MECHANICAL STRENGTH OF TOOTH UNDER DIFFERENT CONDITIONS ENDODONTICALLY



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THESIS ACCEPTANCE CERTIFICATE

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Dedication

To my loving Father who has always guided and supported me, and to my mother without her Prayers I would not have been able to carry out my studies

Certificate of Originality

I hereby declare that this submission is my own work and to the best of my knowledge it contains no materials previously published or written by other person, nor material which to a substantial extent has been accepted for the award of any degree or diploma at RCMS, NUST or at any other educational institute, except where due acknowledgement has been made in the thesis. Any contribution made to the research by others, with whom I have worked at RCMS, NUST or elsewhere, is explicitly acknowledged in the thesis. I also declare that the intellectual content of this thesis is the product of my own work, except for the assistance from others in the project's design and conception or in style, presentation and linguistics which has been acknowledged.

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Wajeeha Batool

List of Abbreviation

Masticatory force	Force created by chewing		
Post and core	Procedure after RCT to provide support to tooth		
Conventional material	Most usually used		
FRC	Fiber reinforced composite		
Occlusion	Normal spatial relation of teeth when jaw is closed		
Ferrule	A cap placed to prevent splitting		
Dowel	A metal post or restorative material for distribution of		
	force and to reduce risk of fracture		

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Abstract

Root canal treatment (also called 'endodontics') is needed when the blood or nerve supply of the tooth (called the 'pulp') is infected through decay or injury. Initially, there is a pain in the early stages of the infection. In some cases tooth may get darken in colour, which mean that the nerve of the tooth has died. The procedure used to recover this, is root canal treatment.

Half of the world's population suffers from dental problems. In modern endodontic treatments, the emphasis is placed far more on cleaning than preparing the root canal system and filling procedures. This does not mean that root canal obturation is less important. Root canal procedure (RCT) is one of the most significant treatments which work for tooth prevention when no other treatment works. However after RCT, tooth may become vulnerable to decay.

It is extensively studied that tooth becomes weaker after root canal treatment, but still this topic is controversial. Life of root canal treated tooth is compromised as compared to healthy tooth in general.

Therefore in this research, we have conducted some experiments using imaged based modeling with Finite Elements to investigate stress analysis in the healthy vs. treated tooth (RCT). It is found out that the stress maps obtained from a treated tooth shows significantly higher stresses compared to the healthy tooth whereas the displacements calculated for healthy tooth are higher compared to the treated tooth. This pertains to the value of elastic modulus used for the treated tooth which is knowingly much higher than the healthy tooth. We have also conducted mathematical modeling for this study in order to validate simulated results. The percentage error is very low between the analytical (mathematical) vs. simulated results, which means our results hold true for this particular study.

CHAPTER 1 Introduction

1.1 Motivation of this Research

Motivation of this research came from daily life observation of dental problems. Dental problems are quite wide spread in common population.

It is important to address that how the tooth strength deteriorates with after the root canal.

Dentist cures infected tooth by medication or filling until the problem is severe. When the tooth is internally infected and damaged, the dentist decides for the root canal treatment. This is very common procedure in dentistry compared to the tooth extraction procedures. Regardless of the adapted procedure, filling material is used in root canal treatment. Some filling materials are considered to be mechanically much suited to the chewing forces generated by our jaws and some are just placeholder. No natter, what kind of filling material is used in the procedure, the tooth life changes from that instant. It could become stronger, or weaker, theoretically it is very well established (Tilashalski et al., 2004). However in terms of experiments, nothing can be said with certainty. If we look at the secondary data, the literature is full of studies, on the behavior of tooth after RCT and the reliability of using RCT as a method of prevention (Caplan and Weintraub, 1997). On the contrary, the issue on how it affects the tooth life and its functioning after RCT is not very clearly addressed.

Several techniques including wet lab, physical testing and FEA have been used to study the consequences for treated and non-treated tooth. However, in terms of mechanics FEA is numerically superior.

In this work Finite Element Analysis is used to measure the mechanical strength of tooth in both conditions, healthy vs. treated. Image processing, numerical modeling and analytical modeling is conducted to address the same. Image based modeling was done by using Simple-ware ScanIP, a software commercially available for processing image and generating model. Numerical analysis is done by using

software Abaqus CAE where it provides the option to fix the model from desired locations including applying load on specific point or area and specifying measures

for stress distribution analysis. To validate the results analytical solution is also calculated and results were compared.

1.2 (Miller et al., 2000) Research question

Mechanical strength of tooth varies under different conditions treated endodontic ally, or non-treated. After RCT a considerable amount of tooth structure is lost which makes tooth weak. Therefore,

"The aim of this research work is to investigate the, strength of treated tooth in comparison to healthy, non-treated tooth"

1.3 Research Objectives

The ultimate aim of this research leads to find out the effect of root canal procedure that how removing pulp inside the tooth is going to make the tooth weak. However, this can only be pursued by knowing that how much force is required to check the stability of tooth.

This is really important to study tooth because more than 90 percent of all systemic diseases produce oral signs and symptoms like AIDS/HIV and Diabetes. Health of mouth and even swab of saliva can tell a lot about one's health. Diseased, crooked, broken or missing teeth make chewing and biting food painful and also effect speech. Through enamel, tooth's root is exposed causing cavities which are painful and sign of tooth decay. In early stages, cavities are easy to cure, but in worst case tooth is numb and cavity is filled.

1.4 Application Areas

Application areas of this research are;

- Dentistry
- Root Canal Treatment

It will help clinicians to determine that how much mechanical strength of tooth is lost during and after the treatment procedure. Only 2% of GDP is spent on health care in Pakistan. Rate of dental caries is not way different from other countries, but cure is low. Even dental problems have increased according to last reported data of 12 years DMFT (Dynamical Mean Field Theory) increased from 0.9 in 1999 to 1.38 in 2003. Above age of 65, 95% people have dental problems, 22% women have bleeding gums and less than 28% children (below 12 years) have healthy gums. Among 10 most malignant cancers, oral cancer is 2nd most common cancer. There is a National health policy but it does not address oral health as much as required and no public policy at all (Harchandani, 2012).

In Pakistan this research will help to improve dental treatments which will help to improve overall health conditions.

1.5 Summary of the Study

Tooth is the only living part which cannot grow itself once fractured. When tooth infection exceeds, one of the way is to remove the infected pulp using root canal treatment (RCT). In this procedure, roots; the live part of tooth providing it nutrition is removed completely. This makes the tooth brittle. Although it is filled with filling material but still it is not as strong as the tooth with no treatment was.

In Root Canal Treatment it is necessary to assure that canals are filled with long lasting filling material, sealed properly and crowned in biomechanical way for long life of tooth.

A root canal is a treatment to repair and save a badly damaged or infected tooth. The procedure involves removing the damaged area of the tooth (the pulp), cleaning and disinfecting it and then filling and sealing it. The common causes affecting the pulp are a cracked tooth, a deep cavity, repeated dental treatment to the tooth or trauma. The term "root canal" comes from cleaning of the canals inside the tooth's root

This research work is conducted to calculate the strength of healthy tooth which has not undergone any treatment and then the tooth which was treated endodonically using procedure like root canal. The difference was numerically estimated at the end using finite element analysis and the results of simulations were validated using an analytical solution of a frustum cone.

CHAPTER 2 Literature Review

2.1 Overview

This chapter describes background and necessary information about tooth and tooth modeling, tooth's mechanical properties, mechanical behavior as well as image based modeling applications in emerging sciences with details on methods and techniques used in this study.

2.2 Tooth Anatomy

Morphologically, tooth may look quite simpler but it is an extremely complex geometry, with varying nerve supplies. There is dental layer which continues developing for five years. Later on with development of tooth it starts disappearing and at the end it forms pearls or islands within the jaw and gingiva.

Hardness varies is different teeth and different areas of same tooth, which is due to orientation and classification of ions. Enamel is the outer most layer of tooth which covers the anatomic crown of tooth. It is hardest structure of body; its hardness is 5 Mohs, on Mohs hardness scale.

Dentin is covered by enamel in crown and in root covered by cementum. Less mineralized, less hard than enamel but is still harder than bone. Highly elastic dentin has 1.6×10^7 psi elastic modulus. Its tensile strength is 6,000 psi and compressive strength is 40,000 psi (Giannini et al., 2004).

Middle of each tooth is hollow to make room for blood vessels in order to keep tooth alive. This collection of soft tissues is called pulp, also containing nerves and blood vessels. Nerves and blood vessels enters the tooth through a hole at very end of the root and travel up through pulp canal to fill up space in the crown called pulp chamber (Masthan, 2010).

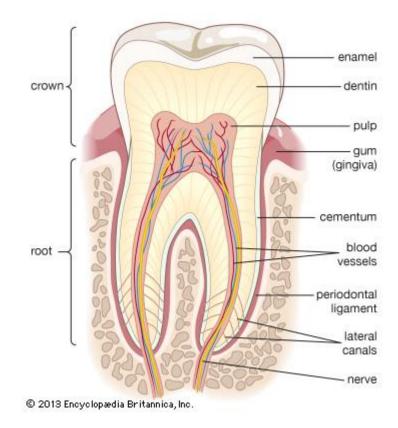


Fig 2.1 Anatomy of Tooth and Root

Wisdom teeth or third molars erupt around age of eighteen, but often removed to prevent displacement. Crown is present on each tooth, projecting into mouth whereas; root is present below gum line, projecting into jaw.

2.3 Modeling and Simulation of Biomaterials

Modeling and simulation is the method to understand the interaction of parts of a system and also the system as whole. In modeling and simulation method, a model is developed, simulated, results are calculated and iterations are continued until satisfactory understanding of results is developed.

Modeling and simulation in endodontic is comparatively a new domain. Whereas in the modeling and simulation of biomaterials is equally important. Any material that interfaces with biological system to evaluate, treat, augment or replace any tissue, organ or function of body is a biomaterial. Examples of other biomaterials include, bone, muscle, composites etc. Biomaterials are used to measure, restore and

Source: (http://www.valenciaperio.com/services/dental-implants/ct-scanner/, 2015)

improve physiological function and enhance survival and quality of life. Biomaterials can be divided into different classes;

- Polymers
- Metals
- Ceramics
- Natural
- Composites

Reality is too complex to model therefore virtual reality models are used. These models are simplified representation of reality. The virtual models can be utilized for simulation, prediction, diagnostics, performance evaluation or control system designs.

2.4 Methods used in literature for image-based numerical modeling of tooth

There are several methods in the literature which can be used for addressing virtual reality models in the field of dentistry. This section is going to first detail all the imaging modalities, including computed tomography, Magnetic resonance imaging and positron emission tomography, then numerical methods used so far, and why Finite Element Method is a specific choice for this study.

2.4.1 Imaging modalities

To create visual representation of the body image modalities are used. These image modalities are used for clinical analysis, medical intervention and to build a biological model of any body organ or part for modeling and simulation. There are different types of imaging modalities which are described briefly in the following sections.

2.4.1.1 Computed Tomography (CT)

Computed tomography is a technique to create detailed images of internal body organs, bones, tissues and blood vessels. CT scans can create images on multiple planes and also three dimensional planes which can be viewed on computer screen (Brenner and Hall, 2007).

CT is cone beam tomography in which c-arm of machine is rotated around the head in 360-degree rotation to capture images in all dimensions. These images are then compiled to make a single 3-D image (Brandt et al., 2006).

CT scanners were first introduced in 1971 with a single detector for brain study under the leadership of Godfrey Hounsfield, an electrical engineer at EMI (Electric and Musical Industries, Itd) (Glenn Jr et al., 1975). Thereafter, it has undergone several changes with increase in number of detectors and decrease in the scan time.

CT scans are used for dental problems, joint disorders and bone problems and many others (Jerusalem et al., 1999).

2.4.1.2 Magnetic Resonance Imaging (MRI)

Magnetic resonance imaging is a technique used in radiology to generate images of body organs. It is used to diagnose and treat diseases by creating detailed images of bone, tissue, and other body structures. This helps to understand anatomy and physiology of the body organs. MRI shows the structure inside the body different than x-ray, CT or ultra sound, which cannot be seen by these technologies. It shows abnormalities using contrast techniques.

Human body is made up of water molecules which comprises of hydrogen and oxygen atoms. Atoms have proton in their center which act as magnet and is sensitive to magnetic field. Whereas MRI scanner contains two magnets, on entering the body first magnet aligns the water molecules which were random earlier. The second magnetic field is turned on and off in series of quick pulses causing the hydrogen atom to alter its position and again going back to relaxed state. Magnetic field is created by passing the electricity (Young et al., 1996).

MRI scans are used for multiple purposes including detection of abnormalities of brain, spinal cord, joints and uterine in women undergoing evaluation for infertility. It is also used to detect certain types of heart problems, injuries, diseases of liver and causes of pelvic pain in women (Lustig et al., 2007). In this study we are not going use MRI scans as most of our focus was on the dense parts of the tooth and not the soft tissue and muscle anatomy. Whereas, MRI is most suited to the environment where we need high resolution scans of soft tissue anatomy as well.

2.4.1.3 **Positron Emission Tomography (PET)**

Positron emission tomography is imaging technique which uses radioactive substance called tracer to detect the disease in the body. PET scans tests shows structure and blood flow in the organs. Tracer is injected into vein and absorbed by body organs and then high lightened by PET scanner. In this way functioning of body organs is observed (Phelps et al., 1988).

Unlike CT and MRI, PET scanner can scan organs to the cellular level. It can show the metabolism of cancer cells, its spread and even response to chemotherapy. CT and MRI can show the shape and size of the organ but PET shows how the organs are functioning (Bailey et al., 2005).

Besides this, PET scanner is also used to detect cancer, heart and central nervous system problems, brain disorder, brain tumors, memory disorders and seizures. Which part of brain is affected by epilepsy, how much sugar brain is up taking can be detected by PET scans and helps to decide the proper treatment in response to any disease or abnormality (Ollinger and Fessler, 1997).

Positron Emission Tomography is more suited for the cancer studies where it is possible to trace the nitrogen based compounds in the body which cause cancer.

2.4.2 Numerical methods

2.4.2.1 Finite Element Method (FEM)

Finite element method is a technique of discretizing any problem into smaller elements to find its approximate solution. These smaller elements or simpler parts are called finite elements which have simple geometrical shapes. These elements when combined are called mesh, finite element mesh (Szabó and Babuska, 2011). Finite element Method (FEM) or alternatively, Finite Element Analysis (FEA) is a method which can be used as simple function for each element such as linear or quadric poly nominal equation to evaluate boundary value problems and partial differential equation (Bathe and Wilson, 1976). Then results from each simple equation are collected and assembled to find approximate solution of whole problem.

Finite element method or finite element analysis is used because it is a widely used method in which order or the elements can be increased for simulation and analysis.

FEM has become most successful method for analysis of dental problems. Tooth has complicated structure and geometry but with help of finite element method accurate representation of geometry is possible. In this method, material properties, loading conditions and other desired properties according to model are assigned and analytical model is created. Later, by using an appropriate engineering suite, realistic and reliable solutions are produced (Van Staden et al., 2006).

With reference to the dental implants, Finite Elements has played a magnificent role in many ways. It is reliable numerical method to measure mechanical strength of different filling materials used to fill the roots of teeth after root canal procedure. Following this in 2005 a 3D Finite Element Analysis was carried out to examine better mechanical properties of dental restoration, specifically customized post system, on already validated experimental work. This study showed that although post types and materials do not affect the stress values but new customized composite materials reduce probability of failure inside the dental regions (Genovese et al., 2005).

After root canal procedure, tooth is usually filled with a restorative material. Filling materials have different elastic properties which play significant role in tooth's durability. Elasticity of material can increase or decrease the strength of stress on tooth. In 2006, finite element models of human maxillary central incisors were developed by a study led by Barjau-Escribano. According to which, depending upon type of prefabricated material, fracture strength and stress distribution was analyzed. 60 human maxillary central incisors were extracted, treated endodontically and were restored; 30 with glass fiber and 30 with stainless steel. After applying theoretical and experimental methods, FE model was generated and 520N and 803N stress was calculated. Restoration material having elastic modulus similar to surrounding (glass fiber) showed less fracture and also allowed further repair (Barjau-Escribano et al., 2006).

People now a days don't just want healthy but beautiful teeth, which is termed as esthetic posts. These esthetic post systems are used for fracture resistance as well as to beautify tooth are very less known yet. To use these posts it is necessary to evaluate their mechanical strength. For this purpose, forty human maxillary, divided into four groups were studied with different restoration materials which were titanium,

quartz fiber, glass fiber and zirconia. Load was applied until tooth fracture. Higher failure rate with quartz and glass fiber were observed as compared to other filling materials (Akkayan and Gülmez, 2002).

In dental research, both experimentally and FEA (bi-dimensional models) are studied to analyze mechanical behavior of different filling materials. In a study, glass fiber was analyzed to compare with either commercial carbon fiber post or a gold alloy cast post. Gold cast post-and-core were observed with great stress at post dentin interface and fiber reinforced composites posts presented high stress in cervical regions. Considering natural tooth as a model, glass fiber composite post induces same stress as the natural tooth (Pegoretti et al., 2002).

Effect of variables like material, shape, modulus of elasticity, diameter, and length are important for tooth modeling. In 2005, these parameters are taken into account by Amussen [reference] who investigated to observe to find stress in dowel restored teeth. Finite element method was used and results concluded that preferred dowel should be wide as allowed by anatomy of tooth, bonded, and parallel sided with high elastic modulus (Asmussen et al., 2005).

Using finite element method, different filling materials can be possibly compared. A study on maxillary incisors was carried out to compare stress distribution among cemented steel, glass and carbon posts. Using FEA, 10 N load was applied on model. Stress calculated for steel post was higher (7.5 MPa) as compared to other materials (3.6 MPa and 5.4MPa). It was concluded that materials more rigid than natural tooth create tension and shear stress; moreover, they do not allow homogeneous stress distribution (Lanza et al., 2005).

To compare durability of glass fiber and Ni-Cr alloy cast post a study was conducted. Eight 3D models of maxillary central incisors were generated to investigate and compare shear stress using ABAQUS/CAE software. At interfacial of post and dentine, glass fiber shows reduced stress as compared to Ni-Cr alloy cast dental post (Madfa et al., 2015).

Fracture resistance ability of filling material can decrease the risk of fracture after treatment. In response to force, behavior of cemented composite was studied in 2016. In this study by occlusal forces stress generated in tooth and indirect

composite resin was calculated to find out fracture resistance ability. 3D FEA was done taking first premolar as model, using cemented composite resin to calculate peak von Mises stress values. Calculated von Mises values concluded that cemented resin reduce the occlusal stress which showed that onlay preparation reduce stress (Mei et al., 2016).

Talking about inlay restorations, these are indirect fillings. Onlay restorations are also indirect fillings except the difference that they are incorporated on the missing are of cusp, which is elevated area of tooth used for grinding. In case of indirect inlays and onlay restorations fracture resistance and fracture mode was studied. Naturally premolars are more susceptible to fractures as compared to molars that is why 55 extracted premolars were studied. A force was applied and a ball of 6mm was kept based on its ability to contact with buccal cusp, palatal cusp and restorations with equal distances. Results evaluated that fractures following inlays were more severe as compared to onlay restorations (Alshiddi and Aljinbaz, 2016).

Another study on tooth modeling came in the year 2001, which deployed brick element technique to model 3D solids and thick shells. Four models of teeth with different properties comprising 3D brick and tetrahedral elements were used to analyze stress. Endodontic post is necessary to preserve maximum dentin structure. Composites have stiffness near to dentin, have gradual decay and their efficient geometry was presented (Joshi et al., 2001).

In 2006, evaluation of natural tooth and maxillary central incisor was carried out to compare their mechanical behavior. Five 3D FE models with different material properties were generated and applied force of 10N to calculate stress distribution (von Mises stress). Accuracy of model was checked by convergence test. It was observed that highly stiff materials resist deformation creating high stress values, whereas material less stiff or having properties similar to natural tooth, cause less stress comparatively (Zarone et al., 2006).

In practice, it is commonly observed that multi-purpose techniques can increase the durability of endodontically treated tooth [reference]. This study was carried out to confirm the phenomenon that multi-post technique usability for the same. A 3D FE model was generated and effect of PCP's was compared to traditionally used filling

materials. As a result, it was seen that multi post technique showed significant reduction in stress ultimately reducing the risk of fracture (Maceri et al., 2007).

Taking previous studies into consideration and observing clinician's priorities, it was concluded that materials having elastic modulus as dentin; like carbon, mineral (fiber posts) and glass are preferred because they do not create as much stress as metallic posts. Furthermore, fiber posts should be as deeply inserted as possible, their diameter not effecting the stress distribution (Boschian Pest et al., 2006).

Tooth's durability is dependent on stress and strain distribution after RCT. For stress and strain to be distributed homogeneously, choice of restorative material cannot be compromised. Eighteen experimental models were generated for FEA to observe best restorative material. 10N load was applied on crown and von Mises stress was selected as stress criteria. Cervical region was affected more because of higher rigidity of filing material. From outer to inner part stress decreased due to material properties of crown, cement, dentin/posts (Sorrentino et al., 2007).

Different regions of tooth have different response to filling material and fracture. Depending on type of crown and restoration procedure, cervical region is more susceptible to fracture. Seven 3D models were created to study the parameters effecting corono-radicular restoration by FEA. Despite of material type, tooth with post coronal ferrule were less subject to stress and placing crown fulfills the purpose of RCT, which is to save cervical region of tooth (Pierrisnard et al., 2002).

Stress concentration can vary in different regions of tooth. Ceramic posts and fiber glass posts were compared in another study [Reference] to find stress concentration according to tooth region. For this purpose, FEA was conducted to calculate shear stress and von Mises stress after load was applied. As a result, in case of glass fiber model, stress concentration occurred in cervical region, whereas ceramic post preserving the dentin, showed stress concentration in proper post adjacent to apical region (Amarante et al., 2008).

Stress distribution on tooth plays important role in tooth fracture as well. If stress is more homogeneously distributed, there are less chances of fracture. That is why materials showing homogeneous stress distribution when load is applied, are considered best which are not specified yet. For this a 3D FEA model was generated

with different restorative materials to calculate von Mises stress in mandibular first molar. Regardless of material properties or loading, cervical region of tooth is more susceptible to fracture. Onlay restoration show better stress distribution than inlays. Materials like gold, having high elastic modulus show better stress concentration (Jiang et al., 2010).

Temperature is an important phenomenon which cannot be ignored while studying restorative materials. Drinking and chewing food expose tooth to hot and cold temperature causing expansion and contraction in response. A study was conducted to investigate the effect of temperature on stress distribution using two different restorative materials, stainless steel and glass fiber. Taking upper right incisors 3D FEA models were generated applying 100N static vertical occlusal force on center of surface of tooth. The result was conducted that stainless steel create more stress as compared to glass fiber, in response to thermal heat (Değer et al., 2015).

By critically reviewing the literature so far, it is observed that finite element analysis has been playing an important role for tooth life and hygiene. There are studies on tooth modeling that deals to determine mechanical strength of teeth.

2.4.2.3 Finite Difference Method (FDM)

FDM is a direct approach to numerical solution to solve partial differential equation as compared to other methods. This method provides more accurate results that need accurate geometrical shapes (Mitchell and Griffiths, 1980). FDM is difficult to use for irregular geometries compared to FEM In equations, derivatives are replaced by algebraic approximations. Final equation has no derivatives but algebraic equation which is used to calculate approximate results (Narasimhan and Witherspoon, 1976).

2.4.2.4 Energy

Energy method is a principle about conservation of energy. This method is based on linear elastic behavior of body in which work done by external forces is equal to stored energy of body is response to load. Specific structure or body deforms itself and reach to the point of minimum potential energy. This behavior is adapted at low temperatures and the lost potential energy is converted into kinetic energy.

For analysis of deformation and stress in elastic structures, principle of minimum energy is used. In this method an expression for potential energy of structure over whole domain is minimized (Straughan, 1992). This method actually expresses the relation between deformation and stress and forces applied externally and internally and also the effect of external factors. In this way, this relation provides governing equations for deformable bodies in structural mechanics. Furthermore, this method also provides approximate solution by skipping through complex parts of the problem and solving the partial differential equations (Joseph, 1966).

2.5 Conventional approaches for material testing:

There are several approaches to test materials. Routine laboratory experiments with help of equipment and machines refer to wet lab techniques.

2.5.1 Wet lab technique

Chemicals, drugs, instruments or tools handled experimentally in laboratory are related to wet lab. For tooth analysis wet lab techniques are also used for different purposes. To measure the strength of tooth, best filling material, effect of crown and tooth's life duration under different conditions is measured and analyzed in wet lab. Material Testing methods for physically measuring the strength includes physical testing procedures which is discussed in the next section.

2.5.2 Physical Testing method

In laboratory, physical testing is done for different materials. Bone can also be tested with machine physically to measure its mechanical strength.

A study was conducted to observe that how ability of posterior teeth is effected by root canal treatment (RCT). For this study, 50 premolars were selected, divided into five groups and load was applied by mechanical test machine (EMIC). For FEA, five 2D models were generated by ANSYS. As a result, it was concluded that composite resins play obvious roll in tooth strengthening because removal of inner dentin during RCT reduce the fracture resistance ability of tooth (Soares et al., 2008).

2.5.3 Why not Traditional methods

A review was conducted that whether endodontic treatment needs restoration. If yes then what type of treatment, material for restoration and which conditions of tooth are to be fulfilled. Although many changes have taken place in RCT procedures, material range and crown type but still basic technique is the same. Root canal treatment (also called 'endodontics') is needed when the blood or nerve supply of the tooth (called the 'pulp') is infected through decay or injury. You may not feel any pain in the early stages of the infection. In some cases your tooth could darken in colour, which may mean that the nerve of the tooth has died (or is dying). This would need root canal treatment.

Posts, (which are a type of dental restoration used either to stabilize a weakened tooth or provide an anchor for a crown) do not strengthen the tooth but provides support to avoid further distortion so should be selected wisely. Adequate strength, modulus of elasticity, retention, biocompatibility, esthetics and retention of tooth should be priorities (Cheung, 2005).

Traditional methods are not preferred in comparison to numerical methods because in traditional methods we have to buy material, equipment and time for testing. It is time and cost consuming process.

In the contrary, simulation is free and effective. We can do the same by computer without physically testing them (Hughes, 2012).

2.5.4 Root Canal Treatment

Root canal treatment (also called 'endodontics') is needed when the blood or nerve supply of the tooth (called the 'pulp') is infected through decay or injury. You may not feel any pain in the early stages of the infection. In some cases your tooth could darken in colour, which may mean that the nerve of the tooth has died (or is dying). This would need root canal treatment. A root canal treatment is required when the pulp of tooth is infected. If the pulp becomes infected, the infection may spread through the root canal system of the tooth. This may eventually lead to an abscess. An abscess is an inflamed area in which pus collects and can cause swelling of the tissues around the tooth. The symptoms of an abscess can range from a dull ache to severe pain, and the tooth may be tender when you bite. If root canal treatment is

not done, the infection will spread and the tooth may need to be taken out. This study will use a root canal treated tooth for the investigation of mechanical strength of the tooth, in different endodontic condition. The summary of the whole chapter is entailed briefly in the next section.

2.6 Summary of the Chapter

Tooth has same structure as bone but unlike bone, tooth is unable to grow itself. Once broken, fractured or deformed, it is unable to heal itself or go back to its natural position. Root canal treatment is adapted to save the tooth with infected pulp which makes the strength of tooth very less. Mechanical strength of tooth is very important to investigate as its main function is grinding and crushing food which takes energy or strength (Van Staden et al., 2006). That is why to measure mechanical strength of treated and non-treated tooth is an important topic to investigate.

Sr. No	Techniques used	Year of publication	Scientist name
1	FEM	2001	(Joshi et al., 2001)
2	FEM	2002	Pierrisnard et al., 2002)
3	Universal testing machine / wet lab	2002	Akkayan and Gülmez, 2002)
4	FE-Experimental work	2002	Pegoretti et al., 2002)
5	FEA	2005	(Genovese et al., 2005)
6	FEA	2005	(Lanza et al., 2005)
7	Survey/ review	2005	(Cheung, 2005)
8	FEA	2005	Asmussen et al., 2005)
9	FEA	2006	(Barjau-Escribano et al., 2006)
10	FEA	2006	(Zarone et al., 2006)
11	FEM	2006	(Boschian Pest et al., 2006)
12	FEA	2007	(Maceri et al., 2007)
13	FEA	2007	(Sorrentino et al., 2007)

Table 2.1 Related Studies till Date

14	FEA / Mechanical test machine	2008	(Soares et al., 2008)
15	FEA	2008	(Amarante et al., 2008)
16	FEA	2010	(Jiang et al., 2010)
17	FEA	2015	(Değer et al., 2015)
18	FEA	2015	(Madfa et al., 2015)
19	FEA	2016	(Mei et al., 2016)
20	Wet lab	2016	(Alshiddi and Aljinbaz, 2016)

CHAPTER 3 Materials and Methods

In this study we have used three fold methodologies. At first computer tomography scan is used for image based modeling using Simpleware suite, then finite element analysis was performed. Later on, the simulation results were validated by using, mathematical equation (analytical solution). First we have reported all the methods which are used in literature and then the proposed methodology is detailed.

3.1 Methods used in literature

In literature different methods has been used including FEM/FEA, wet lab techniques and mechanical testing. All of them have been discussed in detail in chapter 2 - (section 2.4)

3.2 Proposed Methodology

Image based modeling involves light, geometry and materials in the process. Image based approach eliminates the labor which is required for modeling of detailed geometry of objects. It also handles the real world objects by capturing image and applying graphic techniques. (Ref: chapter No. 2)

3.2.1 Why CT?

In dentistry, a standard x-ray does not give enough information required for dental implants. It gives only two dimensional views, which is not sufficient. Computed tomography is a three dimensional technique, in which whole jaw is scanned. This helps the dentist to determine best location, angulation, diameter and length of implants to be placed. CT scan of jaw shows density, volume and anatomy of the tooth which is essential to create exact geometry of tooth for modeling.

3.3 Work flow

Finite element workflow is shown in figure 3.1.

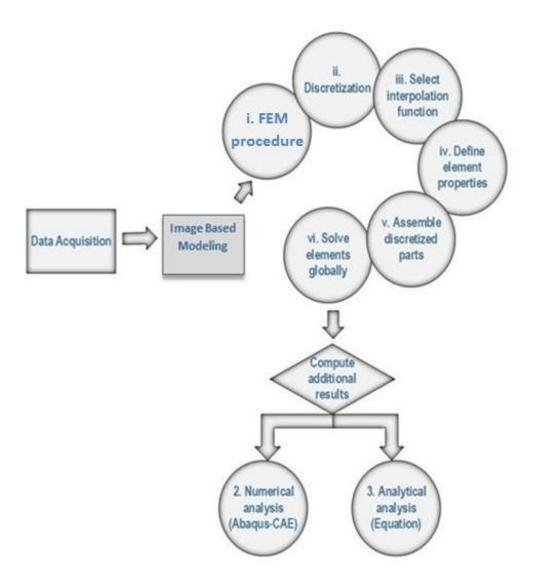


Figure 3.1 Steps of finite element modeling

3.3 Image based modeling

Image based modeling is a sampled representation of geometry. Two dimensional images are captured and gathered to make a three dimensional image to view on computer screen. Data was based on CT scans of human tooth; of two types.

- Healthy tooth (non-treated)
- Root Canal Treated tooth (treated tooth)

• For both types same procedure was adopted and preceded.

3.3.1 Image acquisition

In this research work, computed tomography scans were used. The scans were acquired from an independent researcher, Dr A. Madarati from Madina University Saudi Arabia (KSA) through personal communication. However, the gist of the study was designed independently regardless of the data acquisition. Data set was acquired at a resolution of 19.1 microns using Nikon Custom XCT installed at the University of Manchester UK. Image pixel size was 0.0191mm and object to source distance was 203.55mm. In total there were 998 projections acquired for the non-treated tooth and 1260 projection were acquired for treated tooth. Source Voltage was (kV) =100.000000 and Source Current was (uA) =98.000000.

Data set consisted of two types.

- 1. Human canine with no treatment
- 2. Human canine with root canal treatment.

Computed tomography scans of human canine are obtained for three dimensional finite element analyses.

I. Geometry of non-treated tooth

Geometry of tooth was measured in ABAQUS using a measuring tool, to compare both models as geometry noticeably affects the overall results. Length of non-treated tooth was 14mm and width calculated from upper, middle and lower part was 5.94mm, 4.65mm and 2.01mm. Non treated tooth has pulp in it, which contains nerves and tissues and were assigned the same material properties as pulp using a value from the literature.

II. Geometry of treated tooth

When treated tooth was measured, its length was also 14mm. Width from upper, middle and lower part was calculated as 7.83mm, 6.72mm and 2.01mm; which is different from non-treated tooth. Because it has no pulp in it, therefore, the total slices obtained were 998 slices with no pulp within. Pulp is removed during root

canal treatment, so here material properties of filling material are kept which are different from pulp and near to dentin, specifically of gutta-percha. Therefore we assumed in our analysis that the pulp has been filled in by a restorative dentin material which was given the value of elastic modulus as in the literature.

III. Difference in geometry

Treated and non-treated tooth are different in geometry as analyzed from the measurements. Treated tooth is wider and expanded in size as compared to non-treated one. Treated tooth has no pulp, and instead a dentin material which flattened its size more than a normal tooth. Whereas the non-treated tooth was filled with a pulp, as of normal tooth geometry and mechanics were very different from the non-treated one. This difference in geometry changes the behavior of tooth significantly. Canals volume increases after the tooth is prepared for treatment and canals become slightly wider (Paqué et al., 2009).

3.3.2 Image Processing using Simpleware Ltd.,

In first step simple analysis was conducted to work out methodology. Data set was imported to ScanIP as stack of raw images, keeping X, Y and Z dimensions equal.

Model was down sampled to 10% of the original resolution. This model was thresholded for a new mask generation exactly similar to the normal tooth. Noise was reduced by using morphological filters including median, mean and Gaussian. This was all done in an advanced image processing suite, called Simpleware Ltd. Virtually, to match up with the tooth real anatomy; the three more layers were created by using some dilation and erosion algorithms in Simpleware. At the end the layers were discriminated from each other using the Boolean operation.

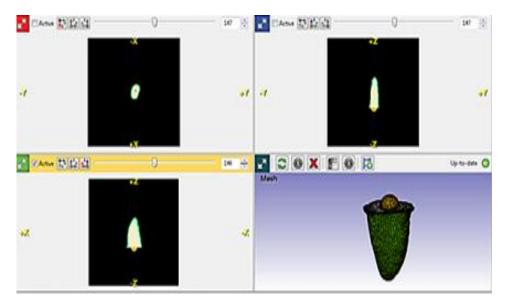


Figure 3.2 Image Processing of Non-Treated Tooth

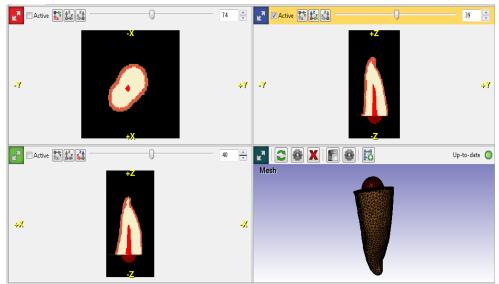


Figure 3.3 Image Processing of Treated Tooth

Original data set as "stacks" was kept to compare the differences or changes in the rendered data during image processing. Furthermore, original data can provide information that how much of original data is lost during image processing which helps in better analysis. More change in original data is actually loss of information which should be avoided as much as possible to get accurate results.

Moreover, extra black background with zero-pixel value was removed by cropping the image from X, Y and Z dimensions. Cropping is done to obtain only related information that is required for modeling and minimizing the computational cost of the solution as well.

As we have discussed above, tooth has a cavity which has pulp in case of normal tooth and a restorative material in case of treated tooth, therefore to highlight cavity as a part of image based model a mask of 1x1x1 pixel was generated. The original dentin was then removed from this new mask.

To remove extra noise around cavity and dentin different filters were used. Median filter was used to increase the image quality and close filter was used to fill the spaces for smoothing.

Natural tooth has different layers around dentin which are not scanned in the data set because CT scans create images on the basis of pixel values. In case of tooth, CT scans captures the bone only and extra layers are to be generated to make tooth's anatomy and mechanical properties as the natural one. To generate extra layers, scans needed padding. Padding is a tool in ScanIP which provides extra back ground, required to create layers around dentin. Following layers have been generated, in the process:

I. Periodontal layer:

Naturally tooth is inserted into periodontal ligament which provides strong fibrous union between tooth bone and root (Elton, 2002). Therefore, by dilation filter a 0.3 mm periodontal layer was generated.

II. Cortical bone:

After periodontal layer, there is cortical bone providing support, protection, storage and release of calcium and reasonable movement (Scott and Turner, 2000). Again using dilation filter a 0.5 mm cortical layer is generated around periodontal layer.

To fulfill the requirement of tooth loading, loading location, a semi-circle of 4mm is generated and kept on surface of tooth above the cavity.

Different smoothing filters were used to make scans fine and ready for better meshing. Also undesired pixels and noise are removed by painting and un-painting on specific slices. Unnecessary pixels were removed and smoothing filters were applied again to make the images more refine. In this way model was generated consisting of four layers compared to the original tooth anatomy for finite element analysis.

3.3.4 Numerical Modeling using FEA

During the numerical modeling procedure, a finite element model of the root of the tooth was created such that it has a reasonable mesh density and realistic material properties. As we know that the results obtained from finite element analysis are usually approximated, therefore to check the accuracy of our results, we performed convergence analysis and sensitivity analysis in our models.

Convergence analysis was performed for four iterations. A direct solver was used to address this which is built-in in Abaqus CAE. In total five optimization cycles were performed for each model.

1. Model Creation

Model was created using Simpleware CAD software, for creating 3D geometry of tooth. For creating geometry of tooth mesh was created.

2. Mesh Generation

In this particular work tetrahedral elements were selected to generate mesh for model because tetrahedral elements give quick results as compared to hexahedral elements.

There are two methods for meshing

Grid meshing - Comprising of 4 steps

Free meshing - Comprising 7 steps

Here we adopted free meshing procedure completing seven steps for each model, treated or non-treated. We have adopted for free meshing algorithm because it take less computational time for mesh convergence.

After mesh generation, model was imported to FEA software Abaqus where boundary condition and load were applied.

2. Boundary conditions

After creating mesh boundary conditions were applied by defining the area where load is to be applied and mentioning constrains.

I. Convergence analysis

Convergence test gives successive approximation of iterative methods, in which it creates sequence of results after which this repeating procedure (with different mesh size) is terminated (Bathe and Wilson, 1976).

Four levels for convergence analysis were achieved (table 3.1) after which the solution for displacement as well as stress analysis converged regardless of how big the mesh gets.

Four models with different number of elements were generated ranging from 42,255 to 68,703 elements on different levels. Variation in number of elements provides different level of complexity, more the number of elements, model will be more complex. Complex model takes more computational power and time but on the other hand results are more refined.

FE model was created and all the parts were dragged into the model. A semicircle was created by using 3D-editing.

Model	Mesh density (Non-treated tooth)	Mesh density (Treated tooth)
1	49,060	29,148
2	55,309	30,165
3	72,943	43,044
4	98,543	77,018

Table 3.1 Mesh Density of Non-Treated and Treated Tooth

Within the Simpleware suite it is possible to adopt a mesh adaptively path by controlling the min and the max edge lengths.

Model	Compound Coarseness for mesh density	Edge length minimum	Edge length maximum	No. of Elements
1	- 45	0.9	1.90	49, 060
2	- 40	0.73	1.73	55, 309
3	- 35	0.56	1.56	72, 943
4	- 30	0.45	1.45	98, 543

Table 3.2 Non-Treated Tooth: Difference in Edge Length and Element Size inDifferent Levels

Similarly for treated tooth, mesh adaptively feature was adopted to control the mesh density and the computation time for each analysis. We started from a very coarser to the very fine mesh and final the solution was converged.

Model	Compound Coarseness for mesh density	Edge length minimum	Edge length maximum	No. of Elements
1	-50	2	4	29, 148
2	- 50	1.5	3	30, 165
3	- 50	0.7	1.7	43, 044
4	- 50	0.4	1.4	77, 018

Table:3.3 Treated Tooth: Difference in Edge Length and Element Size inDifferent Levels

II. Sensitivity to materials

Numerical approximation in computing may lead to round off errors. These may accumulate in a long running simulation and affect the final results. The analyst should test sensitivity of the results to the simulation parameters in the software used.

In dentistry, there are different aspects of filling material and its sensitivity, like biocompatibility of material, its apical stability and handling properties. Materials having mechanical properties like dentin are preferred because they will behave same as dentin (ØRstavik, 2005).Use of different material properties may lead to large errors in the simulations eventually. Table 3.4 and 3.5 shows the values of

elastic modulus used for tooth modeling in this study. All material properties were used as it is, in the simulation experiment.

Material	Modulus of elasticity (MPa)	Poisson ratio	Reference
Pulp	2.07	0.45	(Peyton et al., 1952)
Dentin	18600	0.31	(Rubin et al., 1983)
Periodontal ligament	68.9	0.45	(Weinstein et al., 1980)
Cortical bone	13700	0.3	(Carter and Hayes, 1977)

 Table 3.4
 Material Properties of Non-Treated Tooth

Material	Modulus of elasticity (MPa)	Poisson ratio	Reference
Filling material (Gutta-percha)	0.69	0.45	(Shetty et al., 2013)
Dentin	18600	0.31	(Rubin et al., 1983)
Periodontal ligament	68.9	0.45	(Weinstein et al., 1980)
Cortical bone	13700	0.3	(Carter and Hayes, 1977)

Table 3.5Material properties of treated tooth

Abaqus CAE

In today's era, product simulation is often being performed by engineering groups using niche simulation tools from different vendors. In this case, The Abaqus Unified FEA suite offers powerful and complete solutions for both routine and sophisticated engineering problems including a full vehicle loads, dynamic vibration, multibody systems, impact/crash, nonlinear static, thermal coupling, and acoustic-structural coupling using a common model data structure and integrated solver technology. In this study, we have also used Abaqus for the simulation of tooth mimicking root canal procedures.

Models are imported to Abaqus Boundary conditions applied in ABAQUS, as ENCASTRE. Boundary conditions are to fix the dentin. The whole root, except the upper surface is fixed by applying boundary conditions.

Semicircle was placed on of the dentin as shown in fig (3.3) is selected by angle to apply load in Abaqus CAE. Semicircle is placed for distributed load (Madarati et al., 2010). A load of 100N is applied as pressure (Joshi et al., 2001), on upper part of tooth. The pressure was calculated over the covered area of the semicircle. The covered area of the semicircle was 16.732 mm² and the expected force extracted from literatures was 100. This enabled us to apply a pressure of 5.97 Pa over the covered area of semicircle.



Fig 3.4 Non-Treated Tooth with Semicircle



Fig 3.5 Treated Teeth with Semicircle

After the analysis, results were computed for both Von Misses Stress (MPa) and the Displacements (mm). Results are presented in the table, showing the sequential difference in each model.Displacement and Von Mises stress is calculated after

removing the semicircle, keeping the element with maximum stress value under consideration. In this way we got first convergence test.

3.5 Complex analysis

Complex analysis is basically a replication of simple analysis. Data set is neither resampled nor resized and purely 100% is imported to ScanIP for image processing. For this particular study, models were not developed with full resolution due to limited computational source. So, the results were conducted only with help of 10% resampled data set.

3.6 Mathematical Modeling using Analytical Solution

Mathematical modeling is important requirement while looking for an analytical solution. More than anything else, an *analytical approach* is the use of an *appropriate* process to break a problem down into the smaller pieces necessary to solve it. Each piece becomes a smaller and easier problem to solve. The fundamental premise of the analytical solution is that, it's the only reliable way forward.

Principally, mathematical modeling is applying mathematical equation for solution of real world problems. It uses mathematical model to describe behavior of the system. In mathematical modeling, it is important think out of the box as opposed to glass box (which is thinking inside the box). You can see inside the glass box but not in the black box. Therefore problems in our life are all black boxes, for example tooth infections, tremor modeling, tumor classification and or postural studies are important for life sustainability but they are too complex to get into the root cause. To do this the whole problem first undergo a "make or a break" process to solve for an effort, this process is called as root cause analysis. Skipping root cause analysis is like going to the doctor without diagnosis. Moreover, the numerical solvers available to date, only do what they are told to do therefore a model in terms of its physical as well as mathematical behavior is required. In mathematical modeling, a physical dimension of the problem in hand is sketched and its behavior is predicted using simple mathematical equations. Here in this research work, we have used mathematical model to validate the results obtained from finite element analysis

For the deformation of the uniform objects, axial deformation is usually calculated by using the following formula (for d=0) where d corresponds to diameter.

$$\delta = \frac{PL}{AE}$$
 (1)

Where, for the differential strip shown: $\delta = d\delta$ P = weight of segment y carried by the strip L = dyA = area of the strip

W= weight per unit volume

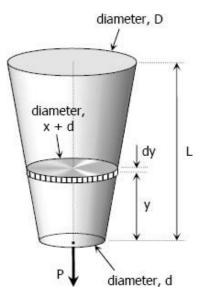


Figure 3.6 Frustum Cone (http://www.mathalino.com/reviewer/mechanicsand-strength-of-materials/solution-to-problem-219-axial-deformation)

In our case the root of tooth is analogous to the cone model with non-zero diameter. Therefore we treated the root as frustum cone (Figure 3.6). Therefore the formula for the non-uniaxial deformation would become:

$$\delta = \frac{WL^2}{6E} \qquad \dots \qquad (2)$$

Equation (2) represents Non uniform deformation formula.

Alternatively, by using varying diameter of the frustum cone, we can also use:

$$\delta = \frac{wL^2(D+0)}{6E(D-0)} - \frac{wL^2(0^2)}{3ED(D-0)}$$
(3)

Either equation 2 or equation 3 is used, the results turns out to be the same. Where L is length of root (without crown) and F is Force applied.

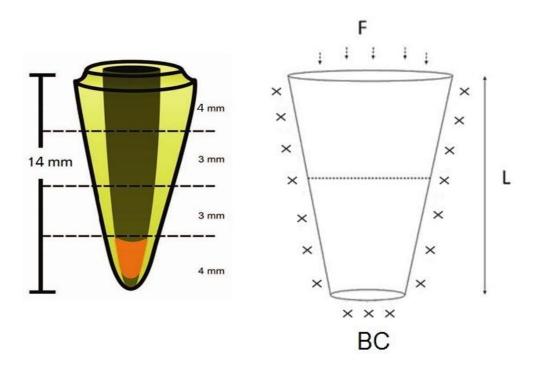


Figure 3.7 (a) Mathematical model of tooth derived from (Bacchi et al., 2013) (b) Tooth as Frustum Cone used in this study

Whereas; **X** symbol shows the area of root where boundary conditions are applied. Because tooth is naturally fixed from all three sides and is free from the crown portion only.

For mathematical validation, tooth is considered as cone because it best explains the geometry of tooth.

The results of the mathematical modeling are presented in the results chapter.

Critical discussion

Tooth is considered as cone to solve mathematical equation for validation of results. All parameters of cone, used in equation are calculated and used to calculate mechanical properties of tooth. Length of tooth is measured vertically, using measuring tool in Simpleware. Furthermore, width of tooth is measured horizontally from three different points.

Measurements	Non-Treated Tooth	Treated tooth
Length	14 mm	14 mm
Width (upper part)	5.94 mm	7.83 mm
Width (middle part)	4.65 mm	6.72 mm
Width (lower part)	2.01 mm	2.01 mm

Length of non-treated and treated tooth is same, whereas both differ in width. Human teeth have more or less same length and width, so when comparing this particular case to other research work on tooth, measurements would be same with minor differences. Although based on tooth type its length and width can vary, whether it is incisor, canine or molar.

In this particular chosen work, length of tooth is important to consider, and the width from lower part. In equation, length of lower part is zero, d=0, but in case of tooth model it is not zero but 2.01 mm.

All the parameters of equation have different effects.

In this particular study, elasticity of materials has an important role. Elasticity has inverse relation with strain, more elastic material have less displacement.

All the materials have same elasticity except the pulp in healthy tooth and filling material (Gutta parcha) in treated tooth. (Material properties are defined in Table 3.4 and 3.5)

Gutta parcha is more elastic then pulp but the treated tooth is showing more displacement. Although it is minor difference is results of displacement (strain), but this can be described by focusing on anatomy of treated tooth. This is because when the canals are treated, to remove pulp and to fill; become more or less wide then their actual size. This change in geometry of canals leads to increased displacement value, even when filling material is more elastic.

45

Stress has direct relation with elasticity of material. Treated tooth is observed to show more stress, whereas healthy tooth has less stress.

We have concluded that all parameters, material properties along with geometry affect the results.

$$\delta = \frac{WL^{2}}{6E}$$

W explains the width per unit volume which can also be written as $w = \delta g$.

L2 is square of tooth's length.

E is elasticity of tooth.

Any parameter among these can affect the results.

$$\delta = \frac{wL^2(D+0)}{6E(D-0)} - \frac{wL^2(0^2)}{3ED(D-0)}$$

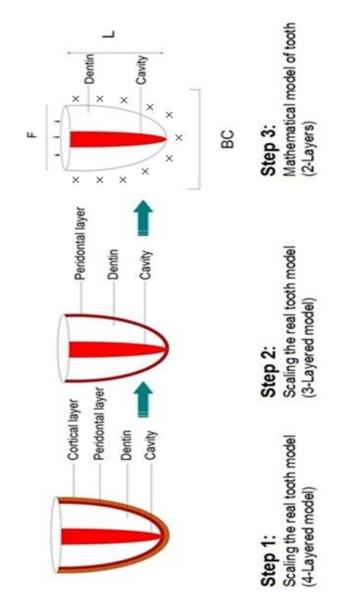


Figure 3.8 Mathematical modeling of tooth reconstructed using MS-Visio 2013

CHAPTER 4

Results

This chapter critically discusses the results obtained from this study. . It also reviews the limitations and the discussions on the acquired results. The rationale of this chapter is organized such that, at first place results obtained from Image based modeling are discussed, then for validation of simulation results, finite element analysis and mathematical modeling are described in the last section.

4.1 Image Acquisition Results

Image based modeling is relatively a newer approach used to create a geometric model from computer graphics. Image based modeling is actually sampled representation of geometry in which set of two dimensional images is used to generate three dimensional model (Quan, 2010). It is of different types based on 2D and 3D images like CT and MRI. In figure 4.1 (a), single slice CT scan of non-treated tooth is presented. Treated tooth figure 4.1 (b) represents single slice of CT scan for modeling.



Figure 4.1 Image Based Modeling Results

In image based modeling, at first an imaging modality is used to obtain a digital model of which the numerical model is developed. Therefore we have divided our image based modeling research in two sections.

1- Image processing results

2- Finite element method

These are discussed in the next section.

4.2 Image processing results

In dentistry, use of image processing technique has steadily increased because of its accuracy. Image processing can enhance the quality of image by filtering. Furthermore it enables to create any part of object (as layers are generated in this model), and to remove extra parts (noise reduction). For this purpose, after acquiring images, image processing is done in this research work.

In Fig 4.3, (a) showing Model 1 is representing mesh, which is increased in next fig 4.3 (b) Model 2. In this way Model 3 and Model 4 are generated with increasing mesh size.

Same procedure for treated tooth is opted, increasing mesh size from Model 1 fig 4.4 (a) to Model 2, fig 4.4 (B) and then Model 3 fig 4.4 (c). Finally Model 4 fig 4.4 (d) with maximum number of elements is having larger mesh size of all four models.

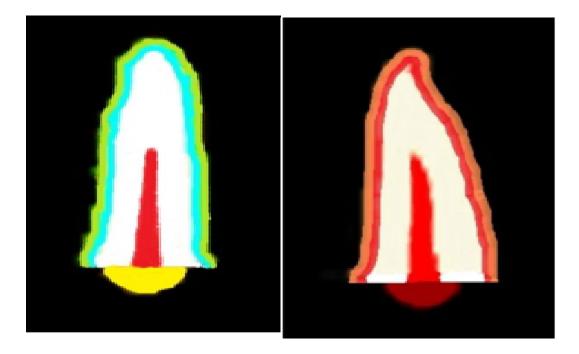
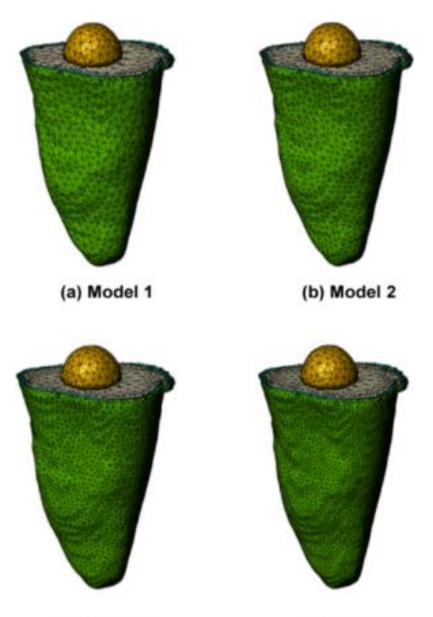


Figure 4.2 (a) Image Processing of Non-Treated Tooth (b) Image Processing of Treated Tooth

4.3 Numerical Modeling (FE) Results

As discussed earlier, It is important to carefully plan finite element analyses. This is because a small mistake in geometry, boundary conditions, choice of material model or simulation parameters can cause the final result to be erroneous. Furthermore, the FEM uses different types of elements in a mesh. Each type of element in a mesh is associated with different assumptions and hence element choice can also influence the results. Commonly used elements include beams, shells, 2D and 3D elements. For prediction of displacement and stress, iterative method based on different mesh size is adapted. This can simplify the problem and can provide analytical solution from which results can be estimated. For this purpose, convergence analysis is done as shown in figure 4.3 and 4.4.



(c) Model 3

(d) Model 4

Figure 4.3 Image Processing Result for Non-Treated Tooth

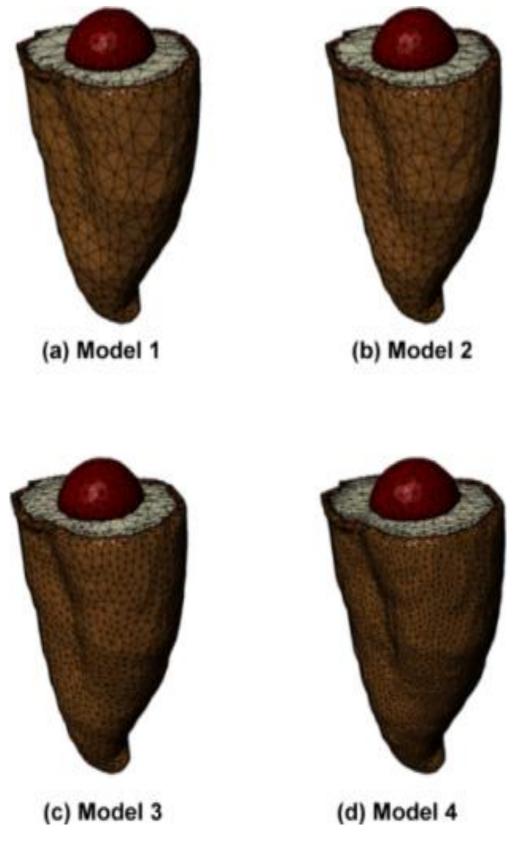


Figure 4.4 Image Processing Result for Treated Tooth

4.3.1 Convergence Analysis Results

It is a common question to ask if the solutions obtained through **FEA** simulations are converged. What does this convergence mean? In this section, therefore convergence analysis results are presented. Here it is important to distinguish between geometric effect and mesh convergence. Especially when meshing a curved surface using straight (or linear) elements will require more elements (or otherwise mesh refinement) to capture the boundary exactly. Mesh refinement leads to significant reduction in errors. Such a refinement can allow an increase in the convergence of solutions without increasing the size of the overall problem being solved.

For reliability of stresses, displacements and deflections, this analysis is to ensure that change in mesh do not affect the overall results. Increasing mesh density is actually mesh refinement, if further mesh refinements do not change the result it is considered that results have been converged.

The graph 4.4; shows displacement on y-axis. Model 1, model 2, model 3 and model 4 are presented on x-axis. Model 1 has lowest mesh density of 49, 060 elements and also lowest displacement of 0.00291mm as compared to other models. With increase in mesh size, the displacement increased respectively. For convergence analysis we have to keep in mind that by increasing mesh density, we should expect a varying behavior such that further increase in the mesh density does not change the overall behavior in terms of stress and deformation results. It is observed from the graph that by increasing mesh density, the displacement kept on increasing (Graph 4.4) unless at model 4 where displacement increased to 0.002931, there was no further change in the displacement was observed on any mesh refinement further and became consistent approximately and no significant change is observed. The solution for model 4 is therefore converged.

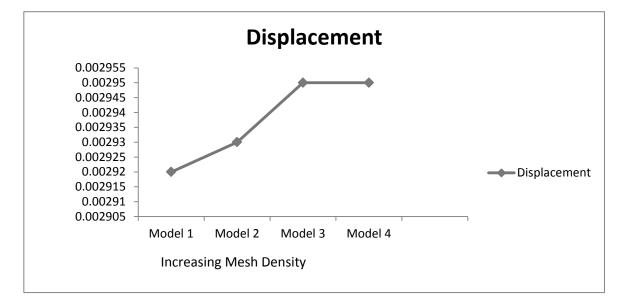
In same way convergence analysis for displacements of treated tooth as shown in graph, fig 4.4 was performed for less errors and more accuracy. Displacement is presented on y-axis and models with varying mesh density are presented on x-axis. Model one with lowest mesh size of all has 0.00331 mm displacement and by

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increasing mesh density further change in overall behavior of model is not observed. As the graph, fig 4.5 shows that by increasing mesh density, to model 4; no noticeable change observed on further mesh refinement. Results obtained from convergence analysis shows that finite elements results from this study holds true as no further mesh refinement is required. Convergence results from stress analysis are also presented in this study for non-treated tooth models, stress is on y-axis and models plotted on x-axis. Model 1 with less mesh elements, comparatively coarser mesh has range of stress as 68.61 MPa and this started decreasing with mesh density. This is a trend opposite to the displacement one. However, it pertains that with increasing mesh density as the mesh becomes more refined there is more resistance in the material to the external forces hence lesser stresses. The results of the extremely fine mesh were later validated by mathematical modeling (discussed in detail in section 4.4).

Model 1 has comparatively a courser mesh having range of stress between 25-30 MPa, 26.08 MPa to be exact and this starts decreasing with mesh density. Mesh density is abbreviated as Model 1, Model 2, Model 3 and Model 4 respectively in ascending order.

Results showed that by applying a pressure of 100 N the value of displacement for non-treated tooth has increased with very minor difference and then converged. Treated tooth showed same displacement pattern of increased value and then converged. Here the displacement is shown in figure for both non-treated and treated tooth.





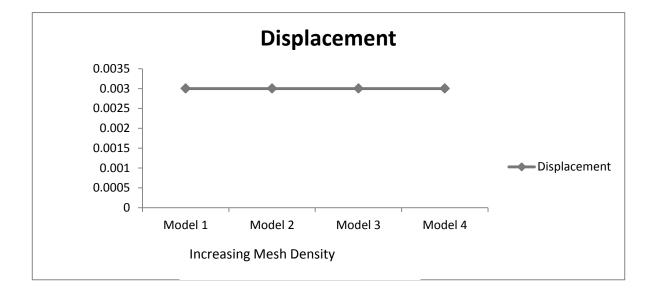


Figure 4.6 Displacement Results for Treated Tooth

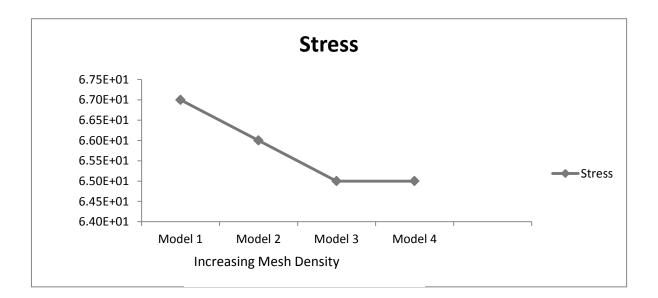


Figure 4.7 Stress Results for Non-Treated Tooth

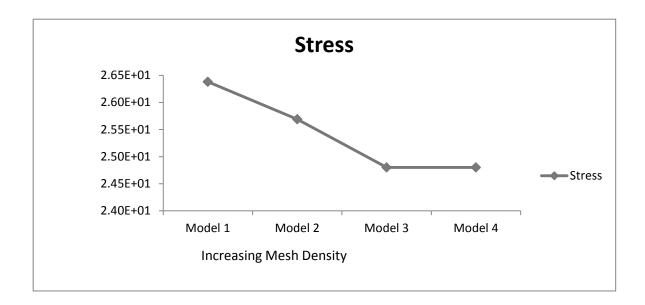
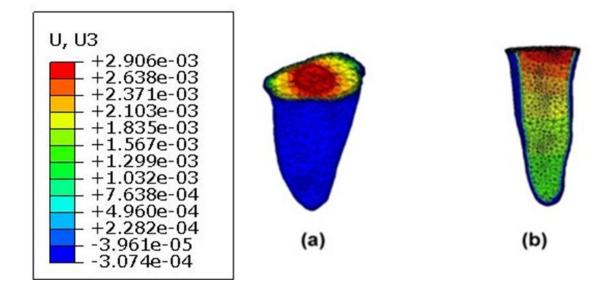


Figure 4.8 Stress Results for Treated Tooth

Similarly, stress patterens for non-treated and treated tooth are presented in Figure 4.9 and 4.10. For non-treated tooth stress value decreased with increase in mesh size and finally converged. Treated tooth showed significant difference in stress value which with minor decrease converged in model 3 and model 4, showing same results.Tooth is a bone so it naturally shows comparitively similar behaviour as bones.





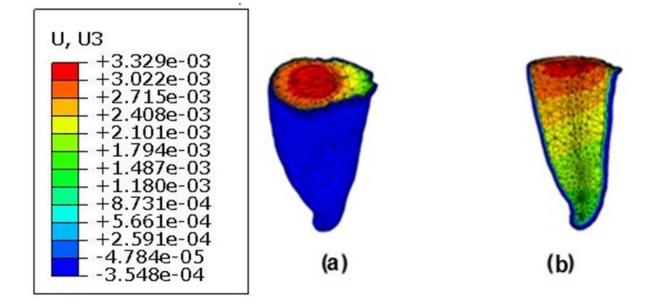


Figure 4.10 Deformation results of treated tooth

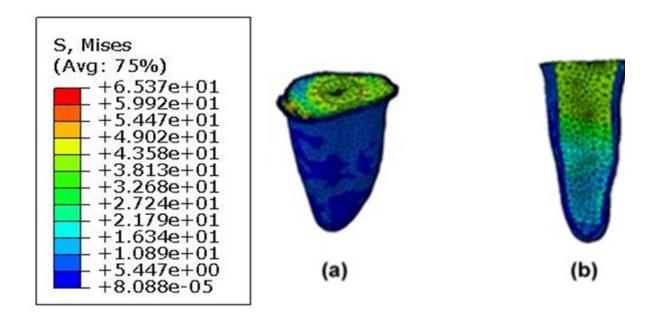


Figure 4.11 Stress results of non-treated tooth

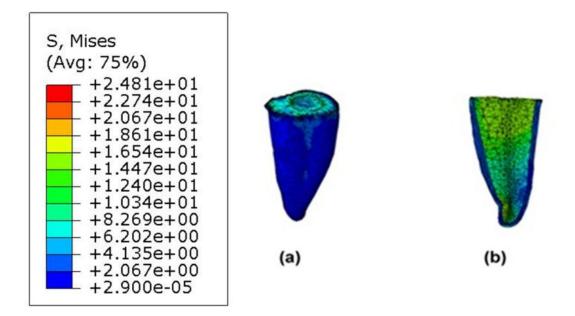


Figure 4.12 Stress results of treated tooth

For deformation and stress results are presented in table with four different levels. Stress distribution among treated and non-treated tooth vary significantly. Model of non-treated tooth showed less stress concentration as compared to treated one, showing that treated tooth is weak.

Model	Convergence	Displacement (Non-treated	Displacement (Treated
	Level	tooth)	tooth)
1	1	0.00292 mm	0.0035 mm
2	2	0.00293 mm	0.00333 mm
3	3	0.00295 mm	0.00334 mm
4	4	0.00290 mm	0.00334 mm

 Table 4.1
 Displacement Pattern between Treated and Non-Treated Tooth

Model	Convergence Level	Stress (Non-treated tooth)	Stress (Treated tooth)
1	1	67.51 MPa	26.38 MPa
2	2	66.24 MPa	25.69 MPa
3	3	65.37 MPa	24.81 MPa
4	4	65.95 MPa	24.84 MPa

 Table 4.2
 Stress Pattern between Treated and Non-Treated Tooth

Meanwhile, the question remains as to what extent can tooth structure loss affect tooth strength? Is it the overall canal space increase or the minimum remaining canal wall thickness that affects tooth strength? It has been suggested that 0.2 to 0.3 mm is the critical canal wall thickness at which condensation forces can lead to perforation or root fracture (Madarati et al., 2010).

From the results obtained in this study, the following can be concluded:

(1) Micro-computed tomography scanning is an efficient and reliable way of looking g at the changes in canal volume as scanned.

(2) The treatment by root canal results in significant tooth structure loss already (therefore, management of such cases should be carefully considered) and our FEA results compliments that. FEA results shows that tooth life is reduced to half looking at the stress contours and deformation plots.

(3) Further studies to correlate tooth structure changes with their resistance to treatment.

4.4 Mathematical Validation of Results

Studies have shown that deformation and stress in uniform bodies is uniformly distributed. In this research, model tooth is a non-uniform object which showed different results from uniform bodies.

Deformation and stress for tooth model are observed to be non-uniform. On one specific point, the body is deformed, although a very minor deformation is observed comparing with the tooth. Similarly, stress is higher in one specific point (red area in figure), and different on other areas of tooth. As tooth model is a non-uniform body,

stress is not uniformly distributed, but varying from one point to another in nonuniform manner. For mathematical validation of results, following equation can be used. This equation is for frustum of cone where d=0.

In this research work, root of tooth is model which is like cone but d is not equal to zero.

$$\delta = \frac{wL^2(D+0)}{6E(D-0)} - \frac{wL^2(0^2)}{3ED(D-0)}$$

The following equation is used to validate the results because it showed more accurate results

$$\delta = \frac{\rho g L^2}{6EA}$$

$$\delta = \frac{2063 \times 9.8 \times (14)^2}{6 \times 18.6 \times 10^9 \times 8}$$

$$\delta = \frac{3962610.4}{892.8 \times 10^9}$$

$$\delta = 0.0044 \ mm$$

Von-Mises Stress (MPa)	Von Mises Stress (MPa) Literature
This study	(Memon et al., 2016)
65.95 MPa (Non-treated tooth)	127 MPa(In Dentin post
24.84 MPa (Treated tooth)	182 MPa (In fiber post)
Displacement (mm)	Displacement (mm)
This study	Literature
0.00290 mm (Non-treated tooth)	0.025 mm (In dentin post)
0.00334 mm (Treated tooth)	0.035 mm (In fiber post)
Force	Force
100 N	100 N

Table 4.3 Comparison of Stress and Displacement Results with Literature

Conclusions:

- This study shows that removing the pulp of tooth during the procedure of root canal treatment, may change the mechanical properties of tooth with time.
- The findings estimate that the stress and deformation results of treated tooth are significantly different from non-treated tooth.
- This study shows that it is required to estimate the more durable and long lasting treatment for tooth, if the pulp becomes infected due to injury or bacterial contamination.
- Results obtained from the non-treated tooth shows that there are more tensile forces on the top of the root which gradually becomes attenuated at the root tip however more compressive forces are found on top of the treated tooth compared to the non-treated one. The base of the treated tooth is full of tensile forces. It should not be ignored that wherever the tensile forces are greater, it makes the material vulnerable to breakage and might cause failure eventually whereas the compressive forces leads to the strength of the material.

CHAPTER 5

Discussion

Previous related studies have found that pulp less treated tooth differs in mechanical properties from healthy tooth having pulp. Removal of pulp leads to dehydration of tooth which changes elasticity of tooth and increases Young's modulus. However dehydration does not make significant change in tensile and compressive strengths of treated tooth. This study shows that mechanical properties of treated tooth differ from non-treated tooth (Huang et al., 1992).

In our research work, mechanical strength of normal vital tooth and treated tooth is measured. Root canal treatment involves removal of pulp which is replaced by filling material. This filling material, gutta-percha has different mechanical properties compared to pulp which affects the mechanical strength of tooth. Gutta-percha is used for filling of root canals (Tamse et al., 1986).

It is important to note here that after comparing our results from the ones published in literature as tabulated in chapter 4. The Von Mises stresses in our study is quite underestimated and might reveal that taking into account more realistic layers, i.e. crown and dynamically processing it, mighthave yielded the results quite close to what is given in the literature (Memos et al., 2016).

Furthermore, from our research work, it is concluded that in result of loading healthy tooth shows less stress whereas treated tooth have increased stress values. Same is the case with deformation results of treated and non-treated tooth. Although deformation results were not very different but still non treated tooth showed less deformation. We can say that in response to loading, treated tooth showed less stress and hence resistance .Healthy tooth with less deformation and more stress values. This indicates that there is difference in mechanical properties of treated as well as non-treated tooth. When a material undergoes a mechanical rub, there is a change in its composition and hence mechanical properties. After root canal significant structure is lost during the process and hence prone to fractures, with the chewing forces.

This research work is helpful in endodontic analyses. It can intimate a specialist how much of the tooth loss could be a threat to overall oral hygiene and health.

Results suggest that root canal treatment may need an optimized and improved procedure where less tooth structure loss is pertinent more, resistance to the bite forces are allowable. In case of tooth disease, early treatment like filling can save tooth from root canal treatment.

Results of this study differ from previous studies in this aspect that finite element analysis is used to measure the mechanical strength of tooth instead of wet lab technique. Same mechanical properties of tooth model were assigned to treated and non-treated tooth, only pulp and filling material had different mechanical properties. This indicated the accuracy of tooth modeling and simulation results which were further validated by mathematical equation.

On basis of this study we suggest that whereas root canal treatment in the only way to save tooth's life, but it makes the tooth weak at the same time. This is probably due to the structural loss of tooth, Root canal treatment can save the life of tooth but weakens its strength overall. Our work confirms that treated tooth is more susceptible to fracture than a healthy one. Healthy, natural tooth is less at risk of fracture.

CHAPTER 6

Future Work

In this section failures encountered in this research are described.

Limitations

- We have used data down sampled to 10% instead of 100% because 100% of data will take a lot of time and computational power as well.
- We have gone through four convergence levels to get accurate results, which is a time consuming process. Furthermore, after four convergence levels, one model with full magnification has generated which could not be run, again due to computational limitation.
- Naturally three layers are present around tooth but here we have generated only two layers to cope up with computational efficiency. We expect that with that one layer (which is layer of cancellous bone), the stress disyribution might be a little different due to more bone mass distributed farther away from the neutral axis of the root.
- Full resolution model encountered different challenges. Due to its large size, magnification and resolution, boundary conditions took a lot of time to get fixed. Even after a lot of time model was not processed and it failed completely.

Future work

- Bigger model might be a better choice for realistic root behavior.
- More realistic models can be generated in future using these models as guidelines.

The behavior studied in this study is mechanically elastic behavior, whereas the processes in nature are usually non-linear and dynamic. Therefore Non-linear finite element analysis can be employed in future for more close to reality results.

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