

# **PERFORMANCE ANALYSIS OF ORTHOGONAL FREQUENCY AND CODE DIVISION MULTIPLEXING (OFCDM)**

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*To my beloved family, and friends*

## ABSTRACT

With the advancements in the multimedia and mobile devices, requirement of high data rate transmission is also increasing. High data rate demand is increasing especially for video conferencing, online gaming and real time broadcasting. The research in the field of wireless communication is focusing new modulation techniques which would provide reliable and high data rate communication. OFDM has provided the high data rate with simple modulation and detection techniques. While the research is progressing to make the communication more reliable, hence increase in the data rate.

Combining the two dimensional spreading with OFDM developed the new modulation scheme named OFCDM, which provides the reliable transmission. OFCDM has two dimensional spreading, in time domain and in frequency domain. Spreading in the frequency domain provides the frequency diversity while spreading in time domain provides the flexible transmission rates. NTT DoCoMo has proposed OFCDM for future fourth (4<sup>th</sup>) generation wireless communication system by testing its prototype model in Japan. So it is getting popular and attractive modulation due to its advantages over OFDM.

In this thesis we will focus on the performance analysis of OFCDM in different fading environments. In this research we will only consider the downlink transmission of OFCDM. So performance analysis of OFCDM is carried out in six SUI channels which are designed for fixed wireless applications. Initially some digital modulation techniques such as OFDM, CDMA, MC-CDMA and multipath fading environments such as Rayleigh, Rician and SUI channels are discussed. Then we will discuss the detection and equalization techniques at the receiver. Performance comparison of OFCDM with OFDM is presented as simulation results. Simulation in term of BER graph is shown to verify the performance in Rayleigh, Rician, and six SUI channels. For detection and equalization at the receiver zero forcing equalization technique is used. Due to presence of Doppler shift in the channel OFCDM also experiences the multi-code interference (MCI). While in this thesis all the simulations are carried out without MCI cancellation technique. Finally we will present the overall results and describes the attractiveness of this modulation technique by comparing the results with OFDM.

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## LIST OF ABBREVIATIONS

<u>Abbreviation</u>	<u>Illustration</u>
ISI	Inter Symbol Interference
OFCDM	Orthogonal Frequency and Code Division Multiplexing
VSF	Variable Spreading Factor
MCI	Multi Code Interference
MMSE	Minimum Mean Square Error
EGC	Equal Gain Combining
MIMO	Multiple Input Multiple Output
AWGN	Additive White Gaussian Noise
SUI	Stanford University Interim
OFDM	Orthogonal Frequency and Code Division Multiplexing
CDMA	Code Division Multiple Access
MC-CDMA	Multi Carrier Code Division Multiple Access
PLC	Power Line Communication
LTE	Long Term Evolution
ICT	Islamabad Capital Territory
FFT	Fast Fourier Transform
IFFT	Inverse Fast Fourier Transform
ICI	Inter Carrier Interference
DS-CDMA	Direct Sequence Code Division Multiple Access
MRC	Maximum Ration Combining
PDF	Probability Density Function
SISO	Single Input Single Output
CP	Cyclic Prefix
BER	Bit Error Rate
QPSK	Quadrature Phase Shift Keying
QAM	Quadrature Amplitude Mapping
SNR	Signal to Noise Ratio
WLAN	Wireless Local Area Network
WiMAX	Worldwide Interoperability for Microwave Access

# 1. Chapter 1 Introduction

## 1.1 Introduction

The 4th generation wireless communication demands the high data rate transmission for broadband especially in the downlink. High data rate requirement is increasing due to advanced and high speed multimedia services. In cellular system downlink rate is always greater than uplink transmission rate. The 4<sup>th</sup> generation technology demands the data rate of 100 Mbps in full mobility with wide area coverage and 1 Gbps for low mobility with low area coverage.

Inter-symbol Interference (ISI) due to Multipath effect is one of the factors that limits the data rate of wireless communication. So OFDM has revolutionized the world by providing high data rate communication by easy technique for ISI cancellation. OFDM is good for high speed wireless communication but it does not provide frequency diversity. While CDMA provide frequency diversity due to spreading code but do not eliminate the ISI. So combining the advantages of both OFDM and CDMA, introduced the new modulation technique named Orthogonal Frequency & Code Division Multiplexing (OFCDM) [1]. CDMA has one dimensional spreading while OFCDM has two dimensional (2-D) spreading, in time and frequency domain. Time domain spreading provides flexible transmission rate while frequency domain spreading provides frequency diversity. Time and frequency domain spreading can be changed flexibly for system to provide variable spreading factor (VSF), resulting flexible transmission rate and flexible frequency diversities. Using spreading factor of  $N = N_T \times N_F = 4 \times 2$ , system can have total 8 code channels by pairing time and frequency domain codes. Thus different number of code channels can be assigned to different users.

Walsh Hadamard codes are used for assigning frequency and time domain spreading codes [2]. Thus all the assigned codes are orthogonal to each other. Packet switching or time multiplexing technique is used for OFCDM in the downlink. It means only one user can access the network at one time instant. So there is no multiple access interference among the users. Each user may be assigned with number of code channels, so interference among the code channels may occur for one user. Multi code interference (MCI) can be occurred due to Doppler shift in time domain and different fading among subcarriers in frequency domain. In Walsh Hadamard numbers of possible codes are always equal to the spreading factor of each

code. So there is limited number of codes available to assign multiple users. In time domain spreading codes all 1's are used as a pilot channel.

At the receiver side hybrid detection [3] technique was proposed for equalization by combining minimum mean square error (MMSE) equalization with MCI cancellation. Hybrid detection performs better than pure MMSE [4], [5] because it eliminates the interference among the coded channels. In the actual system equal gain combining (EGC) is used with time domain spreading and then 3-stage hybrid detection is implemented.

OFCDM was firstly introduced by Yee and Linnartz [6] and NTT DoCoMo in Japan tested its performance in field experiments. NTT DoCoMo is one of the leading mobile communication companies which have largest 3G (WCDMA) network in Japan. Using the bandwidth of 100MHz NTT DoCoMo achieved the data rate of 100 Mbps with mobility of 20 km/h in the downlink without MIMO transmission. Further they have also achieved 5 Gbps of data rate for 12x12 MIMO systems in local area coverage with low mobility [7], [8]. So OFCDM seems to be part of future standards for high data rate wireless communication systems.

## **1.2 Thesis Contribution**

Main aim of this thesis is the performance analysis of OFCDM for different fading environments. List of the contribution in this thesis are described as follows:

1. Thorough study of OFCDM system including its architecture, modulation, demodulation, coding, detection and equalization.
2. Performance analysis of OFCDM system in AWGN and Rayleigh fading channel with single and multiple path delays in presence of zero forcing equalizer.
3. Performance analysis of OFCDM for fixed wireless application by using six SUI (Stanford University Interim) Channels with zero forcing equalizer.

### 1.3 Thesis Outline

Chapter 2 describes the background of some standards and technologies such as OFDM, CDMA, and MC-CDMA. Complete description of these modulation techniques and their advantages are explained. These modulation techniques are basis for understanding OFCDM. In Multipath fading channels we will discuss the two main channels, Rayleigh and Rician fading. This chapter also describes some equalization techniques which are mostly used at the receiver of communication systems. Two main equalization techniques zero forcing and minimum mean square error (MMSE) equalizations are described in this chapter. Chapter 3 discusses thesis contribution in OFCDM. In this chapter OFCDM system model is described with fading environments in which all the simulations are carried out. All the simulation results in term of bit error rate (BER) graph are discussed. Finally chapter 4 describes the conclusion and future work.

## 2. Chapter 2 Background

In this chapter we will discuss the background of wireless channel models for multipath fading environment, physical layer transmission techniques and modulations including OFDM, CDMA, MC-CDMA, and OFCDM. We will also discuss the detection and equalization techniques. Thus all the basic and advanced concepts necessary to understand the OFCDM system are discussed in this chapter.

### 2.1 Transmission Techniques

In this section we will discuss the concepts of physical layer transmission techniques such as OFDM, CDMA, MC-CDMA and OFCDM.

#### 2.1.1 OFDM

OFDM [9] is one of the modulation techniques which have performed very well for high data rate transmission. Then most important concept in OFDM is the orthogonal frequencies which carry the data. So the data is transmitted through parallel subcarriers which can be separated at the receiver due to orthogonality. One of the most important advantages of OFDM is that it minimizes the Inter Symbol Interference (ISI). So low symbol rate is used for transmitting the parallel data stream. OFDM is the physical layer of current wireless and wired standards which support high data rate transmission such as ADSL, Power Line Communication (PLC), Wi-Fi, WiMAX and LTE.

In OFDM frequencies of subcarriers are chosen such that they are all orthogonal to each other. This eliminates the need of any special equalization filter for each subcarrier and the need of inter-carrier guard band. Thus it simplifies the design of transmitter and receiver. For symbol duration of  $T_u$  seconds, subcarrier spacing will be  $\Delta f = 1/T_u$  Hertz. Figure 1 shows the orthogonal subcarriers. Therefore if  $N$  subcarriers are used then the total pass-band bandwidth will be  $B = \Delta f \cdot N$  Hertz. Modulation and Demodulation of data with orthogonal subcarriers in OFDM can be simply realized by IFFT and FFT at transmitter and receiver side. Mathematically OFDM symbol can be expressed as follows:

$$v(t) = \sum_{k=0}^{N-1} X_k e^{\frac{j2\pi kt}{T}}, \quad 0 \leq t < T \quad (2.1)$$

Where  $X_k$  are data symbols,  $N$  is number of subcarriers and  $T$  is the OFDM symbol duration. So each data symbol is modulated with orthogonal subcarrier and making a complex OFDM symbol.

When OFDM symbol is transmitted through multipath channel then it experiences inter symbol interference (ISI). Thus guard interval is inserted between the OFDM symbols to eliminate the ISI. Similarly OFDM also experiences the interference among the subcarriers, which destroys the orthogonality, this effect is known as inter carrier interference (ICI) [10].

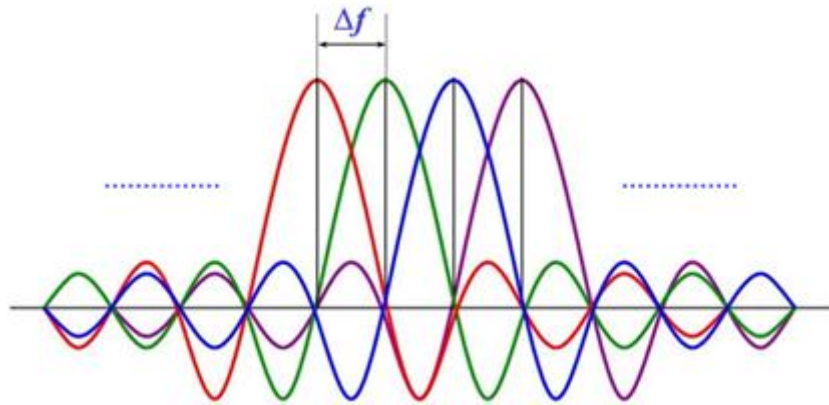


Figure 2.1: Orthogonal subcarriers.

Figure 2.2 shows the OFDM system block diagram. In OFDM system data stream is mapped to symbols with QPSK or M-QAM then it is fed into the IFFT block which converts the frequency domain signal into time domain signal. This time domain signal experiences different interferences such as ISI and ICI. After experiencing the channel fading and noise, receiver converts the time domain signal back to the frequency domain by passing through FFT block [11]. Symbols are then converted into bits after detection.

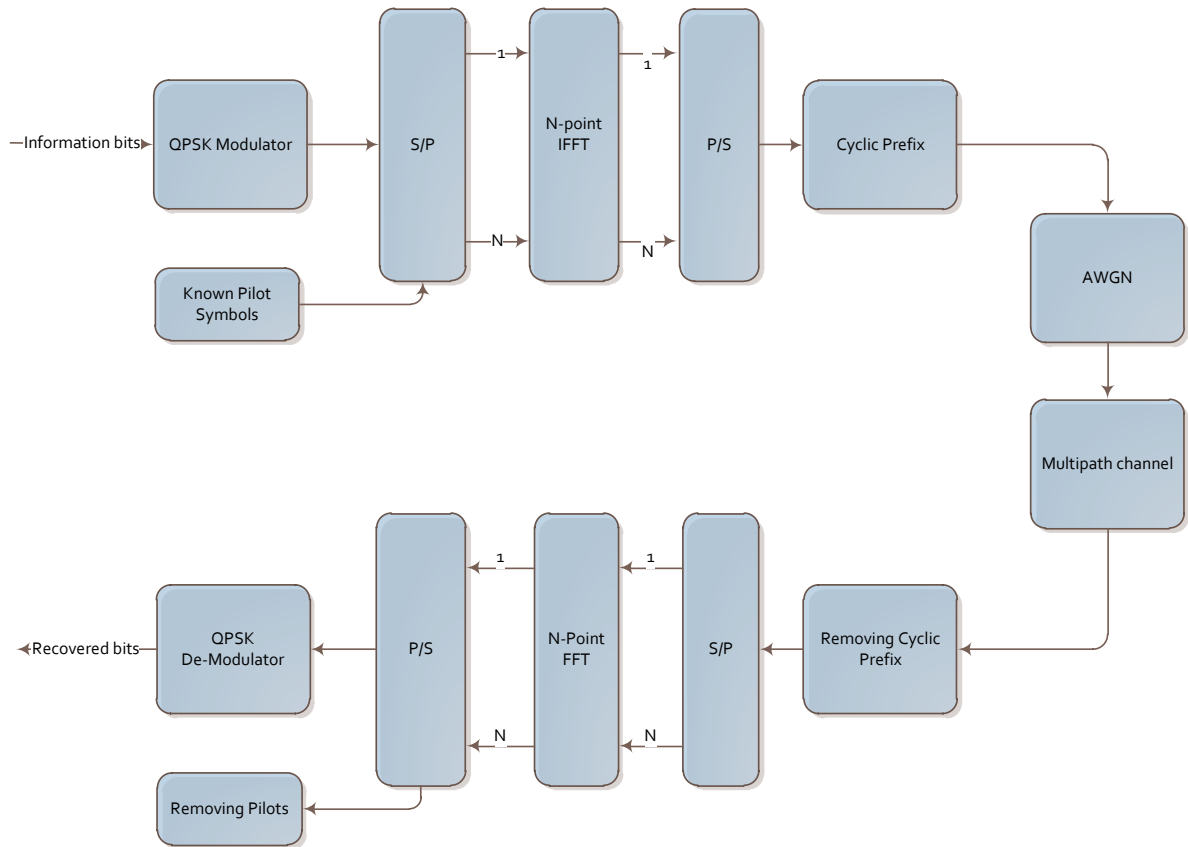


Figure 2.2 OFDM System Block Diagram

### 2.1.2 CDMA

Code Division Multiple Access is also one of the modulations which gained its popularity due to multiple access technique. In this technique multiple users transmit the data at the same time using same frequency and channel by spreading the symbols with orthogonal spreading codes. CDMA allows the users to share same band of frequencies with different spreading code. Each user can easily recover the data by correlating same spreading code with the data. Correlation value outputs the maximum value if same code is used else it outputs zero.

Let's assume the data signal is having pulse duration of  $T_b$  and orthogonal spreading code also known as chip has pulse duration of  $T_c$ . In CDMA system chip rate is always greater than the original signal rate. So data is XOR'ed with chip and modulated to high data rate. This technique is also known as spread spectrum because modulated data rate is higher than the original signal rate. At the transmitter side each user data is spread with orthogonal spreading code then they are multiplexed and transmitted at the same time. Orthogonal codes or chip of length  $L$  can be represented as  $C_k$  where  $C_k = [C_0^k C_1^k \dots C_{L-1}^k]$ , where  $c_k^l$  ( $l =$

0,1,2 ...  $L - 1$ ) is called a chip. All the chips must satisfy the most important property of orthogonality which is the base of CDMA system.

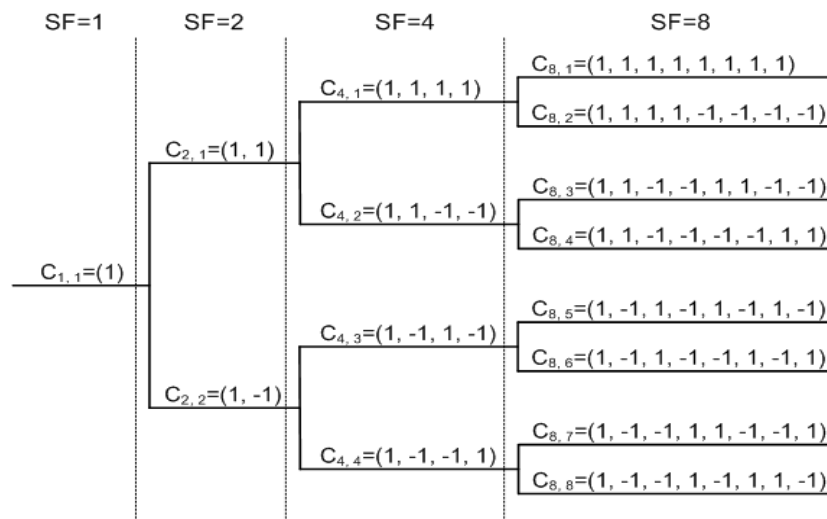


Figure 2.3: Orthogonal variable spreading (OVSF) factor code tree.

Codes used in the CDMA system are also known as Walsh Hadamard code and they can be generated from OVSF code tree as shown in Figure 2.2. One of the property of these code are that they always make square matrix, which means length of code is always equals to the number of orthogonal codes. One of the common spreading techniques is direct sequence code division multiple access (DS-CDMA) [12] in which data is directly multiplied with spreading code and then transmitted. At the receiver side matched filters are used to recover the symbol. CDMA is not suitable in environment which has number of multi paths. Thus in large multi-path interference, detection of CDMA becomes very complex.

### 2.1.3 MC-CDMA

Multi carrier code division multiple access (MC-CDMA) [13]-[14] is the hybrid model of OFDM and CDMA systems. In MC-CDMA data is spread either in time domain or frequency domain and then presented on IFFT block to make an MC-CDMA symbol. So MC-CDMA is divided into two categories. In one category each data symbol is spread in frequency domain and then modulated with OFDM subcarriers, this technique is simply known as MC-CDMA. In second category data symbols are spread in time domain to make data streams and these streams are transmitted parallel through OFDM subcarriers. This technique is known as multi carrier direct sequence code division multiple access (MC-DS-CDMA). So MC-CDMA combines the advantages of both CDMA and OFDM modulations, while it has same drawbacks as OFDM. The one major advantage of MC-CDMA is that symbol rate can be



lowered in each subcarrier, hence the longer symbol duration which makes it easier for quasi-synchronize the transmission [13]. The combination of MIMO technology with MC-CDMA is also studied for multi user environment [15].

### 2.1.4 OFCDM

Orthogonal frequency and code division multiplexing (OFCDM) [17] is also combination of OFDM and CDMA like MC-CDMA, but it contains spreading in both time and frequency domain. Spreading in frequency domain provides the frequency diversity while spreading in time domain provides flexible transmission rates. It fulfills the flexible transmission rate demand of future 4<sup>th</sup> generation system [16]. Two dimensional spreading techniques provide the multi-code channels, which can be assigned to different users. Let's assume that time domain spreading factor  $N_T = 4$  and frequency domain spreading factor  $N_F = 2$ , so there would be total  $N = N_T * N_F = 8$  possible code channels. These code channels can be obtained by pairing the time and frequency domain codes. Figure 2.3 shows the time and frequency domain spreading and possible coded channels. In OFCDM system spreading codes can be generated using Walsh Hadamard codes or OVSF code tree in Figure 2.2. In OFCDM system time domain codes with all 1's are used as pilot channels. Spreading codes with high code distance are assigned with high priority.

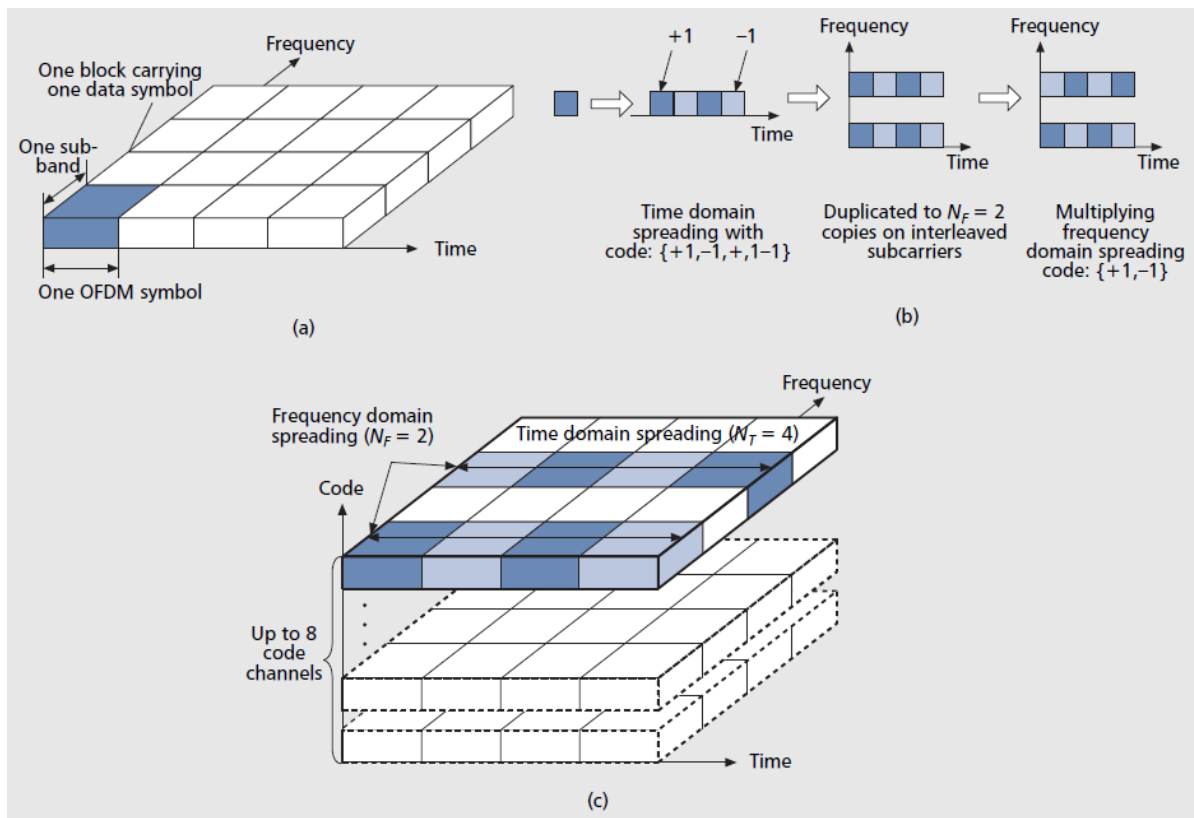


Figure 2.4: OFCDM spreading technique and code channels [17].

In OFCDM downlink system users are served by packet switch or time multiplexing. Thus at any instant only one user can access the network, and multiple code channels can be assigned to one user. So there is no multiuser interference but multi-code interference [18] among one user may occur. In broadband channel orthogonality at the receiver is no longer maintained in time domain due to Doppler shift [19] and in frequency domain due to different fading among subcarriers. Proposed equalization for OFCDM signal at the receiver is realized by hybrid detection [20], which contains multi code interference (MCI) cancellation and minimum mean square error (MMSE) detection. Hybrid detection has performed well compared to pure MMSE and other detection techniques. Further maximum ratio combining (MRC) and equal gain combining (EGC) techniques are also used for detection.

## 2.2 Multipath Fading Channels

In wireless communication system communication between transmit and receive antenna is not always at line of sight. Signal may experience multi path effect in which copies of transmitted signal experiences different reflections. So each copy of signal may face different attenuation and phase change. Receiving antenna may receive copies of same signal for some time instant.

There are two type of fading channels, one is flat fading and another is frequency selective fading. In flat fading channel, coherence bandwidth of the channel is greater than the signal bandwidth. So signal will experience same fading on complete frequency band. In frequency selective fading coherence bandwidth of channel is smaller than signal bandwidth and signal will experience different fading experience on different frequency components. Two well-known channel models Rayleigh and Rician will be discussed as follows.

### 2.2.1 Rayleigh Fading Channel

Rayleigh fading is used as a channel model when there are random scattered signal components with no line of sight. Rayleigh fading model is used when there are many objects in the single path which scatter the signal. When there are much scatter of the signal with no line of sight then channel coefficient are modeled as Rayleigh distribution. Probability density functions of Rayleigh fading [21] with random variable 'R' can be given as follows:

$$p_R(r) = \frac{r}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}}, \quad r \geq 0 \quad (2.2)$$

If there is no dominating component in the scatter then the process will have zero mean. PDF of Rayleigh distribution is shown in Figure 2.4.

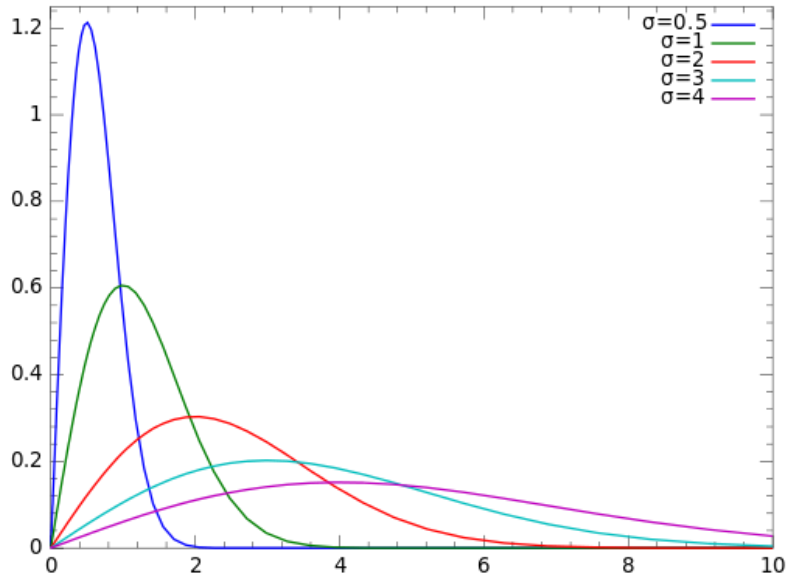


Figure 2.5: PDF of Rayleigh distribution [22].

### 2.2.2 Rician Fading Channel

Rician fading channel model is used as a multipath channel when there is at-least one line of sight signal and it is stronger than other scattered copies of signal. So channel coefficients in such conditions are modeled as Rician distribution. Major component in Rician channel is line of sight. Adding line of sight content in Rayleigh channel makes Rician channel.

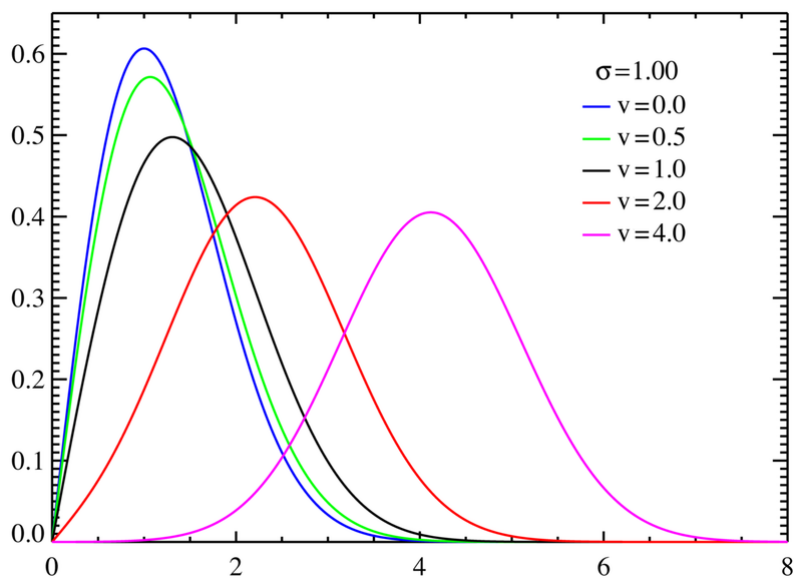


Figure 2.6: PDF of Rice distribution [23].

### 2.3 SUI Channels

Stanford university interim (SUI) [24] channels were developed for fixed wireless applications. Set of six SUI channel models were developed by Stanford University and each model contains 3-tap delays. These models were proposed on the basis of 3 different terrains types (A, B, C). Terrain type A is associated with hilly area and moderate to high tree density. Type B is designed for flat terrain with moderate tree density and type C is designed for flat terrain with light tree density. Four main parameters of SUI channel are Delay spread ( $\mu\text{sec}$ ), Attenuation (dB), Doppler shift (Hz), Power factor K (dB). Table 2.1 shows the SUI channel model based on terrains.

Terrain	SUI Models
C: Flat / Light tree density	SUI-1, SUI-2
B: Flat / Moderate Tree density	SUI-3, SUI-4
A: Hilly / Moderate to Heavy tree density	SUI-5, SUI-6

Table 2.1 SUI Channel models w.r.t terrains.

Table 2.2 and 2.3 shows the parametric model of SUI channels separated by power factor K. Table 2.2 and 2.3 differentiate SUI channels on basis of Delay spread and Doppler shift when K is low and K is moderate or high. For  $K=0$  multipath channel is modeled by Rayleigh fading and for  $K > 0$  channel is modeled as Rician fading.

<b>K = LOW</b>	<b>Delay Spread</b>	Low	Moderate	High
<b>Doppler</b>				
Low		SUI-3		SUI-6
High			SUI-4	SUI-5

Table 2.2 SUI Channel for K = LOW.

<b>K=Moderate/High</b>	<b>Delay Spread</b>	Low	Moderate	High
<b>Doppler</b>		SUI-1, SUI-2		
Low				
High				

Table 2.3 SUI Channels for K = Moderate / High.

## 2.4 Equalization techniques

When the signal reaches at the receiver through multipath channel, there should be a suitable detection technique which should detect and extract the symbol. This detection and extraction of symbol is called equalization. In this section we will discuss the equalization techniques used in digital communication systems. The two main linear equalization techniques are zero forcing and minimum mean square error (MMSE).

### 2.4.1 Minimum mean square error equalizer

Minimum mean square equalizer (MMSE) [25]-[28] equalizer is a type of linear equalization which aims to minimize the difference of transmitted data and equalizer output. In case of frequency selective channel MMSE provides best suitable equalization. One of the advantages of MMSE is that it suppresses the noise and does not let the noise to become infinite as in zero forcing equalization. When noise in the signal is zero then MMSE becomes identical to zero forcing which eliminates the ISI completely. When noise is greater than zero then residual ISI will be observed with some noise at the output of equalizer.

Let's assume that  $h_n$  is the discrete time channel coefficients and  $a_k$  is information symbols, then discrete time channel can be modeled as:

$$y_k = \sum_{n=0}^L h_n a_{k-n} + n_k \quad (2.3)$$

Where  $n_k$  is the Gaussian noise with zero mean and variance of  $N_0/2$ . Equalizer filter having coefficients of  $c_j$  with  $2K+1$  taps can be modeled as:

$$\hat{a}_k = \sum_{j=-K}^K c_j y_{k-j} \quad (2.4)$$

As MMSE tries to minimize the error which is difference between transmitted data and equalized output. MMSE cost function  $J$  can be represented as follows:

$$J = E\{|a_k - \hat{a}_k|^2\} = E\{|\varepsilon_k|^2\} \quad (2.5)$$

where  $E\{\cdot\}$  represents the expected value and  $\varepsilon_k$  is the error. According to orthogonality principle this problem can be solved by taking the filter coefficient  $c_j$  such that it the error should be orthogonal to the received signal  $y_{k-l}$ .

$$E\{\varepsilon_k y_{k-l}\} = 0, \quad |l| \leq K \quad (2.6)$$

Substituting value of  $\varepsilon_k$  gives

$$E\{(a_k - \hat{a}_k)y_{k-l}\} = E\{(a_k - \sum_{j=-K}^K c_j y_{k-j})y_{k-l}\} = 0 \quad (2.7)$$

which is equal to

$$\sum_{j=-K}^K c_j E\{y_{k-j}y_{k-l}\} = E\{a_k y_{k-l}\}, \quad |l| \leq K \quad (2.8)$$

Finally we can write this expression as

$$\sum_{j=-K}^K c_j R_{lj} = R_l, \quad l = -K, \dots, K \quad (2.9)$$

This linear equation can be written in matrix form as

$$C R_{lj} = R_l \quad (2.10)$$

$$C = R_{lj}^{-1} R_l \quad (2.11)$$

Where  $C$  is the column vector of  $2K + 1$  taps coefficients and  $R_{lj}$  is  $(2K + 1) * (2K + 1)$  covariance matrix and  $R_l$  is  $2K + 1$  column vector. Equation 2.10 shows the formula to find the filter coefficients. Figure 2.7 shows the minimum mean square equalization filter.

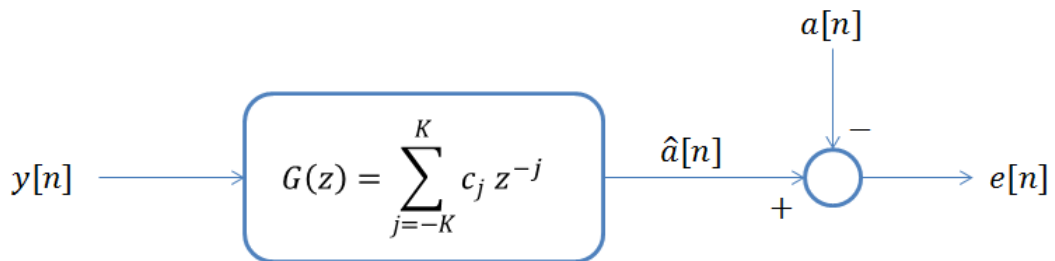


Figure 2.7: MMSE equalization filter.

### 2.4.2 Zero forcing equalization

Zero forcing equalizer [27], [29]-[30] is type of linear equalization which aims to remove the inter-symbol interference (ISI) completely from receive signal. It is the simplest equalization technique which is heavily studied for IEEE 802.11n standard. Zero forcing is very suitable to remove the ISI completely when there is less noise or noise free environment. If some frequencies of the received signal are weak in the presence of noise then it may boost up the noise after equalization to compensate the magnitude.

Zero forcing equalizer is simply the inverse of channel, which can be found by using known pilot symbols. Let's assume that  $S_T(f)$  is the frequency response of known transmitted signal and  $S_R(f)$  is the received signal at the receiving antenna. To find the frequency response of the channel we will divide these two frequency responses.

$$H_E(f) = \frac{S_R(f)}{S_T(f)} \quad (2.12)$$

We can take the inverse Fourier transform of  $H_E(f)$  to get  $h_e$  which provides the filter coefficients to equalize the signal. Actual zero forcing filter has infinite length but it is usually implemented by truncating to make finite length approximation. It can be implemented by using linear transversal filter.

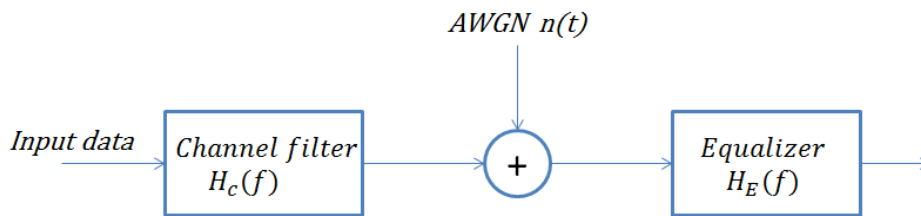


Figure 2.8: Channel and equalization filters.

Figure 2.8 shows the simple model of channel and equalization filter in frequency domain.

The equalization filter  $H_E(f)$  is actually the inverse of channel filter  $H_c(f)$ .

$$H_E(f) = \frac{1}{H_c(f)} \quad (2.13)$$

### 3. Chapter 3 Performance Analysis of OFCDM

In this chapter we will discuss thesis contribution in which performance analysis of SISO-OFCDM system is analyzed. First we will introduce the OFCDM modulation and demodulation and its detection at the receiver side. Then performance analysis of OFCDM in AWGN and Rayleigh fading channel will be discussed. Finally performance analysis of OFCDM will be discussed in six SUI channels. All the performance results will be analyzed by Bit Error rate graph.

#### 3.1 SISO OFCDM

SISO OFCDM system diagram is shown in Figure 3.1. Information bits are first modulated by QPSK modulator. These modulated symbols are converted from serial to parallel stream  $N_B$  and then fed into two dimensional spreading blocks. Parallel symbols are spread with time domain spreading code  $N_T$  and frequency domain spreading code  $N_F$ . These codes are generated from OVSF code tree as shown in Figure 2.2. All 1's in time domain spreading codes are not assigned to data symbols because they are used as pilots. Overall spreading factor can be calculated as:

$$N_S = N_T * N_F \quad (3.1)$$

This overall spreading factor  $N_S$  shows that there are total  $N_S$  code channels available for spreading. One or more than one code channel can be assigned to single user. These code channels are developed by pairing time and frequency domain spreading codes. Let  $K$  be the total number of transmitted groups and each group has separate identical spreading code, then  $K \leq N_S$ . System load factor denoted by  $\zeta$  can be defined as total number of transmitted groups divided by total spreading factor:

$$\zeta = \frac{K}{N_S} \quad (3.2)$$

Let's assume that time domain spreading code matrix is represented as  $C^T(i)$  ( $i = 0,1,2 \dots N_T$ ), where  $C^T(i)$  represents the  $i^{th}$  spreading code and frequency domain spreading code matrix as  $C^F(j)$  ( $j = 0,1,2 \dots N_F$ ) where  $C^F(j)$  represents  $j^{th}$  spreading code. These spreading codes are represented as

$$C^F(j) = [c_0^F(j) \quad c_1^F(j) \quad \dots \quad c_{N_F-1}^F(j)] \quad \text{where } j = 0,1,2, \dots N_F-1 \quad (3.3)$$

$$C^T(i) = [c_0^T(i) \quad c_1^T(i) \quad \dots \quad c_{N_T-1}^T(i)] \quad \text{where } j = 0,1,2, \dots N_T-1 \quad (3.4)$$



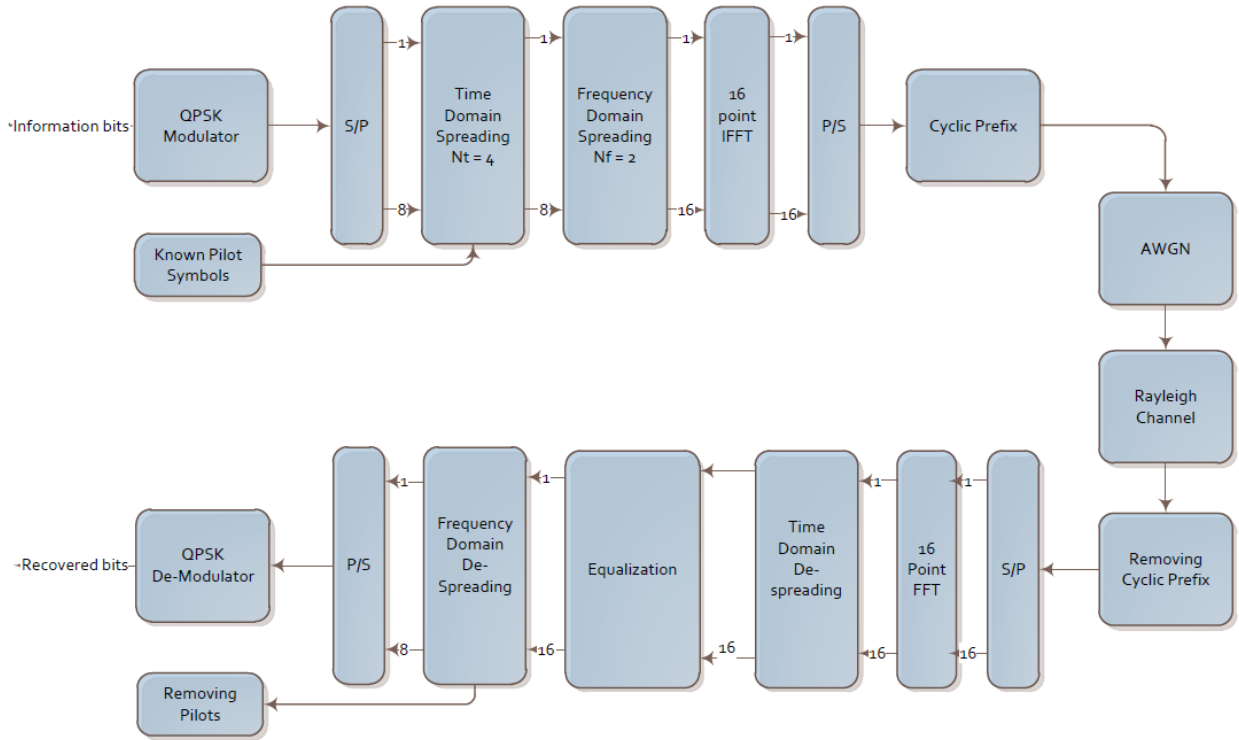


Figure 3.1 OFCDM system block diagram.

2-D spread data is now converted into OFDM symbol by modulating with orthogonal subcarriers. This modulation can be done by using IFFT block. Subcarrier modulated symbols are converted back to serial stream and cyclic prefix (CP) is added. After adding cyclic prefix OFCDM symbol is transmitted through antenna. At the receiving antenna, received symbols are converted from serial to parallel and passed to FFT block to remove the subcarriers. Channel is estimated using pilots and received signal is equalized. Symbols are de-spread with time and frequency domain de-spreader, in which symbols are multiplied with same spreading code and then added to get the original symbols. Finally symbols are converted to serial stream again and QPSK demodulator is used with hard decision method to recover data bits.

In this thesis we focus on the performance analysis of OFCDM compared to OFDM with several assumptions. First we assume that received signal is perfectly synchronized, so we do not need to find the start of the symbol. Second we assume that channel sampling rate is slower than signal rate. OFCDM signal may experience multi-code interference due to Doppler shift. In our system design we did not use any MCI cancellation technique. To represent mathematical model of OFCDM symbol, zeroth data symbol at  $n^{th}$  subcarrier of  $j^{th}$  OFCDM symbol is represented as:

$$D_{n,j}(t) = \sqrt{P} \sum_{k=0}^{K-1} a_{k,n,j} \cdot e^{j2\pi f_m(t-jT)} \quad (3.5)$$

In equation 3.5  $P$  is the signal power and  $a_{k,n,j}$  is the modulated symbol which can be represented as:

$$a_{k,n,j} = d_k c_{N_T,j}^{k_T} c_{N_F,m}^{k_F} \quad (3.6)$$

In equation 3.6  $d_k$  is the data symbol presented on  $K^{th}$  code channel and  $c_{N_T,j}^{k_T} c_{N_F,m}^{k_F}$  are spreading codes of time and frequency domains. So  $d_k$  symbol is first spread with time domain spreading code and then frequency domain spreading code and then allocated to  $n^{th}$  subcarrier. For channel estimation pilot channels are also employed in OFCDM symbols. As all 1's in time domain spreading code is used as pilot channel so pilot symbols at  $n^{th}$  subcarrier of  $j^{th}$  OFCDM symbols is represented as:

$$D_{P,n,j}(t) = \sqrt{P} a_{P,n,j} \cdot e^{j2\pi f_m(t-jT)} \quad (3.7)$$

Where  $a_{P,n,j}$  is represented as:

$$a_{P,n,j} = d_P c_{N_T,j}^0 \quad (3.8)$$

In above equation  $d_P$  is a pilot symbol and  $c_{N_T,j}^0$  is all 1's, so complex transmitted time domain  $N_T$  OFCDM symbol which is combination of data symbols and pilot symbols can be represented as:

$$D(t) = \sum_{n=0}^{M-1} \sum_{j=0}^{N_T-1} [D_{n,j}(t) + D_{P,n,j}(t)] \quad (3.9)$$

In equation 3.9,  $M$  is the total number of subcarriers.

In this thesis performance of the OFCDM system is analyzed in SUI channels with zero forcing equalizer. The reason for adopting zero forcing is that it eliminates the inter-symbol interference (ISI) completely. Its implementation is very simple in which channel inverse is used to equalize the received symbols. Suppose  $P_T(f)$  is the frequency response of the transmitted pilot symbols and  $P_R(f)$  is the frequency response of the received pilot symbols then channel response can be found as:

$$H(f) = \frac{P_R(f)}{P_T(f)} \quad (3.10)$$

The frequency response of the channel  $H(f)$  is used to divide the frequency response of receive OFCDM symbol for equalization. Zero forcing eliminates the ISI completely but it may increase the noise of the equalized signal because it has linear response. Simulation results are obtained in the worst case scenario, in the presence of noise and multi-code interference.

## 3.2 Simulation results

In this section we will provide the simulation results by comparing OFCDM bit error rate (BER) with OFDM. The simulation parameters can be viewed from block diagram shown in figure 3.1. Initially data bits are mapped with QPSK modulator then converted into 8 parallel streams. QPSK modulated symbols are then spread in time domain with spreading factor  $N_T = 4$  and then each parallel time domain stream is spread in frequency domain with spreading factor  $N_F = 2$ . Then each spread symbol is modulated with subcarrier by taking the 16-point IFFT. Finally we add cyclic prefix of 25% and then OFCDM symbol is transmitted through channel. After receiving the OFCDM cyclic prefix is removed, converted from serial to parallel and 16-point FFT is taken to remove subcarriers. Finally frequency and time domain de-spreading is carried out. In this system we have used Zero forcing equalizer. Equalization is done after time domain de-spreading.

### 3.2.1 AWGN and Rayleigh fading channel

Initially OFCDM is compared with OFDM in AWGN channel. Spreading factor of  $N_T = 4$  and  $N_F = 2$  is used for all simulation results and utilizing all eight possible code channels. Bit error rate (BER) graph in AWGN channel is shown in Figure 3.2. The graph shows that there is a huge difference in the performance of OFCDM and OFDM. This is due to spreading technique which replicates each symbol and creates redundancy. Due to this redundancy the symbol detection becomes easy at the receiver side. Redundancy in the OFCDM reduces the data rate and also the bit error rate. To increase the data rate for OFCDM all possible code channels are employed and symbol mapping can be changed from QPSK to 16-QAM, 64-QAM and above.

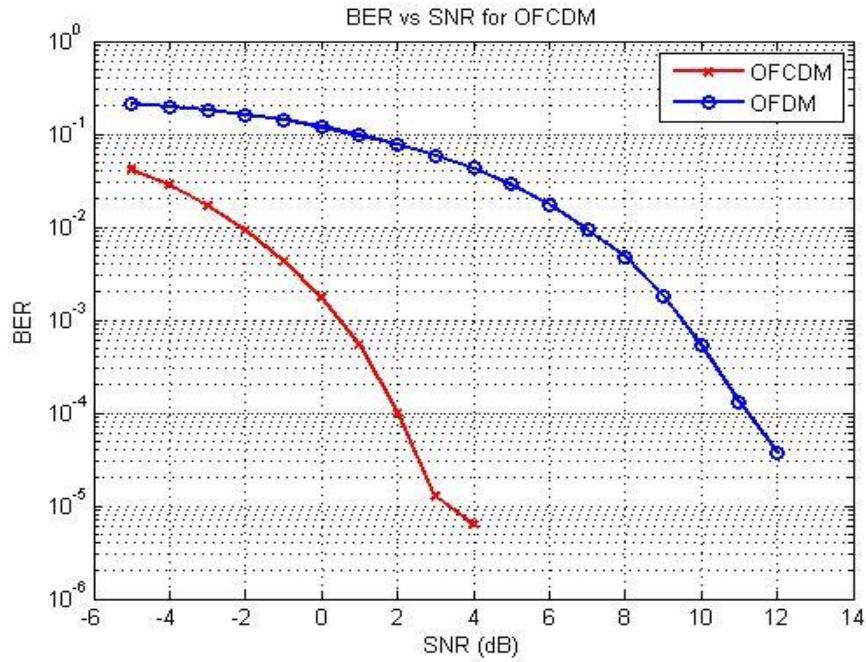


Figure 3.2: OFCDM vs. OFDM in AWGN channel.

Figure 3.3 shows the BER results when spreading factor for OFCDM is increased in AWGN channel. The result shows that by increasing the spreading factor to  $N_T = 8$  and  $N_F = 4$  and employing all 32 possible code channels the bit error rate reduces dramatically. We can see the difference of about 6dB signal to noise ration (SNR) if we consider BER of  $10^{-4}$ .

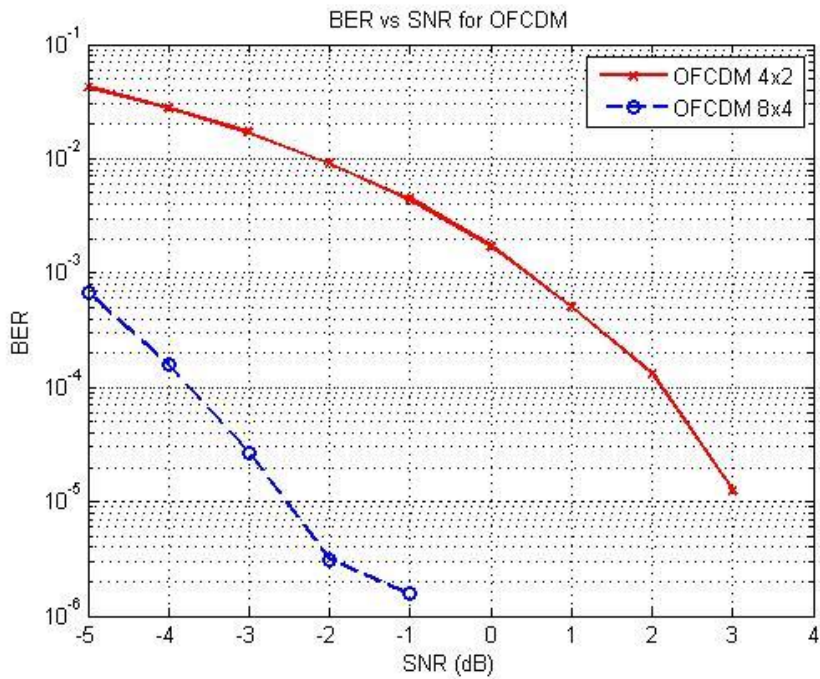


Figure 3.3: OFCDM 4x2 vs. OFCDM 8x4.

Similarly simulation result in single path flat fading Rayleigh channel is shown in figure 3.4. In flat fading all the frequency components faces the same fading. Focusing the bit error rate of  $10^{-4}$  there is a difference of about 6dB SNR.

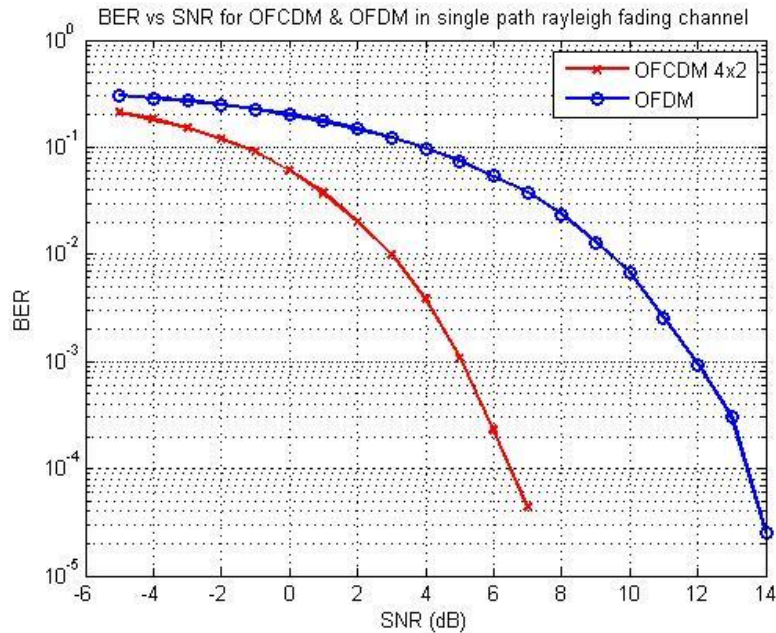


Figure 3.4: OFCDM vs. OFDM in single path Rayleigh fading channel.

Figure 3.5 shows the BER result for six paths Rayleigh fading channel with Doppler shift of 0.02Hz. Path delays of six paths and their gain is shown in table 3.1.

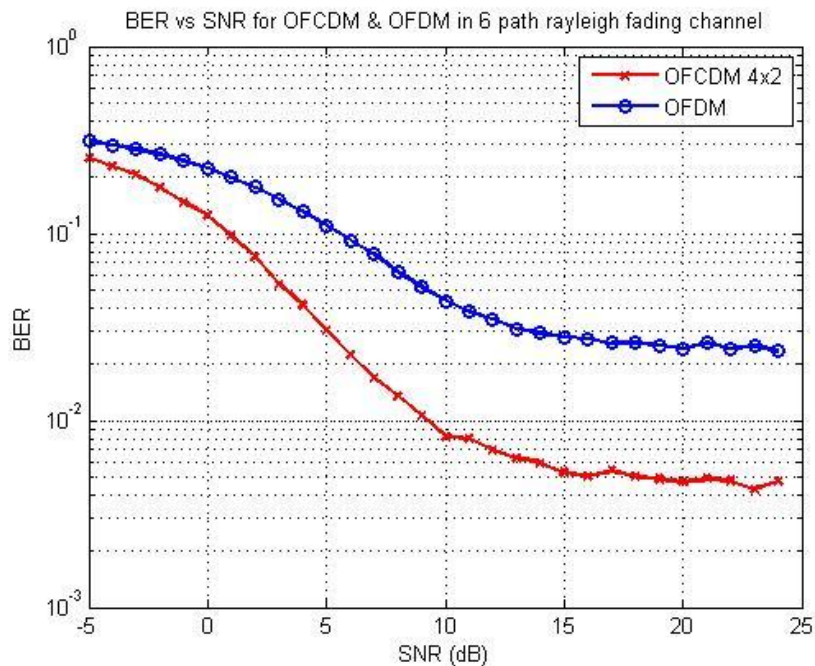


Figure 3.5: OFCDM vs. OFDM in 6 path Rayleigh fading channel.

Thus all the results show that OFCDM has performed well in AWGN, single path Rayleigh and multipath Rayleigh fading channels. Next section will show the simulation results in SUI channel models.

Doppler shift	0.02 Hz
Path vector delays	$[0 \ 0.1e^{-6} \ 0.2e^{-6} \ 0.3e^{-6} \ 4e^{-6} \ 5e^{-6}] \text{ sec}$
Avg. path gain vectors	$[0 \ -4 \ -8 \ -12 \ -16 \ -20] \text{ dB}$

Table 3.1 Path vectors and gains of six paths Rayleigh fading channel.

### 3.2.2 SUI-1 channel model

SUI-1 channel model belongs to terrain C which is associated with flat area and light tree density. Table 3.2 shows the tap delays of SUI-1 channel.

K = 10 Doppler = 0.4 Hz	Tap1	Tap2	Tap3
Delay ( $\mu\text{sec}$ )	0	0.4	0.8
Power (dB)	0	-15	-20

Table 3.2 Parameters of SUI-1 Channel model.

SUI-1 is modeled as Rician fading channels because value is K (power factor) is greater than zero. Figure 3.6 shows the bit error rate result of OFCDM and OFDM in SUI-1 channel model. This figure shows that there is nearly 7dB of difference if we consider BER of  $10^{-4}$ .

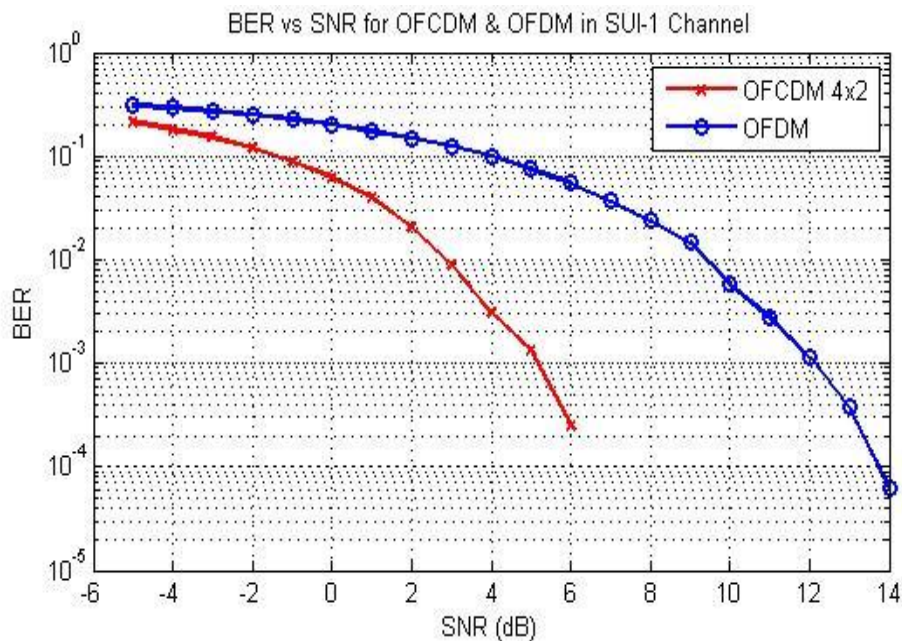


Figure 3.6 OFCDM vs. OFDM in SUI-1 Channel model.

### 3.2.3 SUI-2 channel model

SUI-2 channel also belongs to terrain C with flat area and light tree density model. Values of power factor K and Doppler shift become half and path taps have been changed. Table 3.3 shows the parameters of this channel.

K = 5 Doppler = 0.2 Hz	Tap1	Tap2	Tap3
Delay ( $\mu$ sec)	0	0.4	1.1
Power (dB)	0	-12	-15

Table 3.3 Parameters of SUI-2 channel model.

Due to small difference in the parameters of SUI-1 and SUI-2 channel models there is not much difference in the bit error rate graph results. These both channels also belong to same terrain. Overall OFCDM has performed well compared to OFDM in both graphs. Bit error rate graph for SUI-2 channel is shown in Figure 3.7.

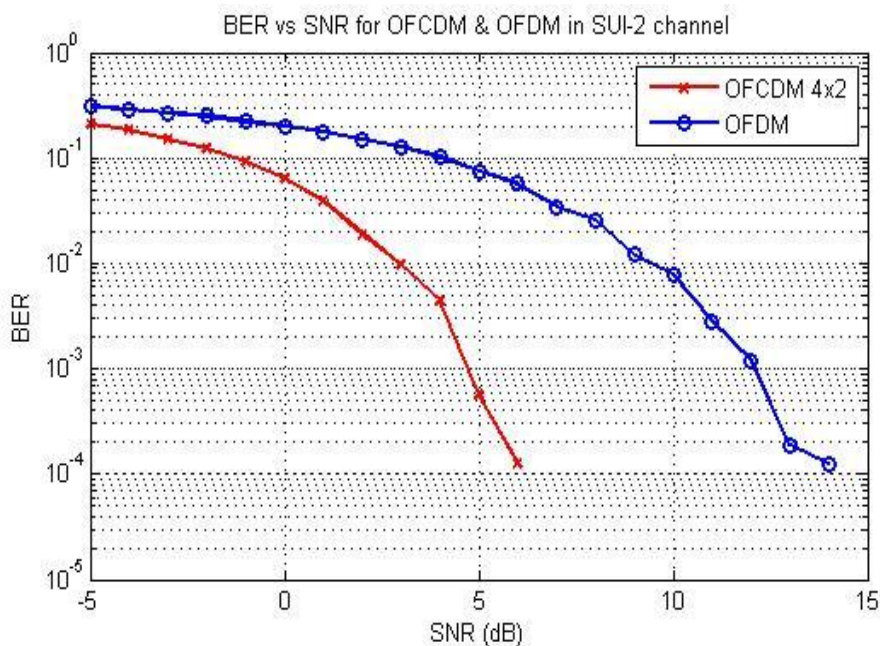


Figure 3.7 OFCDM vs. OFDM in SUI-2 channel model.

### 3.2.4 SUI-3 channel model

SUI-3 channel model has least K (power factor) value and it belongs to terrain B which is associated to flat area and moderate tree density. Table 3.4 shows the tap delays and other parameters.

K = 1 Doppler = 0.4 Hz	Tap1	Tap2	Tap3
Delay ( $\mu$ sec)	0	0.4	0.9
Power (dB)	0	-5	-10

Table 3.4 Parameters of SUI-3 Channel model.

Figure 3.8 shows the bit error rate graph for SUI-3 channel. We can see that the performance of OFCDM compared to OFDM is very good but compared to SUI-1 and SUI-2 OFCDM bit error rate is bit degraded. This is due to fact that valued of K is reduced to 1 and Doppler shift is also increased.

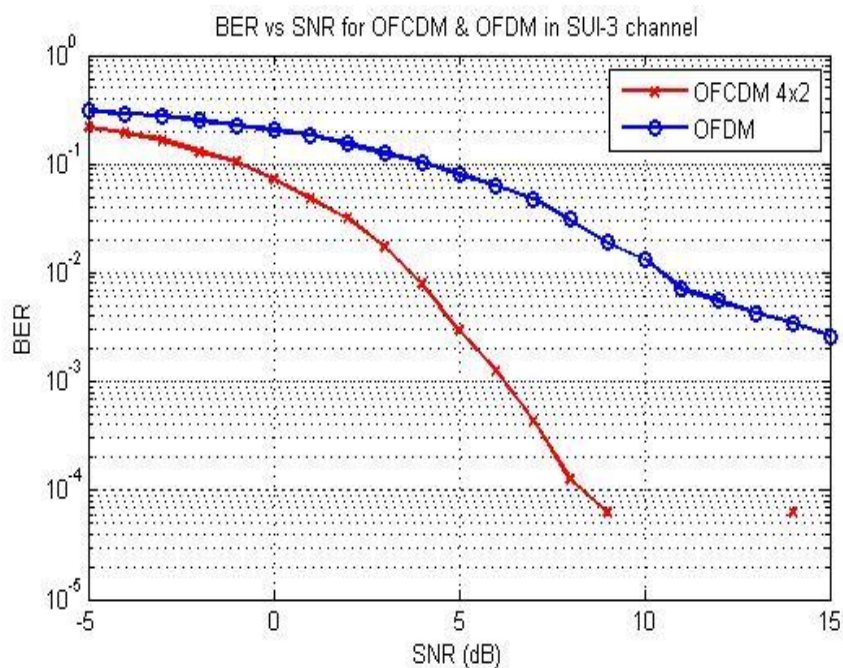


Figure 3.8: OFCDM vs. OFDM in SUI-3 channel.



### 3.2.5 SUI-4 channel model

SUI-4 is modeled as Rayleigh fading channel because value of power factor  $K$  is zero. It also belongs to terrain B which has flat area and moderate tree density. Doppler shift value is dropped to 0.2Hz and tap delay values are increased. Table 3.5 shows the parameters of SUI-4 channel model.

$K = 1$ Doppler = 0.4 Hz	Tap1	Tap2	Tap3
Delay ( $\mu\text{sec}$ )	0	0.4	0.9
Power (dB)	0	-5	-10

Table 3.5 Parameters of SUI-4 channel model.

Figure 3.9 shows the BER graph for SUI-4 channel.

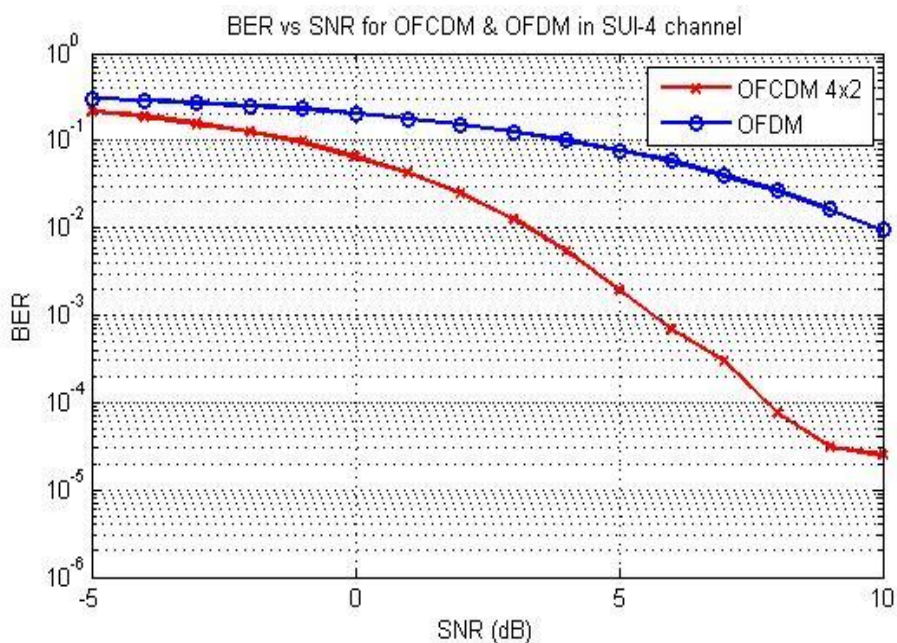


Figure 3.9: OFCDM vs. OFDM in SUI-4 channel.

### 3.2.6 SUI-5 channel model

SUI-5 channel model belongs to terrain A, which is the worst scenario channel having hilly area and moderate to heavy tree density. This channel is also modeled as Rayleigh fading channel and has maximum value of Doppler shift compared to other SUI channels. Table 3.6 shows the SUI-5 parameters.

K = 0 Doppler = 2.0 Hz	Tap1	Tap2	Tap3
Delay ( $\mu\text{sec}$ )	0	4	10
Power (dB)	0	-5	-10

Table 3.6 Parameters of SUI-5 channel model.

Experimental results in Figure 3.10 show that OFCDM has worst bit error rate graph in SUI-5 channel model. There is not much difference in BER of OFCDM and OFDM in this channel mode. This is because of large value of Doppler shift which results in the multi-code interference (MCI) especially in the time domain. MCI destroys the orthogonality of the codes. Thus to get the better results MCI cancellation technique will be required with equalizer.

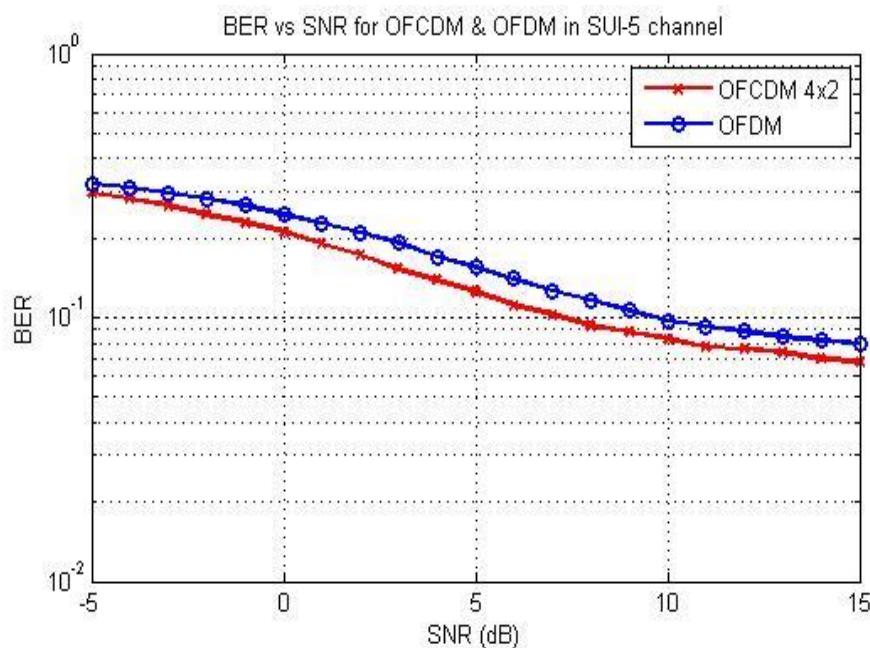


Figure 3.10: OFCDM vs. OFDM in SUI-5 channel.

### 3.2.7 SUI-6 channel model

SUI-6 also belongs to terrain A which is associated with hilly area and moderate to heavy tree density. In this channel model values of tap delays have been increased but compared to SUI-5 Doppler shift is very small. Table 3.7 shows its parameters.

K = 0 Doppler = 0.4 Hz	Tap1	Tap2	Tap3
Delay ( $\mu$ sec)	0	14	20
Power (dB)	0	-10	-14

Table 3.7 Parameters of SUI-6 channel model.

BER result in this channel model is shown in Figure 3.11. It shows that OFCDM has better response than OFDM. Compared to other channel results for OFCDM, it is only better than SUI-5.

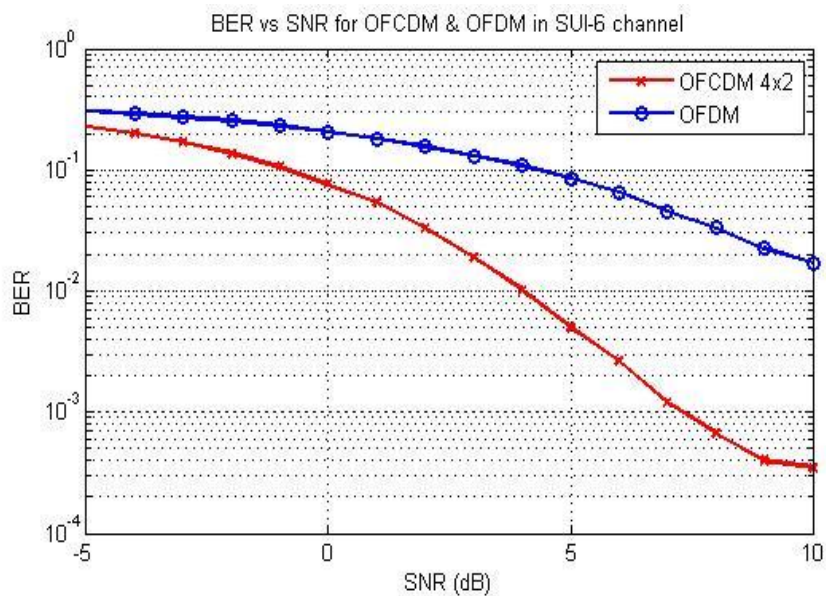


Figure 3.11: OFCDM vs. OFDM in SUI-6 channel.

## 4. Chapter 4 Conclusion and Future work

This chapter will discuss the overall conclusion of the simulation results described in chapter 3. A combined simulation graph will be shown to compare the performance of OFCDM in SUI channels. Further we will discuss the possible future research which can be carried out.

### 4.1 Conclusion

Figure 4.1 shows the combined graph of BER's of OFCDM from all SUI channels. It shows the performance of OFCDM. From this figure we can analyze that OFCDM has best performance in SUI-1 and SUI-2 channels and worst performance in SUI-5 channel. Simulation results show that for broadband fixed wireless applications performance of OFCDM degrades when Doppler shift is increased. This happens due to multi-code interference in time domain spreading code.

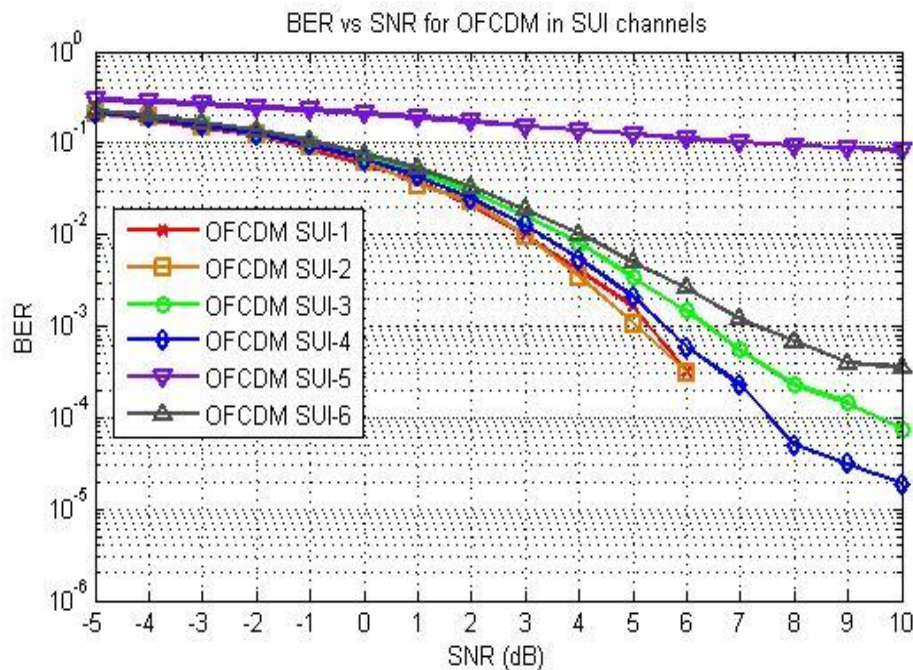


Figure 4.1: OFCDM in six SUI channels.

In the presence of zero forcing equalization the performance of OFCDM is quite promising compared to OFDM. As the graph shows that for SUI-5 channel OFCDM has performed worse, which shows that it needs special equalization and detection at the receiver which should cancel the interference among the spreading code. Main aim of this thesis was to analyze the performance of OFCDM in SUI channels which were designed for fixed wireless applications. Thus OFCDM has proven itself better than OFDM with little complexity at the receiver.

## 4.2 Future work

Orthogonal frequency and code division multiplexing is recommended for future wireless applications especially in the broadband. In the broadband channels its performance is quiet promising in term of BER. For future research and prototype testing it could replace the OFDM which is the physical layer of current wireless standards such as WLAN, WiMAX and LTE. Some future work or possible contributions are discussed as follows:

1. In this thesis simulation model and results were obtained for downlink transmission only. So for further contributions performance analysis of OFCDM can be carried out for uplink transmission only.
2. Performance comparison in six SUI channels can be carried out in future for MIMO transmission of OFCDM.
3. At the receiver side zero forcing equalizer is used for simplicity. In future hybrid detection technique can be used to carry out the simulation results.
4. Large values of Doppler shift with MCI cancellation at the receiver can be used to analyze the performance in worst case scenario.
5. WLAN and WiMAX prototype can be developed based on OFCDM as a physical layer. So real time performance and complexity will be tested based on this prototype system.

## 5. Bibliography

- [1] Yiqing Zhou; Tung-Sang Ng; Jiangzhou Wang; Higuchi, K.; Sawahashi, M., "OFCDM: a promising broadband wireless access technique," *Communications Magazine, IEEE*, vol.46, no.3, pp.38,49, March 2008
- [2] Yiqing Zhou; Jiangzhou Wang; Tung-Sang Ng, "Downlink Transmission of Broadband OFCDM Systems-Part V: Code Assignment," *Wireless Communications, IEEE Transactions on*, vol.7, no.11, pp.4546,4557, November 2008
- [3] Yiqing Zhou; Jiangzhou Wang; Sawahashi, M., "Downlink transmission of broadband OFCDM Systems-part I: hybrid detection," *Communications, IEEE Transactions on* , vol.53, no.4, pp.718,729, April 2005
- [4] N. Yee and J. P. Linnartz, "Wiener filtering of multi-carrier CDMA in Rayleigh fading channel," in *Proc. IEEE PIMRC '94*, vol. 4, Sep. 1994, pp. 1344–1347
- [5] J. P. Linnartz, "Performance analysis of synchronous MC-CDMA in mobile Rayleigh channel with both delay and Doppler spreads," *IEEE Trans. Veh. Technol.*, vol. 50, no. 6, pp. 1375–1387, Nov. 2001.
- [6] N. Yee and J. P. Linnartz, "Multi-carrier CDMA in indoor wireless radio networks," in *Proc. IEEE Int. Symp. Personal, Indoor, and Mobile Radio Commun. (PIMRC'93)*, Sep. 1993, pp. 109–113.
- [7] Achievements in 4G system development, NTT DOCOMO 2007 Attachment 2 "[http://www.nttdocomo.co.jp/english/info/media\\_center/pr/2007/pdf/20070209\\_attachment02.pdf](http://www.nttdocomo.co.jp/english/info/media_center/pr/2007/pdf/20070209_attachment02.pdf)".
- [8] Main technologies of 5Gbps packet transmission experiment, NTT DOCOMO 2007 Attachment1,"[http://www.nttdocomo.co.jp/english/info/media\\_center/pr/2007/pdf/20070209\\_attachment01.pdf](http://www.nttdocomo.co.jp/english/info/media_center/pr/2007/pdf/20070209_attachment01.pdf)"
- [9] Weinstein, S.B., "The history of orthogonal frequency-division multiplexing [History of Communications]," *Communications Magazine, IEEE*, vol.47, no.11, pp.26, 35, November 2009

- [10] Patrick Robertson, Stefan Kaiser, "Analysis of the Loss of Orthogonality through Doppler Spread in OFDM Systems," Global Telecommunications Conference 1999, Rio de Janeiro, Brazil, Vol. IB, pp. 701-706, December 1999
- [11] Charan Langton, "Orthogonal Frequency Division Multiplex (OFDM) Tutorial," Intuitive Guide to Principles of Communications, 2004
- [12] Zhengdong Luo, Junshi Liu, Ming Zhao, Yuanan Liu, and Jinchun Gao, "Double-Orthogonal Coded Space-Time-Frequency Spreading CDMA Scheme," IEEE Journal on selected areas in communications, Vol. 24, No. 6, pp. 1244-1255, June 2006.
- [13] S. Hara and R. Prasad, "Overview of Multicarrier CDMA" IEEE Communications Magazine, Vol. 35, No. 12, pp. 126-133, December 1997.
- [14] A.C. McCormick and E.A. Al-Susa, "Multicarrier CDMA for Future Generation Mobile Communication" IEEE Electronics and Communication Engineering Journal, Vol. 14, No. 2, pp. 52-60, April 2002.
- [15] B. Golkar and F. Danilo-Lemoine, "Space-Time Coding and Spatial Multiplexing in MIMO Multicarrier CDMA," IEEE 18th International Symposium on Personal, Indoor and Mobile Radio Communications, 2007, PIMRC 2007, Athens, Greece, pp. 1-5, 3-7 September 2007.
- [16] Jiangzhou Wang, High-speed Wireless Communications: Ultra-wideband, 3G Long-Term Evolution, and 4G Mobile Systems, Cambridge University Press, 2008.
- [17] Yiqing Zhou; Tung-Sang Ng; Jiangzhou Wang; Higuchi, K.; Sawahashi, M., "OFCDM: a promising broadband wireless access technique," *Communications Magazine, IEEE* , vol.46, no.3, pp.38,49, March 2008
- [18] Yiqing Zhou; Jiangzhou Wang; Tung-Sang Ng, "Downlink Transmission of Broadband OFCDM Systems-Part V: Code Assignment," *Wireless Communications, IEEE Transactions on* , vol.7, no.11, pp.4546,4557, November 2008
- [19] Yiqing Zhou; Jiangzhou Wang; Sawahashi, M., "Downlink transmission of broadband OFCDM Systems-part II: effect of Doppler shift," *Communications, IEEE Transactions on* , vol.54, no.6, pp.1097,1108, June 2006

- [20] Yiqing Zhou; Jiangzhou Wang; Sawahashi, M., "Downlink transmission of broadband OFCDM Systems-part I: hybrid detection," *Communications, IEEE Transactions on* , vol.53, no.4, pp.718,729, April 2005
- [21] J. G. Proakis, *Digital Communications*, Fourth Edition, McGraw Hill, 2000
- [22] Rayleigh distribution,"[http://en.wikipedia.org/wiki/Rayleigh\\_distribution](http://en.wikipedia.org/wiki/Rayleigh_distribution)"
- [23] Rician distribution,"[http://en.wikipedia.org/wiki/Rician\\_distribution](http://en.wikipedia.org/wiki/Rician_distribution)"
- [24] "Channel Models for fixed wireless applications," IEEE 802.16 Broadband wireless access working group, January 2001
- [25] Abdellah, B.; Chouinard, J-Y, "Low complexity linear MMSE equalization, channel decoding and estimation for frequency selective fast fading channels," *Systems, Signal Processing and their Applications (WOSSPA), 2011 7th International Workshop on* , vol., no., pp.79,82, 9-11 May 2011
- [26] Aktan, M.; Dundar, G.; Koca, M., "Low-Power Hardware Efficient MMSE Equalizer Design," *Circuits and Systems for Communications, 2008. ICCSC 2008. 4th IEEE International Conference on* , vol., no., pp.307,311, 26-28 May 2008
- [27] Yi Jiang; Varanasi, M.K.; Jian Li, "Performance Analysis of ZF and MMSE Equalizers for MIMO Systems: An In-Depth Study of the High SNR Regime," *Information Theory, IEEE Transactions on* , vol.57, no.4, pp.2008,2026, April 2011
- [28] Peng Tan; Beaulieu, N.C., "An improved MMSE equalizer for one-dimensional modulation OFDM systems," *Vehicular Technology Conference, 2005. VTC-2005-Fall. 2005 IEEE 62nd* , vol.1, no., pp.157,160, 28-25 Sept., 2005
- [29] Redfern, Arthur J.; Zhou, G.T., "Zero forcing equalization of multiuser time-varying nonlinear systems," *Signals, Systems, and Computers, 1999. Conference Record of the Thirty-Third Asilomar Conference on* , vol.1, no., pp.529,533 vol.1, 24-27 Oct. 1999
- [30] Sato, Y., "Two Extensional Applications of the Zero-Forcing Equalization Method," *Communications, IEEE Transactions on* , vol.23, no.6, pp.684,687, Jun 1975