinforcement ratio

Chapter 1

INTRODUCTION

1.1 General

Engineering professionals are busy trying to improve the quality of RC shear design procedures since long. Unlike flexural failures, shear failures in RC structures are brittle and sudden. Typically shear failures occur with little or no warning. Furthermore, they tend to be less predictable than flexural failures, due to more complex failure mechanisms.

The mechanism of shear failure in RC beams is still under constant debate even after several decades of theoretical and experimental research. This extensive research has given birth to different models/theories and several empirical relations. These encompass sectional models, compression field approach, truss approach with concrete contribution, shear friction and finite element analysis. Even after these great advancements, shear in RC beams appears to be a rather complex problem, still inviting researchers to provide reliable indications for shear design of RC beams. All the above mentioned research highlights the complexity of the problem arises from different phenomena as described below:-

- The arch action or the beam action.
- Cracks propagation.
- Aggregate interlock.
- Dowel action and the steel-concrete bond.
- Beam geometry.
- The material properties.

- The loading conditions.
- The arrangement of transverse and longitudinal reinforcement.
- The axial force (pre-stress actions).
- Ratio a/d.

In 1962, Joint ACI-ASCE Committee 326 report "Shear and Diagonal Tension" discussed the design and behaviour of beams failing due to shear and diagonal tension. The committee considered a database of 194 beam tests without shear reinforcement (predominantly comprising of NSC beams). The database consisted of 130 laboratory specimens subjected to point loads and 64 beams to uniformly distributed loads. Based on the data, following design equation was developed and is included in ACI 318-05.

$$V_c = \left\{1.9\sqrt{f_c'} + 2500\rho_w \frac{V_u d}{M_u}\right\} b_w d \ngtr 3.5\sqrt{f_c'}\, b_w d \text{ and } \frac{V_u d}{M_u} \ngtr 1 \quad \text{ACI Eq 11-5} \qquad 1$$

Or a simplified version as $V_c = \left\{2\sqrt{f_c'}\right\} b_w d$ ACI Eq 11-3 2

This is applicable to both the beams with and without stirrups and $f'_c \leq 10000$ psi.

And for beams with web reinforcement

$V_n = V_c + V_s$ where $V_s = \rho_v f_y bd$ 3

With the advancements in civil engineering and simultaneous follow up by construction industry production of HSC is very much common now a days. HSC has many advantages over NSC like reduction in cross sectional area of the structural element and self weight etc. These advantages have given birth to many attractive uses. Among many other uses HSC is being used in beams as well. HSC is relatively new for the construction industry in Pakistan but it has a potential for extensive use in construction. Our designs are mostly based on ACI code therefore safety and reliability of this shear design equation has to be checked for applicability in HSC beams design due to following two main reasons:-

- The shear design equation is based on data from tests predominantly on NSC.
- The diagonal shear strength is influenced by ductility of concrete. Since ductility decreases with increasing concrete strength, the shear strength of HSC is likely to be less than that of NSC.

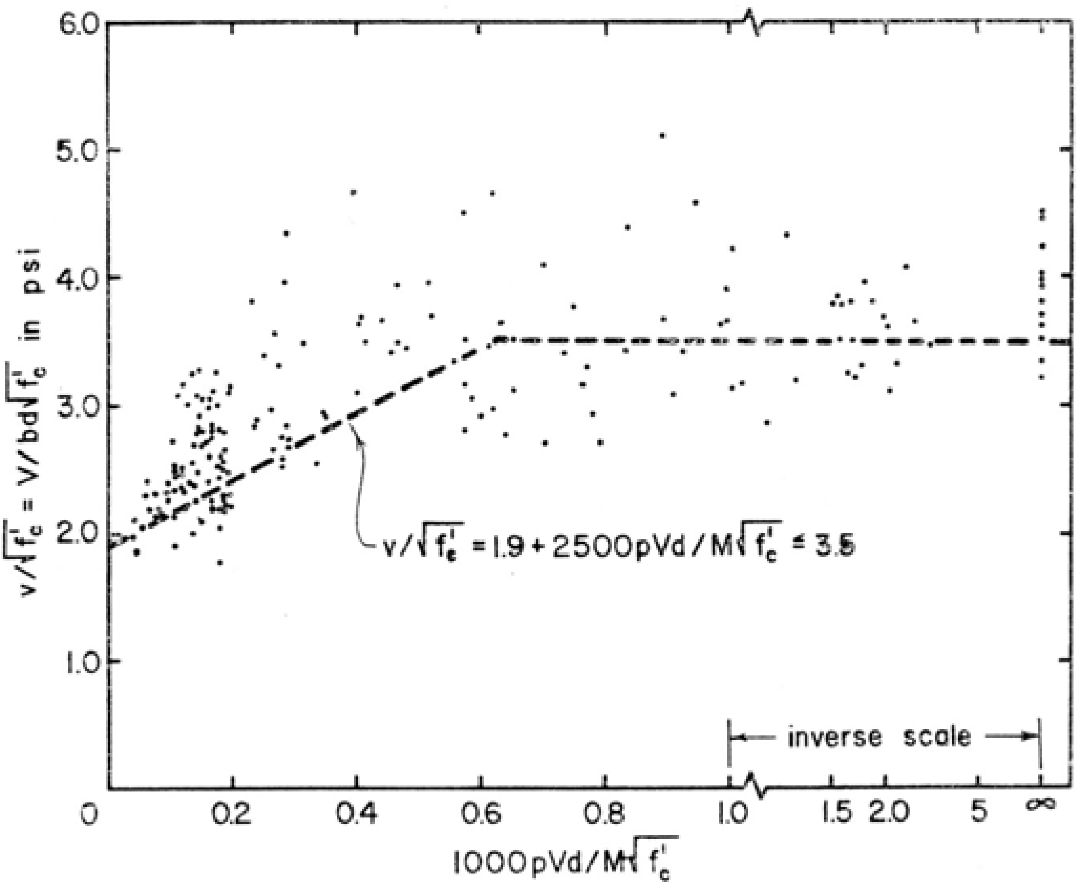


Figure 1.1 Database used in Joint ACI-ASCE Committee 326 Report.

Thus in view of the above a critical analysis of the theoretical approach in HSC shear design vis-à-vis the experimental data is a requirement. All the available data from beams tested for shear resistance will be utilized for prediction of shear resistance of HSC and NSC with and without web reinforcement. Seeing the bright future of HSC in Pakistan's construction industry it is important to understand and investigate the shear strength of HSC beams in this study.

1.2 Research Objectives

This research focuses on improving our understanding of the behavior of shear resistance of HSC beams with and without web reinforcement. The objectives of the study are enumerated below:-

- Critical analyses of the theoretical approach in HSC shear design vis-à-vis the experimental data.
- Shear design equations of ACI design code for concrete structures (ACI-318) which are based on data from tests predominantly on NSC beams has to be checked for safety and reliability in HSC beams.
- Recommendations for future research.

1.3 Research Plan

In order to accomplish the above mentioned objectives a research plan was prepared. The proposed research plan is briefly described below:-

- Effect of axial force will be neglected.
- Research will be limited to simply supported beams.
- Data for both the deep and slender beams will be considered.
- Research will also cover both the HSC and NSC.
- Identification of the factors affecting the shear strength of NSC and HSC beams.
- Collection of available test data from literature for concrete beams failing in shear.

- Based on the collected data, shear strength will be evaluated using Microsoft excel graphical tool and best fit curve technique.
- Provision of recommendations for future research.

Chapter 2

SHEAR IN BEAMS

2.1 General

A beam is a structural member that supports applied loads and its own weight primarily by internal moments and shear. Elastic beam theory is not used in the design of RC beams because compressive stress-strain curve of concrete is non linear and concrete cracks at low tensile stresses. Once a RC beam is loaded on top, it causes bending moments and shear. At any section within the beam, the internal resisting moments and shear are necessary to equilibrate these external loads. Initially the beam remains un-cracked; as the load is increased gradually first the flexural cracks start to appear at the bottom of the beam once the corresponding stresses there reach the tensile strength of the concrete. After the redistribution of stresses either flexural cracks grow further or convert themselves to shear cracks depending upon the weaker beam strength either in flexure or shear. Shear failures are known for their unclear, sudden and brittle nature. That is why designers have adopted the approach of considering the flexure first, leading to size of the section and longitudinal reinforcement details followed by safety check against shear. In this approach the design for shear must ensure that the shear strength equals or exceeds the flexural strength at all points in the beam so as to ensure a ductile failure.

The type of shear failure varies widely with section dimensions, geometry, loading conditions, properties of the member, amount of web reinforcement and relative contribution of arch action or beam action. This is why there is no unique way to design for shear. This will be discussed in detail in the proceeding paragraphs. For a RC beam with stirrups, before cracking, the entire shear is carried by concrete. After the formation of cracks, contributions by aggregate interlock, dowel action of longitudinal steel and stirrups come into play. Every component of this process has a brittle behaviour except stirrups. It is difficult to quantify the individual contributions of these components and is, therefore, lumped together as V_c. Thus V_n is assumed to be sum of V_c and V_s.

2.2 Historical Overview Of Reinforced Concrete

Lime mortar was first used in structures in the Minoan civilization in Crete about 2000 B.C. and is still being used in some areas. This mortar had the disadvantage of being water soluble and could not be used in underwater structures and exposed surfaces/joints. About the third century B.C. Romans resolved this problem by mixing fine sandy volcanic ash with lime mortar. Dome of the Pantheon was the most remarkable concrete structure built in Rome (completed in A.D. 126). Just before A.D. 1800 an English engineer John Smeaton discovered that a mixture of burnt lime stone and clay could be used as water resisting cement. In 1824 Joseph Asdin discovered cement by a mixture of ground lime stone and clay from different quarries and heating it in a kiln. He named it as Portland cement after the Portland stone. Harder clinker formed due to overheating in the process was considered as spoil and discarded till 1845 when I. C. Johnson found that best cement could be made by using this ground clinker. Now a day this material is known as Portland cement. In 1871 Portland cement was produced in Pennsylvania by D. O. Saylor and by T. Millen of South Bend in Indiana. In late 1960s with the development of super plasticizers in Japan and Germany, HSC came in to being and found it’s suitability for use in the production of cast in place structural components for tall buildings.

Perhaps the greatest incentive to the early development of the scientific knowledge of RC came from the work of Joseph Monier in about 1850. Till 1900, the science of RC developed through a series of patents. From 1890 to 1920, practicing engineers gradually gained knowledge of the mechanism of RC, as books, technical articles and codes presented the theories. In 1928 pre-stressed concrete was pioneered by E. Freyssinet. In an 1894 paper to the French Society of Civil Engineers, working stress design method for flexure was developed, which was used universally till 1950.

In 1903 a committee on masonry appointed by American Railway Engineering Association presented specifications for Portland cement concrete. In 1904 first set of building regulations for RC issued in Prussia. Design regulations were issued in U.K., France, Austria and Switzerland between 1907 and 1909. A joint committee of ASCE, ASTM, AREA, AAPCM and ACI presented a preliminary report in 1913 and published final report in 1916 (MacGregor and Wight 2005).

2.3 Development Of Beam Shear Concepts

In 1899 Ritter suggested that after a RC beam cracks due to diagonal tension stresses, it can be idealized as a parallel chord truss with compression diagonals inclined at 45^0 with respect to the longitudinal axis of the beam (*Early truss model*). From 1907 to 1908 Withey in a series of two papers introduced this truss model in USA and highlighted that this approach gave conservative results. In 1909 Talbot confirmed the findings of Withey and based on 106 beam tests, he concluded for the failure of the beams without web reinforcement that:-

> It will be found that the value of *v* [shear stress at failure] will vary with the amount of reinforcement, with the relative length of the beam, and with other factors which affect the stiffness of the beam. . . . In beams without web reinforcement, web resistance depends upon the quality and strength of the concrete. . . . The stiffer the beam the larger the vertical stresses which may be developed. Short, deep beams give higher results than long slender ones, and beams with high percentage of reinforcement [give higher results] than beams with a small amount of metal.

Unfortunately, Talbot's findings about the influence of the *percentage of longitudinal reinforcement* and the *length-to depth ratio* were not reflected in the design equations until much later (ACI-ASCE Committee 445 on shear and torsion).

1920 to 1922 Morsch introduced the use of truss models for *torsion* in a two part publication. These truss models neglected the contribution of the concrete in tension. A German engineer, H. A. Wagner (1929), solved an analogous problem while dealing with the shear design of ''stressed-skin'' aircraft. This approach became known as the *tension field theory*.

The August 1955 shear failure of beams in the warehouse at Wilkins Air Force Depot in Shelby, Ohio, brought into question the traditional ACI shear design procedures. These shear failures, in conjunction with intensified research, clearly indicated that shear and diagonal tension was a complex problem involving many variables and resulted in a return to the forgotten fundamentals.

Kupfer (1964) and Baumann (1972) presented approaches for determining the angle θ assuming that the cracked concrete and the reinforcement were linearly elastic. Methods for determining θ applicable over the full loading range and based on Wagner's procedure were developed by Mitchell and Collins (1974) for members in torsion, and were applied to shear design by Collins (1978). This procedure was called the *compression field theory* (CFT). Collins and Mitchell (1980) abandoned the assumption of linear elasticity and developed the compression field theory (CFT) for members subjected to torsion and shear. Based on extensive experimental investigation, Vecchio and Collins (1982, 1986) presented the *modified compression field theory* (MCFT), which included a rationale for determining the tensile stresses in the diagonally cracked concrete. Schlaich et al. (1987) extended the truss model for beams with uniformly inclined diagonals, all parts of the structure in the form of *strut-and-tie models* (STM) and introduced the concept of *D and B regions*. *Rotating-angle softened-truss model* (RA-STM) which accounts for tensile stresses in diagonally cracked concrete in a different way was developed from 1991 to 1995 by Belarbi and Hsu at the University of Houston. Like the MCFT, this method assumes that the inclination of the principal stress direction (θ) in the cracked concrete coincides with the principal strain direction. For typical elements this angle will decrease as the shear is increased. Hence the name ‘‘rotating angle.’’ Pang and Hsu (1995) limit the applicability of the rotating-angle model to cases where the rotating angle does not deviate from the fixed angle by more than 12^0. *Modified truss model approach*, a combination of the variable-angle truss and a concrete contribution was proposed by Ramirez and Breen in 1991.

2.4 Shear In HSC

History of HSC traces back to late 1960s when super plasticizers were developed in Japan and Germany, which made it possible to achieve both reduced water cement ratio and workability simultaneously. It was found very suitable for use in the production of cast in place structural components for tall buildings. Concrete is defined as high strength purely on the basis of compressive strength at a given age. There is a growing movement to specify the 56 or 90th day strength because HSC contains admixtures that delay the final strength gain. Four basic principles have to be considered in the production of HSC:

- Improved aggregate-matrix bond.
- Reduced porosity.
- Improved compaction.
- Application of internal agents such as silica fumes and plasticizers and external agents such as lateral confinement through internal steel hoops, heat or steam curing, proper handling, and strict quality control.

HSC is a more brittle material than NSC. This means that cracks that form in HSC will propagate more extensively than in NSC. Previous shear tests on HSC have shown a significant difference between the failure planes of HSC and that of NSC. This is due to the fact that cracks tend to propagate through the aggregates in the higher strength concretes rather than around the aggregates as in NSC. The result is a much smoother shear failure surface meaning that the shear carried by aggregate interlock tends to decrease with increasing concrete strength.

Mphonde and Frantz (1984) tested concrete beams without shear reinforcement with varying a/d ratios from 0.015 to 0.036 and concrete strengths ranging from 21 to 103 MPa. They concluded that the effect of concrete strength becomes more significant with smaller a/d ratios and that failures became more sudden and explosive with greater concrete strength. It was also found that there is a greater scatter in the results of specimens with small a/d ratios due to the possible variations in the failure modes.

Elzanaty et al. (1986) based on tastes of 18 beams with concrete strengths, f'c, ranging from 21 to 83 MPa observed a smoother failure plane in the HSC specimens, other variables included ρ and a/d. They observed that shear strength increased with increasing f'c but less than that predicted using the AC1 Code equations and concluded that an increase in the steel ratio led to an increase in the shear capacity of the specimens regardless of concrete strength.

Ahmad et al. (1986) studied the effects of the a/d ratio and longitudinal steel percentage on the shear capacity of beams without web reinforcement. For their tests, the concrete strength was maintained as constant as possible with f_c' in the range of 63 to 70 MPa. Findings were similar to previous experiments with a transition in the failure mode at a/d ratio of approximately 2.5. The envelope involving limits on a/d and ρ which separates shear failures from flexural failures was found to be similar to the envelope for the NSC. However, more longitudinal steel was required to prevent flexural failures. Ahmad et al. found that the shear capacity was proportional to $f_c'^{0.3}$.

2.5 Beam Shear Theories

2.5.1 Classical Beam Theory

By the traditional theory for elastic, homogeneous, un-cracked beams, shear stresses can be calculated using the equation

$$v = \frac{VQ}{Ib} \qquad (4)$$

Where Q = first moment about the centroidal axis of the cross sectional area lying farther from the centroidal axis than the point where shear stress are being calculated.

V = shear on the cross section.

I = moment of inertia of the cross section.

b = width of the member at the cross section.

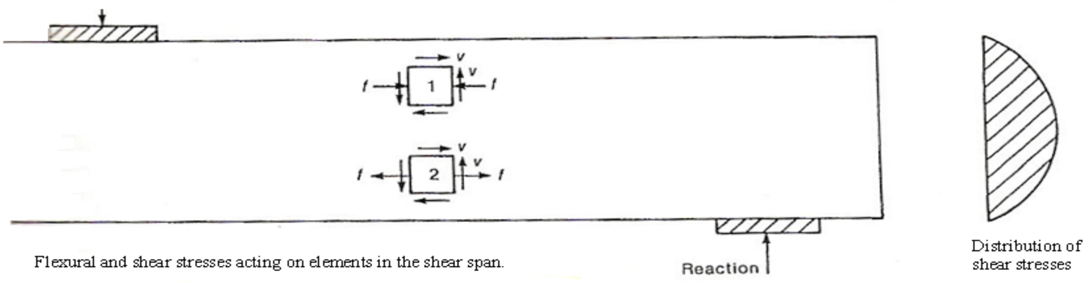


Figure 2.1 Flexural and shear stresses in the shear span for a typical homogeneous rectangular beam (MacGregor and Wight 2005).

The elements shown in figure 2.1 are subjected to combined normal stresses due to flexure and shearing stresses. Equal shearing stresses exit on both the horizontal and vertical planes through the elements. The shear stresses on top and bottom of the elements cause a clockwise couple and those on the other two sides cause anticlockwise. These two couples being equal and opposite in direction cancel out each other. Resulting shear stress distribution is as shown in figure 2.1. The largest and smallest normal stresses acting on such an element are called as principal stresses. Mohr's circle is used to calculate the principal stresses and orientation of the planes on which they act. The orientations of the principal stresses on the elements are shown in figure 2.2. The surfaces on which principal tensile stresses act in the un-cracked beam are shown in figure 2.3. Since the concrete cracks when the principal tensile stresses exceed the tensile strength of the concrete, so the initial cracking pattern should resemble the tensile stress trajectories. Cracking pattern of an actually tested beam is shown in figure 2.4. The vertical cracks are the flexure cracks and diagonal/inclined cracks are termed as inclined cracks, shear cracks or diagonal tension cracks. Such a crack must exist before a beam can fail in shear. Few cracks extend along the reinforcement towards the support weakening the steel concrete bond.

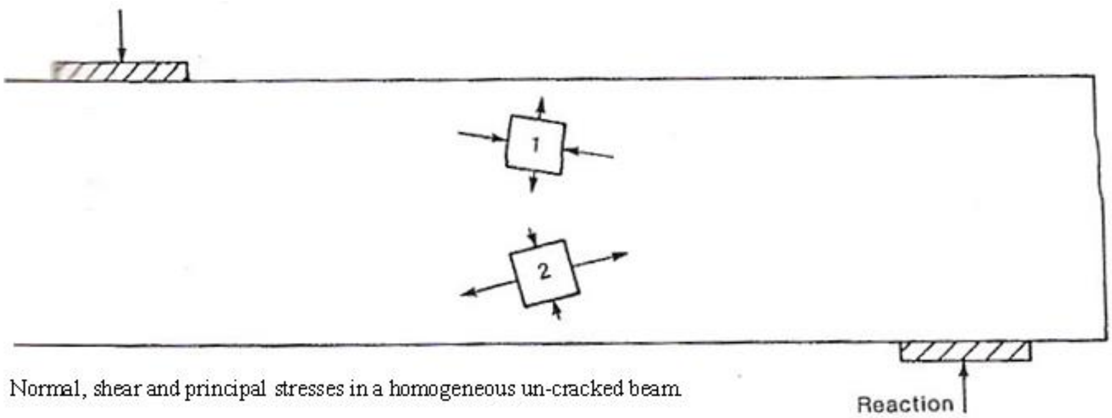


Figure 2.2 Normal, shear and principal stresses in a homogeneous un-cracked beam (MacGregor and Wight 2005).

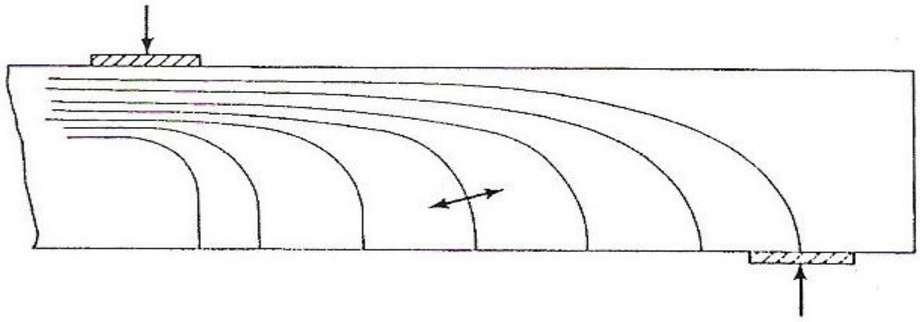


Figure 2.3 Principal compressive stress trajectories in an un-cracked beam (MacGregor and Wight 2005).

In RC beams the similarity between the stress trajectories and the cracking pattern is by no means perfect. In RC beams generally flexure cracks form first followed by a major redistribution of stresses leading to the inclined cracks. That is why onset of inclined cracking cannot be predicted from principal stresses unless shear cracking precedes flexural cracking. This very rarely happens in RC but it does occur in some pre-stressed beams (MacGregor and Wight 2005).

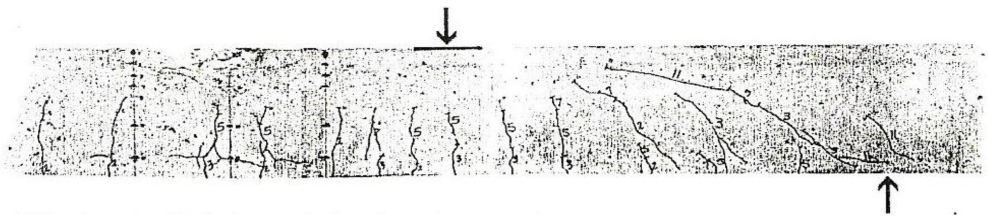


Figure 2.4 Cracking pattern of an actually tested beam.

2.5.2 Average Shear Stress Between Cracks

Consider a beam which is initially cracked, as the load on beam is increased the cracks extend in a vertical direction as shown in figure 2.5. To find the average stress between two cracks it is assumed that the lever arm "jd" remains constant. From the figure 2.5 for the equilibrium of the section of the beam between two cracks, can be written as:-

$$\Delta T = \frac{\Delta M}{jd} \quad \text{or} \quad \Delta T = \frac{V\Delta x}{jd} \tag{5}$$

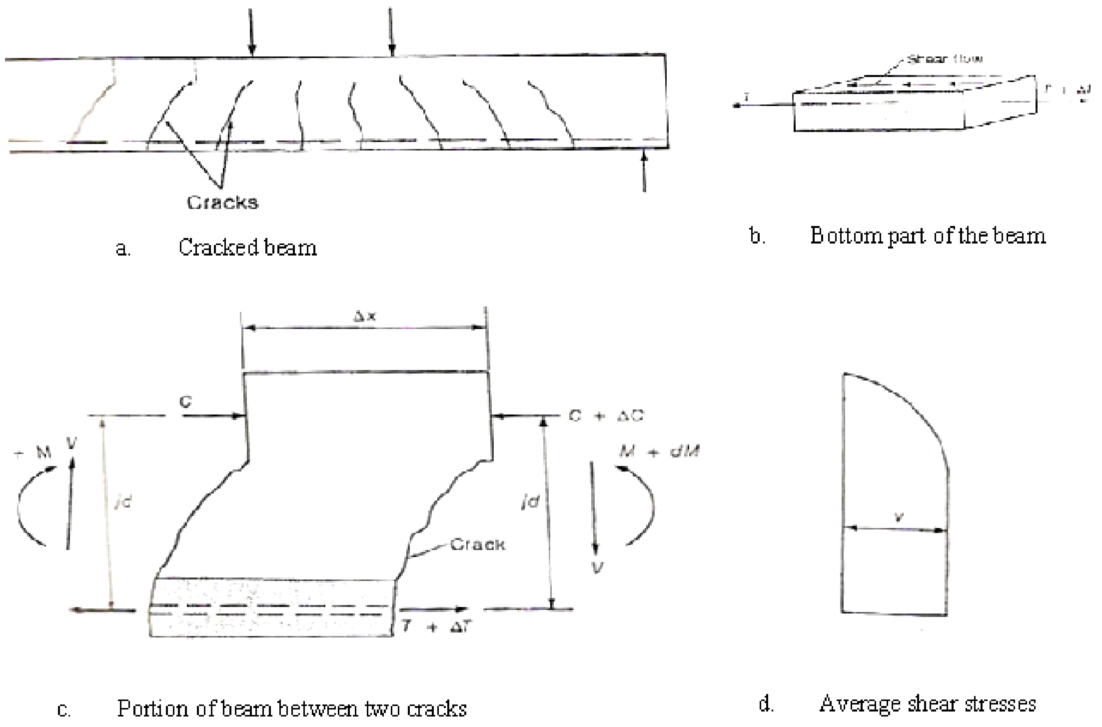


Figure 2.5 Calculation of average stress between cracks (MacGregor and Wight 2005).

From figure 2.5b it is clear that the force ΔT must be transferred by horizontal shear stresses on the top of the element. The average value of these stresses can be calculated as:-

$$\mathcal{V} = \frac{\Delta T}{b_w \Delta x} \quad \text{or} \quad \mathcal{V} = \frac{V}{b_w jd} \qquad 6$$

Since the horizontal shear stresses are equal to vertical shear stresses on the same element so the resulting distribution of shear stresses are shown in figure 2.5.

2.5.3 Concept Of B-Regions And D-Regions

St. Venant's principle suggests that a local disturbance dissipates within one member depth from the point of disturbance. On the bases of this principle it is assumed that D-regions extend about one member depth each way from concentrated load, reaction or abrupt changes in section geometry. The regions between two such regions can be treated as B-regions as shown in figure 2.6. D stands for discontinuity or disrupted and B for Bernoulli, who postulated the linear strain distribution in beams. Longer shear spans carry load by beam action and are referred to as B-regions. Shorter shear spans carry load primarily by arch action involving in plane forces. In general presence of D-regions gives rise to arch action and results in enhanced shear strength. Probably this is the reason for increased shear strength in deep beams having a/d < 2 as shown in figure 3.1. On the other hand B-regions tend to be weaker than corresponding D-regions in shear strength.

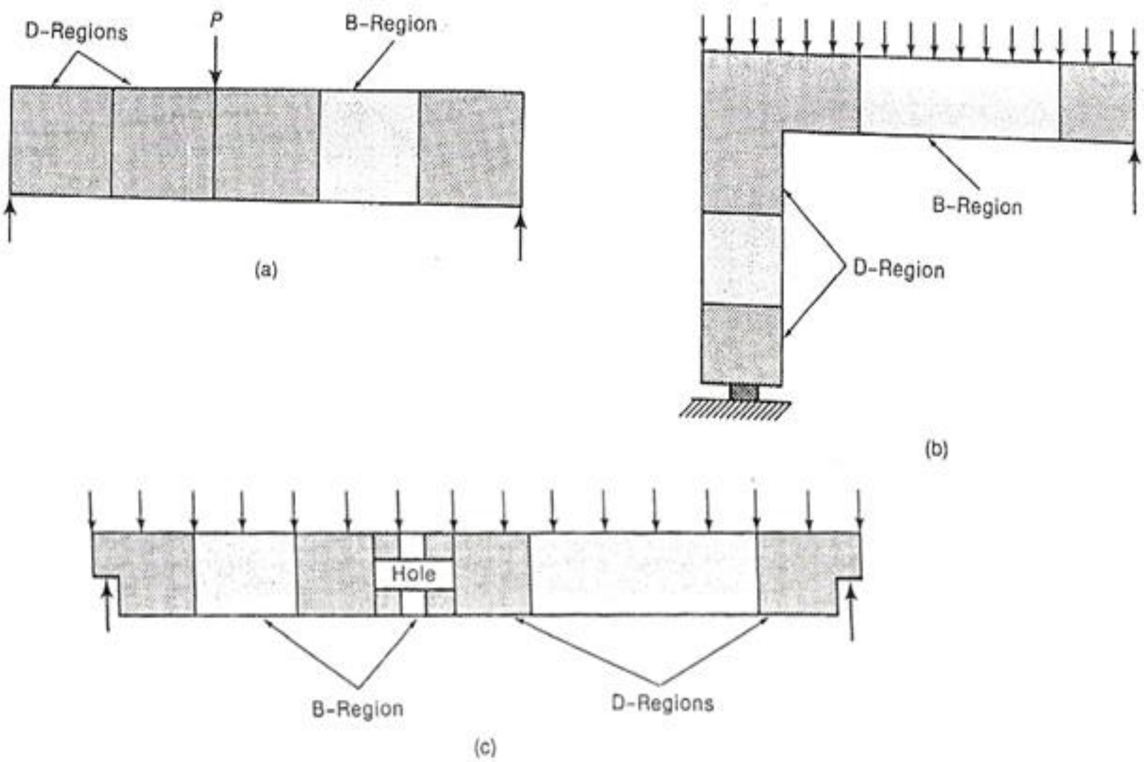


Figure 2.6 D-regions and B-regions (MacGregor and Wight 2005).

2.5.4 Beam Action And Arch Action

From knowledge of the relationship between shear and bar force for the beam shown in figure 2.5 following relation can be achieved:-

$$V = \frac{d}{dx}(M) = \frac{d}{dx}(Tjd) \quad 7$$

This can be expanded as

$$V = \frac{d(T)}{dx} jd + \frac{d(jd)}{dx} T \qquad 8$$

From here two extreme cases can be identified as either "T" or "jd" remains constant. If the lever arm "jd" remains constant then;

$$\mathrm{V} = \mathrm{jd}\frac{\mathrm{d(T)}}{\mathrm{dx}} \qquad 9$$

This is shear flow as shown in figure 2.5. This phenomenon is called as "beam action". Thus for beam action to exist shear flow must exist.

The other extreme can be when T = C, in other words T = C remains constant then;

$$\mathrm{V} = \mathrm{T}\frac{\mathrm{d(jd)}}{\mathrm{dx}} = \mathrm{C}\frac{\mathrm{d(jd)}}{\mathrm{dx}} \qquad 10$$

This occurs when shear flow cannot exist. Two reasons can be responsible for this i.e. either the bond between steel and concrete is lost or shear flow is completely disrupted by inclined cracks. In such a case shear is transferred by "arch action", as is shown in figure 2.6.

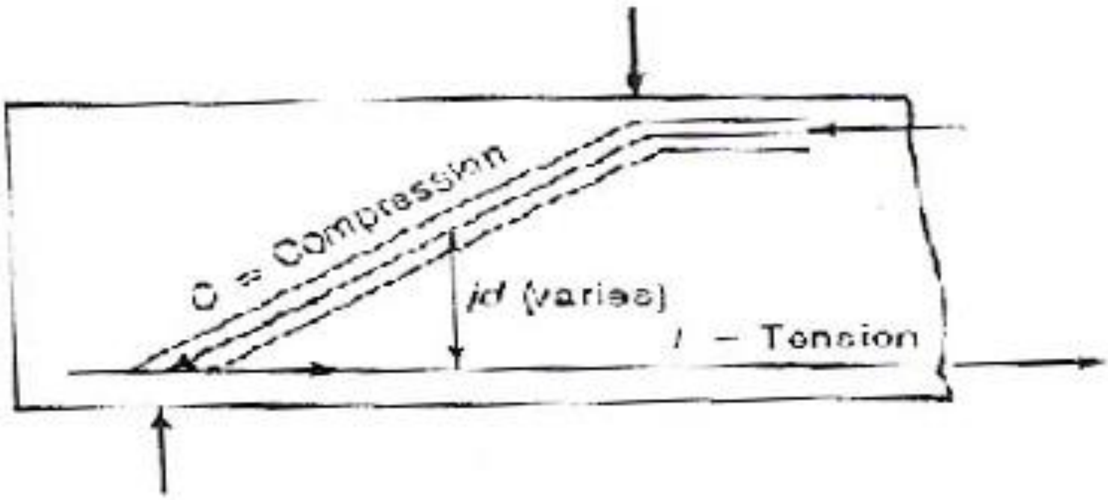


Figure 2.7 Arch action in a beam (MacGregor and Wight 2005).

2.5.5 **Need For Shear Reinforcements**

Consider a beam made of planks as shown in figure 2.9. If these planks are not bonded together and loaded, there will be longitudinal sliding. If they are bonded together (e.g. with stirrups), there will be no sliding in longitudinal direction and horizontal shear stresses will develop as a result. Also if beam is seen from theory of flexure, longitudinal reinforcement is provided to restrain flexural cracks from opening as shown in figure 2.8. An inclined crack opens approximately perpendicular to itself as shown in figure 2.8b. In practice either a combination of flexural and inclined reinforcement (figure 2.8c) or combination of flexural and vertical reinforcement (figure 2.8d) is used to control shear. The inclined or vertical reinforcements are referred to as shear/web reinforcement or stirrups. It is advisable not to use inclined stirrups in seismic areas as they are not effective in resisting reversal of shear resulting from seismic activity, because shear reversals will cause cracking parallel to the inclined stirrups rendering it ineffective.

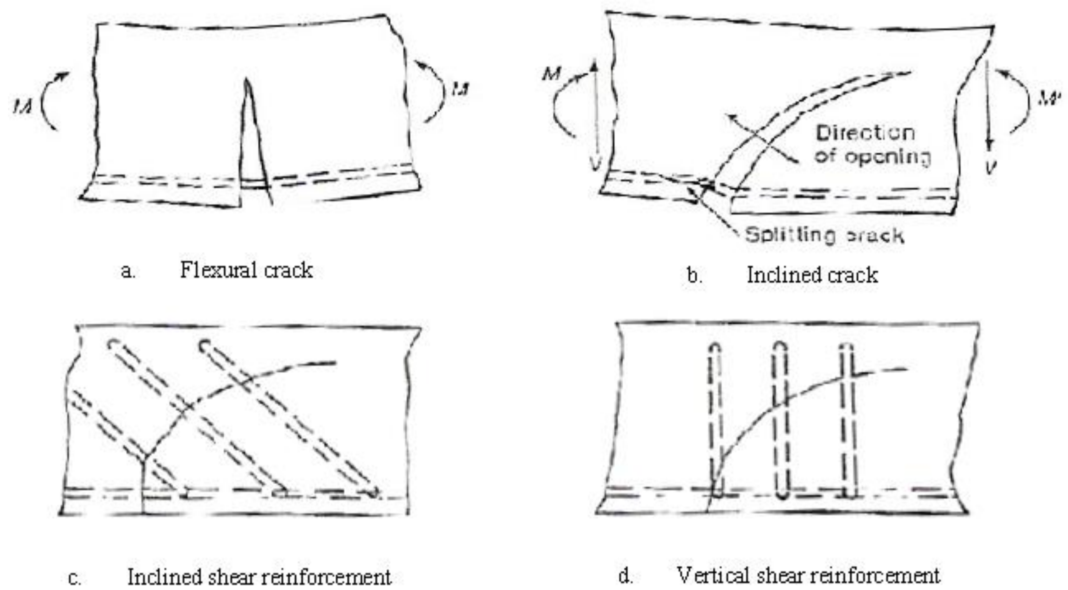


Figure 2.8 Cracks and reinforcements in a RC beam (MacGregor and Wight 2005).

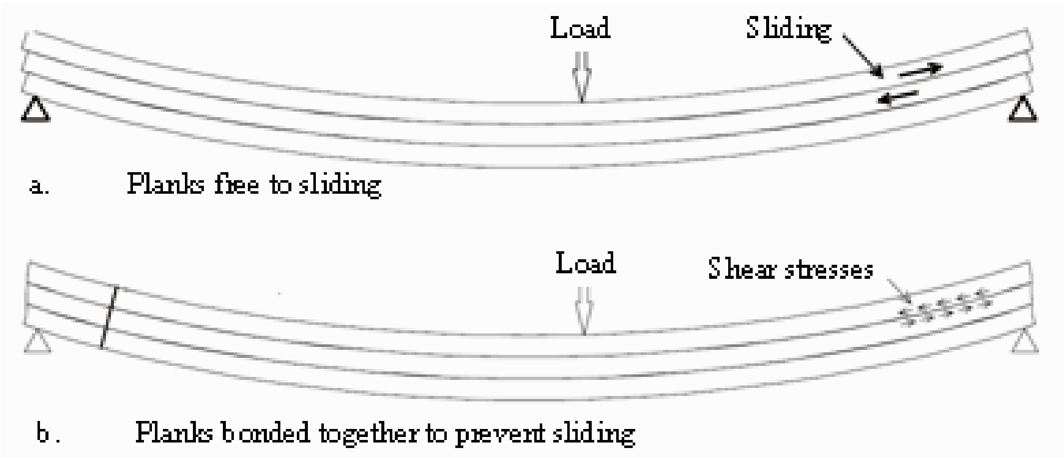


Figure 2.9 Shear stresses developed due restricting longitudinal sliding of planks.

2.5.6 Type Of Web Reinforcements

- Stirrups or ties perpendicular to the longitudinal axis of the beam.
- Stirrups inclined at angle θ to the longitudinal axis of the beam.
- A portion of flexural reinforcement as bent up bars.
- Welded wire fabrics perpendicular to the longitudinal axis of the beam.
- A combination of spirals, circular ties and hoops.

2.5.7 Truss Model

Truss model was originally introduced by Ritter and Morsch in 1899 and 1902 separately. According to this model a cracked RC beam is considered to be a truss with compression and tension force in the top and bottom flanges respectively, vertical tension in the stirrups and inclined compressive forces in the concrete diagonals between the inclined cracks. Several assumptions and simplifications are made to solve this analogous truss. A well known simplification/assumption is yielding of the stirrups before diagonal concrete between the inclined cracks crushes, as failure load reaches (plastic truss model). This model ignores the shear carried by concrete, aggregate interlock and dowel action.

2.5.8 Compression Field Theory

In compression field theory individual compression struts running diagonally from top to the bottom chords between adjacent stirrups are replaced with a series of narrow parallel compression struts that form a continuous compression field. At any given section, the magnitude and direction of the compression stresses are assumed to be constant over the depth of the web. When compression field action is assumed, each stirrup is proportioned in a way to carry the vertical component of the diagonal forces in all of the narrow diagonal struts within a distance of half of the stirrups spacing on either side of a stirrup. It is pertinent to mention that compression field theory ignores the tensile stresses in cracked concrete. Thus beams without stirrups are predicted to have no shear strength according to this theory.

2.5.9 Modified Compression Field Theory

It is a modification of compression field theory which accounts for the influence of tensile stresses in the cracked concrete. This was introduced by Vecchio and Collins in 1986. It also recognises that local stresses in concrete and reinforcement vary from point to point in the cracked concrete.

2.5.10 Shear Friction Method.

It is based on the idea that shear must be transferred across an interface between two surfaces that can slip relative to one another. This shear carrying mechanism is commonly known as aggregate interlock, interface shear transfer or shear friction. It takes shear carried by concrete to be equal to the shear force transmitted across each of a series of shear slip planes. The clamping force needed to mobilize shear friction along the crack is the sum of the components perpendicular to the plane, of the longitudinal steel and stirrups.

Chapter 3

BEHAVIOR OF BEAMS IN SHEAR

3.1 Beams Without Web Reinforcements

In regions of large bending moments, cracks develop almost perpendicular to the axis of the beam. These cracks are called *flexural cracks*. In regions of high shear due to the diagonal tension, the inclined cracks develop as an extension of the flexural crack and are termed *flexure shear cracks* or *inclined cracks*. The slenderness of the beam is very important factor. It can be used to classify beam behaviour i.e. deep, slender and very slender beams and also can be used as an indicator for type of failure. This is excellently described by MacGregor in his book "reinforced concrete mechanics and design" forth edition. Figure 3.1 demonstrates schematically the failure patterns. Beam cross section remains constant as the span is varied. Maximum moment and shear which can be developed correspond to M_n (plotted as horizontal line). Shaded area shows reduction in strength due to shear. The figure clearly divides the shear span into following types:-

Ser	a/d	Type	Arch/Beam action	Final failure
a.	0 - 1	Very short spans	Arch	Anchorage failure
b.	1 - 2.5	Short shear spans	In part arch/beam	A bond failure, or a splitting failure, or a dowel failure, or a shear compression failure
c.	0 - 2	Deep beams	Arch	Compression struts or anchorage failure
d.	2.5 - 6	Slender shear spans	Beam	Inclined cracking
e.	> 6	Very slender shear spans	Beam	Flexural failure

Table 1 Failure Modes Of Beams Without Web Reinforcements.

Very short spans with a/d from 0 to 1 develop inclined cracks joining the load and support. These cracks destroy the horizontal shear flow from longitudinal steel to the compression zone and the behaviour changes from beam action to arch action. The steel reinforcement serves as a tension tie of a tied arch and has uniform force from support to support. The most common failure in such beams is an anchorage failure at the ends of tension tie. Short shear spans with a/d from 1 to 2.5 develop inclined cracks and such beams after redistribution of internal forces are able to carry additional loads, in part by arch action. The failure of such beams will be caused by a bond failure, or a splitting failure, or a dowel failure along the tension reinforcement, or a shear compression failure. Slender shear spans having a/d from 2.5 to 6 develop inclined cracks which disrupt the equilibrium to such an extent that the beam fails at inclined cracking loads. Very slender shear spans with a/d greater than about 6 will fail in flexure prior to the formation of inclined cracks.

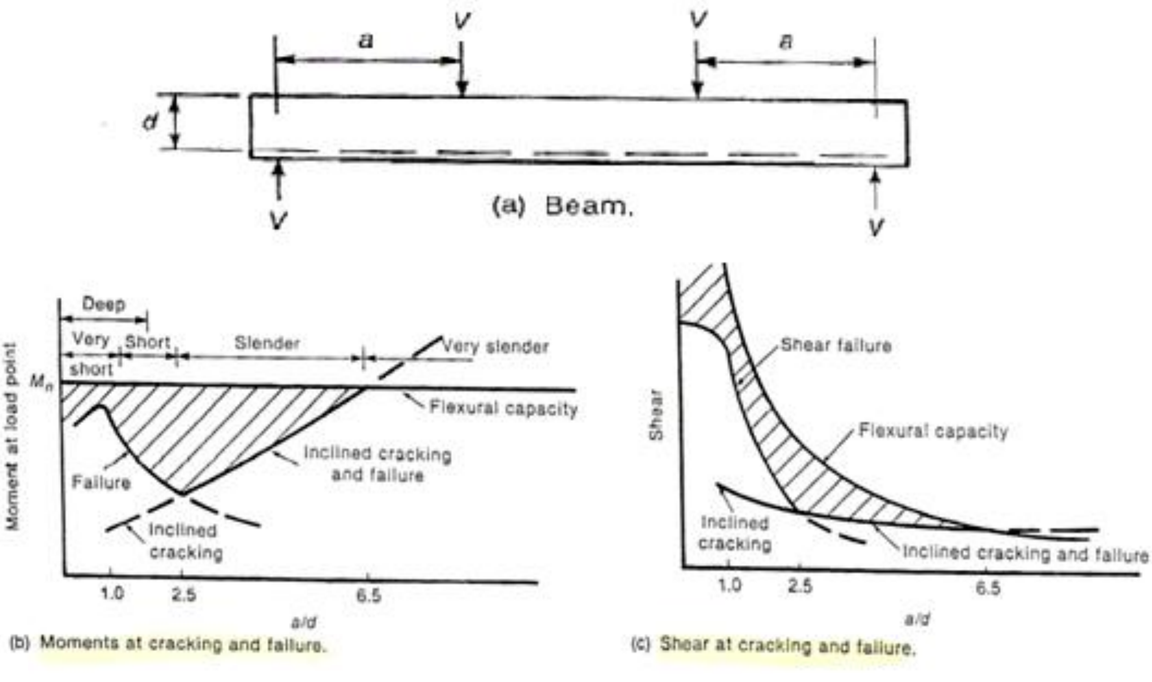
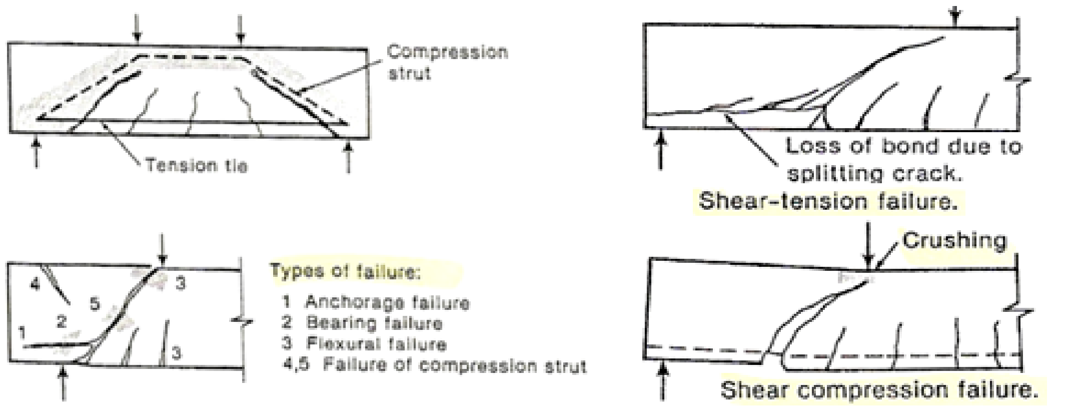


Figure 3.1 Effect of a/d ratio on shear strength of beams without stirrups (MacGregor and Wight 2005).



a. Modes of failure of deep beams (a/d = 0.5 to 2.0)

b. Modes of failure of short shear spans (a/d = 1.5 to 2.5)

Figure 3.2 Modes of failure of beams (MacGregor and Wight 2005).

3.2 Internal Forces In A Beam Without Stirrups

Figure 3.3 gives graphical view of the internal forces being transferred across a shear crack. Where:-

V_{cz} = Shear being transferred in the compression zone

V_{ay} = Vertical component of the shear transferred by aggregate interlock

V_d = Dowel action of the longitudinal reinforcement.

Immediately after inclined cracking as much as 40 to 60 % of the total shear is carried by V_d and V_{ay} together (Taylor 1970). As the crack widens V_a decreases calling for an increase in V_{cz} and V_d. It leads to splitting crack along the longitudinal reinforcement. When this occurs V_d drops, approaching zero, and cracks continue to be wider growing upwards. Figure 3.3 shows that it should be accompanied by gradual decrease in V'_{cz} C'_1. Now at this stage width AB above crack is the only portion left to resist shear and compression. This is the stage when width AB is to less to produce forces needed for equilibrium of the section and section fails. Upon this situation, MacGregor (2005) rightly highlighted the need for considering the tensile force in detailing the bar cut off points and its anchorage. He assumes that about 30% of the shear is transferred in compression zone and 70% across the cracks. In 1970, Taylor reported tests of beams without web reinforcement in which he found that about 25 percent of the shear was transferred by the compression zone, about 25 percent by dowel action of flexural reinforcement and about 50 percent by the aggregate interlock along the crack. Modern shear failure theories assume that a significant amount of the shear is transferred in the web, most of which across the shear cracks.

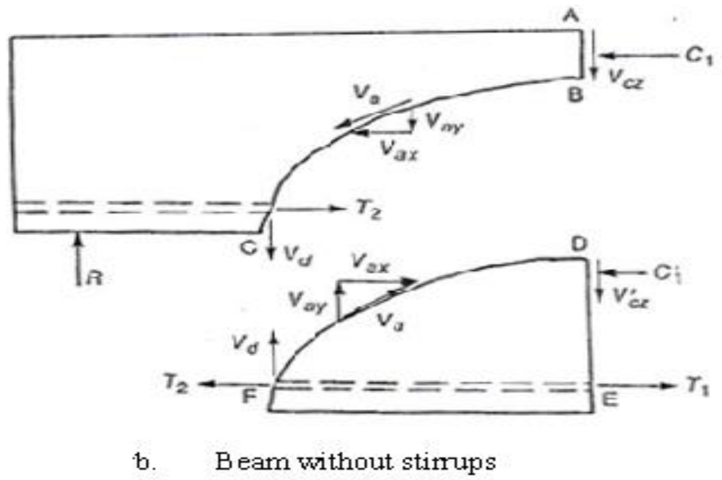


Figure 3.3 Internal forces in a cracked beam without stirrups (MacGregor and Wight 2005).

3.3 Failure Modes Of RC Beams Without Web Reinforcement

3.3.1 Failure By Flexural Cracks

In this type of failure, cracks are mainly vertical and perpendicular to the lines of principal stress. These cracks result from a very small shear stress and a dominant flexural stress. In such a failure mode, a few very fine vertical cracks start to develop in the mid-span area at stresses equal to modulus of rupture. As the external load increases, additional cracks develop in the central region of the span, and the initial cracks widen and extend deeper toward the neutral axis and beyond, with a marked increase in the deflection of the beam. For an under reinforced beam, failure occurs in a ductile manner by initial yielding of the main longitudinal flexural reinforcement. This type of behaviour gives ample warning of the collapse of the beam. The a/d value for this behaviour exceeds 6 in the case of concentrated loading.

3.3.2 Failure By Flexure Shear Cracks

This failure precipitates if the strength of the beam in diagonal tension is lower than its strength in flexure. Slenderness is of intermediate magnitude, with the values varying between 2.5 and 6 for the case of concentrated loading. Such beams can be considered of intermediate slenderness. Cracking starts with the development of a few fine vertical flexural cracks, followed by a concentration of stresses near the head of few of these crack. In time either the flexure crack extends to become flexure shear crack or a new flexure shear crack develops in the un-cracked region over the flexure cracks. This happens normally without ample warning of impending failure, at about d to 2d distance from the face of the support.

3.3.3 Failure By Web Shear Cracks

In thin walled I beams in which a/d is small, shear stresses in the web are higher than flexural stresses. In few extreme such cases and in some pre-stressed beams if the principal tension stresses at the neutral axis exceed those at the bottom flange, a web shear crack occurs there resulting in web crushing.

3.3.4 Shear Compression Failure

In this type of failure, final failure is by crushing of compression zone concrete over the crack.

3.3.5 Shear Tension Failure

This type of failure is by loss of bond due to splitting crack.

3.3.6 Anchorage Failure

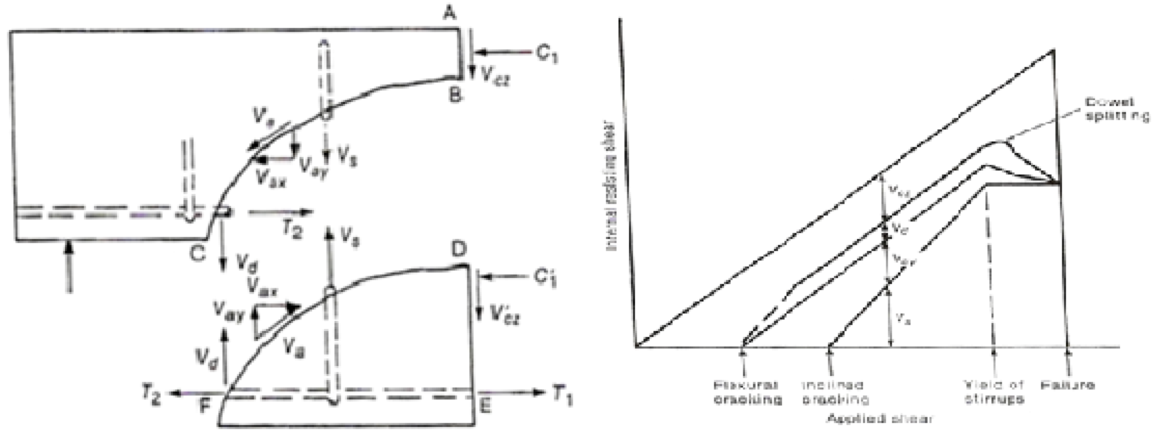
This type of failure is due to anchorage failure at the end of the tension tie (longitudinal reinforcement).

3.4 Beams With Web Reinforcements

Prior to the formation of diagonal cracks, stirrups contribute very little to the shear resistance of the beam. Therefore the pattern of initial cracking and magnitude of external loads causing the initial diagonal cracking are not affected to appreciable extent by the presence of stirrups. When a diagonal crack occurs, there must be a redistribution of internal forces at the cracked section. The external shear previously resisted by concrete web alone is now redistributed partially among the longitudinal tensile reinforcement, aggregate interlock, compression zone of the concrete and stirrups. Stirrups provide dual purpose, firstly by taking part in redistribution of shear at the given section and secondly by restricting the penetration of the diagonal crack into the compression zone. Hence stirrups not only take part in carrying a part of shear but also improve the ability of the concrete compression zone to resist the shear.

3.5 Internal Forces In A Beam With Stirrups

Figure 3.4a gives graphical view of the internal forces being transferred across a shear crack of a beam with stirrups. Notations and the forces are same as that for the case of beam without stirrups with the addition of a new force V_s i.e. shear carried by the stirrups. Prior to the formation of inclined cracks strain in the stirrups is equal to the strain in the neighbouring concrete. Since concrete cracks at very small strain the corresponding strain in stirrups will be negligible. Thus stirrups do not prevent the crack initiation, they come into play only after the cracks have formed. Prior to flexural cracking whole of the shear is carried by concrete. After the formation of flexural cracks and before the inclined cracking, the internal shear is carried in part by V_{cz} , V_d and V_{ay}. As the crack widens V_a decreases and strain in stirrups start increasing. Eventually the stirrups yield and V_s stays constant for higher applied shears. Yielding of stirrups is accompanied by widening of the crack more rapidly. At this stage in time V_{ay} decreases further forcing V_{cz} and V_d to increase at accelerated rate, until a splitting (dowel) failure occurs or the compression zone crushes or the web crushes. Figure 3.4 b explains this process of cracking till failure. It leads to splitting crack along the longitudinal reinforcement. Each of the components of this process has brittle behaviour except V_s.



a. Beam with stirrups b Distribution of internal shears in a beam with stirrups.

Figure 3.4 Internal forces and distribution of internal shears in a beam with stirrups (MacGregor and Wight 2005).

3.6 Failure Modes Of RC Beams With Web Reinforcement

3.6.1 Failure Due To Yielding Of Stirrups

As external load increases after formation of diagonal crack, the web reinforcement and compression zone continue to carry the shear until web reinforcement yields. Failure occurs when compression zone is incapable of carrying further shear. This type of failure is not sudden and gives ample warning of the impending failure.

3.6.2 Stirrup Anchorage Failure

If the stirrups are not well anchored and upper end of diagonal crack reaches very close to the compression zone then anchorage failure may occur before the yielding of stirrups which is not desirable. Common reasons for this may be inappropriate hook arrangement, less lap splicing (deep beams), very high strength steel used in stirrups may develop crack during bending for hook.

3.6.3 Failure Initiated By Crushing Of Web

Compression stresses in compression diagonals in the web of a very thin walled beam may lead to the crushing of the web before any other type of failure occurs.

3.6.4 Failure Of Tension Chord

This type of failure occurs when flexural steel cannot provide the required tension.

3.6.5 Serviceability Failure Due To Excessive Crack Width At Service Load

This type of failure may occur due to water penetrating the beam through the wide inclined cracks and corroding the stirrups.

3.7 Parameters Influencing Shear Capacity Of A Beam

3.7.1 Shear Span To Depth Ratio

Shear span to depth ratio (a/d) is very important factor. It can be used to classify beam behaviour i.e. deep, slender and very slender beams and can also be used as an indicator for type of failure as already discussed in detail earlier. The shear span "a" for concentrated load is the distance between the point of application of the load and the face of support. For distributed loads, the shear span has yet not a very clear definition. The slenderness of the beam is a measure of a/d. More slender the beam, stronger is the tendency toward flexural behaviour. Based on experimental studies

Kani (1964) proposed that the shear strength increases as a/d decreases. In beams with a low a/d, the applied force is transmitted directly to the supports by arch action (compressive struts) of the concrete. If the a/d is greater than 6, however, flexural action is dominant and the shear strength of the beam is not significantly affected by the a/d ratio.

3.7.2 Size Effect

Kani (1967) based on test results showed that there exist a significant size effect on the shear strength of members without transverse reinforcement but ACI Equations (11-3) and (11-5) generally give safe prediction result as shown in figure 3.5. Equation (11-5), in particular, is effective at capturing the effect of a/d at the largest depth. But the important thing to note is Kani chose a high reinforcement ratio of about 2.8% (close to the balanced reinforcement ratio) for all of the size effect beams in his research. Shioya et al. (1989) reaffirmed this fact in a study in which they tested RC members that ranged in depth from 4 to 118 inches. All members were simply supported, did not contain shear reinforcement, were lightly reinforced in flexure (0.4%), and subjected to a uniformly distributed load. In Figure 3.6 the horizontal line corresponds to the shear strength calculated using the traditional shear design expression of the ACI and AASHTO Standard Specifications. The results show that the shear stress at failure decreases as the depth of the member increases. Of particular concern is that members greater than 36 inches deep failed under stresses approximately one-half of the strength calculated by these codes of practice.

Collins et al. 1993 based on tests showed that the size effect disappears when beams without stirrups contain well distributed longitudinal reinforcement. According to ACI committee 445 there is general agreement that the main reason for this size effect is the larger width of diagonal cracks in larger beams; however, there is disagreement on how best to model this phenomenon.

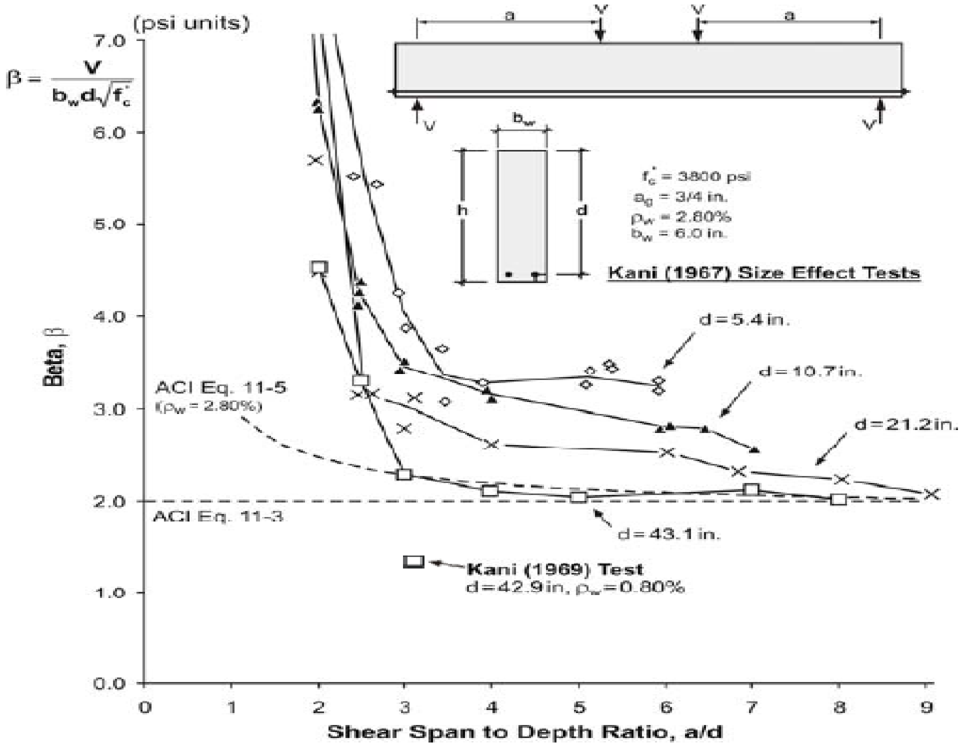


Figure 3.5 Kani's size effect. (Edward G. Sherwood 2008)

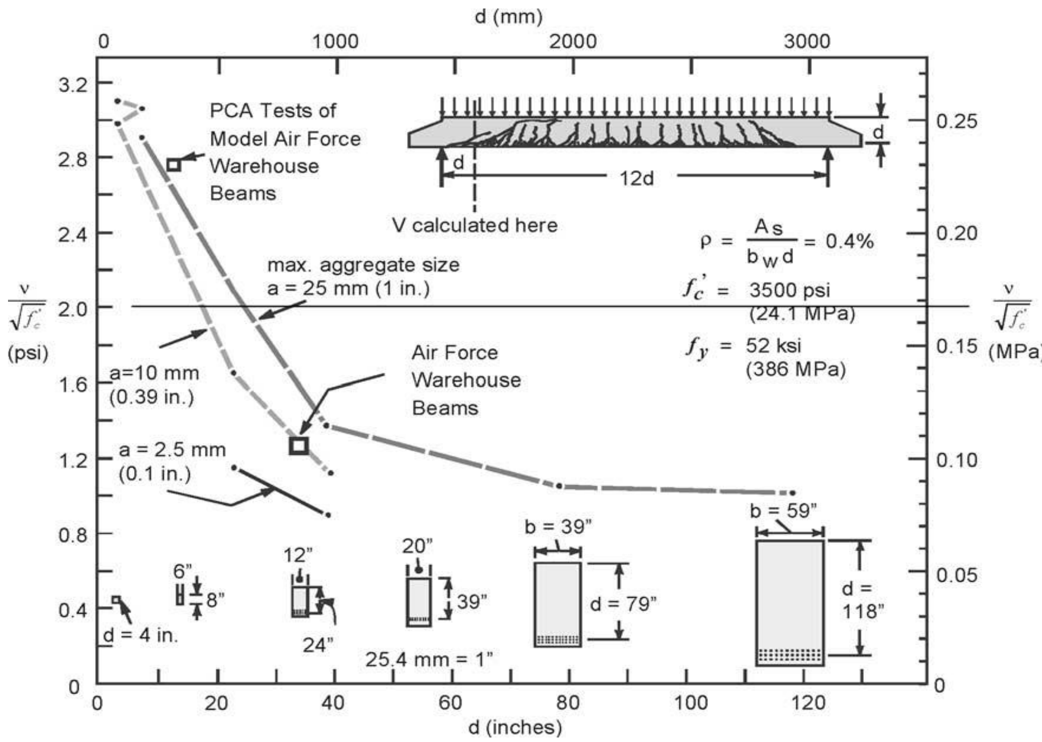


Figure. 3.6 Influence of depth on shear (MacGregor and Wight 2005).

Edward G. Sherwood (2008) reported in his PhD thesis that Collins et. al. (2007) assembled an extensive database of 1849 shear tests as summarized in Figure 3.7. This figure, indicates that only about 1.2% of the tests reported in the literature consisted of slender beams (a/d>2.5) with a very large effective depth. Furthermore, 84% of the shear failures occurred in beams with an effective depth less than 16in. It is apparent, then, that despite the continued interest in shear research since the ACI 318 shear provisions were finalized in 1963, there has been relatively little effort directed at investigating the size effect in shear. It is thus not surprising the size effect and how (or indeed, whether) to account for it in design codes, remains a controversial subject. According to *NCHRP REPORT 549 (2005)* although this depth effect is marked for beams without transverse reinforcement but available test data show little if any depth effect for beams with transverse reinforcement.

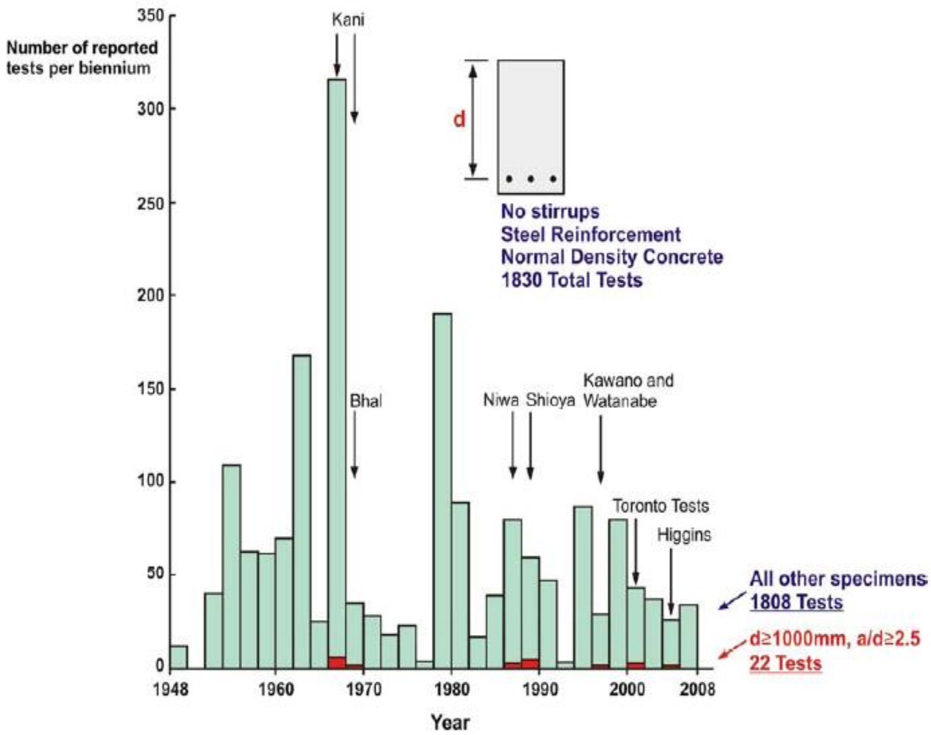


Figure. 3.7 Summary of 60 Years of Shear Research on Members without Stirrups (Edward G. Sherwood 2008).

3.7.3 Longitudinal Reinforcement

Although Kani in 1967 showed that the shear is size dependent but his one of the beam tested (lightly reinforced in longitudinal steel) in 1969 marked on figure 3.5 shows shear dependency on flexural steel as well. The 1973 ASCE-ACI Committee 426 report recognised the influence of longitudinal steel ratio in increasing the crack width and lowering the interface shear transfer. The report suggested that the following equation, incorporating the percentage of longitudinal tension reinforcement, be used to estimate the average shear stress at diagonal cracking:

$$v_c = (0.8 + 100\rho)\frac{\sqrt{f'_c}}{12} \leq 0.192\sqrt{f'_c} \quad \text{(MPa)} \qquad 11$$

3.7.4 Axial Force

It is well known that axial tension decreases the shear strength of members and axial compression increases the shear resistance. For the purpose of this research effect of axial force is out of scope.

3.8 Development Of The ACI Shear Design Method

ACI code is originally based on 45° truss analogy for the RC members with web reinforcement, presented in 1899 by the Swiss engineer Ritter and in 1902 by the German engineer Mörsch. The model allowed designers to calculate tensile stresses in flexural steel and stirrups and compressive stresses in the un-cracked compression zone and struts inclined at 45°. The method presented following simple equation:-

$$v = \frac{V}{b_w jd} = \frac{A_v f_v}{b_w s} \qquad 12$$

V = shear force at a section $\qquad$ b_w = beam width

f_v = stress in stirrups(safe working stress) A_v = area of stirrups

s = stirrup spacing jd = flexural lever arm

ACI simplified the equation by replacing the term jd with d. The equation was criticized for being overly conservative. The model assumed that only transverse reinforcement is effective at carrying shear and a section without stirrups or bent-up bars will have no shear strength which is not 100% true. Extensive research efforts resulted in an empirically derived safe working shear stress relation as given below

$$\frac{V_c}{b_w d} = v_c = 0.03f'_c \qquad 13$$

Now the shear resistance of a RC section was considered as sum of two components, a concrete contribution (V_c) and a web reinforcement contribution (V_s). This method remained in use till early 1960's.

In 1962, ACI Committee 326 summed up the research triggered by the Wilkins Air Force Warehouse roof collapse and recommended a simple, conservative expression of equation 1 for the failure shear based on a curve-fit through 194 experimental data points as shown in Figure 1.1. This well-known expression entered design use through incorporation into the 1963 ACI Design Code, and has remained essentially unchanged since then. A major change was recognition of the influence of longitudinal steel and a/d in a slightly different way. The parameter $\rho_w V_u d/M_u$ was chosen because the stress, f_s, in the flexural steel at shear failure is directly proportional to this parameter, and it was observed that the shear stress at failure decreased as f_s increased. Low values of the term $1000\rho wVd/M(f'c)^{0.5}$ in Figure 1.1 represent sections with small reinforcement ratios and/or subjected to high moment in relation to the shear. The equation 1 is currently Equation (11-5) of the ACI 318 design code 2005. The 1963 ACI 318 Code also included a simplified version of the equation 1 given as equation 2. This Equation is currently Equation (11-3) of the ACI 318 design code 2005 and owing to its simplicity has greater use in practical design situations than does equation 11-5. The ACI 318-05 code also limits $(f'c)^{0.5}$ to 100psi.

3.9 Review Of ACI Code Design

A careful analysis of ACI Equation 11-3 and 11-5 shows that the member depth, M/Vd and ρ_w can be safely neglected and only the concrete strength need be considered in design. Edward G. Sherwood (2008) gave a good review of ACI design equation in his PhD thesis. He pointed out that the largest slender beam (a/d>2.5) in the database used to derive Equation (11-5) of the ACI 318 code had an effective depth, d, of 14.75 in and the average depth for all of the beams was 13.4 in. Hence it is not surprising that the resulting design expression did not account for the size effect. The difficulty in applying empirical design equations to situations outside the scope of the dataset used to derive the equations is apparent, therefore ACI design procedures may be un-conservative when applying to flexural members without stirrups outside database range.

3.10 Existing Equations

The first and simplest approach (Morsch 1909) is to relate the average shear stress at failure to the concrete tensile strength (ASCE-ACI Committee 445 on Shear and Torsion). Experimental results have shown that the average principal tensile stress to cause flexure shear cracking is usually much less than concrete tensile strength. A simple and well-known equation given below is a reasonable lower bound for average shear stress at diagonal cracking for smaller slender beams that are not subjected to axial load and have at least 1% longitudinal reinforcement:-

$$\frac{V_c}{bd} = v_c = 2\sqrt{f_c'} \qquad 14$$

The 1962 ASCE-ACI Committee 326 report presented equation 1 as a more complex empirical equation for calculating the shear capacity of beams without web reinforcement and a simplified version in the form of equation 2. Shear carried by web reinforcement is calculated from equation 3. For a number of reasons, this equation is now considered inappropriate. Even ASCE-ACI Committee 426 suggested that this equation be no longer used.

Zsutty (1971) on the basis of statistical studies of the beam data for selender beams without web reinforcements presented the following empirical equation:

$$v_c = 59 \times \sqrt[3]{\left(f_c' \, \rho_w \, \frac{d}{a}\right)} \qquad (psi) \qquad 15$$

$$v_S = \rho f_y b_w d \qquad 16$$

Bazant and Kim (1984) based on fracture mechanics presented a formula which also included the maximum size of the aggregates apart from other known factors. The formula is applicable to both the beams with and without web reinforcement.

$$V_c = \frac{\sqrt[3]{\rho}}{\sqrt{1+\frac{d}{25d_a}}}\left\{0.833\sqrt{f_c'} + 207\sqrt{\frac{\rho}{\left(\frac{a}{d}\right)^5}}\right\} bd \qquad 17$$

where f_c'' in MPa and d_a = maximum aggregate size

The empirical formula by Okamura and Higai (1980) and Niwa et al. (1986) considers all the main parameters

$$v_c = 0.2\,\frac{\sqrt[3]{\rho}}{\sqrt[4]{d}}\,\sqrt[3]{f_c'}\left(0.75 + \frac{1.4}{a/d}\right) \qquad \text{(MPa)} \qquad 18$$

Where ρ is expressed as a percentage, d in meters and f'_c in MPa. This equation is recommended by ACI committee 445 as one of the more reliable empirical formulas as recent test results on large beams were considered for the size effect.

ACI committee 445 comments on various empirically developed formulas as, “considerable differences exist as a result of the following factors: the uncertainty in assessing the influence of complex parameters in a simple formula; the scatter of the selected test results due to inappropriate tests being considered (for example, bending failures or anchorage failures); the poor representation of some parameters in tests (for example, very few specimens with a low reinforcement amount or high concrete strength); and finally, the concrete tensile strength often not being evaluated from control specimens. These issues limit the validity of empirical formulas and increase the necessity for rational models and theoretically justified relationships.”

ANALYSIS AND DISCUSSION

4.1 General

Shear strength of a RC beam varies with $f_c^{/}$, a/d, section geometry, the amount and arrangement of longitudinal/web reinforcement. A database of 84 beams without web reinforcement and 160 beams with web reinforcement has been collected from the literature. This data is used to evaluate test results and to draw logical conclusions.

4.2 Major Factors Affecting Shear Strength of RC Beams

4.2.1 Concrete Strength f'c

Concrete strength is a major factor influencing shear in RC beams but its use as sole predictor of shear strength is perhaps not justified as shown in figure 4.1.

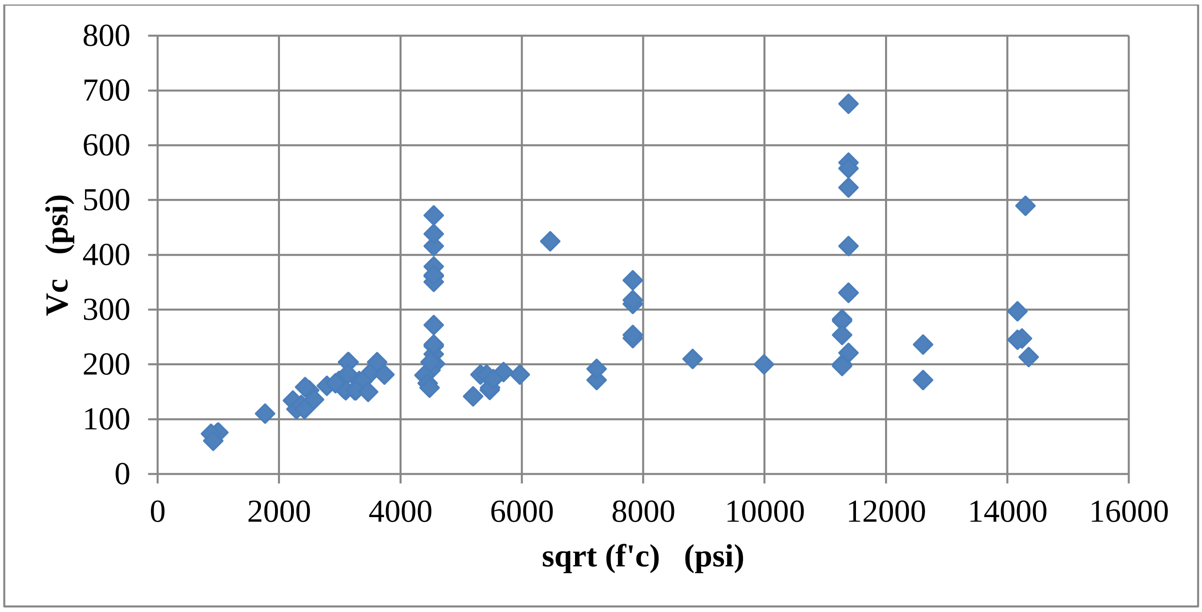


Figure. 4.1 Cracking shear Vc as function of sqrt (f'c)

4.2.2 Slenderness Ratio (a/d)

Slenderness ratio is an important factor for prediction of failure mode. The relationship is inverse as shown in figure 4.2.

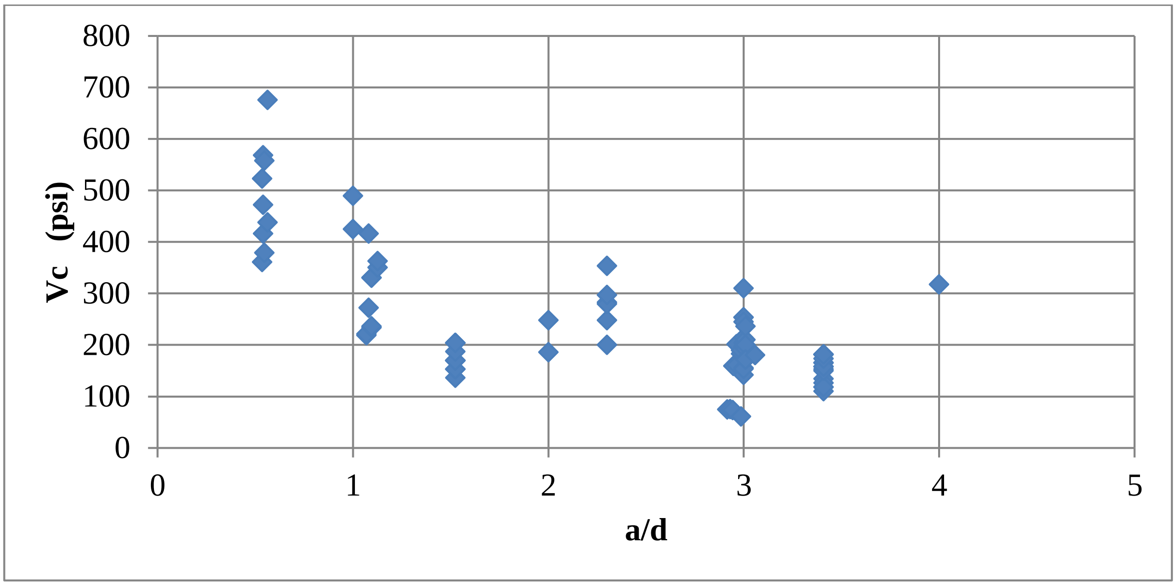


Figure. 4.2 Cracking shear Vc as a function of a/d (beams without web reinforcement)

4.2.3 Dowel Action

Almost all the researchers believe that dowel action is provided by the longitudinal steel and improves the aggregate interlock by reducing the crack width. As the longitudinal steel and surrounding concrete are at same stress level before cracking, dowel action is initiated after the concrete has cracked in flexure. It is believed that dowel action may not be counted as a component of cracking shear strength. The dowel action force can be calculated as Vu – Vc. It can be assumed to be proportional to the longitudinal steel force at failure and inversely proportional to aspect ratio a/d and can be expressed as:-

$$v_d = v_u - v_{cr} = \alpha \times \frac{\rho \times f_y}{a/d} \qquad (psi)$$

4.3 Shear Strength Relationships

4.3.1 Beams Without Web Reinforcement

4.3.1.1 Cracking Shear

The cracking shear strength is defined as the shear stress at the occurrence of the initial crack when dowel action has not yet been initiated. Cracking shear strength for beams without web reinforcement can be estimated as following considering that it is influenced by a/d ratio:-

$$v_c = v_{cr} = \frac{2\times\sqrt{f_c'}}{a/d} \qquad (psi) \qquad 19$$

Plot of above equation with available test data of table 2 is shown in figure 4.3

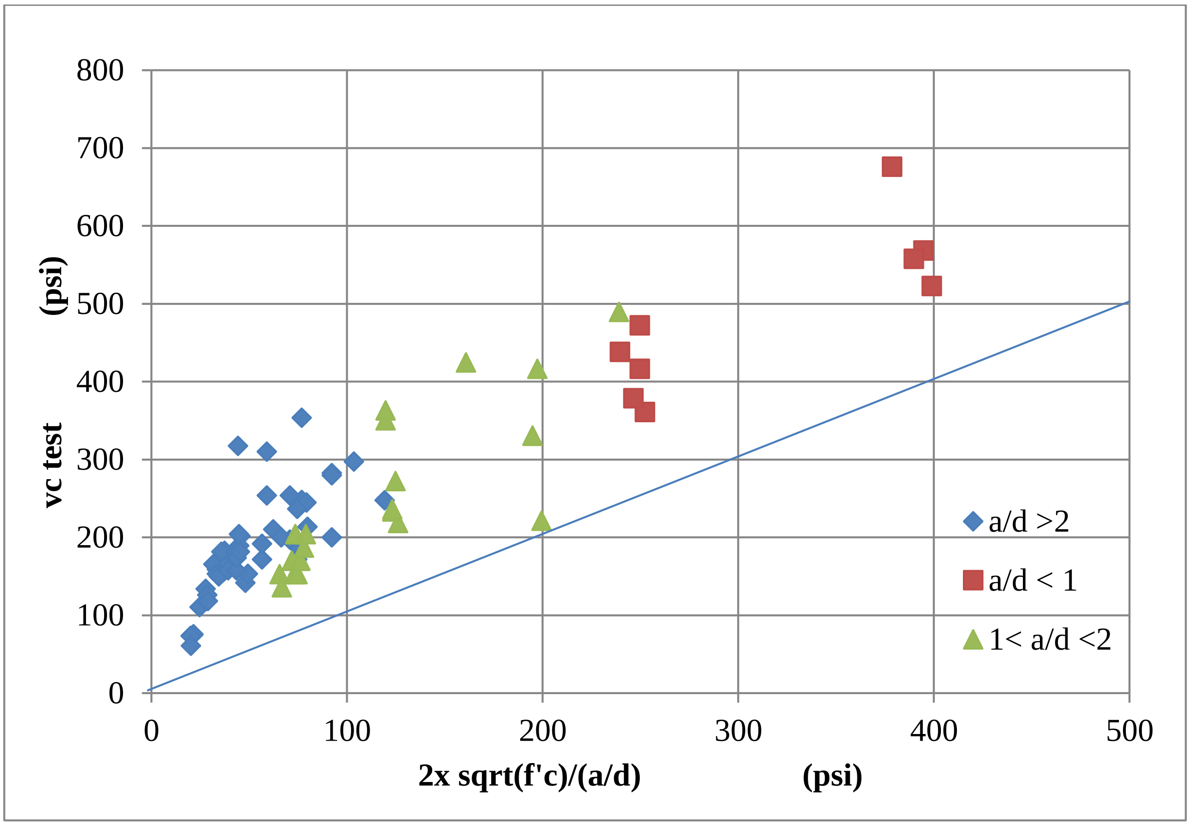


Figure. 4.3 Relation between the proposed cracking shear relation and test data (Beams Without Web Reinforcement)

4.3.1.2 **Ultimate Shear**

The ultimate shear strength is defined as the strength when shear failure occurs in a beam. Due to difficulty in segregation of shear carried by dowel action of longitudinal steel, part played by concrete compression zone and aggregate interlock, these are lumped together as shear carried by dowel action. It can now be calculated as sum of the cracking shear plus dowel action:-

$$v_u = v_{cr} + v_d = \frac{2\times\sqrt{f_c'}}{a/_d} + \alpha \times \frac{\rho\times f_y}{a/_d}$$

A safe and conservative estimate (figure 4.4) based on available test data for beams without web reinforcement can be made using following equation:-

$$v_u = \frac{2\times\sqrt{f_c'}}{a/_d} + 0.2\,\frac{\rho\times f_y}{a/_d} \qquad (psi) \qquad 20$$

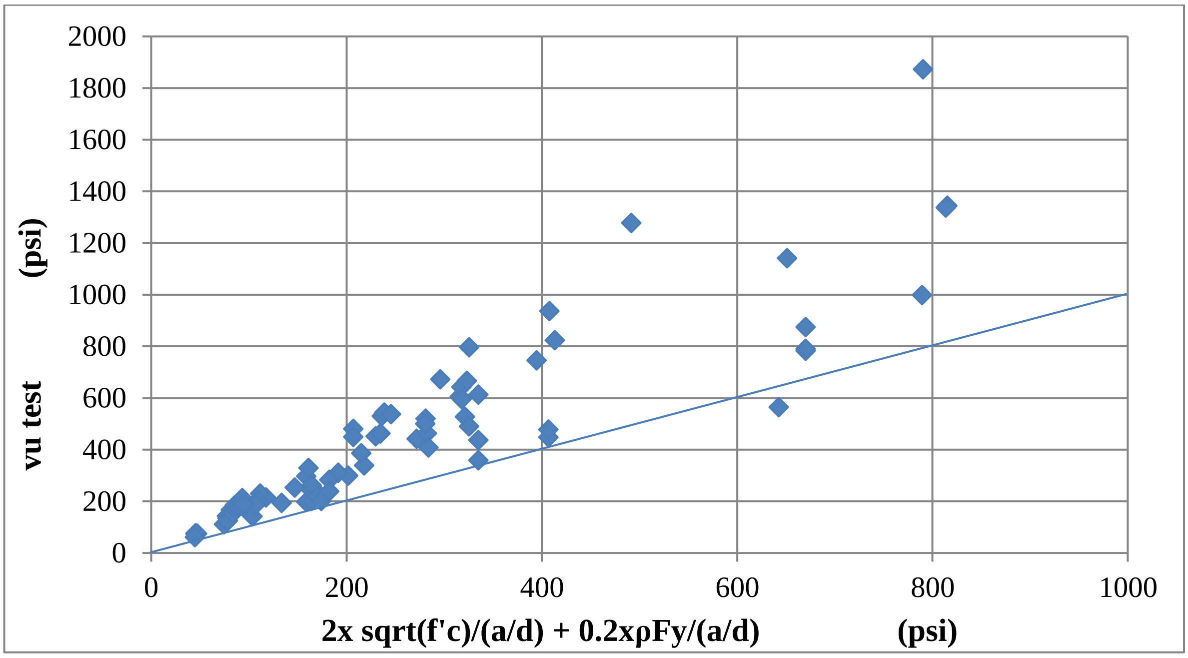


Figure. 4.4 Relation between the proposed ultimate shear relation for beams without web reinforcement and test data

4.3.2 **Beams With Web Reinforcement**

4.3.2.1 **Cracking Shear**

The cracking shear strength is defined as the shear stress at the occurrence of the initial crack when dowel action has not yet been initiated. Cracking shear strength for beams with web reinforcement can also be estimated as following considering that it is same for beams with and without web reinforcement:-

$$v_c = v_{cr} = \frac{2\times\sqrt{f_c'}}{a/d} \qquad (psi) \tag{19}$$

Plot of above equation with available test data of table 3 is shown in figure 4.5

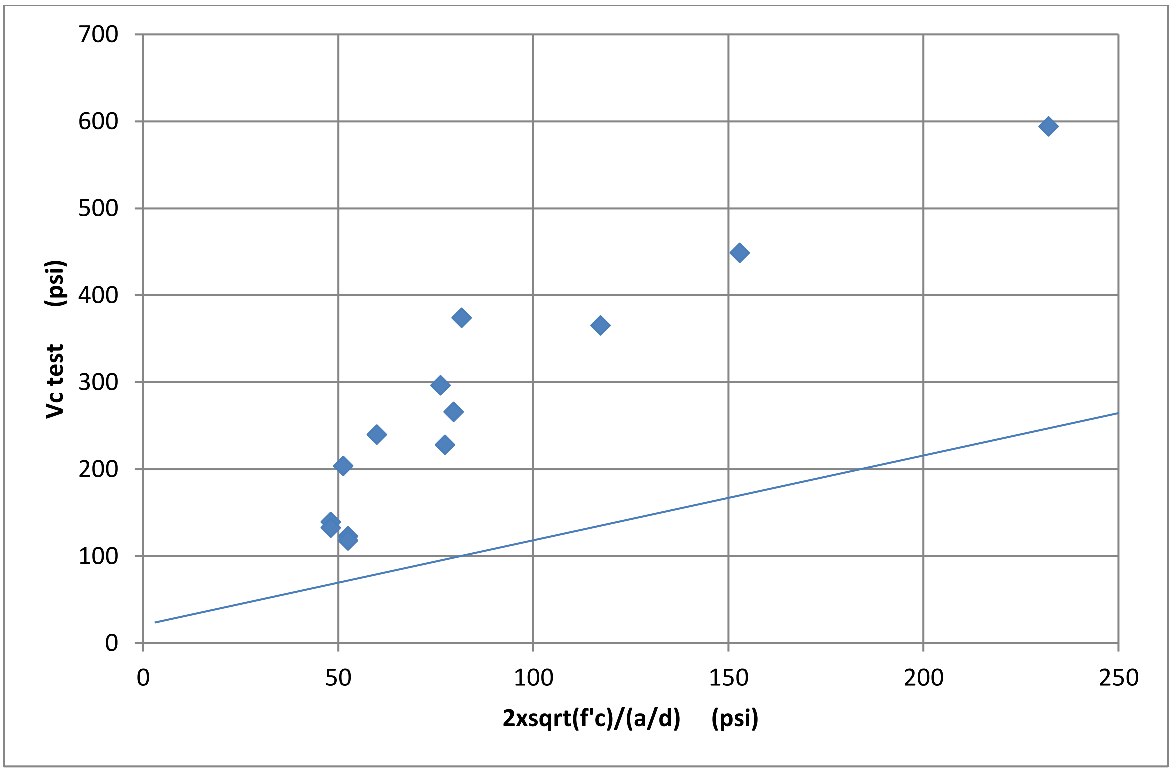


Figure. 4.5 Relation between the proposed cracking shear relation and test data (Beams With Web Reinforcement)

4.3.2.2 **Ultimate Shear**

The ultimate shear strength is defined as the strength when shear failure occurs in a beam. It can now be defined as:-

$$v_u = v_{cr} + v_d + v_s$$

$$V_u = \frac{2 \times \sqrt{f_c'}}{a/d} bd + 0.2 \frac{\rho \times f_y}{a/d} bd + \beta \times A_{vs} f_{vy}$$

A safe and conservative estimate based on available test data for beams with web reinforcement can be made using following equation:-

$$v_u = \frac{2 \times \sqrt{f_c'}}{a/d} bd + 0.2 \frac{\rho \times f_y}{a/d} bd + A_{vs} f_{vy} \qquad 21$$

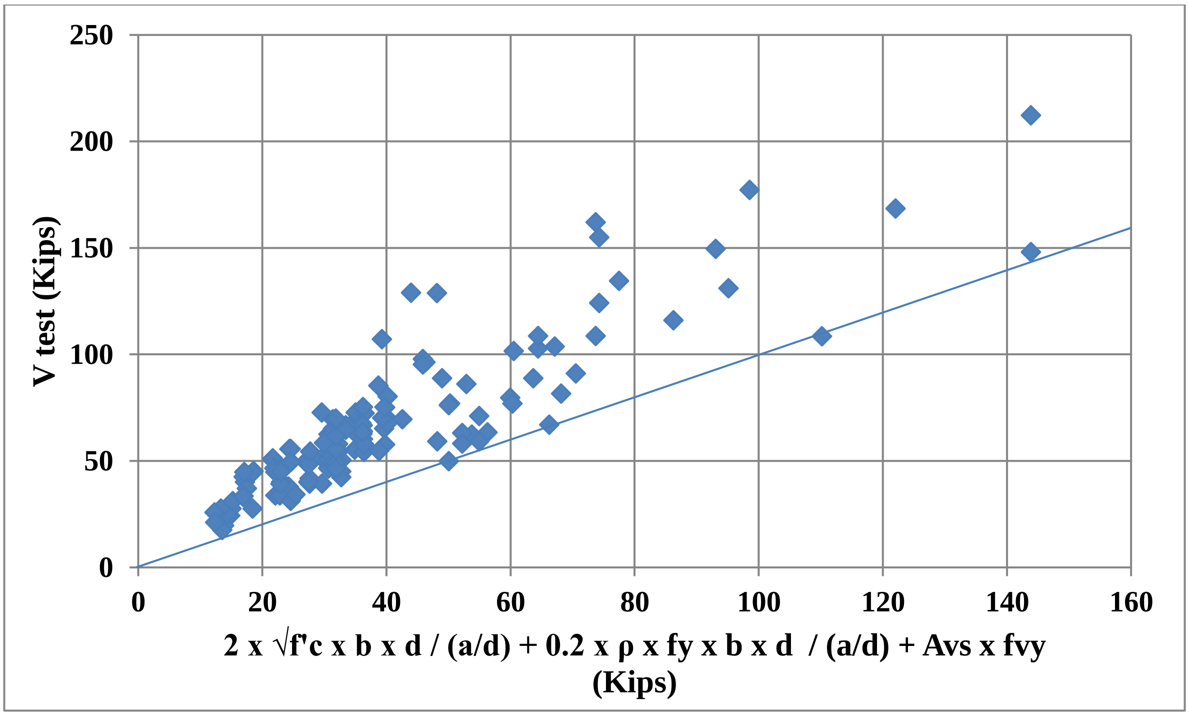


Figure. 4.6 Relation between the proposed ultimate shear relation for beams with web reinforcement and test data

4.4 Conclusions

- Concrete compressive strength is a major but not the sole factor effecting cracking shear strength.
- Shear span to depth ratio is a significant factor affecting shear strength and mode of failure. Shear span to depth ratio is inversely proportional to the shear strength of RC beams.
- Aggregate interlock and concrete compression zone shear strength plays an important role in shear behaviour of RC beams.
- ACI 318 (2005) simplified expression is too conservative in prediction of shear capacity of HSC beams.

4.5 Recommendations

A lot of research all over the world has gone into the shear strength of RC beams but yet this phenomenon is not completely rationalized. Following is suggested to continue the efforts to understand shear mechanics of RC beams:-

- A detailed and well planed research, aimed at better understanding of shear resistance provided by RC beams and creating an indigenous database of RC beams failing in shear. It should investigate shear provided by following:-
  - ✓ Aggregate interlock or matrix bond.
  - ✓ Concrete compression zone.
  - ✓ Flexural steel.
  - ✓ Size effect.
  - ✓ Stirrups.
  - ✓ Shear span to depth ratio.

- There is a marked difference in the ultimate shear strength of deep and slender RC beams. Also these two types have very different failure modes. So the above mentioned research should be carried out separately to investigate the difference between arch action and the beam action.

APPENDIX I

Table 2 Beams Without Web Reinforcement

Beam No	b (in)	d (in)	f'c (psi)	a/d	As (in)	ρ	fy (ksi)	v_c test / v_c calc	v_u test / v_u calc
Eric J. Tompos and Robert J. Frosch 2002									
V18-0	9.0	16.8	5200	3.0	1.58	0.01048	80.0	3.0	1.4
A. Cladera, A. R. Mari 2005									
H 100/1	7.9	14.1	12615	3.0	2.49	0.02240	72.5	3.2	1.3
H 100/5	7.9	14.1	12615	3.0	2.49	0.02240	72.5	2.3	1.6
H 50/1	7.9	14.1	7236	3.0	2.49	0.02240	72.5	3.4	1.2
H 50/5	7.9	14.1	7236	3.0	2.49	0.02240	72.5	3.0	1.6
H 60/1	7.9	14.1	8816	3.0	2.49	0.02240	72.5	3.4	1.3
H 75/1	7.9	14.1	9991	3.0	2.49	0.02240	72.5	3.0	1.2
E. Thorenfeldt and G. Drangsholt									
B 11	5.9	8.7	7830	3.0	0.93	0.01820	72.5	4.3	1.7
B 12	5.9	8.7	7830	2.3	0.93	0.01820	72.5	3.2	1.6
B 13	5.9	8.1	7830	4.0	1.55	0.03230	72.5	7.2	2.0
B 14	5.9	8.1	7830	3.0	1.55	0.03230	72.5	5.3	1.8
B 15	5.9	8.1	7830	2.3	1.55	0.03230	72.5	4.6	1.8
B 21	5.9	8.7	11278	3.0	0.93	0.01820	72.5	3.6	1.9
B 22	5.9	8.7	11278	2.3	0.93	0.01820	72.5	3.0	2.2
B 51	5.9	8.7	14167	3.0	0.93	0.01820	72.5	3.1	1.5
B 52	5.9	8.7	14167	2.3	0.93	0.01820	72.5	2.9	1.6
B 61	11.8	17.4	11277	3.0	3.74	0.01820	72.5	2.8	1.2
B 62	11.8	17.4	11277	2.3	3.74	0.01820	72.5	2.2	2.3
B 65	11.8	16.3	11277	2.3	6.21	0.03230	72.5	3.1	2.3
K. G. MOODY, I. M. VIEST, R. C ELSTNER; and E. HOGNESTAD 1954									
A1	7.0	10.3	4400	3.1	1.56	0.02170	45.0	4.2	1.7
A2	7.0	10.5	4500	3.0	1.58	0.02150	45.0	4.6	1.9
A3	7.0	10.6	4500	3.0	1.64	0.02220	45.0	4.2	2.1
A4	7.0	10.6	4570	3.0	1.76	0.02370	45.0	4.4	1.8
B - 1	6.0	10.6	5320	3.4	1.20	0.01890	45.0	4.2	2.2
B - 10	6.0	10.6	3470	3.4	1.20	0.01890	45.0	4.3	2.1
B - 11	6.0	10.6	5530	3.4	1.20	0.01890	45.0	4.0	2.3
B - 12	6.0	10.6	2930	3.4	1.20	0.01890	45.0	5.2	2.0
B - 13	6.0	10.6	5480	3.4	1.20	0.01890	45.0	3.6	2.1
B - 14	6.0	10.6	3270	3.4	1.20	0.01890	45.0	4.6	1.8
B - 15	6.0	10.6	5420	3.4	1.20	0.01890	45.0	4.2	1.9
B - 16	6.0	10.6	2370	3.4	1.20	0.01890	45.0	4.4	1.7

Beam No	**b (in)**	**d (in)**	**f'c (psi)**	**a/d**	**As (in)**	**ρ**	**fy (ksi)**	***v*c test / *v*c calc**	***v*u test / vu calc**
B - 2	6.0	10.6	2420	3.4	1.20	0.01890	45.0	4.1	1.6
B - 3	6.0	10.6	3740	3.4	1.20	0.01890	45.0	5.1	2.2
B - 4	6.0	10.6	2230	3.4	1.20	0.01890	45.0	4.8	1.9
B - 5	6.0	10.6	4450	3.4	1.20	0.01890	45.0	4.2	2.1
B - 6	6.0	10.6	2290	3.4	1.20	0.01890	45.0	4.2	1.6
B - 7	6.0	10.6	4480	3.4	1.20	0.01890	45.0	4.0	2.0
B - 8	6.0	10.6	1770	3.4	1.20	0.01890	45.0	4.5	1.5
B - 9	6.0	10.6	5970	3.4	1.20	0.01890	45.0	4.0	2.0
B1	7.0	10.5	3070	3.0	1.19	0.01620	45.0	4.2	2.0
B2	7.0	10.6	3130	3.0	1.20	0.01630	45.0	4.9	2.1
B3	7.0	10.6	2790	3.0	1.19	0.01600	45.0	4.5	2.0
B4	7.0	10.7	2430	2.9	1.24	0.01660	45.0	4.8	2.0
C1	7.0	10.6	920	3.0	0.60	0.00810	45.0	3.0	1.4
C2	7.0	10.7	880	2.9	0.62	0.00830	45.0	3.6	1.6
C3	7.0	10.8	1000	2.9	0.60	0.00800	44.0	3.5	1.7
C4	7.0	10.8	980	2.9	0.62	0.00820	45.7	3.5	1.6
III—25a	7.0	21.0	3530	1.5	5.09	0.03460	45.4	2.4	1.4
III—25b	7.0	21.0	2500	1.5	5.09	0.03460	45.4	2.3	1.6
III—26a	7.0	21.0	3140	1.5	6.25	0.04250	43.8	2.8	2.0
III—26b	7.0	21.0	2990	1.5	6.25	0.04250	43.8	2.4	1.9
III—27a	7.0	21.0	3100	1.5	4.00	0.02720	45.7	2.1	2.2
III—27b	7.0	21.0	3320	1.5	4.00	0.02720	45.7	2.2	2.3
III—28a	7.0	21.0	3380	1.5	5.09	0.03460	45.4	2.2	1.6
III—28b	7.0	21.0	3250	1.5	5.09	0.03460	45.4	2.0	1.9
III—29a	7.0	21.0	3150	1.5	6.25	0.04250	43.8	2.8	1.9
III—29b	7.0	21.0	3620	1.5	6.25	0.04250	43.8	2.6	2.1
Ill—24a	7.0	21.0	2580	1.5	4.00	0.02720	45.7	2.0	2.0
Ill—24b	7.0	21.0	2990	1.5	4.00	0.02720	45.7	2.4	2.0
Keun-Hyeok Yang, Heon-Soo Chung, Eun-Taik Lee, Hee-Chang Eun 2003									
L 10-100	6.3	36.8	4553	1.1	2.09	0.00900	116.0	1.7	1.6
L 10-40	6.3	14.0	4553	1.1	0.89	0.01000	116.0	2.9	1.5
L 10-40R	6.3	14.0	4553	1.1	0.89	0.01000	116.0	3.0	2.4
L 10-60	6.3	21.8	4553	1.1	1.33	0.00980	116.0	2.2	1.8
L 10-75	6.3	27.0	4553	1.1	1.78	0.01000	116.0	1.9	1.1
L 10-75R	6.3	27.0	4553	1.1	1.78	0.01000	116.0	1.9	1.3
L 5-100	6.3	36.8	4553	0.5	2.09	0.00900	116.0	1.4	0.9
L 5-40	6.3	14.0	4553	0.6	0.89	0.01000	116.0	1.8	1.8
L 5-60	6.3	21.8	4553	0.5	1.33	0.00980	116.0	1.9	1.3

Beam No	b (in)	d (in)	f'c (psi)	a/d	As (in)	ρ	fy (ksi)	v_c test / v_c calc	v_u test / v_u calc
L 5-60R	6.3	21.8	4553	0.5	1.33	0.00980	116.0	1.7	1.2
L 5-75	6.3	27.0	4553	0.5	1.78	0.01000	116.0	1.5	1.2
UH10-100	6.3	36.8	11383	1.1	2.09	0.00900	116.0	1.1	1.9
UH10-60	6.3	21.8	11383	1.1	1.33	0.00980	116.0	2.1	2.3
UH10-75	6.3	27.0	11383	1.1	1.78	0.01000	116.0	1.7	1.1
UH10-75R	6.3	27.0	11383	1.1	1.78	0.01000	116.0	1.7	1.2
UH5-100	6.3	36.8	11383	0.5	2.09	0.00900	116.0	1.3	1.3
UH5-40	6.3	14.0	11383	0.6	0.89	0.01000	116.0	1.8	2.4
UH5-60	6.3	21.8	11383	0.5	1.33	0.00980	116.0	1.4	1.6
UH5-75	6.3	27.0	11383	0.5	1.78	0.01000	116.0	1.4	1.6
Yuliang Xie, Shuaib H. Ahmad, Tiejun Yu, S. Hino, and W. Chung 1994									
*NNN-1	5.0	8.5	6470	1.0	0.88	0.02071	61.0	2.6	2.0
NHN-1	5.0	8.5	14298	1.0	0.88	0.02071	61.0	2.0	2.6
NHN-2	5.0	8.5	14241	2.0	0.88	0.02071	61.0	2.1	2.2
NHN-3	5.0	8.5	14355	3.0	0.88	0.02071	61.0	2.7	1.5
NNN-2	5.0	8.5	5700	2.0	0.88	0.02071	61.0	2.5	1.5
NNN-3	5.0	8.5	5472	3.0	0.88	0.02071	61.0	3.1	1.5

Table 3 Beams With Web Reinforcement

Beam no	fc' (psi)	b, in	d, in	a/d	As (in)	ρ	fy (ksi)	Asv (in)	fvy (Ksi)	s (in)	Vtest/ Vcalc
Angelakos, Bentz, and Collins 2001											
BM100	6815	11.8	36.4	2.9	3.27	0.00760	79.8	0.11	73.7	11.8	1.53
DB0.530M	4640	11.8	36.4	2.9	2.18	0.00507	79.8	0.22	73.7	23.6	1.23
DB120M	3045	11.8	36.4	2.9	4.36	0.01014	79.8	0.22	73.7	23.6	1.13
DB140M	5510	11.8	36.4	2.9	4.36	0.01014	79.8	0.11	73.7	11.8	1.16
DB165M	9425	11.8	36.4	2.9	4.36	0.01014	79.8	0.11	73.7	11.8	1.68
DB180M	11600	11.8	36.4	2.9	4.36	0.01014	79.8	0.11	73.7	11.8	1.39
A.P. Clark 1951											
A1-1	3575	8.0	15.4	2.3	3.80	0.03100	46.5	0.22	48.0	7.2	1.56
A1-2	3430	8.0	15.4	2.3	3.80	0.03100	46.5	0.22	48.0	7.2	1.48
A1-3	3395	8.0	15.4	2.3	3.80	0.03100	46.5	0.22	48.0	7.2	1.58
A1-4	3590	8.0	15.4	2.3	3.80	0.03100	46.5	0.22	48.0	7.2	1.72
B1 - 1	3388	8.0	15.4	2.0	3.80	0.03100	46.5	0.22	48.0	7.5	1.73
B1 - 2	3680	8.0	15.4	2.0	3.80	0.03100	46.5	0.22	48.0	7.5	1.58
B1 - 3	3435	8.0	15.4	2.0	3.80	0.03100	46.5	0.22	48.0	7.5	1.77
B1 - 4	3380	8.0	15.4	1.9	3.80	0.03100	46.5	0.22	48.0	7.5	1.66
B1 - 5	3570	8.0	15.4	2.0	3.80	0.03100	46.5	0.22	48.0	7.5	1.49
B2-1	3370	8.0	15.4	2.0	3.80	0.03100	46.5	0.22	48.0	3.8	1.88
B2-2	3820	8.0	15.4	2.0	3.80	0.03100	46.5	0.22	48.0	3.8	1.98
B2-3	3615	8.0	15.4	2.0	3.80	0.03100	46.5	0.22	48.0	3.8	2.08
B6-1	6110	8.0	15.4	2.0	3.80	0.03100	46.5	0.22	48.0	7.5	2.20
C1-1	3720	8.0	15.4	1.6	2.53	0.02070	46.5	0.22	48.0	8.0	1.77
C1-2	3820	8.0	15.4	1.6	2.53	0.02070	46.5	0.22	48.0	8.0	1.98
C1-3	3475	8.0	15.4	1.6	2.53	0.02070	46.5	0.22	48.0	8.0	1.58
C1-4	4210	8.0	15.4	1.6	2.53	0.02070	46.5	0.22	48.0	8.0	1.79
C2-1	3430	8.0	15.4	1.6	2.53	0.02070	46.5	0.22	48.0	4.0	1.86
C2-2	3625	8.0	15.4	1.6	2.53	0.02070	46.5	0.22	48.0	4.0	1.92
C2-3	3500	8.0	15.4	1.6	2.53	0.02070	46.5	0.22	48.0	4.0	2.08
C2-4	3910	8.0	15.4	1.6	2.53	0.02070	46.5	0.22	48.0	4.0	1.82
C3-1	2040	8.0	15.4	1.6	2.53	0.02070	46.5	0.22	48.0	8.0	1.53
C3-2	2000	8.0	15.4	1.6	2.53	0.02070	46.5	0.22	48.0	8.0	1.38
C3-3	2020	8.0	15.4	1.6	2.53	0.02070	46.5	0.22	48.0	8.0	1.29
C4-1	3550	8.0	15.4	1.6	3.80	0.03100	46.5	0.22	48.0	8.0	1.63
C6-2	6560	8.0	15.4	1.6	3.80	0.03100	46.5	0.22	48.0	8.0	2.08
C6-3	6480	8.0	15.4	1.6	3.80	0.03100	46.5	0.22	48.0	8.0	2.13
C6-4	6900	8.0	15.4	1.6	3.80	0.03100	46.5	0.22	48.0	8.0	2.08

Beam no	fc' (psi)	b, in	d, in	a/d	As (in)	ρ	fy (ksi)	Asv (in)	fvy (Ksi)	s (in)	Vtest/ Vcalc
D1-1	3800	8.0	15.4	1.2	2.02	0.01630	48.6	0.22	48.0	6.0	1.68
D1-2	3790	8.0	15.4	1.2	2.02	0.01630	48.6	0.22	48.0	6.0	1.99
D1-3	3560	8.0	15.4	1.2	2.02	0.01630	48.6	0.22	48.0	6.0	1.45
D1-6	4010	6.0	12.4	1.9	2.53	0.03420	46.5	0.22	48.0	8.0	1.42
D1-7	4060	6.0	12.4	1.9	2.53	0.03420	46.5	0.22	48.0	8.0	1.46
D1-8	4030	6.0	12.4	1.9	2.53	0.03420	46.5	0.22	48.0	8.0	1.51
D2-1	3480	8.0	15.4	1.2	2.02	0.01630	48.6	0.22	48.0	4.5	1.64
D2-2	3755	8.0	15.4	1.2	2.02	0.01630	48.6	0.22	48.0	4.5	1.75
D2-3	3595	8.0	15.4	1.2	2.02	0.01630	48.6	0.22	48.0	4.5	1.89
D2-4	3550	8.0	15.4	1.2	2.02	0.01630	48.6	0.22	48.0	4.5	1.90
D2-6	4280	6.0	12.4	2.4	2.53	0.03420	46.5	0.22	48.0	6.0	1.56
D2-7	4120	6.0	12.4	2.4	2.53	0.03420	46.5	0.22	48.0	6.0	1.46
D2-8	3790	6.0	12.4	2.4	2.53	0.03420	46.5	0.22	48.0	6.0	1.57
D3-1	4090	8.0	15.4	1.2	3.04	0.02440	48.6	0.22	48.0	3.0	1.81
D4-1	3350	8.0	15.4	1.2	2.02	0.01630	48.6	0.22	48.0	2.3	1.78
D4-1	3970	6.0	12.4	2.4	2.53	0.03420	46.5	0.22	48.0	7.5	1.57
D4-2	3720	6.0	12.4	2.4	2.53	0.03420	46.5	0.22	48.0	7.5	1.47
D4-3	3200	6.0	12.4	2.4	2.53	0.03420	46.5	0.22	48.0	7.5	1.56
D5-1	4020	6.0	12.4	2.4	2.53	0.03420	46.5	0.22	48.0	10.0	1.35
D5-2	4210	6.0	12.4	2.4	2.53	0.03420	46.5	0.22	48.0	10.0	1.45
D5-3	3930	6.0	12.4	2.4	2.53	0.03420	46.5	0.22	48.0	10.0	1.46
E1-2	4375	6.0	12.4	2.0	2.53	0.03420	46.5	0.22	48.0	5.0	1.84
Collins and Kuchma 1999											
BM100	6815	12.2	37.8	2.9	3.26	0.00705	79.8	0.22	73.7	23.6	1.27
BM100D	6815	12.2	37.8	2.9	4.50	0.00973	79.9	0.22	73.7	23.6	1.54
SE100A-M	10295	12.0	37.6	2.5	4.34	0.00960	70.0	0.33	75.7	17.3	1.34
SE100B-M	10875	12.0	37.6	2.5	5.74	0.01270	70.0	0.33	75.7	17.3	1.38
SE50A-M	10730	6.9	18.7	2.7	1.24	0.00960	70.0	0.10	85.8	10.9	1.27
SE50B-M	10730	6.9	18.7	2.7	1.39	0.01076	70.0	0.10	85.8	10.9	1.35
Elzanaty, Nilson, and Slate 1986											
G4	9100	7.0	10.9	4.0	2.51	0.03300	63.0	0.10	55.0	7.5	1.97
G5	5800	7.0	10.9	4.0	1.90	0.02500	63.0	0.10	55.0	7.5	1.74
G6	3000	7.0	10.9	4.0	1.90	0.02500	63.0	0.10	55.0	7.5	1.30
Johnson and Ramirez 1989											
2	5280	12.0	21.2	3.1	6.35	0.02495	76.1	0.10	69.5	10.5	1.00
3	10490	12.0	21.2	3.1	6.35	0.02495	76.1	0.10	69.5	10.5	1.08
4	10490	12.0	21.2	3.1	6.35	0.02495	76.1	0.10	69.5	10.5	1.29

Beam no	fc' (psi)	b, in	d, in	a/d	As (in)	ρ	fy (ksi)	Asv (in)	fvy (Ksi)	s (in)	Vtest/ Vcalc
5	8100	12.0	21.2	3.1	6.35	0.02495	76.1	0.10	69.5	5.3	1.63
7	7400	12.0	21.2	3.1	6.35	0.02495	76.1	0.10	69.5	10.5	1.21
8	7400	12.0	21.2	3.1	6.35	0.02495	76.1	0.10	69.5	10.5	1.11
1	5280	12.0	21.2	3.1	6.35	0.02495	76.1	0.10	69.5	5.3	1.52
Kong and Rangan 1998											
S1-1	8761	10.0	11.9	2.5	3.17	0.02660	65.6	0.06	82.5	0.3	1.67
S1-2	8761	10.0	11.9	2.5	3.17	0.02660	65.6	0.06	82.5	0.3	1.53
S1-3	8761	10.0	11.9	2.5	3.17	0.02660	65.6	0.06	82.5	0.3	1.51
S1-4	8761	10.0	11.9	2.5	3.17	0.02660	65.6	0.06	82.5	0.3	2.04
S1-5	8761	10.0	11.9	2.5	3.17	0.02660	65.6	0.06	82.5	0.3	1.86
S1-6	8761	10.0	11.9	2.5	3.17	0.02660	65.6	0.06	82.5	0.3	1.64
S2-1	9988	10.0	11.9	2.5	3.17	0.02660	65.6	0.06	82.5	0.2	1.87
S2-2	9988	10.0	11.9	2.5	3.17	0.02660	65.6	0.06	82.5	0.2	1.67
S2-3	9988	10.0	11.9	2.5	3.17	0.02660	65.6	0.06	82.5	0.3	1.82
S2-4	9988	10.0	11.9	2.5	3.17	0.02660	65.6	0.06	82.5	0.3	1.58
S2-5	9988	10.0	11.9	2.5	3.17	0.02660	65.6	0.06	82.5	0.3	2.03
S3-1	9284	10.0	12.1	2.5	1.91	0.01576	65.3	0.04	91.7	0.3	2.04
S3-2	9284	10.0	12.1	2.5	1.91	0.01576	65.3	0.04	91.7	0.3	1.74
S3-3	9284	10.0	12.0	2.5	3.17	0.02651	65.6	0.04	91.7	0.3	1.74
S3-4	9284	10.0	12.0	2.5	3.17	0.02651	65.6	0.04	91.7	0.3	1.33
S3-5	9284	10.0	12.2	2.4	4.28	0.03507	64.1	0.04	91.7	0.3	1.84
S3-6	9284	10.0	12.2	2.4	4.28	0.03507	64.1	0.04	91.7	0.3	1.76
S4-1	12026	10.0	22.1	2.4	6.34	0.02866	65.6	0.06	82.5	0.3	1.33
S4-2	12026	10.0	18.1	2.4	5.09	0.02809	62.8	0.06	82.5	0.3	2.67
S4-3	12026	10.0	14.1	2.4	3.82	0.02705	65.3	0.06	82.5	0.3	1.41
S4-4	12026	10.0	11.9	2.5	3.17	0.02660	65.6	0.06	82.5	0.3	1.80
S4-6	12026	10.0	8.1	2.5	2.14	0.02648	64.1	0.06	82.5	0.3	1.99
S5-1	12315	10.0	11.9	3.0	3.17	0.02660	65.6	0.06	82.5	0.3	1.96
S5-2	12315	10.0	11.9	2.7	3.17	0.02660	65.6	0.06	82.5	0.3	1.95
S5-3	12315	10.0	11.9	2.5	3.17	0.02660	65.6	0.06	82.5	0.3	1.70
S5-4	12315	10.0	11.9	2.0	3.17	0.02660	65.6	0.06	82.5	0.3	2.73
S5-5	12315	10.0	11.9	1.8	3.17	0.02660	65.6	0.06	82.5	0.3	2.93
S6-3	9492	10.0	12.0	2.7	3.17	0.02651	65.6	0.04	91.7	0.3	1.46
S6-4	9492	10.0	12.0	2.7	3.17	0.02651	65.6	0.04	91.7	0.3	1.76
S6-5	9492	10.0	12.2	2.6	4.28	0.03507	64.1	0.04	91.7	0.3	2.00
S6-6	9492	10.0	12.2	2.6	4.28	0.03507	64.1	0.04	91.7	0.3	1.93
S7-1	10295	10.0	12.0	3.3	5.09	0.04242	62.8	0.06	82.5	0.2	1.53
S7-2	10295	10.0	12.0	3.3	5.09	0.04242	62.8	0.06	82.5	0.2	1.45

Beam no	**fc' (psi)**	**b, in**	**d, in**	**a/d**	**As (in)**	**ρ**	**fy (ksi)**	**Asv (in)**	**fvy (Ksi)**	**s (in)**	**Vtest/ Vcalc**
S7-3	10295	10.0	12.0	3.3	5.09	0.04242	62.8	0.06	82.5	0.3	1.74
S7-4	10295	10.0	12.0	3.3	5.09	0.04242	62.8	0.06	82.5	0.3	1.93
S7-5	10295	10.0	12.0	3.3	5.09	0.04242	62.8	0.06	82.5	0.4	2.15
S7-6	10295	10.0	12.0	3.3	5.09	0.04242	62.8	0.06	82.5	0.4	2.19
S8-1	10276	10.0	11.9	2.5	3.17	0.02660	65.6	0.06	82.5	0.2	1.95
S8-2	10276	10.0	11.9	2.5	3.17	0.02660	65.6	0.06	82.5	0.2	1.80
S8-3	10276	10.0	11.9	2.5	3.17	0.02660	65.6	0.06	82.5	0.3	2.22
S8-4	10276	10.0	11.9	2.5	3.17	0.02660	65.6	0.06	82.5	0.3	1.90
S8-5	10276	10.0	11.9	2.5	3.17	0.02660	65.6	0.06	82.5	0.3	2.07
S8-6	10276	10.0	11.9	2.5	3.17	0.02660	65.6	0.06	82.5	0.4	2.03
Mustensir Jvaid 2008											
BS 6-1	6633	7.5	8.3	2.5	1.98	0.03200	60.0	0.22	40.0	6.0	2.02
BS 6-2	6700	7.5	8.3	3.9	1.98	0.03200	60.0	0.22	40.0	6.0	2.11
BS 6-3	6758	7.5	8.3	2.5	1.98	0.03200	60.0	0.22	40.0	6.0	1.53
BS 6-4	9300	7.5	8.3	2.5	1.98	0.03200	60.0	0.22	40.0	6.0	1.48
BS 6-5	9700	7.5	8.3	2.5	1.98	0.03200	60.0	0.22	40.0	6.0	1.71
BS 6-6	9975	7.5	8.3	2.5	1.98	0.03200	60.0	0.22	40.0	6.0	1.95
Roller and Russell 1990											
1	17420	14.0	22.0	2.5	5.08	0.01649	68.5	0.10	59.0	8.5	1.01
3	17420	14.0	22.0	2.5	14.04	0.04558	62.5	0.62	66.4	5.0	1.03
4	17420	14.0	22.0	2.5	14.04	0.04558	62.5	0.62	66.4	3.5	1.48
6	10500	18.0	30.0	3.0	9.36	0.01733	67.3	0.22	64.6	15.0	1.61
7	10500	18.0	30.0	3.0	10.16	0.01881	70.1	0.22	64.6	7.8	1.80
8	18170	18.0	30.0	3.0	10.16	0.01881	70.1	0.22	64.6	15.0	0.99
9	18170	18.0	30.0	3.0	12.70	0.02352	70.1	0.22	64.6	7.8	1.38
Sarzam and Al-Musawi 1992											
AL2-H	10919	7.3	9.6	4.0	1.46	0.02072	71.8	0.04	118.9	5.9	2.07
AL2-N	5858	7.3	9.6	4.0	1.46	0.02072	71.8	0.04	118.9	5.9	2.09
AS2-H	10948	7.3	9.5	2.5	1.46	0.02099	71.8	0.04	118.9	5.9	2.43
AS2-N	5655	7.3	9.6	2.5	1.46	0.02072	71.8	0.04	118.9	5.9	2.50
AS3-H	10411	7.3	9.6	2.5	1.46	0.02072	71.8	0.04	118.9	3.9	2.41
AS3-N	5829	7.3	9.6	2.5	1.46	0.02072	71.8	0.04	118.9	3.9	2.62
BL2-H	10977	7.3	9.5	4.0	1.83	0.02619	78.8	0.04	118.9	5.9	2.04
BS2-H	10716	7.3	9.5	2.5	1.83	0.02619	78.8	0.04	118.9	5.9	2.31
BS3-H	10643	7.3	9.5	2.5	1.83	0.02619	78.8	0.04	118.9	3.9	2.36
BS4-H	11615	7.3	9.5	2.5	1.83	0.02619	78.8	0.04	118.9	3.0	2.12
CL2-H	10165	7.3	9.5	4.0	2.28	0.03263	78.8	0.04	118.9	5.9	1.96
CS2-H	10179	7.3	9.5	2.5	2.28	0.03263	78.8	0.04	118.9	5.9	2.28

Beam no	fc' (psi)	b, in	d, in	a/d	As (in)	ρ	fy (ksi)	Asv (in)	fvy (Ksi)	s (in)	Vtest/ Vcalc
CS3-H	10788	7.3	9.5	2.5	2.28	0.03263	78.8	0.04	118.9	3.9	2.26
CS4-H	10977	7.3	9.5	2.5	2.28	0.03263	78.8	0.04	118.9	3.0	2.02
Xie et al 1994											
NHW-1	13461	5.0	8.0	1.0	1.28	0.03200	61.0	0.10	47.0	4.0	2.46
NHW-2	13737	5.0	8.0	2.0	1.28	0.03200	61.0	0.10	47.0	4.0	2.33
NHW-3	14250	5.0	8.0	3.0	1.28	0.03200	61.0	0.10	47.0	4.0	1.77
NHW-3a	13053	5.0	7.8	3.0	1.76	0.04513	61.0	0.10	47.0	3.9	1.64
NHW-3b	14972	5.0	7.8	3.0	1.76	0.04513	61.0	0.10	47.0	3.9	1.83
NHW-4	14335	5.0	7.8	4.0	1.76	0.04513	61.0	0.10	47.0	3.9	1.70
NNW-1	5842	5.0	7.8	1.0	1.76	0.04513	61.0	0.10	47.0	3.0	1.67
NNW-2	5985	5.0	7.8	2.0	1.76	0.04513	61.0	0.10	47.0	2.5	1.50
NNW-3	5909	5.0	7.8	3.0	1.76	0.04513	61.0	0.10	47.0	3.9	1.41
Yoon, Cook, and Mitchell 1996											
H1-N	12615	14.8	25.8	3.3	10.66	0.02800	58.0	0.16	62.4	12.8	1.47
H2-N	12615	14.8	25.8	3.3	10.66	0.02800	58.0	0.16	62.4	12.8	2.20
H2-S	12615	14.8	25.8	3.3	10.66	0.02800	58.0	0.22	62.4	12.8	1.73
M1-N	9715	14.8	25.8	3.3	10.66	0.02800	58.0	0.16	62.4	12.8	1.29
M2-N	9715	14.8	25.8	3.3	10.66	0.02800	58.0	0.22	62.4	9.1	2.08
M2-S	9715	14.8	25.8	3.3	10.66	0.02800	58.0	0.22	62.4	12.8	1.67
N1-N	5220	14.8	25.8	3.3	10.66	0.02800	58.0	0.16	62.4	12.8	1.59
N2-N	5220	14.8	25.8	3.3	10.66	0.02800	58.0	0.16	62.4	12.8	1.68
N2-S	5220	14.8	25.8	3.3	10.66	0.02800	58.0	0.22	62.4	18.3	1.20

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