Mathematical Modeling of Two Dimensional Flows of Nanofluid

By
Aasia Batool

Thesis submitted for the requirements of the degree
MS in **Mathematics**

Department of Mathematics,
School of Natural Sciences,
National University of Sciences and Technology,
H-12, Islamabad, Pakistan
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Aasia Batool
Reg No. 330986

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Supervised by
Dr. Muhammad Asif Farooq

Department of Mathematics,
School of Natural Sciences,
National University of Sciences and Technology,
H-12, Islamabad, Pakistan

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MS THESIS WORK

We hereby recommend that the dissertation prepared under our supervision by: **Aasia Batool**, Regn No. **00000330986** Titled: **"Mathematical Modeling of Two Dimensional Flows of Nanofluid"** accepted in partial fulfillment of the requirements for the award of **MS** degree.

Examination Committee Members

1. Name: PROF. MERAJ MUSTAFA HASHMI Signature: ____________

2. Name: DR. MUJEEB UR REHMAN Signature: ____________

Supervisor's Name: DR. M. ASIF FAROOQ Signature: ____________

Head of Department

30/08/2022
Date

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Dean/Principal

Contents

1 Introduction **1**

1.1 Fluid 1

1.2 Stress 1

1.3 Steady Flow 2

1.4 Unsteady Flow 2

1.5 Uniform Flow 2

1.6 Non-Uniform Flow 2

1.7 Laminar Flow 2

1.8 Turbulent Flow 3

1.9 Compressible flow 3

1.10 Incompressible Flow 3

1.11 Newtonian Fluids 4

1.12 Non-Newtonian Fluids 4

1.13 Viscosity 4

1.14 Kinematic Viscosity 4

1.15 Activation Energy 5

1.16 Viscous Dissipation 5

1.17 Non-Dimensional Parameters 6

1.17.1 Reynolds Number (Re) 6
1.17.2 Prandtl number(Pr) 6
1.17.3 Nusselt number (Nu) 6
1.17.4 Sherwood Number (Sh) 7
1.17.5 Magnetic parameter (M) 7
1.17.6 Skin Friction Coefficient 8
1.17.7 Eckert Number (Ec) 8
1.17.8 Thermal Conductivity 8
1.18 Literature Survey 9

2 Two-Dimensional Slip Flow of Jeffrey Nanofluid Over a Stretching Sheet 11
2.1 Introduction 11
2.2 Basic Equations 11
2.3 Results and Discussions 15

3 Flow of Jeffrey Nanofluid in a Porous Medium Containing Microorganisms 26
3.1 Introduction 26
3.2 Basic Equations 27
3.3 Results and Discussion 29

4 Conclusions/Summary 53

Bibliography 55

List of Figures

2.1 Impact of several values of λ_2 on the velocity curve $\frac{df}{d\eta}$ 17

2.2 Impact of several values of λ_3 on the velocity curve $\frac{df}{d\eta}$ 18

2.3 Impact of several values of β_2 on the velocity curve $\frac{df}{d\eta}$ 19

2.4 Graph of temperature for different choices of Nt 20

2.5 Graph of temperature for different choices of Nb 21

2.6 Graph of temperature for different choices of Ec 22

2.7 Graph of concentration for different choices of Sc 23

2.8 Graph of concentration for different choices of $k1$ 24

2.9 Graph of concentration for different choices of Nt 25

3.1 Impact of several values of λ_2 on the velocity curve $\frac{df}{d\eta}$ 36

3.2 Impact of several values of λ_3 on the velocity curve $\frac{df}{d\eta}$ 37

3.3 Impact of several values of M on the velocity curve $\frac{df}{d\eta}$ 38

3.4 Impact of several values of Kp on the velocity curve $\frac{df}{d\eta}$ 39

3.5 Impact of several values of λ_3 on the temperature curve 40

3.6 Impact of several values of M on the temperature curve 41

3.7 Impact of several values of Nb on the temperature curve 42

3.8 Impact of several values of Pr on the temperature curve 43

3.9 Impact of several values of Rd on the temperature curve 44

3.10 Impact of several values of S on the temperature curve 45
3.11 Concentration curve for three different values of Nt 46
3.12 Concentration curve for three different values of Nb 47
3.13 Concentration curve for three different values of Sc 48
3.14 Impact of several values of Nb on the microorganism curve $\chi(\eta)$ 49
3.15 Impact of several values of Nt on the microorganism curve $\chi(\eta)$ 50
3.16 Impact of several values of Rd on the microorganism curve $\chi(\eta)$ 51
3.17 Impact of several values of Sc on the microorganism curve $\chi(\eta)$ 52

List of Tables

1.1 Few Values of different fluid's viscosities at room temperature 5

1.2 Few values of Prandtl Number for different fluids 7

2.3.1 Effect of several values of parameters λ_2, λ_3 and β_2 on skin friction by considering Pr=7, Sc=0.1, Nb=0.2, Nt=0.1, n=0.2, k_1=0, δ=1, E_1=0.4, Ec=0.1 16

2.3.2 Effect of several values of parameters Ec, Nb and Nt, on Nusselt number by considering , λ_2=4, λ_3=8, Pr=7, Sc=0.2, β_2=0.1, n=0.2, k_1=0, δ=1, E_1=0.4 16

2.3.3 Effect of several values of parameters Sc, Nt and E_1 on Sherwood Number by considering ,λ_2=5, λ_3=5, Pr=7 , Nb=0.2, β_2=0.1, n=0.2, k_1=0, δ=1, Ec=0.2 16

3.3.1 Effect of different parameters on C_{fx} by considering Pr=10, Sc=0, Nb=1, Nt=1, Sb=0.1, Pe=0.9, S=0.1, Rd = 1.5, Nd=1, Ec=0.5 (bvp4c) 32

3.3.2 Effect of different parameters on Nu_x by considering Pr=10, Sc=0.5, Nt=0.1, Sb=0.1, Pe=0.9, Nd=1 (bvp4c) 33

3.3.3 Effect of different parameters on Sh_x by considering Pr=10, Sb=0.1, Pe=0.9, Nd=1 (bvp4c) 34

3.3.4 Effect of different parameters on Nn_x by considering Pr=10, λ_2=2, λ_3=3, kp=0.2 (bvp4c) 35

Abstract

The flourishing advancement in the nano-technology and the thermal engineering, more concentration is required on the thermal properties of nanoparticles that grant numerous uses in technological, industrial and medical sciences. Significant applications of nano-materials incorporate enhanced convective transfer of heat, heat energy, transportation, electronic cooling, solar cells, textiles and automobile etc. On this end present work concerned with 2D, incompressible, steady and no slip condition on Jeffrey nanofluid containing microorganisms using a porous medium. Transfer of mass and heat flow is explained in the existence of viscous dissipation. Assuming similar flow exists, the PDEs are returned in the form of ODEs. The formulated differential system is solved numerically using bvp4c technique. Simulations are achieved for several values of embedded parameters which include thermophoretic parameter (Nt) and Brownian motion parameter (Nb). Parameter's effects are considered in the mathematical model. The graphs for the velocity, concentration, microorganism, and temperature profiles are used to analyse the effects of many variables on the behaviour of fluid flows.

Chapter 1

Introduction

This chapter discusses some basic definitions and non-dimensional parameters.

1.1 Fluid

Under the action of shearing force the substance which has a capability to flow and deforms itself is called a fluid. Shear stress may be small or large.
Liquids and gases are two classes of fluids. Examples: water, mercury, oil, gasoline, petrol, honey, lipstick and air etc.

1.2 Stress

The force per unit area (F/A) is known as stress. It is represented by τ.It can be measured in N/m^2 or Pascal.

1.3 Steady Flow

Steady flow means the velocity of fluid and the other fluid properties at the given location is not changing with time. It can be expressed mathematically as

$$\frac{\partial \xi}{\partial t} = 0,$$

where ξ is any parameter.

1.4 Unsteady Flow

Unsteady flow means the velocity of fluid and the other fluid properties at the given location is changing with time. It can be expressed mathematically as

$$\frac{\partial \xi}{\partial t} \neq 0,$$

where ξ is any parameter.

1.5 Uniform Flow

Uniform flow means the velocity and the fluid properties at a given time are not changing with position.

1.6 Non-Uniform Flow

Non uniform flow means the velocity and the fluid properties at a given time are changing with position.

1.7 Laminar Flow

Laminar flow or streamline flow in tubes occurs when a fluid moves in parallel layers or in stream line, with no disruption between the layers. The laminar flow generally occurs in the fluid flowing with low velocity. Under most practical conditions, the flow in a circular pipe is laminar for $Re < 2000$. Oil flow through a thin tube and blood through capillaries is laminar.

1.8 Turbulent Flow

Fluid particles move randomly or irregular fluctuations and mixing in this type of flow. The turbulent flow behaviour that is seen when a fluid is moving quickly. When $Re > 4000$, the flow is typically turbulent. Common examples of turbulent flow are oil flow in pipe lines and current ocean etc.

1.9 Compressible flow

Compressible fluid is that type of fluid in which the action of pressure and temperature causes change in volume and density. In this flow point to point changes are observed in fluid. A fluid flow is compressible if Mach number is greater than 0.3. Examples: gases.

1.10 Incompressible Flow

Incompressible fluid is that type of fluid in which the action of pressure and temperature causes no change in volume and density. In this flow, density of fluid is constant. A fluid flow is incompressible if Mach number is less than 0.3. In general hydraulic fluids are considered incompressible.

1.11 Newtonian Fluids

Newtonian fluids are that type of fluids which have a relation that the rate of deformation varies directly to the applied shear stress.It is expressed as,

$$\tau = \mu \frac{du}{dy},$$

where μ is the dynamic viscosity and $\frac{du}{dy}$ is the fluid's strain rate. This law is known as Newton's law of viscosity. Examples are glycerol, motor oil and alcohol etc.

1.12 Non-Newtonian Fluids

The non-Newtonian fluids are the fluids that do not follow the Newton's law of viscosity having constant viscosity that is independent of stress. These fluids change their flow behaviour under stress called viscosity. Examples of non-Newtonian fluids are custard, toothpaste, honey, blood and paint etc.

1.13 Viscosity

The Resistance against the motion of fluid is called viscosity. It is expressed as μ. It is also known as dynamic viscosity or absolute viscosity.

1.14 Kinematic Viscosity

It is the internal resistence of fluid to flow. It is defined as the absolute viscosity divided by density. It is represented by the symbol ν. Mathematically

$$\nu = \frac{\mu}{\rho},$$

Table 1.1: Few Values of different fluid's viscosities at room temperature

Substance	μ(mPas)
Air	10^{-2}
Benzene	0.65
Water	1
Molten sodium chloride(1173 K)	1.01
Ethyl alcohol	1.20
Mercury	1.55
Olive oil	100
Castor oil	600
Glycerine(293K)	1500
Honey	10^4
Corn syrup	10^5
Bitumen	10^{11}
Molten glass	10^{15}

1.15 Activation Energy

The smallest amount of energy required to cause a specific chemical reaction or chemical transformation in compounds (or atoms or molecules) is known as activation energy. The unit for measuring the activation energy of a reaction is Joules per mole.

1.16 Viscous Dissipation

The process in which the action of shear forces is transmuted into heat by work done on the fluid's adjacent layers is defined as viscous dissipation, which has a major impact on that fluids which have high viscosity such as polymers and oils.

1.17 Non-Dimensional Parameters

1.17.1 Reynolds Number (Re)

Reynolds number is the result of dividing the inertial force by the viscous force. Mathematically,

$$Re = \frac{uL}{\nu} = \frac{\rho uL}{\mu},$$

here ρ , u and L represents the fluid's density, flow velocity and characteristic length respectively. The kinematic and dynamic viscosities are ν and μ. The Reynolds number (Re) help out to foresee the flow patterns for different fluid flow situations. At low Reynolds number, flow be biased by laminar flow, while at high Reynolds numbers flow shows turbulent behaviour.

1.17.2 Prandtl number(Pr)

The Prandtl number refers to the ratio between molecular diffusivity of momentum and molecular diffusivity of heat. Mathematically,

$$Pr = \frac{\nu}{\alpha} = \frac{c_p\mu}{k},$$

where c_p represents the specific heat and k represents the thermal conductivity. Thermal diffusivity dominates when Pr is less than 1 and kinematic diffusivity dominates when Pr is greater than 1.

1.17.3 Nusselt number (Nu)

It expresses the link between convective hear transfer and conductive heat transfer across a boundry.

$$Nu = \frac{hL}{k},$$

Table 1.2: Few values of Prandtl Number for different fluids

Fluids	Pr
Mixture of noble gases	$0.16 - 0.17$
Air at room temperature	0.71
Water	$1 - 10$
Sea water	13.4
Oils	$50 - 2000$

here h, L and k represents the flow's convective heat transfer coefficient, characteristic length and fluid's thermal conductivity respectively. When convective effects are more than conductive effects then we get large value of Nusselt number i.e, $Nu >> 1$, while $Nu << 1$ when conductive effects are more than convective effects.

1.17.4 Sherwood Number (Sh)

The Sherwood Number is a number which has no dimension. It is described as the ratio between the rate of mass diffusion and the convective mass transfer. The Sherwood Number is defined as

$$Sh = \frac{\text{convective mass transfer}}{\text{mass diffusion rate}} = \frac{k_m}{D/L},$$

1.17.5 Magnetic parameter (M)

The magnetic parameter is expressed as a Lorentz force to the inertial force ratio. It is also called magnetic interaction parameter. Mathematically,

$$M = \frac{\sigma B_0^2}{\rho a},$$

where a is the plate's horizontal velocity, $sigma$ is its electrical conductivity, and $B0$ is the magnetic field's strength.

1.17.6 Skin Friction Coefficient

It is expressed as the ratio of shear stress at the wall to the free stream dynamic pressure.

$$C_f = \frac{\tau_w\big|_{y=0}}{\frac{1}{2}\rho u_w^2},$$

here C_f represents skin friction coefficient, ρ denotes the density of fluid, v is the free stream speed. τ_w is the skin shear stress and $\frac{\rho}{2}u_w^2$ is the free stream dynamic pressure.

1.17.7 Eckert Number (Ec)

The Eckert number is a number which has no dimension. Convective mass transfer divided by the heat dissipation potential is expressed as Eckert number.
The Eckert number expresses the measure of the kinetic energy of the fluid flow relative to the boundary layer enthalpy difference. It is used to characterise heat dissipation in high-speed flows for which viscous dissipation is significant.
Mathematically Eckert number is expressed as :

$$Ec = \frac{\text{Convective mass transport}}{\text{Heat dissipation}} = \frac{u^2}{c_p \Delta T},$$

where u denotes the flow rate, c_p stands for specific heat capacity, and ΔT denotes the temperature difference between the local temperature and the wall.

1.17.8 Thermal Conductivity

It is the relationship between the heat flow rate per unit area $\vec{q}$ and $\vec{\nabla}$T. Fourier's law of heat conduction is another name for this relationship.

$$\vec{q} = -k\vec{\nabla}T,$$

and this is valid for fluids and solids. where $\vec{q}$ represents heat flux, k is the thermal conductivity and $\vec{\nabla}T$ is the temperature gradient. The inverse of thermal conductivity is thermal resistivity.

1.18 Literature Survey

Non-Newtonian fluids ascribe the significance in technological, industrial and medical era of sciences. Scientists have more concentration by scrutinizing distinct rheological mechanism. Many researchers submitted a theory to explain the several properties of gases. Some more fresh investigations reveal the non-Newtonian material's rheological mechanism that can be seen in Refs [1, 2, 3, 4, 5]. The nano-materials are those materials that having a size of 1-100 nm. Many researchers has worked alot on the study of nano-materials and discussed the properties because of significance of applications in different fields. There are numerous implementations in important fields like micro manufacturing, engineering and textiles etc. Nanofluids have a wide range of uses, including in lubricants, electronics, automobiles, thermal sciences, and healthcare. The nano-particles play an important role to enhance the heat transfer rate. The nano-particles increase heat transfer phenomenon when these are added to base fluid. Many researchers have worked and till working on it because of its wide uses in countless fields. Choi and Eastman [6] was the first Mathematician who gave the concept of nanofluid flow. The references for nanofluid are listed in [7, 8, 9]. Khan at al[10] studied the effects of mass and heat transfer on the diffusive flow of nanofluids in porous medium. The mechanism by which the phenomena of heat transmission occurs is thermal radiation. Heat transfer has a great part in machinery efficiency, combustion, reaction kinetics and in solar system. The different fluid properties with thermal radiations discussed in [11]. Activation energy and thermal radiation effects and results are discussed by RamReddy and Naveen[12].[13] summarised the nanopaeticle's convective thermal transport in the existence of slip effects and porous surface. Nanofluids have higher values of thermal conductivity than base fluids reported in Lee et al [14] Ramesh [15] discussed the Poiseuille and Couette flows. Jeffrey fluid flow in porous medium has been discussed by Kumar and Coreseachers [16].

Bio convection nanofluid with slip flow over wavy surface by using the applications of nano-biofuel is explored in[17]. Bioconvection flow with gyrotactic microorganisms is discussed by Khan and Makinde [18]. Hydromagnetic fluid flow in microorganisms is discussed in [19]. [20] investigated the slip flow effects in the existence of microorganisms.

Chapter 2

Two-Dimensional Slip Flow of Jeffrey Nanofluid Over a Stretching Sheet

2.1 Introduction

This work deals with two-dimensional, steady and incompressible slip flow of Jeffrey nanofluid over a stretching sheet. Similarity variables are used to transform (PDEs) into (ODEs). The solutions have been obtained numerically through *bvp4c* technique. The solutions depend on several important parameters such as retardation parameter (λ_3),, thermophoretic parameter (Nt), Prandtl number (Pr), activation energy parameter (E1), Brownian motion parameter (Nb) and Schmidt number (Sc). Behaviour of several interesting parameters on the fluid's flow fields is thoroughly presented and discussed. The results are found in excellent agreement with the existing studies of literature.

2.2 Basic Equations

Steady, incompressible and 2D flow of Jeffrey nanofluid is discussed here. Mass and heat transport flow are explained by viscous dissipation and activation energy. The velocity components are represented by (u,v) in the Cartesian coordinate plane. The formulated equations are given below:

[21]

$$u_x + v_y = 0, \tag{2.1}$$

$$uu_x + vu_y = \frac{\nu}{1+\lambda_2}\left[u_{yy} + \lambda_1\left(uu_x u_{yy} + u_y u_{xy} - u_x u_{yy} + vu_{yyy}\right)\right], \tag{2.2}$$

$$\begin{aligned} uT_x + vT_y = \frac{k_f}{(\rho c_p)_f}(T_{yy}) + \frac{\mu_f}{(\rho c_p)_f\,(1+\lambda_2)}\left[(u_y)^2 + \lambda_1\left(vu_y(u_{yy}) + uu_y u_{yx}\right)\right] + \\ \frac{(\rho c_p)_s}{(\rho c_p)_f}\left[D_B T_y C_y + \frac{D_T}{T_\infty}\left(T_y\right)^2\right], \end{aligned} \tag{2.3}$$

$$uC_x + vC_y = D_B(C_{yy}) - k_0^2\left(C - C_\infty\right)\left(\frac{T}{T_\infty}\right)^n Exp\left[\frac{-E_a}{kT}\right], \tag{2.4}$$

with boundary conditions:

$$\begin{aligned} At\quad y = 0,\quad u = ax + \frac{\beta_1}{1+\lambda_2}\left[u_y + \lambda_1\left(uu_{yx} + v(u_{yy})\right)\right], \\ v = 0, T = T_w, C = C_w, \\ At\quad y \to \infty\quad, u \to 0, T \to T_\infty, C \to C_\infty. \end{aligned} \tag{2.5}$$

In above expressions (x, y), (u, v), ρ_f, μ_f, λ_2, λ_1, ν, , β_1, c_p, D_B, T, T_∞, C, C_∞, T_w, k_0^2, a, $(-1 < n < 1)$, k_f, k, E_a and D_T act for Cartesian coordinates, velocity vectors, density, dynamic viscosity, ratio of relaxation to retardation time, retardation time, kinematic viscosity, velocity slip coefficient, specific heat, Brownian constant, temperature, ambient temperature, concentration of species, ambient concentration, surface temperature, chemical reaction, stretching rate, fitted constant, thermal conductivity, Boltzmann constant, activation energy coefficient and thermophoretic coefficient.

Considering the dimensionless variables:

$$u = axf'(\eta), v = -\sqrt{av}f, \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \eta = \sqrt{\frac{a}{v}}y \tag{2.6}$$

we arrive,

$$\frac{1}{1+\lambda_2}\left(f''' - \lambda_3\left(ff^{(iv)} - f''^2\right)\right) + ff'' - f'^2 = 0, \tag{2.7}$$

$$\frac{1}{Pr}\theta'' + f\theta' + Nb\theta'\phi' + Nt\theta'^2 + \frac{Ec}{1+\lambda_2}\left[f''^2 + \lambda_3\left(f'f''^2 - ff''f'''\right)\right] = 0, \tag{2.8}$$

$$\frac{1}{Sc}\phi'' + f\phi' + \frac{1}{Sc}\frac{Nt}{Nb}\theta'' - k_1\phi\left(1+\delta\theta\right)^n Exp\left[\frac{-E1}{1+\delta\theta}\right] = 0, \tag{2.9}$$

$$\begin{aligned}
&f'(0) = 1 + \frac{\beta_2}{1+\lambda_2}\left[f''(0) + \lambda_3(f'(0)f''(0) - f(0)f'''(0)\right] \\
&f(0) = 0, \theta(0) = 1, \phi(0) = 1, \\
&f'(\infty) \to 0, f''(\infty) \to 0, \theta(\infty) \to 0, \phi(\infty) \to 0.
\end{aligned} \tag{2.10}$$

with λ_3 (retardation parameter), β_2 (slip parameter), Pr (Prandtl number), Nb (Brownian motion parameter), Ec (Eckert number), Nt (thermophoretic parameter), δ (temperature ratio parameter), Sc (Schmidt number), k_1 (chemical reaction parameter), and E_1 (activation energy parameter) which are described as:

$$\begin{aligned}
&\lambda_3 = \lambda_1 a, Pr = \frac{v_f}{\alpha_f}, Ec = \frac{(ax)^2}{c_p(T_w - T_\infty)}, \\
&Nb = \frac{(\rho c_p)_s D_B(C_w - C_\infty)}{(\rho C_p)_f \nu}, Nt = \frac{(\rho c_p)_s D_T(T_w - T_\infty)}{(\rho c_p)_f \nu T_\infty}, \beta_2 = \beta_1\sqrt{\frac{a}{v}}, \\
&Sc = \frac{\nu_f}{D_B}, k_1 = \frac{k_0^2}{a}, \delta = \frac{T_w - T_\infty}{T_\infty}, E_1 = \frac{E_a}{kT_\infty}.
\end{aligned} \tag{2.11}$$

Engineering parameters of interest are defined as

Skin Friction

$$C_{fx} = \frac{\tau_w}{\frac{\rho}{2} u_x^2(x)}; \tau_w = \frac{\mu}{1+\lambda_2}\left[u_y + \lambda_1(uu_{xy} + v(u_y)^2)\right] |y=0, \tag{2.12}$$

Nusselt Number

$$Nu_x = \frac{xq_h}{k\left(T_f - T_\infty\right)}; q_h = -k\frac{\partial T}{\partial y}, \tag{2.13}$$

Sherwood Number

$$Sh_x = \frac{xq_s}{D_B\left(C_f - C_\infty\right)}; q_s = -D_B\frac{\partial C}{\partial y}. \tag{2.14}$$

After implementation of tranformation Eq.(2.12) takes the form

$$\frac{C_{fx} Re_x^{0.5}}{2} = \frac{1}{1+\lambda_2}\left[f''(0) + \lambda_3(f'(0)f''(0) - f(0)f'''(0))\right], \tag{2.15}$$

$$\frac{Nu}{\sqrt{Re_x}} = -\theta'(0), \tag{2.16}$$

$$\frac{Sh}{\sqrt{Re_x}} = -\phi'(0). \tag{2.17}$$

2.3 Results and Discussions

Table (2.2.1) shows computational results for local skin friction. Skin friction is reduced against relaxation to retardation times ratio (λ_2), retardation parameter (λ_3) and slip parameter (β_2).

Table (2.2.2) shows that the Nusselt number decreased by rising the value of thermophoretic parameter(Nt) and increased by increasing the values of Brownian motion parameter (Nb) and Eckert number (Ec).

Table (2.2.3) shows that with incremental values of parameters Schmidt number (Sc) and chemical reaction parameter ($k1$) local Sherwood number decreased while by increasing thermophoretic parameter (Nt) the Sherwood number is increased.

Figures (2.1), (2.2) and (2.3) is plotted to see the impacts of relaxation to retardation times ratio (λ_2), retardation parameter (λ_3) and slip parameter (β_2) on velocity profile. It is seen that an enhancement in relaxation to retardation times ratio (λ_2) and slip parameter (β_2) causes reduction in velocity profile.

Figures (2.4), (2.5) and (2.6) depicts the influence of thermophoretic parameter (Nt), Brownian motion parameter (Nb) and Eckert number (Ec) on the temperature profile. It can be observed that increase in these parameters increase the temperature of the fluid. Figures (2.7), (2.8) and (2.9) are drawn to clarify the effect of Schmidt number (Sc), chemical reaction parameter ($k1$) and thermophoretic parameter (Nt) on concentration curve. The rising values of Schmidt number (Sc), chemical reaction parameter ($k1$) lowers the concentration profile. On the other hand higher the thermophoretic parameter (Nt) turn up the concentration profile.

Table 2.3.1: Effect of several values of parameters λ_2, λ_3 and β_2 on skin friction by considering Pr=7, Sc=0.1, Nb=0.2, Nt=0.1, n=0.2, k_1=0, δ=1, E_1=0.4, Ec=0.1

λ_2	λ_3	β_2	$C_{fx} = \frac{C_{fx}Re_x^{0.5}}{2}$
3	4	0.1	0.9293
4			0.8457
5			0.7822
5	4		0.7822
	5		0.8446
	6		0.9006
	6	0.1	0.9006
		0.2	0.7796
		0.3	0.6912

Table 2.3.2: Effect of several values of parameters Ec, Nb and Nt, on Nusselt number by considering , λ_2=4, λ_3=8, Pr=7, Sc=0.2, β_2=0.1, n=0.2, k_1=0, δ=1, E_1=0.4

Nt	Nb	Ec	$-\theta'(0)$
2	3	0.2	1.3663
2.5			1.2649
3			1.1833
2	2		1.1833
	2.5		1.2780
	3		1.3663
	3	0	1.2649
		0.1	1.3166
		0.2	1.3663

Table 2.3.3: Effect of several values of parameters Sc, Nt and E_1 on Sherwood Number by considering ,λ_2=5, λ_3=5, Pr=7 , Nb=0.2, β_2=0.1, n=0.2, k_1=0, δ=1, Ec=0.2

Sc	Nt	E_1	$-\phi'(0)$
0.5	0.08	0.4	0.5455
1			0.6969
1.5			0.8245
1	0.06		0.6815
	0.08		0.6969
	0.1		0.7103
	0.08	1	0.6969
		2	0.6969
		4	0.6969

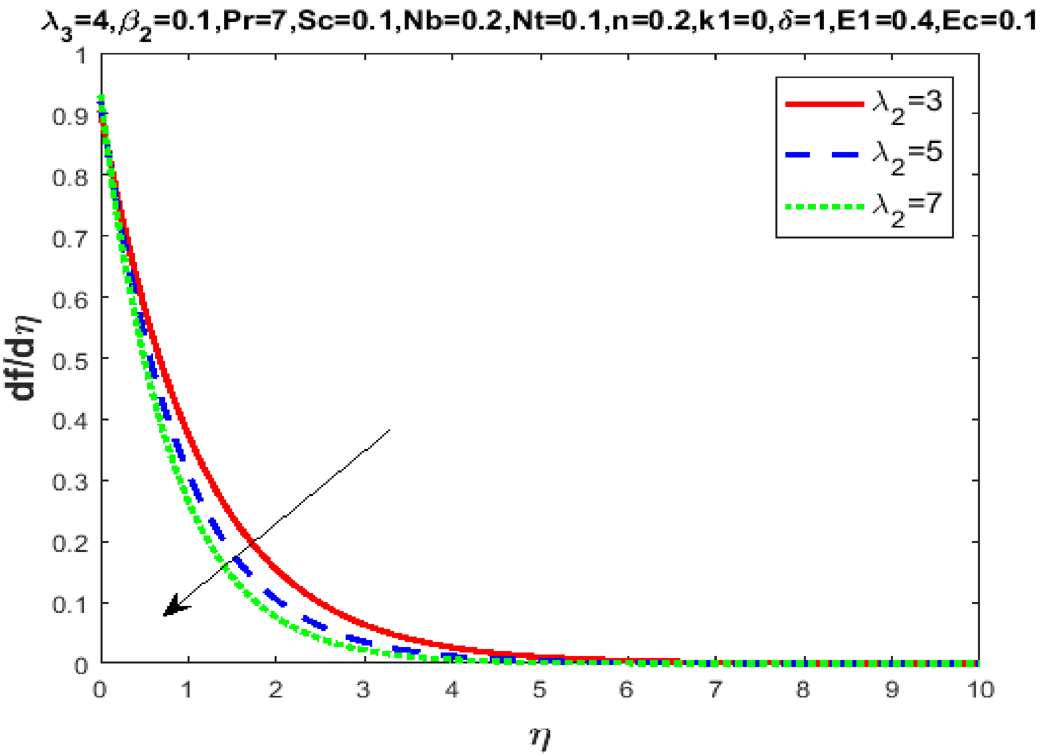

Figure 2.1: Impact of several values of λ_2 on the velocity curve $\frac{df}{d\eta}$

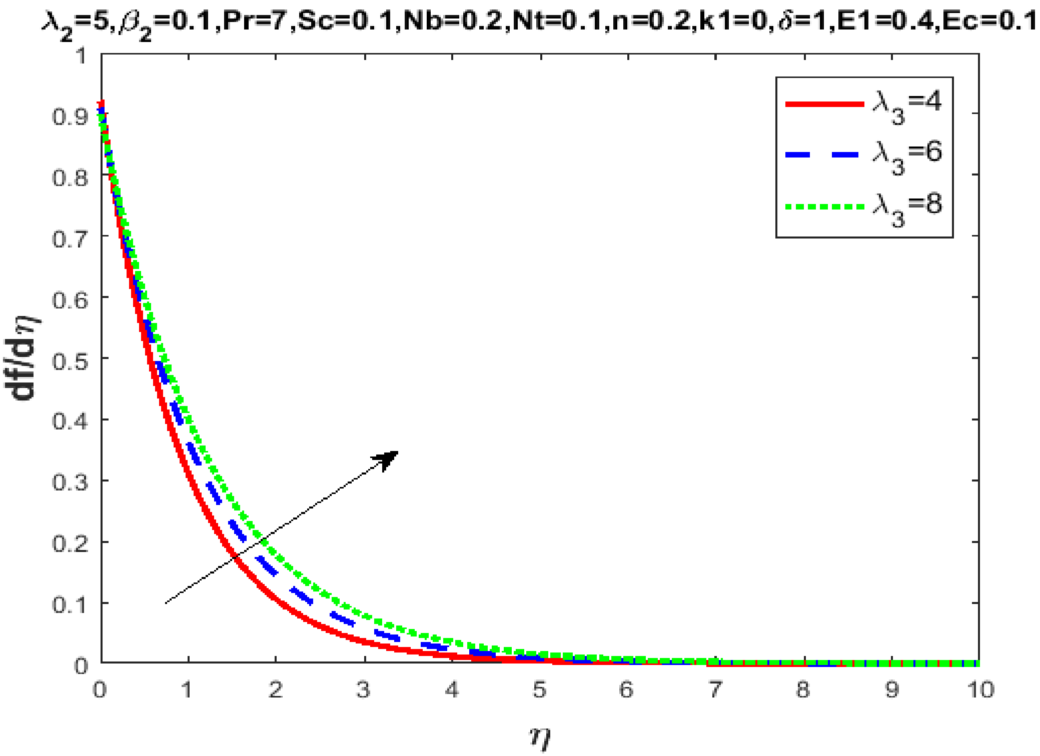

Figure 2.2: Impact of several values of λ_3 on the velocity curve $\frac{df}{d\eta}$

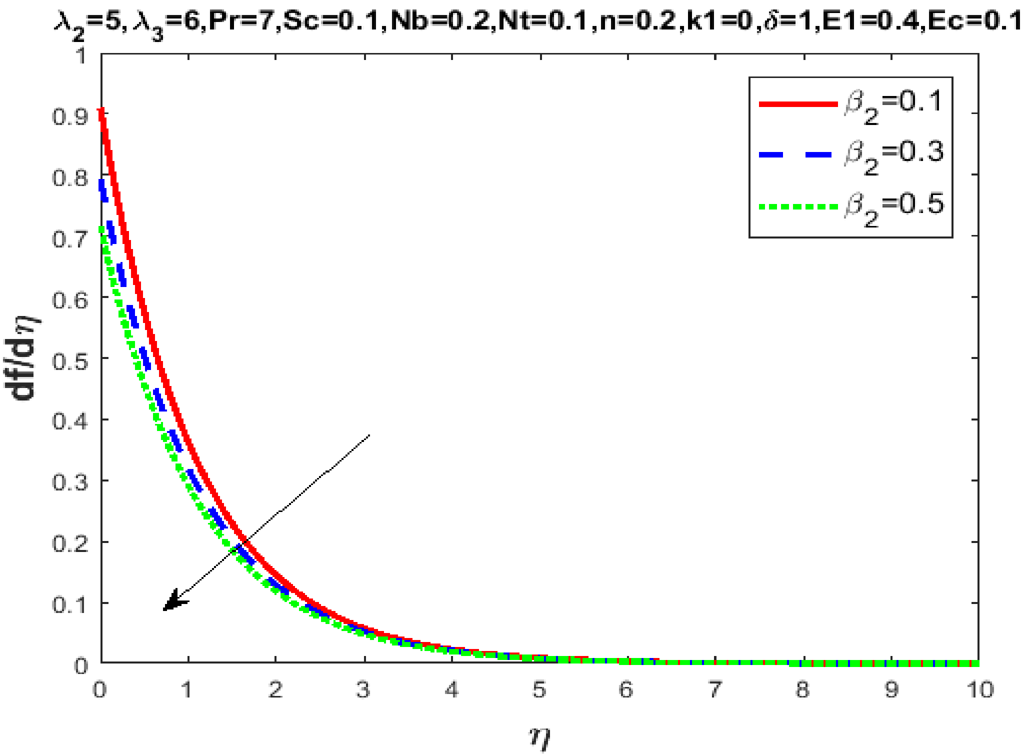

Figure 2.3: Impact of several values of β_2 on the velocity curve $\frac{df}{d\eta}$

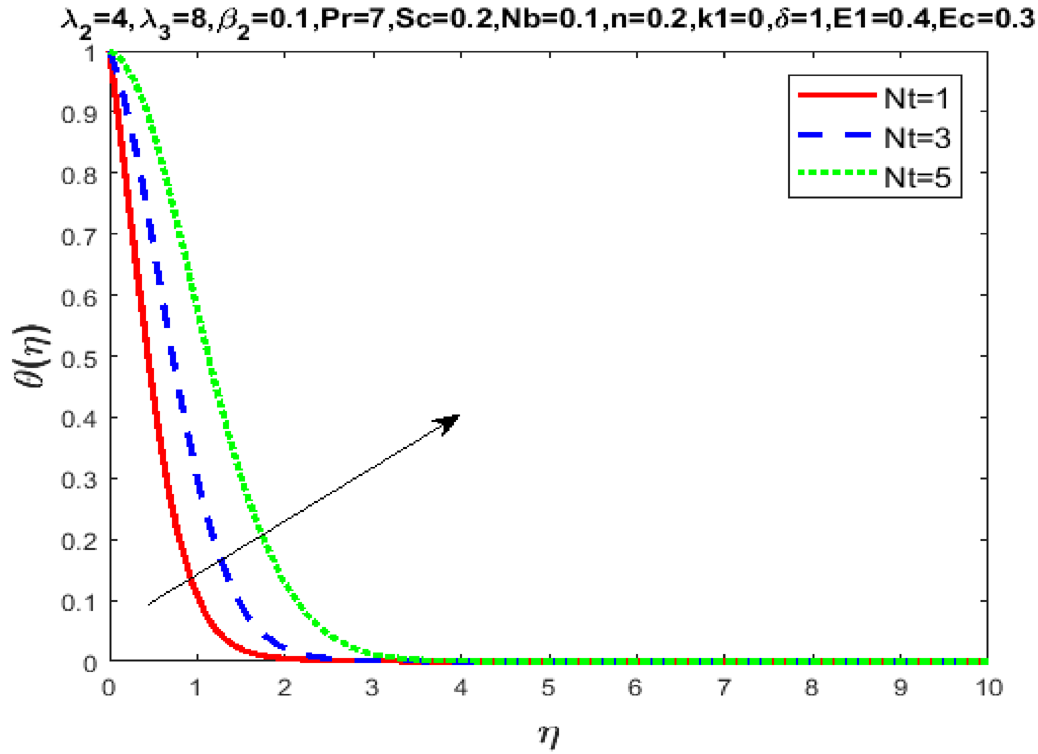

Figure 2.4: Graph of temperature for different choices of Nt

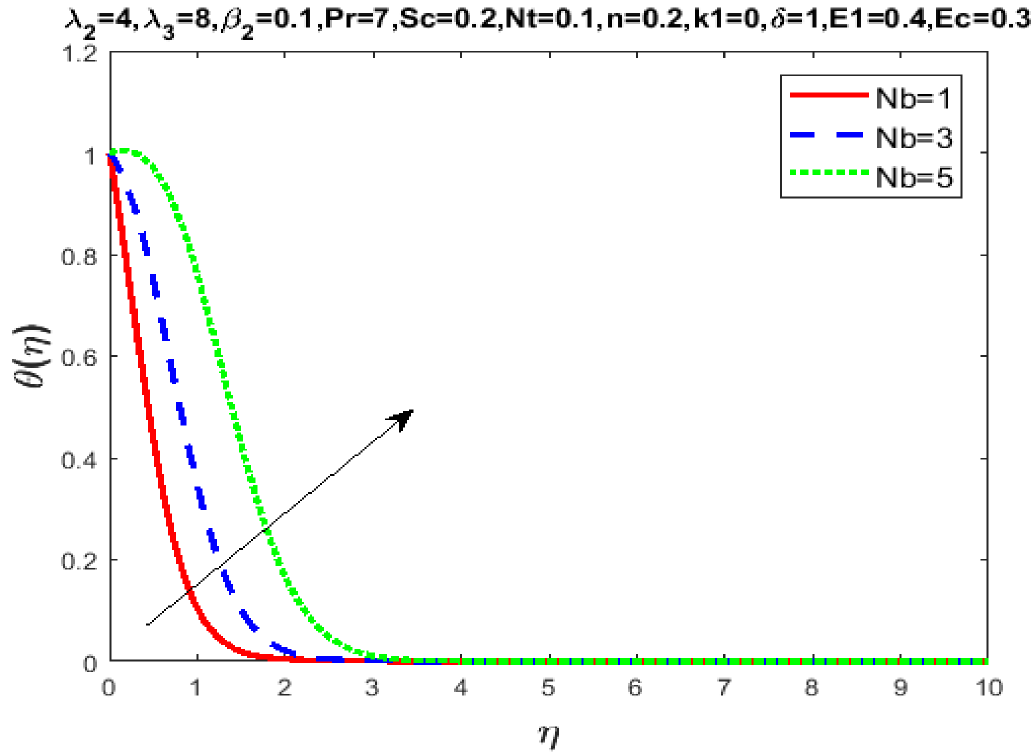

Figure 2.5: Graph of temperature for different choices of Nb

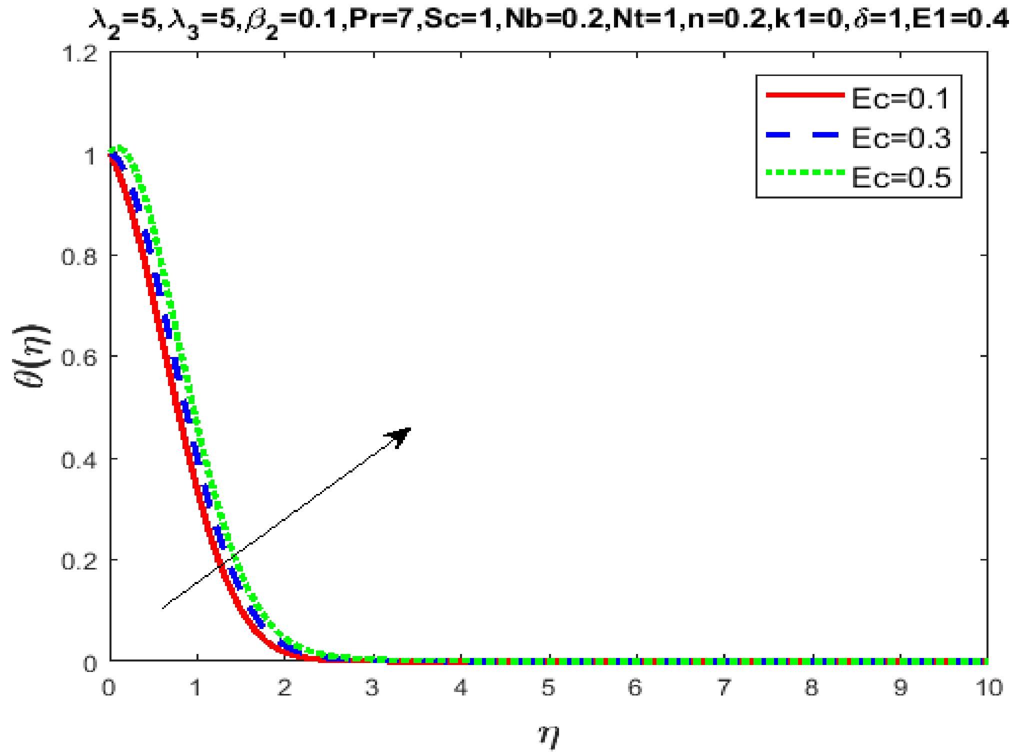

Figure 2.6: Graph of temperature for different choices of Ec

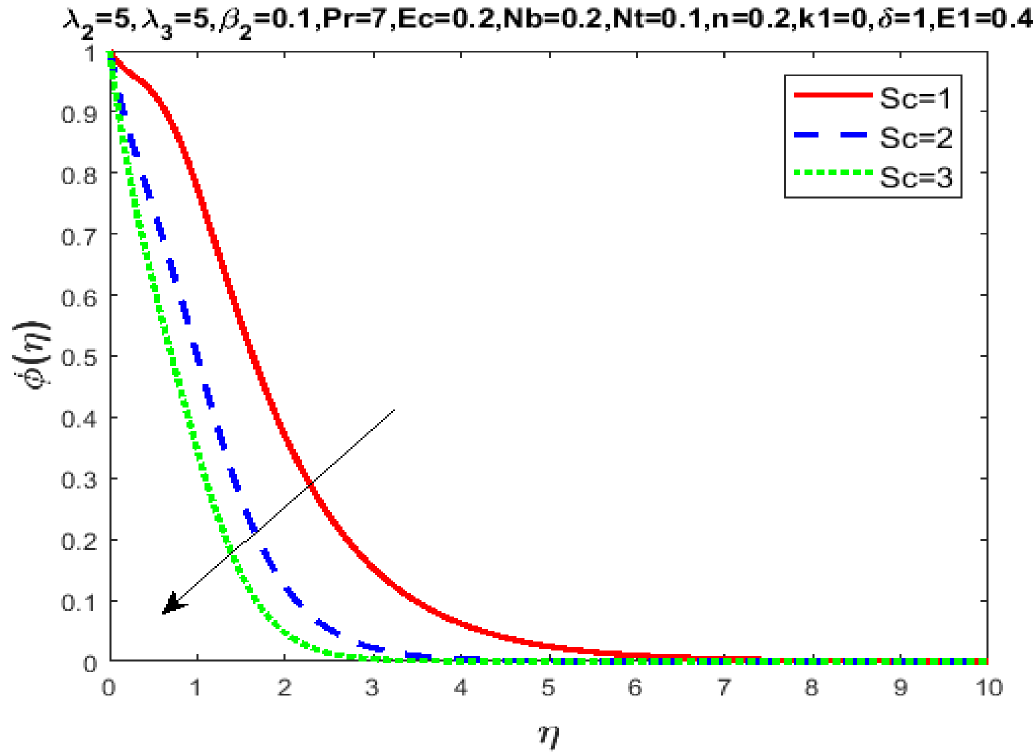

Figure 2.7: Graph of concentration for different choices of Sc

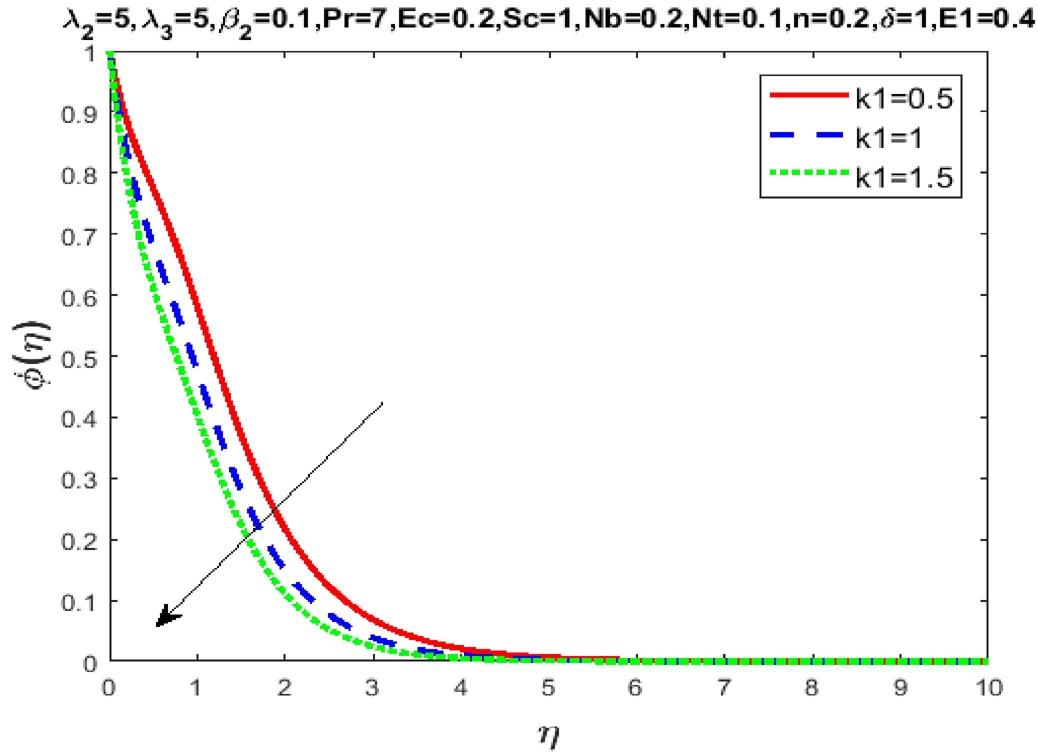

Figure 2.8: Graph of concentration for different choices of $k1$

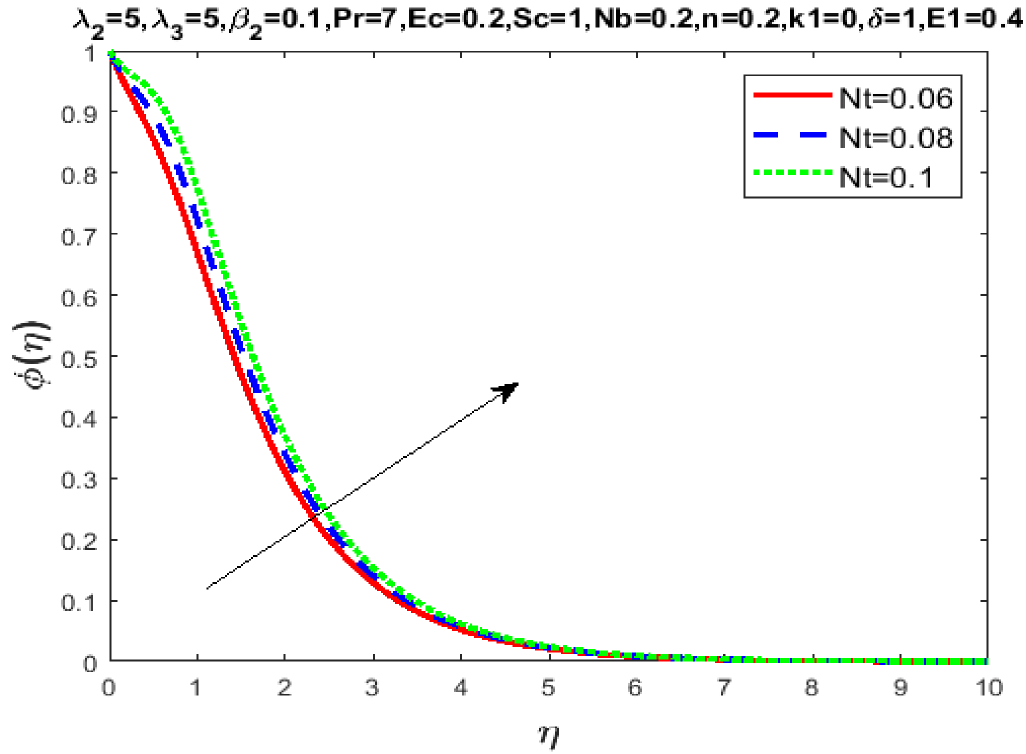

Figure 2.9: Graph of concentration for different choices of Nt

Chapter 3

Flow of Jeffrey Nanofluid in a Porous Medium Containing Microorganisms

3.1 Introduction

Now we will move to an extension to chapter 2. Present work is concerned with 2D, incompressible, steady and no slip flow condition for Jeffrey nanofluid containing microorganisms using a porous medium. Here the impacts of mass and heat transfer flow in the existence of viscous dissipation are elaborated. The (PDEs) have been converted into (ODEs) by using similarity transformations. After converting PDEs into ODEs expression of local skin friction, local Nusselt number, local Sherwood number and density of motile microorganisms are found. The formulated differential system is solved numerically using bvp4c method. Simulations are performed for various values of embedded parameters which include thermophoretic parameter (Nt) and Brownian motion parameter (Nb). Parameters effects are considered in the mathematical model. The graphs for temperature, velocity, microorganisms and concentration curves are constructed to examine the effects of factors on the fluid flows. At the end, we will observe the effect of different parameters with the help of table and graphs.

3.2 Basic Equations

Formulated equations are given below:

$$u_x + v_y = 0, \tag{3.1}$$

$$uu_x + vu_y = \frac{\nu}{1+\lambda_2}\left[u_{yy} + \lambda_1\left(uu_xu_{yy} + u_yu_{xy} - u_xu_{yy} + \nu u_{yyy}\right)\right] - \frac{\sigma\beta_o^2 u}{\rho} - \frac{\mu}{\rho_\infty k^*}u, \tag{3.2}$$

$$\begin{aligned} uT_x + vT_y = \alpha T_{yy} + \frac{\mu_f}{(\rho c_p)_f(1+\lambda_2)}\left[(u_y)^2 + \lambda_1\left(vu_y(u_{yy}) + uu_yu_{yx}\right)\right] + \\ \frac{(\rho c_p)_s}{(\rho c_p)_f}\left[D_BT_yC_y + \frac{D_T}{T_\infty}(T_y)^2\right] - \frac{1}{\rho c_p}q_{ry} + \frac{(T-T_\infty)Q}{\rho c_p}, \end{aligned} \tag{3.3}$$

$$uC_x + vC_y = D_B(C_{yy}) + \frac{D_T}{T_\infty}(T_{yy}), \tag{3.4}$$

$$uN_x + vN_y + \frac{bW_c}{C_w - C_\infty}\left(\frac{\partial}{\partial y}(NC_y)\right) = \frac{\partial}{\partial y}(D_mN_y), \tag{3.5}$$

with boundary conditions:

$$\begin{aligned} &u = ax, v = 0, T = T_w, C = C_w, N = N_w \quad at \quad y = 0, \\ &u \to 0, T \to T_\infty, C \to C_\infty \quad and \quad N \to N_\infty \quad at \quad y \to \infty. \end{aligned} \tag{3.6}$$

In above expressions (x,y), (u,v), ρ_f, μ, λ_2, k_f, a, c_p, D_T, D_B, $(-1 < n < 1)$, λ_1, T, T_∞, T_w, $k*$, σ, β_o, ρ_∞, α, q_r, ν, D_m, Q, b, W_c act for Cartesian coordinates, velocity vectors, density, dynamic viscosity, ratio of relaxation to retadation times, thermal conductivity, stretching rate, specific heat, thermophoretic coefficient, Brownian constsnt, fitted constant, retardation time, temperature, ambient temperature, temperature at wall, Boltzmann constant, Electrical conductivity of fluid, Applied magnetic field, free stream viscosity, thermal diffusivity, radiative heat flux, kinematic viscosity, internal heat generation or absorption, porosity variable, chemotaxis constant, maximum cell swimming speed.

Considering the dimensionless variables:

$$u = axf'(\eta), v = -\sqrt{av}f, \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty},$$
$$\phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, \chi(\eta) = \frac{N - N_\infty}{N_w - N_\infty}, \eta = \sqrt{\frac{a}{v}}y \tag{3.7}$$

we arrive,

$$ff'' - f'^2 + \frac{1}{1+\lambda_2}\left[f''' + \lambda_3\left(f''^2 - ff^iv\right)\right] - Mf' - k_pf' = 0 \tag{3.8}$$

$$\theta'' + Prf\theta' + PrNb\theta'\phi' + PrNt\theta'^2 + Pr\frac{E_c}{1+\lambda_2}\left[f''^2 + \lambda_3(f'f''^2 - ff''f''')\right]$$
$$-\frac{4}{3}PrRd\theta'' - Pr\theta S = 0 \tag{3.9}$$

$$\phi'' + \frac{Nt}{Nb}\theta'' + Scf\phi' = 0 \tag{3.10}$$

$$\chi'' - Sb\chi' - Pe\left[Nd\phi'' + \chi'\phi' + \chi\phi''\right] = 0 \tag{3.11}$$

$$At \;\; \eta = 0,\; f(\eta) = 0, f'(\eta) = 1, \theta(\eta) = 1, \phi(\eta) = 1, \chi(\eta) = 1,$$
$$At \;\; \eta \to \infty,\; f'(\eta) \to 0, f''(\eta) \to 0, \theta(\eta) \to 0, \phi(\eta) \to 0, \chi(\eta) \to 0. \tag{3.12}$$

with λ_3 (retadation parameter), M (dimensionless magnetic variable), Pr (Prandtl number), Pe (Peclet number), Nb (Brownian motion parameter), kp (porosity variable), Nt (Thermophoresis parameter), Sc Schmidt number), Ec (Eckert number), Rd (thermal radiation parameter), S (heat source/sink parameter), Nd (ratio of motile microorganisms) which are defined as:

$$\lambda_3 = \lambda_1 a,\; M = \frac{\sigma\beta_o^2}{\rho a},\; kp = \frac{\nu}{ak*},\; Pr = \frac{\nu_f}{\alpha_f}, Nb = \frac{(\rho c_p)_s D_B (C_w - C_\infty)}{(\rho c_p)\nu},$$
$$Nt = \frac{(\rho c_p)_s D_T (T_W - T_\infty)}{(\rho c_p)_f \nu T_\infty},\quad Ec = \frac{(ax)^2}{c_p(T_W - T_\infty)},\quad Rd = \frac{4\sigma T_\infty^3}{kk*},\quad S = \frac{Q}{\rho c_p a}, \tag{3.13}$$
$$Sc = \frac{\nu_f}{D_B},\quad Pe = \frac{bW_c}{D_m},\quad Nd = \frac{N_\infty}{N_W - N_\infty}$$

Expressions for physical quantities are given below:

$$
\begin{aligned}
C_{fx} &= \frac{\tau_w}{\frac{\rho}{2}u_w^2(x)}; \tau_w = \frac{\mu}{1+\lambda_2}\left[u_y + \lambda_1\left(uu_{xy} + v(u_{yy})\right)\right]|_{y=0} \\
Nu_x &= \frac{xq_h}{k(T_f - T_\infty)}; q_h = -kT_y + q_{ry}, \\
Sh_x &= \frac{xq_s}{D_B(C_f - C_\infty)}; q_s = -D_B C_y, \\
Nn_x &= \frac{xc_s}{D_m(N_f - N_\infty)}; c_s = -D_m N_y.
\end{aligned} \tag{3.14}
$$

After implementation of tranformation Eq.(3.12)takes the form

$$
\begin{aligned}
\frac{C_{fx}Re_x^0.5}{2} &= \frac{1}{1+\lambda_2}\left[f''(0) + \lambda_3\left(f'(0)f''(0) - f(0)f'''(0)\right)\right], \\
\frac{Nu_x}{\sqrt{Re_x}} &= -\left(1 + \frac{4}{3}Rd\right)\theta'(0), \\
\frac{Sh_x}{\sqrt{Re_x}} &= -\phi'(0), \\
\frac{Nn_x}{\sqrt{Re_x}} &= -\chi'(0).
\end{aligned} \tag{3.15}
$$

3.3 Results and Discussion

Table (3.1.1) shows the impacts of different parameters on skin friction. The physical parameter, the local skin friction is reduced against relaxation to retardation times ratio (λ_2), magnetic parameter (M) and porosity variable (kp) while by increasing the values of retardation parameter (λ_3) the value of the local skin friction increases.

Table (3.1.2) shows that the Nusselt number decreased by incremental values of relaxation to retardation times ratio (λ_2), magnetic parameter (M), Brownian motion parameter (Nb), thermal radiation parameter (Rd) and heat sourse (S) and increased by increasing the values of retardation parameter (λ_3).

Table (3.1.3) shows that the Sherwood number delineate an upward trending values against thermophoretic parameter(Nt). By increasing Brownian motion parameter (Nb) Sherwood number also increases and Sherwood number shows anupward trend when Schmidt number (Sc) and heat sourse (S) increases.

Table (3.1.4) shows the density of motile microorganisms describes an upward trend against thermophoretic parameter (Nt), Peclet number (Pe), ratio of motile microorganisms (Nd), thermal radiation parameter (Rd) and Schmidt number (Sc). However a decreasing trend for the local density of motile microorganisms is precieved for arising the Brownian motion parameter (Nb) and a declining tendency for the local density of motile microorganisms is observed for the arising values of the bioconvection Schmidt number (Sb).

Figures (3.1), (3.2), (3.3) and (3.4) illustrate the impacts of relaxation to retardation times ratio (λ_2), retardation parameter (λ_3), porosity variable (kp) and magnetic parameter (M) on the velocity profile. The enhancement in relaxation to retardation times ratio (λ_2), magnetic parameter (M) and porosity variable (kp) cause reduction in velocity curve. While rise in retardation parameter (λ_3) results in an increase of velocity profile.

Figures (3.4), (3.5), (3.6), (3.7), (3.8), (3.9), (3.10), (3.11) and (3.12) examine the results of retardation times ratio (λ_2), retardation parameter (λ_3), magnetic parameter (M), Brownian motion parameter (Nb),ratio of motile microorganisms (Nd), prandtl number (Pr), thermal radiation parameter (Rd) and heat source (S) on the temperature profile. These graphs describe that the temperature of fluid increases by rising the values of these parameters, retardation times ratio (λ_2), magnetic parameter (M), Brownian motion parameter (Nb), ratio of motile microorganisms (Nd), thermal radiation parameter (Rd) and heat source (S). However it is observed that rise in retardation parameter (λ_3) and prandtl number (Pr) reduce the temperature profile.

The influence of the thermophoretic parameter (Nt), Brownian motion parameter (Nb), and heat source (S) on the concentration profile is illustrated in Figures (3.13), (3.14), and (3.15). Rising the values of thermophoretic parameter(Nt) elevates the concentration profile. It can be noted that concentration profile turn down by rising the choices of Brownian motion parameter (Nb) and heat sourse (S).

Figures (3.16), (3.17), (3.18), (3.19), (3.20), (3.21), (3.22) are plotted to appraise the impact of Brownian motion parameter (Nb), Schmidt number (Sc) thermal radiation parameter (Rd), ratio of motile microorganisms (Nd), thermophoretic parameter(Nt), Peclet number (Pe) and bioconvection Schmidt number (Sb). The increasing values of Brownian motion parameter (Nb), thermal radiation parameter (Rd), ratio of motile microorganisms (Nd) and Peclet number (Pe) lower the density of motile microorganisms profile. By rising the values of thermophoretic parameter(Nt) and bioconvection Schmidt number (Sb) illustrates an upward trend of the density of motile microorganisms.

λ_2	λ_3	M	kp	$C_{fx} = \frac{C_{fx} Re_x^{0.5}}{2}$
2	3	0.2	0.2	1.3663
2.5				1.2649
3				1.1833
2	2			1.1833
	2.5			1.2780
	3			1.3663
	3	0		1.2649
		0.1		1.3166
		0.2		1.3663
		0.2	0	1.2649
			0.1	1.3166
			0.2	1.3663

Table 3.3.1: Effect of different parameters on C_{fx} by considering Pr=10, Sc=0, Nb=1, Nt=1, Sb=0.1, Pe=0.9, S=0.1, $Rd = 1.5$, Nd=1, Ec=0.5 (bvp4c)

Table 3.3.2: Effect of different parameters on Nu_x by considering Pr=10, Sc=0.5, Nt=0.1, Sb=0.1, Pe=0.9, Nd=1 (bvp4c)

λ_2	λ_3	Nb	M	S	kp	Rd	Ec	$-(1+\frac{4}{3})Rd)\theta'(0)$
1.8	3	0.1	0.2	0.1	0.2	1.5	0.1	0.5829
2								0.5047
2.2								0.4418
2	2.5							0.3775
	2.7							0.4291
	2.9							0.4755
	3	0.1						0.5047
		0.3						0.4568
		0.5						0.4134
		0.1	0					0.5994
			0.1					0.5487
			0.2					0.5047
			0.2	0.04				0.7660
				0.08				0.6105
				0.12				0.3169
				0.1	0			0.5994
					0.1			0.5487
					0.2			0.5047
					0.2	1		0.6472
						1.5		0.5047
						2		0.2035
						1.5	0	0.5954
							0.1	0.5047
							0.2	0.4118

Table 3.3.3: Effect of different parameters on Sh_x by considering Pr=10, Sb=0.1, Pe=0.9, Nd=1 (bvp4c)

λ_2	λ_3	Sc	Nb	Nt	M	S	kp	Rd	Ec	$-\phi'(0)$
1.5	3	0.5	1	1	0.2	0.1	0.2	1.5	0.5	0.4235
2										0.4203
2.5										0.4449
2	2.5									0.4402
	3									0.4203
	3.5									0.4202
	3	0.5								0.4203
		1								0.6538
		1.5								0.8347
		0.5	1							0.4203
			2							0.4016
			3							0.3916
			1	0						0.3490
				0.5						0.3793
				1						0.4203
				1	0					0.4036
					0.1					0.4110
					0.2					0.4203
					0.2	0				0.3115
						0.1				0.4203
						0.12				0.4699
						0.1	0			0.4036
							0.1			0.4110
							0.2			0.4203
							0.2	1.5		0.4203
								1.7		0.4291
								1.9		0.4513
								1.5	0	0.2860
									0.1	0.3129
									0.2	0.3397

Table 3.3.4: Effect of different parameters on Nn_x by considering Pr=10, λ_2=2, λ_3=3, kp=0.2 (bvp4c)

Sc	Nb	Nt	M	Sb	Pe	S	Rd	Nd	Ec	$-\chi'(0)$
0.5	1	1	0.2	0.1	0.9	0.1	1.5	1	0.5	0.6817
0.6										0.7811
0.7										0.8724
0.5	1									0.6814
	2									0.7811
	3									0.8724
	1	1.1								0.6972
		1.2								0.7117
		1.3								0.7273
		1	0							0.6575
			0.1							0.6699
			0.2							0.6817
			0.2	0.1						0.6817
				0.2						0.6133
				0.3						0.5617
				0.1	0.7					0.6130
					0.8					0.6474
					0.9					0.6817
					0.9	0				0.4902
						0.1				0.6817
						0.12				0.7638
						0.1	1.5			0.6817
							1.7			0.7016
							1.9			0.7456
							1.5	1		0.6817
								1.5		0.8364
								2		0.9905
								1	0.5	0.6817
									0.6	0.7316
									0.7	0.7815

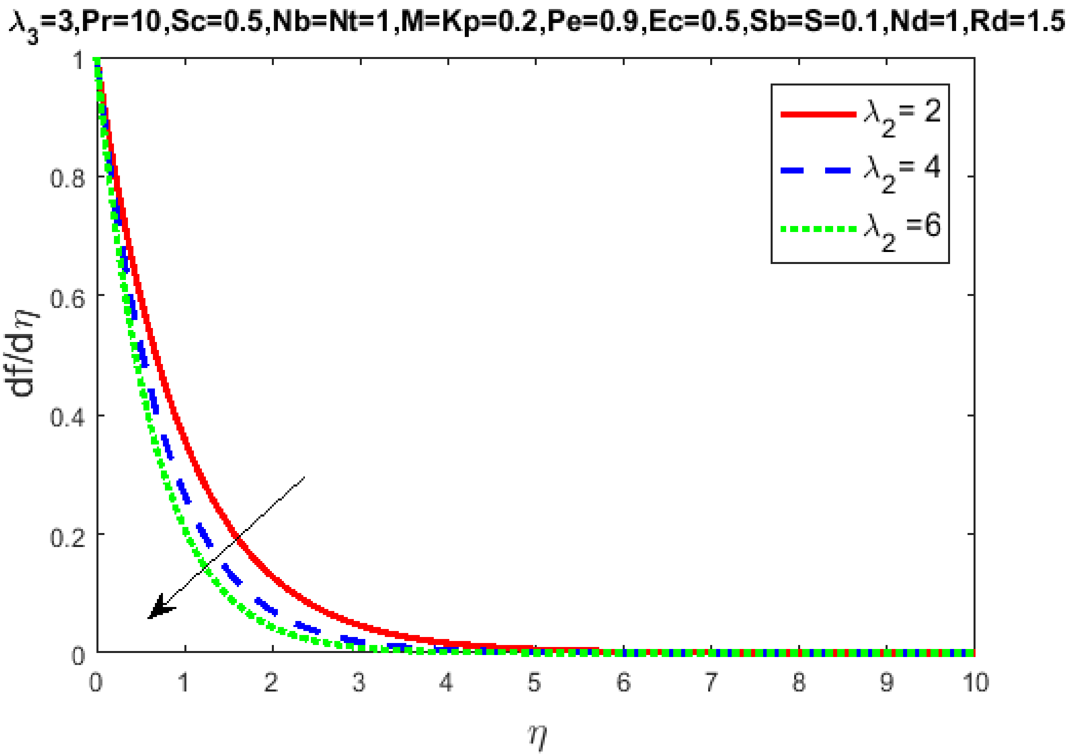

Figure 3.1: Impact of several values of λ_2 on the velocity curve $\frac{df}{d\eta}$

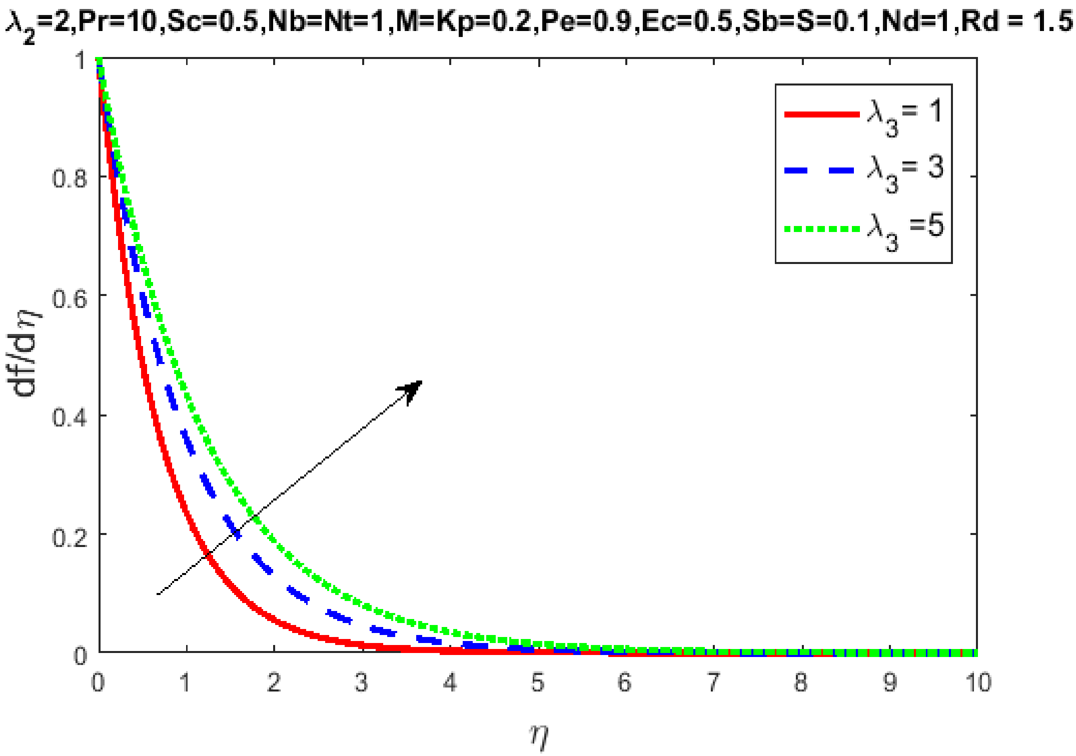

Figure 3.2: Impact of several values of λ_3 on the velocity curve $\frac{df}{d\eta}$

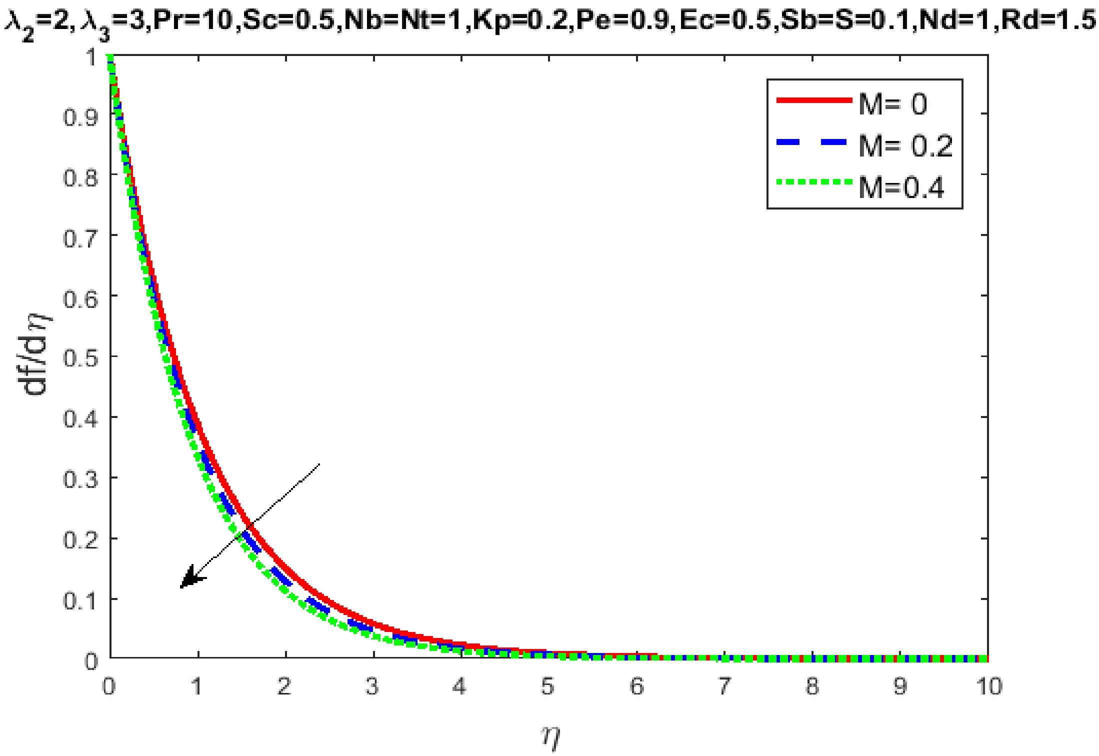

Figure 3.3: Impact of several values of M on the velocity curve $\frac{df}{d\eta}$

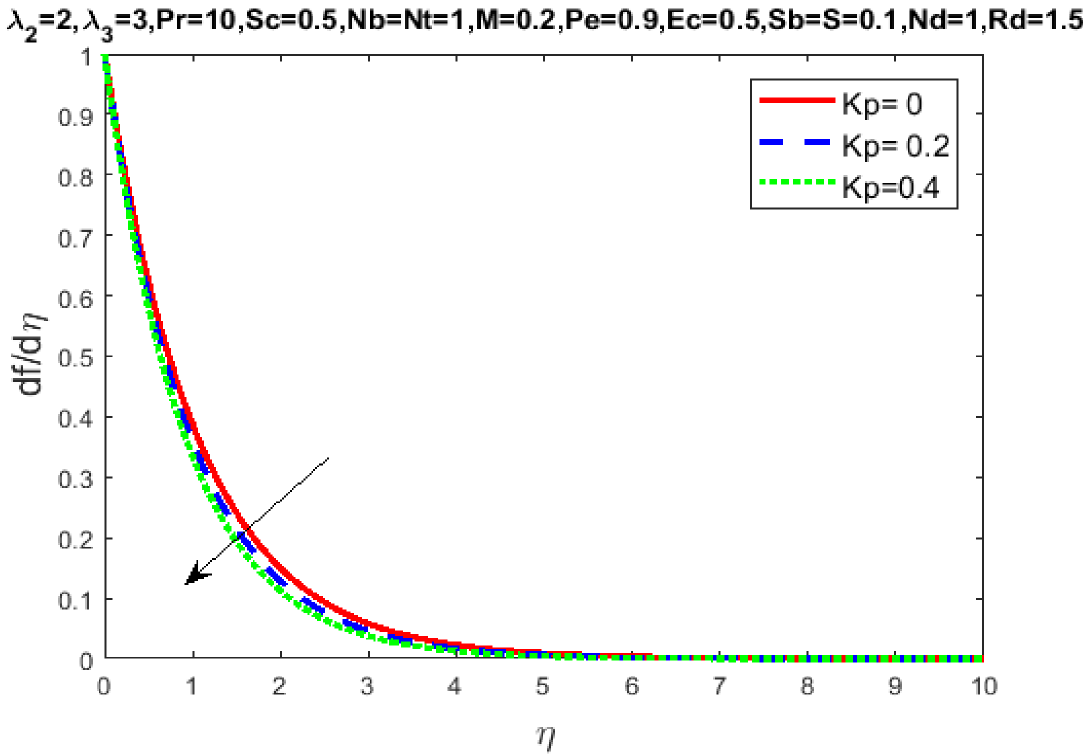

Figure 3.4: Impact of several values of Kp on the velocity curve $\frac{df}{d\eta}$

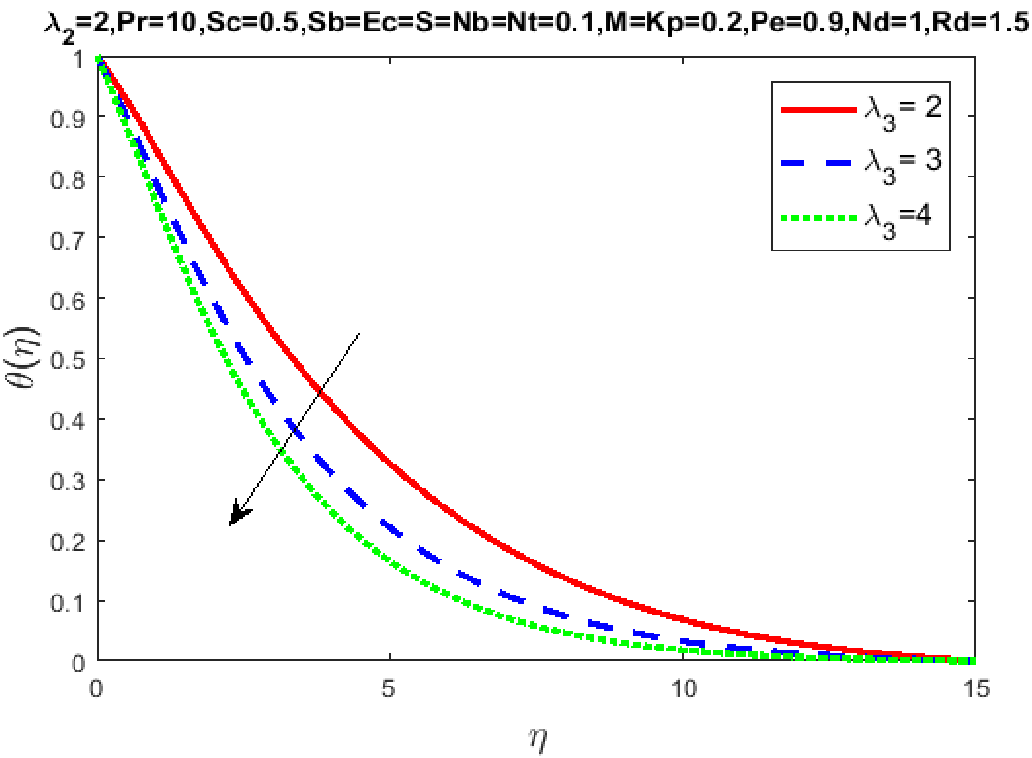

Figure 3.5: Impact of several values of λ_3 on the temperature curve

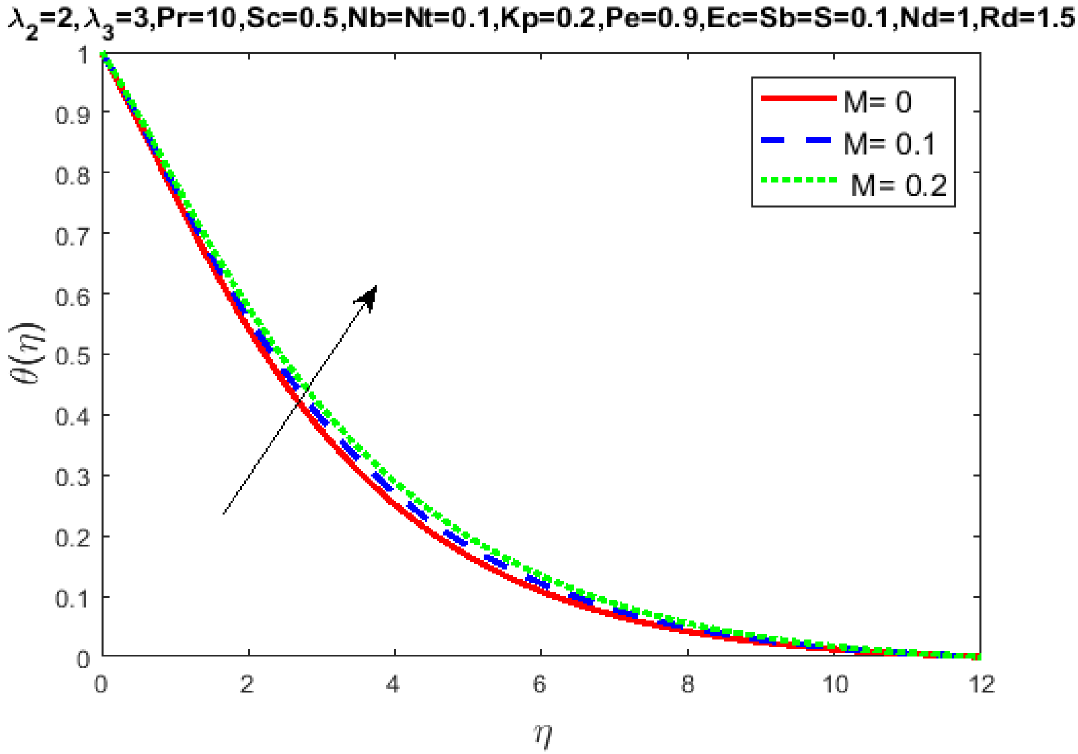

Figure 3.6: Impact of several values of M on the temperature curve

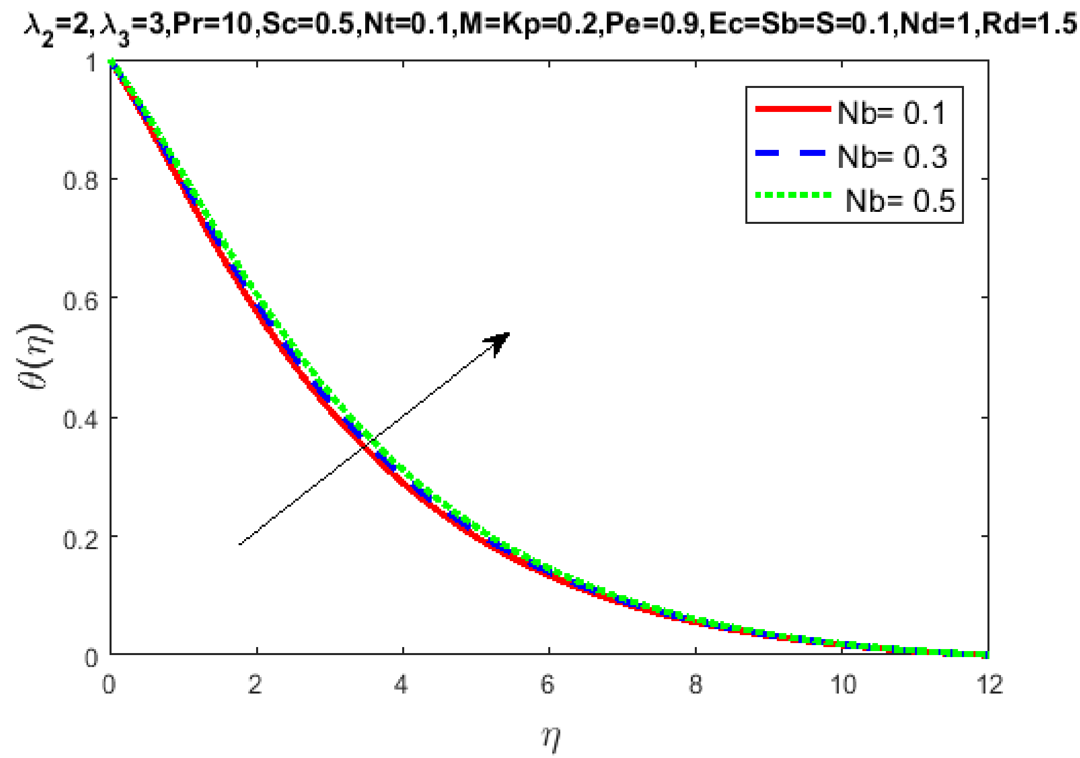

Figure 3.7: Impact of several values of Nb on the temperature curve

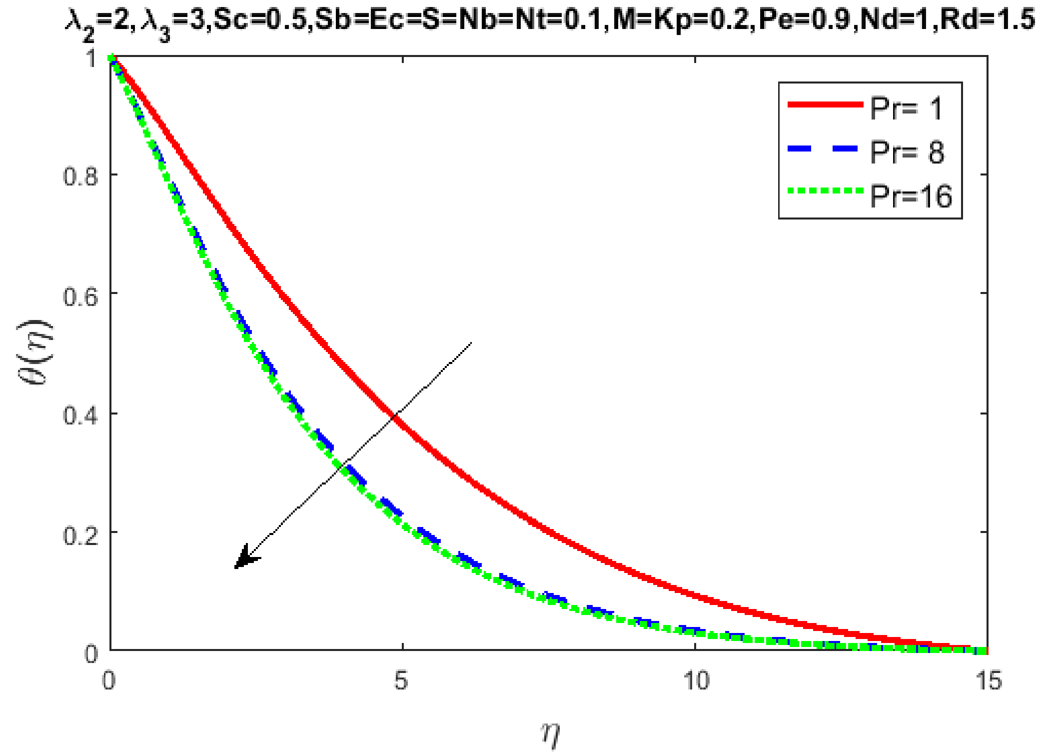

Figure 3.8: Impact of several values of Pr on the temperature curve

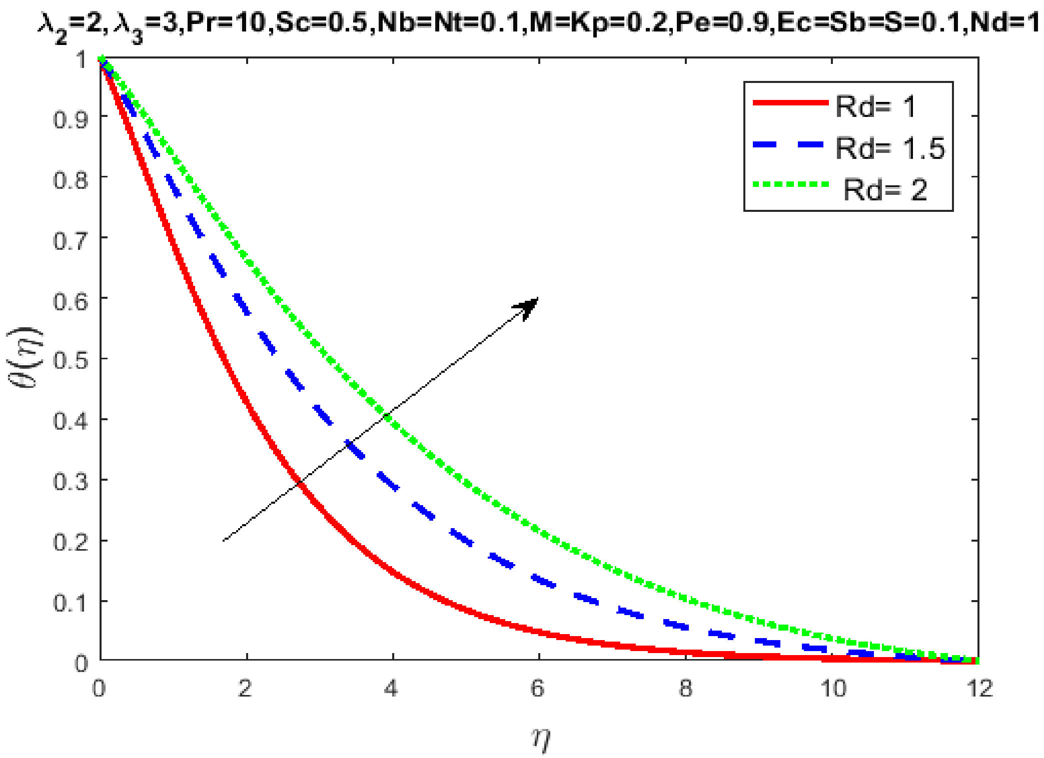

Figure 3.9: Impact of several values of Rd on the temperature curve

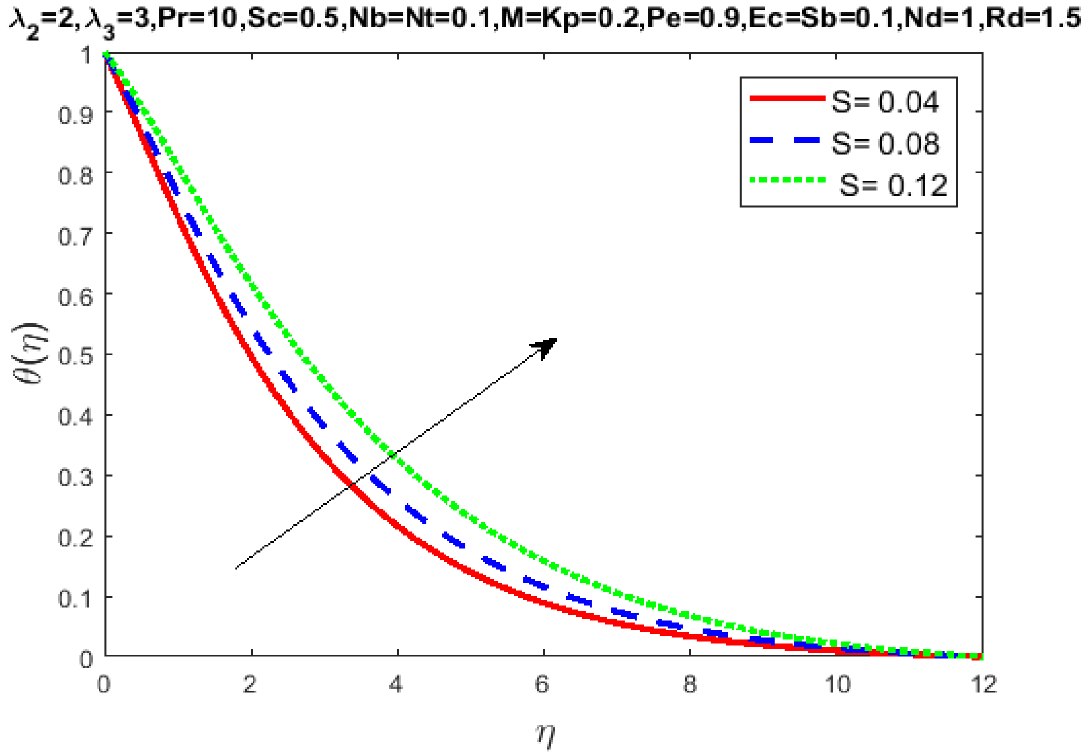

Figure 3.10: Impact of several values of S on the temperature curve

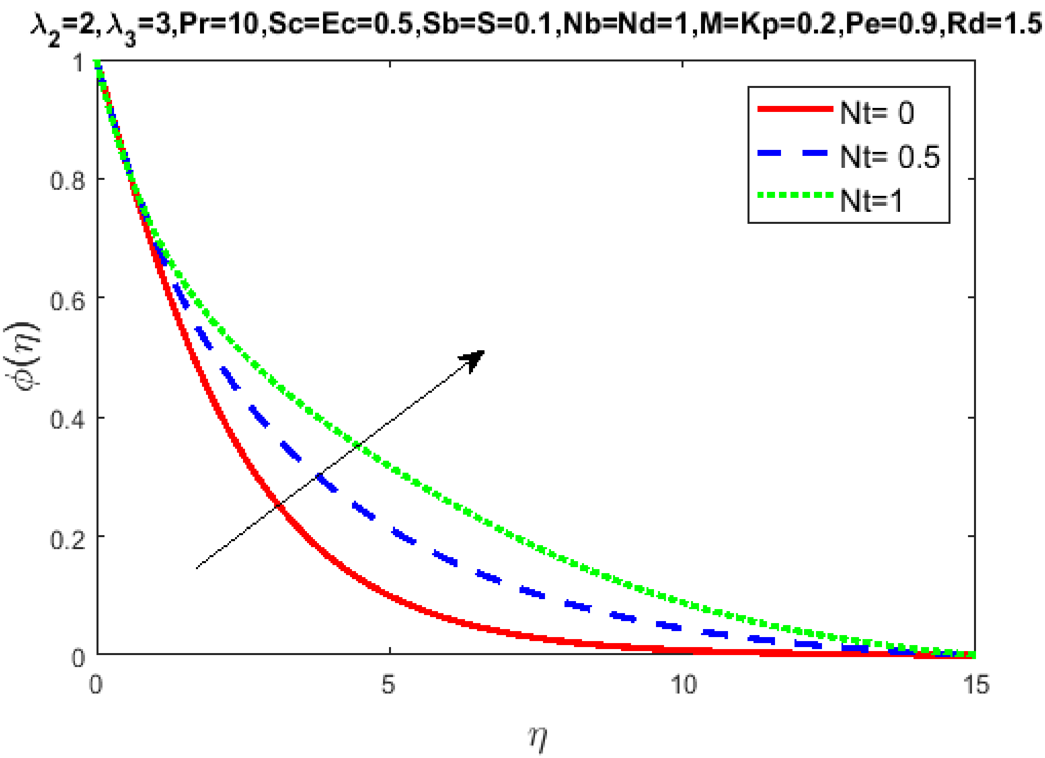

Figure 3.11: Concentration curve for three different values of Nt

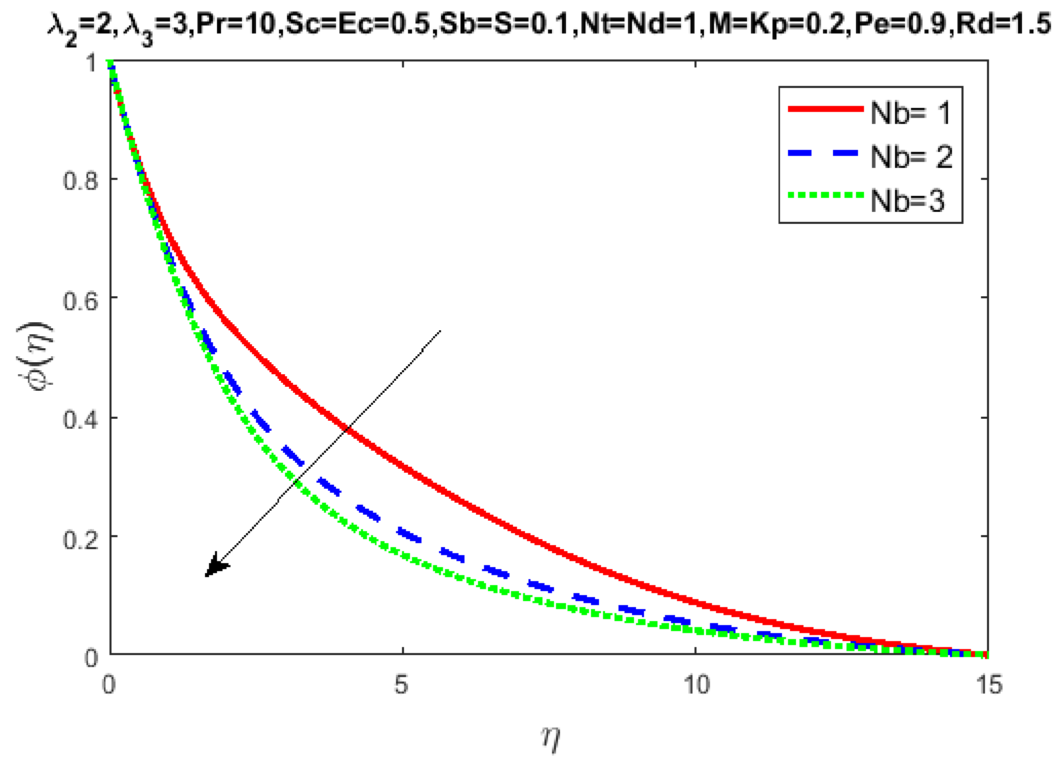

Figure 3.12: Concentration curve for three different values of Nb

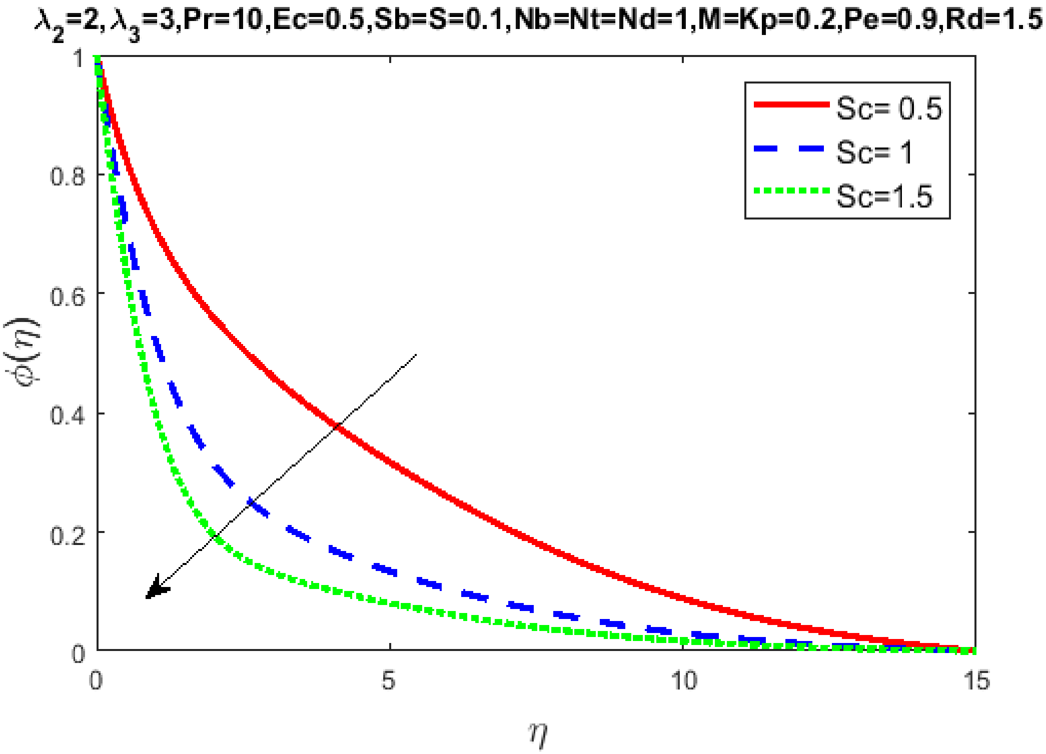

Figure 3.13: Concentration curve for three different values of Sc

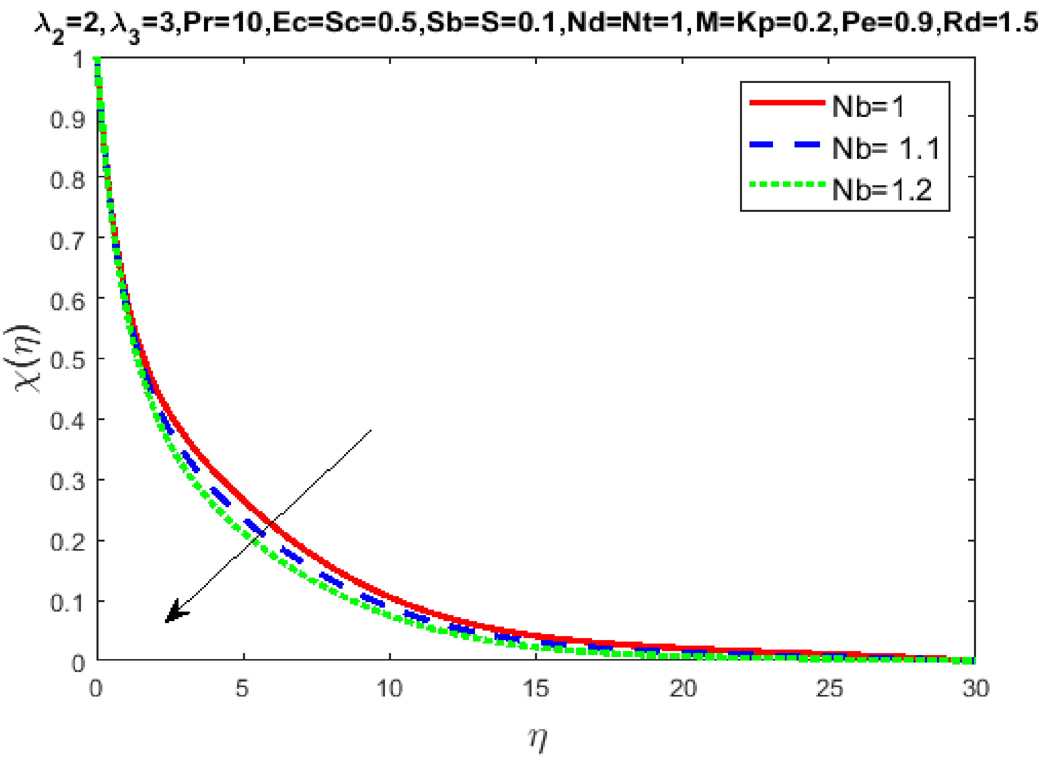

Figure 3.14: Impact of several values of Nb on the microorganism curve $\chi(\eta)$

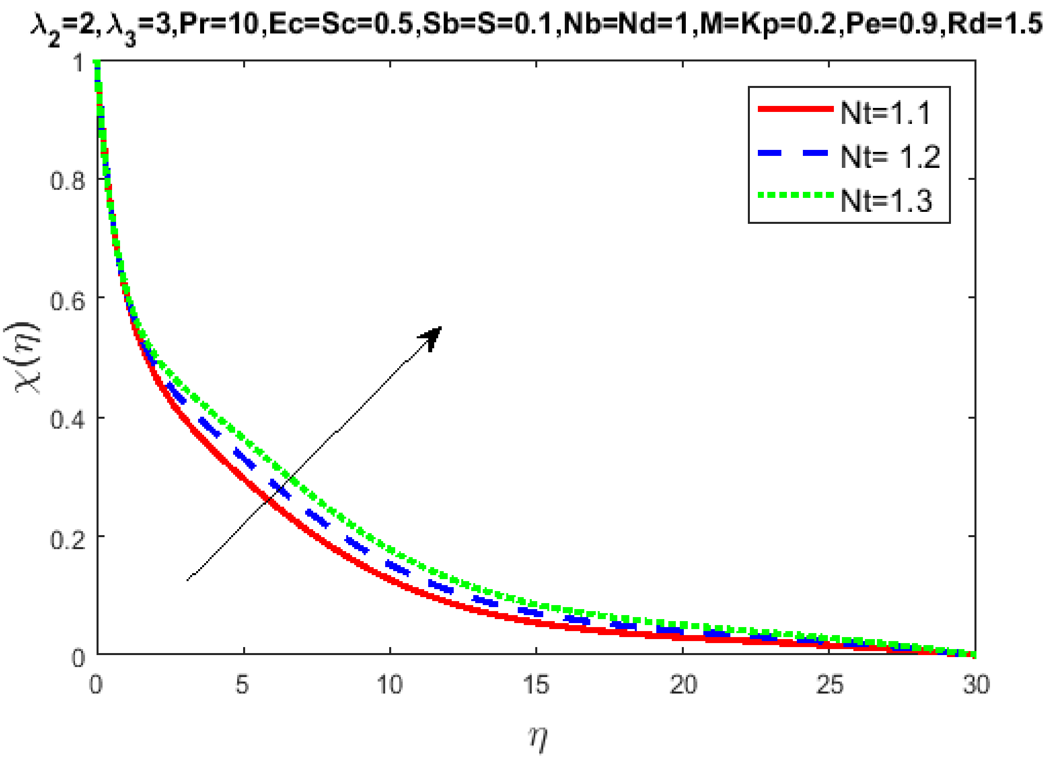

Figure 3.15: Impact of several values of Nt on the microorganism curve $\chi(\eta)$

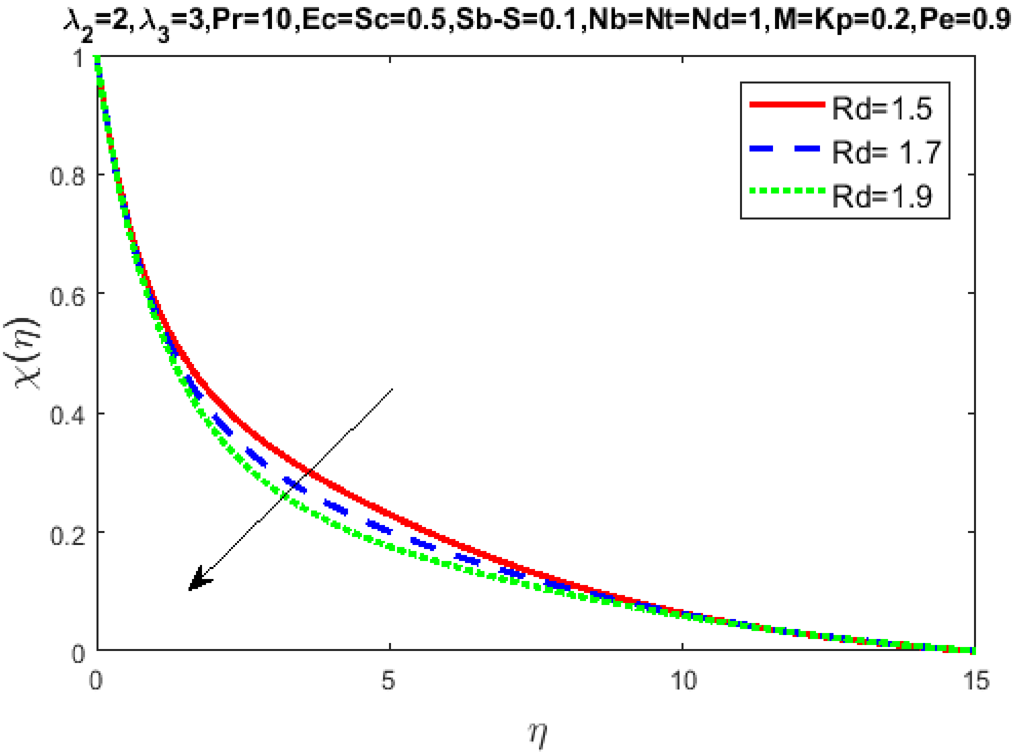

Figure 3.16: Impact of several values of Rd on the microorganism curve $\chi(\eta)$

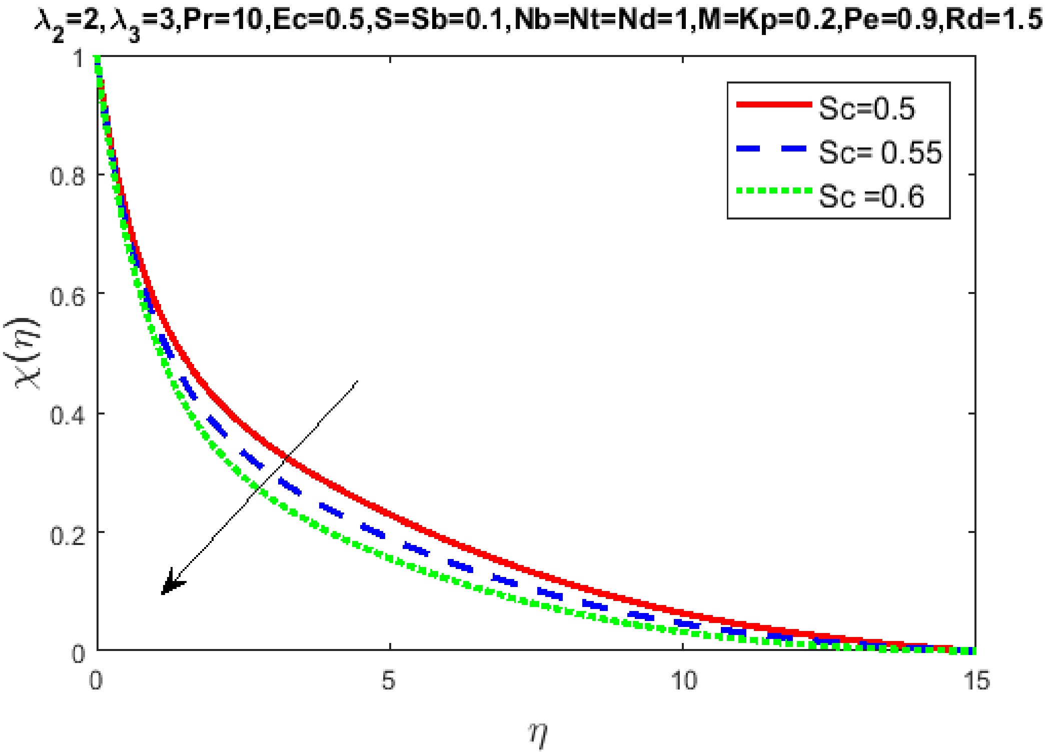

Figure 3.17: Impact of several values of Sc on the microorganism curve $\chi(\eta)$

Chapter 4

Conclusions/Summary

The review work analyses the Jeffrey nanofluid's optimal flow under slip condition that are constrained by a stretched surface. As a novelty, the effects of viscous dissipation and activation energy are also discussed. Numerical solutions based on the bvp4c approach are carried out. The following is a summary of the main findings from this review work:

• When the ratio of relaxation to retardation time parameter increases, the velocity profile improves however when the retardation time parameter increases, the velocity profile degrades.

• Slip increases the amount of resistance to fluid flow.

• With the help of thermophoretic and brownian constant, the temperature profile shows increasing behaviour.

The paper's main concern is a steady MHD fluid of bio nanofluid taking heat radiation and chemical reaction with thermophysical properties. The bvp4c approach-based numerical solutions are applied. The main observations are discussed below:

• The velocity curve is reduced as a result of improvements in the relaxation to retardation times ratio ($lambda2$), the magnetic parameter (M), and the porosity variable (kp). While an increase in the velocity profile is caused by a rise in the retardation parameter ($lambda3$).

• The values of these parameters—the retardation times ratio ($lambda2$), magnetic parameter (M), Brownian motion parameter (Nb), ratio of motile microorganisms (Nd), thermal radiation parameter (Rd), and heat source (S)—increase as the temperature of the fluid rises. However, it is noted that an increase in the prandtl number (Pr) and retardation parameter ($lambda3$) lowers the temperature profile.

• The concentration profile is elevated as the values of the thermophoretic parameter (Nt) increase. It should be noticed that increasing the selections for the Brownian motion parameter (Nb) and heat source (S) causes the concentration profile to decrease.

• The density of motile microorganisms profile is lowered by rising levels of the Brownian motion parameter (Nb), thermal radiation parameter (Rd), ratio of motile microorganisms (Nd), and Peclet number (Pe). The thermophoretic parameter (Nt) and bioconvection (Sb) values rising shows an increase in the density of motile microorganisms.

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