Analysis of Tetraquarks and their production in B_c meson decays.

By

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A dissertation submitted in partial fulfillment of the requirements
for the degree of Master of Science in Physics

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(2022)

FORM TH-4

National University of Sciences & Technology

MS THESIS WORK

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To my adoring parents, who valued my education more than anything else.

Acknowledgements

All praise is due to **ALLAH** with whose favours all good can be accomplished. In addition, we Muslims are indebted to Prophet **Muhammad SAW.**

The dedication of numerous people resulted in this dissertation. I would like to express my profound gratitude to my supervisor, **Dr Ali Paracha** for being the constant source of guidance during my course work and research. Members of my Guidance and Examination Committee (GEC), **Dr. Saadi Ishaaq** and **Dr. Tajammal Khokhar**, have been very supportive throughout my research. For their thoughtful assistance and guidance during my MS programme, I owe a debt of gratitude to each and every one of my Teachers at School of Natural Sciences (SNS).

Apart from academia, I can recall many other faces to whom I feel thankful. I am highly indebted to my parents for being the source of great strength in my life. Their prayers always provide me with the courage and bravery to carry forward. Thanks to my siblings for constantly encouraging me. I'm appreciative of my husband's unwavering support, which enabled me to finish this thesis.

Sincere thanks to my friends especially Benish Altaf, Maryam Athar and Mahpara. They were kind enough to bear me in academic and emotional ups and downs. In the end, I am grateful to all the other people around me who even contributed a bit in my journey of the completion of this thesis.

Maria Iqbal

Abstract

In this thesis, the mass spectra of tetraquark X(5568), observed by the DO collaboration, has been anticipated using the tightly bound diquark model. This tetraquark is found to have the composition of four distinct quark flavors $bdus$. Using the masses and spin-spin interactions within the diquark (anti-diquark), we approximated the mass of not only the lightest $bdus$ tetraquark state, 0^+ and the charged tetraquark state, together with the neutral partners $I = 1$ and $I = 0$, but also the mass of the analogous tetraquark in charm sector, whose composition is $cdus$. The aim of this thesis is to search the tetraquark in the decay of B_c mesons. Using the Naive factorization, the amplitude and the branching ratio for the decay $B_c \to B_s\pi$ is calculated. With the value of the mass for the $bdus$ tetraquark evaluated using diquark model, the branching ratio of the $B_c \to X_{b0}\pi$ decay with the subsequent decay $X_{bo} \to B_s\pi$ is approximated on the same pattern. As of yet, there is no experimental value reported for this branching ratio, thus we will have to wait for future research to test this proposed discovery method.

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Chapter 1

Introduction

Man has been seeking knowledge of the deepest and tiniest aspects of our environment for as long as anybody can remember.The exact nature of matter has been intensively debated throughout the history of natural sciences, and this idea has been refined and expanded upon over time.The view that matter is comprised of a collection of atoms is attributed to Democritus in the 5th century B.C., although this idea was not scientifically explored until the Renaissance [5]. Since then, the notion of the atom has been contested toward the end of the nineteenth century, verified at the start of the twentieth century, and found to have contents of negatively charged electrons, protons having positive charge and neutral neutrons few decades later.However, the mechanism that holds these atom components together was not fully understood. Yukawa proposed the first substantial theory addressing this topic in 1934. (Yukawa was mainly interested in the strong nuclear force and devised an ingenious explanation for its short range. His concept is a combination of particles, forces, relativity, and quantum mechanics that applies to all forces. Yukawa suggested that force is transmitted through particle exchange (called carrier particles). These carrier particles make up the field.) As a result, the meson (or π pion) was proposed and eventually discovered. Dirac proposed a relativistic quantum mechanics framework in 1927 that was designed to represent free electrons, however the equation predicted the presence of a negative energy solution per each positive energy solution, trying to suggest that electrons radiate an endless amount of energy. The discovering of the positron, a positively charged sister of the electron, later explained what appeared to be a disastrous flaw. Dirac's framework for describing free electrons turns out to be a universal property of quantum field theory, in which every particle has an antiparticle with the same mass but opposing charge [6]. In the years after 1930, several more significant contributions were made, including as the finding and classification of neutrinos, which were originally conjectured based only on conservation or symmetry considerations. Also, the alleged strange particles and, afterwards,the baryons emitted by the cosmic rays, later gave rise to the Eightfold Way, a highly successful classification scheme for mesons and baryons. Gell-Mann and Zweig separately proposed the view of quarks as fundamental building block of matter in 1964. They suggested that all hadrons are composed of quarks, providing a comprehension of the Eightfold Way categorization of mesons and baryons [7]. With the inclusion of quarks and the Eightfold Way, particle physics has seen a lot of new research, and in the current view, the Standard Model, briefly discussed in Chapter 2, describes the underlying principles of nature, and we now have a far better grasp of them. Even if the concept of some fundamental building block of matter can be traced back to the ancients, experimental reality is contemporary. The D0 observation of a narrow structure $X(5568)$, constituting four different quark flavours ($bdus$), discovered by the $B_s^0\pi^+$ decay mode has recently sparked a lot of interest. The goal of this thesis is to provide an introduction to the topic of exotic hadrons, mainly tetraquarks, using the diquark-diquark model to estimate the mass spectrum of X(5568) as well as to anticipate a discovery mode for this tetraquark in the B_c decays. An overview of particle physics will be presented, with a focus on tetraquarks and some aspects of the history related to tetraquarks are highlighted. A more in-depth investigation is done on a tetraquark system comprising of a diquark and an antidiquark. The Standard Model, the primary theory that underpins particle physics, is shortly discussed in Chapter 2, followed by a overview of the experimental advancements in the realm of tetraquarks, concluding with an overview of the aspects of tetraquark model utilised in this subject. In chapter 3, the diquark model studied in this thesis is laid out and several numerical fits are made to experimental data. The result of this modeling scheme is presented at the end of this chapter. Non-leptonic charmed B meson decays are discussed in chapter 4. The results from chapter 3 are then utilized to calculate the branching ratio of the charmed B meson decay to the tetraquark X(5568). Finally, in chapter 5, the results are summarized and concluding remarks are given.

Chapter 2

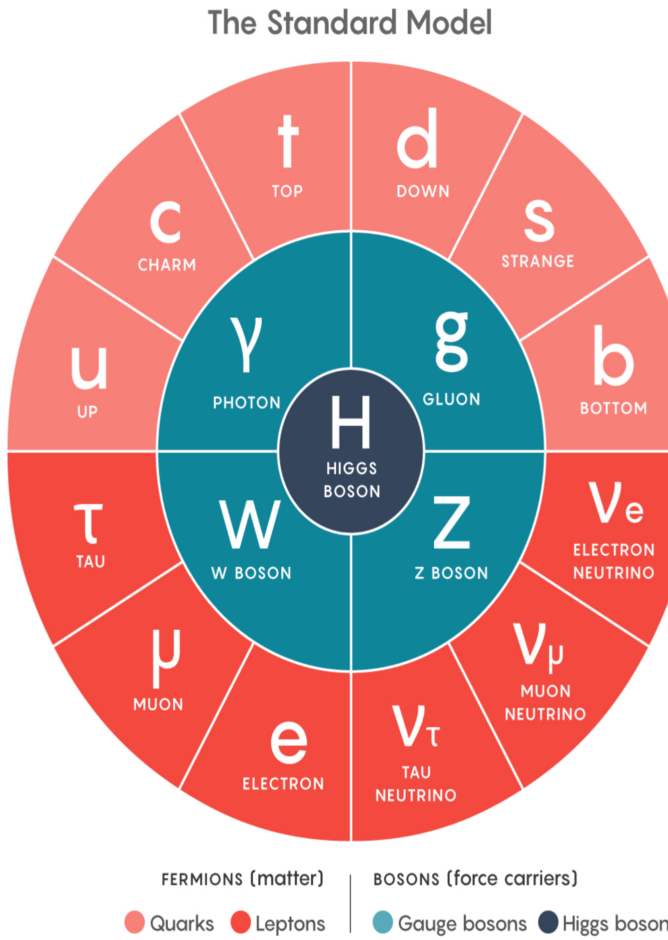
Diquark-diquark Model For Tetraquarks

The main components of matter are referred to as quarks, first suggested by Murray Gell-Mann and George Zweig in 1964 [8]. All the particles are made up of quarks,commonly knowns as Hadrons. We differentiate between baryons and mesons: On one hand, baryons are half-integer spin particles, whereas the most common are made up of a trio of quarks (or three anti-quarks). Mesons, on the other hand, are made up of quark-antiquark pairings and have an integer spin. There are quarks of six different kinds , known as flavors, that are classified into three generations. The first generation includes the light up (u) and down (d) quarks, it has implications for the observable cosmos as well as life on Earth. as the proton (uud), the neutron (udd), and the three pions ($u\bar{d}$, $d\bar{u}$-$d\bar{d}$). The second generation includes the strange (s) and charm (c) quarks, which are several hundred times heavier than the light ones.Lastly, in the third generation, we have the heavy bottom (b) and top (t) quarks, whose masses are hundreds of times that of the light u/d quarks [9].The second and third generations do occur and they are created when the particles of first generation collide either in laboratory or in cosmic rays. Up, down, strange, charm, bottom, and top (from lightest to the heavy ones) quarks, as well as their antiquarks, with six different flavours, has its unique mass and charge. Hadrons are held together by force-carrying particles known as gluons, which clump together in groups of two or three. Ordinary baryons are hadrons that include the proton and neutron of an atom, both of which are built up of three-quark combinations, whereas quark-antiquark couples are used to make hadronic particles known as mesons. Consider quarks to be the LEGO bricks of the subatomic realm, which may be mixed and matched to create more complex structures. More unusual hadrons, according to Gell-Mann, could be created from quark configurations of four or even five quarks, but they were only theoretical until recently. Since such exotic heavy particles decay into more stable byproduct particles in fractions of a second, this is the case. Those byproducts are the ones that appear in particle accelerator detectors, forming separate fingerprints for their heavier precursor particles. However, separating those fingerprints from the noise in the massive volumes of data generated by particle collisions is incredibly difficult. An exploration of the basic theory explaining quarks and quark composites is covered in this chapter. The Standard Model is presented, as well as a review of experimental advances in the field of tetraquarks.

2.0.1 Standard Model

The current view of matter is that all the matter is made up of three types of elementary particles: *leptons*, *quarks*, and $force\ mediators$. The quantum numbers of the leptons and quarks are used to further categorise them. Charge, electron number L_e, muon number L_μ, and tau number L_τ are all occupied by the leptons. The symbols for each lepton are presented in figure below.

There are six anti-leptons that corresponds to such six leptons, with all signs reversed. This results in a total of twelve leptons. The first generation of quarks u and d form an iso-doublet *i.e.* they are allocated isospin $I = \frac{1}{2}$ and $I_3 = -\frac{1}{2}$, [that is why they are termed *up* and *down* quarks]. "New quantum numbers



are assigned to the second and third generations of quarks as follows: s-quark, strangeness S = -1, c-quark, charm C = +1, b-quark, bottomness B = -1, t-quark, topness $T = +1$ ". They are always generated in pairs, resulting in a final state with $S = 0$, $C = 0$, $B = 0$, and $T = 0$, all these quantum numbers remain conserved in electromagnetic and strong interaction. Charge and flavor number are assigned to the quarks, with d, u, s, c, b and t denoting *down*, *up*, *strange*, *charm*, *bottom*, and *top* flavor, respectively. Table 2.2 shows how these are organised, with the symbol for each quark in the first column. Each quark also has a colour assigned to it: red, green, or blue.

Quark type (Flavor)	Electric charge
(u, d)	$(2/3, -1/3)$
(c, s)	$(2/3, -1/3)$
(t, b)	$(2/3, -1/3)$

Gravity, electromagnetism, the weak interaction, and the strong interaction are the four fundamental forces or interactions between particles. The Standard Model is a theory that explains three fundamental interactions: electromagnetic, weak, and strong interactions. The interaction is mediated by a particle known as *gauge bosons* [6].

Gravitation

The force that keeps the planets in orbit draws bodies in a proportionate magnitude to their mass. Newton's law of gravitation or Einstein's general relativity are good descriptions, but there is no generally accepted quantum theory of gravity. This force is neglected when describing particle interactions in particle physics because it plays such a minor role.

The Electromagnetic Interaction

The *photon* is the electromagnetic interaction's mediating gauge boson, acting between charged particles. The interaction produces the force among charged particles, as well as light and other electromagnetic waves. Quantum electrodynamics (QED) is a theory that describes electromagnetic interaction using the photon as a mediating particle.

The Weak Interaction

This interaction occurs between quarks and is based on the *color* of the interacting quarks changing. It is the only interaction that can transform quarks into other quarks, and as the name implies, it is weaker than the other interactions. The weak interaction's mediators are dubbed Z and $W^{\pm}$ bosons.

The Strong Interaction

The strong force acts between quarks, and the *gluon* serves as the intermediary particle. It is in charge of holding quark composites together as well as the nucleon's nucleus binding. Quantum chromodynamics (QCD) is a theory of the strong force that was developed in the 1960s.

Hadrons:

Quarks and gluons are unable to be observed as isolated particles, but they form composite structures *i-e hadrons*. This is due to a phenomenon known as $color\ confinement$, which forces stable hadrons to be solely made up of colorless composites of quarks. If the color attributed to its constituent quarks adds up to zero, a hadron is *colorless*. The quark combination of red, green, and blue, as well as that of a color with the corresponding anticolor, sums up to zero. Hadrons are divided into two types: *mesons* and *baryons*. A quark and an antiquark combine to generate a colourless hadron, named Mesons. Baryons are made up of three quarks that combine to generate colourless hadrons. Apart from mesons and baryons, the quark model does not rule out the possibility of other forms of hadrons with more than three quarks. The classification is not limited to baryons and mesons, but exotic hadrons include tetraquarks and pentaquarks [10]. Tetraquarks, which are made up of two quarks and two antiquarks, pentaquarks, which are made up of four quarks and one antiquark, and glueballs, which are made up solely of gluons, are examples. The main topic of this thesis is tetraquarks and the next section will provide a brief synopsis of the progress of these peculiar hadrons.

2.0.2 Progress in the Field of Tetraquarks

Hadrons containing four or more quarks were proposed in the 1970s, but the first alleged observations did not occur until the turn of the century. Hadrons having four quarks were discovered experimentally by Belle experiment for the first time in 2003 by Japanese physicists, where a resonance peak was observed at (3872.0 ± 0.6)MeV [11] and dubbed it $X(3872)$. Belle-II's discovery of $X(3872)$,which is an exotic hadron, in the decays $B^0 \to J/\psi\pi^+\pi^- K$ in 2003 was completely unexpected. The $X(3872)$ was confirmed as a genuine resonance, rather than a threshold effect, by $BABAR$ [12] , CDF II [13], D0 [14], and afterwards, by the LHCb [15] and CMS [16] (called a "cusp"). Although the BaBar and $D0's$ experimental investigations identified the $X(3872)$ particle in a variety of additional decays, many exotic hadrons are thought to only exist in one decay mode. Later, a large quantity of data had been made available by the experiments CMS, ATLAS and the LHCb on the $X(3872)$ particle, and its current mass is (3871.69 ± 0.17)MeV [17] . When analyzing the $X(3872)$ particle as an exotic particle, one frequent interpretation is that it has two quarks and two antiquarks $c\bar{c}u\bar{u}$ in its quark content and was initially thought to be an unknown excited charmonium particle, but a deeper look at the decay mode $X(3872) \to J/\psi\pi^-\pi^+$ revealed otherwise. It exhibits a non-typical isospin symmetry violation for a charmonium particle. However, it's still a mystery how it connected c and u together. In the study of hadron physics, its discovery marked the start of new age. Hence, the family has been increasing ever since. As a result, tetraquarks (four-quark states), a second layer of strongly interacting particles, arose from the debris of B-meson decays, each carrying a $c\bar{c}$ quark pair.

Many more unusual hadron candidates, having a final state of two heavy quarks with a set of two light antiquarks have been found, since after the discovery of the X(3872) particle. Meanwhile, entries of the Particle Data Group [18] are available for well over two dozen unusual hadrons. They have a variety of values of J^{PC} quantum numbers and can either be charged or neutral. These candidates have been given the labels X, Y, or Z states by the experimental partnerships, and they are combinely referred to as XYZ states [19]. The XYZ system′s trends are hampered by the fact that they incorporate both the short as well as the long distance QCD behaviour, making theoretical propositions challenging. As a result, there are currently several conflicting phenomenological hypothesis for these states. $Z_b^+(10610)$ and $Z_b^+(10650)$ are the two $b\bar{b}$ analogues of the $c\bar{c}$ states that Belle has identified at least [20]. They have valence quark's makeup of $b\bar{b}u\bar{d}$. Physicists at the LHCb confirmed the discovery of the tetraquark Z(4430) in 2014 [21], just a few years after it was first identified in the Belle detector of Japan's KEKB accelerator [22]. In 2016, physicists reviewing data from Fermilab's now-decommissioned Tevatron accelerator from 2002 to 2011 identified another new tetraquark, named X(5568), composed of quarks of four different flavours: up, down, odd, and bottom [23]. Tetraquarks used to be composed of at least two quarks with same flavor, hence X(5568) is an outlier among even this class of exotic particles. Recently, CERN unveiled yet another inclusion to the increasing family of tetraquarks in 2020: a hefty tetraquark comprising two charm quarks and two anti-charm quarks [24]. It was the first particle to be discovered with more than three quarks comprised entirely of one type of quark. It was also the first quark-gluon plasma to be made completely of heavier quarks.

Many models predict that the interior structure of tetraquarks is made up of pairs of diquarks and antidiquarks [25]. A quark-quark pair bound with each other forms a diquark, while an antidiquark comprises of a pair of antiquark-antiquark.They are not colourless in themselves, but they are thought to combine in such a way so as to generate colourless combinations of tetraquarks.

Theoretically, modelling tetraquarks with only heavy quarks is easier because various simplifications may be justified. In Chapter 3, we'll look at a non-relativistic model in which tetraquarks are made up of diquarks and antidiquarks that interact in the same way that conventional quarkonia does.

2.0.3 Quarkonium (S and P wave)

Quarkonium is the name given to the bound system of heavy quarks $Q\bar{Q}$, $Q = c, b$, which includes charmonium $c\bar{c}$ and bottomonium $b\bar{b}$. Quarks, being fermions possessing the spin 1/2, can have their bound system represented as $(Q\bar{Q})_{L,S}$. Now, the value of the spin S can be 0 and 1, while the spin wave function is anti-symmetric and symmetric sequently. If Q and $\bar{Q}$ are thought to be identical fermions with the difference in their charges only, then the Pauli principle can generally be stated as: With the exchange of particles Q and $\bar{Q}$, wave function is antisymmetric. In the particle exchange, when the space coordinates are exchanged a factor $(-1)^L$ is obtained , while we get a factor $(-1)^{S+1}$ in spin coordinates exchange and for charge exchange, we have a factor C (C is called C-parity) [26]. We have the Pauli principle as,

$$(-1)^{L+S+1}C = -1$$

Therefore,

$$C = (-1)^{L+S}$$

So, the result is.

$$\begin{aligned} C = -1 \text{ if } &\quad \text{L + S} \quad \text{odd,} \\ +1 \text{ if } &\quad \text{L + S} \quad \text{even} \end{aligned}$$

Also, $(Q\bar{Q})$ system has its parity given by,

$$P = (-1)(-1)^L = (-1)^{L+1}$$

Using the spectroscopic notation,

$$\mathrm{L} = 0, 1, 2, 3, \cdots$$

Complete Specification of a state is,

$$n^{2S+1}L_J$$

where the principal quantum number is represented by n while J being the total angular momentum. So, the states for $L = 0$ are,

$$\begin{array}{lllll} n & {}^1S_0 & n = 1, 2, & C = +1, \cdots & J^{PC} = 0^{-+} \\ n & {}^3S_1 & n = 1, 2, , & C = -1, \cdots & J^{PC} = 1^{--} \end{array}$$

The ground state is thus a hyperfine doublet *i.e.* $1^1S_0\,(0^{-+})$ and $1^3S_1\,(1^{--})$. States for $L = 1$ are,

$$\begin{array}{lllll} n & {}^1P_J & J = +1, & C = -1, & 1^{+-} \\ n & {}^3P_J & J = 0, 1, 2 & C = 1, & 0^{++}, 1^{++}, 2^{++} \end{array}$$

2.0.4 Tetraquark

Diquark-diantiquark mesons make up tetraquark mesons. A diquark can be in one of two color states: color antisymmetric state $\overline{3}_c$ or symmetric color state 6_c. Similarly, the antidiquark is in one of two color states: antisymmetric color state 3_c or symmetric color state $\overline{6}_c$. Here,

$$
\begin{aligned}
6_c \otimes \overline{6}_c &= 35_c \oplus 1_c \\
\overline{3}_c \otimes 3_c &= 8_c \oplus 1_c \\
\overline{3}_c \otimes \overline{6}_c &= \overline{10}_c \oplus 8_c \\
6_c \otimes 3_c &= 10_c \oplus 8_c
\end{aligned}
$$

The singlet state is formed only by $6_c \otimes \overline{6}_c$ and $\overline{3}_c \otimes 3_c$. All hadrons that have been observed are colour singlets. Similar to how force of repulsion is produced between similar charges by a photon exchange as well as the attraction force among the unlike charges, a gluon exchange generates force of attraction among color singlet states. However, when both the diquark as well as the antidiquark are in a colour symmetric state, they would have repulsive one gluon exchange potential unlike the attractive one in the case when we have both the diquark and antidiquark in color antisymmetric state. In just about any case, the color singlet tetraquark is taken into consideration, with composition of diquark as well as antidiquark exhibiting color triplet states $\overline{3}_c$ and 3_c sequently [26].

Now, diquarks can have either an antisymmetric or symmetric flavor:

$$
\begin{aligned}
[qq] &= \frac{1}{\sqrt{2}}\left(q_i q_j - q_j q_i\right) i, j = u, d, s, c \\
\{qq\} &= \frac{1}{\sqrt{2}}\left(q_i q_j + q_j q_i\right)
\end{aligned}
$$

Same is the case for the flavor states for antidiquark. Pauli principle, for antisymmetric color state $\overline{3}_c$ or 3_c, demands that a diquark or antiquark's wave function be completely symmetric altogether in space, flavour as well as in spin.

$$
\begin{array}{rlrl}
[qq]_{L=0,s=0} & ; P=1 & [qq]_{L=1,s=1} & ; P=-1 \\
\{qq\}_{L=0,s=1} & ; P=1 & \{qq\}_{L=1,s=0} & ; P=-1 \\
[\bar{q}\bar{q}]_{L=0,s=0} & ; P=1 & [\bar{q}\bar{q}]_{L=1,s=1} & ; P=-1 \\
\{\bar{q}\bar{q}\}_{L=0,s=1} & ; P=1 & \{\bar{q}\bar{q}\}_{L=1,s=0} & ; P=-1
\end{array}
$$

2.1 Models for Tetraquark:

To handle the exotic spectroscopy, various explicit kinematic and dynamical procedures have been devised. Hadroquarkonia, hybrids, hadron molecules, and compact diquarks are some of their names. Here's a quick rundown of what they're all about.

2.1.1 Tetraquark as Hadroquarkonia:

The comparability with hydrogen atom prompts this mechanism. A $Q\bar{Q}(Q = c, b)$ forms the hard core of the hadroquarkonium model, which is surrounded by light matter (light $q\bar{q}$ for tetraquarks and qqq for pentaquarks), with the two systems bound by a van der Walls type force. The J/ψ, ψ' or χ_c, degrees of freedom, for example, might be coupled with the light $q\bar{q}$ degrees of freedom to support the observed hadrons [27]. The hard core quarkonium might also be represented in a color-adjoint way, in which case the light degrees of freedom would also comprise of color-octets, producing an overall singlet. A Hadroquarkonium models bear a conceptual flaw in that if the force of binding is not so strong, why does the system maintain stability long enough to qualify as a distinct state? It's unclear why the $Q\bar{Q}$ core and light degrees of freedom do not undergo rearrangement giving rise to two heavy mesons($D\bar{D}^*, B\bar{B}^* etc.$) if the force is strong. This would thus prevent the states $(J/\psi, h_c)\pi\pi$ from appearing in their decays, which are in fact the modes of discovery for many exotic multiquark states.

2.1.2 Tetraquark as Hybrids:

Then there are hybrid models, which have been around since 1994 and are established on QCD-inspired flux-tubes, predicting the light and heavy quarks exotic J^{PC} states combinely. Hybrids, described as hadrons, are composed of valence quarks and gluons, such as $Q\bar{Q}g$. Gluon-dominated states initiate glueballs, having solid QCD predictions but have proven elusive empirically. Non-perturbative gluons, having essentially defined role in creating hybrids, are sort of quasi-particles with $J^{PC} = 1^{+-}$, roughly have a 1 GeV, excitation energy according to recent lattice-QCD computations [28]. The lightest charmonium multiplets would have a mass of roughly 4200 MeV. The Hadron Spectrum Collaboration has conducted extensive research of such hybrids on the lattice, while assuming a heavy pion mass of $m_\pi \sim$ 400 MeV with lattice spacing fixed.This computation identifies a variety of states as charmonium hybrid multiplets, with quantum numbers $J^{PC} = 0^{-+}, 1^{--}, 2^{-+}, 1^{-+}$ and masses (M) in the range $M - M_{\eta_c} \simeq 1200 - 1400$MeV. The state $Y(4260)$, which has a tiny e^+e^- annihilation cross section, had a hybrid interpretation that was provided for the $J^{\mathrm{PC}} = 1^{--}$ state, along similar lines but considerably earlier. Hybrids have also been put forward as providing templates for other exotic hadrons in the meantime. In the fabric of effective field theories, they have been given a more theoretical foundation. However, even in the presence of all of these great theoretical improvements that may one day lead to trustworthy quantitative predictions, in the current investigations, an unambiguous hybrid candidate is still awaiting discovery.

2.1.3 Tetraquark as Hadron molecules:

The tetraquark (pentaquark) states are the bound state of meson-meson (meson-baryon), having residual van der Waals force of attraction initiated by mesonic exchanges according to a popular theory. This idea is supported by the propinquity reflected by the observed exotic hadron masses and their corresponding two-particle thresholds. Following the Heisenberg uncertainty principle, an extremely low binding energy is resulted, which gives exotic hadrons inordinately large hadronic radii. The $X(3872)$, having an S-wave coupling to $D^*\bar{D}$ (and its conjugate) with a binding energy of $\mathcal{E}_X = M_{X(3872)} - M_{D^{*0}} - M_{\bar{D}^0} = +0.01 \pm 0.18\text{MeV}$, is the sterling example of this. Such a hadron molecule elements will be detached by a huge mean square distance $\langle r_X \rangle \propto 1/\sqrt{\mathcal{E}_X} \simeq 10\text{fm}$, where the stated radius correlates to a binding energy of $\mathcal{E}_X = 0.15\text{MeV}$. As opposite to what has been achieved through a number of experiments at the Tevatron and the LHC, this would result in tiny production cross-sections in hadronic collisions . In cases of some theoretical structures, this difficulty is eased by employing a hard (point-like) centre into the hadron molecules. In this way, they offer a similarity to the hadroquarkonium models as mentioned earlier. Rescattering effects are used in others to significantly enhance the cross-sections . The $\mathrm{p_T}$ -spectrum exhibited by these exotic hadrons, under consideration, in the LHC's prompt production data is a signified test. The hadron molecular model can explain some aspects of the current data, such as the unavailability of adequate experimental evidence regarding a quartet of exotic states that are almost mass-degenerate with the X(3872) and contain a light quark-antiquark pair $q\bar{q}, q = u, d$, resulting in the creation of I = 1 and I = 0 multiplets. The diquark image, which is explored below, predicts these multiplets. All isospin configurations, however, won't bond in the molecular image since the main interaction is provided by an exchange of a pion, an isospin-1 meson. As a result, no resonant structure is seen at the$D^0\bar{D}^0$ threshold, indicating that parity conservation prevents a strong interaction coupling of the three pseudo-scalars $D^0\bar{D}^0\pi^0$. On the other hand, for exotics with masses significantly above the pertinent criterion, the argument for hadron molecules is weak.For example, the Zc(3900)+ has a mass of 20 MeV over the $D\bar{D}^*$ threshold, which is also its major decay mode. In the molecular image with regards to hadron, this is hard to accept. In case hadron molecule, theoretical curiosity is still there, with a vast and expanding literature on the subject with ever-increasing horizons, which is cited here in sampling.

2.1.4 Tetraquark as compact Diquark-Antidiquark meson:

Last but not least, there are QCD-based interpretations that characterise tetraquarks and pentaquarks as authentically novel hadron species with a color-non singlet diquark as their basic building block. In the limit of large Nc of QCD, it is demonstrated that these so-called diquark-antidiquark mesons exist as poles in the S-matrix. They might have narrow widths in this approximation, which makes them suitable candidates for multiquark states. The first attempts to investigate multiquark states employing Lattice QCD were made , in which four-quark operator correlations were numerically studied. Using these methods, no evidence of tetraquark states in the sense of S-matrix poles has been found yet. While attempting to establish the signal of a resonance, the background must be strictly controlled.In multiquark state lattice QCD simulations, this is currently not the case. This can be linked to the occurrence of several neighbouring hadronic thresholds as well as lattice-specific difficulties such an unrealistic pion mass. To get definite findings, more sophisticated analytic and computational tools are required [2]. Approximate phenomenological approaches are the way ahead in the absence of trustworthy first principle computations. In this vein, a successful Hamiltonian approach has been widely employed, in which tetraquarks are diquark-antidiquark objects that are bonded together via gluonic interactions (while pentaquarks are diquark-diquark-antiquark objects). This allows for the study of spectroscopy and some characteristics of tetraquark decays. Because it may be used to link the charmonia-like states to their bottomonium-like counterparts via heavy-light diquarks, heavy quark symmetry is beneficial. Diquark models predict an incredibly rich spectra of tetraquarks and pentaquarks, of which only a small portion has been experimentally observed, as shown here. As a result, dynamical selection criteria are desperately needed in diquark models to limit the number of observable states.Because of the complexity of the underlying dynamics, the existing theoretical framework—which subsumes the dynamics within the constraints of the effective Hamiltonian—is obviously insufficient. In the following section, the diquark picture's phenomenology will be examined to see how well it summarizes exotic hadrons and other properties measured in current experimental data.

2.1.5 The Diquark Model:

Diquarks are closely bound coloured objects that serve as the key component for the formation of tetraquark mesons and pentaquark baryons, according to the primary assumption of this paradigm. The diquarks have two key SU(3)-color representations, for which the notation used $[qq]_c$ and $\mathcal{Q}$ are interchangeable. Due to the fact that the quark transformation is triplet 3 of the color SU(3), the direct product $3 \otimes 3 = \bar{3} \oplus 6$ produces the diquarks which are either color anti-triplets $\bar{3}$ or color sextets 6. Using one-gluon exchange as its foundation, the following leading diagram is presented.

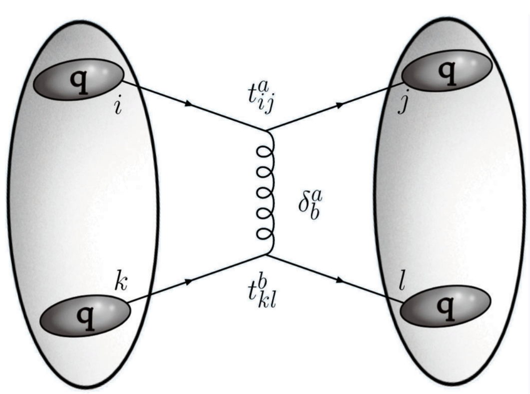


Figure 2.1: Diquark′s One-gluon exchange diagram [1]

The decomposition of the SU(3)-matrices product from 2.1 can be written as

$$t^a_{ij} t^b_{kl} = -\frac{2}{3} \underbrace{\left(\delta_{ij}\delta_{kl} - \delta_{il}\delta_{kj}\right)^a /2}_{\text{antisymmetric: represents } \bar{3}} + \frac{1}{3} \underbrace{\left(\delta_{ij}\delta_{kl} + \delta_{il}\delta_{kj}\right)^b /2}_{\text{symmetric: represents } 6} .$$

The antisymmetric $\bar{3}$ formulation coefficient appears as -2/3 suggesting that the two diquarks are connected with half the strength as that of a quark and an antiquark. In the latter case, coefficient is -4/3. On the contrary, the symmetric 6 has a coefficient of 1/3, which is positive indicating repulsion. This perturbative reasoning is supported by simulations of lattice QCD [29]. Hence, Considering the phenomenology, a diquark is thought to be a antitriplet of $SU(3)_c$ while the antidiquark being a color-triplet. The spectroscopy of both common and exotic hadrons is now generated from two color-triplet fields, quark q_3 with anti-diquark $\bar{\mathcal{Q}}\ or[\bar{q}\bar{q}]_3$, as well as from two color-antitriplet fields, which are antiquark$\bar{q}_{\bar{3}}$ and diquark $\mathcal{Q}\ or[qq]_{\bar{3}}$.

As we have the value of quark spin as -1/2 ,there are two feasible spin configurations for a diquark: spin-0 has two quark spin vectors that are anti-parallel, whereas spin 1 has two quark spin vectors that are aligned, as illustrated in Fig. 2 [30]. The terms "good diquarks" and "bad diquarks" were given to them to denote the fact that in the former instance, the two quarks bound together. However the binding is less strong in the latter scenario.

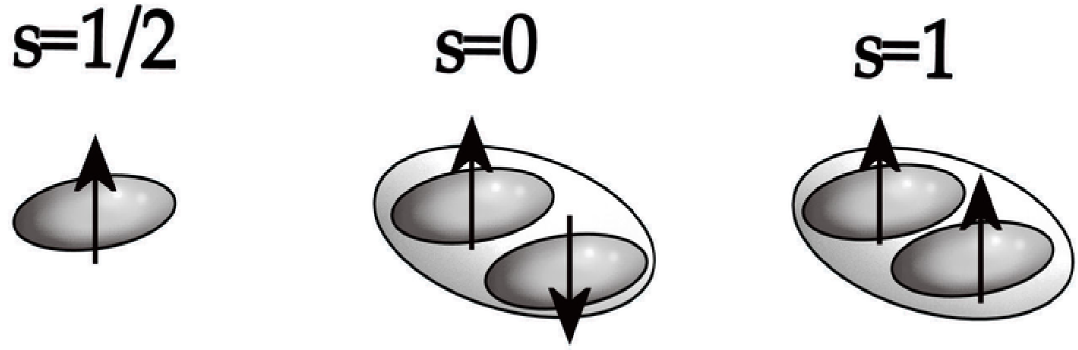


Figure 2.2: Spin of Diquarks [1]

Light diquark lattice simulations show some support for this pattern [29].But in the heavy quark effective theory, for heavy mesons and baryons, it is proved explicitly that the spin degree of freedom of the heavy quark system decouples. We anticipate the same decoupling to hold for our case of heavy-light diquarks $[Q_i q_j]_{\bar{3}}$, here we have $Q_i = c, b$ $q_j = u, d, s$. For the heavy-light diquarks, the configurations of spin-0 and spin-1 are both present.For Diquarks, in the case of heavy baryons (such asΛ_b and Ω_b), composed of one heavy quark with one light diquark, it is required that both the quantum numbers $j^p = 0^+$and $j^p = 1^+$ of the diquark fits into the observed baryonic spectrum. The interpolating diquark operators are generated using heavy-light diquarks and the two spin-states (here $Q = c, b$) [31].

$$
\begin{array}{llll}
\textbf{Scalar} & 0^+: & \mathcal{Q}_{i\alpha} & = \epsilon_{\alpha\beta\gamma}\left(\bar{Q}_c^{\beta}\gamma_5 q_i^{\gamma} - \bar{q}_{i_c}^{\beta}\gamma_5 Q^{\gamma}\right), \\
\textbf{Axial}-\textbf{Vector} & 1^+: & \overrightarrow{\mathcal{Q}}_{i\alpha} & = \epsilon_{\alpha\beta\gamma}\left(\bar{Q}_c^{\beta}\vec{\gamma} q_i^{\gamma} + \bar{q}_{i_c}^{\beta}\vec{\gamma} Q^{\gamma}\right).
\end{array}
$$

Pauli matrices, in the non-relativistic (NR) limit, parametrize these states as:

$$\Gamma^0 = \frac{\sigma_2}{\sqrt{2}} \left(\; Scalar \; 0^+\right)$$

and

$$\vec{\Gamma} = \frac{\sigma_2\vec{\sigma}}{\sqrt{2}} \left(Axial - Vector \; 1^+\right)$$

In order to describe a tetraquark state whose total angular momentum is represented by J here, a state vector will be utilized i-e $\left|Y_{[bq]}\right\rangle = |s_{\mathcal{Q}}, s_{\bar{\mathcal{Q}}}; J\rangle$, where $s_{\mathcal{Q}}$ is the diquark spin while $s_{\bar{\mathcal{Q}}}$ is the antidiquark spin. For $J^{PC}= J^{++}$, 1^{+-}, and 1^{--} states, we use symbols X_J, Z and Y, respectively. Thus, the Pauli forms for the tetraquark with the following diquark-spin and J, the angular momentum for tetraquarks, would be

$$
\begin{aligned}
|0_{\mathcal{Q}}, 0_{\bar{\mathcal{Q}}}; 0_J\rangle &= \Gamma^0 \otimes \Gamma^0, \\
|1_{\mathcal{Q}}, 1_{\bar{\mathcal{Q}}}; 0_J\rangle &= \frac{1}{\sqrt{3}}\Gamma^i \otimes \Gamma_i \ldots, \\
|0_{\mathcal{Q}}, 1_{\bar{\mathcal{Q}}}; 1_J\rangle &= \Gamma^0 \otimes \Gamma^i, \\
|1_{\mathcal{Q}}, 0_{\bar{\mathcal{Q}}}; 1_J\rangle &= \Gamma^i \otimes \Gamma^0, \\
|1_{\mathcal{Q}}, 1_{\bar{\mathcal{Q}}}; 1_J\rangle &= \frac{1}{\sqrt{2}}\varepsilon^{ijk}\Gamma_j \otimes \Gamma_k.
\end{aligned}
$$

When distinguishing between the $c\bar{c}$ and $b\bar{b}$ states, the subscript c or b might be employed.

2.2 Non-Relativistic Hamiltonian of tetraquark

The mass spectrum for tetraquarks is computed using the effective Hamiltonian given below in the non-relativistic limit.

$$H_{\text{eff}} = 2m_{\mathcal{Q}} + H_{SS}^{(qq)} + H_{SS}^{(q\bar{q})} + H_{SL} + H_{LL}, \tag{2.1}$$

where diquark mass is represented by m_Q while the second term depicts spin-spin interactions among quarks (or antiquarks) inside a diquark (or within an anti-diquark), the spin-spin interactions among a quark and an antiquark which belong to two different shells is given by the third term. While the last two terms, which incorporate the tetraquark quantum numbers, represent the spin-orbit and orbit-orbit interactions, sequently. In the S-states, these two words are not present. Consider the condition $Q = c$ and the individual phrases are displayed from H_{eff}.

$$\begin{aligned}
H_{SS}^{(qq)} &= 2\left(\mathcal{K}_{cq}\right)_{\bar{3}}\left[\left(\mathbf{S}_c \cdot \mathbf{S}_q\right) + \left(\mathbf{S}_{\bar{c}} \cdot \mathbf{S}_{\bar{q}}\right)\right], \\
H_{SS}^{(q\bar{q})} &= 2\left(\mathcal{K}_{c\bar{q}}\right)\left(\mathbf{S}_c \cdot \mathbf{S}_{\bar{q}} + \mathbf{S}_{\bar{c}} \cdot \mathbf{S}_q\right) + 2\mathcal{K}_{c\bar{c}}\left(\mathbf{S}_c \cdot \mathbf{S}_{\bar{c}}\right) + 2\mathcal{K}_{q\bar{q}}\left(\mathbf{S}_q \cdot \mathbf{S}_{\bar{q}}\right), \\
H_{SL} &= 2A_{\mathcal{Q}}\left(\mathbf{S}_{\mathcal{Q}} \cdot \mathbf{L} + \mathbf{S}_{\bar{\mathcal{Q}}} \cdot \mathbf{L}\right), \\
H_{LL} &= B_{Q\mathcal{Q}}\frac{L_{\mathcal{Q}\bar{\mathcal{Q}}}\left(L_{\mathcal{Q}\bar{\mathcal{Q}}} + 1\right)}{2}.
\end{aligned}$$

The interactions of the spins of charm c with light quark q in a colour anti-triplet form for one diquark is parametrized by $\left(\mathcal{K}_{cq}\right)_{\bar{3}}$, while for the color-singlet state, where we have two distinct diquarks, the spin interactions in between the quark i and the antiquark $\bar{j}$ are parametrized by $(\mathcal{K}_{i\bar{j}})$. $A_{\mathcal{Q}}$ is the parameter which describe how strong the spin-orbit force is while $B_{\mathcal{Q}}$ specify the intensity of the orbital angular force.

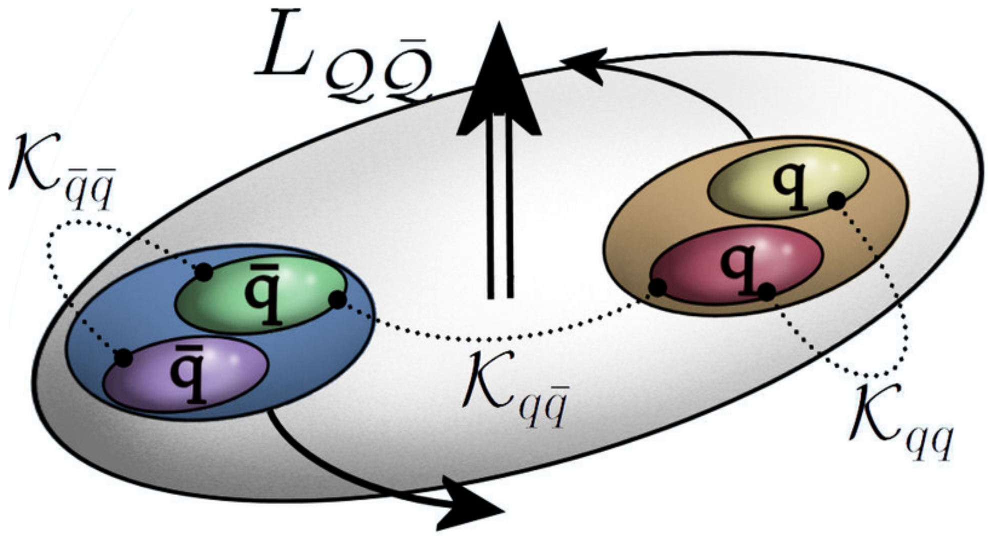


Figure 2.3: Representation of the diquark-diquark interaction [2]

The following is the form of the usual algebra for angular momentum:

$$H_{\text{eff}} = 2m_{\mathcal{Q}} + \frac{B_{\mathcal{Q}}}{2}\langle L^2 - 2a\langle L \cdot S\rangle + 2\kappa_{qc}\left[\langle s_q \cdot s_c\rangle + \langle s_{\bar{q}} \cdot s_{\bar{c}}\rangle\right]$$

$$= 2m_{\mathcal{Q}} - aJ(J+1) + \left(\frac{B_{\mathcal{Q}}}{2} + a\right)L(L+1) + aS(S+1) - 3\kappa_{\text{qc}} + \kappa_{\text{qc}}\left[s_{\text{qc}}\left(s_{\text{qc}}+1\right) + s_{\bar{q}\bar{c}}\left(s_{\bar{q}\bar{c}}+1\right)\right]$$

With some required parameter rescaling, this effective hamiltonian can be used for tetraquark containing a $b\bar{b}$ pair.

2.3 Tetraquark states of S and P-Wave for c$\bar{\text{c}}$ and b$\bar{\text{b}}$

The Fierz transformation relates the two states , one which is in the diquark-antidiquark basis $\left|s_{qQ}, s_{\bar{q}\bar{Q}}; S, L\right\rangle_J$ with another one which is in the $Q\bar{Q}$ *and* $q\bar{q}$ basis $\left|s_{q\bar{q}}, s_{\mathcal{Q}\bar{\mathcal{Q}}}; S', L'\right\rangle_J$. The positive parity S-wave tetraquarks are described by the six states listed in Table 1. Because the L = 0 quantum number defines these states, so for these, the masses are calculated by M_{00} and $\mathcal{K}_{q\mathcal{Q}}$ only, yielding a variety of predictions that may be confirmed with experiments. Table 2 enlist the states for P-wave. In this, L = 1 for the first four states while L = 3 for the fifth one, which implies that it will be considerably heavier.

tags	J^{pC}	$\left\vert s_{qQ}, s_{\bar{q}\bar{Q}}; S, L \right\rangle_J$	$\left\vert s_{q\bar{q}}, s_{Q\bar{Q}}; S', L' \right\rangle_J$	Masses
X_0	0^{++}	$\vert 0,0;0,0\rangle_0$	$\left(\vert 0,0;0,0\rangle_0 + \sqrt{3}\vert 1,1;0,0\rangle_0\right)/2$	$M_{00} - 3\kappa_{qQ}$
X'_0	0^{++}	$(\vert 1,1;0,0\rangle_0$	$\left(\sqrt{3}\vert 0,0;0,0\rangle_0 - \vert 1,1;0,0\rangle_0\right)/2$	$M_{00} + \kappa_{qQ}$
X_1	1^{++}	$(\vert 1,0;1,0\rangle_1 + \vert 0,1;1,0\rangle_1)/\sqrt{2}$	$\vert 1,1;1,L'\rangle_1$	$M_{00} - \kappa_{qQ}$
Z	1^{+-}	$(\vert 1,0;1,0\rangle_1 - \vert 0,1;1,0\rangle_1)/\sqrt{2}$	$(\vert 1,0;1,L'\rangle_1 - \vert 0,1;1,L'\rangle_1)/\sqrt{2}$	$M_{00} - \kappa_{qQ}$
Z'	1^{+-}	$\vert 1,1;1,0\rangle_1$	$(\vert 1,0;1,L'\rangle_1 + \vert 0,1;1,L'\rangle_1)/\sqrt{2}$	$M_{00} + \kappa_{qQ}$
X_2	2^{++}	$\vert 1,1;2,0\rangle_2$	$\vert 1,1;2,L'\rangle_2$	$M_{00} + \kappa_{qQ}$

Table 2.1: The diquark model′s S-wave tetraquark masses and states in two bases. [2]

The masses of a handful of the observed X, Y, Z states can be used to determine the parameters mentioned in the right-hand columns of Tables 1 and 2, with Table 3 showing their numerical outcomes. Heavy quark mass scaling is another way to link few parameters for the sectors $c\bar{c}$ and $b\bar{b}$ [32].

symbols	J^{pC}	$\left\vert s_{qQ}, s_{\bar{q}\bar{Q}}; S, L \right\rangle_J$	$\left\vert s_{q\bar{q}}, s_{Q\bar{Q}}; S', L' \right\rangle_J$	Masses
Y_1	1^{--}	$\vert 0,0;0,1\rangle_1$	$\left(\vert 0,0;0,1\rangle_1 + \sqrt{3}\vert 1,1;0,1\rangle_1\right)/2$	$M_{00} - 3\kappa_{qQ} + B_Q$
Y_2	1^{--}	$(\vert 1,0;1,1\rangle_1 + \vert 0,1;1,1\rangle_1)/\sqrt{2}$	$\vert 1,1;1,L'\rangle_1$	$M_{00} - \kappa_{qQ} + 2a + B_Q$
Y_3	1^{--}	$\vert 1,1;0,1\rangle_1$	$\left(\sqrt{3}\vert 0,0;0,1\rangle_1 - \vert 1,1;0,1\rangle_1\right)/2$	$M_{00} + \kappa_{qQ} + B_Q$
Y_4	1^{--}	$\vert 1,1;2,1\rangle_1$	$\vert 1,1;2,L'\rangle_1$	$M_{00} + \kappa_{qQ} + 6a + B_Q$
Y_5	1^{--}	$\vert 1,1;2,3\rangle_1$	$\vert 1,1;2,L'\rangle_1$	$M_{00} + \kappa_{qQ} + 16a + 6B_Q$

Table 2.2: The diquark model′s P-wave tetraquark masses and states in two bases [1]

The mass inaccuracies owing to parametric uncertainties are predicted to be around 30 MeV on average. Table 4 demonstrates that the charmonium-like sector has significantly more X, Y, Z hadrons found in experiments than the bottomonium-like sector, which has essentially three entries for $Z_b^+(10610)$, $Z_b^+(10650)$ and $Y_b(10891)$.

	charmonium-like	bottomonium-like
M_{00}[MeV]	3957	10630
κ_{qQ}[MeV]	67	23
B_Q[MeV]	268	329
a[MeV]	52.5	26

Table 2.3: Numerical values for H_{eff}

		charmonium-like		bottomonium-like	
Label	J^{PC}	State	Mass [MeV]	State	Mass [MeV]
X_0	0^{++}	–	3756	–	10562
X_0'	0^{++}	–	4024	–	10652
X_1	1^{++}	$X(3872)$	3890	–	10607
Z	1^{+-}	$Z_c^+(3900)$	3890	$Z_b^{+,0}(10610)$	10607
Z'	1^{+-}	$Z_c^+(4020)$	4024	$Z_b^+(10650)$	10652
X_2	2^{++}	–	4024	–	10652
Y_1	1^{--}	$Y(4008)$	4024	$Y_b(10891)$	10891
Y_2	1^{--}	$Y(4260)$	4263	$Y_b(10987)$	10987
Y_3	1^{--}	$Y(4290)$(or $Y(4220)$)	4292	–	10981
Y_4	1^{--}	$Y(4630)$	4607	–	11135
Y_5	1^{--}	–	6472	–	13036

Table 2.4: Experimental as well as from diquark model′s values of hadron masses of X, Y, Z [1]

Based on the values of the parameters in the aforementioned tables, there are a few predictions in the charmonium-like sector that lies in correct range. It should be emphasised that these input values are larger than those previously found [31], especially for the quark-quark couplings inside one diquark,κ_{qQ}. Experiments that assume diquarks are more firmly bound than indicated by the study of the baryons in the diquark-quark picture, get better results. The spectrum presented here accords with that in the modified scheme. [33]. The diquark-antidiquark model has been used as an alternative for computation of tetraquark spectrum [34].

At the moment, there aren't many unique bottomonium-like states. The reason for this is that in the decays of B-hadrons, a number of unique charmonium-like states have been found. The $b\bar{b}$ states are obviously not accessible in this mode.Only the processes like high-energy hadro- and also the electroweak one, can produce them. Tetraquark states containing a single b quark can theoretically be created in the decays of the B_c mesons [3]. We anticipate that all of the LHC experiments will report significant new findings in the domain of exotic spectroscopy incorporating open and hidden heavy quarks since the cross sections of $c\bar{c}$ and $b\bar{b}$ at the LHC are so huge. Exotica creation and decay measurements, like the transverse-momentum distributions and the polarisation information, would help us understand the underlying dynamics. The observation of the tetraquark state $bdus$ under consideration by DO collaboration has sparked a huge interest due to the fact that this would've been the 1st time a tetraquark state with an open b-quark had been discovered. These are predicted in the model of compact tetraquark .[3], as well as the hadron molecular framework [35]. Despite the fact that LHCb has a 20-fold larger B_s^0 sample than D_0, this has yet to be confirmed by the LHCb collaboration. In the next chapter, above mentioned picture of diquark-diquark model is used to estimate the mass of X(5568). The same values will be used in chapter 5 to calculate the branching ratio of the B_c decay to $X_{b0}\pi^+$, which could be the discovery mode of this tetraquark. We'll have to wait for more information from the LHC experiments.

Chapter 3

Mass Spectrum of Lightest X(5568) Tetraquark

The constituent quark model, in its simplified form, calculates the hadron masses from three main ingredients: quark contents, constituent quark masses, and spin-spin interactions [36]. According to it, the color-spin part of the Hamiltonian, given in equation 2.1, which describes the interaction among the components of a hadron would be,

$$H = \sum_i m_i + 2\sum_{i<j} \kappa_{ij}(S_i \cdot S_j) \tag{3.1}$$

and the sum runs over the hadron components. The coefficients κ_{ij} is some effective, representation depending chromo-magnetic couplings, *i.e.* influenced by the flavor of the constituents ij as well as by the pair's particular color state. m_i is the diquark constituent mass, S_i is the quark spin. The spin-spin interaction is presumed to be a contact one in this case [3].

It is unclear how this simple Ansatz can be deduced from the fundamental QCD interaction, specifically how the effect of the spin-independent color forces, which are important for quark confinement, can be epitomised in the integrant masses. However, the equation 3.1 does a good job of describing the spectrum of mesons and baryons, with the parameters having roughly the same values in different scenarios.

S-wave Heavy Diquark

In a diquark, heavy quark $Q(c,b)$ exist in the bound state, set up with light quark $q = (u,d,s)$.

For S-wave, L=0,

$$S = S_q + S_Q$$
$$S^2 = S_q^2 + S_q^2 + 2S_q.S_Q$$

we have,

$$\langle S_q.S_Q \rangle = \frac{\hbar^2}{2}\left[S\left(S+1\right) - \frac{3}{2}\right]$$

The spin-spin interactions that exist among quarks (antiquarks) inside the same diquark (antidiquark), which is tightly bound, are supposed to be the key interactions, in the type-II tetraquark model, which the most recent and most successful one [33].

3.0.1 The Spectrum of $[\bar{\mathbf{b}}\bar{\mathbf{q}}]\,[\bar{\mathbf{s}}\mathbf{q}']$ States

The spins of diquark and antidiquark, $S_{\bar{b}\bar{q}}, S_{sq'}$, total angular momentum represented by J, parity P as well as the charge conjugation C can all be used to classify states. The states are as follows [31]:

i. For $J^{PC} = 0^{++}$:, the two states are:

$$|0^{++}\rangle = \left|0_{\bar{b}\bar{q}}, 0_{sq'}; J=0\right\rangle$$
$$|0^{++\prime}\rangle = \left|1_{\bar{b}\bar{q}}, 1_{sq'}; J=0\right\rangle$$

ii. There can three states having positive parity and the value $J = 1$,

$$|A\rangle = \left|0_{\bar{b}\bar{q}}, 1_{sq'}; J = 1\right\rangle$$
$$|B\rangle = \left|1_{\bar{b}\bar{q}}, 0_{sq'}; J = 1\right\rangle$$
$$|C\rangle = \left|1_{\bar{b}\bar{q}}, 1_{sq'}; J = 1\right\rangle$$

$|A\rangle$ and $|B\rangle$ interchange in charge conjugation, whereas $|C\rangle$ is odd. As a result, the 1^+ complex has two C-odd and one C-even state:

$$\left|1^{++}\right\rangle = \frac{1}{\sqrt{2}}(|A\rangle + |B\rangle)$$
$$\left|1^{+-}\right\rangle = \frac{1}{\sqrt{2}}(|A\rangle - |B\rangle)$$
$$\left|1^{+-\prime}\right\rangle = |C\rangle$$

The state with zero value for both spins cannot occur as $J = 1$, while the state $C = +$ is the only state having both spins equal to one. Thus, in Equation above, the state $|1^{++}\rangle$ has a definite value for the $b\bar{q}$ spin, $S_{b\bar{q}} = 1$.

iii. $J^{PC} = 2^{+1}$, the single state is :

$$\left|2^{++}\right\rangle = \left|1_{\bar{b}\bar{q}}, 1_{sq'}; J = 2\right\rangle$$

2^{++} state has $S_{b\bar{q}} = 1$, too.

Here the composition is: $[\bar{b}\bar{q}]_{3c}[sq']_{\bar{3}\bar{c}}$ with $q \neq q' = d, u$, this indicates that only the couplings κ_{bq} and $\kappa_{sq'}$ will be retained. Hence, the heavy-light diquark would be matched up by the lightest states and the diquark spin values as: $S_{[bq]} = 0, 1$ and $S_{[sq]} = 0$. The 'good diquark' is what is referred to in the later situation [30], and the two resulting states are $J^P = 0^+$ or 1^+, with the 0^+ state being the lightest. The notation, we use, for these particles is,

$$X_{bo} = \left|0_{\bar{b}\bar{q}}, 0_{sq'}\right\rangle, X_{b1} = \left|1_{\bar{b}\bar{q}}, 0_{sq'}\right\rangle \tag{3.2}$$

Considering the above approximation of the type-II tetraquark model, for the states $[\bar{b}\bar{q}][sq']$, the ensuing S-wave mass formula is cumulative in energies of the diquark.

$$M(X_{bS}) = m_{[bq]} + 2\kappa_{bq} S_{\bar{b}} \cdot S_{\bar{q}} + m_{[sq]} + 2\kappa_{sq} S_q \cdot S_{q'}$$

$$= m_{[bq]} + \kappa_{bq}\left(S(S+1) - \frac{3}{2}\right) + m_{[sq]} - \frac{3}{2}\kappa_{sq} \quad (3.3)$$

where S $= S_{[bq]}$.

Mass Spectrum for $\mathbf{X_{b0}}$:

The effective Hamiltonian to deduce the tetraquark mass spectrum for S-state in the non-relativistic limit, using only spin-spin interactions, is as below,

$$H_{eff} = m_{bq} - \frac{3}{2}\kappa_{bq} - \frac{3}{2}\kappa_{sq} + \kappa_{bq}\left[s_{bq}\left(s_{bq}+1\right)\right] + \kappa_{sq}\left[s_{sq'}\left(s_{sq'}+1\right)\right] \quad (3.4)$$

The equation 3.3, the mass formula for S-wave X_{bS}, can be analogized with the relevant tetraquark's mass formula which are $a_0(980)$ [33], $Z_b(10610), Z_b'(10650)$, by substituting $b\bar{s} \to s\bar{s}$ and $b\bar{s} \to b\bar{b}$ [37].
For $a_0(980)$, $b\bar{s}$ is substituted with $s\bar{s}$,

$$a_0(980) = |0_{\bar{s}\bar{q}}, 0_{sq'}\rangle$$

So, the mass formula of $a_0(980)$ becomes,

$$M_{a_0} = m_{[sq]} + m_{[sq]} - 3\kappa_{sq} + \kappa_{sq}\left[s_{sq}\left(s_{sq}+1\right) + s_{sq'}\left(s_{sq'}+1\right)\right]$$

As $s_{sq} = s_{sq'} = 0$,

$$M_{a_0} = 2m_{[sq]} - 3\kappa_{sq} + \kappa_{sq}\left[0\left(0+1\right) + 0\left(0+1\right)\right]$$

$$M_{a_0} = 2m_{[sq]} - 3\kappa_{sq} + \kappa_{sq}\,[0+0]$$

$$M_{a_0} = 2m_{[sq]} - 3\kappa_{sq}$$

$$M_{a_0} = 2\left(m_{[sq]} + \frac{3}{2}\kappa_{sq}\right) \tag{3.5}$$

For tetraquarks $Z_b(10610), Z_b'(10650)$, the substitution $b\bar{s} \to b\bar{b}$ is done.

For $Z_b(10610)$,

$$Z_b = \frac{1}{\sqrt{2}}\left(|1_{\bar{b}\bar{q}}, 0_{bq'}\rangle\right) - \left(|0_{\bar{b}\bar{q}}, 1_{bq'}\rangle\right)$$

$$M_{Z_b} = 2m_{[bq]} - 3\kappa_{bq} + \kappa_{bq}\left[s_{\bar{b}\bar{q}}\left(s_{\bar{b}\bar{q}}+1\right) + s_{bq'}\,(s_{bq'}+1)\right]$$

As $s_{\bar{b}\bar{q}} = (|1,0\rangle)\,, s_{bq'} \;=\; (|0,-1\rangle)\,,$

$$M_{Z_b} = 2m_{bq]} - 3\kappa_{bq} + \kappa_{bq}\,[1\,(1+1) + 0\,(0+1)] + \kappa_{bq}\,[0\,(0+1) + (-1)\,((-1)+1)]$$

$$M_{Z_b} = 2m_{bq]} - 3\kappa_{bq} + \kappa_{bq}\,[2 + (0)] + \kappa_{bq}\,[(0) + ((0)]$$

$$M_{Z_b} = 2m_{bq]} - 3\kappa_{bq} + 2\kappa_{bq}$$

$$M_{Z_b} = 2m_{[bq]} - \kappa_{bq} \tag{3.6}$$

Similarly, for the tetraquark $Z_b'(10650)$,

$$Z_b' = |1_{\bar{b}\bar{q}}, 1_{bq'}\rangle_{J=1}$$

$$M_{Z_b} = 2m_{[bq]} - 3\kappa_{bq} + \kappa_{bq}\left[s_{\bar{b}\bar{q}}\left(s_{\bar{b}\bar{q}}+1\right) + s_{bq'}\,(s_{bq'}+1)\right]$$

As $s_{\bar{b}\bar{q}} \;=\; s_{bq'} \;=\; 1,$

$$M_{Z_b} = 2m_{bq]} - 3\kappa_{bq} + \kappa_{bq}\,[1\,(1+1) + 1\,(1+1)]$$

$$M_{Z_b} = 2m_{bq]} - 3\kappa_{bq} + \kappa_{bq}\,[2+2]$$

$$M_{Z_b} = 2m_{bq]} - 3\kappa_{bq} + 4\kappa_{bq}$$

$$M_{Z'_b} = 2m_{[bq]} + \kappa_{bq} \tag{3.7}$$

Adding equation 3.6 and 3.7, we get

$$m_{[bq]} = \frac{M(Z'_b) + M(Z_b)}{4}$$

Now subtracting equation 3.6 from 3.7, we get

$$\kappa_{bq} = \frac{M(Z'_b) - M(Z_b)}{2}$$

So, after calculating the values of $m_{[bq]}$] and κ_{bq} using the above expression, the mass for the neutral Tetraquark state X_{b0} can be calculated by

$$M(X_{b0}) = m_{[bq]} + \kappa_{bq}\left(-\frac{3}{2}\right) + m_{[sq]} - \frac{3}{2}\kappa_{sq}$$

$$= m_{[bq]} - \frac{3}{2}\kappa_{bq} + m_{[sq]} - \frac{3}{2}\kappa_{sq}$$

Here, the values $s_{\bar{b}\bar{q}}$=$s_{sq'}$=0 are used.

$$M(X_{b_0}) = \left(m_{[bq]} - \frac{3}{2}\kappa_{bq}\right) + \left(m_{[sq]} - \frac{3}{2}\kappa_{sq}\right)$$

Putting the values of mass, $Z_b = 10607.2 \pm 2.0\ MeV$ and $Z'_b = 10652.2 \pm 1.5\ MeV$, the value of $m_{[bq]}$ turns out to be 5314.85 MeV. While the value of κ_{bq} would be about 22.5 MeV. Using the value of 980 $\pm 20 MeV$ for $M(a_0)$ [18], from equation 3.5, it can be found,

$$\left(m_{[sq]} - \frac{3}{2}\kappa_{sq}\right) = 980/2 \simeq 490 MeV$$

With all these values, the mass spectrum for X_{b0} is predicted to be approximately equal to 5771.1 MeV which is roughly 200 MeV higher than that of X(5568) while the $B^+\bar{K}^0$ is only 7 MeV above. In order to be observed as the resonant $B_s\pi$ state, the expected masses for X_{bS} tetraquark states needed to be below the BK threshold.

Mass Spectrum for $\mathbf{X_{b1}}$:

For the mass spectrum of exotic state, $J^P = 1^+$, from Equation (3.4),

$$M(X_{b1}) = m_{bq} - \frac{3}{2}\kappa_{bq} - \frac{3}{2}\kappa_{sq} + \kappa_{bq}\left[s_{bq}\left(s_{bq}+1\right)\right] + \kappa_{sq}\left[s_{sq'}\left(s_{sq'}+1\right)\right]$$

For X_{b1}, the notation used is

$$X_{b1} = \left|1_{\bar{b}\bar{q}}, 0_{sq'}\right\rangle$$

So, for the state X_{b1}, the diquark spin values of $s_{\bar{b}\bar{q}} = 1$ and $s_{sq'} = 0$ are used,

$$M(X_{b1}) = m_{bq} - \frac{3}{2}\kappa_{bq} - \frac{3}{2}\kappa_{sq} + \kappa_{bq}\left[1\left(1+1\right)\right] + \kappa_{sq}\left[0\left(0+1\right)\right]$$

$$M(X_{b1}) = m_{[bq]} + \kappa_{bq}\left(2 - \frac{3}{2}\right) + m_{[sq]} - \frac{3}{2}\kappa_{sq}$$

The expression to calculate the mass spectrum for this state would be,

$$M(X_{b1}) = m_{[bq]} + \frac{1}{2}\kappa_{bq} + m_{[sq]} - \frac{3}{2}\kappa_{sq}$$

Taking the values of m_{bq}, κ_{bq}, $m_{sq} - \frac{3}{2}\kappa_{sq}$ as above [21], the anticipated mass spectrum for state X_{b1} would be,

$$M(X_{b1}) = 5314.85 + 22.5(1/2) + 490$$

$$M(X_{b1}) \simeq 5816.1MeV \quad (J^P = 1^+) \tag{3.8}$$

The X_{b1} state is predicted to decay into $B_s^{*0}\pi^+$ followed by, $B_s^{*0} \rightarrow B_s^0$ and a photon having the energy of 45 MeV is given off in the B_s^* rest frame. Because of its low energy,such photons are unable to be detected at hadron colliders [23]. As a result, the Xb1's observed peak would be moved towards lower invariant masses and would practically overlap with the Xb0's observed peak.

Mass Spectrum for $\mathbf{X_{c1}}$:

On the same lines, one can estimate the mass for S-wave $X_{cs}^{\pm}$. The mass of comparable X_{cs}, having the state $[\bar{c}\bar{q}][sq']$ predicted in the charm sector, decaying into $D_s\pi$, may be estimated using the preceding calculations.

The mass formulas for the relevant tetraquarks $Z_c(3900)$ and $Z_c'(4020)$ are considered for this state. Here, the required substitution is $c\bar{s} \to c\bar{c}$.

For $Z_c(3900)$,

$$Z_c = \frac{1}{\sqrt{2}}\left(|1_{\bar{c}\bar{q}}, 0_{cq'}\rangle\right) - \left(|0_{\bar{c}\bar{q}}, 1_{cq'}\rangle\right)$$

$$M_{Z_c} = 2m_{[cq]} - 3\kappa_{cq} + \kappa_{bq}\left[s_{\bar{c}\bar{q}}\left(s_{\bar{c}\bar{q}} + 1\right) + s_{cq'}\left(s_{cq'} + 1\right)\right]$$

As $s_{\bar{c}\bar{q}} = \left(|1,0\rangle\right), s_{cq'} \ = \ -\left(|0,1\rangle\right),$

$$M_{Z_c} = 2m_{[cq]} - 3\kappa_{cq} + \kappa_{cq}\left[1\left(1+1\right) + 0\left(0+1\right)\right] + \kappa_{cq}\left[0\left(0+1\right) + \left(-1\right)\left(\left(-1\right)+1\right)\right]$$

Hence, for $Z_c(3900)$,

$$M_{Z_c} = 2m_{[cq]} - \kappa_{cq} \tag{3.9}$$

For $Z_c'(4020)$,

$$M_{Z_c'} = \tfrac{1}{\sqrt{2}}\left(|1_{\bar{c}\bar{q}}0_{cq'}\rangle\right) + \left(|0_{\bar{c}\bar{q}}, 1_{cq'}\rangle\right) \text{ or } Z_c' = |1_{\bar{c}\bar{q}}, 1_{cq'}\rangle \text{ [38]}$$

$$M_{Z_c} = 2m_{[cq]} - 3\kappa_{cq} + \kappa_{cq}\left[s_{\bar{c}\bar{q}}\left(s_{\bar{c}\bar{q}} + 1\right) + s_{cq'}\left(s_{cq'} + 1\right)\right]$$

So,

$$M_{Z_c'} = 2m_{[cq]} - 3\kappa_{bq} + \kappa_{cq}\left[1\left(1+1\right) + 0\left(0+1\right)\right] + \kappa_{cq}\left[0\left(0+1\right) + \left(1\right)\left(\left(1\right)+1\right)\right]$$

The mass formula for the tetraquark Z_c would be,

$$M_{Z_c'} = 2m_{[cq]} + \kappa_{cq} \tag{3.10}$$

Adding equation 3.9 and 3.10, we get

$$m_{[cq]} = \frac{M(Z_c') + M(Z_c)}{4}$$

Now subtracting equation 3.9 from 3.10, we get

$$\kappa_{cq} = \frac{M(Z_c') - M(Z_c)}{2}$$

Analogous to equation 3.3, the expression for the mass formula of a generalized charmed X_{cS} state is

$$M(X_{cS}) = m_{[cu]} + \kappa_{cq}\left(S(S+1) - \frac{3}{2}\right) + m_{[sd]} - \frac{3}{2}\kappa_{sq}$$

While, for states X_{c_0} and X_{c_1}, we have

$$M(X_{c_0}) = m_{[cu]} + m_{[sd]} - \frac{3}{2}\kappa_{sq} - \frac{3}{2}\kappa_{cq}$$

$$M(X_{c_1}) = m_{[cu]} + m_{[sd]} - \frac{3}{2}\kappa_{sq} + \frac{1}{2}\kappa_{cq}$$

The value of $m_{[cq]}$ and κ_{cq} comes out to be about 1977.8 MeV and 68.5 MeV respectively,using the values, $Z_c = 3887.1 \pm 2.6$ MeV and $Z_c' = 4024 \pm 1.9$ MeV [18]. While the mass of the exotic states X_{cs}, would be,

$$M(X_{c_0}) \simeq 2365.05 MeV \tag{3.11}$$

$$M(X_{c_1}) \simeq 2502.05 MeV \tag{3.12}$$

Because these states are barely above the DK threshold of $2363 MeV$ as well as the D^*K threshold of 2504 MeV, if the mass estimates are correct, one can expect to find them in the decay channels of DK and D^*K. For $S = 0$ configuration for the light diquark, it means that the diquark is antisymmetric in spin and colour, so the diquark needs to be antisymmetric in flavor configuration too. As a result, the tetraquarks $[\bar{Q}\bar{q}][q'q'']$ relates to the $SU(3)_F$ representation: $\bar{3} \otimes \bar{3} = 3 \oplus \bar{6}$. For tetraquarks, here Q can either be b or c and $q, q', q'' = u, d$. There exist a doubly charged state found in the cham sector with a flavor content of$[\bar{c}\bar{u}][sd] \rightarrow D_s^- \pi^-$. It belongs to the $\bar{6}$. The symmetric 15 representation of $SU(3)_F$, derived from the product: $\bar{3} \otimes 6 = 3 \oplus 15$, corresponds to the doubly charged state in the beauty sector. This entails the existence of so-called bad diquarks, light diquarks with the value S = 1, but such a state is argued to have little binding [30].

Taking the flavor multiplicity of the $X_{b0} = [\bar{b}\bar{q}][sq']$ states into consideration, with $q = q'$ = u, d, we observe that the organisation of these states in an isospin triplet and singlet, respectively, are structurally comparable to $a_0(980)$ and $f_0(980)$ which are the scalar light tetraquarks . Similarly, it is expected that the neutral X_{bo} states would be degenerate in mass.

The state X_{bo} could also decay via $X_{b0}^{(I=0)} \rightarrow B_s + \eta$ but phase space forbids this. As a result, the only option would be a strong decay of $X_{b0}^{(I=0)} \rightarrow B + \bar{K}$, analogous to the scenario of $f_0 \rightarrow K\bar{K}$. If phase space also forbids the latter decay then the isospin violating mixing of this state with $X_{b0}^{(I=1)}$ or through the $\eta - \pi^0$ mixing, would cause the $X_{b0}^{(I=0)} \rightarrow B_s + \pi^0$ decay, similarly to η decay.

The estimates in Eqs. (11–12) for the $I = 0$ $X_{cS}^{\pm}$ state using similar, being only 40–50 MeV over the well-known $Ds0(2317)$ and $Ds1(2460)$ Similarly, the $I = 0$ $X_{cS}^{\pm}$ states yeild the estimates in Eqs. (11 -12). These are only 40-48 MeV over the known states $D_{s0}$and D_{s1} whose masses are 2317 MeV and 2460 MeV. The difference in mass is sufficiently close to the expected error suggesting that X_{cS} can be found with the latter resonances, decaying to $D_s^+\pi^0$ which is resulted from the interactions that cause isospin breaking, either through component mixing of $I = 1$ and $I_3 = 0$ or through mixing of $\eta - \pi^0$.

Chapter 4

Tetraquark Production in Weak Decays of $B_c^{\pm}$ Mesons

A B_c meson consists of a anti b-quark and a c quark and is in pseudoscalar state. Its antiparticle is anti-B_c meson that contains a b-quark and a c-antiquark. The $B_c^{\pm}$ meson is a heavier particle with a mass of 6.2756 GeV/c^2 as the b-quark, it contains, is a massive quark.

4.0.1 B Meson Weak Decays

In The Standard model, the decay of quarks are evoked by weak interactions and mediated by charged W-bosons.As the W-bosons are massive, weak interactions happen at short distances of order $1/m_W$.Based on the structure of charged-current interaction, hadronic weak decays can be categorised into three types : leptonic decays, in which the decaying hadron's quarks annihilate each other and only leptons emerge in the final state; semi-leptonic decays, where in the final state, leptons and hadrons coexist; and non-leptonic decays, where, in the final state, hadrons appear.

As the non-leptonic weak decays are theoretically more complicated, we employ heavy-quark expansion QCD in many two-body B-decays to address them. The confining colour forces among the quarks offer a significant impact on the dynamics of non-leptonic decays wherein only hadrons occur in the final state. The phenomenon of quark rearrangement happens due to the exchange of soft and hard gluon. It leads to the complication in non-leptonic processes. Whereas semileptonic transitions are represented by a few hadronic form factors which parameterize the hadronic matrix elements from quark currents. In the theoretical description of non-leptonic processes, matrix elements of local four-quark operators rather than the current operators are involved. They are much harder to deal with. These strong interaction effects have prevented in the understanding of non-leptonic decays.
Hadronization in case of the decay products in the energetic two-body transition does not happen till they have reached some distance apart from each other. This is because soft gluons are ineffectual in rearranging quarks once they have grouped into color-singlet pairs. The amplitudes of the decay are then supposed to factorize into decay products of hadronic matrix elements of quark current which is color singlet. Many decays of B- mesons involving two-bodies have been analyzed using the factorization approximation. It relates the complexness of amplitudes of non-leptonic decay to the product of decay constants of meson and components of hadronic matrix of current operators, similar to those shown in semi-leptonic decays.The decay constants being the basic hadronic parameters, indicate the potency of the attraction of quark-antiquark within a hadronic state. Because some of them aren't readily available in leptonic or electromagnetic processes, acquiring them from non-leptonic transitions may provide important information [39].

The theoretical problem which is to be resolved here is to calculate the $\mathcal{B.R}$ of B_c meson decays and utilizing the mass predictions from chapter 2 to speculate the branching ratio of neutral $X_{b_0}(5570)^0$ tetraquark state, analyzing it with the values from experiments. in order to calculate the branching ratio for these decay processes, a method known as, naive factorization was proposed. In the decay, this method takes one quark to be the leading one while the other quark is the spectator quark. Only leading order diagrams are used to calculate the interaction and the propagation of the two quarks are considered to be unaffected from each other. This method is helpful in leading to the measurements coming from the experiments, but this method has a major drawback *i.e.* it lacks theoretical basis. QCD Factorization is used as an alternative to the naive factorization ,in calculating the branching ratios and other observables.

4.0.2 Naive Factorization

The main purpose of factorization is to separate observables into perturbatively computable co-efficient functions and to process independent hadronic values.. In the method of factorization, element of hadronic matrix is defined as the product of two components. $\langle h_1 h_2 | H_{eff} | B \rangle = \langle h_2 | J_2 | 0 \rangle \langle h_1 | j_1 | B \rangle$. The first element is proportional to the $B \to h_1$ form factor and the other one is proportional to the h_2 meson decay constant. In the short distance part, the non-leptonic decays of B-mesons are evoked by the weak interaction. This yields the effective four-quark operators. The effective Hamiltonian , in a generalized form, for non-leptonic decay is given as,

$$H_{eff} = \frac{G_F}{\sqrt{2}} V_{q'b} V^*_{q'q} \sum_{i=1}^{10} C_i O_i$$

Where q can be d or s while $V_{q'q}$ represents the CKM factors. O_1, O_2 are current-current operators of tree level. The QCD penguin operators are represented by $O_3, ..., O_6$. $O_7, ...O_{10}$ are electroweak penguin operators.The C_is represent the Wilson coefficients of four quark operators including the QCD correction. The Wilson coefficents of four local quark operators describe the entire perturbative channel dependent component.

4.0.3 Classification of the Quark Operators

The phenomenology of the weak B-decays is dominated by three kinds of operators.

Current-Current Operators

$$O_1^u = (\bar{u}_\alpha b_\alpha)_{V-A}(\bar{q}_\beta u_\beta)_{V-A} \quad O_1^c = (\bar{c}_\alpha b_\alpha)_{V-A}(\bar{q}_\beta c_\beta)_{V-A}$$
$$O_2^u = (\bar{u}_\beta b_\alpha)_{V-A}(\bar{q}_\alpha u_\beta)_{V-A} \quad O_2^c = (\bar{c}_\beta b_\alpha)_{V-A}(\bar{q}_\alpha c_\beta)_{V-A}$$

QCD Penguin Operators

$$O_3 = (\bar{q}_\alpha b_\alpha)_{V-A}\sum_{q'}\left(\bar{q}'_\beta q'_\beta\right)_{V-A} \quad O_4 = (\bar{q}_\beta b_\alpha)_{V-A}\sum_{q'}\left(\bar{q}'_\alpha q'_\beta\right)_{V-A}$$
$$O_5 = (\bar{q}_\alpha b_\alpha)_{V-A}\sum_{q'}\left(\bar{q}'_\beta q'_\beta\right)_{V+A} \quad O_6 = (\bar{q}_\beta b_\alpha)_{V-A}\sum_{q'}\left(\bar{q}'_\alpha q'_\beta\right)_{V+A}$$

Electroweak-Penguin Operators

$$O_7 = \tfrac{3}{2}(\bar{q}_\alpha b_\alpha)_{V-A}\sum_{q'} e_{q'}\left(\bar{q}'_\beta q'_\beta\right)_{V+A} \quad O_8 = \tfrac{3}{2}(\bar{q}_\beta b_\alpha)_{V-A}\sum_{q'} e_{q'}\left(\bar{q}'_\alpha q'_\beta\right)_{V+A}$$
$$O_9 = \tfrac{3}{2}(\bar{q}_\alpha b_\alpha)_{V-A}\sum_{q'} e_{q'}\left(\bar{q}'_\beta q'_\beta\right)_{V-A} \quad O_{10} = \tfrac{3}{2}(\bar{q}_\beta b_\alpha)_{V-A}\sum_{q'} e_{q'}\left(\bar{q}'_\alpha q'_\beta\right)_{V-A}$$

Here α and β are the $SU(3)$ colour indices, $q' = u, d, s, c$. The subscript. $V \pm A \equiv \gamma^\mu (1 \pm \gamma^5)$ which is vector axial current. The quantity $(1 \pm \gamma^5)$ represent the chiral projections. Thus in equations given above, $(\bar{q}_\beta u_\beta)_{V-A} = \bar{q}_\beta \gamma^\mu (1-\gamma^5) u_\beta$ etc.

We apply this facotrization technique in weak interactions, where a meson can be generated directly by a quark current which carry the suitable parity and flavour quantum numbers. For example, considering the transition $\bar{B}^0 \to D^+\pi^-$, the effective Hamiltonian for this process is given as follows

$$H_{eff} = \frac{G_F}{\sqrt{2}}\left\{V_{cb}V_{ud}^*\left\langle D^+\pi^- \left| C_1 O_1 + C_2 O_2 \right| \bar{B}^0\right\rangle\right\}$$

The factorizable part of amplitude for the above process can be written as

$$A_{fact} = \frac{G_F}{\sqrt{2}}\left\{V_{cb}V_{ud}^{*}\left[C_1(\mu)+\frac{1}{N_c}C_2(\mu)\right]\left\langle\pi^{-}\left|\bar{d}\gamma^{\mu}\gamma^{5}u\right|0\right\rangle\left\langle D^{+}\left|\bar{c}\gamma^{\mu}b\right|\bar{B}^{0}\right\rangle\right\}$$
$$A_{fact} = \frac{G_F}{\sqrt{2}}\left\{V_{cb}V_{ud}^{*}a_1\left\langle\pi^{-}\left|\bar{d}\gamma^{\mu}\gamma^{5}u\right|0\right\rangle\left\langle D^{+}\left|\bar{c}\gamma^{\mu}b\right|\bar{B}^{0}\right\rangle\right\}$$

The co-efficient a_1 will be addressed below. For generating a pion from vaccuum through the axial current, the decay constant f_π parametrizes the amplitude[39]. This amplitude is propotionate to the moementum of pion

$$\left\langle\pi^{-}\left|\bar{d}\gamma^{\mu}\gamma^{5}u\right|o\right\rangle = \frac{\iota}{2}f_{\pi}p^{\mu}$$

It appears plausible to assume that the amplitude of energetic weak decays, with the directly formed meson carrying a large momentum, is controlled by its factorizable part. More comprehensive assessment of the kinematics in the decay process given above, can substantiate this assumption: A rapid moving (ud) pair generated in a point-like interaction will hadronize simply after a time granted by its γ factor times a typical hadronization time $\mathcal{T} \sim 1fm/c$, with both of the quarks exiting the interaction region having same direction and with a velocity comparable to that of light. Hadronization happen roughly 20 fm apart from the remaining quarks in the previous case. The (ud) pair , interacting a little with the remaining quarks, behaves like a colourless point like particle inside the interaction region.

One may differentiate three kinds of decays by factorising matrix elements of the four-quark operators incorporated in the effective Hamiltonian [40]. Considering the first class of decays, only a meson which carry charge can be created directly from a color-singlet current such as $\bar{B}^0 \to D^+\pi^-$. The relevant QCD coefficient for these processes is given by, $a_1 = C_1(\mu) + \frac{1}{N_c}C_2(\mu)$

The a_1 factor represents tree diagram. N_c factor here represents the number of quark colors, while $\mu = O(m_b)$ is the scale by which the factorization is considered to be relevant.

The second class includes the transitions in which the meson directly generated from the current is neutral. The case where this transitions occur is $B^- \rightrightarrows K^- J/\psi$. The amplitude for this process is ,

$$A_{fact} = \frac{G_F}{\sqrt{2}} \left\{ V_{cb} V_{cs}^* a_1 \langle J/\psi \left| \bar{c}\gamma^\mu c \right| 0 \rangle \left\langle K^- \left| \bar{s}\gamma^\mu b \right| \bar{B}^- \right\rangle \right\}$$

This is relative to the QCD coefficient

$$a_2 = \frac{1}{N_c} C_1(\mu) + C_2(\mu)$$

The coefficient a_2 is because of the color supressed diagrams.

Transitions where the a_1 and a_2 amplitudes interpose, such as in $B^- \rightarrow D^0\pi^-$, belong to the third class of transitions. Their final state comprises both a neutral and a charged meson. Both of them can be generated by the current using one of the effective Hamiltonian's operators. A combination of a_1 and a_2 is involved in the corresponding amplitudes such as in $B^- \rightarrow \pi^0\pi^-$.

However, we follow the convention of large N_c limit to fix the coefficients from QCD, *i.e.* $a_1 \simeq c_1$ as well as $a_2 \simeq c_2$, where[41]:

$$c_1(\mu) = 1.26 \quad , c_2(\mu) = -0.51 \text{ at } \mu \simeq m_c^2$$

$$c_1(\mu) = 1.12 \quad c_2(\mu) = -0.26 \text{ at } \mu \simeq m_b^2$$

4.0.4 Form Factors for Pseudoscalar (P(p') and Vector V(ϵ, p') Mesons

Here for the above example, there exist a transition from $\bar{B} \rightarrow D^+$ via vector current γ^μ and π^- is generated from a vector through axial current $\gamma^\mu\gamma^5$.

Form factors can be used to express these decays. In the case of $B_c \to P$, where V(P, V) represents a pseudoscalar and a vector meson, respectively, the matrix elements are parametrized in the form of form factors via vector axial-vector as given below [4],

$$\begin{aligned}
\langle P\left(P''\right)\left|V_\mu\right| B_c\left(P'\right)\rangle &= f_+\left(q^2\right) P_\mu + f_-\left(q^2\right) q_\mu \\
\langle V\left(P'', \varepsilon''^*\right)\left|V_\mu\right| B_c\left(P'\right)\rangle &= \epsilon_{\mu\nu\alpha\beta} \varepsilon''^{*\nu} P^\alpha q^\beta g\left(q^2\right) \\
\langle V\left(P'', \varepsilon''^*\right)\left|A_\mu\right| B_c\left(P'\right)\rangle &= -i\left\{\varepsilon_\mu''^* f\left(q^2\right) + \varepsilon''^* \cdot P\left[P_\mu a_+\left(q^2\right) + q_\mu a_-\left(q^2\right)\right]\right\}
\end{aligned}$$

Here, the momentum $P = P' + P''$ while the term is, $q = P' - P''$ and the conventionality $\epsilon_{0123} = 1$ is adopted. The vector current as well as the axial-vector currents are provided as $\bar{\psi}\gamma_\mu\psi'$ and $\bar{\psi}\gamma_\mu\gamma_5\psi'$. The form factors in general parametrization are [42],

$$\begin{aligned}
\langle P\left(p'\right)\left|V_\mu\right| B(p)\rangle =& \left[\left(p+p'\right)_\mu - \frac{m_B^2 - m_P^2}{q^2} q_\mu\right] F_1\left(q^2\right) + \frac{m_B^2 - m_P^2}{q^2} q_\mu F_0\left(q^2\right) \\
\langle V\left(\varepsilon, p'\right)\left|V_\mu - A_\mu\right| B(p)\rangle =& \frac{2i}{m_B + m_V} \epsilon_{\mu\nu\alpha\beta} \varepsilon^{*\nu} p^\alpha p'^\beta V\left(q^2\right) \\
& - \left(m_B + m_V\right)\left[\varepsilon_\mu^* - \frac{\varepsilon^* \cdot q}{q^2} q_\mu\right] A_1\left(q^2\right) + \\
& \frac{\varepsilon^* \cdot q}{m_B + m_V}\left[\left(p+p'\right)_\mu - \frac{m_B^2 - m_V^2}{q^2} q_\mu\right] A_2\left(q^2\right) - \varepsilon^* \cdot q \frac{2m_V}{q^2} q_\mu A_0(q^2)
\end{aligned}$$

where $F_1\left(q^2\right)$ and $F_0\left(q^2\right)$ are form factors related to B to pseudoscalar P transition via vector current V_μ, whereas $V\left(q^2\right), A_1\left(q^2\right)$ and $A_2\left(q^2\right)$ are form factors corresponding to B to vector V transiton by way of vector V_μ current minus the axial vector A_μ current.

Form factors of two kinds are related to each other as,

$$F_1^{B_cP}(q^2) = f_+(q^2), \quad F_0^{B_cP}(q^2) = f_+(q^2) + \frac{q^2}{m_{B_c}^2 - m_P^2} f_-(q^2)$$
$$V^{B_cV}(q^2) = -(m_{B_c} + m_V)\, g(q^2), \quad A_1^{B_cV}(q^2) = -\frac{f(q^2)}{m_{B_c} + m_V}$$
$$A_2^{B_cV}(q^2) = (m_{B_c} + m_V)\, a_+(q^2), \quad A_3^{B_cV}(q^2) - A_0^{B_cV}(q^2) = \frac{q^2}{2m_V} a_-(q^2)$$

We adopt the optional three parameter form for determining the form factor's numerical value:

$$F\left(q^2\right) = \frac{F(0)}{1 - \frac{q^2}{m_{fit}^2} + \delta\left(\frac{q^2}{m_{fit}^2}\right)^2}$$

Where,F can be any of the form factors F_1, F_0 and V, A_0, A_1, A_2, while $[\delta, m_{fit}]$ are shape parameters.

The results for B_c transition form factors is given in the table,

F	$F(0)$	$F(q^2_{\max})$	m_{fit}	δ
$F_1^{B_cB}$	$0.63^{+0.04+0.03}_{-0.05-0.03}$	$0.96^{+0.05+0.08}_{-0.07-0.07}$	$1.19^{+0.09+0.01}_{-0.09-0.01}$	$0.33^{+0.04+0.01}_{-0.04-0.01}$
$F_0^{B_cB}$	$0.63^{+0.04+0.03}_{-0.05-0.03}$	$0.81^{+0.02+0.06}_{-0.03-0.05}$	$1.52^{+0.22+0.02}_{-0.19-0.02}$	$0.52^{+0.16+0.02}_{-0.10-0.02}$
$V^{B_cB^+}$	$3.29^{+0.17+0.32}_{-0.21-0.30}$	$4.89^{+0.19+0.61}_{-0.27-0.53}$	$2.65^{+0.13+0.05}_{-0.14-0.06}$	$1.75^{+0.27+0.10}_{-0.22-0.11}$
$A_0^{B_cB^*}$	$0.47^{+0.01+0.04}_{-0.01-0.04}$	$0.68^{+0.01+0.07}_{-0.02-0.07}$	$0.99^{+0.04+0.04}_{-0.04-0.04}$	$0.31^{+0.03+0.02}_{-0.03-0.02}$
$A_1^{B_cB^*}$	$0.43^{+0.01+0.04}_{-0.01-0.04}$	$0.57^{+0.00+0.06}_{-0.01-0.06}$	$1.16^{+0.07+0.03}_{-0.07-0.03}$	$0.27^{+0.03+0.01}_{-0.03-0.02}$
$F_1^{B_cB_s}$	$0.73^{+0.03+0.03}_{-0.04-0.03}$	$1.01^{+0.02+0.07}_{-0.04-0.06}$	$1.35^{+0.07+0.01}_{-0.08-0.01}$	$0.35^{+0.04+0.00}_{-0.04-0.01}$
$F_0^{B_cB_s}$	$0.73^{+0.03+0.03}_{-0.04-0.03}$	$0.87^{+0.00+0.05}_{-0.02-0.05}$	$1.77^{+0.24+0.04}_{-0.20-0.04}$	$0.60^{+0.23+0.04}_{-0.14-0.04}$
$V^{B_cB_s^+}$	$3.62^{+0.12+0.31}_{-0.15-0.29}$	$4.93^{+0.14+0.53}_{-0.19-0.47}$	$2.94^{+0.11+0.04}_{-0.11-0.05}$	$1.78^{+0.25+0.07}_{-0.21-0.08}$
$A_0^{B_cB_s^*}$	$0.56^{+0.00+0.04}_{-0.01-0.04}$	$0.75^{+0.00+0.07}_{-0.01-0.07}$	$1.13^{+0.03+0.04}_{-0.04-0.04}$	$0.33^{+0.03+0.02}_{-0.03-0.02}$
$A_1^{B_cB_s^*}$	$0.52^{+0.00+0.04}_{-0.01-0.04}$	$0.64^{+0.00+0.06}_{-0.01-0.06}$	$1.33^{+0.07+0.03}_{-0.07-0.03}$	$0.28^{+0.03+0.01}_{-0.03-0.01}$

Table 4.1: $B_c \to B, B^*, B_s, B_s^*$ form factors evaluated in the light front quark model, the uncertainties coming from the decay constants of B_c and that of final state mesons [4]

4.0.5 Branching Ratio

The relative frequency of a certain decay mode is referred as the branching ratio (or branching fraction). For each decay mode, it is required to calculate the branching ratio. The decay rate to a particular decay mode 'j' relative to the value of total decay rate gives the branching ratio BR(j) [43].

$$BR(j) = \frac{\Gamma_j}{\Gamma}$$

The particle's proper lifetime is defined as its lifetime in its rest frame *tau*, which is determined using the value of the total decay rate,

$$\tau = \frac{1}{\Gamma}$$

QCD Factorization

Non-perturbative effects linking the mesons with quarks and gluons, terms of higher order coming from low energy scale interactions, and the long range interactions suitable to a perturbative approach, must all be taken into account when using QCD to compute the branching ratio and other observables. The QCD factoring method is one steo ahead of the Naive factorization method. It provides systematic predictions of non-factorizable sub-leading contributions for various decays in which the factorizable contribution is dominant.

4.1 Non-Leptonic B_c Decays and Search For Tetraquarks

A single W-exchange diagram at the tree level characterises non-leptonic weak decays in the Standard Model. Strong interactions have two effects on this simple picture. Perturbative methods and renormalization-group techniques can account for hard-gluon corrections. They generate new weak vertices that are effective. The long-distance confinement forces create the binding of quarks inside asymptotic hadron states. Separating the two regimes utilizing the operator product expansion (OPE)64 is the primary tool in the computation of non-leptonic amplitudes. The operator product expansion incorporate all long-range QCD effects in the hadronic matrix elements of local four-quark operators. As weak decays involve drastically different time and energy scale and subsequent generation of the final hadron, this treatment appears to be well justified. In the decays of $B^{\pm}$ and B^{0}-mesons, numerous exotic mesons from the XYZ are observed. Not only these, but also many of states belonging to the pentaquark family like, $P_c(4450)^+$ and $P_c(4380)^+$ are observed in the Λ_b-baryon decays. In this chapter, Utilizing this idea, production of the charged $X_{b_0}(5570)^{\pm}$ as well as neutral $X_{b_0}(5570)^{0}$ state of tetraquark is anticipated in weak decays of charmed $B_c^{\pm}$ mesons. It's also worth noting that $B_c^{\pm}$-decays can be a rich source of hidden $c\bar{c}$ tetraquark states that has yet to be discovered via decays such as $B_c^{\pm} \to X(3872)\pi^{\pm}$.

The decays, which are to be taken into consideration, are $B_c^{\pm} \to B_s^0\pi^{\pm},\ B_c^{\pm} \to X_{b_0}^0\pi^{\pm},\ and\ B_c^{\pm} \to X_{b_0}^{\pm}\pi^0$. The process which occur at quark level in these is the weak decay of $c \to su\bar{d}$ and $\bar{b}$ decay as a spectator.

4.1.1 Amplitude For $\mathbf{B_c^+ \to B_s^0\pi^+}$ Decay

For non-leptonic decays, the effective hamiltonian is given by,

$$H_{eff} = \frac{4G_F}{\sqrt{2}} V_{cs}V_{ud}^{*}\left[\mathcal{C}^{(-)}\mathcal{O}^{(-)} + \mathcal{C}^{(+)}\mathcal{O}^{(+)}\right]$$

$$\mathcal{O}^{(\pm)} = [\bar{s}^{\alpha}\gamma_{\mu}P_L c_{\alpha}]\left[\bar{u}^{\beta}\gamma^{\mu}P_L d_{\beta}\right] \pm [\bar{s}^{\alpha}\gamma_{\mu}P_L d_{\alpha}]$$

The above expression contain the products of local four-quark operators which are renormalized at the scale μ while G_F represents the Fermi coupling constant.The elements of quark mixing matrix i-e CKM matrix are denoted by V_{ij}. An alternative operator basis can be used for convenience with $O^{\pm} = (O_1 \pm O_2)/2$ [44], with the corresponding coefficient, here the factor $C_{1,2}(\mu)$ are the Wilson coefficients involving scale of $\mu = m_c, m_b$, $C^{(\pm)} = (C_1 \pm C_2)/2$. QCD penguin contributions have been removed, and the $C^{(\pm)}$ are the renormalization factors from QCD calculated on a scale of momentum corresponding to the mass of b-quark. [45].

α and β represents the color indices, while the term P_L, the chirality, is given by, $P_L = \frac{1}{2}(1-\gamma_5)$, Penguin operators are not shown here due to their very small Wilson coefficients, Only in rare decays, where the tree-level contribution is highly CKM-suppressed or when matrix elements of the $\mathcal{O}_1$ *and* $\mathcal{O}_2$ operators do not participate at all, do the corresponding penguin contributions to weak decay amplitudes become important. The operators $\mathcal{O}_1$ and $\mathcal{O}_2$ are the current operators for this decay. Here, as only a charged meson is generated from a color singlet current so, this process belongs to the first class of decays. We consider tree diagram. For

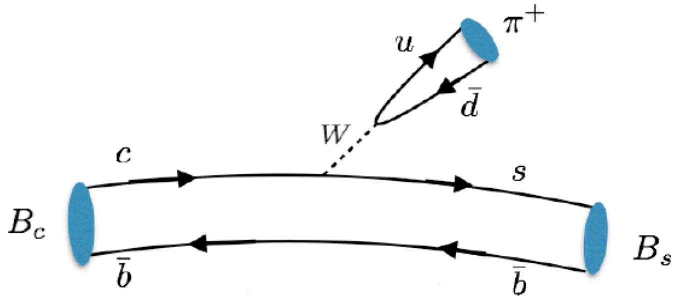


Figure 4.1: Tree Diagram for $B_c \to B_s\pi^+$

this case, the factorizable part of amplitude is written as,

$$A_{fact} = \frac{4G_F}{\sqrt{2}}\left\{V_{cs}V_{ud}^*\left[C_1(\mu) + \frac{1}{N_c}C_2(\mu)\right]\left\langle\pi^+\left|d\gamma^\mu P_L u\right|0\right\rangle\left\langle B_s\left|\bar{s}\gamma_\mu P_L c\right|B_c^+\right\rangle\right\}$$
$$A_{fact} = \frac{4G_F}{\sqrt{2}}\left\{V_{cs}V_{ud}^* a_1\left\langle\pi^+\left|d\gamma^\mu P_L u\right|0\right\rangle\left\langle B_s\left|\bar{c}\gamma_\mu P_L b\right|B_c^+\right\rangle\right\}$$

a_1 represents the tree diagram.

The non-leptonic decay amplitudes can be broken down into three components: the first one being the non-leptonic decay constants while second component is the matrix element of weak current and the third one is the combinations of relevant coefficients. Hadronic matrix elements, in particular, define the non-leptonic decay constant.

$$\langle \pi^+ | d\gamma^\mu P_L u | 0 \rangle = \frac{\iota}{2} f_\pi q^\mu$$

where $d\gamma^\mu P_L u$ is the current and f_π is the leptonic decay constant of the π meson [46]. A composite of two form factors can be used to define the transition matrix element between B_c meson and pseudoscalar meson. This combination varies as the function of the square of the momentum transfer between B_c and the pseudoscalar meson.

For the transition, $B_c \to P(P'')$, where P(P'') is a pseudosclar mesons, through vector current γ_μ , the form factor induced is defined as,

$$\langle P(P'') | V_\mu | B_c(P') \rangle = f_+(q^2) P_\mu + f_-(q^2) q_\mu$$

Here,

$$P = P' + P'' \quad q = P' - P''$$

The transition matrix element can be denoted as

$$\left\langle B_s \left| \bar{c} \gamma_\mu P_L b \right| B_c^+ \right\rangle = \frac{1}{2} \left[f_+(q^2) \left(P_{B_s} + P_{B_c} \right)_\mu + f_-(q^2) \left(P_{B_s} - P_{B_c} \right)_\mu \right]$$

Hence, the amplitude would be,

$$\mathcal{M} \left(B_c^+ \to B_s^0 \pi^+ \right) = \frac{4G_F}{\sqrt{2}} V_{cs} V_{ud}^* \left(\mathcal{C}^{(-)} + \mathcal{C}^{(+)} \right) \tilde{M}$$

We have the factor, $\left(C^{(-)} + C^{(+)} \right) = C_1 \simeq a_1 = 1$

$$\tilde{M} = f_\pi q^\mu \left\langle B_s | \bar{s} \gamma_\mu P_L c | B_c^+ \right\rangle$$

$$= \frac{f_\pi}{4}\left[q^\mu f_+(q^2)\left((P_{B_s}+P_{B_c})_\mu + q^\mu f_-(q^2)\,(P_{B_s}-P_{B_c})_\mu\right)\right]$$

$$= \frac{f_\pi}{4}\left[f_+(q^2)\,(P_{B_s}-P_{B_c})^\mu\,(P_{B_s}+P_{B_c}) + f_-(q^2)\,(P_{B_s}-P_{B_c})^\mu\,(q^2)\,(P_{B_s}-P_{B_c})^\mu\right]$$

$$= \frac{f_\pi}{4}\left[f_+(q^2)\left(P_{B_c}^2-P_{B_s}^2\right) + f_-(q^2)\,(P_{B_s}-P_{B_c})^2\right]$$

$$= \frac{f_\pi}{4}\left[f_+(m_\pi^2)\left(m_{B_c}^2-m_{B_s}^2\right) + f_-(m_\pi^2)m_\pi^2\right]$$

As, $P^2 = m^2$ As the second term is multiplied by m_π^2,it can be neglected. With this, the amplitude becomes,

$$\mathcal{M}\left(B_c^+ \to B_s^0\pi^+\right) = \frac{G_F}{\sqrt{2}}V_{cs}V_{ud}^*\,(C_1)\,f_\pi\left[f_+(m_\pi^2)\left(m_{B_c}^2-m_{B_s}^2\right)\right]$$

The factor $f_\pm(q^2)$ is the vector current form factors. These form factors are assessed at $q^2 = m_\pi^2$. The decay width can be found by the relation,

$$\Gamma\left(B_c^\pm \to B_c^0\pi^\pm\right) = |\mathcal{M}|^2\frac{|\mathrm{p}_\pi|}{8\pi m_{B_c}^2}$$

The term $|\mathrm{p}_\pi|$ here, is the 3-momentum of $\pi^\pm$ which needs to be calculated in the rest frame of $B_c^\pm$-meson. In the B_c meson's rest frame, we have $E_\pi + E_{B_s} = m_{B_c}$ By rearranging and squaring,

$$E_{B_s}^2 = m_{B_c}^2 + E_\pi^2 - 2m_{B_c}E_\pi$$

$$= \frac{1}{2m_{B_c}}\left[m_{B_c}^2 + E_\pi^2 - E_{B_s}^2\right]$$

The energy E and momentum p can be represented by a four-vector p as:

$$p^\mu = (E/c,\mathbf{p}) = (E,\mathbf{p})$$

$$p^2 = p_\mu p^\mu = p_0^2 - \mathbf{p}^2 = E^2 - \mathbf{p}^2$$

On the mass shell, a particle have

$$E^2 = \mathrm{p}^2 + m^2$$

i.e.

$$p^2 = p_\mu p^\mu = m^2$$

$$E_\pi = \frac{1}{2m_{B_c}}\left[m_{B_c}^2 + \mathrm{p}_\pi^2 + m_\pi^2 - P_{B_s}^2 - m_{B_s}^2\right]$$

As the momentum of the two decay products is equal in magnitude but opposite to each other,

$$P_{B_s} = -\mathrm{p}_\pi$$

$$E_\pi = \frac{1}{2m_{B_c}^2}\left[m_{B_c}^2 + m_\pi^2 - m_{B_s}^2\right]$$

After squaring and rearranging,

$$\mathrm{p}_\pi^2 + m_\pi^2 = \left[\frac{1}{2m_{B_c}^2}\right]^2 \left[m_{B_c}^2 + m_\pi^2 - m_{B_s}^2\right]^2$$

$$\mathrm{p}_\pi^2 = \frac{\left(m_{B_c}^2 - (m_{B_s} + m_\pi)^2\right)\left(m_{B_c}^2 - (m_{B_s} - m_\pi)^2\right)}{4m_{B_c}^2}$$

Hence, the 3-momentum of the pion is provided by,[47]

$$\mathrm{p}_\pi = \frac{\sqrt{\left(m_{B_c}^2 - (m_{B_s} + m_\pi)^2\right)\left(m_{B_c}^2 - (m_{B_s} - m_\pi)^2\right)}}{2m_{B_c}}$$

The branching ratio for decay $B_c^\pm \to B_s^0\pi^\pm$ is calculated by,

$$\mathcal{B}\left(B_c^+ \to B_s^0\pi^+\right)\frac{P(\bar{b} \to B_c^+)}{P(\bar{b} \to B_s)}$$

Here, the two fragmentation probabilities, $P(\bar{b} \to B_s)$ and also the $P(\bar{b} \to B_c^+)$, are the production rates of B_c^+ meson and B_s meson in the b-quark jet.

4.1.2 Amplitude for $\mathbf{B_c^+ \to X_{b_0}^0 \pi^+}$ Decay

For the decay $B_c^\pm \to X_{b_0}^{I=0}(5770)\pi^\pm$, the amplitude can be factorized in manmner similar to that of $B_c^\pm \to B_s^0\pi^\pm$.

This amplitude is, (For fig. 4.2)

$$\mathcal{M}\left(B_c^+ \to X_{b0}^{I=0}\pi^+\right) = \frac{4G_F}{\sqrt{2}} V_{cs}V_{ud}^*\left(C_1\right)\tilde{M}$$

$$\tilde{M} = \frac{f_\pi}{m_\pi^2} V_{cs}V_{ud}^*\, q^\mu \left\langle X_{b0}^{I=0}|\bar{s}\gamma_\mu P_L c|B_c^+\right\rangle$$

The state $X_{b_0}^{I=0}$ has $J^P = 0^+$, here the part of charged current having axial vactor is involved in the transition. So, the matrix elements here, would be written in a way similar to that of $B_c^\pm \to B_s^0\pi^\pm$.

$$\mathcal{M}\left(B_c^+ \to X_{b0}^0\pi^+\right) = \frac{G_F}{\sqrt{2}} V_{cs}V_{ud}^*\left(C_1\right)\frac{f_\pi}{m_\pi^2}\left[f_+(m_\pi^2)\left(m_{B_c}^2 - m_{X_b0}^2\right)\right]$$

Here, the corresponding hadronic quantity, $f_+(m_\pi^2)^{B_cX_{b0}}$, is unknown. Because it incorporates the matrix element of axial-current for the transition of a single hadron $\to$ single hadron, QCD sum rules or lattice QCD can be used to calculate it . As both X_{b0}^0 and B_s^0 has the same flavor content, more specifically $\bar{b}s$, so the vector current form factor for this case is expected to be similar to the $f_+(m_\pi^2)^{B_cB_s^0}$. By designating the two form factors ratio as,

$$F\left(X_{b0}/B_s\right) = \left(f_+(m_\pi^2)^{B_cX_{b0}}/f_+(m_\pi^2)^{B_cB_s^0}\right)$$

The relative branching ratios can be formulated in the following way:

$$\frac{\mathcal{B}\left(B_c^\pm \to X_{b0}^{I=0}\pi^\pm\right)}{\mathcal{B}\left(B_c^\pm \to B_s^0\pi^\pm\right)} = F\left(X_{b0}/B_s\right)^2 \frac{(\left(m_{B_c}^2 - m_{X_b0}^2\right)^2 |\mathrm{p}_\pi|^{B_c\to X_{bo}\pi}}{\left(m_{B_c}^2 - m_{B_s}^2\right)^2 |\mathrm{p}_\pi|^{B_c\to B_s\pi}}$$

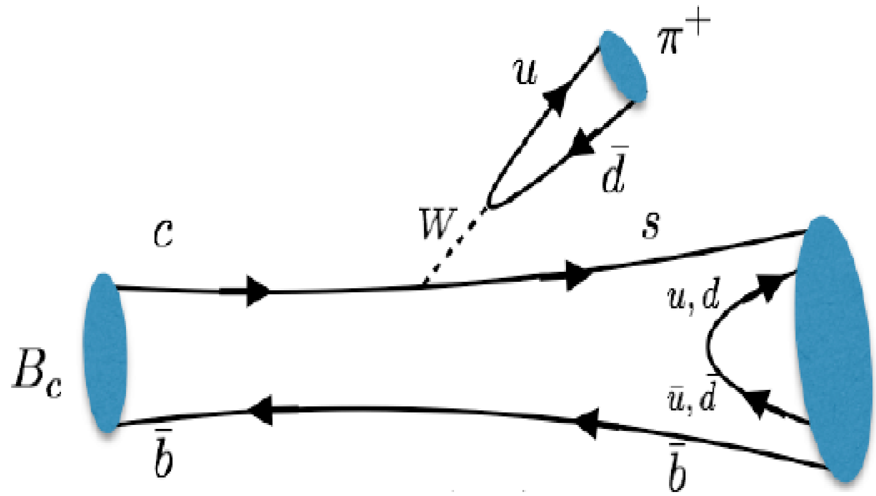


Figure 4.2: Quark Level Diagram for $B_c \to X^0_{bo}\pi^+$

In case of B_c decay into $X^{I=1}_{b0}$, Fig. 4.3, because of the color antisymmetry of the final us pair, only $\mathcal{O}^-$ contributes to the decay amplitude. While for the decay of B_c into $X^{I=0}_{b0}$, not only $\mathcal{O}^-$ contributes but also $\mathcal{O}^+$ equally contributes. For $B^{\pm}_c \to X^{\pm}_{b0}\pi^0 \to (B^0_S\pi^{\pm})\pi^0$ decay, the branching ratio is predicted to be augmented by a factor of $C(-)^2/\left(C(-)+C(+)\right)^2$.

Now consider the decays of B^+_c leading to the bound $c\bar{c}$ tetraquarks, in a similar manner, the quark level active decay is $\bar{b} \to \bar{c}u\bar{d}$. The c-quark in B^+_c would now act as the spectator quark. The decay $B^{\pm}_c \to J/\psi\pi^{\pm}$ is the standard decay for this class.These decays come from two steps, the first one include the excitation of a $q\bar{q}$ pair while the next step involve the quark recombination. The decays like $B^{\pm}_c \to X(3872)^{I=0}\pi^{\pm}$ and $B^{\pm}_c \to X(3872)^{I=1\pm,0}\pi^{0,\pm}$ results from this. The access to the X(3872)'s isosinglet partner $I = 0$ and isotriplet partners $I = 1$ are made allowed by these diagrams. State with I=0, decay into to $J/\psi\omega$ while the other partner of X(3872) I=1, decays to $J/\psi\rho^0$. The charge partner $X(3872)^{\pm}$ can decay to $J/\psi\rho^{\pm}$ and it is possible for it to decay to D^*D, in addition.

Again the amplitude for $B^{\pm}_c \to X(3872)^{I=0}\pi^{\pm}$ can be factorized into components and it would be proportionate to $C^{(-)} + C^{(+)}$. The branching ratio is again

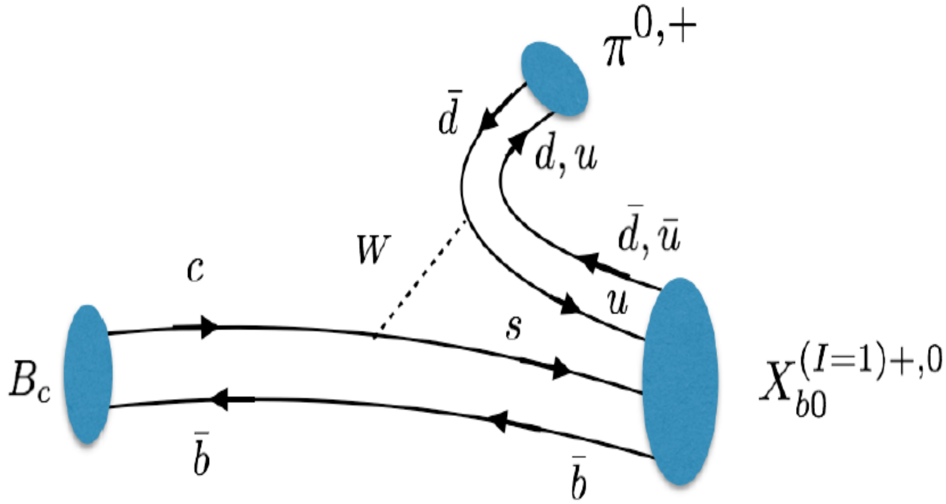


Figure 4.3: Quark Level Diagram for $B_c \rightarrow X_{b0}^{1,0}\pi^{+,0}$[3]

anticipated to be large, similar to that of $B_c^{\pm} \rightarrow J/\psi\pi^{\pm}$.

4.2 Numerical Results and Analysis

Here, the branching ratio of B_c meson for $B_c^+ \rightarrow B_s^0\pi^+$ is evaluated in this section. The branching ratio for this decay has been calculated by the LHCb. All the values taken here are the centre values of the input data, and the numerical numbers we utilised in our computations are listed below [18].

$$m_{B_c} = 6.2756 \pm 0.00011\text{GeV}, m_{B_s} = 5.36677 \pm 0.00024\text{GeV},$$
$$m_\pi = 0.13967 \pm 0.00000035\text{GeV}, \mathrm{p}_\pi = 0.83297\text{GeV}$$
$$f_\pi = 0.140\text{GeV}, |V_{cs}V_{ud}^*| = 96.06105 \times 10^{-2} \pm 0.00000325$$
$$C_1 = 1.1, G_F = 1.17 \times 10^{-5}\text{GeV}^{-2}$$
$$\tau_{B_c} = 0.452 \pm 0.0033 \times 10^{-12}\text{sec}$$
$$\hbar = 6.582 \times 10^{-25}\text{GeVsec},$$
$$m_{X_{bo}} = 5.7711\text{GeV}, m_{X_{b1}} = 5.8161\text{GeV}$$

The value of form factor is $F_1(m_\pi^2) = f_+(m_\pi^2) = 0.737857$ GeV using the three parameter formula, with the required parameters from Table 4.1.

F	$F(0)$	m_{fit}	δ
$F_1^{B_cB_s}$	$0.73^{+0.03+0.03}_{-0.04-0.03}$	$1.35^{+0.07+0.01}_{-0.08-0.01}$	$0.35^{+0.04+0.00}_{-0.04-0.01}$
$F_0^{B_cB_s}$	$0.73^{+0.03+0.03}_{-0.04-0.03}$	$1.77^{+0.24+0.04}_{-0.20-0.04}$	$0.60^{+0.23+0.04}_{-0.14-0.04}$

. The decay width for $B_c^+ \to B_s^0\pi^+$ has the value of 7.43574×10^{-14}GeV. With the ratio of the fragmentation probabilities to be about 0.02 ,a 5.6% branching ratio is obtained for $B_c \to B_s\pi$, this is close to the value from the LHC experiment which is 10% [48]. For the B_c meson decay to the S-state neutral tetraquark X_{b0}, using the mass of tetraquark state esimated by Diquark model, we expected the branching ratio to be large. Assuming a value of $F\left(X_{b0}/B_s\right)^2$ to be 0.5(1), the relative branching ratio would have the value of 0.03217 (0.1287) and the branching ratio for $B_c^\pm \to X_{b0}^{I=0}\pi^\pm$ turns out to be 0.5(1.5)%. The decay $B_c^\pm \to X_{b0}^\pm\pi^0$ with subsequent decay to $(B_S^0\pi^\pm)\pi^0$ is analyzed as well.

Chapter 5

Conclusion and Discussion

LHCb has not confirmed D0′s detection of the X(5568) with $X(5568) \rightarrow B_s^0\pi^\pm$. This fact remains to be checked out that if a state whose constituent quarks are $b\bar{s}u\bar{d}$ did exist in nature, with a distinct mass, with decay pattern and breadth. In this thesis, the diquark-antidiquark model is employed to figure out the mass spectrum of X_{b0}, which is the lowest S-state, and X_{bo}^+, the $J^P = 1^+$ companion, anticipated not only in the bottom sector but also in charm sector. According to the calculations, the lowest state in the b-quark sector has its mass about 5770 MeV, that is roughly less than the BK threshold. While the state X_{bo}^+ could be just beyond this threshold, within the mistakes of our technique, and one must seek the decay $X_{bo}^+ \rightarrow B^+\bar{K}^0$ for it. The X_{bo}^+ may or may not emerge as a resonating $B_s\pi$ state if below threshold [49]. In this thesis, the proposition that the tetraquark states can be sought in the $B_c^\pm$ mesons decays, $B_c^\pm \rightarrow X_{bo}^0\pi^\pm$ and $B_c^\pm \rightarrow X_{bo}^\pm\pi^0$ and also some of these could possibly have a high branching ratio, is considered.
Using the Naive factorization, the amplitude for the decay $B_c^\pm \rightarrow B_s^0\pi^\pm$ is evaluated. For this, we computed the form factors at $q^2 = m_\pi^2$. The branching ratio for this decay is about 5.6% , which turns out to be close to the experimental value *i-e 10* % by LHCb. Using this value, the relative branching ratio for $B_c^\pm \rightarrow X_{bo}^0\pi^\pm$ and $B_c^\pm \rightarrow B_s^0\pi^\pm$ decays turns out to be 0.03217 (0.1287). The branching ratio for

$B_c^{\pm} \to X_{bo}^0 \pi^{\pm}$ decay is 0.5 to 1.5 %. This branching ratio may be measured in the decay mode $B_c^{\pm} \to (B_s^0 \pi^0)\pi^{\pm}$, assuming a good π^0 detection efficiency. Also, an enormous $B_c^{\pm}$ sample is required to be available,in the upcoming LHC experiments. For this, π^0 detection efficiency is needed to be improved in the experiments of the hadron collider such as the LHCb. The background reduction can be done by the help of the detached vertices of the $B_c^{\pm}$ and B_s^0.

Only a small number of B_c decays have been studied thus far [18], and it would be beneficial to make a deliberate effort to grow this database. In the B_c^+ decays,not only there is probability of detecting the tetraquark states of $B_s\pi$ variety but many bound $c\bar{c}$ tetraquark states are anticipated to emerge from the $B_c^+ \to (c\bar{c})(u\bar{d})$ decay. They are followed by the excitation of $q\bar{q}$ from the vacuum. These would give rise to decays like $B_c^{\pm} \to X(3872)^0 \pi^{\pm}$ and $B_c^{\pm} \to X(3872)^{\pm} \pi^0$, and also other analogous tetraquark states. So, in future, these states should be looked for not only in the LHC, but also at Belle-II, if the threshold for $B_c^+ B_c^-$ can be approached by the mass energies centre of e^+e^-. A huge number of potential states can be, therefore, mapped out in the area of $B_c^{\pm}$ decays in to tetraquarks $[cq][\bar{c}\bar{q}']$.

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