Formally Verifying History using Theorem Proving

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Approval

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Abstract

History is considered as an essential part of our life, as it decides our present and future which is primarily based on historical facts. Historical facts are usually documented in books and are justified by the use of informal arguments, which has been acquired through lengthy procedures by historians and they are quite difficult to understand. Despite of the conventional techniques/ methods used by historians to acquire data there is no clear distinction between true historical events and myths. Thus, its a great challenge to ascertain the correctness of reasoning behind a historical fact. A traditional historical procedure comprises of the twelve primitive axioms to ensure the veracity of a historical event.

In this thesis, we overcome this issue by presenting a formal reasoning approach to analyze historical facts. In particular, the approach is based on the formalization of twelve primitive axioms. Using higher-order logic this formalization can in turn be used to reason about the correctness of historical facts within the sound core of a theorem prover. For illustration, the developed reasoning support has been used to reason about some well-known historical facts i.e., three hours blackout at the time of Christ's death.

Dedication

I dedicate this thesis to my loving parents and siblings.

Certificate of Originality

I hereby declare that this submission is my own work and to the best of my knowledge it contains no materials previously published or written by another person, nor material which to a substantial extent has been accepted for the award of any degree or diploma at NUST SEECS or at any other educational institute, except where due acknowledgement has been made in the thesis. Any contribution made to the research by others, with whom I have worked at NUST SEECS or elsewhere, is explicitly acknowledged in the thesis.

I also declare that the intellectual content of this thesis is the product of my own work, except for the assistance from others in the project's design and conception or in style, presentation and linguistics which has been acknowledged.

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Acknowledgment

In the name of Allah, the most Kind and Merciful

I thank Allah for the health and perseverance bestowed upon me during my Research and M.S degree.

Then i would like to thank my parents for their prayers, for altruistically supporting me, for believing in me and motivating me in each and every hard moment. I would like to express my immense gratitude towards my supervisor and mentor Dr. Osman Hasan for providing me a great learning experience. I highly appreciate his skills in the area of formal methods as well as his help in technical writing. I would like to thank Ayesha Gauhar and Ayesha Siddique for their support and guidance.

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Chapter 1

Introduction

1.1 Motivation

History is a record or narrative description, of broadly conceived historic events, which elucidates why and how the past is the key to the present world. Whenever an event is encountered, it is appended to the history stream in a temporal order [21]. However, many historical facts, like the unsolved mysteries behind the Bermuda Triangle [18] and Easter Island [19], do not always have concrete traces of clear evidences. These historical facts usually date back to archival events, e.g., Ellen Austin, USS Cyclops [6], etc. To ascertain the correctness of these facts, we need to establish their existence based on some other events that are known to have occurred. This process requires an extensive background and study of the associated historical facts and the understanding of the methodology to accept the validity of a statement and thus is not a very straightforward task. Moreover, due to the informal nature of validating historical facts, the conclusions drawn by historians are often doubtful, which is quite undesirable given the major influence of historical fats on our society.

1.2 History and Verification

Carrier et al. [20] recently proposed a formal approach to prove history. His approach to rational-empirical history [5] is primarily based on 12 core epistemological assumptions, known as *axioms* of history, and the infamous Baye's Theorem [20]. Figure 1.1 depicts the basic concepts behind Carrier's approach to prove history. The 12 axioms basically describe the widely used terms in history, i.e., hypothesis, evidence and background knowledge, and the procedure to assign probabilities to facts, which are in turn used in the Bayes' theorem to compute the probability of occurrence of an event. This probability value is then used to judge If the event actually occurred or not. This way history can be verified as an act or process of ensuring that histories are well-formed [21]. Carrier further used the Christ Myth [3], related to Jesus, as a case study to demonstrate the effectiveness of the proposed historical event inquiry method.

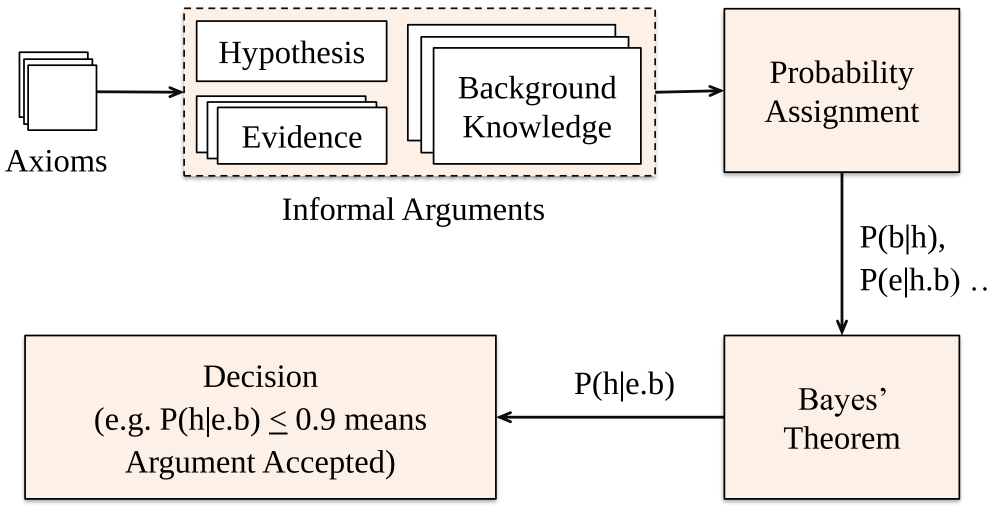

Figure 1.1: Framework for verifying History [5]

Carrier's probabilistic or mathematical approach for verifying history greatly facilitates the process of concluding the correctness of historical facts [2]. However, the correctness of this kind of a manual paper-and-pencil based proof approach cannot be guaranteed. For example, Lecat had identified 1900 errors made by a group of mathematicians [2]. Similarly, the mathematicians of the stature of Littlewood published some faulty proofs [2]. Thus, given the role that history plays in the lives of humans, more rigorous and complete methods for reasoning about historical facts are direly required.

1.3 Formal Verification of History

The history verification is an act or process of ensuring that histories are well-formed [21], i.e., the sequence of events prior to a conclusion confront to some admissible historical fact. In general, this can be achieved by first validating the method used in the proof and then, verifying the proof such that the desired standards are met. Both these concerns characterize the *formal verification*, an automated verification technique for refuting the exactitude of certain historical events [1]. Model checking and theorem proving are the two most widely formal methods. However, the later approach seems to be more suitable because firstly we need to define the axioms and put them in formal way later on modeling of functions will be possible. By using theorem proving, the theorems and definitions, defined for some historical event, can be derived mathematically and solved using automated reasoning techniques [23]. However, it is not the replacement of human understanding or the mathematical proofs but it only expedites the verification process through automation [2].

1.4 Prior State of the Art

In the recent past, many efforts have been made in verifying the historical records but they are focused on medical and biological applications [24]. However, no efforts have been made in verifying the processual history. We are the first to formally verify the processual history, using theorem proving. The mathematical proof usually starts from the postulates, axioms and reaches to the conclusion through some incontrovertible logical steps. For the confirmation of axiomatic proof of the theorem, claimed by the user, the user should enter some basic information which is required for the proof in the system. There exists a number of systems for the formalization of mathematical proofs. For instance, HOL [25], HOL Light [16] and HOL4 [22]. However, HOL4 is transparent tool which has many theorem provers and build-in decision procedures that can establish many unembellished theorems automatically [10].

1.5 Problem Statement

"We are not makers of History, We are made by History."

— Martin Luther King

Present depends on our past, so it is immensely required to distinguish between actual and manipulated historical events. A historical event may comes with discrepancies and to get a single authentic consensus, one has to read hundreds of pages/ document which required a lot of time and continual effort. In addition to verifying the event, the validation of relevant documents are also a gruelling task. Hence a system is required to interpret these historical events more efficiently and accurately.

Mathematics is distinguished from other science because of its precise language and clear rules of argumentation. We can make use of these rules to gain the certain level of confidence regarding the verification of the historical events. To employ mathematics in history, formal methods can be used which contribute towards a complete and accurate analysis of system. The commonly used approaches of formal verification are: Model checking and theorem proving. Due to its profound mathematical reasoning of system specifications, we use HOL4 theorem prover to express the connotation of history in terms of mathematical postulates.

1.6 Proposed Approach

In this thesis, we propose to overcome the above-mentioned limitation by reasoning about historical facts within the sound core of a theorem prover.

With the same motivation, the historical records related to healthcare applications have been formally verified recently [24]. However, to the best of our knowledge no reasoning support for verifying the processual history exists in the literature.

The proposed reasoning approach about history in a theorem prover is primarily based on the Carriers framework [5], depicted in Figure 1.1. The foremost requirement in this regard is formalize the 12 axioms and various other concepts, like hypothesis, evidence, background knowledge, Bayes' theorem etc. We chose the HOL4 theorem prover [22] for this purpose as it supports reasoning about real numbers [7] and probabilities [17]. In this thesis, we mainly present the higher-order-logic formalization of the 12 axioms and the related terms. This formalization can be used along with the formalized Bayes theorem to formally verify historical facts within HOL4. For illustration purposed, we present the formalization of the three hours blackout myth, i.e., the blackout event that is said to have occured at the time of Christ's death, in HOL4.

1.7 Thesis Contributions

The main contributions of this thesis are:

1. **An interpretation method** for mapping the Qualifying Language (QL) to premises and conclusions, with reference to the prior knowledge.

2. A **QL-based elimination method** for eliminating the false premises.

3. **A methodology** for the formalization of historic events using the 12 axioms of history, proposed by Carrier [5], using HOL4.

4. In order to demonstrate the effectiveness of our proposed formalization methodology, we used a Christ myth, i.e., three hour blackout on Jesus crucification, as a **case study**.

1.8 Thesis Organization

Section 2 provides some preliminaries about Carriers approach of proving history [5], HOL4, Theorem Proving and Estimative probability . Then, Section 3 presents our proposed methodology for formal reasoning about history. Section 4 provides the formalization details about the 12 axioms. Section 5 discusses the formally verified properties for the axioms using HOL4 theorem prover. The three hour blackout case study is explained and verified in Section 6. Finally, Section 7 summaries the work and highlights some potential future directions.

Chapter 2

Preliminaries

2.1 Description of Primitive Axioms

According to Richard, there are three basic rules, which we need to follow to prove history. Statments of these rules are; never believe anything you just read, always seek for the primary source of any claim, don't believe on scholars claims who are not the scholar of that era. There are certain steps which needs to follow to analyze the claims credibility. We can analyze through textual format, literacy analysis, source and the last one is historical analysis. But for the proficient historical inquiry Richard proposed twelve axioms whose explanation is given below:

Axiom 1

The evidence on which all the observers are agreed is essential to define principle for rational-empirical history. Any conclusion-derived form the agreed evidence must be free of fallaciousness.

Axiom 2

A consensus must take place among all qualified experts who must agree with the basic principle of REH. Consensus is the correct procedure to find the legitimacy of the argument. The argument went through the process of revision and criticism, the false evidence are trimmed, and at the end the conclusion is in the more refined form.

Axiom 3

We must admit that there is uncertainty in our claim. Confidence comes in certainty not in true and false term. One thing historians soon understand is that we will never know most of the knowledge that actually we want to know.

Axiom 4

After admitting uncertainty, we must accept that there is certain degree of every claim for being true and false. There is a nonzero probability of every claim of being true. There are two type of probabilities one is physical and the other is epistemic. Whenever the author talked about probability, he meant epistemic probability.

Axiom 5

Argument that contains inference which is derived from “possible therefore probably” must be fallacious. This axioms emphasis on Argument from silence, means if document is silent on some issues, historians propose some explanation for that without knowing the actual facts.

Axiom 6

Initial concerns of evidence are depending on the effective consensus of qualified experts. Effective consensus means that it must be having 95% of agreement among the experts. If any scholar claim that the consensus is wrong, then that scholar must constitute the greater burden of evidence to proof his wording.

Axiom 7

Theories are different from facts. Basically facts are actual concrete artifacts where as theories are the stories, which tell that, how the facts are form. We must know the distinction between facts and theories.

Axiom 8

Every premise is important in any argument; even the weakest premise is contributing in the conclusion. Sometimes, the weakest premise weaken the resulting conclusion, no mater how strong other premises are. There are two type of premises, major and minor. Never neglect the minor one. Might possible that the minor premise etiolate the conclusion.

Axiom 9

The strength of any supporting evidence is always proportional to the strength of that claim. Strength of any claim is measured in quantity as well as quality.

Axiom 10

If there is a contradictory claim, which is weaker than the other, than probably weaker one is false. This does not means that the strong claim is true, but it means that the strong claim is asserted as most likely than the other.

Axiom 11

Evidence must support the generalization, and must have more than one example, and cannot be ignored if once supported. The consensus must take place at more than one place to established the final verdict about the argument.

Axiom 12

Confirm before you cite something, citation means that you are supporting that scholar. Cite after double check the material that you are citing. That is not in the evidence, don't assume. If necessary, cite the evidence instead of citing the scholar.

2.2 Estimative Probability

A historical event is primarily composed of three factors; hypothesis (statement under discussion), background (prior) knowledge and evidence, which are the proofs for the historical facts. An evidence is defined in terms of premises and a conclusion and assigned a probabilistic value between 0 and 1. Probabilities are estimated based on the qualifying language (QE), which

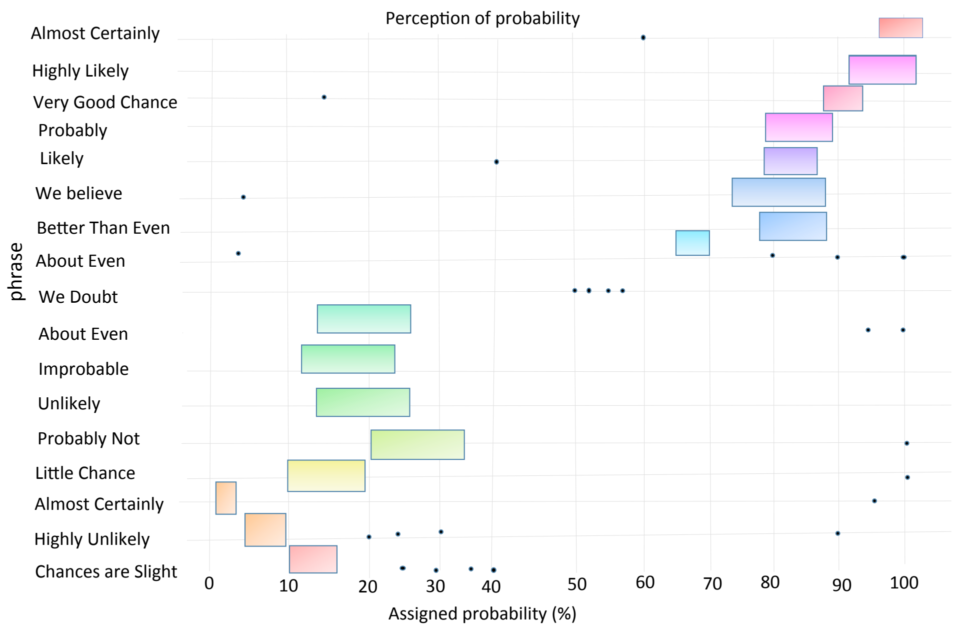

Figure 2.1: Estimated Probability

is composed of key words, like possible, probable, likely, as shown in Figure [15] [26].

2.3 Theorem Proving

To construct and verify a mathematical theorem by the help of computer programs we use theorem proving, which is a formal hardware verification practice. First order, higher order and propositional order are the types of logics on which we can form mathematical theories. Design complexity is increasing day by day so it is preferable to use higher order logic, as it is more communicative and gives more quantifiers than other logics. However, theorem provers assure soundness and correctness for the functions. There are two types of theorem provers, interactive and automated. Former may includes HOL4-Light, HOL, Isabelle, COQ, PVS, and MIZAR. We are going to use HOL to formalize primitive axioms and verify them. For this, HOL is used as it contains prosperous mathematical theories, expressiveness and high degree of programming. HOL can verify the correctness of theories easily. [4].

2.4 HOL4 Theorem Prover

For the implementation of High Order Logic HOL-4 system is used. We use HOL4 for proving theorem in a formal logic environment. To building and proving theorem we need automated reasoning tools and HOL4 is one of them. [10] For proof assistant for higher order logic HOL4 theorem prover is used, which provides such an environment in which we can prove theorems, easy theorems are already established by the theorem provers, the hard ones are need to be proved by the users. HOL provides the platform on which we can do the implementation of combination of deduction, verify the properties for the defined functions. Eight inference rules and five fundamental axioms are the core logic of HOL. [9] Some of the commonly used HOL4 functions and symbols are given in the Table 2.1.

Table 2.1: HOL Notations

Function	**Description**
$\vee$	OR
$\wedge$	AND
$\neg$	Negation
$\Leftrightarrow$	Equality
$\Rightarrow$	implies that
!x.t	for all x that satisfies t
?x.t	there exist some x that satisfies t
&	num to real conversion

Table 2.1: HOL Notations

Function	**Description**
SUC n	natural number successor
h::L	new element h is added in list L
LENGTH L	Length of list L
EL n L	nth element of L List
HD L	Element Head of list L
TL L	Tail of List L
FST (x,y)	returns first element of a pair
SND (x,y)	returns second element of a pair

Chapter 3

Proposed Methodology

The proposed methodology, depicted in Figure 3.1, consists of the following main steps that are implemented using higher-order-logic functions by utilzing the foundational mathematical theories of real and number arithmetic, Booleans, Lists and Pairs:

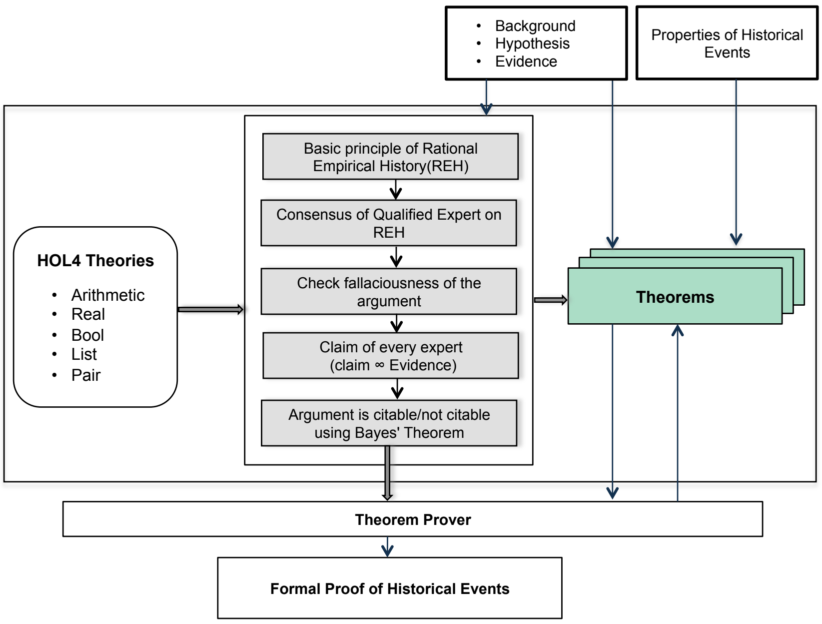

Figure 3.1: Proposed methodology

1. The first step is to formulate the Rational Empirical History (REH) by

classifying the observers with and without evidences.

2. Next, we initiate the consensus process among the qualified experts to revise the argument by eliminating the fallaciousness of the argument.

3. The claim is then obtained from every qualified expert based on the evidence strength supporting it.

4. Finally, the probability of the evidence is obtained based the value of the elements of the qualifying language.

The higher-order-logic functions, for the above-mentioned steps, can be then used to formalize any historical fact and reason about its probabilistic value within the sound core of the HOL theorem prover, which we have used in this work. These probabilities can be formally verified in HOL as theorems, as depicted in Figure 3.1. The main advantage of this work is that the reasoning would be done in a sound environment and thus there is no chance of human error in the proof process. Moreover, the formally verified theorems would contain all the required assumptions explicitly, which would be very useful to understand the constraints about the validity of a historical fact.

Chapter 4

Formalization of Primitive Axioms

In this section, we present the higher-order-logic formalization of the twelve primitive axioms, proposed by Carrier [5], which are explained in Section 2.

4.1 Person's Lsit

The foremost component of our formalization is a person, who is the source of evidence, premise, conclusions, and can be a qualified expert or observer. We propose to model this behavior by a list of pairs, where each element of this list represents a single person. We also formalized functions, given in Table 4.1, that allow us to extract various characteristic of a single person.

Table 4.1: Person's List

Function	**Return Datatype**	**Definition**
Evidence	*Boolean* (Present or not present)	`⊢ ∀ L. evidence L = FST (FST (FST (FST (FST L))))`

Table 4.1: Person's List

Function	**Return Datatype**	**Definition**
Premise	*List of list of real numbers* (Each list of real numbers indicates the probabilistic weights of the premises for a certain evidence)	`⊢ ∀ L. premise L = SND (FST (FST (FST (FST L))))`
Conclusion	*List of real numbers* (Each real number indicates the probabilistic weights of the conclusion of the corresponding premises list)	`⊢ ∀ L. conclusion L = SND (FST (FST (FST L)))`
Qualified expert	*Boolean* (The person is a Qualified Expert or not)	`⊢ ∀ L. qualified_expert L = SND (FST (FST L))`
Observer	*Boolean* (The person is an observer or not)	`⊢ ∀ L. observer L = SND (FST L)`
Qualifying language	*List of real numbers* (Each real number represents estimative probability of word)	`⊢ ∀ L. qualifying_lang L = SND L`

4.2 Functional Flow

The functional flow of the defined functions is explained in the figure. 4.1, assuming that the agreed evidence list is provided. The flow chart demonstrate the functional flow of our methodology.

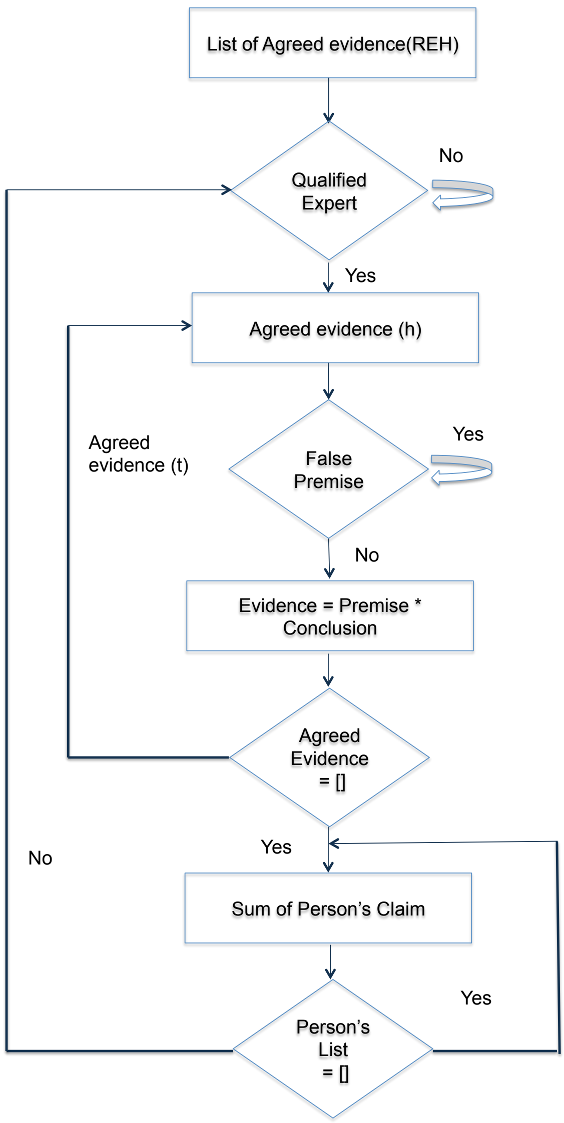

Figure 4.1: Functional Flow

4.3 Definitions

The Figure 4.2 shows how the how consensus is performed and how evidence will be divided.

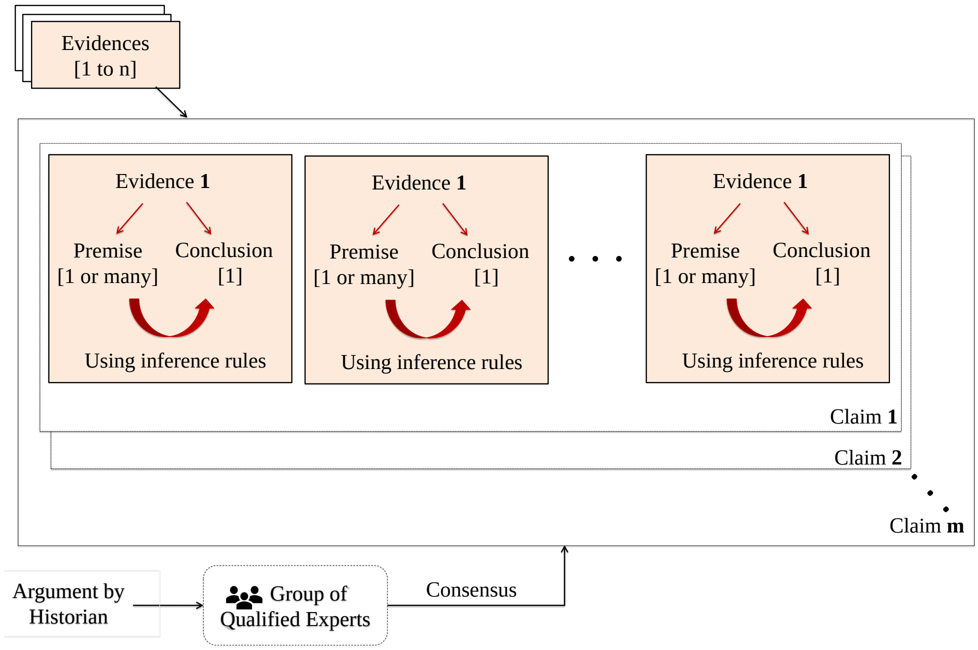

Figure 4.2: Over-view of our approach

Based on the data types, mentioned in Table 4.1, we captured the behavior of the 12 axioms, given in Section 2, in several higher-order-logic functions. According to Axiom 1, given in Section 2, we need to have a list of agreed evidences. We formalized this behaviour using the function `rational_empirical_h- istory`, which takes a person's list L as an input and returns the list of agreed evidences using another function `evidence_recursion`.

Definition 1: *Basic Principle of rational-empirical history*

```
⊢ ∀ L. rational_empirical_history L =
    evidence_recursion L (LENGTH (evidence (HD L)))
```

The function `evidence_recursion` accepts a list L and a parameter n, which represents the length of the list evidences (total number of evidences in Person's list as each person has the same number of evidences), and returns a boolean list indicating that either the evidence is True or False using another function `Allobservers_singleEvidence`.

Definition 2: *For all evidence*

```
⊢ ∀ L. evidence_recursion L 0 = [] ∧
   ∀ L n. evidence_recursion L (SUC n) =
      Allobservers_singleEvidence L n::evidence_recursion L n
```

The function `Allobservers_singleEvidence` recursively checks each evidence against all observers to see whether the evidences against the historical fact are available with all observers or not. It returns a boolean quantity depending on the condition, i.e., if all observers agree on an evidence then the function returns True otherwise False.

Definition 3: *Single agreed evidence against observers*

```
⊢ ∀n. Allobservers_singleEvidence [] n = T ∧
   ∀ h t n. Allobservers_singleEvidence (h::t) n =
      (if observer h then EL n (evidence h)
      else T)
      ∧ Allobservers_singleEvidence t n
```

The function `evidence_acceptance` accepts a persons list L, two variables a and b of datatype real that represent the estimative range for false premises and variables c and d of datatype real representing the estimative range for the false conclusion, along with the variable k that represents the limit to check the correct evidence. This function models the behaviour of Axiom 12 as discussed in Section 2.

Definition 4: *Evidence Acceptance*

```
⊢ ∀ L a b c d k. evidence_acceptance L a b c d k =
   k ≤ persons_claim L a b c d
```

As per Axiom 2, the qualified experts perform a consensus, i.e., the process to eliminate the errors and false evidences. Every expert's claim is assigned some probability value after this process. For this purpose, we use the function `persons_claim`, which accepts a list of persons L, and real numbers a, b, c and d, which represent the estimative ranges for false premises and conclusions.

Definition 5: *Average weightage of qualified expert's claim*

```
⊢ ∀ L a b c d.persons_claim L a b c d =
    persons_claim_val L (LENGTH L) a b c d /
    persons_claim_div L (LENGTH L)
```

The function `persons_claim_val` accepts the list of persons L, an index m of a specific person in the list whose claim weightage is required if he/she is a qualified expert, and real values a, b, c and d as explained above. The claim is obtained by using the function `single_claim`.

Definition 6: *Weightage of qualified expert's claim*

```
⊢ ∀ L a b c d. persons_claim_val L 0 a b c d = 0 ∧
    ∀ L m a b c d. persons_claim_val L (SUC m) a b c d =
    (if qualified_expert (EL m L)
    then single_claim L m a b c d
    else 0) +
    persons_claim_val L m a b c d
```

Similarly, the function `persons_claim_div` recursively counts the total number of qualified experts in a given list of persons L.

Definition 7: *Total qualified experts*

```
⊢ ( ∀ L. persons_claim_div L 0 = 0) ∧
    ∀ L m. persons_claim_div L (SUC m) =
        (if qualified_expert (EL m L) then 1
        else 0) +
        persons_claim_div L m
```

The function `single_claim` accepts the persons list L, a number m (length of agreed evidence) and real numbers a, b, c and d, whose role has been explained above, as the inputs and uses the following function to find the value of the evidence. This way, Definitions 4 - 7 model the behaviour of Axioms 2, 3, 4 and 6, given in Section 2.

Definition 8: *Every expert's claim*

```
⊢ ∀ L m a b c d. single_claim L m a b c d =
evidence_total L (LENGTH (rational_empirical_history L)) m a b
   c d
```

The function `evidence_total` accepts a list L, number n of evidence, the person m and four real numbers real numbers a, b, c and d and returns the average value of the evidence weightage using two other functions `evidence_weightage` and `evidence_divider`.

Definition 9: *Evidence value for single expert*

```
⊢ ∀ L n m a b c d. evidence_total L n m a b c d =
evidence_weightage L n m a b c d / evidence_divider L n m a b
   c d
```

The function `evidence_weightage` accepts the list of persons L, the evidence number n whose premises and conclusion are needed to be checked, a premise number m and the conclusion, and the four real numbers a, b, c and d (explained in Definition 4) and uses another function `check_premise` to check the fallaciousness of the premises and conclusion.

Definition 10: *Weightage of all evidences*

```
⊢ ∀ L m a b c d.
evidence_weightage L 0 m a b c d = 0 ∧
∀ L n m a b c d.
evidence_weightage L (SUC n) m a b c d =
   (if EL n (rational_empirical_history L)
   then check_premise (qualifying_lang (EL m L))
   (EL n (conclusion (EL m L))) a b c d
   else 0) +
```

```
evidence_weightage L n m a b c d
```

The function `evidence_divider` recursively returns the number of true premises along with the corresponding conclusion. The Definitions 8-10 model the behaviour of Axioms 7 and 9.

Definition 11: *Evidence count*

```
⊢ ∀ L m a b c d. evidence_divider L 0 m a b c d = 0 ∧
    ∀ L n m a b c d.
    evidence_divider L (SUC n) m a b c d =
    (if EL n (rational_empirical_history L)
    then divide_by_evi (EL n (premise (EL m L)))
    (qualifying_lang (EL m L))(EL n (conclusion (EL m L))) a b
       c d
    else 0) +
     evidence_divider L n m a b c d
```

The function `check_premise` accepts two lists (premises list, qualifying language list), a real number, representing conclusion probability value corresponding to the premises list and four real numbers (a, b, c and d) explained earlier. The function returns a real value after some manipulation done by the two functions `false_premise` and `check_conclusion`.

Definition 12: *Check value of true premises*

```
⊢ ∀L L1 p a b c d. check_premise L L1 p a b c d =
    if false_premise L L1 a b  check_conclusion p L1 c d
    then premises_val L * p else 0
```

The function `divide_by_evi` accepts four real numbers representing the estimative range for false premises and false conclusion, and two lists (premises list, qualifying language list) and returns a real number as the total value of true premises. The functions `false_premise` and `check_conclusion` determine the fallaciousness of the conclusion and premises by checking if their values lie in a specific range based on Axiom 5, as explained in Section 2.

Definition 13: *True Premises count*

```
⊢ ∀L L1 p a b c d. divide_by_evi L L1 p a b c d =
    if false_premise L L1 a b  check_conclusion p L1 c d
    then 1 else 0
```

Definition 14: *Check fallaciousness of any argument*

```
⊢ ∀L1 a b. false_premise [] L1 a b  T ∧
    ∀ h t L1 a b. false_premise (h::t) L1 a b =
        (if EL a L1  h  h  EL b L1 then F
        else T) ∧
        false_premise t L1 a b

⊢ ∀ p L1 c d. check_conclusion p L1 c d =
    if EL c L1 ≤ p ∧ p ≤ EL d L1 then F else T
```

The function `add_premise` accepts a real list and returns a real value. We used this function to add all premises values. As per Axiom 8, every premises contributes towards the claim value.

Definition 15: *Add premises*

```
⊢ add_premise [] = 0 ∧
    ∀h t. add_premise (h::t) = h + add_premise t
```

Similarly, the following function returns the average value of premises for the given person.

Definition 16: *Average of premises*

```
⊢ ∀ L. premises_val L = add_premise L / &LENGTH L
```

The function `Generalization` models the behavior of Axiom 11. It accepts L, representing generalization samples, x representing the maximum value required to determine the single example weightage and n representing the n^{th} element of the list, and returns a boolean value. If the generalization has more than one example then it returns true otherwise if only one sample is present in the list then it checks if the value of that sample is more than the required value or not and if the value accomplishes the given requirement then it returns true otherwise false.

Definition 17: *Generalization*

```
⊢ ∀ L x n. generalization L x n =
    if LENGTH L > 1 then T
    else if LENGTH L = 1
        then if EL n L ≥ x then T else F
        else F
```

The definitions, presented in this section, model the behavior of 12 axioms, proposed by Carrier's [20], for proving history. Thus, they can be used to formally reason about the correctness of history proofs within the HOL4 theorem prover.

Chapter 5

Properties of Historical Facts

We formally verified various properties for the definitions, given in the previous section, to ensure that they capture the correct behavior.
The following theorem states that the function `Allobservers_singleEvidence` returns false when we have a non empty list of persons, and there is at least one observer (with index n) in the list and at least one of the evidences is false.

5.1 Theorem 1

Statement: *Rational Empirical History*

```
∀ L p n. n < LENGTH L ∧ observer (EL n L) ∧
    (EL p (evidence (EL n L)) = F) ⟹
    (Allobservers_singleEvidence L p = F)
```

The above theorem is valid for a specific observer with index n and we generalize this result for any observer using the existential quantifier as follows:

5.2 Theorem 2

Statement: *Rational Empirical History for all elements*

```
∀ p L. (∃ n. n < LENGTH L ∧ observer (EL n L) ∧
    (EL p (evidence (EL n L)) = F)) ⟹
    (Allobservers_singleEvidence L p = F)
```

Theorem 3 states that the sum of premises is always greater than zero assuming that every element of the list is greater than zero.

5.3 Theorem 3

Statement: *Premises Sum*

```
∀ L. (∀n. EL n L ≥ 0) ⟹ 0 ≤ add_premise L
```

Similarly, the following theorem states that the return value of the function `premises_val` is always less than or equal to 1 provided that the value of every element in the list is less than or equal to 1.

5.4 Theorem 4

Statement: *Average of premises value*

```
∀ L. (∀n. EL n L ≤ 1) ∧ (L≠ [ ]) ⟹ premises_val L ≤ 1
```

Besides ensuring the correctness of the corresponding functions, these formally verified properties also greatly facilitate reasoning about historical facts by reducing the effort in terms of human guidance required for interactive theorem proving.

Chapter 6

Case Study

6.1 Case Study:

To demonstrate the usefulness of our formalization in reasoning about historical facts we consider the famous historical event of a three hour blackout at Christ's death. Gospels reveal a black out for three hours but the provided evidences do not seem to be strong. Thus, its not a very straightforward task to ascertain the validity of this claim. Keeping in view the background knowledge (i.e., the already given information, affirmed information), hypothesis (under discussion statement) and the estimated probabilities of words, defined in Figure 2.1, the premises and conclusions from the Gospels, defined in [5], are given below:

Hypothesis: Black out for three hour at the Christ Death

Background Knowledge: Three-hour solar eclipse is scientifically impossible.

1. **Mark:**

   - **Premise:** Darkness over the whole world from the 6^{th} to the 9^{th} hour (weightage = 0.2).
   - **Conclusion:** Slight chances of blackout (weightage = 0.1).
   - **Evidence weightage:** Premise×Conclusion = (0.2×0.1) = 0.020

2. **Matthew:**
   - **Premise:** Follow Mark's words (weightage = 0.2).
   - **Conclusion:** Slight chances of blackout (weightage = 0.1).
   - **Evidence weightage:** Premise×Conclusion = (0.2×0.1) = 0.020
3. **Luke:**
   - **Premise:** Follow Mark's words (weightage = 0.2).
   - **Conclusion:** Slight chances of blackout (weightage = 0.1).
   - **Evidence weightage:** Premise×Conclusion = (0.2×0.1) = 0.020
4. **John:**
   - **Premise:** No awareness of any darkness occurring at Christ's death (weightage = 0.04).
   - **Conclusion:** No Blackout reported (weightage = 0.03).
   - **Evidence weightage:** Premise×Conclusion = (0.04×0.03) = 0.001

Based on the formal definitions, given in Section 4, we can formalize this scenario in higher-order logic as a person list with 4 elements. This list can then be used with the function `rational_empirical_history` to obtain the list of agreed evidence. Since there is only one evidence and every person is an observer so we formally verified that the list of agreed evidence contains only one element, i.e., true (T) as follows:

Theorem 5: *List of agreed evidences*

```
rational_empirical_history
       [((((([T],[[0.2]]),[0.1]),T),T),
         [0.02; 0.03; 0.04; 0.05; 0.1; 0.2; 0.3; 0.4; 0.5; 0.6;
            0.7; 0.8; 0.9; 1.0]);
        ((((([T],[[0.2]]),[0.1]),T),T),
```

```
        [0.02; 0.03; 0.04; 0.05; 0.1; 0.2; 0.3; 0.4; 0.5; 0.6;
           0.7; 0.8; 0.9; 1.0]);
       (((((([T],[[0.2]]),[0.1]),T),T),
        [0.02; 0.03; 0.04; 0.05; 0.1; 0.2; 0.3; 0.4; 0.5; 0.6;
           0.7; 0.8; 0.9; 1.0]);
       ((((([T],[[0.04]]),[0.03]),T),T),
        [0.02; 0.03; 0.04; 0.05; 0.1; 0.2; 0.3; 0.4; 0.5; 0.6;
           0.7; 0.8; 0.9; 1.0])] = [T]
```

The proof of this theorem was based on the Definitions 1-3. Next we use the function `persons_claim` to verify the average weightage of claims of each qualified expert as follows:

Theorem 6: *Weightage of the claims of qualified expert*

```
persons_claim
      [((((([T],[[0.2]]),[0.1]),T),T),
        [0.02; 0.03; 0.04; 0.05; 0.1; 0.2; 0.3; 0.4; 0.5; 0.6;
           0.7; 0.8; 0.9; 1.0]);
       ((((([T],[[0.2]]),[0.1]),T),T),
        [0.02; 0.03; 0.04; 0.05; 0.1; 0.2; 0.3; 0.4; 0.5; 0.6;
           0.7; 0.8; 0.9; 1.0]);
       ((((([T],[[0.2]]),[0.1]),T),T),
        [0.02; 0.03; 0.04; 0.05; 0.1; 0.2; 0.3; 0.4; 0.5; 0.6;
           0.7; 0.8; 0.9; 1.0]);
       ((((([T],[[0.04]]),[0.03]),T),T),
        [0.02; 0.03; 0.04; 0.05; 0.1; 0.2; 0.3; 0.4; 0.5; 0.6;
           0.7; 0.8; 0.9; 1.0])] 10 11 11 12 = 0.01
```

The proof of the above theorem is based on Definitions 5-7. It is important to know that the estimative probabilities for the premises and conclusions in this example have been acquired from the chart given in Figure 2.1. Considering the background knowledge and evidence, the probability of the claim made by Gospels about darkness is 0.01, which is also in accordance to the scientific fact that the darkness/solar eclipse cannot last for a long time like three hours.

The reasoning shown above is based on our formal definitions, given in Section 4. The proofs were based on simple rewriting steps and thus were automatically done. The usage of a theorem prover ensures sound results and thus is a better option compared to manual paper-and-pencil based analysis, which are human error prone.

6.2 Bayes theorem for History Verification

In his book, Richard uses Bayes Theorem to prove historical events after explaining twelve primitive axioms. The Bayes's theorem is defined in Equation 6.1:

$$P\left(h|e.b\right) = \frac{P\left(h|b\right) * P(e|h.b)}{\left(P\left(h|b\right) * P\left(e|h.b\right)\right) + \left(P\left(\neg h|b\right) * P(e|\neg h.b)\right)} \tag{6.1}$$

The terms used in the above equation are defined in the Table 6.1:

Table 6.1: Bayes' Theorem

Element	**Description**
P	Probability
h	Hypothesis
e	Evidence
b	Background Knowledge
P(h|b)	How typical our description is
P(e|h.b)	probability of expected evidence assuming the description is correct.

Table 6.1: Bayes' Theorem

Element	**Description**
P(¬h|b)	probability of expected evidence assuming the description is false. i.e 1-P(h|b)
P (e|¬h.b)	Consequent probability i.e. probability of the expected evidence provided that the description is false.

The derivation of Equation 6.1 using the basic formula of Bayes Theorem with two variables, conditional probability [12] and chain rule [8]:

$$P\left(X|Y\right) = \frac{P\left(X \cap Y\right)}{P\left(Y\right)} \quad \text{(Conditional Probability)} \tag{6.2}$$

$$P\left(X|Y\right) = \frac{P(X)P(Y|X)}{P(Y)} \quad \text{(Bayes' Theorem)} \tag{6.3}$$

$$P\left(Z|X,Y\right) = \frac{P(X|Z,Y)P(Z|Y)}{P(X|Y)} \quad \text{(Chain Rule)} \tag{6.4}$$

$$P\left(X|Y,\ Z\right) = \ P\ \left(X|Y\right) \quad \text{(Assumption)} \tag{6.5}$$

By putting values of Equation 6.5 in Equation 6.3, we get:

$$P\left(Z|X,Y\right) = \frac{P(X,Y|Z)P(Z)}{P(X,Y)} \tag{6.6}$$

Using Chain Rule in Equation 6.6,

$$P\left(Z|X,Y\right) = \frac{P(Y|Z)P(X|Y,Z)P(Z)}{P(X|Y)P(Y)} \tag{6.7}$$

Using Bayes Theorem as defined in Equation 6.3,

$$P(Z|X,Y) = \frac{P(X|Y,Z)P(Z|Y)}{P(X|Y)} \tag{6.8}$$

Total Probability for Conditional Probability

The denominator in Equation 6.8 is total probability. To find the total probability with respect to three variable. The derivation is as follow:

$$P(X|Y) = P((Z \cup \ Z) \cap X|Y) \tag{6.9}$$

$$= P \ (Z \cap X) \cup (\neg Z \cap X)|Y \tag{6.10}$$

$$= P((Z \cap X)|Y) + ((\neg Z \cap X)\,|Y) \tag{6.11}$$

$$= \frac{P(Z \cap X \cap Y)}{P(Y)} + \frac{P(\neg Z \cap X \cap Y)}{P(Y)} \tag{6.12}$$

$$= P(X|Z \cap Y)\,P(Z|Y) + P(X|\neg Z \cap Y)P(\neg Z|Y) \tag{6.13}$$

Or

$$= P(Z|Y)\,P(X|Z,Y) + P(\neg Z|Y)P(X|\neg Z,Y) \tag{6.14}$$

So the equation 6.8 become in the terms of e (evidence), h (hypothesis), b (background knowledge).

$$P(h|e.b) = \frac{P(h|b) * P(e|h.b)}{(P(h|b) * P(e|h.b)) + (P(\neg h|b) * P(e|\neg h.b))} \tag{6.15}$$

As mentioned in book, [5] the value of term P(h|b) is 0.01, term P(¬h|b) will be 0.99, the term P(e|¬h.b) is 1 [11]. By introduciong the values of the claims provided by four Gospels we found the value of term P(e|h.b) equal to 0.01 which is the core value for reasoning about the historical facts.

If we substitute the values in Bayes Theorem 6.15, we get:

$$P\left(h|e.b\right) = \frac{0.01 * 0.01}{(0.01 * 0.01) + (0.99 * 1)} \tag{6.16}$$

$$P\left(h|e.b\right) = \frac{0.0001}{(0.0001) + (0.99)} \tag{6.17}$$

$$P\left(h|e.b\right) = \frac{0.0001}{0.9901} \tag{6.18}$$

$$P\left(h|e.b\right) = 0.01 \tag{6.19}$$

According to Carrier [5], we can prove this by using Bayes so by seeing equation 6.18, the darkness/solar eclipse can not be possible for a long time like three hours.

Chapter 7

Conclusion

7.1 Summary

Historical facts play a vital role in our daily life, so it is imperative to have a sound method to ascertain their correctness. Many researchers did their best to explore and preserve the history, as it contains the record of past events, but the preserved documents are lengthy to read and understand. Some sort of system is needed to save the human reading time and which is more interactive with the humans. In this thesis, we present a methodology to reason about the historical facts based on twelve primitive axioms presented in Proving History [5]. We have verified some inherent characteristics of these axioms that ensures that our formalization is correct and these verified properties play a vital role in reasoning about historical facts within a theorem prover as well. Three hours blackout myth is used as a case study. Mapping the values on Bayes Theorem which we have obtained by the defined functions, came up with the final outcome. Many philosophers uses formal verification in different fields, but no one ever focuses on using formal verification in historical events. This is for the first time that formal verification is used to reason about the historical facts.

7.2 Future Work

Our proposed approach, related to Formal Verification of History' opens the doors to many new dimensions in the field of theorem proving and model checking. In future, this research can be enhanced by formally verifying the Bayes' theorem having three variables and apply this on wide range of historical facts like existence of Bermuda Triangle [13] or explanation for delusion of 2012 [14].

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