Adaptive Robust Nonlinear Controller Design For Drug Delivery In Chemotherapy Cancer Treatment

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Declaration

I, *Sarwat Akram* declare that this thesis titled "ROBUST NONLINEAR CONTROLLER DESIGN FOR DRUG DELIVERY IN CHEMOTHERAPY CANCER TREATMENT" and the work presented in it are my own and has been generated by me as a result of my own original research.

I confirm that:

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3. Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work
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I'd want to dedicate this thesis to my beloved parents and siblings.

Abstract

Cancer is the second most fatal disease worldwide resulting in millions of deaths worldwide. Numerous complex biological interactions result in the growth of a malignant tumor.The tumor forms as a result of the body's normal cells expanding incorrectly and out of control. Chemotherapy is most widely used treatment for eradicating malignant tumors from the body. Chemotherapy medicine, however, has adverse impacts on the patient's body. The regulation of chemotherapeutic drugs has been suggested in this thesis using adaptive non-linear control methods. For this purpose, a four-state ODE model proposed in the literature has been used. The model's structure is exceedingly nonlinear and chaotic. For the control of chemotherapeutic medication, adaptive controllers using nonlinear control techniques such as ASMC, AISMC, ASTSMC, and ATSMC have been developed. Mathematical analysis based on the Lyapunov stability theory has been addressed to verify the controller stability.The performance of suggested controllers are compared in Simulink/MATLAB. Furthermore, we were not only restricted to MATLAB/SIMULINK, we also performed simulations using the hardware-in-the-loop to gain the confidence regarding our designed controller performance when implemented on actual hardware. The results show that in comparison to the already published control techniques in literature, our ATSMC controller performs best with quick tumor cells removal from body (5% less days to recovery) for minimal amount of drug delivery(46% less dosage drug).

Keywords: Chemotherapy,Cancer tumor, Adaptive Sliding Mode Control, Adaptive Integral Sliding Mode Control,Adaptive Terminal Sliding Mode Control and Adaptive Super Twisting Sliding Mode Control

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List of Abbreviations and Symbols

Abbreviations

LIS	Locked-in Syndrome
QoL	Quality of Life
BCI	Brain-computer Interface
EEG	Electroencephalography, Electroencephalographic
NIRS	Near Infrared Spectroscopy

Chapter 1

Introduction

1.1 Importance of Cancer

Cancer is the deadliest disease worldwide and millions of people die every year due to cancer.Defects in the cell-growth system frequently lead to the unchecked and abnormal expansion of cancer-causing cells. Tumors are made up of some harmful cells, and the presence of the tumor cells encourages the development of immune cells. Major health issues, including mortality, are caused by the immune system's subpar performance and inability to effectively combat the tumor cells. [4],[5]-[6] According to a WHO cancer report, [1] nearly 19.2 million cancer cases are reported annually. According to the data of the National Center for Health Statistics, USA [7] cancer is the second major cause of death in the USA after heart disease,13.3% of the world's population and 14.4% of deaths occur in the Americas. In comparison to other world regions, Asia and Africa have higher rates of certain cancer types associated with poorer prognoses and higher mortality rates, as well as limited access to timely diagnosis and treatment in many countries. This results in higher proportions of cancer deaths in these regions (57.3% and 7.3%, respectively). [1]

In Pakistan, cancer is a significant issue. The prevalence of various cancers affects about 23% of the population. With each accounting for 8% of fatalities, lower respiratory infections and cancer rank highly among the primary causes of death. The number of cancer cases has steadily increased during the past 20 years. The most prevalent among them is breast cancer, which is now very common, as skin cancer, blood cancer, brain tumors, and prostate cancer of all diseases, cancer is the one that is spreading the fastest in Pakistan. Every year, around a million people receive a cancer diagnosis. The figure1.1 below shows the most common cases in Pakistan 2020.

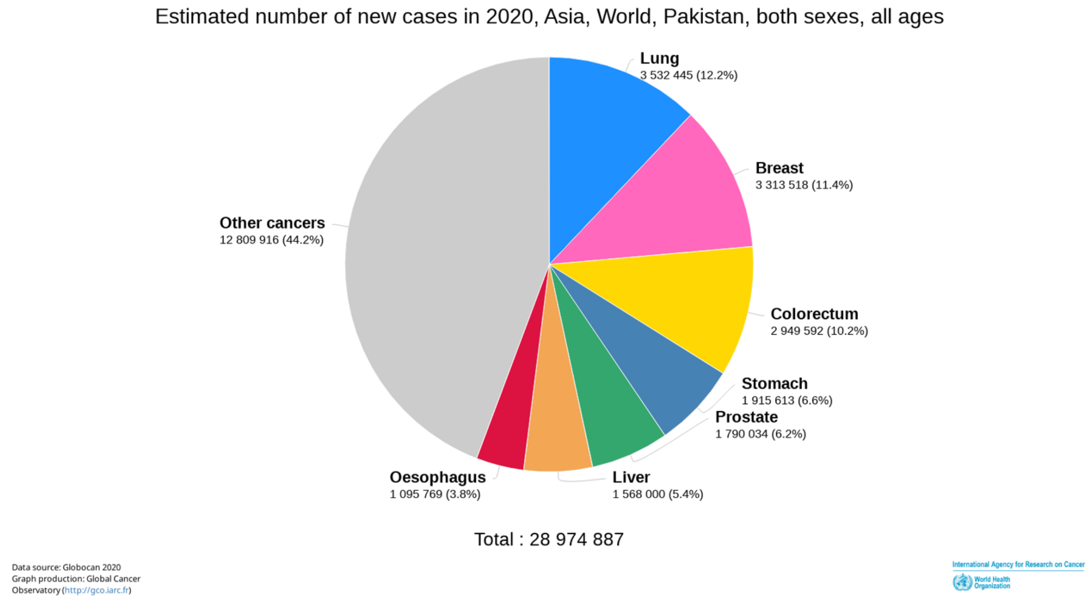

Figure 1.1: Cancer cases in Pakistan [1]

Sadly, Pakistan does not produce any drugs that treat cancer locally; instead, the majority of the country's medications are imported, which drives up the cost of treatment and control. Only relief and some form of control are intended by the affordable locally produced drug. Only 40% of Pakistanis have access to quality healthcare, leaving the country critically lacking in medical facilities for treating cancer patients. Most patients (60%) lack access to diagnostic and treatment resources. Many Pakistanis develop advanced stages of cancer, which reduces their likelihood of being cured, because palliative care, the treatment intended to relieve cancer symptoms, is lacking in that country.

The treatment of cancer is pricey. One of the finest ways to remove tumor cells from the body is through chemotherapy. Chemotherapy treatment involves the removal of tumor cells by injecting less amount of drug in less number of days. The chemotherapy method will reduce the expense of cancer treatment, more patients will be treated in less time. Chemotherapy will reduce the death rate, which will enhance patient health. More people can be treated in hospitals, which is an indirect benefit. As a result hospital workload will be reduced.

1.2 Cancer Treatment Methods

The removal of cancer cells from the body can be accomplished via a variety of treatment methods, including radiotherapy, immunotherapy, surgery, and chemotherapy.

Cancer can be treated with radiotherapy either before it spreads or after it has been detected. In radiotherapy, radiation is utilized to eliminate cancer cells. radiation was selected because it can either kill cancer cells or stop their growth by causing DNA damage. In radiotherapy, powerful energy beams like X-rays are employed. These cells are severely damaged and either stop dividing or pass away, which means they separate from the body. The removal of cancer cells by RT may take days or weeks.

The cancer-fighting ability of the immune system is enhanced by immunotherapy. The body's immune system helps the body fight off diseases and infections. Organs, lymphatic system components, and white blood cells are some of their constituents. Immunotherapy is a type of biological therapy. Drugs originating from living beings are used as a form of treatment for cancer patients receiving biological therapy. immunotherapy is advised since it makes use of the patient body's immune system to fight cancer. Cancer can spread because the patient immune system doesn't recognize it as an invasion. Immunotherapy can support the patient immune system in detecting and battling cancer.

Surgery is used for cancer diagnosis, staging, and treatment. The discomfort or problems brought on by cancer can also be palliated (relieved) through surgery. Sometimes a single technique can achieve more than one of these goals. Sometimes, numerous surgeries may be needed over time.

Chemotherapy is used in conjunction with other therapies like surgery, radiation, and hormone therapy to eradicate malignant cells from the body. One or more anti-cancer medications are used as part of the chemotherapy treatment. The chemotherapy drugs are put into the body through a catheter tube placed in an artery or vein supplying blood to the tumor, or in a cavity/ body part near the tumor site. Although chemo is an effective treatment method, however, it has many side effects associated with it. The side effects can vary from mild which can be treated easily, to serious side effects that adversely affect the patient health and recovery process.

Side Effects of Chemotherapy

All of the body's rapidly dividing and growing cells are affected by chemotherapy. This applies to both cancerous and healthy cells, such as the bone marrow's production of new blood cells and the cells that make up the mouth, stomach, skin, hair, and reproductive organs. Chemotherapy's adverse effects are brought on by damaged normal cells. Chemotherapy treatment is toxic in nature we have to inject the correct amount of drug if the correct amount of drug is not injected then the patient's health would suffer. Chemotherapy treatment involves the removal of healthy cells along with cancer cells which results in a rapid decline in patient health. The type and dosage of medications given, as well as how a patient reacts from one treatment cycle to the next, determines the severity of the side effect. The majority of side effects are temporary and manageable. Once treatment is stopped and the normal, healthy cells return, they typically start to get better gradually. Chemotherapy can occasionally result in persistent side effects. These could involve harm to the patient kidneys, reproductive organs, nerves, heart, lungs, or nerve endings. To minimize the side effects it is extremely important to administer the medication in the right amounts to the

1.3 Problem Statement

There is a need to design an efficient controller for effective drug delivery in chemotherapy treatment, thus ensuring quick recovery and improved patient health. The controller should be robust and adaptive to the variations in the system as well.

1.4 Benefits of Purposed Control Techniques

- Patient health under chemotherapy treatment will be improved.
- Patient recovery time will be improved.
- The cost of cancer treatment will be reduced.
- The indirect benefits include more patients can be treated in given hospital facilities and the hospital burden will be reduced.

1.5 Prior State of the Art

Various work has been done previously for the control of drug dosage for chemotherapy treatment. Current state-of-the-art include the work [8], in which Synergetic and State feedback nonlinear control techniques are used along with fuzzy on a four-state model. However, the control design is not robust and not adaptive as well.

1.6 Research Goals and Objectives

- To study and understand various mathematical models used in literature for the chemotherapy drug delivery mechanism.

- Implement already used control techniques in the literature and reproduce the simulation results published in the literature.

- Understand the Robust control techniques and implement them in MATLAB/Simulink.

- Study and understand Adaptive Nonlinear control techniques and implement them on MATLAB/Simulink.

- Compare our results with the already published results in the literature.

- Implement the designed controller using Hardware-in-the-loop (HIL) and compare the results with MATLAB/Simulink results.

1.7 Methodology

Different Nonlinear Control approaches have been used in this study to eliminate tumor cells from the body in fewer days and with a smaller overall treatment dosage. To do this, we must first study and comprehend robust nonlinear control techniques before putting them into use in MATLAB/SIMULINK. To achieve better outcomes, we adapted our controller by utilizing adaptive law and compared our findings to prior research in the literature. The Stability Analysis was done using the Lyapunov-based stability method. We performed simulations using hardware-in-the-loop and compared the outcomes to those obtained using MATLAB/Simulink because we were not solely limited to MATLAB/SIMULINK.

Figure 1.2 shows the research's chosen workflow.

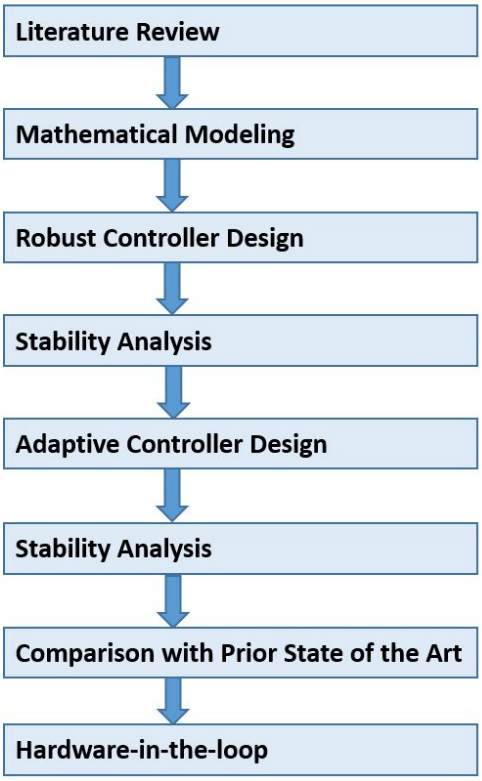

Figure 1.2: METHODOLOGY

1.8 Contribution

The robust nonlinear controller is designed for effective drug delivery in chemotherapy treatment, which is adaptive to the variations in the systems. We have not found such an adaptive controller in literature so far implemented for the problem at hand.

1.9 Organization of Thesis

Chapter 2: Tumor Model

In Chapter 2 Cancer details and different treatment methods for cancer are discussed, four–the state mathematical tumor model is discussed in detail. **Chapter 3: Literature Review**

Chapter 2 discusses the details of cancer, various cancer treatments, and a four-state mathematical tumor model in great depth.

Chapter 4: Control dynamics of Non-Linear Tumor Model

There is a discussion of several nonlinear control techniques both with and without adaptation.

Chapter 5: Hardware-In-the-Loop

In Chapter 5, the specifics of the HIL implementation are covered.

Chapter 6: Simulations and Results

Results and Simulations are discussed in Chapter 6.

Chapter 7: Conclusion

Chapter 7 discusses the results of the research and potential directions for further work.

Chapter 2

Literature Review

2.1 Background

Early 20th-century researchers began studying cancer, and it is still a hot research area. Malignant cells grow uncontrollably and abnormally, which frequently happens as a result of defects in the cell-growth process. Tumors are made up of some harmful cells, and the presence of the tumor cells encourages the development of immune cells. Major health problems, leading to death, are brought on by the immune system's subpar functioning and inability to fight tumor cells.

2.2 Literature on Cancer Treatment Methods

As briefly discussed in chapter 1, there are various treatment methods for cancer. In this section, we discuss those treatment methods in more detail.

2.2.1 Immunotherapy

In recent years, immunotherapy treatments that enhance the immune system's capacity to identify and destroy malignancy have benefited an increasing number of cancer patients. These novel medications, which include immune checkpoint inhibitors and CAR T-cell therapies, have had remarkable and long-lasting effects in a few patients. Rarely, immunotherapy treatment has caused tumors in people with advanced cancer to vanish. Like any medical procedure, immunotherapy can have side effects, many of which are brought on when the immune system, which has been engaged to combat cancer, also begins to assault the body's healthy cells and tissues. For different people, there are different side effects. The type of cancer, its stage, the type of immunotherapy the patient is receiving, and the amount will all have an effect on the symptoms the patient experiences and how the patient feels. Immunotherapy may endure for a long time, and side effects can occur both during and after treatment at any moment. Doctors are unable to predict the occurrence of side effects, their severity, or when they may manifest. Understanding the warning signs and what to do if a patient start having problems is therefore crucial. All immunotherapy treatments come with a few common side effects.

2.2.2 Radiotherapy

While radiation is safe, there may be additional short or long-term side effects. This could happen if radiation damages healthy cells in addition to the cancer cells that are being treated. The sorts of side effects patients may experience and how severe they are can vary depending on the patient's general health, the amount of radiotherapy provided, the area of the patient's body being treated, and any additional cancer treatments patient may be receiving. Some patients receiving radiation therapy experience little, if any, adverse effects and can resume their regular activities. Others go into greater depth on adverse outcomes. Patients frequently experience radiation side effects a few weeks into their treatment. Even after the course of treatment is through, they continue for a time. Some of the frequent side effects include fatigue, dry mouth and/or mouth sores, nausea and/or vomiting, bowel issues like diarrhea, urinary issues like urgency or incontinence, lymphoedema (swelling in soft tissue), hair loss, and infertility. Fatigue is when a patient feels exhausted and worn out. It could start quickly or take time to grow. A patient might feel worn out differently from someone else receiving the same dose of radiation therapy in the same body location because different patient experience exhaustion in various ways. The patient may experience other side effects from radiation therapy depending on the location of the body that is treated. Most of them are temporary and fade with time, however, some, like infertility, may be lifelong.

2.2.3 Surgery

Like all cancer treatments, surgery has advantages, disadvantages, and adverse effects. How severe and specific side effects are for each patient varies depending on the type and location of cancer, the procedure used, any additional therapies the patient underwent before the surgery, such as chemotherapy and radiation therapy, the patient's general health, and any warning signs the patient experienced before the procedure. Many procedures today are less invasive than they used to be. Surgery frequently allows for a quicker recovery for patients and has milder side effects. After surgery, there are numerous ways to deal with physical side effects including disc comfort. However, the majority of surgeries have an impact on the body and might have detrimental effects in the short and long term. Minimizing side effects is a crucial part of cancer care and treatment. This is supportive care, often known as palliative care.

2.2.4 Chemotherapy

Chemotherapy is an effective treatment for many cancer types. But it frequently results in side effects, just like other cancer treatments. It's critical to be informed of possible chemotherapy side effects so patients can be alert to them.

Depending on the medication or medication combination recommended, patients who are receiving chemotherapy may encounter side effects. Side effects are caused by various medications differently. Additionally, each patient's experience is unique. Even when taking the same medication, not all patients will have the same adverse effects. Additionally, if the patient uses the same medication again, they can experience different adverse effects than previous people in the past.

The most common side effect of chemotherapy is fatigue. Even if the patient receives enough sleep, fatigue is when the patient feels tired. It is the most typical adverse reaction to chemotherapy.

Hair falls Not all chemotherapy treatments, although some do result in hair loss. The patient body's hair may grow slowly over time or in big clumps. Usually, hair loss begins several weeks during chemotherapy. One to two months into treatment, it usually gets worse. Pain. Chemotherapy can occasionally be painful. Several types of pain can result from nerve damage, such as headaches, muscle discomfort, stomach pain, burning, numbness, or shooting pains that typically affect the fingers and toes.

Any cancer-related pain will be treated by first addressing its underlying cause. Additional to chemotherapy itself, there may be other causes of pain. If chemotherapy-related pain is present, medical professionals may manage it by:

- Providing painkiller drugs.
- Modifying the dosage of particular medications..
- Blocking pain signals from the nerves to the brain through spinal manipulations or nerve blocks..

The cells in the mouth and throat can suffer damage from chemotherapy. Mucositis, a condition brought on by this, results in painful sores in certain locations. Typically, mouth sores appear 5 to 14 days following treatment. It's crucial to keep an eye out for infection in these wounds.

Some chemotherapy treatments result in sloppy or watery stools. Patients can avoid being dehydrated by either preventing diarrhea or treating it quickly (losing too much body fluid). It also aids in the prevention of other health issues. Constipation can result after chemotherapy. This entails either insufficient bowel motions or challenging bowel movements. Constipation can also be brought on by other medications, such as painkillers. By getting regular exercise, eating well-balanced meals, and drinking adequate fluids, patients can reduce their risk of constipation. Chemotherapy can make the patient nauseous and make the patient throw up. Before and after every chemotherapy dose, several drugs are administered with preventing the goal of preventing nausea and vomiting.

The spongy material inside of a patient's bones is called the patient's bone marrow. It produces fresh blood cells. Because chemotherapy alters this procedure, having too few blood cells may have negative effects on the patient.

Following chemotherapy, the number of blood cells often recovers to normal. Low blood cell counts can, however, become problematic during treatment and need to be carefully monitored. Some medications harm the nerves. Nerve or muscle problems such as Loss of balance, Tingling, Burning, swaying or trembling, stiff neck or headache, issues with normal vision, hearing or walking, and clumsiness may occur. Usually, following treatment or with a lower chemotherapy dose, these symptoms improve. After chemotherapy, it may take 6 to 12 months for symptoms to subside. Some adverse consequences may last a lifetime. After chemotherapy, some people have problems focusing and thinking clearly. Chemo brain is a common term used by cancer

2.3 Mathematical Modeling of Tumor Model

Since it is known that interactions between various body cells are crucial for the growth of tumors, a variety of spatial and non-spatial mathematical models have been presented to shed light on the dynamic behavior of cancer cells and their interactions with the surrounding ones. [9] In addition to other biological dimensions, these models include interactions at the molecular, cellular, and tissue levels. While ODE models provide a much more easy research base, spatial models frequently take the form of cellular automata or partial differential equations (PDEs). To control the death rate due to cancer previously different work has been done using One-state, Two-state, three-state, and four-state tumor mathematical models. In the one-state Model[6] only the behavior of tumor cells was discussed. In the two-state model[10] two clone model for tumor, regrowth is considered in which a small population of immune-resistant cancer cells that are either initially present or that form and expand unchecked by the activity of killer cells is what causes tumor regrowth. Further three-state tumor model describes the dynamics of acute Leukemia cells[11]
Another three-state tumor mathematical model uses tumor cells, hunting predator cells, and resting predator cells[12], Several tumor progression-related subjects have been studied using the four-state tumor model. The model focuses on the interactions at the tumor site between immune, normal, and tumor cells. site.[8],[13],[14],[15]

2.3.1 One-state Mathematical Model

The paper [6] proposes a control model for chemotherapy drug delivery schedule using a one-dimensional tumor growth mathematical model. The model imposes a constraint on the growth of tumor size such that it must decreases once the chemotherapy treatment starts. In this model, no interaction of the tumor cell with immune cells, healthy cells, or with drugs is considered. The system is treated using the well-known numerical solution technique known as control parametrization, resulting in a non-linear programming problem. In this paper, only tumor cells were discussed.

2.3.2 Two-state Mathematical Model

A two-dimensional Mathematical Model of tumor growth that discusses the relationship between the tumor and immune cells[10]
To reduce the rate of expansion of the main immunogenic tumor cell population, The mice received a very modest amount (0.004%) of the tumor cell population. The model also predicts that the initial quantity of implanted tumor cells will impact the size of the tumor following dormancy as well as the eventual recurrence of the tumor. In this two-state model, only immune and tumor cells were discussed.

2.3.3 Three-state Mathematical Model

The three-dimensional cancer model deals with normal cells, leukemic cells, and the number of chemotherapy agents [11]
To control the number of therapeutic agents delivered to the patient, two types of therapy functions—the monotonic function and the non-monotonic function—have been taken into account in this study. Three-dimensional cancel model in which tumor and immune cells are not defined.

Another work has been done using the mathematical model of acute leukemia [16]
Using the concept of acute leukemia, the manuscript uses monotonic and non-monotonic therapy functions to describe various treatment effects on both normal and leukemic cells. the goal of utilizing a safe dose of a complete chemotherapeutic drug to kill the leukemic cells while preserving a safe amount of normal cells.

A three-dimensional cancer model dealing with tumor cells, hunting predator cells, and resting predator cells[12]
The main objective was to stop the growth of tumor cells at a point where they would no longer be harmful, to keep the hunting predator cells at their highest level, and to keep the resting predator cells at 40% of the hunting predator cells. Chemotherapy and its effects on various cell types have been developed. A three-dimensional cancer model is used which is not very accurate. Moreover, the nonlinear controllers used are not very robust.

2.3.4 Four-state Mathematical Model

The non-linear mathematical model for tumor growth proposed by De Pillis and Radunskaya was chosen for chemotherapy drug control. [5],[4]
The interaction of normal, tumor and immune cells at the tumor site is the focus of the model. Different work has been done using this model the details of the work done are mentioned below.

The mathematical model of tumor growth with immune response and chemotherapy serves as the foundation for this paper's phase-space analysis[4]
The primary objective was to demonstrate that all orbits are constrained and must converge to one of several potential equilibrium sites. As a result, the basin of attraction in which an orbit begins determines its long-term behavior Using numerical experiments, it is shown that optimal control therapy can move the system into a desired basin of attraction, whereas the addition of a drug term to the system can move the solution trajectory into a desired basin of attraction to the solutions of the model with a time-varying drug term approach to the solutions of the system without the drug once treatment has stopped.

The paper [8] in which an It is addressed how normal, tumor, and immune cells interact with the tumor site in a four-dimensional cancer model. The main goal was to reduce the tumor cells from the body in less number of days, by injecting less amount medicine for this purpose Non-linear Control techniques were used. In this four-state model, the Nonlinear control techniques used are not very robust.

[14] a four-dimensional mathematical model in which the logistic growth law is used to simulate both tumor and host cells. Tumor cells can encourage the development of immune cells, and these immune cells then kinetically kill out the tumor cells. In an updated model of brain tumor, several nonlinear control techniques have been applied to the therapeutic agent to reduce the number of brain tumor cells, maintain a safe number of normal cells, maintain immune cells above a certain value, and ensure the use of the correct dosage of the drug for the therapy. The robust non-linear control techniques are not applied, but the four-state mathematical model is precise.

A four-dimensional model that includes the dynamics of normal cells, immune cells, tumor cells, and drug dosage[13]
The controller is designed such that the tumor cells achieve their reference value of 0 in

the shortest amount of time by delivering the drug in an appropriate amount, maintaining a safe number of normal cells, and keeping the immune cells above a particular value. Variable structured-based nonlinear control algorithms have been used for this purpose. The main drawback was that the Controller design is not adaptive to the internal or external variations in the system.

A logistic growth law is used in this model to simulate both the tumor cells and the host cells. Immune cells can be stimulated to develop by tumor cells, and once they do, the immune cells kinetically destroy the tumor cells. Two nonlinear control algorithm-based controllers were developed for the therapeutic agent in an updated mathematical model of a brain tumor to reduce the tumor cells, maintain a safe number of healthy cells, keep the immune cells above a specific level, and ensure the correct dosage of the drug during the therapy.[15] The robust non-linear control techniques are not implemented, but the four-state mathematical model is accurate.

Chapter 3

The Non-Linear Tumor Model Control Dynamics

3.1 Nonlinear Tumor Model

De Pillis and Radunskaya [4],[5] created the mathematical model of the nonlinear four-state ODE for tumor progression, and it has been used to examine several features of tumor development, including cancer dormancy, creeping through, and immune surveillance escape. The selected model focuses on how the surrounding immune, normal, and tumor cells interact. Tumor cells within the body encourage the growth of immune cells. The reaction of these immune cells is insufficient to compete with the rapidly dividing tumor cell. At the tumor location, normal and tumor cells are continually competing for the same scarce resources. Using ordinary differential equations, the model is depicted in eq 3.1.1.

$$\begin{aligned}
\dot{N} &= j_2 N\,(1 - g_2 N) - h_4 TN - v(1 - e^{-M})N \\
\dot{T} &= j_1 T(1 - g_1 T) - h_2 IT - h_3 TN - v_2(1 - e^{-M})T \\
\dot{I} &= o + \frac{\rho IT}{\alpha + T} - h_1 IT - i_1 I - v_1(1 - e^{-M})I \\
\dot{M} &= u(t) - i_2 M
\end{aligned} \tag{3.1.1}$$

In the above model (3.1.1) N(t) represent normal cells, T(t) represents tumor cells,I(t) represents immune cells at time t.

M(t) is the amount of chemotherapeutic medication discovered in the patient's bloodstream at the tumor location, and u(t) is the medication dosage that was given to the patient at time t. $N(1-g_2N)$,$T(1-g_1T)$, $\frac{\rho IT}{\alpha+T}$ correspondingly indicate the logistic growth of normal cells, tumors, and immune cells. Where $j_1$and j_2 are the rates at which normal and malignant cells develop. The carrying capacities of normal and malignant cells are g_1 and g_2, respectively. The decay of normal cells, or h_4T(t)N(t), results from the competition of normal and malignant cells for the same local resources, h_2IT Represents several tumor cells killed by immune cells, h_3TN Represents several tumor cells killed by normal cells.$i_1I$represents immune cells per capita death, I_2M represents the chemo drug per capita death . h_1I(t)T(t),$v_3(1-e^{-M})N$ represents the interaction of tumor and immune cells. Represents the number of Normal cells killed by Chemo drug u,$v_2(1-e^{-M})T$ Represents the number of Tumor cells killed by Chemo drug u, $v_1(1-e^{-M})I$ Represents the number of Immune cells killed by Chemo drug u. Immune cells age at a pace of I_1 in the absence of tumor cells. The cells that died as a result of chemotherapeutic drug treatment are designated as a_i(i = 1, 2, 3). Below table 3.1 shows the Values of Normalized Parameter

Parameters	**Description**	**Values**
v_1	Immune cells killed due to chemo drug	0.2
v_2	Tumor cells killed due to chemo drug	0.3
v_3	Normal cells killed due to chemo drug	0.1
g_1	Carrying capacity of Tumor cells	1
g_2	Normal cells Carrying capacity	1
h_1	Tumor cells killed fractional Immune cells	1
h_2	Immune cells killed fractional tumor cells	0.5
h_3	Normal cells killed fractional tumor cells	1
h_4	tumor cells killed fractional normal cells	1
i_1	immune cells Per capita death	0.2
i_2	chemo drug Per capita death	1
j_1	Growth rate of tumor cells growth rate per unit	1.5
j_2	Normal cells growth rate per unit	0.33
o	Steady source rate of immune cells	0.33
ρ	Response rate of Immune cells	0.01
α	Threshold rate of Immune cells	0.3

Table 3.1: Normalized Parameter Values[4]

Additionally, we can modify the model by substituting $N(t), T(t), I(t) and M(t)$ by x_1,x_2,$x_3$and x_4 we get:

$$\begin{aligned}
\dot{x_1} &= j_2 x_1(1 - g_2 x_1) - h_4 x_2 x_1 - v_3(1 - e^{-x4})x_1 \\
\dot{x_2} &= j_1 x_2(1 - g_1 x_2) - h_2 x_3 x_2 - h_3 x_2 x_1 - v_2(1 - e^{-x4})x_2 \\
\dot{x_3} &= o + \frac{\rho x_3 x_2}{\alpha + x_2} - h_1 x_3 x_2 - i_1 x_3 - v_1(1 - e^{-x4})x_3 \\
\dot{x_4} &= u(t) - i_2 x_4
\end{aligned} \tag{3.1.2}$$

There are three possible possibilities for the system's equilibrium points in the absence of chemotherapeutic medications.

• Two unstable dead equilibrium points exist in the system where normal cells are not present.

• There exists a stable equilibrium point at [1, 0, 1.65] with N = 1, T = 0, and I =

1.65. After chemotherapy has successfully eliminated a tumor from the body, this is the system's intended equilibrium state.

- The system has a coexisting equilibrium point with N = 0.44, T = 0.56, and I = 0.44, where both tumour and normal cells can be found.

Figure 3.1 depicts the system's co-existing equilibrium condition before the injection of chemotherapeutic medicines. The system approaches equilibrium after approximately 138-140 days, as shown in the graph.

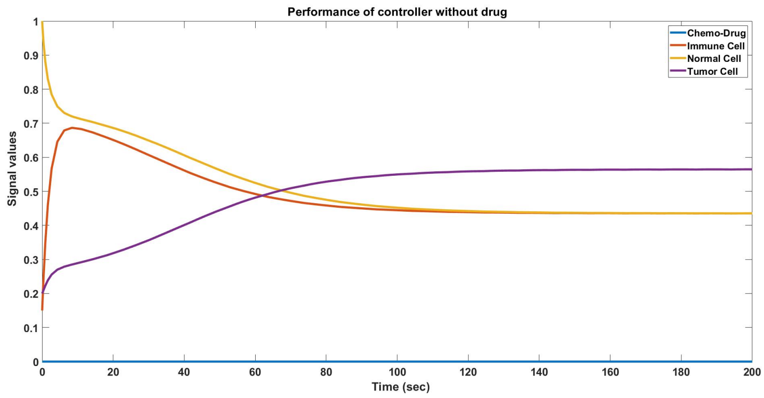

Figure 3.1: Performance of Controller without drug .

Considering the above tumor model we will be using Robust Nonlinear control techniques and then we will be adapting the controller by using adaptive law.

3.2 Robust Controller Design

In this section Robust Control techniques SMC, ISMC, TSMC, and ST-SMC are discussed.

3.2.1 Sliding Mode Control

Sliding mode control a robust controller that has benefits such as finite time convergence, minimal steady-state error, cheap processing costs, and ease of use. The sliding surface is chosen in the first stage, after which the control rule is developed to direct the system toward the sliding surface. Figure 3.2 shows a visual representation of SMC. It demonstrates how the variable initially has a value of Xo, converges to the sliding surface due to the control action, and then eventually reaches the intended value.

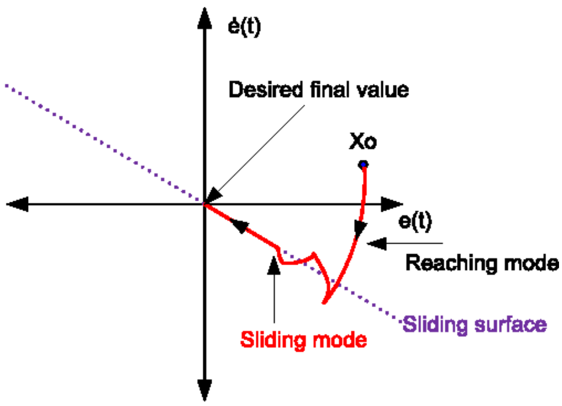

Figure 3.2: Sliding Mode Control Phase [2]

We define error terms as follows to track all states to their target values:

$$\begin{aligned} e_1 &= x_1 - x_{1ref} \\ e_2 &= x_2 - x_{2ref} \\ e_3 &= x_3 - x_{3ref} \\ e_4 &= x_4 - x_{4ref} \end{aligned} \tag{3.2.1}$$

In above eq 3.2.1 $x_1, x_2, x_3 and x_4$ are the state varibles and $x_{1ref}, x_{2ref}, x_{3ref}, x_{4ref}$ are the desired values.

Taking time derivative of error terms in 3.3.1

$$\begin{aligned} \dot{e}_1 &= \dot{x}_1 - \dot{x}_{1ref} \\ \dot{e}_2 &= \dot{x}_2 - \dot{x}_{2ref} \\ \dot{e}_3 &= \dot{x}_3 - \dot{x}_{3ref} \\ \dot{e}_4 &= \dot{x}_4 - \dot{x}_{4ref} \end{aligned} \tag{3.2.2}$$

The Sliding surface is defined as

$$S = c_1 e_1 + c_2 e_2 + c_3 e_3 + c_4 e_4 \tag{3.2.3}$$

here $c_1, c_2, c_3 and c_4$ are constant design parameters of the sliding surfaces which can have any positive constant value.

Taking time derivative of S

$$\dot{S} = c_1 \dot{e}_1 + c_2 \dot{e}_2 + c_3 \dot{e}_3 + c_4 \dot{e}_4 \tag{3.2.4}$$

putting values of $\dot{e}_1, \dot{e}_2, \dot{e}_3, \dot{e}_4$ in eq 3.3.5

$$\begin{aligned}\dot{S} \quad &= c_1(\dot{x}_1 - \dot{x}_{1ref}) + c_2(\dot{x}_2 - \dot{x}_{2ref}) \\ &+ c_3(\dot{x}_3 - \dot{x}_{3ref}) + c_4(\dot{x}_4 - \dot{x}_{4ref}\end{aligned} \tag{3.2.5}$$

$$\begin{aligned}\dot{S} \quad &= c_1\dot{x}_1 - c_1\dot{x}_{1ref} + c_2\dot{x}_2 - c_2\dot{x}_{2ref} \\ &+ c_3\dot{x}_3 - c_3\dot{x}_{3ref} + c_4\dot{x}_4 - c_4\dot{x}_{4ref}\end{aligned} \tag{3.2.6}$$

putting values of $\dot{x}_1, \dot{x}_2, \dot{x}_3, \dot{x}_4$ in eq 3.2.6

$$\begin{aligned}\dot{S} \quad &= c_1(j_2x_1)(1 - g_2x_1) - h_{44}x_2x_1 - v_3(1 - e^{-x4})x_1 - c_1\dot{x}_{1ref} + c_2(j_1x_2(1 - g_1x_2) \\ &-h_{22}x_3x_2 - h_{33}x_2x_1 - v_2(1 - e^{-x4})x_2) - h_2\dot{x}_{2ref} + h_3(o + \tfrac{\rho x_3x_2}{\alpha + x_2} \\ &-h_{11}x_3x_2 - i_1x_3 - v_1(1 - e^{-x4})x_3) - h_3\dot{x}_{3ref} + c_4(u(t) - i_2x_4) - h_4\dot{x}_{4ref}\end{aligned} \tag{3.2.7}$$

Simplifying above eq 3.2.7

$$\begin{aligned}\dot{S} \quad &= c_1(j_2x_1 - j_2g_2x_1^2 - h_{44}x_2x_1 - v_3x_1 + v_3e^{-x_4}x_1) - c_1\dot{x}_{1ref} + c_2(j_1x_2 - j_1x_2^2g_1 \\ &-h_{22}x_3x_2 - h_{33}x_2x_1 - v_2x_2 + e^{-x_4}x_2) - g_2\dot{x}_{2ref} + c_3(o + \tfrac{\rho x_3x_2}{\alpha + x_2} - h_{11}x_3x_2 - i_1x_3 \\ &-v_1(1 - e^{-x_4})x_3) - c_3\dot{x}_{3ref} + c_4(u(t) - i_2x_4) - c_4\dot{x}_{4ref}\end{aligned} \tag{3.2.8}$$

Simplifying above eq3.2.8

$$\begin{aligned}\dot{S} \quad &= c_1 j_2 x_1 - c_1 j_2 g_2 x_1^2 - c_1 h_{44} x_2 x_1 - c_1 v_3 x_1 + c_1 v_3 e^{-x_4} x_1 - c_1 \dot{x}_{1ref} + c_2 j_1 x_2 - c_2 j_1 x_2^2 g_1 \\ &-h_{22} x_3 x_2 - c_2 h_{33} x_2 x_1 - c_2 v_2 x_2 + c_2 e^{-x_4} x_2 - c_2 \dot{x}_{2ref} + c_3 o + c_3 \tfrac{\rho x_3 x_2}{\alpha + x_2} \\ &-c_3 h_{11} x_3 x_2 - c_3 i_1 x_3 - c_3 v_1 x_3 - c_3 e^{-x4} x_3 v_1 - c_3 \dot{x}_{3ref} + c_4 u(t) - c_4 i_2 x_4 - c_4 \dot{x}_{4ref}\end{aligned} \tag{3.2.9}$$

We consider $\dot{S}$=-ksign(S) the SMC's reaching law. When the state is far from the switching manifold, a law known as the power rate reaching law speeds up reaching. k, which can be any constant positive value, is a design parameter. Following is a definition of the Signum function:

$$\begin{aligned} sign(S) &= -1 \quad if \quad S < 0 \\ &= 0 \quad if \quad S = 0 \\ &= 1 \quad if \quad S > 0 \end{aligned} \tag{3.2.10}$$

$$\begin{aligned}-ksign(S) \quad &= c_1 j_2 x_1 - c_1 j_2 g_2 x_1^2 - c_1 h_{44} x_2 x_1 - c_1 v_3 x_1 + c_1 v_3 e^{-x_4} x_1 - c_1 \dot{x}_{1ref} + c_2 j_1 x_2 \\ &-c_2 j_1 x_2^2 g_1 - h_{22} x_3 x_2 - c_2 h_{33} x_2 x_1 - c_2 v_2 x_2 + c_2 e^{-x_4} x_2 - c_2 \dot{x}_{2ref} + c_3 o \\ &+c_3 \tfrac{\rho x_3 x_2}{\alpha + x_2} - c_3 h_{11} x_3 x_2 - c_3 i_1 x_3 - c_3 v_1 x_3 - c_3 e^{-x4} x_3 v_1 - c_3 \dot{x}_{3ref} + c_4 u(t) \\ &-c_4 i_2 x_4 - c_4 \dot{x}_{4ref}\end{aligned} \tag{3.2.11}$$

Rearranging equation 3.2.11

$$\begin{aligned}-c_4 u(t) \quad &= c_1 j_2 x_1 - c_1 j_2 g_2 x_1^2 - c_1 h_{44} x_2 x_1 - c_1 v_3 x_1 + c_1 v_3 e^{-x_4} x_1 - c_1 \dot{x}_{1ref} + c_2 j_1 x_2 \\ &-c_2 j_1 x_2^2 g_1 - h_{22} x_3 x_2 - c_2 h_{33} x_2 x_1 - c_2 v_2 x_2 + c_2 e^{-x_4} x_2 - c_2 \dot{x}_{2ref} + c_3 o \\ &+c_3 \tfrac{\rho x_3 x_2}{\alpha + x_2} - c_3 h_{11} x_3 x_2 - c_3 i_1 x_3 - c_3 v_1 x_3 - c_3 e^{-x4} x_3 v_1 - c_3 \dot{x}_{3ref} + ksign(S) \\ &-c_4 i_2 x_4 - c_4 \dot{x}_{4ref}\end{aligned} \tag{3.2.12}$$

eq3.2.12 will give control input u:

$$\begin{aligned} u(t) &= \tfrac{-1}{c_4}[c_1 j_2 x_1 - c_1 j_2 g_2 x_1^2 - c_1 h_{44} x_2 x_1 - c_1 v_3 x_1 + c_1 v_3 e^{-x_4} x_1 - c_1 \dot{x}_{1ref} + c_2 j_1 x_2 - c_2 j_1 x_2^2 g_1 \\ & -h_{22} x_3 x_2 - c_2 h_{33} x_2 x_1 - c_2 v_2 x_2 + c_2 e^{-x_4} x_2 - c_2 \dot{x}_{2ref} + c_3 o + c_3 \tfrac{\rho x_3 x_2}{\alpha + x_2} \\ & -c_3 h_{11} x_3 x_2 - c_3 i_1 x_3 - c_3 v_1 x_3 - c_3 e^{-x4} x_3 v_1 - c_3 \dot{x}_{3ref} + ksign(S) - c_4 i_2 x_4 - c_4 \dot{x}_{4ref}] \end{aligned} \quad (3.2.13)$$

For stability analysis, let us consider following Lyapunov candidate function:

$$V = \frac{1}{2} S^2 \quad (3.2.14)$$

$$\dot{V} = S\dot{S} \quad (3.2.15)$$

putting $\dot{S} = -ksign(S)$in eq 3.2.15:

$$\dot{V} = -S(ksign(S)) \quad (3.2.16)$$

Lyapunov analysis shows that the proposed controller meets the stability conditions, which ensures the convergence of errors to zero in finite time and asymptotic stability of the system.

3.2.2 Integral Sliding Mode Control

To order to lower the steady-state error and reduce the chattering effect integral terms of errors are added to the sliding surface in ISMC a variation of sliding mode control. The integral of error terms are defined as follows:

$$\begin{aligned} e_5 &= \int x_1 - x_{1ref} dt \\ e_6 &= \int x_2 - x_{2ref} dt \\ e_7 &= \int x_3 - x_{3ref} dt \\ e_8 &= \int x_4 - x_{4ref} dt \end{aligned} \quad (3.2.17)$$

In above eq 3.2.17 $x_1, x_2, x_3 and x_4$ are the state varibles and $x_{1ref}, x_{2ref}, x_{3ref}, x_{4ref}$ are the desired values.

Taking time derivative of integral error terms in eq 3.2.17

$$\begin{aligned} e_5 &= x_1 - x_{1ref} = e_1 \\ e_6 &= x_2 - x_{2ref} = e_2 \\ e_7 &= x_3 - x_{3ref} = e_3 \\ e_8 &= x_4 - x_{4ref} = e_4 \end{aligned} \tag{3.2.18}$$

Sliding Surface is defined as

$$S = f_1 e_1 + f_2 e_2 + f_3 e_3 + f_4 e_4 + f_5 e_5 + f_6 e_6 + f_7 e_7 + f_8 e_8 \tag{3.2.19}$$

here $f_1, f_2, f_3, f_4 f_5, f_6, f_7, f_8$ are constant design parameters of the sliding surfaces which can have any positive constant value. Taking time derivative of S

$$\dot{S} = f_1 \dot{e}_1 + f_2 \dot{e}_2 + f_3 \dot{e}_3 + f_4 \dot{e}_4 + f_5 \dot{e}_5 + f_5 \dot{e}_6 + f_7 \dot{e}_7 + f_8 \dot{e}_8 \tag{3.2.20}$$

putting values of $\dot{e}_1, \dot{e}_2, \dot{e}_3, \dot{e}_4$ in eq3.2.20

$$\dot{S} = f_1(\dot{x}_1 - \dot{x}_{1ref}) + f_2(\dot{x}_2 - \dot{x}_{2ref}) + f_3(\dot{x}_3 - \dot{x}_{3ref}) + f_4(\dot{x}_4 - \dot{x}_{4ref}) + f_5 e_1 + f_6 e_2 + f_7 e_3 + f_8 e_4 \tag{3.2.21}$$

$$\begin{aligned} \dot{S} \quad &= f_1 \dot{x}_1 - f_1 \dot{x}_{1ref} + f_2 \dot{x}_2 - f_2 \dot{x}_{2ref} + f_3 \dot{x}_3 - f_3 \dot{x}_{3ref} \\ &+ f_4 \dot{x}_4 - f_4 \dot{x}_{4ref} + f_5 e_1 + f_6 e_2 + f_7 e_3 + f_8 e_4 \end{aligned} \tag{3.2.22}$$

putting values of $\dot{x}_1, \dot{x}_2, \dot{x}_3, \dot{x}_4$ in eq3.2.22

$$\begin{aligned}\dot{S} \quad =& f_1[j_2x_1(1-g_2x_1) - h_4x_2x_1 - v_3(1-e^{-x4})x_1 - f_1\dot{x}_{1ref} + f_2[j_1x_2(1-g_1x_2) - h_2x_3x_2 \\ &-h_3x_2x_1 - v_2(1-e^{-x4})x_2 - f_2\dot{x}_{2ref} + f_3[o + \tfrac{\rho x_3x_2}{\alpha+x_2} - h_1x_3x_2 - i_1x_3 - v_1(1-e^{-x4})x_3] \\ &-f_3\dot{x}_{3ref} + f_4[u(t) - i_2x_4] - f_4\dot{x}_{4ref} + f_5e_1 + f_6e_2 + f_7e_3 + f_8e_4\end{aligned} \tag{3.2.23}$$

Reaching law of SMC $\dot{S}$= -ksign(S) here k is a design parameter and can have any constant positive value.

Signum function is defined as follows:

$$\begin{aligned} sign(S) &= -1 \quad if \quad S<0 \\ &= 0 \quad if \quad S=0 \\ &= 1 \quad if \quad S>0 \end{aligned} \tag{3.2.24}$$

putting value of $\dot{S}$ in eq 3.2.23:

$$\begin{aligned}\text{-}ksign(S) \quad =& f_1[j_2x_1(1-g_2x_1) - h_4x_2x_1 - v_3(1-e^{-x4})x_1 - f_1\dot{x}_{1ref} + f_2[j_1x_2(1-g_1x_2) \\ &-h_2x_3x_2 - h_3x_2x_1 - v_2(1-e^{-x4})x_2] - f_2\dot{x}_{2ref} + f_3[o + \tfrac{\rho x_3x_2}{\alpha+x_2} - h_1x_3x_2 - i_1x_3 \\ &-v_1(1-e^{-x4})x_3] - \dot{x}_{3ref}f_3 + f_4[u(t) - i_2x_4] - f_4\dot{x}_{4ref} + f_5e_1 + f_6e_2 + f_7e_3 + f_8e_4\end{aligned} \tag{3.2.25}$$

Simplifying eq 3.2.25:

$$\begin{aligned}\text{-}ksign(S) \quad =& f_1[j_2x_1 - g_2x_1^2 - h_4x_2x_1 - v_3x_1 + v_3e^{-x4}x_1] - f_1\dot{x}_{1ref} + f_2[j_1x_2 - j_1g_1x_2^2 \\ &-h_2x_3x_2 - h_3x_2x_1 - v_2x_2 - v_2e^{-x4}x_2] - f_2\dot{x}_{2ref} + f_3[o + \tfrac{\rho x_3x_2}{\alpha+x_2} - h_1x_3x_2 \\ &-i_1x_3 - v_1x_3 - v_1e^{-x4}x_3] - \dot{x}_{3ref}f_3 + f_4[u(t) - i_2x_4] - f_4\dot{x}_{4ref} + f_5e_1 \\ &+f_6e_2 + f_7e_3 + f_8e_4\end{aligned} \tag{3.2.26}$$

Simplifying eq3.2.26 further will give:

$$\begin{aligned}
\text{-}ksign(S) =& f_1 j_2 x_1 - f_1 g_2 x_1^2 - f_1 h_4 x_2 x_1 - f_1 v_3 x_1 + f_1 v_3 e^{-x4} x_1 - f_1 \dot{x}_{1ref} + f_2 j_1 x_2 \\
& - f_2 j_1 g_1 x_2^2 - f_2 h_2 x_3 x_2 - f_2 h_3 x_2 x_1 - f_2 v_2 x_2 - f_2 v_2 e^{-x4} x_2 - f_2 \dot{x}_{2ref} + f_3 o \\
& + f_3 \tfrac{\rho x_3 x_2}{\alpha + x_2} - f_3 h_1 x_3 x_2 - f_3 i_1 x_3 - f_3 v_1 x_3 - f_3 v_1 e^{-x4} x_3 - \dot{x}_{3ref} f_3 + f_4 u(t) \\
& - f_4 i_2 x_4 - f_4 \dot{x}_{4ref} + f_5 e_1 + f_6 e_2 + f_7 e_3 + f_8 e_4
\end{aligned} \tag{3.2.27}$$

Simplifying above eq3.2.27

$$\begin{aligned}
\text{-}f_4 u(t) =& f_1 j_2 x_1 - f_1 g_2 x_1^2 - f_1 h_4 x_2 x_1 - f_1 v_3 x_1 + f_1 v_3 e^{-x4} x_1 - f_1 \dot{x}_{1ref} + f_2 j_1 x_2 \\
& - f_2 j_1 g_1 x_2^2 - f_2 h_2 x_3 x_2 - f_2 h_3 x_2 x_1 - f_2 v_2 x_2 - f_2 v_2 e^{-x4} x_2 - f_2 \dot{x}_{2ref} + f_3 o \\
& + f_3 \tfrac{\rho x_3 x_2}{\alpha + x_2} - f_3 h_1 x_3 x_2 - f_3 i_1 x_3 - f_3 v_1 x_3 - f_3 v_1 e^{-x4} x_3 - \dot{x}_{3ref} f_3 + ksign(S) \\
& - f_4 i_2 x_4 - f_4 \dot{x}_{4ref} + f_5 e_1 + f_6 e_2 + f_7 e_3 + f_8 e_4
\end{aligned} \tag{3.2.28}$$

eq 3.2.28 will give our control our control input u

$$\begin{aligned}
u(t) =& -\tfrac{1}{f_4}[f_1 j_2 x_1 - f_1 g_2 x_1^2 - f_1 h_4 x_2 x_1 - f_1 v_3 x_1 + f_1 v_3 e^{-x4} x_1 - f_1 \dot{x}_{1ref} + f_2 j_1 x_2 \\
& - f_2 j_1 g_1 x_2^2 - f_2 h_2 x_3 x_2 - f_2 h_3 x_2 x_1 - f_2 v_2 x_2 - f_2 v_2 e^{-x4} x_2 - f_2 \dot{x}_{2ref} + f_3 o \\
& + f_3 \tfrac{\rho x_3 x_2}{\alpha + x_2} - f_3 h_1 x_3 x_2 - f_3 i_1 x_3 - f_3 v_1 x_3 - f_3 v_1 e^{-x4} x_3 - \dot{x}_{3ref} f_3 + ksign(S) \\
& - f_4 i_2 x_4 - f_4 \dot{x}_{4ref} + f_5 e_1 + f_6 e_2 + f_7 e_3 + f_8 e_4]
\end{aligned} \tag{3.2.29}$$

In order to check the stability we take following lyapunov function:

$$V = \frac{1}{2} S^2 \tag{3.2.30}$$

After taking time derivative above eq becomes 3.2.31

$$\dot{V} = S\dot{S} \tag{3.2.31}$$

Putting value of $\dot{S}$

$$\dot{V} = S(-ksign(S) \tag{3.2.32}$$

Lyapunov analysis shows that the proposed controller meets the stability conditions, which ensures the convergence of errors to zero in finite time and asymptotic stability of the system

3.2.3 Terminal Sliding Mode Control

Compared to SMC, terminal sliding mode control (TSMC) makes better use of nonlinear functions in the sliding surface to ensure error convergence to zero in a finite amount of time and higher accuracy in acquiring and maintaining the terminal sliding surface. It has a fairly straightforward implementation, can accommodate model uncertainty, is robust to internal and external perturbations/disturbances, and ensures parametric invariance. The incorporation of a specific nonlinear factor in the system dynamics, which considerably enhances the convergence property, is the key component of TSMC [17]

In contrast to more conventional nonlinear controllers like backtracking, Lyapunov redesign, etc., it also offers the advantage of model order reduction. For the regulation of nonlinear systems, terminal SMC has been used in [17],[18],[19]. Considering the tracking errors of all the states defined in eq 3.2.1

Taking time derivative of error terms

$$\begin{aligned}
\dot{e}_1 &= \dot{x}_1 - \dot{x}_{1ref} \\
\dot{e}_2 &= \dot{x}_2 - \dot{x}_{2ref} \\
\dot{e}_3 &= \dot{x}_3 - \dot{x}_{3ref} \\
\dot{e}_4 &= \dot{x}_4 - \dot{x}_{4ref}
\end{aligned} \tag{3.2.33}$$

Taking the sliding surface as follows:

$$S = f_1e_1 + f_2e_2 + f_3e_3 + f_4e_4 + f_5(\int e_1dt)^{\frac{p_1}{q_1}} + f_6(\int e_2dt)^{\frac{p_2}{q_2}} + f_7(\int e_3dt)^{\frac{p_3}{q_3}} + f_8(\int e_4dt)^{\frac{p_4}{q_4}} \tag{3.2.34}$$

In above eq design cofficients are $f_1, f_2, f_3, f_4, f_5, f_6, f_7$ and f_8 and their value can be any positive real number.

positive odd numbers are $p_1, p_2, p_3, p_4, q_1, q_2, q_3$ and q_4 such that:

$1 < \frac{p_i}{q_i} < 2$ and i=1,2,3,4...

Taking above eq3.2.34 time derivative

$$\begin{aligned} \dot{S} =& f_1\dot{e}_1 + f_2\dot{e}_2 + f_3\dot{e}_3 + f_4\dot{e_4} + f_5(\textstyle\int e_1dt)^{\frac{p_1}{q_1}-1}(\frac{p_1}{q_1})e_1 + f_6(\int e_2dt)^{\frac{p_2}{q_2}-1}(\frac{p_2}{q_2})e_2 \\ &+ f_7(\textstyle\int e_3dt)^{\frac{p_3}{q_3}-1}(\frac{p_3}{q_3})e_3 + f_8(\int e_4dt)^{\frac{p_4}{q_4}-1}(\frac{p_4}{q_4})e_4 \end{aligned} \tag{3.2.35}$$

letting the above terms as:

$$\begin{aligned} A &= (\int e_1dt)^{\frac{p_1}{q_1}-1}(\frac{p_1}{q_1})e_1 \\ B &= (\int e_2dt)^{\frac{p_2}{q_2}-1}(\frac{p_2}{q_2})e_2 \\ C &= (\int e_3dt)^{\frac{p_3}{q_3}-1}(\frac{p_3}{q_3})e_3 \\ D &= (\int e_4dt)^{\frac{p_4}{q_4}-1}(\frac{p_4}{q_4})e_4 \end{aligned} \tag{3.2.36}$$

putting values of $\dot{e}_1$,$\dot{e}_2$,$\dot{e}_3$,$\dot{e}_4$ in eq 3.2.35

$$\begin{aligned} \dot{S} =& f_1(\dot{x}_1 - \dot{x}_{1ref}) + f_2(\dot{x}_2 - \dot{x}_{2ref}) + f_3(\dot{x}_3 - \dot{x}_{3ref}) + f_4(\dot{x}_4 - \dot{x}_{4ref}) + f_5A \\ &+ f_6B + f_7C + f_8D \end{aligned} \tag{3.2.37}$$

Simplifying eq 3.2.37

$$\begin{aligned} \dot{S} =& f_1\dot{x}_{1ref} - f_1\dot{x}_{1ref} + f_2\dot{x}_2 - f_2\dot{x}_{2ref} + f_3\dot{x}_3 - f_3\dot{x}_{3ref} + f_4\dot{x}_4 - f_4\dot{x}_{4ref} + f_5A \\ &+ f_6B + f_7C + f_8D \end{aligned} \tag{3.2.38}$$

putting $\dot{x}_1, \dot{x}_2, \dot{x}_3, \dot{x}_4$ in eq 3.2.38

$$
\begin{aligned}
\dot{S} \quad &= f_1(j_2x_1(1-g_2x_1) - h_4x_2x_1 - v_3(1-e^{-x4})x_1) - f_1\dot{x}_{1ref} + f_2(j_1x_2(1-g_1x_2) \\
&-h_2x_3x_2 - h_3x_2x_1 - v_2(1-e^{-x4}x_2) - f_2\dot{x}_{2ref} + f_3(o + \tfrac{\rho x_3x_2}{\alpha+x_2} - h_1x_3x_2 - i_1x_3 \\
&-v_1(1-e^{-x4}x_3) - f_3\dot{x}_{3ref} + f_4(u(t) - d_2x_4) - f_4\dot{x}_{ref} + f_5A + f_6B + f_7C + f_8D
\end{aligned} \tag{3.2.39}
$$

Simlplifying eq 3.2.39

$$\begin{aligned}\dot{S} \quad =& f_1(j_2x_1 - j_2g_2x_1^2 - f_4x_2x_1 - v_3x_1 + v_3e^{-x_4}x_1) - f_1\dot{x}_{1ref} + f_2(j_1x_2 - j_1x_2^2g_1 - h_2x_3x_2 \\ & -h_3x_2x_1 - v_2x_2 + e^{-x_4}x_2) - g_2\dot{x}_{2ref} + f_3(o + \tfrac{\rho x_3x_2}{\alpha+x_2} - h_1x_3x_2 - i_1x_3 - v_1(1-e^{-x4})x_3) \\ & -f_3\dot{x}_{3ref} + f_4(u(t) - i_2x_4) - f_4\dot{x}_{4ref} + f_5A + f_6B + f_7C + f_8D\end{aligned} \tag{3.2.40}$$

$$\begin{aligned}\dot{S} \quad =& f_1j_2x_1 - f_1j_2g_2x_1^2 - f_1h_4x_2x_1 - f_1v_3x_1 + f_1v_3e^{-x_4}x_1 - f_1\dot{x}_{1ref} + f_2j_1x_2 \\ & -f_2j_1x_2^2g_1 - f_2h_2x_3x_2 - f_2h_3x_2x_1 - f_2v_2x_2 + f_2e^{-x_4}x_2 - f_2\dot{x}_{2ref} + f_3o \\ & +f_3\tfrac{\rho x_3x_2}{\alpha+x_2} - f_3h_1x_3x_2 - f_3i_1x_3 - f_3v_1x_3 - f_3e^{-x4}x_3v_1 - f_3\dot{x}_{3ref} + f_4u(t) \\ & -f_4i_2x_4 - f_4\dot{x}_{4ref} + f_5A + f_6B + f_7C + f_8D\end{aligned} \tag{3.2.41}$$

We consider $\dot{S}$=-ksign(S) the SMC's reaching law. When the state is far from the switching manifold, a law known as the power rate reaching law speeds up reaching. k, which can be any constant positive value, is a design parameter. Following is a definition of the Signum function:

$$\begin{aligned} sign(S) \ &= -1 \quad if \quad S < 0 \\ &= 0 \quad if \quad S = 0 \\ &= 1 \quad if \quad S > 0 \end{aligned} \tag{3.2.42}$$

putting $\dot{S}$=$-ksign(S)$in eq 3.2.41

$$\begin{aligned}\text{-}ksign(S) \quad =& f_1j_2x_1 - f_1j_2g_2x_1^2 - f_1h_4x_2x_1 - f_1v_3x_1 + f_1v_3e^{-x_4}x_1 - f_1\dot{x}_{1ref} + f_2j_1x_2 - f_2j_1x_2^2g_1 \\ & -h_2x_3x_2 - f_2h_3x_2x_1 - f_2v_2x_2 + f_2e^{-x_4}x_2 - f_2\dot{x}_{2ref} + f_3o + f_3\tfrac{\rho x_3x_2}{\alpha+x_2} - f_3h_1x_3x_2 \\ & -f_3i_1x_3 - f_3v_1x_3 - f_3e^{-x4}x_3v_1 - f_3\dot{x}_{3ref} + f_4u(t) - f_4i_2x_4 - f_4\dot{x}_{4ref} \\ & +f_5A + f_6B + f_7C + f_8D\end{aligned} \tag{3.2.43}$$

Rearranging eq 3.2.43:

$$\begin{aligned}
\text{-}f_4u(t) \quad =& f_1j_2x_1 - f_1j_2g_2x_1^2 - f_1h_4x_2x_1 - f_1v_3x_1 + f_1v_3e^{-x_4}x_1 - f_1\dot{x}_{1ref} + f_2j_1x_2 \\
& -f_2j_1x_2^2g_1 - h_2x_3x_2 - f_2h_3x_2x_1 - f_2v_2x_2 + f_2e^{-x_4}x_2 - f_2\dot{x}_{2ref} + f_3o \\
& +f_3\tfrac{\rho x_3x_2}{\alpha+x_2} - f_3h_1x_3x_2 - f_3i_1x_3 - f_3v_1x_3 - f_3e^{-x4}x_3v_1 - f_3\dot{x}_{3ref} - ksign(S) \\
& -f_4i_2x_4 - f_4\dot{x}_{4ref} + f_5A + f_6B + f_7C + f_8D
\end{aligned} \tag{3.2.44}$$

From eq 3.2.44 we will get control input u:

$$\begin{aligned}
u(t) \quad =& \tfrac{-1}{f_4}[f_1j_2x_1 - f_1j_2g_2x_1^2 - f_1h_4x_2x_1 - f_1v_3x_1 + f_1v_3e^{-x_4}x_1 - f_1\dot{x}_{1ref} + f_2j_1x_2 \\
& -f_2j_1x_2^2g_1 - h_2x_3x_2 - f_2h_3x_2x_1 - f_2v_2x_2 + f_2e^{-x_4}x_2 - f_2\dot{x}_{2ref} + f_3o \\
& +f_3\tfrac{\rho x_3x_2}{\alpha+x_2} - f_3h_1x_3x_2 - f_3i_1x_3 - f_3v_1x_3 - f_3e^{-x4}x_3v_1 - f_3\dot{x}_{3ref} - ksign(S) \\
& -f_4i_2x_4 - f_4\dot{x}_{4ref} + f_5(\textstyle\int e_1dt)^{\frac{p_1}{q_1}-1}(\tfrac{p_1}{q_1})e_1 + f_6(\textstyle\int e_2dt)^{\frac{p_2}{q_2}-1}(\tfrac{p_2}{q_2})e_2 \\
& +f_7(\textstyle\int e_3dt)^{\frac{p_3}{q_3}-1}(\tfrac{p_3}{q_3})e_3 + f_8(\textstyle\int e_4dt)^{\frac{p_4}{q_4}-1}(\tfrac{p_4}{q_4})e_4
\end{aligned} \tag{3.2.45}$$

Consider the following proposed Lyapunov function for stability analysis

$$V = \frac{1}{2}S^2 \tag{3.2.46}$$

Taking the eq3.2.46 time derivative

$$\dot{V} = S\dot{S} \tag{3.2.47}$$

putting $\dot{S}$ in eq 3.2.47

$$\dot{V} = -S(ksign(S) \tag{3.2.48}$$

Lyapunov analysis shows that the proposed controller meets the stability conditions, which ensures the convergence of errors to zero in finite time and asymptotic stability of the system.

3.2.4 SuperTwisting Sliding Mode Control

First, we must choose a sliding surface for the Super Twisting mode-based Sliding Mode Control. Although there are other methods for choosing sliding surfaces, tracking faults is the simplest. Error is defined as the difference between actual and desired values. Tracking of errors terms are defined as followed

$$\begin{aligned} e_1 &= x_1 - x_{1ref} \\ e_2 &= x_2 - x_{2ref} \\ e_3 &= x_3 - x_{3ref} \\ e_4 &= x_4 - x_{4ref} \end{aligned} \tag{3.2.49}$$

In above eq 3.2.49 $x_1, x_2, x_3 and x_4$ are the state varibles and $x_{1ref}, x_{2ref}, x_{3ref}, x_{4ref}$ are the desired values.
Taking time derivative of error terms in eq 3.2.49:

$$\begin{aligned} \dot{e}_1 &= \dot{x}_1 - \dot{x}_{1ref} \\ \dot{e}_2 &= \dot{x}_2 - \dot{x}_{2ref} \\ \dot{e}_3 &= \dot{x}_3 - \dot{x}_{3ref} \\ \dot{e}_4 &= \dot{x}_4 - \dot{x}_{4ref} \end{aligned} \tag{3.2.50}$$

Sliding Surface is defined as:

$$S = f_1 e_1 + f_2 e_2 + f_3 e_3 + f_4 e_4 \tag{3.2.51}$$

here f_1, f_2, f_3, f_4 are constant design parameters of the sliding surfaces which can have any positive constant value.
Taking time derivative of eq 3.2.51

$$\dot{S} = f_1 \dot{e}_1 + f_2 \dot{e}_2 + f_3 \dot{e}_3 + f_4 \dot{e}_4 \tag{3.2.52}$$

putting values of $\dot{e}_1, \dot{e}_2, \dot{e}_3, \dot{e}_4$ in eq 3.2.52

$$\dot{S} = f_1(\dot{x}_1 - \dot{x}_{1ref}) + f_2(\dot{x}_2 - \dot{x}_{2ref}) + f_3(\dot{x}_3 - \dot{x}_{3ref}) + f_4(\dot{x}_4 - \dot{x}_{4ref}) \tag{3.2.53}$$

Simplifying eq 3.2.53

$$\dot{S} = f_1\dot{x}_1 - f_1\dot{x}_{1ref} + f_2\dot{x}_2 - f_2\dot{x}_{2ref} + f_3\dot{x}_3 - f_3\dot{x}_{3ref} + f_4\dot{x}_4 - f_4\dot{x}_{4ref} \tag{3.2.54}$$

putting values of $\dot{x}_1, \dot{x}_2, \dot{x}_3, \dot{x}_4$ in eq 3.2.54

$$\begin{aligned}\dot{S} &= f_1[j_2x_1(1-g_2x_1) - h_4x_2x_1 - v_3(1-e^{-x4})x_1] - f_1\dot{x}_{1ref} + f_2[j_1x_2(1-g_1x_2) \\ &\quad -h_2x_3x_2 - h_3x_2x_1 - v_2(1-e^{-x4})x_2] - f_2\dot{x}_{2ref} + f_3[o + \tfrac{\rho x_3x_2}{\alpha+x_2} - h_1x_3x_2 \\ &\quad -i_1x_3 - v_1(1-e^{-x4})x_3] + f_4[u(t) - i_2x_4] - f_4\dot{x}_{4ref}\end{aligned} \tag{3.2.55}$$

Simplifying eq 3.2.55

$$\begin{aligned}\dot{S} =& f_1[j_2x_1(1-g_2x_1) - h_4x_2x_1 - v_3(1-e^{-x4})x_1] - f_1\dot{x}_{1ref} + f_2[j_1x_2(1-g_1x_2) \\ & -h_2x_3x_2 - h_3x_2x_1 - v_2(1-e^{-x4})x_2] - f_2\dot{x}_{2ref} + f_3[o + \tfrac{\rho x_3x_2}{\alpha+x_2} - h_1x_3x_2 \\ & -i_1x_3 - v_1(1-e^{-x4})x_3] - \dot{x}_{3ref}f_3 + f_4[u(t) - i_2x_4] - f_4\dot{x}_{4ref}\end{aligned} \tag{3.2.56}$$

$$\begin{aligned}\dot{S} =& f_1[j_2x_1 - g_2x_1^2j_2 - h_4x_2x_1 - v_3x_1 + v_3e^{-x4}x_1] - f_1\dot{x}_{1ref} + f_2[j_1x_2 - j_1g_1x_2^2 \\ & -h_2x_3x_2 - h_3x_2x_1 - v_2x_2 + v_2e^{-x4}x_2] - f_2\dot{x}_{2ref} + f_3[o + \tfrac{\rho x_3x_2}{\alpha+x_2} - h_1x_3x_2 \\ & -i_1x_3 - v_1x_3 + v_1e^{-x4}x_3] - \dot{x}_{3ref}f_3 + f_4[u(t) - i_2x_4] - f_4\dot{x}_{4ref}\end{aligned} \tag{3.2.57}$$

$$\begin{aligned}\dot{S} =& f_1j_2x_1 - f_1g_2x_1^2j_2 - f_1h_4x_2x_1 - f_1v_3x_1 + f_1v_3e^{-x4}x_1 - f_1\dot{x}_{1ref} + f_2j_1x_2 - f_2j_1g_1x_2^2 \\ & -f_2h_2x_3x_2 - f_2h_3x_2x_1 - f_2v_2x_2 + f_2v_2e^{-x4}x_2 - f_2\dot{x}_{2ref} + f_3o + f_3\tfrac{\rho x_3x_2}{\alpha+x_2} - f_3h_1x_3x_2 \\ & -f_3i_1x_3 - f_3v_1x_3 + f_3v_1e^{-x4}x_3 - \dot{x}_{3ref}f_3 + f_4u(t) - f_4i_2x_4 - f_4\dot{x}_{4ref}\end{aligned} \tag{3.2.58}$$

putting $\dot{S} = 0$ in eq 3.2.58

$$\begin{aligned} 0 \quad &= f_1 j_2 x_1 - f_1 g_2 x_1^2 j_2 - f_1 h_4 x_2 x_1 - f_1 v_3 x_1 + f_1 v_3 e^{-x4} x_1 - f_1 \dot{x}_{1ref} + f_2 j_1 x_2 - f_2 j_1 g_1 x_2^2 \\ &- f_2 h_2 x_3 x_2 - f_2 h_3 x_2 x_1 - f_2 v_2 x_2 + f_2 v_2 e^{-x4} x_2 - f_2 \dot{x}_{2ref} + f_3 o + f_3 \tfrac{\rho x_3 x_2}{\alpha + x_2} - f_3 h_1 x_3 x_2 - f_3 i_1 x_3 \\ &- f_3 v_1 x_3 + f_3 v_1 e^{-x4} x_3 - \dot{x}_{3ref} f_3 + f_4 u(t) - f_4 i_2 x_4 - f_4 \dot{x}_{4ref} \end{aligned} \tag{3.2.59}$$

Rearranging eq 3.2.59

$$\begin{aligned} -f_4 u(t) \quad &= f_1 j_2 x_1 - f_1 g_2 x_1^2 j_2 - f_1 h_4 x_2 x_1 - f_1 v_3 x_1 + f_1 v_3 e^{-x4} x_1 - f_1 \dot{x}_{1ref} + f_2 j_1 x_2 - f_2 j_1 g_1 x_2^2 \\ &- f_2 h_2 x_3 x_2 - f_2 h_3 x_2 x_1 - f_2 v_2 x_2 + f_2 v_2 e^{-x4} x_2 - f_2 \dot{x}_{2ref} + f_3 o + f_3 \tfrac{\rho x_3 x_2}{\alpha + x_2} - f_3 h_1 x_3 x_2 \\ &- f_3 i_1 x_3 - f_3 v_1 x_3 + f_3 v_1 e^{-x4} x_3 - \dot{x}_{3ref} f_3 - f_4 i_2 x_4 - f_4 \dot{x}_{4ref} \end{aligned} \tag{3.2.60}$$

eq 3.2.60 will give control input u

$$
\begin{aligned}
u(t) \quad &= -\tfrac{1}{f_4}[f_1 j_2 x_1 - f_1 g_2 x_1^2 j_2 - f_1 h_4 x_2 x_1 - f_1 v_3 x_1 + f_1 v_3 e^{-x4} x_1 - f_1 \dot{x}_{1ref} + f_2 j_1 x_2 - f_2 j_1 g_1 x_2^2 \\
&\quad - f_2 h_2 x_3 x_2 - f_2 h_3 x_2 x_1 - f_2 v_2 x_2 + f_2 v_2 e^{-x4} x_2 - f_2 \dot{x}_{2ref} + f_3 o + f_3 \tfrac{\rho x_3 x_2}{\alpha + x_2} - f_3 h_1 x_3 x_2 \\
&\quad - f_3 i_1 x_3 - f_3 v_1 x_3 + f_3 v_1 e^{-x4} x_3 - \dot{x}_{3ref} f_3 - f_4 i_2 x_4 - f_4 \dot{x}_{4ref}]
\end{aligned}
\tag{3.2.61}
$$

Next the switching control for supertwisting sliding mode

u_{sw} is defined as

$$u_{sw} = -K_1 \left|S\right|^{\alpha} sign(S) + u_1 \tag{3.2.62}$$

$$\dot{u}_1 = -K_2 sign(S) \tag{3.2.63}$$

Taking integral of eq 3.2.62

$$u_1 = -k_2 \int sign(S) dt \tag{3.2.64}$$

putting u_1 from eq 3.2.64 in eq 3.2.62 k_1 and k_2 are given in [17]:

$$u_{sw} = -k_1 \left|S\right|^{\alpha} sign(S) - k_2 \int sign(S) dt \tag{3.2.65}$$

The overall control law for ST-SMC can be written as:

$$u_{ST-SMC} = u_{eq} + u_{sw} \tag{3.2.66}$$

putting values u_{eq} from eq 3.2.61 and $u_{sw}$from eq3.2.65 in eq 3.2.66:

In order to check the stability of the controller following conditions should be fulfilled:

$$\begin{aligned} u_{ST-SMC} &= \tfrac{-1}{f_4}[f_1 j_2 x_1 - f_1 g_2 x_1^2 j_2 - f_1 h_4 x_2 x_1 - f_1 v_3 x_1 + f_1 v_3 e^{-x4} x_1 - f_1 \dot{x}_{1ref} + f_2 j_1 x_2 \\ & -f_2 j_1 g_1 x_2^2 - f_2 h_2 x_3 x_2 - f_2 h_3 x_2 x_1 - f_2 v_2 x_2 + f_2 v_2 e^{-x4} x_2 - f_2 \dot{x}_{2ref} + f_3 o \\ & +f_3 \tfrac{\rho x_3 x_2}{\alpha + x_2} - f_3 h_1 x_3 x_2 - f_3 i_1 x_3 - f_3 v_1 x_3 + f_3 v_1 e^{-x4} x_3 - \dot{x}_{3ref} f_3 - f_4 i_2 x_4 \\ & -f_4 \dot{x}_{4ref}] - K_1 \left|S\right|^{\alpha} sign(S) - k_2 \int sign(S) dt \end{aligned} \tag{3.2.67}$$

1. V should be positive definite
2. V should be radially unbounded
3. $\dot{V}$ should be negative definite

Defining Lyapunov candidate function as:

$$V = \tfrac{1}{2} S^2 \tag{3.2.68}$$

Taking time derivative of eq3.2.68

$$\dot{V} = S\dot{S} \tag{3.2.69}$$

$$\dot{V} = S[f_1 \dot{e}_1 + f_2 \dot{e}_2 + f_3 \dot{e}_3 + f_4 \dot{e}_4] \tag{3.2.70}$$

Putting value of $\dot{e}_4$ from eq 3.2.50 in to eq 3.2.70:

$$\dot{V} = S[f_1 \dot{e}_1 + f_2 \dot{e}_2 + f_3 \dot{e}_3 + f_4 [\dot{x}_4 - \dot{x}_{4ref}]] \tag{3.2.71}$$

putting value of $\dot{x}_4$ from eq 3.1.2 in to eq 3.2.71:

$$\dot{V} = S[f_1\dot{e}_1 + f_2\dot{e}_2 + f_3\dot{e}_3 + f_4[u(t) - i_2x_4 - \dot{x}_{4ref}]] \tag{3.2.72}$$

Simpling above eq 3.2.72

$$\dot{V} = S[f_1\dot{e}_1 + f_2\dot{e}_2 + f_3\dot{e}_3 + f_4u(t) - f_4i_2x_4 - f_4\dot{x}_{4ref}] \tag{3.2.73}$$

Putting value of u(t) from eq 3.2.67in to eq 3.2.73:

$$\begin{aligned}\dot{V} \quad =&S[f_1\dot{e}_1 + f_2\dot{e}_2 + f_3\dot{e}_3 + f_4\tfrac{-1}{f_4}[f_1 j_2 x_1 - f_1 g_2 x_1^2 j_2 - f_1 h_4 x_2 x_1 - f_1 v_3 x_1 + f_1 v_3 e^{-x4} x_1 - f_1 \dot{x}_{1ref} \\ &+ f_2 j_1 x_2 - f_2 j_1 g_1 x_2^2 - f_2 h_2 x_3 x_2 - f_2 h_3 x_2 x_1 - f_2 v_2 x_2 + f_2 v_2 e^{-x4} x_2 - f_2 \dot{x}_{2ref} + f_3 o \\ &+ f_3 \tfrac{\rho x_3 x_2}{\alpha + x_2} - f_3 h_1 x_3 x_2 - f_3 i_1 x_3 - f_3 v_1 x_3 + f_3 v_1 e^{-x4} x_3 - \dot{x}_{3ref} f_3 - f_4 i_2 x_4 \\ &- f_4 \dot{x}_{4ref} + k_1 |S|^{\alpha} sign(S) + k_2 \textstyle\int sign(S)dt] \\ &- f_4 i_2 x_4 - f_4 \dot{x}_{4ref}]\end{aligned} \tag{3.2.74}$$

Simplifying eq 3.3.85:

$$\dot{V} \leq \quad \mathrm{S}[-k_1 |S|^{\alpha} sign(S) - k_2 \textstyle\int sign(S)dt)] \tag{3.2.75}$$

The developed controller is stable and $\dot{V}$ is a negative definite variable according to equation 3.2.75. The stability analysis suggested in [20] demonstrates that the proposed controller meets the stability condition $\dot{V} \leq 0$, this also explains how in a fixed amount of time, all errors eventually converge to zero.

3.3 Adaptive Controller Design

In order to get more accurate results controllers are adapted using adaptive law.

3.3.1 Adaptive Sliding Mode Control

Error terms are defined as :

$$\begin{aligned} e_1 &= x_1 - x_{1ref} \\ e_2 &= x_2 - x_{2ref} \\ e_3 &= x_3 - x_{3ref} \\ e_4 &= x_4 - x_{4ref} \end{aligned} \tag{3.3.1}$$

In above eq 3.3.1 $x_1, x_2, x_3 and x_4$ are the state varibles and $x_{1ref}, x_{2ref}, x_{3ref}, x_{4ref}$ are the desired values.

Taking time derivative of error terms in 3.3.1

$$\begin{aligned} \dot{e}_1 &= \dot{x}_1 - \dot{x}_{1ref} \\ \dot{e}_2 &= \dot{x}_2 - \dot{x}_{2ref} \\ \dot{e}_3 &= \dot{x}_3 - \dot{x}_{3ref} \\ \dot{e}_4 &= \dot{x}_4 - \dot{x}_{4ref} \end{aligned} \tag{3.3.2}$$

$$S = c_1 e_1 + c_2 e_2 + c_3 e_3 + c_4 e_4 \tag{3.3.3}$$

here $c_1, c_2, c_3 and c_4$ are constant design parameters of the sliding surfaces which can have any positive constant value.

Taking time derivative of S

$$\dot{S} = c_1 \dot{e}_1 + c_2 \dot{e}_2 + c_3 \dot{e}_3 + c_4 \dot{e}_4 \tag{3.3.4}$$

putting values of $\dot{e}_1, \dot{e}_2, \dot{e}_3, \dot{e}_4$ in eq 3.3.3

$$\dot{S} = c_1(\dot{x}_1 - \dot{x}_{1ref}) + c_2(\dot{x}_2 - \dot{x}_{2ref}) + c_3(\dot{x}_3 - \dot{x}_{3ref}) + c_4(\dot{x}_4 - \dot{x}_{4ref}) \tag{3.3.5}$$

$$\begin{aligned} \dot{S} \quad &= c_1\dot{x}_1 - c_1\dot{x}_{1ref} + c_2\dot{x}_2 - c_2\dot{x}_{2ref} \\ &+ c_3\dot{x}_3 - c_3\dot{x}_{3ref} + c_4\dot{x}_4 - c_4\dot{x}_{4ref} \end{aligned} \tag{3.3.6}$$

putting values of $\dot{x}_1, \dot{x}_2, \dot{x}_3, \dot{x}_4$ in above eq 3.3.6.

$$\begin{aligned} \dot{S} \quad &= c_1(j_2x_1)(1 - g_2x_1) - h_4x_2x_1 - v_3(1 - e^{-x4})x_1 - c_1\dot{x}_{1ref} + c_2(j_1x_2(1 - g_1x_2) \\ &- h_2x_3x_2 - h_3x_2x_1 - v_2(1 - e^{-x4})x_2) - h_2\dot{x}_{2ref} + h_3(o + \tfrac{\rho x_3 x_2}{\alpha + x_2} \\ &- h_1x_3x_2 - i_1x_3 - v_1(1 - e^{-x4})x_3) - h_3\dot{x}_{3ref} + c_4(u(t) - i_2x_4) - h_4\dot{x}_{4ref} \end{aligned} \tag{3.3.7}$$

Simplifying eq 3.3.7

$$\begin{aligned}\dot{S} \quad &= c_1(j_2x_1 - j_2g_2x_1^2 - h_4x_2x_1 - v_3x_1 + v_3e^{-x_4}x_1) - c_1\dot{x}_{1ref} + c_2(j_1x_2 - j_1x_2^2g_1 \\ &-h_2x_3x_2 - h_3x_2x_1 - v_2x_2 + e^{-x_4}v_2x_2) - c_2\dot{x}_{2ref} + c_3(o + \tfrac{\rho x_3x_2}{\alpha+x_2} - h_1x_3x_2 - i_1x_3 \\ &-v_1(1-e^{-x4})x_3) - c_3\dot{x}_{3ref} + c_4(u(t) - i_2x_4) - c_4\dot{x}_{4ref}\end{aligned} \tag{3.3.8}$$

Taking j_2 as adaptive parameter j_2=δ_1

$$\begin{aligned}\dot{S} \quad &= c_1\delta_1x_1 - c_1\delta_1g_2x_1^2 - c_1h_4x_2x_1 - c_1v_3x_1 + c_1v_3e^{-x_4}x_1 - c_1\dot{x}_{1ref} + c_2j_1x_2 - c_2j_1x_2^2g_1 \\ &-h_2x_3x_2c_2 - c_2h_3x_2x_1 - c_2v_2x_2 + c_2e^{-x_4}x_2v_2 - c_2\dot{x}_{2ref} + c_3o + c_3\tfrac{\rho x_3x_2}{\alpha+x_2} \\ &-c_3h_1x_3x_2 - c_3i_1x_3 - c_3v_1x_3 - c_3e^{-x4}x_3v_1 - c_3\dot{x}_{3ref} + c_4u(t) - c_4i_2x_4 - c_4\dot{x}_{4ref}\end{aligned} \tag{3.3.9}$$

for the convergence of the system, defining the adaptive law which can estimate the parameters as :

$$\tilde{\delta}_i = \hat{\delta_i} - \delta_i \tag{3.3.10}$$

we can take i=1,2

from adaptive law we can conclude value of δ_1

$$\delta_1 = \hat{\delta_1} - \tilde{\delta}_1 \tag{3.3.11}$$

for stability analysis we take lypanov candid function

$$V = \frac{1}{2}S^2 + \frac{1}{2\eta}\delta_1^2 \tag{3.3.12}$$

Taking Time derivative of eq 3.3.12

$$\dot{V} = S\dot{S} + \frac{1}{\eta}\tilde{\delta}_1\dot{\hat{\delta}}_1 \tag{3.3.13}$$

Putting value of $\dot{S}$

$$\begin{aligned}\dot{V} =& c_1\delta_1 x_1 - c_1\delta_1 g_2 x_1^2 - c_1 h_4 x_2 x_1 - c_1 v_3 x_1 + c_1 v_3 e^{-x_4} x_1 - c_1\dot{x}_{1ref} + c_2 j_1 x_2 - c_2 j_1 x_2^2 g_1 \\ & -h_2 x_3 x_2 c_2 - c_2 h_3 x_2 x_1 - c_2 v_2 x_2 + c_2 e^{-x_4} x_2 v_2 - c_2\dot{x}_{2ref} + c_3 o + c_3\tfrac{\rho x_3 x_2}{\alpha + x_2} \\ & -c_3 h_1 x_3 x_2 - c_3 i_1 x_3 - c_3 v_1 x_3 - c_3 e^{-x4} x_3 v_1 - c_3\dot{x}_{3ref} + c_4 u(t) - c_4 i_2 x_4 - c_4\dot{x}_{4ref} \\ & +\tfrac{1}{\eta}\tilde{\delta}_1\dot{\hat{\delta}}_1\end{aligned} \tag{3.3.14}$$

putting value of δ_1 from eq 3.3.11

$$\begin{aligned}\dot{V} =& S[c_1[\hat{\delta_1} - \tilde{\delta}_1]x_1 - c_1[\hat{\delta_1} - \tilde{\delta}_1]g_2 x_1^2 - c_1 h_4 x_2 x_1 - c_1 v_3 x_1 + c_1 v_3 e^{-x_4} x_1 - c_1\dot{x}_{1ref} + c_2 j_1 x_2 \\ & -c_2 j_1 x_2^2 g_1 - h_2 x_3 x_2 c_2 - c_2 h_3 x_2 x_1 - c_2 v_2 x_2 + c_2 e^{-x_4} x_2 v_2 - c_2\dot{x}_{2ref} + c_3 o \\ & +c_3\tfrac{\rho x_3 x_2}{\alpha + x_2} - c_3 h_1 x_3 x_2 - c_3 i_1 x_3 - c_3 v_1 x_3 - c_3 e^{-x4} x_3 v_1 - c_3\dot{x}_{3ref} + c_4 u(t) - c_4 i_2 x_4 - c_4\dot{x}_{4ref}] \\ & +\tfrac{1}{\eta}\tilde{\delta}_1\dot{\hat{\delta}}_1\end{aligned} \tag{3.3.15}$$

Seperate terms having $\tilde{\delta}_1$:

$$\begin{aligned}\dot{V} =& S[c_1\hat{\delta_1} x_1 - c_1\hat{\delta_1} g_2 x_1^2 - c_1 h_4 x_2 x_1 - c_1 v_3 x_1 + c_1 v_3 e^{-x_4} x_1 - c_1\dot{x}_{1ref} + c_2 j_1 x_2 \\ & -c_2 j_1 x_2^2 g_1 - h_2 x_3 x_2 c_2 - c_2 h_3 x_2 x_1 - c_2 v_2 x_2 + c_2 e^{-x_4} x_2 v_2 - c_2\dot{x}_{2ref} + c_3 o \\ & +c_3\tfrac{\rho x_3 x_2}{\alpha + x_2} - c_3 h_1 x_3 x_2 - c_3 i_1 x_3 - c_3 v_1 x_3 - c_3 e^{-x4} x_3 v_1 - c_3\dot{x}_{3ref} + c_4 u(t) \\ & -c_4 i_2 x_4 - c_4\dot{x}_{4ref}] - S c_1\tilde{\delta}_1 x_1(1 - g_2 x_1) + \tfrac{1}{\eta}\tilde{\delta}\dot{\hat{\delta}}_1\end{aligned} \tag{3.3.16}$$

Taking $\frac{1}{\eta}\tilde{\delta}_1$ Common from eq 3.3.16:

$$\begin{aligned}\dot{V} =& S[c_1\hat{\delta_1} x_1 - c_1\hat{\delta_1} g_2 x_1^2 - c_1 h_4 x_2 x_1 - c_1 v_3 x_1 + c_1 v_3 e^{-x_4} x_1 - c_1\dot{x}_{1ref} + c_2 j_1 x_2 \\ & -c_2 j_1 x_2^2 g_1 - h_2 x_3 x_2 c_2 - c_2 h_3 x_2 x_1 - c_2 v_2 x_2 + c_2 e^{-x_4} x_2 v_2 - c_2\dot{x}_{2ref} + c_3 o \\ & +c_3\tfrac{\rho x_3 x_2}{\alpha + x_2} - c_3 h_1 x_3 x_2 - c_3 i_1 x_3 - c_3 v_1 x_3 - c_3 e^{-x4} x_3 v_1 - c_3\dot{x}_{3ref} + c_4 u(t) \\ & -c_4 i_2 x_4 - c_4\dot{x}_{4ref}] + \tfrac{1}{\eta}\tilde{\delta}[\dot{\hat{\delta}}_1 - S\eta c_1 x_1(1 - g_2 x_1)\end{aligned} \tag{3.3.17}$$

Simplifying eq 3.3.17

$$\begin{aligned}\dot{V} \quad =&S[c_1\hat{\delta_1}x_1 - c_1\hat{\delta_1}g_2x_1^2 - c_1h_4x_2x_1 - c_1v_3x_1 + c_1v_3e^{-x_4}x_1 - c_1\dot{x}_{1ref} + c_2j_1x_2 \\ &-c_2j_1x_2^2g_1 - h_2x_3x_2c_2 - c_2h_3x_2x_1 - c_2v_2x_2 + c_2e^{-x_4}x_2v_2 - c_2\dot{x}_{2ref} + c_3o \\ &+c_3\tfrac{\rho x_3x_2}{\alpha+x_2} - c_3h_1x_3x_2 - c_3i_1x_3 - c_3v_1x_3 - c_3e^{-x4}x_3v_1 - c_3\dot{x}_{3ref} + c_4u(t) \\ &-c_4i_2x_4 - c_4\dot{x}_{4ref}] + \tfrac{1}{\eta}\tilde{\delta}[\dot{\hat{\delta}}_1 - S\eta c_1x_1 + S\eta c_1g_2x_1^2]\end{aligned} \tag{3.3.18}$$

Considering the boundedness of parameter estimation, so in$\dot{V}$ the adaptive update laws are designed as:

$$\dot{\hat{\delta}}_1 \quad =\eta Proj(\hat{\delta_1}, +Sc_1x_1 - Sc_1g_2x_1^2) \tag{3.3.19}$$

To get the varying parameters bounded, adaptation parameters are redefined as follows:

$$\dot{\hat{\delta}}_1 {=} {+}S\eta f_1x_1 - S\eta f_1g_2x_1^2 \tag{3.3.20}$$

Taking eq 3.3.19 in to eq 3.3.16 then eq 3.3.16 can be simplified as :

$$\begin{aligned}\dot{V} \leq \quad & S[c_1\hat{\delta_1}x_1 - c_1\hat{\delta_1}g_2x_1^2 - c_1h_{44}x_2x_1 - c_1v_3x_1 + c_1v_3e^{-x_4}x_1 - c_1\dot{x}_{1ref} + c_2j_1x_2 \\ &-c_2j_1x_2^2g_1 - h_2x_3x_2c_2 - c_2h_3x_2x_1 - c_2v_2x_2 + c_2e^{-x_4}x_2v_2 - c_2\dot{x}_{2ref} + c_3o \\ &+c_3\tfrac{\rho x_3x_2}{\alpha+x_2} - c_3h_1x_3x_2 - c_3i_1x_3 - c_3v_1x_3 - c_3e^{-x4}x_3v_1 - c_3\dot{x}_{3ref} + c_4u(t) \\ &-c_4i_2x_4 - c_4\dot{x}_{4ref}] + \tfrac{1}{\eta}\tilde{\delta}[\dot{\hat{\delta}}_1 - Sf_1x_1 + Sf_1x_1^2g_2]\end{aligned} \tag{3.3.21}$$

Defining Signum Function as

$$\begin{aligned} sign(S) \; &= -1 \quad if \quad S < 0 \\ &= 0 \quad\; if \quad S = 0 \\ &= 1 \quad\; if \quad S > 0 \end{aligned} \tag{3.3.22}$$

putting $\dot{S}{=}{-}ksign(S)$in eq 3.3.9

$$\begin{aligned} -ksign(S) \quad &= c_1\hat{\delta_1}x_1 - c_1\hat{\delta_1}g_2x_1^2 - c_1h_4x_2x_1 - c_1v_3x_1 + c_1v_3e^{-x_4}x_1 - c_1\dot{x}_{1ref} + c_2j_1x_2 - c_2j_1x_2^2g_1 \\ &\quad -h_2x_3x_2c_2 - c_2h_3x_2x_1 - c_2v_2x_2 + c_2e^{-x_4}x_2v_2 - c_2\dot{x}_{2ref} + c_3o + c_3\tfrac{\rho x_3x_2}{\alpha+x_2} \\ &\quad -c_3h_1x_3x_2 - c_3i_1x_3 - c_3v_1x_3 - c_3e^{-x4}x_3v_1 - c_3\dot{x}_{3ref} + c_4u(t) - c_4i_2x_4 - c_4\dot{x}_{4ref} \end{aligned} \tag{3.3.23}$$

$$\begin{aligned} -c_4u(t) \quad &= c_1\hat{\delta_1}x_1 - c_1\hat{\delta_1}g_2x_1^2 - c_1h_4x_2x_1 - c_1v_3x_1 + c_1v_3e^{-x_4}x_1 - c_1\dot{x}_{1ref} + c_2j_1x_2 - c_2j_1x_2^2g_1 \\ &\quad -h_2x_3x_2c_2 - c_2h_3x_2x_1 - c_2v_2x_2 + c_2e^{-x_4}x_2v_2 - c_2\dot{x}_{2ref} + c_3o + c_3\tfrac{\rho x_3x_2}{\alpha+x_2} \\ &\quad -c_3h_1x_3x_2 - c_3i_1x_3 - c_3v_1x_3 - c_3e^{-x4}x_3v_1 - c_3\dot{x}_{3ref} + ksign(S) - c_4i_2x_4 - c_4\dot{x}_{4ref} \end{aligned} \tag{3.3.24}$$

eq 3.3.24 will give control input u

$$\begin{aligned} u(t) \quad &= \tfrac{-1}{c_4}[c_1\hat{\delta_1}x_1 - c_1\hat{\delta_1}g_2x_1^2 - c_1h_4x_2x_1 - c_1v_3x_1 + c_1v_3e^{-x_4}x_1 - c_1\dot{x}_{1ref} + c_2j_1x_2 - c_2j_1x_2^2g_1 \\ &\quad -h_2x_3x_2c_2 - c_2h_3x_2x_1 - c_2v_2x_2 + c_2e^{-x_4}x_2v_2 - c_2\dot{x}_{2ref} + c_3o + c_3\tfrac{\rho x_3x_2}{\alpha+x_2} \\ &\quad -c_3h_1x_3x_2 - c_3i_1x_3 - c_3v_1x_3 - c_3e^{-x4}x_3v_1 - c_3\dot{x}_{3ref} + ksign(S) - c_4i_2x_4 - c_4\dot{x}_{4ref}] \end{aligned} \tag{3.3.25}$$

putting eq 3.3.23 in eq 3.3.21 and consedring properties of sgn(·) from eq 3.3.22, eq 3.3.21 can be simplified as :

$$\dot{V} \leq -k\,|S| \leq 0 \tag{3.3.26}$$

Thus, it is demonstrated that the system as a whole is asymptotically stable and the developed controller satisfies the Lyapunov stability criteria.

3.3.2 Adaptive Integral Sliding Mode Control

Taking Integral of Error terms defined in eq 3.3.1:

$$\begin{aligned} e_5 &= \int x_1 - x_{1ref} dt \\ e_6 &= \int x_2 - x_{2ref} dt \\ e_7 &= \int x_3 - x_{3ref} dt \\ e_8 &= \int x_4 - x_{4ref} dt \end{aligned} \tag{3.3.27}$$

In above eq 3.3.27 $x_1, x_2, x_3 and x_4$ are the state varibles and $x_{1ref}, x_{2ref}, x_{3ref}, x_{4ref}$ are the desired values.

Taking time derivative of error terms in 3.3.27 :

$$\begin{aligned} \dot{e}_5 &= x_1 - x_{1ref} = e_1 \\ \dot{e}_6 &= x_2 - x_{2ref} = e_2 \\ \dot{e}_7 &= x_3 - x_{3ref} = e_3 \\ \dot{e}_8 &= x_4 - x_{4ref} = e_4 \end{aligned} \tag{3.3.28}$$

defining the sliding surface as :

$$S = f_1 e_1 + f_2 e_2 + f_3 e_3 + f_4 e_4 + f_5 e_5 + f_6 e_6 + f_7 e_7 + f_8 e_8 \tag{3.3.29}$$

here $f_1, f_2, f_3, f_4 f_5, f_6, f_7, f_8$ are constant design parameters of the sliding surfaces which can have any positive constant value. Taking the time derivative of S:

$$\dot{S} = f_1 \dot{e}_1 + f_2 \dot{e}_2 + f_3 \dot{e}_3 + f_4 \dot{e}_4 + f_5 \dot{e}_5 + f_5 \dot{e}_6 + f_7 \dot{e}_7 + f_8 \dot{e}_8 \tag{3.3.30}$$

putting values of $\dot{e}_1, \dot{e}_2, \dot{e}_3, \dot{e}_4$ from eq3.3.30 and simplifying eq 3.3.30 will give:

$$\begin{aligned}\dot{S} &= f_1\dot{x}_1 - f_1\dot{x}_{1ref} + f_2\dot{x}_2 - f_2\dot{x}_{2ref} + f_3\dot{x}_3 - f_3\dot{x}_{3ref} \\ &+f_4\dot{x}_4 - f_4\dot{x}_{4ref} + f_5e_1 + f_6e_2 + f_7e_3 + f_8e_4\end{aligned} \tag{3.3.31}$$

putting values of $\dot{x}_1, \dot{x}_2, \dot{x}_3, \dot{x}_4$in above eq 3.3.31

$$\begin{aligned}\dot{S} &=f_1j_2x_1 - f_1g_2x_1^2 - h_4x_2x_1 - v_3(1-e^{-x4})x_1 - f_1\dot{x}_{1ref} + f_2[j_1x_2(1-g_1x_2) - h_2x_3x_2 \\ &-h_3x_2x_1 - v_2(1-e^{-x4})x_2 - f_2\dot{x}_{2ref} + f_3[o + \tfrac{\rho x_3x_2}{\alpha+x_2} - h_1x_3x_2 - i_1x_3 - v_1(1-e^{-x4})x_3] \\ &-f_3\dot{x}_{3ref} + f_4[u(t) - i_2x_4] - f_4\dot{x}_{4ref} + f_5e_1 + f_6e_2 + f_7e_3 + f_8e_4\end{aligned} \tag{3.3.32}$$

Simplifying eq 3.3.32

$$\begin{aligned}\dot{S} &=f_1[j_2x_1(1-g_2x_1) - h_4x_2x_1 - v_3(1-e^{-x4})x_1 - f_1\dot{x}_{1ref} + f_2[j_1x_2(1-g_1x_2) - h_2x_3x_2 \\ &-h_3x_2x_1 - v_2(1-e^{-x4})x_2] - f_2\dot{x}_{2ref} + f_3[o + \tfrac{\rho x_3x_2}{\alpha+x_2} - h_1x_3x_2 - i_1x_3 - v_1(1-e^{-x4})x_3] \\ &-\dot{x}_{3ref}f_3 + f_4[u(t) - i_2x_4] - f_4\dot{x}_{4ref} + f_5e_1 + f_6e_2 + f_7e_3 + f_8e_4\end{aligned} \tag{3.3.33}$$

$$\begin{aligned}\dot{S} &=f_1[j_2x_1(1-g_2x_1) - h_4x_2x_1 - v_3(1-e^{-x4})x_1 - f_1\dot{x}_{1ref} + f_2[j_1x_2(1-g_1x_2) - h_2x_3x_2 \\ &-h_3x_2x_1 - v_2(1-e^{-x4})x_2] - f_2\dot{x}_{2ref} + f_3[o + \tfrac{\rho x_3x_2}{\alpha+x_2} - h_1x_3x_2 - i_1x_3 - v_1(1-e^{-x4})x_3] \\ &-\dot{x}_{3ref}f_3 + f_4[u(t) - i_2x_4] - f_4\dot{x}_{4ref} + f_5e_1 + f_6e_2 + f_7e_3 + f_8e_4\end{aligned} \tag{3.3.34}$$

Taking j_2 as adaptive parameter j_2=δ_1

$$\begin{aligned}\dot{S} &=f_1\delta_1x_1 - f_1\delta_1x_1^2j_2 - f_1h_4x_2x_1 - f_1v_3x_1 + f_1v_3e^{-x4}x_1 - f_1\dot{x}_{1ref} + f_2j_1x_2 \\ &-f_2j_1g_1x_2^2 - f_2h_2x_3x_2 - f_2h_3x_2x_1 - f_2v_2x_2 + f_2v_2e^{-x4}x_2 - f_2\dot{x}_{2ref} + f_3o \\ &+f_3\tfrac{\rho x_3x_2}{\alpha+x_2} - f_3h_1x_3x_2 - f_3i_1x_3 - f_3v_1x_3 - f_3v_1e^{-x4}x_3 - \dot{x}_{3ref}f_3 + f_4u(t) \\ &-f_4i_2x_4 - f_4\dot{x}_{4ref} + f_5e_1 + f_6e_2 + f_7e_3 + f_8e_4\end{aligned} \tag{3.3.35}$$

for the convergence of the system, defining the adaptive law which can estimate the parameters as we can take i=1,2,3...

$$\tilde{\delta_i}=\hat{\delta_i}-\delta_i \tag{3.3.36}$$

from adaptive law we can conclude value of δ_1

$$\delta_1=\hat{\delta_1}-\tilde{\delta}_1 \tag{3.3.37}$$

for stability analysis we take lypanov candid function

$$V=\frac{1}{2}S^2+\frac{1}{2\eta}\delta_1^2 \tag{3.3.38}$$

Taking Time derivative of V

$$\dot{V}=S\dot{S}+\frac{1}{\eta}\tilde{\delta}_1\dot{\hat{\delta}}_1 \tag{3.3.39}$$

Putting value of $\dot{S}$

$$\begin{aligned}\dot{V}\quad =&\text{S}[f_1\delta_1x_1-f_1g_2x_1^2j_2-f_1h_4x_2x_1-f_1v_3x_1+f_1v_3e^{-x4}x_1-f_1\dot{x}_{1ref}+f_2j_1x_2\\&-f_2j_1g_1x_2^2-f_2h_2x_3x_2-f_2h_3x_2x_1-f_2v_2x_2+f_2v_2e^{-x4}x_2-f_2\dot{x}_{2ref}+f_3o\\&+f_3\tfrac{\rho x_3x_2}{\alpha+x_2}-f_3h_1x_3x_2-f_3i_1x_3-f_3v_1x_3-f_3v_1e^{-x4}x_3-\dot{x}_{3ref}f_3+f_4u(t)\\&-f_4i_2x_4-f_4\dot{x}_{4ref}+f_5e_1+f_6e_2+f_7e_3+f_8e_4]+\tfrac{1}{\eta}\tilde{\delta}_1\dot{\hat{\delta}}_1\end{aligned} \tag{3.3.40}$$

putting value of δ_1 from eq 3.3.37

$$\begin{aligned}\dot{V} =& \mathrm{S}[f_1[\hat{\delta_1} - \tilde{\delta}_1]x_1 - f_1 g_2 x_1^2[\hat{\delta_1} - \tilde{\delta}_1] - f_1 h_4 x_2 x_1 - f_1 v_3 x_1 + f_1 v_3 e^{-x4} x_1 - f_1 \dot{x}_{1ref} + f_2 j_1 x_2 \\ & -f_2 j_1 g_1 x_2^2 - f_2 h_2 x_3 x_2 - f_2 h_3 x_2 x_1 - f_2 v_2 x_2 + f_2 v_2 e^{-x4} x_2 - f_2 \dot{x}_{2ref} + f_3 o \\ & + f_3 \tfrac{\rho x_3 x_2}{\alpha + x_2} - f_3 h_1 x_3 x_2 - f_3 i_1 x_3 - f_3 v_1 x_3 - f_3 v_1 e^{-x4} x_3 - \dot{x}_{3ref} f_3 + f_4 u(t) \\ & -f_4 i_2 x_4 - f_4 \dot{x}_{4ref} + f_5 e_1 + f_6 e_2 + f_7 e_3 + f_8 e_4] + \tfrac{1}{\eta}\tilde{\delta}_1 \dot{\hat{\delta}}_1\end{aligned} \tag{3.3.41}$$

Seperate the terms having $\tilde{\delta}_1$:

$$\begin{aligned}\dot{V} =& \mathrm{S}[f_1 \hat{\delta_1} - f_1 g_2 x_1^2 \hat{\delta_1} - f_1 h_4 x_2 x_1 - f_1 v_3 x_1 + f_1 v_3 e^{-x4} x_1 - f_1 \dot{x}_{1ref} + f_2 j_1 x_2 \\ & -f_2 j_1 g_1 x_2^2 - f_2 h_2 x_3 x_2 - f_2 h_3 x_2 x_1 - f_2 v_2 x_2 + f_2 v_2 e^{-x4} x_2 - f_2 \dot{x}_{2ref} + f_3 o \\ & + f_3 \tfrac{\rho x_3 x_2}{\alpha + x_2} - f_3 h_1 x_3 x_2 - f_3 i_1 x_3 - f_3 v_1 x_3 - f_3 v_1 e^{-x4} x_3 - \dot{x}_{3ref} f_3 + f_4 u(t) \\ & -f_4 i_2 x_4 - f_4 \dot{x}_{4ref} + f_5 e_1 + f_6 e_2 + f_7 e_3 + f_8 e_4] - S f_1 \tilde{\delta}_1 x_1 (1 - g_2 x_1) + \tfrac{1}{\eta}\tilde{\delta}\dot{\hat{\delta}}_1\end{aligned} \tag{3.3.42}$$

Taking $\frac{1}{\eta}\tilde{\delta}_1$ common from eq 3.3.42:

$$\begin{aligned}\dot{V} =& S[f_1 \hat{\delta_1} x_1 - f_1 g_2 x_1^2 \hat{\delta_1} - f_1 h_4 x_2 x_1 - f_1 v_3 x_1 + f_1 v_3 e^{-x4} x_1 - f_1 \dot{x}_{1ref} + f_2 j_1 x_2 \\ & -f_2 j_1 g_1 x_2^2 - f_2 h_2 x_3 x_2 - f_2 h_3 x_2 x_1 - f_2 v_2 x_2 + f_2 v_2 e^{-x4} x_2 - f_2 \dot{x}_{2ref} + f_3 o \\ & + f_3 \tfrac{\rho x_3 x_2}{\alpha + x_2} - f_3 h_1 x_3 x_2 - f_3 i_1 x_3 - f_3 v_1 x_3 - f_3 v_1 e^{-x4} x_3 - \dot{x}_{3ref} f_3 + f_4 u(t) \\ & -f_4 i_2 x_4 - f_4 \dot{x}_{4ref} + f_5 e_1 + f_6 e_2 + f_7 e_3 + f_8 e_4] + \tfrac{1}{\eta}\tilde{\delta}_1[\dot{\hat{\delta}}_1 - S\eta f_1 x_1 (1 - g_2 x_1)]\end{aligned} \tag{3.3.43}$$

Simplifying eq 3.3.43

$$\begin{aligned}\dot{V} =& S[f_1\hat{\delta_1}x_1 - f_1g_2x_1^2\hat{\delta_1} - f_1h_4x_2x_1 - f_1v_3x_1 + f_1v_3e^{-x4}x_1 - f_1\dot{x}_{1ref} + f_2j_1x_2 \\ & -f_2j_1g_1x_2^2 - f_2h_2x_3x_2 - f_2h_3x_2x_1 - f_2v_2x_2 + f_2v_2e^{-x4}x_2 - f_2\dot{x}_{2ref} + f_3o \\ & +f_3\frac{\rho x_3x_2}{\alpha+x_2} - f_3h_1x_3x_2 - f_3i_1x_3 - f_3v_1x_3 - f_3v_1e^{-x4}x_3 - \dot{x}_{3ref}f_3 + f_4u(t) \\ & -f_4i_2x_4 - f_4\dot{x}_{4ref} + f_5e_1 + f_6e_2 + f_7e_3 + f_8e_4] + \frac{1}{\eta}\widetilde{\delta}_1[\dot{\hat{\delta}}_1 - S\eta f_1x_1 + S\eta f_1g_2x_1^2]\end{aligned} \tag{3.3.44}$$

Considering the boundedness of parameter estimation, so in $\dot{V}$ the adaptive update laws are designed as:

$$\dot{\hat{\delta}}_1 = \eta Proj(\hat{\delta_1}, +Sf_1x_1 - Sf_1g_2x_1^2) \tag{3.3.45}$$

To get the varying parameters bounded, adaptation parameters are redefined as follows:

$$\dot{\hat{\delta}}_1 = +S\eta f_1x_1 - S\eta f_1g_2x_1^2 \tag{3.3.46}$$

Taking eq 3.3.45 into eq 3.3.43 eq 3.3.43 can be written as :

$$\begin{aligned}\dot{V} \leq\ & S[f_1\hat{\delta_1}x_1 - f_1g_2x_1^2\hat{\delta_1} - f_1h_4x_2x_1 - f_1v_3x_1 + f_1v_3e^{-x4}x_1 - f_1\dot{x}_{1ref} + f_2j_1x_2 \\ & -f_2j_1g_1x_2^2 - f_2h_2x_3x_2 - f_2h_3x_2x_1 - f_2v_2x_2 + f_2v_2e^{-x4}x_2 - f_2\dot{x}_{2ref} + f_3o \\ & +f_3\frac{\rho x_3x_2}{\alpha+x_2} - f_3h_1x_3x_2 - f_3i_1x_3 - f_3v_1x_3 - f_3v_1e^{-x4}x_3 - \dot{x}_{3ref}f_3 + f_4u(t) \\ & -f_4i_2x_4 - f_4\dot{x}_{4ref} + f_5e_1 + f_6e_2 + f_7e_3 + f_8e_4] + \frac{1}{\eta}\widetilde{\delta}[\dot{\hat{\delta}}_1 - Sf_1x_1 + Sf_1x_1^2g_2]\end{aligned} \tag{3.3.47}$$

Reaching law of SMC $\dot{S}$= -ksign(S) here k is a design parameter and can have any constant positive value.Signum function is defined as follows:

$$\begin{aligned} sign(S) &= -1 \quad if \quad S<0 \\ &= 0 \quad if \quad S=0 \\ &= 1 \quad if \quad S>0 \end{aligned} \tag{3.3.48}$$

putting value of $\dot{S}$ in eq 3.3.35 :

$$\begin{aligned} -ksign(S) =& f_1\hat{\delta_1}x_1 - f_1g_2x_1^2\hat{\delta_1} - f_1h_4x_2x_1 - f_1v_3x_1 + f_1v_3e^{-x4}x_1 - f_1\dot{x}_{1ref} + f_2j_1x_2 \\ & -f_2j_1g_1x_2^2 - f_2h_2x_3x_2 - f_2h_3x_2x_1 - f_2v_2x_2 + f_2v_2e^{-x4}x_2 - f_2\dot{x}_{2ref} + f_3o \\ & +f_3\frac{\rho x_3x_2}{\alpha+x_2} - f_3h_1x_3x_2 - f_3i_1x_3 - f_3v_1x_3 - f_3v_1e^{-x4}x_3 - \dot{x}_{3ref}f_3 + f_4u(t) \\ & -f_4i_2x_4 - f_4\dot{x}_{4ref} + f_5e_1 + f_6e_2 + f_7e_3 + f_8e_4 \end{aligned} \tag{3.3.49}$$

$$\begin{aligned} u(t) =& -\frac{1}{f_4}[f_1\hat{\delta_1}x_1 - f_1g_2x_1^2\hat{\delta_1} - f_1h_4x_2x_1 - f_1v_3x_1 + f_1v_3e^{-x4}x_1 - f_1\dot{x}_{1ref} + f_2j_1x_2 \\ & -f_2j_1g_1x_2^2 - f_2h_2x_3x_2 - f_2h_3x_2x_1 - f_2v_2x_2 + f_2v_2e^{-x4}x_2 - f_2\dot{x}_{2ref} + f_3o \\ & +f_3\frac{\rho x_3x_2}{\alpha+x_2} - f_3h_1x_3x_2 - f_3i_1x_3 - f_3v_1x_3 - f_3v_1e^{-x4}x_3 - \dot{x}_{3ref}f_3 + ksign(S) \\ & -f_4i_2x_4 - f_4\dot{x}_{4ref} + f_5e_1 + f_6e_2 + f_7e_3 + f_8e_4] \end{aligned} \tag{3.3.50}$$

Putting eq 3.3.49 in eq 3.3.47 and consedring properties of sign(.) from eq 3.3.48 eq 3.3.47 can be written as :

$$\dot{V} \leq -k\,|S| \leq 0 \tag{3.3.51}$$

Thus, it is demonstrated that the system as a whole is asymptotically stable and the developed controller satisfies the Lyapunov stability criteria.

3.3.3 Adaptive Terminal Sliding Mode Control

Consedring the tracking of errors defined in eq 3.3.1.Sliding Surface is defined as:

$$S = f_1e_1 + f_2e_2 + f_3e_3 + f_4e_4 + f_5(\int e_1dt)^{\frac{p_1}{q_1}} + f_6(\int e_2dt)^{\frac{p_2}{q_2}} + f_7(\int e_3dt)^{\frac{p_3}{q_3}} + f_8(\int e_4dt)^{\frac{p_4}{q_4}} \tag{3.3.52}$$

where $f_1, f_2, f_3, f_4, f_5, f_6, f_7$ and f_8 are the gains of the controller which are positive real numbers.

$p_1, p_2, p_3, p_4, q_1, q_2, q_3 and q_4$ are the gains which are positive odd numbers.

$1 < \frac{p_i}{q_i} < 2$ and i=1,2,3,4...
After taking time derivative eq 3.3.52 becomes

$$\begin{aligned}\dot{S} \quad =& f_1\dot{e}_1 + f_2\dot{e}_2 + f_3\dot{e}_3 + f_4\dot{e_4} + f_5(\int e_1 dt)^{\frac{p_1}{q_1}-1}(\tfrac{p_1}{q_1})e_1 \\ &+ f_6(\int e_2 dt)^{\frac{p_2}{q_2}-1}(\tfrac{p_2}{q_2})e_2 + f_7(\int e_3 dt)^{\frac{p_3}{q_3}-1}(\tfrac{p_3}{q_3})e_3 \\ &+ f_8(\int e_4 dt)^{\frac{p_4}{q_4}-1}(\tfrac{p_4}{q_4})e_4\end{aligned} \tag{3.3.53}$$

letting above terms as :

$$\begin{aligned}A &= (\int e_1 dt)^{\frac{p_1}{q_1}-1}(\frac{p_1}{q_1})e_1 \\ B &= (\int e_2 dt)^{\frac{p_2}{q_2}-1}(\frac{p_2}{q_2})e_2 \\ C &= (\int e_3 dt)^{\frac{p_3}{q_3}-1}(\frac{p_3}{q_3})e_3 \\ D &= (\int e_4 dt)^{\frac{p_4}{q_4}-1}(\frac{p_4}{q_4})e_4\end{aligned} \tag{3.3.54}$$

putting values of $\dot{e}_1$,$\dot{e}_2$,$\dot{e}_3$,$\dot{e}_4$ from eq 3.3.2 in eq 3.3.53

$$\begin{aligned}\dot{S} \quad =& f_1(\dot{x}_1 - \dot{x}_{1ref}) + f_2(\dot{x}_2 - \dot{x}_{2ref}) + f_3(\dot{x}_3 - \dot{x}_{3ref}) + f_4(\dot{x}_4 - \dot{x}_{4ref}) + f_5 A \\ &+ f_6 B + f_7 C + f_8 D\end{aligned} \tag{3.3.55}$$

$$\begin{aligned}\dot{S} \quad =& f_1\dot{x}_{1ref} - f_1\dot{x}_{1ref} + f_2\dot{x}_2 - f_2\dot{x}_{2ref} + f_3\dot{x}_3 - f_3\dot{x}_{3ref} + f_4\dot{x}_4 - f_4\dot{x}_{4ref} \\ & +f_5A + f_6B + f_7C + f_8D\end{aligned} \tag{3.3.56}$$

putting $\dot{x}_1, \dot{x}_2, \dot{x}_3, \dot{x}_4$ in above eq

$$\begin{aligned}\dot{S} \quad =& f_1(\delta_1 x_1(1-g_2x_1) - h_4x_2x_1 - v_3(1-e^{-x4})x_1) - f_1\dot{x}_{1ref} + f_2(j_1x_2(1-g_1x_2) - h_2x_3x_2 \\ & -h_3x_2x_1 - v_2(1-e^{-x4}x_2) - f_2\dot{x}_{2ref} + f_3(o + \tfrac{\rho x_3x_2}{\alpha+x_2} - h_1x_3x_2 - i_1x_3 - v_1(1-e^{-x4}x_3) \\ & -f_3\dot{x}_{3ref} + f_4(u(t) - d_2x_4) - f_4\dot{x}_{ref} + f_5A + f_6B + f_7C + f_8D\end{aligned} \tag{3.3.57}$$

Simlplifying eq 3.3.57

$$\begin{aligned}\dot{S} \quad =& f_1(\delta_1x_1 - \delta_1g_2x_1^2 - f_4x_2x_1 - v_3x_1 + v_3e^{-x_4}x_1) - f_1\dot{x}_{1ref} + f_2(j_1x_2 - j_1x_2^2g_1 - h_2x_3x_2 \\ & -h_3x_2x_1 - v_2x_2 + e^{-x_4}x_2v_2) - g_2\dot{x}_{2ref} + f_3(o + \tfrac{\rho x_3x_2}{\alpha+x_2} - h_1x_3x_2 - i_1x_3 - v_1(1-e^{-x4})x_3) \\ & -f_3\dot{x}_{3ref} + f_4(u(t) - i_2x_4) - f_4\dot{x}_{4ref} + f_5A + f_6B + f_7C + f_8D\end{aligned} \tag{3.3.58}$$

Taking adaptive parameter $=j_2$ so we can take $j_2 = \delta_1$

$$\begin{aligned}\dot{S} \quad =& f_1\delta_1x_1 - f_1\delta_1g_2x_1^2 - f_1h_4x_2x_1 - f_1v_3x_1 + f_1v_3e^{-x_4}x_1 - f_1\dot{x}_{1ref} + f_2j_1x_2 \\ & -f_2j_1x_2^2g_1 - f_2h_2x_3x_2 - f_2h_3x_2x_1 - f_2v_2x_2 + f_2e^{-x_4}x_2v_2 - f_2\dot{x}_{2ref} + f_3o \\ & +f_3\tfrac{\rho x_3x_2}{\alpha+x_2} - f_3h_1x_3x_2 - f_3i_1x_3 - f_3v_1x_3 - f_3e^{-x4}x_3v_1 - f_3\dot{x}_{3ref} \\ & +f_4u(t) - f_4i_2x_4 - f_4\dot{x}_{4ref} + f_5A + f_6B + f_7C + f_8D\end{aligned} \tag{3.3.59}$$

for the convergence of the system, defining the adaptive law which can estimate the parameters as we can take i=1,2

$$\tilde{\delta}_i = \hat{\delta_i} - \delta_i \tag{3.3.60}$$

from adaptive law we can conclude value of δ_1

$$\delta_1 = \hat{\delta_1} - \tilde{\delta}_1 \tag{3.3.61}$$

for stability analysis we take lypanov candid function

$$V = \frac{1}{2}S^2 + \frac{1}{2\eta}\delta_1^2 \tag{3.3.62}$$

Taking Time derivative of V

$$\dot{V} = S\dot{S} + \frac{1}{\eta}\tilde{\delta}_1\dot{\hat{\delta}}_1 \tag{3.3.63}$$

Putting $\dot{S}$ in above eq:

$$\begin{aligned}\dot{V} =& \mathrm{S}[f_1\delta_1 x_1 - f_1\delta_1 g_2 x_1^2 - f_1 h_4 x_2 x_1 - f_1 v_3 x_1 + f_1 v_3 e^{-x_4} x_1 - f_1 \dot{x}_{1ref} + f_2 j_1 x_2 - f_2 j_1 x_2^2 g_1 \\ & -h_2 x_3 x_2 - f_2 h_3 x_2 x_1 - f_2 v_2 x_2 + f_2 e^{-x_4} x_2 v_2 - f_2 \dot{x}_{2ref} + f_3 o + f_3 \tfrac{\rho x_3 x_2}{\alpha + x_2} - f_3 h_1 x_3 x_2 \\ & -f_3 i_1 x_3 - f_3 v_1 x_3 - f_3 e^{-x4} x_3 v_1 - f_3 \dot{x}_{3ref} + f_4 u(t) - f_4 i_2 x_4 - f_4 \dot{x}_{4ref} \\ & +f_5 A + f_6 B + f_7 C + f_8 D] + \tfrac{1}{\eta}\tilde{\delta}_1 \dot{\hat{\delta}}_1\end{aligned} \tag{3.3.64}$$

putting value of δ_1 from eq 3.3.61

$$\begin{aligned}\dot{V} =& \mathrm{S}[f_1[\hat{\delta_1} - \tilde{\delta}_1] x_1 - f_1[\hat{\delta_1} - \tilde{\delta}_1] g_2 x_1^2 - f_1 h_4 x_2 x_1 - f_1 v_3 x_1 + f_1 v_3 e^{-x_4} x_1 - f_1 \dot{x}_{1ref} + f_2 j_1 x_2 \\ & -f_2 j_1 x_2^2 g_1 - h_2 x_3 x_2 - f_2 h_3 x_2 x_1 - f_2 v_2 x_2 + f_2 e^{-x_4} x_2 v_2 - f_2 \dot{x}_{2ref} + f_3 o + f_3 \tfrac{\rho x_3 x_2}{\alpha + x_2} \\ & -f_3 h_1 x_3 x_2 - f_3 i_1 x_3 - f_3 v_1 x_3 - f_3 e^{-x4} x_3 v_1 - f_3 \dot{x}_{3ref} + f_4 u(t) - f_4 i_2 x_4 - f_4 \dot{x}_{4ref} \\ & +f_5 A + f_6 B + f_7 C + f_8 D] + \tfrac{1}{\eta}\tilde{\delta}_1 \dot{\hat{\delta}}_1\end{aligned} \tag{3.3.65}$$

Seperate the terms having $\tilde{\delta}_1$

$$\begin{aligned}\dot{V} =& S[f_1 \hat{\delta_1} x_1 - f_1 \hat{\delta_1} g_2 x_1^2 - f_1 v_3 x_1 + f_1 v_3 e^{-x_4} x_1 - f_1 \dot{x}_{1ref} + f_2 j_1 x_2 - f_2 j_1 x_2^2 g_1 - h_2 x_3 x_2 \\ & -f_2 h_3 x_2 x_1 - f_2 v_2 x_2 + f_2 e^{-x_4} x_2 v_2 - f_2 \dot{x}_{2ref} + f_3 o + f_3 \tfrac{\rho x_3 x_2}{\alpha + x_2} - f_3 h_1 x_3 x_2 - f_3 i_1 x_3 - f_3 v_1 x_3 \\ & -f_3 e^{-x4} x_3 v_1 - f_3 \dot{x}_{3ref} + f_4 u(t) - f_4 i_2 x_4 - f_4 \dot{x}_{4ref} + f_5 A + f_6 B + f_7 C + f_8 D] \\ & -S f_1 \tilde{\delta}_1 x_1 (1 - g_2 x_1) + \tfrac{1}{\eta}\tilde{\delta}\dot{\hat{\delta}}_1\end{aligned} \tag{3.3.66}$$

Taking $\frac{1}{\eta}\tilde{\delta}_1$ common from eq 3.3.66

$$\begin{aligned}\dot{V} = \; & \mathrm{S}[f_1\hat{\delta_1}x_1 - f_1\hat{\delta_1}g_2x_1^2 - f_1h_4x_2x_1 - f_1v_3x_1 + f_1v_3e^{-x_4}x_1 - f_1\dot{x}_{1ref} + f_2j_1x_2 - f_2j_1x_2^2g_1 \\ & -h_2x_3x_2 - f_2h_3x_2x_1 - f_2v_2x_2 + f_2e^{-x_4}x_2v_2 - f_2\dot{x}_{2ref} + f_3o + f_3\tfrac{\rho x_3x_2}{\alpha+x_2} - f_3h_1x_3x_2 \\ & -f_3i_1x_3 - f_3v_1x_3 - f_3e^{-x4}x_3v_1 - f_3\dot{x}_{3ref} + f_4u(t) - f_4i_2x_4 - f_4\dot{x}_{4ref} \\ & +f_5A + f_6B + f_7C + f_8D] + \tfrac{1}{\eta}\widetilde{\delta}[\dot{\hat{\delta}}_1 - S\eta f_1x_1(1-g_2x_1)]\end{aligned} \tag{3.3.67}$$

Simplifying eq 3.3.67:

$$\begin{aligned}\dot{V} = \; & \mathrm{S}[f_1\hat{\delta_1}x_1 - f_1\hat{\delta_1}g_2x_1^2 - f_1h_4x_2x_1 - f_1v_3x_1 + f_1v_3e^{-x_4}x_1 - f_1\dot{x}_{1ref} + f_2j_1x_2 - f_2j_1x_2^2g_1 \\ & -h_2x_3x_2 - f_2h_3x_2x_1 - f_2v_2x_2 + f_2e^{-x_4}x_2v_2 - f_2\dot{x}_{2ref} + f_3o + f_3\tfrac{\rho x_3x_2}{\alpha+x_2} - f_3h_1x_3x_2 \\ & -f_3i_1x_3 - f_3v_1x_3 - f_3e^{-x4}x_3v_1 - f_3\dot{x}_{3ref} + f_4u(t) - f_4i_2x_4 - f_4\dot{x}_{4ref} + f_5A \\ & +f_6B + f_7C + f_8D] + \tfrac{1}{\eta}\widetilde{\delta}[\dot{\hat{\delta}}_1 - S\eta f_1x_1 + S\eta f_1g_2x_1^2]\end{aligned} \tag{3.3.68}$$

Considering the boundedness of parameter estimation, so in$\dot{V}$ the adaptive update laws are designed as:

$$\dot{\hat{\delta}}_1 = \eta Proj(\hat{\delta}_1, +Sf_1x_1 - Sf_1g_2x_1^2) \tag{3.3.69}$$

To get the varying parameters bounded, adaptation parameters are redefined as follows:

$$\dot{\hat{\delta}}_1 = S\eta f_1x_1 - S\eta f_1g_2x_1^2 \tag{3.3.70}$$

Taking eq 3.3.69 into eq 3.3.67 eq 3.3.67 can be written as

$$\begin{aligned}\dot{V} \leq \ & \text{S}[f_1\hat{\delta}_1x_1 - f_1\hat{\delta}_1g_2x_1^2 - f_1h_4x_2x_1 - f_1v_3x_1 + f_1v_3e^{-x_4}x_1 - f_1\dot{x}_{1ref} + f_2j_1x_2 \\ & -f_2j_1x_2^2g_1 - h_2x_3x_2 - f_2h_3x_2x_1 - f_2v_2x_2 + f_2e^{-x_4}x_2v_2 - f_2\dot{x}_{2ref} + f_3o \\ & +f_3\frac{\rho x_3x_2}{\alpha+x_2} - f_3h_1x_3x_2 - f_3i_1x_3 - f_3v_1x_3 - f_3e^{-x4}x_3v_1 - f_3\dot{x}_{3ref} + f_4u(t) \\ & -f_4i_2x_4 - f_4\dot{x}_{4ref} + f_5A + f_6B + f_7C + f_8D] + \frac{1}{\eta}\tilde{\delta}[\dot{\hat{\delta}}_1 - Sf_1x_1 + Sf_1x_1^2g_2]\end{aligned} \tag{3.3.71}$$

Reaching law of SMC $\dot{S}$= -ksign(S) here k is a design parameter and can have any constant positive value.Signum function is defined as follows

$$\begin{aligned} sign(S) &= -1 \quad if \quad S < 0 \\ &= 0 \quad if \quad S = 0 \\ &= 1 \quad if \quad S > 0 \end{aligned} \tag{3.3.72}$$

putting $\dot{S}$=$-ksign(S)$in eq 3.3.59

$$\begin{aligned} \text{-}ksign(S) \ &= f_1\hat{\delta}_1x_1 - f_1\hat{\delta}_1g_2x_1^2 - f_1h_4x_2x_1 - f_1v_3x_1 + f_1v_3e^{-x_4}x_1 - f_1\dot{x}_{1ref} + f_2j_1x_2 - f_2j_1x_2^2g_1 \\ & -h_2x_3x_2 - f_2h_3x_2x_1 - f_2v_2x_2 + f_2e^{-x_4}x_2v_2 - f_2\dot{x}_{2ref} + f_3o + f_3\frac{\rho x_3x_2}{\alpha+x_2} - f_3h_1x_3x_2 \\ & -f_3i_1x_3 - f_3v_1x_3 - f_3e^{-x4}x_3v_1 - f_3\dot{x}_{3ref} + f_4u(t) - f_4i_2x_4 - f_4\dot{x}_{4ref} \\ & +f_5A + f_6B + f_7C + f_8D(\frac{p_4}{q_4})e_4 \end{aligned} \tag{3.3.73}$$

Rearranging eq 3.3.73:

$$\begin{aligned}
\text{-}f_4 u(t) \quad =& f_1\hat{\delta_1}x_1 - f_1\hat{\delta_1}g_2x_1^2 - f_1h_4x_2x_1 - f_1v_3x_1 + f_1v_3e^{-x_4}x_1 - f_1\dot{x}_{1ref} + f_2j_1x_2 - f_2j_1x_2^2g_1 \\
& -h_2x_3x_2 - f_2h_3x_2x_1 - f_2v_2x_2 + f_2e^{-x_4}x_2v_2 - f_2\dot{x}_{2ref} + f_3o + f_3\tfrac{\rho x_3x_2}{\alpha+x_2} - f_3h_1x_3x_2 \\
& -f_3i_1x_3 - f_3v_1x_3 - f_3e^{-x4}x_3v_1 - f_3\dot{x}_{3ref} - f_4i_2x_4 - f_4\dot{x}_{4ref} + +f_5A \\
& +f_6B + f_7C + f_8D + ksign(S)
\end{aligned} \tag{3.3.74}$$

from eq3.3.74 we can get our control input u

$$\begin{aligned}
u(t) \quad =& \tfrac{-1}{f_4}[f_1\hat{\delta_1}x_1 - f_1\hat{\delta_1}g_2x_1^2 - f_1h_4x_2x_1 - f_1v_3x_1 + f_1v_3e^{-x_4}x_1 - f_1\dot{x}_{1ref} + f_2j_1x_2 \\
& -f_2j_1x_2^2g_1 - h_2x_3x_2 - f_2h_3x_2x_1 - f_2v_2x_2 + f_2e^{-x_4}x_2v_2 - f_2\dot{x}_{2ref} + f_3o \\
& +f_3\tfrac{\rho x_3x_2}{\alpha+x_2} - f_3h_1x_3x_2 - f_3i_1x_3 - f_3v_1x_3 - f_3e^{-x4}x_3v_1 - f_3\dot{x}_{3ref} + ksign(S) \\
& -f_4i_2x_4 - f_4\dot{x}_{4ref} + +f_5A + f_6B + f_7C + f_8D]
\end{aligned} \tag{3.3.75}$$

Putting eq 3.3.73 in eq3.3.71

and consedring the properties of sign(·) from eq3.3.72, eq 3.3.71 can be simplified as :

$$\dot{V} \leq -k\,|S| \leq 0 \tag{3.3.76}$$

It is demonstrated as a consequence that the system as a whole is asymptotically stable and that the developed controller complies with the requirements for Lyapunov stability.

3.3.4 Adaptive Super Twisting Sliding Mode Control

Defining Sliding Surface as:

$$S = f_1 e_1 + f_2 e_2 + f_3 e_3 + f_4 e_4 \tag{3.3.77}$$

here f_1, f_2, f_3, f_4 are constant design parameters of the sliding surfaces which can have any positive constant value.

Taking time derivative of eq 3.3.76

$$\dot{S} = f_1 \dot{e}_1 + f_2 \dot{e}_2 + f_3 \dot{e}_3 + f_4 \dot{e}_4 \tag{3.3.78}$$

putting values of $\dot{e}_1, \dot{e}_2, \dot{e}_3, \dot{e}_4$from eq 3.3.2 in eq 3.3.78

$$\dot{S} = f_1(\dot{x}_1 - \dot{x}_{1ref}) + f_2(\dot{x}_2 - \dot{x}_{2ref}) + f_3(\dot{x}_3 - \dot{x}_{3ref}) + f_4(\dot{x}_4 - \dot{x}_{4ref}) \tag{3.3.79}$$

Simplifying eq 3.3.79

$$\dot{S} = f_1\dot{x}_1 - f_1\dot{x}_{1ref} + f_2\dot{x}_2 - f_2\dot{x}_{2ref} + f_3\dot{x}_3 - f_3\dot{x}_{3ref} + f_4\dot{x}_4 - f_4\dot{x}_{4ref} \tag{3.3.80}$$

putting values of $\dot{x}_1, \dot{x}_2, \dot{x}_3, \dot{x}_4$ in eq 3.3.79

$$\begin{aligned}\dot{S} \quad &= f_1[j_2x_1(1-g_2x_1) - h_4x_2x_1 - v_3(1-e^{-x4})x_1] - f_1\dot{x}_{1ref} + f_2[j_1x_2(1-g_1x_2) - h_2x_3x_2 \\ &-h_3x_2x_1 - v_2(1-e^{-x4})x_2] - f_2\dot{x}_{2ref} + f_3[o + \tfrac{\rho x_3x_2}{\alpha+x_2} - h_1x_3x_2 - i_1x_3 - v_1(1-e^{-x4})x_3] \\ &+f_4[u(t) - i_2x_4] - f_4\dot{x}_{4ref}\end{aligned} \tag{3.3.81}$$

Simplifying eq 3.3.80

$$\begin{aligned}\dot{S} \quad &= f_1[j_2x_1(1-g_2x_1) - h_4x_2x_1 - v_3(1-e^{-x4})x_1] - f_1\dot{x}_{1ref} + f_2[j_1x_2(1-g_1x_2) - h_2x_3x_2 \\ &-h_3x_2x_1 - v_2(1-e^{-x4})x_2] - f_2\dot{x}_{2ref} + f_3[o + \tfrac{\rho x_3x_2}{\alpha+x_2} - h_1x_3x_2 - i_1x_3 - v_1(1-e^{-x4})x_3] \\ &-\dot{x}_{3ref}f_3 + f_4[u(t) - i_2x_4] - f_4\dot{x}_{4ref}\end{aligned} \tag{3.3.82}$$

$$\begin{aligned}\dot{S} \quad &= f_1[j_2x_1 - g_2x_1^2j_2 - h_4x_2x_1 - v_3x_1 + v_3e^{-x4}x_1] - f_1\dot{x}_{1ref} + f_2[j_1x_2 - j_1g_1x_2^2 \\ &-h_2x_3x_2 - h_3x_2x_1 - v_2x_2 + v_2e^{-x4}x_2] - f_2\dot{x}_{2ref} + f_3[o + \tfrac{\rho x_3x_2}{\alpha+x_2} - h_1x_3x_2 - i_1x_3 - v_1x_3 \\ &+v_1e^{-x4}x_3] - \dot{x}_{3ref}f_3 + f_4[u(t) - i_2x_4] - f_4\dot{x}_{4ref}\end{aligned} \tag{3.3.83}$$

Taking j_2 adaptive parameter so we can write j_2=δ_1

$$\begin{aligned}\dot{S} \quad &= f_1\delta_1x_1 - f_1g_2x_1^2\delta_1 - f_1h_4x_2x_1 - f_1v_3x_1 + f_1v_3e^{-x4}x_1 - f_1\dot{x}_{1ref} + f_2j_1x_2 - f_2j_1g_1x_2^2 \\ &-f_2h_2x_3x_2 - f_2h_3x_2x_1 - f_2v_2x_2 + f_2v_2e^{-x4}x_2 - f_2\dot{x}_{2ref} + f_3o + f_3\tfrac{\rho x_3x_2}{\alpha+x_2} - f_3h_1x_3x_2 - f_3i_1x_3 \\ &-f_3v_1x_3 + f_3v_1e^{-x4}x_3 - \dot{x}_{3ref}f_3 + f_4u(t) - f_4i_2x_4 - f_4\dot{x}_{4ref}\end{aligned} \tag{3.3.84}$$

for the convergence of the system, defining the adaptive law which can estimate the parameters as :

$$\tilde{\delta_i}=\hat{\delta_i}-\delta_i \tag{3.3.85}$$

we can take i=1,2

from adaptive law we can conclude value of δ_1

$$\delta_1=\hat{\delta_1}-\tilde{\delta_1} \tag{3.3.86}$$

$$\begin{aligned}\dot{S}\quad &= f_1\hat{\delta_1}x_1 - f_1g_2x_1^2\hat{\delta_1} - f_1h_4x_2x_1 - f_1v_3x_1 + f_1v_3e^{-x4}x_1 - f_1\dot{x}_{1ref}\\ &+ f_2j_1x_2 - f_2j_1g_1x_2^2 - f_2h_2x_3x_2 - f_2h_3x_2x_1 - f_2v_2x_2 + f_2v_2e^{-x4}x_2\\ &- f_2\dot{x}_{2ref} + f_3o + f_3\tfrac{\rho x_3x_2}{\alpha+x_2} - f_3h_1x_3x_2 - f_3i_1x_3 - f_3v_1x_3\\ &+ f_3v_1e^{-x4}x_3 - \dot{x}_{3ref}f_3 + f_4u(t) - f_4i_2x_4 - f_4\dot{x}_{4ref}\end{aligned} \tag{3.3.87}$$

for stability analysis we take lypanov candid function

$$V=\frac{1}{2}S^2+\frac{1}{2\eta}\delta_1^2 \tag{3.3.88}$$

Taking Time derivative of V

$$\dot{V}=S\dot{S}+\frac{1}{\eta}\tilde{\delta}_1\dot{\hat{\delta}}_1 \tag{3.3.89}$$

Putting $\dot{S}$ in above eq:

$$\begin{aligned}\dot{V} =& S[f_1\delta_1 x_1 - f_1 g_2 x_1^2\delta_1 - f_1 h_4 x_2 x_1 - f_1 v_3 x_1 + f_1 v_3 e^{-x4} x_1 - f_1\dot{x}_{1ref} + f_2 j_1 x_2 \\ & - f_2 j_1 g_1 x_2^2 - f_2 h_2 x_3 x_2 - f_2 h_3 x_2 x_1 - f_2 v_2 x_2 + f_2 v_2 e^{-x4} x_2 - f_2\dot{x}_{2ref} + f_3 o \\ & + f_3 \tfrac{\rho x_3 x_2}{\alpha + x_2} - f_3 h_1 x_3 x_2 - f_3 i_1 x_3 - f_3 v_1 x_3 + f_3 v_1 e^{-x4} x_3 - \dot{x}_{3ref} f_3 + f_4 u(t) \\ & - f_4 i_2 x_4 - f_4 \dot{x}_{4ref}] + \tfrac{1}{\eta}\tilde{\delta}_1\dot{\hat{\delta}}_1\end{aligned} \tag{3.3.90}$$

Putting value of δ_1 from eq 3.3.86 in eq 3.3.90

$$\begin{aligned}\dot{V} =& S[f_1[\hat{\delta_1} - \tilde{\delta}_1]x_1 - f_1 g_2 x_1^2[\hat{\delta_1} - \tilde{\delta}_1] - f_1 h_4 x_2 x_1 - f_1 v_3 x_1 + f_1 v_3 e^{-x4} x_1 - f_1\dot{x}_{1ref} \\ & + f_2 j_1 x_2 - f_2 j_1 g_1 x_2^2 - f_2 h_2 x_3 x_2 - f_2 h_3 x_2 x_1 - f_2 v_2 x_2 + f_2 v_2 e^{-x4} x_2 \\ & - f_2\dot{x}_{2ref} + f_3 o + f_3 \tfrac{\rho x_3 x_2}{\alpha + x_2} - f_3 h_1 x_3 x_2 - f_3 i_1 x_3 - f_3 v_1 x_3 + f_3 v_1 e^{-x4} x_3 - \dot{x}_{3ref} f_3 \\ & + f_4 u(t) - f_4 i_2 x_4 - f_4 \dot{x}_{4ref}] + \tfrac{1}{\eta}\tilde{\delta}_1\dot{\hat{\delta}}_1\end{aligned} \tag{3.3.91}$$

Seperate the terms having $\tilde{\delta}_1$

$$\begin{aligned}\dot{V} =& [S f_1\hat{\delta_1} x_1 - f_1 g_2 x_1^2\hat{\delta_1} - f_1 h_4 x_2 x_1 - f_1 v_3 x_1 + f_1 v_3 e^{-x4} x_1 - f_1\dot{x}_{1ref} + f_2 j_1 x_2 - f_2 j_1 g_1 x_2^2 \\ & - f_2 h_2 x_3 x_2 - f_2 h_3 x_2 x_1 - f_2 v_2 x_2 + f_2 v_2 e^{-x4} x_2 - f_2\dot{x}_{2ref} + f_3 o + f_3 \tfrac{\rho x_3 x_2}{\alpha + x_2} - f_3 h_1 x_3 x_2 \\ & - f_3 i_1 x_3 - f_3 v_1 x_3 + f_3 v_1 e^{-x4} x_3 - \dot{x}_{3ref} f_3 + f_4 u(t) - f_4 i_2 x_4 - f_4 \dot{x}_{4ref}] \\ & - S f_1\tilde{\delta}_1 x_1(1 - g_2 x_1) + \tfrac{1}{\eta}\tilde{\delta}\dot{\hat{\delta}}_1\end{aligned} \tag{3.3.92}$$

Taking $\frac{1}{\eta}\tilde{\delta}_1$ common from eq 3.3.92:

$$\begin{aligned}\dot{V} =& S[f_1\hat{\delta} x_1 - f_1 g_2 x_1^2\hat{\delta}_1 - f_1 h_4 x_2 x_1 - f_1 v_3 x_1 + f_1 v_3 e^{-x4} x_1 - f_1\dot{x}_{1ref} + f_2 j_1 x_2 - f_2 j_1 g_1 x_2^2 \\ & - f_2 h_2 x_3 x_2 - f_2 h_3 x_2 x_1 - f_2 v_2 x_2 + f_2 v_2 e^{-x4} x_2 - f_2\dot{x}_{2ref} + f_3 o + f_3 \tfrac{\rho x_3 x_2}{\alpha + x_2} - f_3 h_1 x_3 x_2 \\ & - f_3 i_1 x_3 - f_3 v_1 x_3 + f_3 v_1 e^{-x4} x_3 - \dot{x}_{3ref} f_3 + f_4 u(t) - f_4 i_2 x_4 - f_4 \dot{x}_{4ref}] \\ & + \tfrac{1}{\eta}\tilde{\delta}[\dot{\hat{\delta}}_1 - S\eta f_1 x_1(1 - g_2 x_1)]\end{aligned} \tag{3.3.93}$$

Simplifying eq 3.3.93

$$\begin{aligned}\dot{V} =&S[f_1\hat{\delta}x_1 - f_1g_2x_1^2\hat{\delta}_1 - f_1h_4x_2x_1 - f_1v_3x_1 + f_1v_3e^{-x4}x_1 - f_1\dot{x}_{1ref} + f_2j_1x_2 - f_2j_1g_1x_2^2 \\ &-f_2h_2x_3x_2 - f_2h_3x_2x_1 - f_2v_2x_2 + f_2v_2e^{-x4}x_2 - f_2\dot{x}_{2ref} + f_3o + f_3\tfrac{\rho x_3x_2}{\alpha+x_2} - f_3h_1x_3x_2 \\ &-f_3i_1x_3 - f_3v_1x_3 + f_3v_1e^{-x4}x_3 - \dot{x}_{3ref}f_3 + f_4u(t) - f_4i_2x_4 - f_4\dot{x}_{4ref}] \\ &+\tfrac{1}{\eta}\widetilde{\delta}[\dot{\hat{\delta}}_1 - S\eta f_1x_1 + S\eta f_1g_2x_1^2]\end{aligned} \tag{3.3.94}$$

Considering the boundedness of parameter estimation, so in$\dot{V}$ the adaptive update laws are designed as

$$\dot{\hat{\delta}}_1 =\eta_1 Proj(\hat{\delta}_1, +Sf_1x_1 - Sf_1g_2x_1^2) \tag{3.3.95}$$

To get the varying parameters bounded, adaptation parameters are redefined as follows:

$$\dot{\hat{\delta}}=+\eta Sf_1x_1 - S\eta f_1g_2x_1^2 \tag{3.3.96}$$

From eq 3.3.94 and eq 3.3.95 we can write :

$$\begin{aligned}\dot{V} \leq\ & \mathrm{S}[f_1\hat{\delta}x_1 - f_1g_2x_1^2\hat{\delta}_1 - f_1h_4x_2x_1 - f_1v_3x_1 + f_1v_3e^{-x4}x_1 - f_1\dot{x}_{1ref} + f_2j_1x_2 - f_2j_1g_1x_2^2 \\ &-f_2h_2x_3x_2 - f_2h_3x_2x_1 - f_2v_2x_2 + f_2v_2e^{-x4}x_2 - f_2\dot{x}_{2ref} + f_3o + f_3\tfrac{\rho x_3x_2}{\alpha+x_2} - f_3h_1x_3x_2 \\ &-f_3i_1x_3 - f_3v_1x_3 + f_3v_1e^{-x4}x_3 - \dot{x}_{3ref}f_3 + f_4u(t) - f_4i_2x_4 - f_4\dot{x}_{4ref}]\end{aligned} \tag{3.3.97}$$

u_{sw} can be written as

$$u_{sw}= -K_1\,|S|^{\alpha}\,sign(S) + u_1 \tag{3.3.98}$$

Taking integral of u_1

$$\dot{u}_1 = -K_2 sign(S) \tag{3.3.99}$$

$$u_1 = -k_2 \int sign(S)dt \tag{3.3.100}$$

$$u_{sw} = -k_1 \left|S\right|^{\alpha} sign(S) - k_2 \int sign(S)dt \tag{3.3.101}$$

from eq 3.3.97and 3.3.98 we can write

$$\begin{aligned} -k_1 \left|S\right|^{\alpha} sign(S) - k_2 \int sign(S)dt \quad &= f_1\hat{\delta}_1 x_1 - f_1 g_2 x_1^2 \hat{\delta}_1 - f_1 h_4 x_2 x_1 - f_1 v_3 x_1 + f_1 v_3 e^{-x4} x_1 \\ &-f_1 \dot{x}_{1ref} + f_2 j_1 x_2 - f_2 j_1 g_1 x_2^2 - f_2 h_2 x_3 x_2 - f_2 h_3 x_2 x_1 \\ &-f_2 v_2 x_2 + f_2 v_2 e^{-x4} x_2 - f_2 \dot{x}_{2ref} + f_3 o + f_3 \tfrac{\rho x_3 x_2}{\alpha + x_2} \\ &-f_3 h_1 x_3 x_2 - f_3 d_1 x_3 - f_3 v_1 x_3 + f_3 v_1 e^{-x4} x_3 - \dot{x}_{3ref} f_3 \\ &+f_4 u(t) - f_4 i_2 x_4 - f_4 \dot{x}_{4ref} \end{aligned} \tag{3.3.102}$$

simplifying above eq 3.3.102

$$\begin{aligned} -f_4 u(t) \quad &= f_1\hat{\delta}_1 x_1 - f_1 g_2 x_1^2 \hat{\delta}_1 - f_1 h_4 x_2 x_1 - f_1 v_3 x_1 + f_1 v_3 e^{-x4} x_1 - f_1 \dot{x}_{1ref} + f_2 j_1 x_2 \\ &-f_2 j_1 g_1 x_2^2 - f_2 h_2 x_3 x_2 - f_2 h_3 x_2 x_1 - f_2 v_2 x_2 + f_2 v_2 e^{-x4} x_2 - f_2 \dot{x}_{2ref} + f_3 o \\ &+f_3 \tfrac{\rho x_3 x_2}{\alpha + x_2} - f_3 h_1 x_3 x_2 - f_3 i_1 x_3 - f_3 v_1 x_3 + f_3 v_1 e^{-x4} x_3 - \dot{x}_{3ref} f_3 + f_4 u(t) \\ &-f_4 i_2 x_4 - f_4 \dot{x}_{4ref} + k_1 \left|S\right|^{\alpha} sign(S) + k_2 \int sign(S)dt \end{aligned} \tag{3.3.103}$$

Simplifying further will give u(t)

$$\begin{aligned} u(t) \quad &= \tfrac{-1}{f}{}_4 [f_1\hat{\delta}_1 x_1 - f_1 g_2 x_1^2 \hat{\delta}_1 - f_1 h_4 x_2 x_1 - f_1 v_3 x_1 + f_1 v_3 e^{-x4} x_1 - f_1 \dot{x}_{1ref} + f_2 j_1 x_2 \\ &-f_2 j_1 g_1 x_2^2 - f_2 h_2 x_3 x_2 - f_2 h_3 x_2 x_1 - f_2 v_2 x_2 + f_2 v_2 e^{-x4} x_2 - f_2 \dot{x}_{2ref} + f_3 o \\ &+f_3 \tfrac{\rho x_3 x_2}{\alpha + x_2} - f_3 h_1 x_3 x_2 - f_3 i_1 x_3 - f_3 v_1 x_3 + f_3 v_1 e^{-x4} x_3 - \dot{x}_{3ref} f_3 - f_4 d_2 x_4 \\ &-f_4 \dot{x}_{4ref} + k_1 \left|S\right|^{\alpha} sign(S) + k_2 \int sign(S)dt] \end{aligned} \tag{3.3.104}$$

Putting value of u in eq 3.3.97 we get

$$\begin{aligned}\dot{V} \leq \quad & S[f_1\hat{\delta}x_1 - f_1 g_2 x_1^2 \hat{\delta}_1 - f_1 h_4 x_2 x_1 - f_1 v_3 x_1 + f_1 v_3 e^{-x4} x_1 - f_1 \dot{x}_{1ref} + f_2 j_1 x_2 - f_2 j_1 g_1 x_2^2 \\ & -f_2 h_2 x_3 x_2 - f_2 h_3 x_2 x_1 - f_2 v_2 x_2 + f_2 v_2 e^{-x4} x_2 - f_2 \dot{x}_{2ref} + f_3 o + f_3 \tfrac{\rho x_3 x_2}{\alpha + x_2} - f_3 h_1 x_3 x_2 \\ & -f_3 i_1 x_3 - f_3 v_1 x_3 + f_3 v_1 e^{-x4} x_3 - \dot{x}_{3ref} f_3 + f_4 \tfrac{-1}{f_4}[f_1 \hat{\delta}_1 x_1 - f_1 g_2 x_1^2 \hat{\delta}_1 - f_1 h_4 x_2 x_1 - f_1 v_3 x_1 \\ & +f_1 v_3 e^{-x4} x_1 - f_1 \dot{x}_{1ref} + f_2 j_1 x_2 - f_2 j_1 b_1 x_2^2 - f_2 h_2 x_3 x_2 - f_2 h_3 x_2 x_1 - f_2 v_2 x_2 + f_2 v_2 e^{-x4} x_2 \\ & -f_2 \dot{x}_{2ref} + f_3 o + f_3 \tfrac{\rho x_3 x_2}{\alpha + x_2} - f_3 h_1 x_3 x_2 - f_3 i_1 x_3 - f_3 a_1 x_3 + f_3 v_1 e^{-x4} x_3 - \dot{x}_{3ref} f_3 - f_4 i_2 x_4 \\ & -f_4 \dot{x}_{4ref} + k_1 |S|^{\alpha} sign(S) + k_2 \textstyle\int sign(S) dt] - f_4 i_2 x_4 - f_4 \dot{x}_{4ref}]\end{aligned} \tag{3.3.105}$$

Simplifying eq 3.3.105

$$\dot{V} \leq \quad S[-k_1 |S|^{\alpha} sign(S) - k_2 \textstyle\int sign(S) dt)] \tag{3.3.106}$$

The system as a whole is shown to be asymptotically stable in the eq 3.3.106 above, and the constructed controller is shown to meet the Lyapunov stability condition.

Chapter 4

Results and Discussion

In this Chapter, the results of MATLAB implementation are discussed. Firstly the comparison of the robust controller is done with the already work done, Secondly, the comparison of Adaptive controllers is done with the already work done. The comparison was done on the bases of tracking tumor cells in less number of days and with the intake of medicine in less number of days. The x-axis represents time, while the y-axis represents medicine dosage. All of the tumor mathematical model's states correspond to various cell types, and values are employed in normalized form. The simulation can last up to 200 days at most. The table below lists the initial circumstances for normal, tumor, immune cell, and chemotherapeutic medication.4.1.

State	**Initial Condition**
N(0)/$x_1(0)$	1
T(0)/$x_2(0)$	0.2
I(0)/$x_3(0)$	0.15
M(0)/$x_4(0)$	0

Table 4.1: Initial conditions of the states

4.1 Comparison of Robust Controllers

In the accompanying Figure 4.1 the comparison of the controllers (SMC, ISMC, STSMC, and TSMC) is done with Synergetic and State Feedback Control. From Figure 4.1 we can see that TSMC outperforms other controllers, as normal and tumor cells are tracking to their reference value within 60 days, immune cell is tracking to their reference value within 62 days, and drug at the site is being delivered within 20 days, respectively. On the other side, synergetic control is underperforming.

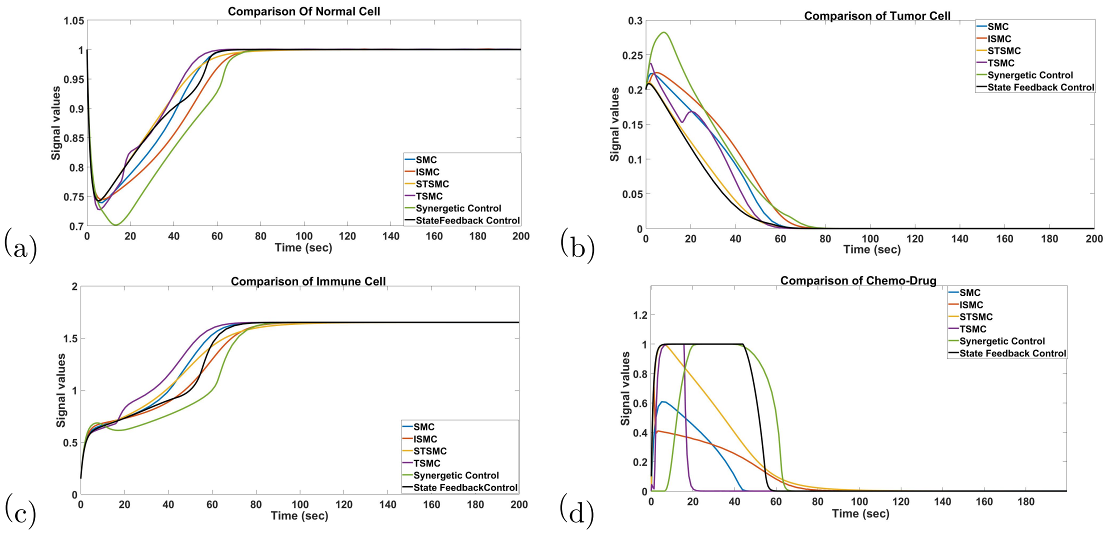

Figure 4.1: Comparison of Controllers

The following figure 4.2 compares the drug delivery scenario. We can see that the delivery of the medication takes around 18 days in TSMC and 62 days in Synergetic Control.

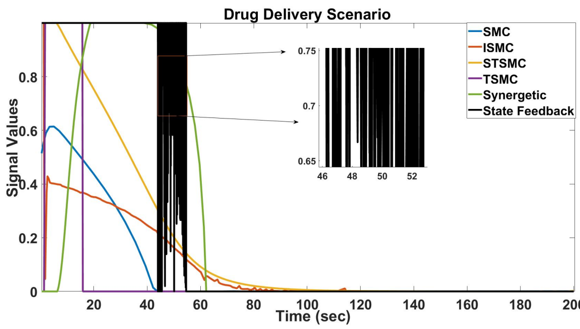

Figure 4.2: Drug Delivery scenario

The table below 4.2, compares various controllers.. From the table 4.2 we can conclude that TSMC is performing best as the tumor cells are tracking to their reference value within 60 days and the amount of drug dosage was 14.31.

Table 4.2: Comparison of several controllers

Techniques	Time of Convergence	Error at steady state	Total drug dosage
SMC	62days	No	16.59
ISMC	70days	No	18.25
TSMC	60days	No	14.31
STSMC	60days	No	35.16
Synergetic Control	60days	No	24.0000 [8]
Statefeedback	68days	No	14.8637 [8]

4.2 Comparison of Adaptive Controllers

According to Figure 4.3, ATSMC is outperforming other controllers because normal, tumor cells are tracking to their reference value within 57 days and immune cells are tracking in 60 days, and drug at the site is being delivered within 20 days, respectively.

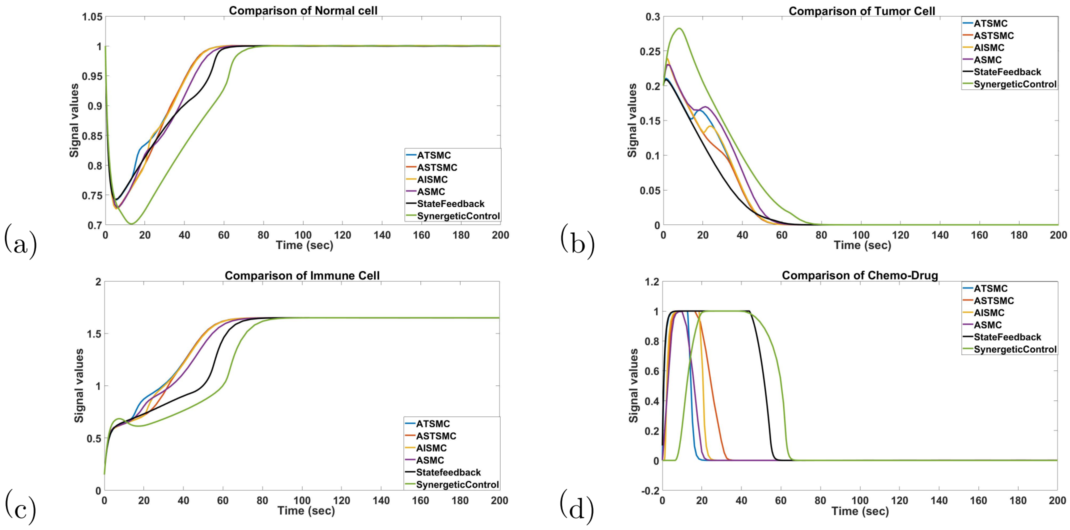

Figure 4.3: Comparison of Controllers

Figure 4.4 compares the drug delivery scenario for the above-mentioned controllers. In ATSMC, the medication is supplied in 17 days; in Synergetic Control, it takes 62 days.

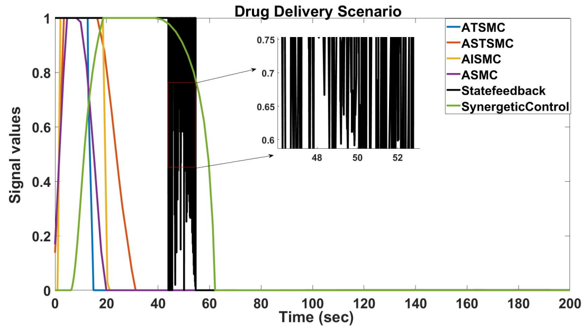

Figure 4.4: Drug Delivery scenario

The comparison of adaptive controllers with synergetic and state feedback controllers is shown in the table 4.3 below. As the tumor cell is tracking to the reference values in 57 days and the total drug was 13.62, we can say that ATSMC is performing at its best.

Table 4.3: Comparison of various controllers

Techniques	**Time of convergence**	**Error**	**Total drug dosage**
ASMC	60days	No	13.17
AISMC	58days	No	17.96
ATSMC	57days	No	13.62
ASTSMC	58days	No	22.31
Synergetic Control	60days	No	24.0000 [8]
Statefeedback	68days	No	14.8637 [8]

4.3 Comparison of Controllers

In this section, the results of robust controllers and adaptive controllers are compared.

4.3.1 Comparison of SMC and ASMC

The Comparison of SMC with ASMC is shown in the below figure 4.5 from the figure it's clear that ASMC is producing better results as compared to SMC. In ASMC normal and immune cells are tracking to their reference value within 60 days tumor cells are tracking to their reference value within 61 days drug at the site is being delivered within 22 days.

While in SMC normal cells are tracking to their reference value within 62 days, tumor cells are tracking to their reference value within 63 days immune cells are tracking to their reference value within 65 days and the drug at the site is being delivered within 43 days.

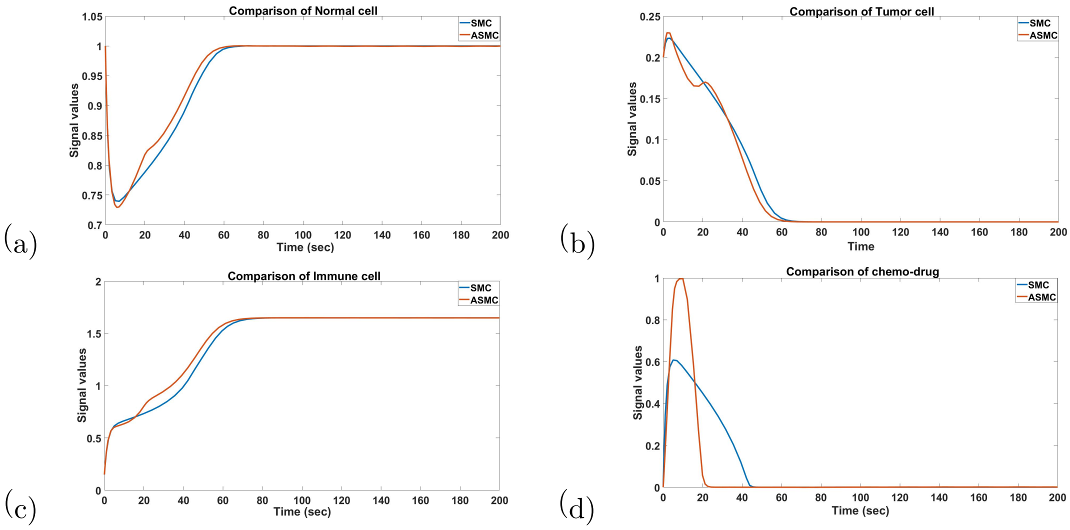

Figure 4.5: Comparison of SMC and ASMC

4.3.2 Comparison of ISMC and AISMC

The comparison of ISMC and AISMC is displayed in figure 4.6 below. From the figure, it is obvious that AISMC is outperforming normal cells are tracking to their reference value with in 58 days, tumor cells are tracking to their reference value within 60 days, immune cells are tracking within 58 days, and drug at the site is being delivered within 23 days,while in ISMC, immune cells track to their reference value in 82 days, tumor cells track to their reference value in 67 days, and normal cells track to their reference value in 65 days drug at the site is being delivered within 81 days.

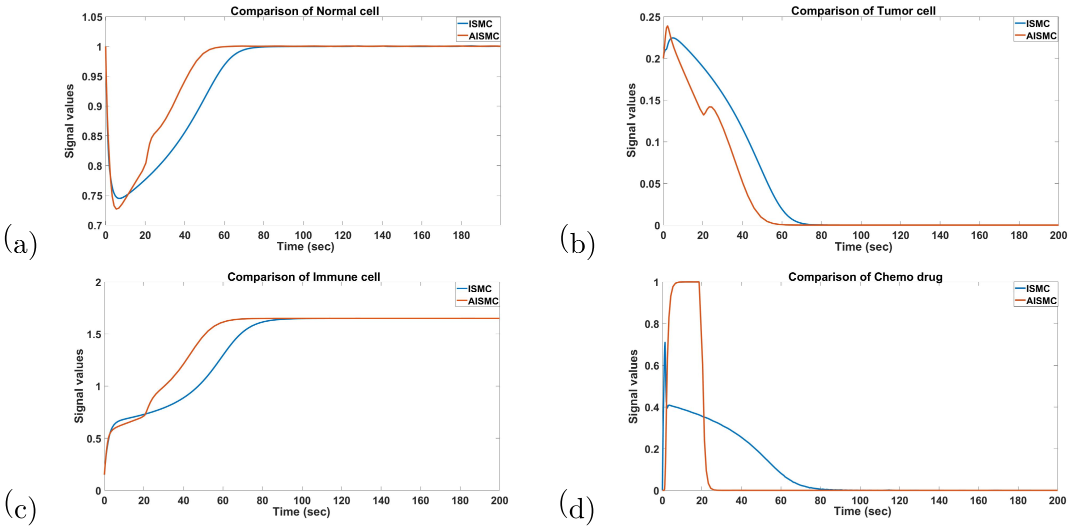

Figure 4.6: Comparison of ISMC and AISMC

4.3.3 Comparison of STSMC and ASTSMC

The figure 4.7 below shows the comparison of STSMC and ASTSMC from the figure 4.7 we can see that ASTSMC outperforms as normal and tumor cells are tracking to their reference value with in 58 days, immune cells are tracking to their reference value within 60 days and the drug at the tumor site is being delivered within 37 days.In STSMC, immune cells track to their reference value in 100 days, tumor cells track to their reference value in 62 days, normal cells track to their reference value in 80 days and drug at the site is being delivered within 100 days.

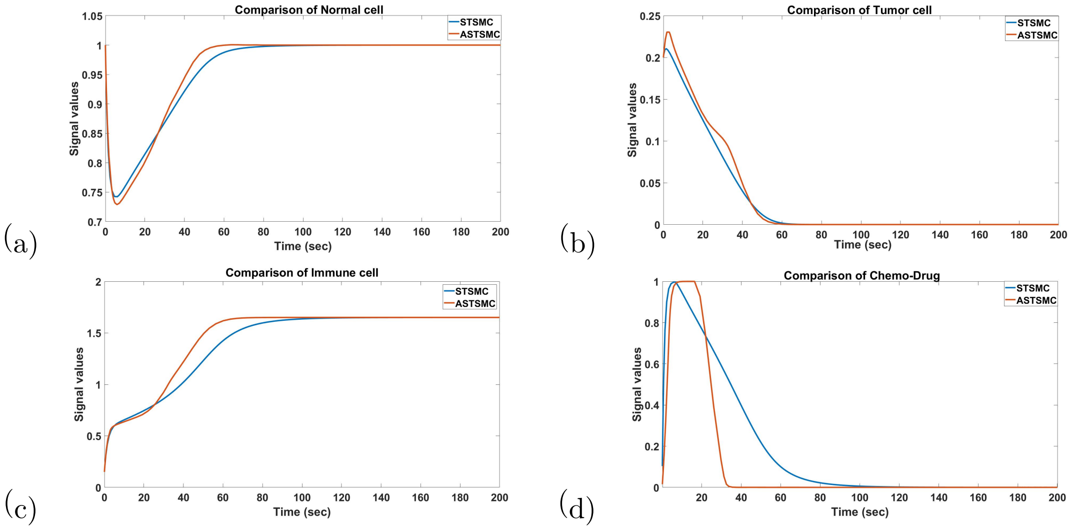

Figure 4.7: Comparison of STSMC and ASTSMC

4.3.4 Comparison of TSMC and ATSMC

The comparison between TSMC and ATSMC is depicted in the image below 4.8. The figure makes it obvious that ATSMC surpasses TSMC in terms of results as normal, tumor and immune cells are tracking to their reference value within 57 days and the drug at the tumor site is being delivered within 20 days.In TSMC normal and tumor cells are tracking to their reference value within 60 days, the immune cell is tracking to their reference value within 62 days, and the drug at the site is being delivered within 20 days, respectively.

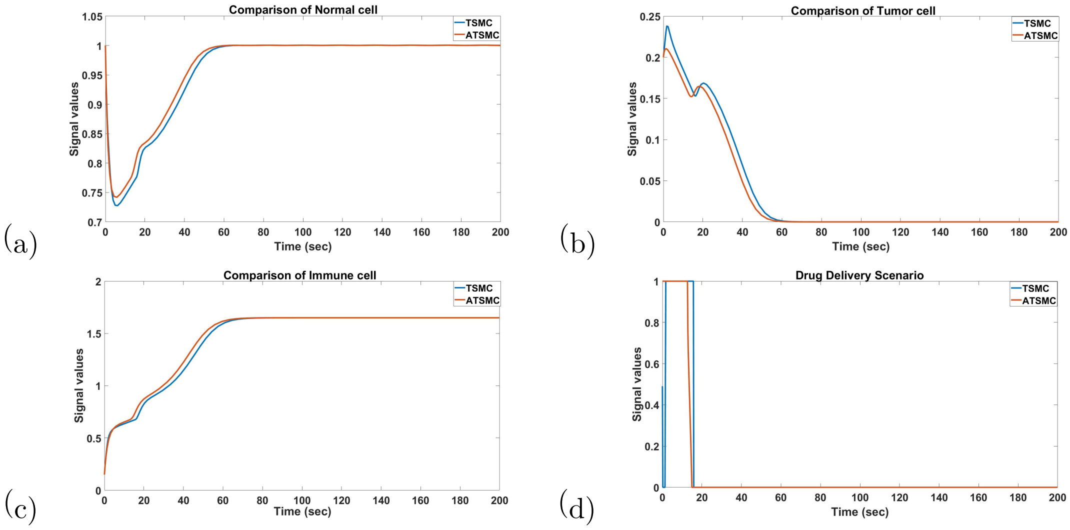

Figure 4.8: Comparison of TSMC and ATSMC

4.3.5 Comparison of Drug Delivery Scenario

Figure 4.9 depicts the drug delivery situation for all cases. The medication was given to SMC and ASMC for a total of 42 and 20 days. The medicine is being given by the controller at ISMC and AISMC for a total of about 120 and 21 days. Figure 4.9 show the drug delivery scenario for both cases. ST SMC and ASTSMC received the medicine for a total of 100 and 35 days respectively. The controller at TSMC and ATSMC is administering the medication for a total of roughly 18 and 17 days, respectively.

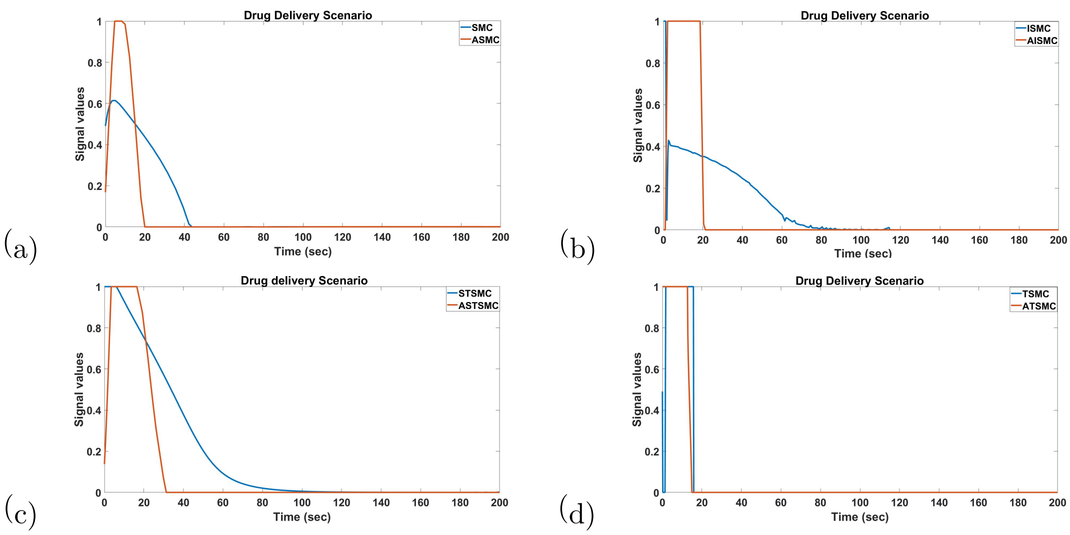

Figure 4.9: Comparison Of Drug delivery Scenario

4.4 Hardware-in-the-loop

Hardware-in-the-loop (HIL) simulation is a type of real-time simulation. HIL simulation shows how the controller responds in real-time to realistic virtual stimuli.

4.4.1 HIL Combination

HIL is the combination of Software in the loop and Processor in the Loop. While the system is processed on the processor in the loop, the simulation work is done on the software in the loop.[21],[22]
MATLAB/Simulink has been used to depict the performance of the suggested controller, and the HIL setup has been used for experimental analysis. The setup of the launch pad is shown in the Figure 4.10. The performance of the controllers as illustrated in

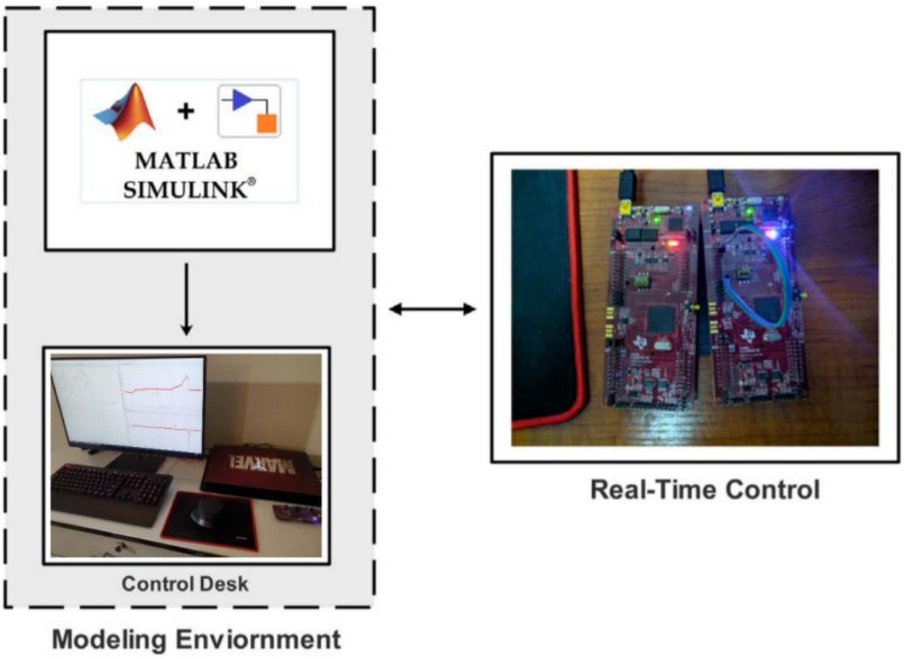

Figure 4.10: Hardware-In-The-Loop setup [3]

Figure 4.11 is further validated by the use of real-time controller hardware in a loop (C-HIL) experiment. These C-HIL tests are useful to confirm how the signed controller will operate in a real-world system. The model is simulated in MATLAB/Simulink, and the launchpad is a C2000 Delfino Microcontroller F28397D Launchpad that generates the control signals. From the figure4.11 we can say that in ASMC normal cells are tracking to their reference values within 63 days, tumor cells are tracking within 64 days, immune cells are tracking in 79 days and the drug at the site is being delivered within 45 days. In AISMC normal cells are tracking within 62 days, tumor cells are tracking within 63 days, immune cells are tracking within 78 days and the drug at the site is being delivered within 43 days however in ASTSMC normal cells are tracking with in 65 days, tumor cells are tracking within 65 days, immune cells are tracking within 80 days and drug at the site is being delivered within 70 days. On the other hand in ATSMC normal cell is tracking to their reference value within 60 days, tumor cells are tracking within 58 days immune in 63 days and the drug at the tumor site is being delivered within 37 days, whereas in MATLAB normal and tumor cells are tracking within 57 days, immune cells are tracking in 60 days, and drug at the site is being delivered within 20 days respectively. The delay in HIL simulation occurs due to the delay occurs when running in the simulation.

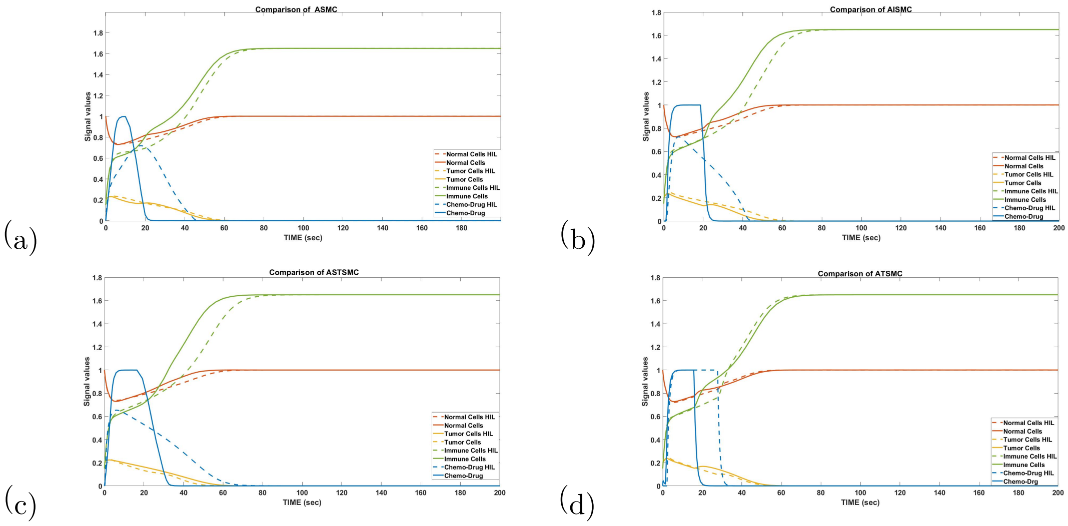

Figure 4.11: Comparison of Controllers

4.4.2 Benefits of HIL

1. Cost saving.

2. HIL testing can include scenarios that would be too risky or difficult to evaluate in a real-world setting. The HIL tests can be repeated. The HIL testing procedure is highly automated and supports multithreading, enabling numerous tests to run concurrently, and accelerating the development process.

3. Easily implemented on both simple and complex Systems.

4. HIL Simulations give confidence that the controller will work on the actual Hardware as well.

Chapter 5

Conclusion and Future work

5.1 Conclusion

1. Designed and implemented robust nonlinear controllers in MATLAB/Simulink.
2. Designed and implemented the adaptive nonlinear controllers in MATLAB/Simulink.
3. Results showed that Adaptive Terminal Sliding Mode Controller (ATSMC) performed better than other controllers.
4. In comparison to the already published nonlinear controllers for the said problem:
   - ATSMC tracked tumor cells to the reference value with 46% less drug dosage
   - ATSMC tracked healthy cells to reference value quickly with 5% less days.
5. Less drug dosage will improve the overall patient health since the side effects faced will be less.
6. Reduce treatment cost.
7. .HIL Simulations give confidence that the controller will work on the actual Hardware as well.

5.2 Future work

1. Future work can be done by applying barrier-based Non-linear Control Techniques.
2. Future work can be done by putting the proposed controller on actual platforms.

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