Design and Fabrication of Limited Slip Differential for Formula Student race car

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In Partial Fulfillment
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Bachelors in Mechanical Engineering

by

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EXAMINATION COMMITTEE

We hereby recommend that the dissertation prepared under our supervision by: Ali Ghaffar.Titled: "Design and Fabrication of Limited Slip Differential for Formula Student race car" be accepted in partial fulfillment of the requirements for the award of Bachelors in Mechanical Engineering degree with (___ grade).

Committee Chair: Dr. ShahidIkramullah Butt Affiliation	____________________ Dated:
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Abstract

Without a limited slip the acceleration of a racing car be affected if one of the tires is in the air or comes into contact with a slippery surface. In this paper different designs for a converting the open differential of the Daihatsu Mira into a limited slip differential were considered. The amount of torque transferred was calculated using various formulas. Due to the limited space of the differential housing available and material and manufacturing limitations the breakaway torque could only be increased to a certain extent. The dimensions of the different parts required to convert the open differential into a limited slip are given in the report for manufacturing purposes.

Preface

Formula Student is an engineering competition held annually in the UK.Student teams from different universities around world. Each team designs, builds, tests, and races a small-scale formula style racing car. The cars are judged on a number of criteria as listed below. It is run by the Institution of Mechanical Engineers and uses the same rules as the original Formula SAE with supplementary regulations.

Judging:

The cars are judged by industry specialists on the following criteria:

Static events:

1. Engineering Design
2. Cost & Sustainability
3. Business Presentation
4. Technical Inspection

Dynamic Events:

1. Skid pan
2. Autocross/Sprint
3. Acceleration
4. Endurance
5. Fuel economy

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Originality report

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Introduction

The project was to design a limited slip differential for Formula Student Race Car. The aim of the project was to design a limited slip differential that would fit in the differential housing already available. There were certain set parameters that were to be followed while designing the limited slip differential. FEM analyses were performed after the initial calculations and CAD drawings, to ensure that the design was feasible. In this report, calculations are shown which gave reasonable results, as shown by the FEM analyses.

Literature review

Different websites and books were reviewed in order to gain an understanding of the limited slip differential and to perform calculations.

Some of the books reviewed were:

Shigley's Mechanical Engineering Design (9th edition, Budynas and Nisbett)

STANDARD HANDBOOK OF MACHINE DESIGN by Joseph Shingley

A Textbook of Machine Design (Khurmi and Gupta)

DIFFERENTIAL:

A differential is a mechanical device used in automobiles. In a vehicle, the differential usually consists of a set of gears that allows each of the driving wheels to rotate. The gears transmit the rotating motion of the driveshaft or drive train to the half shafts and split power to each of the driving axle shafts. Front wheel drive cars have differential between the front wheels (Fig 2). Rear wheel drive cars have a differential between the rear wheels (Fig 3). Part-time four-wheel-drive systems do not have a differential between the front and rear wheels; instead of this, they are locked together which makes the front and rear wheels turn at the same average speed. This is the reason why such vehicles are difficult to turn on concrete after the four-wheel-drive system is engaged. All-wheel drive (AWD) cars have three differentials, one on the front axle, one on the rear and one in the center (Fig.4). In 4 wheel drive (4WD) vehicles there are two differentials, one in the rear axle and one in the front axle (Fig.5). The purpose of a differential is to allow the wheels of vehicle to spin at different speeds as well as to allow the transfer of torque to the two wheels. When a car is turning a corner its outer wheels need to turn at a larger radius (Fig.1) and a larger distance as compared to the inner wheels. This requires then to turn at a faster rate in order to keep up with the inner tires. If the tires were connected by a simple solid shaft and not a differential, then the outer tires would be forced to rotate at the same speed as the inner tires because they are connected directly by the shaft. This would cause the outer tires to get dragged along which damages them and affects performance. When a differential is used it allows the outer tire to turn faster and the inner tires to turn slower.

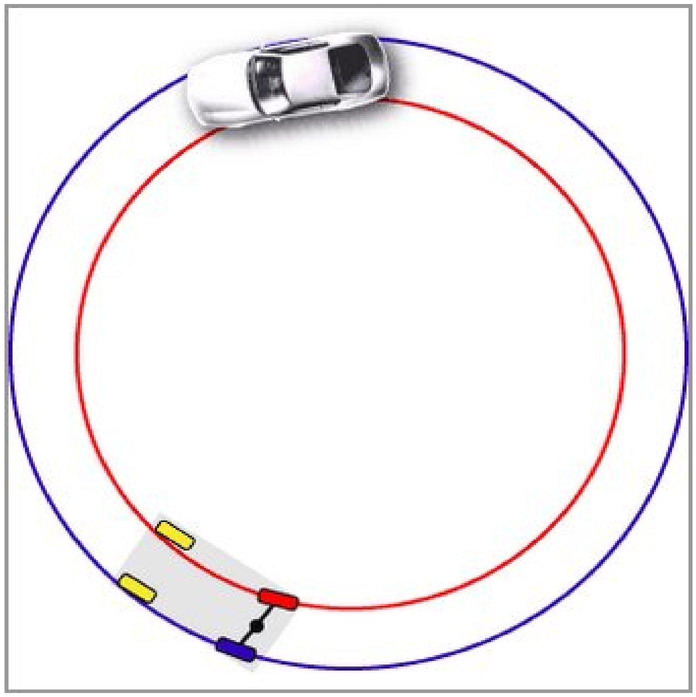


Figure 1 Car turning in a circle

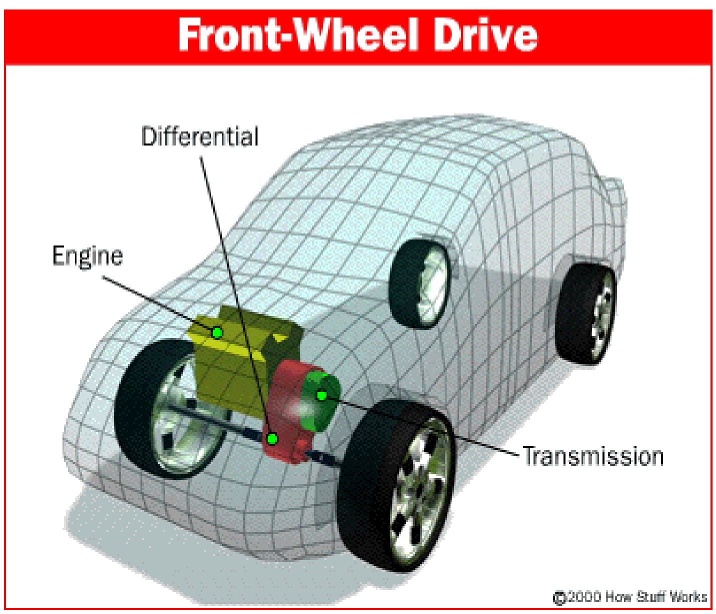


Figure 2Front Wheel drive car configuration

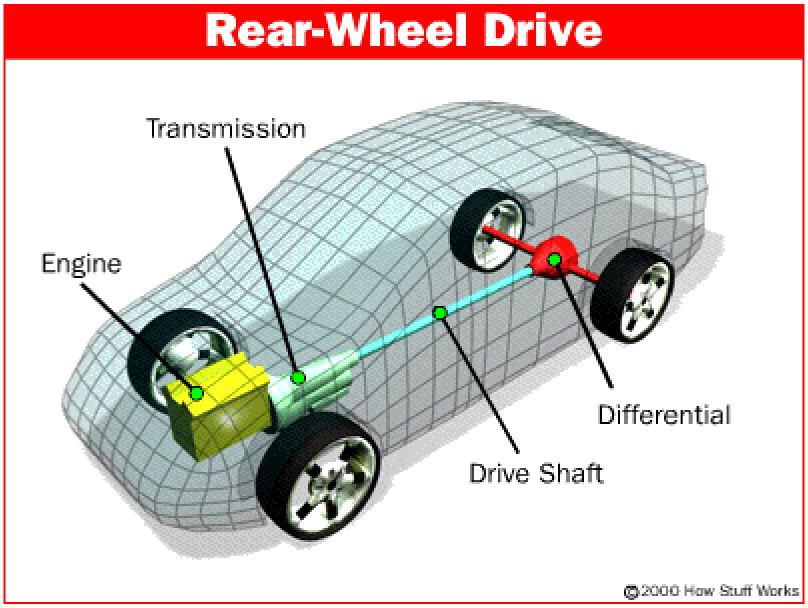


Figure 3Rear wheel drive car configuration

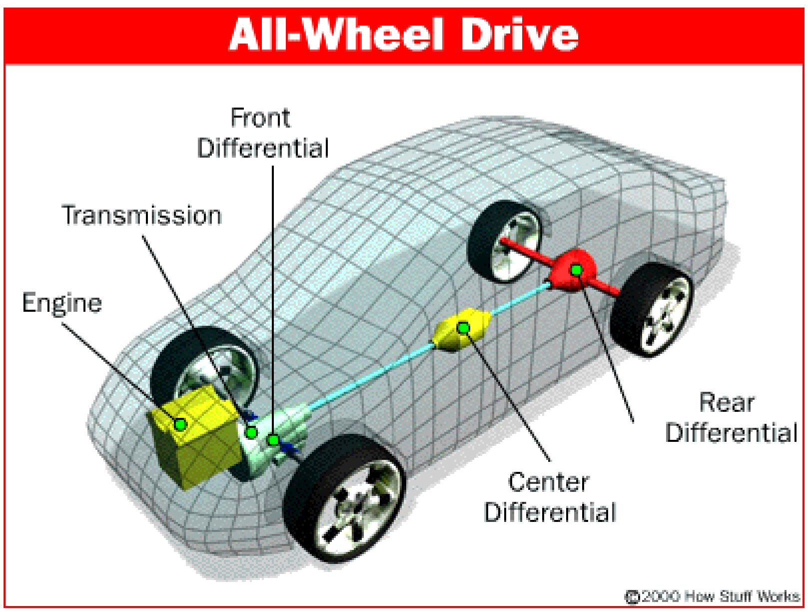


Figure 4All-wheel drive car configuration

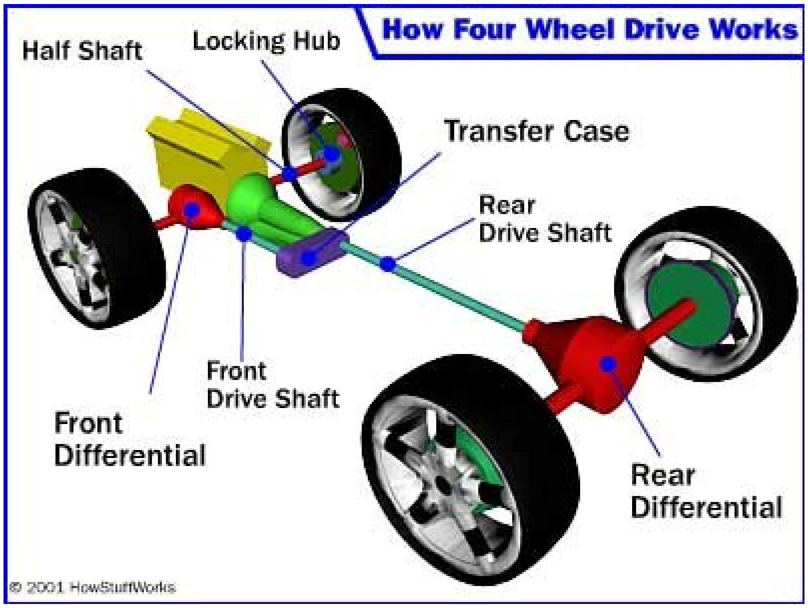


Figure 54WD configuration

OPEN DIFFERENTIAL

An open differential is the simplest kind of differential that comes standard with many vehicles.

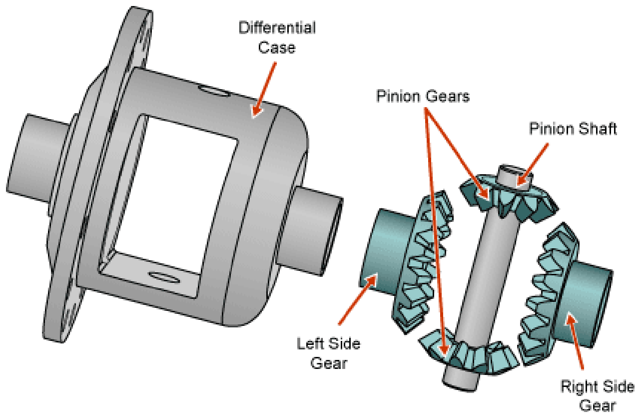


Figure 6 Exploded view of an open differential and its housing

As shown in the figure the open differential has a casing which has two sets of bevel gears in it. The smaller gears are called pinion or spider gears. They are mounted on a pinion shaft with bearings. They can rotate freely on the bearings. The larger gears are called side gears they cannot rotate freely rather they rotate with the half shafts on which they are mounted with splines. The side gears have internal while the half shafts have external splines. The housing has a ring or crown gear that is fixed to the differential housing with bolts as shown below. Its components are listed below:

Differential Case

- Holds the differential Gears
- Also carries the gear so surname is ‘carrier’

Differential Side Gears

- Transfers power from the case to the axel shafts

Differential Pinion Gear

- Allows each side gear to moveindependently of the carrier
- Crawls over the side gears;often-called a ‘spider gear’

Differential Pin

- Holds the differential pinion gears in place
- Locks the side gears in place
- Retains C-clips in side gears for some applications

Gear

- Attached to the case via bolts to make the case spin

Pinion Gear

- Attached to the housing via bearings
- Transfers power to the gear through the driveshaft

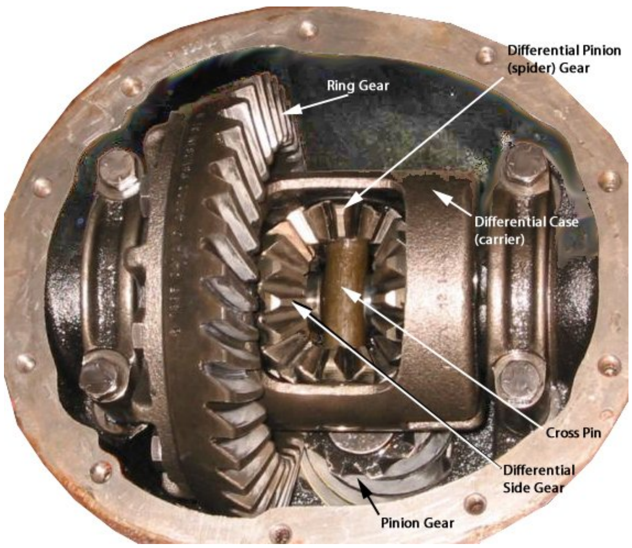


Figure 7An open differential in its housing with

the ring gear bolted to the differential case

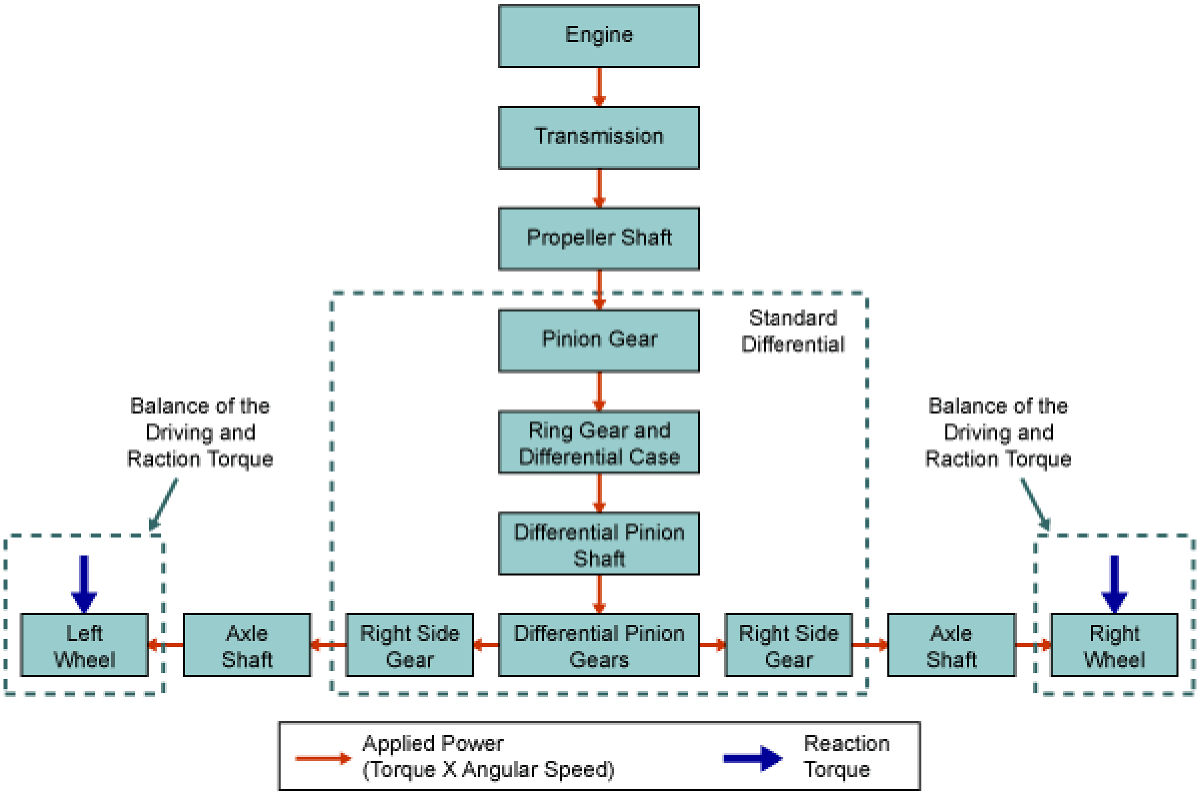


Figure 8Open differential function block diagram

The torque is transmitted from the engine to the differential via transmission and the propeller shaft. At the end of the propeller shaft is a pinion gear which meshes with the ring or crown gear of the differential casing. When the ring or crown gear rotates it rotates the differential casing. The following conditions could happen:

In a straight line:

This causes the spider gears to rotate with the housing. They press down on the side gears causing them to rotate at the same speed as if they are connected by a solid rod. The torque is split evenly between the twotires.

Around a corner or when on wheel is on a surface with less traction:

The spider gears start walking on the side gears. The outer tire or the tire on the slippery surface starts rotating faster. The inner tire is rotating slower. The torque from the engine is split equally in this case as well.

The biggest problem with the open differential is that it splits torque evenly under all conditions.

For example consider a situation in which one tire is on a good friction surface (such as a road) or is stuck and the other is in the air. The torque required to spin the tire in the air is very less because it is free as it is not in contact with any surface. Very less torque will go to this wheel. Since the open differential splits the torque evenly, the same amount of torque goes to the other wheel. Thus the vehicle has little to no torque and remains stuck. The open differential always applies the same torque to both wheels, and the maximum amount of torque is limited to the greatest amount that will not make the wheels slip. It doesn't take much torque to make a tire slip in the air. And when the wheel with good traction is only getting the very small amount of torque that can be applied to the wheel with less traction, the car isn't going to move very much or at all.

Another situation is when the vehicle is on a slippery surface or a surface with less traction. Again torque required to make the wheel on the slippery surface is less so it starts slipping. And the same low value of torque is transferred to the other wheel. Since torque determines the amount of force with which the vehicle pushes against ground (hence its acceleration), using an open differential is disadvantageous in motorsport.

Consider a vehicle that is moving on a road. And it happens to drive over a part of the road where one half of the road is dry and the other is slippery (covered with ice or snow, or is wet etc.). Let's assume that the one of the wheels of the car travels over the dry patch of the road and the other over the slippery surface. The tire that move over the slippery surface requires much less torque to make it spin. Let's assume this value is 6 N.m. The open differential will transmit 6 N.m to the other tire as well.

Limited slip differential

Limited slip differentials employ different mechanisms to permit normal differential action during turns, while also solving the trouble of slippage. When one powered tire slips, the LSD transfers more torque to the non-slipping wheel. The more common LSDs achieve this with clutches or a fluid-filled housing. Following are some the types of LSD which are commonly used on passenger cars:

1. Fixed value
2. Torque sensitive
3. Speed sensitive
4. Electronically controlled

1) Fixed value

In this differential the maximum torque difference between the two outputs is a fixed value at all times regardless of torque input to the differential or speed difference between the two outputs. Typically this differential used spring loaded clutch assemblies.

2) Torque sensitivity (HLSD)

This type includes helical gear limited-slip differentials and clutch, cone (an alternative type of clutch) where the engagement force of the clutch is a function of the input torque applied to the differential (as the engine applies more torque the clutches grip harder).

Torque sensing LSDs respond to driveshaft torque, so that the more driveshaft input torque present, the harder the clutches, cones or gears are pressed together, and thus the more closely the drive wheels are coupled to each other.

a) Clutch, cone-type, or plate LSD

The clutch type has a stack of thin clutch-discs, half of which are coupled to one of the drive shafts, the other half of which are coupled to the spider gear carrier. The clutch stacks may be present on both drive shafts, or on only one. If on only one, the remaining drive shaft is linked to the clutched drive shaft through the spider gears. In a cone type the clutches are replaced by a pair of cones which are pressed together achieving the same effect.

3) Speed sensitivity

Speed-sensitive differentials limit the torque difference between the outputs based on the difference in speed between the two output shafts. Thus for small output speed differences the differential’s behavior may be very close to an open differential. As the speed difference increase the limiting torque increases. This results in different dynamic behavior as compared to a torque sensitive differential.

a) Viscous (VLSD)

Figure 9Nissan 240SX Viscous LSD

The viscous type is generally simpler because it relies on hydrodynamic friction from fluids with high viscosity. Silicone-based oils are often used. Here, a cylindrical chamber of fluid filled with a stack of perforated discs rotates with the normal motion of the output shafts. The inside surface of the chamber is coupled to one of the driveshafts, and the outside coupled to the differential carrier. Half of the discs are connected to the inner, the other half to the outer, alternating inner/outer in the stack. Differential motion forces the interleaved discs to move through the fluid against each other. In some viscous couplings when speed is maintained the fluid will accumulate heat due to friction. This heat will cause the fluid to expand, and expand the coupler causing the discs to be pulled together resulting in a non-viscous plate to plate friction and a dramatic drop in speed difference. This is known as the hump phenomenon and it allows the side of the coupler to gently lock. In contrast to the mechanical type, the limiting action is much softer and more proportional to the slip, and so is easier to cope with for the average driver.

4) Electronic

An electronic limited-slip differential will typically have a planetary or bevel gear set similar to that of an open differential and a clutch pack similar to that in a torque sensitive differential. In the electronic unit the clamping force on the clutch is controlled externally by a computer or other controller. This allows the control of the differential's limiting torque, to be controlled as part of a total chassis management system. An example of this type of differential is Subaru's DCCD used in the 2011 Subaru WRX STi.

Torsen differential

We started with the Torsen T-1 differential. It has three pairs of Invex gears (helical gears with spur gears on both ends) and two helical gears assembled in the configuration shown in the figure below. It multiplies the torque on one wheel times the torque bias ratio (TBR) and sends transfers it to the other wheel. For example if a Torsen differential has a TBR of 2 it will multiply the torque on the wheel with less traction but the biggest problems were that:

1) Helical gears are very difficult to manufacture

2) If one wheel is off the ground then it gets almost negligible torque (close to zero). This torque value multiplied with any number is very small if not zero.

So we decided to shift to the cone clutch differential.

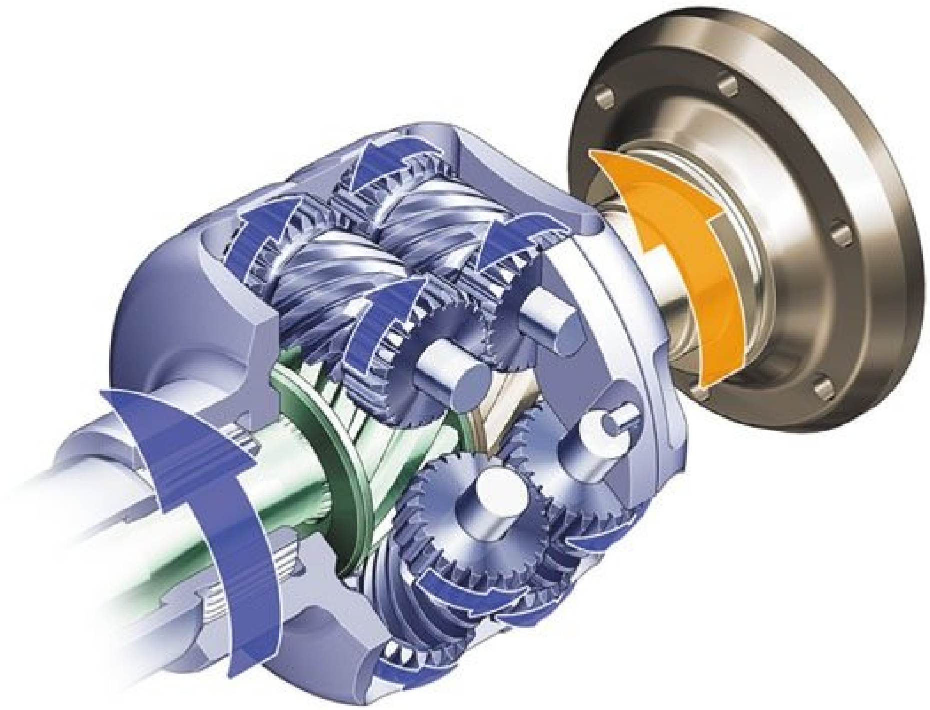


Figure 10Torsen T1 LSD

Cone clutch limited slip differential

A cone clutch differential is a type of limited slip differential. It is a variation of the clutch type limited slip differential. It uses cone clutches to transfer torque to the wheels. These cones are located at the back of the side gears. These cones are splined and fit onto the splines on the journal bearing behind the side gears. When the tire that has less traction tries to slip it is prevented from doing so by the cone clutches which are designed so that they are wedged into their conical housing thus locking the two half shafts. Helical compression springs are located between the side gears to wedge the clutches into the differential case. The tire with less grip would need relatively more torque (i.e. breakaway torque) to break free or break away and start spinning at a different speed as opposed to a simple open differential.

A cone clutch limited slip differential uses the friction produced by cone-shaped axle gears to provide improved traction.

Under rapid acceleration or when one wheel loses traction. The differential pinion gears, as they drive the cones, push outward on the cone gears. This action increases friction between the cones and case, driving the wheels with even greater torque.

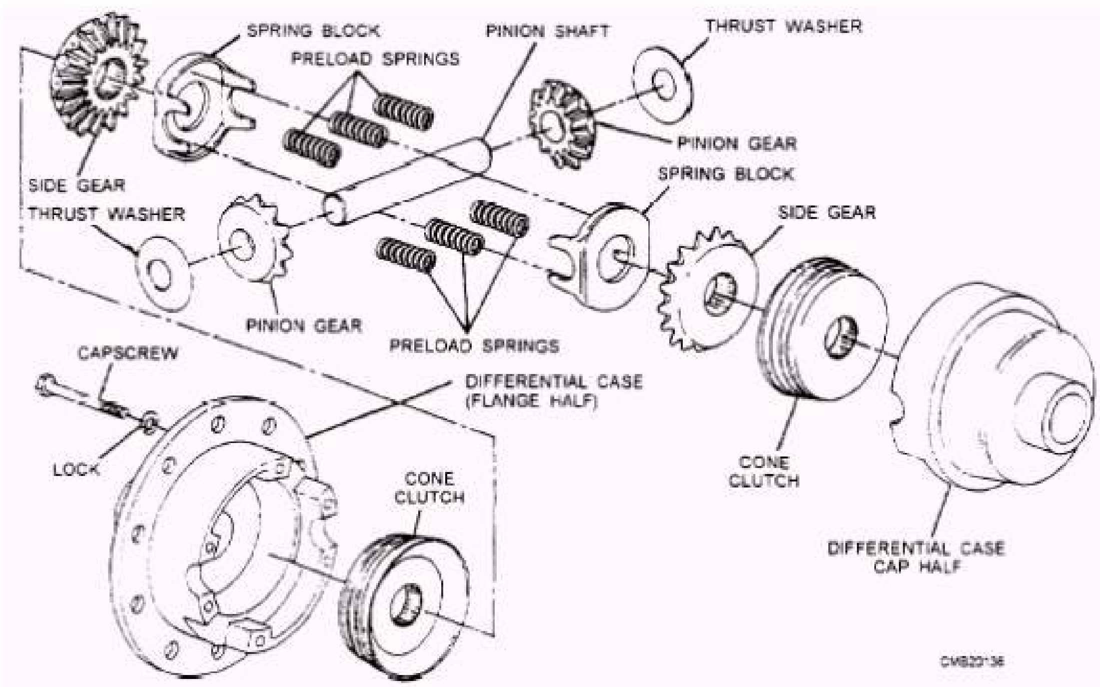


Figure 11Cone clutch differential

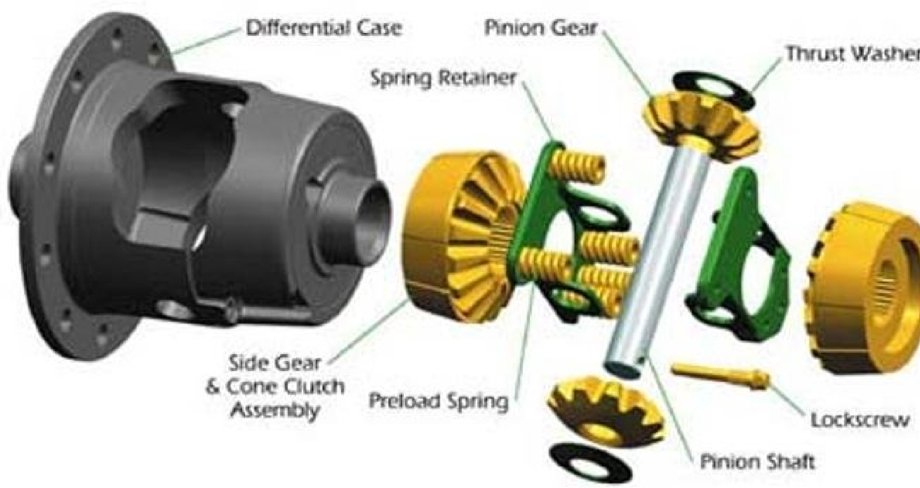


Figure 12Cone clutch differential

The advantage of the cone clutch differential is that its components are relatively easier to fabricate and the modification of an open differential required in order to turn it into a cone clutch differential is limited slip differential is relatively lesser and simpler. The side gears would require modification. Cones with internal splines would need to be manufactured. Springs and two steel plates would also need to be added. Furthermore the housing/ differential casing would require some machining.

A cone clutch consists of a cup that is either keyed or is splined onto one of the shafts, a cone slides axially on splines or keys on the mating shaft, and a helical spring usually hold the clutch in engagement. In a cone clutch differential the cone clutches are wedged into the "cup" by springs as shown (Fig). The cone angle 'α' and the diameter (D) and face width (d) of the cone are some of the more important design parameters.Thewedging effect reduces rapidly if larger cone angles are used.

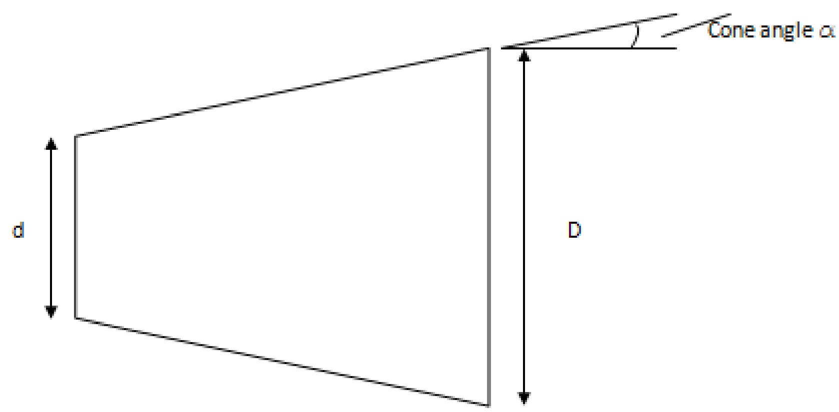


Figure 13 Cone of limited slip differential

The theory of uniform wear has been used. This theory assumes that the wear induced during service is constant and uniform over the entire surface area of the cone clutch.

According to Shigley’s Mechanical Engineering Design by Joseph Shingley by Budynas and Nisebett

1) Torque transmitted by the cone clutch:

The maximum amount of torque (in Nm) that can be transferred by a cone clutch is given by:

$$T_{cone-clutch} = \int rfpdA = \int_{d/2}^{D/2} (rf)(p_a \frac{d}{2r})(\frac{2\pi rdr}{\sin\alpha})$$
$$= \frac{\pi fp_a d}{\sin\alpha} \int_{d/2}^{D/2} rdr = \frac{\pi fp_a d}{8\sin\alpha}(D^2 - d^2)$$

Where:

P_a is pressure applied in pascals (Pa)

f is coefficient of static friction

D is diameter of base of cone in meters

d is diameter of smaller end of cone in meters

$\alpha$is the cone angle in degrees

$fpdA$ is the differential friction force.

The actuating force F (in newtons) required to transfer this torque T is given by:

$$F = \frac{4T_{cone-clutch}\sin\alpha}{f(D+d)}$$

The following values for the different parameters were used:

Coefficient of friction	0.2	
P_a (hard steel /hard steel)	1300000	N/m^2
Diameter of top of cone (d)	0.054	m
Diameter of base of cone (D)	0.0581	m
Height/length of cone	0.016	m
Cone angle	7.301249	degrees
Surface area of cone (not including the top and bottom surfaces)	2.84×10^{-3}	m^2

Torque (Nm)	Force (N)	Pressure (formula) N/m^2
20	453.4741207	1.30E+06
19	430.8004146	1.24E+06
18	408.1267086	1.17E+06
17	385.4530026	1.11E+06
16	362.7792965	1.04E+06
15	340.1055905	9.78E+05
14	317.4318845	9.13E+05
13	294.7581784	8.48E+05
12	272.0844724	7.82E+05
11	249.4107664	7.17E+05
10	226.7370603	6.52E+05
9	204.0633543	5.87E+05
8	181.3896483	5.22E+05
7	158.7159422	4.56E+05
6	136.0422362	3.91E+05
5	113.3685302	3.26E+05
4	90.69482413	2.61E+05
3	68.0211181	1.96E+05
2	45.34741207	1.30E+05
1	22.67370603	6.52E+04
0	0	0.00E+00

Table 1Max torque transmitted by cone clutchand the corresponding force required

Th above table shows the maximumm torque that can be transmitted upto th maxximum pressure limit of 1.3 MPa and the corrseponding force required for this torque. This is also the breakaway torque.

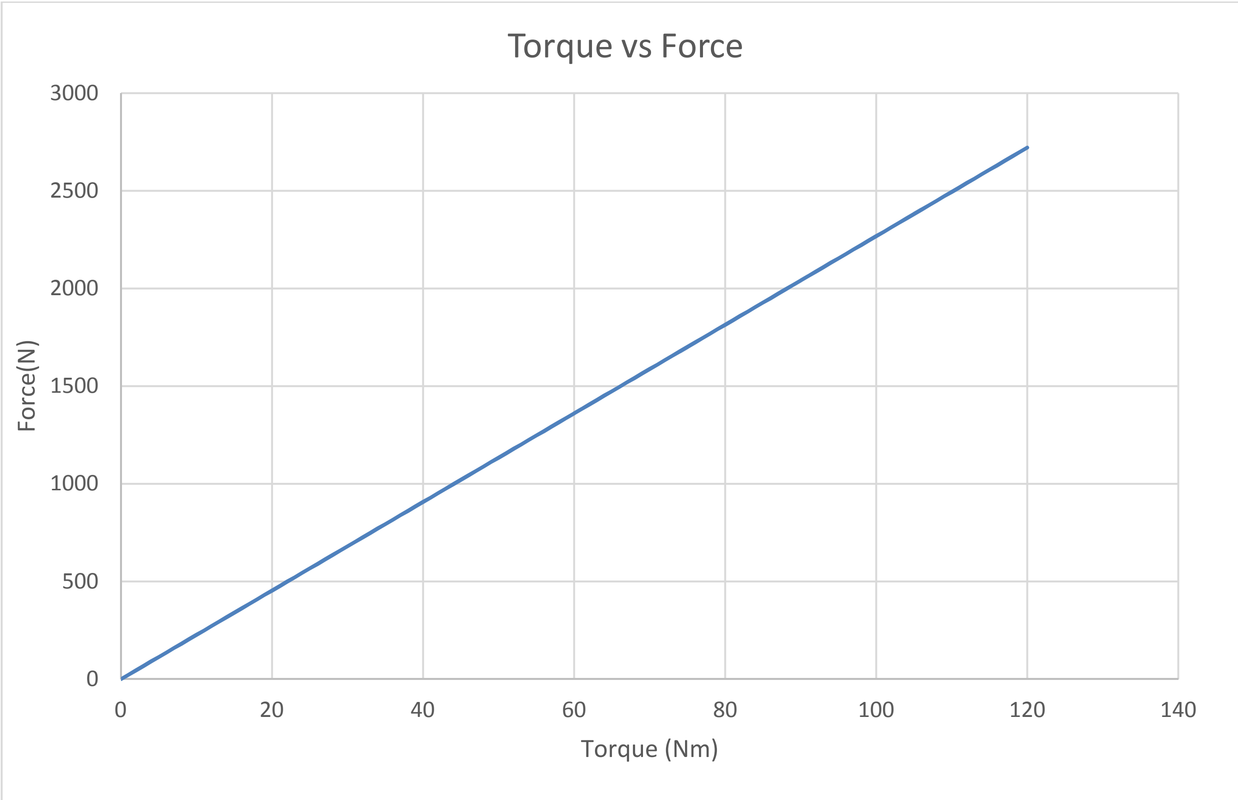


Figure 14Comparison of torque transmitted and force applied

Torque is directly proportional to the actuating force. In order for more torque to be transmitted, the magnitude of the actuating force must be increased.

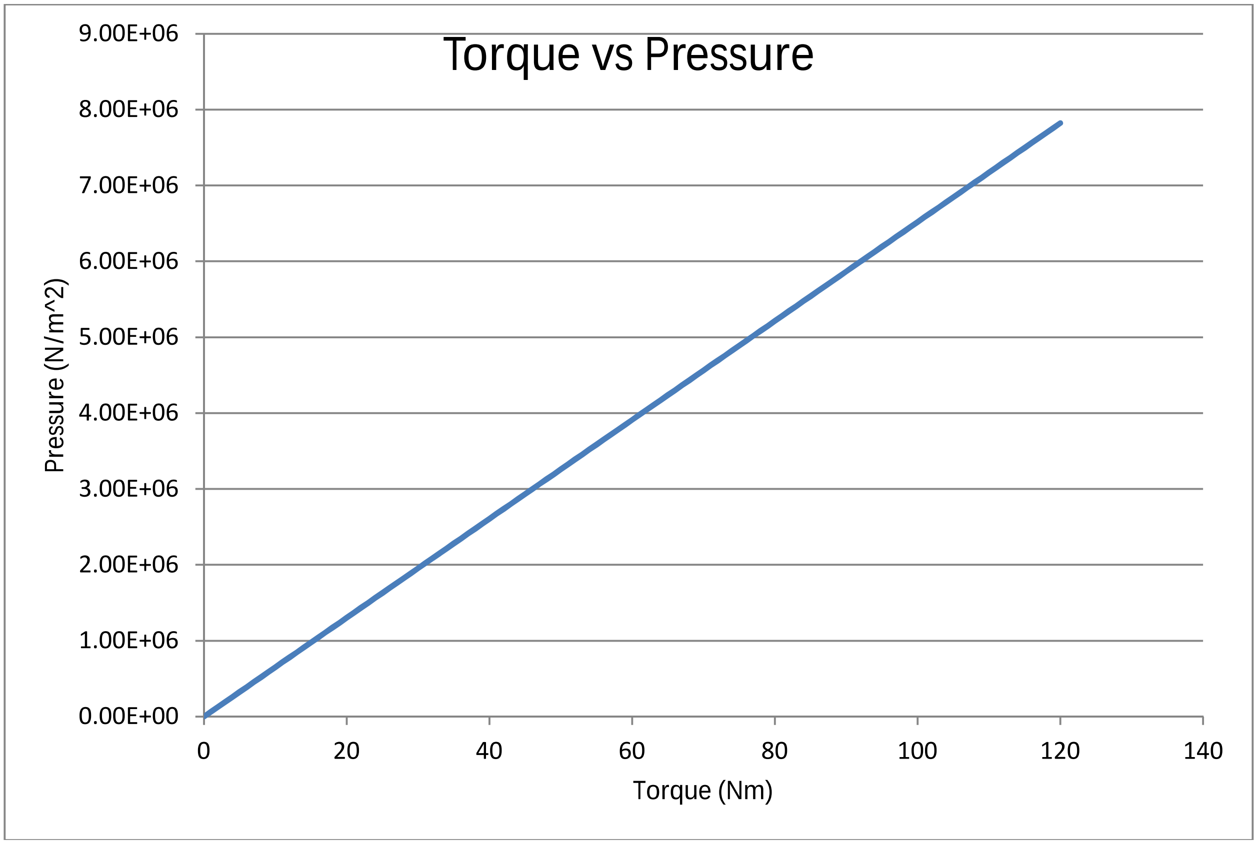


Figure 15Comparison of torque transmitted and pressure applied

The greater the torque to be transmitted, the greater is the pressure applied on the surface area of the cone by the "cup" into which it is pressed.

From the values obtained through calculations it can be observed that the maximum torque transferable by the cone clutch within the maximum applied pressure range is 20 Nm.

2) Torque transferred the face of the side gear:

Apart from the torque transferred from the cone clutch, torque is also transmitted due to friction between the spring block and the surface of the side gears.

In order to calculate the torque added by the face of the side gear the area of the flat face of the side gear in contact with the spring block was first calculated. This came out to be 498 mm^2. This area was then converted into the area of an equivalent ring for ease of calculations. Setting the area of the inner diameter of this assumed ring equal to the hole in the centre of the side gear it was determined that the dimensions of the ring would be as follows:

Inner diameter of washer $d = 0.0256$ m

Outer diameter of washer $D = 0.0359$ m

Area of washer $A = 4.98 \times 10^{-6}\ m^2$

Maximum pressure $P_a = 1.3$ MPa

The maximum torque that can be transmitted by this area is given by:

$$T_{side-gear-face} = \frac{Ff(D+d)}{4}$$

Where

f is coefficient of friction between spring block an face of side gear

D is the outer diameter in meters

d is inner diameter in meters

F is force applied in newtons

The pressure applied on the face of the side gear is given by

$$P_a = \frac{F}{A}$$

The values of F are the same as used for the cone clutch.

Force (formula) N	Washer torque (Nm)	Washer Pa (MPa)
453.4741207	1.394432921	0.911486594
430.8004146	1.324711275	0.865912264
408.1267086	1.254989629	0.820337935
385.4530026	1.185267983	0.774763605
362.7792965	1.115546337	0.729189275
340.1055905	1.045824691	0.683614945
317.4318845	0.976103045	0.638040616
294.7581784	0.906381399	0.592466286
272.0844724	0.836659753	0.546891956
249.4107664	0.766938107	0.501317627
226.7370603	0.697216461	0.455743297
204.0633543	0.627494814	0.410168967
181.3896483	0.557773168	0.364594638
158.7159422	0.488051522	0.319020308
136.0422362	0.418329876	0.273445978
113.3685302	0.34860823	0.227871648
90.69482413	0.278886584	0.182297319
68.0211181	0.209164938	0.136722989
45.34741207	0.139443292	0.091148659
22.67370603	0.069721646	0.04557433
0	0	0

Table 2Force applied on side gear face and corresponding torque and max pressure

Thus torque transferred by face of side gear: T=1.394 Nm

Total torque:

$T_{total} = T_{cone\ clutch} + T_{side\ gear\ face}$

= 20 Nm +1.394 Nm

= 21.394 Nm

Thus the breakaway torque or the maximum torque each wheel can transmit is 21.394 Nm.

We also varied the cone height to find the effect of increasing the height on the cone angle and torque transferred. The results were as follows:

Height m	**Cone angle (degrees)**	**surface area m^2**	**Torque Nm**
0.001	63.99665	0.00040	2.92782
0.002	45.70732	0.00050	3.67630
0.003	34.34610	0.00064	4.66408
0.004	27.13514	0.00079	5.76953
0.005	22.29363	0.00095	6.93662
0.006	18.86359	0.00112	8.13888
0.007	16.32305	0.00128	9.36276
0.008	14.37278	0.00145	10.60079
0.009	12.83178	0.00163	11.84852
0.01	11.58513	0.00180	13.10319
0.011	10.55674	0.00197	14.36298
0.012	9.69444	0.00214	15.62665
0.013	8.96131	0.00232	16.89333
0.014	8.33054	0.00249	18.16238
0.015	7.78221	0.00267	19.43335
0.016	7.30124	0.00284	20.70589
0.017	6.87600	0.00302	21.97971
0.018	6.49736	0.00319	23.25461
0.019	6.15809	0.00337	24.53042
0.02	5.85238	0.00354	25.80700
0.021	5.57549	0.00372	27.08425

Height m	Cone angle	surface area m^2	Torque Nm
0.022	5.32355	0.00389	28.36208
0.023	5.09334	0.00407	29.64041
0.024	4.88216	0.00424	30.91917
0.025	4.68777	0.00442	32.19833
0.026	4.50823	0.00459	33.47782
0.027	4.34190	0.00477	34.75762
0.028	4.18740	0.00494	36.03769
0.029	4.04349	0.00512	37.31801
0.03	3.90913	0.00529	38.59854
0.031	3.78341	0.00547	39.87928
0.032	3.66550	0.00565	41.16019
0.033	3.55472	0.00582	42.44127
0.034	3.45042	0.00600	43.72250
0.035	3.35207	0.00617	45.00386
0.036	3.25916	0.00635	46.28535
0.037	3.17125	0.00653	47.56696
0.038	3.08796	0.00670	48.84867
0.039	3.00893	0.00688	50.13048
0.04	2.93384	0.00705	51.41237
0.041	2.86241	0.00723	52.69436
0.042	2.79436	0.00740	53.97642
0.043	2.72948	0.00758	55.25855
0.044	2.66753	0.00776	56.54075
0.045	2.60834	0.00793	57.82302
0.046	2.55171	0.00811	59.10534
0.047	2.49749	0.00828	60.38772
0.048	2.44552	0.00846	61.67015
0.049	2.39567	0.00864	62.95263

Table 3 Variation of surface area and torque with increasing cone height

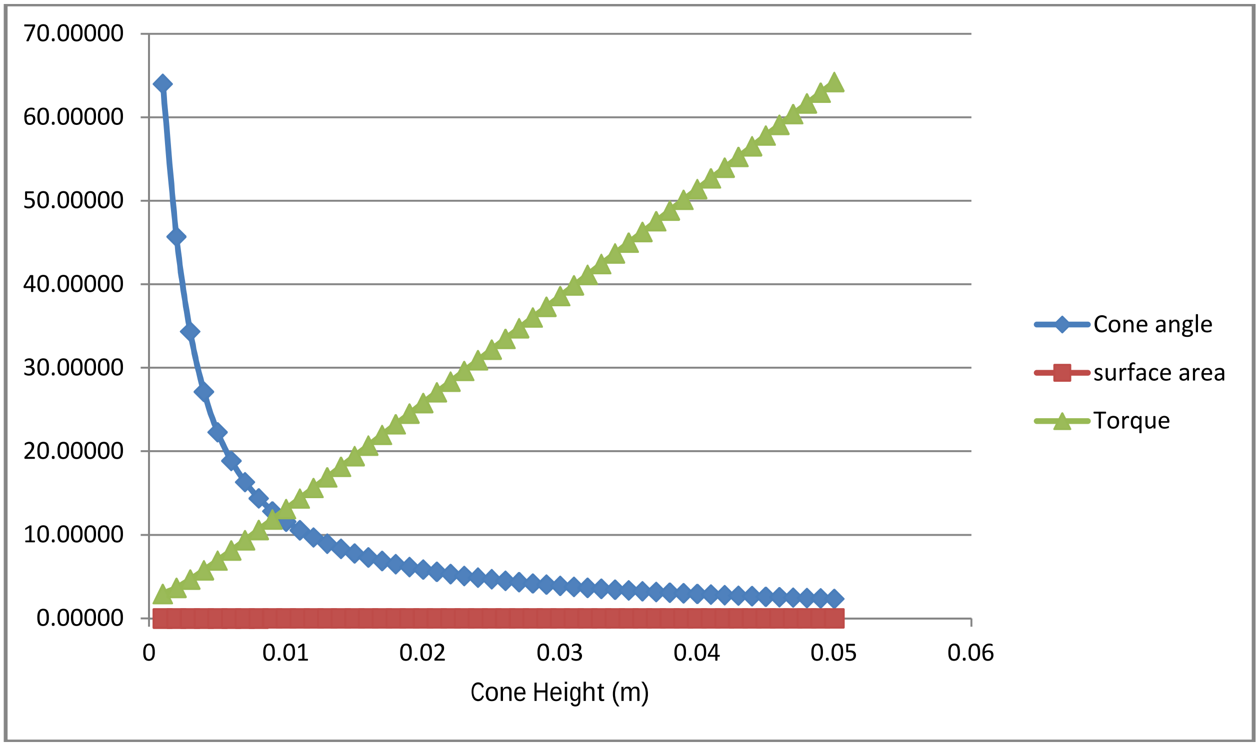


Figure 16Variation of surface area and torque with increasing cone height

It can be seen from the calculated values and their graphical representation that increasing the height of the cone:

1) Decreases the cone angle α

2) Increases the surface area of the cone clutch

3) Increases the torque transfer

3) Design of spring:

The material we used was Carbon Steel. Some of its properties and other materials used in making springs are given below:

Material	*Allowable shear stress (τ) MPa*			*Modulus of rigidity (G) kN/m^2*	*Modulus of elasticity (E) kN/mm^2*
	Severe service	*Average service*	*Light service*		
1. Carbon steel					
(*a*) Upto to 2.125 mm dia.	420	525	651		
(*b*) 2.125 to 4.625 mm	385	483	595		
(*c*) 4.625 to 8.00 mm	336	420	525		
(*d*) 8.00 to 13.25 mm	294	364	455		
(*e*) 13.25 to 24.25 mm	252	315	392	80	210
(*f*) 24.25 to 38.00 mm	224	280	350		
2. Music wire	392	490	612		
3. Oil tempered wire	336	420	525		
4. Hard-drawn spring wire	280	350	437.5		
5. Stainless-steel wire	280	350	437.5	70	196
6. Monel metal	196	245	306	44	105
7. Phosphor bronze	196	245	306	44	105
8. Brass	140	175	219	35	100

Figure 17 Properties of materials used for springs

We chose carbon steel, as the spring material due to its high value of allowable shear stress.

Design parameters and calculations of spring:

1) Outer diameter of spring /Diameter of hole Do =14.26 mm

2) Maximum allowable shear stress τ=651 MPa (Assuming light service)

3) Number of active turns N_a =4

4) Modulus of Rigidity G=80 GPa

5) Operating length $x_{operating}$=18.8 mm

5) Pitch p= 5.44 mm

6) Wire diameter d= 1.9 mm

7) Coil mean diameter D= D_o -d =12.36 mm

8) Spring index C=6.505

9) Wahl factor $K_W = \frac{4C-1}{4C-4} + \frac{0.615}{C}$ =1.2308

10) Axial Load $W = \frac{\pi d^2 \tau}{8K_W C}$ =115.2672 N

11) Deflection δ= $\frac{8WD^3 n}{Gd^4}$ = 6.68 mm

12) Total number of turns $N_t = N_a + 2 = 6$

13) Solid length L_S=(n+2)d = 11.4 mm

14) Free Length L_f= p x n + d = 25.56 mm

Squared and Ground ends

15) Spring constant k= 115.267/6.68=14.255 N/mm

Four such springs will be used which will apply a combined force of 115.26 x 4 =164.04 N

As the table above shows in order to transmit 21.6384 Nm of torque, a force of 453.47 N is required. This force is applied by springs. We designed four springs each of which would apply 115 N of force.

We could not fabricate the cone clutch differential even though we had performed the design and calculations. In order to fix the cone clutch on the back of the gear we would have to create splines on the inside of the cone and on the outside of the journal bearing which is present behind the side gears. The journal bearing already has splines on the inside and creating splines on its outer surface would reduce its area of cross section significantly, leaving only a thickness of roughly 1mm making it susceptible to structural failure.

Thus we decided to move on to another design which was the phantom grip locking differential.

PHANTOM GRIP LIMITED SLIP DIFFERENTIAL:

A Phantom Grip limited slip differential is a modification of the typical open differential into a LSD. The modification includes inserting two plates of steel with holes in them between the side gears of the differential. Springs are inserted into the holes. These springs push against the two plates in opposite directions. When the plates are pushed outwards, each plate presses against the respective side gear. This effectively locks the side gears together. Thus if one tire encounters a slippery surface, it will not slip immediately rather torque will be transferred to the tire with more traction by increasing the breakaway torque (i.e. torque required to make a wheel slip). The amount of torque transferred depends on how much force the springs are applying on the plates and thus the side gears. The greater the force applied by the springs, the greater the breakaway torque and greater is the torque transferred.

Figure 18Plates and springs of Phantom Grip

Figure 19Assembled Phantom Grip

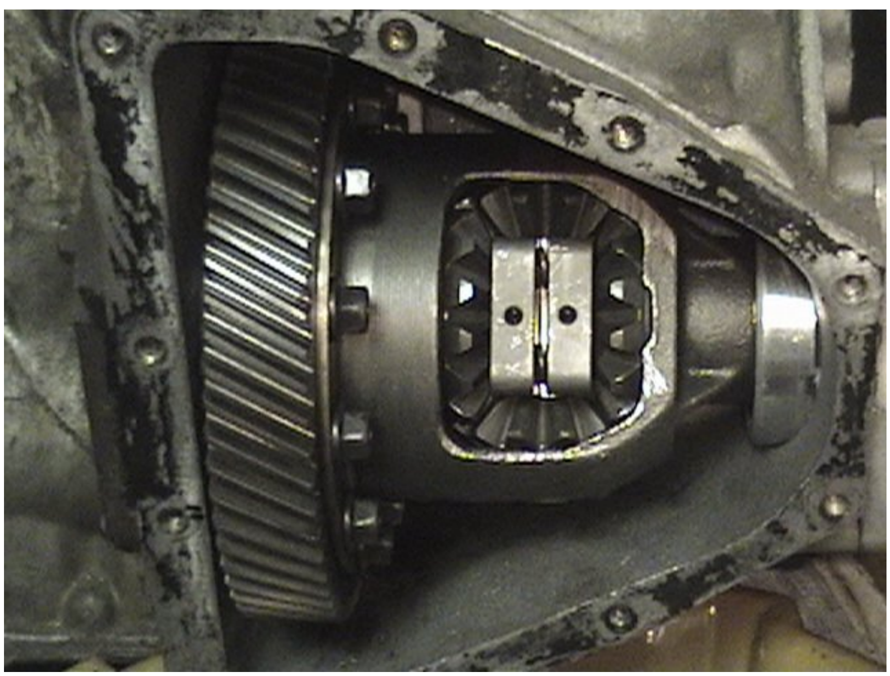


Figure 20Phantom Grip installed in a differential

In order to increase the torque transmitted by the friction between the washer behind the side gear and the housing, the external diameter of the washer was increased to 0.053 m which is the diameter of the bottom side of the side gear.

1) Torque transmitted by washer

For torque transmitted by friction between washer and housing:

Inner diameter of washer d=0.037 m

External diameter of washer D= 0.053 m

Maximum pressure limit $P_{a,\max} =$ 2 MPa

Area of washer $A = \frac{\pi}{4}(D^2 - d^2)$ = 0.001131 m^2

Coefficient of friction between washer and housing f=0.2

The corresponding actuating force is calculated by the formula below:

$$F = \frac{4T}{f(D+d)} \quad \text{(N)}$$

The pressure at the corresponding force is given by:

$$P_a = \frac{F}{A} \quad \text{(Pa)}$$

The values of actuating force and applied pressure calculated are shown on the next page:

Torque T (Nm)	Actuating Force F(N)	Applied Pressure (MPa)
10.2	2266.67	2.00
10.1	2244.44	1.98
10	2222.22	1.96
9.9	2200.00	1.95
9.8	2177.78	1.93
9.7	2155.56	1.91
9.6	2133.33	1.89
9.5	2111.11	1.87
9.4	2088.89	1.85
9.3	2066.67	1.83
9.2	2044.44	1.81
9.1	2022.22	1.79
9	2000.00	1.77
8.9	1977.78	1.75
8.8	1955.56	1.73
8.7	1933.33	1.71
8.6	1911.11	1.69
8.5	1888.89	1.67
8.4	1866.67	1.65
8.3	1844.44	1.63
8.2	1822.22	1.61
8.1	1800.00	1.59
8	1777.78	1.57
7	1555.56	1.38
6	1333.33	1.18
5	1111.11	0.98
4	888.89	0.79
3	666.67	0.59
2	444.44	0.39
1	222.22	0.20
0	0.00	0.00

Table 4Torque transferred by washer

Maximum torque that can be transferred by the washer within the pressure limits:

Twasher=10.2 Nm at a force of F=2266.67 N.

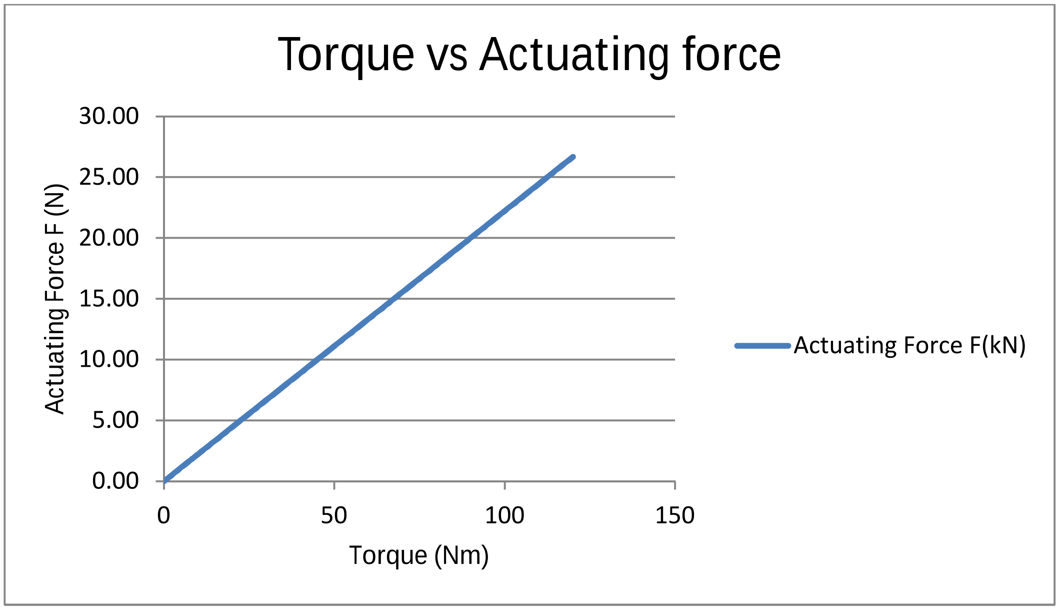


Figure 21Torque vs actuating force

It is evident from the graph that increasing the torque transferred increases the actuating force F which need to be applied to transmit the torque.

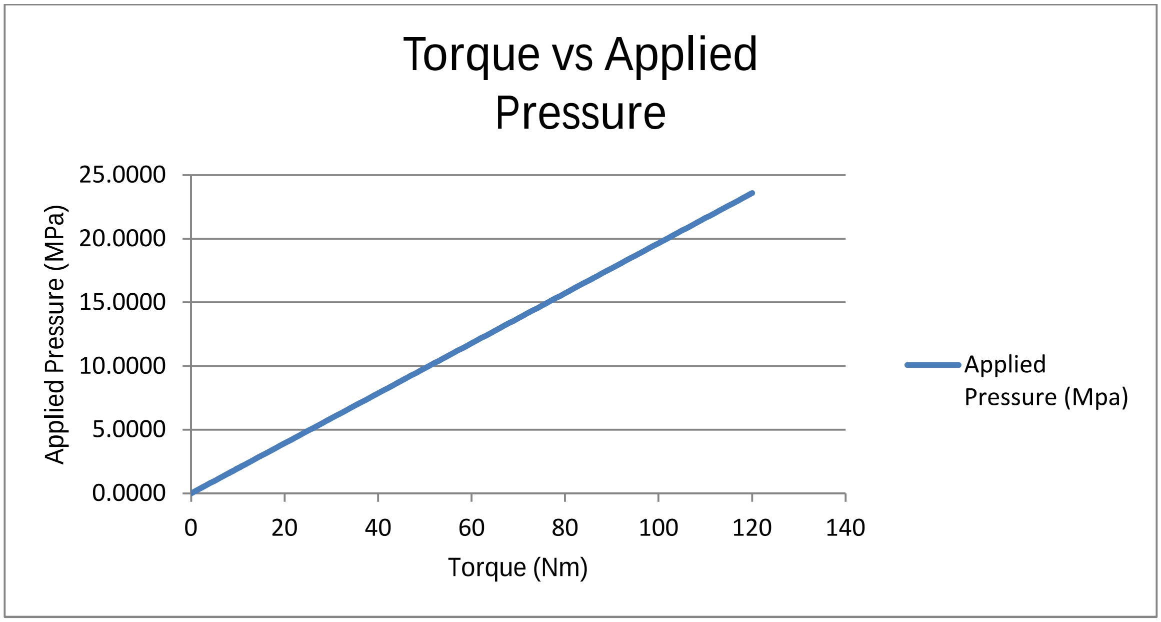


Figure 22Torque vs Applied pressure

2) Torque transmitted by front of side gear

We calculated the front area of the side gear and converted its area to that of a circular object such as a washer or ring having the inner diameter equal to the diameter of the hole in the side gear and then used the formula of area to calculate its outer diameter for the corresponding area and then the torque that would be transmitted by it. This was done because the formulas available were

Inner diameter of equivalent washer d=0.256 m

Outer diameter of equivalent washer D= 0.356 m

Coefficient of friction between steel plates and side gear face f= 0.2

Torque was calculated for the force being applied by the following formula:

$$T = \frac{Ff}{4}(D + d)$$

The values of force were the same as used above.

The values of torque and applied pressure are given below:

Torque (Nm)	Actuating Force (N)	Applied Pressure Pa (MPa)
3.00	975.16	1.95
2.9	942.66	1.89
2.8	910.15	1.82
2.7	877.65	1.76
2.6	845.14	1.69
2.5	812.64	1.63
2.4	780.13	1.56
2.3	747.63	1.50
2.2	715.12	1.43
2.1	682.61	1.37
2	650.11	1.30
1	325.05	0.65
0	0.00	0.00

Table 5Torque transmitted and corresponding actuauting force

Maximum torque that can be transferred by the face of side gear within the pressure limits:

$T_{side\ gear\ face}$ =3.00 Nm at a force of F=975.16 N

Four springs will apply a force F_{spring}= 975.16/4 =243.79

But in order to easily manufacture a spring a spring index should be at least 5.

Spring calculations:

Do	14.26 mm
active turns n	6
G (MPa)	80
pitch p (mm)	4.17
τ (MPa)	595

$$W = \frac{\pi d^3 \tau}{8 K_S D} \quad \text{(N)}$$

Where d is diameter of wire in mm

τ is shear stress is MPa

K_S is shear stress factor

D is mean diameter of coil in mm

Maximum deflection is given by:

$$\delta = \frac{8WD^3n}{Gd^4} \quad \text{(mm)}$$

Where W is axial force in Newtons

D is mean coil diameter in meters

n is number of active turns

G is shear modulus in GPa

wire dia d (mm)	Mean coil diaD (mm)	C=D/d	Ks=1+(1/2C)	Axial load W (N)	δ/n(mm)	Deflection δ (mm)
2.37	11.89	5.017	1.100	237.892	1.2675	7.6047

$x_{operaitng}$=18.8 mm

$\delta = x_{free} - x_{operaitng}$

6.33 mm=x_{free}- 18.8 mm

x_{free} =25.13

pitch p =4.08 mm

total number of turns = 8

solid length L_S =16.59 mm

Free length L_f =25.14 mm

According to the calculations for a spring index of 5 the maximum force one spring can apply is 237.892 N.

So four springs will apply951.568 N.

By interpolating we find that if a force of 951.568 N is applied then the maximum torque that can be transferred by the friction between the washer and the housing is 4.28 Nm and the maximum torque that can be transferred by the face of the side gear is 2.93 Nm.

So total torque transferred:

$T_{total}= T_{washer}+T_{side\ gear\ face}$

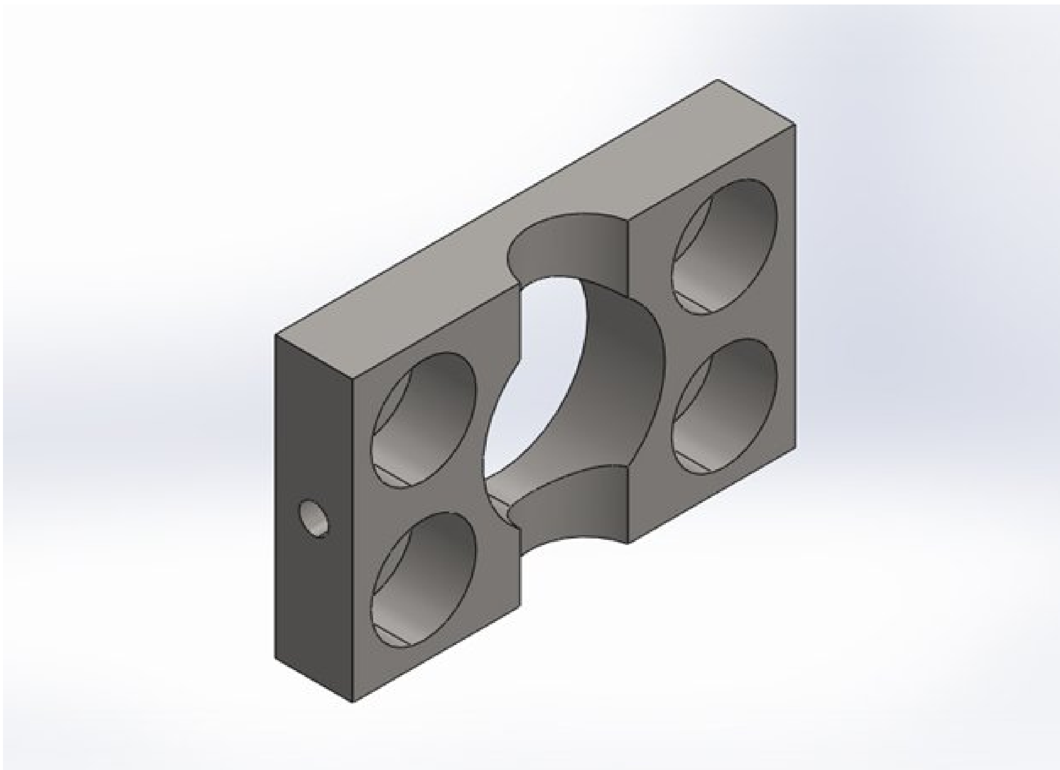
= 4.28 Nm + 2.93 Nm

= 7.21 Nm

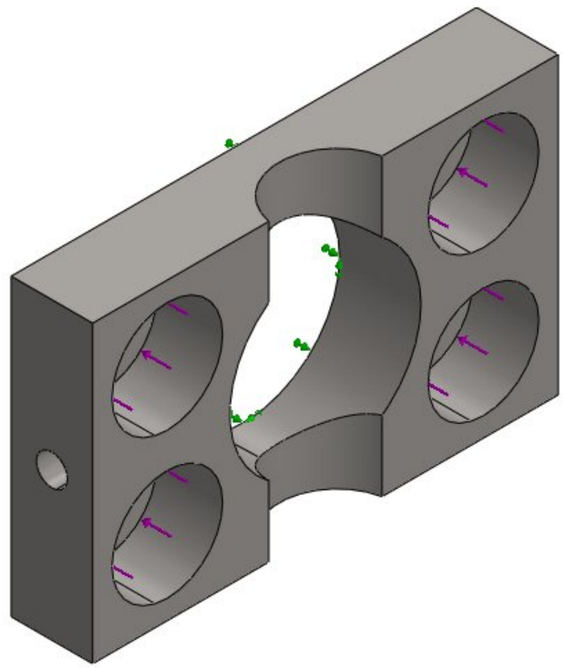
Which is 6.008 % of the total torque (which is 120 Nm)

Simulation of phantom grip

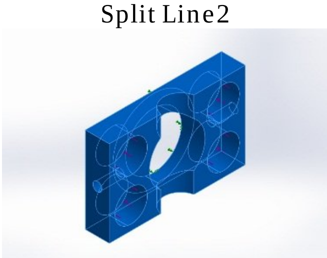
Date: Wednesday, June 10, 2015
Designer: Ali Ghaffar
Study name: phantom grip left plate final
Analysis type:Static



Model Information



Model name: phantom grip_left
Current Configuration: Default

<table>
<tr><th colspan="4">Solid Bodies</th></tr>
<tr><th>Document Name and Reference</th><th>Treated As</th><th>Volumetric Properties</th><th>Document Path/Date Modified</th></tr>
<tr><td>Split Line2
</td><td>Solid Body</td><td>Mass:0.096832 kg
Volume:1.23118e-005 m^3
Density:7865 kg/m^3
Weight:0.948953 N</td><td>D:\Academic\Semester 8\Final year project\Drawings\bevel gear cone\phantom grip_left.SLDPRT
Jun 03 09:53:28 2015</td></tr>
</table>

Study Properties

Study name	phantom grip left plate final
Analysis type	Static
Mesh type	Solid Mesh
Thermal Effect:	On
Thermal option	Include temperature loads
Zero strain temperature	298 Kelvin
Include fluid pressure effects from SolidWorks Flow Simulation	Off
Solver type	FFEPlus
Inplane Effect:	Off
Soft Spring:	Off
Inertial Relief:	Off
Incompatible bonding options	Automatic
Large displacement	Off
Compute free body forces	On
Friction	Off
Use Adaptive Method:	Off
Result folder	SolidWorks document (D:\Academic\Semester 8\Final year project\Drawings\bevel gear cone)

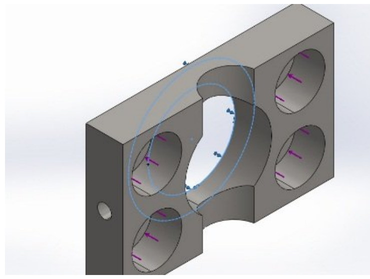
Units

Unit system:	SI (MKS)
Length/Displacement	mm
Temperature	Kelvin
Angular velocity	Rad/sec
Pressure/Stress	N/m^2

Material Properties

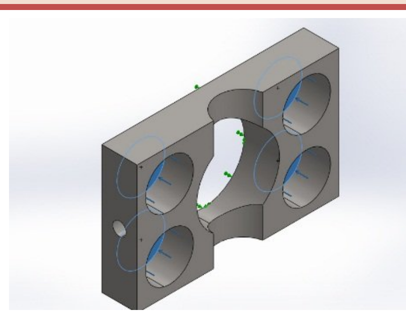
Model Reference	Properties	Components
	Name: ANSI 4041 Model type: Linear Elastic Isotropic Default failure criterion: Max von Mises Stress Yield strength: 4.171e+008 N/m^2 Tensile strength: 6.55e+008 N/m^2 Compressive strength: 6.55e+008 N/m^2 Elastic modulus: 2e+011 N/m^2 Poisson's ratio: 0.29 Mass density: 7865 kg/m^3 Shear modulus: 8e+010 N/m^2 Thermal expansion coefficient: 1.23e-005 /Kelvin	SolidBody 1(Split Line2)(phantom grip_left)
Curve Data:N/A		

Loads and Fixtures

Fixture name	Fixture Image	Fixture Details
Fixed-2		Entities: 1 face(s) Type: Fixed Geometry

Resultant Forces

Components	X	Y	Z	Resultant
Reaction force(N)	998.801	-0.0019534	0.00766385	998.801
Reaction Moment(N·m)	0	0	0	0

Load name	Load Image	Load Details
Force-1		Entities: 4 face(s) Type: Apply normal force Value: 250 N

Mesh Information

Mesh type	Solid Mesh
Mesher Used:	Standard mesh
Automatic Transition:	Off
Include Mesh Auto Loops:	Off
Jacobian points	4 Points
Element Size	1.70359 mm
Tolerance	0.0851794 mm
Mesh Quality	High

Mesh Information - Details

Total Nodes	30227
Total Elements	18451
Maximum Aspect Ratio	7.5326
% of elements with Aspect Ratio < 3	99.3
% of elements with Aspect Ratio > 10	0
% of distorted elements(Jacobian)	0
Time to complete mesh(hh;mm;ss):	00:00:03
Computer name:	1-PC

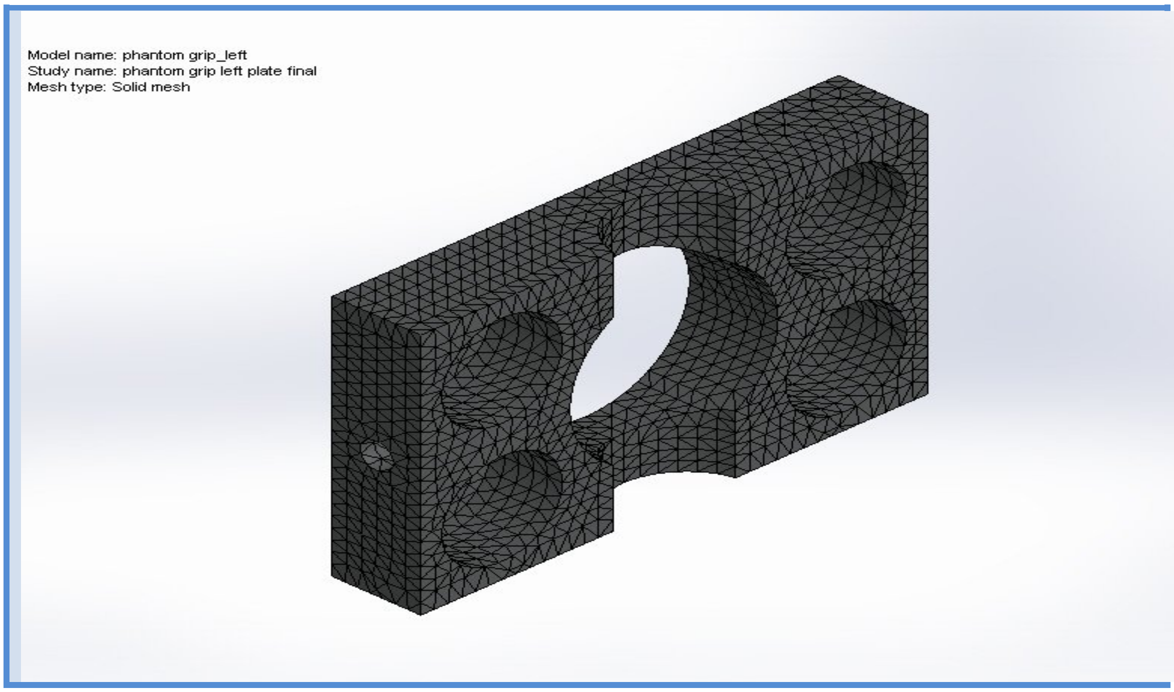
Resultant Forces

Reaction Forces

Selection set	Units	Sum X	Sum Y	Sum Z	Resultant
Entire Model	N	998.801	-0.0019534	0.00766385	998.801

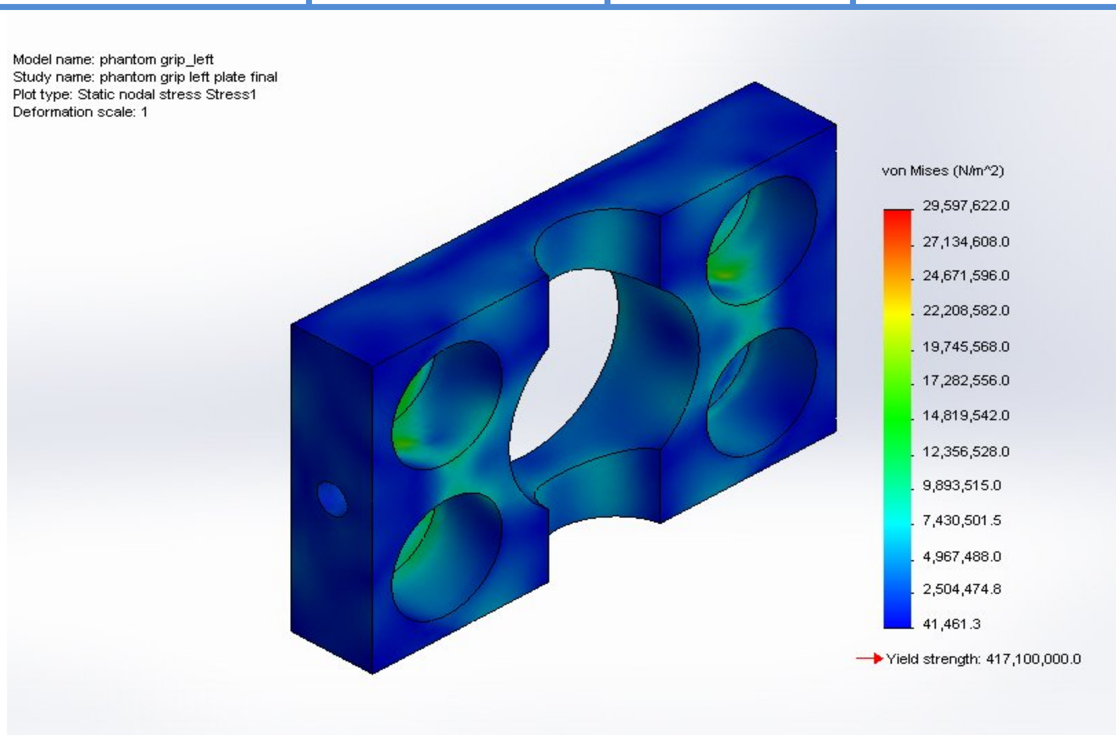
Reaction Moments

Selection set	Units	Sum X	Sum Y	Sum Z	Resultant
Entire Model	N·m	0	0	0	0



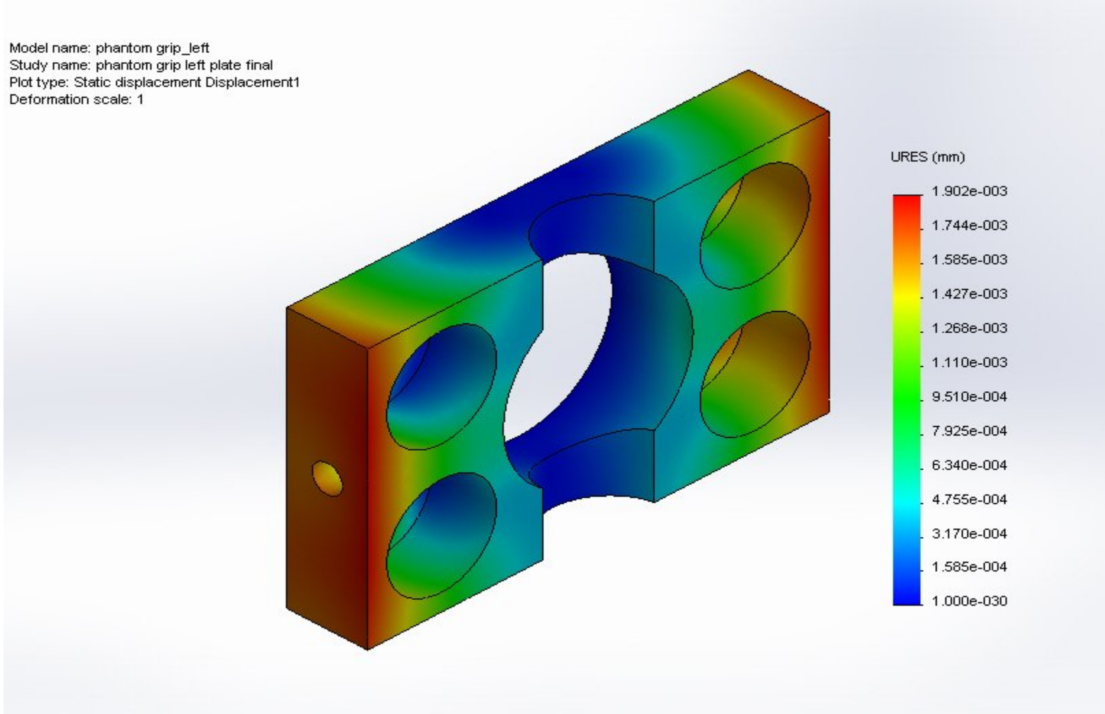
Study Results

Name	Type	Min	Max
Stress1	VON: von Mises Stress	41461.3 N/m^2 Node: 2363	2.95976e+007 N/m^2 Node: 23182



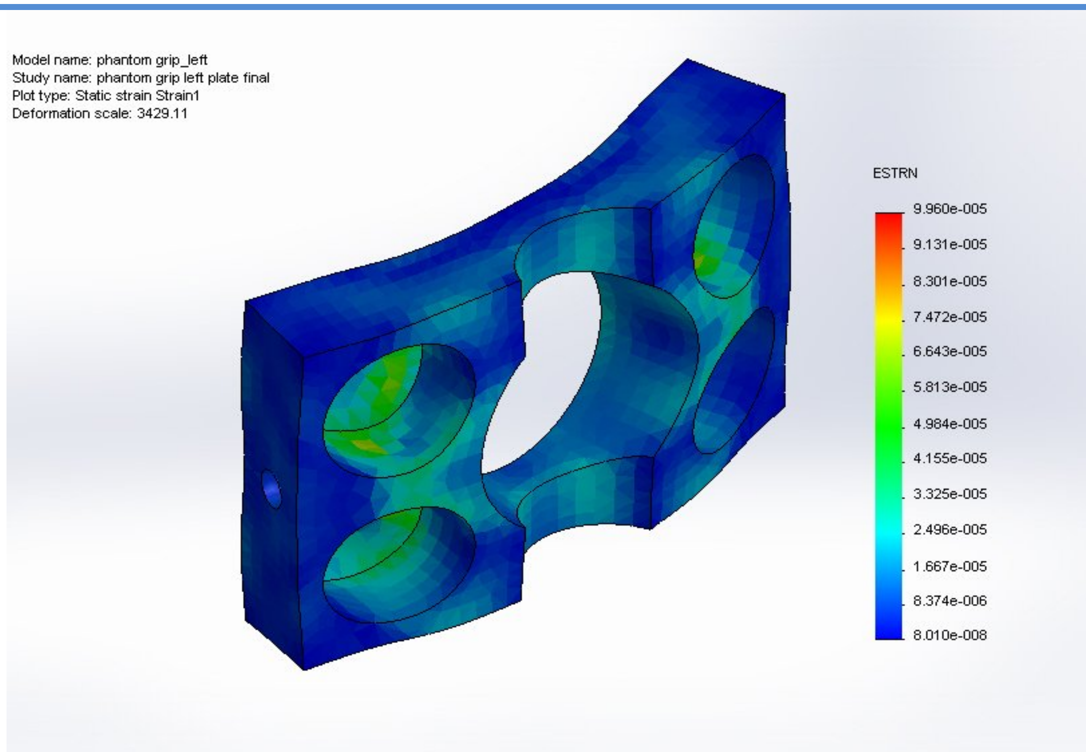
phantom grip_left-phantom grip left plate final-Stress-Stress1

Name	Type	Min	Max
Displacement1	URES: Resultant Displacement	0 mm Node: 503	0.00190204 mm Node: 1765



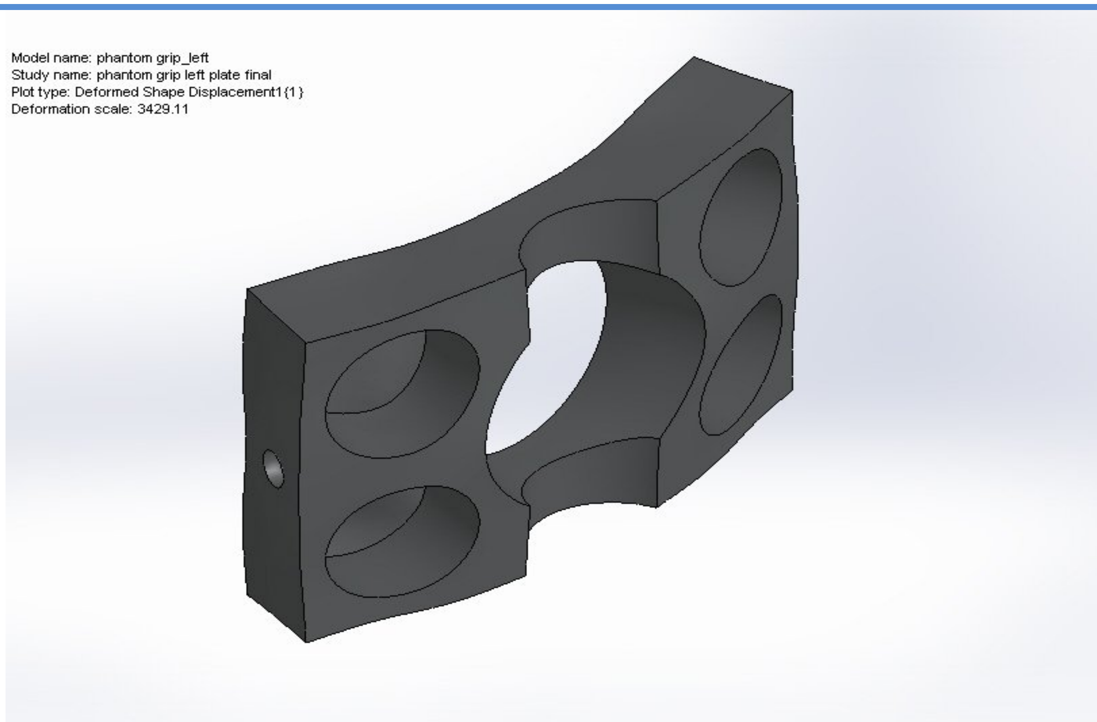
phantom grip_left-phantom grip left plate final-Displacement-Displacement1

Name	Type	Min	Max
Strain1	ESTRN: Equivalent Strain	8.01025e-008 Element: 10279	9.96014e-005 Element: 5141



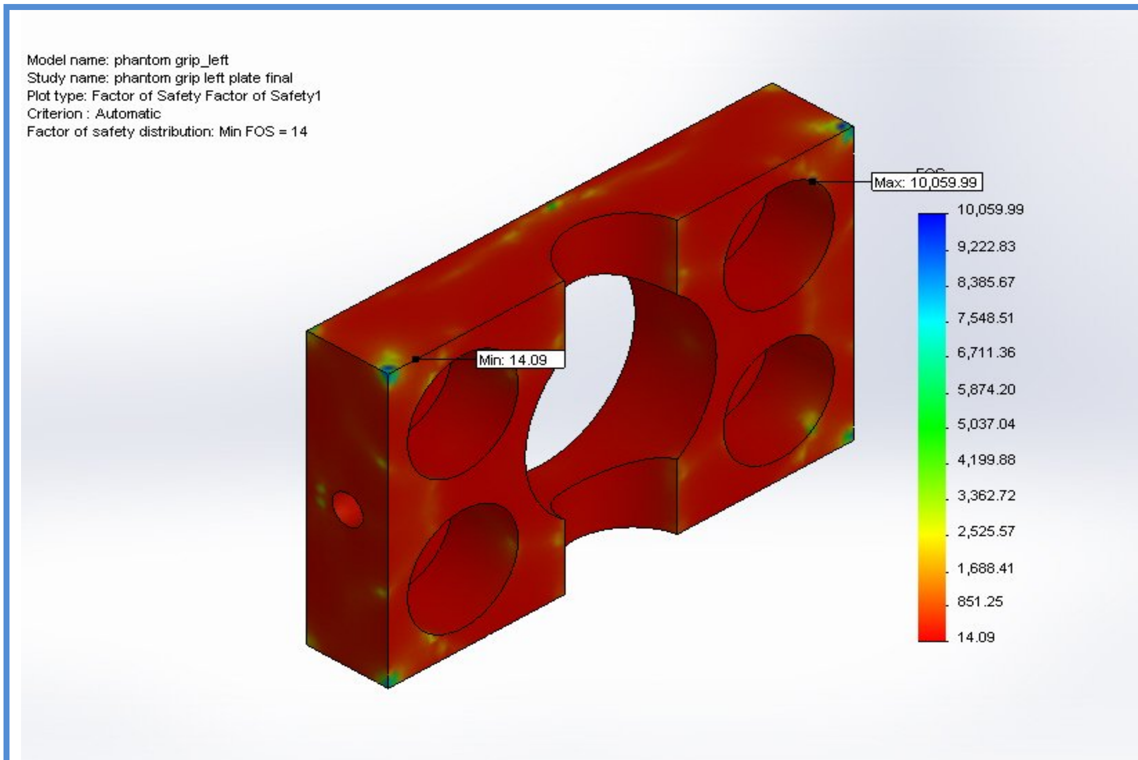
phantom grip_left-phantom grip left plate final-Strain-Strain1

Name	Type
Displacement1{1}	Deformed Shape



phantom grip_left-phantom grip left plate final-Displacement-Displacement1{1}

Name	Type	Min	Max
Factor of Safety1	Automatic	14.0923 Node: 23182	10060 Node: 2363



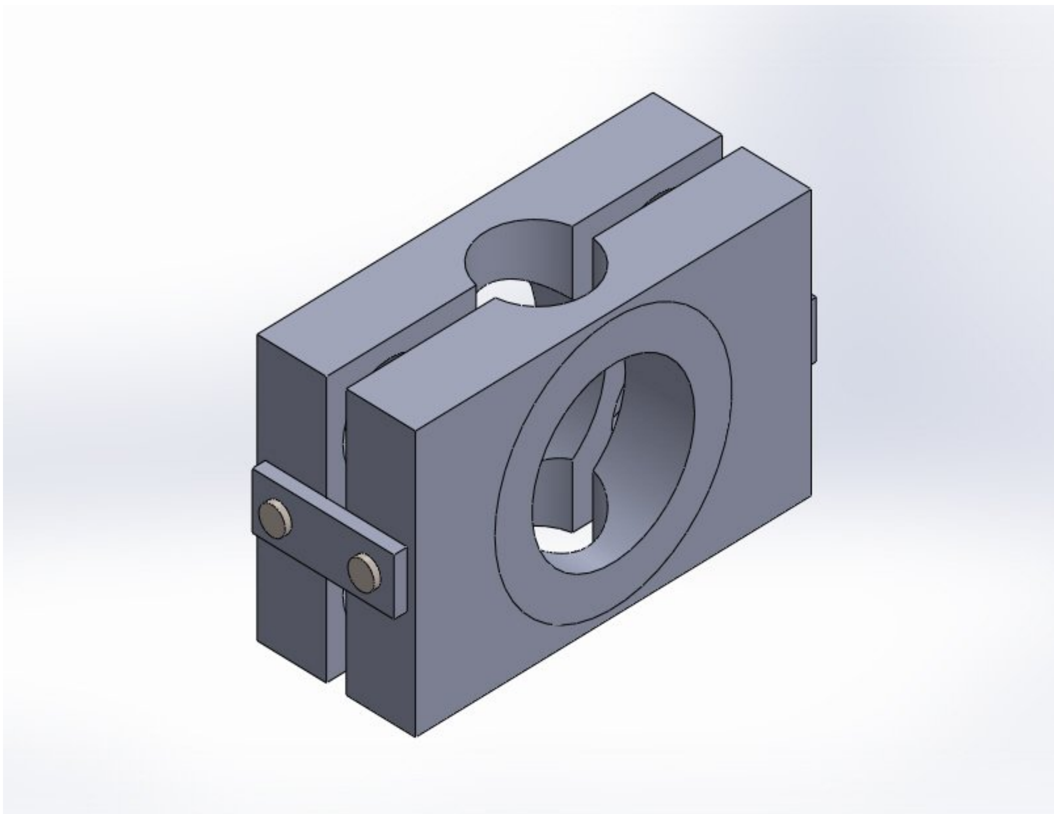
phantom grip_left-phantom grip left plate final-Factor of Safety-Factor of Safety1

Conclusion

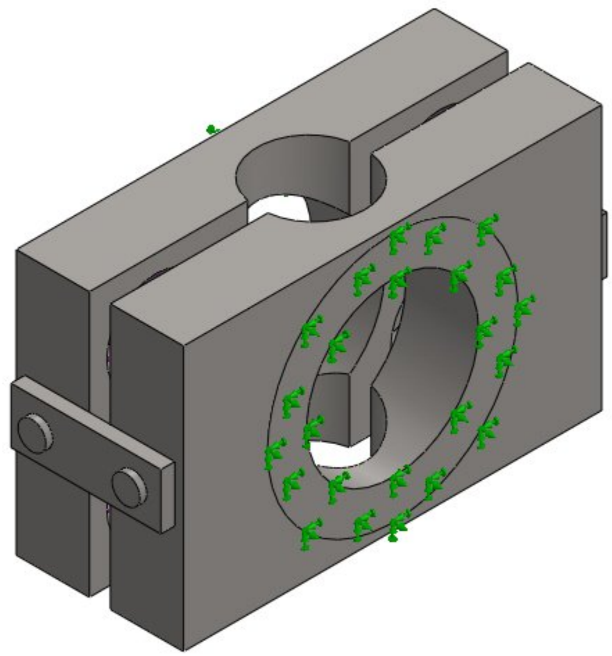
The deflection and stresses in each of the side plates is under plastic limit. The stresses produced in this structure is in safe limits with a suitable factor of safety.

Simulation of phantom grip Assembly

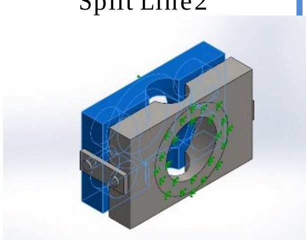
Date: Wednesday, June 10, 2015
Designer: Ali Ghaffar
Study name: Assembly Analysis
Analysis type:Static

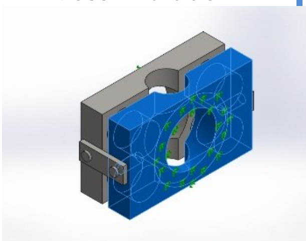
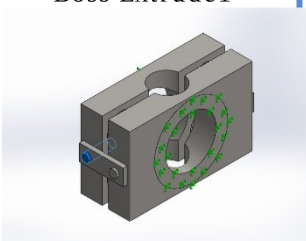
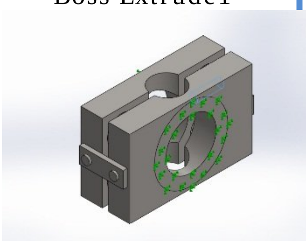
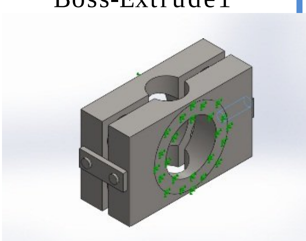
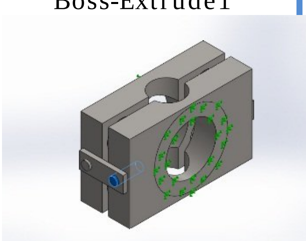


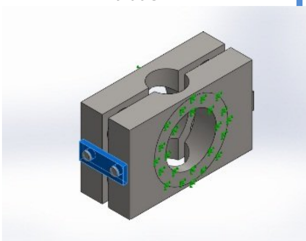
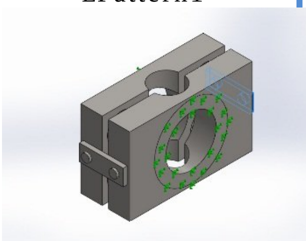
Model Information



Model name: phantom grip assambly
Current Configuration: Default

Solid Bodies			
Document Name and Reference	Treated As	Volumetric Properties	Document Path/Date Modified
Split Line2 	Solid Body	Mass:0.096832 kg Volume:1.23118e-005 m^3 Density:7865 kg/m^3 Weight:0.948953 N	D:\Academic\Semester 8\Final year project\Drawings\bevel gear cone\phantom grip_left.SLDPRT Jun 10 01:57:36 2015

Boss-Extrude2 	Solid Body	Mass:0.0957985 kg Volume:1.21804e-005 m^3 Density:7865 kg/m^3 Weight:0.938825 N	D:\Academic\Semester 8\Final year project\Drawings\bevel gear cone\phantom grip_right.SLDPRT Jun 10 03:08:27 2015
Boss-Extrude1 	Solid Body	Mass:0.00128485 kg Volume:1.63363e-007 m^3 Density:7865 kg/m^3 Weight:0.0125915 N	D:\Academic\Semester 8\Final year project\Drawings\bevel gear cone\retaining pin.SLDPRT Jun 03 17:15:41 2015
Boss-Extrude1 	Solid Body	Mass:0.00128485 kg Volume:1.63363e-007 m^3 Density:7865 kg/m^3 Weight:0.0125915 N	D:\Academic\Semester 8\Final year project\Drawings\bevel gear cone\retaining pin.SLDPRT Jun 03 17:15:41 2015
Boss-Extrude1 	Solid Body	Mass:0.00128485 kg Volume:1.63363e-007 m^3 Density:7865 kg/m^3 Weight:0.0125915 N	D:\Academic\Semester 8\Final year project\Drawings\bevel gear cone\retaining pin.SLDPRT Jun 03 17:15:41 2015
Boss-Extrude1 	Solid Body	Mass:0.00128485 kg Volume:1.63363e-007 m^3 Density:7865 kg/m^3 Weight:0.0125915 N	D:\Academic\Semester 8\Final year project\Drawings\bevel gear cone\retaining pin.SLDPRT Jun 03 17:15:41 2015

LPattern1 	Solid Body	Mass:0.00231022 kg Volume:2.93735e-007 m^3 Density:7865 kg/m^3 Weight:0.0226402 N	D:\Academic\Semester 8\Final year project\Drawings\bevel gear cone\retaining strip.SLDPRT Jun 03 17:13:57 2015
LPattern1 	Solid Body	Mass:0.00231022 kg Volume:2.93735e-007 m^3 Density:7865 kg/m^3 Weight:0.0226402 N	D:\Academic\Semester 8\Final year project\Drawings\bevel gear cone\retaining strip.SLDPRT Jun 03 17:13:57 2015

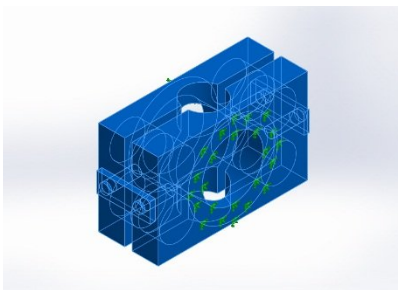
Study Properties

Study name	assambly
Analysis type	Static
Mesh type	Solid Mesh
Thermal Effect:	On
Thermal option	Include temperature loads
Zero strain temperature	298 Kelvin
Include fluid pressure effects from SolidWorks Flow Simulation	Off
Solver type	FFEPlus
Inplane Effect:	Off
Soft Spring:	Off
Inertial Relief:	Off
Incompatible bonding options	Automatic
Large displacement	Off
Compute free body forces	On
Friction	Off
Use Adaptive Method:	Off
Result folder	SolidWorks document (D:\Academic\Semester 8\Final year project\Drawings\bevel gear cone)

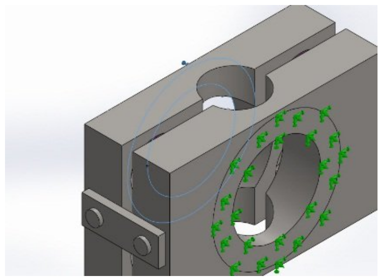
Units

Unit system:	SI (MKS)
Length/Displacement	mm
Temperature	Kelvin
Angular velocity	Rad/sec
Pressure/Stress	N/m^2

Material Properties

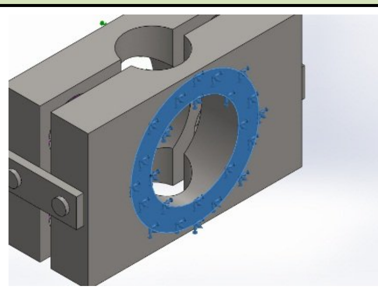
Model Reference	Properties	Components
	Name: ANSI 4041 Model type: Linear Elastic Isotropic Default failure criterion: Max von Mises Stress Yield strength: 4.171e+008 N/m^2 Tensile strength: 6.55e+008 N/m^2 Compressive strength: 6.55e+008 N/m^2 Elastic modulus: 2e+011 N/m^2 Poisson's ratio: 0.29 Mass density: 7865 kg/m^3 Shear modulus: 8e+010 N/m^2 Thermal expansion coefficient: 1.23e-005 /Kelvin	SolidBody 1(Split Line2)(phantom grip_left-1), SolidBody 1(Boss-Extrude2)(phantom grip_right-1), SolidBody 1(Boss-Extrude1)(retaining pin-1), SolidBody 1(Boss-Extrude1)(retaining pin-2), SolidBody 1(Boss-Extrude1)(retaining pin-3), SolidBody 1(Boss-Extrude1)(retaining pin-4), SolidBody 1(LPattern1)(retaining strip-1), SolidBody 1(LPattern1)(retaining strip-2)
Curve Data:N/A		

Loads and Fixtures

Fixture name	Fixture Image	Fixture Details
Fixed-1		Entities: 1 face(s) Type: Fixed Geometry

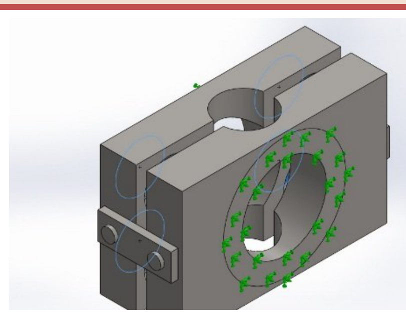
Resultant Forces

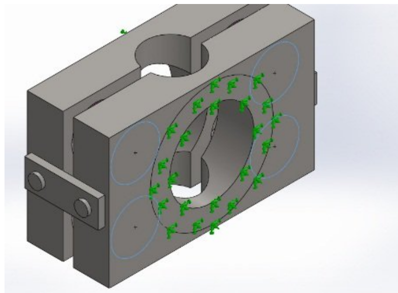
Components	X	Y	Z	Resultant
Reaction force(N)	766.804	0.552707	-4.54869	766.818
Reaction Moment(N·m)	0	0	0	0

Fixture name	Fixture Image	Fixture Details
Fixed-2		Entities: 1 face(s) Type: Fixed Geometry

Resultant Forces

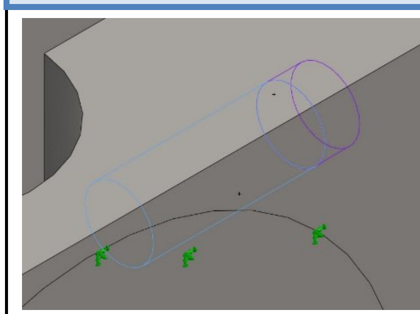
Components	X	Y	Z	Resultant
Reaction force(N)	-766.73	-0.55272	4.54868	766.743
Reaction Moment(N·m)	0	0	0	0

Load name	Load Image	Load Details
Force-1		Entities: 4 face(s) Type: Apply normal force Value: 250 N

Force-2		Entities: 4 face(s) Type: Apply normal force Value: 250 N

Connector Definitions

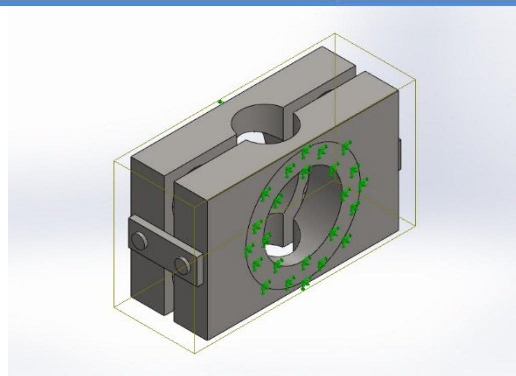
Pin/Bolt/Bearing Connector

Model Reference	Connector Details	Strength Details
 Pin Connector-1	Entities: 2 face(s) Type: Pin Connection type: With key (No rotation) Connection type: With retaining ring (No translation)	No Data

Connector Forces

Type	X-Component	Y-Component	Z-Component	Resultant
Axial Force (N)	-0	-0	-8.1767	-8.1767
Shear Force (N)	-34.174	0.14641	0	34.174
Torque (N·m)	0	0	2.8546e-005	2.8546e-005
Bending moment (N·m)	0.00034171	0.12288	0	0.12288

Contact Information

Contact	Contact Image	Contact Properties
Global Contact		Type: Bonded Components: 1 componen t(s) Options: Compatibl e mesh

Mesh Information

Mesh type	Solid Mesh
Mesher Used:	Standard mesh
Automatic Transition:	Off
Include Mesh Auto Loops:	Off
Jacobian points	4 Points
Element Size	4.50383 mm
Tolerance	0.225192 mm
Mesh Quality	High

Resultant Forces

Reaction Forces

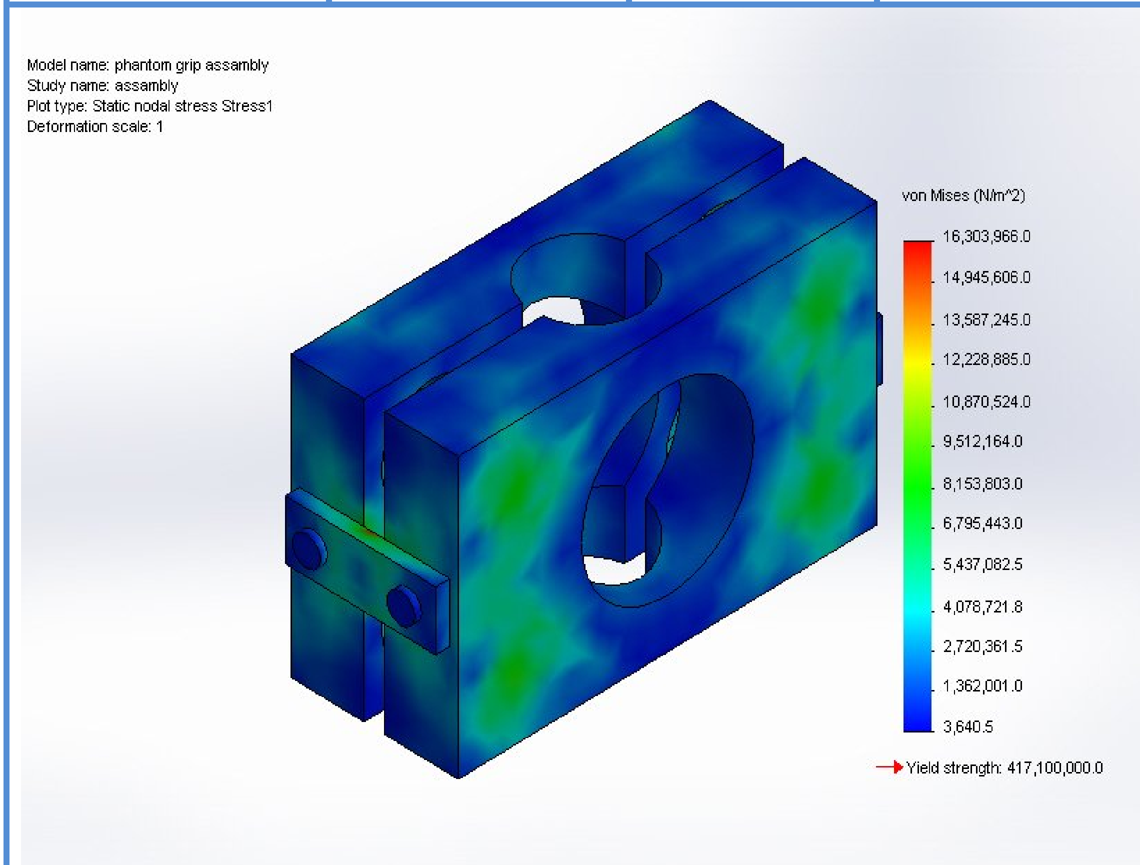
Selection set	Units	Sum X	Sum Y	Sum Z	Resultant
Entire Model	N	0.0743589	-1.60933e-005	1.66893e-006	0.0743589

Reaction Moments

Selection set	Units	Sum X	Sum Y	Sum Z	Resultant
Entire Model	N·m	0	0	0	0

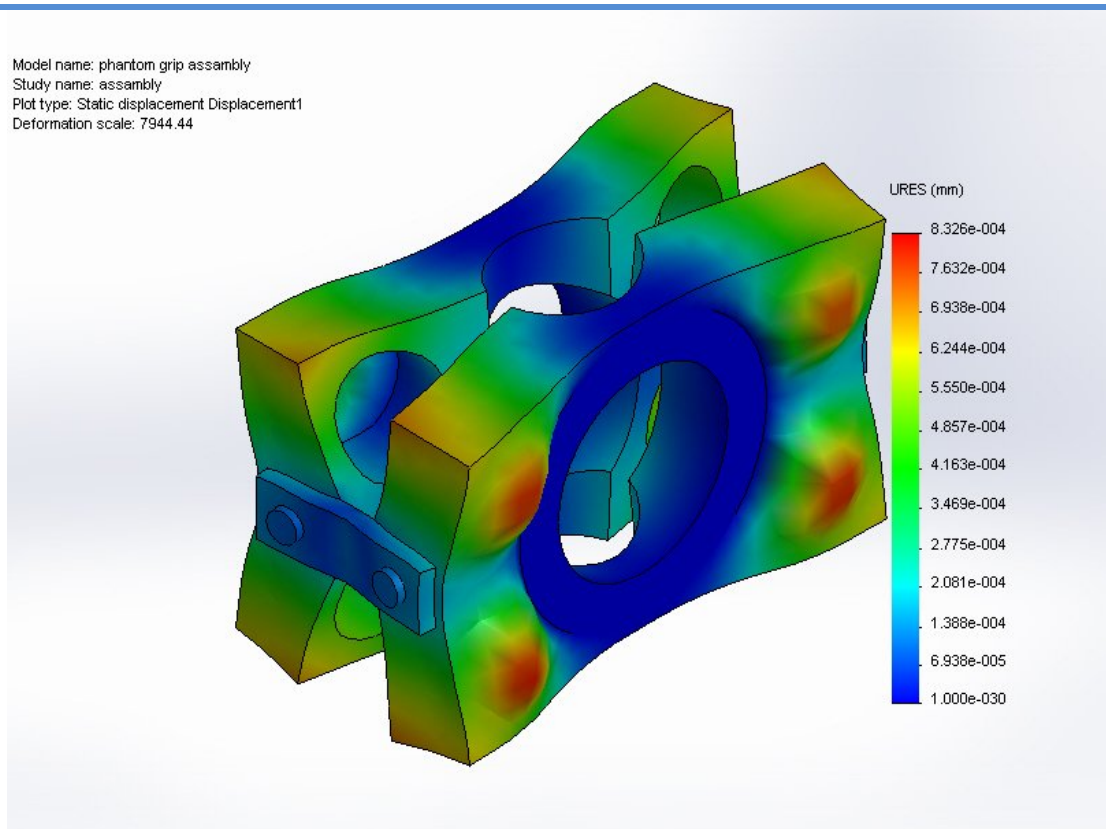
Study Results

Name	Type	Min	Max
Stress1	VON: von Mises Stress	3640.49 N/m^2 Node: 7573	1.6304e+007 N/m^2 Node: 8336



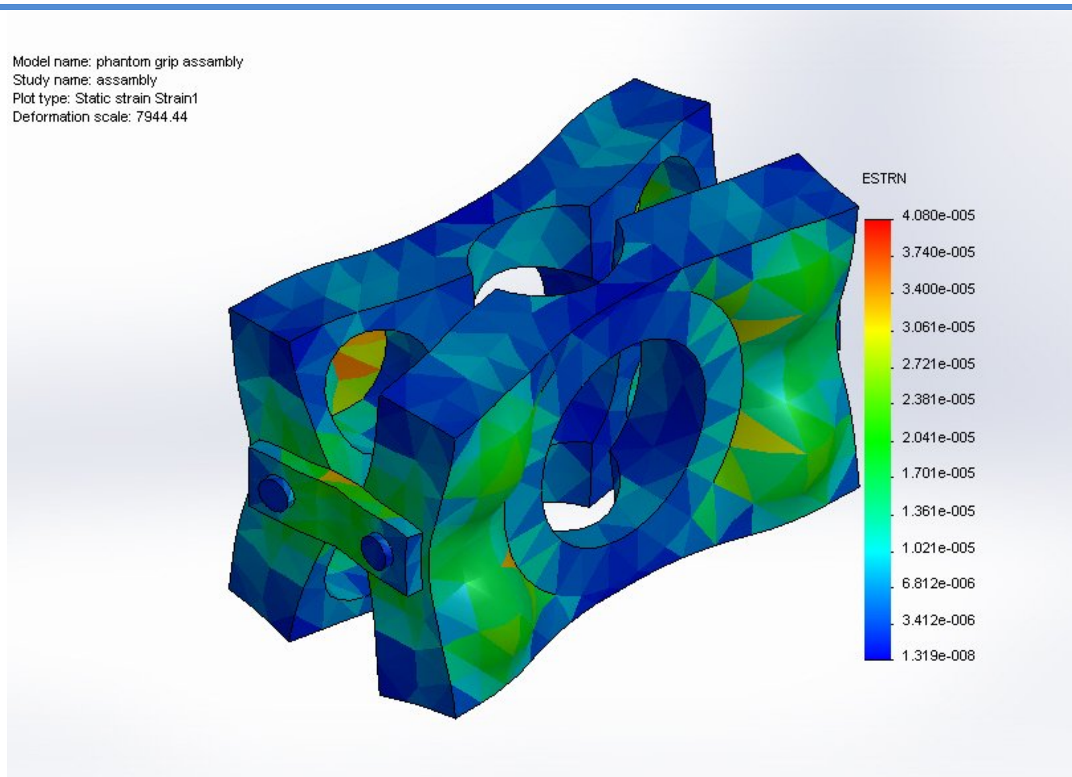
phantom grip assambly-assambly-Stress-Stress1

Name	Type	Min	Max
Displacement1	URES: Resultant Displacement	0 mm Node: 225	0.000832574 mm Node: 4840



phantom grip assambly-assambly-Displacement-Displacement1

Name	Type	Min	Max
Strain1	ESTRN: Equivalent Strain	1.31916e-008 Element: 4021	4.08032e-005 Element: 4646



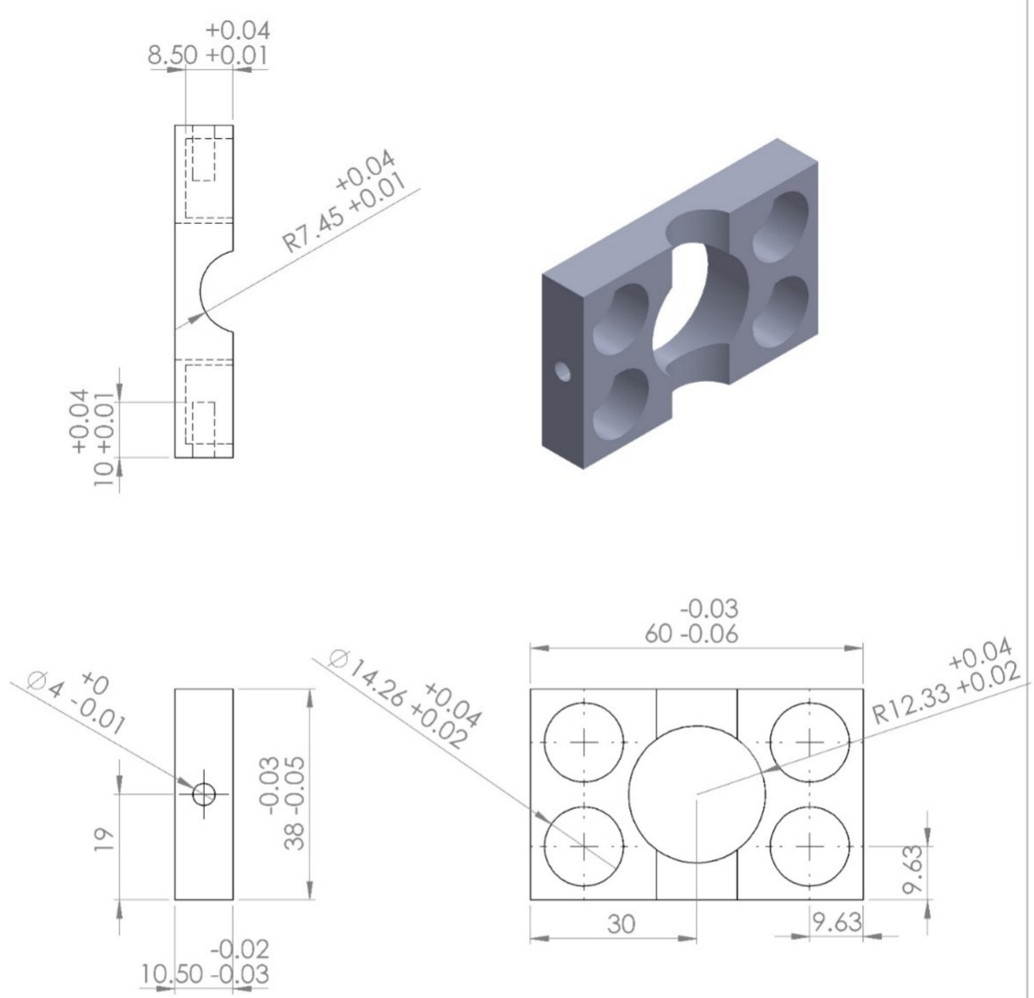
phantom grip assambly-assambly-Strain-Strain1

Conclusion

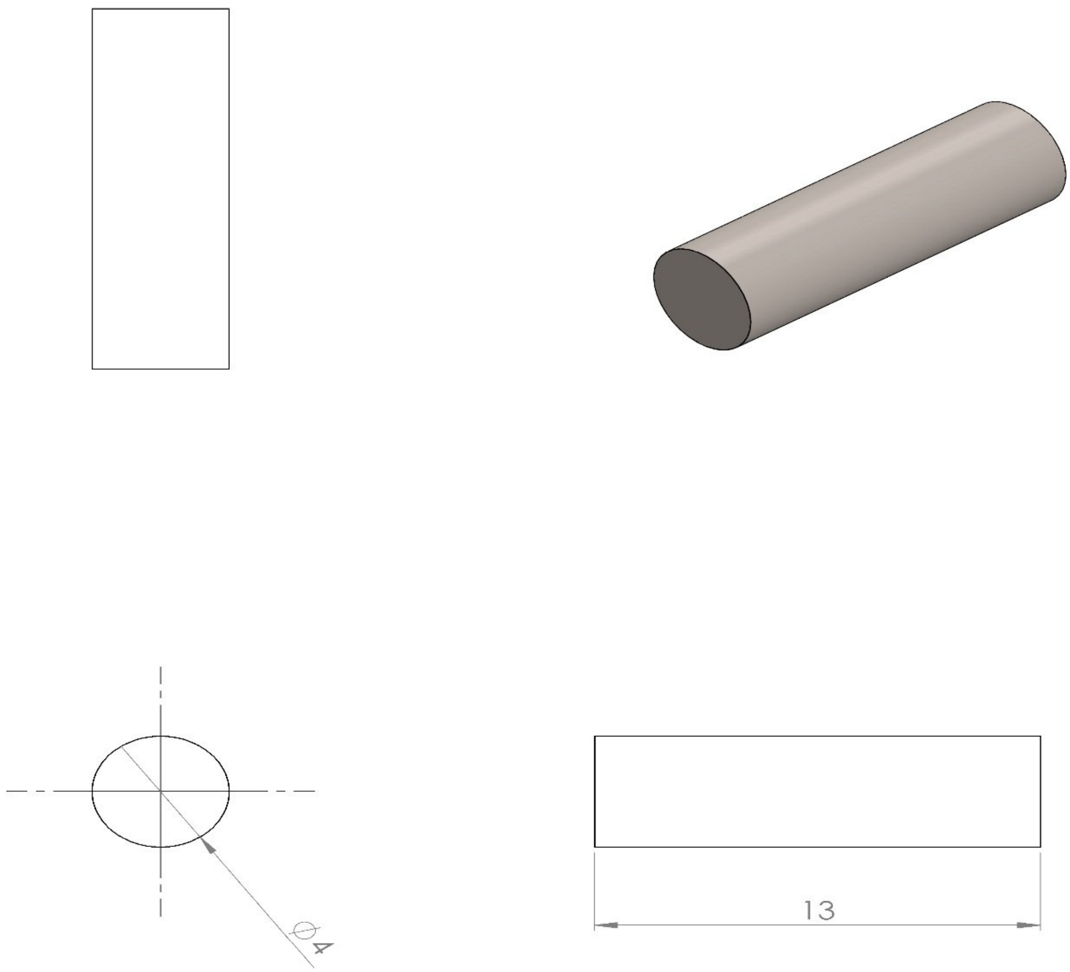
The whole assembly works perfectly fine under stress. The maximum stress and displacements are well under safe limits with suitable factor of safety.

Drawings

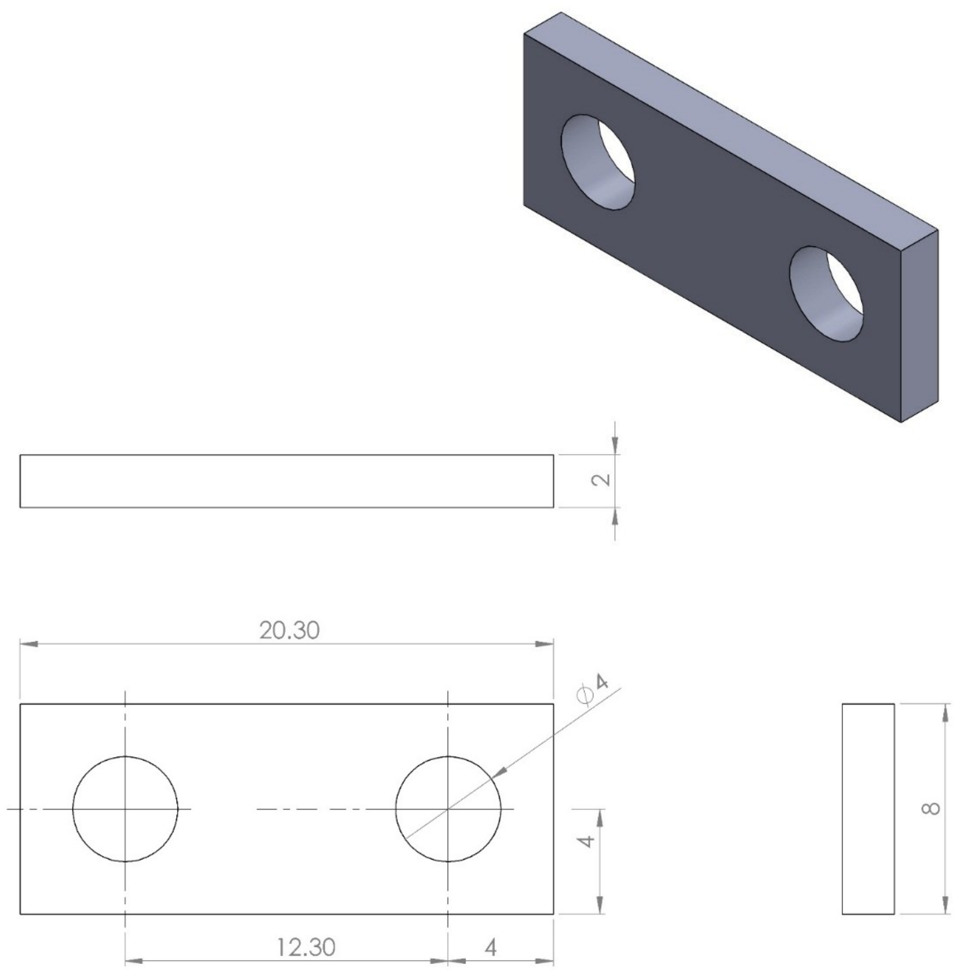
1. Side plate



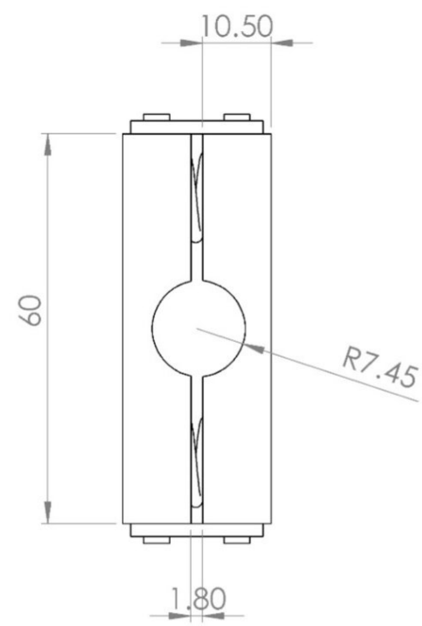
2. Retaining Pin

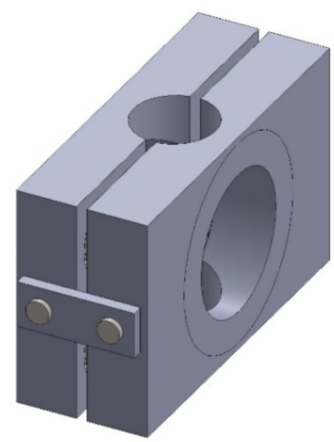


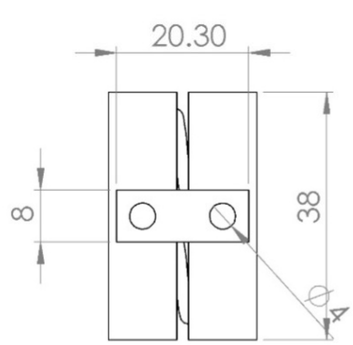
3. Retaining Strip

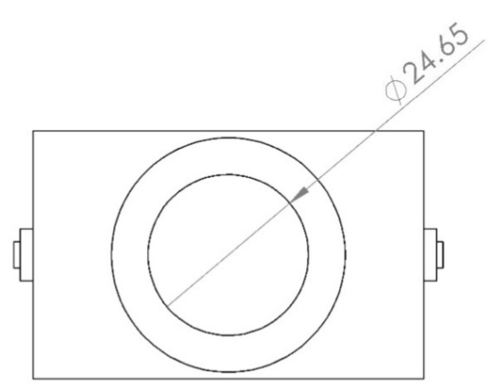


4. Assembly Drawing of Phantom Grip LSD









5. Exploded View