PARAMETRIC STUDY OF SUSPENSION SYSTEM FOR A LOW MASS VEHICLE

by

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بسم الله الرحمن الرحيم

In the name of Allah, the most

Beneficent and the most Merciful

DECLARATION

I hereby declare that I have developed this thesis entirely on the basis of my personal efforts under the sincere guidance of my supervisor Prof. Dr. M. Afzaal Malik. All the sources used in this thesis have been cited and the contents of thesis have not been plagiarized. No portion of the work presented in thesis has been submitted in support of any application for any other degree of qualification to this or any other university or institute of learning.

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ABSTRACT

Automobile Industry is currently focused in developing and discovering ways to improve fuel economy of the vehicles, keeping in view the rapid consumption of world's energy reserves. There may be several solutions to this problem, like altering the automobile engine to operate on some alternate fuel or altogether reshaping the automobile engine functionality which can generate comparable power output with less fuel consumption, but currently the most popular approach in the industry is to develop light weight vehicles, which seems to be a more appropriate solution to the said problem.

Though light weight vehicles may seem to be an appropriate solution for improved fuel economy but there is a drastic change in the dynamic behavior of such vehicles. Vehicle Dynamics is usually gauged in terms of ride and handling of the vehicle. Suspension along with the tires defines the vehicle dynamics. Mass of the vehicle also plays a vital role in the vehicle dynamics, and reducing the mass may result in significant degradation of vehicle's dynamic performance. Suspension System performs the job of not only damping the vibrations coming from the road but also it supports vehicle's weight, keeps tires in contact with the road to maintain traction and also keeps the wheels in the desired orientation. Thus both ride and handling maybe controlled by controlling the Suspension System parameters. The Sprung Mass of a light vehicle varies significantly during operation. Therefore, in order to achieve appropriate dynamic performance from a light weight vehicle its suspension must be designed accordingly.

This thesis presents a study on the effects of Suspension System on a Low Mass Vehicle by changing its parameters to achieve appropriate Ride Performance. Both Traditional and Bond Graph modeling techniques are used to model and analyze the vehicle system, and a comparison between the two techniques is also presented to decide which technique is more suitable for the modeling and analysis of vehicle dynamics. Simulations are carried out in MATLAB and 20-sim software for evaluating the ride performance and based on these simulation a frame work for designing a Low Mass Vehicle is defined.

ACKNOWLEDGEMENTS

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DEDICATION

To my Family and Teachers

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CHAPTER 1: INTRODUCTION

This chapter mainly explains the problem statement by first discussing the challenges faced by the current automobile industry mainly regarding the energy crisis and the technological reforms being carried out by the industry to keep pace with the changing market requirements. Low Mass Vehicle is then presented as a remedy to the problem and the implications of vehicle weight reduction on its ride performance is then discussed in detail. Role of vehicle suspension system is then discussed in reference to its contribution in the improvement of ride performance. At the end chapter wise distribution of the thesis report is explained.

1.1 Pioneers of Automobile

Italian Enrico Bernardi and the two Germans Karlz Benz and Gottlieb Daimler are considered as the pioneers of the automobile. Barnardi in 1882 invented a petrol fuelled combustion engine “Motrice Pia” (named after his daughter) and fitted it into his son’s tricycle to claim the first motorcycle. At the same time the two Germans got the patents for the invention of their respective petrol engines. Karl Benz (25^{th} October, 1882) and Gottlieb Daimler (16^{th} December, 1883) [22].

1.2 Current Automobile Sector

Automobile Industry stands as one of the worlds largest and the most prominent established sector assisting mankind with its diverse applications to achieve excellence and satisfaction in so many jobs performed daily. Be it a family car, a motor bike, a passenger bus, a crane used in construction or a tank to combat the enemy. The applications and demand of automobiles are ever increasing and the automobile sector is striving hard not to just meet up all these challenges but also adapt to the changing requirements.

By the passage of time the automobile sector has transformed into a global market as more and more partners joined the auto industry from around the world. Still it is considered that the automotive industry is dominated by a small group of companies like Ford, Suzuki, Toyota, Nissan, Honda, GM, BMW, Volkswagen and Mitsubishi etc.

In automobiles, four wheeled ground vehicles are the most in demand worldwide. China, Japan, USA, Germany and South Korea are the leading car producing countries. Twentieth Century revolutionized the auto industry by introducing so many technological enhancements related to vehicle’s performance enhancement and passenger safety. Ever changing global trends have altogether changed the perception of the automobiles and the leading automotive industries are striving hard for maintaining their shares and come up to the costumers' expectations.

1.3 Challenges for Automobile Sector

Conventional cars either run on petrol or on diesel, which eventually comes from the fossil fuel reserves of our planet. Increasing population and thus the demand for more energy is indicating towards an end of these fuel reserves in the near future. Car manufacturers have shifted their efforts to cope with this prevailing energy crisis situation and the emphasis is mainly on the following three possible solutions:

- New Propulsion Technologies
- Alternate Fuel Resources
- Vehicle Weight Reduction

1.4 Coping up with Challenges

Considering these options the auto sector has come up with the idea of hybrid vehicles and most of the manufacturers have shifted their efforts towards developing hybrid vehicles and that too lighter in weight. Alternate fuels are being searched to suit the automotive requirements, which mainly consist of Biodiesel, CNG, Electric Battery Power, Ethanol, Hydrogen and LPG.

GM, DC and BMW have established a collaboration which has resulted in a breakthrough in hybrid technology [23]. Nissan and BMW are planning for their Electric Vehicles in near future. Ford has already introduced the world's first Plug-In Fuel Cell Hybrid Car. Fiat has developed an alternate fuel prototype vehicle and aims to achieve performance close to that of traditional petrol engines.

1.5 The Low Mass Vehicle

Whatever the fuel maybe and whatever the propulsion technology being implemented, the weight factor will always affect the fuel consumption. Therefore, most of the manufacturers are focusing on light weight solutions which tend to improve the fuel economy and reduce hazardous emissions from the vehicle into the environment.

According to [24], Audi's second Generation Sports car will use carbon-fiber reinforced plastic to make it 25% lighter than the current model and PSA is looking forward to use aluminum for its vehicles to reduce the weight and increase power-train fuel efficiency.

According to [23] Ford has planned to use Nanotechnology to develop strong light weight materials for the automobiles manufacturing, whereas, Mazda has introduced their new Mazda-2 which is 100 kg lighter than the previous incarnation and Honda has introduced Honda Insight as 40% light in weight than its Mild Steel counterpart.

Light weight solutions tend to solve the problem as far as the fuel economy and environmental issues are concerned, but this remedy does not completely satisfy the consumer requirements as several questions arise about the passenger safety and the vehicle's performance. A Low Mass Vehicle (LMV) is usually made by using lighter materials like aluminum, high strength lighter steel and carbon composites reinforced plastic and has faced acute criticism that whether the LMV ensures passenger and highway safety or not [20].

1.6 Safety Issues of Low Mass Vehicle

According to [20] the author points towards the questions which arise concerning the safety aspects of LMV are of the nature that whether there exists any relationship between the weight of the vehicle with that of the passengers' safety, and whether the materials used in LMV are capable of providing comparable strength to the LMV as Mild Steel provides to a standard vehicle. Automotive Engineers and experts have come up to satisfactory answers for these questions, and are convinced that LMV are safer.

Recent research has shown that vehicle size and its design are related to its safety rather than its weight. Crash avoidance mechanisms and active measures can also improve vehicle's safety. Also when considering the laws of physics it is easy to handle lighter vehicles as compared to heavy ones due to the factor of momentum. Rollover safety for the vehicle can be addressed by proper vehicle design.

Regarding the concern related to lighter materials used in the automobiles, several tests are analysis have revealed that the lighter materials like aluminum, high strength light steel and carbon composite reinforced plastic which are now used in place of Mild Steel are more efficient in absorbing energy during a crash . Thus the questions about the safety aspects of LMV are already undertaken satisfactorily and LMV is considered as a safe ride for the passengers.

1.7 Performance of the Low Mass Vehicle

When we talk about the performance of an automobile, we are basically talking about its 'Ride and Handling' characteristics, where Ride is related to the comfort experienced by the passengers as the vehicle travels on the road and Handling refers to the control and safety of the vehicle and its passengers during cornering and maneuvering. Optimum Performance of a vehicle is dependent on the overall vehicle design including its aerodynamic profile; however the main components which affect the ride and handling of a vehicle are mainly the Vehicle Mass and its distribution, the Tires and the Suspension System.

Tires are the contact points of the vehicle with the road and the Tire-Road Interaction forms the basis of the vehicle's performance as it results in the transfer of road disturbances to the vehicle via its suspension and its linkages, and also performs the vehicle's maneuvering as per the directions set by the driver through the steering. Tire-Contact Patch, the material of the tire, its geometry and shape, its pressure, the caster and the camber angle all play a vital role in defining the Tire-Road Interaction, which ultimately results in vehicle's performance characteristics. The Traction of the Vehicle, its Cornering Performance, Roll Stability, Slip, Over-Steer and Under-Steer are the main factors which are affected by the Tire-Road Interaction.

Besides the Tire-Road Interaction, the Suspension System is also the defining parameter for the Ride and Handling Characteristics of the Vehicle. The road disturbances are transferred to the vehicle and the passengers via suspension system. These disturbances are in the form of vibrations and termed as the 'Dose' which not only cause discomfort to the passengers but also cause harm to any on-board electronic equipment.

Despite tire-road interaction and suspension system, there is another factor which plays a very important role in defining the automobile's performance and that is the mass of the automobile. Weight of a vehicle is usually distributed as the 'Sprung Mass' and the 'Un-Sprung Mass'. Sprung Mass is the weight of the vehicle supported by the Suspension System, where as the weight of tires and the wheels is termed as the Un-Sprung Mass. Whenever a vehicle traverse a road the vibrations from the road are first transmitted to the U-Sprung Mass then to the Sprung Mass. A proper proportion of Sprung to Un-Sprung Mass is usually maintained so that good ride performance is achieved. Greater the Un-Sprung Mass, more the vibrations transmitted to the Sprung Mass. Therefore, when the weight of a vehicle is reduced, main reduction of weight is carried out from the Sprung Mass, which disturbs the

Sprung to Un-Sprung Mass ratio and results in a ‘Degraded Ride Performance’. Here we will look into the details of variation of sprung mass of the LMV in context of a passive suspension design.

1.8 Suspension and Ride Performance

The Suspension System connects the wheels of the automobile with its body and serves the following purposes [3]:

- Ensures proper tire-Road Contact for better road grip / handling
- Attenuates the Vibrations coming from the wheels to ensure better ride
- Supports the weight of the automobile
- Maintains wheels alignment

There are three fundamental components of a suspension system, springs, dampers and anti-sway bars. Springs support the sprung mass of the vehicle and absorb the road shocks; springs are characterized by their spring rate which defines the spring as soft or hard. Spring-Rate is defined as the weight needed to compress a spring by a certain amount. Dampers basically reduce the magnitude of vibrations by converting the Kinetic Energy of the suspension into heat which is ultimately dissipated through Hydraulic Fluid and hence control the transient response of sprung mass. Anti-Sway bars are also called the Anti-Roll bars and basically provide roll resistance and thereby determine the lateral load transfer rate distribution. Higher Roll-Stiffness for rear wheels result in Under-Steer and Higher Roll-Resistance for front wheels result in Under-Steer.

Suspensions are available in different combinations of these components and maybe different for front and rear wheels of an automobile. Several factors may play a role in the selection of a specific suspension configuration like, whether the automobile is front wheels driven or rear wheel driven, size and dimensions of the vehicle, type of vehicle such as car, jeep or truck etc. Usually an automobile has different suspensions for Front and Rear wheels. Usually Independent Suspension System is used for Front wheels, whereas for Rear Wheels both Dependant and Independent Suspensions are used. Suspension parameters can be adjusted according to the user requirements for example, suspension systems with softer springs provide good ride quality however handling is compromised, however for stiffer springs handling is better for degraded ride quality [25].

Suspension of the vehicle plays a vital role in determining the Ride of the vehicle. Ride of a vehicle is considered to be comfortable if the passengers seated inside the vehicle do not experience harmful vibrations while the vehicle traverses on a road, and this is more likely to happen if the Natural Frequency of the vehicle suspension is in the range of 1 to 1.5 Hz as passengers start to feel discomfort when vibrations reach the frequency of about 2 Hz. Research has revealed that there are some specific vibration frequencies which are harmful to humans and therefore, must be avoided by designing appropriate suspensions to achieve better ride performance. As for example, frequencies in the range of about 0.5 to 0.85 Hz cause motion sickness. Also frequencies in the range of 18 to 20 Hz are not suitable for head and the neck regions [2].

As far as Ride comfort is considered, it depends upon the complete vehicle design rather than the design of suspension only, still the Natural Frequency of the Suspension System is considered the most important factor in achieving good ride performance.

Designing a proper Suspension System for a Low Mass Vehicle is a challenging task. Several changes may be required to adapt in the existing suspension designs to achieve optimum performance. Given the relationships we can work out the suspension stiffness for the desired frequencies of the LMV to achieve a good ride, however, due to weight reduction several problems occur as the payload to weight ratio increases, which results is undesired handling characteristics. Especially the problem occurs when we have to deal with the roll resistance, the ground clearance and the breaking performance. The problem is in dealing with the shifting of Inertias during maneuvering and in order to cater for these problems the stiffness of the springs is increased which in turn affects the Ride performance of the vehicle.

1.8.1 Types and Classification of Suspension

According to [3], here exist several suspension system designs for both Front and Rear Wheels in the category of Independent and Dependant Type Suspension Systems.

Independent Type Suspension Systems used for Front Wheels include:

- MacPherson Strut
- Double-Wishbone / A-arm Suspension System
- Multi-Link Suspension
- Trailing Arm Suspension

Dependant Type Suspension Systems for Rear Wheels include:

- Solid-Axle
- Solid-Axle Coil Spring
- Four Bar Type
- De Dion

Independent Type Rear Suspension Systems for Rear Wheels include:

- Swing Axle
- Trailing-Arm Rear Suspension
- Semi-Trailing Arm

These are some of the usually used suspension designs. However the most commonly used suspensions is the MacPherson Strut for Independent Front Wheels. This design was made in 1947 by Earle S. MacPherson of the General Motors. MacPherson Strut consists of a Coil Spring and a Shock Absorber packaged together in a single unit. Besides MacPherson Strut another design which is mostly used for the Front Wheels falling in the Independent Suspension category is the A-arm or Double-Wishbone Suspension System. Double-Wishbone Suspension design consists of a coil spring and a shock absorber mounted on the wishbones. This design provides a better control for the Camber, Castor and Roll.

For Rear Wheels in the category of Dependant Suspension the Suspension commonly used are the Solid Axle (Leaf Spring) and the Solid Axle Coil Spring. For Independent Rear Suspension all the Independent Front Wheel Suspensions can be used but in a simplified form i.e. they do not include the Steering Rack used for turning the wheels.

There exist other specialized suspension designs like multi-link suspensions and the Bose Suspension System etc. The Bose Suspension System is considered to revolutionize the suspension technology as it constitutes an electromagnetic suspension system for the automobiles. Bose Suspension System replaces the conventional spring-shock absorber arrangement with a Linear Electromagnetic Motor (LEM) along with Amplifiers. This design provides greater control to the user over the maneuvers like cornering and braking and enhances both Ride and Handling performance significantly.

1.9 Problem Statement

In view of the prevailing energy crisis, the automobile industry has to carry out technological reforms in their products to meet market requirements. As discusses earlier, the current industry trend is towards Hybrid and Light Weight Vehicles.

This thesis is intended towards the issues related to the 'Ride Performance' of light weight vehicles and discusses the implications of weight reduction on vehicle's ride performance and focuses on the role of suspension system to achieve optimum ride performance by studying the changes in ride behavior due to the changes in suspension parameters.

1.10 Objectives

As the topic states "Parametric Design Study of the Suspension System of a Low Mass Vehicle", therefore, it is required to conduct a parametric study for establishing the ride characteristics of a LMV. Although the industry is adapting the light weight solution but the term 'Low Mass Vehicle' does not have any known existence. Therefore, there is a need to characterize a vehicle as a "LMV" and establish a framework for its design.

1.11 Methodology

In this thesis first the mathematical models of a vehicle are developed using Newton's Method and Bond Graph Method. As the thesis focuses on the ride performance of the vehicle, therefore, Full Car model is ignored, only Quarter Car and Half Car Models are developed. Since the thesis is dealing with the ride performance of the vehicle and 'Road' plays a vital role in defining the ride of a vehicle, therefore, Random Road Profiles are also generated and given as input to the vehicle models in order to simulate the actual behavior of the vehicle.

Vehicle's Ride Performance of a highway vehicle is then simulated using both the Newton and the Bond Graph Method. Weight reduction of the vehicle is then carried out and the changes in the ride performance are observed. Based on the simulation results the criteria for characterizing a vehicle as 'Low Mass Vehicle' is then established. After characterizing the LMV, parameters of the suspension system are tuned to study the changes in the ride behavior of the LMV.

Based on the results suggestions are made for the future research work to optimize the performance of the LMV by manipulating the Suspension System.

1.12 Literature Review

The topic under consideration requires a diverse range of topics to be covered in the Literature Review like; Vehicle Dynamics, Engineering Mechanics, Modeling and Simulation, Bond Graph Method, Vibration Analysis of Automotive Systems, Random Vibrations and Fourier Transforms etc.

Review of the automotive history and the current automotive industry trends was conducted using the references [20], [21], [22], [23] & [24].

Theory related to the Vehicle Dynamics and the Suspension System was studied using the references [1], [2], [3], [4], [5] & [6]. This covered the topics related to Basics of Vehicle Dynamics, Ride and Handling and Suspension Systems of the Automobiles.

Vehicle Models development and analysis using the Newton's Method required the study of the references [7] & [4]. This included the Modeling and Ride Response of the Quarter Car and the Half Car Models. Whereas, Modeling and Simulation of the Vehicle Models using Bond Graph Method required the study of: [8], [9], [10] & [11].

Literature Reviewed for the development of Road Models included [6] & [14], whereas, Vehicle Simulation with Random Road Inputs required the review of: [12], [13], [15], [16], [17], [18] & [19].

1.13 Thesis Distribution

In light of the Literature Review it was possible to identify the tasks required to be performed to achieve the objectives mentioned above. A plan was formulated to address the problem in a systematic manner so that the solution approach is clearly understood.

The thesis distribution scheme in different Chapters is as such:

Chapter 1: (Introduction) covers the basics of vehicle suspension system and vehicle ride performance, defines the problem statement and the objectives and also explains the literature survey carried for the completion of thesis.

Chapter 2: (Physical System) discusses the Quarter Car Model and the Half Car Model in detail. Their significance in the Ride Response and the types of road inputs required to study the ride response.

Chapter 3: (Mathematical Modeling) discusses the Mathematical Modeling of Vehicle using Traditional Methods and the Bond Graph Method. Road Modeling is also included in this chapter.

Chapter 4: (Numerical Simulation) includes the Simulation of the models developed in Frequency and Time Domain.

Chapter 5: (Results and Discussion) presents the outcome of the thesis, i.e. the characterization and design framework for a LMV and also suggests the future research work to optimize the performance of the LMV by manipulating the Suspension System.

CHAPTER 2: PHYSICAL SYSTEM

2.1 Introduction

This chapter discusses the physical system, the inputs and the outputs considered for achieving the objectives of the study. Since the study is related to the vehicle dynamics, therefore, the physical system under consideration is an automobile, which can be expressed in different forms for studying different phenomenon. In this study, the automobile needs to be considered from the perspective of the study of ride behavior, therefore, in this chapter ride specific representation of the automobile, the inputs and outputs are considered.

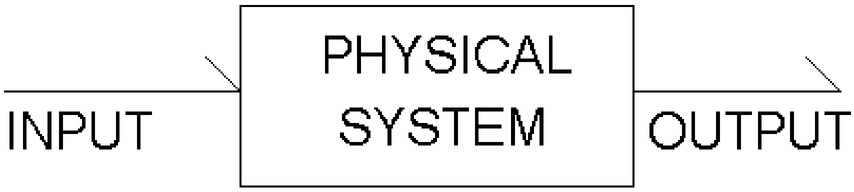

Figure 2.1: Physical System

2.2 Physical System for Studying Ride Performance

In this study an automobile is the Physical System under consideration, but the important thing is the representation of automobile to address the ride analysis problem. Although a vehicle moving on a road is considered to have 6 degrees of freedom as shown in figure 2.2. (Figure courtesy [1]):

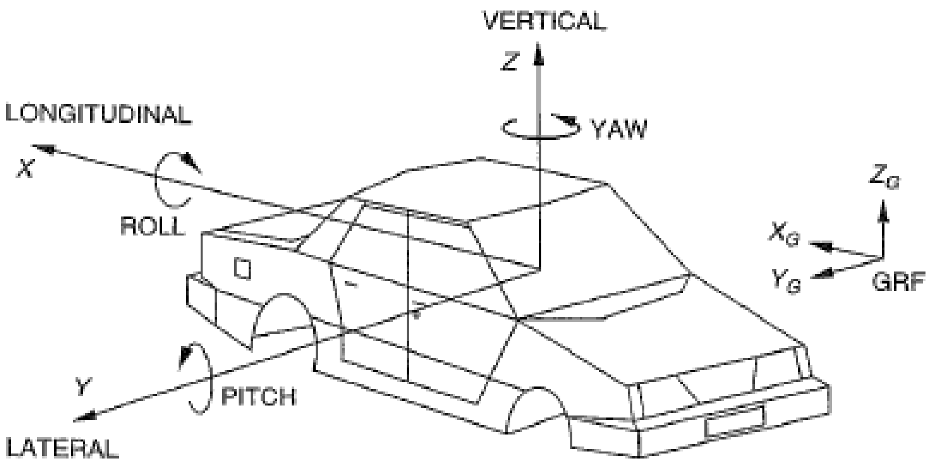

Figure 2.2: Degrees of Freedom of a Car

1. Forward and Reverse Movement (Along Longitudinal Axis)
2. Left and Right Movement (Along Lateral Axis)
3. Bounce Movement (Along Vertical Axis)
4. Roll Movement (About Longitudinal Axis)
5. Pitch Movement (About Lateral Axis)
6. Yaw Movement (About Vertical Axis)

A Full Car Model is ideally required to study all the aspects of a vehicle's dynamics, which covers both the ride and the handling performance of the vehicle. However, out of the six mentioned types of movements only Pitch, Bounce and Roll movements are ideally considered for the study of vehicle's ride behavior. Roll movements are usually ignored for the study of ride behavior by assuming that road is smooth and the input on all wheels is the same. If only the Bounce and Pitch movements are to be considered then Quarter Car and the Half Car Models are usually required for studying the ride behavior of the vehicle.

Both the Quarter Car and Half Car models give specific information about the vehicle which is essential to compute the vehicle ride response. The Quarter Car model gives information about the vertical dynamics of a vehicle only (pitch movement), whereas the Half Car model gives information about two types of movements depending on the choice of vehicle axis to formulate the model. For example, in order to obtain information about bounce and the pitch movements the Half Car model is setup along the lateral axis and for calculating roll it is setup along the longitudnal axis.

Since the study is related to the effects of changes in the suspension parameters on the LMV, therefore the Suspension System is also included in the Physical System Represenation.

2.2.1 The Quarter Car Model

The Quarter Car Model shown in figure 2.3 is a 2- Degrees of Freedom (DOF) system, according to the general rule for calculating the number of DOF of a system given by [7]. i.e.

Number of DOF of a Mechanical System =

No. of Masses × No. of possible types of motion of each Mass **(2.1)**

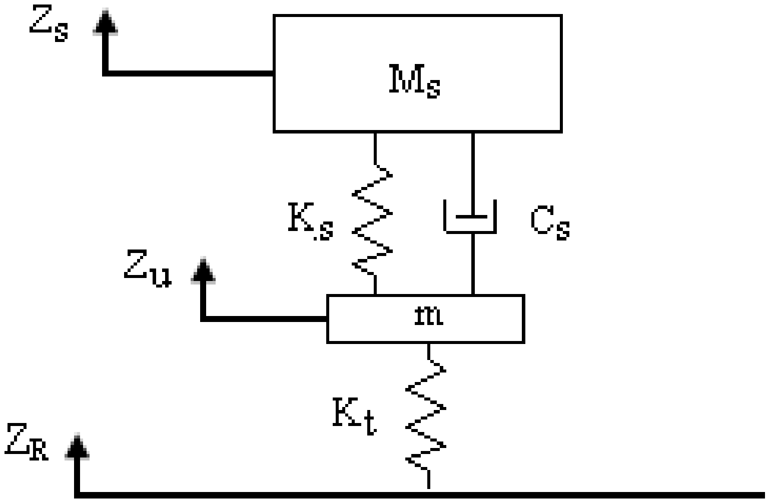

Figure 2.3: Schematic of Quarter Car Model

M_s: Sprung Mass

m: Un-Sprung Mass

K_s: Suspension Stiffness

C_s: Suspension Damping

K_t: Tire Stiffness

Z_s: Sprung Mass Displacement

Z_u: Un-Sprung Mass Displacement

Z_R: Road Input

The Quarter Car Model shows the quarter of a car with Sprung Mass taken equal to one fourth of the total Sprung Mass of the Vehicle, K_s and C_s representing the spring and damper of the Suspension attached between a single Wheel and the Sprung Mass. Un-Sprung Mass represents the mass of a single wheel. Tire is represented by K_t only, i.e. reflecting the stiffness of the tire only. Damping of the tire is usually not considered as its value is significantly less as compared to the overall damping of the system.

The Quarter Car model shown in figure 2.3 is a relatively simpler model showing a passive suspension system. Several Active and Semi-active suspension designs are also being used in modern cars for the improvement of vehicle dynamics. In reality the suspension system consists of three components (spring, damper and anti-roll bar), however, this schematic shows only the spring and the damper for the sake of simplicity. Also the components of the suspension are practically non-linear, however for basic study of the behavior of a vehicle these components are assumed to be linear. Non-linearity can be incorporated later in a more advanced stage

2.2.2 The Half Car Model

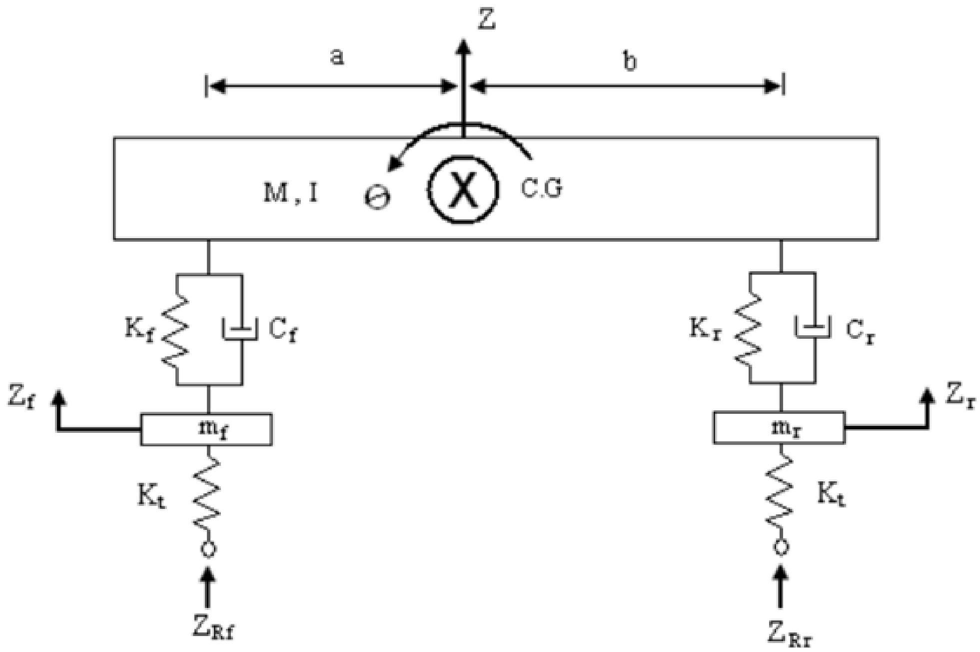

Figure 2.4: Schematic of Half Car Model

The Half Car Model shown in figure 2.4 is a 4-DOF System, according to the general rule for calculating the number of DOF of a system given by equation (2.1).

The Half-Car Model shown in figure 2.4 is set up along the lateral axis in order to study the Pitch and the Bounce movements as the roll movement is not considered for evaluating the ride response of the vehicle. In the said schematic the Sprung Mass refers to half of the total Sprung Mass of the Vehicle. Whereas, 'm_f' and 'm_r' correspond to the mass of front wheel and rear wheel respectively.

M: Sprung Mass

I: Pitch Inertia of Sprung Mass

C.G: Centre of Gravity

K_f: Front Suspension Stiffness

C_f: Front Suspension Damping

K_r: Rear Suspension Stiffness

C_r: Rear Suspension Damping

m_f: Un-Sprung Mass Front

m_r: Un-Sprung Mass Rear

Z: Bounce Movement

Ɵ: Pitch Movement

a: Distance between Front Axle and C.G

b: Distance between Rear Axle and C.G

Z_f : Front Sprung-Mass Displacement

Z_r: Rear Sprung-Mass Displacement

K_t: Tire Stiffness

Z_{Rf}: Road Input Front

Z_{Rr}: Road Input Rear

Like the Quarter Car schematic this Half Car schematic also shows a relatively simpler model consisting of passive suspension systems. The constituents of the suspension system are also assumed to be linear for simplicity which is actually non-linear in nature. The Half Car model shown above can also be simplified to a 2- DOF system by representing the tire and suspension collectively in the form of Ride Rate. This simplification may give the basic idea about vehicle's bounce and pitch behavior, but the model may not be able to give complete details related to the movement of sprung masses. Different configurations of the Half Car can be modeled depending upon the information required for simulation. Increasing the components and including non-linearity improves the approximation of the system but requires more computing power and time.

2.3 System Inputs

Whenever a vehicle traverses a road, disturbances in the form of vibrations are transmitted all over the vehicle and reach the passengers. These vibrations may cause discomfort to the passengers and effect the functionality of several constituent assemblies of the vehicle specially the electronic counterparts. These vibrations are caused by road roughness and also the on board sources of the vehicle for example the engine [2]. However, the vibrations coming from the road are considered to be the most significant in defining the performance of a vehicle.

2.3.1 The Road

The input from road is the result of its roughness, and the presence of several irregularities like potholes, cracks and bumps etc. In vehicle dynamics the term 'Road Profile' refers to the road irregularities and is described as the elevation profile along the wheel tracks over which the vehicle travels [2].

According to [14], 'A profile is a two-dimensional slice of the road surface, taken along an imaginary line'. Road Profile is characterized as lateral and longitudinal according to the line along which it is taken. In vehicle dynamics problems, the longitudinal road profile is considered as it shows the design grade, the road roughness and its texture. Practically instruments called 'Road Profilers' are used to measure road roughness. These profilers generate a series of number representing the elevation profile, which are then fed to the computer for processing and generating a road profile.

Input from the road is stochastic in nature and falls in the category of Static Random Gaussian Process [17], therefore a road profile can be described either by profile itself or by its statistical properties [2]. Power Spectral Density (PSD) function is the most appropriate represenation for the statistical data of a road profile.

'PSD of a road profile is defined as a plot of the amplitudes versus spatial frequency'. Where, spatial frequency is expressed as the 'wavenumber' with units of [cycles/meter]. Roads are classified according to their PSD as Class-A to Class-E as smooth to rough as given in ISO 8608 standard.

In order to simulate the performance of a vehicle these road profiles need to be included in the mathematical model of the vehicle. One way to include the road model is to use the data from the profilers which is more accurate, otherwise approximate road models can also be generated in spatial and time domain using their PSD according to ISO 2613, which is also a very useful method.

Road profiles can be taken as Parallel Tracks over which the vehicle moves. The difference in the parallel tracks elevation generates the roll movements which is usually ignored in case of highway vehicles, where the roads are usually smooth and the difference in elevation between the two profiles is negligible. In this case the parallel track models are replaced by Single Track Model, where it represents only the bounce and pitch movement of the vehicle.

In this study only the bounce and pitch movements are under consideration, therefore, road is approximated as a Single Track Model and given to the Quarter Car and Half Models to study the Ride Performance.

2.4 System Outputs

Since bounce and pitch movements of the automobile are the outcomes of quarter car and half car models, therefore Bounce and Pitch are included in the outputs for the physical system under consideration.

As discussed earlier, ride response of a LMV is required to be evaluated in this study. According to [15], occupant comfort is directly associated with the ride performance of the vehicle and in order to quantify the ride perception of humans, it is possible to measure the ride characteristics of a road vehicle and relate it to available vibration evaluation mehtods. Therefore, in order to evaluate vibration severity motion needs to be weighted according to relevant importance of different characteristics of the excitation including the magnitude, the frequency, axis and duration.

According to ISO 2631 standard, Vibration Dose Value (VDV) is obtained by first measuring accelations at specific contact points of the occupant, then the acceleration cartesian components after decomposition using fourier series are multiplied by defined frequency weighting values and axis factors.

Where,

(2.2)

According to [18], "VDV is primarily a measurement procedure used to report the relative severity of complex vibration exposures, being preferred to other measures due to its use of the duration and variability of the motion."

Where, '$i = 1,2,3$' correspond to the three axis for the measurement of acceleration. And 'a_i' correspond to the respective components of accelerations along the three axis. Figure 2.5 shows the Ride Index Calculation, (Figure courtesy [15]).

As shown below in the figure 2.5 'a_x, a_y & a_z' signify the components of acceleration measured along the three axis and 'W_d & W_b' signify the frequency weighting factors and 'Axis Factor' is given the value of 1.0 as described in the ISO 2631 standard.

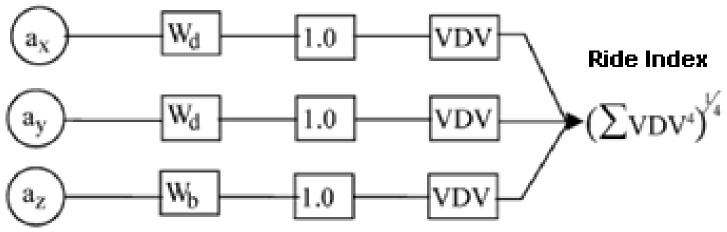

Figure 2.5: Ride Index Calculation

From all the discussion so far represented in this chapter, The Inputs, Outputs and the Physical System selected for our study are shown below in figure 2.6.

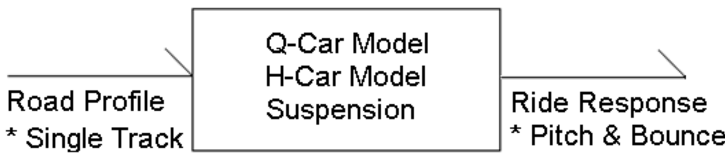

Figure 2.6: Physical System with inputs and outputs

CHAPTER 3: MATHEMATICAL MODELING

This chapter discusses different modeling techniques which are being applied to model the vehicle response. This chapter presents the system, the input and output in the mathematical form as defined in chapter 2, so that simulation of the vehicle can be carried out. The Quarter Car and the Half Car Models are derived in this chapter using these different modeling techniques and a comparison between these techniques is also presented in order to identify the most suitable modeling technique for this study. Road Model is also developed so that vehicle response can be evaluated in a more realistic manner.

3.1 Modeling Techniques

Modeling of Vehicle Systems generally fall in the category of multi body systems (MBS), consisting of finite number of interconnected rigid bodies [10]. In order to derive differential equations of such systems several modeling methods are traditionally available in classical mechanics, each with distinct formulation methodology. Although these traditional modeling techniques are readily applied for the modeling of physical systems, however it becomes difficult to model when the complexity of the system increases.

According to [6] modeling of large systems using Traditional Mathematical Modeling methods is laborious and error-prone, unless otherwise the methods are computerized.

The Bond Graph Method is now being frequently applied for modeling such systems because of several advantages it has over the traditional modeling techniques. This text uses both the techniques to model the Vehicle System, and presents a comparison between the two modeling techniques.

3.1.1 Traditional Modeling Techniques

Traditional Modeling Techniques consist of several Mathematical and Graphical Methods. The Newton's Method, the Lagrange Method, the Newton-Euler and the Euler-Lagrange Methods are well known Mathematical Modeling Methods.

The Newton's Method is based on the concept of balance in the forces and the moments. The Lagrange Method is based on the formulation of system's Kinetic Energy and Potential Energy. Euler Equations are based on conservation of Momentum and the Energy Methods are based on conservation of system's energy.

Several Graphical Methods are also applied to model Physical Systems as they are easily understandable and communicable to the individuals [8]. The Block Diagrams and The Signal Flow Graphs are examples of traditionally used graphical methods used for modeling physical systems.

3.1.2 Bond Graph Methodology

Bond Graph method is a graphical technique for modeling multi-domain dynamic systems and an essential feature of Bond Graph Method is its application for modeling multi-disciplinary engineering systems due to its capability of representing the interaction between different energy domains.

The main difference between the traditionally available Graphical Modeling Methods and the Bond Graph Method is that the Block Diagrams and the Signal Flow Graphs represent only the computational structure; however the Bond Graphs reflect the actual physical system, composed of subsystems, components or basic elements that interact by exchanging energy [8].

Bond Graph method represents a physical system using a small set of ideal elements combined through external ports representing power flow, thus giving information about the energy interaction between the system elements [10]. The elements can also be selected from different energy domains thereby allowing to model multi-domain systems, which is a significant advantage of the Bond Graph method over the traditional modeling techniques.

In order to show the power flow between the elements of a multi-domain system Bond Graph uses generalized power and energy variables. The energy variables associated with the energy storage elements are then selected as the State Variables to develop the State Space formulation for the system [9], which results in a more meaningful representation of a physical system unlike the traditional modeling techniques, where the choice of the state variables is dependent upon the modeler.

Using the Bond Graph method different sub systems or system components can be modeled separately and due to the modular nature can be incorporated quite easily into other systems.

Bond Graph method has widely been used in the modeling of vehicle systems, because of the advantages discussed above. According to [19] either four quarter-car models or two half-car models coupled together have been considered as a resultant full-car model.

Loucas [11] used the Bond Graph method to develop a model to predict the dynamic behavior of a truck, showing integration of quasi-static engine model with the drive train and the vehicle dynamics model. Then they used an algorithm to reduce a model with 55 elements to just 18 elements to achieve identical predictions with improved computational efficiency.

In Bond Graph Technique for linear mechanical systems Inertia element (I) is used for representing Mass, Resistor Element (R) is used for representing damper, Compliance (C) is used for representing spring with compliance taken as inverse or spring stiffness and Source of Flow (S_f) is used for representing the input from the road.

Directional Bonds are used to link the elements with each other, which show the instantaneous energy flow between them. Each bond has two power conjugated variables associated with it called, effort (e) and flow (f), Causal Strokes are then applied at the ends of the bonds to show the direction of flow of energy between the element ports [8].

Power = Effort × Flow

These Power Variables are used to express the power associated with different energy domains, i.e. in case of electrical systems the term Effort stands for Voltage and Flow stands for Current and their product yields the Power i.e. P=VI. In case of Mechanical Translational Systems, Effort represents Force while Flow represents Velocity. For Mechanical Rotational Systems Effort represents Torque and Flow represents Angular Velocity.

Energy is obtained by taking Integral of Power. Similarly, Energy Variables are obtained by taking Integral of Power Variables. Therefore, in case of Mechanical Translational Systems Momentum (p) and Displacement (q) are the Energy Variables obtained by taking Integrals of Force and Velocity respectively.

A fully augmented bond graph model is obtained when all the bonds are named in the graph (i.e., numbering of bonds), reference power directions are assigned and causal sense for the pair of effort and flow are assigned [9].

3.2 Mathematical Models Using Traditional Technique

In this section, the Quarter Car and the Half Car models are developed using the Newton's Method.

3.2.1 The Quarter Car Model

The Quarter Car Model is derived from the schematic of the Quarter Car shown in the figure 2.3. The Differential Equations for the Quarter Car Model developed using the Newton's method are:

- - **(3.1)**

- - **(3.2)**

By arranging the equations (3.3) & (3.4) in general matrix form:

(3.3)

We get,

- -
- - **(3.4)**

3.2.2 The Half-Car Model

The Half Car Model is derived from the schematic of the Half Car shown in figure 2.4. The differential equations for the Half Car Model developed using the Newton's method are:

- - - - - - **(3.5)**

- - - - - - - - **(3.6)**

- - - **(3.7)**

- - - - - **(3.8)**

By arranging the equations (3.5) to (3.8) in general matrix form, we get:

- - - - - - - - -

(3.9)

3.3 Mathematical Models Using Bond Graph Technique

In this section, the Quarter Car and the Half Car models are developed using the Bond Graph Method.

3.3.1 The Quarter Car Model

The Quarter Car Model is now represented using Bond Graph Technique in figure 3.1.

A fully augmented bond graph of a Quarter Car model is shown in figure 3.2.

State-Space Formulation is developed using Bond Graph Methodology by first identifying the Energy Storage Elements and then the Energy Variables associated with them are selected as State-Variables [9]. I-Elements and C-Elements are the Energy Storage Elements, whereas the momentum (p) and displacement (q) are taken as the Energy Variables for 'I' and 'C' elements respectively.

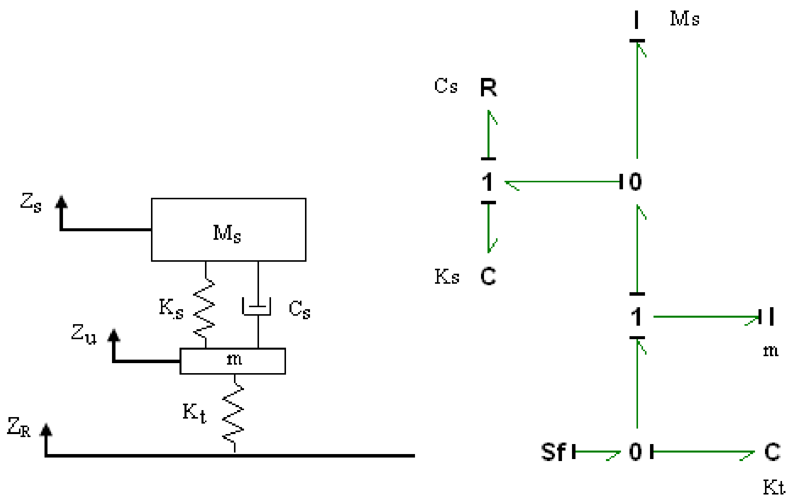

Figure 3.1: Schematic and Bond Graph of a Quarter-Car Model

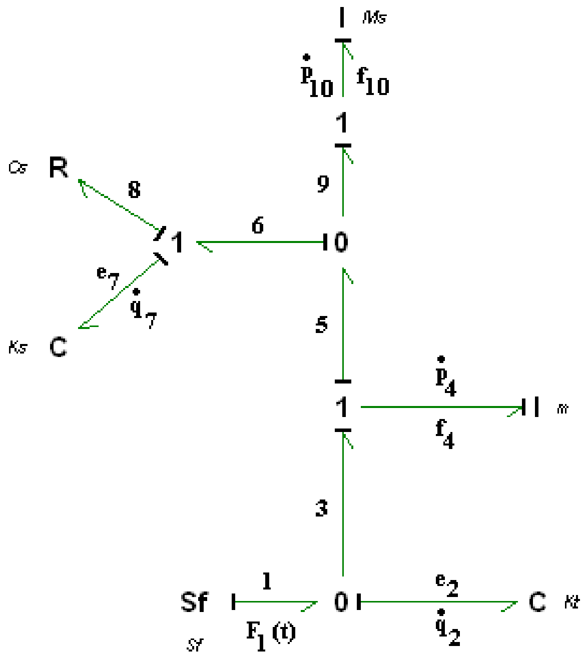

Figure 3.2: Fully Augmented Bond Graph of Quarter Car

For the Quarter Car model shown in figure 3.1, the State-Space Formulation is shown in equation (3.10). Where, State Variables are ‘p10’, ‘q7’, ‘p4’, ‘q2’ and input is F_1(t).

$$\begin{bmatrix} \dot{p}_{10} \\ \dot{q}_7 \\ \dot{p}_4 \\ \dot{q}_2 \end{bmatrix} = \begin{bmatrix} R_S/I_{MS} & 1/C_S & R_S/I_m & 0 \\ -1/M_S & 0 & 1/I_m & 0 \\ -R_S/I_{MS} & -1/C_S & -R_S/I_m & 1/C_t \\ 0 & 0 & -1/I_m & 0 \end{bmatrix} \begin{bmatrix} p_{10} \\ q_7 \\ p_4 \\ q_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} F_1(t)$$

(3.10)

p_{10} Momentum of Sprung Mass

q_7 Deflection of Suspension Spring

p_4 Momentum of un-sprung mass

q_2 Deflection experienced by tire due to its stiffness

3.3.2 The Half Car Model

Bond Graph of the Half Car model shown in figure 3.3 is complex and it is a tedious task to derive its equations like those for the Quarter Car Model as shown in equation (3.10). Therefore, the Bond Graph of Half Car is simulated in 20-sim software for ride analysis.

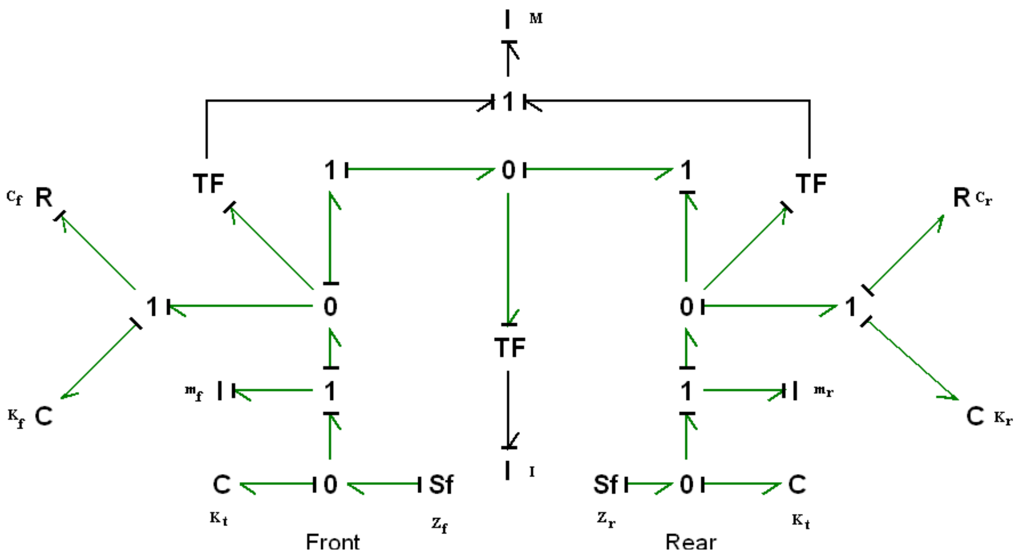

Figure 3.3: Bond Graph of a Half-Car Model

3.4 The Road Model

After modeling the physical system, now it is required to model the inputs to the system so that system can be analyzed for solving the problem. The primary input to the system as discussed earlier is the road, and that too in the form of a single track model.

3.4.1 Road Classification

As discussed in the chapter 2, road is stochastic in nature and is represented by its PSD. Also as per the definition of PSD, it is a function of wave number (spatial frequency). According to [2] & [7] the PSD of roads show a characteristic drop in magnitude with the wave number. According to [6] Random Road Profiles can be approximated by PSD as:

$$-^{-} \qquad \textbf{(3.11)}$$

Ω: Wave Number

w: Waviness

PSD(Ω_o): Value of PSD at reference wave number (Ω_o) [m^2 / (rad/m)]

$\Omega_o = 1$ [rad/m]

Where roads can be classified into classes from A to E by setting ω =2, according to the ISO 8608 standard. Each class is defined by the value of PSD(Ω_o), i.e. for

Class-A (Very Good) PSD (Ω_o) = 1×10^{-6} [m^2 / (rad/m)]

Class-B (Good) PSD (Ω_o) = 64.75×10^{-6} [m^2 / (rad/m)]

Class-C (Average) PSD (Ω_o) = 128.5×10^{-6} [m^2 / (rad/m)]

Class-D (Poor) PSD (Ω_o) = 192.25×10^{-6} [m^2 / (rad/m)]

Class-E (Very Poor) PSD(Ω_o) = 256×10^{-6} [m^2 / (rad/m)]

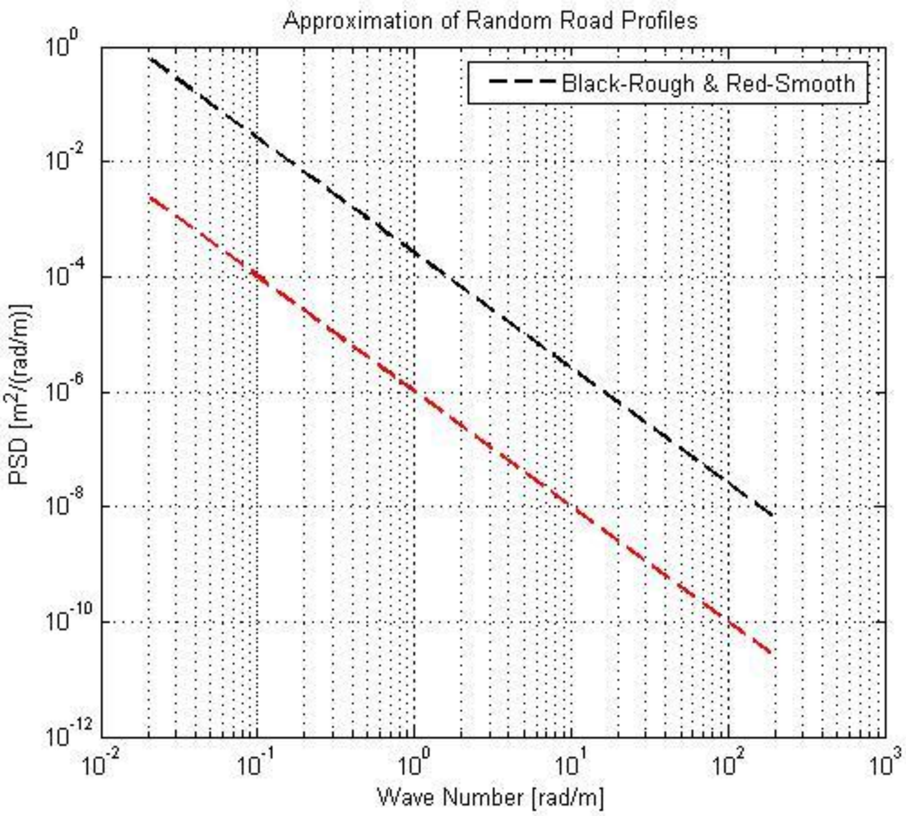

Figure 3.4: Road Classification as per ISO 8608

Figure 3.4 shows the classification of roads as smooth and rough as per ISO 8608 standard based on the corresponding reference PSD values. This figure also shows the relationship between the PSD and the wave number as discussed earlier, as the drop in road PSD can clearly be observed with the increase in wave number on the log-log scale.

The code for generating the plot shown in figure 3.4 is shown in 'Appendix A'.

3.4.2 Road Profile Generation

A random road profile can be approximated by a superposition of infinite sine waves with randomly varying parameters i.e. Amplitude, Frequency & Phase. Road Profiles can be generated both as a function of space and as a function of time. MATLAB is used to generate the road profiles by superposition of 200 sine waves of randomly varying parameters. Increasing the number of sine waves improves the approximation of road profile at the cost of processing time for the simulation; therefore, 200 sine waves are superimposed to generate the road profiles.

3.4.3 Random Road Profile as a function of Space $Z_R(s)$

\- **(3.12)**

φ: Phase Angle

N: Number of sine waves

(3.13)

And — **(3.14)**

For generating the random road profile A, φ & Ω are given random values based on the ranges determined below, and PSD (Ω_o) is given the value corresponding to the class of road. Length of the profile is taken to be 30 meters.

Road imposes frequencies to the vehicle in a wide range, but according to [2] the range of temporal frequency to study ride behavior of the vehicle is taken as [0 to 25] Hz. However, the temporal frequency range taken for profile generation is [0 to 100] Hz. i.e.

Temporal Frequency (f) = [0 to 100] [Hz]

Circular Frequency (ω) = 2πf [rad/sec]

Wave Number (Ω) = ω / v

v = vehicle speed

φ = [0 to 2π] [rad]

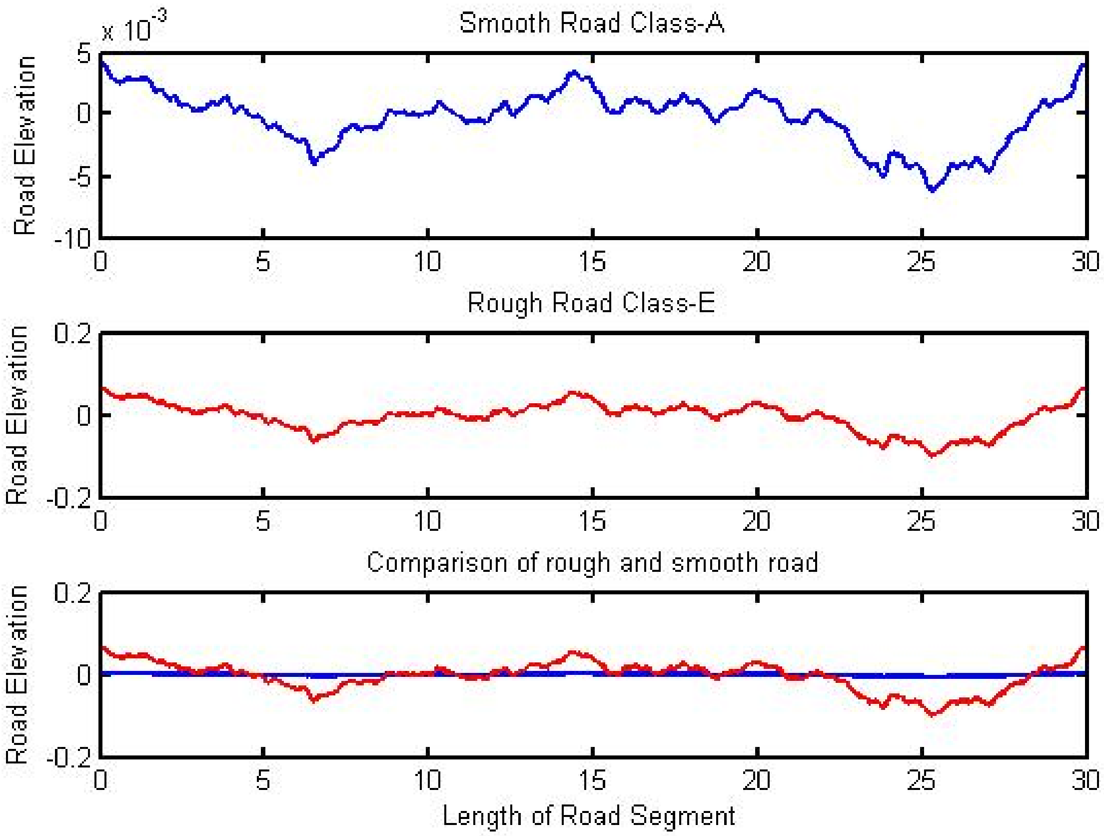

Figure 3.5: Road Profile as function of Space

Code for generation of road profile as a function of displacement is shown in 'Appendix B'.

3.4.4 Random Road Profile as a function of time $Z_R(t)$

Random Road Profile can also be obtained as a function of time in a similar fashion. In order to do that equation (3.12) needs to be changed from spatial to time domain. Such that the equation for road profile in time domain $Z_R(t)$ comes out to be:

- **(3.15)**

In equation (3.15) spatial frequency is replaced by circular frequency (ω) and distance (s) is replaced by time (t). Rest of the parameters is kept the same as in equation (3.12). For the same ranges as specified earlier random road profile is plotted using MATLAB for time duration of 30 seconds.

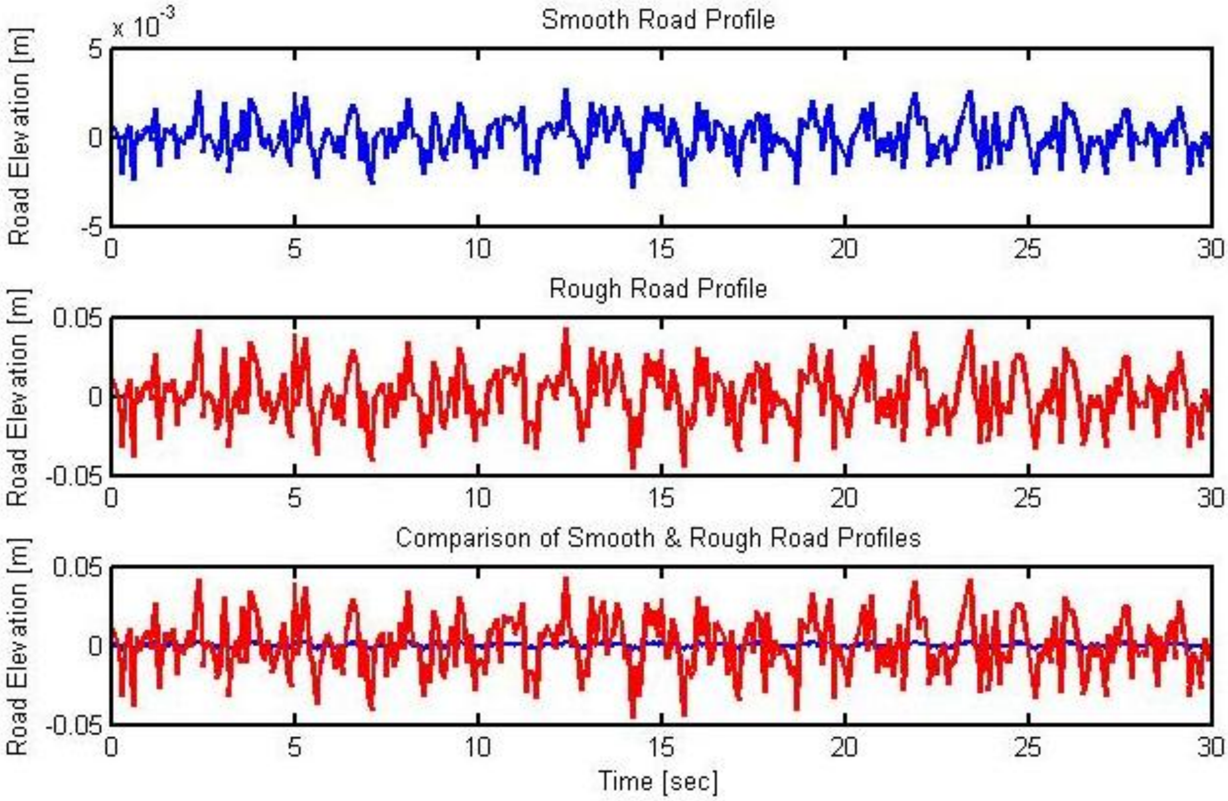

Figure 3.6: Road Profile as function of Time

Code for generation of road profile as a function of time is shown in ‘Appendix C’.

CHAPTER 4: NUMERICAL SIMULATION AND RESULTS

This chapter uses the so far developed models of the Quarter Car, the Half Car and the Road to simulate the ride behavior of a highway vehicle. Simulations are then carried out for studying the vehicle ride behavior when mass of the vehicle is reduced. Suspension parameters are adjusted to cater for ride degradation due to mass reduction and based on these simulations Framework for Designing a Low Mass Vehicle is also presented.

4.1 Vehicle Data

Vehicle data for simulation of ride behavior of a normal highway vehicle is taken from [10] for a Renault Clio RL 1.1 car.

Sr.	Parameter	Value
1	Kerb Weight	8100 [N]
2	Wheel Base	1.650 [m]
3	Distance from C.G to Front Axes	0.916 [m]
4	Distance from C.G to Rear Axes	1.556 [m]
5	Un-Sprung Mass at each wheel	38.42 [kg]
6	Suspension Stiffness	14 900 [N/m]
7	Suspension Damping	475 [N - sec/m]
8	Tire Stiffness	150 000 [N/m]
9	Pitch Inertia	2443.26 [Kg/m^2]

Table 4.1: Vehicle Data for a normal Highway Vehicle

From the data given in table 4.3, Sprung Mass can be calculated by subtracting the total un-sprung mass from the total mass of the vehicle.

Total Un-Sprung Mass	=	153.7	[Kg]
Total Sprung Mass	=	672.0	[Kg]
Sprung Mass for H-Car	=	336.0	[Kg]
Sprung Mass for Q-Car	=	168.0	[Kg]

4.2 Natural Frequencies

In order to study the ride behavior of the vehicle, first the Natural Frequencies of both the Quarter Car and Half Car models are required to be calculated, and then system response is checked for forced harmonic inputs with their frequencies matching the system's Natural Frequencies. Considering equations (3.4) & (3.9) for the Quarter Car and Half Car model respectively, Free Vibration Equations are obtained when all the inputs are taken as zero. Natural Frequencies are calculated from the free vibration equations when the damping is neglected, i.e. Un-Damped System [4].

Characteristic Equation of a system is given by [7]:

$$\det([K] - \omega^2 [M]) \tag{4.1}$$

Where, 'ω' refers to the Natural Frequency. For Quarter Car Model (2 DOF System) there are two Natural Frequencies, and for Half Car Model (4 DOF System) there are four Natural Frequencies.

Natural Frequencies for the Quarter-Car Model are calculated using MATLAB.

$\omega 1 = 9.42$ (rad/sec) = 1.5 Hz

$\omega 2 = 65.5$ (rad/sec) = 10.4 Hz

Here '$\omega 1$' refers to the Natural Frequency of the Sprung Mass, where as '$\omega 2$' refers to the Natural Frequency of Un-Sprung Mass, also called the Wheel Hop Frequency.

Natural Frequencies for the Half-Car Model are calculated using MATLAB.

$\omega 1 = 9.42$ (rad/sec) = 1.5 Hz

$\omega 2 = 6.30$ (rad/sec) = 1.0 Hz

$\omega 3 = \omega 4 = 65.5$ (rad/sec) = 10.4 Hz

Here, $\omega 3$ & $\omega 4$ have same value because front and rear suspensions have same stiffness and damping, and the wheels being used are also the same.

4.3 Simulating The Resonance

Now the Forced Response of the Quarter Car and Half Car models is plotted by taking sinusoidal Road Inputs. For Quarter Car $Z_R = 0.08 \text{ Sin } (\omega \times t)$, and for Half Car $Z_{Rf} = 0.08 \text{ Sin } (\omega \times t)$ and $Z_{Rr} = 0.08 \text{ Sin } ((\omega \times t)+d)$. Models developed from both the Traditional and Bond Graph Technique show same results.

The Phenomenon of Resonance Occurs when the Forced Input Frequency matches with the System's Natural Frequency. As can be seen from the plots shown in figure 4.1, figure 4.2, figure 4.3, figure 4.4, figure 4.5 and figure 4.6. The Displacement and Pitch Response Amplitudes of the Sprung Mass increase drastically when forced input frequency is taken equal to the Natural Frequency of the System.

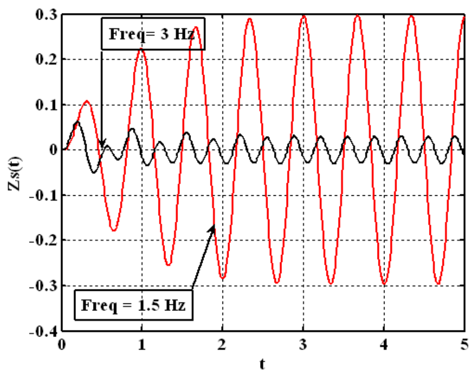

Figure 4.1: Zs of Quarter Car for ω1

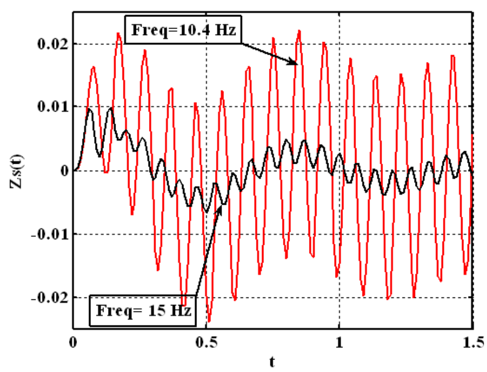

Figure 4.2: Zs of Quarter Car for ω2

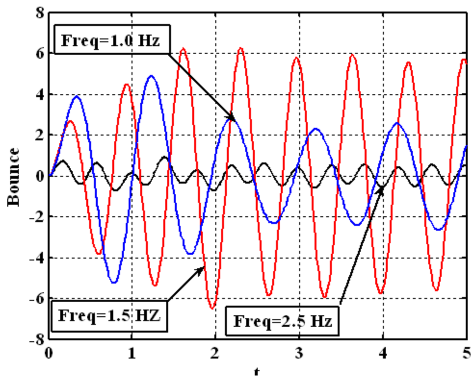

Figure 4.3: Bounce of H-Car for ω1 & ω2

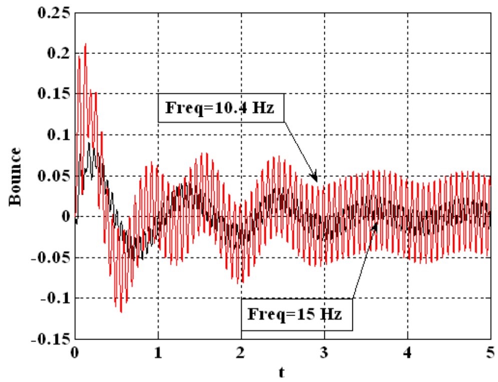

Figure 4.4: Bounce of H-Car for ω3 & ω4

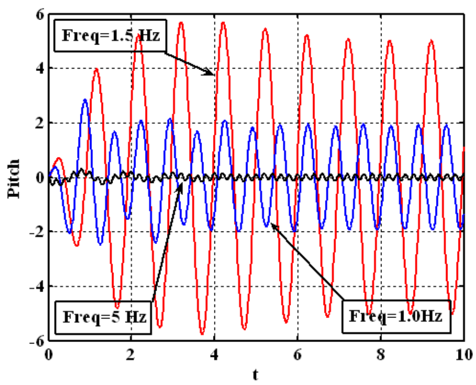

Figure 4.5: Pitch of H-Car for ω1 & ω2

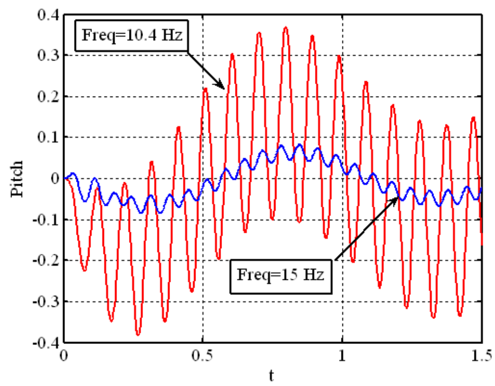

Figure 4.6: Pitch of H-Car for ω3 & ω4

MATLAB codes for generating resonance simulation of Quarter Car and Half Car are shown in 'Appendix D' and 'Appendix E' respectively.

4.4 Response of Quarter Car

System Response consists of two parts, the Transient Response (Natural Response) and the steady-state response (Forced Response). Transient Response dies out before the System attains its steady-state. In order to evaluate the transient response either transfer function or the state space model is required for the system.

4.4.1 Transfer Function

For frequency response analysis of Linear Time Invariant (LTI) and Single Input Single Output (SISO) systems, Transfer Function representation is the most convenient approach. Since in this problem the system is assumed as linear, therefore system analysis using Transfer Function approach can also be performed.

Transfer Function of the Quarter Car can be obtained by first taking the Laplace Transform of the Model Equations (3.2) & (3.2).

(4.2)

(4.3)

Eliminating $X_u(S)$ from equation (4.3) and (4.4) and obtaining $X_s(S)/X_r(S)$ gives the Transfer Function.

(4.4)

Once the Transfer Function is obtained, it can be entered into MATLAB to analyze the Transient and Frequency Response of the system.

4.4.2 Transient Response of the Quarter Car

Time Response of a system consists of Transient and Steady-State Response. Where the Transient Response is also called the homogenous solution of the system and dies out before the System attains its steady-state. Transient Response of the Quarter Car is plotted using a Unit-Step Input.

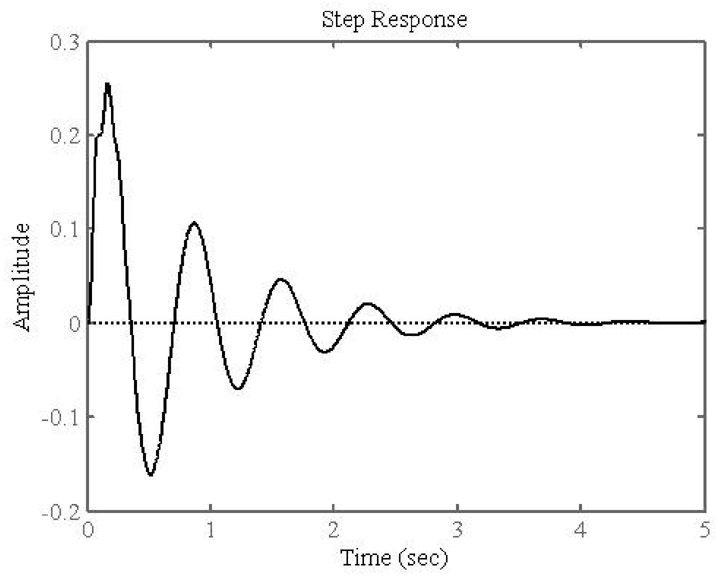

Figure 4.7: Step Response of Q-Car

Step Response of the Quarter Car is shown by figure 4.7 to show the Transient Response. This response contains the information like, Peak Time, Rise Time, Percent Overshoot and Settling time and gives the designer an idea about the speed with which the system attains its steady state. Transient Response will be discussed at the end of the chapter using 20-sim software. See Appendix F for MATLAB code to generate figure 4.7 and figure 4.8.

4.4.3 Steady-State Response of the Quarter Car

Stead-State response signifies the system behavior when time approaches infinity, i.e. when the transients have died out and the system responds to the forced input.

4.4.1 Transmissibility

Forced response of the Quarter Car on the said frequency range is thus required to get the basic information about the ride behavior of the vehicle. In order to achieve that, we need to plot the Response Gain (Amplitude Ratio of Sprung Mass to the Road Input) against the mentioned frequency range.

A closed-form solution of the two equations (3.1) and (3.2) for the steady-state harmonic motion is thus required [2]. Response Gain (Z_S/Z_R) is thus given as:

(4.2)

Equation (4.2) is complex in nature as it contains both real and imaginary terms. In order to obtain Response Gain to be plotted against the frequency, it is thus required to evaluate both the imaginary and real terms present in the numerator and the denominator. Therefore, the magnitude of numerator and denominator is determined by taking the square root of the sum of both the real and imaginary terms.

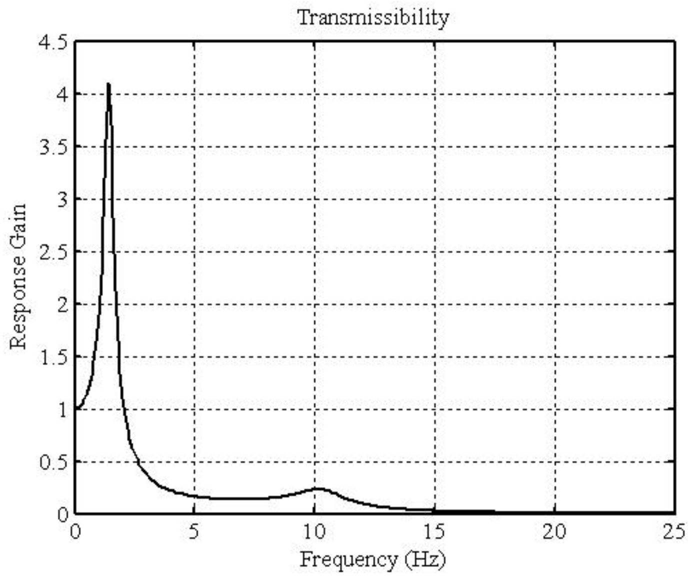

Figure 4.8: Response Gain Vs. Frequency for Q-Car Model

Plot of the Response Gain against the Road Input Frequency is shown in the figure 4.8. Where the first peak corresponding to '$\omega 1$' signifies the resonance of sprung mass, where as the second relatively smaller peak corresponding to '$\omega 2$' refers to the 'Wheel Hop Resonance'.

As can be seen from the figure 4.8 the response gain at '$\omega 1$' is greater than that at '$\omega 2$' corresponding to the response gains of sprung and the un-sprung mass respectively.

Response Gain for the dynamic system is also called the "Transmissibility". According to [2]:

"Transmissibility is the non-dimensional ratio of response amplitude to excitation amplitude for a system in steady-state forced vibration."

4.4.4 Frequency Response of the Quarter Car

The Quarter Car model provides a basic insight into the vehicle ride behavior. As the vehicle travels on a road it encounters vibrations in a wide frequency range. In order to study the ride behavior of the vehicle, road vibrations in the range of [0 to 25 Hz] are usually considered [2].

The computer code for the frequency response of Quarter Car is presented in ‘Appendix F’.

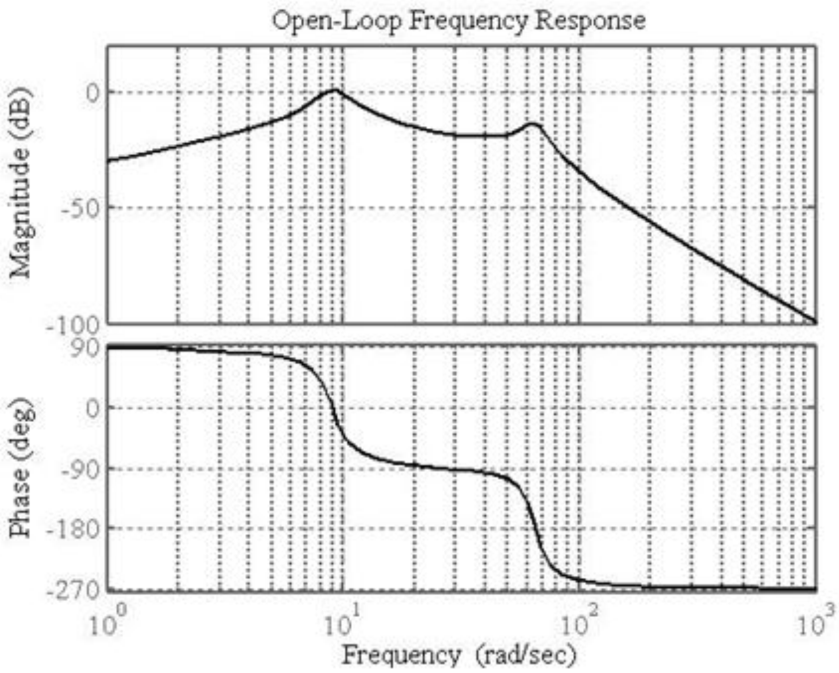

Figure 4.9: Bode Diagram of Q-Car

The Bode Diagram shows the system response in terms of system gain and phase shift. Where system gain is the ratio of output to input amplitude and phase shift is the difference between the phase of output and input. The two plots shown in figure 4.9 represent the Gain Frequency Response and the Phase Frequency Response.

For the system under consideration the information regarding the gain frequency response is important rather than the phase frequency response. Therefore, in rest of the text, phase frequency response will not be considered.

The first plot of the figure 4.9 shows the gain frequency response of the system, showing Magnitude ratio in terms of dB, where 1 dB = 20 $\log_{10}$ (x), and the frequency on a log scale. The plot shows increase in the system gain for low frequency range and drop in gain for high frequency range. The two peaks show the location of system natural frequencies as calculated earlier. The response gain curve can be changed by manipulating sprung mass and suspension stiffness as bode plots do not represent the damping of the system.

4.5 Effect of Mass Reduction on Vehicle Ride Performance

For an automobile the ride response mainly depends on its natural frequencies, where the natural frequencies are determined based on system mass and stiffness. Natural Frequency is given by the formula:

(4.6)

From equation (4.6) it is obvious that natural frequency is directly proportional to Stiffness and is inversely proportional to Mass. According to [2], in order to achieve good ride performance for a normal highway vehicle the range of '$\omega 1$' should be between [1.0 – 1.5 Hz] and '$\omega 2$' should be around 10 [Hz]. Therefore, for varying sprung mass the value of '$\omega 1$' should remain in the prescribed range of '$\omega 1$'.

There is another very important factor which needs to be addressed properly while studying the vehicle response. That is the 'Static Deflection' of the Suspension, which is basically responsible for defining the ride height of the vehicle. Static Deflection (SD) is given by:

(4.7)

Basically Static Deflection stands for the allowable stroke for the suspension to bear the sprung mass. From the equation (4.7) it is obvious that for a given suspension stiffness it depends on the sprung mass of the vehicle. For normal highway vehicles the load capacity is less as compared to the heavy vehicles, therefore, suspension stroke required is less as compared to the heavy trucks. According to [2] the value of Static Deflection for normal highway cars should be kept around [5 – 6 in], whereas for heavy vehicles like trucks it is kept around 10 inches.

For the given vehicle data the value of SD comes out to be 4.35 [in] at its kerb weight. As the payload of vehicle increases, natural frequency of its sprung mass decreases as depicted in equation (4.6). There is a limit to the maximum value of the payload which refers to the fully laden condition of the vehicle. The maximum value of the payload should be such that the value of '$\omega 1$' is not less than 1.0 [Hz] and the value of SD is not more than 6 [in].

Using equation (4.6), putting in the value of sprung mass natural frequency equal to 1.0 [Hz], the value for ‘Ms’ for the Quarter Car turns out to be 377 [Kg]. Original value of sprung mass was 168 [Kg], this means a payload of 209 [Kg] (for Q-Car) will cause the value of sprung mass natural frequency to decrease to 1.0 [Hz].

Now using equation (4.7) and putting in the value of SD equal to 6 [in], the value of ‘Ms’ comes out to be 231.5 [Kg], this means a payload of 63.5 [Kg] (for Q-Car) is the maximum allowed for the vehicle. For Full Car maximum payload allowed is 254 [Kg]. It is a general practice to design the vehicle for the fully laden condition, therefore at its fully laden weight the natural frequency of its sprung mass should be less than or equal to 1.0 Hz and the value of SD should be equal to 6 [in].

Now let us plot the response curves for the Quarter Car for kerb mass and fully laden mass. For fully laden weight the value for sprung mass natural frequency reduces to 1.27 [Hz].

System response for its kerb weight and for its fully laden condition is represented in figure 4.10, figure 4.11 and figure 4.12.

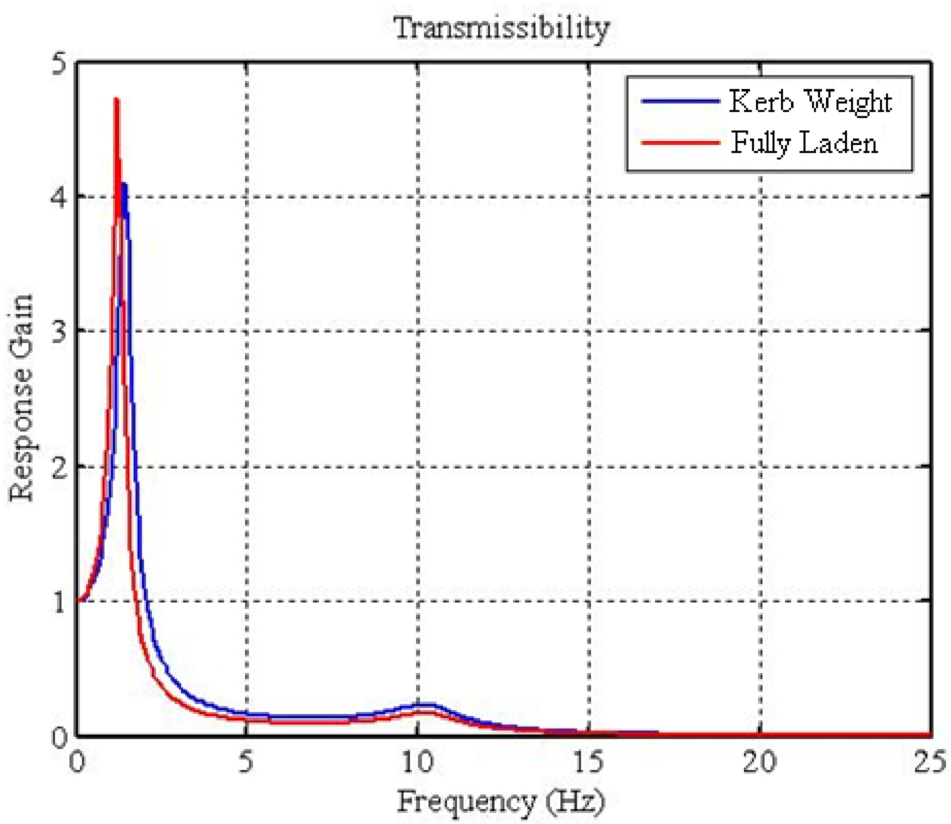

Figure 4.10: Transmissibility for Kerb and fully Laden Weight

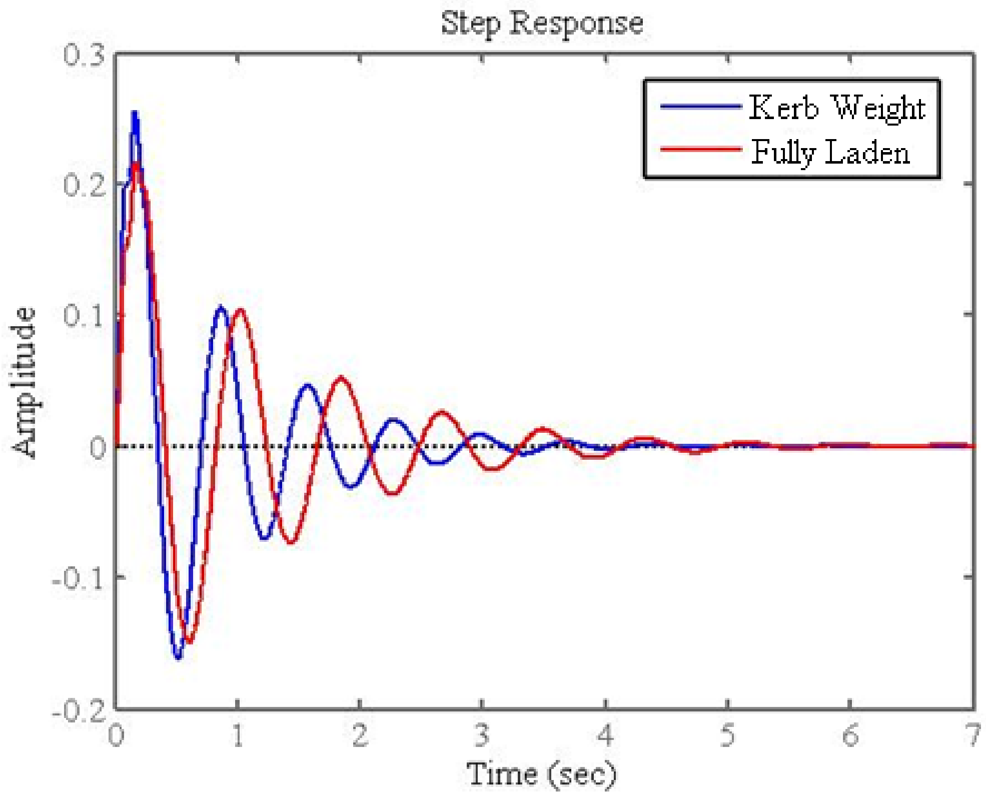

Figure 4.11: Step Response for Kerb and Fully Laden Weight

The 'blue curve' is the response curve for the vehicle kerb weight and the 'red curve' is the response curve for fully laden weight. Figure 4.10 shows that at fully laden weight '$\omega 1$' has reduced, and the gain value has increased followed by sudden damping and ride improvement. It is also obvious that response gain at wheel hop resonance has also decreased.

Figure 4.11 shows that the amplitude level has decreased for increased payload and the settling time has increased mainly because of the increased system inertia.

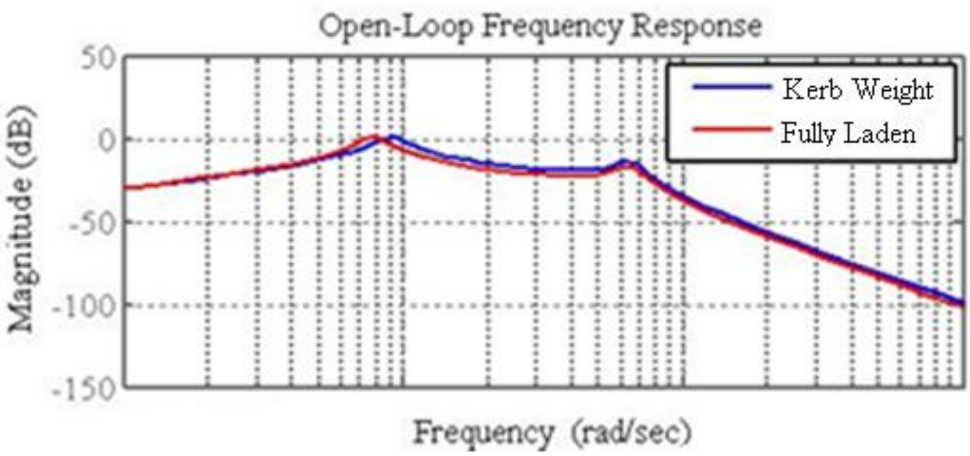

Figure 4.12: Bode Diagram for Kerb and Fully Laden Weight

Let us now focus on the Low Mass Vehicle. In a LMV the weight of the vehicle is reduced by several means to basically improve fuel efficiency, but with implications on its ride performance.

Reduction	Ms [Kg]	Ks [N/m]	ω1	ω2
	168.0	14900	1.49	10.42
10% Ms	151.2	14900	1.58	10.42
20% Ms	134.4	14900	1.67	10.42
30% Ms	117.6	14900	1.79	10.42
40% Ms	100.8	14900	1.93	10.42
50% Ms	84.0	14900	2.12	10.42

Table 4.2: Effects of Mass Reduction on System Natural Frequencies

As discussed earlier the ride performance basically depends upon the value of system natural frequencies, now let's see what happens to the ride performance when the sprung mass is reduced.

As can be seen from table 4.2 that 'ω1' changes with change in 'Ms' this is mainly because the fact that 'ω1' represents the natural frequency of the sprung mass and its value is decreasing by decreasing the sprung mass value and keeping the suspension stiffness constant. Also by decreasing 'Ms' the ratio of sprung to un-sprung mass has also changed, because only the sprung mass value has changed and the un-sprung value is kept constant. This is the reason for the constant value of 'ω2'. In order to keep the value of 'ω1' within the prescribed range of [1.0 to 1.5 Hz] suspension stiffness needs to be reduced. The prescribed value of 'ω2' is 10.0 Hz therefore, un-sprung mass is not changed.

4.6 Suspension Parametric Study for a Low Mass Vehicle

Suspension consists of springs, dampers and anti-sway bars. As discussed in first chapter this study is limited to the ride performance therefore only springs and dampers will be considered.

4.6.1 Tuning Suspension Stiffness

Table 4.2 clearly shows that reduction in sprung mass causes the sprung mass natural frequency to shift beyond the allowable limit of 1.5 [Hz]. According to equation (4.6) the sprung mass natural frequency will decrease if stiffness is reduced. This means if the value of sprung mass is decreased then in order to maintain ride performance the suspension stiffness must be reduced. But according to equation (4.7), SD increases if 'Ks' reduces, however, if both 'Ms' & 'Ks' are reduced in same proportion then the value of SD is not changed.

Reduction	Ms [Kg]	Ks [N/m]	ω1	ω2	Ms/Mu
	168.0	14900	1.49	10.42	4.37
10%	151.2	13410	1.50	10.38	3.93
20%	134.4	11920	1.50	10.33	3.49
30%	117.6	10430	1.50	10.28	3.06
40%	100.8	8940	1.50	10.23	2.62
50%	84.0	7450	1.50	10.19	2.18

Table 4.3: Adjustment of Suspension Stiffness to Compensate Vehicle Mass Reduction

Table 4.3 depicts the values of 'Ks' in order to compensate for reduced sprung mass, these dropping values do not affect the Static Deflection of the Suspension, as both 'Ks' and 'Ms' are reduced in same proportion. If the value of 'Ks' is decreased by a percentage that is more than that of 'Ms' then it is a possibility that SD may accede the prescribed range and vehicle's handling gets affected.

Now it will be interesting to see the response curves for original and reduced values of 'Ms' & 'Ks'. Referring to the values of 'Ms' & 'Ks' given in "Table 4.3", Transmissibility and Step Response of the Quarter Car is plotted, shown in figure 4.13 & figure 4.14 respectively.

Where the black curve shows the original response, the blue curve shows the response for the 30% reduction of 'Ms' & 'Ks', and the red curve shows the response for the 50% reduction of 'Ms' & 'Ks'.

Although the adjustment of 'Ks' has resulted in achieving the values of 'ω1' and 'ω2' similar to the original response as shown in figure 4.7, however, the ride response characteristics seem to be different. As can be seen from figure 4.13, the ride response has deteriorated for the frequency range [2.0 - 12.0 Hz]. Also the transmissibility at wheel hop resonance has increased.

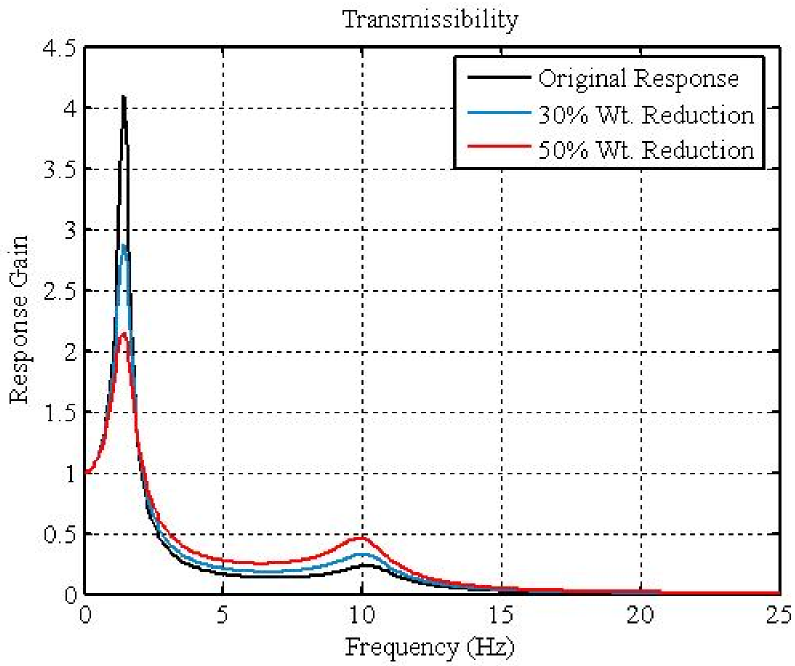

Figure 4.13: Transmissibility Curve for ‘Ks’ adjustment to compensate ‘Ms’ reduction

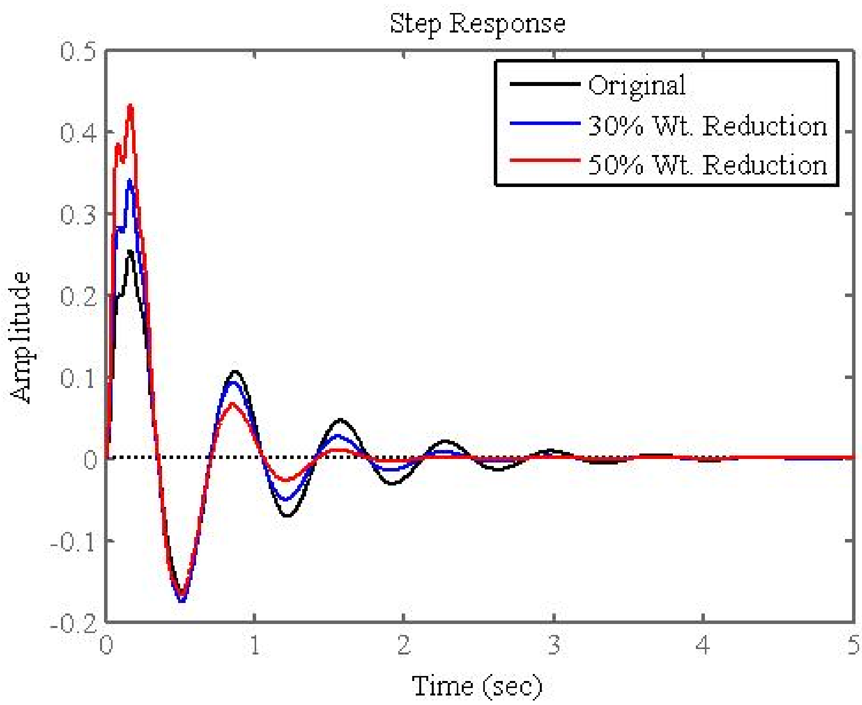

Figure 4.14: Step Response for ‘Ks’ adjustment to compensate ‘Ms’ reduction

4.6.2 Tuning Suspension Damping

Let us have a look at the ride response of the Quarter Car by changing the value of suspension damping. Considering the data of the vehicle for no weight reduction, plots are taken for increasing and decreasing suspension damping ‘Cs’ by 30%.

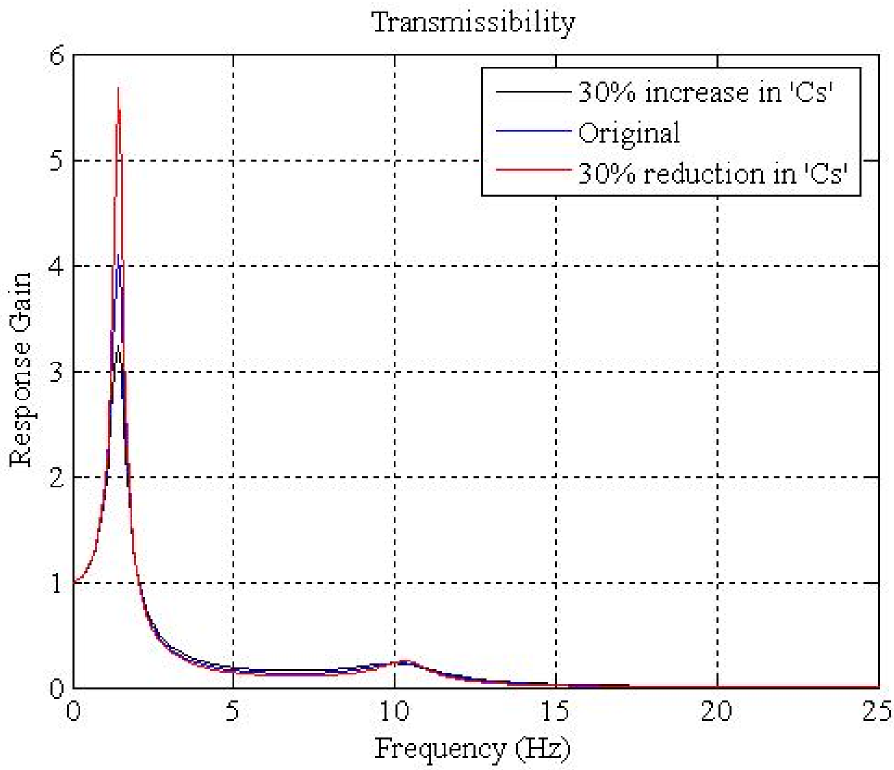

Figure 4.15: Adjustment of suspension Damping and Transmissibility

As can be seen from figure 4.15, increasing or decreasing the damping of Suspension does not really affect the natural frequencies; however the amplitudes of the response gains are affected. Increasing the damping causes the amplitude of sprung mass response at '$\omega 1$' to reduce, also attenuation is seen in the range of [2.0 – 10.0 Hz].

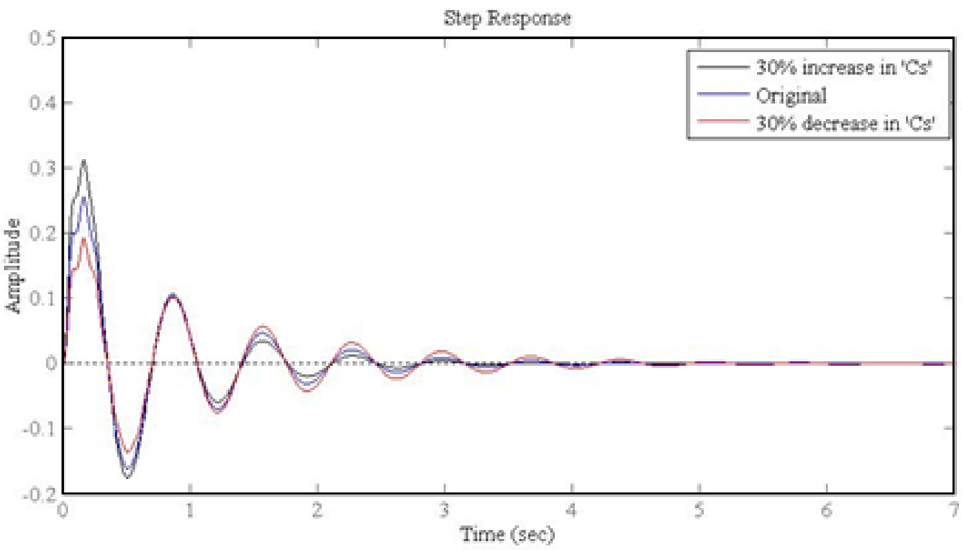

Figure 4.16: Adjustment of suspension Damping and Step Response

From figure 4.16 it is observed that increasing the suspension damping reduces the settling time.

Referring to the red response curve of figure 4.13 representing the plot for data of 50% weight reduction as shown in table 4.3, as discussed earlier there is ride degradation. Let us see if response can be improved using the 'Suspension Damping'.

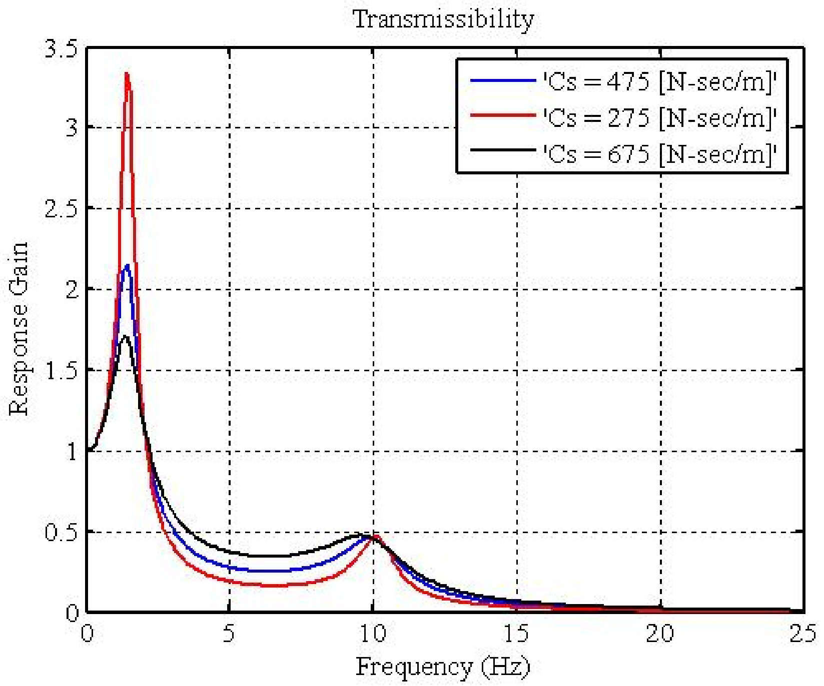

Figure 4.17: Adjustment of Suspension Damping to Cater for Mass Reduction

As can be seen from figure 4.17, increasing the value of 'Cs' continues to degrade the ride response in the frequency range of [2.0 to 12.0 Hz] however Response at 'ω1' has improved. On the other hand decreasing the value of 'Cs' results in improving the response in the frequency range of [2.0 to 12.0 Hz] however the response at 'ω1' and 'ω2' has degraded. Also it can be seen that increasing suspension damping results in slight shift of 'ω2' towards 'ω1'.

This phenomenon is due to system's Damping Ratio. Where Damping Ratio 'ξ' is defined as:

(4.8)

Before the mass reduction was carried out the value of damping ratio was 'ξ = 0.15'. As the values of 'Ms' and 'Ks' were decreased by 30% the value of damping ration increased to 'ξ = 0.21' and when the values of 'Ms' and 'Ks' were reduced to 50% the value of damping ration had increased to '$\xi = 0.3$'. That is why its value was decreased to obtain improved ride response. The adjusted value of 'C_s' resulted in a damping ratio of '$\xi = 0.17$'

4.7 Simulating Ride Response of the LMV

In light of all the discussion and analysis presented above, let us now address the Ride Response of a Low Mass Vehicle. Table 4.1 presents the data for a normal highway vehicle. Its response is presented in figure 4.7, figure 4.8 & figure 4.9. The response will be treated as a reference, and the said vehicle will be converted to a Low Mass Vehicle and the response of the LMV will then be adjusted with respect to this reference.

As discussed earlier, generally the highway vehicles are designed for best ride performance for fully laden conditions. Therefore, let us consider the fully laden conditions of the considered vehicle.

Payload (PL) = 260 [Kg]

Sprung Mass (Ms) = 240 [Kg]

Un-Sprung Mass (Mu) = 38.42 [Kg]

Suspension Stiffness (Ks) = 8000 [N/m]

Ms/Mu = 1.56

$\xi = 0.2$

For the Quarter Car Model, Natural Frequency of the Fully Laden Mass is calculated using equation (4.6) and the SD is calculated using equation (4.7) by plugging in the value of mass as sum of sprung mass and payload.

$\omega 1 = 1.27$ [Hz]

SD = 0.1532 [m] ~ 6 [in]

This shows the vehicle being considered satisfies the conditions for the range of '$\omega 1$' and 'SD' for the fully laden condition.

Now let us consider the Kerb Weight conditions for the vehicle by plugging in the value of sprung mass in the equation (4.7).

$\omega 1 = 1.83$ [Hz]

This shows that for kerb mass the condition for 'ω1' is violated. Such a vehicle is called the Low Mass Vehicle, which in fully laden condition satisfies the condition for 'ω1' but violates for the kerb weight condition.

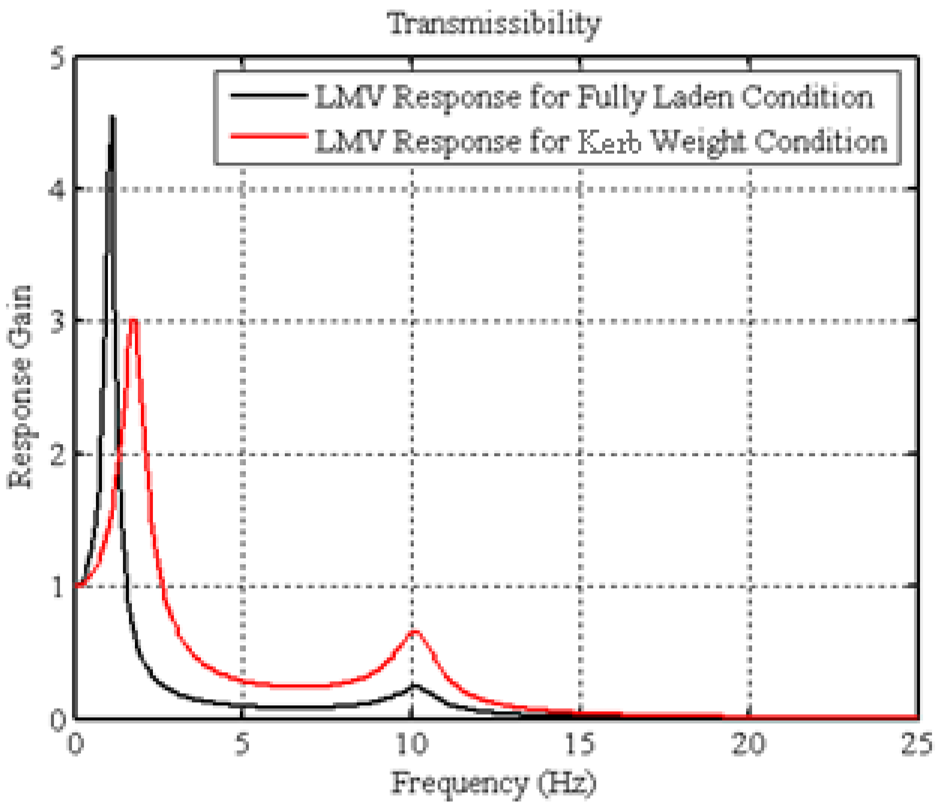

Figure 4.18: Response Gain Comparison of LMV for Fully Laden and Kerb wt.

Figure 4.18 shows the response comparison of the LMV for the Fully Laden and Kerb Weight Conditions. Black curve shows the response for fully laden condition. As can be seen that for kerb weight condition the value of 'ω1=1.83 [Hz]', which is a violation of the allowable range for 'ω1'. The response of the vehicle shows significant degradation for kerb weight condition as can be clearly seen in the figure 4.18. The figure depicts the degradation in ride response with decrease in payload, which is a result of change in Damping Ratio as shown by the equation (4.8).

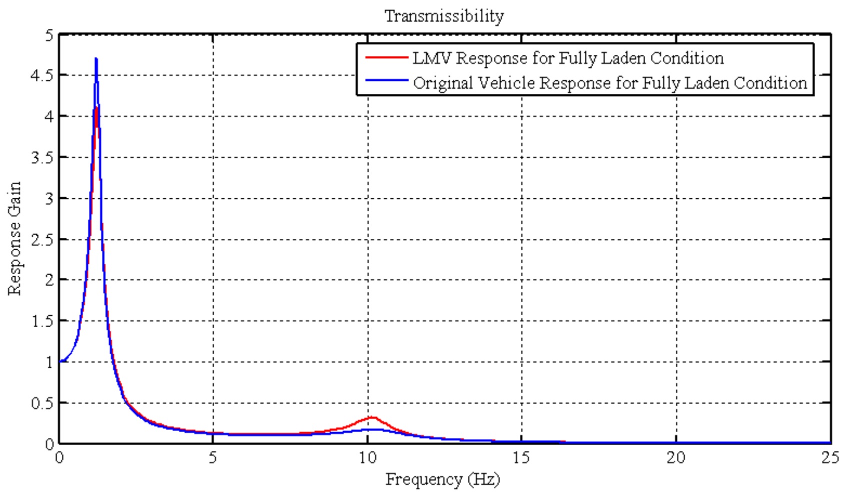

Figure 4.19: Response Gain Comparison for Fully Laden Condition

Figure 4.19 shows a clear match in the response curves of the original and the LMV for the Fully Laden Conditions except for the response at ‘ω2’. For LMV the value of response at ‘ω2’ is greater as compared to the original vehicle. Response at this point can be improved by implementing an active suspension.

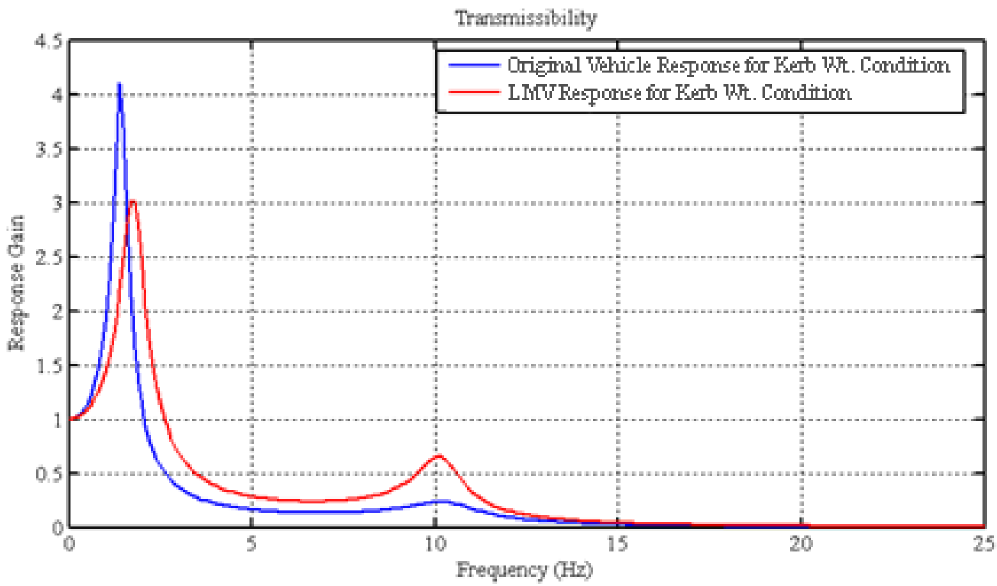

Figure 4.20: Response Gain Comparison for Kerb Weight Condition

Figure 4.20 shows the response gain comparison of the kerb weight condition for the original and the LMV. As it is clearly seen that the response curve of the LMV for kerb weight condition is nowhere close to the original vehicle’s response. In order to improve the response for LMV at kerb weight, passive suspension system does not seem to work, and solutions like active and semi-active suspension need to be studied for improving the response for given conditions.

4.8 Time Response of Quarter Car of Original Vehicle for Random Road Input

As random road profile has been modeled in the previous chapter, now this profile can be used as an input to the Quarter Car State Space Model and the responses of the original and LMV can be compared.

Figure 4.21 shows the Time Response of Quarter Car of the Original Vehicle for Random Road input. Here 'Z_r(t)' shows the Random Road Input as modeled in the previous chapter, in this plot the road input is taken for Class-A road i.e. the smooth road. Since we are dealing with a ride problem and that too of a highway vehicle therefore, only smooth road will be considered as an input to the system. Rough road input is worth considering for problem involving the handling of the vehicle. 'Z_u(t)'is the un-sprung displacement caused by the random road Input and 'Z_s(t)' is the sprung mass displacement.

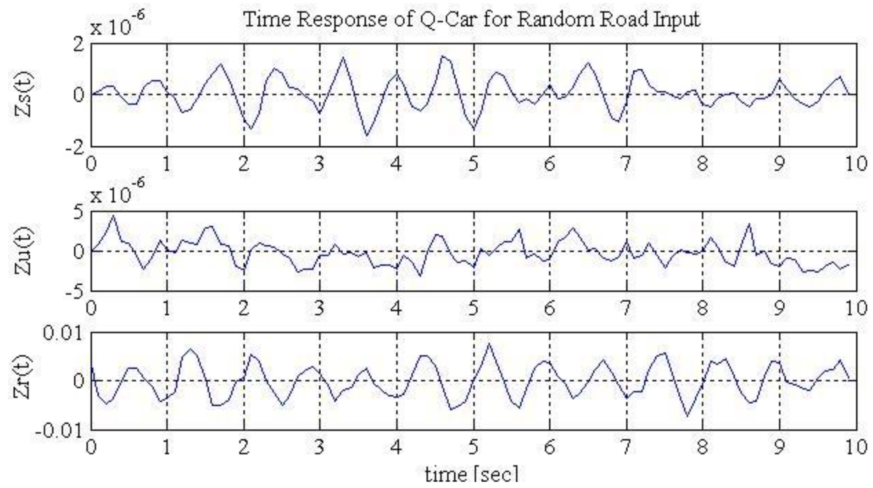

Figure 4.21: Time Response of Original Vehicle for Random Road Input

4.9 Time Response of Quarter Car of LMV for Random Road Input

Now time response of the LMV is simulated for Fully Laden and Kerb Weight Conditions for random roads. Where the blue curves show the response for fully laden condition and the red curve shows the response for kerb weight condition.

As calculated earlier, for fully laden condition sprung mass for the Quarter Car including the total payload is taken equal to 125 [Kg], and for the kerb weight condition the sprung mass is taken equal to 60 [Kg]. Suspension Stiffness and the Suspension Damping remain constant at 8000[N/m] and 275 [N-sec/m] respectively.

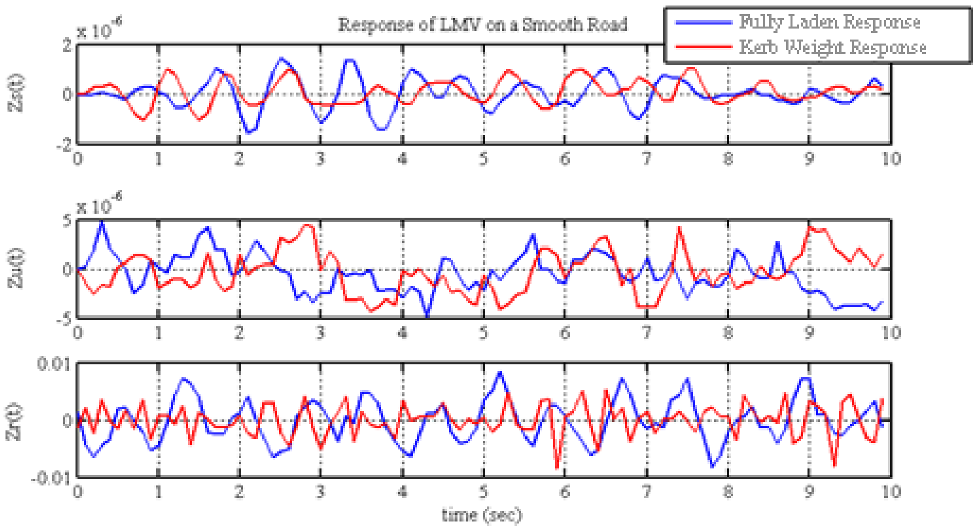

Figure 4.22: Time Response of LMV for Smooth Road

As can be seen from figure 4.22 the two responses are different from each other. However, the difference cannot be quantified using this information as figure 4.20 shows clear difference between the fully laden and kerb weight response of the LMV. Statistical tools need to be implemented to quantify these results. Bond Graph method can be used to retrieve more details from the two responses.

4.9 Quarter Car Response Using Bond Graph

Now Bond Graph Method will be used for simulating the Quarter Car response for the Low Mass Vehicle. Bond Graph has the capability to solve complex systems, which cannot be dealt easily using the traditional techniques therefore; Bond Graph technique will also be used to simulate the Half Car Model.

MATLAB code for generating the figures 4.21 and figure 4.22 is given in Appendix G.

4.9.1 Transient Response

First let us discuss the transient response of the original vehicle as mentioned earlier in section 4.4.2. Bond Graph of the Quarter Car is already shown in figure 3.1 and figure 3.2. Now in order to simulate the Quarter Car response for Random Road Input the amended Bond Graph is shown in figure 4.23. A random source is added to the model as shown below.

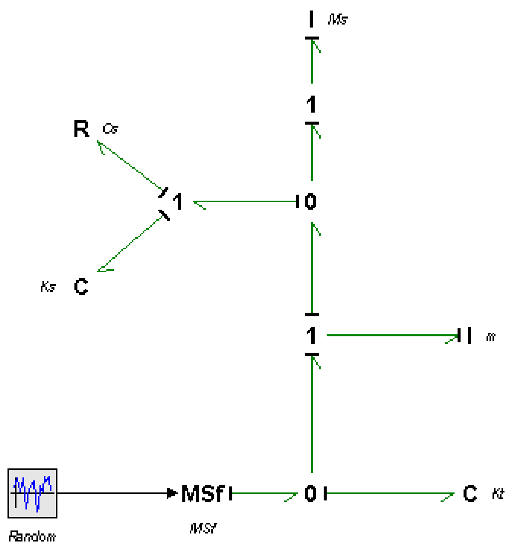

Figure 4.23: Bond Graph of Q-Car for 20-sim Software

In order to formulate the Transient Response this bond graph model is input to the 20-sim software. By giving the values to the elements vehicle is defined. First the data for original vehicle is entered as shown in table 4.1. Then the frequency response is defined in the simulator of the 20-sim software. Velocity of the Random Source is taken as the Input and the velocity of the sprung mass is taken as the output. 20-sim software then generates a Transfer Function and the state-space of the Quarter Car, which we have already developed in sections 4.4.2 and 3.3 respectively.

20-sim generates the Step Response of the said vehicle using the entered data as shown in figure 4.23.

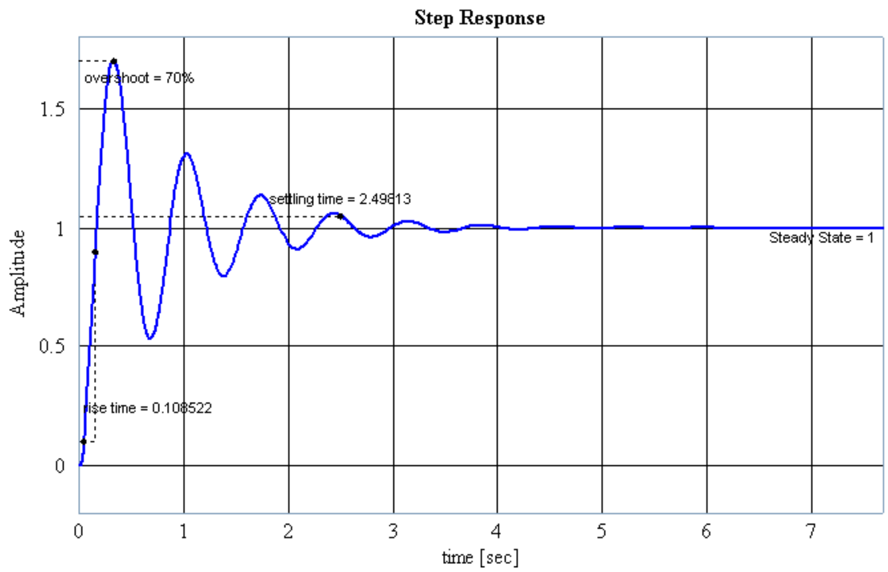

Figure 4.24: Step Response of Original Vehicle Q-Car Model using Bond Graph

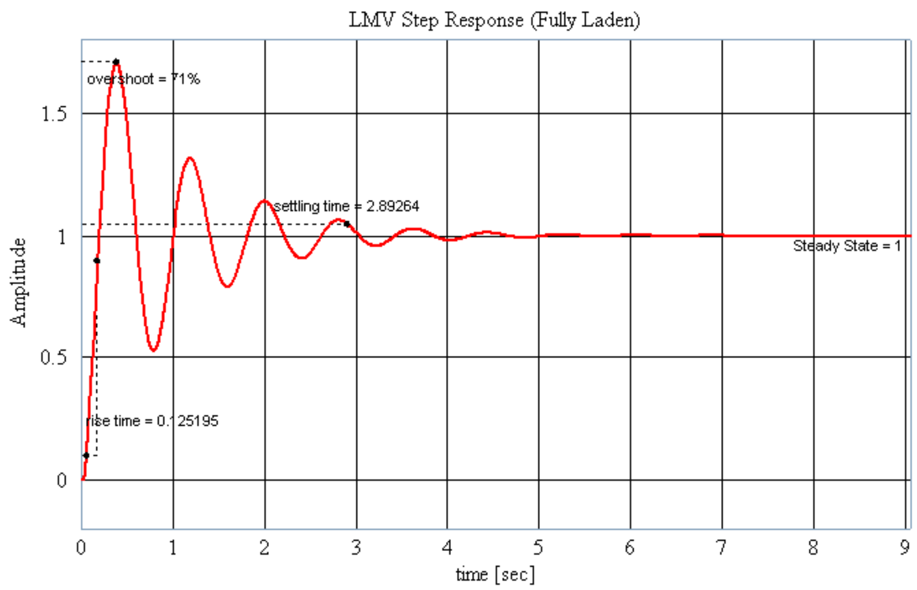

Figure 4.25: Step Response of LMV Q-Car Model for Fully Laden Condition

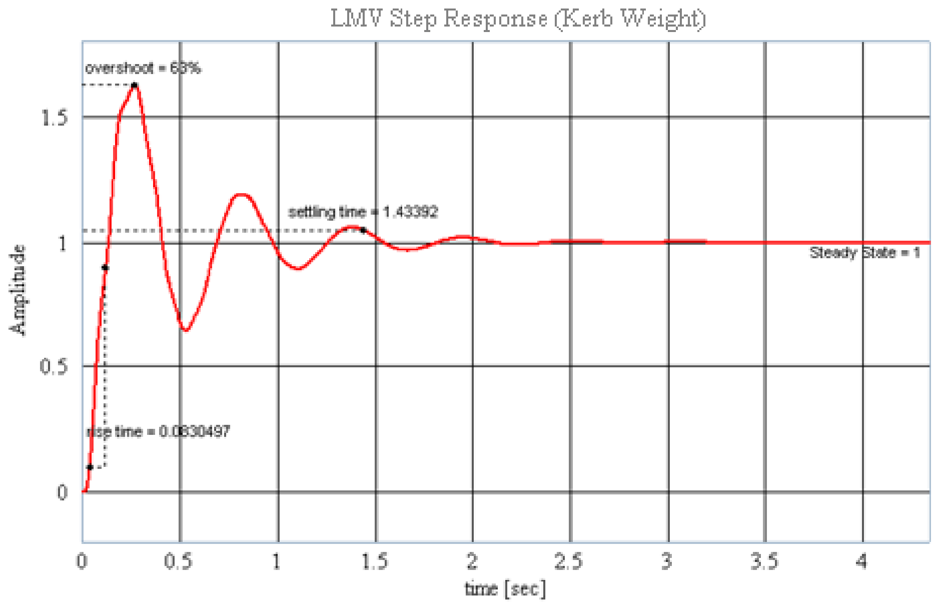

Figure 4.26: Step Response of LMV Q-Car Model for Kerb Weight Condition

Transient responses shown in the three plots figure 4.24, figure 4.25 and figure 4.26 are compared in the table 4.4. As can be seen clearly from the table and the figures that the transient response of the Original and the LMV (Fully Laden) is similar, where as that of the LMV (Kerb Weight) is significantly different.

Transients of the LMV (Kerb Weight) die out very quickly as compared to the other two responses.

Transient Response	Original Vehicle	LMV (Fully Laden)	LMV (Kerb Weight)
Rise Time [sec]	0.108	0.125	0.083
Settling Time [sec]	2.49	2.89	1.434
% OS	70 %	71 %	63 %

Table 4.4: Comparison of Transient Response of Original and LMV

4.9.2 Response of LMV Quarter Car for Random Road Input

Using 20-sim software an accelerometer is attached to the sprung mass to measure the vertical accelerations resulting from random road input. Referring to the figure 4.22 the amplitude for smooth road is in the range of ± 0.01 m. Now the resulting bond graph is shown in figure 4.27.

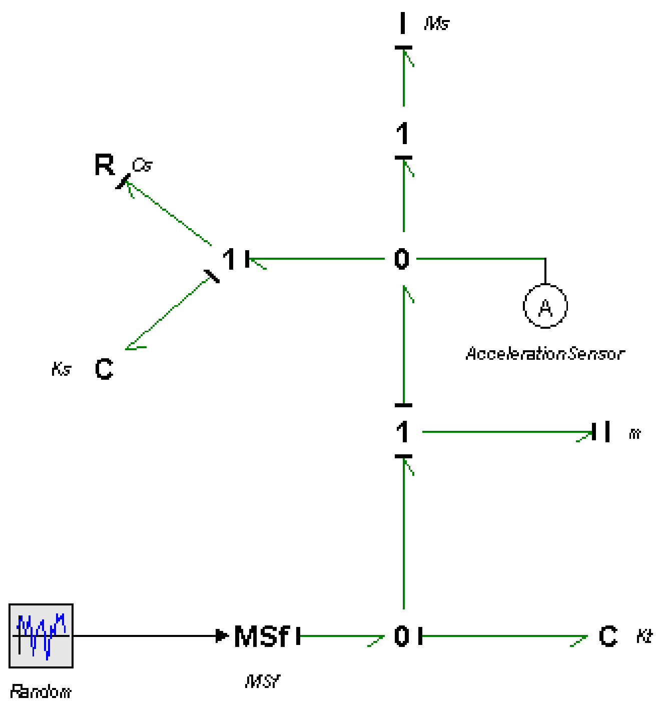

Figure 4.27: Bond Graph for measuring Accelerations on Sprung Mass

Entering the vehicle data for fully laden condition of the LMV, the following acceleration response is observed using the 20-sim software or the sprung mass.

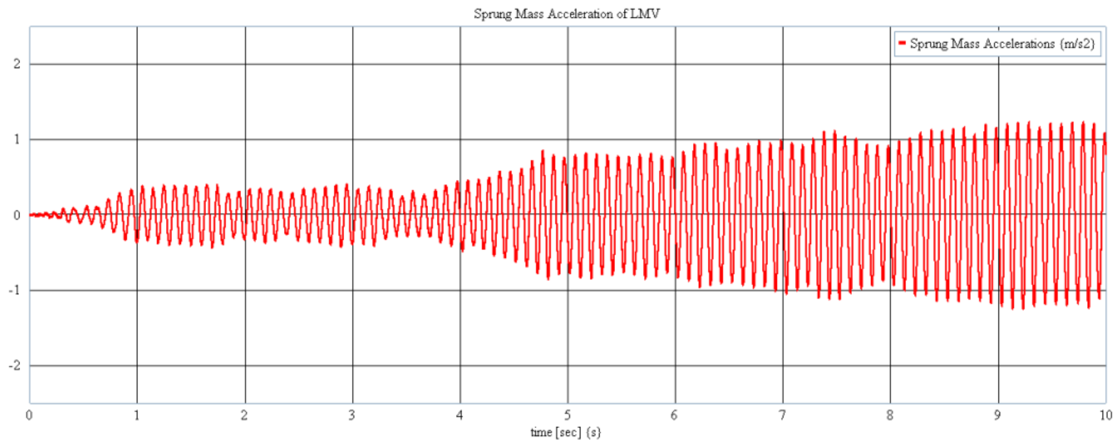

Figure 4.28: Sprung Mass Acceleration for Fully Laden Condition

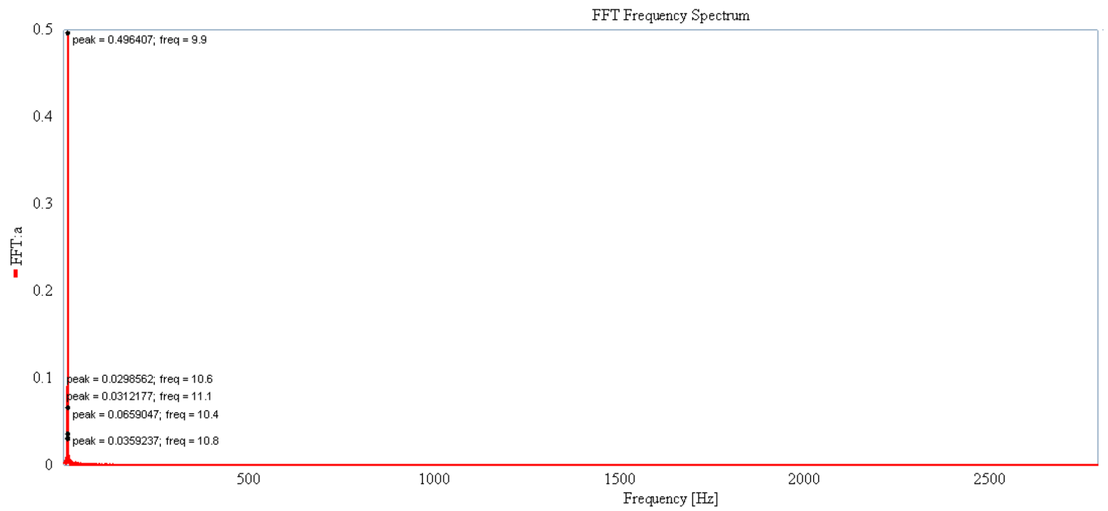

Figure 4.29: FFT Frequency Spectrum of LMV Sprung Mass Acceleration for Fully Laden Condition

FFT spectrum shows that the maximum peak occurs at the frequency of 9.9 [Hz], i.e. around the un-sprung mass natural frequency. Value of Maximum acceleration is 0.496 [m-sec^{-2}]. Other acceleration peaks also occur near the same range specifying greater acceleration value in higher frequency range.

FFT spectrum for kerb weight condition is also obtained in similar fashion which shows similar behavior of greater acceleration values near un-sprung mass natural frequency signifying higher acceleration in high frequency range.

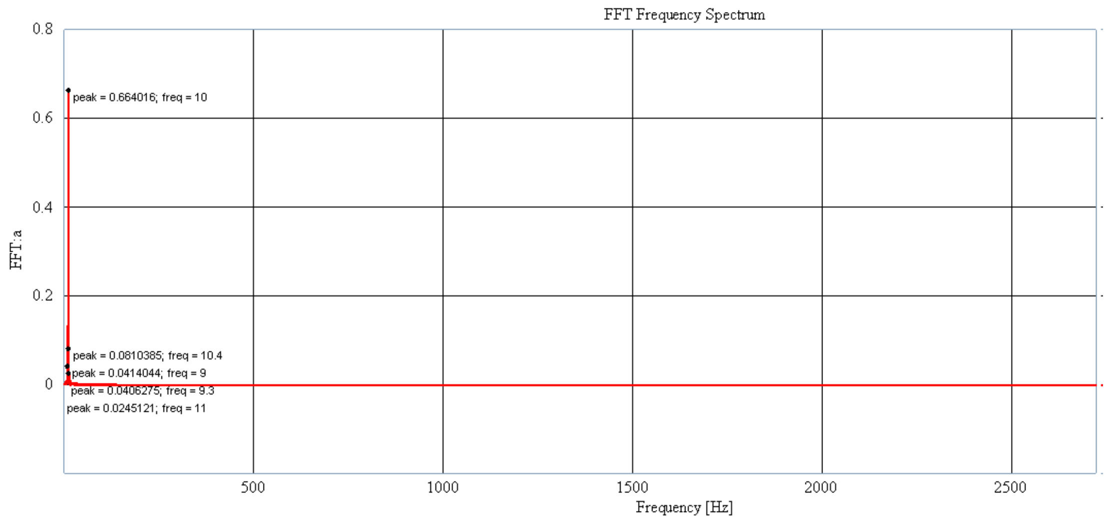

Figure 4.30: Sprung FFT Frequency Spectrum of LMV Sprung Mass Acceleration for Kerb Weight Condition

Figure 4.30 Shows the FFT spectrum of LMV Sprung Mass accelerations for kerb weight condition. Maximum value for acceleration is 0.664 [m-sec^{-2}] at 10.0 [Hz].

This shows clearly that for kerb weight condition the ride is degraded as acceleration levels are raised for higher frequency range.

4.10 Half Car Response Using Bond Graph

As discussed earlier, it is a tedious task to evaluate the response of Half Car using traditional techniques therefore, Bond Graph Method is applied to simulate its response.

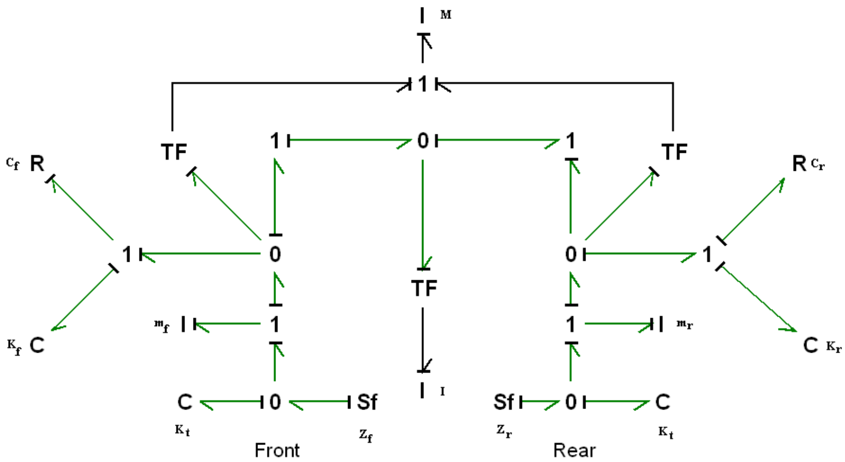

Figure 4.31: Half Car Bond Graph

20-sim software is used to simulate the response of Half Car for LMV. Inputs to the system are taken as the velocities of the two road inputs at front and rear wheel and the velocities of the sprung mass (linear and angular) are taken as outputs. The values of parameters of the designed LMV are plugged in the Half Car Model and first the response for fully laden condition is simulated and then for the kerb mass condition.

4.10.1 Response of LMV Using Bond Graph for Fully Laden Condition

Values of the LMV parameters for fully laden condition are input in the 20-sim software. Transfer Function thus obtained is shown in figure 4.32.

$$\frac{1585\ s^4 + 9.222e{+}004\ s^3 + 7.536e{+}006\ s^2 + 3.604e{+}008\ s + 5.242e{+}009}{s^7 + 37.34\ s^6 + 8296\ s^5 + 2.708e{+}005\ s^4 + 1.74e{+}007\ s^3 + 4.918e{+}008\ s^2 + 9.662e{+}008\ s + 1.405e{+}010}$$

Figure 4.32: Transfer Function of LMV Half Car Model for Fully Laden Condition

Frequency Response is computed for the said system using 20-sim software. Bode Diagram is generated as shown in figure 4.33.

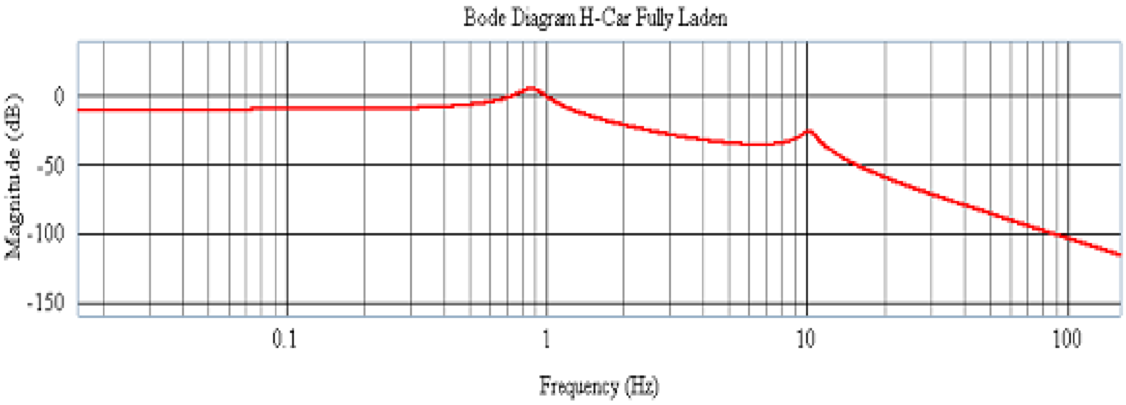

Figure 4.33: Bode Diagram of LMV Half Car for Fully Laden Condition

Transient Response of the LMV Half Car Model for fully laden condition is then computed using the 20-sim software. Figure 4.34 shows the Step Response of the LMV Half Car for Fully Laden Condition.

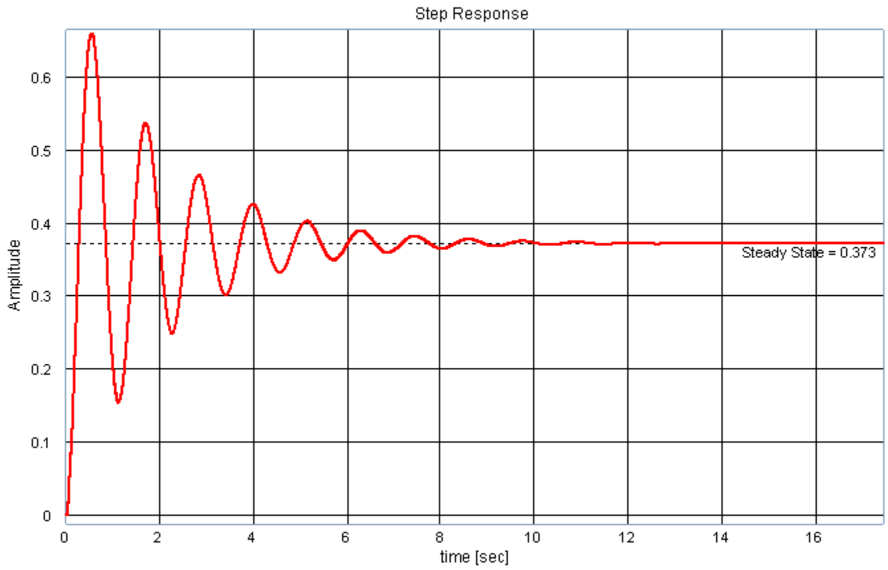

Figure 4.34: Step Response of LMV Half Car for Fully Laden Condition

4.10.2 Response of LMV Using Bond Graph for Kerb Weight Condition

LMV sprung mass value is changed for its kerb weight and a new Transfer Function is obtained as shown in figure 4.35.

$$\frac{3302\, s^4 + 1.921e{+}005\, s^3 + 1.57e{+}007\, s^2 + 7.508e{+}008\, s + 1.092e{+}010}{s^7 + 38.51\, s^6 + 8364\, s^5 + 2.81e{+}005\, s^4 + 1.793e{+}007\, s^3 + 5.176e{+}008\, s^2 + 2.013e{+}009\, s + 2.928e{+}010}$$

Figure 4.35: Transfer Function of LMV Half Car for Curb Weight Condition

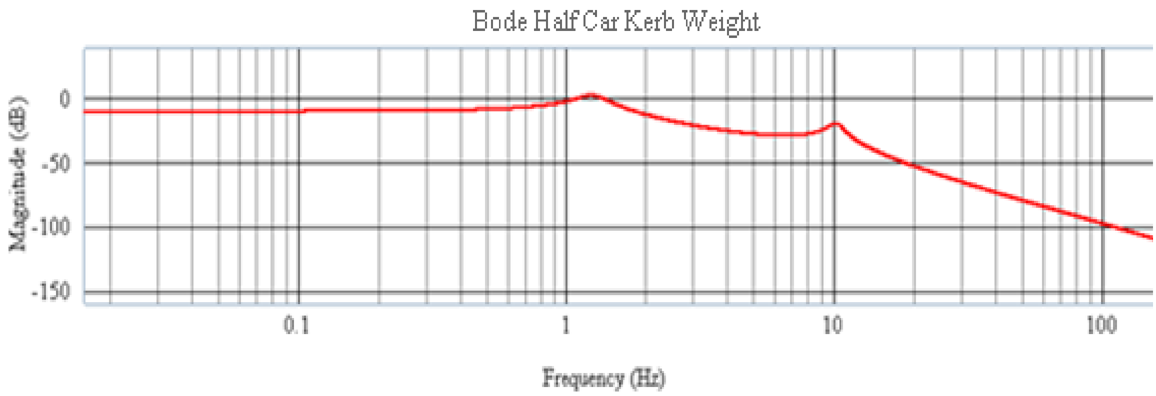

Figure 4.36: Bode Diagram of LMV Half Car Model for Curb Weight Condition

20-sim then generates the Bode Diagram for the curb weight condition of LMV. “Fig 4.36” shows the Bode Diagram for the LMV Half Car Model. Transient Response is then represented by the Step Response generated by 20-3im as shown in the "Fig 4.37" for the curb weight condition for LMV Half Car Model.

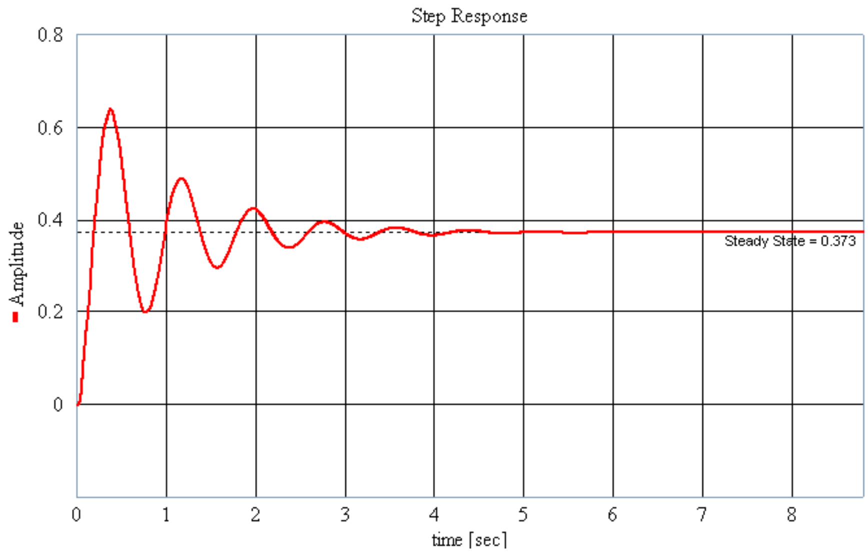

Figure 4.37: Step Response of LMV Half Car Model for Curb Weight Condition

NOTE:

Tables 4.2, 4.3 and 4.4 are formulated using the MATLAB code given in Appendix F.

CHAPTER 5: CONCLUSIONS AND RECOMMENDATIONS

This Chapter presents the output of the research and study done so far. The main objective of the study was to study the effects of suspension parameters on the ride performance of a Low Mass Vehicle. Since the term "Low Mass Vehicle" does not have any known existence, therefore, characterization of vehicles as a LMV is also addressed in the study and this chapter also presents the characterization of vehicles as LMV and also the frame work for designing a LMV. This chapter also includes the recommendations which indicate the areas to be addressed for future research in the topic.

5.1 Characteirzation of a Low Mass Vehicle

In view of the prevailing energy crisis the automobile industry is focusing towards light weight vehicles for improving fuel efficiency. Light weight materials are being utilized and dimensions of the vehicles are being optimized to get rid of unnecessary weight from the vehicles. Though weight reduction seems to be a straight forward solution for the problem, but there are implications on the performance of the vehicle.

From the previous discussion on the effect of mass on the performance of the vehicle, it is established that suspension needs to be optimized to cater for vehicle mass reduction. Suspension stiffness and damping are tuned to optimize the ride performance of the vehicle. As discussed earlier by changing the sprung mass of the vehicle the natural frequency '$\omega 1$' for the sprung mass gets shifted and the range of '$\omega 1$'prescribed for good ride is [1.0 to 1.5 Hz]. Suspension is tuned to keep '$\omega 1$' in the mentioned range to cater for weight reduction. Another important factor is the vehicle payload. For full load condition called the fully laden condition and without any payload called the kerb weight condition. Value of '$\omega 1$' also changes for a vehicle with changing payload, but for both fully laden and kerb weight conditions the value should remain in the prescribed range.

The study has revealed that for light weight vehicles payload becomes significant, and for fully laden condition ride quality is good, but when the payload is removed the ride deteriorates and the value of '$\omega 1$'increases beyond 1.5 [Hz] limit. Thus vehicle characterization as a LMV is given as:

"When the difference in Sprung Mass Natural Frequency of a Vehicle is more than 25% for its Fully Laden and Kerb Weight Condition, then the Vehicle is classified as a LMV "

5.2 Framework for Designing a Low Mass Vehicle

After characterization of a vehicle as a LMV, now the framework for designing a LMV is presented in light of all the simulations and the results from previous chapter. There are several parameters involved in the designing of the LMV and all of them need to be considered for the designing of the LMV.

- Sprung Mass
- Suspension Stiffness
- Suspension Damping
- Static Deflection
- Natural Frequency Range for Sprung Mass
- Vehicle Payload

All of these mentioned parameters have been studied thoroughly in the previous chapter, now based on this study a framework for designing a LMV is presented below:

1) First of all the range for the value of 'Sprung Mass Natural Frequency' needs to be defined, which in our case is defined as [1.0 – 1.5] Hz.
2) The value of 'Static Deflection' for the suspension needs to be defined, which in our case is defined as 6 [in].
3) For fully laden condition the value of suspension stiffness should be adjusted such that the value of sprung mass natural frequency is approximately equal to 1.0 [Hz].
4) Value of damping ratio increases with decrease in sprung mass, therefore, its value should be kept around 0.2 for fully laden conditions, keeping in view the fact that its value will increase for curb weight condition.

5.3 Conclusions

Whenever weight reduction is carried out in a normal vehicle to make it a LMV, the natural frequency of its sprung mass reduces, which can result in Ride deterioration if its value decreases than the allowable limit. Therefore, in order to cater for the mass reduction, suspension stiffness also needs to be reduced in the same proportion as that of the sprung mass, so that static deflection of the suspension is not changed. Although this adjustment of suspension stiffness keeps the value of sprung mass natural frequency at the desired value, but results in ride degradation for the frequency range between the natural frequencies of sprung mass and wheel hop. Deterioration in this range is handled by decreasing the value of suspension damping, but this reduction in damping causes the response gain at wheel hop resonance to increase. This increase in wheel hop resonance can be addressed by several active and semi active damping techniques.

From the simulations carried out in the previous chapter it is observed that at curb weight condition the ride is significantly deteriorated and cannot be optimized by using passive suspension. In case of LMV active or semi-active suspension is necessary to improve ride performance.

Generally active or semi-active suspensions do not improve ride significantly but are utilized primarily to improve handling performance, but for the specific case of a LMV active or semi-active suspension is a must as it is the only option for improving ride of the LMV at its curb weight condition.

5.4 Recommendations for Future Work

For this study suspension was treated as linear with passive spring and damper. It is recommended that for future research nonlinearities may be included in the suspension and the horizon of the problem may be broadened for tackling handling performance as well. Several active and semi-active techniques are being applied for improving the performance of the vehicles, mainly in the area of handling performance, however in case of a LMV these techniques may also contribute significantly in improving the ride performance. Therefore, it is suggested that for future research active and semi-active suspension systems may also be included in the study.

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APPENDIX

Appendix A:

MATLAB code for representing different classes of roads according to ISO 8608 standard.

```
% This program shows the log-log plot of the Road PSD
% for Rough & Smooth Road with increase in Wave Number
clear all
speed=120;                  % [km/hr]
v=speed*(5/18);             % [m/sec]
f=[0:0.1:1000];             % Temporal Frequency [Hz]
OMG=2*pi*f/v;               % Spatial Frequency [rad/m]
A=1e-6;                     % Smooth Road, Class-A, Red
E=256e-6;                   % Rough Road, Class-E, Black
PSD_E=E.*(OMG).^(-2);
PSD_A=A.*(OMG).^(-2);
loglog(OMG,PSD_E, 'black','linewidth',1.5,'linestyle','--');
hold on
loglog(OMG,PSD_A,'r','linewidth',1.5,'linestyle','--');
hold on;
title('Approximation of Random Road Profiles');
xlabel('Wave Number [rad/m]');
ylabel('PSD [m^2/(rad/m)]');
legend('Black-Rough & Red-Smooth');
grid on
```

Appendix B:

MATLAB code for generating Road Profile as a function of displacement.

```
% This program generates the Random Road Profile as a function of displacement
L=[0:0.1:29.9];                 % Length of the Road Segment [m]
z=zeros(1,300);                 % Space allocation for addition of sine waves
kph=100;                        % Speed [Km/Hr]
sp=kph*(5/18);                  % Speed [m/sec]
Hz=100;                         % Max. value of temporal freq considered [Hz]
ang=2*pi;                       % Max. value of phase angle considered [rad]
wn=2*pi*Hz/sp;                  % Max. value of spatial freq considered [rad/sec]
del=wn/300;                     % Loop for adding sine waves to obtain Random Road Profile
for i=1:200                     % No. of sine Waves
   wav(i)=wn*rand;              % Random generation of spatial freq.
   ph(i)=ang*rand;              % Random generation of phase angle.
   B(i)=2*(10e-5)*(wav(i)^-2)*del;
   A(i)=sqrt(B(i));
   y=A(i)*sin((wav(i)*L)+ph(i));        % Sine Wave generation
   z=z+y;                               % Addition of Sine waves
end
plot(L,z,'r')                           % Road Profile
title('Random Road Profile');
xlabel('Length of Road Segment');
ylabel('Road Elevation');
```

Appendix C:

MATLAB code for generating road profile in time domain.

```
% This Program generates the road profile in time domain
t=[0:0.1:29.9];              % Time Interval [sec]
z=zeros(1,300);              % Space allocation for addition of sine waves
kph=60;                      % Speed [Km/Hr]
sp=kph*(5/18);               % Speed [m/sec]
Hz=100;                      % Max. Value of Temporal Frequency [Hz]
ang=2*pi;                    % Max. Value of  Phase Angle. [rad]
wn=2*pi*Hz/sp;               % Max. Value of Spatial Frequency [rad/meter]
del=wn/1000;
for i=1:100                  % No. of sine Waves
   f(i)=Hz*rand;             % Random generation of Temporal Freq [Hz]
   ph(i)=ang*rand;           % Random generation of phase angle [rad]
   wav(i)=wn*rand;           % Random generation of Spatial Freq.
   B(i)=2*(1e-6)*(wav(i)^-2)*del;
   A(i)=sqrt(B(i));
   y=A(i)*sin((2*pi*f(i)*t)+ph(i));% Single Sine wave
   z=z+y;                    % Addition of Sine waves for Road Profile
end
plot(t,z,'k')                % Plot of Road Profile in time domain
title('Random Road Profile');
xlabel('Time [sec]');
ylabel('Road Elevation [m]');
```

Appendix D:

MATLAB code for Simulating Resonance of Quarter Car

```
% Code for generating steady-state response of the Quarter Car Model:
tspan=[0:0.01:7];                % [Hz]
y0=[0.05;0;0.0;0.0];             % Initial Conditions: Zs(0)=0.01m & DZs(0)=1.0m/sec
                                 % Using solver for 2nd Order ODEs 'ode23'
[t,y]=ode23('solveODE',tspan,y0);
A=0.1;                           % Amplitude of Sine wave [m]
Hz=2;                            % Temporal Frequency [Hz]
w=2*pi*Hz;                       % Cyclic Frequency [rad/sec]
Z=A*sin(w*t);
subplot(3,1,1)
plot(t,y(:,1));
grid on
xlabel('t');
ylabel('Zs(t)');
hold on;
subplot(3,1,2);
plot(t,y(:,3));
grid on
xlabel('t');
ylabel('Zu(t)');
hold on;
subplot(3,1,3);
plot(t,Z);
grid on;
xlabel('t');
ylabel('Zr');
hold on;
```

```
function f = solveODE(t,y)
A=0.1;                  % Amplitude of Sine wave [m]
Hz=2;                   % Temporal Frequency [Hz]
w=2*pi*Hz;              % Cyclic Frequency [rad/sec]
Ms=168.0;               % Sprung Mass [Kg]
Mus=38.42;              % Un-Sprung Mass [Kg]
Ks=14900;               % Suspension Stiffness [N/m]
Cs=475;                 % Suspension Damping [N-sec/m]
Kt=150000;              % Tire Stiffness [N/m]
Ct=0;                   % Tire Damping [N-sec/m]
a1=Cs/Ms;
a2=Ks/Ms;
a3=1/Mus;
a4=Cs/Mus;
a5=(Cs+Ct)/Mus;
a6=Ks/Mus;
a7=(Ks+Kt)/Mus;
f=zeros(4,1);
f(1)=y(2);
f(2)=-(a1*y(2))+(a1*y(4))-(a2*y(1))+(a2*y(3));
f(3)=y(4);
f(4)=(A*sin(w*t))+(a4*y(2))-(a5*y(4))+(a6*y(1))-(a7*y(3));
```

Appendix E:

MATLAB code for Simulating Resonance of Half Car

```
% This code solves the Steady-State Response
% of the Half Car Model using the ODE Solver 'ode23'
tspan=[0:0.01;30];                    % Time Span [sec]
y0=[0; 0; 0; 0; 0; 0; 0; 0];          % Initial Conditions
[t,y]=ode23('solveHC',tspan,y0);
v=100;                                % Speed [Km/Hr]
s=v*5/18;                             % Speed [m/sec]
A=0.15;                               % Amplitude of Sine wave [m]
w=200;                                % [rad/sec]
wb=1.650;                             % Wheel Base [m]
d=wb/s;
subplot(3,2,1);
plot(t,y(:,1));
grid on;
xlabel('t');
ylabel('Z(t)');
hold on;
subplot(3,2,2);
plot(t,y(:,3));
grid on;
xlabel('t');
ylabel('theeta');
hold on;
subplot(3,2,3);
plot(t,y(:,5));
grid on;
xlabel('t');
```

```
ylabel('Zf(t)');
hold on;
subplot(3,2,4);
plot(t,y(:,7));
grid on;
xlabel('t');
ylabel('Zr(t)');
hold on;
subplot(3,2,5);
plot(t,A*sin(w*t));
grid on;
hold on;
subplot(3,2,6);
plot(t,A*sin(w*(t+d)));
grid on;
hold on;

function f = solveHC(t,y)
f=zeros(8,1);
A=0.15;             % Amplitude of Sine wave [m]
w=200;              % Cyclic Frequency [rad/sec] f=w/2*pi [Hz]
v=100;              % Speed [Km/Hr]
s=v*5/18;           % Speed [m/sec]
M=412.85;           % Sprung Mass [Kg]
mf=38.42;           % Un-Sprung Mass Front [Kg]
mr=38.42;           % Un-Sprung Mass Rear [Kg]
I=1221.68;          % Moment of Inertia
kf=14900;           % Suspension Stiffness Front [N/m]
kr=14900;           % Suspension Stiffness Rear [N/m]
```

```
cf=475;                         % Suspension Damping Front [N-sec/m]
cr=475;                         % Suspension Damping Rear [N-sec/m]
kt=150000;                      % Tire Stiffness [N/m]
wb=1.650;                       % Wheel Base [m]
a=0.916;                        % Dist of CG from Front [m]
b=1.556;                        % Dist of CG from Rear [m]
d=wb/s;                         % Time delay [sec]
XIF=A*(sin(w*t));
XIR=A*(sin(w*(t+d)));
a2=(a^2); b2=b^2;
%Defining Constants and their relations for the matrices
f(1)=y(2);
f(2)=((a*kf-b*kr)*y(3)/M)+(kf*y(5)/M)+(cf*y(6)/M)+(kr*y(7)/M)+(cr*y(8)/M)-
((kf+kr)*y(1)/M)-((cf+cr)*y(2)/M)-((b*cr-a*cf)*y(4)/M);
f(3)=y(4);
f(4)=((a*kf-b*kr)*y(1)/I)-((b*cr-a*cf)*y(2)/I)-((a2*kf+b2*kr)*y(3)/I)-
((a2*cf+b2*cr)*y(4)/I)-(a*kf*y(5)/I)-(a*cf*y(6)/I)+(b*kr*y(7)/I)+(b*cr*y(8)/I);
f(5)=y(6);
f(6)=(cf*y(2)/mf)-(a*cf*y(4)/mf)-(cf*y(6)/mf)+(kf*y(1)/mf)-(a*kf*y(3)/mf)-
((kf+kt)*y(5)/mf)+(kt*A*sin(w*t));
f(7)=y(8);
f(8)=(cr*y(2)/mr)+(b*cr*y(4)/mr)-(cr*y(8)/mr)+(kr*y(1)/mr)+(b*kr*y(3)/mr)-
((kr+kt)*y(7)/mr)+(kt*A*(sin(w*(t+d))));
```

Appendix F:

MATLAB code for Frequency Response of the Quarter Car Model

```
% This code is meant for generating the frequency response of the Quarter Car Model.
Ms=168;                 % Sprung Mass [Kg]
Ks=14900;               % Suspension Stiffness [N/m]
Kt=150000;              % Tire Stiffness [N/m]
Mu=38.42;               % Un-sprung Mass [Kg]
Cs=475;                 % Suspension Damping [N-m/sec]
% Transmissibility
K1=Kt/Ms;
K2=Ks/Ms;
C=Cs/Ms;
M=Mu/Ms;
f=[0:0.1:25];           % Frequency Range [Hz]
w=2*pi*f;               % [rad/sec]
a=K1*K2;
b=K1*C*w;
a2=a^2;
b2=b.^2;
N=sqrt(a2+b2);
c1=(K1+(K2*M)+K2)*(w.^2);
d1=M*(w.^4)+a-c1;
c2=K1*C*w;
c3=(1+M)*C*(w.^3);
d2=c2-c3;
D=sqrt((d1.^2)+(d2.^2));
G=N./D;
% Transfer Function
num1=Kt*Cs;
```

```
den1=Ms*Mu;
den2=(Ms+Mu)*Cs;
den3=((Kt*Ms)+(Ms+Mu)*Ks);
den4=Kt*Cs;
den5=Kt*Ks;
num=[num1 Ks];
den=[den1 den2 den3 den4 den5];
TF=tf(num,den);
[r,p,k]=residue(num,den)
[A,B,C,D]=tf2ss(num,den)
% Plots
% Transmissibility
figure(1)
plot(f,G,'r'); grid on; hold on;
title('Transmissibility')
xlabel('Frequency (Hz)')
ylabel('Response Gain')
% Bode Diagram
figure(2)
bode(TF,'r'); hold on; grid on;
title('Open-Loop Frequency Response')
% Step Response
figure(3)
step(A,B,C,D);
hold on;
```

Appendix G:

MATLAB code for the time response of Quarter Car for Random Road Input

```
clear all;
tspan=[0:0.1:9.9];              % [sec]
y0=[0;0;0.0;0.0];               % Initial Conditions: Zs(0)=0.01m & DZs(0)=1.0m/sec
[t,y]=ode23('solveODE',tspan,y0);
                                % Using solver for 2nd Order ODEs 'ode23'
subplot(3,1,1)
plot(t,y(:,1));
grid on
xlabel('t');
ylabel('Zs(t)');
hold on;
subplot(3,1,2);
plot(t,y(:,3));
grid on
xlabel('t');
ylabel('Zu(t)');
hold on;
z=zeros(1,100);                 % Space allocation for addition of sin waves
kph1=100;                       % [Km/Hr]
sp1=kph1*(5/18);                % [m/sec]
Hz1=100;                        % [Hz]
ang1=360;
wn1=2*pi*Hz1/sp1;
del1=wn1/100;
for i=1:100                     % No. of sin Waves
   f1(i)=Hz1*rand;              % Random generation of freq [0 to 200] Hz
   ph1(i)=ang1*rand;            % Random generation of phase angle [0 to 360] deg
```

```
    wav1(i)=wn1*rand;          % Wave Number range [0 to 75] acc to freq
    B1(i)=2*(1e-6)*(wav1(i)^-2)*del1;
    A1(i)=sqrt(B1(i));
    y1=A1(i)*sin((2*pi*f1(i)*tspan)+ph1(i));
    z=z+y1;
end
subplot(3,1,3);
plot(tspan,z);
grid on;
xlabel('t');
ylabel('Zr(t)');
hold on;
```