POST-BUCKLING ANALYSIS OF UNDERWATER VESSELS

By

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In the name of Allah,

The most Beneficent and the

most Merciful

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ABSTRACT

Most of the underwater vessels are designed to avoid buckling. This in turn adds to the weight and hence cost and performance of the vessels. This cost can be reduced if nonlinear and post buckling behavior of the vessels is carried out. In certain cases, the stiffness in the system can be introduced that will avoid large deformations and the resulting plastic strain. One form of this stiffness can come from structures contained by the vessels. In this way the lower buckling mode of the vessel can be filtered out and it will be able to withstand higher buckling pressures.

Extensive work has been done on the postbuckling and buckling behavior of vessels to find out the critical load and the postbuckling strength of the structure. The objective of this research is to study the nonlinear buckling behavior of shell within shell structures .The postbuckling behavior of the outer shell will be studied resulting in the contact between the two cylinders and the critical buckling load of the complete model will be found out. The lower buckling mode will be filtered out by the introduction of nonlinear stiffness from the contained structure. Interaction of the vessel with the contained structure will also be studied.

The system was modeled in ANSYS and linear buckling analyses was carried out first to determine the Eigen buckling modes of the individual cylinders. Complete nonlinear solution using the complete model was obtained. Geometric (large deformation), and element (contact) nonlinearities were included in the analysis. The buckling load of the structure was found out.

As a result of nonlinear buckling analysis, we found out the outer cylinder has a finite postbuckling strength .Because of which it deformed and came in contact with the inner cylinder and the complete system collapsed at the critical load of the structure. The buckling load is higher than the buckling load of the individual cylinders which is an indication that the first buckling mode has been filtered out.

DECLARATION

I hereby declare that I have developed this thesis entirely on the basis of my personal efforts under the sincere guidance of my supervisor Dr.Hasan Aftab Saeed and Dr. Abdul Rauf. All the sources used in this thesis have been cited and the contents of this thesis have not been plagiarized. No portion of the work presented in this thesis has been submitted in support of any application for any other degree of qualification to this or any other university or institute of learning.

Maryam Zareef

Dedicated to my parents, siblings, parent in-laws and my loving husband

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1 CHAPTER 1

1.1 Introduction

Underwater vessels are of vital use in marine systems. They are designed as shell structures. Underwater vessels experience external loads and in resisting these loads, they are subjected to buckling. Buckling is a structural instability that arises when stiffness matrix of a structure becomes singular. This singular matrix then gives rise to buckling loads/pressures and corresponding buckling modes. The number of buckling modes depends on the number of degrees of freedom in the system. Buckling instability usually occurs at loads at which the stress in the structure is far below the yield limit of the material.

Buckling of a shell depends upon geometry, material properties and the load applied. Finite Element Methods are used to carry out analysis on linear and nonlinear buckling behavior of structures and softwares like ANSYS, ABAQUS and NASTRAN etc. are used for implementation.

1.2 Aim of Study

This research is about finding out the critical buckling load of inner and outer shells using ANSYS. The aim is to understand why an Eigenvalue Buckling analysis, a Nonlinear Buckling analysis and post buckling analysis is required to find out about the critical buckling load of the cylinder and post buckled behavior. The objective is to find out how the lower buckling modes are filtered because of the interaction of two shells with each other.

1.3 Methodology

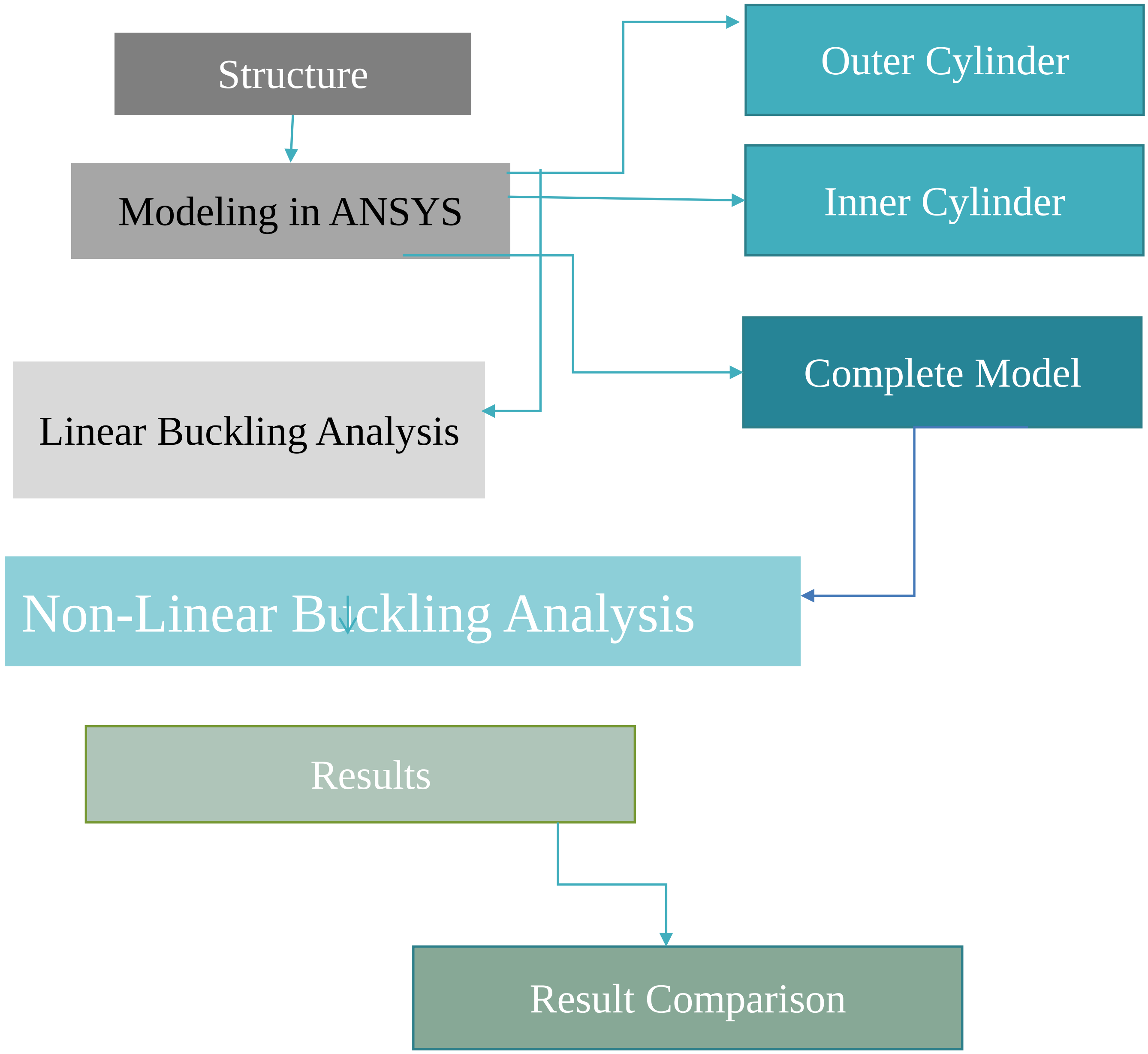


Figure 1: Methodology

1.4 Proposal

First of all, Buckling Analysis will be performed for both outer and inner shells. Critical buckling loads and the respected buckling mode shape will be found out using ANSYS. It will give buckling load value which will be higher as it does incorporate the effects of any type of nonlinearity .These numerical results will be compared with theoretical results.

Non-linear buckling analysis will be done in order to include geometric and contact nonlinearities. For nonlinear buckling analysis, surface to surface contact will be generated between the two shells. Based on the results obtained, post buckling behavior studies will be carried out. Using the stiffness of inner shell, lowest buckling mode will be filtered out .Now the vessel will buckle at a higher buckling load without adding any cost and weight to the system.

1.5 Scope of Study

The objective of this research is to have a better understanding of post-buckling behavior of underwater vessels. The lower buckling modes will be filtered out by the introduction of nonlinear stiffness from the contained structure. Interaction of the vessel with the contained structure will also be studied. The system will be modeled using ANSY and linear buckling analyses will be performed first to determine the buckling modes and respected mode shapes of the system. Complete nonlinear solution will then be obtained to study postbuckling behavior of the outer shell and the buckling load of the composite structure. Geometric (large deformation), element (contact) nonlinearities will be included in the analysis. Stiffness of the system will be such adjusted as to filter out lower buckling modes and no cost to the system.

2 CHAPTER 2

LITERATURE REVIEW

Buckling and post-buckling behavior of cylinder shells have been studied extensively by the researchers using different type of structures, loading conditions, boundary conditions and nonlinearities.

Buckling is a failure that results when compressive stresses exceed a yield criterion. Buckling is related to establishment of load carrying capacity. Therefore buckling, nonlinear buckling and post buckling analysis are importance both computationally and experimentally. Nonlinear buckling analysis leads to a post-buckling analysis. Post buckling analysis helps us to study the behavior of the structure after reaching the critical value. It tells us whether the structure will be able to carry more load or will collapse without taking any more load. If it continues to carry load, it results in another buckling stage.

Because of extensive use of underwater vessels, a lot of theoretical and experimentation investigations have been carried regarding post buckling analysis. Lorenz, Timoshenko and Southwell [1]performed the first theoretical investigation for axially loaded configurations and were based on simplifying assumptions. But there was a huge difference in analytical and experimental values. So simplifying assumptions were reviewed and removed. Further investigations incorporated the effect of pre-buckling deformations, effect of boundary conditions and geometric nonlinearities. It was concluded that geometric imperfections are one of the main reasons which result in discrepancies between experimental and theoretical value of buckling load b. Moreover, it was explained through investigations that critical buckling load tells us about how much load a structure can carry. Now extensive work is being done to carry out post buckling analysis with more accuracy with the usage of advance experimental and analytical techniques.

Material and geometric nonlinearities affect the buckling behavior of a structure, nonlinear and postbuckling behavior of the cylindrical shells and hence the critical value of pressure. Thickness and geometric imperfections effectively reduce the buckling strength as compared to prebuckling as indicated by A.Combescure [2] in his study on nonlinear buckling behavior of cylinders.

Tao Zeng [3] studied how cylindrical shells buckle after reaching critical pressure. He incorporated large deflation in his study along with geometric imperfection. Cylinders under axial compressions were also a part of the study. It was concluded that imperfection parameter greatly influence buckling strength whereas Werner Schneider [4] verified that the buckling capacity of the shells reduces significantly when they experience external pressure with equivalent of geometric imperfections.

The use of composites is increasing in everyday life especially in structures mainly because of their property of being light in weight and still providing more strength. Despite the fact they have high strength doesn't prevent them from the influence of imperfections. In composites, delamination occurs due to imperfections and the result is reduced buckling strength and ultimately the failure as depicted by Tafreshi [5] while working post-buckling behavior of composite cylinders experiencing uniform external pressure. Jian-Feng Ma [6] worked using bamboos as cylindrical shells. The structure of the bamboos is a combination of reinforced fibers and a hollow core. Fibers provide strength and better load carrying ability whereas hollows core contributes to weight reduction. Because of such composition, the mechanical properties varies for fiber core to core .Fibers have a high value of young 'modulus as compared to the core. Because of this, bamboo can carry twice the load as compared to a cylinder as predicted by nonlinear analysis.

Buckling phenomenon is dependent on the geometry of the structure. P.Jasion [7] performed buckling analysis based on effects of geometry .He compared cylindrical shells and barreled shells. From results of his FEM analysis, that barreled shells showed more stability as compared to conventional cylinder when exposed to external pressure concluded that cylindrical shells are less stable as compared to barreled ones and the reason behind it is the meridional radius. Meridional radius is in inverse relationship with the load bearing capacity of the structure. For barreled structure, this value was low in comparison with cylinder .That is why they can carry more load.

Geometric and material imperfections were studied by Lee [8] as a part of his research. The reports gives theoretical and experimental results showing that the buckling strength of the pressure hull decreases with the increase in magnitude of imperfections.

Researchers from University of Portsmouth, UK are extensively working on buckling and postbukling behavior of thick and thin cylinders under different type of loadings. Andrew P.F

Little [9] presented design charts to find buckling pressure. These charts were based on theoretical, experimental and numerical results. Theoretical calculations were done using Von-Mises Formula and Windenburg thinness ratio and FEM analysis in ANSYS provided the numerical solution.

Analytical solution provided a higher value of buckling pressure as it didn’t include any imperfection. But there was a small difference between experimental and theoretical values. The charts can be used for cylindrical shells having initial ovality this ovality should not exceed the value of 0. 16 times the thickness of the shell.

Carl T.F.Ross [10] experimented on thick conical cylinders and concluded they were buckling inelastically. He also worked on buckling of thick circular cylinders assuming to be nearly perfect and experiencing uniform hydrostatic pressure [11].

B.Prabu [12] worked on buckling analysis of thin shells under the influence of external pressure He used different types of geometric nonlinearities in his work. Based on the results from ANSYS, it was established that local imperfection does not decrease the capacity of the shell to carry load but if the magnitude of geometric imperfection is increased, it effects the critical buckling load significantly. These imperfections can by caused by manufacturing processes and effect the load carrying capacity of shells. R.Lo Frano [13] performed experiments on circular tubes using different values of diameter and thickness. Based on results, it was showed that the collapse behavior depends upon the material properties of the structure and if there is any out of roundness present in the shell, it will reduce the buckling strength.

Whereas Hien Luong T.Nguyen [14] determined that the variation in the thickness is a possible cause of load carrying capacity reduction so effect of thickness must be considered while designing the cylinder.

Buckling of functionally graded shells when exposed to external pressure was studied by A.H.Sofiyev [15]. The mechanical properties of these shells are effected by the thickness variation .Change in thickness is due to change in volume fraction of the components. Thus volume fraction is a key factor influencing the buckling load of these structures. Theoretical and numerical calculations also supported the consequence of volume fraction on buckling capacity.

C T F Ross [16] presented the experimental results for conical shells using ring stiffeners and compared the results with numerical and theoretical results including material and geometric nonlinearity.

Stiffeners in cylinders are used to increase the buckling resistance of the shells. SREELATHA P.R [17] studied the incorporation of stiffeners in shells.

Most of the buckling, non-buckling and postbukling analysis include the use of FEM method and softwares like ANSYS, ABAQUS etc. My research involves the filtration of lower buckling modes by the introduction of nonlinear stiffness from the contained structure. The application of this new technique will result in lighter and more efficient structure. So far not much have been done on using the contact stiffness from the contained structure and very little is available from the literatures on the topic. I will be modeling and analyzing my model in ANSYS including geometric and contact nonlinearity and will study the linear buckling, nonlinear buckling and postbukling behavior of outer shell and the interaction of two shells.

3 BUCKLING AND POST BUCKLING

3.1 Buckling

Buckling is mode of failure in which a structure fails suddenly when it experiences compressive loads. When a structure is compressed, it doesn't deform too much until the load reaches its critical value. At this point, the structure goes through a large deformation and may fail to bear any load. Such behavior is known as buckling.

Different load levels can cause different buckling modes but mostly the lowest buckling mode is of interest. Structural loads influence the stiffness of the structure. Positive stress stiffness is induced by tensile load which increases the overall stiffness of the structure as it is added to the elastic stiffness of the structure. Whereas, a compressive load decreases the overall stiffness of the structure as it produces a negative stress stiffness and therefore is subtracted from the elastic stiffness of structure. If this subtraction results in making the value of the overall stiffness of the system then buckling takes place. And when buckling occurs, small increment in load can cause huge deformation and thus instability.

3.2 Types of Buckling Analysis

Buckling analysis is of two types

1. Linear or Eigen Buckling
2. Nonlinear Buckling Analysis

3.2.1 Linear Buckling Analysis

Linear Buckling analysis or "Eigen buckling Analysis" give us the loads and associated mode shapes at which the structure will buckle. The lowest value at which structure will become unstable is the critical load .This analysis does not provide any information regarding post buckled shape.

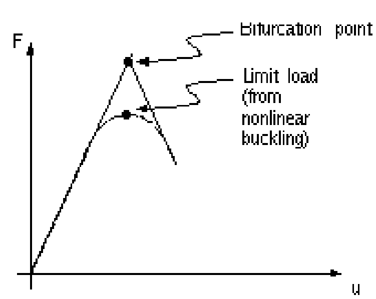


Figure 2: Linear Buckling [18]

Linear buckling analysis finds out the eigenvalues of a structure for given load and boundary conditions. The eigenvalues are obtained by using the following formula [19]

$$([\boldsymbol{k}] + \boldsymbol{\lambda}[\boldsymbol{s}])\{\boldsymbol{\Psi}\} = \mathbf{0} \ldots\ldots\ldots\ldots\ldots\ldots \textbf{(1)}$$

Where

[k] = initial stiffness matrix

[s]= stress matrix

λ = eigenvalue

{Ψ} = Eigenvector

Linear buckling does not include any material, geometric and element nonlinearities which exist in real structures. Thus linear buckling analysis predicts a higher value of buckling load as imperfections and nonlinear effects significantly decrease the buckling load.

For this reason, the results of linear buckling analysis must be cautiously used keeping in mind that the actual buckling load may be lower than that predicted values.

3.2.2 Nonlinear Buckling Analysis

Nonlinear buckling analysis provides information about buckling behavior of real structure. Nonlinear effects and imperfections are included in this analysis. It gives accurate values of buckling loads and is used to study the postbukling behavior of the structures.

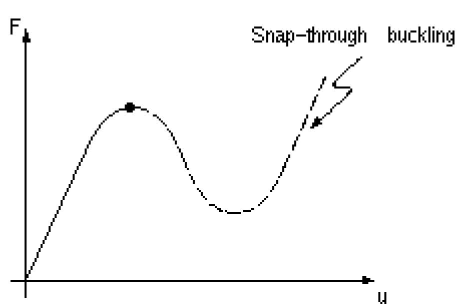


Figure 3: Nonlinear load deflection curve [18]

Nonlinear Buckling analysis is used to analyze the behavior of structures in real world. In this method, load is increased gradually until the structure becomes unstable at a certain load level.

The equilibrium equation given as

$$\{\boldsymbol{F}\} = [\boldsymbol{K}]\{\boldsymbol{U}\}\ldots\ldots\ldots\ldots\ldots\ldots \quad \textbf{(2)}$$

Where

[K] = Global stiffness matrix

{U} = Displacement vector

[F] =Load vector

This equation in terms of stiffness can be rewritten as

$$\{\boldsymbol{U}\} = \{\boldsymbol{F}\}/[\boldsymbol{K}]\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad \textbf{(3)}$$

This method uses an iterative process to find displacement matrix since load and stiffness matrices are unknown. Structure becomes unstable when stiffness approaches zero within this iterative process. In ANSYS, indication of instability is an unconverged solution representing that buckling has occurred and the structure cannot bear any more .The last unconverged solution just before the system diverges gives us the value of buckling load.

3.2.2.1 Structural Nonlinearities

Nonlinear Analysis involves nonlinear stress strain relationships which results in nonlinear load displacement relationship. These nonlinearities can be because of the geometry, large deformations, change in constraints or change of loading conditions. Thus structural nonlinearities can be divided into Geometric, Material and Contact nonlinearity.

3.2.2.1.1 Geometric Non linearity

Geometric nonlinearity can arise because of large strain, small strain, rotations and reduction in stability of structure. If a structure undergoes very large deformations, its changing configuration can cause the structure to behave nonlinearly. An example would be the fishing rod shown in Figure 2.

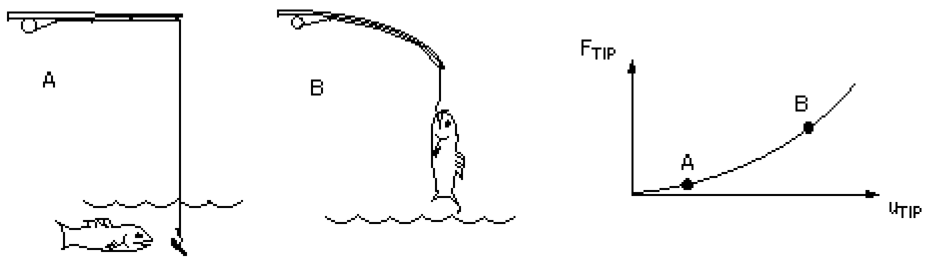


Figure 4: A fishing rod demonstrates geometric nonlinearity [20]

Geometric non-linearity can be classified in two types,

3.2.2.1.1.1 Large Deflection and rotation

When a structure shows a significant change in its original dimensions, orientation and loading direction as compared to its minimum dimension then it is experiencing large deformation so it comes under the category of large deflection ad rotation analysis.

3.2.2.1.1.2 Stress Stiffening

Stress in a particular direction can change stiffness in some other direction, which results in stress stiffening. A structure which shows significant stiffness in tension and no stiffness when compressed shows that it is experiencing stress stiffening.

3.2.2.1.2 Material Non linearity

Material nonlinearity can arise because of time independent or time dependent behaviors. Time independent behavior include plasticity whereas time dependent behavior comprises of creep and viscoelastic / viscoplastic behavior. In our analysis, we are assuming material to be linear and elastic so we won't be discussing material nonlinearity.

3.2.2.1.3 Contact Non linearity

When two elements come in contact, progressive displacement can result in either self or element -to-element contact. This gives rise to contact nonlinearity. In contact nonlinearity, stiffness of the structure significantly decrease when contact is engaged or disengaged.

3.2.3 Buckling Load Safety Factor (BLF)

The buckling load safety factor BLF is actually ratio of buckling load and applied load. It is a number by which the applied load is multiplied to obtain the buckling load magnitude. BLF is given as [21]

$$BLF = \frac{Buckling\ Load}{Applied\ Load} \ldots\ldots\ldots\ldots\ldots \quad (4)$$

If BLF is in between 0 and 1 then it means buckling is predicted.

3.3 Post Buckling

Post buckling stage is an extension of the buckling stage. After the load reaches its critical value, it's a possibility that the load value may not vary or it may start to decrease, while deformation increases. If the structure continues to take more load to continue increasing deformation, it results in a second buckling cycle. Post buckling analysis is non-linear in nature, so it gives much more information about the behavior of the structure after it has reached it critical value.

4 Eigenvalue Buckling Analysis

4.1 Critical Length of the Cylinder

The complete model is composed of inner and outer shell with a surface to surface contact between them. Before starting the analysis, it is essential to find out the critical length of the outer cylinder to determine whether it lies in the category of short, intermediate or long cylinder as given in [22].

For short cylinder

$$\frac{L}{R_o} < 1.72\sqrt{\frac{t}{R_o}}$$

For intermediate cylinder

$$L < L_o$$

For long cylinder

$$L > L_c$$

Critical length of the cylinder is given as

$$\boldsymbol{L_c = 1.1 D_0 \sqrt{\frac{D_0}{t}}} \quad \textbf{(5)}$$

Where

D_O = Outer diameter of the cylinder = 100mm

t = Wall thickness of the cylinder = 2mm

So

$$L_c = 1.1 * 0.1 * \sqrt{\frac{0.1}{0.002}}$$

$$L_c = 778\ mm$$

Modelling the outer shell with the effective length of 800 mm which is greater than critical length so this makes it a long cylinder.

4.2 Material Properties:

The model consists of inner and outer cylinder .Both shells were modelled with mild steel having following material properties

Young's Modulus, E = 200e9 Pa

Poisson's Ratio, $\nu = 0.3$

Yield strength, Y = 250 MPa

Tensile strength = 400MPa

4.3 FE Model of Outer Shell

4.3.1 Geometric Dimensions

Outer Diameter, D_o = 100 mm

Effective Length of Cylinder, L = 800 mm

Thickness, t = 2 mm

4.3.2 Element Type

FEA model of outer shell was modelled in the global Cartesian coordinate system using Shell 181. Shell181 is used for thin to comparatively-thick shells. This element has four nodes. Each node allows translations in x, y, and z directions, and rotations about the x, y, and z axes. Mapped meshing with quadrilaterals was used.

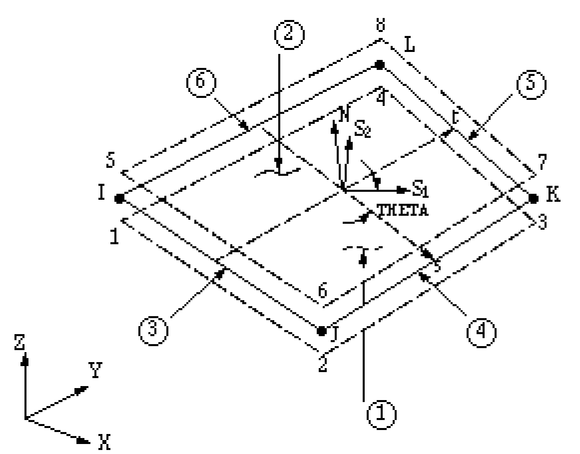


Figure 5: Shell 181 Element [23]

4.3.3 Boundary Conditions

There are different types of boundary conditions like Clamped, simply supported and fixed as given in [24].Boundary conditions effect the solution of short and medium length cylinders but for long cylinders, boundary conditions doesn't influence the solution.

Boundary conditions being used are known as "Classical Boundary Conditions" in which the cylinder is simply supported at edges in such a way that translations in x, y and z direction were zero on left end ($U_x = U_y = U_z = 0$) and on right end, translations in x, and y direction were zero ($U_x = U_y = 0$).Also, these constraints best represent the effects of end caps. After boundary conditions, hydrostatic pressure having magnitude of 20MPa was applied on the outer cylinder.

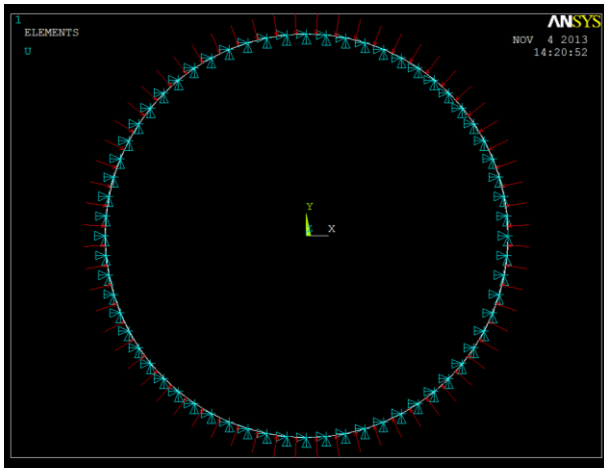


Figure 6 : Front View of the Constrained Ends

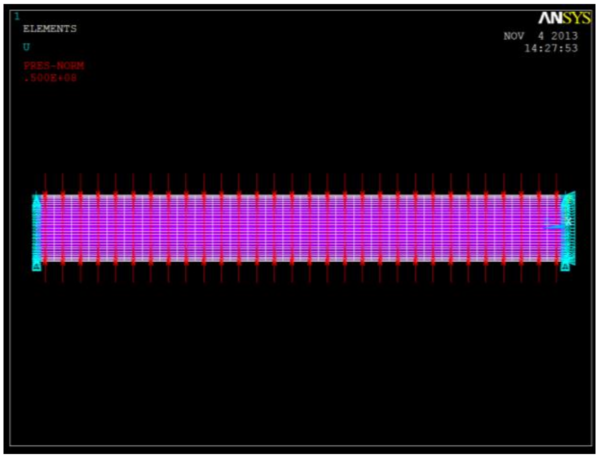


Figure 7: Boundary Conditions (Ux = Uy = Uz = 0 on right end & Ux = Uy = Uz = 0 on left end)

Figure 8: Isometric View of Outer Cylinder with Boundary Conditions and applied Load

4.3.4 Eigen Buckling Analysis

A linear static analysis followed by an Eigenvalue Buckling analysis like [25] was then performed and mesh was refined several times until the error between the computational and theoretical solution was minimum of 0.55%.The no of element divisions on circumference was 6 and 30 longitudinally with 1902 total no of elements. This gave the first two lowest numerical values of buckling modes and respected mode shapes.

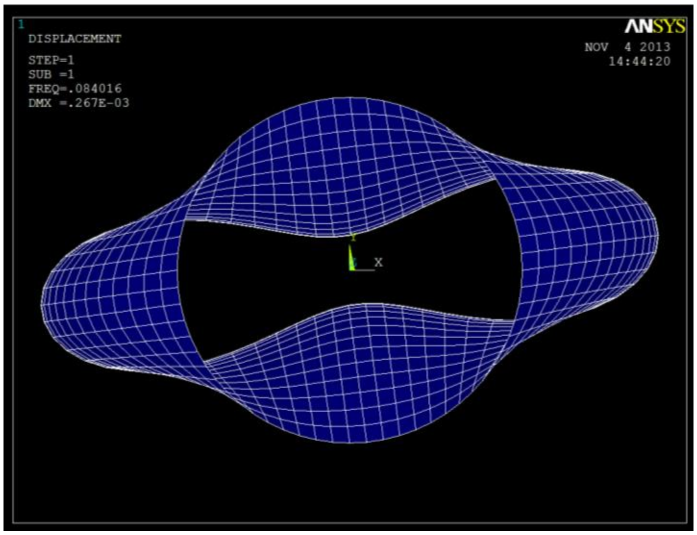


Figure 9 : Mode Shape 1

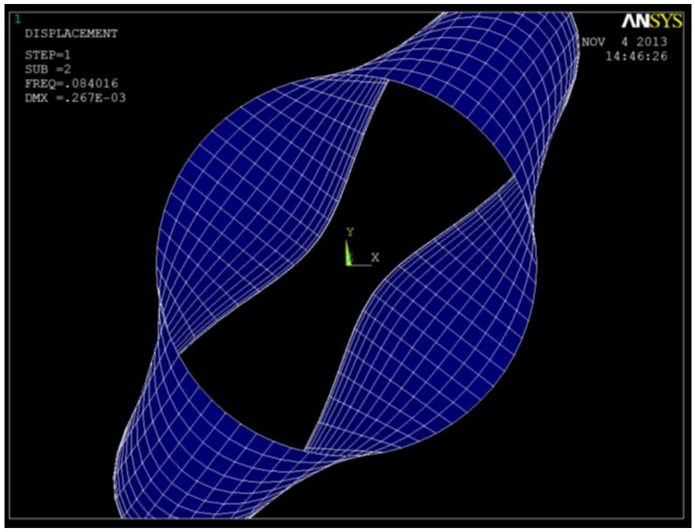


Figure 10 : Mode Shape 2

4.3.5 Comparison Between theoretical and Analytical Buckling Pressure for Outer Shell

Von Mises gives the theoretical solution for a thin, long and simply supported cylinder under hydrostatic pressure as given in [26] as

$$\boldsymbol{P_{cr}} = \frac{E\left(\frac{t}{a}\right)}{\left[n^2-1+0.5\left(\frac{\pi a}{l}\right)^2\right]} \times \left\{\frac{1}{\left[n^2\left(\frac{l}{\pi a}\right)^2+1\right]^2} + \frac{t^2}{12a^2(1-\nu^2)}\left[n^2-1+\left(\frac{\pi a}{l}\right)^2\right]^2\right\} \quad \text{........................ (6)}$$

Where

P_{cr} = Buckling Pressure

A = Mean radius of circular shell

E = Modulus of Elasticity

ν = Poisson's ratio

l = effective length of the cylinder

n = No. of circumferential lobes

Using the values of geometric dimensions and material properties and using 2 circumferential lobes in equation, the minimum buckling pressure comes out to be 4.177 MPa. From ANSYS, we got the value of critical buckling pressure as 4.200 MPa. Thus the difference between the theoretical and analytical value of buckling 0.55%.

4.4 FE Model of Inner Shell

After analyzing and calculating the buckling load for outer shell, Inner shell was modelled and analyzed.

4.4.1 Geometric Dimensions for Inner Shell

Inner shell is a cylinder with a flange at each end. The dimensions are

Outer Diameter, D_o =92 mm

Total Length of Cylinder, L = 800 mm

Cylinder Thickness, t = 2 mm

Flange Thickness= 25mm

Using the dimensions, found out the critical length of the inner cylinder using equation (5) to be 686.4 mm. As $L > L_c$ so this also lies in the category of long cylinder.

4.4.2 Element Type

The main section of the cylinder was modelled using SHELL 181 just like the outer shell. The flanges were mapped meshed with quadrilaterals using 6 divisions circumferentially and 30 longitudinally.

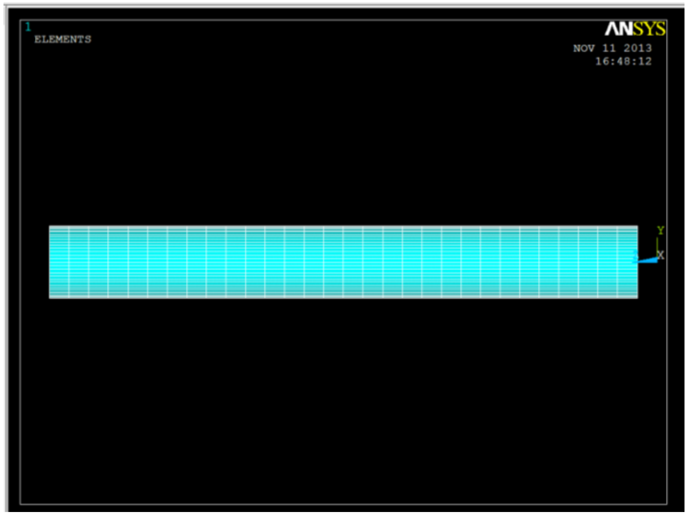


Figure 11: Main Section of inner cylinder

Flanges were modelled using SOLID 185. SOLID185 is used for 3D solid modeling of structures. This element has eight nodes .Each node allows translations in the x, y, and z directions. It has the capability to incorporate effects of large deflection and strain, stress stiffening, plasticity.

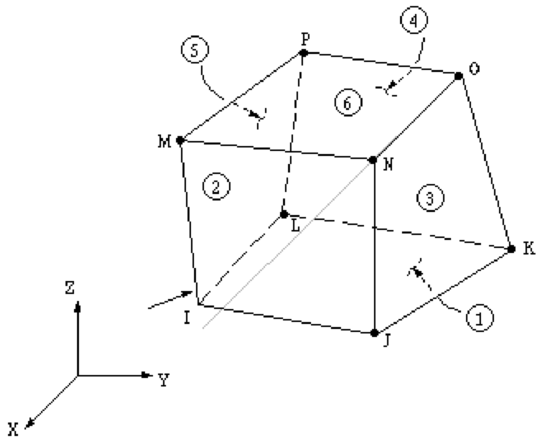


Figure 12: SOLID185 3-D Structural Solid [27]

Flanges were meshed using hexahedral. The no of element divisions were 16 circumferentially, 4 longitudinally and 3 through thickness. The total no of elements for flanges were1536.

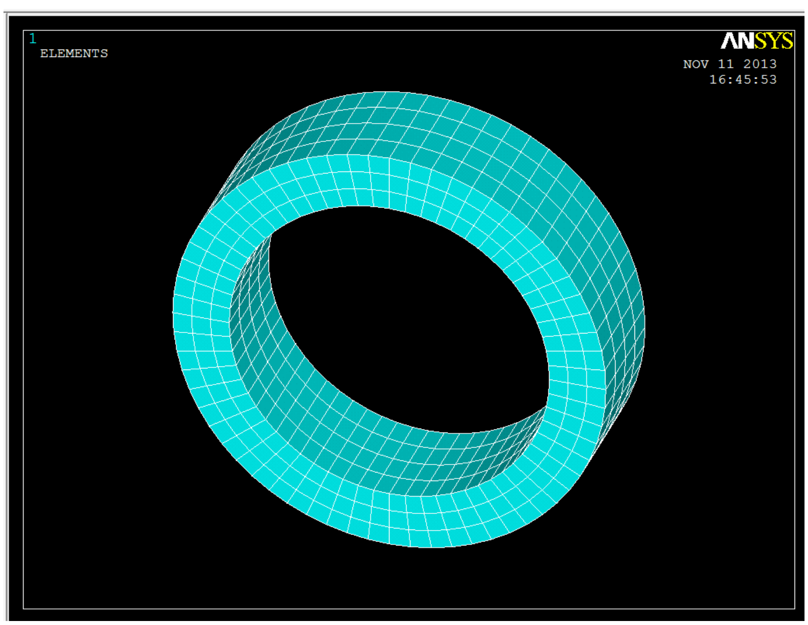


Figure 13: Hex meshing of flange

4.4.3 Boundary Conditions and Load

Boundary conditions were applied after the model was complete. The ends were simply supported just like the outer cylinders. A uniform pressure of 20MPa was applied as hydrostatic pressure.

4.4.4 Eigen Buckling Analysis

An Eigenvalue Buckling analysis was then performed and mesh was refined several times until the error between the computational and theoretical solution was minimum. This gave the first two lowest numerical values of buckling modes and respected mode shapes. The critical buckling pressure was 5.25MPa.

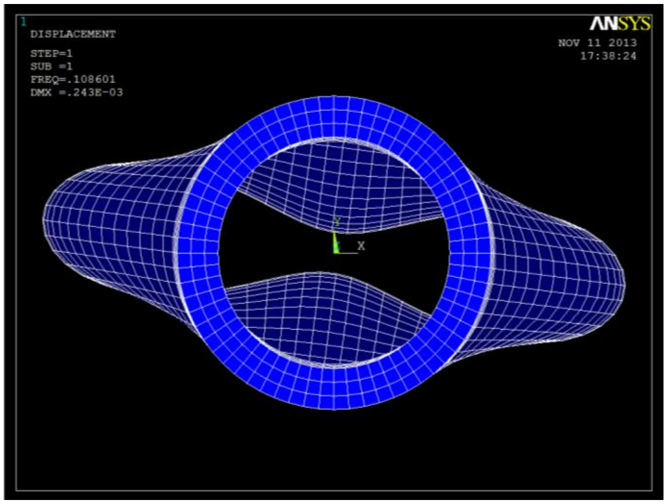


Figure 14: Mode Shape 1

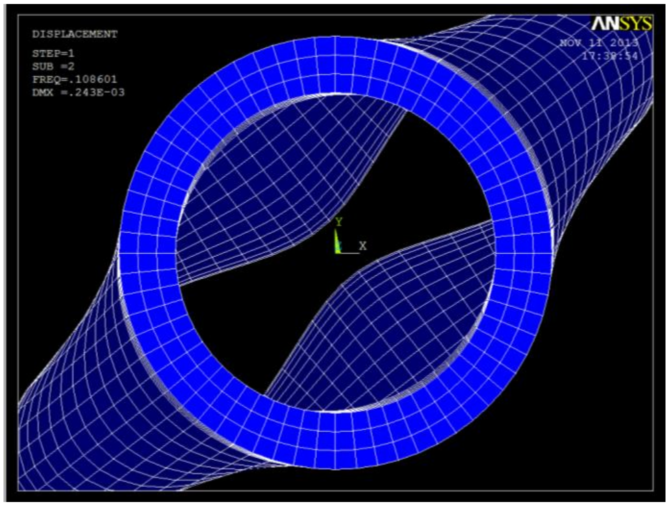


Figure 15: Mode Shape 2

4.4.5 Comparison Between theoretical and Analytical Buckling Pressure for Outer Shell

Using equation 6, the value of theoretical buckling load for the inner shell was calculated using the geometric dimensions and material properties. This value came out to be 5.07MPa.The difference between the theoretical and computational value was 3.55%.

4.4.6 Conclusion from Linear Buckling Analysis

Linear buckling analysis was performed on inner as well as outer cylinders. Based on the analysis, it was concluded that the analytical results were in accordance with the theoretical results and small differences as expected were observed. The lowest critical load of the outer shell came out to be 4.2 MPa and for inner shell, it was 5.25MPa.These values will help us in determining whether the complete model will fail above these values as we expect or otherwise. The linear buckling results of outer shell will be used to generate a geometric imperfection in outer cylinder of the complete shell.

5 Nonlinear Buckling Analysis

After the individual analysis of both the shells was done, it was time to model and analyze the complete structure. In complete model, outer shell and inner shell were combined in such a way that contact was established between them.

5.1 Contact between Outer and Inner Shell

Our aim is to achieve a higher buckling load as compared to individual cylinders by engaging he contact between two cylinders. In doing so, the outer cylinder will not only provide protection to the inner cylinder but also will enhance the buckling strength of the overall system by coming in contact with the inner cylinder. The overall stiffness of the system will be higher as a result of which the lower buckling mode will be filtered out.

For the complete model, we designed a symmetric frictionless surface to surface contact between the two cylinders. In symmetric contact, each surface acts as a target as well as contact surface. Symmetric contacts helps in convergence.

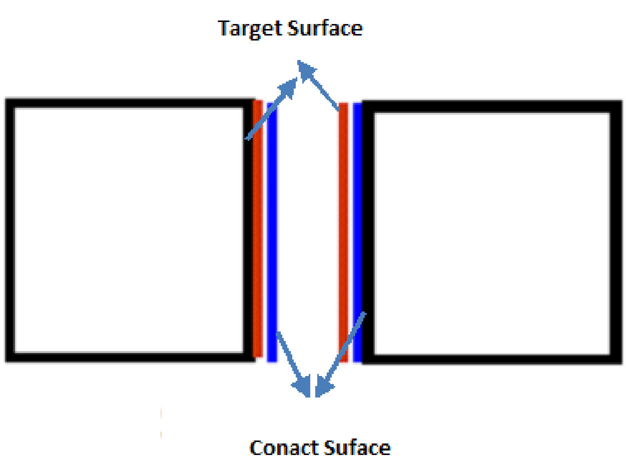


Figure 16: Symmetric Contact

Both the cylinders were considered as flexible so a flexible-flexible contact was established. The elements used for target and contact were Target 170 and Contact 174 respectively. TARGET 170 is used for representing 3-D target surfaces. This target surface is paired with a contact surface using shared real constant set. Any translational or rotational displacement can be imposed on the target segment. There are different segment types for this element, 4-node quad were used in our model.

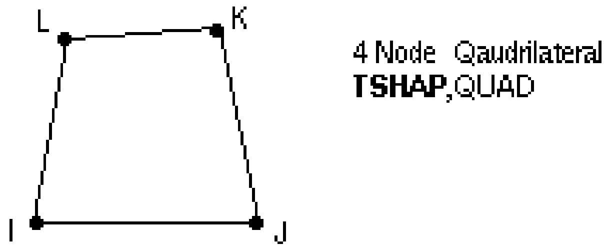


Figure 17: Target170 Segment type [28]

This target surface was paired with Contact 174. CONTA174 represented contact and sliding between 3-D target surface and a deformable contact surface, defined by it. It has three degrees of freedom at each node. It allows translations in the nodal x, y, and z directions. It is located on the surfaces of 3D shell elements. Midside nodes were dropped to match the geometry of the underlying shell elements.

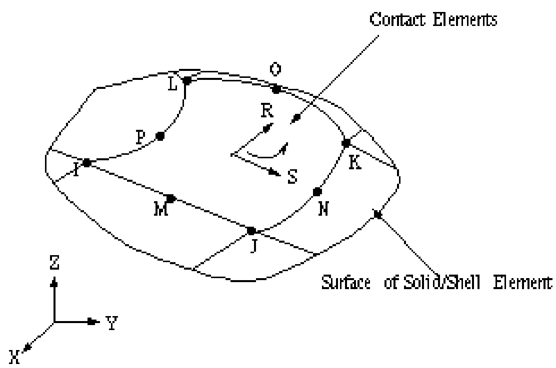


Figure 18: Contact 174 [29]

5.1.1 Contact Status

Contact status was checked to verify that the contact was established. The contact status is given as:

Deformable-deformable contact pair identified by real constant set 4 and contact element type 6 has been set up.

Contact algorithm: Augmented Lagrange method

Contact detection at: Gauss integration point

Default contact stiffness factor FKN: 1.0000

The resulting contact stiffness: 0.56177E+14

Default penetration tolerance factor FTOLN: 0.10000

The resulting penetration tolerance: 0.71203E-03

Average contact surface length: 0.15329E-01

Average contact pair depth: 0.71203E-02

Pinball region factor PINB: 1.0000

The resulting pinball region : 0.71203E-02

All selected contact pairs are initially open. Rigid body motion can occur.

For surface-to-surface contact elements, augmented Lagrangian method was selected. This method is an iterative series of penalty updates to find the exact Lagrange multipliers. The augmented Lagrangian method is preferred to use as it offers better conditioning and is less sensitive to the magnitude of the contact stiffness coefficient.

5.1.2 Established Contact

The complete model was created in ANSYS using the dimensions of inner and outer shells and symmetric contact between them .There is a gap of 2mm between the shells. The complete model is shown below

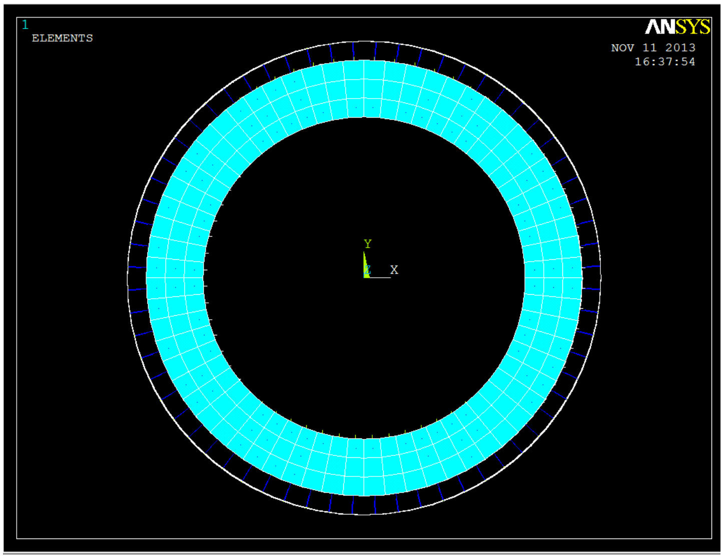


Figure 19: Contact between outer and inner shell

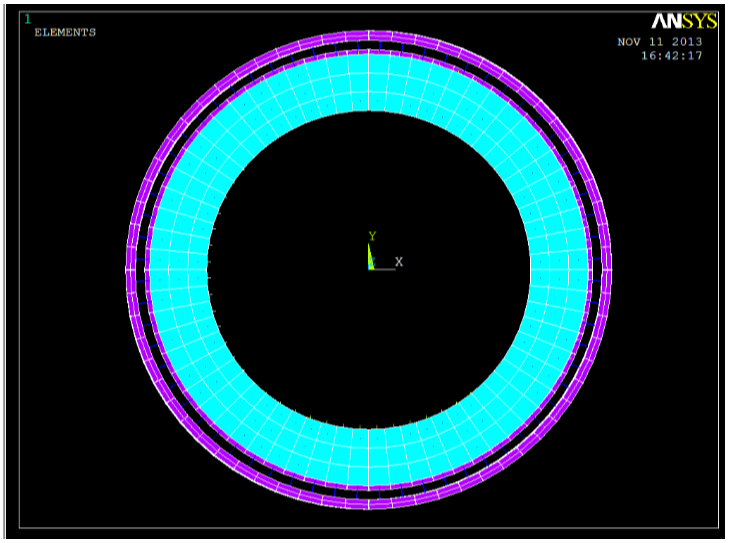


Figure 20: Contact between shells with thickness

5.2 Boundary Conditions and Hydrostatic Pressure

After the contact was established, constraints were applied. Outer shell was constrained exactly the same way as it was during the linear buckling analysis. But in complete model, only the inner side of flanges were simply supported. These boundary conditions also represent end caps so end caps were designed physically. Hydrostatic pressure having magnitude of 20MPa was applied to the outer shell only. This pressure will cause the outer shell to deform and then come in contact with inner shell. Their interaction will result in overall increase in the stiffness of the geometry and as a result the model will buckle at higher buckling load as compared to the critical buckling load of any of the shells.

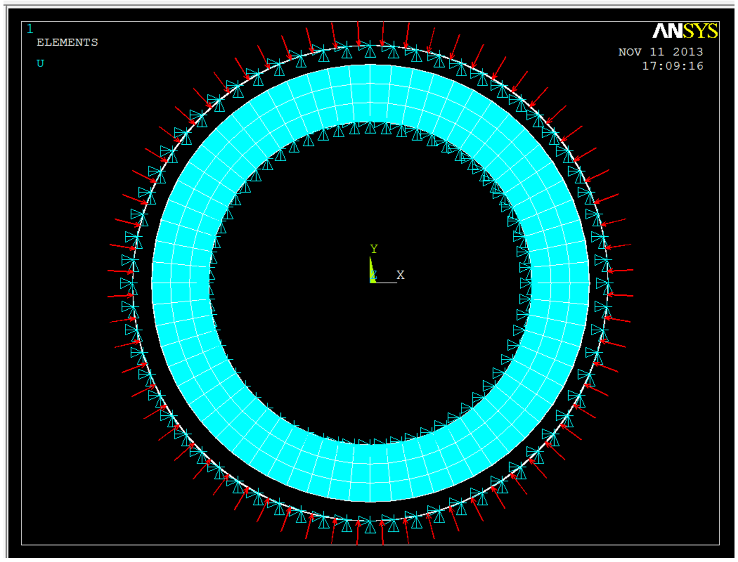


Figure 21: Boundary Conditions and Hydrostatic pressure on complete model (Front view)

5.1 Introduction of Geometric Imperfection (Out of Circularity)

Linear analysis doesn't include any imperfections so it solutions is actually for an ideal structure but real structures have imperfections. As discussed earlier, there can be different type of imperfections. Contact nonlinearity was already introduced as the result of establishment of contact between the two cylinders. We are assuming our model to be linearly elastic so we are not introducing any material nonlinearity. Geometric nonlinearity was introduced in the outer cylinder of the model as ovality. Outer cylinder was chosen from the complete model and linear buckling analysis was performed as earlier.

The nodal displacements produced as a result of linear buckling analysis were used to create geometric imperfection. Scale factor of "1" was used to update the coordinates of the complete nonlinear geometry. This introduced out of circularity in the model. "e" represents the eccentricity in the cylinder.

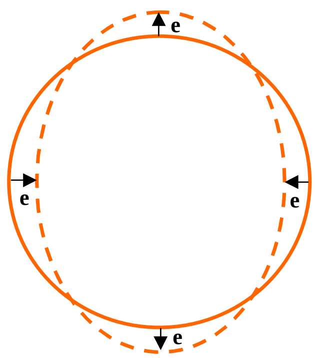
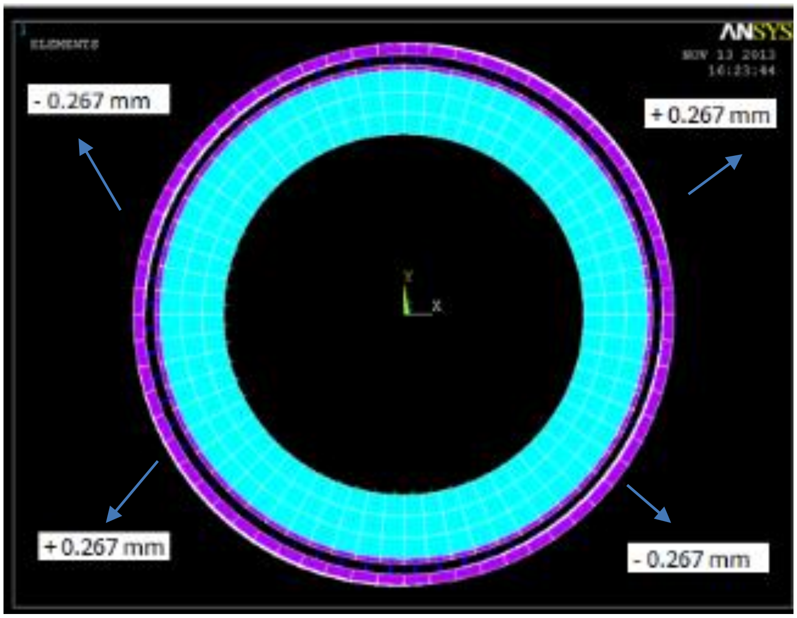


Figure 22: Out of Circularity [8]



Now the model has contact and geometric nonlinearity.

5.2 Nonlinear Buckling Analysis

After the complete geometry was modelled, contact established, boundary conditions and load applied and geometric imperfection was introduced, nonlinear large displacement static analysis was performed [30].Large displacement with prestress was on. Minimum no of sub steps chosen were 20 and maximum no of iterations were 1000.Analysis was performed and the critical buckling load was found out. Based on these results, the critical buckling pressure of composite cylinder.

6 Results

6.1 Buckling Load

The nonlinear buckling analysis provided us with the critical value of the buckling load of the structure. The buckling load came out to be 7.440MPa.This value is greater than the individual buckling load of the cylinders. The deformed geometry is shown below

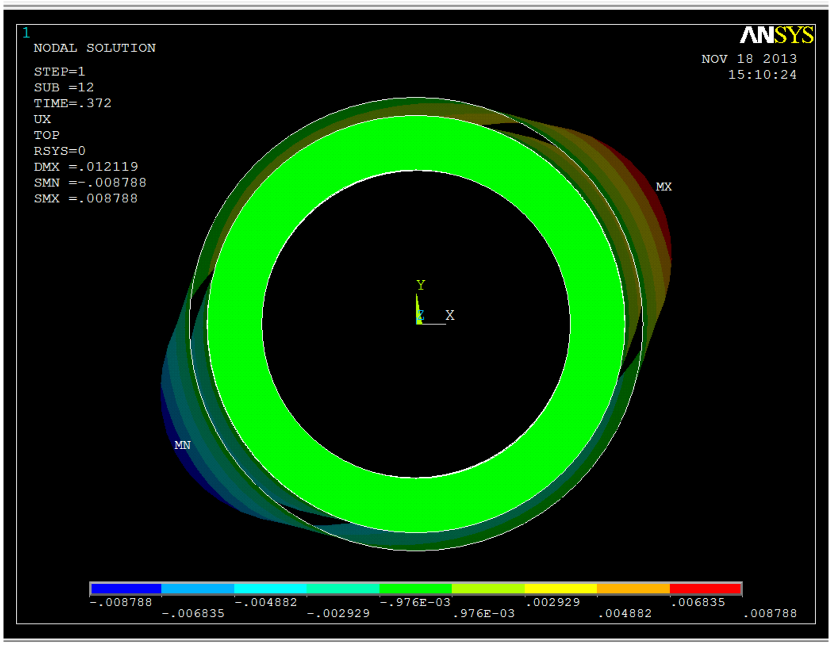


Figure 23: Front View of the deformed structure

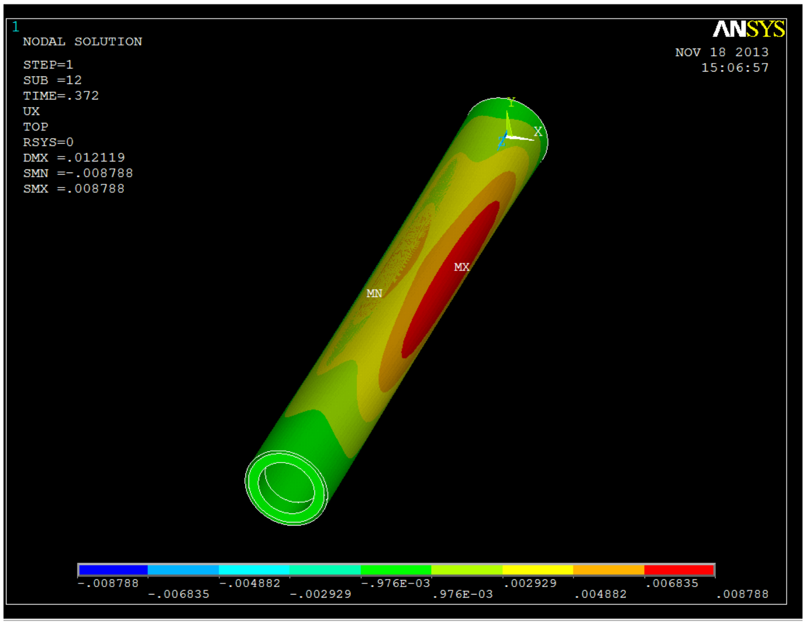
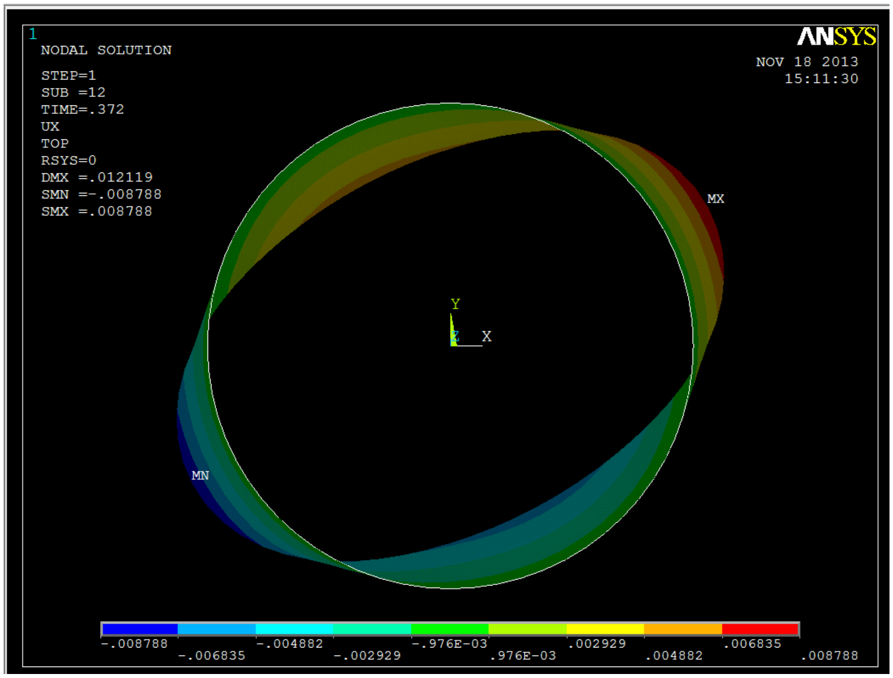


Figure 24: Oblique View of the Deformed Structure

The maximum deformation produced in the structure was 12.119 mm. The contour plots for displacement for inner and outer cylinders were generated to check whether the inner cylinder is deforming when it comes in contact with the outer cylinder. The displacement plot for outer cylinder is shown below



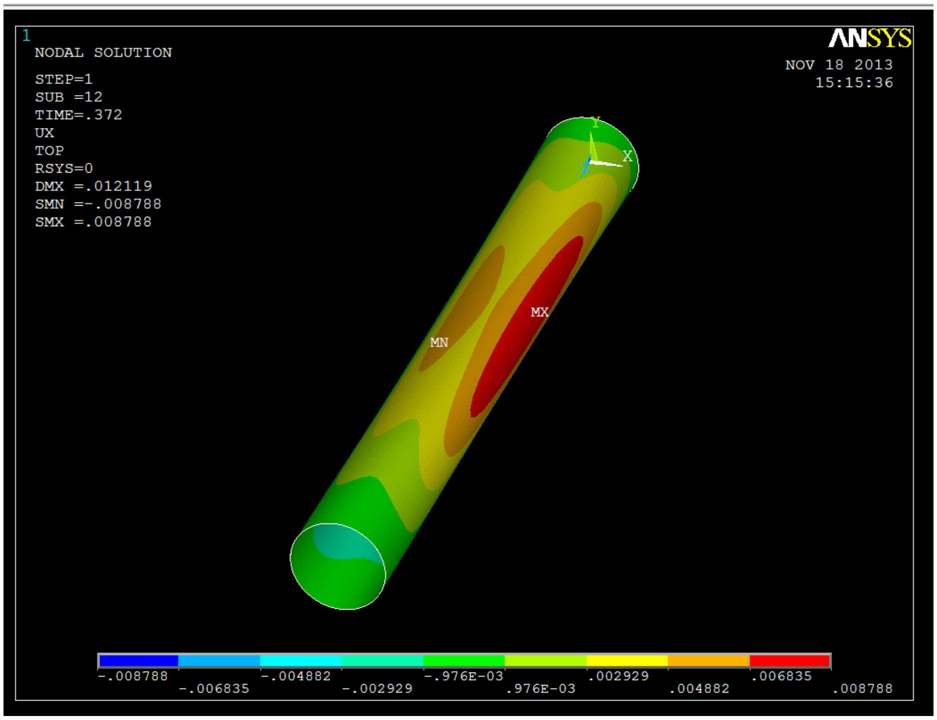
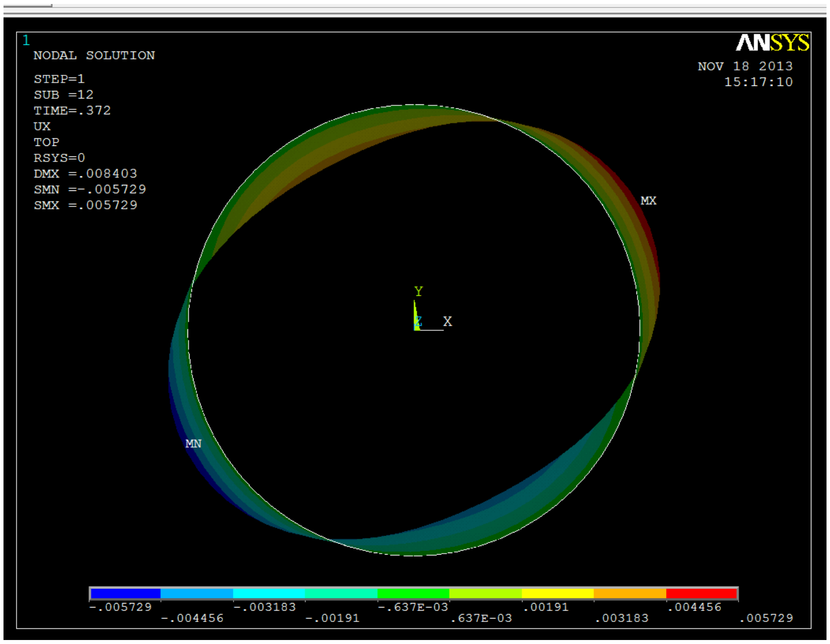


Figure 25 : Deformation plot for outer cylinder



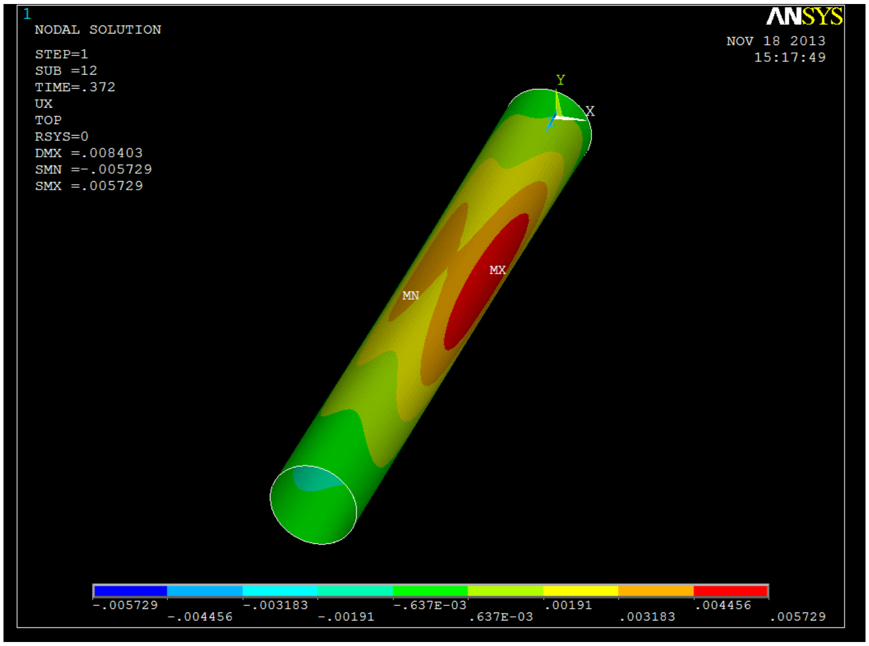


Figure 26: Deformation plot for inner cylinder

Thus the deformation plots show us the maximum value of deformation in outer cylinder to be 12.119 mm whereas maximum deformation in the inner cylinder is 8.788mm.This means that when the load is applied on the outer cylinder, it buckles at its critical value. After reaching the critical value, it continues to deform resulting in a postbuckling path. Thus the outer cylinder has a finite postbuckling strength. It deforms until it comes in contact with the unpressurized inner cylinder closing the gap between them. The inner cylinder acts to support the outer cylinder and resist further collapse. Physically the contact forces counter the external pressure and reduce the compressive hoop stress.

Eventually those same contact forces generate compressive stress in the inner cylinder, and as the loading increases eventually the inner cylinder buckles until the whole structure reaches it critical value. The critical value of the structure was 7.400Mpa.To verify our results, we performed nonlinear buckling analysis again with applied loads of 30MPa up to 50MPa.The critical buckling load and the displacements in inner and outer cylinders were found. Based on these results in table 1, it was concluded the critical values for the structure was from 7.4 to 7.6 MPa and the deflection behavior was similar in all cases. So we can approximate the buckling load of the cylinder by taking the average of all the values. So the buckling pressure of the structure will be 7.486MPa.

Table 1: Critical Loads and Deflection for different values of applied load

Sr. No	Applied Load (MPa)	Critical Buckling Load (MPa)	Maximum Deflection Outer Cylinder (mm)	Maximum Deflection Inner Cylinder (mm)
1	20	7.40-0	12.119	8.788
2	30	7.440	12.121	8.405
3	40	7.603	13.171	9.45
4	50	7.500	12.808	9.36

Buckling behavior of nanotubes under different type of loadings was studied by Shima **[31]**. On analyzing volume change in single walled and doubled walled nanotubes with indices (7,7), (12,12),(7,7)@(12,12), he observed that the collapse pressure of (7,7) swnt was 2.4 GPA and 7 GPa. When the two tubes were combines, they collapsed at a pressure of 10GPa which was higher than the collapse pressure of individual tubes. So it was concluded

"This means that the outer tube acts as a "protection shield" and the inner tube supports the outer one and increases its structural stability; this interpretation is consistent with the optical spectroscopic measurement [147]" [31]

The results provides a good reference to compare my results with but Shima has worked on post buckling behavior of the complete structure whereas my work is more about the postbuckling behavior of outer shell and interaction of two shells.

6.2 Experimental Setup

A simple experiment can be designed to find the collapse pressure of the structure. For experimental setup, two cylinders should be machined to get the desired length of 0.8 m and 0.75 m. First of all, experiment will be performed on individual cylinders and then on the complete structure .For composite structure, a blind flange will be welded with the inner cylinder to close the ends. To maintain the gap between inner and outer cylinder and to seal the outer cylinder, four "O" rings will be inserted between the cylinders at each end using circular flanges. This will result in complete model with proper sealing and closed ends.

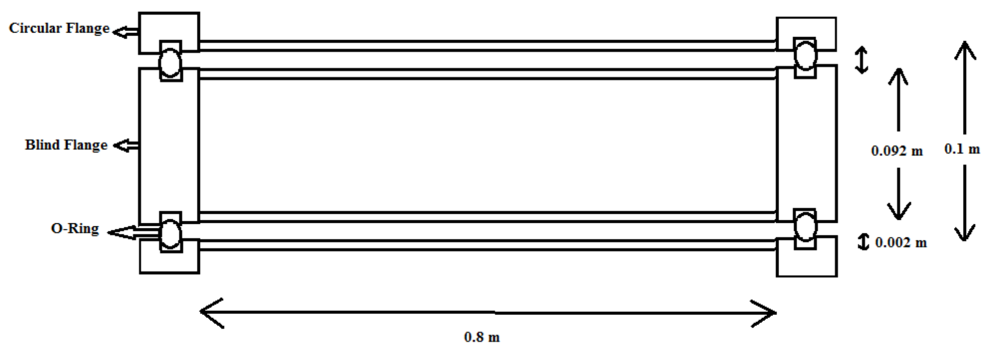


Figure 27: Complete Model for Testing

To perform the experiment, we will be using the same procedure as used by Ross [26], the equipment used was

- High Pressure Tank
- Hydraulic Pump
- Pressure Gauge

6.2.1 Test Procedure:

Consider a high pressure tank which can sustain pressure up to 60 MPa where the individual cylinders and then complete structure will be placed individually. Pressure pump will be used to increase the pressure manually with an increment of 1MPa.The lid at the top of the tank should be screwed tightly. There is a bleed valve attached to the lid of the tank. It should be opened to expel the air trapped inside by gradually pumping the water as shown in Fig 28. After removing the air, the bleed valve should be closed to seal the system. Hydraulic pump will be used to increase the pressure in the tank. Pressure gauge should be monitored carefully to find out collapse pressure. Failure will be accompanied by a banging sound and the sudden fall in pressure. Collapsed cylinders and complete structure can be seen after releasing the pressure and by removing the lid.

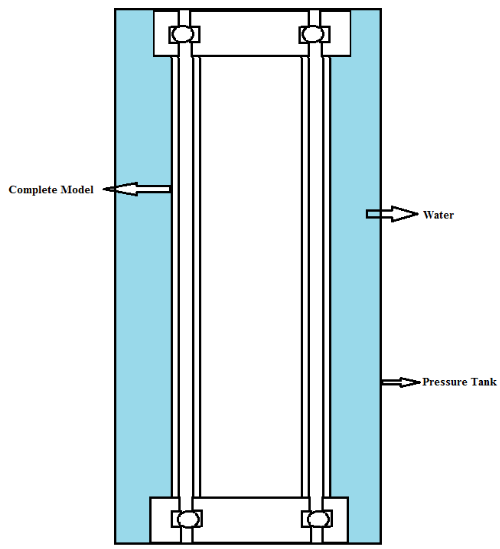


Figure 28: Test Setup

6.3 Effect of Geometric imperfection

Many researchers have worked on geometric imperfections and their effect on buckling strength.it has been concluded that the buckling strength of the system decreases when magnitude of imperfection increases. We have introduced a scale “1” of imperfection in our analysis which induced a maximum deflection of 0.91mm in the outer cylinder of the structure. Lee [8] worked on effects of imperfection in a pressure hull.

OOR (in)	ANSYS (psi)
1	9796
2	8450
3	7950
4	6950

He performed nonlinear analysis in Ansys using imperfection magnitude of 1”, 2”, 3”,4” and conclude ttha evalue of buckling load decreased. In out cas,e we aplied a magintude of 0.546mm,0.637mm,0.728mm,0.819mm,0.91mm and 1.001mm.

Table 2: Geometric Imperfection VS Buckling Load

Sr. No	Applied Load (MPa)	Scale	Critical Buckling Load (MPa)
1	20	0.6	8.660
2	20	0.7	8.020
3	20	0.8	7.603
4	20	0.9	7.500
5	20	1	7.440
6	20	1.1	7.260

From the results, it was concluded that the magnitude of buckling load is decreasing but it is not decreasing as a rapidly as in a single cylinder representing the pressure hull. Thus buckling load is not very sensitive to geometric imperfections.

7 Conclusions and Recommendations

We performed linear buckling analysis and nonlinear buckling analysis. Based on linear and nonlinear buckling analysis we concluded,

- It is important to find out the critical length of the structure to find out whether it is a long, short or intermediate cylinder.
- Boundary condition effects the solution of short and intermediate cylinder but doesn't influence in case of long cylinder.
- Linear buckling analysis over predicts the buckling value. It is mostly higher than the theoretical value as the structure is assumed to be linear and elastic and no imperfection is considered.
- Buckling Load factor found out as a result of Linear buckling analysis gives us an idea about selection of factor of safety to avoid buckling.
- The buckling load for outer cylinder came out to be 4.2MPa and it was 5.25Mpa for inner cylinder.
- Buckling load for inner and outer cylinder was more than the theoretical values.
- Nonlinear analysis is required to find out the actual behavior of the structure and the postbuckling behavior of the outer cylinder.
- Contact and geometric imperfection was introduced to perform nonlinear buckling analysis. The nonlinear analysis for the structure gave us the buckling load value of 7.495MPa which was greater than the individual buckling load of the inner structure.
- We have filtered out the first buckling load of the individual cylinders.
- The interaction of the inner cylinder with the outer cylinder has resulted in higher value of nonlinear stiffness and thus higher buckling load.
- We have studies that postbuckling behavior of the outer shell which shows that it has a postbuckling strength.
- The structure is undergoing very small deflection until critical load.

Based on our analysis, we are recommending future work as follows

- This analysis should be carried out for short and intermediate cylinders where boundary conditions play a significant role.

- Analytical study to verify that nonlinear stiffness of the structure is increased or not for different types of loadings and boundary conditions.
- Material nonlinearities should be incorporated in the analysis to find out any plastic deformation in the system.
- Calculation of plastic strain as a result of avoiding large deformation.
- Analysis of the structure after introducing friction in the system.
- We have filtered out first buckling mode, analytical study to find out how many modes can be filtered out.
- Proper sealing of the system.

Bibliography

[1] N. J.HOFF, "THE PERPLEXING BEHAVIOR OF THIN CIRCULAR," 1966.

[2] G. A.Combescure, "Nonlinear Buckling of Cylinders Under External Pressure with Nonaxisymmetric Thickness Imperfections using the COMI Axisymmetric Shell Element," *International Journal of Solids and Structures,* vol. 38, pp. 6207-6226, 2001.

[3] L. W. Tao Zeng, "Post-buckling Analysis of Stiffened Braided Cylindrical Shells Under Combined External Pressure and Axial Compression," *Composite Structures,* vol. 60, pp. 455-466, 2003.

[4] A. B. Werner Schneider*, "Consistent Equivalent Geometric Imperfections for the Numerical Buckling Strength Verification of Cylindrical Shells Under Uniform External Pressure," *Werner Schneider*, Andreas Brede,* vol. 43, pp. 175-188, 2005.

[5] A. Tafreshi, "Delamination Buckling and Postbuckling in Composite Cylindrical Shells Under External Pressure," *Thin-Walled Structures,* vol. 42, pp. 1379-1404, 2004.

[6] W.-y. C. L. Z. D.-h. Z. Jian-Feng Ma, "Elastic Buckling of Bionic Cylindrical Shells Based on Bamboo," *Journal of Bionic Engineering,* vol. 5, pp. 231-238, 2008.

[7] K. P.Jasion, "Elastic Buckling of Barrelled Shell Under External Pressure," *Thin-Walled Structures,* vol. 45, pp. 393-399, 2007.

[8] H. C. Lee, "Buckling Analysis of a Submarine with Hull Imperfections," Rensselaer Polytechnic Institute, Connecticut, April 2007.

[9] C. T. S. X. Andrew P.F.Little, "Inelastic buckling of geometrically imperfect tubes under external hydrostatic pressure," *Journal of Ocean Technology,* vol. 3, pp. 75-90, 2008.

[10] D. S. T. J. Carl T.F.Ross, "Inelastic Buckling of Thick-Walled Circular Conical Shells Under External Hydrostatic Pressure," *Ocean Engineering,* vol. 26, pp. 1297-1310, 1999.

[11] A. S. G. X. a. A. P. Carl T.F.Ross, "Buckling of Near-Perfect Thick-Walled Circular Cylinders Under External Hydrostatic Pressure," *Journal of Ocean Technology,* vol. 4, pp. 85-108, 2009.

[12] N. B.Prabu, "Finite Element Analysis of Buckling of Thin Cylindrical Shell Subjected to Uniform External Pressure," *Journal of Solid Mechanics,* vol. 1, pp. 148-158, 2009.

[13] G. R.Lo Frano, "Experimental Evidence of Imperfection Influence on the Buckling of Thin Cylindcl Shell Under Uniform External Pressure," *Nuclear Engineering and Design,* vol. 239, pp. 193-200, 2009.

[14] I. T. Hien Luong T.Nguyen, "Buckling Under the External Pressure of Cylindrical Shells with Variable Thickness," *International Journal of Solids and Structures,* vol. 46, pp. 4163-4168, 2009.

[15] N. M. A.H.Sofiyev, "Buckling of FGM Hybrid Truncated Conical Shells Subjected to Hydrostatic Pressure," *Thin-Walled Structures,* vol. 47, pp. 61-72, 2009.

[16] C. K. I. M. A. E. C T F Ross, "Non-linear general instability of ring-stiffened conical shells under," *Journal of Physics: Conference Series 305,* 2011.

[17] A. M. SREELATHA P.R, "Linear and Nonlinear Buckling Analysis of Stiffened Cylindrical Submarine Hull," *International Journal of Engineering Science and Technology (IJEST),* 2012.

[18] [Online]. Available: http://mostreal.sk/html/guide_55/g-str/GSTR7.htm.

[19] R. G. X.L. Zhao, Structural Failure and Plasticity, 2000, p. 872.

[20] [Online]. Available: http://mostreal.sk/html/guide_55/g-str/GSTR8.htm.

[21] [Online]. Available: http://help.solidworks.com/2012/English/SolidWorks/cworks/Buckling_Load_Factor.htm.

[22] D. R. Moss, Presuure Vessel Design Manual, 2004, p. 86.

[23] [Online]. Available: http://ans2.vm.stuba.sk/html/elem_55/chapter4/ES4-181.htm.

[24] J. a. J.M.Rotter, Buckling of thin metal shells, Spon Press, 2004, p. 158.

[25] [Online]. Available: http://www.mece.ualberta.ca/tutorials/ansys/IT/Buckling/Buckling.html.

[26] C. T. S. G. X. Andrew P.F.Little, "Desiging Under Pressure," *Journal of Ocean Technology,* vol. 3, no. 1, p. 78, 2008.

[27] [Online]. Available: http://www.ansys.stuba.sk/html/elem_55/chapter4/ES4-185.htm.

[28] [Online]. Available: http://mostreal.sk/html/elem_55/chapter4/ES4-170.htm.

[29] [Online]. Available: http://www.ansys.stuba.sk/html/elem_55/chapter4/ES4-174.htm.

[30] [Online]. Available: http://www.youtube.com/watch?v=mDPCMEhxJm4.

[31] H. Shima, "Buckling of Carbon Nanotubes: A State of the Art Review," *materials,* vol. 5, 2012.

[32] W. S. Marco Gettel, "Buckling Strength Verification of Cantilevered Cylindrical Shells Subjected to Transverse Load using Eurocode 3," *Journal of Constructional Steel Research,* vol. 63, pp. 1467-1478, 2007.

[33] S. A. Hayder A.Rasheed, BUCKLING AND POSTBUCKLING STRUCTURES, vol. 1, Imperial College Press, 2008, pp. 259-307.

[34] [Online]. Available: http://www.mece.ualberta.ca/tutorials/ansys/IT/Buckling/Buckling.html.

[35] D. C. K. P. K. S. A. T. T. Y. I. H. K. S. V. G. A. K. J. Arvanitidis, "Pressure screening in the interior of primary shells in double-wall carbon," 2004.

Appendix A- Nonlinear Buckling Analysis Results

Table 3: Buckling Load for Structure (Applied Load = 30 MPa)

Set	Time/Freq	Load Step	Substeps
1.	0.10000	1	1
2.	0.14500	1	2
3.	0.19000	1	3
4.	0.21250	1	4
5.	0.23500	1	5
6.	0.24259	1	6
7.	0.24601	1	7
8.	0.24701	1	8
9.	0.24801	1	9

Table 4: Buckling Load for Structure (Applied Load = 40 MPa)

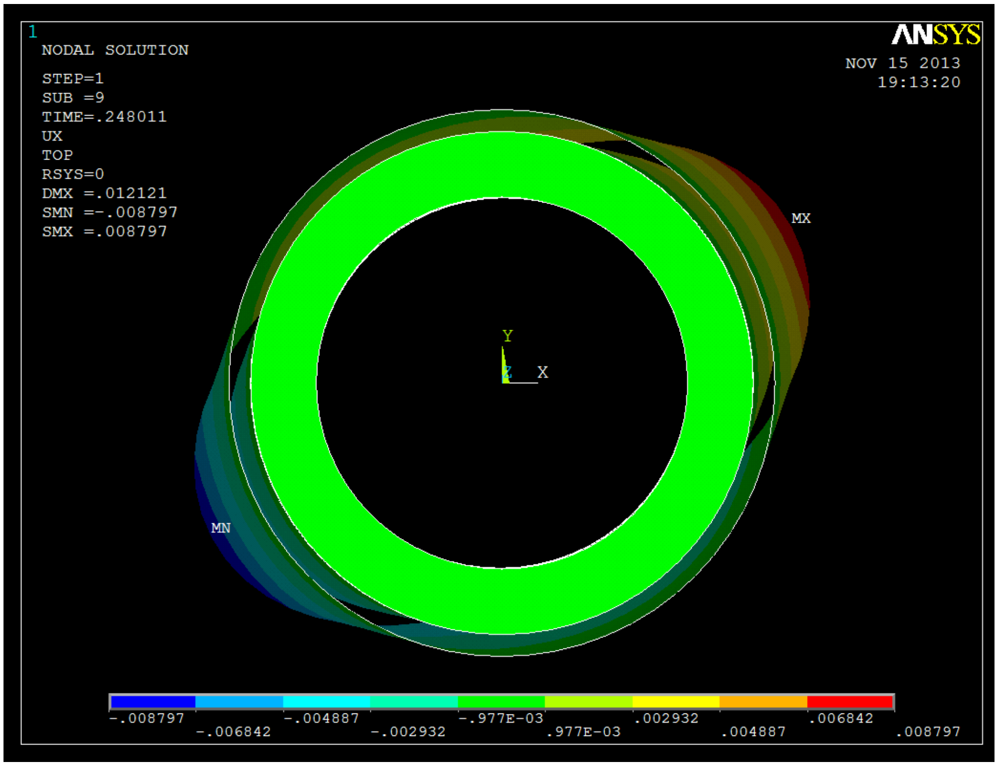
Set	Time/Freq	Load Step	Substeps
1.	0.10000	1	1
2.	0.15000	1	2
3.	0.16250	1	3
4.	0.16813	1	4
5.	0.17375	1	5
6.	0.17797	1	6
7.	0.17897	1	7
8.	0.17997	1	8
9.	0.18097	1	9
10.	0.18197	1	10
11.	0.18297	1	11
12.	0.18447	1	12
13.	0.18672	1	13
14.	0.19009	1	14

Table 5: Buckling Load for Structure (Applied Load = 50 MPa)

Set	Time/Freq	Load Step	Substeps
1.	0.50000E-01	1	1
2.	0.10000	1	2
3.	0.12500	1	3
4.	0.15000	1	4

Highlighted values represent the buckling Load factor. Buckling load is calculated by multiplying this value with applied load.

Appendix B- Deformation Plots for Different Load Values



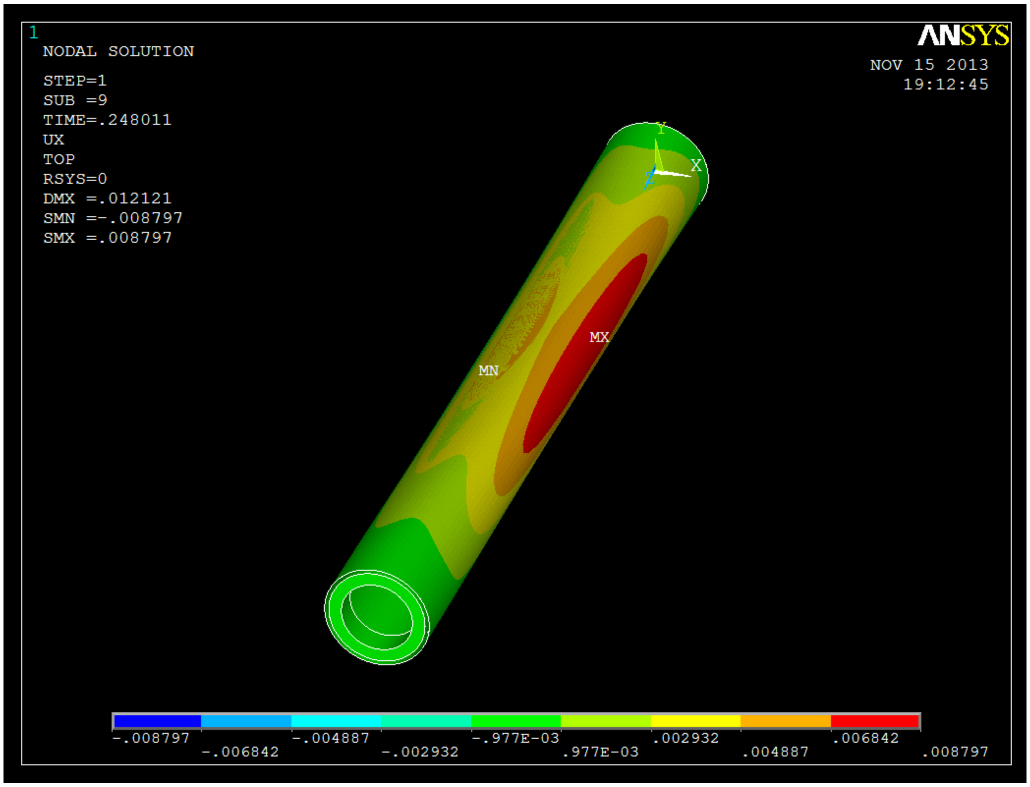
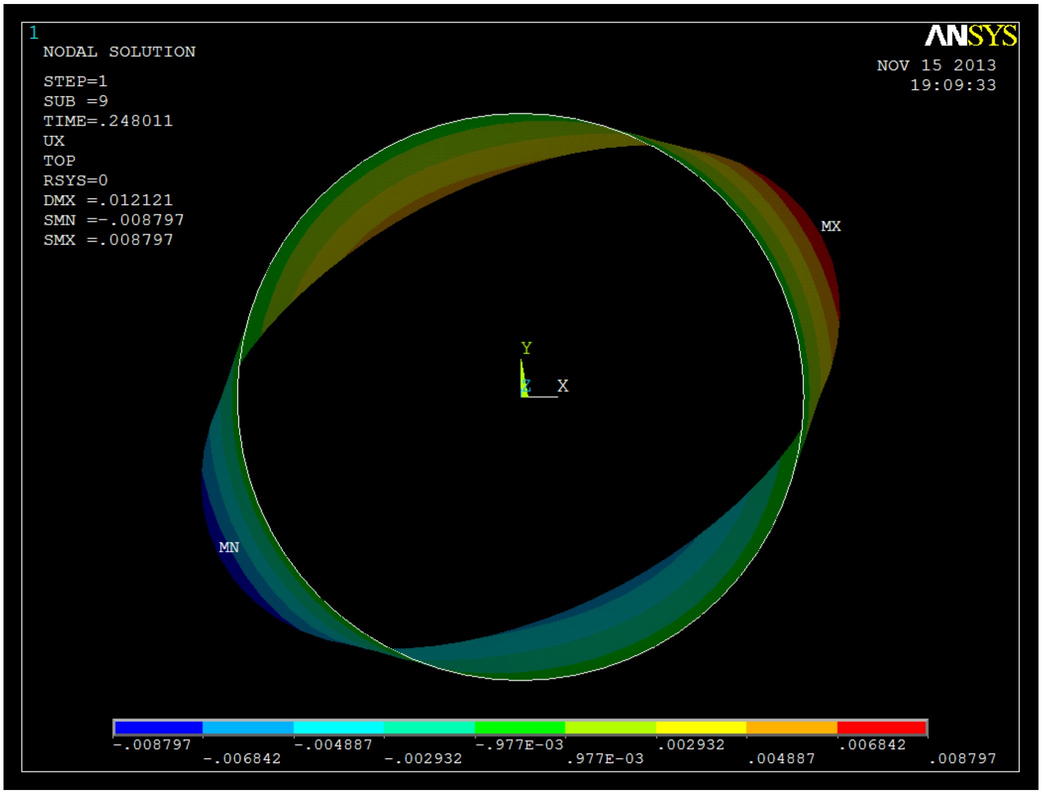


Figure 29: Deformation Plot (30MPa)



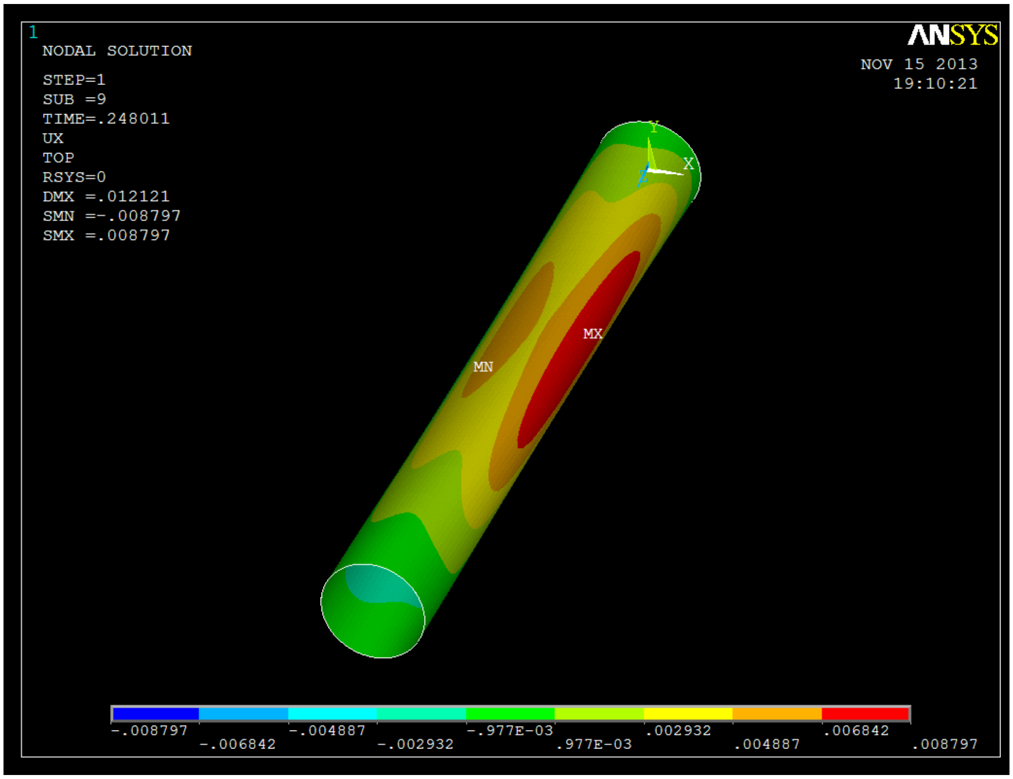
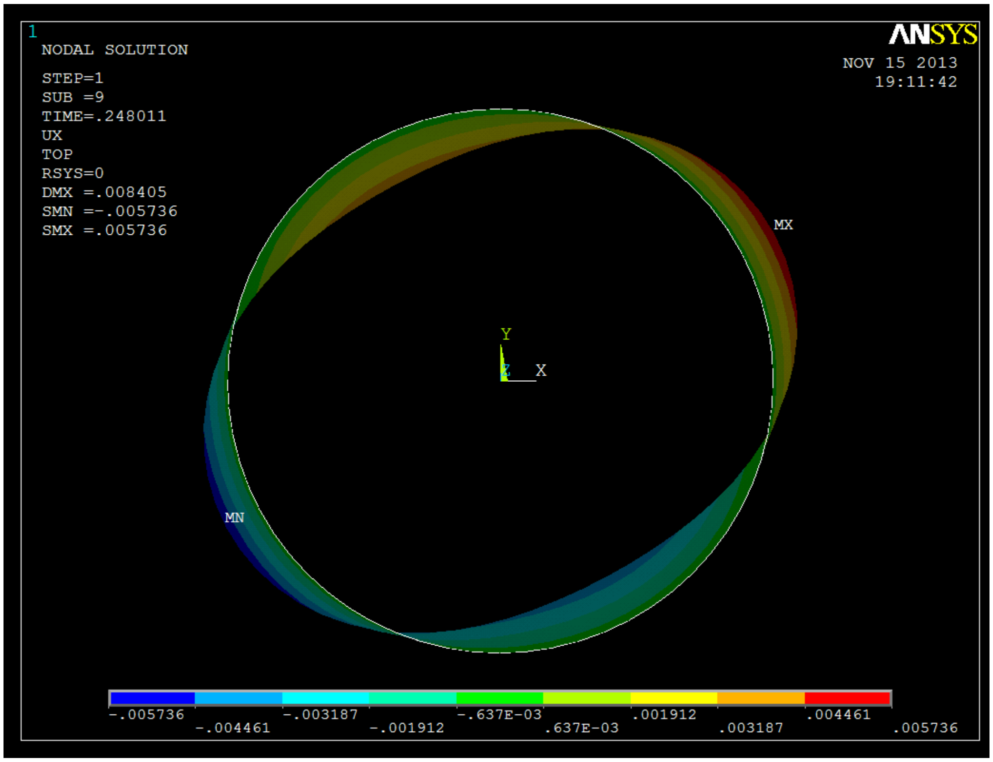


Figure 30: Deformation Plot for outer cylinder (30MPa)



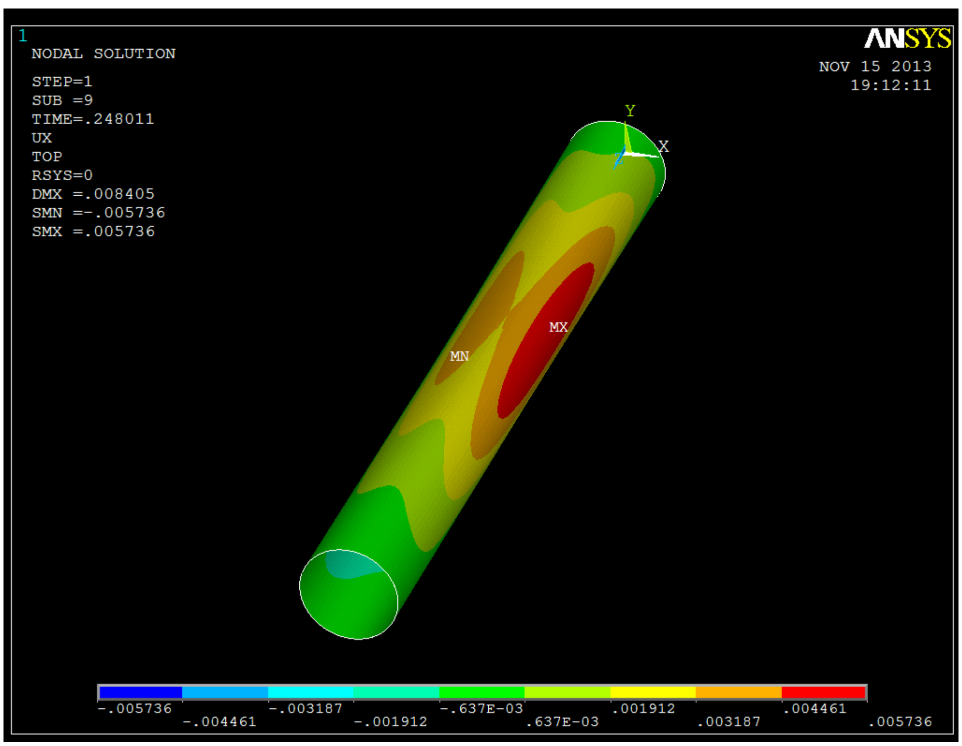
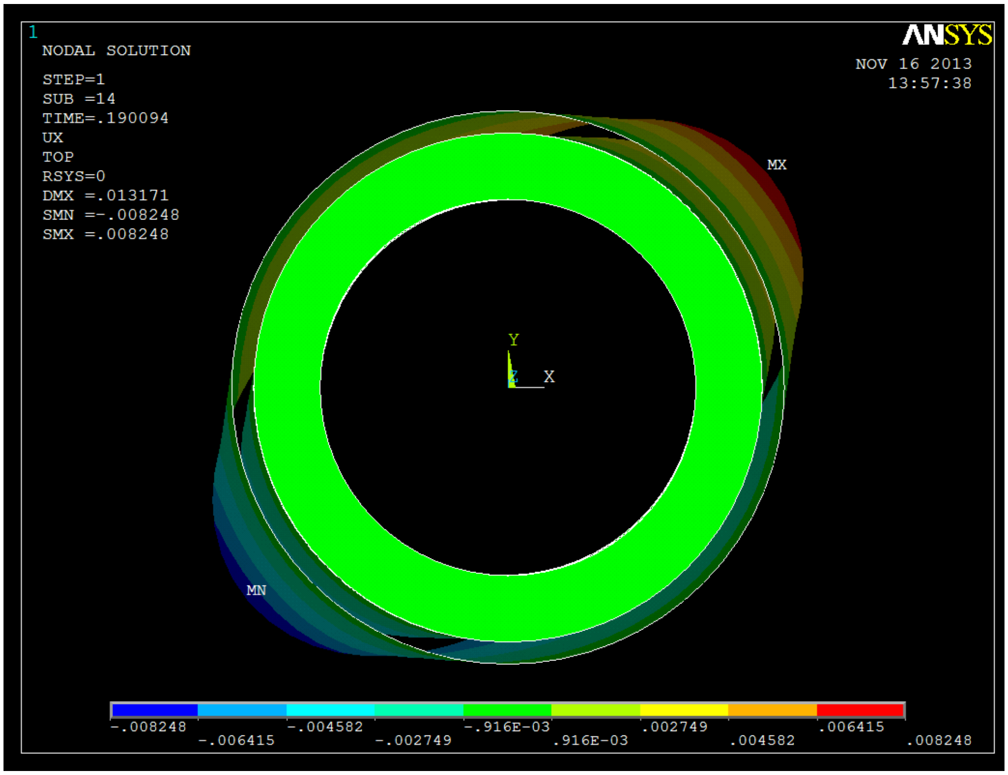


Figure 31: Deformation Plot for inner cylinder (30MPa)



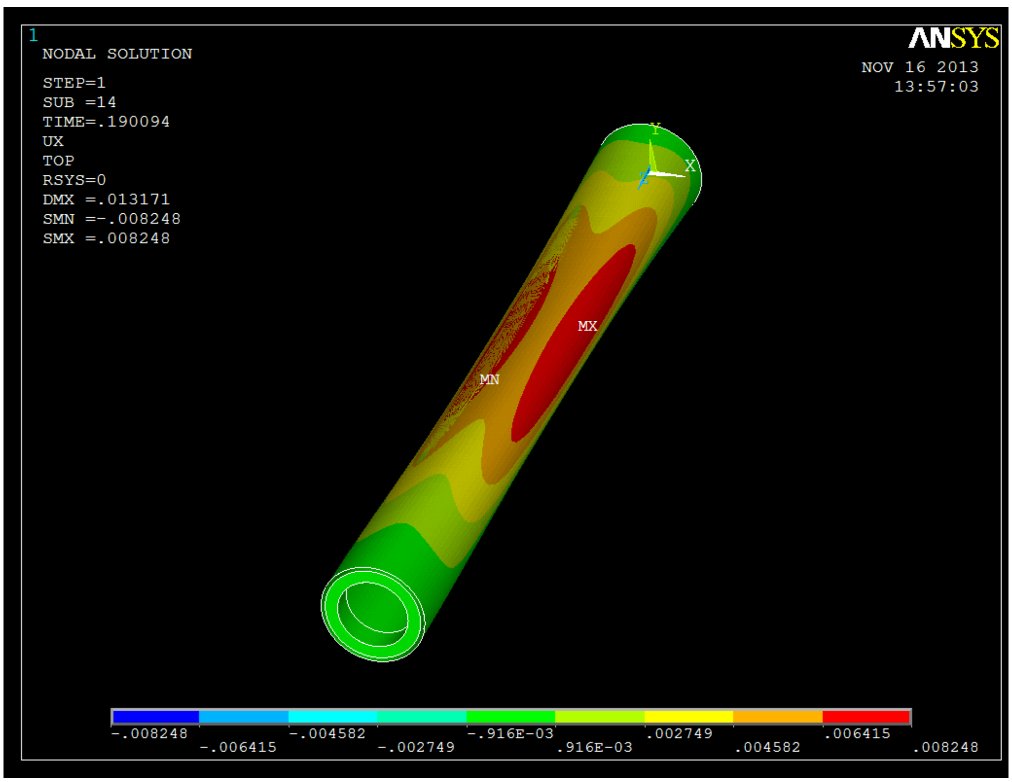
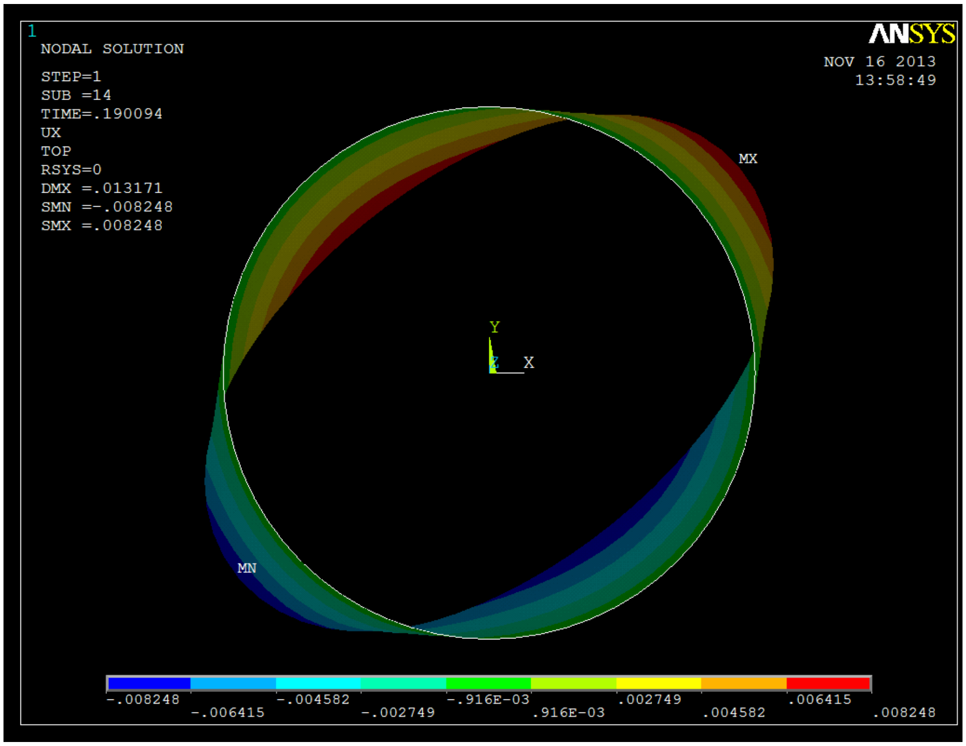


Figure 32: Deformation Plot (40 MPa)



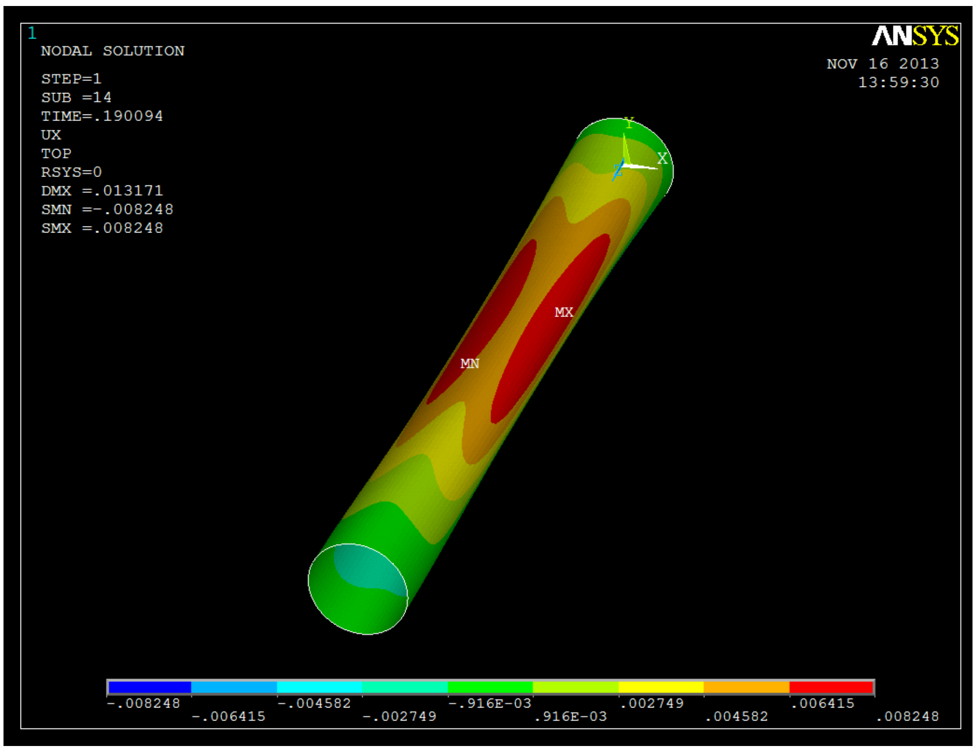


Figure 33: Deformation Plot for outer cylinder (40 MPa)

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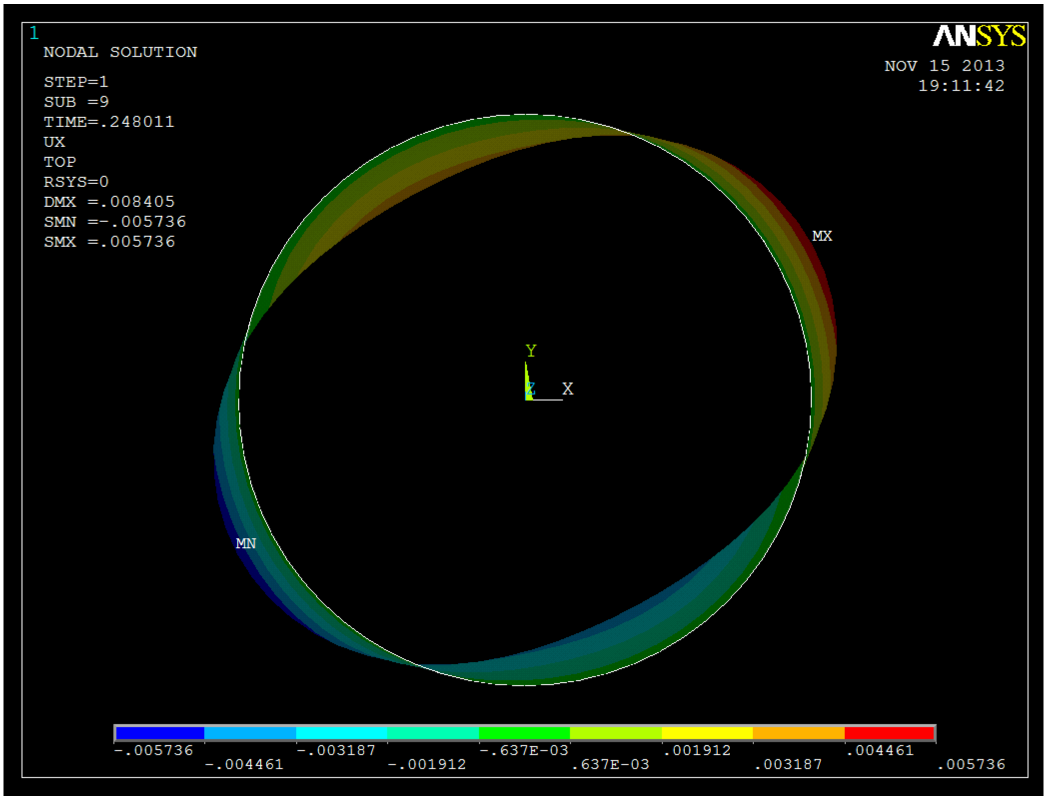


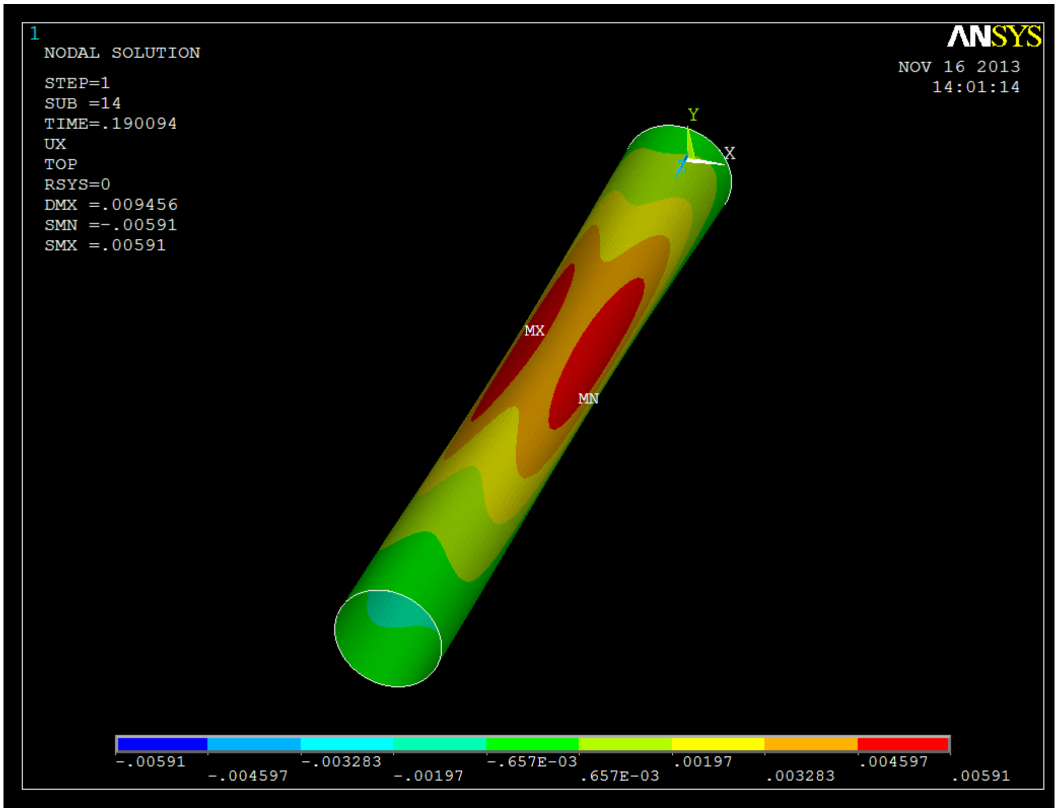
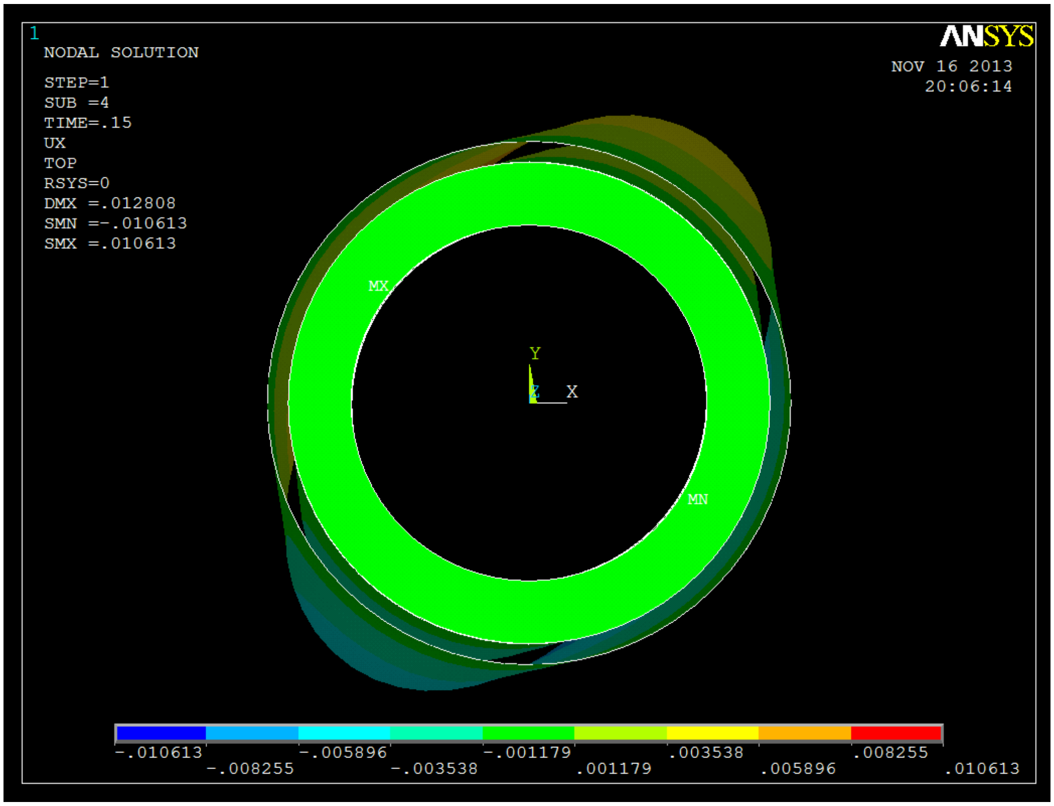


Figure 34: Deformation plot for inner cylinder (40 MPa)



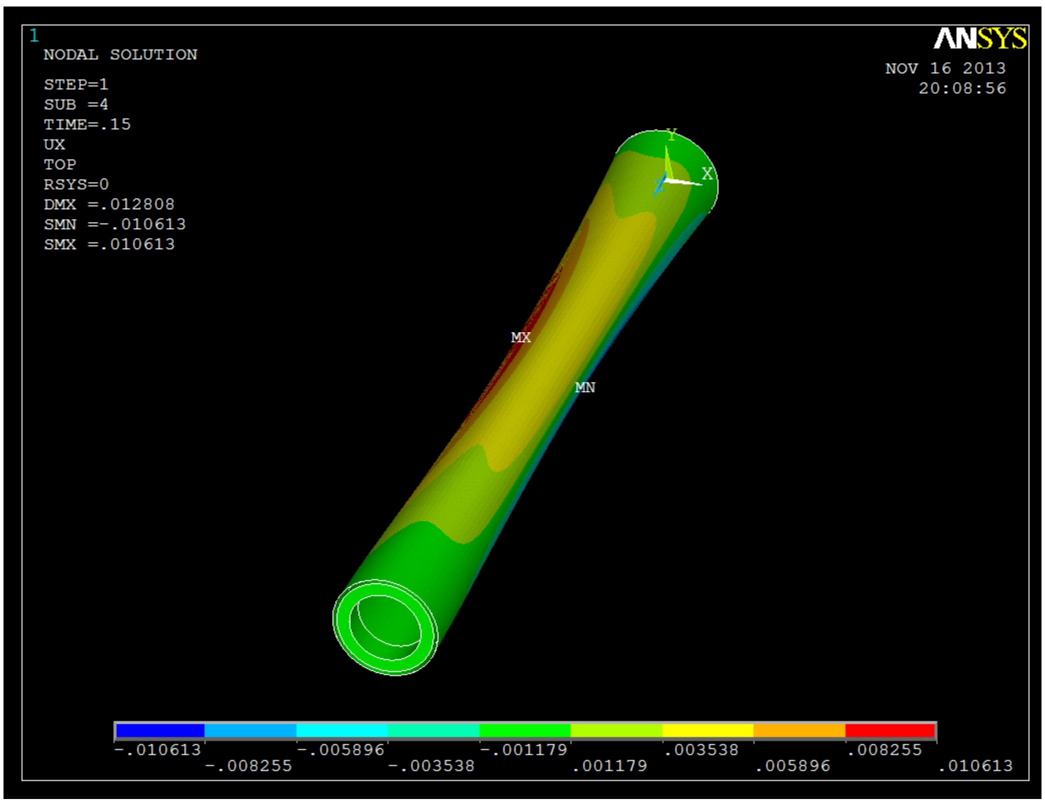
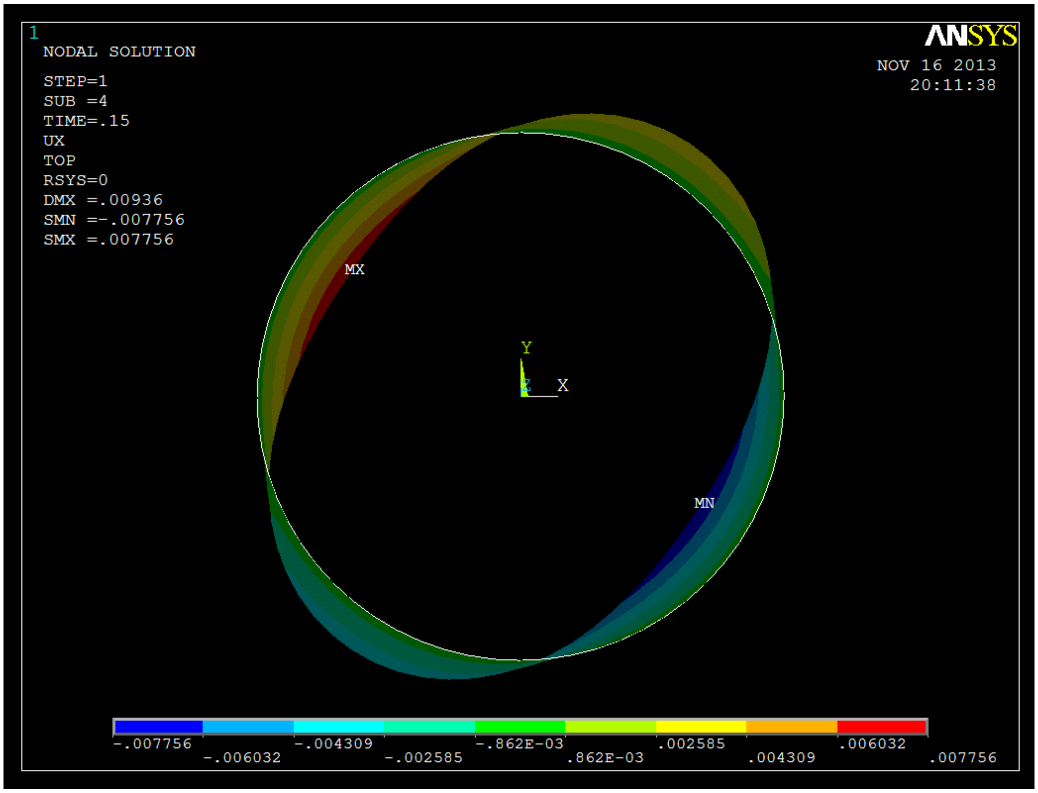


Figure 35: Deformation plot (50 MPa)



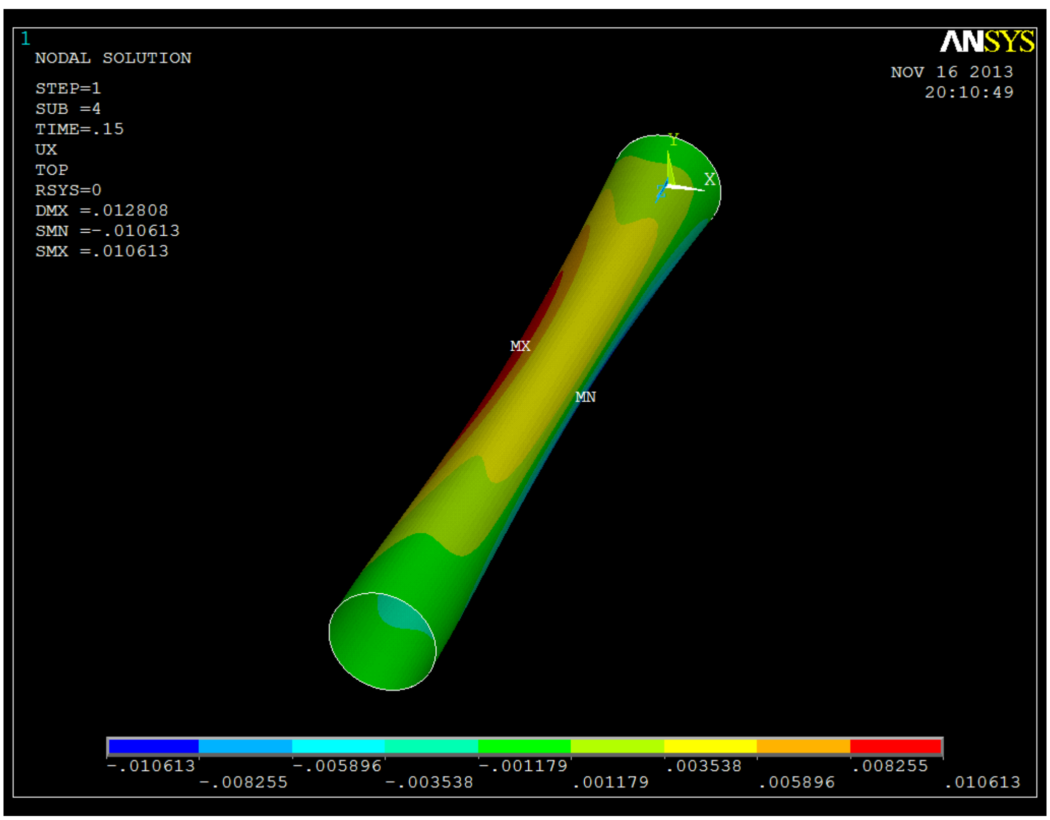


Figure 36: Deformation plot for outer cylinder (50 MPa)

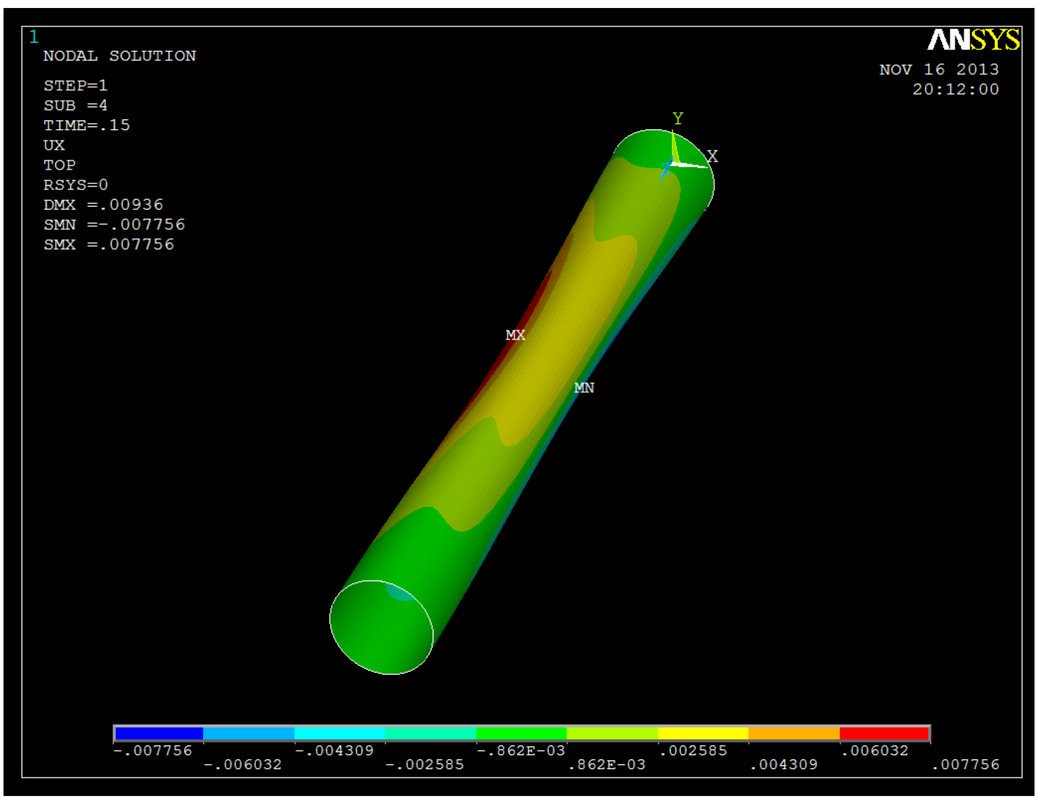


Figure 37: Deformation Plot for inner cylinder (50MPa)

Appendix C: Buckling Load for Different Values of Geometric Imperfections

-Table 6: Buckling Load for Structure (Scale =0.6)

Set	Time/Freq	Load Step	Substeps
1.	0.50000E-01	1	1
2.	0.10000	1	2
3.	0.17500	1	3
4.	0.25000	1	4
5.	0.32500	1	5
6.	0.40000	1	6
7.	0.40938	1	7
8.	0.41875	1	8
9.	0.42227	1	10
10.	0.42578	1	11
11.	0.43105	1	12
12.	0.43303		13

]

Table 7: Buckling Load for Structure (Scale = 0.7)

Set	Time/Freq	Load Step	Substeps
1.	0.10000	1	1
2.	0.20000	1	2
3.	0.30000	1	3
4.	0.40000	1	4
5.	0.40100		5

Table 8: Buckling Load for Structure (Scale = 0.9)

Set	Time/Freq	Load Step	Substeps
1.	0.50000E-01	1	1
2.	0.10000	1	2
3.	0.17500	1	3
4.	0.27500	1	4
5.	0.37500		5

Table 9: Buckling Load for Structure (Scale = 1.1)

Set	Time/Freq	Load Step	Substeps
1.	0.1000	1	1
2.	0.20000	1	2
3.	0.3000	1	3
4.	0.35000	1	4
5.	0.32500	1	5
6.	0.35625	1	6
7.	0.35725	1	7
8.	0.3825	1	8
9.	0.3925	1	9
10.	0.36075	1	10
11.	.36300	1	11