

Vehicle Modeling and Performance Evaluation Using Active torque Distribution

By

Rizwan Latif

2011-NUST-MS PhD-37



Submitted to the Department of Mechanical Engineering in fulfillment of the requirements for the degree
of

**MASTER OF SCIENCE
IN
MECHANICAL ENGINEERING**

Thesis Supervisor

Lt Col Dr. Aamer Ahmad Baqai

College of Electrical & Mechanical Engineering
National University of Sciences and Technology (NUST)
2014



***IN THE NAME OF ALLAH THE MOST GRACIOUS
THE MOST MERCIFUL***

To Aisha

DECLARATION

I hereby declare that the research paper titled “**VEHICLE MODELLING AND PERFORMANCE EVALUATION USING ACTIVE TORQUE DISTRIBUTION**” is my own hard work and to the best of my knowledge does not contain any materials which was previously published or was written by another person, nor material which to a certain extent has been accepted for the award of any degree or diploma at NIIT or any other education institute, except where due acknowledgment, is made in the thesis. Any other contribution made to the research by others, with whom I have worked at NIIT or elsewhere, is explicitly acknowledged in the thesis.

I also declare that the intellectual content of this thesis is the product of my own work, except to the extent that assistance from others in the project’s design and conception or in style, presentation and linguistic is acknowledged. I also verified the originality of contents through plagiarism software.

Author Name: Rizwan Latif

Signature: _____

ACKNOWLEDGEMENT

I would like to thank Almighty Allah, who is the most beneficent and the greatest forgiver, who guided me in through every difficulty and my parents that due to their efforts, hard work and prayers today I exist.

At the beginning I was totally unaware what to do but thanks to Sir Raja Amir who lead me to the correct direction and extended every support to me any time I needed it. I would like to express my gratitude to Dr. Aamer Ahmad Baqai, Dr Imran Shafi and Dr. Hasan Aftab, who co-operated with me and helped me in every aspect of my thesis. I would also like to thank all the teachers who taught me in my two years of education and made me capable of doing all this.

It was all possible because of the support of my friends morally and technically specially Adnan. I would also like to thank Nabeel, Ajmal and Awais who helped me a lot in my studies. And more importantly I express my extreme gratitude towards my family, my sons Hassan and Mahad. I want to thank my wife especially as her patience, support, and, tolerance was the driving force for me. Without her I would have not been able to do all this work.

Contents

INTRODUCTION

1.1	MOTIVATION	2
1.2	PROBLEM STATEMENT	3
1.3	PROPOSED SOLUTION	4
1.4	OUT LINE OF REPORT	5

2. LITERATURE REVIEW AND BACKGROUND KNOWLEDGE

2.1	LITERATURE REVIEW	7
2.2	BACKGROUND	13
2.2.1	DYNAMIC SYSTEM MODELLING	13
2.2.2	Vehicle Coordinate Systems	14
a.	Earth Fixed Coordinate System	14
b.	Vehicle Fixed Coordinate System	15
2.2.3	Lumped Mass	16
2.2.4	Wheel Model	16
a.	Complex Physical Model:	16
b.	Similarity Method:	17
c.	Simple Physical Model:	17
d.	Empirical Tire Model:	17
2.2.4.1	Wheel Ground Contact Point Velocities	18
2.2.4.2	Wheel Slip and Tire Side Slip Angle	19
2.2.4.3	Friction Co-efficient Calculation	20
2.2.4.4	Calculation of Friction Forces	20
2.3	THE COMPLETE VEHICLE MODEL	22
a.	Handling Model	22
b.	Ride Model	25
c.	Model Verification	26
2.4	CHAPTER SUMMARY	27

3. PROPOSED METHODOLOGY

3.1	YAW MOMENT CONTROL THROUGH TORQUE TRANSFER	29
3.2	CENTRAL TRANSFER CASE TORQUE TRANSFER	30
a.	Strategy 1: Switching Corrective Torque b/w Front and Rear Axles	32

b.	Strategy 2: Corrective Torques: Add to the Front and Subtract to Rear Axle_____	32
c.	Strategy 3: Addition / Subtraction of Torque only to Front Axle _____	33
d.	Strategy 4: Addition / Subtraction of Torque only to Rear Axle_____	33
3.4	ACTIVE TORQUE DISTRIBUTION STRATEGIES THROUGH LATERAL ACCELERATION 34	
3.5	CHAPTER SUMMARY _____	36
4. Results and Discussion		
4.1	SELECTION OF TEST MANEUVERS _____	38
a.	J-Turn Test: _____	38
b.	Double Lane Change (DLC):_____	38
4.2	SIMULATION OF ROAD SURFACE CONDITIONS _____	38
4.3	IMPLEMENTATION OF CONTROL STRATEGIES _____	38
a.	J-Turn Test _____	39
(1)	Oversteer Case: _____	40
(1)	Understeer Case: _____	43
a.	Double Lane Change (DLC):- _____	46
4.4	BEST STRATEGY _____	51
4.5	CONTROLLER SWITCHING _____	52
	CHAPTER SUMMARY _____	54
5. Conclusion and Future Work		
5.1	CONCLUSION _____	56
5.2	FUTURE WORK _____	56

LIST OF ABBREVIATIONS

VDC	Vehicle Dynamic Control
FL	Front Left Wheel
FR	Front Right Wheel
RL	Rear Left Wheel
RR	Rear Right Wheel
V_W	Wheel Velocity
V_x	Vehicle Velocity in Longitudinal Direction
V_y	Vehicle Velocity in Lateral Direction
$\dot{\psi}$	Yaw Rate
l_f	Distance from Vehicle CoG to Front Wheel
l_R	Distance from Vehicle CoG to Rear Wheel
l	Base of Vehicle
d_r	Distance b/w two rear wheels
d_f	Distance b/w two front wheels
F_{WL}	Force on Wheel in Longitudinal Direction
F_{WS}	Force on Wheel in Lateral Direction
F_X	Friction Force in x-axis
F_Y	Friction Force in y-axis
m_{CoG}	Mass of Vehicle at CoG
h_{CoG}	Height from ground to CoG
J_Z	Mass Moment of Inertia in z-axis
J_w	Wheel Mass Moment of Inertia

J_x	Mass Moment of Inertia in x-axis
J_y	Mass Moment of Inertia in y-axis
K_p	Proportional Gain
K_i	Integral Gain
K_d	Differential Gain

LIST OF FIGURES

FIG 1.1	The Functioning of Yaw Control System	3
FIG 2.1	Vehicle in Earth fixed coordinate system	15
FIG 2.2	Vehicle Fixed Coordinate System	15
FIG 2.3	Velocity Components throughout the Vehicle	18
FIG 2.4	Wheel Slip Calculation	19
FIG 2.5	Direction of Friction Forces	21
FIG 2.6	Vehicle Handling Model	23
FIG 2.7	Rotational Dynamics of Wheel	24
FIG 2.8	Vehicle Ride Model	25
FIG 2.9	Comparison of Yaw Rate of 10 DOF Model against of CarSim	26
FIG 2.10	Comparison of Lateral Acceleration of 10 DOF Model against of CarSim	27
FIG 3.1	Block Diagram of Vehicle Dynamic Control via Active Torque Distribution	29
FIG 3.2	Schematic of Vehicle in Various Scenarios and Adopted Sign Conventions in Steady State	31
FIG 3.3	Block Diagram of Vehicle Dynamic Control via Active Torque Distribution	34
FIG 4.1	J-Turn Steer Input	39
FIG 4.2	Path Followed by 10 DOF Model	39
FIG 4.3	Comparison of Yaw Rate in Oversteer using Strategy 1	41
FIG 4.4	Comparison of Yaw Rate in Oversteer using Strategy 2	41
FIG 4.5	Comparison of Yaw Rate in Oversteer using Strategy 4	42
FIG 4.6	Comparison of Yaw Rates with different Strategies	42
FIG 4.7	Tire Longitudinal Slip in Oversteer case	43
FIG 4.8	Comparison of Yaw Rate in Understeer using Strategy 1	45
FIG 4.9	Comparison of Yaw Rate in Understeer using Strategy 2	45
FIG 4.10	Comparison of Yaw Rate in Understeer using Strategy 3	46
FIG 4.11	Comparison of Yaw Rates with different Strategies	46

FIG 4.12	Tire Longitudinal Slips in Understeer Case	47
FIG 4.13	Design Path for DLC	48
FIG 4.14	Design Path of Vehicle at 15 m/s on snow	49
FIG 4.15	Yaw Rate -Reference value vs. No VDC	49
FIG 4.16	Yaw Rate with VDC	50
FIG 4.17	Slips of Wheels with NO VDC	51
FIG 4.18	Slips at Wheels with VDC	51
FIG 4.19	Yaw Rate –With and w/o VDC at Ice	52
FIG 4.20	Switching Controller in Understeer Case	54
FIG 4.21	Switching Controller in Over Steer Case	54

LIST OF TABLES

Table 4.1	Different parameters with Oversteer case	53
Table 4.2	Different parameters with Understeer case	53

ABSTRACT

During sharp maneuvers, to control the yaw stability of a vehicle, the available solutions may be either brake based or lifting of gas pedal. These systems help in controlling the braking and driving forces that operate on the left and right wheels in such a way that a driver has a direct control over these cornering forces. As a result, they facilitate an appreciably safer and more enjoyable driving, however the vehicle systems based on brakes, on sharp maneuvers, have been exposed to decline the longitudinal performance, and impose an understeer or oversteer behavior, as their system conforms to more stability and less performance.

An alternate to the system described above is Active Torque Distribution system using either electronically controlled differential or electronically controlled central transfer case. Until now much work was done on torque distribution between right and left wheels using electronic control differentials. A limited amount of work on torque distribution between front and rear axles was done using electronically control transfer case. This research uses the second approach due to following reasons:-

- a. Use of active differential is costly as compared to a electronically controlled central transfer case (as proposed in this research)
- b. Loss of energy/power as in the case of differential braking will be solved by using this strategy.

In conducting this research, a non-linear ten degree-of-freedom vehicle model incorporating a non-linear tire model was adopted and simulated in the MATLAB environment. Using this model, various VDC torque management architectures as well as choices of feedback controllers were studied. For the purposes of yaw stability control design, the desired or reference performance of the vehicle was obtained from a neutrally steered vehicle model. Standard test maneuvers such as J-turn test and Double Lane Change (DLC) were simulated to evaluate the effectiveness of the proposed torque distribution strategies. The simulation results indicated that all VDC torque management strategies were generally very effective in tracking the reference yaw rate of the vehicle on both dry and slippery surface conditions

CHAPTER # 1

INTRODUCTION

MOTIVATION

PROBLEM STATEMENT

PROPOSED SOLUTION

OUTLINE OF THE REPORT

1.1 Motivation

In Four Wheels Drive, vehicle performance and safety on critical driving conditions are the most concerned areas for the researchers. In the last two decades, there have been extensive developments in the field of automotive passive safety such as seatbelts and airbags, improved crumple zones in car bodies, increased use of high strength steel, etc. These developments are useful in mitigating the harm in an accident but they do not help to prevent the accident. Further, these safety features are very beneficial in frontal impact crashes but are less effective in side impact and rollovers. With the emergence of automobile industry, vehicle dynamics control (VDC) systems have been developed to enhance safety and control of vehicles. The control of drivers over their vehicle is the primary concern of vehicle dynamics control systems, which seek to prevent unintended vehicle behavior through active control of different assemblies and help the drivers to maintain control of their vehicles. A few major systems that have become common equipment in production passenger vehicles are, Anti-lock Brake Systems (ABS) to prevent lock-up of wheel, and traction control systems to prevent the drive wheels from losing grip when accelerating [1]. In addition to ABS and traction control, many automotive companies have been developing Electronic Stability Control (ESC) systems or yaw control systems to prevent the vehicle from spins, skids, and rollovers [2]. Rollover prevention, collision warning and avoidance, and preparation for an impending accident by adjusting seat belts and seat positions are additional examples of control systems for safety purposes. The main function of electronic stability control is to provide improved performance, stability and control not only when the vehicle is accelerating and braking but also during cornering and avoiding obstacles.

During cornering, a stability control system or yaw control system compares desired motion/behavior with the vehicle's actual one. When the stability control system detects any discrepancy between them, it automatically applies brakes to individual wheels to control the direction of the vehicle back in line with the driver's intent. The driver's intention is calculated from the speed of the vehicle and driver's commanded steering wheel angle. Here yaw control/differential braking could be one of the possible control input.

1.2 Problem Statement

The problem statement can be very skillfully judged through the figure below. In Fig. 1.1 the lower curve shows that when a steering input is given to a vehicle on a dry and had a high tire-road friction coefficient road, the trajectory that the vehicle would follow. In this case the high friction coefficient is able to provide the lateral force required by the vehicle to negotiate the curved road. If the vehicle speed were too high and coefficient of friction were small, then the vehicle would be incapable of following the nominal motion required by the driver - it would instead travel on a trajectory of larger radius (smaller curvature), as shown in the upper curve of Fig 1.1. It happens generally with the vehicle with no additional measure to overcome this problem. A solution to this which an average driver makes is, either to lift the gas pedal or apply brakes. In both the cases the vehicle has to face deterioration in longitudinal speed.

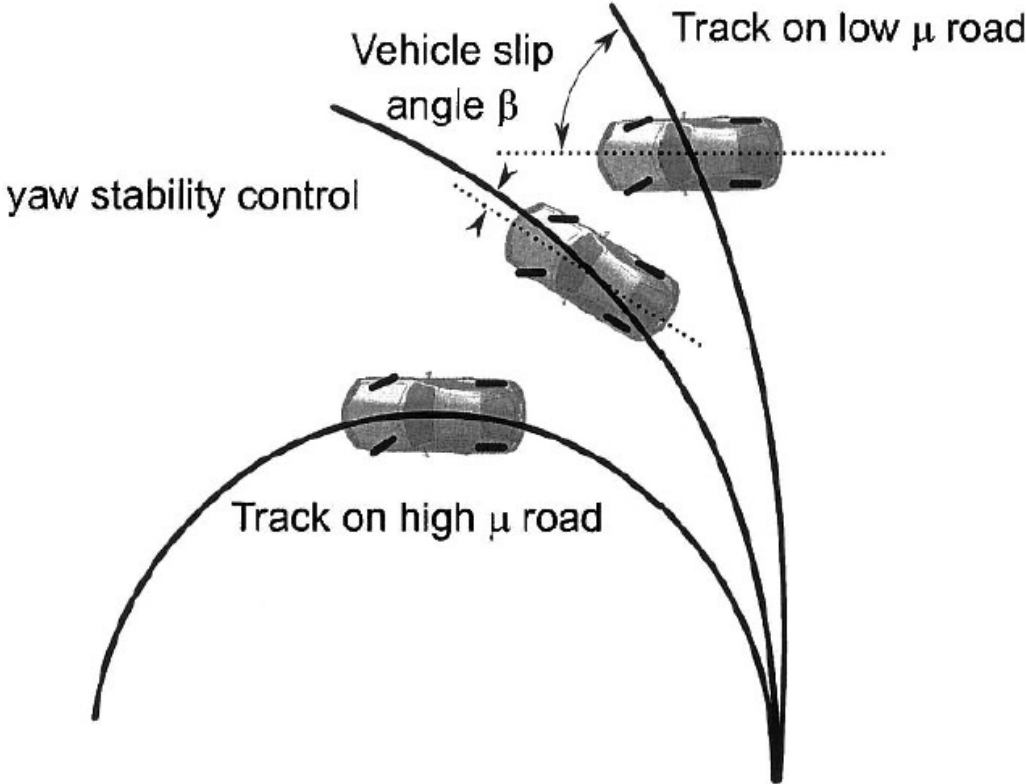


Figure 1.1: The Functioning of Yaw Control System

1.3 Proposed Solution

The solution to above mentioned problem is the incorporation of Electronic or Yaw Stability Control Systems. The aim of the yaw control system is to restore the yaw velocity of the vehicle as much as possible to the nominal motion commanded by the driver. If the coefficient of friction is very small, it might not be possible to entirely achieve the nominal yaw rate motion that would be achieved by the driver on a high friction coefficient road surface. In this case, the yaw control system would partially succeed by making the vehicle's yaw rate closer to the expected nominal yaw rate, as shown by the middle curve in said figure.

Three types of stability control systems have been proposed and developed for yaw control:

Differential Braking systems which utilize the ABS brake system on the vehicle to apply differential braking between the right and left wheels to control yaw moment.

Active Steer systems which modify the driver's steering angle input and add a correction steering angle to the wheels.

Active Torque Distribution systems which utilize active differentials and all wheel drive technology to independently control the drive torque distributed to each wheel and thus provide active control of both traction and yaw moment.

By large, the first system i.e. differential braking systems is the main focus of researchers and have been in use on several production vehicles. Steer-by-wire systems have not received attention much. These advancements try to enhance both vehicle stability and performance during maneuvers by imposing—with differential braking—an understeering or oversteering behavior to the vehicle. The brake-based systems, however, have been exposed to decline the longitudinal performance of the vehicle, particularly during strong acceleration curves. Also there have been found loss of power/energy during differential braking and alternates are needed to be explored. Active torque distribution system is gaining attention of researchers in the recent past and are likely to be available in production cars in the future due to main two following reasons and are the main emphasis in this thesis work:-

- a. Major role in avoiding accidents
- b. Due to this quality ESC is now mandatory in USA

1.4 Out Line Of Report

The following paragraphs describe the outline of this thesis. The organization of chapters and the contributions of each chapter are explained in brief:

In **Chapter 2**, brief review of previous work is given. Since the advent of vehicle its control via different strategies had started. This chapter mentioned the work done up till now with relevance to this research work.

Chapter 3 covers the mathematical modeling of a complete vehicle.

In **Chapter 4**, the proposed stability control strategies for controller design incorporating *central transfer case* is mentioned in detail.

Chapter 5 discusses computer simulation results obtained from implementing the different torque distribution strategies discussed in Chapter 4. The chapter begins with the selection of test maneuvers for simulations. The simulation results for different torque distribution strategies and parameters like controller gains and controller efforts are compared.

Chapter 6 concludes the thesis and suggests directions for future research to further improve certain performance characteristics of independent torque distribution systems.

CHAPTER # 2

LITERATURE REVIEW & BACKGROUND KNOWLEDGE

2.1 Literature Review

This section elaborates the existing work done on vehicle yaw stability control. Since the dawn of vehicle technology the researchers are working on stability control of this moving machine. The history of directional stability of wheeled platforms is as old as Second World War when this technology was incorporated in aircrafts. The start of 80's saw the advent of Anti Lock Braking System (ABS) and since then a much work is done in this regard. A few selective but effective works done for vehicle yaw stability control via active torque distribution is appended below:

J. Ackermann and W. Sienel [4], in 1993 introduced the idea of changing, via an electronic control system, a car's actions. This phenomenon has opened the way to the so-called global chassis control, where the mutual execution of the existing actuators such as steering, brakes, and traction is used to organize the performance of the vehicle. They use a linear model, a robust control law with the feedback of the yaw rate to front wheel steering. Action is to take the front steering angle as an input that adds the required controller gain, feed in the steering dynamics (plant). Outputs are the lateral acceleration at the front axle, the sideslip angle, and the yaw rate. Global chassis control research area may give an assurance a gateway towards the future intelligent vehicles, where the driver's preferences and inclinations will be sensed by the control systems. These sensors will ultimately control the vehicle's dynamic behavior close to the driver's intentions. Having controlling the vehicle's dynamic behavior the first and foremost priority of such a system will always be guaranteeing and enforcing the safety of the passengers thereafter a tradeoff between the safety and performance specially on cornering.

K. Sawase et al [5] describes two of latest developments made by Mitsubishi Motors: the ASC (Active Stability Control) system and the AYC (Active Yaw Control) system. A. Van Zanten [1] introduced methodology that is based on the basic principal of ESC. The critical driving situations sometimes overburdened the drivers with the stabilizing task. Coefficient of the friction of the road and the grip reserves of the tires are the two important parameters that cannot be judged by an average driver. The drivers are usually troubled by the distorted vehicle behavior in critical driving situations; as a result, a thought-out and well-considered reaction of the driver cannot be anticipated. To counter these hazards ESC has to be designed to stabilize the vehicle

even in situations with panic reactions and driving failures like exaggerated steering. These systems help in controlling the braking and driving forces that operate on the left and right wheels in such a way that a driver has a direct control over these cornering forces. As a result, they facilitate an appreciably safer and more enjoyable driving. ESC is a computerized technology that enhances the safety of vehicle stability by detecting any loss of traction (skidding). During cornering, ESC detects any discrepancy between them, it automatically applies brakes to individual wheels to control the direction of the vehicle back in line with the driver's intent. The driver's intention is calculated from the speed of the vehicle and driver's commanded steering wheel. The ESC system in use, no doubt has wide application towards the safety of passenger cars but it does not improve a vehicle's cornering performance; instead it helps to minimize the loss of control. Another limitation of the vehicle systems based on brakes, however, on sharp maneuvers, have been made to decline the longitudinal performance, as their system conforms to more stability and less performance

Due to the limitation of ESC as mentioned above, an alternative to brake-based solutions is provided by the use of new generation of torque biasing devices for the yaw stability of a vehicle. This system can be used to control the mechanism which can actively distribute the driving torque between the front and rear axle or between right and left tires to improve stability and performance. A brief work done by these researchers are as follows:

Stephen J. Hallowell et al [6] uses electric motors on each wheel to control the amount of torque by controlling the speed of the individual wheel. A High Mobility Multipurpose Wheeled Vehicle (HMMWV) is used for this purpose. To power each wheel, four independently controlled 100 horsepower Unique Mobility motors are used, and a fifth motor is attached to a diesel power plant as a generator. The torque control law consists two portions - one stability controller and second one is traction controller. In this system the inputs are longitudinal acceleration, steer angle, and wheel angular velocities. Based on measured wheel angular velocity, the traction control law examines the wheel acceleration. Indication of slipping or locked wheel shows that the wheel acceleration is not within specified bounds to counter it a fixed torque decrease factor is assigned to bring wheel acceleration within bounds. The idea for

installation of electric motors is much improved but cannot be used for passenger car being light in weights and less spacious than HMMWV.

F. Assadian and M. Hancock [7] takes the idea of active differential placed in RWD vehicle. The paper uses a linear bicycle model for the control development. For the simulation purpose it uses a 10 DOF vehicle model on simulator. The idea of vehicle model used is of RWD (rear wheel drive). The performance is controlled via torque distribution at the rear wheel. The controllers used in this case are proportional–integral–derivative controller (*PID controller*), Single input single output controller (SISO) H_∞ , and Multiple input single output controllers (MISO) H_∞ . Results of these two controllers were compared. As it uses a bicycle model complete vehicle dynamics cannot be predicted. In a bicycle model the radius of turn is much longer than the wheel base of the bicycle. So other angles/effects can be assumed to be very small and their effect can be neglected. Although for research purposes bicycle model has been used by a number of researchers, but many forces that are essential part of full vehicle dynamics like air drag force, rolling resistance etc are neglected in such a model. The output parameter which computes the vehicle behavior was the yaw rate as reference or output. It was shown that generally H_∞ controllers have better robust performance characteristic when compared to the traditional PID controllers. The main reason for this performance difference, are two folds: first the solution of the H_∞ controllers are derived analytically while the PID gains are tuned utilizing a numerical optimization technique with no guarantee for a global minimum solution. When the order of the dynamic system increases, the PID strategy is incapable of shaping all the dynamic system poles. Furthermore the relative performance difference of the proposed H_∞ strategies is highly dependent to the actuator bandwidth.

Junmin Wang et al [8] use a nonlinear 3 DOF vehicle model. Forces acting on a vehicle were calculated through Dugoff Tire model. Damrongrit Piyabongkarn, et al [9] gave the idea which is now under development by the renowned car manufacturers like Audi, Mercedes etc. In this paper the selected system discussed is for a 4WD vehicle whereas the vehicle is basically a RWD vehicle. The main logic behind this research is of active differential methodology. The main emphasis on this research is about torque distribution in the rear differential that controls the behavior of vehicle during cornering. An LSD (limited slip differential) equipped with a

controller is used in the rear differential which distributed the torque between rear right and rear left vehicle. It works on the methodology of an on demand 4WD that switches between 4WD to RWD according to the limitation defined for the yaw rate of the vehicle. Again the main idea is of transferring the torque from side to side. The torque-biasing models were generated in Matlab/Simulink environment to evaluate the effectiveness of the proposed model with controllers. In order to perform the co – simulation a full vehicle model developed in CarSim was modified.

Thereafter the researchers using the same methodology i.e. distribution of torque between right/left to left/right according to the situation. The main difference that they brought to their research was of using the different control strategies like M. Canale et al [10]. They used the same torque distribution methodology. An enhanced internal model control (IMC) technique is used to design the feedback controller. A feed forward control contribution has been added to improve the transient behavior by giving rise to a two degree of freedom (bicycle) structure. An accurate 14 degrees of freedom non-linear model of a segment D car is used to show improvements in stability in demanding conditions such as mu-split braking and damping properties in reversal steer, understeering characteristics, and low friction step steer maneuvers.

A sliding mode control algorithm is used by Shuen Zhao et al [11] basing on the same methodology as mentioned above. For the simplicity of work they use only 4 DOF for the validation of their result and a number of assumptions like longitudinal slip ratio is ignored and change of the tire forces induced by the weight shift, is not considered. With these assumptions and lesser DOF model, one cannot completely grasp the picture of using this control strategy.

Avesta Goodarzi and Ebrahim Esmailzadeh [12] use a fuzzy logic algorithm on an 8 DOF model. Electric motors are used for each tire and controlled by a fuzzy logic based controller. The problem with using motors is the same that it cannot be used in passengers' cars. Riccardo Marino et al [13] use a 7 DOF vehicle model with rear wheel drive (RWD) vehicle. The torque distribution was carried out between rear left and rear right tires. Results were validated through Simulator CarSim. Control methodology used in this case was based on Lyapunov techniques.

M. Canale et al [14] in their research for the torque biasing, a comparison is made between two control strategies. The research emphasis on a robust control technique to guarantee the system stability that overcomes the system uncertainties and the wide range of operating situations, which are typical of the automotive context. Two different feedback controllers, basing on sliding mode and internal model control methodologies, are designed, and their performances are compared using CarSim by means of extensive simulation tests. Both strategies performed satisfactory during different tests. Sliding Mode controller proved to be more effective than the IMC controller except it shows a chattering effect on the control input. Stability in demanding oversteering conditions, like braking in a high speed turn, may be worse than the uncontrolled case in both control techniques. So suggestions were made that a controller be designed by mixing technologies of both control strategies.

M. Canale and L. Fagiano [15] to show effectiveness in the vehicle stability context, use a Nonlinear Model Predictive Controller (NMPC) methodology to improve vehicle yaw rate dynamics by means of a rear active differential. A single track model, which incorporated only vehicle sideslip angle and yaw rate, is catered for. Results were validated on simulation using a simulink based model.

Matteo Corno et al [16] describe a critical drawback of braking via active differential in their research. While braking in this situation, vehicle experiences a yaw motion especially at very high speed even on a straight line driving. To counter such a motion they devised a controller using Linear Parametrically Varying (LPV) technique to overcome such problems. A 7 DOF two track model was used to check the performance of a vehicle.

Hai Yu et al [17] in 2009 use a 2 DOF linear bicycle model for steady state vehicle handling property analysis and active yaw dynamics controller design for torque distribution between rear right and left wheels. An 8 DOF vehicle model on simulink was used for simulation purposes. An LQG (Linear Quadratic Gaussian) feedback control law is used for torque distribution.

Another comparison of techniques made by Hongliang Zhou [18] in 2010, in which a sliding mode control methodology combined with back stepping control algorithm using a 7 DOF vehicle model were used for yaw stability control of a vehicle. Results were then validated on ve-DYNA, a commercial multibody vehicle dynamics simulator. Sliding mode control and back stepping combined strategies were used to design an observer to calculate the longitudinal forces acting on the tire that further helps in calculating the forces acting on the vehicle body.

Basing on above literature review, torque distribution between the right and left tires proved to be very effective for traction control especially when moving on strong acceleration maneuvers. For this purpose the vehicle is either FWD or RWD with required active differential that distributed the torque between the two wheels. It is obvious from above research that torque is being distributed between sides to side and is good only for the FWD or RWD vehicles, and when it comes to the AWD, heavy electric motors are being used that are not feasible using in passenger cars. Another approach that uses the mythology of torque distribution between front and rear axle is equally effective. Although it does not directly make the torque distribution at wheels but its effectiveness is equally guaranteed on axles. If the weight transfer is more towards the rear axle the vehicle will understeer like in the case of accelerating the vehicle or during cornering. Likewise the vehicle tends to oversteer while braking as weight is shifted ahead. By transfer of torque to front and rear axle can counter this effect of weight shifting. The research on this aspect of torque distribution between front and rear is limited.

Giulio Panzani et al [19] presented a new approach in this regards in their paper in Dec 2010. The vehicle used in this research is a FWD vehicle that is on – demand 4WD. Simulation is done on CarSim simulator. Instead of two controllers used by [10] this research is based on a single electronically controlled central transfer case. A hydraulic switch is used which cutoff the torque distribution when the vehicle requires more stability thereby making it a FWD vehicle. The said controller establishes a tradeoff between stability and performance during strong acceleration maneuvers. The threshold is based on vehicle sideslip angle, when the angle crosses a predefined limit the hydraulic switch turns off and the vehicle becomes a FWD. This research uses the stiffness of transfer case as control variable. As said earlier the threshold value is sideslip angle. There are two observer used i.e. stability supervisor and acceleration supervisor. When the

vehicle during cornering reaches predefined value of sideslip angle, the controller calculates the min value of transfer case stiffness and all the torque generated, shifted to front wheel and vehicle while becomes FWD i.e. at its max stability. When the vehicle does not cross the required threshold value, the controller calculates the max value of transfer case stiffness and vehicle enjoys a tradeoff between stability and performance.

Taking along the same idea of central transfer case, this research work mainly focus on 4WD vehicle, with high CoG (Centre of Gravity), those are more prone to rollover during strong acceleration maneuver. In this research a 10 DOF vehicle model with considerable CoG height is used to study the handling characteristics of a vehicle. The modeling would be done on MATLAB. The research work mentioned above mainly uses torque distribution amongst right and left wheels during strong acceleration maneuvers except [19]. This research focuses on torque distribution between front and rear axle. Unlike [19] which uses a FWD that incorporates an electronic controller to make the vehicle on – demand 4WD, this research uses a 4WD drive vehicle, in which torque will be distributed on seeing the oversteer and understeer behavior of the vehicle. Instead of vehicle side slip angle. The threshold in this research would be vehicle desired yaw rate. A controller will be designed that will increase or decrease the amount of torque in front or rear axle accordingly. The research will be carried on both low friction and/or high speed. In the end this analytical model will be compared with commercially available software CarSim.

2.2 BACKGROUND

2.2.1 DYNAMIC SYSTEM MODELLING

Approach to be followed

Mainly two types of reference behavior are used.

- a. Steady state
- b. Neutral steered

This research work uses a neutral steer model as comparison. This research also focuses on a 10 Degree of Freedom vehicle model which deals completely with ride and handling characteristics of a vehicle. Control over vehicle will be done via central transfer case instead of differential as in case of earlier approaches. The research also focuses on vehicle'

understeer and oversteer behavior at sharp maneuver. The vehicle's behavior would also be judged by giving a step input and DLC (Double Lane Change).

This research work is mainly two prongs: vehicle dynamics and control theory application. There are mainly two types of control systems: open loop (without feedback) and close loop (with feedback). Every control system must guarantee first the stability of the system. This research work will use a control strategy that will be optimum and effective as well due to main two reasons:-

- c. Use of active differential is costly as compared to a electronically controlled central transfer case (as proposed in this research)
- d. Loss of energy/power as in the case of differential braking will be solved by using this strategy.

In this chapter, a 10 DOF nonlinear lumped parameter vehicle model, opted and simulated in the MATLAB environment, is described. The model is used for the analysis of the active torque control systems.

To understand the dynamics of the complete vehicle, there is a requirement to know some basics of vehicle dynamics beforehand.

2.2.2 Vehicle Coordinate Systems

a. Earth Fixed Coordinate System

Vehicle behavior and trajectory during the course of a maneuver can be elaborated with respect to a right-hand orthogonal axis system fixed on the earth. It is generally preferred to correspond with the vehicle fixed coordinate system at the start point of the movement [20]. The coordinates (see Figure 2.1) are:

X - Straight travel

Y - Travel to the right

Z - Vertical travel (positive upward)

α - Heading angle (angle between x and X in the ground plane)

γ - Course angle (angle between the vehicle's velocity vector and X axis)

β - Sideslip angle (angle between x axis and the vehicle velocity vector)

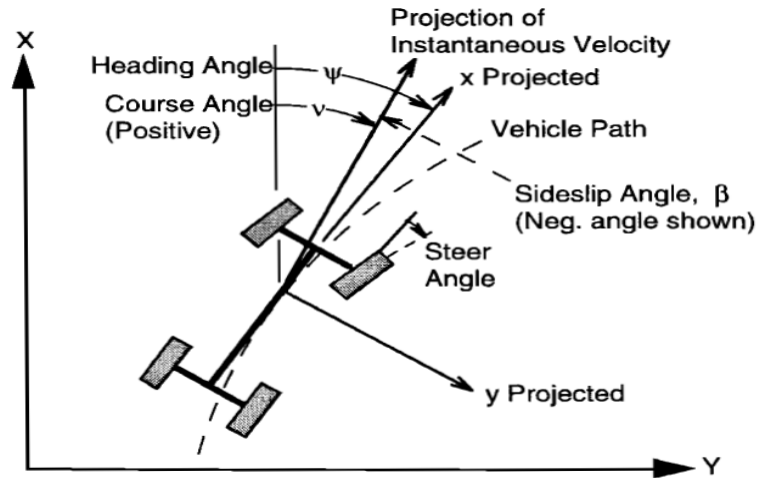


Figure 2.1: Earth fixed coordinate system of a Vehicle [20]

b. Vehicle Fixed Coordinate System

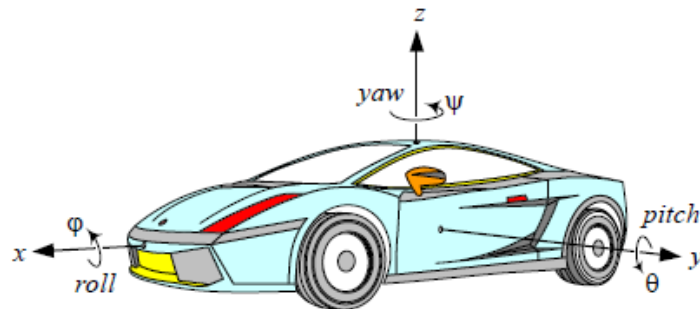


Figure 2.2 Fixed Coordinate System of a Vehicle [20]

Vehicle handling concerns two basic issues, controlling the direction of motion of the vehicle and to stabilize the course of the vehicle against external instabilities. For the understanding of vehicle dynamics it is critical to find out which forces that affects on the vehicle. When accelerating, braking and turning a vehicle is under influence of forces in longitudinal, transversal and vertical axes. A vehicle has six degrees of freedom, these three translation axes mentioned before and also rotation about these axes. The vehicle axis can be defined with reference to a right-hand orthogonal coordinate system, as mentioned above, which originates at the Center of Gravity (CoG) and travels with the vehicle. The coordinates may be defined as:

x - Straight and on the longitudinal plane of symmetry

- y – At the right side of the vehicle
- z – Upward: with respect to the vehicle
- - Roll velocity about the x axis
- θ - Pitch velocity about the y axis
- Yaw velocity about the z axis

2.2.3 Lumped Mass

A vehicle is made up of many components distributed within it, yet, all components move together. For example, under braking, the complete vehicle slows down as a unit; so we can represent it as one whole mass i.e. lumped mass and this mass is located at its center of gravity (CoG) with appropriate properties of mass and inertia. In modeling purposes one mass is sufficient for acceleration, braking, and most turning analyses. For ride analysis, this mass can further be divided into two parts, the wheels are denoted as "unsprung masses", and the lumped mass representing the body is the "sprung mass".

For representation of single mass, the vehicle is considered to be as a mass concentrated at its center of gravity (CoG) as shown in Fig 2.2. With appropriate rotational moments of inertia, the point mass at the CoG is dynamically equivalent to the vehicle itself for all motions in which it is reasonable to assume the vehicle to be rigid.

2.2.4 Wheel Model

Wheel model constitutes the bases of any vehicle model. Generally it consists of three parts: model development, its performance analysis and then controlling it. The aim of pneumatic tire is to help in providing a smooth ride; but the main purpose of an automotive pneumatic tire is to transmit moments and forces in all three mutually perpendicular directions for vehicle directional control. Various tests have been performed and mathematical models have been developed in an attempt to appreciate and foretell the generation of these forces. The models have been divided into four different classifications [21],

- a. **Complex Physical Model:** These models intend to model tire performance in greater detail rather than vehicle performance. These models are concerned with cost, temperature, tire wear, traction, life etc. These models typically use finite element modeling techniques.

- b. **Similarity Method:** Model based on this method were useful earlier in the tire force modeling process but have found less use recently as they have been outdated by the utility afforded by other models.
- c. **Simple Physical Model:** Uses simple mechanical representation. They are possible close from solution like Brush Model. These models have been developed and improved over the recent years but have not yet found their way into dynamic simulation activities.
- d. **Empirical Tire Model:** Also referred to as semi empirical tire model. Empirical tire models deal exclusively with the steady-state behavior of the tire. They relate the kinematic and physical properties of tires to the development of tractive forces at the contact between the road way surface and the tire. Examples under this category are a widely used Magic tire model, Dugoff tire model, model developed by Burckhardt [7] etc. Magic tire model uses almost as many as 85 parameters for calculation of tire behavior. It calculates longitudinal slip but neglect the effect of lateral (side) slip generated in a tire. Dugoff uses the combined slip for calculation purposes but the friction is assumed to be constant in the model, which disables its ability to also characterize the longitudinal force accurately for longitudinal slip. This research work uses the Burckhardt approach that incorporates the effect of resultant slip which is the combination of both longitudinal and lateral slip and also calculates the friction coefficient from the resultant slip instead of a fix value. Jacob [22] in his research states that Burckhardt shows the real behavior of tire more accurately.

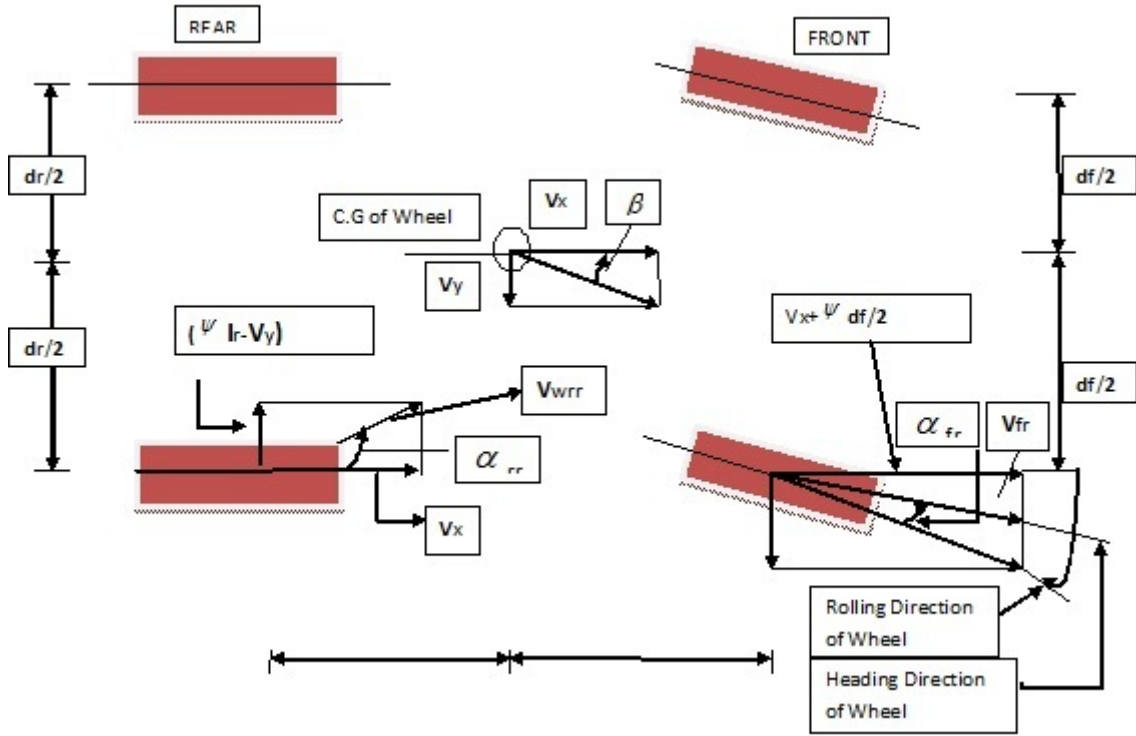


Figure 2.3: Velocity Components throughout the Vehicle [23]

2.2.4.1 Wheel Ground Contact Point Velocities

Here we will discuss the velocity of the wheels with reference to some fixed reference point – the so called wheel ground contact point velocity or wheel velocity v_w . There are several methods to calculate the velocities at the wheel ground contact points but we will calculate using longitudinal and lateral component of vehicle velocity (Fig 2.3).

$$V_{WFL} = \left(V_x - \square \cdot \frac{d_f}{2} \right) \cos \delta_f + \left(V_y + \square \cdot l_f \right) \sin \delta_f \quad (2.1)$$

$$V_{WFR} = \left(V_x + \square \cdot \frac{d_f}{2} \right) \cos \delta_f + \left(V_y + \square \cdot l_f \right) \sin \delta_f \quad (2.2)$$

$$V_{WLR} = \left(V_x - \square \cdot \frac{d_r}{2} \right) \quad (2.3)$$

$$V_{WRR} = \left(V_x + \square \cdot \frac{d_r}{2} \right) \quad (2.4)$$

2.2.4.2 Wheel Slip and Tire Side Slip Angle

Wheel slip is the relative motion between a tire and the road surface it is moving on as shown in fig 2.4.

	Braking	Driving
	$v_R \cos \alpha \leq v_W$	$v_R \cos \alpha \geq v_W$
Longitudinal Slip	$s_L = (v_R \cos \alpha - v_W) / v_W$	$s_L = (v_R \cos \alpha - v_W) / v_R \cos \alpha$
Side Slip	$s_S = v_R \sin \alpha / v_W$	$s_S = \tan \alpha$

(2.5)

The longitudinal slip s_L must always be between -1 and 1. The resultant wheel slip is the geometrical sum of the longitudinal and side slip.

$$s_{Res} = \sqrt{s_L^2 + s_S^2} \quad (2.6)$$

Here $v_R = R\omega$ will be used in subsequent equations

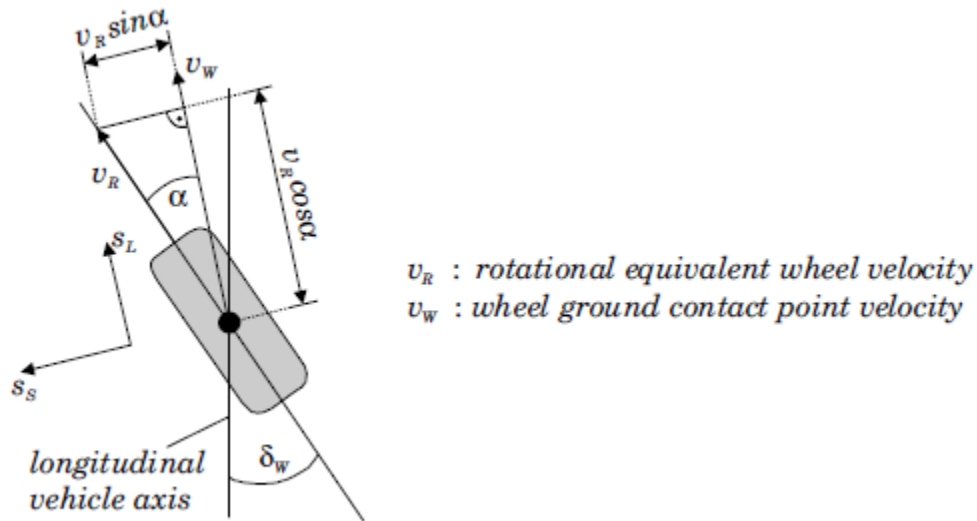


Figure 2.4: Wheel Slip Calculation [23]

The tire slip angle (α) is defined to be the angle between the direction of heading of the tire and the direction of travel, as shown in fig 2.4. The tire slip angle for each wheel can be derived from lateral and longitudinal components of the velocity at the wheel with respect to the CoG of the vehicle.

$$\alpha_{FL} = \delta_w - \tan^{-1} \left(\frac{V_{y^+} \cdot l_F}{V_{x^-} \cdot \frac{d_F}{2}} \right) \quad (2.7)$$

$$\alpha_{FR} = \delta_w - \tan^{-1} \left(\frac{V_{y^+} \cdot l_F}{V_{x^+} \cdot \frac{d_F}{2}} \right) \quad (2.8)$$

$$\alpha_{RL} = -\tan^{-1} \left(\frac{V_{y^-} \cdot l_R}{V_{x^-} \cdot \frac{d_R}{2}} \right) \quad (2.9)$$

$$\alpha_{RR} = -\tan^{-1} \left(\frac{V_{y^-} \cdot l_R}{V_{x^+} \cdot \frac{d_R}{2}} \right) \quad (2.10)$$

2.2.4.3 Friction Co-efficient Calculation

The friction or adhesion co-efficient μ is defined as the ratio of the frictional force acting in the wheel plane F_{fric} and the wheel ground contact force F_Z :

$$\mu = \frac{F_{fric}}{F_Z} \quad (2.11)$$

The calculation of friction forces which is used in this work; instead of constant value of μ can be carried out using Burckhardt approach:

$$\mu(s_{Res}) = c1 \cdot (1 - e^{-c2 \cdot s_{Res}}) - c3 \cdot s_{Res} \quad (2.12)$$

Where $c1$, $c2$ and $c3$ are the parameter sets for friction co-efficient characteristics for various road surfaces. The resultant slip s_{Res} is directed in the same direction as the resultant co-efficient μ_{Res} . By this one can derive the longitudinal and lateral friction co-efficient.

$$\mu_L = \mu_{Res} \cdot \frac{s_L}{s_{Res}} \quad \text{and} \quad \mu_L = k_s \cdot \mu_{Res} \cdot \frac{s_S}{s_{Res}} \quad (2.13)$$

Due to the presence of tread profile the friction behavior can also be dependent on direction; this can be expressed by introducing an attenuation factor k_s for the lateral friction co-efficient. Common low profile tires have an attenuation factor of between 0.9 and 0.95.

2.2.4.4 Calculation of Friction Forces

In the direction v_W :

$$F_{WLFL} = \mu_{LFL} \cdot F_Z \quad , \quad F_{WLFR} = \mu_{LFR} \cdot F_Z \quad (2.14)$$

$$F_{WLRL} = \mu_{LRL} \cdot F_Z \quad , \quad F_{WLRR} = \mu_{LRR} \cdot F_Z \quad (2.15)$$

In the direction at right angles to v_W :

$$F_{WSFL} = \mu_{SFL} \cdot F_Z \quad , \quad F_{WSFR} = \mu_{SFR} \cdot F_Z \quad (2.16)$$

$$F_{WSRL} = \mu_{SRL} \cdot F_Z \quad , \quad F_{WSRR} = \mu_{SRR} \cdot F_Z$$

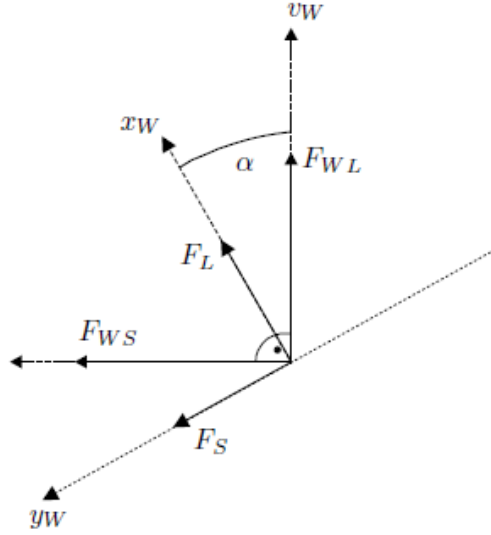


Figure 2.5: Direction of Friction Forces [23]

Transforming into the wheel co-ordinate system (x_W, y_W):

$$\text{In the direction } x_W : \quad F_{LFL} = F_{WLFL} \cdot \cos\alpha_{FL} + F_{WSFL} \cdot \sin\alpha_{FL} \quad (2.17)$$

$$F_{LFR} = F_{WLFR} \cdot \cos\alpha_{FR} + F_{WSFR} \cdot \sin\alpha_{FR} \quad (2.18)$$

$$F_{LRL} = F_{WLRL} \cdot \cos\alpha_{RL} + F_{WSRL} \cdot \sin\alpha_{RL} \quad (2.19)$$

$$F_{LRR} = F_{WLRR} \cdot \cos\alpha_{RR} + F_{WSRR} \cdot \sin\alpha_{RR} \quad (2.20)$$

$$\text{In the direction } y_W : \quad F_{SFL} = F_{WSFL} \cdot \cos\alpha_{FL} - F_{WLFL} \cdot \sin\alpha_{FL} \quad (2.21)$$

$$F_{SFR} = F_{WSFR} \cdot \cos\alpha_{FR} - F_{WLFR} \cdot \sin\alpha_{FR} \quad (2.22)$$

$$F_{SRL} = F_{WSRL} \cdot \cos\alpha_{RL} - F_{WLRL} \cdot \sin\alpha_{RL} \quad (2.23)$$

$$F_{SRR} = F_{WSRR} \cdot \cos\alpha_{RR} - F_{WLRR} \cdot \sin\alpha_{RR} \quad (2.24)$$

The longitudinal and lateral friction forces can now be transformed from the wheel coordinate system to the vehicle/undercarriage coordinate system. For the wheel on the rear axle no transformation is required as the wheel plane lies parallel to the longitudinal vehicle axis:

$$F_{XRL} = F_{LRL} \quad , \quad F_{YRL} = F_{SRL} \quad (2.25)$$

$$F_{XRR} = F_{LRR} \quad , \quad F_{YRR} = F_{SRR} \quad (2.26)$$

For the wheels on the front axle the forces are transformed by the wheel turn/steer angle δ_w :

$$F_{XFL} = F_{LFL} \cdot \cos\delta_F - F_{SFL} \cdot \sin\delta_F \quad , \quad F_{YFL} = F_{SFL} \cdot \cos\delta_F + F_{LFL} \cdot \sin\delta_F \quad (2.27)$$

$$F_{XFR} = F_{LFR} \cdot \cos\delta_R - F_{SFR} \cdot \sin\delta_R \quad , \quad F_{YFR} = F_{SFR} \cdot \cos\delta_R + F_{LFR} \cdot \sin\delta_R \quad (2.28)$$

2.3 The Complete Vehicle Model

To predict the exact behavior of the vehicle one needs to have a vehicle dynamics model that is accurate to design the control system for yaw control. If the performance of the vehicle is not calculated during designing stage, it can lead to a threatening maneuver and improper handling behavior such as rollover. The dynamic behavior and the safety of the vehicle can be investigated beforehand with the help of mathematical models of a complete vehicle. The computer simulation tools help in investigating the vehicle dynamic behavior and safety, without the need to built or tests a vehicle which is very expensive. A maximum DOF vehicle model is required to predict the study of handling, ride, and performance of four wheel vehicles.

The vehicle model can further be classified into two sub – model as described below. The assumptions made here are - To represent the sprung and unsprung masses one lumped mass is used - The vehicle body is being modeled as rigid - The inner and outer steer angle is taken as one steer angle – It is also assumed that tires have a contact with the ground all the time.

a. Handling Model

The handling model as shown in Fig 2.6 is made up of 7 DOF. The 4 DOF correspond to the spin of each wheel. The remaining 3 DOF corresponds to vehicle body: longitudinal, lateral and yaw motions.

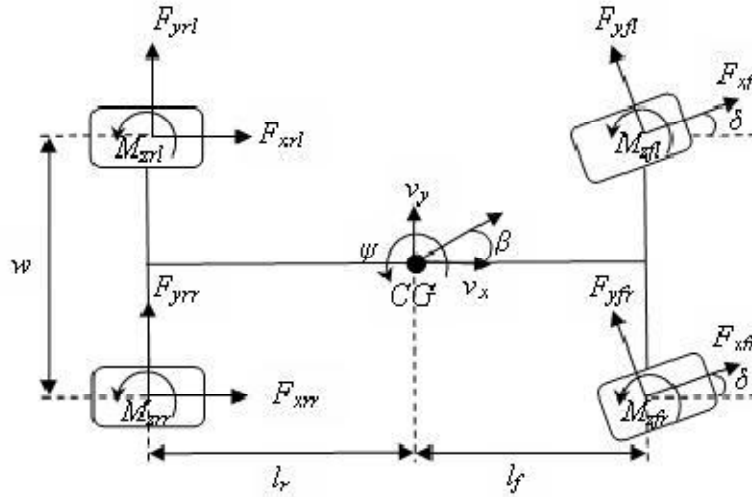


Figure 2.6: Vehicle Handling Model [24]

These equations are given by:

Summation of forces in longitudinal direction

Inertial acceleration a_x at the centre of gravity of the vehicle in x axis comprises two terms. The two terms are the acceleration which is due to the motion along longitudinal direction, and the centripetal acceleration,

$$a_x = \dot{V}_x - \dot{\psi} \cdot V_y \quad (2.29)$$

Applying Newton's Second Law of motion

$$a_x = \frac{1}{m_{CoG}} (F_{XFL} + F_{XFR} + F_{XRL} + F_{XRR}) - \dot{\psi} \cdot V_y \quad (2.30)$$

Summation of Forces in lateral direction

Inertial acceleration a_y at the centre of gravity of the vehicle in y axis comprises two terms. The two terms are the acceleration which is due to the motion along lateral direction, and the centripetal acceleration,

$$a_y = \dot{V}_y + \dot{\psi} \cdot V_x \quad (2.31)$$

Applying Newton's Second Law of motion

$$a_y = \frac{1}{m_{CoG}} (F_{YFL} + F_{YFR} + F_{YRL} + F_{YRR}) + \dot{\psi} \cdot V_x \quad (2.32)$$

Summation of Moment for Yawing

$$\ddot{\psi} = \frac{1}{J_Z} [l_F(F_{YFR} + F_{YFL}) - (F_{YRL} + F_{YRR})l_R + (F_{XRR} - F_{XRL})\frac{d_R}{2} + (F_{XFR} - F_{XFL})\frac{d_F}{2}] \quad (2.33)$$

Calculation of Wheel Ground Contact Forces

The normal forces acting on the four wheels can be derived by:

$$F_{ZFL} = m_{CoG} \cdot \left(\frac{l_R}{l} g - \frac{h_{CoG}}{l} a_x \right) \cdot \left[\frac{1}{2} - \frac{h_{CoG} \cdot a_y}{d_F \cdot g} \right] \quad (2.34)$$

$$F_{ZFR} = m_{CoG} \cdot \left(\frac{l_R}{l} g - \frac{h_{CoG}}{l} a_x \right) \cdot \left[\frac{1}{2} + \frac{h_{CoG} \cdot a_y}{d_F \cdot g} \right] \quad (2.35)$$

$$F_{ZRL} = m_{CoG} \cdot \left(\frac{l_F}{l} g + \frac{h_{CoG}}{l} a_x \right) \cdot \left[\frac{1}{2} - \frac{h_{CoG} \cdot a_y}{d_R \cdot g} \right] \quad (2.36)$$

$$F_{ZRR} = m_{CoG} \cdot \left(\frac{l_F}{l} g + \frac{h_{CoG}}{l} a_x \right) \cdot \left[\frac{1}{2} + \frac{h_{CoG} \cdot a_y}{d_R \cdot g} \right] \quad (2.37)$$

Summation of Torque about Wheels

Each wheel in the vehicle model will have an angular acceleration corresponding to the torque on the wheel. The wheel angular velocity, ω represents the degree of freedom of the spin of the tire as shown in Fig 2.7. The equations for summation of torque about wheel axle for each wheel are given as.

$$J_{\omega FL} \dot{\omega}_{FL} = T_{FL} - R_{FL} F_{WFL} \quad (2.38)$$

$$J_{\omega FR} \dot{\omega}_{FR} = T_{FR} - R_{FR} F_{WFR} \quad (2.39)$$

$$J_{\omega RL} \dot{\omega}_{RL} = T_{RL} - R_{RL} F_{WRL} \quad (2.40)$$

$$J_{\omega RR} \dot{\omega}_{RR} = T_{RR} - R_{RR} F_{WRR} \quad (2.41)$$

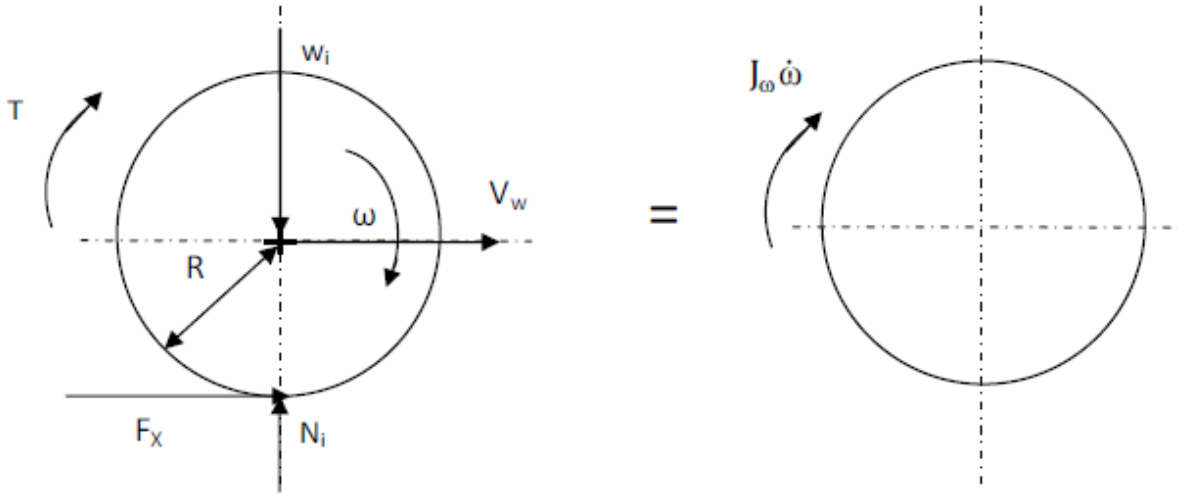


Figure 2.7: Rotational Dynamics of Wheel [24]

$$\ddot{Z} = \frac{1}{M_s} [(F_{ZFL} + F_{ZFR}) + (F_{ZRL} + F_{ZRR}) - \frac{1}{l_R} m_{CoG} a_x h_{CoG}] \quad (2.43)$$

c. Model Verification

The 10 Degrees of Freedom model was verified using nonlinear multibody simulator software CarSim. Our model is made using MATLAB codes only. The trend between the simulation and experiment results as shown in Fig 2.9 and 2.10 is almost the same with acceptable error. This error is due to the simplification in the vehicle dynamics model compared to the CarSim. The modeling done in this research is for 10 DOF vehicle model and suspension effect is not catered for whereas the same is very much there in the case of CarSim model. Yaw rate and lateral acceleration were plotted using J-Turn Steer input.

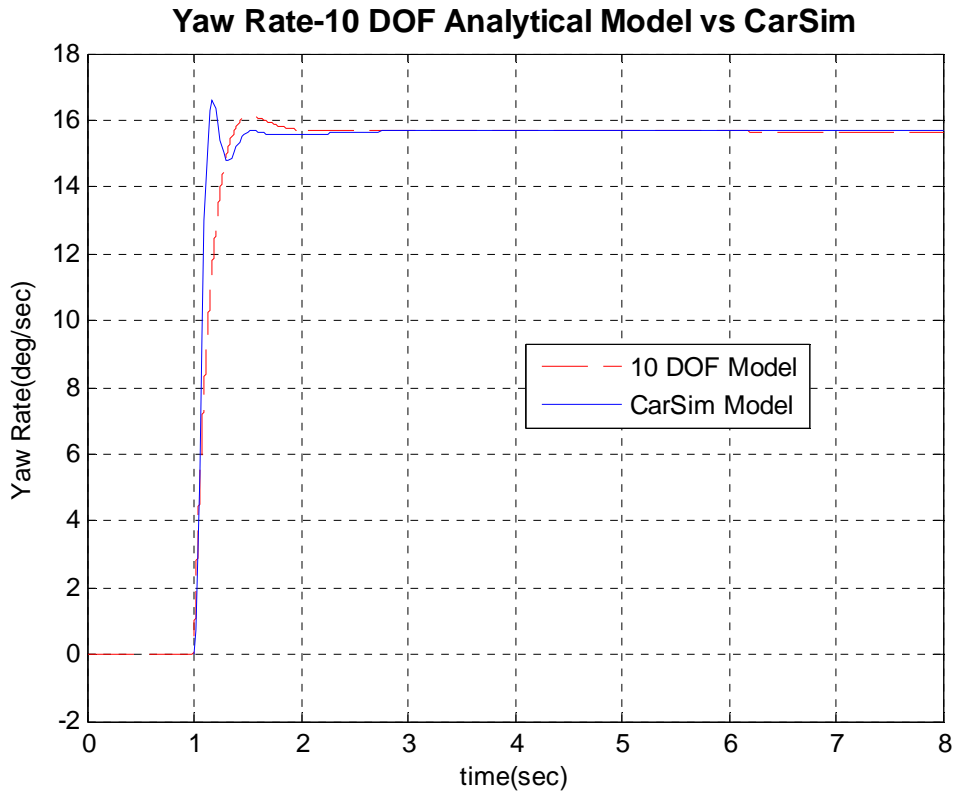


Figure 2.9: Comparison of Yaw Rate of 10 DOF Model against of CarSim

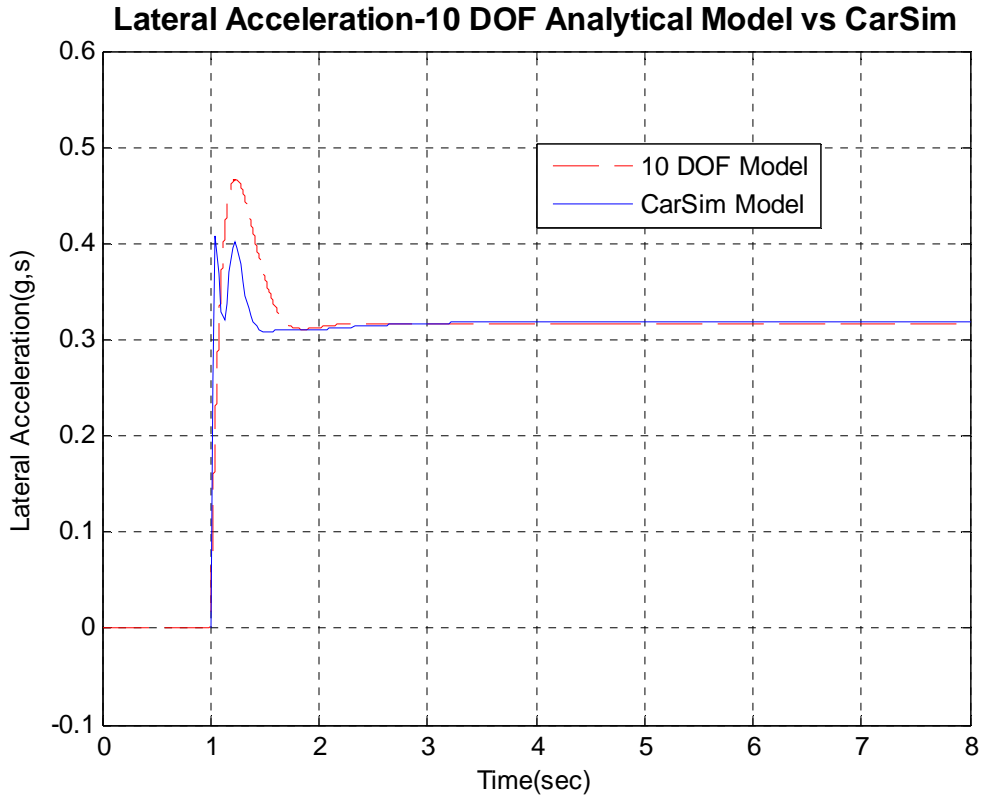


Figure 2.10: Comparison of Lateral Acceleration of 10 DOF Model against of CarSim

2.4 Chapter Summary

This chapter described the literature review and modeling efforts on producing as possible the behavior of individual components. In modeling it started with the wheel model which constituted the basis of any vehicle model. The purpose of modeling any system on any software is to save efforts and time to draw first the model and then extract required data out of it. Due to the fact that there is a huge difference between different vehicles in terms of structure and kinematics, such models are very specific, and special know-how is required to alter a model for use with a different vehicle type. Modeling in this research work was carried on using MATLAB, and validation was done with the help of multibody software simulator CarSim.

CHAPTER # 3

POPOSED METHODOLOGY

Control Architecture

Active Torque Distribution Strategies through Yaw Rate

Active Torque Distribution Strategies through Lateral Acceleration

CONTROL ARCHITECTURE

In this section the technique for torque distribution between front and rear wheels will be described in detail. The concept of a controller in central transfer case may be introduced that will shift the amount of torque as per the requirement to front or rear axle. The main function of this controller may be defined as it controls the Yaw moment of the vehicle through torque transfer.

3.1 Yaw Moment Control through Torque Transfer

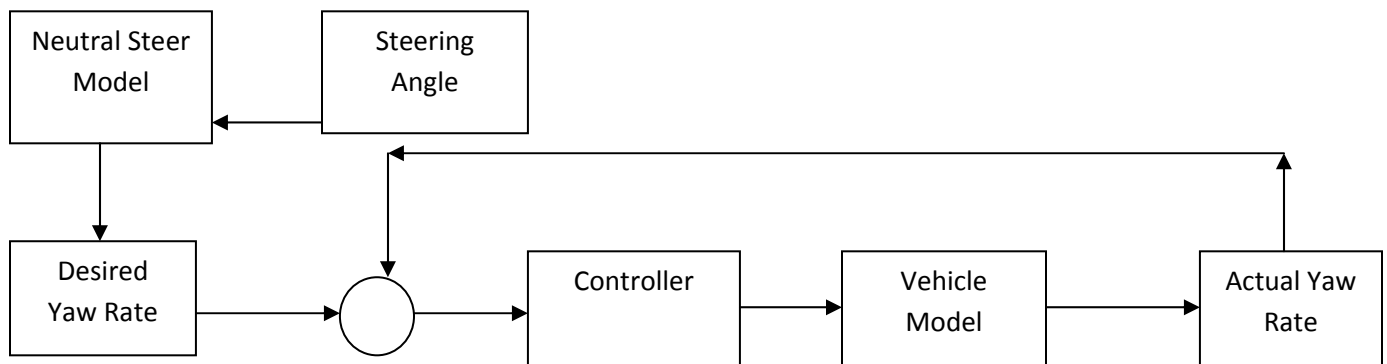


Figure 3.1: Block Diagram of Vehicle Dynamic Control via Active Torque Distribution

In this section, a physics- based description of the yaw moment control as achieved by a central transfer case torque distribution is described. The idea behind understeering and oversteering of a vehicle is that as more torque is transferred to the front, the effect of longitudinal forces on the front wheels increases. Due to this effect, the longitudinal slip of the front axle grows while that of the rear axle drops. This also leads to a decrease in the lateral forces generated by the front tires as compared to the rear ones as explained by the friction ellipse [20] & [23]. Due to this increase in longitudinal forces an understeering effect is induced in the vehicle. If the torque to the rear is increased and keeping in mind the same phenomenon an oversteering effect is induced in the vehicle. In other words we can say that the vehicle understeers when the driving torque on the rear wheels is reduced in comparison to that of the front wheels. The longitudinal forces on the front wheels increase while those on the rear wheels decrease. Consequently, the lateral forces generated by the front wheels decrease while those of the rear wheels increase.

3.2 Central Transfer Case Torque Transfer

This approach of torque distribution involves Central Transfer Case torque transfer i.e. addition or subtraction of corrective torques to the individual axles. This approach doesn't necessarily constrain the total torque to a constant value that is produced by the engine rather this approach provides an additional degree of freedom in torque distribution thus allowing independent torque control of each axle. In this research, this approach has been closely studied and implemented in simulations. The choice of appropriate feedback control variables (yaw rate) that go with this approach will be described in detailed. In this approach torque will be added or subtracted to the front and rear axle. The total torques on the front and rear axles of the vehicle are given, respectively, by:

$$T_F = T_{FR} + T_{FL} \quad (3.1)$$

$$T_R = T_{RR} + T_{RL} \quad (3.2)$$

Where $T_{FR}, T_{FL}, T_{RR}, T_{RL}$, are the individual torques acting on the individual wheels.

The difference between the actual yaw rate and the desired yaw rate is an obvious measure of deviation of the vehicle from its desired course and hence can be used to create the corrective yaw moment using an appropriate controller. In this work, the required torque, ΔT , that would be added or subtracted to the speed control torques of the individual axles for generating the desired yaw moment, is evaluated from a PID type function of yaw rate error i.e. e_r and is given by:

$$e_r = r - r_{des}$$
$$\Delta T_r = K_p e_r + K_i \int e_r dt + K_d \frac{de_r}{dt} \quad (3.3)$$

Let us consider the vehicle in different scenarios, including left or right hand turning and understeering or oversteering behavior. The conditions can be expressed mathematically for the two cases (figure 3.2) as follows:

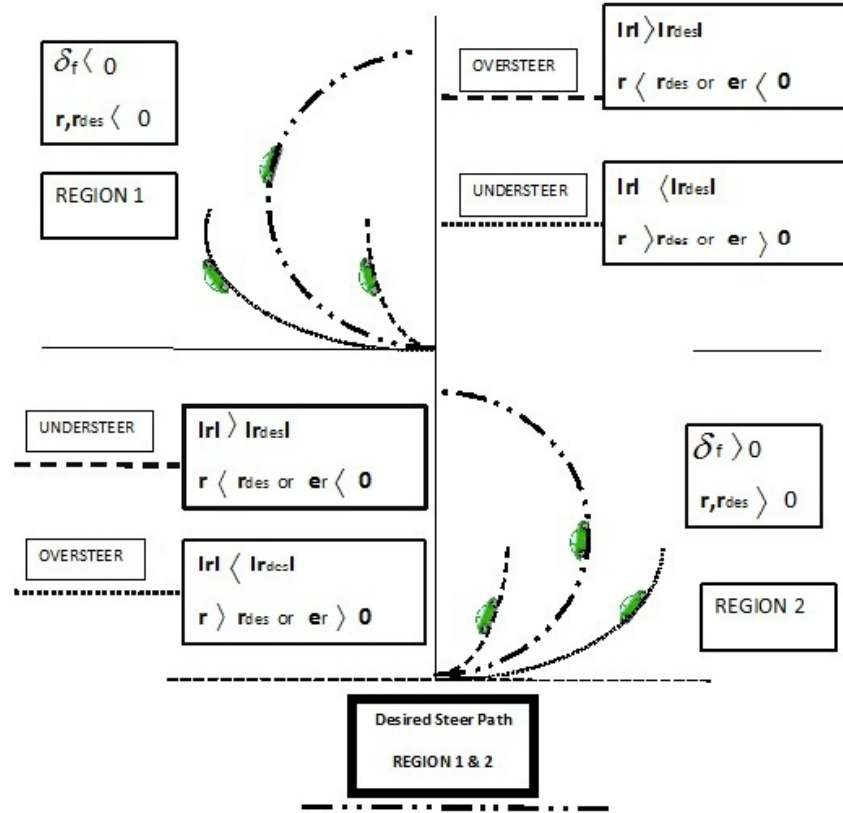


Figure 3.2: Schematic of Vehicle in Various Scenarios and Adopted Sign

Case 1: $\delta_w < 0$ or $r, r_{des} < 0$

For the right hand turn of the vehicle as shown in region 1 of Figure 3.2, the steering angle is negative. The desired and actual yaw rates are also negative as per the sign convention adopted. Oversteering occurs when $e_r < 0$, and the understeer occurs when $e_r > 0$.

Case 2: $\delta_w > 0$ or $r, r_{des} > 0$

For the left hand turn of the vehicle as shown in region 2 of Figure 3.2, the steering angle is positive. The desired and actual yaw rates are also positive as per the sign convention adopted. Understeering occurs when $e_r < 0$, and the oversteering occurs when $e_r > 0$.

With these left and right maneuvers, the following torque distribution strategies are conceived to achieve the corrective desired yaw moment. The strategies apply the torques to the front and rear axles.

a. Strategy 1: Switching Corrective Torque b/w Front and Rear Axles

Yaw rate control torque is applied to the front or rear axle of the vehicle depending on the sign of the yaw rate error. In the right hand turn and the steering angle is negative, when oversteering condition for a negative yaw rate error, the drive torque on the front axle is increased while for an understeering condition for a positive yaw rate error, the drive torque on the rear axle is increased. In the left hand turn and the steering angle is positive, when understeering condition for a negative yaw rate error, the drive torque on the rear axle is increased while for an oversteering condition for a positive yaw rate error, the drive torque on the front axle is increased.

Mathematically, these are described as follows:

When $\delta_w < 0$

$$e_r > 0 \quad T_{R_{new}} = T_R + |\Delta T_r| \quad (\text{Understeering}) \quad (3.4)$$

$$T_{F_{new}} = T_F \quad (3.5)$$

$$e_r < 0 \quad T_{F_{new}} = T_F + |\Delta T_r| \quad (\text{Oversteering}) \quad (3.6)$$

$$T_{R_{new}} = T_R \quad (3.7)$$

When $\delta_w > 0$

$$e_r > 0 \quad T_{F_{new}} = T_F + |\Delta T_r| \quad (\text{Oversteering}) \quad (3.8)$$

$$T_{R_{new}} = T_R \quad (3.9)$$

$$e_r < 0 \quad T_{R_{new}} = T_R + |\Delta T_r| \quad (\text{Understeering}) \quad (3.10)$$

$$T_{F_{new}} = T_F \quad (3.11)$$

b. Strategy 2: Corrective Torques: Add to the Front and Subtract to Rear Axle

In this strategy, half the corrective torque is added to the front axle and half of them are subtracted from the rear axle and vice versa depending upon the steering angle and yaw rate error.

When $\delta_w < 0$

$$e_r > 0 \quad T_{R_{new}} = T_R + |\Delta T_r/2| \quad (\text{Understeering}) \quad (3.12)$$

$$T_{F_{new}} = T_F - |\Delta T_r/2| \quad (3.13)$$

$$e_r < 0 \quad T_{R_{new}} = T_R - |\Delta T_r/2| \quad (\text{Oversteering}) \quad (3.14)$$

$$T_{F_{new}} = T_F + |\Delta T_r/2| \quad (3.15)$$

When $\delta_w > 0$

$$e_r > 0 \quad T_{R_{new}} = T_R - |\Delta T_r/2| \quad (\text{Oversteering}) \quad (3.16)$$

$$T_{F_{new}} = T_F + |\Delta T_r/2| \quad (3.17)$$

$$e_r < 0 \quad T_{R_{new}} = T_R + |\Delta T_r/2| \quad (\text{Understeering}) \quad (3.18)$$

$$T_{F_{new}} = T_F - |\Delta T_r/2| \quad (3.19)$$

c. Strategy 3: Addition / Subtraction of Torque only to Front Axle

In this strategy, the corrective torque is applied only to the front axle depending upon the sign of yaw rate error, while no corrective torque is applied to rear axle:

When $\delta_w < 0$

$$e_r > 0 \quad T_{R_{new}} = T_R \quad (\text{Understeering}) \quad (3.20)$$

$$T_{F_{new}} = T_F - (\Delta T) \quad (3.21)$$

$$e_r < 0 \quad T_{R_{new}} = T_R \quad (\text{Oversteering}) \quad (3.22)$$

$$T_{F_{new}} = T_F + (\Delta T) \quad (3.23)$$

When $\delta_w > 0$

$$e_r > 0 \quad T_{(R_{new})} = T_R \quad (\text{Oversteering}) \quad (3.24)$$

$$T_{(F_{new})} = T_F + (\Delta T) \quad (3.25)$$

$$e_r < 0 \quad T_{(R_{new})} = T_R \quad (\text{Understeering}) \quad (3.26)$$

(3.27)

$$T_{(F_{new})} = T_F - (\Delta T)$$

d. Strategy 4: Addition / Subtraction of Torque only to Rear Axle

In this strategy, the corrective torque is applied only to the rear axle depending upon the sign of yaw rate error, while no corrective torque is applied to front axle:

When $\delta_w < 0$

$$e_r > 0 \quad T_{(F_new)} = T_F \quad \text{(Understeering)} \quad (3.28)$$

$$T_{(R_new)} = T_R + (\Delta T) \quad (3.29)$$

$$e_r < 0 \quad T_{(F_new)} = T_F \quad \text{(Oversteering)} \quad (3.30)$$

$$T_{(R_new)} = T_R - (\Delta T) \quad (3.31)$$

When $\delta_w > 0$

$$\delta_{\square} > 0 \quad \delta_{(\square\square\square\square)} = \delta_{\square} \quad \text{(Oversteering)} \quad (3.32)$$

$$\delta_{(\square\square\square\square)} = \delta_{\square} - (\square\square) \quad (3.33)$$

$$\delta_{\square} < 0 \quad \delta_{(\square\square\square\square)} = \delta_{\square} \quad \text{(Understeering)} \quad (3.34)$$

$$\delta_{\square} = \delta_{\square} + (\square\square) \quad (3.35)$$

3.4 Active Torque Distribution Strategies through Lateral Acceleration

Steering
Angle



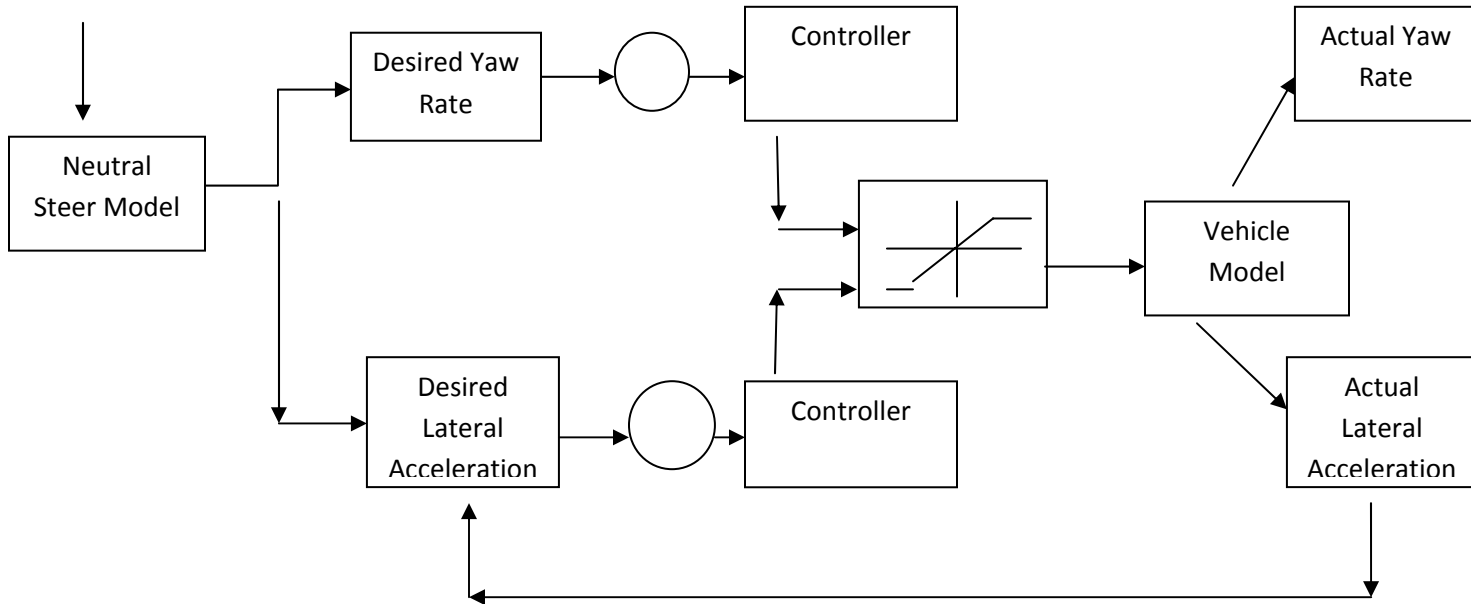


Figure 3.3: Block Diagram of Vehicle Dynamic Control via Active Torque Distribution

The same strategies that have been explained in last section may be enhanced taking lateral acceleration as control parameter. The error computed using lateral acceleration can also be incorporated in the same way as yaw rate error as shown in Fig 3.3. When lateral acceleration is incorporated as a feedback variable, lateral acceleration error can be calculated by

$$e_{\dot{\psi}} = \dot{\psi}_{\text{Desired}} - \dot{\psi}_{\text{Actual}} \quad (3.36)$$

Where desired value of lateral acceleration can be calculated as

$$\dot{\psi}_{\text{Desired}} = \frac{a_{\text{Desired}}}{v} \left(\frac{57.3 * v}{1000} + \frac{a_{\text{Desired}} * v}{g} \right) \quad (3.37)$$

Thereafter the required torque can be evaluated by using a PID type function

$$\Delta T_{\text{req}} = K_p e_{\dot{\psi}} + K_i \int e_{\dot{\psi}} dt + K_d \frac{de_{\dot{\psi}}}{dt} \quad (3.38)$$

Now the same value of differential torque can be added along with the value of required yaw rate torque in any of the strategy mentioned earlier. For example we can take only one strategy numerated earlier in order to clarify our technique. Strategy 1 mentioned above is mathematically expressed as:-

$$\text{When } \dot{\psi}_{\text{Desired}} < 0$$

$$\square_{-} > 0 \quad T_{R_{new}} = T_R + |\Delta T_r| + |\Delta T_a| \quad (\text{Understeering}) \quad (3.39)$$

$$T_{F_{new}} = T_F \quad (3.40)$$

$$e_r < 0 \quad T_{F_{new}} = T_F + |\Delta T_r| + |\Delta T_a| \quad (\text{Oversteering}) \quad (3.41)$$

$$T_{R_{new}} = T_R \quad (3.42)$$

When $\delta_w > 0$

$$e_r > 0 \quad T_{R_{new}} = T_R + |\Delta T_r| + |\Delta T_a| \quad (\text{Oversteering}) \quad (3.43)$$

$$T_{F_{new}} = T_F \quad (3.44)$$

$$e_r < 0 \quad T_{R_{new}} = T_R + |\Delta T_r| + |\Delta T_a| \quad (\text{Understeering}) \quad (3.45)$$

$$T_{F_{new}} = T_F \quad (3.46)$$

3.5 Chapter Summary

This chapter described the different control strategies keeping in view the vehicle dynamic behavior. Generation of the corrective yaw moment between front and rear torque transfer was described with the physics-based explanation of variations in longitudinal forces achieved through torque transfer. Based on the physical consequence of longitudinal force distribution, two approaches of distribution of torque to each axle of the vehicle were identified and explained.

The focus was on the proposed approach of central transfer case torque transfer that provided an additional degree of freedom in torque distribution thereby allowing independent torque control of each axle. Four torque distribution strategies for achieving the yaw moment control through each of the feedback control variables: yaw rate and lateral acceleration. The four strategies could also be applied a combined feedback of yaw rate and lateral acceleration for generating the desired corrective yaw moment.

CHAPTER # 4

RESULTS & DISCUSSION

In this chapter, simulation results will be presented using different torque distribution strategies discussed in Chapter 4. The vehicle and tire data taken from a C-Class Hatchback car, available in Appendix A of [25], is used in the simulations. The selection of suitable test maneuvers for analysis is discussed first in Section 4.1.

4.1 Selection of Test Maneuvers

In order to analyze the effectiveness of the proposed control strategies, following standard test maneuvers were considered and appropriately modified as explained below:-

a. J-Turn Test:

J-turn test is conducted to discuss the performance characteristics of this system like its tracking ability in a sudden steer angle change (step steer). A typical step steer profile of steering wheel angle (SWA) of 1rad with the step applied at 1st sec and achieving the required steering-wheel angle in 0.15 sec.

b. Double Lane Change (DLC):

Another steering test i.e. standard Double Lane Change (DLC) also known as “moose test” is used to identify the vehicle dynamics. Here it is used to verify the effectiveness of proposed control strategy. At 1st sec the vehicle takes a turn of 1 radians and after 3rd sec it takes a turn in opposite direction of 1 radians.

4.2 Simulation of Road Surface Conditions

Different road surface conditions (different coefficients of friction between road and tires, μ), calculated using Burckhardt Tire Model, were simulated to evaluate the effectiveness of proposed Control Strategies. Control strategies were verified on J-Turn Test on wet Asphalt surface (μ (approx) = 0.5) and DLC was conducted on two surfaces i.e. snow (μ (approx) = 0.1) and ice (μ (approx) = 0.05). Subsequent sections show the proofs of these tests.

4.3 Implementation of Control Strategies

In this section control strategies mentioned in chapter 4 are incorporated using two test maneuvers described in section 5.1. First we will take on J-Turn Test.

a. J-Turn Test

J-turn test is conducted to discuss the performance characteristics of this system like its tracking ability in a sudden steer angle change (step steer). The angle made by our vehicle model as described earlier is shown in Fig 4.1. The path followed by the vehicle in our model is also shown in Fig 4.2.

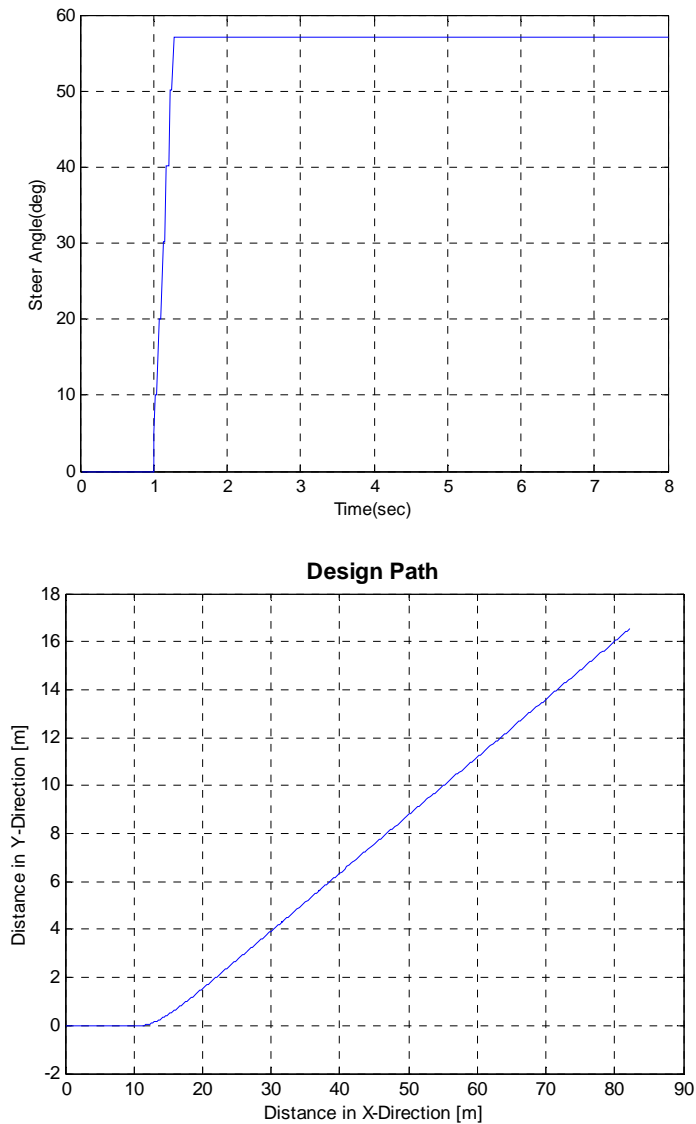


Figure 4.2: Path Followed by 10 DOF Model

The initial speed of the vehicle is 15 m/s (54 km/hour) and it enters the turn with the same speed. Now we will discuss the different strategies applied on an understeer and oversteer vehicle.

(1)Oversteer Case: Understeer and oversteer are the sensitivities of a vehicle to steering. In oversteer case a car turns more than the amount commanded by a driver. The yaw rate in oversteering of the car is more than the desired and in this case the error between the actual and desired one is negative in our research. Strategies 1, 2 and 4 for oversteer case were applied here and the result for the time history of yaw rate as compared to a perfect neutral steer vehicle, where the error is zero, has been shown in Fig 4.3, 4.4 and 4.5. These plots clearly show the effectiveness of our torque control strategies on oversteer situation. With the help of yaw rate controller and the lateral acceleration controller, our vehicle effectively tracks the reference value that is of a neutral steered vehicle. Fig 4.6 shows the comparison of all three strategies. Another important value to be mentioned here is tire side slip. As mentioned earlier that the longitudinal value of tire side slip must remain between -1 and 1. Fig 4.7 clearly shows the same. In these strategies torque was being transferred to the front axle to overcome the effect of oversteer, so more torque ahead but slip did not cross the limit. Strategy 1 was used here. Fig 4.8 shows the roll, pitch and sprung mass velocity of this controlled vehicle, which are quite satisfactory.

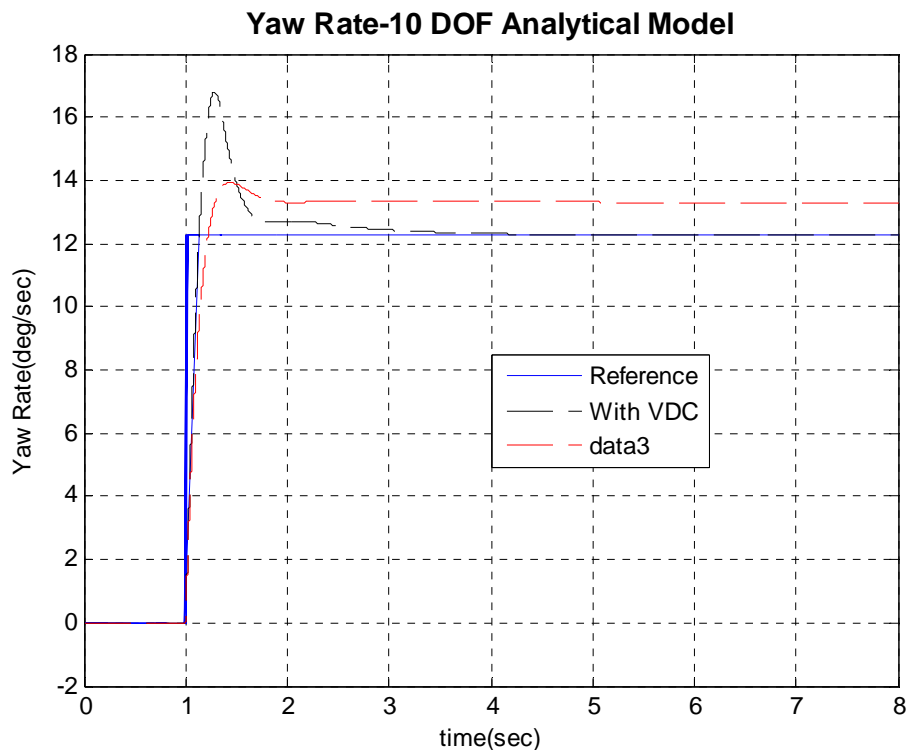


Figure 4.3: Comparison of Yaw Rate in Oversteer using Strategy 1/3

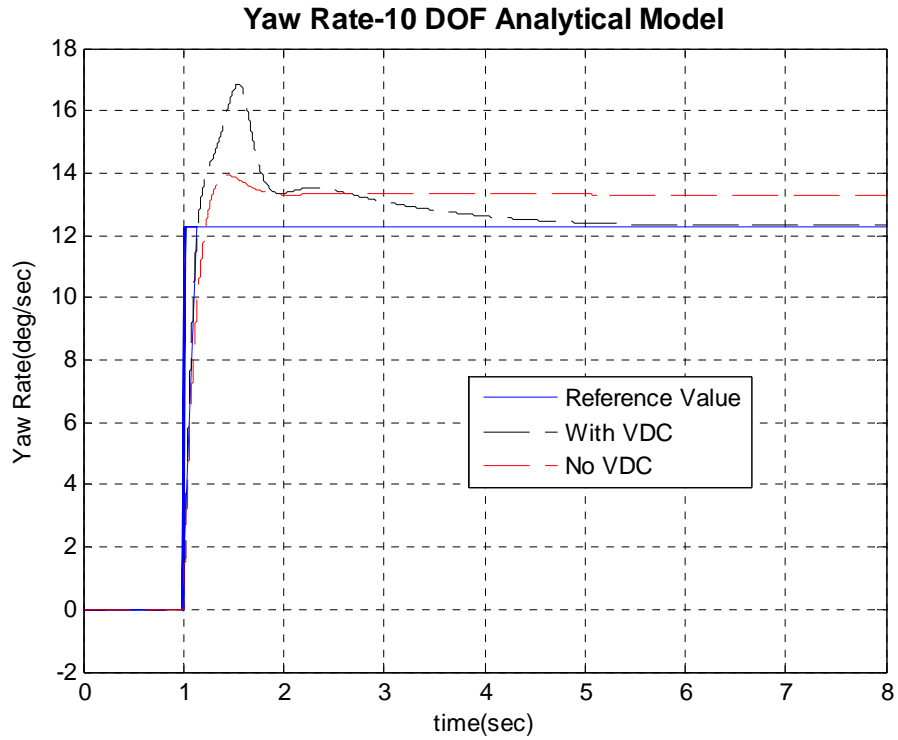


Figure4.4: Comparison of Yaw Rate in Oversteer using Strategy 2

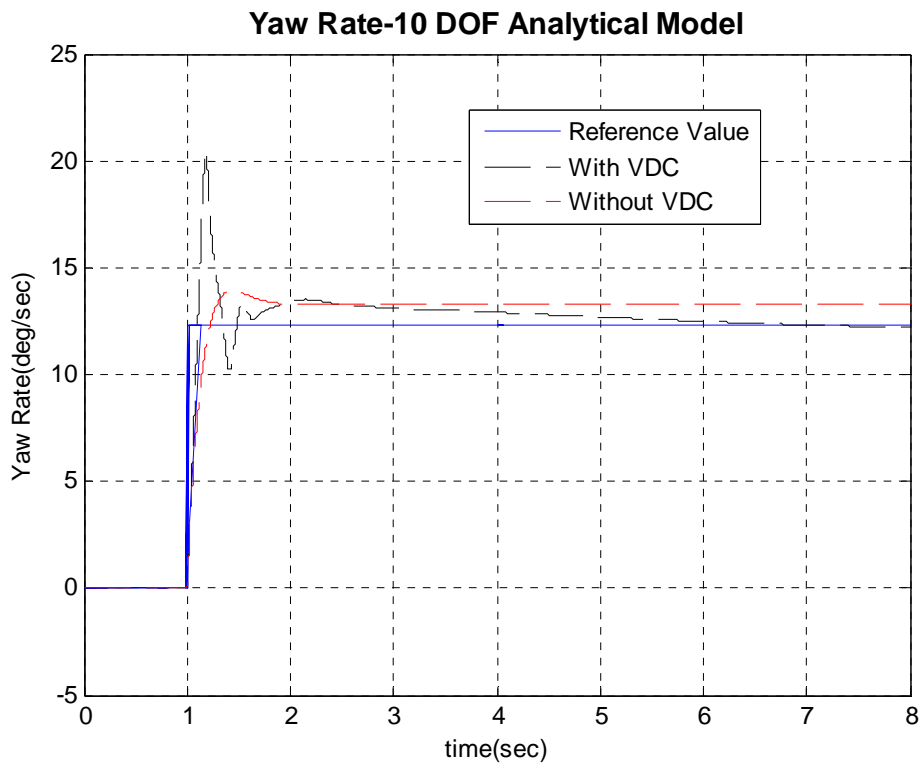


Figure4.5: Comparison of Yaw Rate in Oversteer using Strategy 4

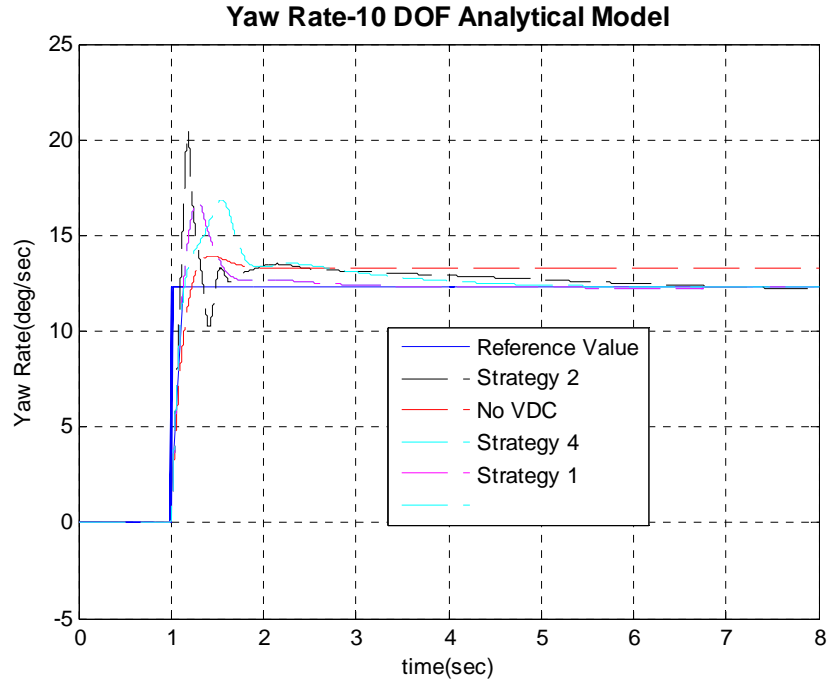


Figure 4.6: Comparison of Yaw Rates with different Strategies

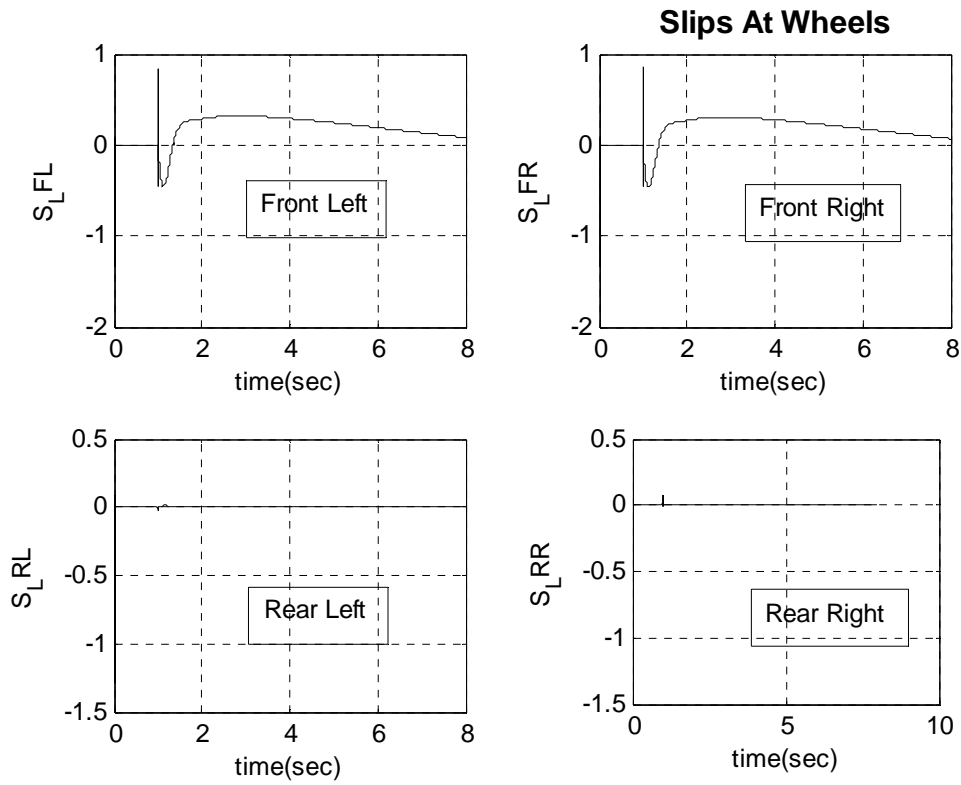


Figure 4.7: Tire Longitudinal Slip in Oversteer case

In fig 4.7 when control strategy is applied and torque is distributed to front axle as described by above mentioned strategies, one can see a fluctuation in the value of tire slip at 1 sec of the vehicle ride. That is because the VDC was incorporated that amount of time. These values are calculated using strategy 3 i.e. Addition / Subtraction of torque only to front axle. As described earlier that the value of longitudinal slip should always be remained within -1 and 1, the same can be observed by this plot.

(1) Understeer Case: In understeer case a car turns less than the amount commanded by a driver. In this case the error between the actual and desired one is positive in our research. Strategies 1, 2 and 3 pertaining to understeer case were applied here and the result for the time history of yaw rate as compared to a perfect neutral steer vehicle, where the error is zero, has been shown in Fig 4.8 ,4.9 and 4.10. These plots clearly show the effectiveness of our torque control strategies on understeer situation. With the help of yaw rate controller and the lateral acceleration controller, our vehicle effectively tracks the reference value that is of a neutral steered vehicle. Fig 4.11 shows a comparison of all three control strategies pertaining to understeer conditions. Fig 4.12 shows the longitudinal slips of tires that are well within the limit.

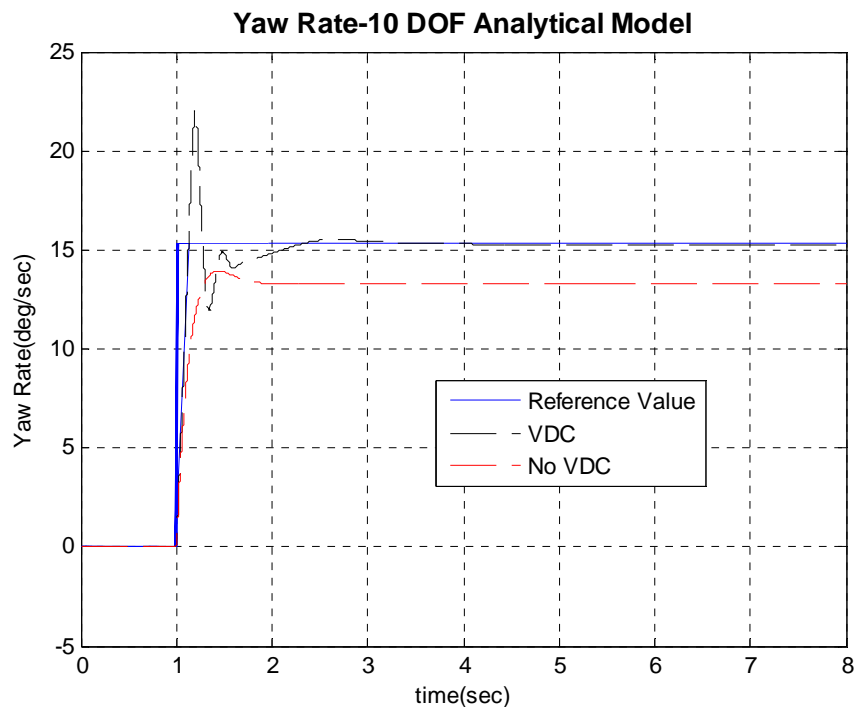


Figure 4.8: Comparison of Yaw Rate in Understeer using Strategy 1

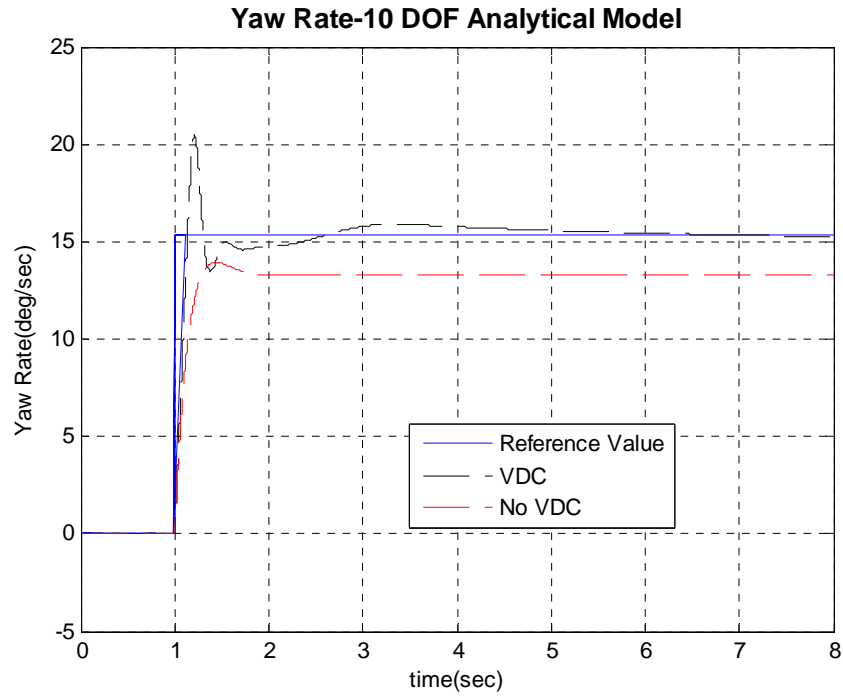


Figure 4.9: Comparison of Yaw Rate in Understeer using Strategy 2

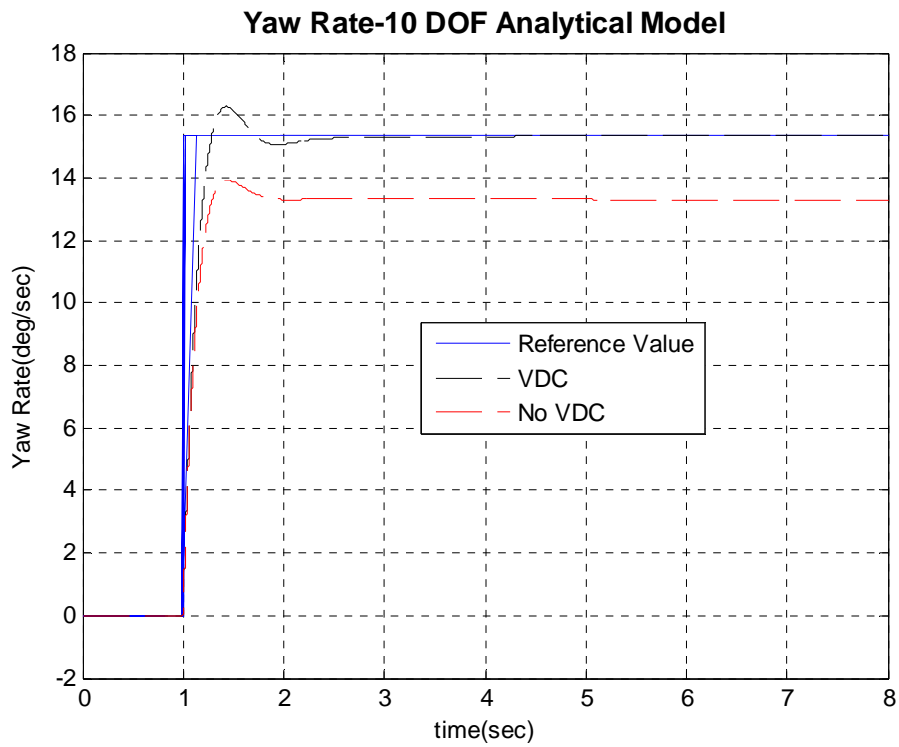


Figure 4.10: Comparison of Yaw Rate in Understeer using Strategy 3

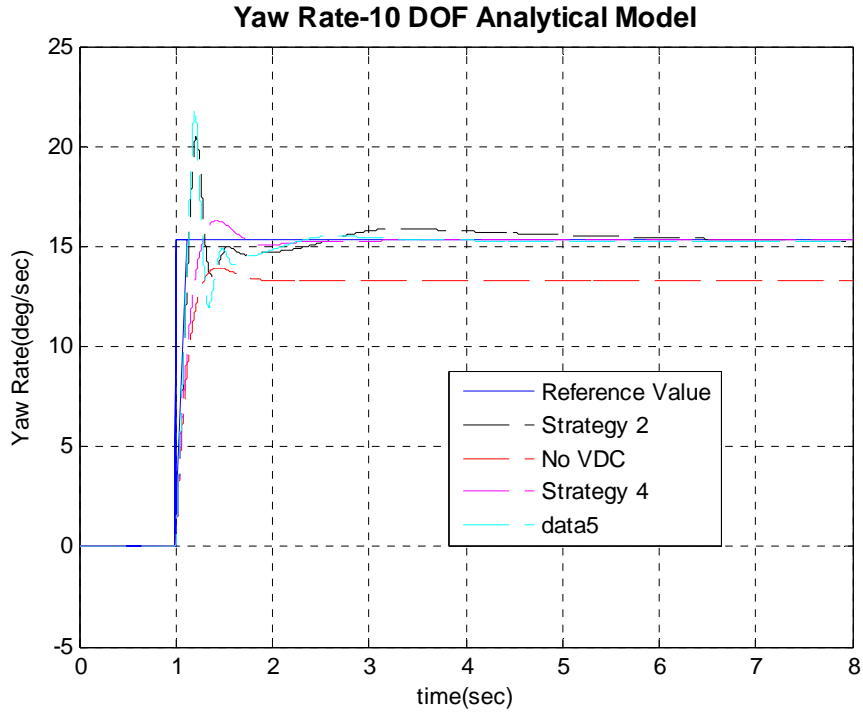


Figure 4.11: Comparison of Yaw Rates with different Strategies

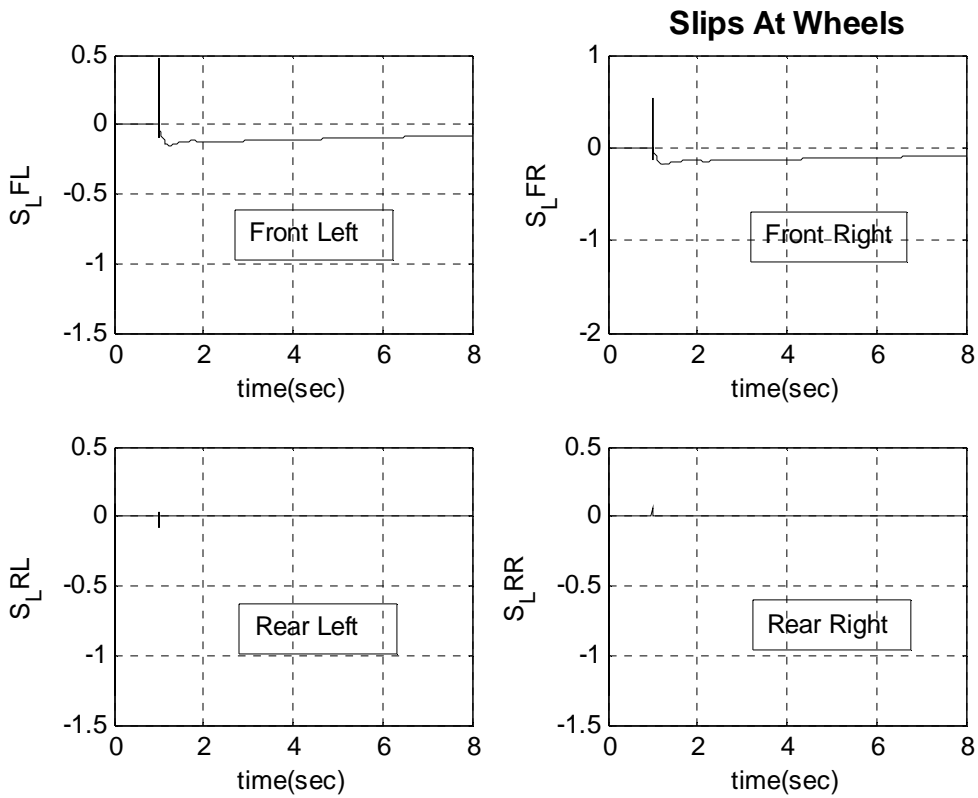


Figure 4.12: Tire Longitudinal Slips in Understeer Case

Strategy 3 is implemented here where torque is subtracted from the front wheel whereas no torque is added or subtracted from rear axle

a. Double Lane Change (DLC):- DLC test as elaborated in section 4.1b is implemented here through our vehicle model on the proposed control strategies. Surface chosen to conduct this test is snow which has a very low friction of coefficient. At this type of surface, during turns of angle 1 radians (57.3 degrees) and at speed of 15 m/s (54 km/hr), there are very likely chances that our vehicle will lose track and may go out of control. The same trend is proved in subsequent paras that our vehicle at 15 m/s lose track and went out of control. The proposed control strategy in these circumstances is required to keep the vehicle on its original path and helps the driver to regain control over his vehicle. In normal circumstances the path as shown in fig. 4.13 will be followed by a vehicle in DLC. On such driving conditions as described above our vehicle has deviated the path as defined but with the help of Control Strategy it has regained the required yaw rate. The design path as shown in fig 4.15 clearly shows the deviation in path. In fig. 4.16 our vehicle shows yaw rate that initially follows the prescribed path but later it does not only detrack but also becomes out of control. Our vehicle enters the DLC with the speed of 11 m/s but as it reaches the speed of 15 m/s it loses control and detracks. At this moment the vehicle is RWD. With the help of our control strategy 1, the corrective torque is switched from rear to front and with the help of PID controller it regains control and starts following original path that has been initially assigned to it.

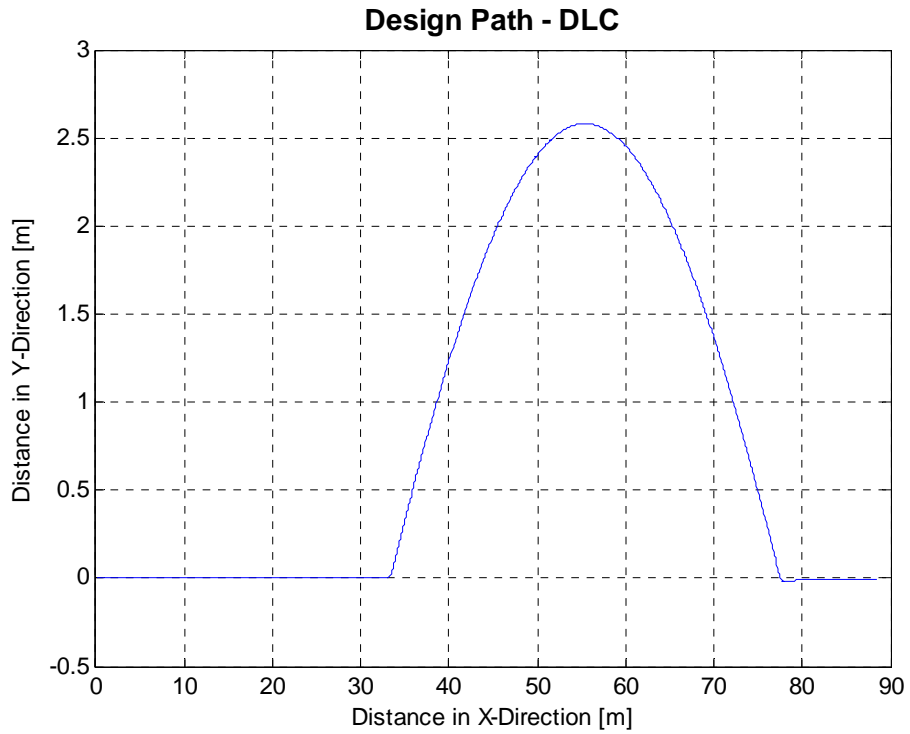


Fig. 4.13: Design Path for DLC

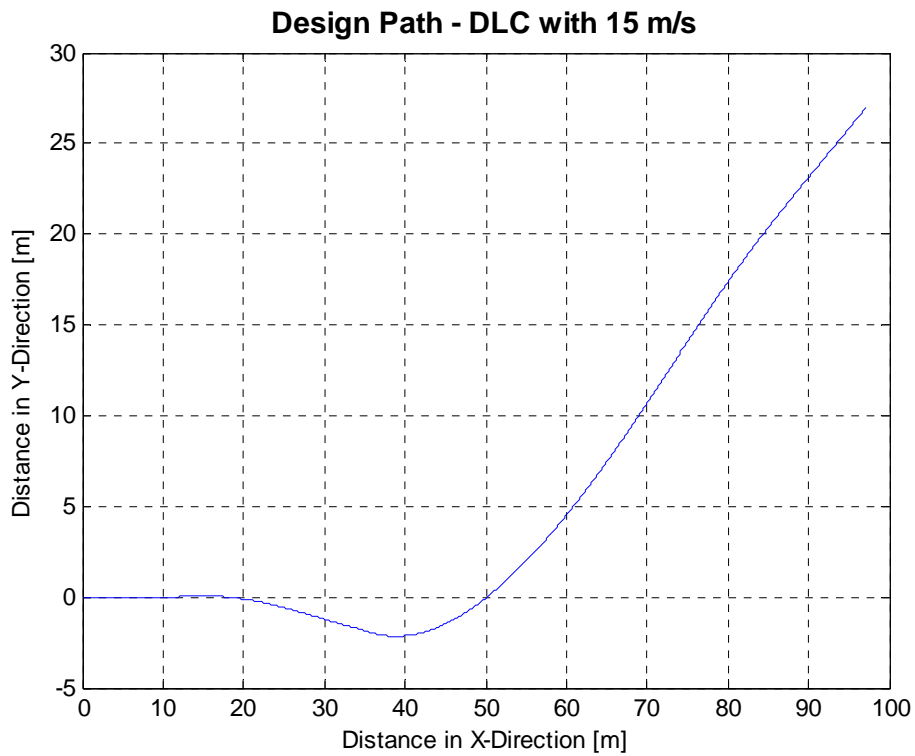


Fig. 4.14: Design Path of Vehicle at 15 m/s on snow

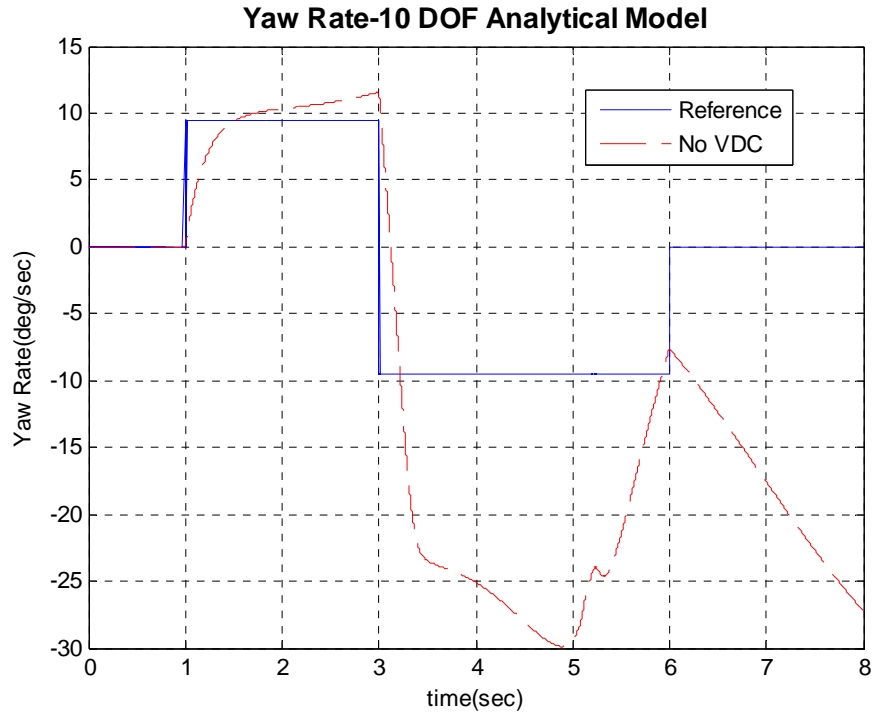


Fig. 4.15: Yaw Rate -Reference value vs. No VDC

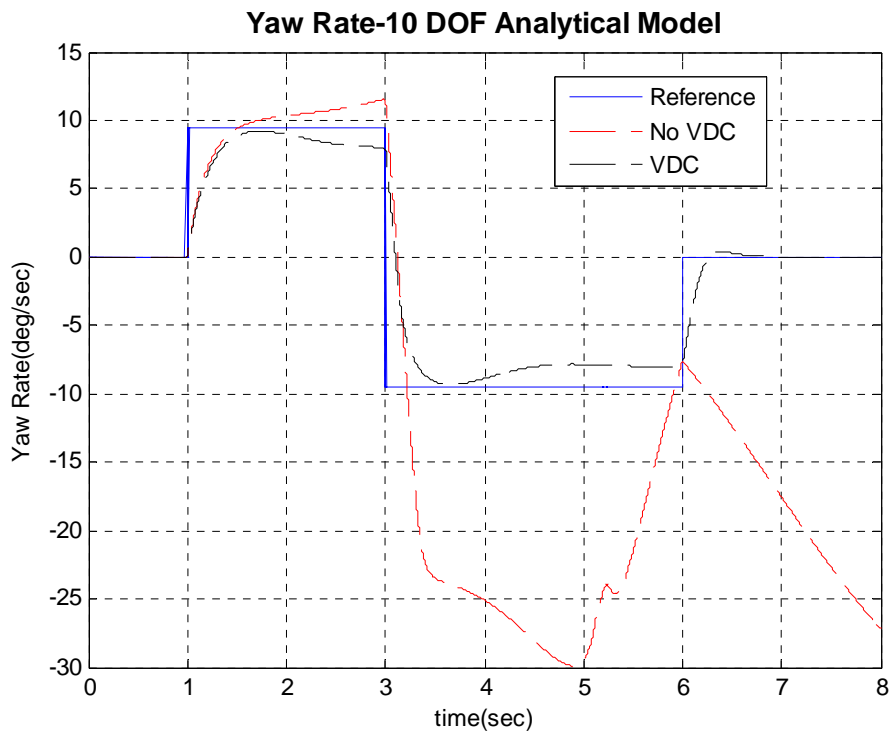


Fig. 4.16: Yaw Rate with VDC

Fig. 4.16 compares the all three yaw rate, the reference value, the yaw rate with NO VDC in red dotted line and with the help of our VDC strategy the vehicle has started following our reference value. Another aspect i.e. slips at wheel is shown in fig 4.18. It is the slip calculated when the vehicle is getting out of control at 15 m/s. Although our vehicle is not following the path in such driving conditions but the fig shows the validation of our model as well that the slip remains under the prescribed limit i.e. between -1 and 1. After 6 sec when our vehicle gets detrack there is a significant jump in the rear wheels. No unusual deviation is found in front wheels. With the help of our vehicle dynamic control when the torque is switched between rear and front wheel as also elaborated in case of yaw rate, the slips as shown in fig 4.19, not only remain in limit but also one can see a smooth travel in wheels.

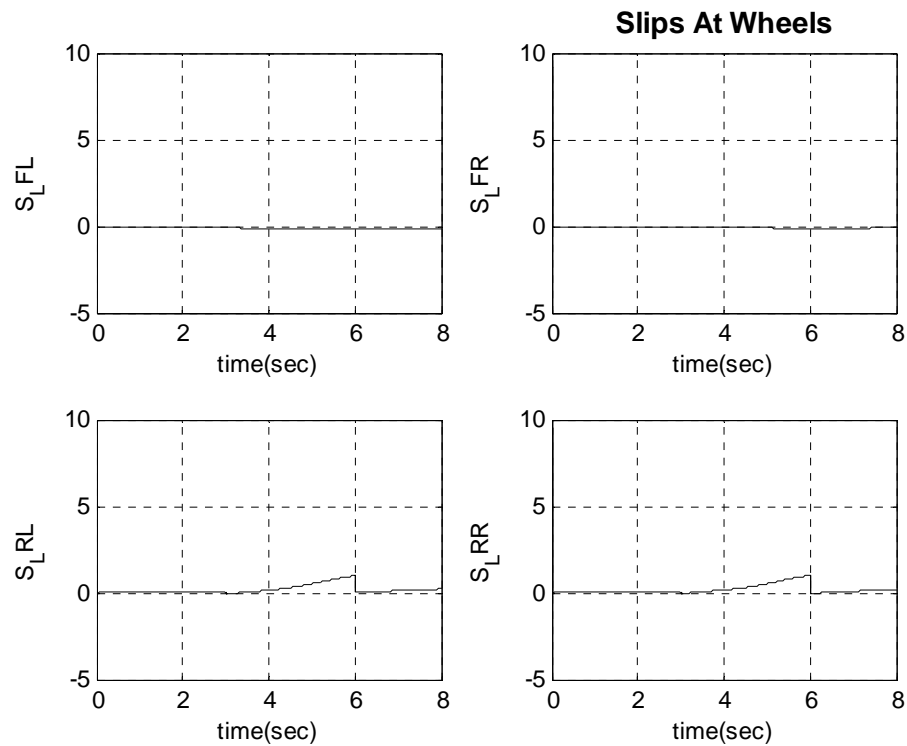


Figure 4.17: Slips of Wheels with NO VDC

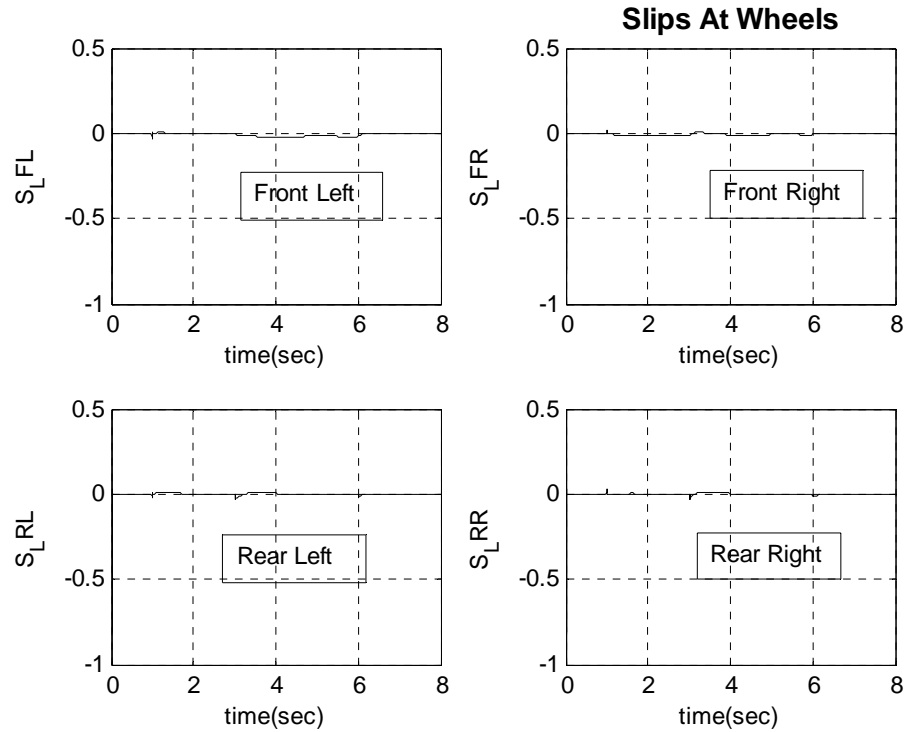


Figure 4.18: Slips at Wheels with VDC

Using the same turn maneuver i.e. DLC is now implemented on another road condition i.e. ice. At 6 m/sec the vehicle lost control and detrack from the original path. Thanks to our VDC system that by incorporating the controller input, the vehicle does not only start following the original track but also its speed is enhanced from 6 m/sec to 8 m/sec. Fig 4.20 shows the results.

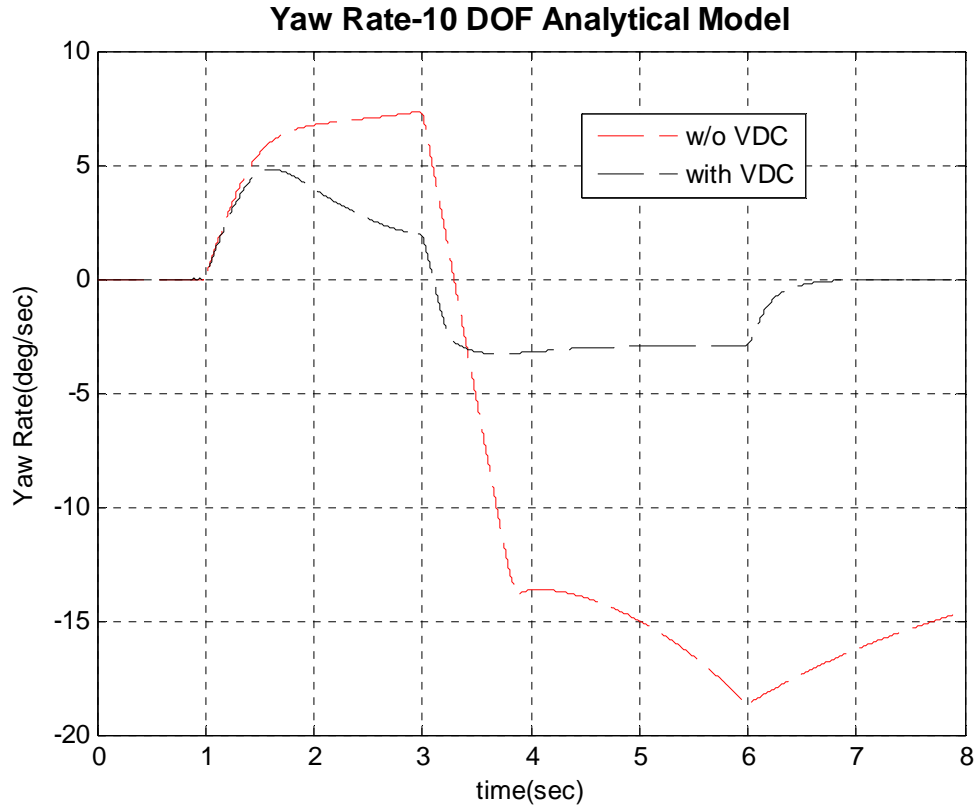


Figure 4.19: Yaw Rate –With and w/o VDC at Ice

4.4 Best Strategy

To evolve the best strategy amongst the four mentioned above, a few basic parameters have been calculated. The table 4.1 pertains to the oversteer case and 4.2 shows the results of understeer case. For DLC maneuver in both surface conditions i.e. snow and ice, strategy 3 proved to be very effective. As also obvious from table 4.1& 4.2, strategy is 3 shown to be very effective. It is concluded that overall strategy 3 i.e. Addition/Subtraction of corrective torque from front axle only, for torque distribution between front and rear axle, is recommended to be the best strategy.

Strategies	Overshoot Ref Value=13.94	Settling Time	Control effort required Min Torque – Max Torque (Nm)
1/3	0.026	2.5	-5.73e4,6.1865e3 (front)
2	0.0291	4.5	-5.594e4,4.278e3 (front)
3	-	-	
4	0.0659	5.25	-4.923e4,5.87e3 (rear)

Table 4.1: Different parameters with oversteer case

Strategies	Overshoot Ref Value=15.35	Settling Time	Control effort required Min Torque – Max Torque (Nm)
1	0.0665	2	-3.93e4,3.5089e3 (rear)
2	0.0513	5	-1.009e3,2.473e4 (front) -2.473e4,1.009e3 (rear)
³ / ₄	0.0093	1.5	-1.533e4,172.0897 (front)
4	-	-	

Table 4.1: Different parameters with oversteer case

4.5 Controller Switching

As mentioned earlier that the error between the actual and desired yaw rates is an obvious measure of vehicle deviation from original path. This could be the feedback variable to instigate the controller for required purpose. Whenever there would be an error generated in the system the controller will activate to eliminate the error. Fig 4.21 and 4.22 illustrate the Controller switching into the system in understeer and oversteer cases respectively. In understeer case when the error is greater than zero our controller activates and in oversteer case when the error is negative our controller activates. The black line in the figure shows the activation of the

controller with the generation of error. These plots were drawn using strategy 3 i.e. addition/subtraction of corrective torques only from front axle.

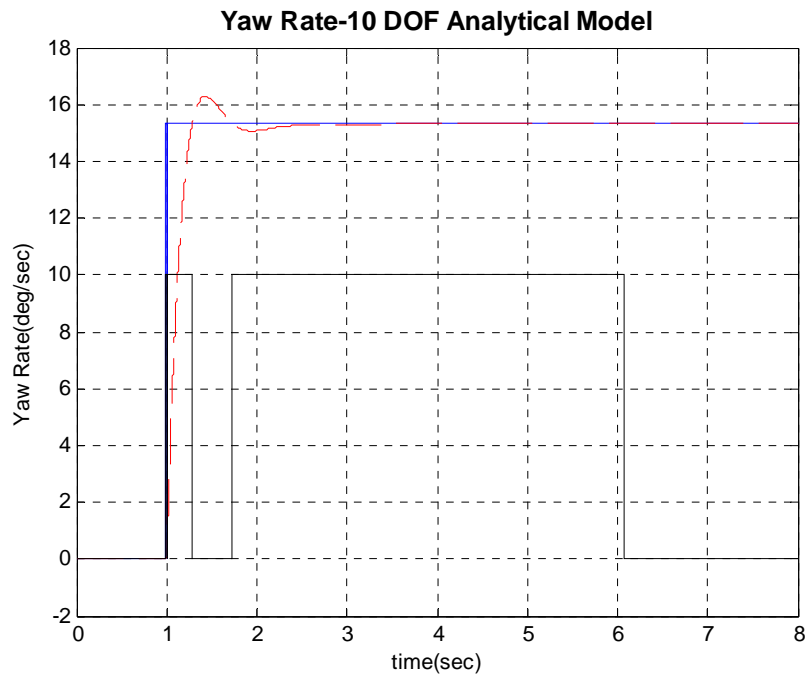


Figure 4.20: Switching Controller in understeer case using strategy 3

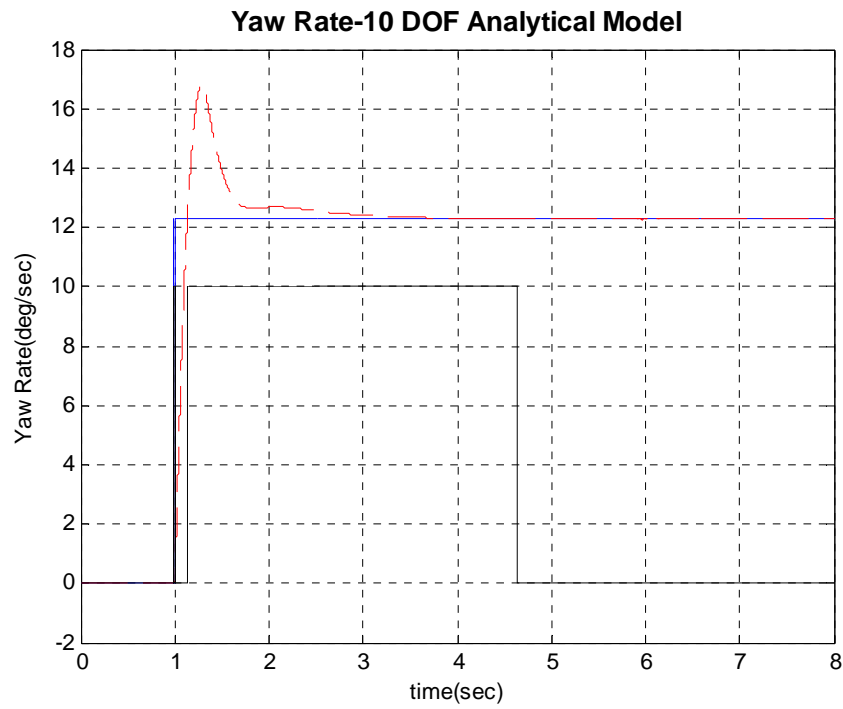


Figure 4.21: Switching Controller in Oversteer case using strategy 3

Chapter Summary

The chapter presented the simulation results obtained from the different yaw moment control strategies. Standard test maneuvers such as J-turn test and Double Lane Change were appropriately modified and simulated for evaluating the effectiveness of proposed torque distribution strategies. Effects of road conditions and speed controller were studied while analyzing the performance of each torque distribution strategy, comparing different strategies and comparing yaw moment control through different feedback control techniques. The results showed a good yaw moment control and limiting the tire sideslip within their prescribed limit.

CHAPTER # 5

CONCLUSION & FUTURE WORK

5.1 CONCLUSION

A limited but effective research was conducted on independent torque distribution management systems. In conducting this research, a non-linear ten degree-of freedom vehicle model and a non-linear tire model using Burckhardt approach were adopted. Various vehicle dynamics control architectures for oversteer and understeer were studied and applied to the vehicle model developed using the desired states obtained from the developed neutral steer model of the vehicle. Based on the physical consequence of longitudinal force distribution, an approach of distribution of torque between front and rear differential using central transfer case was identified. Four torque distribution strategies for achieving the yaw moment control through each of the feedback control variables: yaw rate and lateral acceleration and a strategy combining yaw rate and lateral acceleration control were developed. The simulation responses of the basic vehicle dynamics model include components due to initial conditions of vehicle states, steering input and applied individual wheel torques. Standard test maneuvers such as J-turn and Double Lane Change were appropriately modified and simulated for evaluating the effectiveness of proposed torque controller and were compared with a non linear simulator CarSim. Effects of road conditions, controller gains and speed controller were studied while analyzing the performance of each torque distribution strategy, comparing different strategies and evaluating yaw moment control through different feedback control variables. The yaw rate controller was found to be effective in tracking of yaw rate of the vehicle on slippery surface conditions. Tire sideslip of the value remained very small and always remained in between the prescribed values.

5.2 FUTURE WORK

In this research a PID controller i.e. bang bang control theory is applied. Vehicle dynamics and changing its dynamics via Electronic Stability Control has opened new ways where driver's inclinations will be sensed by ESC and vehicle dynamics will be modified accordingly. This research was focused on yaw rate control and in different driving conditions i.e. wet asphalt and on snow, our control strategies worked efficiently. The performance of the system can be further enhanced by doing more research focused on the following areas:-

Torque development systems or the controller used in this research was a switching torque strategy i.e. using a PID controller that require fine-tuning of control parameters and gain scheduling to handle different regimes of operation. More robust and non-linear controllers can be designed instead, to control non-linearities and uncertainties in the model more effectively. Control theories as mentioned in chapter 2 may be incorporated to further check the performance of these control strategies.

REFERENCES:

- [1] A. Van Zanten, “Bosch ESP systems: 5 years of experience,” *SAE Trans.*, vol. 109, no. 7, pp. 428–436, 2000.
- [2] “Insurance Institute for Highway Safety”, June 2010.
- [3] Rajamani, Rajesh. *Vehicle Dynamics and Control*. s.l. : Springer, 2005. pp. 221-256
- [4] J. Ackermann and W. Sienel, “Robust yaw damping of cars with front and rear wheel steering,” *IEEE Trans. Control Syst. Technol.*, vol. 1, no. 1, pp. 15–20, Mar. 1993
- [5] Sawase et al: “Development of Active Yaw Control System”, *Journal of JSAE*, Vol. 50, No. 11, 1996
- [6] Stephen J. Hallowell and Laura R. Ray, “All-Wheel Driving using Independent Torque Control of each Wheel”, *IEEE Trans.*, ISIE, 2003.
- [7] F. Assadian and M. Hancock, “A Comparison of Yaw Stability Control Strategies for the active Differential”, *IEEE Trans.*, ISIE, June 20 – 23 2005.
- [8] Junmin Wang and Raul G. Longoria, “Coordinated Vehicle Dynamics Control with Control Distribution”, *American Control Conference Minneapolis, Minnesota, USA*, June 14-16, 2006
- [9] Damrongrit Piyabongkarn, et al. “Dynamic Modeling of Torque-Biasing Devices for Vehicle Yaw Control”, *Journal of JSAE*, 2006-01-1963, 2006.
- [10] M. Canalea, L. Fagianoa, M. Milanesea, P. Borodani, “Robust vehicle yaw control using an active differential and IMC techniques”, *Control Engineering Practice*, 2007
- [11] Shuen Zhao, Yinong Li , Ling Zheng and Shaobo Lu, “Vehicle Lateral Stability Control Based on Sliding Mode Control”, *IEEE International Conference on Automation and Logistics August 18 - 21, 2007, Jinan, China*
- [12] Avesta Goodarzi and Ebrahim Esmailzadeh, “Design of a VDC System for All-Wheel Independent Drive Vehicles”, *IEEE/ASME Trans on Mechatronics*, vol. 12, No. 6, December 2007

- [13] Riccardo Marino, Scalzi Stefano, Cinili Fabio, “A Nonlinear Semiactive Rear Differential Control in Rear Wheel Drive Vehicles”, Proceedings of the 26th Chinese Control Conference July 26-31, 2007, Zhangjiajie, Hunan, China
- [14] M. Canale, L. Fagiano, A. Ferrara, C. Vecchio, “A Comparison Between IMC and Sliding Mode Approaches to Vehicle Yaw Control”, American Control Conference Westin Seattle Hotel, Seattle, Washington, USA June 11-13, 2008
- [15] M. Canale and L. Fagiano, “Vehicle yaw control using a fast NMPC approach”, 47th IEEE Conference on Decision and Control Cancun, Mexico, Dec. 9-11, 2008
- [16] Matteo Corno, Mara Tanelli, Ivo Boniolo, Sergio M. Savaresi, “Advanced Yaw Control of Four-wheeled Vehicles via Rear Active Differential Braking”, Joint 48th IEEE Conference on Decision and Control and 28th Chinese Control Conference Shanghai, P.R. China, December 16-18, 2009
- [17] Hai Yu, Wei Liang, Ming Kuang and Ryan McGee, “Vehicle Handling Assistant Control System via Independent Rear Axle Torque Biasing”, American Control Conference, Hyatt Regency Riverfront, St. Louis, MO, USA. June 10-12, 2009
- [18] Hongliang Zhou and Zhiyuan Liu, “Vehicle Yaw Stability-Control System Design Based on Sliding Mode and Backstepping Control Approach”, IEEE Trans on Vehicular Technology, vol 59, No 7, Sep 2010
- [19] Giulio Panzani et al, “Designing On-Demand Four-Wheel-Drive Vehicles via Active Control of the Central Transfer Case”, IEEE Trans on Intelligent Transport System, vol. 11, No. 4, December 2010
- [20] Gillespie D. Thomas *Fundamentals of Vehicle Dynamics*. s.l. : SAE Inc. pp. 8-9
- [21] Pacejka, Hans. B. *Tyre and vehicle dynamics*. second. s.l. : Oxford: Butterworth Heinemann, 2002. pp. 172-197
- [22] Jacob Svendenius, “Tire Modeling and Friction Estimation” PhD thesis, Department of Automatic Control Lund University Lund, April 2007
- [23] U. Kiencke and L. Nielsen. “Automotive Control System”. second. s.l. : Springer, 2005. pp. 304 - 327.

[24] Joga Dharma Setiawan, Mochamad Safarudin and Amrik Singh, “Modeling, simulation and Validation of 14 DOF Full Vehicle Model

[25] **Genta, G.** *Motor Vehicle Dynamics, Modeling and Simulation, Series on Advances in Mathematics for Applied Sciences.* Vol. 43.16