Noise Constrained Incremental Least Mean Square Algorithm

Author
Usman Hameed
00000203922

Supervisor
Dr. Sajid Gul Khawaja

Co-Supervisor
Dr. Omer Bin Saeed

MASTERS

in

COMPUTER ENGINEERING

Department of Computer Engineering

COLLEGE OF ELECTRICAL & MECHANICAL ENGINEERING
NATIONAL UNIVERSITY OF SCIENCES & TECHNOLOGY

ISLAMABAD

August, 2019

Noise Constrained Incremental Least Mean Square Algorithm

Author

Usman Hameed

00000203922

A thesis submitted in partial fulfillment of the requirements

for the degree of MS

in the

Computer Engineering

Thesis Supervisor: Dr. Sajid Gul Khawaja

Thesis Supervisor's Signature: ________________________________

DEPARTMENT OF COMPUTER ENGINEERING

COLLEGE OF ELECTRICAL & MECHANICAL ENGINEERING

NATIONAL UNIVERSITY OF SCIENCES & TECHNOLOGY

ISLAMABAD

August, 2019

Declaration

I hereby certify that I have developed this thesis titled as "Noise Constrained Incremental Least Mean Square" entirely on the basis of my personal efforts under the sincere guidance of my supervisor Dr. Sajid. Gul. Khawaja All of the sources used in this thesis have been cited and contents of this thesis have not been plagiarized. No portion of the work presented in this thesis has been submitted in support of any application for any other degree of qualification to this or any other university or institute of learning.

Signature of Student

Usman Hameed

00000203922

Language Correctness Certificate

This thesis has been read by an English expert and is free of typing, syntax, semantic, grammatical and spelling mistakes. Thesis is also according to the format given by the university.

Signature of Student

Usman Hameed

00000203922

Signature of Supervisor

Dr. Sajid Gul Khawaja

Copyright Statement

.

In dedication

To my ***father Abdul Hameed Tahir****:*

for encouraging and supporting me to achieve this daunting task.

To my ***mother Mrs. Hameed Tahir****:*

for making me be who I am.

Acknowledgments

I am thankful to my Creator ALMIGHTY ALLAH Who gave me an opportunity to use best of my knowledge and encouraged me to carry out this work. Without His blessings I wouldn't have achieved this accomplishment. Whosoever guided me throughout the course of my thesis, whether my parents or any other individual was His will, so indeed none be worthy of praise but Him.

I owe my deepest gratitude to my thesis supervisor, ***Dr. Sajid Gul Khawaja*** for his patient guidance, motivation and appreciation throughout this work. My deep gratitude goes to him for showing endless faith in me. I am much esteemed to have worked under his supervision.

I am also highly obliged and grateful to my thesis co-supervisor ***Dr. Omer Bin Saeed*** and committee members ***Dr. Shoab Ahmad Khan*** and ***Dr. Usman Akram*** for their valuable feedback in this research work.

I am grateful to my parents for their prayers and moral support. Their wish to see me achieve greater heights in my education and career has motivated me

I express my gratitude to my friends for actively encouraging me and for all the support they had provided during the course of this study. Additionally I would like to express my gratitude towards everyone who provided me assistance in any stage of this thesis accomplishment.

Abstract

We proposed a noise constrained based distributed adaptive estimation algorithm for wireless sensor network, based on the incremental scheme. The Least Mean Square (LMS) Algorithm's cost function is modified by using noise variance on every nodes, and noise-variance's knowledge is used for estimation the parameter of interest. This modification result to improve convergence speed of the algorithm keeping the steady mean square error minimized. Theoretical Mean and Steady State Analysis are performed for the convergence of the algorithm and steady state mean square error. In Mean analysis the step size limit of the proposed algorithm define, and in steady state analysis the steady state mean square error define. Under different scenarios experimental results show the superiority of the proposed Noise Constrained Incremental LMS over non-constrained ILMS.

Key Words: Adaptive distributed algorithm, Least Mean Square, Noise Constraint, Incremental distributed scheme, Steady State Analysis, Mean Analysis

Table of Contents

List of Tables

List of Figures

Chapter 1

Introduction

1.1 Wireless Sensors Networks(WSNs)

Wireless sensors network consist a group of dedicated sensing nodes which are spatially dispersed. These dedicated nodes measure information and monitor the real time environmental physical condition. The sensing node collect some information from the environment. These information could effect by some sort of noise which are present in the environment. Different type of noise model used in the wireless sensor network such as Gaussian Noise, Laplacian Noise, Uniform Noise, Exponential noise etc. Additive White Gaussian Noise(AWGN) is the common type of noise which effects wireless sensor network.

AWGN is common noise which effect every real time stochastic system. This noise is additive which means it add the system model, and the white means it effect the all possible frequencies of the system and constant noise add in a system and Gaussian shows, it follow the normal distribution.

The AWGN model is simple and due to this we easily observe the underlying behavior of the system. AWGN produced through the different environment changes and weather condition.It also comes though natural noise such as vibration changes and thermal vibration. In wireless sensor Network, there are two network models used, Centralized and distributed Network.

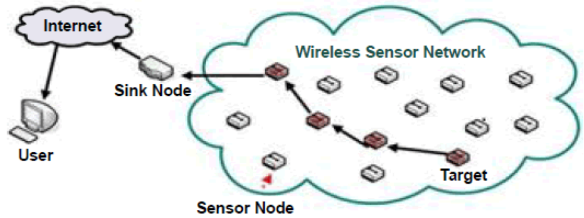

Figure 1.1: Information flow in WSNs

Centralized Network

In centralized wireless sensor network the all sensing nodes collects information individually from the environment and then send their information to the central processing node for processing the information and estimated the desired result as shown in fig 1.2. When estimation complete on the central nodes then the results send back to the all nodes in the network. Due to manage huge amount of data and communicating between the nodes a powerful central processor is required for the Centralized network. Therefor communication overhead increased for the large network (more than 1000 sensors).

Distributed Network

In distributed network the sensing node collect data and share their information to the neighboring node as shown in fig 1.3. In this network no central processor used the nodes direct communicate to the others nodes. The main objective of this network is the estimation is accurate on each node in the network.

In centralized network when the central node fail then the whole network collapse but as compared to this the distributed network robust to node failure. Large communication overhead saved at the cost of signal processing at sensors, battery power used optimally for sub-optimal but acceptable results. The nodes depend exclusively on their local infor-

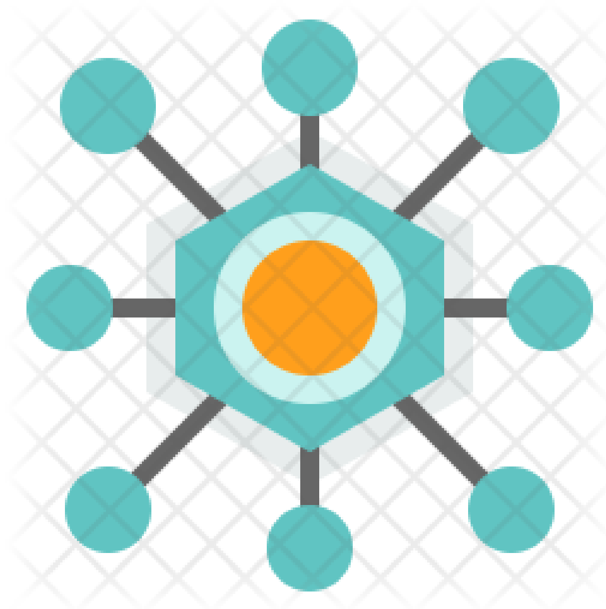

Figure 1.2: Centralized Network

mation and relationships with their instant neighbors in the distributed solution. There is a significant reduction in the quantity of processing and communication [2]-[3].

1.2 Distributed Processing

Distributed Processing is the extraction of data from distributed nodes over the geographic area. The sensing node could collect the noisy information associated with some parameter of interest. In distributed processing there are two schemes of cooperation used for communicating between the nodes.

A) Incremental Scheme

B) Diffusion Scheme

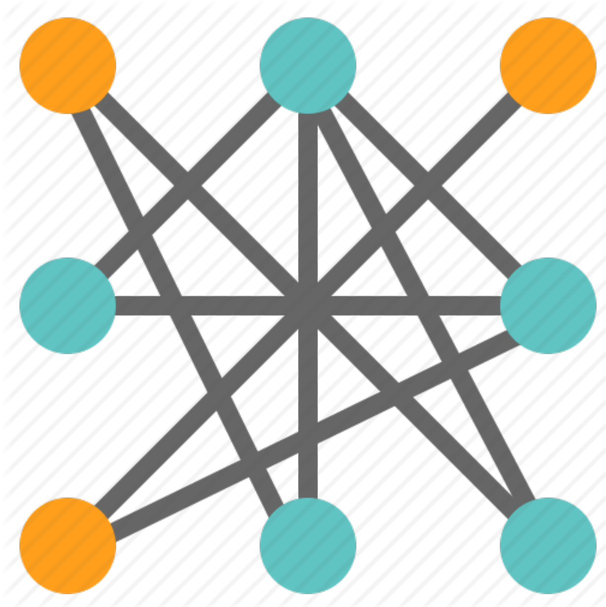

Figure 1.3: Distributed Network

1.2.1 Incremental Scheme

In incremental Scheme the node collect information from the environment and send their estimation result to the adjacent node. Sequential manner follow from the flow of information between nodes over the whole network.

1.2.2 Diffusion Scheme

In Diffusion Scheme the sensing nodes collect information from the environment and send their estimation result to the all node of the network follow some network topology. In Diffusion scheme, excessive communication done between the nodes.

In our research we work on the Distributed network because it robust to node failure. By using incremental scheme for cooperation between the nodes the Communication overhead reduced.

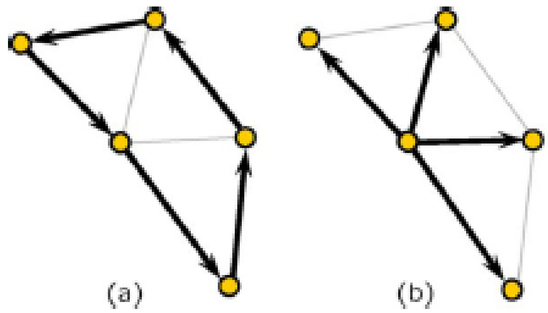

Figure 1.4: Cooperation schemes a) Incremental b) diffusion

1.3 Real Time Application of the Wireless Sensor Network

1.3.1 Temperature Sensing

We take an example of wireless sensor network where the nodes collect local temperature from the environment. every node monitor temperature T_i and the objective of the network is to approach the average temperature of the whole network. The node follow some specific manner for communicating the neighbouring nodes [4][5]), The outcome of the combination is a new measurement of these nodes, following is the mathematical model is applied on every node.

$x_1(i) \leftarrow \alpha_1 x_1(i-1) + \alpha_2 x_2(i-1) + \alpha_5 x_5(i-1)$ (node 1)

Due to this the nodes present in the network slowly approach the average temperature T of the network.

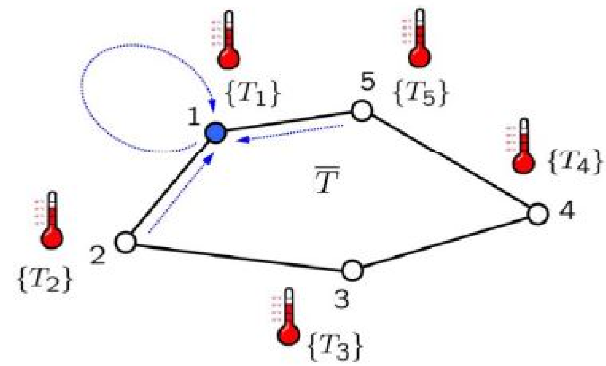

Figure 1.5: Temperature Sensing In a Distributed Network [1].

1.3.2 Monitoring Chemical concentration

In another example where some source diffuse some chemical to the environment and the sensing nodes collect this information and try to estimate the concentration of this chemical in air or in water (see figure. 1.6).

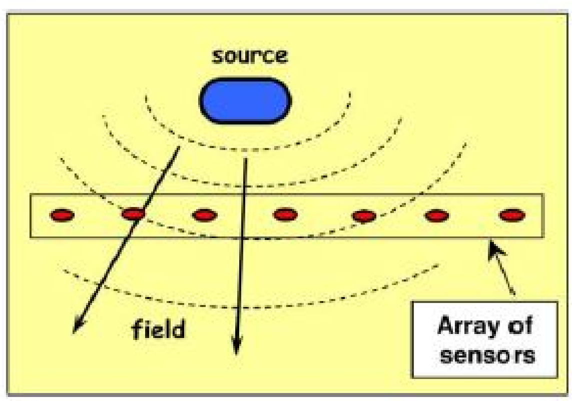

Figure 1.6: Monitoring Chemical Concentration by using diffusion Scheme[1].

This diffusion model are translated into a diffusion equation which is given below:

$\frac{\partial c(x,t)}{\partial t} = \theta_1 \frac{\partial^2 c(x,t)}{\partial x^2} + \theta_2 \frac{\partial c(x,t)}{\partial x} + \theta_3 c(x,t) + u(x,t)$

where c(x,t) refers to chemical concentration at some location x in time t [6], [7] .

The foundation of future control networks and data communication will be such distributed networks connecting laptops, PCs, sensors, actuators and cell phones. Applications range from sensor networks to agricultural accuracy, target location, environmental monitoring, management of disaster relief, smart spaces and medical applications [2], [7] - [8], ([4] [5], and [9]).

We see different real time application that are translated into some mathematical model. These model can be adaptive or non-adaptive. In WSNs most commonly adaptive model used for estimation of some the parameter of interest.

1.4 Least Mean Square (LMS) Model

LMS model is commonly used adaptive algorithm in most of the research. The basic idea behind of this algorithm is that the estimated weights approach to the optimum weights of the actual output. The wights update through stochastic gradient descent algorithm. In this algorithm the error feed back to the adaptive system for optimum value convergence given in fig 1.7.

The gradient descent method given below: $w_i = w_{i-1} + \alpha E(J(w))$

Where w_i is the updated weights, w_{i-1} is the previous weights, α show the fixed and variable step size of the LMS algorithm and $J(w)$ show the cost function of the LMS algorithm.

1.5 Background

All the past researches use adaptive algorithm which make use of unconstrained parameters. However, the NCLMS algorithm used constrained parameters but all nodes converge with same steady state estimation and the NCDLMS worth on diffusion network and more computations occur because every node handles the whole network nodes parameters. So

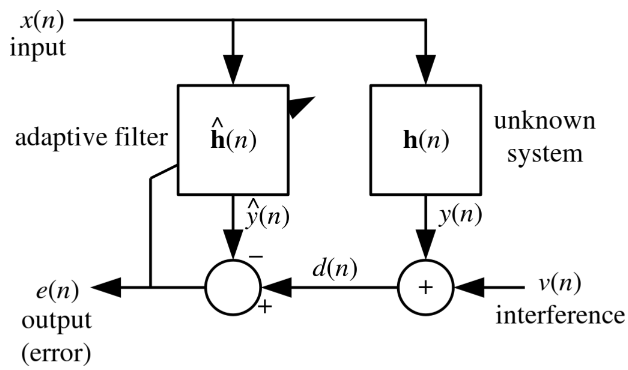

Figure 1.7: Adaptive LMS Model

we work on Incremental scheme in which node communicate to the next adjacent neighbor and all nodes collaborate in cyclic pattern.

1.6 Problem Statement

This brings us to the problem statement of my thesis which is:

"Derive and Analyze a Noise Constrained Least Mean Square Algorithm based on the Incremental scheme for a distributed Wireless Sensor Network."

In the current era a lot of research has been carried out on the development of adaptive algorithms which can be applied on distributed networks to extract meaningful information of parameters which are of interest. In our research, we plan to work on adaptive LMS algorithm to limit the parameter noise variance in distributed network by using incremental strategy. The noise variance is not required in Wiener solution (theory perspectives) but there are some benefits for understanding the adaptive solution. Robbins-Monro approach is used to minimized the error for noise variance constraint. After applying NCLMS algorithm in incremental distributed network, we work on analysis part, and extensive simulation performed which will compare our algorithm with other adaptive algorithms.

1.7 Aims and Objective

Aims and objectives of the research are as follows:

- Detailed analysis and comparison of different methodologies proposed for distributed network.
- Identification and tracking of noisy linear FIR channel.
- Effect of variable step size in adaptive algorithm for estimation of noise parameter
- Overcome the trade off between convergence speed and state state mean square error of the adaptive algorithm.

1.8 Structure of the thesis

This chapter briefly introduces the problem tackled in the thesis. The rest of the thesis is structured in the following fashion:

Chapter 2 discusses some of the previous LMS algorithm techniques used over distributed network and some noise-constrained algorithm.

Chapter 3 understand the problem formulation over the distributed network using incremental technique for node communication and presents the proposed algorithm which use noise parameter for estimation

Chapter 4 in this chapter whole analysis present to check the stability of the proposed algorithm.Three type of analysis perform i.e. mean analysis, transient state and steady state analysis

Chapter 5 All simulation done in this chapter.

Finally the thesis is concluded in Chapter 6. Furthermore, guidelines and future prospects for the proposed algorithm and analysis part have also been highlighted in this chapter.

Chapter 2

Literature review

We start this chapter by using of stochastic- gradient method to develop the theory of adaptive network. These method derive from steepest-descent solution by replacing the gradient vector and Hessian matrix with some approximation.Different approximation helps to build different algorithm with performance and some degree of complexity. The resultant algorithm called stochastic-gradient algorithms.

There are two main purpose by using stochastic-gradient algorithm. First they avoid to use the exact signal statistics such as co-variances matrix, but this method achieve this feature by using learning mechanism that enable them to measure this type of signal statistics. Second this method own a tracking mechanism. These two main reason are behind the wide spread of this method.There are following stochastic-gradient algorithms, but our main focus on just least mean square most of the research circulate around this concept.

1. The Least mean Square(LMS) algorithm.
2. The affine projection algorithm(APA).
3. The recursive least-square algorithm(RLS).
4. The Normalized Least mean Square(NLMS) algorithm.

We focus on least mean square(LMS) in our research. The LMS is a class of adaptive filter in which tap weights updates through stochastic gradient descent algorithm and meet the real or actual values. As compare with error which is the difference between actual and estimated values the wights updates. The basic objective of this algorithm to predict the desired signal by using information of the actual values and error. The LMS algorithm was invented in 1960.

In [10] first time work on step size of LMS algorithm. The basic LMS has fixed step size and in this research the variable step size introduced. When mean square error increased the step size decreased and when mean-square error decreased the step size increased. Due to this the Variable step size least mean square(VSSLMS) perform better as compared to previous LMS algorithm.The complete algorithm derived and presented. The mean and mean-square analysis performed for the robustness of the algorithm with theory results compared with simulation results.

In [11] worked on modified step size. when large step size used the convergence speed increased but mean-square error also increased therefor three step size used for convergence of the algorithm.Three different feedback system introduced with gradient descent of different steps size. The proposed algorithm obtain low mean-square error and fast convergence.This system is used for multi user detection but due to this the hardware complexity of the detector increased.

In [12] worked on constrained adaptive algorithm for finite impulse response channel estimation. The channel noise variance is used in the gradient descent algorithm. The Langrange and Robbin-Munro method is used to reduced the conventional mean-square error and minimized the effect of noise information in the basic gradient descent algorithm. The resulting algorithm was a variable step size (VSS) and called Noise constrained Least Mean Square(NCLMS) algorithm. This algorithm worked better all previous VSS algorithm.

In [13] worked on error Nonlinearities of adaptive algorithm and developed a unified approach for transient analysis. They presented the new performance result without restriction of regression data are Gaussian or not. The complete energy conservation relation derived for the adaptive algorithm with avoiding the explicit recursion of weigh error vectors.

In [1] adaptive algorithm research enhanced and apply the adaptive algorithm over distributed network.The incremental scheme used for communication between the nodes over adaptive network where each nodes pass their information to the adjacent neighbors. The proposed algorithm enforce the problem of linear estimation in cooperative fashion where each nodes pas information throughout the network.The mean and steady state analysis performed and robustness of the performed algorithm subjected through theory nad simulation results.

In [14] the previous ILMS algorithm updates and used diffusion scheme for cooperation between the nodes. In diffusion scheme we see the whole network as a single unit. The advancement of this network the transient analysis presented and learning curve derived.

In [15] presented distributed LMS for Consensus-Based-In Network adaptive processing. In this work the wireless Network sensor used for the online parameter estimation and tracking of non stationary signals.

In [16] presented new variation in previous adaptive algorithm over distributed network. In this work the fixed step size converted into variable step size (VSS). These techniques apply both incremental and diffusion fashion. The proposed algorithm works better then previous algorithm.The simulation results presented and compare with Incremental Least Mean Square (ILMS) and Diffusion Least Mean Square (DLMS).The proposed algorithm shown that VSSLMS with adaptive combiners provides a simplified solution than that of the Diffusion LMS algorithm.

In [17] efficient adaptive combination strategy used for distributed estimation over diffusion network.To derive the proposed adaptive combiner the concept of minimum variance unbiased estimation was used in a systematic way. The complete mean and mean square analysis performed to check the stability and robustness of the algorithm. The theoretical analysis show the better approximation of practical performance.

In [18] addressed the problem of reliability of observation in distributed adaptive network. Then we propose a new distributed incremental LMS algorithm which consider the reliability of observation.Two phases included in the proposed algorithm a training phase estimating phase. In first phase every nodes estimated the observation noise variance and unknown parameter; and in second phase according to its observation noise variance the step-size parameter for each node is adjusted. Finally the proposed algorithm improves the performance of distributed incremental least mean square(DILMS).

In [19] works shows the effect of noise variances used in the diffusion least mean square algorithm(DLMS). they used the noise variance and change the step size into variable form. the mean and steady state analysis performed to show the robustness of the algorithm. The proposed algorithm worked better when the noise information not available at the sensing nodes.The comparison performed between DLMS, VSSDLMS and purposed NCDLMS and simulation show that the resulting algorithm worked better.

In [20] proposed the transient analysis of the Incremental combination of two least mean square(LMS) algorithm.In this work the previous proposed combiner redesigned to improve the overall combination performance.

In [21] proposed constrained algorithm over adaptive distribution network using diffusion scheme. Complete mean and mean square analysis performed. The mean square analysis lead to the transient and steady state analysis where the steady state error cater fall. Theoretical and simulation results compared. This work compared with past DLMS, and purposed algorithm worked better in both convergence and steady state error.

In [22] proposed a variable step size diffusion least mean square. They change the step size presented in previous DLMS into variable form. The complete analysis performed. And te results was better from DLMS algorithm.

In [23] proposed improved LMS system for time varying application by using hybrid variable and fixed step size. The algorithm worked better then previous non hybrid system.

In [24] improved the least mean square(LMS) and normalized least mean square(NLMS) algorithm by improving convergence speed and stability of both algorithm.

In [25] addresses the performance of LMS algorithm by changing step size and apply on different iterations. This paper basically a performance analysis of lMs algorithm in changing scenarios.

In [26] proposed a multi rate least mean square algorithm and compare the performance of proposed algorithm with past multi rate LMS algorithm.

In [27] proposed a adaptive algorithm over distributed network which lean and track the non stationary data by using diffusion scheme for cooperation between the sensing nodes.Laplacian Regularized (LR) LMS and diffusion adaptation LR LMS used when a similarity found on the sensing nodes and can be utilized for the respective distributed and centralized cases.

In [28] presented a unified analysis approach for the variable step size algorithm. In past many variable step size algorithm presented but there was a drawback that is the analysis not performed in closed form. The complete mean square analysis performed ehich leads the steady state and transient analysis. The simulation and theory analysis compared all the VSS adaptive algorithm

2.1 Summary of the Literature Review

This is a summary of our literature review which shows that different researcher works on different aspects. Either they have worked on distributed network , constrained and step size. We works on all three aspects. Our algorithm is noise constrained and apply over distributed adaptive algorithm using incremental scheme and having variable step size.

Algorithm	Date	Distributed Network	Constrained	Step Size(α)
NCLMS	2001	No	constrained	variable
ILMS	2007	Yes using Incremental Scheme	Non- Constrained	fixed
DLMS	2008	Yes using Diffusion Scheme	Non- Constrained	fixed
NCDLMS	2013	Yes using Diffusion Scheme	Constrained	Variable
VSS LMS analysis	2017	No	Both	Variable
Diffusion LMS analysis	2018	Yes using Diffusion Scheme	Non-Constrained	Fixed
Proposed NCILMS	2019	Yes using Incremental Scheme	Constrained	variable

Table 2.1: Summary Of The Literature Review

Chapter 3

Proposed Methodology

There have been a lot of work in literature on incremental scheme in order to solve the problem of estimation over distributed adaptive network. In this scheme the cost function is decoupled with the cost function of the other nodes in the network.By using adaptive algorithm to minimize the cost function based on incremental scheme [1].

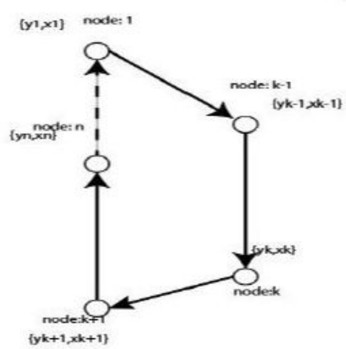

Figure 3.1: Adaptive network of N nodes.

3.1 Problem Statement

Let us consider a distributed network of N sensor nodes communicating with each other using the incremental scheme to estimate a parameter w_0 of size $(M \times 1)$ as shown in Fig 3.1. In the incremental scheme, each node k updates its estimate using the estimate from node (k-1) and then passes it on to node (k+1). Each node collects data and uses a regression vector x_k of size $(1 \times M)$ which helps in scalar measurement of ${y_k}^{(i)}$ that are related by:

$$y_k^i = \mathbf{x}_k^i \mathbf{w_0} + n_k\,(i) \tag{3.1}$$

where n_k is zero mean stationary Gaussian noise with variance $\sigma^2_{n_k}$. The scalar measurement and the regression vector that are collected from each nodes are transformed to a matrix form.

$$\begin{aligned} X &= col x_1, x_2, ..., x_N (N \times M) && (3.2) \\ Y &= col y_1, y_2, ..., y_N (N \times 1) && (3.3) \end{aligned}$$

The information of x_k and y_k is used to estimate w_0 with an iterative update w_k^i at node K. By assuming that every k^{th} node communicates and shares information only with their neighbor at every time instant i. The objective of the adaptive algorithm is to minimize the cost function given by

$$J_k(w) = \mathbb{E}[(y_k - x_k w)^2] \tag{3.4}$$

w is the estimated weight of optimum weight $w0$. The gradient decent algorithm is given as:

$$w_i = w_{i-1} - \mu \frac{\sigma}{\sigma_w} J_k(w) \tag{3.5}$$

after solving above equation its simplifies by:

$$w_i = w_{i-1} - \mu(R_{y,x_k} - R_{x_k} w_{i-1}) \tag{3.6}$$

where $R_{y,x}$ is cross-correlation between y_k and x_k, and R_{x_k} is auto-correlation of x_k. A more better solution for estimation of w_k in [29] and simply incremental LMS algorithm is given by:

$$\Phi_0^{(i)} = w_{i-1} \tag{3.7}$$

$$\Phi_k^i = \Phi_{k-1}^i + \mu x_{k,i}(y_k^i - x_k \Phi_{k-1}^i]) \tag{3.8}$$

$$w_i = \Phi_N^i \tag{3.9}$$

where Φ is intermediate updates of weight which are passed through one sensor node to the next immediate neighbor following the incremental scheme.

3.1.1 Proposed Changes In the Incremental Least Mean Square Algorithm

- The step size μ used in ILMS algorithm defined by (3.8) and (3.9) are fixed.
- This leads to trade-off between speed and steady-state error.
- Introducing a constraint in cost function, by improving this trade-off at the cost of increased computational complexity.
- This conversion helps adaptive algorithm to converge faster while keeping the steady state error minimized.

The above-mentioned derivation leads us to a noise constrained algorithm which is presented in the next section.

3.2 Proposed Noise Constraint Incremental Least Mean Square Algorithm

In this section we discuss our proposed noise constrained least mean square algorithm (NCLMS) which makes use of derivation from previous section. Let us consider the following equation

$$y_k^i = w_0^T x_k^i + n_k \tag{3.10}$$

where x_k is zero mean stationary input process with co-variance $R = E[x_k x_k^T]$ and n_k is also a zero mean stationary noise with variance σ_n^2. Both n_k and x_k are uncorrelated.

The cost function can be decompose as:

$$J(w) = \sum_{k=1}^{N} J_k(w) \tag{3.11}$$

and each $J_k(w)$ is given by:

$$J_k(w) = \mathbb{E}\,|y_k - w^T x_k|^2 \tag{3.12}$$

3.2.1 Modified Cost Function

Now consider a constrained minimization problem that incorporates the knowledge of noise variance σ_n^2. We need to minimize $J_k(w)$ over w subject to constrained $J(w) = \sigma_n$, the

Lagrangian for this problem is:

$$J_{k1}(w,\lambda) = J_k(w) + \lambda(J_k(w) - \sigma_n^2) \tag{3.13}$$

Where λ is Lagrangian multiplier. The critical value of k(w, λ) are (w, λ):w=wo.

There is no fair critical value of w because critical λ is not unique so this is a potential problem for an adaptive algorithm. To avoid this problem we subtract $\lambda\sigma_n^2$ from (3.13) to get augmented Lagrangian [12] resulting in the following equation.

$$J_{k2}(w,\lambda) = J_k(w) + \gamma\lambda(J_k(w) - \sigma_n^2) - \gamma\lambda^2 \tag{3.14}$$

where γ is just constant term, now the unique critical value for $J_k2(w,\lambda)$ is $(w,\lambda) = (w0, 0)$. Keeping in view the equation 3.12 the cost function comes out to be:

$$J_k2(w,y,x,\lambda) = J_k(w,y,x) + \gamma\lambda(J_k(w,y,x) - \sigma_n^2) - \gamma\lambda^2 \tag{3.15}$$

we consider a training sequences of x_k, y_k which is provided to every node. In such cases a better solution of root finding algorithm for extracting critical values (wo,0) is Robbin-Monro(RM) algorithm [30].

$$w_{k+1}^i = w_k^i + \alpha\frac{\sigma}{\sigma_w}J(w,y,x,\lambda) \tag{3.16}$$

$$\lambda_{k+1} = \lambda_k - \beta\frac{\sigma}{\sigma_{\lambda_k}}J(w,y,x,\lambda) \tag{3.17}$$

where α and β are just positive step size and negative and positive sign of the partial derivative terms chose to stable the values of (wo,0) of the system.

By solving the partial derivative of (3.16) and (3.17) we get:

$$\begin{aligned}
\frac{\sigma}{\sigma_w} J_{k2} &= (2+2\gamma\lambda)(y_k - w^T x_k)(x_k) && (3.18)\\
\frac{\sigma}{\sigma_{\lambda_k}} J_{k2} &= \gamma((y_k - w^T x_k)^2 - \sigma_n^2) - 2\gamma\lambda_k && (3.19)
\end{aligned}$$

Now substituting the partial derivatives terms (3.19) and (3.18) in equation (3.16) and (3.17) we get:

$$\begin{aligned}
w_{k+1} &= w_k + \alpha(1+\gamma\lambda_k)(y_k - w^T x_k)x_k && (3.20)\\
\lambda_{k+1} &= \lambda_k(1-\beta) + \frac{\beta}{2}((y_k - w^T x_k) - \sigma_n^2) && (3.21)
\end{aligned}$$

Using this constrained knowledge in (3.7) we have a better practical adaptive solution shown below:

$$\begin{aligned}
\epsilon_0^i &= \epsilon_{i-1} && (3.22)\\
\epsilon_k^i &= \epsilon_{k-1}^i + \alpha_k \mathbf{x}_{k,i}^* (d_k^i - x_{k,i}\epsilon_{k-1}^i) && (3.23)\\
\alpha_k &= \alpha(1+\gamma\lambda_k^i) && (3.24)\\
\lambda_k^i &= (1-\beta)\lambda_{k-1}^i + \frac{\beta}{2}\left(e_k^2(i) - \sigma_{n,k}^2\right) && (3.25)\\
\epsilon_i &= \epsilon_N^i && (3.26)
\end{aligned}$$

When compared this algorithm with ILMS , where step size α_k is not a fixed value, it changes with every instant of time i and every k^{th} node. Moreover, (3.23) has an extra term of Lagrange multiplier whose value changes from node to node according to (3.25). Thus to conclude the proposed NCILMS algorithm is mathematically shown in equations

(3.22) to (3.26).

Chapter 4

Analysis Of the Proposed Algorithm

In the previous section we derive a Noise Constraint Incremental Least Mean Square Algorithm, Now we addresses the stability and robustness of the algorithm through some analysis. Theoretical analysis are performed for the convergence of the algorithm and steady state mean square error. The analysis show the robustness of the algorithm and check stability in different scenario. Two type of analysis performed in our work.

- Mean Analysis
- Steady State Analysis

In mean analysis the step size limit of our proposed algorithm defined.

In steady state analysis the steady state mean square error equation derived, and due to this equation we check the robustness of the proposed algorithm by comparing theoretical and simulation results of steady mean square error.

To find the actual value of the system the following equation used. In this equation the optimum weights is used for finding the true observation.

$$y_k(i) = x_{k,i}w^0 + n_k(i) \tag{4.1}$$

In the above equation $n_k(i)$ show noise with noise variance $\sigma_{n,k}$, This linear model is shown in eq (4.1) are used in many real time application [6],[31],[32], [33], [34]. The following analysis also handle non stationary data where w_o change with respect to time ([32],[35]).

4.1 Mean Analysis

In order to check the stability of the algorithm mean analysis is carried out and we interested in mean square deviation (MSD) [28] and data realization vectors in (3.2) and (3.3). Subtracting wo from equation (3.23) we get:

$$\varphi_{k+1} = (I - \alpha_k x_k x_k^T)\varphi_k - \alpha_k n_k x_k \tag{4.2}$$

Applying the expectation operator on (3.25), (3.24) and (4.2) equations. In (4.2), when we apply the data assumption, we can separate Φ_k from other variables. After these changes the term further simplifies and which is given below:

$$\begin{aligned} \mathbb{E}[\varphi_{k+1}] &= [I - \alpha_k R]\varphi_k & (4.3)\\ \mathbb{E}[\alpha_k] &= \alpha(1 + \gamma\lambda_k) & (4.4)\\ \mathbb{E}[\lambda_{k+1}] &= (1-\beta)\lambda_k + \frac{\beta}{2}J_k & (4.5) \end{aligned}$$

where $J_k = \mathbb{E}[e_k^2]$-σ_n^2.

From [1] we noticed that the incremental LMS algorithm is stable when weights are in the unit circle[36]. Therefor Lagrange multiplier effect the stability of the algorithm. In this case algorithm is stable when each node holds the given term:

$$(I_M - \alpha_k(1 + \gamma\,\mathbb{E}[\lambda_k])R_{x,k}) \to 0, n \to \infty \tag{4.6}$$

which holds true if

$$0 < \alpha_k < \frac{2}{(1 + \gamma_{\mathrm{NC}}\mathbb{E}\left[\lambda_{k,i}\right])\,\theta_{\max}\left(\mathbf{R}_{\mathbf{x},k}\right)},\ \ 1 \le k \le N \tag{4.7}$$

where θ_{max} shows the maximum eigenvalues of $R_{x,k}$. The step size limit depends on the stability of the λ_k we use all parameters arbitrary and usually its depends on signal to noise ratio (SNR)

4.2 Mean Square Analysis

The mean square analysis follows a specific learning behavior for the curve. This analysis leads to the two main analysis, transient and steady state analysis. By starting this analysis we take some terms which used in the further analysis. We define some weights error which shown as given below

$$\begin{aligned}
\Phi_k^i &= w^0 - \varphi_k^i(wight - error - vector - at - time - i) && (4.8)\\
e_{a,k}(i) &= x_{k,i}\Phi_{k-1}^i(a - priori - error) && (4.9)\\
e_{p,k}(i) &= x_{k,i}\Phi_k^i(a - posteriori - error) && (4.10)\\
e_k(i) &= d_k(i) - x_{k,i}\Phi_{k-1}^i(output - error) && (4.11)
\end{aligned}$$

$$e_k(i) = y_k(i) - x_{k,i}\mathit{\Phi}_{k-1}^i = x_{k,i}w^0 + n_k(i) = e_{a,k}(i) + n_k(i) \qquad (4.12)$$

Hence,$E|e_k(i)|^2 = E|e_{a,k(i)}|^2 + \sigma_{n,k}^2$, so that evaluating $E|e_{a,k(i)}|^2$ is useful for evaluating $E|e_k(i)|^2$.

$$\begin{aligned}
\eta_k &= E||\Phi^(i)_{k-1}||^2(MSD) && (4.13)\\
\mathit{\Psi}_k &= E|e_{a,k}(i)|^2(EMSE) && (4.14)
\end{aligned}$$

$$\eta_k = E||\Phi^i_{k-1}||^2_I, \Psi_k = E|e_{a,k}(i)|^2_{R_{u,k}} \tag{4.15}$$

$$e^{\sum}_{a,k}(i) = x_{k,i}\sum\Phi^i_{k-1} and e^{\sum}_{p,k}(i) = x_{k,i}\sum\Phi^i_k \tag{4.16}$$

when apply the expectation operation on the variable step size equations the resultant is given by:

$$E[\lambda_k] = (1-\beta)E[\lambda_{k-1}] + \frac{\beta}{2}(E[e^2_k - \sigma^2_k]) \tag{4.17}$$

$$\alpha^i_k = \alpha(1+\gamma\lambda'_k) \tag{4.18}$$

$$\Phi^i_{k-1}, \Phi^i_k, e^{\sum}_{a,k}(i), e^{\sum}_{p,k}(i) \tag{4.19}$$

$$\Phi^i_k = \Phi^i_{k-1} - \alpha_k x_{k,i} e_k(i) \tag{4.20}$$

Multiplying the previous equation from the left by $xk, i\sum$ gives

$$x_{k,i}\sum\Phi^i_k = x_{k,i}\sum\Phi^i_{k-1} - \alpha_k||x_{k,i}||^2_{\sum} e_k(i) \tag{4.21}$$

so that from the definitions (4.16)

$$e^{\sum}_{p,k}(i) = e^{\sum}_{a,k}(i) - \alpha_k||x_{k,i}||^2_{\sum} e_k \tag{4.22}$$

and, subsequently

$$e_k(i) = \frac{1}{\alpha_k}\frac{(e^{\sum}_{a,k}(i) - e^{\sum}_{p,k}(i))}{||x_{k,i}||^2_{\sum}} \tag{4.23}$$

Substituting (4.23) into (4.20) and rearranging terms,

$$||\Phi_k^i||^2_{\sum} + \frac{|e^{\sum}_{a,k}(i)|^2}{||x_{k,i}||^2_{\sum}} = ||\Phi_{k-1}^i||^2_{\sum} + \frac{|e^{\sum}_{p,k}(i)|^2}{||x_{k,i}||^2_{\sum}} \tag{4.24}$$

$$||\Phi_k^i||^2_{\sum} = ||\Phi_{k-1}^i||^2_{\sum} - \alpha_k e^{\sum}_{a,k} e_k - \alpha_k e_k e^{\sum}_{a,k} + \alpha_k^2 ||x_k||^2_{\sum} . |e_k|^2 \tag{4.25}$$

Using (4.12) and taking expectation of both sides gives:

$$E||\Phi_k^i||^2_{\sum} = E||\Phi_{k-1}^i||^2_{\sum} - \alpha_k E e^{\sum}_{a,k} e_k - \alpha_k E e_k e^{\sum}_{a,k} + \alpha_k^2 \sigma^2_{n,k} E||x_k||^2_{\sum} + \alpha_k^2 E||x_k||^2_{\sum} . |e_{a,k}|^2 \tag{4.26}$$

$$\begin{aligned} E||\Phi_k^i||^2_{\sum} =& E||\Phi_{k-1}^i||^2_{\sum} \\ & - \alpha_k E \Phi^*_{k-1} \sum x_k^* x_k \Phi_{k-1} \\ & - \alpha_k E \Phi^*_{k-1} x_k^* x_k \sum \Phi_{k-1} \\ & + \alpha_k^2 \sigma^2_{n,k} E||x_k||^2_{\sum} + \alpha_k^2 E \Phi^*_{k-1} \sum x_k^* x_k \Phi_{k-1} \end{aligned} \tag{4.27}$$

Now, given that $||x||^2_A + ||x||^2_B = ||x||^2_{A+B}$, the previous equation can be rewritten more compactly as

$$E||\Phi_k||^2_{\sum} = E(||\Phi_k||^2_{\sum'}) + \alpha_k^2 \sigma^2_{n,k} E||x_k||^2_{\sum} \tag{4.28}$$

in terms of stochastic weighting matrix

$$\sum{}' = \sum - \alpha_k (x_k^* x_k \sum + \sum x_k^* x_k) + \alpha_k^2 ||x_k||^2_{\sum} x_k^* x_k \tag{4.29}$$

Invoking the independence of the regression data x_k allows us to write

$$E(||\Phi_k||^2_{\sum'}) = E(||\Phi_k||^2_{E\sum'}) \tag{4.30}$$

So that (4.28) and (4.29) become

$$E||\Phi^i_k||^2_{\sum} = E||\Phi^i_{k-1}||_{\sum"} + \alpha^2_k \sigma^2_{n,k} E||x_k||^2_{\sum} \tag{4.31}$$

where $\sum" = E\sum'$ is given by

$$\sum' = \sum -\alpha_k E(x^*_k x_k \sum + \sum x^*_k x_k) + \alpha^2_k E||x_k||^2_{\sum} x^*_k x_k \tag{4.32}$$

and $\sum"$ is now deterministic matrix.

$$Ex^*_k x_k = R_{x,k}, E||x_k||^2_{\sum} = Tr(R_{x,k}\sum), andE||x_k||^2_{\sum} x^*_k x_k \tag{4.33}$$

In [32] paper, we assume Gaussian data for simplicity. Thus, assume that the x_k arise from a circular Gaussian distribution and introduce the eigen decomposition $R_{x,k} = X_k \Lambda_k X^*_k$, where Λ_k is unitary and is a diagonal matrix with the eigenvalues of $R_{x,k}$. Introduce further the transformed quantities
$\overline{\Phi_k} = X^*_k \Phi_k, \overline{\phi_{k-1}} = X^*_k \Phi_{k-1}, \overline{x_k} = x_k X_k$
$\overline{\sum} = X^*_k \sum X_k, \overline{\sum"} = X^*_k \sum" X_k$
Since X_k is unitary, we have that $\phi_k = X^*_k \Phi_k$ and $\phi_{k-1} = X^*_k \Phi_{k-1}$, so that (4.2) and (4.32) can be rewritten in the equivalent forms

$$E||\overline{\Phi^i_k}||^2_{\overline{\sum}} = E||\overline{\Phi^i_{k-1}}||_{\overline{\sum"}} + \alpha^2_k \sigma^2_{n,k} E||\overline{x_k}||^2_{\sum} \tag{4.34}$$

$$\overline{\sum^{"}} = \overline{\sum} - \alpha_k E(\overline{\mathrm{x_k^*}}\overline{\mathrm{x_k}}\overline{\sum} + \overline{\sum}\overline{\mathrm{x_k^*}}\overline{\mathrm{x_k}}) + \alpha_k^2 E||\overline{\mathrm{x_k}}||^2_{\overline{\sum}}\overline{\mathrm{x_k^*}}\overline{\mathrm{x_k}} \tag{4.35}$$

The moments we need to evaluate are now $E||\overline{\mathrm{x_k}}||^2_{\overline{\sum}}, E(\overline{\mathrm{x_k^*}}\overline{\mathrm{x_k}}) and E||\overline{\mathrm{x_k}}||^2_{\overline{\sum}}\overline{\mathrm{x_k^*}}\overline{\mathrm{x_k}}$. The first two moments are straightforward since

$$E||\overline{\mathrm{x_k}}||^2_{\overline{\sum}} = Tr(\Lambda_k \overline{\sum}), and E(\overline{\mathrm{x_k^*}}\overline{\mathrm{x_k}}) = \Lambda_k \tag{4.36}$$

The third moment is given for Gaussian regressors by [32]

$$E||\overline{\mathrm{x_k}}||^2_{\overline{\sum}}\overline{\mathrm{x_k^*}}\overline{\mathrm{x_k}} = \Lambda_k Tr(\overline{\sum}\Lambda_k) + \gamma \Lambda_k \overline{\sum}\Lambda_k \tag{4.37}$$

where $\gamma = 1$ for circular complex data and $\gamma = 2$ for real data. Substituting (4.36) and (4.37) into the variance relation (4.2), (4.35) leads to

$$E||\overline{\Phi^{\mathrm{i}}_{\mathrm{k}}}||^2_{\overline{\sum}} = E||\overline{\Phi^{\mathrm{i}}_{\mathrm{k-1}}}||_{\overline{\sum^{"}}} + \alpha_k^2 \sigma^2_{n,k} Tr(\Lambda_k \overline{\sum}) \tag{4.38}$$

$$\overline{\sum^{"}} = \overline{\sum} - \alpha_k \Lambda_k \overline{\sum} + \overline{\sum}\Lambda_k + \alpha_k^2 \Lambda_k Tr(\overline{\sum}\Lambda_k) + \gamma \Lambda_k \overline{\sum}\Lambda_k \tag{4.39}$$

$\overline{\sigma} = diag(\overline{\sum})$, $\overline{\sigma}' = diag(\overline{\sum}')$,$\lambda_k = diag(\Lambda_k)$

where the $diag()$ notation will be used in two ways: $\Lambda = diag(\lambda)$ is a diagonal matrix whose entries are those of the vector λ , $\lambda = diag(\Lambda)$ and is a vector containing the main diagonal of Λ .

Using the diagonal notation, expression (4.39) can be rewritten in terms of $(\overline{\sigma}, \lambda_k)$ as'

$$\overline{\sigma'} = (I - 2\alpha_k \Lambda_k + \gamma \alpha_k^2 \Lambda_k^2)\overline{\sigma} + \alpha_k^2(\lambda_k^T)\lambda_k = \overline{\mathrm{F_k}}\overline{\sigma} \tag{4.40}$$

where $\overline{\mathrm{F_k}}$ is defined by

$$\overline{\mathrm{F_k}} = I - 2\alpha_k\Lambda_k + \gamma\alpha_k^2\Lambda_k + \alpha_k^2\lambda_k\lambda_k^T \tag{4.41}$$

Moreover,$g_k = \mu_k^2\sigma_{n,k}^2(\lambda_k^T$ expression (4.38) becomes

$$E||\overline{\Phi_{\mathrm{k}}^{\mathrm{i}}}||_{\overline{\sigma}_k}^2 = E||\overline{\Phi}_{k-1}^i||_{\overline{\mathrm{F_k}}\overline{\sigma}_k} + g_k\overline{\sigma_{\mathrm{k}}} \tag{4.42}$$

$$\begin{aligned}
E||\overline{\Phi}_1^i||_{\overline{\sigma}_1}^2 &= E||\overline{\Phi}_N^i||_{\overline{\mathrm{F}}_1\overline{\sigma}_1} + g_1\overline{\sigma}_1 \\
E||\overline{\Phi}_2^i||_{\overline{\sigma}_2}^2 &= E||\overline{\Phi}_1^i||_{\overline{\mathrm{F}}_2\overline{\sigma}_2} + g_2\overline{\sigma}_2 \\
E||\overline{\Phi}_{k-2}^i||_{\overline{\sigma}_{k-2}}^2 &= E||\overline{\Phi}_{k-3}^i||_{\overline{\mathrm{F}}_{k-2}\overline{\sigma}_{k-2}} + g_{k-2}\overline{\sigma}_{k-2} \\
E||\overline{\Phi}_N^i||_{\overline{\sigma}_N}^2 &= E||\overline{\Phi}_{N-1}^i||_{\overline{\mathrm{F}}_N\overline{\sigma}_N} + g_N\overline{\sigma}_N
\end{aligned} \tag{4.43}$$

Choosing $\overline{\sigma_{\mathrm{k-2}}} = \overline{\mathrm{F}}_{k-2}\overline{\sigma}_{k-2}$, we could write $||\overline{\Phi_{\mathrm{k-1}}^{\mathrm{i}}}||_{\overline{\sigma}_{k-1}}^2$ in term of $||\overline{\Phi^{\mathrm{i}}}_{k-2}||_{\overline{\sigma}_{k-2}}^2$ as follow:

$$E||\overline{\Phi}_{k-2}^i||_{\overline{\mathrm{F}}_{k-1}\overline{\sigma}_{k-1}}^2 = E||\overline{\Phi}_{k-3}^i||_{\overline{\mathrm{F}}_{k-2}\overline{\mathrm{F}}_{k-1}\overline{\sigma}_{k-1}} + g_{k-2}\overline{\mathrm{F}}_{k-1}\overline{\sigma}_{k-2} \tag{4.44}$$

$$E||\overline{\Phi}_{k-1}^i||_{\overline{\sigma}_k-1}^2 = E||\overline{\Phi}_{k-3}^i||_{\overline{\mathrm{F}}_{k-2}\overline{\mathrm{F}}_{k-1}\overline{\sigma}_{k-1}} + g_{k-2}\overline{\mathrm{F}}_{k-1}\overline{\sigma}_{k-1} + g_{k-1}\overline{\sigma}_{k-1} \tag{4.45}$$

for node $k-1$ the same iteration we get:

$$\begin{aligned} E||\overline{\Phi}_{k-1}^{i}||_{\overline{\sigma}_{k-1}}^{2} &= E||\overline{\Phi}_{k-3}^{i}||_{\overline{\mathrm{F}}_{k}....\overline{\mathrm{F}}_{N}\overline{\mathrm{F}}_{1}....\overline{\mathrm{F}}_{k-1}\overline{\sigma}_{k-1}} \\ &+ g_{k}\overline{\mathrm{F}}_{k+1}....\overline{\mathrm{F}}_{N}\overline{\mathrm{F}}_{1}....\overline{\mathrm{F}}_{k-1}\overline{\sigma}_{k-1} \\ &+ g_{k+1}\overline{\mathrm{F}}_{k+2}....\overline{\mathrm{F}}_{N}\overline{\mathrm{F}}_{1}....\overline{\mathrm{F}}_{k-1}\overline{\sigma}_{k-1} \\ &. \\ &. \\ &. \\ &+ g_{k-2}\overline{\mathrm{F}}_{k-1}\overline{\sigma}_{k-1} + g_{k-1}\overline{\sigma}_{k-1} \end{aligned} \tag{4.46}$$

(4.46) by defining $\Pi_{k,l}$ as follows:

$$\Pi_{k,l} = \overline{\mathrm{F}}_{k+l-1}\overline{\mathrm{F}}_{k+l}....\overline{\mathrm{F}}_{N}\overline{\mathrm{F}}_{1}....\overline{\mathrm{F}}_{k-1} l = 1, 2, 3, ...N \tag{4.47}$$

So:

$$E||\overline{\Phi}_{k-1}^{i}||_{\overline{\sigma}_{k-1}}^{2} = E||\overline{\Phi}_{k-1}^{i}||_{\Pi_{k,l}\overline{\sigma}_{k-1}} + a_{k}\overline{\sigma}_{k} \tag{4.48}$$

where:

$$a_{k} = g_{k}\Pi_{k,2} + g_{k+1}\Pi_{k,3} + ... + g_{k-2}\Pi_{k,N} + g_{k-1} \tag{4.49}$$

where $g_{k} = \alpha_{k}^{2}\sigma_{n,k}^{2}\lambda_{k}^{T}$

4.3 Steady State Analysis

The mean square analysis follows a specific learning behavior for the curve that leads to the steady-state mean squared error. This error is defined through two different terms Mean Square Deviation (MSD) and Excess Mean Square Error (EMSE), depending on what value is chosen for the weighting matrix.

$$E||\overline{\Phi}_{k-1}||^2_{\overline{\sigma}_{k-1}} = E||\overline{\Phi}_{k-1}||_{\Pi_{k,l}\overline{\sigma}_{k-1}} + a_k\overline{\sigma}_k \tag{4.50}$$

By simplifying we obtain:

$$E||\overline{\Phi}_{k-1}||^2_{(I-\Pi_{k,l}\sigma_{k-1})\overline{\sigma}_{k-1}} = a_k\overline{\sigma}_{k-1} \tag{4.51}$$

According to $\eta_k = E||\overline{\Phi}_{k-1}||^2_q$ and $\zeta_k = E||\overline{\Phi}_{k-1}||^2_{\lambda_k}$

$$\eta_k = a_k(I - \Pi_{k,i})^{-1}q(MSD) \tag{4.52}$$

Likewise, by choosing σ_{k-1} that satisfies $(I - \Pi_{k,i})\overline{\sigma}_{k-1} = \lambda_k$, we can achieve an equation for steady state EMSE quantity, i.e.,

$$\zeta_k = a_k(I - \Pi_{k,i})^{-1}\lambda(EMSE) \tag{4.53}$$

(4.52)to(4.53) show the steady state behaviors of MSD and EMSE

Chapter 5

Simulation Result

In this section we will discuss the experimentation that has been carried out for the testing and validation of our proposed algorithm. We performed different simulations to check the quality of the proposed NCILMS algorithm. The performance measure for comparison has been taken as MSD. Moreover, the results have been calculated for different values of signal to noise ratio (SNR).

In our network we take N equal to 20 and M equal to 5 . The constrained parameter α,β, γ are random selected. λ is the Lagrangian operator.We use AWGN Model for experimentation

There are two types of performance comparison that are considered.

- MSD: Mean Square Deviation
- EMSE: Excess Mean Square Error

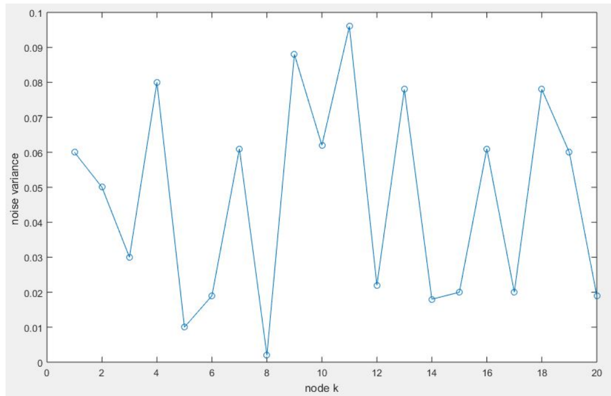

Figure 5.1: Observation noise power profile at each node.

we compare our results with non-constrained ILMS algorithm. The experimental setup contains 20 nodes in the network and the length of unknown vector is kept at 5. The noise at one node is independent to the other nodes so noise variance σ_n is different at every node. In ILMS the step size has been fixed to 0.02. For the NCILMS algorithm, α, β, are fixed at at 0.0027, 0.01 and 14.1 respectively.

Furthermore, initial Lagrange multiplier λ is fixed at 0.996. The simulation is carried out at average SNR of 20db. The results have been recorded after 1000 iteration and 100 Monte Carlo runs. With respect to time/iteration i, the estimated weights w approach to optimum weights $w0$ and the Lagrange multiplier λ approaches to zero.

Fig 5.2, 5.3 and 5.4 shows the comparative results of our algorithm against non-constrained ILMS at SNR 10, 20 and 30 dB it can be seen from the graph that our proposed algorithm outperforms the non-constrained ILMS algorithm.The proposed algorithm converge faster as compared to the incremental least mean square algorithm keeping the steady state mean square error same.

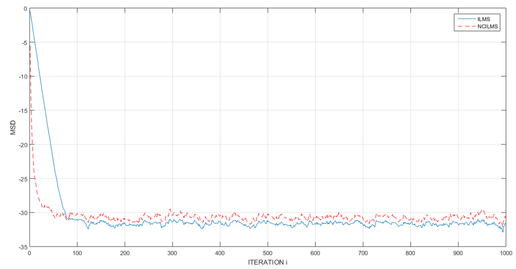

Figure 5.2: MSD comparison of distributed ILMS with adaptive combiners,NCILMS for a network of 20 nodes.

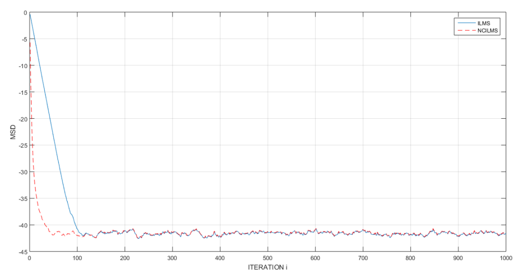

Figure 5.3: MSD comparison of distributed ILMS with adaptive combiners,NCILMS for a network of 20 nodes.

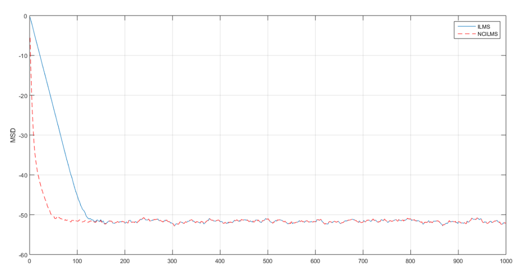

Figure 5.4: MSD comparison of distributed ILMS with adaptive combiners,NCILMS for a network of 20 nodes.

In figures 5.5 we use another performance measure excess mean square error (EMSE) of proposed NCILMS algorithm with non-constrained ILMS algorithm at SNR 10db. It can be seen in this figure that performance of our proposed algorithm is better as compared to ILMS algorithm.

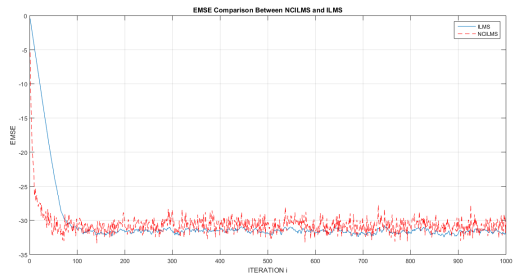

Figure 5.5: EMSE comparison of distributed ILMS with adaptive combiners,NCILMS for a network of 20 nodes at SNR 10db.

The performance of NCILMS improves at SNR 20db and 30db clearly shown in figure 5.6 and figure 5.7. The convergence rate is better than other ILMS algorithm.

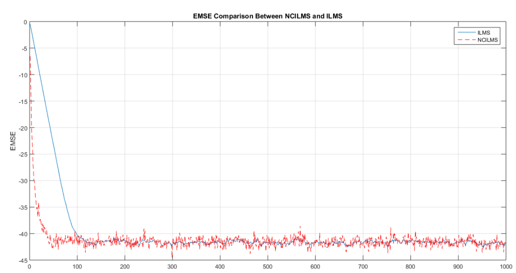

Figure 5.6: EMSE comparison of distributed ILMS with adaptive combiners,NCILMS for a network of 20 nodes at SNR 10db.

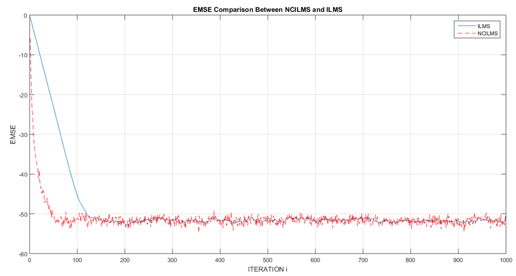

Figure 5.7: EMSE comparison of distributed ILMS with adaptive combiners, NCILMS for a network of 20 nodes at SNR 20db.

when taking the step size value large the convergence speed fast . but the steady state mean square error maximize also. and when taking the values of step size small then the convergence speed slow , but steady state mean square error minimize. Our proposed algorithm works better in both situation as shown in fig. 5.8 and fig. 5.9 and overcome this trade off by using noise variance in modified cost function.

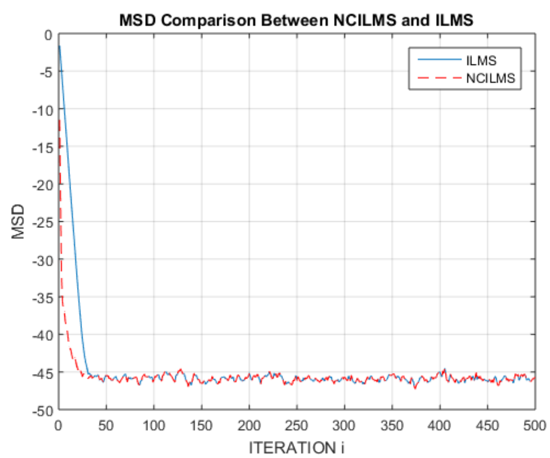

Figure 5.8: MSD comparison of distributed ILMS with adaptive combiners, NCILMS for a network of 20 nodes at SNR 20db using step size 0.01.

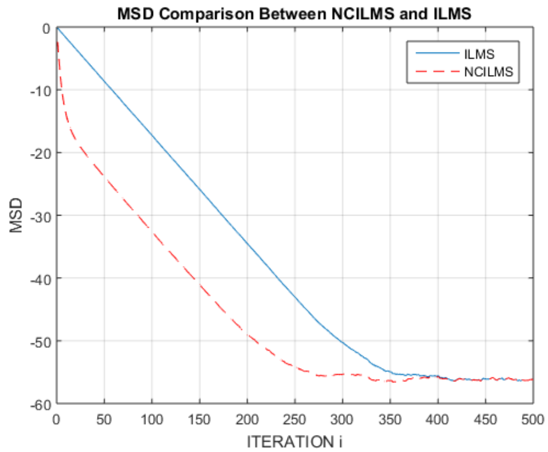

Figure 5.9: MSD comparison of distributed ILMS with adaptive combiners, NCILMS for a network of 20 nodes at SNR 20db using step size 0.001.

In table (5.1) the steady state mean square error of theory and simulation results shown. This show that our algorithm is excellent in different scenario when the noise variance is accurately measured. We clearly see the minor difference in the MSD values of simulation results and the MSD values derived from the theoretical equation (4.52) which addresses our proposed algorithm robustness.

SNR(dB)	α	β	γ	λ	Sim(dB)	Th(dB) equ(4.52)
20	0.01	0.01	14.1	0.9	-40.87	-40.84
30	0.01	0.01	10.3	1.7	-50.95	-50.77
30	0.001	0.01	14.1	0.98	-52.75	-52.74
20	0.01	0.01	12	1.9	-45.22	-44.32

Table 5.1: Comparison Between Theory And Simulation results of Steady State Mean Square Error

Chapter 6

Conclusion And Future Work

In this work we have proposed a noise constrained incremental least mean square algorithm. The complete derivation of the algorithm is given in (3.2) and in order to check the stability of the algorithm mean analysis has also been carried out.The step size on every nodes has been derived and verify the step size by analysis. The steady state analysis completed derived and the results show in result section.The results indicate that our proposed algorithm works better and its convergence rate is also faster as compared with previous ILMS algorithm with the steady state mean square error minimized. Noise constrained LMS algorithm is derived for WSN based on the incremental scheme. Mean analysis and steady-state mean square analysis have been for the proposed algorithm. Simulation and theory results found to be matching for the steady-state analysis

In future we perform complete mean square analysis for the algorithm, including learning behavior during the transient stage. we will perform the analysis where the noise variance not accurately measure.

References

[1] Cassio G Lopes and Ali H Sayed. Incremental adaptive strategies over distributed networks. *IEEE Transactions on Signal Processing*, 55(8):4064–4077, 2007.

[2] Deborah Estrin, Lewis Girod, Greg Pottie, and Mani Srivastava. Instrumenting the world with wireless sensor networks. In *icassp*, volume 1, pages 2033–2036. Citeseer, 2001.

[3] Mati Wax and Thomas Kailath. Decentralized processing in sensor arrays. *IEEE transactions on acoustics, speech, and signal processing*, 33(5):1123–1129, 1985.

[4] John Tsitsiklis and Michael Athans. Convergence and asymptotic agreement in distributed decision problems. *IEEE Transactions on Automatic Control*, 29(1):42–50, 1984.

[5] Lin Xiao and Stephen Boyd. Fast linear iterations for distributed averaging. *Systems & Control Letters*, 53(1):65–78, 2004.

[6] Lorenzo A Rossi, Bhaskar Krishnamachari, and C-CJ Kuo. Distributed parameter estimation for monitoring diffusion phenomena using physical models. In *2004 First Annual IEEE Communications Society Conference on Sensor and Ad Hoc Communications and Networks, 2004. IEEE SECON 2004.*, pages 460–469. IEEE, 2004.

[7] Dan Li, Kerry D Wong, Yu-Hen Hu, and Akbar M Sayeed. Detection, classification, and tracking of targets. 2002.

[8] David Culler, Deborah Estrin, and Mani Srivastava. Overview of wireless sensor networks. *IEEE Computer, Special Issue in Sensor Networks*, 37(8):41–49, 2004.

[9] Demetri P Spanos, Reza Olfati-Saber, and Richard M Murray. Approximate distributed kalman filtering in sensor networks with quantifiable performance. In *Proceedings of the 4th international symposium on Information processing in sensor networks*, page 18. IEEE Press, 2005.

[10] Raymond H Kwong and Edward W Johnston. A variable step size lms algorithm. *IEEE Transactions on signal processing*, 40(7):1633–1642, 1992.

[11] Byung Goo Choi, Yong Wan Park, and Jeong Hee Choi. The adaptive least mean square algorithm using several step size for multiuser detection. In *Vehicular Technology Conference Fall 2000. IEEE VTS Fall VTC2000. 52nd Vehicular Technology Conference (Cat. No. 00CH37152)*, volume 6, pages 2822–2825. IEEE, 2000.

[12] Yongbin Wei, Saul B Gelfand, and James V Krogmeier. Noise-constrained least mean squares algorithm. *IEEE Transactions on Signal Processing*, 49(9):1961–1970, 2001.

[13] Tareq Y Al-Naffouri and Ali H Sayed. Transient analysis of adaptive filters with error nonlinearities. *IEEE Transactions on Signal Processing*, 51(3):653–663, 2003.

[14] Cassio G Lopes and Ali H Sayed. Diffusion least-mean squares over adaptive networks: Formulation and performance analysis. *IEEE Transactions on Signal Processing*, 56(7):3122–3136, 2008.

[15] Ioannis D Schizas, Gonzalo Mateos, and Georgios B Giannakis. Distributed lms for consensus-based in-network adaptive processing. *IEEE Transactions on Signal Processing*, 57(6):2365–2382, 2009.

[16] Muhammad Omer Bin Saeed, Azzedine Zerguine, and Salam A Zummo. Variable step-size least mean square algorithms over adaptive networks. In *10th International*

Conference on Information Science, Signal Processing and their Applications (ISSPA 2010), pages 381–384. IEEE, 2010.

[17] Noriyuki Takahashi, Isao Yamada, and Ali H Sayed. Diffusion least-mean squares with adaptive combiners: Formulation and performance analysis. *IEEE Transactions on Signal Processing*, 58(9):4795–4810, 2010.

[18] Amir Rastegarnia, Mohammad Ali Tinati, and Azam Khalili. A distributed incremental lms algorithm with reliability of observation consideration. In *2010 IEEE International Conference on Communication Systems*, pages 67–70. IEEE, 2010.

[19] Muhammad O Bin Saeed, Azzedine Zerguine, and Salam A Zummo. A robust lms adaptive algorithm over distributed networks. In *2011 Conference Record of the Forty Fifth Asilomar Conference on Signals, Systems and Computers (ASILOMAR)*, pages 547–550. IEEE, 2011.

[20] Luiz FO Chamon and Cássio G Lopes. Transient performance of an incremental combination of lms filters. In *21st European Signal Processing Conference (EUSIPCO 2013)*, pages 1–5. IEEE, 2013.

[21] Muhammad O Bin Saeed, Azzedine Zerguine, and Salam A Zummo. A noise-constrained algorithm for estimation over distributed networks. *International Journal of Adaptive Control and Signal Processing*, 27(10):827–845, 2013.

[22] Muhammad O Bin Saeed, Azzedine Zerguine, and Salam A Zummo. A variable step-size strategy for distributed estimation over adaptive networks. *EURASIP Journal on Advances in Signal Processing*, 2013(1):135, 2013.

[23] Farqad Y Farhan and Siddeeq Y Ameen. Improved hybrid variable and fixed step size least mean square adaptive filter algorithm with application to time varying system

identification. In *2015 10th System of Systems Engineering Conference (SoSE)*, pages 94–98. IEEE, 2015.

[24] Divya Sharma and Rashpinder Kaur. Improvement in convergence speed and stability of least mean square and normalized least mean square algorithm. In *2015 2nd International Conference on Computing for Sustainable Global Development (INDIACom)*, pages 1496–1500. IEEE, 2015.

[25] Rachana Nagal, Pradeep Kumar, and Poonam Bansal. Performance analysis of least mean square algorithm for different step size parameters with different filter order and iterations. In *2015 International Conference on Recent Developments in Control, Automation and Power Engineering (RDCAPE)*, pages 326–331. IEEE, 2015.

[26] Adem Ukte and Aydin Kizilkaya. Comparing the performances of least mean squares based multirate adaptive filters. In *2016 26th International Conference Radioelektronika (RADIOELEKTRONIKA)*, pages 325–329. IEEE, 2016.

[27] May Zar Lin, Manohar N Murthi, and Kamal Premaratne. Regularized lms and diffusion adaptation lms with graph filters for non-stationary data. In *2017 IEEE 7th International Workshop on Computational Advances in Multi-Sensor Adaptive Processing (CAMSAP)*, pages 1–5. IEEE, 2017.

[28] Muhammad Omer Bin Saeed. Lms-based variable step-size algorithms: A unified analysis approach. *Arabian Journal for Science and Engineering*, 42(7):2809–2816, 2017.

[29] Ali H Sayed. Adaptive networks. *Proceedings of the IEEE*, 102(4):460–497, 2014.

[30] Marie Duflo. *Random iterative models*, volume 34. Springer Science & Business Media, 2013.

[31] Lin Xiao, Stephen Boyd, and Sanjay Lall. A scheme for robust distributed sensor fusion based on average consensus. In *IPSN 2005. Fourth International Symposium on Information Processing in Sensor Networks, 2005.*, pages 63–70. IEEE, 2005.

[32] Ali H Sayed. *Fundamentals of adaptive filtering*. John Wiley & Sons, 2003.

[33] Jarvis Haupt and Robert Nowak. Signal reconstruction from noisy randomized projections with applications to wireless sensing. In *IEEE/SP 13th Workshop on Statistical Signal Processing, 2005*, pages 1182–1187. IEEE, 2005.

[34] Habib Benali, Mélanie Pélégrini-Issac, and Frithjof Kruggel. Spatio-temporal covariance model for medical images sequences: Application to functional mri data. In *Biennial International Conference on Information Processing in Medical Imaging*, pages 197–203. Springer, 2001.

[35] Nabil R Yousef and Ali H Sayed. A unified approach to the steady-state and tracking analyses of adaptive filters. *IEEE Transactions on Signal Processing*, 49(2):314–324, 2001.

[36] Federico S Cattivelli and Ali H Sayed. Diffusion lms strategies for distributed estimation. *IEEE transactions on signal processing*, 58(3):1035–1048, 2009.