CONTINUOUS MODE ANALYSIS OF TOOLS FOR QUANTUM TELEPORTATION

by

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This Dissertation is dedicated to my parents

for their love and prayers.

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Abstract

We study continuous mode model of photon state. We also study the models of photo detectors. We develop the continuous mode model for real world photon detectors which consider the effects of variation in efficiency of detector depending upon its spectral function. We study the nature of entanglement in different Bell states and develop continuous mode form of Bell states. We verify the behavior of Bell states by finding the coincidence between detectors acting as Bell state analyzers. Entanglement is the phenomenon which plays key role in teleportation. Once we varify that nature of entanglement remains preserved in our continuous mode model of Bell state, we teleport an unknown continuous mode state using our continuous mode model of photon states and photo detectors. We calculate the three-fold coincidence among four detectors while two detectors act as Bell state analyzers and other two are used for detection of teleported state. We find that three-fold coincidence probability curves are in accordance with expected values.

Contents

List of Figures

Chapter 1

Introduction

Quantum information and computation involves the study of processes used for manipulating the information while making use of systems that obey laws of quantum physics. This branch of science extends back to the start of 20^{th} century when scientists were unable to understand and explain various phenomena on the basis of existing rules and principles of physics. For example, it was observed that light behaves like both waves and particle. It was known from ancient times that light has wave behaviour but discovery of the fact that light also has particle like properties jolted the understanding of physical world. This is where quantum mechanics took birth and concept of photon was introduced. It was learned that when light was used to find the position of a particle at micro level, its velocity changed during the measurement process. At micro level such a change can not be ignored when energy of photon is greater compared to mass of particle. This change in velocity during measurement of position led to the fact that both position and momentum of a micro particle can not be measured together perfectly. This result is a direct consequence of wave-particle duality of light. In macroscopic world, objects posses greater mass and hence momentum of an object does not changes due to position measurement and we can measure momentum and position simultaneously.

Once the formalism for quantum mechanics was established, major focus of quantum mechanics remaind on the study of properties of matter at microscopic level. On the other end, information theories remaind focused on data processing rather than information extraction. So extraction of information from micro particles in context of quantum mechanics constitutes the subject matter of quantum information science. We can divide it furthur into quantum computation and quantum information theory. Quantum computation deals with different algorithms for problem solving and quantum information theories are related to obtaining and processing information [1].

In our thesis we make use of quantum mechanical system to transfer information stored in a quantum state from one point to another. Entangled state is used as tool of communication between two points. Once the state is transfered, information is extracted by performing quantum measurement. Result of measurement tells us about the state transfered. This type of communication is called quantum teleportation.

In section 1.1 we describe the difference between classical and quantum information arising due to different nature of classical and quantum mechanical systems. In section 1.2 we explain the qubits which are used to represent quantum information. In section 1.3 we explain the measurement process using measurement operators. We also explain the properties of these measurement operators in this section. In section 1.4 we describe the quantum entanglement which lies at the heart of most quantum communication processes. In section 1.5 we explain the process of quantum teleportation between two distant points.

1.1 Classical Information vs Quantum Information

Claude Shannon, in 1940 [2], pointed that major problem in communication is producing message at destination exactly or approximately similar to one sent from source. Methods and tools used in a particular communication process may vary greatly depending upon nature of communication process. While studying communication we often come across term signalling. Signalling in this case refers to the physical disturbance or perturbation in transmission media used for transmitting information between two points via communication channel. Shannon developed the models for communication and laid the basis for information theory.

After Shannon's work, it was realised that information theory is not limited only to transmission of information but it can also play important role in understanding of some other phenomena like transformation and storage of information. It is not only possible to transmit information from one place to another but it is also possible to transmit information from one moment to other in time by storing it in some storage device [2].

Now a days, classical information is not the only subject of interest but quantum and biological information is also being studied extensively. These types of information are hidden in some real physical systems and can be associated with a particular property of system. Classical information is processed (stored, transmitted, transformed) by physical phenomena which obey the laws of classical physics. Similarly quantum information is processed with the help of quantum systems which follow rules of quantum mechanics. Classical and quantum systems differ from each other and similarly classical and quantum information are different from each other in many aspects. Classical information is stored in a system which has definite and defined state. We can clone the particular state of system. We are able to measure the state without disturbing or changing the state. On the other hand quantum information is stored as some property of quantum system. For example, quantum information can be stored in polarization of photon or spin of an atomic and sub-atomic particle. These system have unique properties like superposition of states and entanglement. These properties can not be understood or explained with the help of classical physics and thus have no classical analogy. The quantum state associated with quantum information can not be cloned and we can not measure the state to obtain information without altering it.

Study of quantum information theory covers: (i) classical information transmission through quantum channels; (ii) quantum information transmission through quantum channels; (iii) quantum entanglement effect on transmission of information; (iv) informational properties during process of quantum measurement.

Quantum communication is accomplished with the help of a source, quantum channel and a recipient. Source provides the quantum system in required quantum state containing information in it. Quantum channel or path transmits the information from one point to another. Recipient is one that receives the signal and get information out of it by decoding signal. Quantum channel transfers the information from one spatial point to another point as well from one moment to another moment of time. Information transmitted by quantum communication processes is not always correct and may have altered due to some errors. However various methods have also been established to remove these errors.

1.2 Qubits

Qubit is used to represent the quantum information just like bit is used to represent classical information [3]. Classical bit can be regarded as a physical object having one stable and distinct state of two possible states for system. We conventionally represent a bit with the help of binary code of 0 and 1. A bit also represents the classical information amount so we need many bits to encode information created by classical system. States produced by classical information sources are always distinguishable and stable among many possible output states. Qubit is quantum counter part of a bit. It is a quantum system with two possible quantum states. Examples of such systems include up and down spins of an electron, an atom with excited or unexcited state and a photon with possible horizontal and vertical polarizations. Qubit is traditionally represented by state vectors $|0\rangle$ and $|1\rangle$. $|0\rangle$ and $|1\rangle$ are orthogonal states and play similar role as 0 and 1 plays in classical systems, but their role in two cases is not exactly same. A classical bit will exist only in 0 or 1 but a qubit can exist in state $|0\rangle$, $|1\rangle$ or in superposition of both states. If $|A\rangle$ represent the state of qubit then superposition of two state will be written as

$$|A\rangle = \alpha |0\rangle + \beta |1\rangle , \tag{1.2.1}$$

α and β are complex numbers having sum of modulus square of each equal to 1. Thus depending upon the value of α and β a qubit can have a number of possible states. These many possible states associated with qubit provide basis to believe that qubit can store greater information as compared to classical bits. Information stored in qubit can be obtained by doing measurement on qubit [4]. Measurement of qubit is explained in next section.

1.3 Measurement on Qubit

Measuring a qubit will reveal the information encoded in qubit. For a given space of quantum state $|\psi\rangle$ have orthonormal base $V^n = |x_i\rangle$, we have set of measurement operators as M_m for n number of possible measurements with $m = 1, 2, ...n$. Such a set of operators will only qualify as quantum measurement operators if they fulfill two conditions which are given as

- Product of all operators $M^\dagger M$ must be positive.
- These operators must obey completeness relation $\sum M^\dagger M = 1$.

Using these measurement operators we can get the *probability for measurement of m* as

$$p_m = \langle\psi|M_m^\dagger M_m|\psi\rangle. \tag{1.3.1}$$

Other thing we can measure using these measurement operators is post measurement state. We can find state after measurement $|\psi\rangle'$ as

$$\begin{aligned} |\psi\rangle' &= \frac{1}{\sqrt{p_m}} M\,|\psi\rangle \\ &= \frac{1}{\sqrt{\langle\psi|M_m^\dagger M_m|\psi\rangle}} M\,|\psi\rangle\,. \end{aligned} \tag{1.3.2}$$

There are many possibilities for a quantum measurement, however we will take the measurement in basis of state. Thus we will write the projector operator $M_m^\dagger = |x_m\rangle\langle x_m|$ for pure state $|x_m\rangle$. Writing the state $|\psi\rangle$ as $|\psi\rangle = \sum x_i\,|x_i\rangle$ and with $M_m^\dagger M_m = |x_i\rangle\langle x_i|$ we can easily see that probability for $|\psi\rangle$ to be in state $|x_m\rangle$ is $|x_m|^2$. Similarly state after measurement will be

$$|\psi\rangle' = \frac{x_m}{|x_m|}\,|x_m\rangle\,. \tag{1.3.3}$$

Let a qubit is in superposition of states $|0\rangle$ and $|1\rangle$ having state $|Q\rangle$ as

$$|Q\rangle = \alpha\,|0\rangle + \beta\,|1\rangle\,, \tag{1.3.4}$$

where α and β are *complex probability amplitude* satisfying the relation

$$|\alpha|^2 + |\beta|^2 = 1,$$

$|\alpha|^2$ is the probability of finding the qubit in $|0\rangle$ state where as $|\beta|^2$ gives the probability of finding qubit in state $|1\rangle$. So according to relation $|\psi\rangle'{=}\frac{x_m}{|x_m|}\,|x_m\rangle$ possible states of qubit after measurement are

$$\begin{cases} \frac{\alpha}{|\alpha|}\,|0\rangle \\ \frac{\beta}{|\beta|}\,|1\rangle \end{cases} \tag{1.3.5}$$

As the qubit is in superposition of $|0\rangle$ and $|1\rangle$, measurement will result in collapse of the state $|\psi\rangle$ either in $|0\rangle$ and $|1\rangle$. Operators for measurement of qubit state $|Q\rangle$ will be $M_0 = |0\rangle\langle 0|$ and $M_1 = |1\rangle\langle 1|$. Applying M_0 will result in collapse of state $|Q\rangle$ into $|0\rangle$ with probability $p_0 = |\alpha|^2$. Application of M_1 will give $|1\rangle$ with probability $p_1 = |\beta|^2$. This measurement reduces the quantum state into classical situation with bits 0 or 1. However there is little difference between the two cases. In classical bits we are sure about the information encoded in 0 or 1 bit but in quantum measurement we are not certain if information is encoded in $|0\rangle$ or $|1\rangle$ due to probabilities associated with measurement result. In order to explain this property we consider different possible cases given below

- If $\alpha = 0$ and $\beta = 1$ then $p_0 = 0$ and $p_1 = 1$.
- If $\alpha = \frac{1}{\sqrt{2}}$ and $\beta = \frac{1}{\sqrt{2}}$ then $p_0 = p_1 = \frac{1}{2}$.
- If If $\alpha = \frac{1}{\sqrt{3}}$ and $\beta = \frac{2}{\sqrt{3}}$ then $p_0 = \frac{1}{3}$ and $p_1 = \frac{2}{3}$.
- If $\alpha = 1$ and $\beta = 0$ then $p_0 = 1$ and $p_1 = 0$.

In case 1 we are sure that qubit will collapse in bit 1 giving information. In case 2 we have equal probability for 0 and 1 bits. In case 3 chances for qubit to collapse in bit 1 are twice then bit 0. Case 4 is opposite of case

1 and we have 100% certainty that qubit will collapse in bit 1. These four results have been obtained by using operators M_0 and M_1. Here we get an additional insight about measurement of qubit. If we concentrate on case 3 we see that when we use operator M_0 we have $\frac{1}{3}$ chances of obtaining 0 bit and $\frac{2}{3}$ are the chances that measurement will fail. Similarly chances of failure for measurement while using M_1 are $\frac{1}{3}$ and those of success are $\frac{2}{3}$. Failure of measurement means that if we use operator M_m for input state $|Q\rangle$, there is probability $|x_m|^2$ for state to collapse in $|x_m\rangle$ and chances that this collapse will not occur are $1 - |x_m|^2$. When output is not $|x_m\rangle$ then no information is obtained from measurement and measurement is said to be failed. As a result of failed measurement on qubit, information is lost irreversibly and qubit is annihilated [5–7].

1.4 Entanglement

In quantum mechanics, a system comprising of subsystems, sometimes exhibits a very interesting phenomenon called entanglement. It is perhaps most prominent difference between classical and quantum physics. In classical physics we can corelate two particles by preparing the system in a particular state and then we can communicate this state to a distant observer to prepare the same state. These corelation can fully be understood and explained using classical probability distribution. However in quantum mechanics situation is completely different and corelations can not be explained simply by using laws of classical physics [8, 9].

In 1935 Einstein, Podolsky and Rosen [10] suggested a thought experiment describing incompleteness of laws of quantum mechanics. This thought experiment is normally known as EPR paradox. EPR paradox was widely discussed and it was finally understood with the help of non local property of quantum mechanics. It was proposed that two particles can interact with each other even if they are very far apart from each other and they can not behave independently. Such interacting particles are called as entangled particles and this phenomenon of long distance interaction is called entanglement.

Later on experiments also confirmed this non local and long distance interaction between particles [11]. Thus the focus on entanglement shifted from being a fundamental quantum mechanics problem to its utilization in different applications. During 1980s and 1990s, different quantum information processes including quantum teleportation, quantum cryptography and quantum algorithms for quantum computations were proposed and quantum entanglement lied at the heart of all these processes. Since then quantum entanglement is considered to be a very important resource in quantum communication and information processes.

Suppose we have two qubits A and B. If both qubits are in state $|0\rangle$ then state of system can be written as $|0\rangle_A |0\rangle_B$. Similarly if qubits are in state $|1\rangle$ the state of system will be $|1\rangle_A |1\rangle_B$. If we express system in superposition of two states then

$$|\psi\rangle = \frac{|0\rangle_A |0\rangle_B + |1\rangle_A |1\rangle_B}{\sqrt{2}}. \tag{1.4.1}$$

This state $|\psi\rangle$ is an example of entangled state [12].

Classical and quantum correlations are different in nature. Classical correlations are usually consequences of conservation laws. Such an example is decay of a particle at rest. When a particle decays, two newly formed particles fly apart in opposite directions as required by law of conservation of momentum. If we measure the momentum of one particle, we will know the momentum of other as according to conservation of momentum both particles has equal momentum but opposite directions. Momentum of both the particles is independent from each other and measurement of one has no effect on other. Measuring the momentum of one and finding the momentum of other is direct consequence of law of conservation of momentum. This is an example of classical correlation between two particles. Now we see at the quantum correlation between two particles. Let us we have an entangled state $|\psi\rangle$ given as

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow_1\downarrow_2\rangle + |\downarrow_1\uparrow_2\rangle), \tag{1.4.2}$$

here $|\uparrow\rangle$ and $|\downarrow\rangle$ represent an electron with spin up and down respectively. Similar to that of classical correlation, when we perform measurement on particle 1 and find that particle 1 is in spin up state we will immediately know that particle 2 is in spin down state and vice versa. Difference between two cases lies in the superposition of quantum states which makes the outcome of measurement completely random. This means that before measurement we do not know if the measurement will result in $|\uparrow_1\rangle$ or $|\downarrow_1\rangle$. As soon as particle 1 collapses in $|\uparrow_1\rangle$ or $|\downarrow_1\rangle$ it appears that immediate transfer of information between two particles has been taken place and particle 2 simultaneously collapses in corresponding $|\downarrow_2\rangle$ or $|\uparrow_2\rangle$. This transfer of information occurs even when particles are very far apart.

Apparently this phenomenon violates theory of special relativity which limits the speed of information transfer and states that no information can travel faster than speed of light. However it is safe to say that no useful information is travelled between two points. This can be assumed because when entangled state is prepared, both particles (entangled) interact with each other at some point during creating entanglement. After entanglement when two particles move apart quantum state of system (containing two subsystems) spreads over large spatial region and thus no information is traveled between two subsystems during measurement. Entangled state of system is a complete description and contains all necessary information about subsystems present within. Entanglement is purely quantum mechanical phenomenon and has no counter part in classical mechanics.

Entanglement plays important role in quantum communication processes like quantum teleportation. Quantum teleportation is discussed in detail in next section.[13, 14]

1.5 Quantum Teleportation

Term teleportation has been borrowed from parapsychology where it means transporting things or persons from one place to another with the help of mental powers. This term became popular in science fiction where transport between two points was instantaneous. However this dream is still to be fulfilled. Special theory of relativity does not allow the transportation of an object faster than speed of light. Basic idea behind quantum teleportation is that we do not need to transport material objects between two points but there may already exist such objects and we just have to arrange them in proper way and we transmit state of a system. To achieve this we follow a set of instructions which are present in quantum mechanical wave function of system with maximum information regarding an object. This is like measuring the wave function of system at one place and then sending information about measurement to some other places by using classical means of communication and there this information can be used to reconstruct original wave function. We imagine that Alice (sender) wants to teleport a state to Bob (receiver). It is not necessary for Alice to know the state to be teleported. Problem on Bob's end is that he can not reconstruct the state unless Bob's system is related to Alice's system. This relation between two systems is created by entanglement. Entanglement plays key role in quantum teleportation. Due to to entanglement between two systems (Alice's and Bob's), measurement on one system affects the other system. So the state teleported to Bob from Alice directly depends upon this measurement process.

Bennett et al. (1993) proposed a scheme for teleportation of polarization state of photon [15]. We take the polarization state of photon to be

$$|\phi_1\rangle = \alpha\,|H_1\rangle + \beta\,|V_1\rangle\,, \tag{1.5.1}$$

where H_1 and V_1 are horizontal and vertical polarization of photon labelled as 1. α and β are complex numbers and satisfy the relation $|\alpha|^2 + |\beta|^2 = 1$. Teleportation is done with the help of entangled photons in polarization degree of freedom. State of such an entangled system is

$$\left|\Psi^-\right\rangle = \frac{1}{\sqrt{2}}(|H_2V_3\rangle - |V_2H_3\rangle). \tag{1.5.2}$$

Photon 1 is sent to Alice and photon 2 is sent to Bob. Our task here is to transfer state of photon 1 to photon 3 present at Bob's end. To complete this task, Alice has to do measurement on photon 1 along with photon 2. This measurement destroys the state of both Photon 1 and 2. In next step Alice communicates her output of measurement to Bob through classical channel. Once Bob knows the result of measurement carried out by Alice, he is in a position to determine which operation he has to perform on his

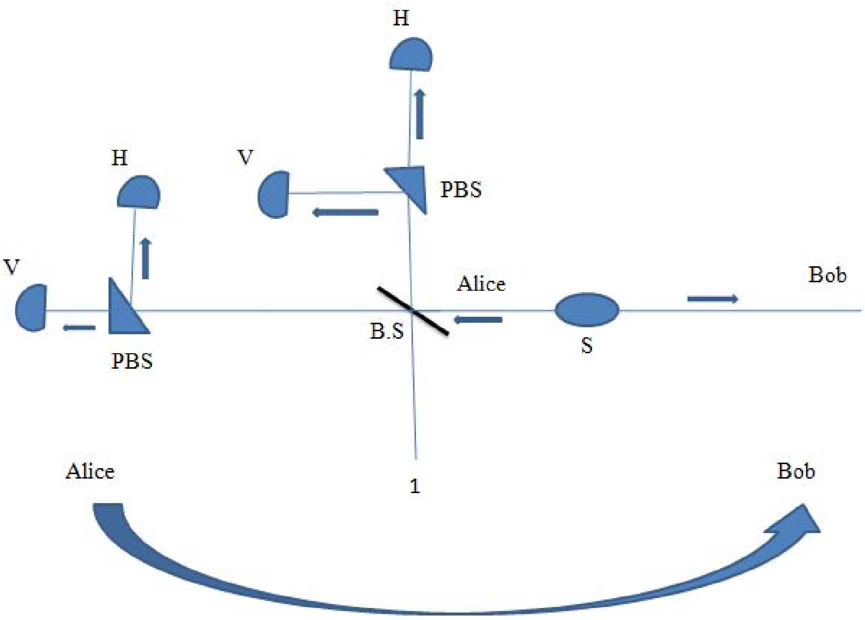

Figure 1.1: The scheme of quantum teleportation. A source labelled as S produces a pair of entangled photons. One photon is sent to Bob and other to Alice. Alice also has a photon in path 1 whose polarization state Alice wants to teleport. Alice performs Bell state measurement (BSM) using a 50 : 50 beam splitter and four detectors labelled as H_{a}, V_{a}, H_{b}, V_{b}. Polarization beam splitter (PBS) placed before detectors differentiate between horizontal and vertical polarization and directs them to respective detectors.

photon in order to get the state of photon 1. Measurement process followed by Alice is Bell state measurement (BSM). During measurement we combine photon 1 and photon 2 on 50 : 50 beam splitter. Mathematically we write the combined state of three photons after beam splitter application and little rearrangement

$$\begin{aligned} |\Psi_{123}\rangle = |\phi_1\rangle \otimes \left|\Psi^-_{23}\right\rangle = \frac{1}{2}[(\alpha\,|V_3\rangle - \beta\,|H_3\rangle)\Phi^+_{12} + (\alpha\,|V_3\rangle + \beta\,|H_3\rangle)\Phi^-_{12} \\ (-\alpha\,|H_3\rangle + \beta\,|V_3\rangle)\Psi^+_{12} + (-\alpha\,|H_3\rangle - \beta\,|V_3\rangle)\Psi^-_{12}]. \end{aligned} \tag{1.5.3}$$

Photon 1 and Photon 2, after passing through beam splitter, are in mixture of all Bell states. Equation (1.5.3) giving complete state of system collapses to only one part depending upon measurement result. State of photon 3 present at Bob's end is given in round brackets. Careful inspection of equation (1.5.3) reveals that fourth term is identical to state of photon 1. Other three terms can be made identical to state of photon 1 by using different operations and changing the signs of α and β. During measurement Alice has to distinguish among four Bell states. Using linear optical setup, Alice can only distinguish between $|\Psi^+\rangle$ and $|\Psi^-\rangle$ where as $|\Phi^+\rangle$ and $|\Phi^-\rangle$ can not be distinguished. Due to indistinguishability of $|\Phi^+\rangle$ and $|\Phi^-\rangle$ Bob can not know which operations he has to perform. Thus Teleportation will only work for 50% of times. However we can also give away the results when Alice detects $|\Psi^+\rangle$ and thus we will say teleportation is successful only in 25% of cases when detectors clicks for $|\Psi^-\rangle$ [16–19].

Teleportation has emerged as an alternative way of communication and is of great interest for scientists as it may lead to faster communication compared to usual classical communication means.

1.6 Thesis Outline

Our aim is to develop a model for quantum teleportation that incorporates practical limitations imposed by inefficiencies of apparatus and environmental interference during teleportation. In this thesis, we develop continuous mode spectral representation of state obtained from parametric down conversion process. We develop a continuous mode model of detector which responds to spectral distribution on input state and also incorporate effects of limited efficiencies. This continuous mode model is a base model to incorporate quantum memories in quantum communication setups.

In chapter 2 we have given an overview of Dirac notation of states and density operators. We describe the sources used to produce entangled photon pairs. We explain the process of BSM. Further we explain detector models including practical imperfections. Then we describe how a single mode state is converted to a continuous mode state.

In chapter 3 we convert the single mode state obtained from parametric down conversion into continuous mode state. We develop detector model whose efficiency is different for different modes of incident state. In the end we apply continuous mode model for verification of entanglement in continuous mode photon state and find the three fold coincidence probability to check the success of quantum telportation.

In chapter 4 we conclude our thesis.

Chapter 2

PRELIMINARIES

In quantum communication, major focus lies on the development of different techniques and models which allow better and more practical ways for doing different quantum communication processes such as quantum teleportation, quantum key distribution and quantum dense coding. Various models have been proposed over the time for carrying out these processes. Some of these models use quantum mechanical properties of photons to achieve the goals while others use electrons,protons and atoms to get quantum mechanical environment.

We use a linear optical setup for quantum communication in which we take entangled photons as our resource. We consider limited efficiency for detectors used during measurement and extraction of information stored in photons. Model for single mode photons has already been developed and we give an over view of previously developed models. We also incorporate the previously developed continuous mode model state of photon in our continuous mode model of detectors and photons.

In this chapter we give the background knowledge of setup we use for continuous mode teleportation. For this purpose we explain the preliminaries of quantum mechanics. In section 2.1 we explain Dirac representation of quantum mechanics. In section 2.2 we describe density matrix and its properties. In section 2.3 we give details of photon sources used to create entanglement. In section 2.4 we explain maximally entangled Bell's states and process of BSM. In section 2.5 we give the details of already developed photon detector models with practical limitations. In section 2.6 we explain how can we convert the single mode photon number state into continuous mode continuous state.

2.1 Dirac Representation of Quantum Mechanics

A quantum mechanical system is represented by state vectors. These state vectors are Hilbert space elements and represent physical state in which system exists. These state vectors can be written in different bases. This is similar to a situation where a vector is written in components in different coordinate systems. A vector does not depend upon the coordinate system which is used to represent vector in its components. Similarly a quantum state does not depend upon bases used to represent it. Dirac introduced the notation of kets, bras and bra-ket. These terms are explained as follows [20]:

Kets

If a system has state vector ψ then we represent it as $|\psi\rangle$ in Dirac notation and call it ket-vector or just ket for simplicity. These kets are elements of Hilbert space which we also call ket-space.

Bras

It is a well known fact of linear algebra that we can associate a dual space with any vector space. So if we have a ket state vector $|\psi\rangle$, dual space element associated with it is represented as $\langle\psi|$. This is called bra vector or just bra for convenience. Thus for any ket vector $|\psi\rangle$ there will be unique bra vector $\langle\psi|$.

Properties of Kets and Bras

If we have a ket vector $|\psi\rangle$ and corresponding bra vector $\langle\psi|$ then ket and bra vector are related as

$$|\psi\rangle^\dagger = \langle\psi| \qquad \langle\psi|^\dagger = |\psi\rangle \tag{2.1.1}$$

For bras and kets we have one to one correspondence.

$$a\,|\psi\rangle + b\,|\phi\rangle \leftrightarrow a^*\,\langle\psi| + b^*\,\langle\phi| \tag{2.1.2}$$

with a and b being complex numbers. Multiplication of a ket vector with complex number results in another ket vector. Same is true for bra vector.

Bra-ket

Inner product is represented by $\langle|\rangle$ symbol in Dirac notation. Inner product is also called as scalar product. Inner product (ψ, ϕ) is represented as $\langle\psi|\phi\rangle$.

$$(\psi, \phi) = \langle\psi|\phi\rangle \tag{2.1.3}$$

Operators

An operator is defined as to be a mathematical rule which when operates on a ket $|\psi\rangle$ changes it to another ket $|\psi'\rangle$ of same space. Similarly when an operator operates on bra $\langle\phi|$, it changes it to another bra $\langle\phi'|$ of same space.

$$\hat{A}\,|\psi\rangle = |\psi'\rangle \qquad \langle\phi|\,\hat{A} = \langle\phi'| \tag{2.1.4}$$

Projection Operator

An operator is said to be projection operator if it is hermitian and is equal to own square.

$$\hat{A}^\dagger = \hat{A} \qquad \hat{P}^2 = \hat{P} \tag{2.1.5}$$

2.2 Density Matrix

A density operator helps us to distinguish between mixed and pure states of a quantum mechanical system system. Pure states have maximum information stored within and can be written as superposition of basis vectors in an orthonormal basis. Mixed states need a statistical approach and explain ensemble of states (pure). For example a quantum system can exist in any pure state $|\psi_1\rangle$, $|\psi_2\rangle$ and $|\psi_3\rangle$ having probabilities of p_1, p_2 and p_3. Density operator also describes the composite system existing in entangled state. Density operator of pure state $|\psi\rangle$ is defined as

$$\rho = |\psi\rangle \langle\psi| \,. \tag{2.2.1}$$

Density operator for mixture of pure states is defined as

$$\rho = \sum_k p_k |\psi_k\rangle \langle\psi_k| \,, \tag{2.2.2}$$

where p_k represent the probability of each pure state. Density matrix provides us a way to represent the system with mixed states. Example of such system may be the one which contains two subsystems interacting with each other [21].

2.2.1 Properties of Density Operators

- Trace of density operator is unity.

$$Tr(\rho) = 1. \tag{2.2.3}$$

- Density operator must be non negative $\rho \geq 0$.
- Density operator must be Hermitian $\rho^\dagger = \rho$.

2.3 Photon Sources

Quantum communication can be achieved by using atoms and electrons but practically photons are the information carrier in most of the long distance communication processes. Now a days, quantum communication is considered to be most advanced technology for processing quantum information. For example, many quantum communication processes like teleportation and cryptography have been successfully achieved in laboratories. There are different techniques to generate entangled photons for quantum communication. These processes use entangled photons as resource. One Such technique is Parametric Down Conversion (PDC).

2.3.1 Parametric Down Conversion

It is a process in which a photon incident on a nonlinear crystal of nonlinearity $\chi^{(2)}$($\chi^{(2)}$ represents second order nonlinearity) is down converted into two photons having lower individual frequencies and momenta as compared to incident photon but sum of frequencies and momenta of daughter photons is equal to frequency and momentum of original incident photon. It is a non linear process. In quantum communication processes like quantum teleportation, entangled photons are used. Now a days PDC is used as standard tool for creating photons in entangled states.

Two photons produced after down conversion are traditionally labelled as signal and idler photons. Process is governed by the conditions given by

$$\begin{aligned} \omega_p &= \omega_s + \omega_i, \\ \boldsymbol{k_p} &= \boldsymbol{k_s} + \boldsymbol{k_i}, \end{aligned} \tag{2.3.1}$$

where ω_p is frequency of parent photon and ω_s and ω_i are frequencies of signal and idler photon respectively. Similarly $\boldsymbol{k_p}$, $\boldsymbol{k_s}$ and $\boldsymbol{k_i}$ represent wave-vectors for parent, signal and idler photon respectfully. $\boldsymbol{k_p} = \boldsymbol{k_s} + \boldsymbol{k_i}$ is called "phase matching condition".

Parametric down conversion of photons is a probabilistic process. This means that every photon passing through crystal will not decay in two photons. Usually efficiency of down conversion is very small and is of the order of 10^{-6} [22]. Schematic representation of PDC is shown in Fig (2.1)

A photon can be down converted either by spontaneous PDC or stimulated PDC. In stimulated PDC [23] we send a beam of higher frequency to interact with non linear crystal of non linearity $\chi^{(2)}$. This beam is known as pump beam. We send another beam of lower frequency that interacts with the pump beam inside crystal. This second beam is called as signal beam. Signal beam is amplified and pump beam is depleted. During this process another beam called "idler" beam is generated because of energy conservation. Frequency of idler beam is the difference of frequency of pump and signal beam.

Spontaneous PDC involves single incident beam of photons called signal beam. Photon in signal beam interacts with non linear crystal and spontaneously decays into two daughter photons which are conventionally termed as signal and idler photons.

In addition to frequencies and wave vectors, polarization is another property which can be associated with down converted photons. Based on polarization of down converted photons there are two types of PDC which are

- Parametric down conversion type-I
- Parametric down conversion type-II

Parametric Down Conversion Type-I

Signal and idler photons produced as a result of PDC type-I have identical polarization. For example, If signal photon has horizontal polarization denoted as H then idler photon will also have horizontal polarization. Similarly if signal photon has vertical polarization V then idler will also be in vertical polarization state. State of photons produced in PDC type-I can be written as

$$\begin{aligned} \left|\phi^{+}\right\rangle &= \frac{1}{\sqrt{2}}(|H_{\mathrm{s}}H_{\mathrm{i}}\rangle + |V_{\mathrm{s}}V_{\mathrm{i}}\rangle). \\ \left|\phi^{-}\right\rangle &= \frac{1}{\sqrt{2}}(|H_{\mathrm{s}}H_{\mathrm{i}}\rangle - |V_{\mathrm{s}}V_{\mathrm{i}}\rangle). \end{aligned} \tag{2.3.2}$$

In type I PDC, polarization of signal and idler photon is perpendicular to the polarization of pump photon. For example if pump photon has ordinary polarization then signal and idler photons will have extra ordinary polarization [24]. Extra-ordinary polarization is parallel to optical axis of crystal used to down convert the pump photon and ordinary polarized photons have polarization orthogonal to optical axis direction.

Parametric Down Conversion Type-II

In PDC type-II, signal and idler photons have polarization orthogonal to each other. If signal photon has horizontal polarization then idler photon

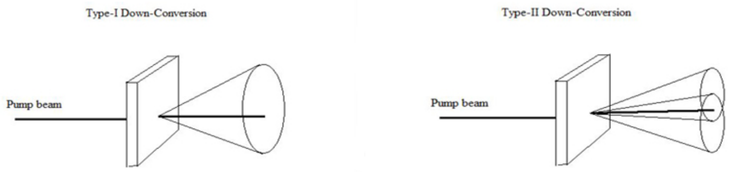

Figure 2.1: shows parametric down conversion process. Photons coming out of crystal form cones. In parametric down conversion type I both photons belonging to single pair appear on same cone with same polarization of both photons. In type II parametric down conversion photons produced as a result of down conversion appears in different cones with one photon having horizontal polarization and other with vertical polarization.

will possess vertical polarization and vise versa. Entangled states prepared by PDC type-II can be represented as follows

$$\begin{aligned} |\psi^+\rangle &= \frac{1}{\sqrt{2}}(|H_s V_i\rangle + |V_s H_i\rangle). \\ |\psi^-\rangle &= \frac{1}{\sqrt{2}}(|H_s V_i\rangle - |V_s H_i\rangle). \end{aligned} \tag{2.3.3}$$

Down converted photons (signal and idler) emerge from crystal into two cones. One cone contains photons with horizontal polarization and other with vertical polarization. Condition of momentum conservations makes the photons of each pair to lie on opposite side of pump beam. Entangled pairs of photons exist at the points where both cones intersect each other [24].

2.4 Bell States and Bell State Measurement

Bell state measurement (BSM) is a key process in quantum communication. It helps to observe and understand the interference of entangled photons.

2.4.1 Bell States

These states are named after the scientist John S. Bell. Bell states are maximally entangled states. Let us have two photons which are entangled to each other in polarization degree of freedom. This means as soon as we know the polarization of photon 1, we can tell about polarization of photon

2 provided with the information of entangled state. For a two qubit system, there are four Bell states (Maximally entangled states)

$$
\begin{aligned}
|\psi_{12}^{+}\rangle &= \frac{1}{\sqrt{2}}(|H_1V_2\rangle + |V_1H_2\rangle). \\
|\psi_{12}^{-}\rangle &= \frac{1}{\sqrt{2}}(|H_1V_2\rangle - |V_1H_2\rangle). \\
|\phi_{12}^{+}\rangle &= \frac{1}{\sqrt{2}}(|H_1H_2\rangle + |V_1V_2\rangle). \\
|\phi_{12}^{-}\rangle &= \frac{1}{\sqrt{2}}(|H_1H_2\rangle + |V_1V_2\rangle).
\end{aligned} \tag{2.4.1}
$$

These states have been used as basic tool in different quantum communication processes. One such example is quantum teleportation.

2.4.2 Bell State Measurement

Different quantum communication processes use entangled states as source. Information stored in these entangled states is obtained by performing BSM. Bell state measurement is an optical process that helps to distinguish between Bell states. However, all the four Bell states can not be distinguished using BSM based upon linear optical setup. We can distinguish between $|\psi^{+}\rangle$ and $|\psi^{-}\rangle$ but $|\phi^{+}\rangle$ and $|\phi^{-}\rangle$ can not be differentiated using linear optical processes. Fig (2.2) shows the experimental setup for BSM.

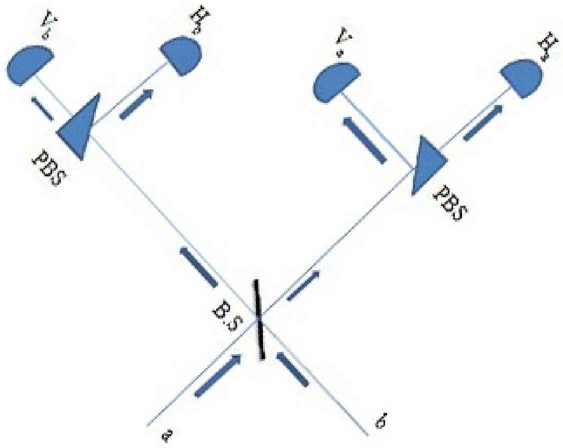

Figure 2.2: shows the model for performing Bell state measurement. Photons present in path a and b are incident on a 50 : 50 beam splitter where these photon interact each other and are reflected or transmitted by beam splitter. These photons then fall upon polarising beam splitters which transmit or reflect photons depending upon their polarization. Finally photons are detected at detectors which are labelled as H_a, V_a, H_b, V_b.

A 50 : 50 beam splitter is one for which we have equal reflectance r and transmittance t. $|r|$ and $|t|$ for 50 : 50 beam splitter is $\frac{1}{\sqrt{2}}$. Let I, R and T be the amplitudes of incident, reflected and transmitted beams (classical treatment) then beam splitter action follows

$$R = rI. \qquad\qquad T = tI. \tag{2.4.2}$$

We assume beam splitter to be lossless so incident beam intensity must be equal to sum of intensities of reflected and transmitted beams:

$$|I|^2 = |T|^2 + |R|^2, \tag{2.4.3}$$

which is possible if

$$|r|^2 + |t|^2 = 1. \tag{2.4.4}$$

While treating quantum mechanically, we replace complex amplitudes of beams (incident, reflected and transmitted) by respective annihilation operators $\hat{a}_k$ (where $k = i, r, t$). Similar to that of classical treatment we can write in quantum treatment

$$\hat{a}_r = r\hat{a}_i. \qquad \hat{a}_t = t\hat{a}_i. \tag{2.4.5}$$

These operators of every field must obey the following commutation relations

$$\begin{aligned}[][\hat{a}_k, \hat{a}_l^\dagger] &= \delta_{kl}. \\ [\hat{a}_k, \hat{a}_l] &= 0. \\ [\hat{a}_k^\dagger, \hat{a}_l^\dagger] &= 0.\end{aligned} \tag{2.4.6}$$

Since operators in equation (2.4.5) do not satisfy these commutation relations so these are not providing complete quantum description of beam splitter action. This problem can be solved by considering that in classical treatment of beam splitter we may take incident beam only on one input port, but in quantum mechanics, we cannot leave any input port empty. Even if we do not take incident photons on both inputs we will have to take vacuum on one input port.

Now beam splitter action on quantum operator can be written as

$$\hat{a}_r \xrightarrow{\hat{U}_B} \frac{1}{\sqrt{2}}(\hat{a}_i + \iota\hat{a}_0), \qquad \hat{a}_t \xrightarrow{\hat{U}_B} \frac{1}{\sqrt{2}}(\iota\hat{a}_i + \hat{a}_0), \tag{2.4.7}$$

where $\hat{a}_0$ represents operator for vacuum [25].

For the measurement process 50 : 50 beam splitter is used. Two photons in entangled state enter the 50 : 50 beam splitter through spatial modes a and b and leave in output channels leading to polarization beam splitter. If $|\psi^-\rangle$ is incident on beam splitter then two photons will appear in different outputs and if any one of the $|\psi^+\rangle, |\phi^\pm\rangle$ is incident then both photons will appear in same output. Polarization beam splitter shown in figure separates horizontal and vertical polarization. If a state is incident and detector H_a and V_b or H_b and V_a give clicks then it will be indication that $|\psi^-\rangle$ was incident state. For $|\psi^+\rangle$ we will get coincidence between H_a and V_a or H_b and V_b. For $|\phi^\pm\rangle$ we will get two clicks at same detectors which means we will get H_aH_a, V_aV_a, H_bH_b, V_bV_b clicks [26].

Possibility of different outcomes for two photon after being incident at 50 : 50 beam splitter is shown in Fig (2.3).

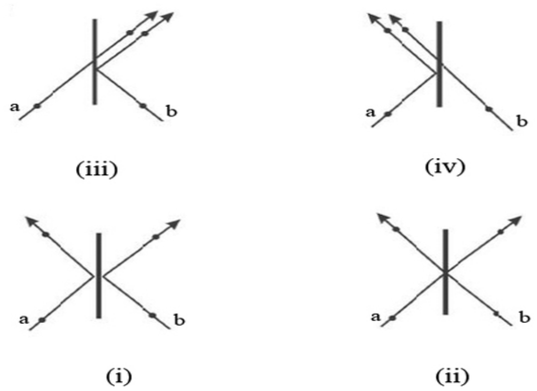

Figure 2.3: shows four possible outcomes for photons incident on 50 : 50 beam splitter. In case (i) both photons are reflected from beam splitter. In case (ii) both photons coming in path a and b are transmitted. In case (iii) photon in path a is transmitted and photon in path b is reflected. In case (iv) photon on path a is reflected and photon in path b is transmitted.

In case of $|\psi^-\rangle$ state, photons after beam splitter action appear on different sides of beam splitter which can easily be shown as

$$\left|\psi^-\right\rangle = \frac{1}{\sqrt{2}}(|H_{\mathrm{a}}V_{\mathrm{b}}\rangle - |V_{\mathrm{a}}H_{\mathrm{b}}\rangle) \xrightarrow{\hat{U}_B} \frac{\iota}{\sqrt{2}}(|H_{\mathrm{c}}V_{\mathrm{d}}\rangle + |V_{\mathrm{c}}H_{\mathrm{d}}\rangle), \tag{2.4.8}$$

where beam splitter operation is defined as

$$\begin{aligned} |H_{\mathrm{a}}\rangle &\xrightarrow{\hat{U}_B} \frac{1}{\sqrt{2}}(|H_{\mathrm{c}}\rangle + \iota\,|H_{\mathrm{d}}\rangle). \\ |H_{\mathrm{b}}\rangle &\xrightarrow{\hat{U}_B} \frac{1}{\sqrt{2}}(|H_{\mathrm{c}}\rangle - \iota\,|H_{\mathrm{d}}\rangle). \\ |V_{\mathrm{a}}\rangle &\xrightarrow{\hat{U}_B} \frac{1}{\sqrt{2}}(|V_{\mathrm{c}}\rangle + \iota\,|V_{\mathrm{d}}\rangle). \\ |V_{\mathrm{b}}\rangle &\xrightarrow{\hat{U}_B} \frac{1}{\sqrt{2}}(|H_{\mathrm{c}}\rangle - \iota\,|V_{\mathrm{d}}\rangle). \end{aligned} \tag{2.4.9}$$

Similarly for $|\psi^+\rangle$ and $|\phi^\pm\rangle$ we get

$$\left|\psi^+\right\rangle = \frac{1}{\sqrt{2}}(|H_{\mathrm{a}}V_{\mathrm{b}}\rangle + |V_{\mathrm{a}}H_{\mathrm{b}}\rangle) \xrightarrow{\hat{U}_B} \frac{1}{\sqrt{2}}(|H_{\mathrm{c}}V_{\mathrm{c}}\rangle + |V_{\mathrm{d}}H_{\mathrm{d}}\rangle). \tag{2.4.10}$$

And

$$\left|\phi^{+}\right\rangle = \frac{1}{\sqrt{2}}(|H_{\rm a}H_{\rm b}\rangle - |V_{\rm a}V_{\rm b}\rangle) \xrightarrow{\hat{U}_B} \frac{1}{2\sqrt{2}}(|H_{\rm c}H_{\rm c}\rangle + |H_{\rm d}H_{\rm d}\rangle + |V_{\rm c}V_{\rm c}\rangle + |V_{\rm d}V_{\rm d}\rangle),$$
$$\left|\phi^{-}\right\rangle = \frac{1}{\sqrt{2}}(|H_{\rm a}H_{\rm b}\rangle - |V_{\rm a}V_{\rm b}\rangle) \xrightarrow{\hat{U}_B} \frac{1}{2\sqrt{2}}(|H_{\rm c}H_{\rm c}\rangle + |H_{\rm d}H_{\rm d}\rangle - |V_{\rm c}V_{\rm c}\rangle - |V_{\rm d}V_{\rm d}\rangle), \tag{2.4.11}$$

which show that photons appear on the same side of beam splitter.

2.5 Detector Model

For all phenomena related to Quantum optics we need photon detectors to perform experiments. Photon detectors are devices that give information about presence of photon by generating signals. Usually theoretical models of photon detectors are concerned with photon numbers but practically photo-detector models also take into account other degree of freedoms like spectral, polarization and spatial degrees of freedom. Now restricting ourselves to photon number degree of freedom, we discuss ideal model for such detectors.

2.5.1 Photon Number Resolving Detectors

The ideal photo detector can respond to the photon number of incident state, and can distinguish between different number states. Mathematically, we can define such detectors by a projection operator

$$\hat{\Pi}_n = |n\rangle\langle n|, \tag{2.5.1}$$

where n is the number of photons in incident state. Since $\sum_{j=0}^{\infty}\hat{\Pi}_j = 1$ so this set of operators follow POVM theorem. After performing measurement at input state, output state is written as

$$\hat{\rho}_{\rm out} = \hat{\Pi}_n\hat{\rho}_{\rm in}\hat{\Pi}_n. \tag{2.5.2}$$

In output state, here, normalization factor has been ignored for the sake of simplicity. The probability for detection of n photons, when input state is ρ, is

$$p = Tr[\hat{\Pi}_n\rho] = \langle n|\rho|n\rangle. \tag{2.5.3}$$

From the condtion $\sum_{n=0}^{\infty}\hat{\Pi}_n = 1$, it is ensured that probabilty sum is also 1.

2.5.2 Non-Photon Number Resolving Detectors

In real world, however, we do not have ideal detectors and imperfections are always present. Detectors available at present can not give information about number of photons in incident state. These are called as “non-photon number resolving detectors”. They only give information about the presence or absence of photons. Signal generated is usually an electric pulse. In the early days, Geiger counters served as photon detectors. Every time counter detected a photon, it gave sound of click. This terminology is still in use and detector outcomes are usually labelled as clicks and no-clicks. Non-photon number resolving detectors give two outputs depending upon if a photon is present or not i.e a click if one or more photons are incident and a no-click if no photons are incident on detector.

Model for non-photon number resolving detectors is established by considering detector to be an ideal photon number resolving detector and then taking trace over all possible outcomes which will then give desired output. Projection operators for these detectors are written as

$$\hat{\Pi}(\text{click}) = \sum_{n=1}^{\infty} |n\rangle \langle n| \,. \tag{2.5.4}$$

$$\hat{\Pi}(\text{no-click}) = |0\rangle \langle 0| \,. \tag{2.5.5}$$

We can see from above two equations that the condition $\hat{\Pi}(\text{click})+\hat{\Pi}_0(\text{no-click}) = \hat{I}$ is also satisfied. Output state in this case is written as

$$\hat{\rho}_{\text{out}} = \sum_{n=1}^{\infty} \hat{\Pi}_n \hat{\rho}_{\text{in}} \hat{\Pi}_n. \tag{2.5.6}$$

This is the output state representing the presence of photons hence giving a click. Output showing no click can be written as

$$\hat{\rho}_{\text{out}} = \hat{\Pi}_0 \hat{\rho}_{in} \hat{\Pi}_0. \tag{2.5.7}$$

Probability for registering a click is

$$p = Tr[\hat{\Pi}_n \rho] = \sum_{n=1}^{\infty} \langle n|\rho|n\rangle. \tag{2.5.8}$$

Detector models discussed so far are ideal and detector imperfections like non unit efficiency have not been included in these models. In next section we discuss detector model with finite efficiency.

2.5.3 Non-Unit Efficiency Detector

We now describe a model that explains only those detectors which exhibit photon loss during detection process. Such detectors are called as “finite efficiency” or “lossy” detectors. How a photon in a detector is lost depends upon the working mechanism of detector but we can say that each photon that enters the detector has probability η of giving click. We can explain this with the help of an example: Consider we have 3 baskets a, b and c. In basket “a” we have $n = x + y$ balls (x and y representing detected and lost balls respectively) . Baskets “b” and “c” are labelled as “detected” and “lost”. Let the probability for x balls to go in basket b (representing detected) is η then probability for finding x balls in “b” and y balls in “c” is

$$p(x, y) = \binom{n+m}{n}\eta^n(1-\eta)^m. \tag{2.5.9}$$

POVM for such detectors can be written as

$$\hat{\Pi}_n = \sum_{m=0}^{\infty}\binom{n+m}{n}\eta^n(1-\eta)^m \left|n+m\right\rangle\left\langle n+m\right|. \tag{2.5.10}$$

This is similar to a situation where we have placed a beam splitter with efficiency or transmission coefficient η before an ideal detector as shown in Fig (2.4).Reflected photons are considered to be lost where as transmitted photons are detected with the help of ideal photon number detector.

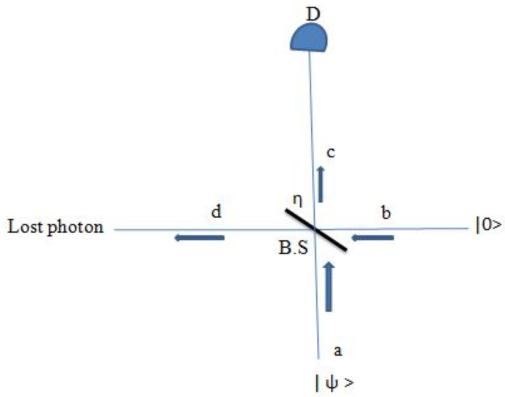

Figure 2.4: Model for faulty detector: A beam splitter with transmission coefficient η is placed before an ideal detector. At one input we have vacuum as input state and other input photons are incident. Photons transmitted by beam splitter are detected while those which are reflected are lost. This is similar to a situation where a detector can not detect all photons due to its efficiency limitations and some of he photons are lost during measurement.

In next section we discuss continuous mode quantum states.

2.6 Continuous mode quantum states

So far we have assumed in our discussion that all photons are identical and indistinguishable. Although these photons follow some distribution in frequency space but due to their indistinguishability, we can represent a photon state in dirac notation simply by $|1\rangle$. In this representation exact frequency distribution of photons is ignored. If we suppose that photons are not completely indistinguishable then this representation is not enough and we will need some representation that gives information about individual distribution of photons. So we can write single photon frequency representation as

$$|1\rangle \rightarrow \int f(\omega)\,|1\rangle_{\omega}\,d\omega = \int f(\omega)\hat{a}^{\dagger}_{\omega}\,|0\rangle\,d\omega, \tag{2.6.1}$$

here $f(\omega)$ is normalized distribution function. It gives information about distribution of photon in frequency space. $|1\rangle_{\omega}$ is single photon state with single frequency ω and $\hat{a}^\dagger_\omega$ is creation operator for single frequency photon. Integration is over entire frequency space. Spectral distribution function $f(\omega)$ can be Lorentzian or Gaussian depending upon the nature of photon source [27].

Bell states in continuous mode frequency representation can be written as

$$\begin{aligned}
\left|\psi^+_{12}\right\rangle &= \frac{1}{\sqrt{2}}[\int f(\omega_1,\omega_2)\left|H_1^{(\omega_1)}V_2^{(\omega_2)}\right\rangle d\omega_1 d\omega_2 + \int g(\omega_1,\omega_2)\left|V_1^{(\omega_1)}H_2^{(\omega_2)}\right\rangle d\omega_1 d\omega_2],\\
\left|\psi^-_{12}\right\rangle &= \frac{1}{\sqrt{2}}[\int f(\omega_1,\omega_2)\left|H_1^{(\omega_1)}V_2^{(\omega_2)}\right\rangle d\omega_1 d\omega_2 - \int g(\omega_1,\omega_2)\left|V_1^{(\omega_1)}H_2^{(\omega_2)}\right\rangle d\omega_1 d\omega_2],\\
\left|\phi^+_{12}\right\rangle &= \frac{1}{\sqrt{2}}[\int f(\omega_1,\omega_2)\left|H_1^{(\omega_1)}H_2^{(\omega_2)}\right\rangle d\omega_1 d\omega_2 + \int g(\omega_1,\omega_2)\left|V_1^{(\omega_1)}V_2^{(\omega_2)}\right\rangle d\omega_1 d\omega_2],\\
\left|\phi^-_{12}\right\rangle &= \frac{1}{\sqrt{2}}[\int f(\omega_1,\omega_2)\left|H_1^{(\omega_1)}H_2^{(\omega_2)}\right\rangle d\omega_1 d\omega_2 + \int g(\omega_1,\omega_2)\left|V_1^{(\omega_1)}V_2^{(\omega_2)}\right\rangle d\omega_1 d\omega_2],
\end{aligned} \tag{2.6.2}$$

$f(\omega_1,\omega_2)$ and $g(\omega_1,\omega_2)$ are joint spectral amplitudes of two photons represented by label 1 and 2.

In this chapter we have revised all the basic concepts and preliminary knowledge of processor required for developing continuous mode model of long distance quantum communication. In next chapter we develop our theory for modeling of resources in long distance communication in context of continuous mode treatment.

Chapter 3

Continuous mode Analysis of Quantum Communication Tools

Sources used in quantum communication generally produce continuous mode fields. Parametric down conversion, for example, is such a process that produce continuous mode fields. This means that photons produced during PDC may not be identical and may have different frequency modes. Presence of different modes in signal makes it important to study and develop the continuous mode models of detectors in order to enables us to understand the behaviour of entangled photon states which are closer to practical sources used in different quantum communication processes like quantum teleportation.

In this chapter we give continuous mode treatment of different tools used in quantum teleportation. In section 3.1 we develop $|\psi^+\rangle$ state obtained from PDC. In section 3.2 we convert the $|\psi^+\rangle$ in continuous mode frequency representation with entanglement in polarization degree of freedom. In section 3.3 we develop continuous mode projection operator for a detector which has variable efficiency depending upon the mode of input photon state during measurement process. In section 3.4 we have verified the entanglement nature in continuous mode photon state. In section 3.5 we give the result of two fold coincidence for entanglement verification. In section 3.6 we calculate three fold coincidence for quantum teleportation. In section 3.7 we give results for three fold coincidence probability.

3.1 Practical Photon Sources

Bell states $|\psi^{\pm}\rangle$ and $|\phi^{\pm}\rangle$ are four photon entangled states exhibiting entanglement between members of single pair. Sources used to produce entanglement through parametric down conversion, however, do not produce single pair on demand but there is probability that many photons pairs are created. We have used parametric down conversion type II for obtaining $|\psi^{+}\rangle$ state. Parametric down conversion is described with the help of Lie algebra $\mathrm{SU}(1,1)$. Analysis for $|\psi^{-}\rangle$ has already been done and here we develop $|\psi^{+}\rangle$ using $\mathrm{SU}(1,1)$ algebra. PDC involves a non linear crystal with second order non linearity (χ^2) which is pumped by a coherent field and each pump photon spontaneously decays into two photons (a pair is generated). If the two photons produced during down conversion are identical then we call it "degenerate down conversion" and if photons are non identical then it is called "nondegenerate PDC". SU(1,1) algebra is described by set of operators $\hat{K}_x, \hat{K}_y, \hat{K}_z$ and these operators follow following commutation relations

$$[\hat{K}_x, \hat{K}_y] = -\iota\hat{K}_z. \quad [\hat{K}_y, \hat{K}_z] = \iota\hat{K}_x. \quad [\hat{K}_z, \hat{K}_x] = -\iota\hat{K}_y. \tag{3.1.1}$$

For degenerate PDC, where identical photons are produced these generators have the form

$$\begin{aligned} \hat{N}_x^{(j)} &= \frac{1}{4}(\hat{c}_j^{\dagger}\hat{c}_j^{\dagger} + \hat{c}_j\hat{c}_j). \\ \hat{N}_y^{(j)} &= \frac{1}{4\iota}(\hat{c}_j^{\dagger}\hat{c}_j^{\dagger} - \hat{c}_j\hat{c}_j). \\ \hat{N}_z^{(j)} &= \frac{1}{4}(\hat{c}_j^{\dagger}\hat{c}_j + \hat{c}_j\hat{c}_j^{\dagger}). \end{aligned} \tag{3.1.2}$$

For non degenerate PDC, where non identical photons are produced these generators take the form

$$\begin{aligned} \hat{N}_x^{(ij)} &= \frac{1}{2}(\hat{c}_i^{\dagger}\hat{c}_j^{\dagger} + \hat{c}_i\hat{c}_j). \\ \hat{N}_y^{(ij)} &= \frac{1}{2\iota}(\hat{c}_i^{\dagger}\hat{c}_j^{\dagger} - \hat{c}_i\hat{c}_j). \\ \hat{N}_z^{(ij)} &= \frac{1}{2}(\hat{c}_i^{\dagger}\hat{c}_i + \hat{c}_j\hat{c}_j^{\dagger}). \end{aligned} \tag{3.1.3}$$

Pair creation rate in PDC depends upon the non linearity (χ^2) of crystal, strength of pump field and interaction time. State obtained as a result of down conversion does not simply represent a photon pair but it is a superposition of vacuum and photon pairs from single pair to higher number of pairs. Quantum state obtained from PDC can be written as

$$\left|\psi^+\right\rangle = \exp\left[\iota\chi(\hat{a}_{\mathrm{H}}^{\dagger}\hat{b}_{\mathrm{V}}^{\dagger} + \hat{a}_{\mathrm{H}}\hat{b}_{\mathrm{V}})\right] \otimes \exp\left[\iota\chi(\hat{a}_{\mathrm{V}}^{\dagger}\hat{b}_{\mathrm{H}}^{\dagger} + \hat{a}_{\mathrm{V}}\hat{b}_{\mathrm{H}})\right]\left|vac\right\rangle. \tag{3.1.4}$$

This state is not in normal ordered form. In quantum optics usually we work with normal ordered states. Advantage of normal ordered form is that we can easily see if a term will give zero when applied to vacuum state[28], so we will write this state obtained by PDC in normal ordered state. We accomplish normal ordering by introducing operators K_+, K_-, K_0. These operators are defined as (for first term in Eq 3.1.4)

$$\begin{aligned} K_+ &= \hat{a}_{\mathrm{H}}^{\dagger}\hat{b}_{\mathrm{V}}^{\dagger}. \\ K_- &= \hat{a}_{\mathrm{H}}\hat{b}_{\mathrm{V}}. \\ K_0 &= \frac{1}{2}(\hat{a}_{\mathrm{H}}^{\dagger}\hat{a}_{\mathrm{H}} + \hat{b}_{\mathrm{V}}^{\dagger}\hat{b}_{\mathrm{V}} + 1). \end{aligned} \tag{3.1.5}$$

Normal ordering formula is

$$\exp\left[\alpha_+K_+ + \alpha_0K_0 + \alpha_-K_-\right] = \exp\left[A_+K_+\right]\exp\left[lnA_0K_0\right]\exp\left[A_-K_-\right]. \tag{3.1.6}$$

By looking at equation (3.1.4), we can see that

$$\alpha_\pm = \iota\chi. \qquad \alpha_0 = 0. \tag{3.1.7}$$

And further

$$\begin{aligned} A_\pm &= \frac{(\frac{\alpha_\pm}{\theta})\sinh\theta}{\cosh\theta - (\frac{\alpha_0}{2\theta})\sinh\theta}, \\ A_0 &= (\cosh\theta - (\frac{\alpha_0}{2\theta})\sinh\theta)^{-2}, \end{aligned} \tag{3.1.8}$$

where θ is given as

$$\theta = ((\alpha_0/2)^2 - \alpha_+\alpha_-)^{\frac{1}{2}}. \tag{3.1.9}$$

Since $\alpha_0 = 0$ in our case, so

$$\theta = \chi. \tag{3.1.10}$$

Thus we will have

$$\begin{aligned} A_+ &= \iota \tanh \chi. \\ A_- &= \iota \tanh \chi. \\ A_0 &= (\cosh \chi)^{-2}. \end{aligned} \tag{3.1.11}$$

Now we can write

$$\exp[\iota\chi(\hat{a}_{\mathrm{H}}^{\dagger}\hat{b}_{\mathrm{V}}^{\dagger} + \hat{a}_{\mathrm{H}}\hat{b}_{\mathrm{V}})] = \exp[\iota \tanh \chi \hat{a}_{\mathrm{H}}^{\dagger}\hat{b}_{\mathrm{V}}^{\dagger}] \exp[ln[(\cosh\chi)^{-2}]\frac{1}{2}(\hat{a}_{\mathrm{H}}^{\dagger}\hat{a}_{\mathrm{H}} + \hat{b}_{\mathrm{V}}^{\dagger}\hat{b}_{\mathrm{V}} + 1)] \\ \exp[\iota \tanh \chi \hat{a}_{\mathrm{H}}\hat{b}_{\mathrm{V}}]. \tag{3.1.12}$$

To make equation simple let us assume

$$\begin{aligned} \iota \tanh \chi &= \phi(\chi). \\ -ln[\cosh \chi^{-2}] &= \rho(\chi). \end{aligned} \tag{3.1.13}$$

Thus first term in equation (3.1.4) can be written as

$$\exp[\iota\chi(\hat{a}_{\mathrm{H}}^{\dagger}\hat{b}_{\mathrm{V}}^{\dagger} + \hat{a}_{\mathrm{H}}\hat{b}_{\mathrm{V}})] = \exp[\phi(\chi)\hat{a}_{\mathrm{H}}^{\dagger}\hat{b}_{\mathrm{V}}^{\dagger}] \exp[\rho(\chi)\frac{1}{2}(\hat{a}_{\mathrm{H}}^{\dagger}\hat{a}_{\mathrm{H}} + \hat{b}_{\mathrm{V}}^{\dagger}\hat{b}_{\mathrm{V}} + 1)] \\ \exp[\phi(\chi)\hat{a}_{\mathrm{H}}\hat{b}_{\mathrm{V}}] |vac\rangle . \tag{3.1.14}$$

For second term in equation (3.1.4) we can write generators as

$$\begin{aligned} K_+ &= \hat{a}_{\mathrm{V}}^{\dagger}\hat{b}_{\mathrm{H}}^{\dagger}. \\ K_- &= \hat{a}_{\mathrm{V}}\hat{b}_{\mathrm{H}}. \\ K_0 &= \frac{1}{2}(\hat{a}_{\mathrm{V}}^{\dagger}\hat{a}_{\mathrm{V}} + \hat{b}_{\mathrm{H}}^{\dagger}\hat{b}_{\mathrm{H}} + 1). \end{aligned} \tag{3.1.15}$$

By following similar steps as done before, we get

$$\exp[\iota\chi(\hat{a}_{\mathrm{V}}^{\dagger}\hat{b}_{\mathrm{H}}^{\dagger} + \hat{a}_{\mathrm{V}}\hat{b}_{\mathrm{H}})] = \exp[\phi(\chi)\hat{a}_{\mathrm{V}}^{\dagger}\hat{b}_{\mathrm{H}}^{\dagger}] \exp[\rho(\chi)\frac{1}{2}(\hat{a}_{\mathrm{V}}^{\dagger}\hat{a}_{\mathrm{V}} + \hat{b}_{\mathrm{H}}^{\dagger}\hat{b}_{\mathrm{H}} + 1)] \\ \exp[\phi(\chi)\hat{a}_{\mathrm{V}}\hat{b}_{\mathrm{H}}] |vac\rangle , \tag{3.1.16}$$

So normal ordered state produced by parametric down conversion can be written as

$$|\psi_+\rangle = \exp[\phi(\chi)(\hat{a}_{\mathrm{H}}^{\dagger}\hat{b}_{\mathrm{V}}^{\dagger} + \hat{a}_{\mathrm{V}}^{\dagger}\hat{b}_{\mathrm{H}}^{\dagger})] \exp[\rho(\chi)] \\ \exp[\frac{1}{2}\rho(\chi)(\hat{a}_{\mathrm{H}}^{\dagger}\hat{a}_{\mathrm{H}} + \hat{b}_{\mathrm{V}}^{\dagger}\hat{b}_{\mathrm{V}} + \hat{a}_{\mathrm{V}}^{\dagger}\hat{a}_{\mathrm{V}} + \hat{b}_{\mathrm{H}}^{\dagger}\hat{b}_{\mathrm{H}})] \exp[\phi(\chi)(\hat{a}_{\mathrm{H}}\hat{b}_{\mathrm{V}} + \hat{a}_{\mathrm{V}}\hat{b}_{\mathrm{H}})] |vac\rangle . \tag{3.1.17}$$

As last two terms do not have any effect on vacuum so we are left with

$$|\psi_{+}\rangle = \exp\left[\rho(\chi)\right]\exp\left[\phi(\chi)(\hat{a}^{\dagger}_{\mathrm{H}}\hat{b}^{\dagger}_{\mathrm{V}} + \hat{a}^{\dagger}_{\mathrm{V}}\hat{b}^{\dagger}_{\mathrm{H}})\right]|vac\rangle . \quad (3.1.18)$$

This is the state we obtain from PDC which also include vacuum along with photon number states.

3.2 Continuous mode Analysis of Entangled State

State obtained after normal ordering in previous section represents photons in single mode. As described previously we can convert a single mode photon state in continuous mode state in frequency representation according to Eq 2.6.1. Accordingly we can write $|\psi^{+}\rangle$ given in Eq 3.1.18 in continuous mode frequency representation as

$$\left|\psi^{+}\right\rangle = \exp\left[\rho(\chi)\right]\exp\left[\phi(\chi)(\int\int(f(\omega,\omega')\hat{a}^{\dagger}_{\mathrm{H}}(\omega)\hat{b}^{\dagger}_{\mathrm{V}}(\omega') + g(\omega,\omega')\hat{a}^{\dagger}_{\mathrm{V}}(\omega)\hat{b}^{\dagger}_{\mathrm{H}})d\omega d\omega')\right] |vac\rangle , \quad (3.2.1)$$

here $f(\omega,\omega')$ and $g(\omega,\omega')$ are joint spectral functions and integral is from $-\infty$ to ∞. We can also write it as

$$\left|\psi^{+}\right\rangle = \exp\left[\rho(\chi)\right]\sum_{n=0}^{\infty}\frac{(\phi(\chi))^{n}}{n!}\prod_{i=1}^{n}\int\int(f(\omega_i,\omega'_i)\hat{a}^{\dagger}_{\mathrm{H}}(\omega_i)\hat{b}^{\dagger}_{\mathrm{V}}(\omega'_i) + g(\omega_i,\omega'_i)\hat{a}^{\dagger}_{\mathrm{V}}(\omega_i)\hat{b}^{\dagger}_{\mathrm{H}}) d\omega'_i d\omega_i |vac\rangle . \quad (3.2.2)$$

After applying creation operator and writing state in horizontal and vertical polarization we get

$$\left|\psi^{+}\right\rangle = \exp\left[\rho(\chi)\right]\sum_{n=0}^{\infty}(\phi(\chi))^{n}\prod_{i=1}^{n}\int\int f(\omega_i,\omega'_i)\left|H_{\mathrm{a}}(\omega_i)\right\rangle\left|V_{\mathrm{b}}(\omega'_i)\right\rangle d\omega'_i d\omega_i + \int\int g(\omega_i,\omega'_i)\left|V_{\mathrm{a}}(\omega_i)\right\rangle\left|H_{\mathrm{b}}(\omega'_i)\right\rangle d\omega'_i d\omega_i . \quad (3.2.3)$$

This is the continuous mode entangled state obtained from PDC.

3.3 Continuous mode Detector Model

For Continuous mode input state having multiphoton effect, Projection operator for non-photon number discriminating detector is

$$\hat{\Pi}_{\text{click}} = \sum_{n=1}^{\infty} \prod_{i=1}^{n} \int_{-\infty}^{\infty} \varepsilon(\omega_i) \left| n(\omega_i) \right\rangle \left\langle n(\omega_i) \right| d\omega_i, \tag{3.3.1}$$

here $\varepsilon(\omega_i)$ represents the efficiency of detector. In terms of Horizontal and Vertical polarization we write it as

$$\begin{aligned} \hat{\Pi}_{\text{click}} = \quad & \sum_{n=1}^{\infty} \prod_{i=1}^{n} \int_{-\infty}^{\infty} \varepsilon_H(\omega_i) \left| H(\omega_i) \right\rangle \left\langle H(\omega_i) \right| d\omega_i \otimes \\ & \sum_{m=1}^{\infty} \prod_{j=1}^{m} \int_{-\infty}^{\infty} \varepsilon_V(\omega_j') \left| V(\omega_j') \right\rangle \left\langle V(\omega_j') \right| d\omega_j', \end{aligned} \tag{3.3.2}$$

3.4 Verification of Continuous mode State

We have converted $|\psi^+\rangle$ state into continuous mode frequency representation in Eq. (3.4.1). To check if this continuous mode conversion correctly represents the $|\psi^+\rangle$ state in continuous mode, we perform Bell state measurement. Entangled photons from PDC source are incident on beam splitter. We introduce time difference between the arrival times of two photons at beam splitter. This can be achieved by placing the beam splitter closer to one input. This will make possible for photons to arrive at beam splitter at different times. This time delay adds phase to input state. Input state can be written as

$$\begin{aligned} \left|\psi^+\right\rangle = \exp\left[\rho(\chi)\right] \sum_{n=0}^{\infty} (\phi(\chi))^n \prod_{i=1}^{n} \int \int f(\omega_i, \omega_i') \left| H_{\text{a}}(\omega_i) \right\rangle \left| V_{\text{b}}(\omega_i') \right\rangle \exp\left[\iota\omega_i t\right] d\omega_i' d\omega_i + \\ \int \int g(\omega_i, \omega_i') \left| V_{\text{a}}(\omega_i) \right\rangle \left| H_{\text{b}}(\omega_i') \right\rangle \exp\left[\iota\omega_i t\right] d\omega_i' d\omega_i. \end{aligned} \tag{3.4.1}$$

When photons from two inputs arrive at beam splitter, there are four possible outcomes. If we label photons in accordance with their spatial modes a and b while output spatial modes are labelled as c nd d, then after reaching at beam splitter both photons will either appear in output c with photon a being reflected and photon b being transmitted. Similarly if photon a is transmitted and photon b is reflected then both photons will end up in output d. If both photons are transmitted then each out put will have one photon. Similarly for situation in which both photons are reflected each output receives one photon. In order to verify $|\psi^+\rangle$ state, we place two detectors on either side of 50 : 50 beam splitter and check coincidence between two detectors(as shown in fig).

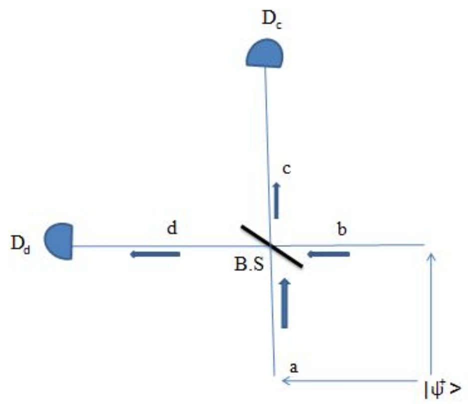

Figure 3.1: shows the setup used for verifying if the input state is $|\psi^+\rangle$ or not. If there is no time difference between photon arrival at beam splitter, both photons appear on same side of beam splitter which means we do not get any coincidence between two detectors D_{c} and D_{d}. This confirms that input state is $|\psi^+\rangle$.

At zero time difference between photons arrival at beam splitter, we do not get coincidence between detectors. With increasing time difference between photons, probability of coincidence increases thus we well have a dip at zero time delay.

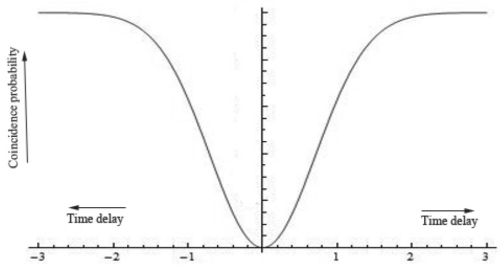

Figure 3.2: shows the expected behaviour of $|\psi^+\rangle$ state. At zero time difference coincidence is between detector placed on either side of beam splitter is zero. As the time difference increases coincidence of detector increases and then become constant thus we get a dip at zero time difference.

After beam splitter action continuous mode $|\psi^+\rangle$ state obtained from PDC is

$$|\psi\rangle = \frac{1}{2}(\exp[\rho(\chi)]\sum_{n=0}^{\infty}(\phi(\chi))^n\prod_{i=1}^{n}(\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\exp[\iota\omega_i t](f(\omega_i,\omega_i')(|H_{\mathrm{b}}(\omega_i)V_{\mathrm{b}}(\omega_i')\rangle - \iota\,|H_{\mathrm{b}}(\omega)V_{\mathrm{a}}(\omega_i')\rangle + \iota\,|H_{\mathrm{a}}(\omega_i)V_{\mathrm{b}}(\omega_i')\rangle + |H_{\mathrm{a}}(\omega_i)V_{\mathrm{a}}(\omega_i')\rangle) + g(\omega_i,\omega_i') (|V_{\mathrm{b}}(\omega_i)H_{\mathrm{b}}(\omega_i')\rangle - \iota\,|V_{\mathrm{b}}(\omega_i)H_{\mathrm{a}}(\omega')\rangle + \iota\,|V_{\mathrm{a}}(\omega_i)H_{\mathrm{a}}(\omega_i')\rangle + |V_{\mathrm{a}}(\omega_i)H_{\mathrm{a}}(\omega_i')\rangle))d\omega_i d\omega_i')). \tag{3.4.2}$$

Probability of coincidence is given by

$$P_{\mathrm{coinc}} = \langle\psi|\hat{\Pi}|\psi\rangle, \tag{3.4.3}$$

where $\hat{\Pi}$ is continuous mode detector, for which projection operator is written as

$$\hat{\Pi} = \sum_{m=1}^{\infty}\prod_{j=1}^{m}\int_{-\infty}^{\infty}\varepsilon_{\mathrm{H}}(\Omega_j)\,|H_{\mathrm{a}}(\Omega_j)\rangle\,\langle H_{\mathrm{a}}(\Omega_j)|\,d\Omega_j\otimes \sum_{m'=1}^{\infty}\prod_{k=1}^{m'}\int_{-\infty}^{\infty}\varepsilon_{\mathrm{V}}(\Omega_k')\,|V_{\mathrm{a}}(\Omega_k')\rangle\,\langle V_{\mathrm{a}}(\Omega_k')|\,d\Omega_k' \tag{3.4.4}$$

After applying projection operator and using the relation $\langle m(\omega_i)|n(\omega_j)\rangle =$

$\delta_{mn}\delta(\omega_i - \omega_j)$ we are left with

$$P_{\text{coinc}} = \frac{1}{4}\exp\left[4\rho(\chi)\right]\sum_{m=1}^{\infty}(\phi(\chi))^m\prod_{j=1}^{m}(\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(\varepsilon_{\text{H}}(\Omega_j)\varepsilon_{\text{V}}(\Omega'_j)|f(\Omega_j,\Omega'_j)|^2$$
$$+\varepsilon_{\text{H}}(\Omega_j)\varepsilon_{\text{V}}(\Omega'_j)f(\Omega_j,\Omega'_j)g^*(\Omega'_j,\Omega_j)\exp\left[-\iota\Omega_j t\right]\exp\left[\iota\Omega'_j t\right]+$$
$$\varepsilon_{\text{H}}(\Omega_j)\varepsilon_{\text{V}}(\Omega'_j)g(\Omega_j,\Omega'_j)f^*(\Omega'_j,\Omega_j)\exp\left[\iota\Omega_j t\right]\exp\left[-\iota\Omega'_j t\right]+$$
$$\varepsilon_{\text{H}}(\Omega_j)\varepsilon_{\text{V}}(\Omega'_j)|g(\Omega_j,\Omega'_j)|^2)d\Omega_j d\Omega'_j). \tag{3.4.5}$$

Now we find the coincidence between detectors H_{a} and V_{b}. Projection operator for this process is

$$\hat{\Pi} = \sum_{m=1}^{\infty}\prod_{j=1}^{m}\int_{-\infty}^{\infty}\varepsilon_{\text{H}}(\Omega_j)\left|H_{\text{a}}(\Omega_j)\right\rangle\left\langle H_{\text{a}}(\Omega_j)\right| d\Omega_j\otimes$$
$$\sum_{m'=1}^{\infty}\prod_{k=1}^{m'}\int_{-\infty}^{\infty}\varepsilon_{\text{V}}(\Omega'_k)\left|V_{\text{b}}(\Omega'_k)\right\rangle\left\langle V_{\text{b}}(\Omega'_k)\right| d\Omega'_k. \tag{3.4.6}$$

Coincidence probability in this case will be

$$P_{\text{coinc}} = \frac{1}{4}\exp\left[4\rho(\chi)\right]\sum_{m=1}^{\infty}(\phi(\chi))^m\prod_{j=1}^{m}(\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(\varepsilon_{H}(\Omega_j)\varepsilon_{\text{V}}(\Omega'_j)|f(\Omega_j,\Omega'_j)|^2$$
$$+\varepsilon_{\text{H}}(\Omega_j)\varepsilon_{\text{V}}(\Omega'_j)f(\Omega_j,\Omega'_j)g^*(\Omega'_j,\Omega_j)\exp\left[-\iota\Omega_j t\right]\exp\left[\iota\Omega'_j t\right]+$$
$$\varepsilon_{\text{H}}(\Omega_j)\varepsilon_{\text{V}}(\Omega'_j)g(\Omega_j,\Omega'_j)f^*(\Omega'_j,\Omega_j)\exp\left[\iota\Omega_j t\right]\exp\left[-\iota\Omega'_j t\right]+$$
$$\varepsilon_{\text{H}}(\Omega_j)\varepsilon_{\text{V}}(\Omega'_j)|g(\Omega_j,\Omega'_j)|^2)d\Omega_j d\Omega'_j). \tag{3.4.7}$$

This behaviour given in Eq. (3.4.5) and Eq. (3.4.7) is similar to that of $|\psi^+\rangle$ Bell state hence we can say that during conversion from single mode to continuous mode nature of entanglement remained preserved.

3.5 Results

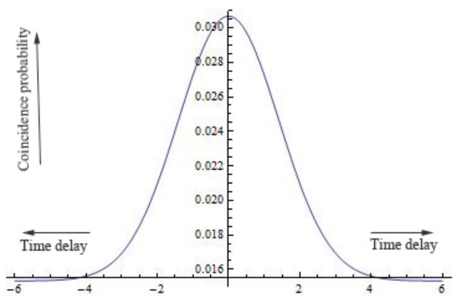

Figure 3.3: shows the variation in probability of coincidence of detectors H_a and V_a with time when Bell state $|\psi^+\rangle$ is incident.

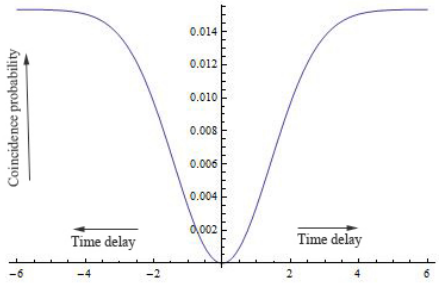

Figure 3.4: shows the variation in probability of coincidence of detectors H_a and V_b with time when Bell state $|\psi^+\rangle$ is incident.

We have used detectors with 70% efficiency having Gaussian distribution over frequency domain. In Fig (3.3) at zero time difference we have maximum probability for coincidence between detectors H_a and V_a. As the time difference increases probability of coincidence decreases and then become constant. In Fig (3.4) at zero time difference we have minimum probability for coincidence between detectors H_a and V_b. As the time difference increases probability of coincidence increases and then become constant. This behavior is similar to what we get for single mode entangled $|\psi^+\rangle$ state.

3.6 Teleportation and Three Fold Coincidence

Quantum teleportation is a process which involves transfer of a quantum state from one point to some other distant point in the absence of some communication channel connecting initial and final point. Let Alice and Bob created a pair of entangled photons in laboratory. Bob gets one photon from pair and take it with him to some distant point leaving one photon with Alice. None of the Alice or Bob know the state of photon they posses. Now Alice has to send an unknown state $|\phi\rangle$ to Bob. Alice will send this state $|\phi\rangle$ to Bob by making it to interact with the photon which Alice has obtained from entangled pair. Alice will perform Bell state measurement on her two qubits (photons) and will communicate with Bob through Classical channel to tell the result of her measurement. After knowing the result of Alice's measurement, Bob can recover the state $|\phi\rangle$ after performing appropriate operation.

In order to check if we have successfully teleported the required state we find three fold coincidence. This three fold coincidence involves three detectors D_{a}, D_{c} and D_{+}. Schematic diagram is shown in figure. Combined state of system including state to be teleported and state obtained from PDC source is

$$\begin{aligned}|\Psi\rangle = \exp[\rho(\chi)]\sum_{n=0}^{\infty}(\phi(\chi))^{n}\prod_{i=1}^{n}(\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(f(\omega_i,\omega_i')S(\omega'')\alpha\,|H_{\text{c}}(\omega'')H_{\text{a}}(\omega_i)V_{\text{b}}(\omega_i')\rangle \\ +g(\omega_i,\omega_i')S(\omega'')\alpha\,|H_{\text{c}}(\omega'')V_{\text{a}}(\omega_i)H_{\text{b}}(\omega_i')\rangle + f(\omega_i,\omega_i')S(\omega'')\beta\,|V_{\text{c}}(\omega'')H_{\text{a}}(\omega_i)V_{\text{b}}(\omega_i')\rangle \\ +g(\omega_i,\omega_i')S(\omega'')\beta\,|V_{\text{c}}(\omega'')V_{\text{a}}(\omega_i)H_{\text{b}}(\omega_i')\rangle)\exp[\iota\omega''t]d\omega d\omega_i' d\omega'',\end{aligned} \tag{3.6.1}$$

$\exp[\iota\omega''t]$ is the phase due to time difference introduced between photons of channel a and channel c in order to observe the variation in coincidence probability with change in time difference. After the interaction of photons in channel a and channel c

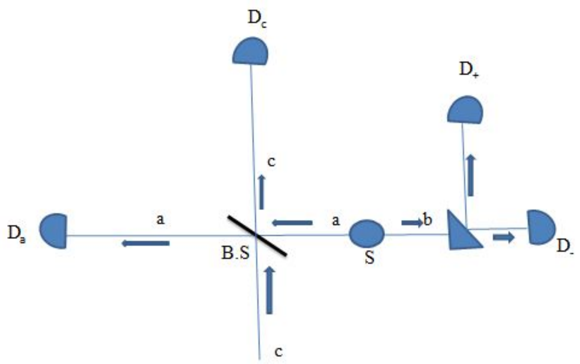

Figure 3.5: explains the measurement process for finding three fold coincidence. Click of D_{+} indicate successful teleportation. Click of D_{-} indicate the cases in which teleportation is not completed successfully.

at 50 : 50 beam splitter state becomes

$$\begin{aligned}|\Psi\rangle = \exp[2\rho(\chi)]\sum_{n=0}^{\infty}(\phi(\chi))^n\prod_{i=1}^{n}(\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\exp[\iota\omega''t](f(\omega_i,\omega_i')\\ S(\omega'')\alpha(\frac{|H_{\rm a}(\omega'')\rangle+\iota\,|H_{\rm c}(\omega'')\rangle}{\sqrt{2}}\frac{|H_{\rm a}(\omega_i)\rangle-\iota\,|H_{\rm c}(\omega_i)\rangle}{\sqrt{2}}\,|V_{\rm b}(\omega_i')\rangle\rangle\rangle)\\ +g(\omega_i,\omega_i')S(\omega'')\alpha(\frac{|H_{\rm a}(\omega'')\rangle+\iota\,|H_{\rm c}(\omega'')\rangle}{\sqrt{2}}\frac{|V_{\rm a}(\omega_i)\rangle-\iota\,|V_{\rm c}(\omega_i)\rangle}{\sqrt{2}}\,|H_{\rm b}(\omega_i')\rangle\rangle)\\ +(f(\omega_i,\omega_i')S(\omega'')\beta(\frac{|V_{\rm a}(\omega'')\rangle+\iota\,|V_{\rm c}(\omega'')\rangle}{\sqrt{2}}\frac{|H_{\rm a}(\omega_i)\rangle-\iota\,|H_{\rm c}(\omega_i)\rangle}{\sqrt{2}}\,|V_{\rm b}(\omega_i')\rangle\rangle\rangle)\\ +g(\omega_i,\omega_i')S(\omega'')\beta(\frac{|V_{\rm a}(\omega'')\rangle+\iota\,|V_{\rm c}(\omega'')\rangle}{\sqrt{2}}\frac{|V_{\rm a}(\omega_i)\rangle-\iota\,|V_{\rm c}(\omega_i)\rangle}{\sqrt{2}}\,|H_{\rm b}(\omega_i')\rangle\rangle)\\ d\omega_i d\omega_i' d\omega'').\end{aligned} \tag{3.6.2}$$

Mathematically Coincidence is found as

$$P_{\rm coin} = \langle\Psi|\hat{\Pi}|\Psi\rangle. \tag{3.6.3}$$

Projection operator for finding coincidence between $D_{\rm a}$, $D_{\rm c}$ and D_+ is

$$\begin{aligned}\hat{\Pi} =& \sum_{n=1}^{\infty}\prod_{i=1}^{n}\int_{-\infty}^{\infty}\varepsilon(\Omega_i)\,|n_{\rm a}(\Omega_i)\rangle\,\langle n_{\rm a}(\Omega_i)|\,d\Omega_i\\ &\otimes\sum_{m=1}^{\infty}\prod_{j=1}^{m}\int_{-\infty}^{\infty}\varepsilon(\Omega_j')\,|m_{\rm c}(\Omega_i')\rangle\,\langle m_{\rm c}(\Omega_i')|\,d\Omega_i'\\ &\otimes\sum_{l=1}^{\infty}\prod_{k=1}^{l}\int_{-\infty}^{\infty}\varepsilon(\Omega_l'')\,|+(\Omega_l'')\rangle\,\langle+(\Omega_l'')|\,d\Omega_l'',\end{aligned} \tag{3.6.4}$$

where $|n_{\rm a}(\Omega_i)\rangle\,\langle n_{\rm a}(\Omega_i)|$ and $|n_{\rm c}(\Omega_i')\rangle\,\langle n_{\rm c}(\Omega_i')|$ are defined in such a way that they detect photons in channel a and c with out distinction of horizontal and vertical polarization which means their measurement result does not depend upon photon polarization. Detector D_+ clicks if the teleported state is correct and identical to initial state.

Detectors $D_{\rm a}$ and $D_{\rm c}$ act as Bell state analyzers. When the state of incident photons is projected to $|\psi^+\rangle$ Bell state, teleported state will be identical to initial original state and hence D_+ will give a click. As $|\psi^+\rangle$ state does not give click at D_a and D_c at zero time delay between arrival of photon in channel a and channel c so result will be a dip in probability curve at zero time delay. As the time delay increases from zero, probability for coincidence between D_a and D_c increases and hence probability for coincidence of D_a, D_c and D_+ increases.

We can write $|H_\mathrm{b}(\omega_i')\rangle$ and $|V_\mathrm{b}(\omega_i')\rangle$ in terms of $\{|+\rangle, |-\rangle\}$ basis as follows

$$\begin{aligned} |H_\mathrm{b}(\omega_i')\rangle &= \frac{|+(\omega_i')\rangle + |-(\omega_i')\rangle}{\sqrt{2}}, \\ |V_\mathrm{b}(\omega_i')\rangle &= \frac{|+(\omega_i')\rangle - |-(\omega_i')\rangle}{\sqrt{2}}. \end{aligned} \tag{3.6.5}$$

Putting values of H_b and V_b and taking projection of state after beam splitter using operator in equation (3.6.4)

$$\begin{aligned} |\Psi\rangle' = &\frac{\exp[2\rho(\chi)]\sum_{n=1}^{\infty}(\phi(\chi))^n\prod_{i=1}^{n}(\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\exp[\iota\omega''t]}{2\sqrt{2}} \\ &(-\varepsilon(\omega_i)\varepsilon(\omega_i')\varepsilon(\Omega'')f(\omega_i,\omega_i')S(\omega'')\alpha\,|H_\mathrm{a}(\omega'')H_\mathrm{c}(\omega_i)+(\omega_i')\rangle + \\ &\varepsilon(\omega_i)\varepsilon(\omega_i')\varepsilon(\Omega'')f(\omega_i,\omega_i')S(\omega'')\alpha\,|H_c(\omega'')H_a(\omega_i)+(\omega_i')\rangle \\ &-\varepsilon(\omega_i)\varepsilon(\omega_i')\varepsilon(\Omega'')g(\omega_i,\omega_i')S(\omega'')\alpha\,|H_\mathrm{a}(\omega'')V_\mathrm{c}(\omega_i)+(\omega_i')\rangle + \\ &\varepsilon(\omega_i)\varepsilon(\omega_i')\varepsilon(\Omega'')g(\omega_i,\omega_i')S(\omega'')\alpha\,|H_\mathrm{c}(\omega'')V_\mathrm{a}(\omega_i)+(\omega_i')\rangle \\ &-\varepsilon(\omega_i)\varepsilon(\omega_i')\varepsilon(\Omega'')f(\omega_i,\omega_i')S(\omega'')\beta\,|V_\mathrm{a}(\omega'')H_\mathrm{c}(\omega_i)+(\omega_i')\rangle + \\ &\varepsilon(\omega_i)\varepsilon(\omega_i')\varepsilon(\Omega'')f(\omega_i,\omega_i')S(\omega'')\beta\,|V_\mathrm{c}(\omega'')H_\mathrm{a}(\omega_i)+(\omega_i')\rangle \\ &-\varepsilon(\omega_i)\varepsilon(\omega_i')\varepsilon(\Omega'')g(\omega_i,\omega_i')S(\omega'')\beta\,|V_\mathrm{a}(\omega'')V_\mathrm{c}(\omega_i)+(\omega_i')\rangle + \\ &\varepsilon(\omega_i)\varepsilon(\omega_i')\varepsilon(\Omega'')g(\omega_i,\omega_i')S(\omega'')\beta\,|V_\mathrm{c}(\omega'')V_\mathrm{a}(\omega_i)+(\omega_i')\rangle\,d\omega_i d\omega_i' d\omega''). \end{aligned} \tag{3.6.6}$$

This is the state in which only those terms have survived that will give us clicks on detectors D_a, D_c and D_+ giving confirmation of success of teleportation. During measurement detectors D_a and D_c detected the photons with out considering the polarization degree of freedom which means photons with horizontal and vertical polarization were treated equally.

Now probability of coincidence $\langle\Psi|\hat{\Pi}|\Psi\rangle$ is

$$P_{\text{coin}} = \frac{\exp\left[2\rho(\chi)\right]\overline{\exp\left[2\rho(\chi)\right]}\sum_{n=1}^{\infty}(\phi(\chi))^{n}(\overline{\phi(\chi)})^{n}\prod_{i=1}^{n}(\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}}{8}$$

$$(\varepsilon(\omega_i)\varepsilon(\omega_i')\varepsilon(\Omega'')|\alpha|^2|f(\omega_i,\omega_i')|^2|S(\omega'')|^2-$$

$$\varepsilon(\omega_i)\varepsilon(\omega_i')\varepsilon(\Omega'')|\alpha|^2 f(\omega_i,\omega_i')f^*(\omega'',\omega_i')S(\omega'')S^*(\omega_i)\exp\left[\iota\omega''t\right]\exp\left[-\iota\omega_i t\right]$$

$$+\,\varepsilon(\omega_i)\varepsilon(\omega_i')\varepsilon(\Omega'')|\alpha|^2|f(\omega_i,\omega_i')|^2|S(\omega'')|^2-$$

$$\varepsilon(\omega_i)\varepsilon(\omega_i')\varepsilon(\Omega'')|\alpha|^2 f(\omega_i,\omega_i')f^*(\omega'',\omega_i')S(\omega'')S^*(\omega_i)\exp\left[\iota\omega''t\right]\exp\left[-\iota\omega_i t\right]$$

$$+\,\varepsilon(\omega_i)\varepsilon(\omega_i')\varepsilon(\Omega'')|\alpha|^2|g(\omega_i,\omega_i')|^2|S(\omega'')|^2-$$

$$\varepsilon(\omega_i)\varepsilon(\omega_i')\varepsilon(\Omega'')\alpha^*\beta f(\omega_i,\omega_i')g^*(\omega'',\omega_i')S(\omega'')S^*(\omega_i)\exp\left[\iota\omega''t\right]\exp\left[-\iota\omega_i t\right]$$

$$+\,\varepsilon(\omega_i')\varepsilon(\Omega'')+|\alpha|^2|g(\omega_i,\omega_i')|^2|S(\omega'')|^2-$$

$$\varepsilon(\omega_i)\varepsilon(\omega_i')\varepsilon(\Omega'')\alpha^*\beta f(\omega_i,\omega_i')g^*(\omega'',\omega_i')S(\omega'')S^*(\omega_i)\exp\left[\iota\omega''t\right]\exp\left[-\iota\omega_i t\right]$$

$$+\,\varepsilon(\omega_i)\varepsilon(\omega_i')\varepsilon(\Omega'')|\beta|^2|f(\omega_i,\omega_i')|^2|S(\omega'')|^2-$$

$$\varepsilon(\omega_i)\varepsilon(\omega_i')\varepsilon(\Omega'')\beta^*\alpha g(\omega_i,\omega_i')f^*(\omega'',\omega_i')S(\omega'')S^*(\omega_i)\exp\left[\iota\omega''t\right]\exp\left[-\iota\omega_i t\right]$$

$$+\,\varepsilon(\omega_i)\varepsilon(\omega_i')\varepsilon(\Omega'')|\beta|^2|f(\omega_i,\omega_i')|^2|S(\omega'')|^2-$$

$$\varepsilon(\omega_i)\varepsilon(\omega_i')\varepsilon(\Omega'')\beta^*\alpha g(\omega_i,\omega_i')f^*(\omega'',\omega_i')S(\omega'')S^*(\omega_i)\exp\left[\iota\omega''t\right]\exp\left[-\iota\omega_i t\right]$$

$$+\,\varepsilon(\omega_i)\varepsilon(\omega_i')\varepsilon(\Omega'')|\beta|^2|g(\omega_i,\omega_i')|^2|S(\omega'')|^2-$$

$$\varepsilon(\omega_i)\varepsilon(\omega_i')\varepsilon(\Omega'')|\beta|^2 g(\omega_i,\omega_i')g^*(\omega'',\omega_i')S(\omega'')S^*(\omega_i)\exp\left[\iota\omega''t\right]\exp\left[-\iota\omega_i t\right]$$

$$+\,\varepsilon(\omega_i)\varepsilon(\omega_i')\varepsilon(\Omega'')|\beta|^2|g(\omega_i,\omega_i')|^2|S(\omega'')|^2-$$

$$\varepsilon(\omega_i)\varepsilon(\omega_i')\varepsilon(\Omega'')|\beta|^2 g(\omega_i,\omega_i')g^*(\omega'',\omega_i')S(\omega'')S^*(\omega_i)\exp\left[\iota\omega''t\right]\exp\left[-\iota\omega_i t\right]$$

$$d\omega_i d\omega_i' d\omega''). \quad (3.6.7)$$

Incident state will be projected on $|\psi^+\rangle$ in only 25% of cases. In 75% of cases incident state will be projected on one of the $|\psi^-\rangle$, $|\phi^+\rangle$, $|\phi^-\rangle$ states. When the incident state is projected on $|\phi^+\rangle$ or $|\phi^-\rangle$, neither D_+ nor D_- clicks. Thus D_- will click only in 25% percent cases. At zero time delay D_a, D_c and D_- will click if projected state is $|\psi^-\rangle$. When photon in channel "a" and "c" have time delay then we have four possible outcomes for D_{a} and D_{c} while D_- click as shown in fig. (2.3). We can see that probability for coincidence of D_-, D_{a} and D_{c} is 25%. Hence we expect a straight line for coincidence probability of D_-, D_{a} and D_{c}.

Coincidence among $D_{\rm a}$,$D_{\rm c}$ and D_{-} is

$$P_{\rm coin} = \frac{\exp{[2\rho(\chi)]}\overline{\exp{[2\rho(\chi)]}}\sum_{n=1}^{\infty}(\phi(\chi))^n(\phi(\chi))^n\prod_{i=1}^{n}(\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}}{8}$$

$$(\varepsilon(\omega_i)\varepsilon(\omega_i')\varepsilon(\Omega'')|\alpha|^2|f(\omega_i,\omega_i')|^2|S(\omega'')|^2-$$

$$\varepsilon(\omega_i)\varepsilon(\omega_i')\varepsilon(\Omega'')\alpha|^2 f(\omega_i,\omega_i')f^*(\omega'',\omega_i')S(\omega'')S^*(\omega_i)\exp{[\iota\omega''t]}\exp{[-\iota\omega_i t]}$$

$$+\,\varepsilon(\omega_i)\varepsilon(\omega_i')\varepsilon(\Omega'')|\alpha|^2|f(\omega_i,\omega_i')|^2|S(\omega'')|^2-$$

$$\varepsilon(\omega_i)\varepsilon(\omega_i')\varepsilon(\Omega'')|\alpha|^2 f(\omega_i,\omega_i')f^*(\omega'',\omega_i')S(\omega'')S^*(\omega_i)\exp{[\iota\omega''t]}\exp{[-\iota\omega_i t]}$$

$$+\,\varepsilon(\omega_i)\varepsilon(\omega_i')\varepsilon(\Omega'')|\alpha|^2|g(\omega_i,\omega_i')|^2|S(\omega'')|^2+$$

$$\varepsilon(\omega_i)\varepsilon(\omega_i')\varepsilon(\Omega'')\alpha^*\beta f(\omega_i,\omega_i')g^*(\omega'',\omega_i')S(\omega'')S^*(\omega_i)\exp{[\iota\omega''t]}\exp{[-\iota\omega_i t]}$$

$$+\,\varepsilon(\omega_i)\varepsilon(\omega_i')\varepsilon(\Omega'')|\alpha|^2|g(\omega_i,\omega_i')|^2|S(\omega'')|^2+$$

$$\varepsilon(\omega_i)\varepsilon(\omega_i')\varepsilon(\Omega'')\alpha^*\beta f(\omega_i,\omega_i')g^*(\omega'',\omega_i')S(\omega'')S^*(\omega_i)\exp{[\iota\omega''t]}\exp{[-\iota\omega_i t]}$$

$$+\,\varepsilon(\omega_i)\varepsilon(\omega_i')\varepsilon(\Omega'')|\beta|^2|f(\omega_i,\omega_i')|^2|S(\omega'')|^2+$$

$$\varepsilon(\omega_i)\varepsilon(\omega_i')\varepsilon(\Omega'')\beta^*\alpha g(\omega_i,\omega_i')f^*(\omega'',\omega_i')S(\omega'')S^*(\omega_i)\exp{[\iota\omega''t]}\exp{[-\iota\omega_i t]}$$

$$+\,\varepsilon(\omega_i)\varepsilon(\omega_i')\varepsilon(\Omega'')|\beta|^2|f(\omega_i,\omega_i')|^2|S(\omega'')|^2+$$

$$\varepsilon(\omega_i)\varepsilon(\omega_i')\varepsilon(\Omega'')\beta^*\alpha g(\omega_i,\omega_i')f^*(\omega'',\omega_i')S(\omega'')S^*(\omega_i)\exp{[\iota\omega''t]}\exp{[-\iota\omega_i t]}$$

$$+\,\varepsilon(\omega_i)\varepsilon(\omega_i')\varepsilon(\Omega'')|\beta|^2|g(\omega_i,\omega_i')|^2|S(\omega'')|^2-$$

$$\varepsilon(\omega_i)\varepsilon(\omega_i')\varepsilon(\Omega'')|\beta|^2 g(\omega_i,\omega_i')g^*(\omega'',\omega_i')S(\omega'')S^*(\omega_i)\exp{[\iota\omega''t]}\exp{[-\iota\omega_i t]}$$

$$\varepsilon(\omega_i)\varepsilon(\omega_i')\varepsilon(\Omega'')|\beta|^2|g(\omega_i,\omega_i')|^2|S(\omega'')|^2-$$

$$\varepsilon(\omega_i)\varepsilon(\omega_i')\varepsilon(\Omega'')|\beta|^2 g(\omega_i,\omega_i')g^*(\omega'',\omega_i')S(\omega'')S^*(\omega_i)\exp{[\iota\omega''t]}\exp{[-\iota\omega_i t]}$$

$$d\omega_i d\omega_i' d\omega''). \quad (3.6.8)$$

This is the coincidence probability of $D_{\rm a}$, $D_{\rm c}$ and D_{-}.

3.7 Results

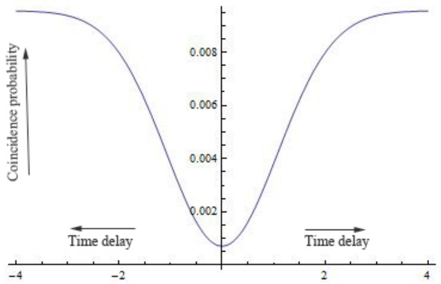

Figure 3.6: shows the variation in probability of coincidence of detectors D_{a}, D_{c} and D_{+} with time.

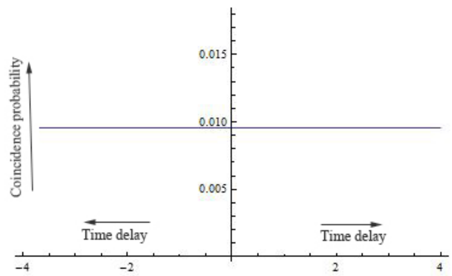

Figure 3.7: shows the variation in probability of coincidence of detectors D_{a}, D_{c} and D_{-} with time.

Fig (3.6) shows the variation in probability of coincidence of detectors D_{a}, D_{c} and D_{+}. As the time delay between two photons increases, coincidence probability also increases and then become constant. In Fig (3.7) we see a straight line for coincidence of detectors D_{a}, D_{c} and D_{-}. In this case coincidence probability does not change with time. These results are in agreement with experimental results [29]. Thus we have successfully developed continuous mode quantum teleportation setup with practical sources.

Chapter 4

Conclusion

Quantum communication processes involve processes like entanglement which do not have any classical counter part. Entanglement is co-relation that may exist between two quantum objects in such a way that quantum state of one objects affects the state of other object. Photons, atoms and electrons are the examples which can be classified as quantum objects. Parametric down conversion is a process which can provide us with entangled pair of photons. Parametric down conversion causes an incident photon to split in to two photons of lower frequencies. In practical set up incident field may have photons with different frequencies. Thus parametric down conversion leads to entangled photons having frequencies over a range of frequency spectrum. The presence of different modes of frequency makes it necessary to study continuous mode effects in photon sources in order to give complete description of real world quantum communication processes.

In this thesis we have reviewed the continuous mode treatment for photon sources that produce the entangled pair of photons. We developed the $|\psi^+\rangle$. We have extended the continuous mode analysis of Bell's state and verified the behaviour of our developed $|\psi^+\rangle$ state using linear optical setup. We incorporated already developed continuous mode representation of detector models and developed model for imperfect detectors. In our model we have incorporated the effects of limited efficiency. Our model of continuous mode photon state obtained from PDC and detectors successfully showed that the behaviour of continuous mode Bell's state is consistant with single mode

Bell's state. We performed continuous mode teleportation and calculated three fold coincidence in order to check success of teleportation.

This continuous mode treatment leads us to frame work of quantum memories which is our next goal.

Bibliography

[1] Masahito Hayashi. *Quantum Information.* Springer-Verlag Berlin Heidelberg, 1 edition, 2006.

[2] Dan C. Marinescu. *Classical and Quantum Information.* 1 edition, February 2011.

[3] Mark M. Wilde. *Quantum Information Theory.* cambridge University Press, 2013.

[4] Christopher G. Timpson. *Quantum Information Theory and the Foundations of Quantum Mechanics.* Oxford University Press, 2012.

[5] Emmanuel Desurvire. *Classical and Quantum Information Theory: An Introduction for the Telecom Scientist.* Cambridge University Press, 2009.

[6] Michael A. Nielsen and Isaac L. Chuang. *Quantum Computation and Quantum Information.* Cambridge University Press, Octuber 2000.

[7] Michel Le Bellac. *Quantum Physics.* Cambridge University Press, 2011.

[8] Jacob Dunningham and Vlatko Vedral. *Introductory Quantum Physics and Relativity.* Imperial College Press, December 2010.

[9] Jaeger Gregg. *Entanglement, Information, and the Interpretation of Quantum Mechanics.* Springer-Verlag Berlin Heidelberg, 2009.

[10] A. Einstein, B. Podolsky, and N. Rosen. Can quantum-mechanical description of physical reality be considered complete? *Phys. Rev.*, 47:777–780, May 1935.

[11] Alain Aspect, Jean Dalibard, and Gérard Roger. Experimental test of Bell's inequalities using time- varying analyzers. *Phys. Rev. Lett.*, 49:1804–1807, Dec 1982.

[12] Masahito Hayashi, Satoshi Ishizaka, Akinori Kawachi, Gen Kimura, and Tomohiro Ogawa. *Introduction to Quantum Information Science.* Springer-Verlag Berlin Heidelberg, 2015.

[13] Maximilian A. Schlosshauer. *Decoherence and the Quantum-To-Classical Transition.* Springer-Verlag Berlin Heidelberg, 2007.

[14] Ingemar Bengtsson and Karol Zyczkowski. *Geometry of Quantum States An Introduction to Quantum Entanglement.* Cambridge University Press, 2007.

[15] Charles H. Bennett, Gilles Brassard, Claude Crépeau, Richard Jozsa, Asher Peres, and William K. Wootters. Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels. *Phys. Rev. Lett.*, 70:1895–1899, Mar 1993.

[16] Harry Paul. *Introduction to Quantum Optics: From Light Quanta to Quantum Teleportation.* Cambridge University Press, 1 edition, June 2004.

[17] Anirban Pathak. *Elements of Quantum Computation and Quantum Communication.* CRC Press, 2013.

[18] Anton Zeilinger. *Dance of the Photons: From Einstein to Quantum Teleportation.* Farrar, Straus and Giroux, 2010.

[19] Furusawa Akira and Loock Peter van. *Quantum Teleportation and Entanglement: A Hybrid Approach to Optical Quantum Information Processing.* Willey publishers, 2011.

[20] Nouredine Zettili. *Quantum Mechanics Concepts and Applications.* Willey publishers, second edition, 2009.

[21] Peter Lambropoulos and David Petrosyan. *Fundamentals of Quantum Optics and Quantum Information*. Springer-Verlag Berlin Heidelberg, 2007.

[22] GordonW. F. Drake, editor. *Handbook of Atomic, Molecular, and Optical Physics*. Springer; 2nd edition, 2005.

[23] Zhe-Yu Jeff Ou. *Multi-Photon Quantum Interference*. Springer US, 2007.

[24] Gernot Alber, Thomas Beth, Michal Horodecki, Pawel Horodecki, Ryszard Horodecki, Martin Rtteler, Harald Weinfurter, Reinhard Werner, and Anton Zeilinger. *Quantum Information*. Springer Tracts in Modern Physics. Springer-Verlag Berlin Heidelberg, 1 edition, 2001.

[25] Christopher Gerry and Peter Knight. *Introductory Quantum Optics*. The University of Cambridge, 2004.

[26] S. P. Walborn, W. A. T. Nogueira, S. Pdua, and C. H. Monken. Optical bell-state analysis in the coincidence basis. *EPL (Europhysics Letters)*, 62(2):161, 2003.

[27] Peter P. Rohde and Timothy C. Ralph. Frequency and temporal effects in linear optical quantum computing. *Phys. Rev. A*, 71:032320, Mar 2005.

[28] Howard M. Wiseman and Gerard J. Milburn. *Quantum Measurement and Control*. Cambridge University Press, 2014.

[29] Dik Bouwmeester, Jian-Wei Pan, Klaus Mattle, Manfred Eibl, Harald Weinfurter, and Anton Zeilinger. Experimental quantum teleportation. *Nature*, 390:575–579, 1997.