إِسْرِواللهِ الرَّحْمِنِ الرَّحِبِ

IN THE NAME OF ALLAH ALMIGHTY, THE MOST BENEFICIENT, THE MOST MERCIFUL, AND THE MOST COMPASSIONATE

## **DEDICATION**

This work is dedicated to my parents whose prayers and encouragement made me do this and to all those people who went off their way to help me to bring out the best in me and to make me what I am today.

Thank you

### ACKNOWLEDGEMENT

In the name of *Allah Almighty*, Most Gracious, Most Merciful who has blessed us with the physical and mental capabilities to complete the assigned project.

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AQEEL AHMAD

## **DECLARATION**

We hereby declare and affirm that the thesis titled "Analysis of Design Parameters on the Performance of SC-CP DFE Communication System" is neither whole nor as a part thereof has been copied out from any source (except data,). It is further declared that I developed this report entirely on the basis of our personal efforts made under the sincere guidance of our project supervisor and due to help of Almighty Allah.

If any part of this project proved to be copied or found be a part of some other, we shall stand by the consequences.

No portion of this work presented in this report has been submitted in support of any application for any degree and qualification of this or any other university or institute of learning. If found we will stand responsible.

AQEEL AHMAD

# **TABLE OF CONTENTS**

ABSTRACT	L
CHAPTER 1	2
INTRODUCTION	2
ORGANIZATION	3
CHAPTER 2	5
CHANNEL CHARACTERISTICS AND CHANNEL ESTIMATION	5
MOBILE CHANNEL CHARACTERISTICS	5
Fading due to Multipath Effects	5
Flat Fading	5
Frequency Selective Fading	7
Fading effects due to Doppler Spread	3
Fast Fading	3
Slow Fading	)
CHANNEL IMPULSE RESPONSE ESTIMATION	)
CHAPTER 3 11	l
ANTI-MULTIPATH APPROACHES11	l
SINGLE CARRIER WITH TIME DOMAIN EQUALIZATION	l
ORTHOGONAL FREQUENCY DIVISION MULTIPLEXING (OFDM)	2
SINGLE CARRIER MODULATION WITH FREQUENCY DOMAIN EQUALIZER PROCESSING (SC-FDE)	1
Why Use SC-FDE?	7
DECISION FEEDBACK EQUALIZER	3
CHAPTER 4 20	)
ITERATIVE BLOCK DFE (IBDFE)	)
ARCHITECTURE OF IBDFE	)
PARAMETER ESTIMATION	1
CHAPTER 5	7
ANALYSIS AND PERFORMANCE EVALUATIONS	7
VARIATIONS IN IBDFE	7

CASE 1	
CASE 2	
CASE 3	
Training Transmission Format	
SIMULATIONS AND RESULTS	
IMPLEMENTATION IN A WIN32 C/C++ ENVIRONMENT	
CHAPTER 6	
Conclusion	
FUTURE WORK	
REFERENCES	

### Abstract

Orthogonal Frequency Division Multiplexing (OFDM) has gained enormous popularity in the recent years due to its inherent ability to counter multipath effects and support high data rates. Its implementation has been made simpler through the efficient calculation of Discrete Fourier Transform (DFT) via the Fast Fourier Transform (FFT) algorithm. However, timing and frequency synchronization issues in OFDM affect its performance severely. In addition, it suffers from high Peak to Average Power ratio (PAPR) at the transmitter and requires the transmitter to operate with a large linear range.

To alleviate these problems, a Single Carrier with Frequency Domain Equalization (SC-FDE) is considered which gives performance comparable to OFDM. Timing and frequency synchronization is not a big problem in SC-FDE. Various frequency domain equalizers have been implemented which include Linear Equalizer, Decision Feedback Equalizer (DFE) and Iterative Block DFE. In order to avoid the error propagation phenomenon of the DFEs and to increase the robustness of the Equalizer, block based techniques are proposed. We took an overview of these DFEs, and analyzed the performance of IBDFE under different channel constraints and using different types of PN sequences. The simulations show that how IBDFE perform when training sequences are sent and when they are absent.

## **Chapter 1**

## Introduction

This thesis analyses various frequency domain equalization techniques for a single carrier communication system which has been proposed as an alternative to OFDM to combat multipath fading. As the need for higher data rates is growing, there is an increase in the requirement of greater signal bandwidth. If the signal bandwidth becomes greater than the coherence bandwidth of the channel, the signal will experience frequency selective fading. In order to mitigate the effects of multipath, various techniques are in use. These include Single Carrier with Time Domain Equalization and OFDM.

Single Carrier modulation with time domain equalization has been in use in voice band telephone modems where the channel dispersion is not a dominant factor. But as the channel dispersion and hence the multipath delay spread increases, the time domain equalizer grows more and more complex rendering it unfavorable to be used.

Orthogonal Frequency Division Multiplexing (OFDM) is another technique which provides us with low complexity and offers a good solution for multipath fading. The reason being, that OFDM transmits multiple slowly modulated carriers in parallel. This is done by taking IDFT of the signal at the transmitter side which modulates the data symbols on orthogonal subcarriers. For each sub-carrier, the channel appears to be frequency flat causing only one attenuation and phase rotation. At the receiver side the data can be recovered by doing a simple DFT operation followed by a single-tap equalizer. OFDM with its simplicity is a very efficient anti-multipath approach. However, due to the orthogonal sub-carriers, its Peak to Average Power ratio (PAPR) is very high, requiring predistortion techniques or large back off for the power amplifier. Lately, Frequency Domain Equalization has been suggested to be used with Single Carrier Modulation which gives the performance comparable to OFDM with almost the same overall complexity with no PAPR problem [1]. This comparison involves the Frequency Domain Linear Equalizer. Further studies involve Frequency Domain Decision Feedback Equalizer (FD-DFE) [2], [3] which in fact is a Hybrid DFE as the Forward Filter is in the Frequency Domain whereas the Feedback Filter is in the Time Domain. This Equalizer shows a little improvement in the performance at high SNR values at the expense of increase in complexity. Another FDE technique has been presented which has both the filters in the frequency domain. This is the Iterative Block DFE (IBDFE) [4] which gives us a filter with lower complexity than the FD-DFE.

This thesis explores some of these equalization techniques. A complete Single Carrier Communication system has been implemented which includes the blocks of Timing and Frequency Synchronization, hence, they are not assumed perfect. Channel Estimation algorithm [5] has also been added to the algorithm. Algorithms in [3] and [4] have been simulated and their performances are compared in various scenarios i.e. AWGN, Multipath Channel and Rayleigh Faded Channel. Also we have simulated the new IBDFE algorithm and analyzed the effect of different design parameters on its performance (for example the transmitted and detected signal powers and the correlation coefficients). And compared the pros and cons of IBDFE's performance when training data is sent.

After all the simulation results we implemented the IBDFE on a hardware platform. Currently the IBDFE is designed for a WIN32 platform in C/C++ Environment.

### **Organization**

The thesis is organized in the following sections to easily make the readers understand;

- Chapter 2 elaborates the characteristics of a mobile channel including different types of fading. It further explains an algorithm which is used for the estimation of slow fading channels.
- **Chapter 3** gives an overview of the different anti multipath approaches, which include the Time Domain Equalization, OFDM, SC-FDE and Decision feedback Equalizer, and their advantages and disadvantages
- **Chapter 4** briefly describes the architecture of an IBDFE and the estimation of its parameters
- Chapter 5 makes an observation on the parameters estimated for IBDFE and makes new estimations to make IBDFE perform under three different cases. This chapter also presents the results obtained from simulations of the communication system under the constraints introduced, and implementation of IBDFE in a WIN32 C/C++ environment is presented
- Chapter 6 concludes the thesis with some suggestion of Future Work to improve IBDFE

## **Chapter 2**

## **Channel Characteristics and Channel Estimation**

The mobile channels in use today frequently experience multipath and other Doppler spread degradations. These degradations can be characterized in certain categories [12].

### **Mobile Channel Characteristics**

Wireless mobile communications are characterized by two types of fading effects: Large scale fading and small scale fading. The average signal power attenuation due to motion over large areas is called large scale fading. Its statistics express the path loss as a function of distance. Small scale fading is the phenomenon of changes in the signal amplitude and phase due to small changes in the spatial separation between the transmitter and the receiver. This type of fading is constituted of two mechanisms: time spreading of the channel and time variant nature of the channel. In this context, few terms are worth defining; Delay spread, Coherence Bandwidth, Doppler spread, and Coherence time.

Delay Spread is a type of distortion which is caused when an identical signal arrives at different times at its destination. The signal usually arrives in a number of different paths and with different angles of arrival. The time difference between the arrival moment of the first multipath component, usually the line of sight component, and the last one, is called delay spread.

Coherence bandwidth is a statistical measure of the range of frequencies over which the channel can be considered as "flat" i.e. a channel which passes all spectral components with approximately equal gain and linear phase. In other words, coherence bandwidth is

the range of frequencies over which two frequency components have a strong potential for amplitude correlation.

Doppler spread  $B_D$  is a measure of the spectral broadening caused by the time rate of change of the mobile radio channel and is defined as the range of frequencies over which the received Doppler spectrum is essentially non-zero. When a pure sinusoidal tone of frequency  $f_C$  is transmitted, the received signal spectrum, called the Doppler spectrum, will have components in the range  $f_c - f_d$  to  $f_c + f_d$ , where  $f_d$  is the Doppler shift. The amount of spectral broadening depends on  $f_d$  which is a function of the relative velocity of the mobile, and the angle  $\theta$  between the direction of motion of the mobile and direction of arrival of the scattered waves.

Coherence Time is the time interval during which two data points spaced by a certain distance in time experience sufficient correlation after passing through the channel. In other words, it is the measure of the staticness of the channel. Typically for mobile communications, it is defined as

$$T_C = \frac{0.423}{f_d} \tag{2.1}$$

#### **Fading due to Multipath Effects**

#### Flat Fading

In this type of fading, the multipath structure of the channel is such that the spectral characteristics of the transmitted signal are preserved at the receiver. However, the strength of the received signal changes with time, due to fluctuations in the gain of the channel caused by multipath.

If the channel gain changes over time, a change of amplitude occurs in the received signal. Over time, the received signal r(t) varies in gain, but the spectrum of the transmission is preserved. In a flat fading channel, the reciprocal bandwidth of the transmitted signal is much larger than the multipath time delay spread of the channel, and  $h_b(t,\tau)$  can be approximated as having no excess delay (i.e. a single delta function with  $\tau = 0$ ). Flat fading channels are also known as amplitude varying channels and are sometimes referred to as narrow band channels, since the bandwidth of the applied signal is narrow as compared to the channel flat fading bandwidth. Hence, a signal undergoes flat fading if

$$B_{S} \ll B_{C}$$
  
and  $T_{S} \gg \sigma_{A}$ 

Where  $T_s$  is the reciprocal bandwidth (e.g. symbol period) and  $B_s$  is the bandwidth, respectively, of the transmitted modulation, and  $\sigma_r$  and  $B_c$  are the rms delay spread and coherence bandwidth, respectively, of the channel.

#### **Frequency Selective Fading**

Frequency selective fading is due to the time dispersion of the transmitted symbols within the channel. Thus the channel includes intersymbol interference (ISI). Viewed in the frequency domain, certain frequency components in the received signal spectrum have greater gains than others.

For frequency-selective fading, the spectrum S(f) of the transmitted signal has a bandwidth which is greater than the coherence bandwidth  $B_c$  of the channel. Viewed in the frequency domain, the channel becomes frequency selective, where the gain is different for different frequency components. Frequency selective fading is caused by multipath delays which exceed the symbol period of the transmitted symbol. Frequency selective fading channels are also called wideband channels because the bandwidth of the signal s(t) is wider than the bandwidth of the channel impulse response. As time varies, the channel varies in gain and phase across the spectrum of s(t), resulting in time varying distortion in the received signal r(t). Hence, a signal undergoes frequency selective fading if

$$B_{S} > B_{C}$$
  
and  $T_{S} < \sigma_{T}$ 

#### Fading effects due to Doppler Spread

#### **Fast Fading**

In a fast fading channel, the channel impulse response changes rapidly within the symbol duration, i.e. the coherence time of the channel is smaller than the symbol period of the transmitted signal. This causes frequency dispersion (also called time-selective fading) due to Doppler spreading, which leads to signal distortion. Viewed in the frequency domain, signal distortion due to fast fading increases with increasing Doppler spread relative to the bandwidth of the transmitted signal. Therefore, a signal undergoes fast fading if

$$T_{S} > T_{C}$$
  
and  $B_{S} < B_{D}$ 

A flat fading, fast fading channel is a channel in which the amplitude of the delta function varies faster than the rate of change of the transmitted baseband signal. In the case of a frequency selective fast fading channel, the amplitude, phase, and time delays of any one of the multipath components vary faster than the rate of change of the transmitted signal. Usually, fast fading only occurs for very low data rates.

#### **Slow Fading**

In a slow fading channel, the channel impulse response changes at a rate much slower than the transmitted baseband signal s(t). In this case, the channel maybe assumed to be static over one or several reciprocal bandwidth intervals. In the frequency domain this implies that the Doppler spread of the channel is much less than the bandwidth of the baseband signals. Therefore, a signal undergoes slow fading if

$$T_{S} \ll T_{C}$$
 and  $B_{S} \gg B_{D}$ 

The velocity of the mobile (or the velocity of the objects in the channel) and the baseband signaling determines whether a signal undergoes fast fading or slow fading.

Channel Estimation is required for all the equalizers. Various techniques are used to estimate the channel response, some of which include [5] and [6]. We found the technique of [6] to be very efficient and we will describe it in detail.

### **Channel Impulse Response Estimation**

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This algorithm assumes channel to be quasi-static which is perfectly valid in our case as the frequency domain equalization techniques are tested on the assumption that the signal duration is less than or equal to the coherence time of the channel. Hence, we are dealing with the slow fading case.

Let us define the following:

.

$$r(0) \triangleq [r(0) \ r(1) \ \cdots \ r(N-1)]^{T}$$
 (2.2)

$$\mathbf{h} \stackrel{\text{\tiny def}}{=} \begin{bmatrix} h_0 & h_1 & \cdots & h_{K-1} \end{bmatrix}^1 \tag{2.3}$$

9

$$W(v) \triangleq diag\{1, e^{j2\pi v/N}, e^{j2\pi 2v/N}, ..., e^{j2\pi (N-1)v/N}\}$$
(2.4)

$$\mathbf{n} \triangleq \begin{bmatrix} n(0) & n(1) & \cdots & n(N-1) \end{bmatrix}^{\mathrm{T}}$$
(2.5)

$$S \triangleq \begin{pmatrix} s(0) & s(-1) & \cdots & s(-K+1) \\ s(1) & s(0) & \cdots & s(-K+2) \\ \vdots & & & \\ s(N-1) & s(N-2) & \cdots & s(N-K) \end{pmatrix}$$
(2.6)

where  $h_0, h_1, ..., h_{K-1}$  give the instantaneous path gains,  $\{s(k): k = -N_g, -N_g + 1, ..., N-1\}$ are the samples of the transmitted training symbol( including the cyclic prefix part),  $\{r(k): k = 0, 1, ..., N-1\}$  the corresponding received samples (excluding the cyclic prefix part),  $\{n(k): k = 0, 1, ..., N-1\}$  the noise samples, and v the normalized frequency offset. The received sample vector can then be expressed as

$$r(0) = e^{j\phi}W(v) \cdot S \cdot h + n$$
 (2.7)

If we use the frequency offset estimate  $\hat{\mathcal{V}}$  , the ML channel impulse response estimate can be realized by

$$\hat{\mathbf{h}} = [\mathbf{S}^{H}, \mathbf{S}]^{-1} \mathbf{S}^{H}, \mathbf{W}^{H}(\hat{\mathbf{v}}), \mathbf{r}(0)$$
 (2.8)

In this channel estimation, the knowledge of the maximum delay spread of the channel is required.

## **Chapter 3**

## **Anti-Multipath Approaches**

We will examine some of the existing anti multipath approaches which are very popular these days

### Single Carrier with Time Domain Equalization

A conventional anti-multipath approach which was pioneered in voiceband telephone modems and has been applied in many other digital communications systems is to transmit a single carrier, modulated by data using Quadrature Phase Shift Keying (QPSK) or Quadrature Amplitude Modulation (QAM) and to use an adaptive equalizer at the receiver to compensate for the Inter-Symbol Interference (ISI) [2]. Its main components are one or more transversal filters for which the number of adaptive tap coefficients is on the order of the number of data symbols spanned by the multipath. For example, for a 20µs delay spread we would require a transversal filter with at least 100 taps and at least several hundred multiplication operations per data symbol. For tens of mega symbols per second and more than about 30-50 symbols ISI, the complexity and required digital processing speed become exorbitant, and this time-domain equalization approach becomes unattractive. Figure 3-1 shows the basic block diagram of a Time Equalization System.



Figure 3-1: Block diagram of Time Domain Equalization System

### **Orthogonal Frequency Division Multiplexing (OFDM)**

One way to mitigate the frequency-selective fading seen in a wide band channel is to use a multicarrier technique which subdivides the entire channel into smaller sub-bands, or subcarriers. Orthogonal frequency division multiplexing (OFDM) is a multicarrier modulation technique which uses orthogonal subcarriers to convey information. It transmits multiple modulated sub-carriers in parallel [2], each occupies only a very narrow bandwidth. Since the channel affects only the amplitude and phase of each sub-carrier, equalization of each sub-carrier's gain and phase does compensation for frequency selective fading. In the frequency domain, since the bandwidth of a subcarrier is designed to be smaller than the coherence bandwidth, each sub-channel is seen as a flat fading channel which simplifies the channel equalization process. In the time domain, by splitting a high-rate data stream into a number of lower-rate data stream that are transmitted in parallel, OFDM resolves the problem of ISI in wide band communications. More technical details on OFDM are at [8] and [13].

In summary, OFDM has the following advantages:

- For a given channel delay spread, the implementation complexity is much lower than that of a conventional single carrier system with time domain equalizer. Spectral efficiency is high since it uses overlapping orthogonal subcarriers in the frequency domain. Thus requiring high power amplifier for backing off.
- Modulation and demodulation are implemented using inverse discrete Fourier transform (IDFT) and discrete Fourier transform (DFT), respectively, and fast Fourier transform (FFT) algorithms can be applied to make the overall system efficient.
- Capacity can be significantly increased by adapting the data rate per subcarrier according to the signal-to-noise ratio (SNR) of the individual subcarrier.

Because of these advantages, OFDM has been adopted as a modulation of choice by many wireless communication systems such as wireless LAN (IEEE 802.11a and 11g) and DVB-T (Digital Video Broadcasting-Terrestrial).

However, it suffers from the following drawbacks [3]:

- High peak-to-average power ratio (PAPR): The transmitted signal is a superposition
  of all the subcarriers with different carrier frequencies and high amplitude peaks
  occur because of the superposition.
- High sensitivity to frequency offset: When there are frequency offsets in the subcarriers, the orthogonality among the subcarriers breaks and it causes intercarrier interference (ICI).
- A need for an adaptive or coded scheme to overcome spectral nulls in the channel: In the presence of a null in the channel, there is no way to recover the data of the subcarriers that are affected by the null unless we use rate adaptation or a coding scheme.

Typically, the FFT block length M is at least 4-10 times longer than the maximum impulse response span. One reason for this is to minimize the fraction of overhead due to the insertion of a cyclic prefix at the beginning of each block. The cyclic prefix is a repetition of the last data symbols in a block. Its length in data symbols exceeds the maximum expected delay spread. The cyclic prefix is discarded at the receiver. Its purpose is to:

- Prevent contamination of a block by ISI from the previous block
- Make the received block appear to be periodic with period **M**



Figure 3-2: Block Diagram of SC-FDE and OFDM

### Single Carrier Modulation with Frequency Domain Equalizer Processing (SC-FDE)

An SC system transmits a single carrier, modulated, for example, with QAM at a high symbol rate [2]. For broadband multipath channels, conventional time domain equalizers are impractical because of the complexity (very long channel impulse response in the time domain). Frequency domain equalization (FDE) is more practical for such channels. Single carrier with frequency domain equalization (SC-FDE) technique is another way to fight the frequency-selective fading channel. Frequency domain linear equalization in an SC system is simply the frequency domain analog of what is done by a conventional linear time domain equalizer. It delivers performance similar to OFDM with essentially the same overall complexity, even for long channel delay. For channels with severe delay spread, frequency domain equalization is computationally simpler than corresponding time domain equalization for the same reason OFDM is simpler: because equalization is performed on a block of data at a time and the operations on this block involve an efficient FFT operation and a simple channel inversion operation. Figure 3-2 shows the block diagram of SC-FDE and compares it with that of OFDM.

In the transmitter of SC-FDE, we add a cyclic prefix (CP), which is a copy of the last part of the block, to the input data at the beginning of each block in order to prevent inter-block interference (IBI) and also to make linear convolution of the channel impulse response look like a circular convolution. Apart from the CP, a PN sequence can also be added, and since it is repeated for every block of data, it performs the same work as that of the CP. It should be noted that circular convolution problem exists for any FDE since multiplication in the DFT-domain is equivalent to circular convolution in the time domain. When the data signal propagates through the channel, it linearly convolves with the channel impulse response and thus channel filtering and equalization should have the same type of convolution either linear or circular convolution. One way to resolve this problem is to add a CP or other PN sequence in the transmitter that will make the channel filtering look like a circular convolution FDE.

Another way is not to use CP or PN sequence but perform an "overlap and save" method in the frequency domain equalizer to emulate the linear convolution. However, the use of cyclic prefix gives protection form inter-block interference which should not be neglected. For either OFDM or SC-FDE broadband wireless systems operating in severe outdoor multipath environments, typical values of FFT size could be 256 - 1024. SC-FDE receiver transforms the received signal to the frequency domain by applying DFT and does the equalization process in the frequency domain. Most of the well-known time domain equalization techniques, such as minimum mean-square error (MMSE) equalization, decision feedback equalization, and turbo equalization, can be applied to the FDE After the equalization, the signal is brought back to the time domain via IDFT and detection is performed.



Figure 3-3: Differences between SC-FDE and OFDM

Comparing the two systems in Figure 3-2, it is interesting to find the similarity between the two. Overall, they both use the same communication component blocks and the only difference between the two diagrams is the location of the IDFT block. Thus, one can expect the two systems to have similar link level performance and spectral efficiency.

However, there are distinct differences that make the two systems perform differently as illustrated in Figure 3-3. In the receiver, OFDM performs data detection on a per-subcarrier basis in the frequency domain whereas SC-FDE does it in the time domain after the additional IDFT operation. Because of this difference, OFDM is more sensitive to a null in the channel spectrum and it requires channel coding or power/rate control to overcome this deficiency. Also, the duration of the modulated time symbols are expanded in the case of OFDM with parallel transmission of the data block during the elongated time period.

Also notable is that a frequency domain receiver processing SC modulated data shares a number of common signal processing functions with an OFDM receiver as can be seen in Figure 3-2.



Figure 3-4: Differences between SC-FDE and OFDM symbols

Overlap-add or Overlap-save techniques could also be used to avoid the extra overhead of the cyclic prefix.

### Why Use SC-FDE?

The use of SC modulation and FDE by processing the FFT of the received signal has several attractive features:

- SC modulation has reduced peak-to-average ratio requirements from OFDM, thereby allowing the use of less costly power amplifiers.
- Its performance with FDE is similar to that of OFDM, even for very long channel delay spread.
- Frequency domain receiver processing has a similar complexity reduction advantage to that of OFDM: complexity is proportional to log of multipath spread.
- Coding while desirable, is not necessary for combating frequency selectivity as it is in adaptive OFDM
- A further sensitivity of OFDM, not shared to the same degree by single carrier, is phase noise and frequency offsets, due to the close spacing in frequency of its subcarriers. This sensitivity leads to tighter local oscillator requirements for OFDM systems.

• SC modulation is a well-proven technology in many existing wireless and wireline applications, and its RF system linearity requirements are well known.

A comparison of the various anti-multipath schemes is given in Table 2.1

	OFDM	SC-FDE	SC-TDE
Signal PAPR	High	Low	Low
Computational Complexity	Low	Low	High
Coding Requirement	Strict	Flexible	Flexible

Table 3-1 A comparison of Anti-multipath schemes

### **Decision Feedback Equalizer**

A decision feedback equalizer (DFE) is a nonlinear equalizer that uses previous detector decision to eliminate the ISI on pulses that are currently being demodulated. In other words, the distortion on a current signal that was caused by previous signals is subtracted. Figure3-5 shows a simplified block diagram of a DFE where the forward filter and the feedback filter can each be a linear filter. The nonlinearity of the DFE comes from the nonlinear characteristic of the detector that provides an input to the feedback filter after making a decision. The basic idea of a DFE is that if the values of the symbols previously detected are known, then ISI caused by the previous symbols can be canceled out exactly by subtracting past symbol values after multiplying with appropriate feedback and feedforward filter coefficients. The forward and feedback filter coefficients can be adjusted simultaneously to satisfy a criterion of minimizing error, such as minimizing the MSE. The main advantage of a DFE implementation is the feedback filter, which is working to remove ISI, and thus its output is free of channel noise.



Figure 3-5: Block Diagram of a simple DFE

The following chapters will elaborate the architecture and function of DFE, with some specific algorithms of computing FF and FB filter coefficients. In our work we will discuss the Iterative Block DFE and will make some variation in it to check whether it enhances or worsen the performance of IBDFE.

## **Chapter 4**

### **Iterative Block DFE (IBDFE)**

### Architecture of IBDFE

Figure 4-1 shows the general architecture of the IBDFE. The DFT is computed over successive blocks having **P** sample. Unlike an ordinary FD-DFE it consists of two parts, feedforward filter coefficients  $C_p^{(l)}$  and the feedback filter coefficients  $B_p^{(l)}$  where p=0,1,2 3,....P-1.  $C_p^{(l)}$  partially equalizes for the interference while the residual interference is equalized by the  $B_p^{(l)}$ . Note that the DFT of received signal block is point to point multiplied by the  $C_p^{(l)}$  to get  $Z_p^{(l)}$  at  $I^{\text{th}}$  iteration,

$$Z_{\rm p}^{(l)} = C_{\rm p}^{(l)} * R_{\rm p}^{(l)}$$
 for p=0,1,2,....P-1 (3.1)

Similarly feedback is fed through  $Y_p^{(l)}$  which was obtained by point to point multiplication of the detected data at previous iteration and  $B_p^{(l)}$  (feedback coefficients)

$$Y_p^{(l)} = B_p^{(l)} \hat{S}_p^{(l-1)}$$
(3.2)

And the detected signal at the  $I^{\text{th}}$  iteration is

$$U_{\rm p}^{(l)} = Y_{\rm p}^{(l)} + Z_{\rm p}^{(l)}$$
(3.3)

Where  $Y_p$  <sup>(*l*)</sup> minimizes the interference caused by the precursors and the postcursors of the global impulse response. Since  $Y_p$  <sup>(*l*)</sup> depends on the detected at previous iteration, so for the first iteration no detected data is available, thus;

$$d_{\rm p}^{(0)} = 0, \qquad {\rm p} = 0, 1, 2, \dots, {\rm M} - 1$$
 (3.4)

In the end, IFFT is taken to obtain the time domain vector signal at the decision point

$$u^{(l)} = \frac{1}{P} W^{H} U^{(l)}$$
(3.5)

The block diagram of IBDFE is shown in Figure 4-1a and Figure 4-1b.



Figure 4-1a: Basic Architecture of IBDFE (Feed forward Part)



Figure 4-1b: Basic Architecture of IBDFE (Feed Back Part)

To design the filter [4] proposes two approaches, Hard Detection IBDFE (HD-IBDFE) and Soft Detection IBDFE (SD-IBDFE). In HD-IBDFE the FB filter uses the hard detected data at the previous iteration as input, and FF and FB filters are designed to minimize the resulting MSE, while SD-IBDFE does not uses feedback filter, instead it generates  $Y_p$ <sup>(/)</sup> based on some statistical computation.

The mean square error at the decision point can be computed as follows

$$J_{HD}^{(l)} = E\left[\widetilde{|d_n^{(l)} - d_n|^2}\right] = \frac{1}{P} \sum_{i=0}^{P-1} E\left[|u_i^{(l)} - s_i|^2\right]$$
(3.6)

Considering the above diagram the equation  $J_{HD}^{(l)}$  can be written as

$$J_{HD}^{(l)} = \frac{1}{P^2} \sum_{p=0}^{P-1} E\left[ |C_P^{(l)} + B_p^{(l)} \hat{S}_P^{(l-1)} - S_P|^2 \right]$$
(3.7)

After minimizing this equation by applying the gradient method with respect to the FB filter coefficients, we get

$$B_{p}^{(l)} = -\frac{r_{S_{p},\hat{S}_{p}^{(l-1)}} \times (H_{p}C_{p}^{(l)} - \gamma^{(l)})}{M_{\hat{S}_{p}^{(l-1)}}}$$
with  $p = 0, 1, 2, ..., P - 1$ 
(3.8)

and

Where  $M_{s_p}$  and  $M_{s_p}^{(l-1)}$  are the signal powers in frequency domain, and can also be calculated using parseval's theorem as:

 $\gamma^{(l)} = \sum_{p=0}^{P-1} H_p C_p^{(l)}$ 

$$M_{S_p} = E[|S_p|^2]$$
(3.10)

And

$$M_{\hat{S}_{P}^{(l)}} = E[|\hat{S}_{P}^{(l)}|^{2}]$$
(3.11)

And in the similar way minimizing the gradient with respect to the forward filter coefficients, we obtain,

$$C_{p}^{(l)} = \frac{H_{p}^{*}}{M_{W} + M_{S_{p}} \left(1 - \frac{|r_{S_{p},\hat{S}_{p}^{(l-1)}}|^{2}}{M_{\hat{S}_{p}^{(l-1)}}M_{S_{p}}}\right) |H_{p}|^{2}}$$
(3.12)

23

(3.9)

#### **Parameter Estimation**

For the design of both IBDFEs, we need to know the FD response of the channel. A technique for channel estimation has been presented in [14]. HD-IBDFE, receiver also requires the estimate of powers  $M_{sp}$  and  $M_{sp}^{(l-1)}$ , and the correlation coefficient  $r_{spsp}^{(l-1)}$ .

 $M_{s_p}$  and  $M_{s_p}^{(l-1)}$  can be calculated using Parseval's theorem. Also if we assume that we know distribution of the data then we can calculate the average power as:

$$M_{S_p} = \sum \left| \rho_i * \left| A_i \right|^2 \right| \tag{3.13}$$

Where  $\boldsymbol{\rho}_i$  is the probability of  $\boldsymbol{i}^{th}$  symbol and  $\boldsymbol{A}_i$  is the constellation point.

For the special case when data comes in an equally likely way then equation (3.13) can be simplified as

$$M_{S_p} = \sum |A_i|^2 / N$$
 (3.14)

Where **N** is the number of constellation points.

If we have detected signal then the correlation can be calculated as

$$r_{S_{p}\hat{S}_{P}^{(l-1)}} = E\left[S_{p}\hat{S}_{P}^{(l-1)*}\right]$$
(3.15)

24

In [4] a first estimate can be obtained by correlating the equalized received signal and the detected data signal, namely

$$\hat{r}_{1}^{(l)} = \frac{1}{P} \sum_{p=0}^{P-1} \frac{R_{P}}{H_{P}} \hat{S}_{P}^{(l-1)*}$$
(3.16)

By putting the variance of the reciprocal of the channel DFT coefficients

$$M_{H} = E\left[\left|\frac{1}{H_{p}}\right|^{2}\right]$$
(3.17)

we can compute the variance of the estimate equation (3.16) as follows

$$E\left[\left|\hat{r}_{1}^{(l)} - r_{S_{p},\hat{S}_{p}^{(l-1)}}\right|^{2}\right] = E\left[\frac{1}{P}\sum_{p=0}^{P-1}\left|\frac{W_{p}\hat{S}_{p}^{(l-1)*}}{H_{p}}\right|^{2}\right]$$
$$= M_{W}M_{H}\frac{1}{P}\sum_{p=0}^{P-1}M_{\hat{S}_{p}^{(l-1)}}$$
(3.18)

Thus therefore, for a very dispersive channel,  $M_H$  will produce a significant increase in the estimate of correlation. A more reliable estimate can be obtained by taking into account only the frequencies whose channel response gain is larger than a given specific threshold  $H_{TH}$ .

The following set of frequencies are defined

$$\boldsymbol{\zeta} = \left\{ \boldsymbol{p} : \left| \boldsymbol{H}_{\boldsymbol{p}} \right| \geq \boldsymbol{H}_{\boldsymbol{T}\boldsymbol{H}} \right\}$$

Where  $H_{TH}$  is a specific threshold of which only the greater points of the channel's response is accounted.

$$|H_{TH}|^{2} = \frac{M_{W}}{M_{S_{p}}}$$
(3.19)

A new estimate of the correlation is now:

$$\hat{r}_{2}^{(l)} = \frac{1}{P_{\varsigma}} \sum_{p \in \varsigma} \frac{R_{p}}{H_{p}} \hat{S}_{p}^{(l-1)*}$$
(3.20)

A good correlation estimate can also be made such that the probability of overestimating  $|r_{S_P,\hat{S}_P^{(l-1)}}|$  is sufficiently small.

$$r_{3}^{(l)} = (1 - \eta) \frac{1}{P_{\varsigma}} \sum_{p \in \varsigma} \frac{R_{p}}{H_{p}} S_{p}^{(l-1)*}$$
 for  $0 \le \eta < 1$  (3.21)

## **Chapter 5**

## **Analysis and Performance Evaluations**

### Variations in IBDFE

The SC-FDE performs a little better than Linear Equalizer in high SNR conditions at the expense of increase in complexity. It is because of that, it can cancel the postcursors of the ISI. The IBDFE, on the other hand, because of its feedback mechanism, has the ability to cancel both the precursors and the postcursors of ISI. Also in the IBDFEs the error propagation is limited to one block only, and as it has the feedback filter in the frequency domain, hence decreasing the complexity as compared to the other existing DFEs.

It should be noted that the complexity of the receiver may vary as the dispersion of the channel varies. We can say that, as dispersion increases, the complexity gain of the IBDFE structure over TD-DFE and H-DFE will increase. Because of the fact that longer FB filters are needed and the Frequency domain implementation has a larger impact on complexity of IBDFE. Besides, for channels with a low or moderate dispersion, very simple FB filters can be used, and the complexity reduction provided by the IBDFEs may be negligible.

Our study of the IBDFE will circulate about the following three points:

- 1. When Training Data is not sent and no Channel Information is known
- 2. When Training Data is not sent and full knowledge of Channel characteristics is known
- 3. When Training Data is sent.

We will discuss these one by one.

#### CASE 1

When Training Data is not sent and no Channel Information is known. The IBDFE will be behaving like just a blind Equalizer in Frequency Domain. More information about this type of equalizers can be found in [15]. Equation (3.15) show that the correlation between the transmitted data and the detected data at the previous iteration can be computed only when training data is available, but the estimation techniques based on the training data will lower the bandwidth efficiency. We can make an estimate of the correlation between the transmitted data and the detected data by using equations (3.16), (3.20) and (3.21). Here we used equation (3.21) for our work.

On the other side we also need to know the Channel Impulse Response in FD, for which some techniques of estimating channels are proposed in papers like [9] and [14].

#### CASE 2

It happens in very rare cases when we have the full knowledge of channel's parameters, for example communication in a closed room or a hall may be provided with full knowledge of channel. if we exactly know the channel's impulse response and the statistical parameters like mean white noise, white noise variance, delay spread, multipaths and their gains, and power delay profile of Rayleigh channel are known then we need not to send training data. In these circumstances IBDFE performs best and the effect of correlation coefficient is minimized.

We used the equations (4.1) and (4.2) for the computation of feedback and feedforward filter coefficients instead of using equations (3.8) and (3.12):

$$C_p = \frac{H_p^*}{M_W} \tag{4.1}$$

$$B_p = -\left(H_p C_p - \gamma\right) \tag{4.2}$$

#### CASE 3

As discussed earlier for the IBDFE, the filter coefficients are totally dependant on the correlation factor between the transmitted and the detected data. Improved performance of the IBDFE is now subject to the proper estimation of the correlation factor. If correlation coefficient is underestimated, then the FB vector will be less effective in canceling out the ISI, and this will results in a slower convergence of the equalizer. On the other side, if is the correlation coefficient is overestimated, the FB filtered vector will tend to cancel ISI entirely. Now, since the actual detected data are not as reliable as predicted by the correlation estimate, the cancellation process will introduce more errors, which will worsen the performance.

Since this design scheme does not use a training sequence, we may not get good correlation estimation, as the channel conditions become more dispersive. If we send a training sequence the correlation factor becomes easy to calculate. We then use equation (3.15) to compute the correlation.

#### **Training Transmission Format**

There are no specific constraints for the composition of training sequence format. In fact, the training sequence may be of the same modulation scheme as that of the transmitted signal. For example, if the modulation of the transmitted data is QPSK then the training sequence should be randomly generated sequence with the QPSK constellation mapping.

Since we have assumed that the channel remains the same for at least the duration of a single block transmission, the training symbol hence designed will deliver us an accurate result that can be used for the filter optimization and the same filter calculations will suffice for the rest of the symbols in the frame as they have experienced nearly the same channel. The transmission format for the data will now be as shown in Figure 5-1.

 Training	Data	Data	Data	Data	Data	Data	Training	Data	Data	
 Block1	Block1	Block2	Block3	Block4	Block5	Block6	Block2	Block7	Block8	

Figure 5-1: Training transmission format for IBDFE

We previously assumed that the channel remains the same at least for the duration of a block, but it will change for other block transmission. So it is necessary that we send a training block after send a specific number of blocks to make an estimate of the channel again. As the IBDFE is trained for the training block it will filter the Data Block1 with very tiny amount of errors, Data Block2 with a little greater number of errors and so on for the following blocks. So increasing the number of Data Blocks between the Training Blocks will increase in bit Error Rate. And reducing the number of Data Blocks between the training blocks will enhance the performance of the system. But this will be provided on the cost of Low quantity of data transferred per unit time (i.e., Low bandwidth efficiency). In this scenario we have to choose an optimum point with reasonable Data carriage and BER loss. For our System we used to send eight Data Blocks between two Training Blocks.

### Simulations and Results

The above mentioned frequency domain equalizers (HD-IBDFE) has been simulated in MATLAB using a Rayleigh fading channel. We assumed the channel to be time invariant, at least for the duration of one block and the channel estimate is assumed ideal. We used QPSK transmission and size of DFT is set equal to the block size of P=128, and the PN sequence is of length of L=16, thus the data length of 112.



Figure 5-2: BER of IBDFE with different channel constraints



Figure 5-3: BER of IBDFE with different PN sequences

For an HD-IBDFE, Figure 5-2 shows the average BER of a transmitted verses the average signal to noise ratio (SNR) at the receiver input after the fourth iterations. Note that when no channel information is given and we do not have the training sequence sent then IBDFE performs a little coarser. In this case the FD equalizer just behaves like a blind equalizer [15], and channel estimation is done as proposed in [14] and [9]. And if we have a perfect knowledge of channel parameters (like white noise mean and variance, power delay profile of Rayleigh channel etc.) signal to noise ratio can further be enhanced. In this case we use the Equation (4.1) and (4.2) for the computation of FF and FB coefficients.

We used an estimated channel to give the resulting curve, it can be noticed that sending the training sequence improves the BER. [14] and [9] proposes a channel estimation technique that can also be used with DFE. Equation (3.15) can be used to directly compute the correlation between detected data and the transmitted data when we have the training sequence sent.

In Figure 5-3 plot for the different PN sequences are shown. The two special cases are considered Zero Padding and CP. it should be noted that the PN extension yields a reduced bit error rate (BER) with respect to the CP extension, since in the latter case, data detection errors affect both the information data and the CP extension, and same is for the Zero padding over PN extension.

It should be noted that the complexity of the receiver may vary as the dispersion of the channel varies. We can say that, as dispersion increases, the complexity gain of the IBDFE structure over TD-DFE and H-DFE will increase. Because of the fact that longer FB filters are needed and the Frequency domain implementation has a larger impact on complexity of IBDFE. Besides, for channels with a low or moderate dispersion, very simple FB filters can be used, and the complexity reduction provided by the IBDFEs may be negligible.

#### Implementation in a WIN32 C/C++ Environment

Although the Iterative Block Decision Feedback Equalizer offers very low complexity for being in frequency domain, but still having the feed back in such a way that it transforms data from FD to TD and from TD to FD twice makes it complex to implement in any of the hardware platform like DSP kit or FPGA. We used a WIN32 platform to implement the IBDFE system for its better computation capability.

A C/C++ code is written for the IBDFE in a WIN32 environment and for the computation of DFT and IDFT we used the specific library of Microsoft "fftw.lib". In order to use FFTW effectively, one needs to learn one basic concept of FFTW's internal structure: FFTW does not use a fixed algorithm for computing the transform, but instead it adapts the DFT algorithm to details of the underlying hardware in order to maximize performance. Hence, the

computation of the transform is split into two phases. First, FFTW's planner "learns" the fastest way to compute the transform on our machine. The planner produces a data structure called a plan that contains this information. Subsequently, the plan is executed to transform the array of input data as dictated by the plan. The plan can be reused as many times as needed. In typical high performance applications, many transforms of the same size are computed and, consequently, a relatively expensive initialization of this sort is acceptable. More information on FFTW is given in [16].

The basic usage of FFTW to compute a one-dimensional DFT of size N is simple, and it typically looks something like this code:

```
#include <fftw3.h>
....
{
fftw_complex *in, *out;
fftw_plan p;
....
in = (fftw_complex*) fftw_malloc(sizeof(fftw_complex) * N);
out = (fftw_complex*) fftw_malloc(sizeof(fftw_complex) * N);
p = fftw_plan_dft_ld(N, in, out, FFTW_FORWARD, FFTW_ESTIMATE);
....
fftw_execute(p); /* repeat as needed */
....
fftw_destroy_plan(p);
fftw_fmed/arbit fftw_fmed/arbit;
```

Figure 5-4: C/C++ code structure for calculating DFT

First is to allocate the input and output arrays. Using fftw\_malloc, which behaves like malloc except, that it properly, aligns the array when SIMD instructions are available.

The data is an array of type fftw\_complex, which is by default a double composed of the real (in[i][0]) and imaginary (in[i][1]) parts of a complex number. The next step is to

create a plan, which is an object that contains all the data that FFTW needs to compute the FFT. This function creates the plan:

#### fftw\_plan fftw\_plan\_dft\_1d(int n, fftw\_complex \*in, fftw\_complex \*out, int sign, unsigned flags)

The first argument, n, is the size of the transform we are trying to compute. The size n can be any positive integer, but sizes that are products of small factors are transformed most efficiently (although prime sizes still use an  $O(n \log n)$  algorithm). The next two arguments are pointers to the input and output arrays of the transform. These pointers can be equal, indicating an in-place transform. The fourth argument, sign, can be either FFTW\_FORWARD (-1) or FFTW\_BACKWARD (+1), and indicates the direction of the transform; technically, it is the sign of the exponent in the transform. It is negative for DFT and positive for IDFT.

The flags argument is usually either FFTW\_MEASURE or FFTW\_ESTIMATE. FFTW\_MEASURE instructs FFTW to run and measure the execution time of several FFTs in order to finds the best way to compute the transform of size n. This process takes some time (usually a few seconds), depending on our machine and on the size of the transform. FFTW\_ESTIMATE, on the contrary, does not run any computation and just builds a reasonable plan that is probably sub-optimal.

Once the plan has been created, we can use it as many times as we need for transforms on the specified in/out arrays, computing the actual transforms via fftw\_execute(plan):

void fftw\_execute(const fftw\_plan plan);

When we are done with the plan, we deallcate it by calling fftw\_destroy\_plan(plan)

Arrays allocated with fftw\_malloc should be deallocated by fftw\_free rather than the ordinary free.

The implementation will be conducted on the DSP board (like TMS320 or BlackFinn) we need to shift our code to their compatible format. The FFT transform can be performed by some builtin controller and complex class of the Visual DSP is used

## **Chapter 6**

#### Conclusion

The need for high data rates is increasing day by day, which requires a greater bandwidth, which results in the channel fading. To overcome these problems of frequency selective and multipath fading we analyzed various techniques that were proposed, to provide reliable data communication. Single Carrier with Time domain Equalizer processing at the receiver is one. This approach compensates for the multipath phenomenon for small channel impulse responses. However, as the impulse response gets larger the complexity of time domain equalizer grows, rendering it improper for use in communication systems. Orthogonal Frequency Division Multiplexing is another technique used for this purpose. This has been popular recently and has been employed in various communication systems due to its good performance and simplicity of structure. It transmits the data by sending orthogonal sub-carriers in parallel by using FFT operator. At the receiver the data is retrieved by demodulating the received signal with an IFFT operator and equalizing it with one-tap equalizer. There are however, some drawbacks associated with this mode of communication. OFDM exhibits high Peak to Average Power ratio and therefore requires a large linear range of the amplifier. In addition it is very sensitive to carrier frequency offsets. Single Carrier modulation with Frequency Domain Equalization has been proposed to give performance comparable to OFDM with almost the same overall complexity. It is a topic of recent interests and various comparisons have been done on the performance of the two schemes. Iterative Block DFE has a reduced complexity and gives better performance than the two but is highly dependant on the correlation factor, which if not estimated correctly can worsen the performance. We examined the performance of IBDFE in three cases.

No training seq. No channel Info

No training seq. full channel Info With training seq.

Also we examined the performance of IBDFE with different types of PN sequences. We showed that IBDFE performs best with Zero Padding and when we have full channel knowledge and if we have sent training sequence.

### Future Work

IBDFE performs better than any of the existing equalizers; however it needs to be optimized more, to give high data rates. IBDFE performance is dependant on the proper detection of the previously detected signal. Some mechanism is needed to sense the next or future values; in this way our IBDFE would be able to enhance performance. Also IBDFEs should be designed in the way to work for the multiple users or a MIMO system. In this scenario IBDFE not only to compete for the ISI but also for the interference between the multiple carriers of a MIMO system. Thus IBDFE will require some other mechanism for the compensation of ICI along with the feedback mechanism. Also there is a need to design a new training sequence for the synchronization of timing and frequency offset.

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