# Analysis of Fully Compliant Straight Line Mechanism by Pseudo Rigid Body Model

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National University of Sciences and Technology, Islamabad
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# Analysis of Fully Compliant Straight Line Mechanism by Pseudo Rigid Body Model

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by

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| | |
|---|---|
| $b$ | Length of coupler point P from mid of coupler link |
| $c$ | Distance from the neutral plane to the top/bottom plane of link |
| $c_\theta$ | Parametric angle coefficient |
| E | Young modulus of elasticity |
| $\Xi$ | Ratio of length of small length flexure pivot and length of pseudo rigid body link |
| $F$ | Force at coupler point P |
| $h$ | Thickness of a segment |
| $h_{ij}$ | Kinematic coefficient for a four bar |
| I | Moment of inertia |
| $K$ | Spring constant |
| $K_\theta$ | Stiffness coefficient |
| $K_i$ | Spring constant at respective characteristic pivot |
| $L_i$ | Length of flexible member |
| $l_i$ | Length of small length flexure pivot |
| M | Moment |
| $P$ | Force |
| $q$ | Generalized coordinate |
| $r_i$ | Length of pseudo rigid body link |
| $S_y$ | Yield strength |
| $\sigma_c$ | Critical stress |
| $\sigma_{top}$ | Stress at top surface of a link |
| $\sigma_{bottom}$ | Stress at bottom surface of a link |
| T | Torque |
| $t$ | In-plane thickness of a segment |
| $\theta_0$ | Deflection angle of compliant member |
| $\Theta$ | Pseudo rigid body angle |
| $\theta_2$ | Crank link angle from global x-axis |
| $\theta_3$ | Coupler link angle from global x-axis |

| | |
|---|---|
| $\theta_4$ | Rocker link angle from global x-axis |
| $\theta_{i0}$ | Initial angle / position of a link |
| $\delta\theta$ | Virtual angular displacement |
| $V$ | Potential energy |
| $\delta W$ | Virtual work |
| $X_p$ | Displacement of coupler point P in x-axis |
| $\delta z$ | Virtual displacement |

# ABSTRACT

A compliant mechanism can be defined as flexible structure, which uses elastic deformation to achieve force and motion transmission. These mechanisms are different from the classic linkage mechanisms with regard to the fact that they rely on elastic deformation for functionality. Hence any analysis of compliant mechanism requires a study of mechanics of deformable bodies, in addition to kinematics. Because of the same very reason, analysis and synthesis of compliant mechanisms has been the subject of significant study in the research community, which led to a number of design approaches for developing compliant mechanisms. Pseudo Rigid Body Model is one of the approaches. In this work, modeling and analysis of compliant straight line mechanism, based on Roberts approximate straight line mechanism, is explored. The Pseudo Rigid Body Model is used to model the compliant mechanism. Its motion and force-displacement relationship are predicted by Pseudo Rigid Body Model and finally, a commercial finite element code (ANSYS) capable of nonlinear analysis is used to model the mechanism with its flexible configuration. The theoretical results based on PRBM are compared with FEA results of compliant model, which suggest that PRBM reduces the complexity of analysis and is a desirable method for approximating an initial design phase of compliant mechanism.

# CHAPTER – 1

# INTRODUCTION AND BACKGROUND

In this chapter, compliant mechanisms definition and classification is presented along with previous work on design and analysis methods used for these mechanisms. Furthermore, software's dedicated to compliant mechanisms are also presented.

## 1.1 Mechanism

A mechanism is a device which transforms motion to some desirable pattern and typically develops very low forces and transmits little power. Reuleaux [1876] has defined mechanism as; it is an assemblage of resistant members, connected by movable joints, to form a closed kinematic chain with one link fixed and having the purpose of transforming motion. Traditional mechanisms consist of rigid (non-deformable) members connected at movable joints. Forces are applied at the joints to provide motion for rigid members. Examples include a reciprocating engine piston, a crane hoist, and a vice grip, etc. A rigid mechanism simply transfers energy from the input to the output. Since energy is conserved between the input and output, the output force may be much larger than the input force (mechanical advantage), but the output displacement is much smaller than the input displacement (geometric advantage) or vice versa [Howell. 2001].

Analyses of rigid-body mechanisms are well understood since closed form solution are available. However, there are many other applications where some of the members are intentionally designed to be relatively compliant compared with other members. Such mechanisms are referred as compliant mechanisms.

## 1.2 Compliant Mechanisms

A compliant mechanism also transfers or transforms motion, force or energy. However, they gain at least some of their mobility from the deflection of flexible members rather than from movable joints only [Howell, 2001]. The example of compliant mechanism is shown in figure 1.1.

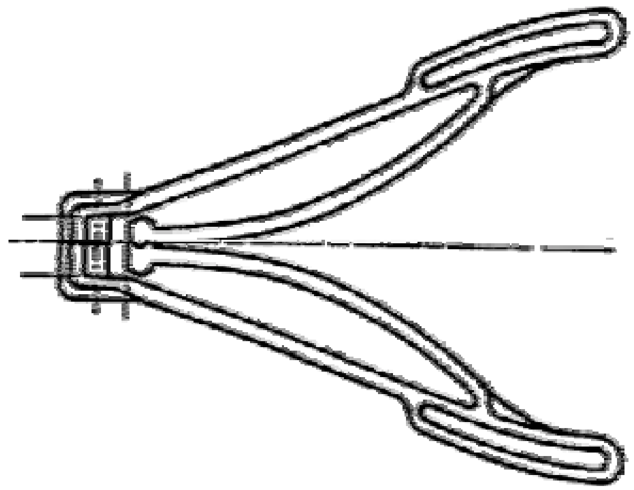


**Figure 1.1: Compliant Crimping Mechanism**

Lobontiu [2002] has defined compliant mechanisms in more precise manner. He stated, it is a mechanism that is composed of at least one member (link) that is sensibly deformable (flexible or compliant) compared to other rigid links. Figure 1.2 shows that a rigid-body mechanism (top) becomes a compliant mechanism (bottom) when the connecting rod is clamped at the sliding block.

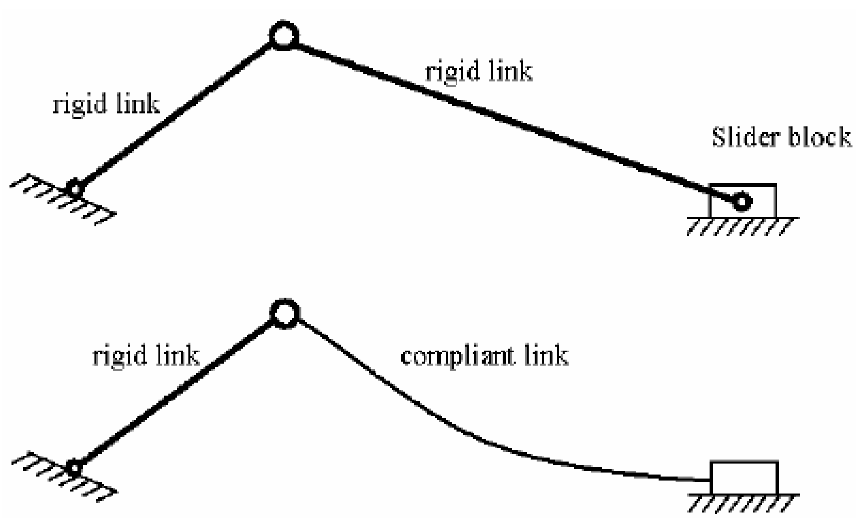



**Figure 1.2: Rigid body mechanism (top), Compliant mechanism (bottom)**

At the macro scale several innovative compliant mechanisms have been created such as the no assembly stapler and windshield wiper shown in figure 1.3 and 1.4 [Flexsys. Inc]. At the micro-scale, compliant mechanisms have been implemented as displacement amplification transmissions for comb-drive actuators as depicted in figure 1.5 [Kota, 2001, Kota *et.al.*, 2003] and in novel precision 6 DOF manipulation stages with nano-scale displacement resolution and mm range developed by the Culpepper group at MIT, figure 1.6 [Kim *et.al.*, 2004].


**Figure 1.3: Macro-scale monolithic Stapler developed by Kota group at University of Michigan using compliant mechanism**


**Figure 1.4: Monolithic Windshield wiper developed by Kota group at University of Michigan using compliant mechanisms**

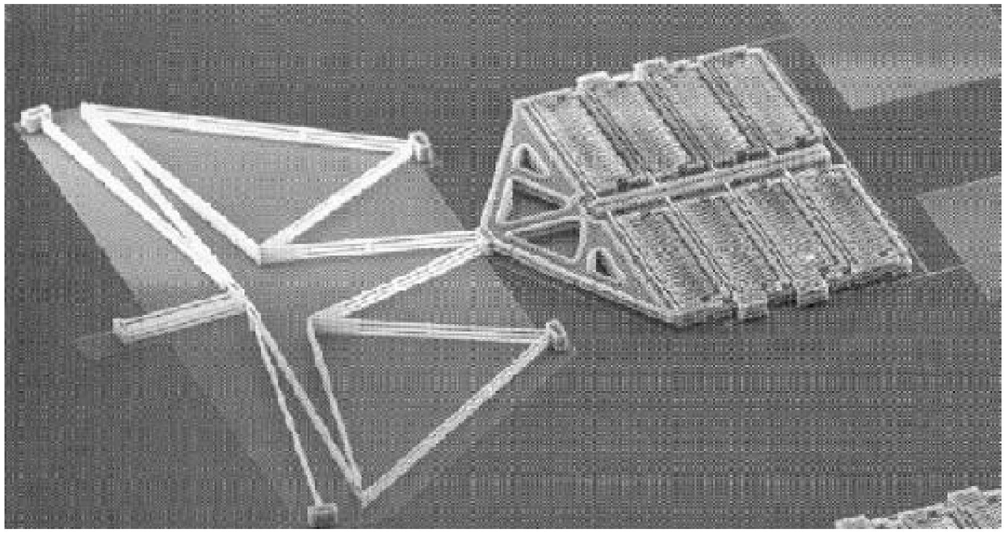


**Figure 1.5: SEM image of a compliant mechanism attached to a comb-drive electrostatic actuator. The compliant mechanism has a geometric advantage of 20. The entire device occupies an area of 350 µm by 350 µm [Kota, 2001 & Kota *et.al.,* 2003]**

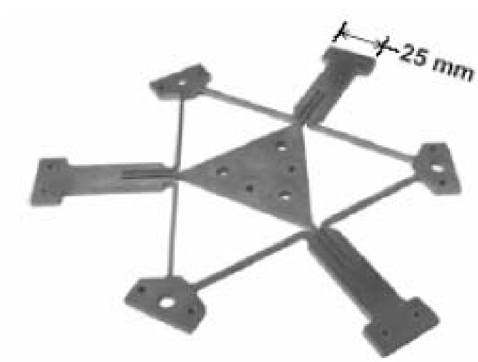



**Figure 1.6: Hex-Flex 6-DOF Nanomanipulator [Kim *et.al.*, 2004]**

These mechanisms can be fully compliant or partially compliant. A fully compliant mechanism is one that has no rigid body joints. A partially compliant mechanism is one that has some compliant members and some non-compliant joints like pin joint etc [Wittwer, 2001].

### 1.2.1 Compliant Mechanism – Design and Analysis Background

The idea of using deflecting members to gain motion and energy storage as opposed to rigid members connected through kinematic joints is nothing new. Mother Nature has used compliance since the beginnings of life in things such as plants, bird wings and legs of small insects. Inventors, inspiring from nature, have used deflections in their mechanism designs [Jensen *et.al.*, 1998]. Mankind has used compliance in catapults and bows which have existed for thousands of years. Moreover, many new applications of compliant technology required more advanced materials in order to be viable.

Compliant mechanisms also possess some unique challenges that needed to be solved before they could be truly useful as replacements for many rigid-body mechanisms. Some of these challenges included finding new ways to analyze large deflections, finding ways to relate compliant mechanism kinematics to rigid-body kinematics and others.

In order for compliant mechanisms to be useful as replacements for many rigid-body mechanisms, the ability to analyze deflections beyond the linear range was needed. Many models used to analyze deflections in beams, such as those taught in strengths of materials or machine design courses make use of simplifying assumptions which limit their usefulness to small deflections. While these assumptions may be perfectly valid and justified in many applications, some compliant mechanisms experience large deflections which undermine the linear models' accuracy [Brandon, 2003 & Levent, 2005].

#### 1.2.1.1 Bernoulli – Euler Equation

Euler was the first to quantify the deflection of flexible beams with the development of the Bernoulli-Euler equation in 1744. This equation, later, was solved for large deflections of cantilever beams using elliptical integrals. However, these solutions had very limited applications and were difficult to use. Further research in this area has included finding large deflections of beams with various geometries and developing methods of large-deflection finite element analysis. Hill and Midha [Jensen *et.al.*, 1998, Wittwer, 2001] also addressed the numerical analysis of large-deflection beams.

### 1.2.1.2 Graphical Method

Graphical methods are also used for compliant mechanism synthesis. Compliant mechanisms using compliant segments with both end forces and end moments are investigated, along with three-dimensional compliant mechanisms. Optimization to the design of compliant mechanisms is also introduced. The effects of compliant members on mechanical advantage in a mechanism have also been investigated later. A system of classification and nomenclature for compliant mechanisms has also been established to aid in the naming and analysis of compliant mechanisms in 1994 [Jensen *et.al.*, 1998].

### 1.2.1.3 Pseudo Rigid Body Model

Historically, the most common method of compliant mechanism design has been trial and error. However, the conception of the pseudo-rigid-body modeling technique has successfully opened the way for simple design and analysis of many compliant mechanisms [Howell *et.al*., 1996]. Howell and Midha [Howell *et.al*., 1995] have introduced this idea. The premise is that a beam which derives its motion from bending can be modeled as a rigid beam with a torsional spring or springs and pin joint(s) at calculated positions. The positions of the pin joints and the torsional springs are calculated depending upon the end and loading conditions. The compliant members of any mechanism are modeled by using different basic models defined in PRBM and these are standard cantilever beam, fixed-guided segment, small-length flexural pivot, fixed-fixed beam etc, with different loading conditions. The torsional spring constants are calculated as a function of the bending moment of inertia and the material properties. The pseudo-rigid-body model also provides methods for calculating stresses. The pseudo-rigid-body model is limited in some respects because models for all compliant mechanisms have not been developed but even then it allows many compliant mechanisms to be designed and analyzed much more easily than was possible in the past. In recent years, work has been focused on methods of synthesizing new compliant mechanisms. Ananthasuresh and Frecker [Frecker *et.al.*, 1997] presented work on applying topological synthesis to the design of compliant mechanisms.. In this method, a computer-driven optimization routine is used to find the right configuration of flexible members to accomplish a certain task [Pucheta *et.al.,* 2008].

### 1.2.2 Advantages of Compliant Mechanisms

An advantage of compliant mechanisms is the reduction in the total number of parts required to accomplish a certain task. This reduces manufacturing and assembly time and cost [Howell, 2001]. Some mechanisms may even be constructed of one piece. This kind of design has recently been given a new name, which is "Design for No Assembly" (DNA). Virtually, any product with multiple mechanical parts performing a motion function can be considered for a no-assembly design approach, utilizing compliant mechanisms [Kota, CSDL]. In Figure 1.4, a windshield wiper design made by FlexSys Inc can be seen. The top image shows a 15 piece conventional wiper, and the bottom shows a single-piece compliant wiper design that can be injection molded in one-step. The advantage of this wiper is that, it evenly distributes blade pressure and can conform to any windshield, from flat to highly curved [Flexsys. Inc].

Since compliant mechanisms have fewer movable joints, need for lubrication is less. Also because of that, mechanism precision is increased and vibration, noise and backlash is reduced.

As compliant mechanisms are made up of flexible members, the deflection of these members can be used as an advantage during design. For example, constant force mechanisms [Boyle, 2001 & Weight, 2001].

Weight issue is generally significant in some applications, such as aerospace. Compliant mechanisms are lighter than rigid link mechanisms synthesized for the same purpose. Another advantage of compliant mechanisms is the ease with which they are miniaturized. Simple microstructures, actuators and sensors are widely used in MEMS applications [Howell, 2001].

### 1.2.3 Disadvantages of Compliant Mechanisms

The main disadvantage of compliant mechanisms is the difficulty of analyzing and designing them. Knowledge of mechanism analysis and synthesis methods and the deflection of flexible members is required [Howell, 2001].

It is necessary to have good understanding of mechanism theory and strength of materials, and their interaction in a complex situation. Fatigue analysis is another issue. Since compliant segments are often loaded cyclically, those members must be designed to have sufficient fatigue life to perform their prescribed functions.

Energy storage of the flexible members can be a disadvantage for some mechanisms. For example, if mechanism's task is to transfer energy from input to output, not all of input energy is transferred, but some is stored in the mechanism.

The motion from the deflection of compliant links is also limited by the strength of the deflecting members. Furthermore, a compliant link cannot produce a continuous rotational motion which is possible with a pin joint. Compliant links that remain under stress for long periods of time or at high temperatures may experience stress relaxation or creep [Howell, 2001].

### 1.2.4 Compliant MEMS

Micro-electro-mechanical systems (MEMS) integrate electrical circuitry with mechanical devices having dimensions measured in microns [Wittwer, 2001]. Several methods of MEMS fabrication exist. The most common one is surface micromachining. Surface micromachining takes place on a silicon wafer using techniques similar to those used for integrated circuit manufacturing [Howell, 2001]. Although many researchers have used deflections to gain motion, some have specifically studied the use of deflection in MEMS. Ananthasuresh applied topological synthesis to the design of compliant MEMS [Ananthasuresh research group]. He described the principal benefits and challenges. Some examples of design of compliant MEMS using the topological synthesis method and pseudo-rigid-body models can be found in literature [Jensen *et.al.*, 1998].

The advantages of compliance in MEMS [Howell, 2001] are, compliant mechanisms:

(a) Can be fabricated in a plane.

(b) Require no assembly.

(c) Require less space and are less complex.

(d) Have less need for lubrication.

(e) Have reduced friction and wear.

(f) Have less clearance due to pin joints, resulting in higher precision.

(g) Integrate energy storage elements (springs) with the other components.

There are also some challenges associated with designing compliant MEMS. The performance is highly dependent on the material properties, yet the design is limited to a few materials that are compatible with the fabrication methods. Also, the fabrication method itself creates some problems. For example, “stiction” occurs in surface micromachining. The machined structure remains stuck to the substrate and large force is necessary to move it [Jensen *et.al.*, 1998].

## 1.3 Design Software’s

### 1.3.1 MSC ADAMS

ADAMS can be used for flexible body simulation of a mechanical design. ADAMS is useful for early simulation of the effects of flexibility in mechanical systems when detailed finite element representations is not available. In ADAMS a parametric flexible body representation of a component can be built and analyzed. Changes can be made to the flexible body and effect of the changes can be evaluated [MSC Softwares Product].

### 1.3.2 PennSyn

PennSyn 1.0 is software developed by the research group of Ananthasuresh, in University of Pennsylvania [Ananthasuresh Research Group]. It is implemented in Matlab 5.3 (release 11). It has an easy-to-use menu with help buttons for each step. It generates compliant topologies, animates the resulting motion, creates an IGES file of the solution, and stores the solution for later use.

### 1.3.3 TOPOPT

TOPOPT is a web-based topology optimization program developed by Dmitri Tcherniak, Ole Sigmund, Thomas A. Poulsen and Thomas Buhl [TOPOPT website, Dmitri *et.al.*, 2001]. The TOPOPT program solves the general topology optimization problem of distributing a given amount of material in a design domain subject to load and support conditions, such that the stiffness of the structure is maximized. The restrictions on the TOPOPT program are the following:

(a) Two dimensions.
(b) Rectangular design domains.
(c) 1000 square elements (= 1000 design variables).
(d) 100 design iterations.

The short-term goal of the web-based topology optimization program TOPOPT is to develop a simple to use topology optimization tool that can be used in the education of engineers, architects and other structural designers and to investigate the use of web-browsers as interfaces to CAD programs.

The long-term goal is to develop an interface to a multi-purpose topology optimization program that can be used to solve general structural design problems, MEMS design problems and other topology optimization problems involving multiple physical domains [TOPOPT website, Dmitri *et.al.*, 2001].

### 1.3.4 OPTISHAPE

OPTISHAPE is software developed by Quint Technologies [Quint Corporation website]. It is based on the structural topology optimization using the homogenization method. This theory has been applied to the various kinds of problems such as static problem, eigenvalue problem and frequency response problem.

### 1.3.5 CSDL

In order to numerically implement the design theory, an optimization and analysis software based on Matlab platform has been developed by research students of Sridhar Kota, in University of Michigan [Kota, CSDL].

The compliant mechanism is discretized with six degree of freedom frame finite elements. The energy efficiency of the system is maximized under the reasonable physical and geometrical boundary conditions. The specification of the mechanical advantage and geometric advantage can be reached after the convergent iterations. A promising result with great efficiency can be found if the topology and initial conditions are reasonable.

After the optimization process, dynamic analysis will be performed so that a full understanding for the performance of compliant mechanism can be achieved. The analyses include natural frequency and mode, static force analysis, dynamic responses, spectrum analysis, sensitivity analysis and equivalent spring characteristic analysis.

### 1.3.6 ANSYS

ANSYS can be used for flexible body simulation of a mechanical design. In ANSYS a parametric flexible body representation of a component can be built and analyzed. Changes can be made to the flexible body and effect of the changes can be evaluated in terms of static force analysis, dynamic responses, spectrum analysis and sensitivity analysis.

# CHAPTER – 2
# COMPLIANT MECHANISMS DESIGN METHODOLOGY & PSEUDO RIGID BODY MODEL CONCEPTS

Aim of this chapter is to introduce some basic concepts. Theory presented here will be used in the following chapter to come up with a synthesis and analysis method of compliant straight line mechanism.

## 2.1 Pseudo Rigid Body Model

The Pseudo-Rigid-Body Model (PRBM) plays an important part in the design and analysis of compliant mechanisms. This section gives a brief overview and then discusses the nomenclature and equations for several different types of flexible segments [Howell, 2001].

### 2.1.1 Overview

The design and analysis of compliant mechanisms can be complicated. Traditionally, the large non-linear deflections have caused significant difficulties in the design of compliant mechanisms. Techniques such as finite element analysis (FEA) and elliptic integrals provide accurate information, but make design very complicated. Fortunately, the development of the PRBM has greatly increased the speed and ease in which compliant mechanisms can be designed. The PRBM allows for the approximation of the force-deflection characteristics of flexible segments. Thus, the PRBM is intended to be an intermediate design tool, allowing for the rapid design and analysis of first generation compliant mechanisms. Afterwards, techniques such as FEA and other numerical methods can be used to refine the designs. The PRBM becomes a tool to take beginning ideas to refined designs [Weight, 2001].

The power of the PRBM comes from its ability to model compliant members using rigid members that have the same force-deflection characteristics as the original member. Continuous work in developing the PRBM has shown it to accurately model the behavior of compliant mechanism in displacement, force, velocity, and acceleration. Thus, designers can draw results from the vast number of traditional rigid body mechanism design and analysis tools.

For each type of flexible segment, several parameters are defined. The first of these is the ***characteristic pivot***. The characteristic pivot is the center of the arc created by the path of the end of the beam. This pivot lies on the flexible beam and is represented as a pin joint in the PRBM. Equations for the position of this characteristic pivot is given for each type of flexible segment and is easily determined. Additionally, the variable $\Theta$ is the ***pseudo-rigid-body angle*** and equations relating it to the end angle of the beam are presented.

The strain energy stored in each flexible member is represented by a torsional spring with a spring constant of $K$. This spring constant is determined by geometric and material properties and is also dependent upon the type of flexible member. Formulas for each of these spring constants will be given.

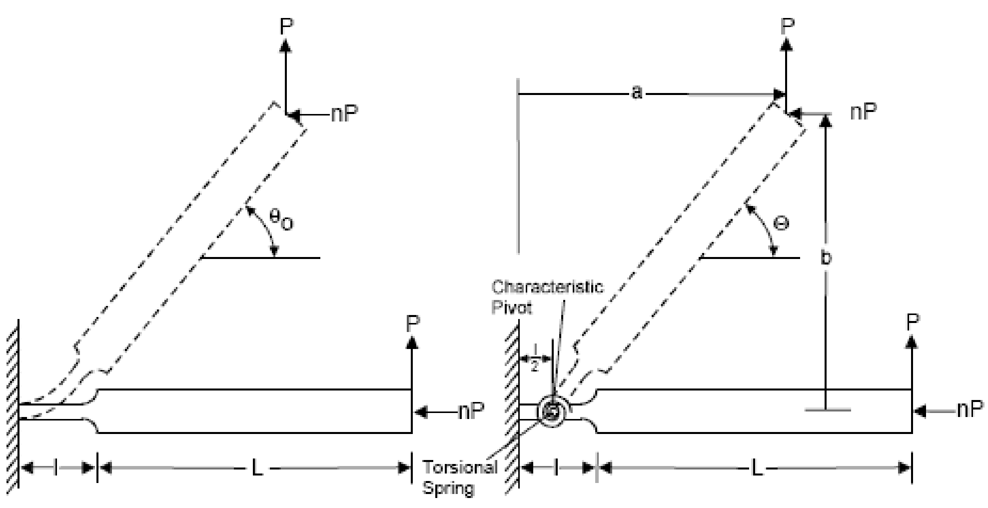



**Figure 2.1: (a) Small Length Flexure Pivot (b) Equivalent PRBM [Howell, 2001]**

These are the main parameters that will be described below. Each section gives a diagram for the flexible segment and its corresponding PRBM diagram, the characteristic pivot, and the torsional spring constant. Additionally, the formula for the maximum stress in each flexible element will be given.

### 2.1.2 Small Length Flexure Pivot

A small-length flexural pivot is one in which a large beam is grounded or pinned through a smaller beam, as illustrated in Figure 2.1a. Typically, a small-length flexural pivot satisfies the following conditions:

$$L \gg l \tag{2.1}$$

$$(EI)_L \gg (EI)_l \tag{2.2}$$

Where, $L$ is the length of large beam and $l$ is the length of small length flexure pivot. The characteristic pivot is located at the center of the small-length flexural pivot as shown in Figure 2.1b. For small-length flexural pivots, the basic equations are

$$\Theta = \theta o \tag{2.3}$$

$$K = (EI)_l / l \tag{2.4}$$

$$T = K\Theta \tag{2.5}$$

Where, $\Theta$ is *pseudo rigid body angle* and $\theta$o is compliant mechanism deflection angle. $E$, Young modulus of elasticity and $I$ is moment of inertia.

The $x$ and $y$ coordinates of the end of the beam can be found through

$$a = l / 2 + (L + l / 2)\cos\Theta \tag{2.6}$$

$$b = (L + l/2)\sin\Theta \tag{2.7}$$

The stress (σ) equations for this flexible segment are

$$\sigma_{top} = \frac{-(Pa + nPb)c}{I} - \frac{nP}{A} \tag{2.8}$$

$$\sigma_{bottom} = \frac{(Pa + nPb)}{I} - \frac{nP}{A} \tag{2.9}$$

Where, $A$ is cross sectional area, $P$ is force and σ is stress with subscript top and bottom to indicate stress at top surface and vice versa.

The equations presented here are sufficient in most cases. At times, the size of the small-length flexural pivot is small enough that the spring constant can be ignored if other larger torques are present. This special case small-length flexural pivots are called living hinges.

### 2.1.3 Cantilever Beam with Force at End

A second type of flexible segment is a cantilever beam with a force at the end. Figure 2.2a show the cantilever beam with its PRBM and corresponding parameters (Figure 2.2b).

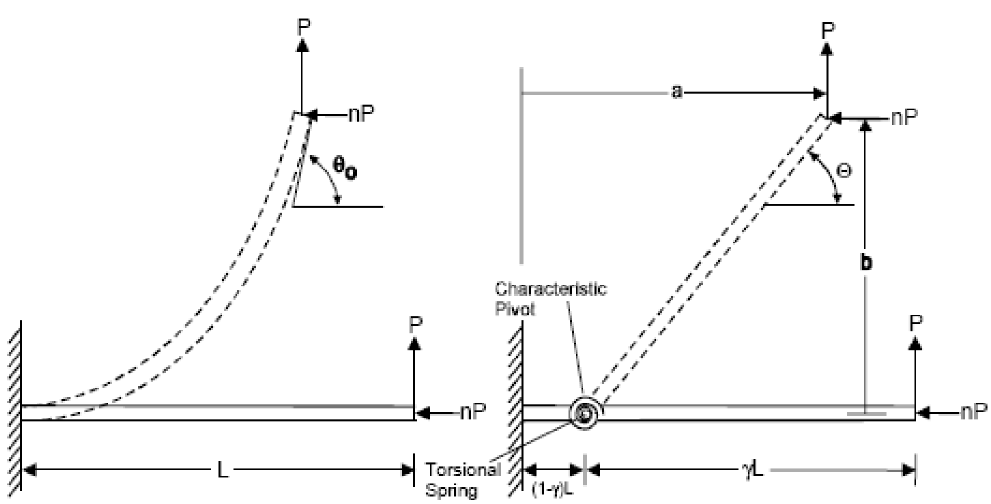



**Figure 2.2: (a) Cantilever beam with force at end**
**(b) Equivalent PRBM**
**[Howell, 2001]**

The characteristic pivot is located a distance $\gamma L$ from the free end where $\gamma$ is the *characteristic radius factor*. The value of the characteristic radius factor is a function of the direction of the applied force and can be expressed in terms of *n* as follows:

$$\gamma = 0.841655 - 0.0067807n + 0.000438n2; \quad (0.5<n<10.0) \qquad (2.10)$$

$$\gamma = 0.852144 - 0.0182867n; \quad (-1.8316<n<0.5) \qquad (2.11)$$

$$\gamma = 0.912364 + 0.0145928n; \quad (-5<n<-1.8216) \qquad (2.12)$$

For values of n between -0.5 and 1.0,

$$\gamma_{avg} = 0.85 \qquad (2.13)$$

There is some slight deviation between the pseudo rigid body angle and the actual angle of the beam. This variation is almost linear and is compensated through

$$\theta_0 = c_\theta\, \Theta \qquad (2.14)$$

where,

$$c_\theta \approx 1.24 \qquad (2.15)$$

$$K = \gamma K_\theta\, EI / L \qquad (2.16)$$

$$K_\theta \approx 2.65 \qquad (2.17)$$

The value $K_\theta$ is called the *stiffness coefficient*, $c_\theta$ is *parametric angle coefficient* and *L* is length of compliant member. The approximation given in Equation (2.17) is accurate in most cases. However, more accurate values are given in Howell (2001). With the spring constant, force and torque calculations can be made according to Equation (2.5).

The $x$ and $y$ coordinates of the end of the beam can be found through

$$a = L - \gamma L(1 - \cos\Theta) \tag{2.18}$$

$$b = \gamma L \sin\Theta \tag{2.19}$$

The stress in the beam can be calculated from Equation (2.8) and Equation (2.9), which are the same equations used for the small-length flexible segment.

### 2.1.4 Cantilever Beam with End Moment Loading

A cantilever beam is often loaded with an end moment. Figure 2.3 show this loading configuration along with its PRBM. The equations for this configuration are identical to the previous configuration. However, there are some differences in the values of the parameters. The characteristic radius factor is

$$\gamma = 0.7346 \tag{2.20}$$

The parametric angle coefficient is

$$c_\theta \approx 1.5164 \tag{2.21}$$

and stiffness coefficient is

$$K_\theta \approx 2.0643 \tag{2.22}$$

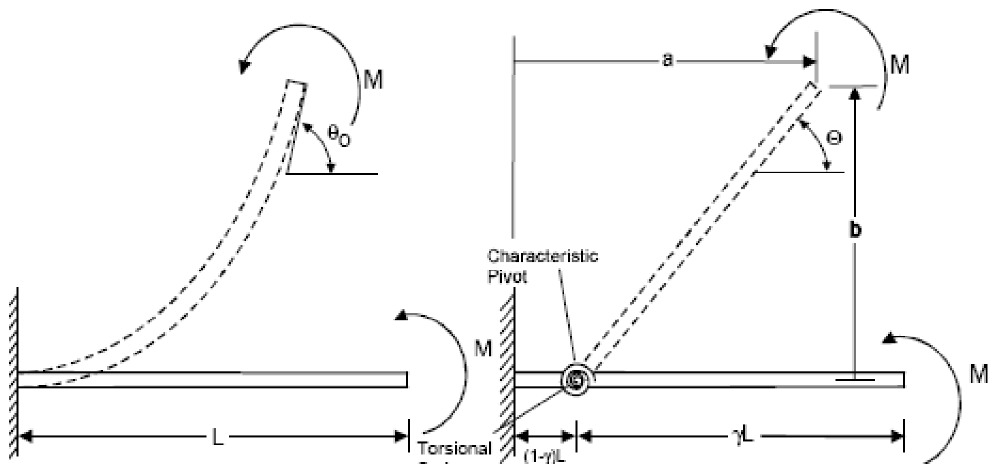



**Figure 2.3: (a) Cantilever beam with end moment loading**
**(b) Equivalent PRBM**
**[Howell, 2001]**

### 2.1.5 Fixed-Guided Beam Segment

A common type of flexible segment is the fixed-guided beam. This beam consists of a beam fixed on one end, while the other end is kept perpendicular to ground during displacement. This is commonly used for mechanisms such as parallel and folded beam mechanisms and is illustrated in Figure 2.4.

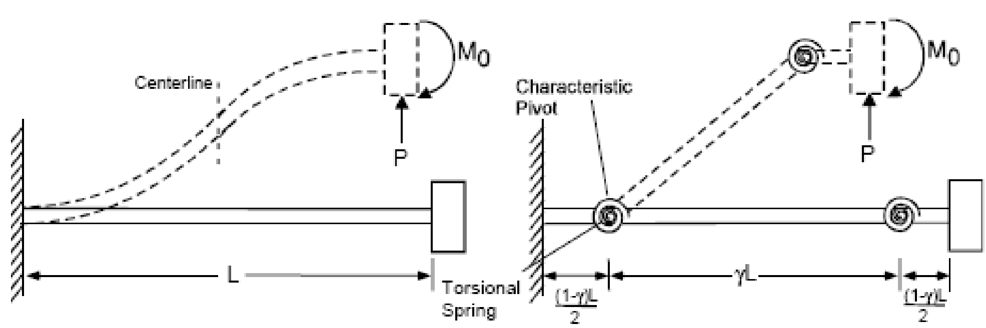



**Figure 2.4: (a) Fixed guided beam segment**
**(b) Equivalent PRBM**
**[Howell, 2001]**

Close observation of the fixed-guided beam shows that the curvature is zero at the middle due to symmetry. It is also known that the curvature at the end of a cantilever beam with an end load is also zero. Therefore, the fixed-guided beam can be modeled as two cantilever beams each half the length of the original beam. Thus, the PRBM is easily found.

The parameters for this beam are similar to previous parameters. The characteristic radius factor is

$$\gamma = 0.8517 \tag{2.23}$$

Since the beam has a constant end angle, the parametric angle coefficient is trivial and

$$c_\theta = 0 \tag{2.24}$$

The spring constant is found to be

$$K = 2\gamma K_\theta \, EI / l \tag{2.25}$$

From observation, it can be seen that the spring constant for the fixed-guided beam is twice that for cantilever beams. This indicates that the overall stiffness of the fixed-guided beam is four times that of the cantilever beam.

Since the end of the beam is constrained, a reactionary moment, $M_0$ is created. The formula for this moment is

$$M_0 = Pa/2 \tag{2.26}$$

or

$$M_0 = \frac{Pa}{2}[1 - \gamma(1 - \cos\Theta)] \tag{2.27}$$

The maximum stress occurs at the ends of the beam where the moment is largest and has a value of

$$\sigma_{max} = \frac{Pac}{2I} \tag{2.28}$$

Where, c is distance from center of a beam to its surface, *I* is moment of inertia and *P* is force.

### 2.1.6 Fixed-Fixed Beam Segment

In fixed-guided beam, beam maintains constant end angle. In fixed-fixed beam, ends are not constrained to stay at the same angle and are allowed to move as needed. Its pseudo rigid body model is same as fixed-guided beam. This model is easy to implement in compliant mechanism design and is useful in the initial design phase. Once a compliant mechanism design has been developed, it can further be refined and tested [Howell, 2001].

## 2.2 Type Synthesis of Compliant Mechanism

Type synthesis may be defined as the process of determining possible mechanism structures to perform a given task or combination of tasks without regard to the dimensions of the components. Type synthesis is performed to select a mechanism type before carrying out dimensional synthesis, which is the process of choosing mechanism dimensions to create a finished mechanism design.

The first step of the design process is the formulation of a mathematical model to represent the structure of a mechanism. In rigid-body kinematics, graph theory provides a mathematically rigorous representation of a mechanism structure through the use of matrices. The matrix representation for compliant mechanisms builds on the foundation established in rigid-body kinematics by adding information regarding segment type and the connectivity between segments to the matrices that represent a mechanism's topology [Murphy *et.al.*, 1994 & Howell, 2001].

The type synthesis technique consists of finding a number of possible mechanism configurations, including kinematic inversions of each type, which can solve the particular problem. The mechanism configuration which will most easily solve the problem can then be chosen [Howell, 2001].

## 2.3 Rigid-Body Replacement Synthesis

Compliant mechanism synthesis can easily be accomplished by rigid-body replacement synthesis [Howell, 2001]. This synthesis approach is useful when a compliant mechanism is to be used to perform a traditional rigid body mechanism task, for example path or motion generation, without concern for the energy storage in the flexible member or segment. Once the kinematic geometry is determined, the structural properties of the flexible members may be chosen according to the allowable stresses and input requirements. In rigid body replacement synthesis rigid body equations are applied directly to the pseudo rigid body model because only the kinematics of the mechanism is an issue, it may also be called as “Kinematic Synthesis”.

Rigid body equations can be applied directly to a model but problem lies in determining the suitable pseudo rigid body model for a compliant mechanism and evaluating the feasibility of the resulting model. It is necessary because synthesis may yield configurations that are adequate for rigid body but are not suitable for compliant mechanism, such as flexure pivot, it does not deflect through a complete rotation. Therefore it can not be used at a place where complete rotation is desirable [Howell, 2001].

# CHAPTER – 3
# MODELING AND ANALYSIS OF COMPLIANT STRAIGHT LINE MECHANISM

Aim of this chapter is to introduce a modeling concept that can be used for analysis of compliant mechanism. Pseudo-rigid-body model is used to model compliant mechanisms that are generated from Roberts approximate straight line mechanism by using rigid body replacement synthesis. The analysis is carried out using the same concept as given in the literature, but they are used to define utility of PRBM as an initial design tool by comparing its result with commercial finite element code i.e. ANSYS, to benchmark accuracy of PRBM and its usefulness in compliant mechanism modeling.

## 3.1 Special Mechanisms

There are some unusual mechanisms which can meet common needs of mechanical engineering problems. These special mechanisms can serve a range of purposes like generating straight lines, transferring torque between non-coaxial shafts, self centering steering and mechanical punched card readers. Some mechanisms have special motion characteristics different from those of generic mechanisms. These mechanisms are used for special purposes and few particular categories of motion. These mechanisms are unusual enough to be called as special mechanisms. Straight line mechanisms are one of the special mechanisms. These are of two types:

- Approximate straight line mechanisms.
- Exact straight line mechanisms.

### 3.1.1 Why Roberts Mechanism

Roberts approximate straight line mechanism is a symmetrical four bar linkage. Its construction is different from contemporaries, in the sense that, this mechanism has an extension to the coupler at the coupler mid point. This extension is perpendicular to the line joining the two adjacent joints. The end point of the coupler extension generates an approximate straight line for the motion between the fixed points. The reason for selecting it, which also makes it different from other straight line mechanisms, is that it does not have crossing links. This enables the fully compliant design to be fabricated in a single layer as one part, requiring no assembly. The schematic of the mechanism is shown in Figure 3.1

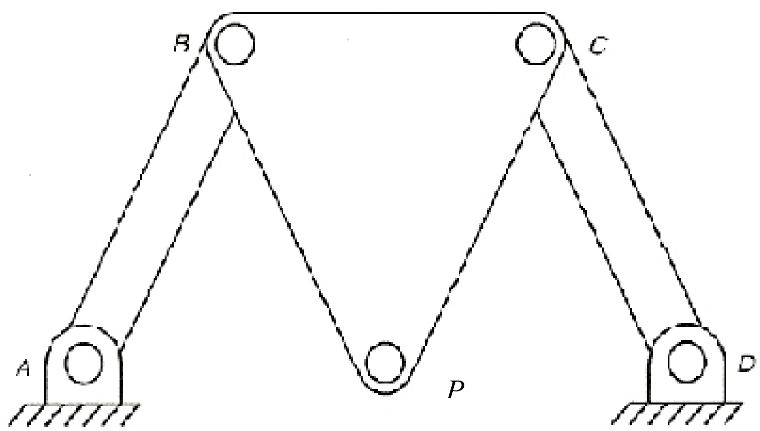



**Figure 3.1: Roberts Approximate Straight Line Mechanism**

The mechanism designed by the Richard Roberts has the proportions of lengths of member of the linkages as

$$AB = BP = CP = CD \tag{3.1}$$

$$AD = 2BC \tag{3.2}$$

The same Roberts design proportions are used as constraints in developing its compliant counterparts.

## 3.2 Modeling of Compliant Straight Line Mechanism

### 3.2.1 Segment Type

The constraints that define the Roberts equivalent compliant counterpart are mention at Equations (3.1) and (3.2). The other limitations are, mechanism must be fully compliant with fixed connection of flexible segment with ground and coupler link or use flexural pivot between ground, side links and coupler link, without these later limitations the resultant compliant segment enumerations of Roberts mechanism, which is essentially a four bar chain, would result in 135 nonisomorphic compliant chains after the application of type synthesis as define by Murphy for four bar mechanism [Murphy *et.al.*, 1994 & Howell, 2001]. These above mentioned constraints have led us to only two configurations which are fully compliant.

### 3.2.2 Rigid Body Replacement Synthesis

At this stage a rigid body replacement synthesis is applied to Roberts approximate straight line mechanism. In first model, the pin joints are replaced by small-length flexural pivots and in second model, rigid links at the side are replaced with flexible segments. This has given us the two models of fully compliant straight line mechanism as shown in Figure 3.2. The pseudo rigid body model is then applied to generate equivalent PRBM models to these fully compliant mechanisms and shown in Figure 3.3.

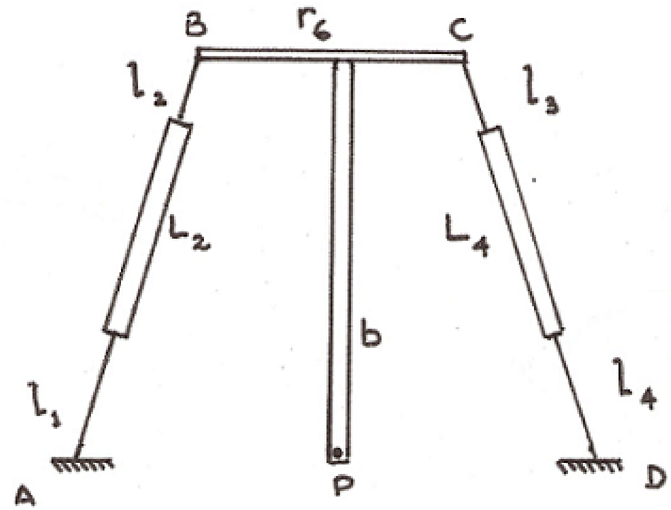



**Figure 3.2a: Model 1 – Fully Compliant Straight Line Mechanism with Four Small Length Flexure Pivots (*$l_1$, $l_2$,$l_3$ and $l_4$* are Small length flexure pivots, *$L_2$* and *$L_4$* are rigid segments)**

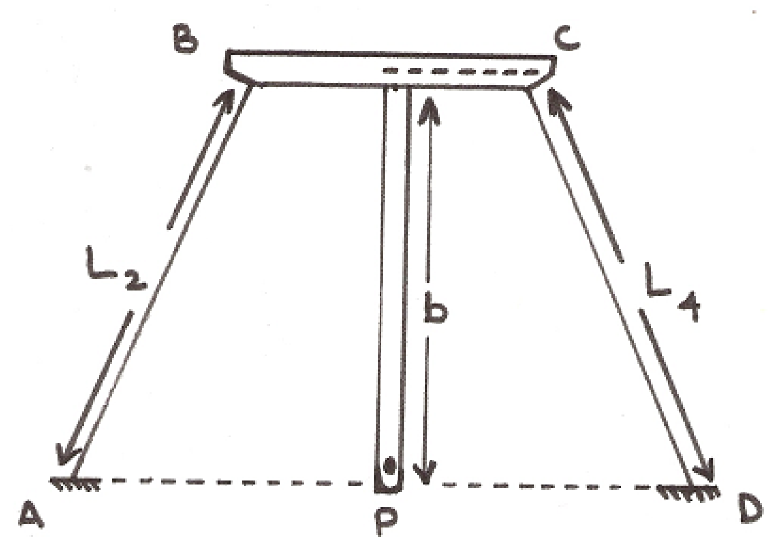



**Figure 3.2b:** **Model 2 – Fully Compliant Straight Line Mechanism with Fixed-Fixed segments at the side ($L_2$ and $L_4$ are flexible segments)**

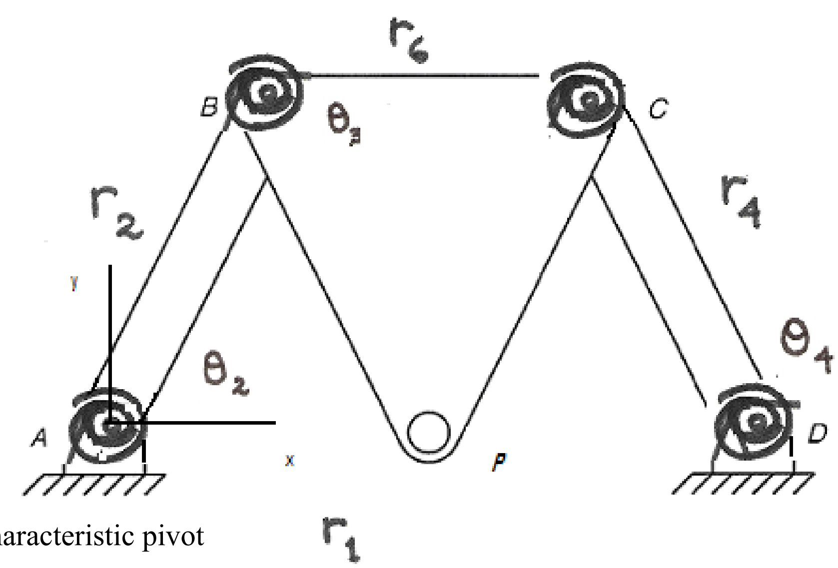



**Figure 3.3a:** **Equivalent Pseudo Rigid Body Model**

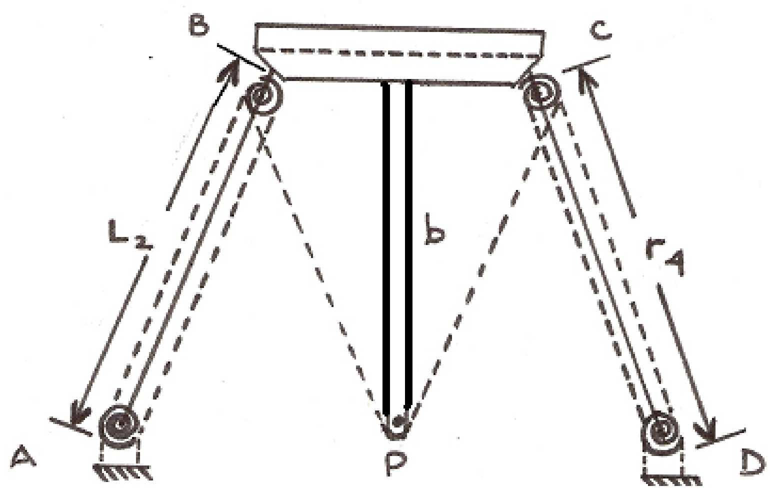



**Figure 3.3b: Dashed lines are indicating Pseudo Rigid Body Model Solid Lines are indicating Compliant Mechanism with Two flexible segments (Fixed-Fixed segment type)**

The appropriate lengths of the flexible segments were determined by using the pseudo rigid body model as discussed in chapter 2. This is presented in detail in later sections.

## 3.3 Theoretical Model

### 3.3.1 Position Analysis – Closed Form Equations

The variables used in the equation are shown in Figure 3.4. The known variables for the problem are $r_2$, $r_6$, $r_3$, $r_4$ and $r_1$. The initial position condition is $\theta_3$ zero degree. The crank angle $\theta_2$ is the input and is measured from $r_1$ i.e. ground.

The law of cosine is used to determine the length $\delta$ (i.e. BD) and the internal angles $\beta$, $\phi$ and $\lambda$.

$$\delta^2 = (r_1^2 + r_2^2 - 2\, r_1 r_2 \cos\theta_2)^2 \tag{3.3}$$

$$\beta = a\cos(r_1^2 + \delta^2 - r_2^2)/2r_1\delta \tag{3.4}$$

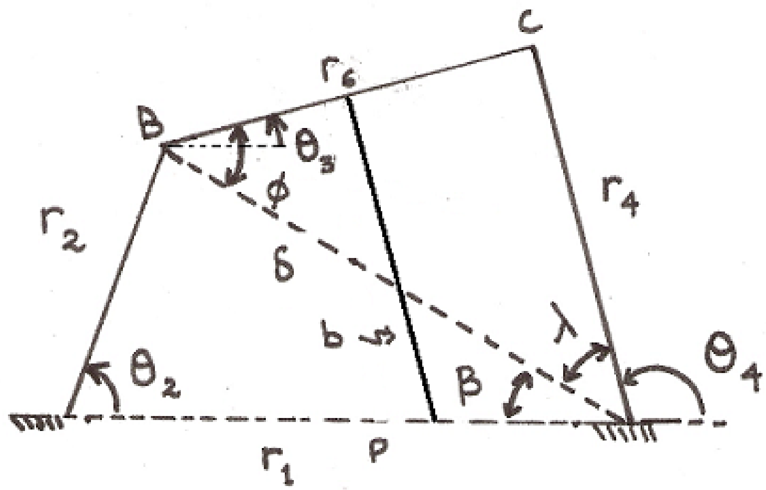



**Figure 3.4: Roberts Approximate Straight Line Mechanism Skeleton**

$$\lambda = a\cos(r_4^2 + \delta^2 - r_6^2)/2r_4\delta \tag{3.5}$$

$$\phi = a\cos(r_6^2 + \delta^2 - r_4^2)/2r_6\delta \tag{3.6}$$

The link angle $\theta_3$ and $\theta_4$ are

$$\theta_3 = \phi - \beta \tag{3.7}$$

$$\theta_4 = \Pi - (\beta + \lambda) \tag{3.8}$$

The equation relating the displacement of coupler point P with the link angles is

$$Xp = r_2(\cos\theta_2 - \cos\theta_{20}) + (r_6/2)\cos\theta_3 + b\sin\theta_3 \tag{3.9}$$

Where, $\theta_{20}$ is initial position of crank and $b$ is the distance of coupler point P from the mid of coupler link, $r_6$.

$$b^2 = (\overline{BP})^2 - (r_6/2)^2 \tag{3.10}$$

### 3.3.2 Principle of Virtual Work

The principle of virtual work [Howell, 2001] and the PRBM can be used to determine the static force for a given deflection. It is assumed that all forces referred throughout this work are static unless stated otherwise.

To determine the static force for a given deflection, equations must be developed relating displacement, compliant member deflection, and static input force. Using the principle of virtual work and the PRBM, a fictitious or virtual displacement ($\delta z$) can be made from which the virtual work ($\delta W$) can be calculated from

$$\delta W = F.\delta z \tag{3.11}$$

Similarly, virtual work due to moment can be calculated from

$$\delta W = M.\delta\theta \tag{3.12}$$

Where $\delta W$ is the virtual work due to the moment, $M$, and virtual angular displacement $\delta\theta$. An equation for forces can be found by taking the derivative of potential energy, ($V$), with respect to the generalized coordinate, ($q$). This would results in

$$\delta W = -\frac{dV}{dq}\delta q \tag{3.13}$$

Summing the virtual works in Equation (3.11) to (3.13) results in

$$\delta W = \sum_i F_i.\delta z_i + \sum_j M_j.\delta\theta_j - \sum \frac{dV_k}{dq_k}dq_k \tag{3.14}$$

Having established equations for virtual work, the principle of virtual work can be applied. The principle of virtual work can be stated as (Paul, 1979) [Weight, 2001]:

The net virtual work of all active forces is zero if and only if an ideal mechanical system is in equilibrium.

This principle allows equation (3.14) to be set equal to zero. If all virtual displacements are written in terms of the generalized coordinate, equation (3.14) reduces to

$$(\sum_i F_i.A + \sum_j M_j.B - \sum \frac{dV_k}{dq_k})dq_k = 0 \tag{3.15}$$

Where $A$ and $B$ are vectors that changes the linear and angular displacements into terms of the generalized coordinate. If $\delta q_k$ is assumed to be zero (hence the fictitious displacement), then the remaining equation can be solved for the unknown force or moment.

The method of virtual work was applied to the pseudo rigid body model shown in Figure 3.3. The variable $\theta_2$ was chosen to be the generalized coordinate. Equations for the virtual work associated with each torsional spring were developed. An equation was developed relating an unknown static force at coupler point $P$ in the horizontal direction. These equations were summed, and the principle of virtual work was applied. The force, $F$, was solved and results in

$$F = \frac{-K_1(\theta_2-\theta_{20}) - K_2[(\theta_2-\theta_{20})-(\theta_3 - {}_{30})] + K_2[(\theta_2-\theta_{20})-(\theta_3-\theta_{30})]h_{32} - K_3[(\theta_4-\theta_{40})-(\theta_3-\theta_{30})]h_{42} + K_3[(\theta_4-\theta_{40})-(\theta_3-\theta_{30})]h_{32} - K_4(\theta_4-\theta_{40})h_{42}}{r_2\sin\theta_2 + h_{32}(\frac{r_6}{2}\sin\theta_3 - b\cos\theta_3)} \tag{3.16}$$

Where,

$\theta_{io}$ is initial position angle, $K_i$ is spring constant, $h_{32}$ , $h_{42}$ are kinematic co-efficients for four bar mechanisms and are

$$h_{32} = \frac{d\theta_3}{d\theta_2} = \frac{r_2 \sin(\theta_4 - \theta_2)}{r_3 \sin(\theta_3 - \theta_4)} \tag{3.17}$$

$$h_{42} = \frac{d\theta_4}{d\theta_2} = \frac{r_2 \sin(\theta_3 - \theta_2)}{r_4 \sin(\theta_3 - \theta_4)} \tag{3.18}$$

The Equation (3.16) is valid for both compliant mechanisms PRB model shown in Figure 3.3a and 3.3b.

Since $K_1$, $K_2$, $K_3$, $K_4$ are equal in both models. The Equation (3.16) would become

$$F = \frac{-K(2-h_{32})\Delta\theta_2 + (2h_{32} - 1 - h_{42})\Delta\theta_4 + (2h_{42} - h_{32})\Delta\theta_3}{r_2 \sin\theta_2 + h_{32}(\frac{r_6}{2}\sin\theta_3 - b\cos\theta_3)} \tag{3.19}$$

Where,

$$\Delta\theta_i = (\theta_i - \theta_{i0}) \tag{3.20}$$

This equation tells how force $F$ is related to link lengths, spring constants and angle of the mechanisms as defined in Figure 3.3.

### 3.3.3 Stress Analysis

Stress analysis on flexures, whether it's a small length flexure pivot or it is a flexible link, is easy to carryout. The main issue in compliant mechanism is to ensure that the stress occurred at the flexure should remain with in the yield strength limits of a material used in mechanism. The maximum stress occurs when the flexures are subjected to the maximum deflection.

To analyze the stresses in mechanism, it is necessary to first look at the stress in one of the mechanism links. The critical stress, $\sigma_c$, in a small length flexible pivot under bending can be determined from Equation (3.21) and $\sigma_c$, at characteristic pivots of flexible segment can be determined from Equation (3.21).

$$\sigma_c = \frac{Mc}{I} \tag{3.21}$$

Where,

$M$ = bending moment.

$c$ = distance from the neutral plane to the top / bottom plane ($h/2$).

$I$ = moment of inertia of the cross-section of flexible segment ($bh^3/12$).

Using PRBM, the bending moment is found to be

$$M_i = K_i.\Delta\Theta_i \tag{3.22}$$

Where,

$K_i$ is PRBM spring constant,

$\Delta\Theta_i$ is Pseudo rigid body angle

In both of our PRB models $\Theta_i$ is equal to $\theta_i$ as parametric angle coefficient $c_\theta$ is negligible, which is consistent with PRBM Equations (2.3) and (2.24).

The values for $\Delta\Theta$ for each pivot in the compliant straight line mechanism is defined as

$$\Delta\Theta_1 = \theta_2 - \theta_{20} \tag{3.23}$$

$$\Delta\Theta_2 = (\theta_2 - \theta_{20}) - (\theta_3 - \theta_{30}) \tag{3.24}$$

$$\Delta\Theta_3 = (\theta_4 - \theta_{40}) - (\theta_3 - \theta_{30}) \tag{3.25}$$

$$\Delta\Theta_4 = (\theta_4 - \theta_{40}) \tag{3.26}$$

Substituting Equation (3.22), $I$ and $c$ in Equation (3.21), we get critical stress at each pivot in pseudo rigid body model of small length flexure pivot compliant mechanism and fixed-fixed flexible segment.

$$\sigma_c = \frac{6K_i\Theta_i}{b_i h_i^2} \tag{3.27}$$

As we know, the limit condition for mechanism is

$$\sigma_c = S_y \tag{3.28}$$

Substituting Equation (3.28) and (2.4) in (3.27) results in maximum deflection angle or in other words limits of deflection angle for model 1 (small length flexure pivot)

$$\Theta_i = S_y \frac{b_i h_i^2}{6K_i} \tag{3.29}$$

For model 2 (Fixed-Fixed segement), by substituting Equation (3.28) and (2.25) in Equation (3.27), yields

$$\Theta_i = S_y \frac{b_i h_i^2}{6K_i} \tag{3.30}$$

### 3.3.4 Material Selection

Different types of material can be used in compliant mechanism design. Material selection is generally governed by flexibility concerns as large deflection is desirable in these mechanisms. The most important thing to remember in selecting a material for compliant mechanism is that stiffness and strength, ductility and flexibility, are not equivalent. Brittle material can also be used to construct compliant mechanisms if their geometry is made such that they are not overstressed [Howell, 2001].

In selecting a material for our model, there is only one concern and that is mechanical performance of compliant straight line mechanism, which is defined as the displacement range of coupler point *P*. To obtain a large displacement range, material should allow a great elastic deformation without undergoing plastic stress region. This means that the material must have large ratio between the elastic limit i.e. yield strength ($S_y$) and modulus of elasticity ($E$). A trade-off of selection between the materials with the low $E$ value and the high $S_y/E$ value should be conducted before a final material is selected. According to the list of material shown in Table 3.1, the material of polypropylene is selected.

**Table 3.1: Material Properties for Several Material [Howell, 2001]**

| Material | Young Modulus, $E$ (GPa) | Yield Strength, $S_y$ (MPa) | $(S_y/E)$*1000 |
|---|---|---|---|
| Steel (1010 hot rolled) | 207 | 179 | 0.87 |
| Steel (4140 Q&T @ 400) | 207 | 1641 | 7.9 |
| Aluminum (1100 annealed) | 71.7 | 34 | 0.48 |
| Aluminum (7075 heat treated) | 71.7 | 503 | 7 |
| Titanium (Ti-35A annealed) | 114 | 207 | 1.8 |
| Titanium (Ti-13 heat treated) | 114 | 1170 | 10 |
| Beryllium copper (CA 170) | 128 | 1170 | 9.2 |
| Polycrystalline silicon | 169 | 930 | 5.5 |
| Polyethylene (HDPE) | 1.4 | 28 | 20 |
| Nylon (type 66) | 2.8 | 55 | 20 |
| Polypropylene | 1.4 | 34 | 25 |
| Kevlar (82 vol %) in epoxy | 86 | 1517 | 18 |
| E-glass (73.3 vol%) in epoxy | 56 | 1640 | 29 |

## 3.4 Model 1 – Analysis Results

In model 1, by rigid body replacement synthesis of Roberts approximate straight line mechanism, a fully compliant mechanism with four small length flexure pivots was developed as shown in Figure 3.5. There are some assumptions, which are linked with its Pseudo Rigid Body Model. The assumptions are as under and will help in determining the length of PRBM link.

**(a)** The flexure pivot length is much smaller than the corresponding PRBM link length. The same is defined in Equation (2.1)

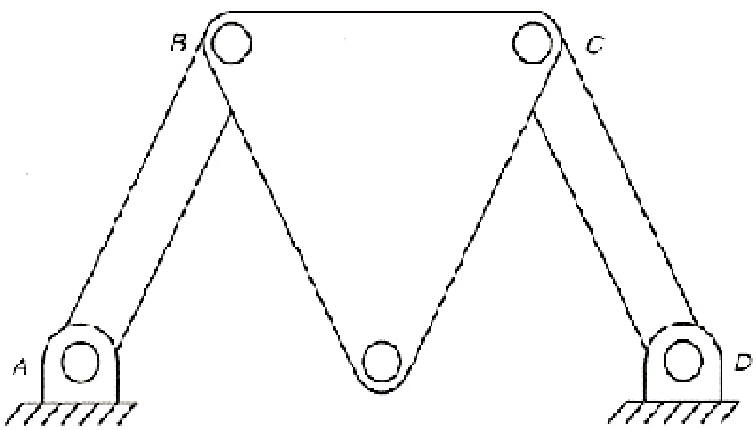
$$L \gg l \tag{2.1}$$

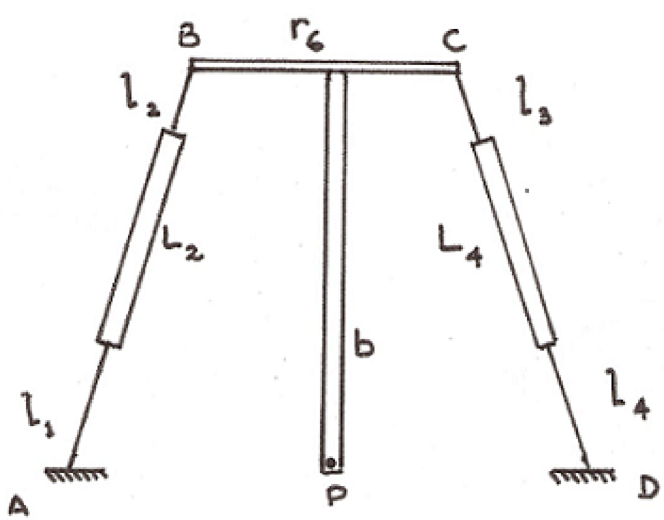
Mathematically,

$$l_i = \Xi \,.\, r_i \tag{3.31}$$

Where, $\Xi$, is the ratio of $l_i$ over $r_i$ and $r_i$ will be either one of the two PRBM link lengths or an average of corresponding link lengths, it defined as

$$r_{avg} = \frac{r_i + r_{i+1}}{2} \tag{3.32}$$



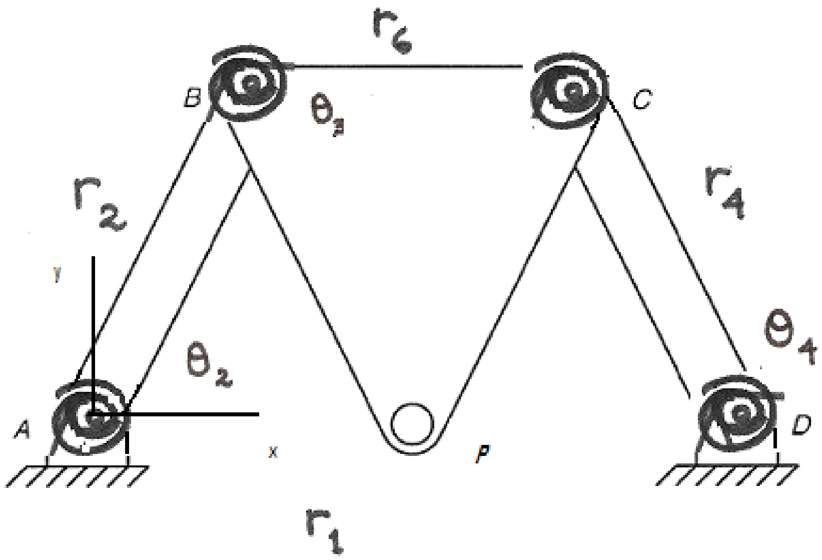



**Figure 3.5:** **(a) Roberts Approximate Straight line Mechanism**
**(b) Compliant Mechanism with four Small Length Flexure Pivots after Rigid Body Replacement Synthesis**
**(c) Equivalent Pseudo Rigid Body Model**

(b) Commonly the value of $\Xi$ is 0.10. This value was used in calculating link lengths.

(c) The cross-section of compliant segment is assumed to be of rectangular shape, as deflection is dependent on material and deflection. Therefore, moment of inertia *I* of small length flexure is assumed 2.4mm$^4$. By keeping width, *h* fixed at1.5mm, *t*, the in-plane thickness turned out to be 8.75mm.

(d) The segment between two small length flexures ($L_2$, $L_4$) are assumed to be rigid but of same material as of small length flexures.

The Figure 3.5a shows rigid body diagram. A fully compliant mechanism is obtained by replacing each pin joint with small length flexure pivot as shown in Figure 3.5b. The pseudo rigid body model is shown in Figure 3.5c, with torsional spring at all joints to represent the strain energy associated with the deflection of the flexible segment. The small length flexure pivots are located such that their centers are at the location of the pin joint.

### 3.4.1 Link Lengths and Initial Position

The variables for link lengths of model 1 is shown in Figure 3.5 (a to c) and length dimension are given in Table 3.2. The PRBM link lengths r2 and r4 are calculated from

$$r_2 = \frac{l_1}{2} + L_2 + \frac{l_2}{2} \tag{3.33}$$

$$r_4 = \frac{l_3}{2} + L_4 + \frac{l_4}{2} \tag{3.34}$$

Where, $L_2$, and $L_4$ are rigid link lengths and $l_1$, $l_2$, $l_3$, $l_4$ are small flexural link length. The lengths $r_2$ and $r_4$ are pivot to pivot (characteristic pivots) distance.

The length $r_6$ of PRBM is same as of rigid body mechanism ( $\overline{BC}$ ). The coupler point *P* length (*b*) is calculated through Equation (3.10).

**Table 3.2: Link Lengths and Initial Position of Mechanism–Model 1**

| Rigid Body Mechanism | | Compliant Mechanism | | PRBM | |
|---|---|---|---|---|---|
| **Designation** | **Length /Angle** | **Designation** | **Length / Angle** | **Designation** | **Length / Angle** |
| AB | 50 mm | $l_1$ | 5 mm | $r_2$ | 55 mm |
| | | $L_2$ | 50 mm | | |
| | | $l_2$ | 5 mm | | |
| BP% | 50 mm | - | - | - | - |
| CP% | 50 mm | - | - | - | - |
| CD | 50 mm | $l_3$ | 5 mm | $r_4$ | 55 mm |
| | | $L_4$ | 50 mm | | |
| | | $l_4$ | 5 mm | | |
| BC | 32 mm | $L_6$ | 32 mm | $r_6$ | 32 mm |
| AD | 64 mm | $L_1$ | 64 mm | $r_1$ | 64 mm |
| $b$ | 47.37 mm | $b$ | 57.82 | $b$ | 52.6 mm |
| $\theta_{20}$ | 71.33° | $\theta_{20}$ | 74.53° | $\theta_{20}$ | 73.08° |
| $\theta_{30}$ | 0° | $\theta_{30}$ | 0° | $\theta_{30}$ | 0° |
| $\theta_{40}$ | 108.66° | $\theta_{40}$ | * | $\theta_{40}$ | 106.91° |
| Legend: | %' - Length that are according to constraints define in Equation (3.1) and used for calculating Coupler point length | | | | |
| | *' - Not measured in FEM | | | | |

The MATLAB code (Appendix – A) is used to determine the initial position parameters of PRBM Model 1, Table 3.2.

### 3.4.2 Force – Displacement Relationship using Pseudo Rigid Body Model

The MATLAB routine is used for determining the force – displacement relation of pseudo rigid body model shown in Figure 3.5c, the code is appended as Appendix-A. The maximum displacement was limit by yield strength of polypropylene. The input parameters are shown in Table 3.3

**Table 3.3: Input Variables in MATLAB for PRBM – Model 1**

| Varaiable | Value |
|---|---|
| $r_1$ | 64 mm |
| $r_2$ | 55 mm |
| $r_3$ | 55 mm |
| $r_6$ | 32 mm |
| Yield Strength of Polypropylene, *Sy* | 34 Mpa |
| Young Modulus of Polypropylene, *E* | 1.4 Gpa |
| Moment of Inertia, *I* | 2.4 $mm^4$ |
| Thickness of small length flexure pivot, *h* | 1.5 mm |
| Length of small length flexure pivot, *l* | 5 mm |
| In-plane thickness of small length flexure Pivot, *t* | 8.75 mm |

The input was crank angle $\theta_2$, the limits of $\theta_2$ for calculating force displacement relationship was determined by Equation (3.30). Spring constant, K, was determined 0.689 N-m by Equation (2.7). Force and displacements were calculated by Equation (3.16 to 3.18) and (3.9), respectively. Stresses at each pivot were calculated by Equation (3.27). The results are shown in Table 3.4.

**Table 3.4: PRBM Results of Model 1 - Fully Compliant Mechanism with Small Length Flexure Pivot**

| $\theta_2$ | Force, F (mN) | Displacement, Xp (mm) | Stress at Pivot 1 (Pascal) | Stress at Pivot 2 (Pascal) | Stress at Pivot 3 (Pascal) | Stress at Pivot 4 (Pascal) |
|---|---|---|---|---|---|---|
| 68.452 | 5198.3 | 8.3282 | -1.70E+07 | -3.37E+07 | -3.29E+07 | -1.62E+07 |
| 68.624 | 5008.1 | 8.0253 | -1.64E+07 | -3.24E+07 | -3.17E+07 | -1.56E+07 |
| 69.197 | 4373.1 | 7.0128 | -1.43E+07 | -2.83E+07 | -2.77E+07 | -1.37E+07 |
| 69.713 | 3800.1 | 6.0975 | -1.24E+07 | -2.46E+07 | -2.41E+07 | -1.19E+07 |
| 70.286 | 3161.6 | 5.0757 | -1.03E+07 | -2.04E+07 | -2.01E+07 | -9.97E+06 |
| 70.859 | 2520.8 | 4.0487 | -8.17E+06 | -1.63E+07 | -1.61E+07 | -7.98E+06 |
| 71.431 | 1877.4 | 3.0164 | -6.07E+06 | -1.21E+07 | -1.20E+07 | -5.97E+06 |
| 71.947 | 1295.9 | 2.0825 | -4.18E+06 | -8.34E+06 | -8.29E+06 | -4.13E+06 |
| 72.52 | 646.61 | 1.0393 | -2.08E+06 | -4.15E+06 | -4.14E+06 | -2.07E+06 |
| 73.036 | 59.222 | 0.095193 | -1.90E+05 | -3.80E+05 | -3.80E+05 | -1.90E+05 |
| 73.093 | -6.2358 | -0.010023 | 20000 | 40001 | 40002 | 20001 |
| 73.666 | -663.07 | -1.0658 | 2.12E+06 | 4.25E+06 | 4.26E+06 | 2.13E+06 |
| 74.182 | -1258 | -2.0217 | 4.01E+06 | 8.05E+06 | 8.09E+06 | 4.06E+06 |
| 74.755 | -1923.5 | -3.0904 | 6.11E+06 | 1.23E+07 | 1.24E+07 | 6.22E+06 |
| 75.27 | -2526.9 | -4.0586 | 8.00E+06 | 1.61E+07 | 1.63E+07 | 8.19E+06 |
| 75.786 | -3135 | -5.0331 | 9.89E+06 | 1.99E+07 | 2.02E+07 | 1.02E+07 |
| 76.302 | -3747.9 | -6.0141 | 1.18E+07 | 2.38E+07 | 2.42E+07 | 1.22E+07 |
| 76.817 | -4366.3 | -7.0019 | 1.37E+07 | 2.77E+07 | 2.83E+07 | 1.42E+07 |
| 77.39 | -5060.1 | -8.1081 | 1.58E+07 | 3.20E+07 | 3.28E+07 | 1.65E+07 |
| 77.505 | -5199.7 | -8.3304 | 1.62E+07 | 3.29E+07 | 3.37E+07 | 1.70E+07 |

The PRBM predicted a maximum stress of 33.7 MPa at about 8.3 mm. when mechanism moves from its initial position to 8.3 mm in +ve x-axis, the max stress is developed at pivot 2 and in reverse direction pivot 3 has shown a maximum stress. The force displacement relationship is approximately linear, Figure 3.6

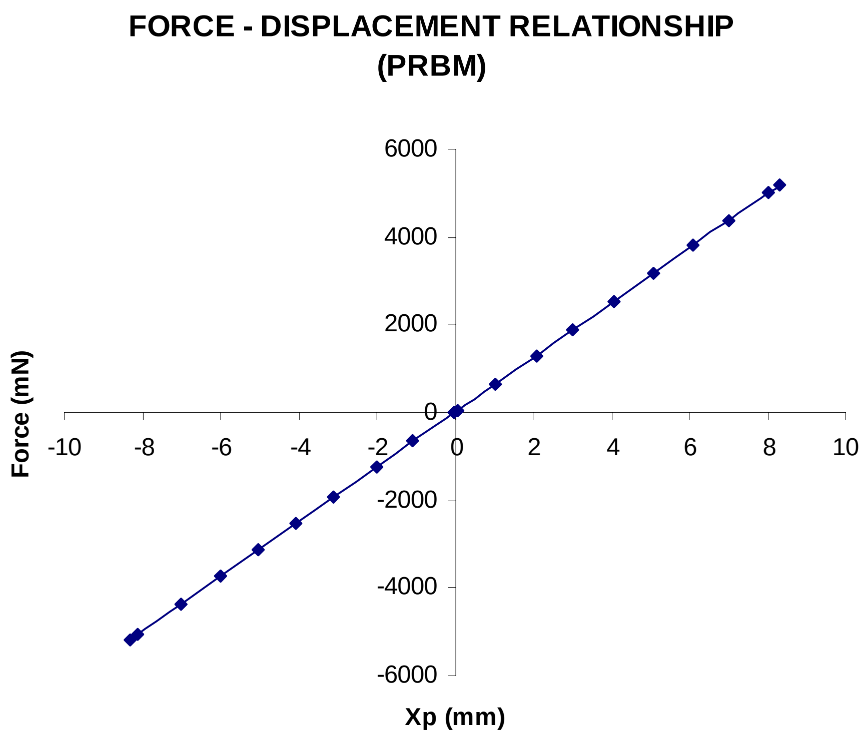



**Figure 3.6: Force – Displacement Relationship by using PRBM - Model 1**

### 3.4.3 Force – Displacement Relationship of Compliant Mechanism using Finite Element Analysis

The finite element analysis software used for force – displacement analysis of compliant mechanism, shown in Figure 3.5b, is ANSYS. The compliant straight line mechanism was analyzed as a 2D model with the material Polypropylene. The finite element model consisted of two-dimensional beam elements i.e. beam 3, Figure 3.7. The analysis employed a nonlinear solver that accounted for large deflections and stress stiffening. Table 3.5 enlists the input variables for FEA. Displacements were applied to the coupler point *P* in 1 mm increments. The force and stress were registered at each step. The force – displacement values of compliant mechanism measured by FEA are given in Table 3.6.

**Table 3.5: Input Variables for FEA - Model 1**

| Varaiable | Value |
|---|---|
| $L_i$ | 50 mm |
| $l_i$ | 5 mm |
| $r_6$ | 32 mm |
| Yield Strength of Polypropylene, *Sy* | 34 Mpa |
| Young Modulus of Polypropylene, *E* | 1.4 Gpa |
| Moment of Inertia, *I* | 2.4 $mm^4$ |
| Thickness of fixed-fixed segment, *h* | 1.5 mm |
| $\theta_{20}$ | 74.37° |
| Length of coupler point P, *b* | 57.82 |
| In-plane thickness of small length flexure Pivot, *t* | 8.75 mm |

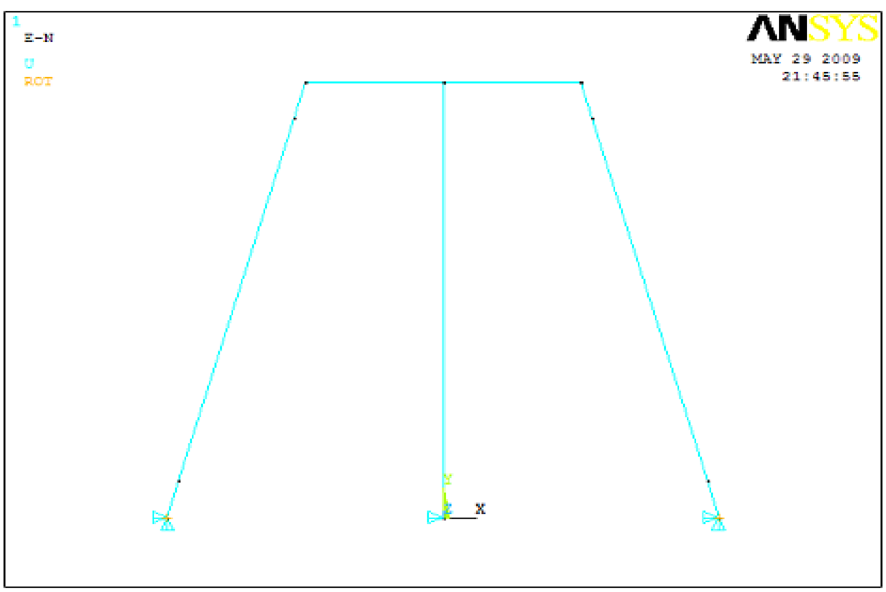



**Figure 3.7: Finite Element Model with 2D Beam Element (BEAM 3) - Model 1**

**Table 3.6: FEA Results of Model 1 - Fully Compliant Mechanism with Four Small Length Flexure Pivots**

| Force, F (mN) | Displacement, Xp (mm) | Stress (MPa) |
|---|---|---|
| 4912.11 | 8.5 | 33.5625 |
| 4622.36 | 8 | 31.5746 |
| 4042.42 | 7 | 27.5935 |
| 3463.35 | 6 | 23.623 |
| 2885 | 5 | 19.6628 |
| 2307.27 | 4 | 15.7124 |
| 1730.02 | 3 | 11.7713 |
| 1153.15 | 2 | 7.83916 |
| 596.512 | 1 | 3.91553 |
| -596.511 | -1 | 3.93095 |
| -1153.15 | -2 | 7.85451 |
| -1730.02 | -3 | 11.7711 |
| -2307.26 | -4 | 15.6809 |
| -2885 | -5 | 19.5845 |
| -3463.34 | -6 | 23.4821 |
| -4042.42 | -7 | 27.3741 |
| -4622.36 | -8 | 31.2607 |
| -4912.15 | -8.5 | 33.2486 |

The FEA predicted a maximum stress of 33.5 MPa at about 8.5 mm. The force displacement relationship is approximately linear and is shown in Figure 3.8.

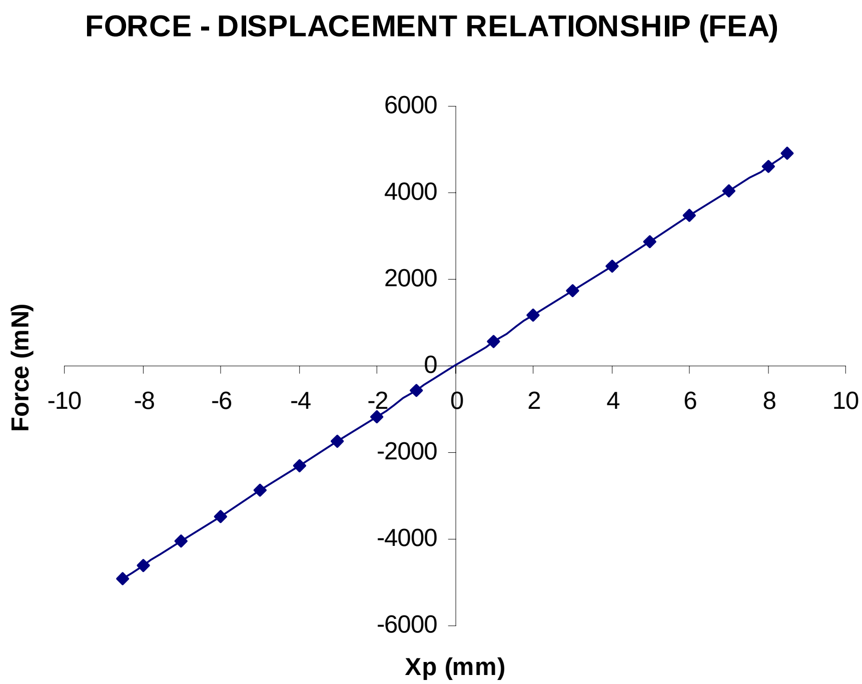



**Figure 3.8: Force – Displacement relationship of Compliant Mechanism using ANSYS - Model 1**

### 3.4.4 Force – Displacement Relationship Comparison

The comparison of PRBM results and FEA results of compliant mechanism was carried out, Figure 3.9. The FEA has given a 33.2 MPa at 8.5 mm where as PRBM predicted 34 MPa at 8mm. The variation is small as FEA results are close to what is predicted through PRBM. Table 3.7 enlists the spring constant calculated from Figure 3.9. The spring constant of PRBM is calculated theoretically from Equation (2.4) and is found out to be 689 N-mm. On the other hand FEA uses 596 N-mm as geometric and material properties are input to the solver. The spring constant of PRBM and FEA are having a difference of approximately 13%. Both FEA and PRBM have exhibited approximately linear force-displacement behaviour.

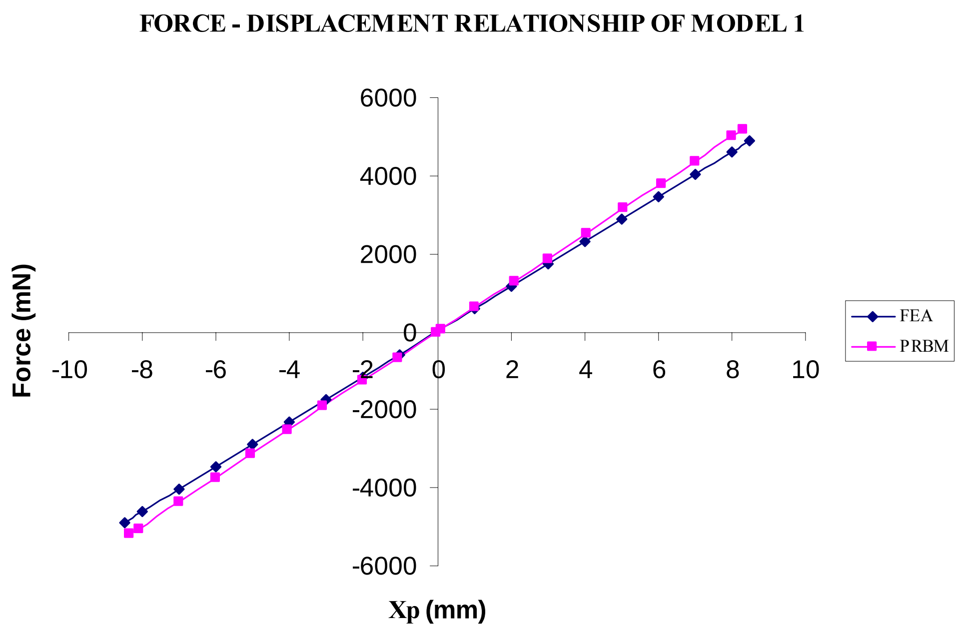



**Figure 3.9: Force Displacement Relationship of Model 1 – Comparison of FEA and PRBM Results**

**Table 3.7: Comparison of Spring Constant of FEA and Predicted by PRBM**

| Model | Spring Constant (N-m) |
|---|---|
| PRBM | 0.689 |
| FEA | 0.596 |

## 3.5 Model 2 – Analysis Results

In model 2, a fully compliant mechanism with two flexible segments was developed by rigid body replacement synthesis, Figure 3.10. These flexible segments were modeled as Fixed-Fixed segment because this segment model allows a change in beam end angle, as already discussed in section 2.1.5. There are some assumptions, which are linked with its Pseudo Rigid Body Model. The assumptions are as under and will help in determining the length of PRBM link and compliant member lengths.

(a) The length of flexible segment, *L*, is

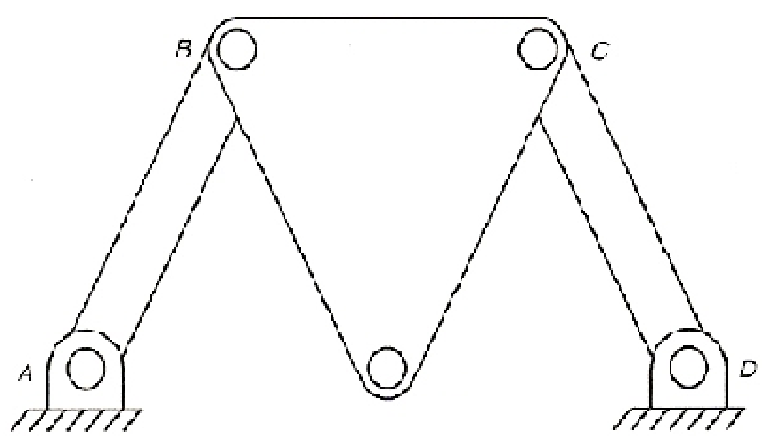
$$L_i = \frac{r_i}{\gamma} \tag{3.36}$$

Where, $L_i$ is flexible link length and $r_i$ is PRBM link length or effective link length, γ is PRBM characteristic radius factor.

(b) γ is assumed to be 0.85.

(c) $K_\theta$ is approximated as 2.65.

(d) The links of compliant mechanism are assumed as of rectangular shape, as deflection is dependent on material and geometry. Therefore, moment of inertia, *I,* of small length flexure is assumed 2.4mm$^4$. By keeping width, *h* fixed at1.5mm, *t*, the in-plane thickness turned out to be 8.75mm.



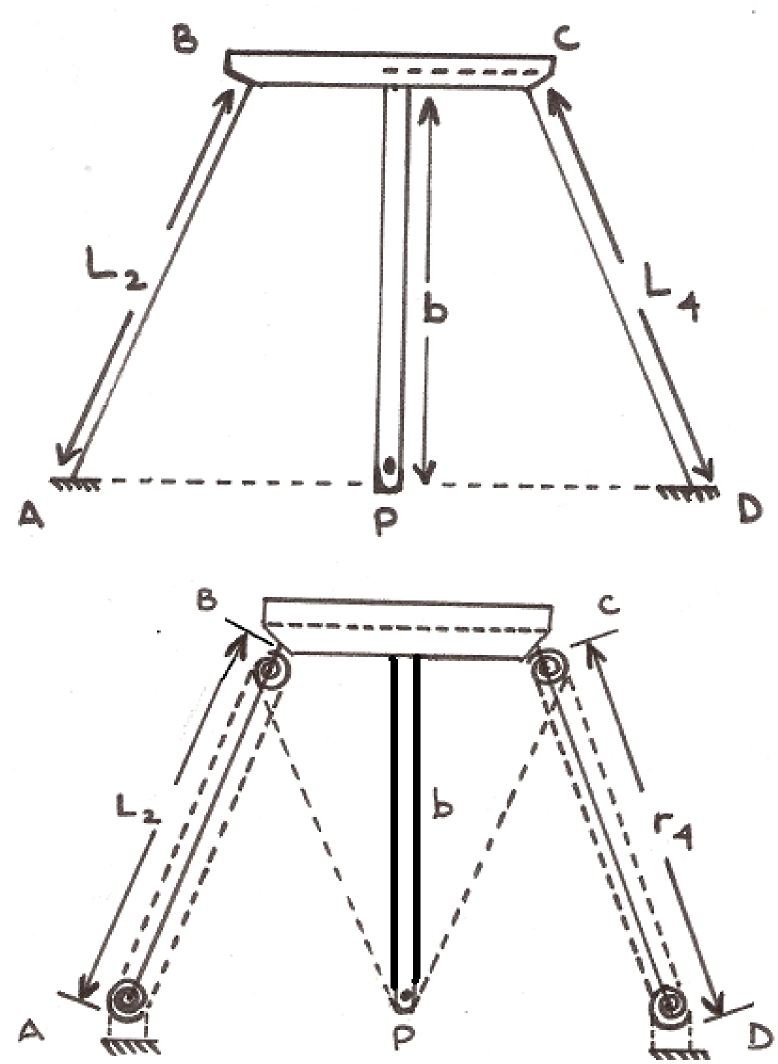



**Figure 3.10: (a) Roberts Approximate Straight Line Mechanism**

**(b) Fully Compliant Mechanism with Fixed-Fixed Segment**

**(c) Equivalent Pseudo Rigid Body Model**

The Figure 3.10a shows rigid body diagram. A fully compliant mechanism is shown in Figure 3.10b. The pseudo rigid body model is shown in Figure 3.10c, with torsional spring at all joints to represent the strain energy associated with the deflection of the flexible segment. The pivots are located at the pin joint.

### 3.5.1 Link Lengths and Initial Position

The variables for link lengths are shown in Figure 3.10(a to c). Table 3.8 enlists the length dimensions with initial position of mechanism. In this model rigid body length dimensions were taken as PRBM link lengths and flexible segment lengths were calculated by Equation (3.36). The MATLAB code (Appendix – B) is used to determine the initial position parameters of PRBM for Model 2.

**Table 3.8: Link Lengths and Initial Position of Mechanism**

| **Rigid Body Mechanism** | | **Compliant Mechanism** | | **PRBM** | |
|---|---|---|---|---|---|
| **Designation** | **Length /Angle** | **Designation** | **Length / Angle** | **Designation** | **Length / Angle** |
| AB | 50 mm | $L_2$ | 58.82 mm | $r_2$ | 50 mm |
| $\mathrm{BP}^{\%}$ | 50 mm | - | - | - | - |
| $\mathrm{CP}^{\%}$ | 50 mm | - | - | - | - |
| CD | 50 mm | $L_4$ | 58.82 mm | $r_4$ | 50 mm |
| BC | 32 mm | $L_6$ | 32 mm | $r_6$ | 32 mm |
| AD | 64 mm | $L_1$ | 64 mm | $r1$ | 64 mm |
| $b$ | 47.37 mm | $b$ | 56.60 mm | $b$ | 47.37 mm |
| $\theta_{20}$ | 71.33° | $\theta_{20}$ | 74.21° | $\theta_{20}$ | 71.33° |
| $\theta_{30}$ | 0° | $\theta_{30}$ | 0° | $\theta_{30}$ | 0° |
| $\theta_{40}$ | 108.66° | $\theta_{40}$ | * | $\theta_{40}$ | 108.66° |
| Legend: | %' - Length that are according to constraints define in Equation (3.1) and used for calculating Coupler point length | | | | |
| | *' - Not measured in FEM | | | | |

### 3.5.2 Force – Displacement Relationship using Pseudo Rigid Body Model

The MATLAB routine is used for determining the force – displacement relation of pseudo rigid body model shown in Figure 3.10c, the code is appended as Appendix-B. The maximum displacement was limit by yield strength of polypropylene. The input parameters are shown in Table 3.9.

**Table 3.9: Input Variables in MATLAB for PRBM - Model 2**

| Varaiable | Value |
|---|---|
| $r_1$ | 64 mm |
| $r_2$ | 50 mm |
| $r_3$ | 50 mm |
| $r_6$ | 32 mm |
| Yield Strength of Polypropylene, *Sy* | 34 Mpa |
| Young Modulus of Polypropylene, *E* | 1.4 Gpa |
| Moment of Inertia, *I* | 2.4 $mm^4$ |
| Thickness of small length flexure pivot, *h* | 1.5 mm |
| In-plane thickness of small length flexure Pivot, *t* | 8.75 mm |

The input was crank angle, $\theta_2$, the limits of $\theta_2$ for calculating force displacement relationship was determined by Equation (3.30). Force and displacements were calculated by Equation (3.16 to 3.18) and (3.9), respectively. Stresses at each pivot were calculated by Equation (3.27). The results are shown in Table 3.10.

**Table 3.10: PRBM Results of Model 2 - Fully Compliant Mechanism with Fixed-Fixed Flexible Segment**

| $\theta_2$ | Force, F (mN) | Displacement, Xp (mm) | Stress at Pivot 1 (Pascal) | Stress at Pivot 2 (Pascal) | Stress at Pivot 3 (Pascal) | Stress at Pivot 4 (Pascal) |
|---|---|---|---|---|---|---|
| 59.273 | 5700.4 | 18.85 | -1.70E+07 | -3.35E+07 | -3.13E+07 | -1.48E+07 |
| 59.788 | 5456.4 | 18.08 | -1.63E+07 | -3.21E+07 | -3.00E+07 | -1.42E+07 |
| 60.476 | 5132.2 | 17.049 | -1.53E+07 | -3.02E+07 | -2.84E+07 | -1.35E+07 |
| 61.164 | 4808.9 | 16.014 | -1.43E+07 | -2.83E+07 | -2.67E+07 | -1.27E+07 |
| 61.794 | 4513.3 | 15.061 | -1.34E+07 | -2.65E+07 | -2.51E+07 | -1.20E+07 |
| 62.481 | 4191.3 | 14.016 | -1.25E+07 | -2.46E+07 | -2.34E+07 | -1.13E+07 |
| 63.112 | 3896.5 | 13.053 | -1.16E+07 | -2.29E+07 | -2.18E+07 | -1.05E+07 |
| 63.742 | 3601.8 | 12.086 | -1.07E+07 | -2.11E+07 | -2.02E+07 | -9.80E+06 |
| 64.429 | 3280.3 | 11.025 | -9.73E+06 | -1.92E+07 | -1.85E+07 | -8.99E+06 |
| 65.06 | 2985.4 | 10.048 | -8.85E+06 | -1.75E+07 | -1.69E+07 | -8.23E+06 |
| 65.69 | 2690.2 | 9.0656 | -7.96E+06 | -1.57E+07 | -1.52E+07 | -7.45E+06 |
| 66.32 | 2394.4 | 8.078 | -7.07E+06 | -1.40E+07 | -1.36E+07 | -6.67E+06 |
| 66.95 | 2097.9 | 7.0848 | -6.18E+06 | -1.22E+07 | -1.19E+07 | -5.87E+06 |
| 67.581 | 1800.5 | 6.0859 | -5.29E+06 | -1.05E+07 | -1.03E+07 | -5.07E+06 |
| 68.211 | 1502.2 | 5.0811 | -4.41E+06 | -8.74E+06 | -8.59E+06 | -4.25E+06 |
| 68.841 | 1202.5 | 4.0701 | -3.52E+06 | -6.99E+06 | -6.89E+06 | -3.42E+06 |
| 69.471 | 901.49 | 3.0526 | -2.63E+06 | -5.23E+06 | -5.18E+06 | -2.57E+06 |
| 70.102 | 598.83 | 2.0284 | -1.74E+06 | -3.47E+06 | -3.45E+06 | -1.72E+06 |
| 70.732 | 294.32 | 0.99717 | -8.53E+05 | -1.70E+06 | -1.70E+06 | -8.47E+05 |
| 71.305 | 15.718 | 0.053256 | -45392 | -90775 | -90758 | -45375 |
| 71.362 | -12.243 | -0.041482 | 3.53E+04 | 7.07E+04 | 7.07E+04 | 3.54E+04 |
| 71.419 | -40.222 | -0.13628 | 1.16E+05 | 2.32E+05 | 2.32E+05 | 1.16E+05 |
| 71.992 | -321.1 | -1.0879 | 9.23E+05 | 1.85E+06 | 1.86E+06 | 9.31E+05 |
| 72.565 | -604.09 | -2.0463 | 1.73E+06 | 3.48E+06 | 3.50E+06 | 1.76E+06 |
| 73.138 | -889.39 | -3.0117 | 2.54E+06 | 5.11E+06 | 5.16E+06 | 2.59E+06 |
| 73.769 | -1206.1 | -4.0822 | 3.43E+06 | 6.91E+06 | 7.01E+06 | 3.53E+06 |
| 74.342 | -1497 | -5.0637 | 4.23E+06 | 8.56E+06 | 8.71E+06 | 4.39E+06 |
| 74.914 | -1790.9 | -6.0535 | 5.04E+06 | 1.02E+07 | 1.04E+07 | 5.26E+06 |
| 75.487 | -2088.1 | -7.0519 | 5.85E+06 | 1.19E+07 | 1.22E+07 | 6.15E+06 |
| 76.06 | -2388.9 | -8.0597 | 6.66E+06 | 1.36E+07 | 1.40E+07 | 7.05E+06 |
| 76.633 | -2693.6 | -9.0772 | 7.46E+06 | 1.53E+07 | 1.58E+07 | 7.97E+06 |
| 77.149 | -2971.5 | -10.002 | 8.19E+06 | 1.68E+07 | 1.74E+07 | 8.80E+06 |
| 77.722 | -3284.7 | -11.04 | 9.00E+06 | 1.85E+07 | 1.92E+07 | 9.75E+06 |
| 78.295 | -3603 | -12.09 | 9.80E+06 | 2.02E+07 | 2.11E+07 | 1.07E+07 |
| 78.811 | -3894.1 | -13.045 | 1.05E+07 | 2.18E+07 | 2.29E+07 | 1.16E+07 |
| 79.326 | -4190.1 | -14.012 | 1.13E+07 | 2.34E+07 | 2.46E+07 | 1.25E+07 |
| 79.899 | -4525.2 | -15.099 | 1.21E+07 | 2.52E+07 | 2.66E+07 | 1.35E+07 |
| 80.415 | -4833 | -16.091 | 1.28E+07 | 2.68E+07 | 2.84E+07 | 1.44E+07 |
| 80.931 | -5147.1 | -17.097 | 1.35E+07 | 2.84E+07 | 3.03E+07 | 1.53E+07 |
| 81.389 | -5432.2 | -18.003 | 1.42E+07 | 2.99E+07 | 3.20E+07 | 1.62E+07 |
| 81.905 | -5760.2 | -19.038 | 1.49E+07 | 3.16E+07 | 3.39E+07 | 1.72E+07 |

The PRBM predicted a maximum stress of 33.9 MPa at about 19 mm. when mechanism moves from its initial position to 19 mm in +ve x-axis, the max stress is developed at pivot 2 and in opposite i.e. –ve x-axis direction pivot 3 has shown a maximum stress. The force-displacement relationship is approximately linear and is shown in Figure 3.11.

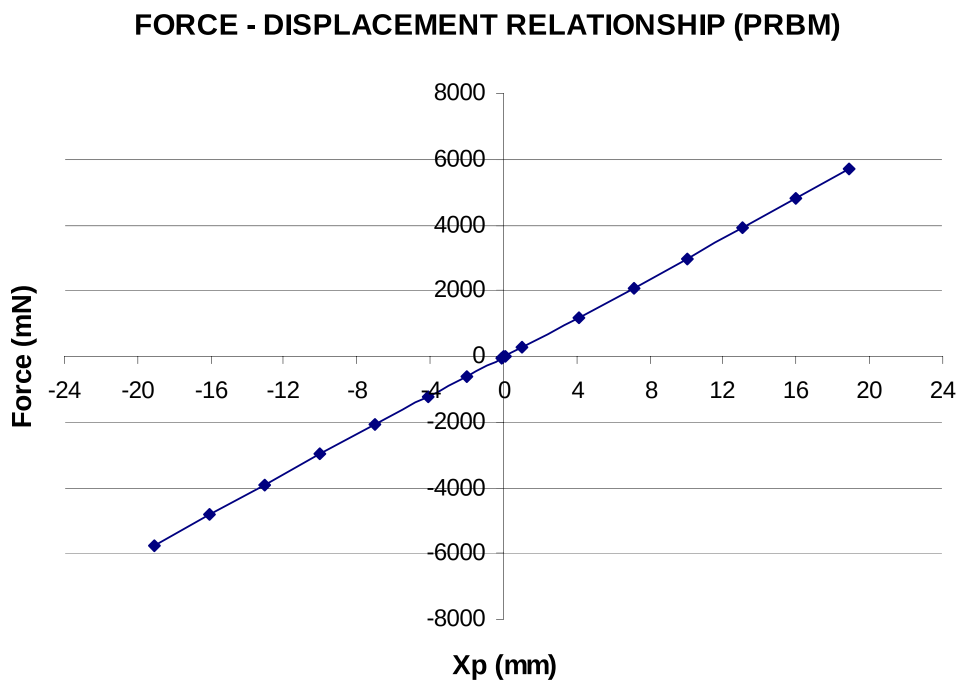



**Figure 3.11: Force – Displacement Relationship by using PRBM - Model 2**

### 3.5.3 Force – Displacement Relationship of Compliant Mechanism using Finite Element Analysis

The finite element analysis software ANSYS is used for force – displacement relationship of fixed-fixed segment, Figure 3.10b. The compliant straight line mechanism was analyzed as a 2D model with the material Polypropylene. The finite element model consisted of two-dimensional beam elements i.e. beam 3, Figure 3.12. The analysis employed a nonlinear solver that accounted for large deflections and stress stiffening. Table 3.11 enlists the input variables for FEA. Displacements were applied to the coupler point *P* in 1 mm increments. The force and stress were registered at each step. The force – displacement values of compliant mechanism measured by FEA are given in Table 3.12.

**Table 3.11: Input Variables for FEA – Model 2**

| Varaiable | Value |
|---|---|
| $L_i$ | 58.8235 mm |
| $r_6$ | 32 mm |
| Yield Strength of Polypropylene, *Sy* | 34 Mpa |
| Young Modulus of Polypropylene, *E* | 1.4 Gpa |
| Moment of Inertia, *I* | 2.4 mm$^4$ |
| Thickness of fixed-fixed segment, *h* | 1.5 mm |
| $\theta_{20}$ | 74.2167º |
| Length of coupler point P, *b* | 56.6057 |
| In-plane thickness of small length flexure Pivot, *t* | 8.75 mm |

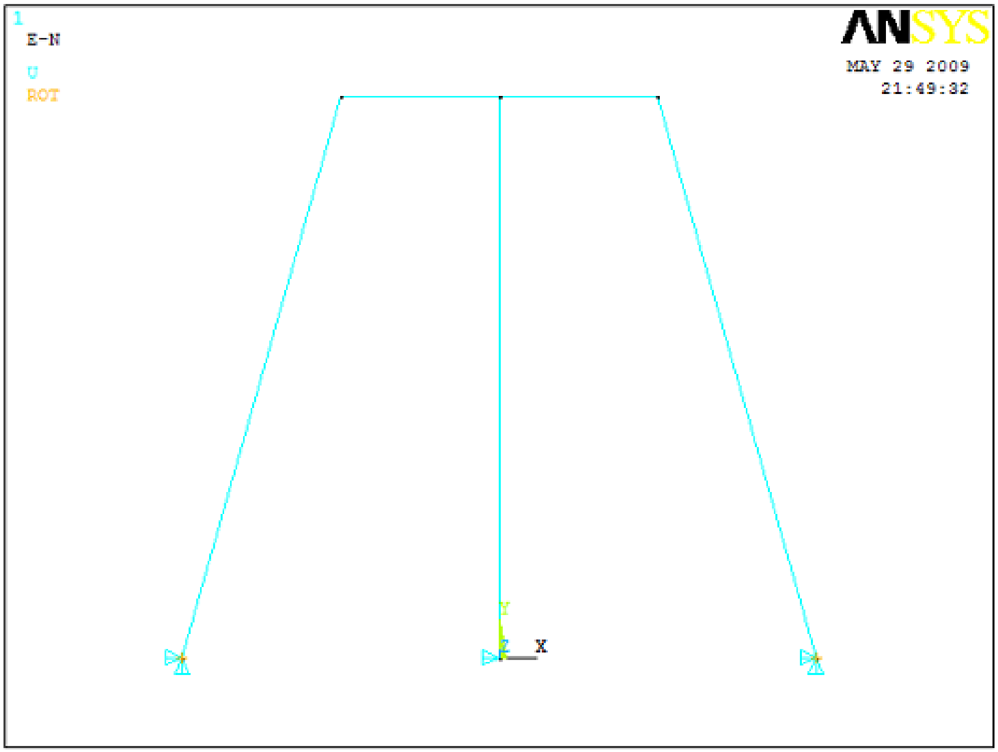



**Figure 3.12: Finite Element Model with 2D Beam Element (BEAM 3) – Model 2**

The FEA predicted a maximum stress of 33.8 MPa at about 20 mm. The force displacement relationship is approximately linear and is shown in Figure 3.13. The maximum displacement this mechanism shown was 20 mm at approximately 34 MPa.

**Table 3.12: FEA Results of Model 2 - Fully Compliant Mechanism with Fixed-Fixed Segment**

| Force, F (mN) | Displacement, Xp (mm) | Stress (MPa) |
|---|---|---|
| 5189.52 | 20 | 33.853 |
| 4922.3 | 19 | 31.3374 |
| 4656.68 | 18 | 29.628 |
| 4392.32 | 17 | 27.9249 |
| 4129.03 | 16 | 26.2282 |
| 3868.1 | 15 | 24.5377 |
| 3606.01 | 14 | 22.8541 |
| 3344.96 | 13 | 21.1771 |
| 3084.75 | 12 | 19.5067 |
| 2825.34 | 11 | 17.8431 |
| 2566.55 | 10 | 16.1862 |
| 2308.35 | 9 | 14.5362 |
| 2050.64 | 8 | 12.8931 |
| 1793.37 | 7 | 11.2569 |
| 1536.49 | 6 | 9.62765 |
| 1279.91 | 5 | 8.0054 |
| 1023.61 | 4 | 6.39017 |
| 767.518 | 3 | 4.782 |
| 511.594 | 2 | 3.1809 |
| 255.769 | 1 | 1.5869 |
| -255.769 | -1 | 1.56603 |
| -511.591 | -2 | 3.12476 |
| -767.518 | -3 | 4.67617 |
| -1023.61 | -4 | 6.22027 |
| -1279.91 | -5 | 7.75706 |
| -1536.47 | -6 | 9.28654 |
| -1793.36 | -7 | 10.8087 |
| -2050.63 | -8 | 12.3236 |
| -2308.35 | -9 | 14.8313 |
| -2566.55 | -10 | 16.3316 |
| -2825.33 | -11 | 17.8248 |
| -3084.79 | -12 | 19.3107 |
| -3344.96 | -13 | 21.7894 |
| -3606 | -14 | 22.261 |
| -3868.1 | -15 | 24.7255 |
| -4129.02 | -16 | 26.1829 |
| -4392.28 | -17 | 27.6332 |
| -4656.66 | -18 | 29.0764 |
| -4922.28 | -19 | 31.5127 |
| -5189.52 | -20 | 33.9419 |

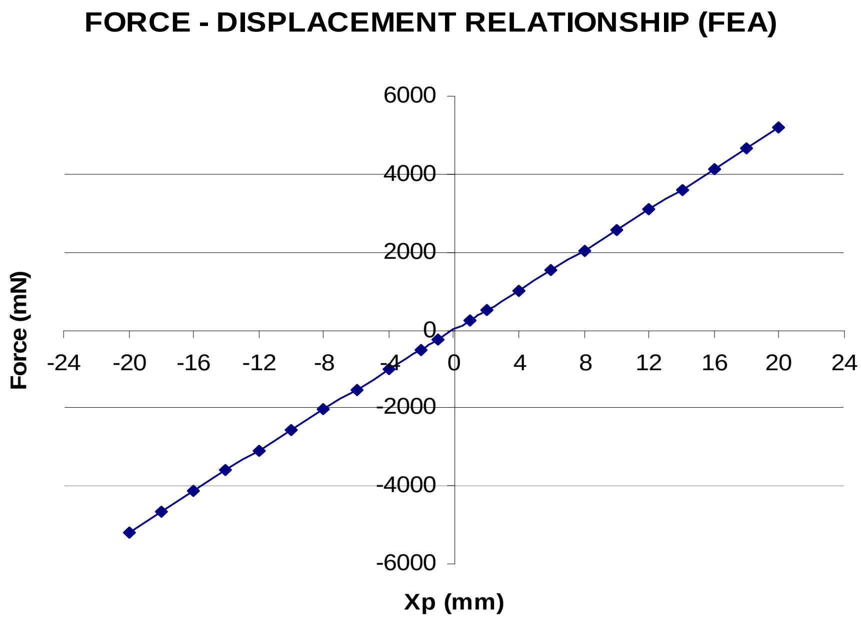



**Figure 3.13: Force – Displacement Relationship of Compliant Mechanism using ANSYS – Model 2**

### 3.5.4 Force – Displacement Relationship Comparison

The comparison of PRBM and FEA results of compliant mechanism was carried out and shown in Figure 3.14. The FEA has given a 34 MPa at 20 mm where as PRBM predicted 34 MPa at 19mm. The variation is small as FEA results are close to what is predicted through PRBM. Table 3.13 enlists the spring constant calculated from Figure 3.14. The spring constant of PRBM is calculated theoretically from Equation (2.25) and is found out to be 264.92 N-mm. On the other hand FEA uses 255.76 N-mm. The spring constant of PRBM and FEA are having a difference of approximately 4%. Both FEA and PRBM have exhibited approximately linear force-displacement behaviour.

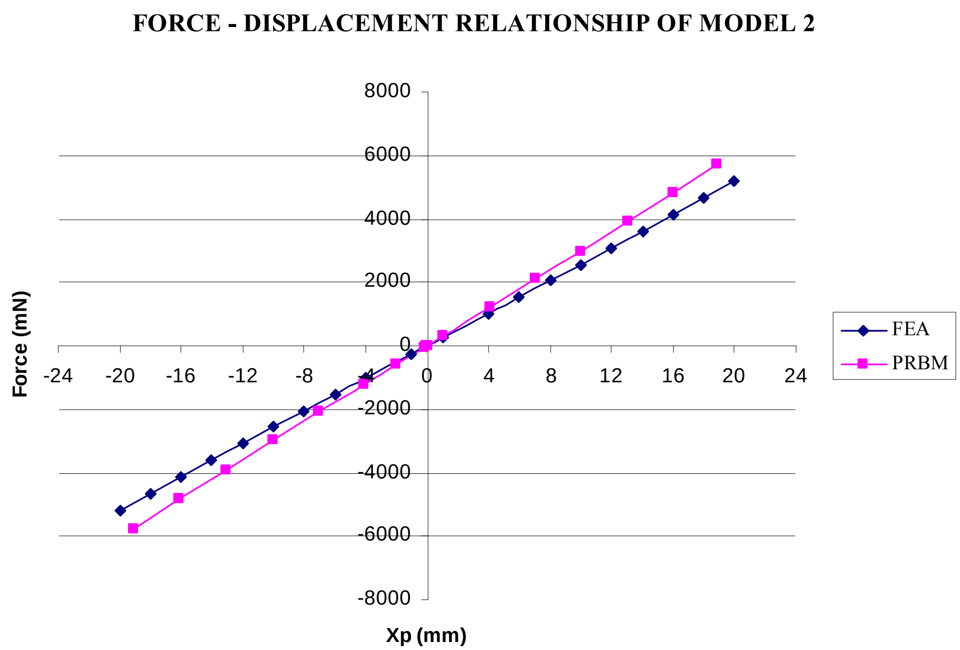



**Figure 3.14:** **Force Displacement Relationship of Model 2 – Comparison of FEA and PRBM Results**

**Table 3.13: Comparison of Spring Constant of FEA and Predicted by PRBM**

| Model | Spring Constant (N-m) |
|---|---|
| PRBM | 0.264 |
| FEA | 0.255 |

## 3.6 Comparison among Model 1 and Model 2

The behaviour of compliant mechanism is depending on constraints on which it is designed, its geometry i.e. shape, size and type of segment or flexible member, and material used. Therefore, designing a mechanism for particular task is acceptable only and only if, it meets its constraints and perform task as desired.

The models shown in Figure 3.5 and 3.10 are generated from same rigid body mechanisms, they are fully compliant, monolithic, but behave differently. Model 1, which is made of four small length flexure pivots, shows a displacement of 8.3mm at 34 MPa with maximum force of 5199 mN, Table 3.4. Whereas, model 2, shows a displacement of 19 mm at 34 MPa with maximum force of 5760 mN, in their PRBM analysis. The comparison is shown in Figure 3.15.

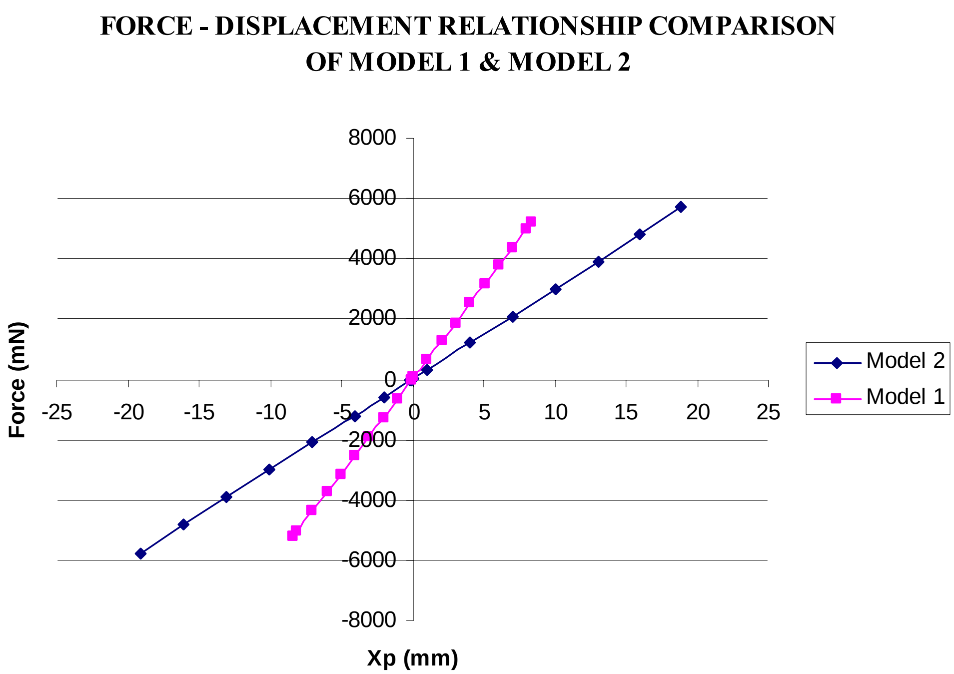



**Figure 3.15: Behavioural Comparison of Model 1 and Model 2**

The difference is because of their geometry, one (Model 1) is made of four flexible joints and other (Model 2) is having two flexible links replacing rigid side links. In model 1, small length flexure has not only shown decrease in displacement but also shown an increased stiffness that result in an increased force for particular displacement. This behaviour of flexible joint is in line with the benchmarking given by Trease Brian in his work [Trease *et.al.*, 2005], which states, flexible joints are stiff and offer limited deflection than flexible member or link . The force – displacement relation is shown to be linear in both models that suggest these can be incorporated in larger systems by modeling them as a linear spring.

# CHAPTER – 4
# DISCUSSION AND CONCLUSION

## 4.1 Review of Purpose

Analysis of compliant mechanisms has many difficulties, as both kinematic and deflection theories are needed. Synthesis is even harder, as many constraints are introduced. The purpose of this study was to present a suitable approach that can be used for synthesizing a compliant mechanism from rigid body mechanism and to analyze it with a method that is less complex and easy to use at initial design stage.

Pseudo rigid body model was used to transform a rigid body mechanism into a complaint mechanism and then it is analyzed. The pseudo rigid body model has the advantage that, rigid body kinematics theory is applicable on the mechanism. But, it has some limitations. Accuracy of the end coordinates of the flexible link is dependent on the position of the torsional spring and loading conditions. So, either tables for loading conditions have to be used or good assumptions have to be made. Also, there is the disadvantage that accuracy is lost at some point. But, study reveals that for many cases, end point coordinates of the flexible link is accurate for even large angles for example, for perpendicular force acting at the end of a flexible beam; Pseudo-rigid body model gives accurate results up to a deflection angle of 77 degrees [Howell 2001].

## 4.2 Summary

A full example was also presented in this study; converting a rigid body mechanism i.e. Roberts approximate straight line mechanism, into fully compliant monolithic mechanisms. As a result, two models are made with the help of rigid body replacement synthesis, their force – displacement relation and stresses are predicted through PRBM.

During modeling and analysis some assumptions were made. The position of the characteristic pivots, $\gamma$, for small length flexure and fixed-fixed segments were assumed to be same for all loading conditions. The torsional spring constant coefficient, $K_\theta$ , for fixed-fixed segement was assumed to be same for all loading conditions. This modeling and analysis approach using Pseudo-rigid body model has proved itself very useful, which is evident from Figures 3.9 and 3.14.

The finite element model of both the compliant mechanism has been made. Since the dimension (thickness and width of flexible members) relatively small, 2-D beam elements are used. For the boundary conditions, flexible links are fixed to the ground and displacement is introduced at the coupler point *P*, until the flexible link reaches its maximum deflection position corresponding to its yield strength. The PRBM approximation is compared with the results obtained from commercial finite element analysis software, ANSYS. The behaviour of both compliant mechanism models were also compared to gain an insight about the effects of configuration / geometry on performance of a mechanism.

## 4.3 Conclusion

The idea of using deflecting members to gain motion and energy storage as opposed to rigid members connected through kinematic joints is not new to the human race e.g. catapults, bows etc. The area of modern compliant mechanisms first studied by Professor Ashok Midha. Although many have contributed to the knowledge but Midha may be considered the father of modern compliant mechanisms [Howell, 2001]. Compliant mechanisms possess some unique challenges that needed to be solved before they could be truly useful as replacements for many rigid-body mechanisms. Some of these challenges included finding new ways to analyze large deflections, finding ways to relate compliant mechanism kinematics to rigid-body kinematics and others. Many models used to analyze deflections in beams but their usefulness is limited to small deflection. Whereas as, some compliant mechanisms experience large deflections which undermine the linear models' accuracy. Historically, the most common method of compliant mechanism design has been trial and error. However, the conception of the pseudo-rigid-body modeling technique has successfully opened the way for simple design and analysis of many compliant mechanisms. The PRBM has the ability to easily evaluate the motion and force –

displacement characteristics for a particular configuration, which is the critical element in designing compliant mechanism for product application.

## 4.4 Future Work

As a future work, design software using the principals presented in this thesis can be developed. Living hinges (Flexible joints) are important to design a fully compliant mechanism, so related work about their design method and fatigue life can be performed (work about living hinges in literature are limited and mostly experimental). Pseudo rigid body models presented in chapter 2 are only capable of modeling constant cross-section flexible segments; varying size segment models can be developed and investigated to enhance the application of PRBM in designing compliant mechanisms.

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# APPENDIX – A

# MATLAB CODE FOR MODEL – 1

```
%This Program Computes Force, Displacement and Stress
%for Case 1- Fully Compliant Mechanism,
%with Four Small Length Flexure Pivots Flexible Segement
%%%%%%%%%%%
%Geometry Parameter Inputs which are used to calculate initial position
clear all
format short
r_1 = 64/1000;  %PRBM Lengths in meters
r_2 = 55/1000;
r_4 = 55/1000;
r_6 = 32/1000;
r_3 = 55/1000;
theta_2_0 = acos(((r_2)^2+(r_6)^2-(r_3)^2)/(2*r_2*r_6));    %radians
theta_2_0_degree = theta_2_0*(180/pi);

delta_initial = sqrt((r_1)^2+(r_2)^2-(2*(r_1)*(r_2))*cos(theta_2_0));  % distance in meters
lambda_initial = acos(((r_4)^2+(delta_initial)^2-(r_6)^2)/(2*(r_4)*(delta_initial))); % angle in radians
beta_initial = acos(((r_1)^2+(delta_initial)^2-(r_2)^2)/(2*(r_1)*(delta_initial)));  % angle in radians
phi_initial = acos(((r_6)^2+(delta_initial)^2-(r_4)^2)/(2*(r_6)*(delta_initial)));  % angle in radians
theta_3_0 = ((phi_initial)-(beta_initial)); %Intial angle theta 3 in radians
theta_3_0_degree = theta_3_0*(180/pi);
theta_4_0 = (pi-((beta_initial)+(lambda_initial)));   %Initial angle theta 4 in radians
theta_4_0_degree = theta_4_0*(180/pi);
b = sqrt((r_3)^2-((r_6)/2)^2);    % b is distance of coupler point P from coupler link

%This part of program compute Angle Theta limits,
%Range of theta for Theta_2

Sy = 34e+6;   %Yield Strength of Material i.e. polypropylene in Pascals or N/m2
E = 1.4e+9;   %Young Modulus of Material i.e. polypropylene in Pascals or N/m2
Gamma = 0.85;  %Characteristic Radius Factor PRBM
K_theta = 2.65;   %Stiffness co-efficient PRBM
L = 5/1000;   %Length of Flexible segment
t = 8.75/1000;   %In-Plane depth of Flexible segment in meters
```

```
h = 1.5/1000;   %Thickness of Flexible segement in meters
I = (t*(h)^3)/12 %Moment of Inertia
K_1 = ((E*I)/L)  %Torsional Spring Constant of Spring 1
K_2 = ((E*I)/L)  %Torsional Spring Constant of Spring 2
K_3 = ((E*I)/L)  %Torsional Spring Constant of Spring 3
K_4 = ((E*I)/L)  %Torsional Spring Constant of Spring 4
Theta_Range = (((t*(h)^2)*Sy)/(6*K_1));   %Range of Theta_2 around its initial value in Radians
% theta_2_0-theta range =< Theta 2 => theta_2_0+theta range
theta_range_degree = Theta_Range*(180/pi);

%This Part of Program will compute Force and Displacement

range_1 = (theta_2_0-(Theta_Range));
range_2 = (theta_2_0+(Theta_Range));
theta_2 = [range_1:.001:range_2]
theta_2_degree = theta_2*(180/pi);

%This incremental theta_2 will give total displacement of Coupler P

delta = sqrt((r_1)^2+(r_2)^2-((2*(r_1)*(r_2))*cos(theta_2))); % distance in meters


 for i=1:1:length(theta_2),

   beta(i) = acos(((r_1)^2+(delta(i))^2-(r_2)^2)/(2*(r_1)*delta(i)));  % angle in radians
   beta_degree = beta(i)*(180/pi);
   lemda(i) = acos(((r_4)^2+(delta(i))^2-(r_6)^2)/(2*(r_4)*delta(i)));  % angle in radians
   lemda_degree = lemda(i)*(180/pi);
   phi(i) = acos(((r_6)^2+(delta(i))^2-(r_4)^2)/(2*(r_6)*delta(i)));  % angle in radians
   phi_degree = phi(i)*(180/pi);
   theta_3(i) = (phi(i)-beta(i)); %Subsequent angle theta 3 in radians
   theta_3_degree = theta_3(i)*(180/pi);
   theta_4(i) = (pi-(beta(i)+lemda(i))); %Subsequent angle theta 4 in radians
   theta_4_degree = theta_4(i)*(180/pi);
   h_3_2(i) = ((r_2)*sin(theta_4(i)-theta_2(i)))/((r_6)*sin(theta_3(i)-theta_4(i)));
   h_4_2(i) = ((r_2)*sin(theta_3(i)-theta_2(i)))/((r_4)*sin(theta_3(i)-theta_4(i)));
   Xp(i) = (((r_2)*(cos(theta_2(i))-cos(theta_2_0)))+((r_6)/2)*(cos(theta_3(i))-
           cos(theta_3_0))+(b*(sin(theta_3(i)))))*1000;
   a(i) = -K_1 *(theta_2(i)-theta_2_0)-K_2*((theta_2(i)-theta_2_0)-(theta_3(i)-theta_3_0)) +
           K_2*((theta_2(i)-theta_2_0)-(theta_3(i)-theta_3_0))*h_3_2(i);
```

```
bb(i) = - K_3*((theta_4(i)-theta_4_0)-(theta_3(i)-theta_3_0))*h_4_2(i);
c(i) = K_3*((theta_4(i)-theta_4_0)-(theta_3(i)-theta_3_0))*h_3_2 (i)- K_4*(theta_4(i)-
        theta_4_0)*h_4_2(i);
d(i) = ( r_2 * sin (theta_2(i)) + (r_6/2)*sin (theta_3(i))* h_3_2(i)  -  b * cos(theta_3(i))*h_3_2(i) );

F(i) = ((a(i) + bb (i) + c (i)) / d(i))*1000;

% Stress calculation at each pivot

mt_1(i) = K_1 * (theta_2(i)-theta_2_0);
mt_2(i) = K_2*((theta_2(i)-theta_2_0)-(theta_3(i)-theta_3_0));
mt_3(i) = K_3*((theta_4(i)-theta_4_0)- (theta_3(i)-theta_3_0) );
mt_4(i) = K_4 *(theta_4(i)-theta_4_0);
stress_1(i) = ((mt_1(i)  *  ( h/2))/ I);
stress_2(i) = ((mt_2(i)  *  ( h/2))/ I);
stress_3(i) = ((mt_3(i)  *  ( h/2))/ I);
stress_4(i) = ((mt_4(i)  *  ( h/2))/ I);

 end


plot(Xp,F), xlabel('Xp (millimeters)'), ylabel('F  (mN)')
grid on

table = [theta_2_degree', F', Xp', stress_1', stress_2', stress_3', stress_4' ]
%%%%%%%%%%%%%%%%%%Program ends%%%%%%%%%%%%%%%%
```

# APPENDIX – B

# MATLAB CODE FOR MODEL – 2

```
%This Program Computes Force, Displacement and Stress
%for Case 2- Fully Compliant Mechanism,
%with Fixed-Fixed Flexible Segement
%%%%%%%%%%%
%Geometry Parameter Inputs which are used to calculate initial position

clear all
format short

r_1 = 64/1000;  %PRBM Lengths in meters
r_2 = 50/1000;
r_4 = 50/1000;
r_6 = 32/1000;
r_3 = 50/1000;
theta_2_0 = acos(((r_2)^2+(r_6)^2-(r_3)^2)/(2*r_2*r_6));    %radians
theta_2_0_degree = theta_2_0*(180/pi);

delta_initial = sqrt((r_1)^2+(r_2)^2-(2*(r_1)*(r_2))*cos(theta_2_0));  % distance in meters
lambda_initial = acos((((r_4)^2+(delta_initial)^2-(r_6)^2)/(2*(r_4)*(delta_initial)));  % angle in radians
beta_initial = acos((((r_1)^2+(delta_initial)^2-(r_2)^2)/(2*(r_1)*(delta_initial)));  % angle in radians
phi_initial = acos((((r_6)^2+(delta_initial)^2-(r_4)^2)/(2*(r_6)*(delta_initial)));  % angle in radians
theta_3_0 = ((phi_initial)-(beta_initial)); %Intial angle theta 3 in radians
theta_3_0_degree = theta_3_0*(180/pi);
theta_4_0 = (pi-((beta_initial)+(lambda_initial)));   %Initial angle theta 4 in radians
theta_4_0_degree = theta_4_0*(180/pi);
b = sqrt((r_3)^2-((r_6)/2)^2);    % b is distance of coupler point P from coupler link

%This part of program compute Angle Theta limits,
%Range of theta for Theta_2

Sy = 34e+6;   %Yield Strength of Material i.e. polypropylene in Pascals or N/m2
E = 1.4e+9;   %Young Modulus of Material i.e. polypropylene in Pascals or N/m2
Gamma = 0.8517;  %Characteristic Radius Factor PRBM
K_theta = 2.65;   %Stiffness co-efficient PRBM
```

```
L = r_2/Gamma;   %Length of Flexible segment
t = 8.75/1000;   %In-Plane depth of Flexible segment in meters
h = 1.5/1000;   %Thickness of Flexible segement in meters
I = (t*(h)^3)/12; %Moment of Inertia
K_1 = ((2*Gamma*K_theta*E*I)/L);  %Torsional Spring Constant for Spring 1
K_2 = ((2*Gamma*K_theta*E*I)/L);  %Torsional Spring Constant for Spring 2
K_3 = ((2*Gamma*K_theta*E*I)/L);  %Torsional Spring Constant for Spring 3
K_4 = ((2*Gamma*K_theta*E*I)/L);  %Torsional Spring Constant for Spring 4
Theta_Range = (((t*h^2)*Sy)/(6*K_1));   %Range of Theta_2 around its initial value in Radians

% theta_2_0-theta range/2 =< Theta 2 => theta_2_0+theta range/2

theta_range_degree = Theta_Range*(180/pi);

%This Part of Program will compute Force and Displacement

range_1 = (theta_2_0-(Theta_Range));
range_2 = (theta_2_0+(Theta_Range));
theta_2 = [range_1:.001:range_2]
theta_2_degree = theta_2*(180/pi);

%This incremental theta_2 will give Total displacement of Coupler P

delta = sqrt((r_1)^2+(r_2)^2-((2*(r_1)*(r_2))*cos(theta_2))); % distance in meters


 for i=1:1:length(theta_2),

   beta(i) = acos(((r_1)^2+(delta(i))^2-(r_2)^2)/(2*(r_1)*delta(i)));  % angle in radians
   beta_degree = beta(i)*(180/pi);
   lemda(i) = acos(((r_4)^2+(delta(i))^2-(r_6)^2)/(2*(r_4)*delta(i)));  % angle in radians
   lemda_degree = lemda(i)*(180/pi);
   phi(i) = acos(((r_6)^2+(delta(i))^2-(r_4)^2)/(2*(r_6)*delta(i)));  % angle in radians
   phi_degree = phi(i)*(180/pi);
   theta_3(i) = (phi(i)-beta(i)); %Subsequent angle theta 3 in radians
   theta_3_degree = theta_3(i)*(180/pi);
   theta_4(i) = (pi-(beta(i)+lemda(i))); %Subsequent angle theta 4 in radians
   theta_4_degree = theta_4(i)*(180/pi);
   h_3_2(i) = ((r_2)*sin(theta_4(i)-theta_2(i)))/((r_6)*sin(theta_3(i)-theta_4(i)));
   h_4_2(i) = ((r_2)*sin(theta_3(i)-theta_2(i)))/((r_4)*sin(theta_3(i)-theta_4(i)));
```

```
   Xp(i) = (((r_2)*(cos(theta_2(i))-cos(theta_2_0)))+((r_6)/2)*(cos(theta_3(i))-
            cos(theta_3_0))+(b*(sin(theta_3(i)))))*1000;


a(i) = -K_1 *(theta_2(i)-theta_2_0)-K_2*((theta_2(i)-theta_2_0)-(theta_3(i)-theta_3_0)) +
            K_2*((theta_2(i)-theta_2_0)-(theta_3(i)-theta_3_0))*h_3_2(i);
bb(i) = - K_3*((theta_4(i)-theta_4_0)-(theta_3(i)-theta_3_0))*h_4_2(i);
c(i) = K_3*((theta_4(i)-theta_4_0)-(theta_3(i)-theta_3_0))*h_3_2 (i)- K_4*(theta_4(i)-
            theta_4_0)*h_4_2(i);
d(i) = ( r_2 * sin (theta_2(i)) + (r_6/2)*sin (theta_3(i))* h_3_2(i)  -  b * cos(theta_3(i))*h_3_2(i) );

F(i) = ((a(i) + bb (i) + c (i)) / d(i))*1000;

% Stress calculation at each pivot

mt_1(i) = K_1*(theta_2(i)-theta_2_0);
mt_2(i) = K_2*((theta_2(i)-theta_2_0)-(theta_3(i)-theta_3_0));
mt_3(i) = K_3*((theta_4(i)-theta_4_0)- (theta_3(i)-theta_3_0) );
mt_4(i) = K_4*(theta_4(i)-theta_4_0);
stress_1(i) = ((mt_1(i)*( h/2))/I);
stress_2(i) = ((mt_2(i)*( h/2))/I);
stress_3(i) = ((mt_3(i)*( h/2))/I);
stress_4(i) = ((mt_4(i)*( h/2))/I);

 end

plot(Xp,F), xlabel('Xp (millimeters)'), ylabel('F  (mN)')
grid on

table = [theta_2_degree', F', Xp', stress_1', stress_2', stress_3', stress_4' ]
%%%%%%%%%%%%%%%%%Program ends%%%%%%%%%%%%%%%%
```

# APPENDIX – C

## ANSYS APDL FOR MODEL – 1

```
/Batch by Arslan
/COM, ANSYS release 10
finish
/CLEAR,NOSTART
/prep7

ET,1,BEAM3

KEYOPT,1,6,1
KEYOPT,1,9,0
KEYOPT,1,10,0
R,1,13.125,2.46,1.5, , , ,
R,2,43.75,91.14,5, , , ,

MPTEMP,,,,,,,,
MPTEMP,1,0
MPDATA,EX,1,,1400
MPDATA,PRXY,1,,.3
MP,EX,2,20000E6

TYPE,1
MAT,1
REAL,1

n,1,-32,0
n,2,-30.67,4.82
n,5,-17.33,53.01
n,6,-16,57.82
n,14,32,0
n,13,30.67,4.82
n,10,17.33,53.01
n,9,16,57.82
e,1,2
e,5,6
e,9,10
e,13,14

TYPE,1
MAT,1
REAL,2

n,3,-30.67,4.82
n,4,-17.33,53.01
n,11,30.67,4.82
n,12,17.33,53.01
e,3,4
e,11,12
```

```
TYPE,1
MAT,2
REAL,2

n,7,-16,57.82
n,8,16,57.82
n,15,0,57.82
n,16,0,0
e,7,15
e,15,8
e,15,16


NUMMRG,NODE, , , ,LOW
NUMCMP,NODE

/REPLOT
/SOLU

ANTYPE,4
TRNOPT,FULL
LUMPM,0
D,1, , , , , ,ALL, , , , ,
D,8, , , , , ,ALL, , , , ,

D,10, ,1, , , ,UX, , , , ,
NLGEOM,1
SSTIF,ON
OUTRES,ERASE
OUTRES,ALL,ALL
KBC,0
RESCONTRL,DEFINE,ALL,ALL,1
TIME,1
LSWRITE,1,

DDELE,10,ALL
D,10, ,2, , , ,UX, , , , ,
NLGEOM,1
SSTIF,ON
OUTRES,ERASE
OUTRES,ALL,ALL
KBC,0
RESCONTRL,DEFINE,ALL,ALL,1
TIME,2
LSWRITE,2,

DDELE,10,ALL
D,10, ,3, , , ,UX, , , , ,
NLGEOM,1
SSTIF,ON
OUTRES,ERASE
OUTRES,ALL,ALL
KBC,0
RESCONTRL,DEFINE,ALL,ALL,1
```

```
TIME,3
LSWRITE,3,

DDELE,10,ALL
D,10, ,4, , , ,UX, , , , ,
NLGEOM,1
SSTIF,ON
OUTRES,ERASE
OUTRES,ALL,ALL
KBC,0
RESCONTRL,DEFINE,ALL,ALL,1
TIME,4
LSWRITE,4,

DDELE,10,ALL
D,10, ,5, , , ,UX, , , , ,
NLGEOM,1
SSTIF,ON
OUTRES,ERASE
OUTRES,ALL,ALL
KBC,0
RESCONTRL,DEFINE,ALL,ALL,1
TIME,5
LSWRITE,5,

DDELE,10,ALL
D,10, ,6, , , ,UX, , , , ,
NLGEOM,1
SSTIF,ON
OUTRES,ERASE
OUTRES,ALL,ALL
KBC,0
RESCONTRL,DEFINE,ALL,ALL,1
TIME,6
LSWRITE,6,

DDELE,10,ALL
D,10, ,7, , , ,UX, , , , ,
NLGEOM,1
SSTIF,ON
OUTRES,ERASE
OUTRES,ALL,ALL
KBC,0
RESCONTRL,DEFINE,ALL,ALL,1
TIME,7
LSWRITE,7,

DDELE,10,ALL
D,10, ,8, , , ,UX, , , , ,
NLGEOM,1
SSTIF,ON
OUTRES,ERASE
OUTRES,ALL,ALL
KBC,0
RESCONTRL,DEFINE,ALL,ALL,1
```

```
TIME,8
LSWRITE,8,

DDELE,10,ALL
D,10, ,-1, , , ,UX, , , , ,
NLGEOM,1
SSTIF,ON
OUTRES,ERASE
OUTRES,ALL,ALL
KBC,0
RESCONTRL,DEFINE,ALL,ALL,1
TIME,9
LSWRITE,9,

DDELE,10,ALL
D,10, ,-2, , , ,UX, , , , ,
NLGEOM,1
SSTIF,ON
OUTRES,ERASE
OUTRES,ALL,ALL
KBC,0
RESCONTRL,DEFINE,ALL,ALL,1
TIME,10
LSWRITE,10,

DDELE,10,ALL
D,10, ,-3, , , ,UX, , , , ,
NLGEOM,1
SSTIF,ON
OUTRES,ERASE
OUTRES,ALL,ALL
KBC,0
RESCONTRL,DEFINE,ALL,ALL,1
TIME,11
LSWRITE,11,

DDELE,10,ALL
D,10, ,-4, , , ,UX, , , , ,
NLGEOM,1
SSTIF,ON
OUTRES,ERASE
OUTRES,ALL,ALL
KBC,0
RESCONTRL,DEFINE,ALL,ALL,1
TIME,12
LSWRITE,12,

DDELE,10,ALL
D,10, ,-5, , , ,UX, , , , ,
NLGEOM,1
SSTIF,ON
OUTRES,ERASE
OUTRES,ALL,ALL
KBC,0
RESCONTRL,DEFINE,ALL,ALL,1
```

```
TIME,13
LSWRITE,13,

DDELE,10,ALL
D,10, ,-6, , , ,UX, , , , ,
NLGEOM,1
SSTIF,ON
OUTRES,ERASE
OUTRES,ALL,ALL
KBC,0
RESCONTRL,DEFINE,ALL,ALL,1
TIME,14
LSWRITE,14,

DDELE,10,ALL
D,10, ,-7, , , ,UX, , , , ,
NLGEOM,1
SSTIF,ON
OUTRES,ERASE
OUTRES,ALL,ALL
KBC,0
RESCONTRL,DEFINE,ALL,ALL,1
TIME,15
LSWRITE,15,

DDELE,10,ALL
D,10, ,-8, , , ,UX, , , , ,
NLGEOM,1
SSTIF,ON
OUTRES,ERASE
OUTRES,ALL,ALL
KBC,0
RESCONTRL,DEFINE,ALL,ALL,1
TIME,16
LSWRITE,16,

LSSOLVE,1,16,1
SOLVE

/POST1

ETABLE,SMAX.I,NMISC, 1
ETABLE,SMAX.J,NMISC, 3
ETABLE,SMIN.I,NMISC,2
ETABLE,SMIN.J,NMISC, 4
ETABLE,MFORX.I,SMISC, 1
ETABLE,MFORX.J,SMISC,7
ETABLE,MFORY.I,SMISC,2
ETABLE,MFORY.J,SMISC, 8

ETABLE,REFL

/POST26

ESOL,2,1, ,NMISC,1,SMAX.I
```

```
ESOL,3,2, ,NMISC,3,SMAX.J
ESOL,4,1, ,NMISC,2,SMIN.I
ESOL,5,2, ,NMISC,4,SMIN.J
RFORCE,6,10,F,X,FORCE
NSOL,7,10,U,X,DISP
```

# APPENDIX – D

# ANSYS APDL FOR MODEL – 2

```
/Batch by Arslan
/COM, ANSYS release 10
Finish
/CLEAR,NOSTART

/prep7

ET,1,BEAM3

KEYOPT,1,6,1
KEYOPT,1,9,0
KEYOPT,1,10,0
R,1,13.125,2.46,1.5, , , ,
R,2,43.75,91.14,5, , , ,

MPTEMP,,,,,,,,
MPTEMP,1,0
MPDATA,EX,1,,1400
MPDATA,PRXY,1,,0.3

MP,EX,2,20000E6

TYPE,1
MAT,1
REAL,1
n,1,-32,0
n,2,-16,56.6057
n,5,16,56.6057
n,6,32,0

e,1,2
e,5,6

TYPE,1
MAT,2
REAL,2

n,3,-16,56.6057
n,4,16,56.6057
n,7,0,56.6057
n,8,0,0
e,3,7
e,7,4
e,7,8

NUMMRG,NODE, , , ,LOW
```

```
NUMCMP,NODE
/REPLOT

/SOLU
ANTYPE,4
TRNOPT,FULL
LUMPM,0
D,1, , , , , ,ALL, , , , ,
D,4, , , , , ,ALL, , , , ,

D,6, ,1, , , ,UX, , , , ,
NLGEOM,1
SSTIF,ON
OUTRES,ERASE
OUTRES,ALL,ALL
KBC,0
RESCONTRL,DEFINE,ALL,ALL,1
TIME,1
LSWRITE,1,

DDELE,6,ALL
D,6, ,2, , , ,UX, , , , ,
NLGEOM,1
SSTIF,ON
OUTRES,ERASE
OUTRES,ALL,ALL
KBC,0
RESCONTRL,DEFINE,ALL,ALL,1
TIME,2
LSWRITE,2,

DDELE,6,ALL
D,6, ,3, , , ,UX, , , , ,
NLGEOM,1
SSTIF,ON
OUTRES,ERASE
OUTRES,ALL,ALL
KBC,0
RESCONTRL,DEFINE,ALL,ALL,1
TIME,3
LSWRITE,3,

DDELE,6,ALL
D,6, ,4, , , ,UX, , , , ,
NLGEOM,1
SSTIF,ON
OUTRES,ERASE
OUTRES,ALL,ALL
KBC,0
RESCONTRL,DEFINE,ALL,ALL,1
TIME,4
LSWRITE,4,

DDELE,6,ALL
D,6, ,5, , , ,UX, , , , ,
```

```
NLGEOM,1
SSTIF,ON
OUTRES,ERASE
OUTRES,ALL,ALL
KBC,0
RESCONTRL,DEFINE,ALL,ALL,1
TIME,5
LSWRITE,5,

DDELE,6,ALL
D,6, ,6, , , ,UX, , , , ,
NLGEOM,1
SSTIF,ON
OUTRES,ERASE
OUTRES,ALL,ALL
KBC,0
RESCONTRL,DEFINE,ALL,ALL,1
TIME,6
LSWRITE,6,

DDELE,6,ALL
D,6, ,7, , , ,UX, , , , ,
NLGEOM,1
SSTIF,ON
OUTRES,ERASE
OUTRES,ALL,ALL
KBC,0
RESCONTRL,DEFINE,ALL,ALL,1
TIME,7
LSWRITE,7,

DDELE,6,ALL
D,6, ,8, , , ,UX, , , , ,
NLGEOM,1
SSTIF,ON
OUTRES,ERASE
OUTRES,ALL,ALL
KBC,0
RESCONTRL,DEFINE,ALL,ALL,1
TIME,8
LSWRITE,8,

DDELE,6,ALL
D,6, ,9, , , ,UX, , , , ,
NLGEOM,1
SSTIF,ON
OUTRES,ERASE
OUTRES,ALL,ALL
KBC,0
RESCONTRL,DEFINE,ALL,ALL,1
TIME,9
LSWRITE,9,

DDELE,6,ALL
D,6, ,10, , , ,UX, , , , ,
```

```
NLGEOM,1
SSTIF,ON
OUTRES,ERASE
OUTRES,ALL,ALL
KBC,0
RESCONTRL,DEFINE,ALL,ALL,1
TIME,10
LSWRITE,10,

DDELE,6,ALL
D,6, ,11, , , ,UX, , , , ,
NLGEOM,1
SSTIF,ON
OUTRES,ERASE
OUTRES,ALL,ALL
KBC,0
RESCONTRL,DEFINE,ALL,ALL,1
TIME,11
LSWRITE,11,

DDELE,6,ALL
D,6, ,12, , , ,UX, , , , ,
NLGEOM,1
SSTIF,ON
OUTRES,ERASE
OUTRES,ALL,ALL
KBC,0
RESCONTRL,DEFINE,ALL,ALL,1
TIME,12
LSWRITE,12,

DDELE,6,ALL
D,6, ,13, , , ,UX, , , , ,
NLGEOM,1
SSTIF,ON
OUTRES,ERASE
OUTRES,ALL,ALL
KBC,0
RESCONTRL,DEFINE,ALL,ALL,1
TIME,13
LSWRITE,13,

DDELE,6,ALL
D,6, ,14, , , ,UX, , , , ,
NLGEOM,1
SSTIF,ON
OUTRES,ERASE
OUTRES,ALL,ALL
KBC,0
RESCONTRL,DEFINE,ALL,ALL,1
TIME,14
LSWRITE,14,

DDELE,6,ALL
D,6, ,15, , , ,UX, , , , ,
```

```
NLGEOM,1
SSTIF,ON
OUTRES,ERASE
OUTRES,ALL,ALL
KBC,0
RESCONTRL,DEFINE,ALL,ALL,1
TIME,15
LSWRITE,15,

DDELE,6,ALL
D,6, ,16, , , ,UX, , , , ,
NLGEOM,1
SSTIF,ON
OUTRES,ERASE
OUTRES,ALL,ALL
KBC,0
RESCONTRL,DEFINE,ALL,ALL,1
TIME,16
LSWRITE,16,

DDELE,6,ALL
D,6, ,17, , , ,UX, , , , ,
NLGEOM,1
SSTIF,ON
OUTRES,ERASE
OUTRES,ALL,ALL
KBC,0
RESCONTRL,DEFINE,ALL,ALL,1
TIME,17
LSWRITE,17,

DDELE,6,ALL
D,6, ,18, , , ,UX, , , , ,
NLGEOM,1
SSTIF,ON
OUTRES,ERASE
OUTRES,ALL,ALL
KBC,0
RESCONTRL,DEFINE,ALL,ALL,1
TIME,18
LSWRITE,18,

DDELE,6,ALL
D,6, ,19, , , ,UX, , , , ,
NLGEOM,1
SSTIF,ON
OUTRES,ERASE
OUTRES,ALL,ALL
KBC,0
RESCONTRL,DEFINE,ALL,ALL,1
TIME,19
LSWRITE,19,

DDELE,6,ALL
D,6, ,20, , , ,UX, , , , ,
```

```
NLGEOM,1
SSTIF,ON
OUTRES,ERASE
OUTRES,ALL,ALL
KBC,0
RESCONTRL,DEFINE,ALL,ALL,1
TIME,20
LSWRITE,20,

DDELE,6,ALL
D,6, ,-1, , , ,UX, , , , ,
NLGEOM,1
SSTIF,ON
OUTRES,ERASE
OUTRES,ALL,ALL
KBC,0
RESCONTRL,DEFINE,ALL,ALL,1
TIME,21
LSWRITE,21,

DDELE,6,ALL
D,6, ,-2, , , ,UX, , , , ,
NLGEOM,1
SSTIF,ON
OUTRES,ERASE
OUTRES,ALL,ALL
KBC,0
RESCONTRL,DEFINE,ALL,ALL,1
TIME,22
LSWRITE,22,

DDELE,6,ALL
D,6, ,-3, , , ,UX, , , , ,
NLGEOM,1
SSTIF,ON
OUTRES,ERASE
OUTRES,ALL,ALL
KBC,0
RESCONTRL,DEFINE,ALL,ALL,1
TIME,23
LSWRITE,23,

DDELE,6,ALL
D,6, ,-4, , , ,UX, , , , ,
NLGEOM,1
SSTIF,ON
OUTRES,ERASE
OUTRES,ALL,ALL
KBC,0
RESCONTRL,DEFINE,ALL,ALL,1
TIME,24
LSWRITE,24,

DDELE,6,ALL
D,6, ,-5, , , ,UX, , , , ,
```

```
NLGEOM,1
SSTIF,ON
OUTRES,ERASE
OUTRES,ALL,ALL
KBC,0
RESCONTRL,DEFINE,ALL,ALL,1
TIME,25
LSWRITE,25,

DDELE,6,ALL
D,6, ,-6, , , ,UX, , , , ,
NLGEOM,1
SSTIF,ON
OUTRES,ERASE
OUTRES,ALL,ALL
KBC,0
RESCONTRL,DEFINE,ALL,ALL,1
TIME,26
LSWRITE,26,

DDELE,6,ALL
D,6, ,-7, , , ,UX, , , , ,
NLGEOM,1
SSTIF,ON
OUTRES,ERASE
OUTRES,ALL,ALL
KBC,0
RESCONTRL,DEFINE,ALL,ALL,1
TIME,27
LSWRITE,27,

DDELE,6,ALL
D,6, ,-8, , , ,UX, , , , ,
NLGEOM,1
SSTIF,ON
OUTRES,ERASE
OUTRES,ALL,ALL
KBC,0
RESCONTRL,DEFINE,ALL,ALL,1
TIME,28
LSWRITE,28,

DDELE,6,ALL
D,6, ,-9, , , ,UX, , , , ,
NLGEOM,1
SSTIF,ON
OUTRES,ERASE
OUTRES,ALL,ALL
KBC,0
RESCONTRL,DEFINE,ALL,ALL,1
TIME,29
LSWRITE,29,

DDELE,6,ALL
D,6, ,-10, , , ,UX, , , , ,
```

```
NLGEOM,1
SSTIF,ON
OUTRES,ERASE
OUTRES,ALL,ALL
KBC,0
RESCONTRL,DEFINE,ALL,ALL,1
TIME,30
LSWRITE,30,

DDELE,6,ALL
D,6, ,-11, , , ,UX, , , , ,
NLGEOM,1
SSTIF,ON
OUTRES,ERASE
OUTRES,ALL,ALL
KBC,0
RESCONTRL,DEFINE,ALL,ALL,1
TIME,31
LSWRITE,31,

DDELE,6,ALL
D,6, ,-12, , , ,UX, , , , ,
NLGEOM,1
SSTIF,ON
OUTRES,ERASE
OUTRES,ALL,ALL
KBC,0
RESCONTRL,DEFINE,ALL,ALL,1
TIME,32
LSWRITE,32,

DDELE,6,ALL
D,6, ,-13, , , ,UX, , , , ,
NLGEOM,1
SSTIF,ON
OUTRES,ERASE
OUTRES,ALL,ALL
KBC,0
RESCONTRL,DEFINE,ALL,ALL,1
TIME,33
LSWRITE,33,

DDELE,6,ALL
D,6, ,-14, , , ,UX, , , , ,
NLGEOM,1
SSTIF,ON
OUTRES,ERASE
OUTRES,ALL,ALL
KBC,0
RESCONTRL,DEFINE,ALL,ALL,1
TIME,34
LSWRITE,34,

DDELE,6,ALL
D,6, ,-15, , , ,UX, , , , ,
```

```
NLGEOM,1
SSTIF,ON
OUTRES,ERASE
OUTRES,ALL,ALL
KBC,0
RESCONTRL,DEFINE,ALL,ALL,1
TIME,35
LSWRITE,35,

DDELE,6,ALL
D,6, ,-16, , , ,UX, , , , ,
NLGEOM,1
SSTIF,ON
OUTRES,ERASE
OUTRES,ALL,ALL
KBC,0
RESCONTRL,DEFINE,ALL,ALL,1
TIME,36
LSWRITE,36,

DDELE,6,ALL
D,6, ,-17, , , ,UX, , , , ,
NLGEOM,1
SSTIF,ON
OUTRES,ERASE
OUTRES,ALL,ALL
KBC,0
RESCONTRL,DEFINE,ALL,ALL,1
TIME,37
LSWRITE,37,

DDELE,6,ALL
D,6, ,-18, , , ,UX, , , , ,
NLGEOM,1
SSTIF,ON
OUTRES,ERASE
OUTRES,ALL,ALL
KBC,0
RESCONTRL,DEFINE,ALL,ALL,1
TIME,38
LSWRITE,38,

DDELE,6,ALL
D,6, ,-19, , , ,UX, , , , ,
NLGEOM,1
SSTIF,ON
OUTRES,ERASE
OUTRES,ALL,ALL
KBC,0
RESCONTRL,DEFINE,ALL,ALL,1
TIME,39
LSWRITE,39,

DDELE,6,ALL
D,6, ,-20, , , ,UX, , , , ,
```

```
NLGEOM,1
SSTIF,ON
OUTRES,ERASE
OUTRES,ALL,ALL
KBC,0
RESCONTRL,DEFINE,ALL,ALL,1
TIME,40
LSWRITE,40,


LSSOLVE,1,40,1
SOLVE


/POST1

ETABLE,SMAX.I,NMISC, 1
ETABLE,SMAX.J,NMISC, 3
ETABLE,SMIN.I,NMISC,2
ETABLE,SMIN.J,NMISC, 4
ETABLE,MFORX.I,SMISC, 1
ETABLE,MFORX.J,SMISC,7
ETABLE,MFORY.I,SMISC,2
ETABLE,MFORY.J,SMISC, 8

ETABLE,REFL

/POST26
ESOL,2,2, ,NMISC,1,SMAX.I
ESOL,3,2, ,NMISC,3,SMAX.J
ESOL,4,2, ,NMISC,2,SMIN.I
ESOL,5,2, ,NMISC,4,SMIN.J
RFORCE,10,6,F,X,FORCE
NSOL,11,6,U,X,DISP
```