PREDICTING THE RESPONSE OF RC BEAM AGAINST IMPACT LOADING WHILE INCORPORATING LARGE DISPLACEMENT

By

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Islamabad, Pakistan

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Abstract

A formulation for analyzing beams experiencing impact loading, taking into account their large displacement behavior, is developed using a three-node beam element based on displacements. Ever since computers were first applied to nonlinear structural analysis, various nonlinear beam elements have been proposed. The substantial number of publications on nonlinear analysis of beam structures is partly attributed to the availability of different nonlinear formulations. However, it remains unclear which formulation is the most effective. It is worth noting that developing a comprehensive nonlinear beam formulation is not a simple task when considering a beam element. This study aims to address this issue by proposing a formulation to accurately predict the nonlinear behavior of beams subjected to impact loading. To validate the proposed formulation, it is cross-verified against experimental data from various research studies available in the literature through statistical analysis. Further, a comparative study is conducted with previously available literature models and with those predicted by the proposed large displacement solution model. This comparison ensures the efficiency and reliability of the model

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CHAPTER 1: INTRODUCTION

1.1. Impact loading

In impact loading, a sudden and intense force or load is applied to a structure member for a short time period. Typically, such loading is characterized by its rapid and dynamic nature, resulting in high levels of stress and strain on the material or structure.

Impact loading can arise in numerous situations and applications, such as:

1. Engineering and Structural Analysis: In engineering, impact loading is considered when designing structures to withstand unforeseen events, such as collisions, explosions, or heavy impacts.

2. Sports and Safety Equipment: In the design and testing of safety equipment, such as helmets, padding, and body armor, impact loading is crucial in order to avoid injury to athletes, workers, or individuals in high-impact situations.

3. Aerospace and Defense: For aircraft, spacecraft, and military vehicles, impact loading is crucial to the design process in order to ensure that they will withstand potential collisions, bird strikes, and other kinds of impacts.

4. Industrial and Machinery Applications: Industrial machinery can be subject to impact loading during start-up, shutdown, or sudden changes in load conditions, requiring appropriate engineering and design in order to prevent catastrophic failure..

Considering the dynamic nature of impact loading and the rapid changes in momentum, its effects can differ from those of static loading. In order for structures and materials to cope with such transient loading conditions effectively, they must be carefully designed and tested.

Researchers and engineers utilize a variety of simulation techniques to understand the response of materials and structures against impact loading conditions, ensuring reliability, safety, and performance in real-world environments. Examples include finite element analysis, computer modeling, and physical testing.

1.2. RC beam response against impact loading

This study is focusing to investigate the behavior of Reinforced Concrete against impact loading, it is important to discuss the different loading conditions that will influence the behavior of the beams.

a. **Low Rate and Extended Duration Scenario:** In the presence of a low rate of impact loading over an extended period of time, RC beams will predominantly exhibit flexural behavior. As a consequence, the main mode of deformation will be bending, in which the beam undergoes curvature as a result of the load. The figure-1 demonstrates a visual representation of this flexural behavior.
b. **High Rate and Short Duration Scenario:** A RC beam, on the other hand, will exhibit predominant local shear behavior when experience high rate of impact loading over a small period of time. The deformation pattern implies that the concrete and steel reinforcement will be sheared along the length of the beam. In order to make the explanation of this local shear behavior more understandable, a visual rendition is provided.
c. **Middle Rate of Impact Loading Scenario:** In cases where impact loading rates fall between the extremes of low and high, RC beams will fail through flexural and shear failure patterns. As a result of a beam experiencing both bending and localized shear effects simultaneously, this flexural-shear interaction occurs. As part of this study, a comprehensive analysis of the flexural and shear failure patterns will be conducted.

This research aims to forecast the non-linear flexural behavior of RC beams under a variety of impact loading conditions. Non-linearity refers to the fact that the response of the beam may not follow a linear relationship with the load applied, and a thorough understanding of this behavior is vital to the design of resilient structures that can resist dynamic loads.

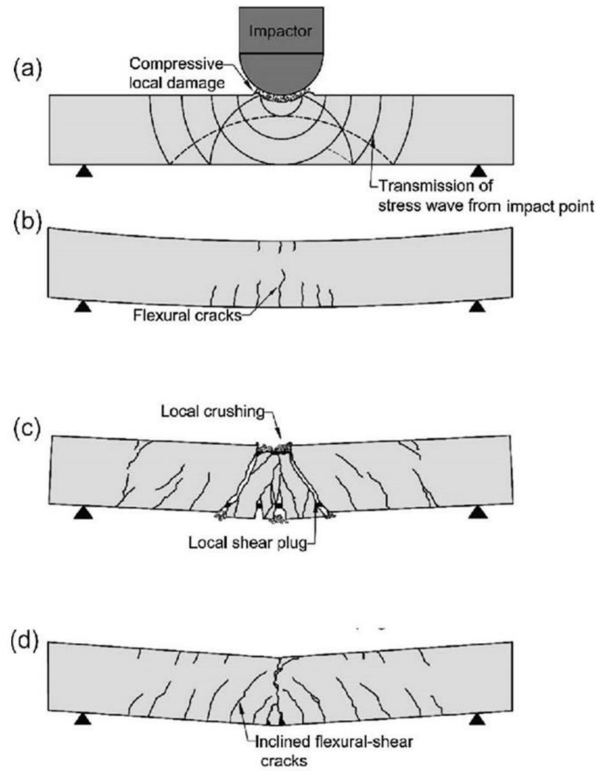

Figure 1.1 RC beam response against impact loading

1.3. Methods for Evaluating the Effects of Impact loading

As discussed previously, understanding the impact response of structures is crucial for a variety of engineering and safety applications. Whenever a structure experience impact loading, such as a collision, blast, or sudden external force, its behavior and capacity to withstand such loads become critical factors. Researchers and engineers need to analyze how the structure deforms, absorbs energy, and reacts during an impact in order to ensure its safety, performance, and durability.

1.3.1. Empirical Method:

To estimate the behavior of a structure against impact loading, the empirical method is a practical approach that relies on experimental data and observations. In this method, the structure is physically tested on a scaled-down model or on an actual structure. A structural engineer subjects the structure to controlled impact loads in order to accurately measure various parameters, including displacements, accelerations, and stress distributions. As a result of analyzing the experimental data, they advance a better understanding of the performance of the structure during an impact.[1]

An empirical approach is particularly useful when dealing with complex or non-linear systems for which analytical or numerical approaches may prove challenging due to the lack of comprehensive theoretical models. For accurate and reliable data on the impact response of real-world structures, empirical testing is essential. Because real-world structures have complexities that are difficult to capture through equations alone, empirical testing is essential for the development and improvement of impact models. Furthermore, empirical data can serve as a basis for validating and improving analytical and numerical models.

1.3.2. Analytical Method:

In the analytical method, mathematical equations and principles are used to determine how a structure will respond to an impact load. Engineers develop mathematical models that simulate the behavior of structures under impact conditions. Models based on differential equations and calculus are used to describe the structural response based on fundamental principles of mechanics and physics. [2]

This approach is suitable for systems that are well-defined and relatively simple, where mathematical equations can be used to accurately describe the underlying physical phenomena. These models can be used by engineers to predict the behavior of structures without the need for physical experimentation. Using analytical solutions, one can gain insight into the variation in key parameters, such as deformation, stress, and energy absorption, as a result of different impact scenarios. Analytical methods may, however, have limitations in situations involving complex structures or situations where the underlying physics are highly non-linear or difficult to model mathematically. When such situations arise, numerical methods are often more effective.

1.3.3. Numerical Method:

Numerical methods involve solving complex equations describing the functioning of structures against impact loads through the use of computer simulations and mathematical models. It involves the use of computer software to generate a virtual representation of the structure. In the simulation, they specify the geometry, the properties of the material, the boundary conditions, and the impact scenario.[3]

As the numerical method is highly versatile and can be applied to complex geometries and material properties, it lends itself well to the analysis of real-world structures exposed to complex impact scenarios. Simulations can be run multiple times with varying parameters to gain a comprehensive interpretation of how the structure behaves under different conditions. The use of numerical simulations is particularly valuable for predicting outcomes for which the use of empirical or analytical methods is difficult or impractical. The results provide valuable insight into critical factors including failure modes, stress concentrations, and potential damage, which assists in the design and optimization of structures for specific impact scenarios.

As a result, each method for determining the impact response of structures has distinct advantages and applications. Empirical methods provide direct experimental data, which ensures the accuracy and validity of models. Analytical methods provide quick and accurate predictions for simpler systems that have well-known equations. The numerical method is well suited for handling complex and real-world structures, providing detailed and comprehensive information about the behavior of these structures. It is possible to ensure the safety, efficiency, and robustness of structures subjected to impact loads by judiciously combining these methods.

1.4. Large Displacement

In structural mechanics, the "large displacement solution" is a sophisticated method of analyzing structures that undergo significant deformations, where the conventional linear elastic theory is no longer sufficient. Structures can exhibit nonlinear behavior when subjected to substantial loads or displacements, which means that the relationship between applied forces and resulting deformations is not linear.[4]

The following points can be highlighted within the context of the large displacement solution:

a. **Geometric Nonlinearity**: Traditional linear analysis assumes that displacements and strains are small, which means that the structure's geometry will remain relatively unchanged during loading. This assumption, however, is no longer valid when dealing with large displacements. In such a case, geometric nonlinearity becomes important, and the deformation of the structure can have a significant effect on its overall shape and geometry.
b. **Changes in Stiffness:** Structures can experience significant stiffness changes under large displacements. Linear analysis typically assumes that stiffness remains constant regardless of the deformation. However, due to changes in shape and material behavior, stiffness properties may vary as the structure deforms.
c. **Nonlinear Behavior of Deformations:** Large displacement solutions consider the nonlinear relationship between applied forces and resulting deformations. During larger displacements, the material behavior may become nonlinear, and the relationship between stress and strain may deviate from Hooke's law, describing linear elastic behavior.
d. **Precise Results:** When geometric nonlinearity is taken into account, as well as changes in stiffness during loading, the large displacement solution provides more accurate and precise results than linear analysis. By using it, engineers can more accurately predict what will happen to structures when they are subjected to significant deformations.
e. **Practical Applications:** A large displacement solution is especially relevant when analyzing structures experience extreme loads, for example bridges under earthquake forces, tall buildings under winds, or aerospace structures subjected to aerodynamic forces.

In summary, the large displacement solution is a complex approach that takes into account geometric nonlinearities and changes in stiffness during loading. Taking these factors into consideration, this method provides more precise and accurate results when analyzing structures that undergo significant deformations. A linear elastic analysis would lead to inaccurate predictions in situations where extreme loads are applied to structures, ensuring safety and reliability.

CHAPTER 2: LITERATURE REVIEW

2.1. Introduction

Critical evaluation of present literature regarding analysis of rigid-plastic behavior of structure under dynamic loading conditions is present in this chapter. This study focuses on contentious issues in the literature that have underscored the importance of this investigation. It is important to understand how ductile elasto-plastic structures respond to extreme loads, such as pulses, impulses, and impacts. It may be difficult to get understanding of dynamic plastic behavior of a system by using numerical methods such as finite element and finite difference. An approximate model, namely rigid-perfectly plastic, has been developed to get more clear understanding of the response. The simplified rigid-plastic model has enabled closed-form, theoretical solutions to simple structural members, shedding light on plastic deformation and energy absorption mechanisms.

Calculating the rigid-plastic response of more complex structural members and systems requires computational methods. To determine dynamic response of structures before rigid-plastic idealization, it was not possible to use a reliable and efficient computer program. As an alternative, mathematical programming has proven to be valuable in developing a structured and unified discrete mathematical and computational approach to rigid-plastic dynamic problems. In this study, the dynamic rigid-plastic response of beam systems is considered using a computer-oriented formalism based on linear complementarity problems (LCPs).

For the purpose of validating the formalism, the study compares the LCP predictions for one dimensional beams under impact with closed-form theoretical solutions. An extensive literature review on rigid-plastic beams subjected to extreme loads is also presented, and numerical challenges encountered during the LCP solution of impulsively loaded and impacted beams prompted the inclusion of plastic shear deformation in the model. Using existing literature, this chapter emphasizes the importance of considering flexural deformation when solving problems agonist extreme dynamic loads.

2.2. Methods for investing dynamic behaviour of RC beams against impact loads

2.2.1. Experiments Studies

Bhatti AQ and Kishi N (2009)[5] in their work, the focus was on developing a simple elastoplastic analysis method for shear-failure-type RC beams. In this paper, twelve rectangular RC beams with simple supports were subjected to a fall-weight impact test and a 3-D finite element analysis. An impact load of 400 kg was applied to the beams at their midpoint using a steel weight. A non-linear finite element analysis code, LS-DYNA, was being applied in this study. FE analysis was found to accurately predict mid-span displacement, crack patterns, and impact force histories for RC beams on their side surfaces based on the results of the study.

Adhikary S Das, Li B, and Fujikake K (2012)[6]: objective of their study was to get a better understanding of how RC beams react to changes in loading rates. A comprehensive test program was conducted to measure load vs mid span displacement, strains at longitudinal reinforcement mid-points, accelerations along specimen lengths and crack profiles. Increasing loading rates resulted in increases in absorption energy, stiffness, peak load and strain rate, as shown in the present study.

Fujikake and Bing Li (2009)[7] in their research conducted in 2009, In this study, reinforced concrete (RC) beams were examined in terms of their impact response under a variety of loads. An analytical model capable of forecasting max mid-span deflection and impact load was developed by the researchers to assess the damage levels resulting from these impacts. For the experiment, they used specimens of reinforced concrete beams with under-reinforced sections and transverse reinforcements sufficient to cause overall flexural failure. The experimenters conducted drop hammer tests to determine the impact of steel longitudinal reinforcement and drop height on the response of the beams. The experimental impact responses were analyzed using a mass-spring-damper system with 2 degrees of freedom that accounted for loading rate effects. For RC beams which fail in flexural failure, these results were well in arrangement with the experimental data. Research helps improve structural design and safety by better forecasting the response of RC beams against impact loads.

2.2.2. Analytical Models

In their research conducted in 2018, Wuchao Zhao and Jiang Qian[8], RC beams against impact load can be predicted using a straightforward and novel method developed by the researchers. Based on four fundamentals first one is contact law, second is energy conservation, third one is impulse-momentum theorem and fourth is wave theory, they used their approach to explain the phenomenon. To establish a relationship between force and deflection for the RC beam, they also integrated conventional beam theory and layered section theory. As part of the calculation, strain rate was also taken into account. 143 impact tests were conducted to demonstrate the accuracy of their proposed method in estimating maximum midspan deflection under impact loads. This aspect of anti-impact design may be applied effectively despite the overestimation in predicting peak impact force.

The mid-span deflection was evaluated;

$$\mathrm{E_{kstab}} + (\mathrm{M} + \mathrm{m})\mathrm{g_{smax}} = \int_0^{smax} \mathrm{F(s)ds} \qquad (1)$$

$\mathrm{E_{kstab}}$ is kinetic energy, M in above equation is impact mass, m is effective calculated mass of beam, g is gravity of acceleration, $smax$ is maximum mid-span deflection, and F is strength of beam under drop-weight loading at mid-span.

2.2.3. Numerical Study

In 2016, Adhikary and Bing Li[9] conducted research to assess the feasibility of numerical investigation as an alternative to experimental studies when studying RC beams against impact loads. Experiments on structural members are often complicated and costly to conduct for determining the effects of a variety of parameters. In order to validate their finite-element analysis results, the researchers first compared them to experimental results. The impacts of various parameters on the dynamic increase factor (DIF) of maximum resistance and the failure mode of RC beams were then evaluated using a numerical parametric analysis. It was discovered that longitudinal strengthening in beams had a substantial impact on the DIF of maximum resistance. DIF was greater in beams with little longitudinal reinforcement than in beams with significant longitudinal reinforcement. Furthermore, the longitudinal reinforcement ratio was important in changing the failure mode of under-reinforced beams, especially at high loading rates (e.g., 2 m/s), when the failure mode switched from flexure to shear during static loading. Furthermore, beams with a considerable quantity of transverse reinforcement had a lower DIF than beams with a modest amount. Longitudinal reinforcement yield strength became a major component influencing the shift from flexure to shear failure modes at high loading rates.

It was in 2023 that Azam and Asad[10]: linear complementarity problem (LCP) was developed as a computer tool for examining the dynamic behavior of RC beams under impact. This improved model can correctly forecast the behavior of RC beams to impact loads, providing significant insights for engineering design and performance.

2.3. Nonlinear dynamic behaviour of rigid plastic structure

In 1958, Symonds and Mentelin[11] transverse impulsive pressure loading and axial constraints were used to explore the plastic deformation of simply supported and clamped beams. The shift from fundamental beam behavior to catenary effects happens as deflections increase due to finite axial forces. Study authors emphasize the unreality of disregarding axial forces when deflections are greater than beam thickness. Without taking into account axial forces, continuous beam deflections can be significantly smaller than predicted. In addition to presenting valuable curves, the authors illustrate how the beam's span-depth ratio and its intensity determine the reduction in deflection. The findings of this study contribute to our understanding of beam behavior and have significant consequences for engineering and structural design.

A study by Taijiro Nonaka[12] (1977) analyzes the deformation behavior of affixed beams with and without restrictions against axial displacement at the ends. A focused mass is carried in the center of the beam, and the beams are subjected to blast loading. Previously, Nonaka developed theoretical predictions for the deformation of restraint beams of cross-section forced against axial displacements. Furthermore, he approximated the eventual deformation of a stationary beam without axial constraints using the findings from his prior work on a totally clamped beam. This study compares experimental data to theoretical predictions and finds a high level of agreement, indicating the credibility and validity of these models. Nonaka's experimental study contributes to a better understanding of the behavior of firm beams under blast loads, both with and without axial constraints, and has practical implications for structural engineers.

Symonds and Jones[13] (1976), provide a comprehensive discussion of beams subjected to impulsive loading, with a focus on clamped beams subjected to end rotations and axial displacements. The yield stress is examined in this review as a function of strain rate and the effects of small finite transverse displacements. On the basis of this existing knowledge, the authors present different solutions that are resulting from rigid-plastic analysis and combine these factors in an estimated manner. The formulas are validated by conducting experiments on mild steel beams while relating outcomes with deflections calculated from formulas. Literature review provides a better understanding of beam behavior and can be applied to engineering applications, aiding in the design of structures to withstand impulsive loading scenarios, while considering the complex interaction between material properties and beam geometry.

Vaziri[14] (1985) develop an expressions calculating maximum permanent deflection of beams, which are valid over a wide dynamic range, including pseudostatic step loads as well as high-pressure impulsive loads. The significant influence of geometry changes on the beam's response, even for small deflections, emphasizes the importance of considering the beam's shape and dimensions when analyzing its behavior under particular loading conditions. In addition, the study introduces a new approach by combining the response expressions in order to establish isoresponse relationships, which allows engineers to visualize the beam's deflection patterns under a variety of loading scenarios and geometries. The research significantly enhances our understanding of beam behavior and provides valuable tools to design structures that are capable of withstanding varied pressure pulse loads, thus contributing to safer and more efficient engineering practices.

According to Liu and Jones[15] (1988), rigid, perfectly plastic clamped beams can undergo transverse shear and bending when subjected to impulsive loads caused by mass impacts at any point along the beam's length. They examines the effect of finite deflections on beam's performance under various loading scenarios.

The study findings of Symonds[16] and Mentel, Nonaka[17], Symonds, P. S and Jones, Vaziri, and Liu, J. H[18] and Jones indicate that when displacements reach the depth of the beam, membrane forces become a dominant influence on the beam's behavior. In a transition from bending action to membrane response, there is a significant stiffening effect, which alters drastically the nature of deformation. Accordingly, the displacements resulting from bending may be much smaller than what can be obtained from bending alone. It is important to take into account membrane forces when analyzing structural systems, especially in scenarios where displacements are comparable to the beam's dimensions.

2.4. Mathematical programming

The primary goal of this study is to assess the suitability of mathematical programming for modeling nonlinear dynamics and to develop computer-oriented techniques for numerically solving dynamic problems in stiff plastic, planar-framed structures. Mathematical programming involves the theory and techniques associated with optimizing an objective function while adhering to specific constraints. The mathematical programming technique in this study is used to simulate the physical behavior of nonlinear dynamics in rigid plastic systems and presents efficient algorithms that can be implemented on computers to solve dynamic problems in planar framed structures numerically. This study focuses on the study of plane framed structures, which are known for their structural integrity and their applicability in a variety of engineering fields. In mathematical programming, objective functions are formulated to optimize certain performance criteria while accounting for the constraints imposed by physical properties, geometry, and boundary conditions.

Furthermore, the goal of this study is to offer a full knowledge of the dynamic response of stiff plastic planar framed structures. The project's goal is to provide algorithms that allow for efficient calculations and numerical simulations that generate accurate forecasts of how these structures will react when subjected to various dynamic loading situations. It is acknowledged in the study that it is important to accurately represent material behavior, particularly plastic deformation, in order to accurately predict the structural response to extreme loads. As a result of harnessing the power of mathematical programming, this study strives to improve our understanding of structural behavior under dynamic conditions, contributing to the design of more resilient and efficient structures as well as improving engineering practices.

Maier[19] demonstrated in 1984 offers exceptional mathematical formalism for effectively representing response of discrete elastoplastic structures. The study provides valuable insight into the development of this method and its accurate behavior for analyzing structural problems in the future. Mathematical programming provides researchers and engineers with a powerful tool for accurately simulating the response of elastoplastic structures, allowing them to better understand their behavior under quasi-static loading. This discovery will pave the way for further advancements in structural analysis, resulting in better engineering practices and more resilient, efficient, and robust structures. The implications of this research go beyond its immediate findings, placing mathematical programming as a cornerstone in the field of structural engineering and providing novel approaches to the analysis of various structural systems.

In a groundbreaking work published in 2021, Rodrigo Pierott[20] and Ahmed W.A. Hammad introduced an optimize model for calculating the size of RC beams against extreme load. In their model, strength of concrete (f_{ck}), area of cross-section, and diameter of reinforcement bars are considered as design variables, resulting in realistic representation of optimization process. By determining reinforcement layouts, defining f_{ck} values, and minimizing construction costs, the research seeks to minimize construction costs, while ensuring structural integrity. With Rodrigo Pierott and Ahmed W.A. Hammad's sophisticated finite element method program and longitudinal reinforcement database generator, essential data can be obtained for stress and strain analysis and for code compliance. By adopting an Evolutionary Algorithm, the optimization problem can be effectively solved, resulting in significant cost savings of 3.63% to 17.07% over existing researches. The study's goal is to promote the design of RC beams by providing useful insights into cost-effective and structurally optimum solutions, hence contributing to the progress of engineering techniques in the construction sector.

According to Azam and Moiz[21] in 2021, a linear complementarity method was establish to predict the behavior of rigid-plastic structures when experience impact loads. Lemke's algorithm was used for calculating the linear complementarity problems (LCP). They propose more authentic approach to kinetics and kinematics to enhance the efficiency of LCP formulation. In order to test the accuracy of the proposed technique, the obtained numerical findings are compared to previously known experimental data in the literature.

In their 2023 study, Azam and Asad[22] We provide a unique computational framework for assessing the dynamic response of shear critical reinforced concrete beams against impact loads based on a linear complementarity problem (LCP). The researchers offer comprehensive analysis of impacted simply supported beams. Built on results of 46 reinforced concrete beams in the study's database that experienced flexure shear or shear failures, the model was demonstrated to be a highly predictive system (R^2 = 95%). Additionally, the efficiency of the formulation has been tested against ABAQUS, a commercial software program, showing that its computational performance is competitive. In this study, structural analysis is significantly advanced, offering valuable insights and an efficient tool for analyzing RC structures subjected to impact loads, resulting in a potential improvement in engineering design and performance.

2.5. Effect of material elasticity

In their 1984 paper, P. S. Symonds and W. T. Fleming[23] examine the distortions of line structural element i.e. beam having a mass at its corner and exposed to pulse loading for short duration of time. This study examines crucial assumption of neglecting elastic strains, where elastic moduli are considered to be infinite in "elementary rigid-plastic theory" for dynamic structural response problems. The researchers assessed its validity by comparing numerical solutions obtained from previously available computer program that are based on advance finite element with a slightly modified rigid-plastic solution that takes into account large deflections. In addition, they employ a simplified elastic-plastic approach that artificially separates elastic and plastic actions for a deeper understanding. The study's findings provide considerable insight into the impact of elastic stresses on RC beam behavior subjected to pulse loading, allowing for a more detailed explanation of the limitations and significance of rigid-plastic theory in the context of dynamic structural behavior.

The study conducted by P. S. Symonds and Charles W. G. Frye[24] in 1988 confirmed that structures experience large dynamic loads, resulting in extensive plastic deformation, are less likely to be influenced significantly by material elasticity. According to their findings, rigid plastic analysis will be employe to determine a structure's dynamic response under two circumstances: first, when the input energy surpasses the maximum elastic energy the structure is capable of storing, and second, when pulse durations are significantly shorter than the elastic structure's natural period. In these situations, where plastic deformation dominates the behavior and elastic effects play a relatively minor role, rigid plastic analysis is an effective technique for assessing the dynamic behavior of structure.

2.6.Summary

The literature review indicates that examination of rigid-plastic structural systems against extreme loading conditions is a topic of considerable in field of structural engineering. For simple structural members, approximate models, such as rigid-perfectly plastic, have provided valuable insights; however, more complex systems require computational methods to obtain rigid-plastic responses. In rigid-plastic structures, mathematical programming, specifically the Linear Complementarity Problem (LCP), is an efficient and robust method for simulating linear dynamics.

During this study, reinforced concrete (RC) beams will be examined under impact loading to determine their dynamic response. Our approach will build on recent research by Azam and Asad[25], by incorporating a numerical formulation based on the Linear Complementarity Problem (LCP). Furthermore, we will improve the model by accounting for substantial displacement effects in order to gain a more complete knowledge of the behavior of RC beams under excessive dynamic loads. Finally, the study hopes to give further insight into the structural behavior of RC structures under impact loads, as well as credible forecasts that will aid in the advancement of our understanding of these structures[26].

CHAPTER 3 METHODOLOGY

A mathematical model for analyzing the dynamic behavior of beams subjected to impact loads is offered. The goal is to acquire a better understanding of how these structures respond when subjected to unexpected and severe loading situations. In order to achieve this, we use the Linear Complementarity Problem (LCP) method, which provides a robust mathematical framework for simulating the performance of rigid-plastic systems under extreme dynamic conditions. Incorporating a solution that addresses large displacements is the utmost significant feature of our formulation. During impact loading, structures can experience significant deformations, and neglecting these large displacement effects could result in inaccuracies in the analysis. With our model, we aim to obtain more accurate results by taking into account large displacements solution (LDS), providing a comprehensive understanding of how beams respond to dynamic forces.

3.1 Dynamic Rigid-Plastic Model Considering Large Displacement

The initial goal of our research is to create a numerical model capable of reliably predicting the behavior of reinforced concrete (RC) beams under impact loads. To that end, we will create a impler model of a simply supported RC beam subjected to a certain impact. A finite number of components will be discretized to ease numerical analysis of the beam, as illustrated in Figure 3.

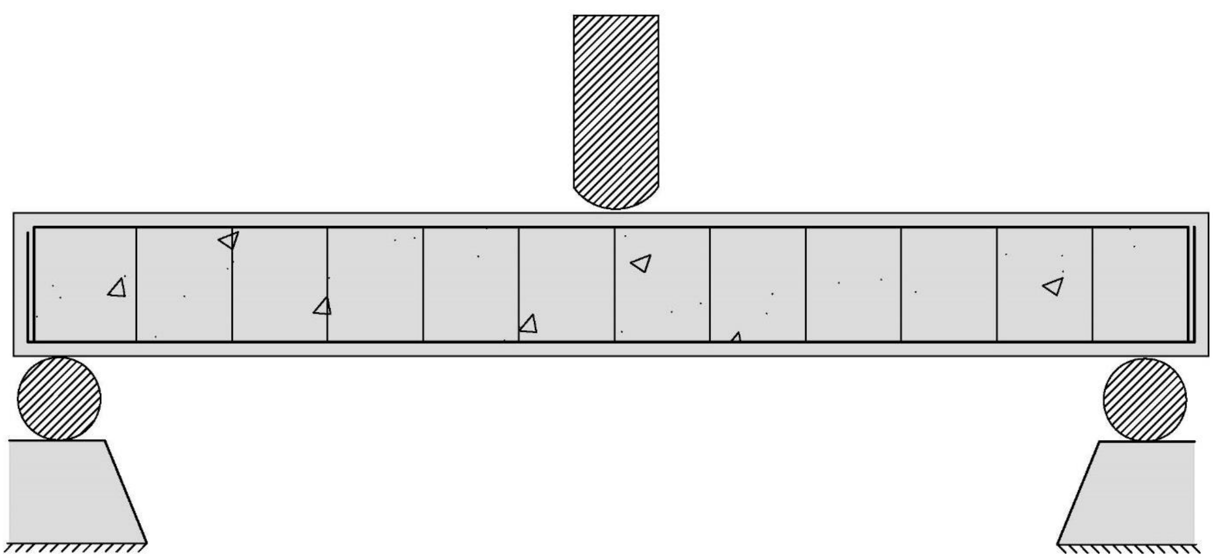

Figure 3.1 Impact loading on a simply-supported beam

The proposed model will take into account the dynamic behavior of an RC beam subjected to an impact load at a specified impact point. To thoroughly investigate the behavior of the beam under such stress conditions, the continuous structure will be split into distinct pieces. As a consequence of these elements, we will be able to properly estimate the behavior of the beam in the presence of impact loads using mathematical approaches and computational algorithms.

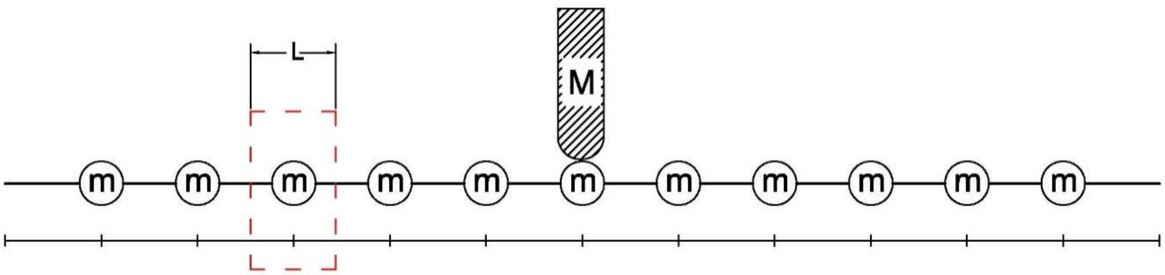

Figure 3.2 Impact loading on discretized simply-supported beams

Figure 4 depicts the displacement and deformation of a planar frame element M with a length of primary L. The element's original location in its local coordinate system is referenced..

- F'_M are representing end forces
- d'_M are the displacement

By p, we depict the rigid body rotation of structural member.

c= *cosp*, *s*=sin*p* (2)

It depicts the interaction or interconnection force between the end forces of the member or element (I = l, 2,...,SM) and from which the element has been withdrawn for structural system. Figure 4 is an example of a planar beam or frame element where SM = 6 and F are the equivalent forces. In addition, there are several indepentent member forces X (j = 1,2,...,M). The six forces F may always be written as a set of three independent forces X for a planar element. As a result, Sm= 3 when compared to a planar element.

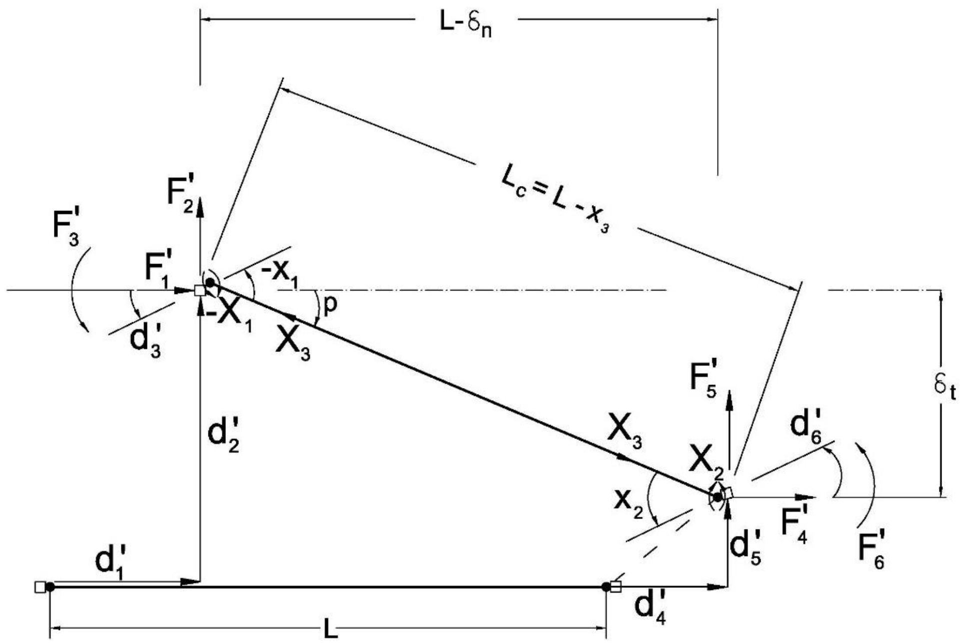

Figure 3.3 Planar Member in deformed Configuration

The forces F or are found at the terminals of the elements. These forces, in concert with loads spread throughout the element's length, create stress resultants or generalized stresses along its longitudinal axis. For a planar element, an axial force is X1, a transverse shear force is X2, and a bending moment is X3 at position X. The member forces F calculate the stress resultants (X). Assuming that the element travels with no change in geometry and that there is no load on the element, the following correlations exist between the two sets of member forces:

$$\begin{bmatrix} F^{1'} \\ F^{2'} \\ F^{3'} \\ F^{4'} \\ F^{5'} \\ F^{6'} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ -L^{-1} & -L^{-1} & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -L^{-1} & -L^{-1} \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ -1 & 0 \\ 0 & -1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \pi_n \\ \pi_t \end{bmatrix} \quad (3)$$

Or

$$F'_M = A_M^T \; X_M \text{ - } A_M^T \; \pi_M \quad (4)$$

Where

$$F'_M = A_M^T \; X_M \text{ - } A_M^T \; \pi_M \quad (5)$$

$\pi_M = Z_M\ X_M$ (6)

- π_M is additional axial force
- π_T is additional shear force
- A_M^T, is the equilibrium matrix in original position
- Z_M depends on current position of element

$p = \tan^{-1} \frac{\delta_t}{L-\delta_n}$

(7)

$L_C^2 = (L - x_3)^2 = (L-\delta_n)^2 + \delta_t{}^2$ (8)

When the law of kinematics is applied to the member M, it is discovered that the independent member deformations x_m, with respect to the displacements of the member ends d'_M. The equation from Figure 4 will become

$$\begin{bmatrix} x_1 + (p - \frac{\delta_t}{L}) \\ x_2 - (p - \frac{\delta_t}{L}) \\ x_3 + (\delta_n - x_3) \end{bmatrix} = \begin{bmatrix} 0 & -L^{-1} & -1 & 0 & L^{-1} & 0 \\ 0 & L^{-1} & 0 & 0 & -L^{-1} & 1 \\ 1 & 0 & 0 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} d'_1 \\ d'_2 \\ d'_3 \\ d'_4 \\ d'_5 \\ d'_6 \end{bmatrix} \quad (9)$$

Or

$X_M + X_{\pi M} = A_M\ d'_M$ (10)

$x_{\pi 1}$ and $x_{\pi 2}$ are added rotational distortions while $x_{\pi 3}$, is an extra axial distortion

$$\begin{bmatrix} x_{\pi 1} \\ x_{\pi 2} \\ x_{\pi 3} \end{bmatrix} = \begin{bmatrix} \frac{S}{L_c} & -\frac{1}{L} + \frac{C}{L_c} \\ -\frac{S}{L_c} & \frac{1}{L} - \frac{C}{L_c} \\ 1 - c & s \end{bmatrix} \begin{bmatrix} \delta_n \\ \delta_t \end{bmatrix} + \begin{bmatrix} R_{x1} \\ R_{x2} \\ R_{x3} \end{bmatrix} \quad (11)$$

Or

$x_{\pi M} = Z_M^T\ \delta_{\pi M} + R_{\pi M}$ (12)

$$R_{x1} = -R_{x2} = p - \frac{Lsinp}{L_c} \quad (13)$$

$$R_{x3} = \text{-L (1- cos}p) \quad (14)$$

3.2 Nodal Governing System

According to Teixeira de Freitas and Lloyd Smith, it is conceivable to utilize the description of a single member M to generate matching descriptions for the complete structural system. Based on D'Alembert's Principle, this may be extended to a kinetic-kinematic description as well. Inertia forces can be attributed to unconnected masses, and u represents future motions. The nodal kinetic-kinematic description for the structural system will become

$$\begin{bmatrix} 0 & A^T & A_d^T & A_\pi^T & A_o^T \\ A & 0 & 0 & 0 & 0 \\ A_d & 0 & 0 & 0 & 0 \\ A_\pi & 0 & 0 & 0 & 0 \\ A_o & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} q \\ -X \\ \mu \\ \pi \\ \lambda \end{bmatrix} = \begin{bmatrix} Q = 0 \\ x + x_\pi \\ \mu \\ \delta_\pi \\ \delta \end{bmatrix} \quad (16)$$

Where,

$$\pi = ZX \quad (17)$$

$$x_\pi = Z^T \delta_\pi + R_x, \quad (18)$$

$$\mu = -m\ddot{u} \quad (19)$$

A, A_d, A_π, and A_0 consist of unchanging basics that are based on the geometry of the construction in its original position. Equations (16–19) are applicable to dislocations of any magnitude.

3.3 Material Model

The non-holonomic plasticity relations (20) are shows in of generalized stress S and generalized strain rates s at the precarious parts of structural system.

- X are independent member forces
- $\dot{x}_p$ are independent deformation rates
- F' are member end forces
- d'_p are velocities

Therefore, the following transformations must be implemented:

$$\dot{x}_p = T\,\dot{s} \tag{20}$$

$$S = T^t X. \tag{21}$$

In our structural analysis model, we have selected to utilize a material model known as the "rigid plastic model." This model allows us to study the behavior of materials under certain loading conditions. At each node in our system, two separate possibilities can arise in reply to the applied load [27].

The first option is that the node remains in a state of rigidity, meaning it does not experience any permanent deformation. In such cases, the node's behavior can be characterized as a point on the graph, positioned along the y-axis. This point designates that the deformation at the node is essentially zero.

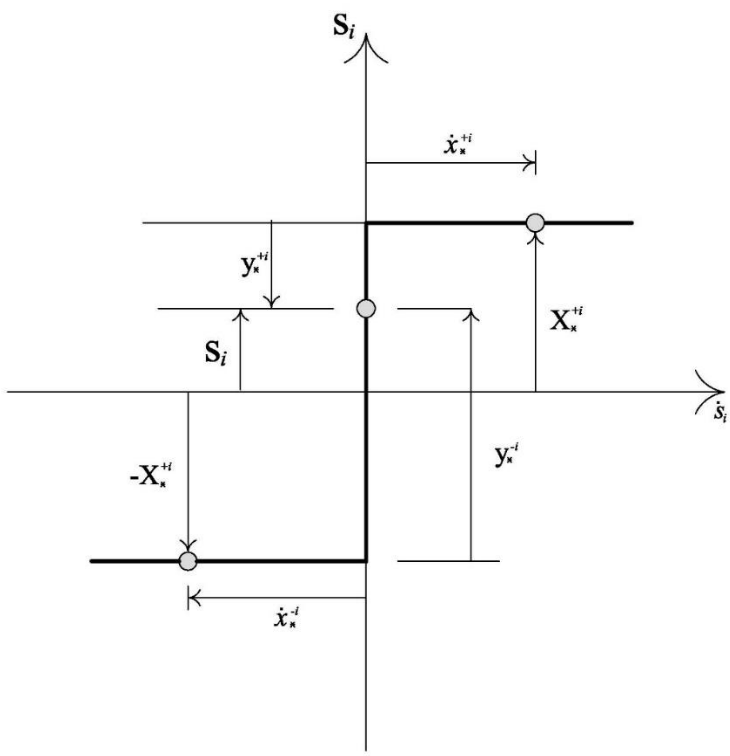

Figure 3.4 Material model

To comprehend and pinpoint this specific point on the graph, we introduce a new variable termed as "y." The variable "y" acts as a limit that command the location of the point along y-axis and assists us in accurately characterizing the node's response to the applied load when it remains rigid.

A second possibility is that the node experiences plastic deformation. This scenario involves the node moving sideways horizontal line. In this material model, this curve represents the region of plastic deformation. Plastic deformation occurs when the node undergoes a change in shape or size that cannot be recovered. The variable 'x' is introduced to capture and quantify this behavior accurately. Nodes are measured along the horizontal surface of the plastic deformation curve using the variable "x".

Depending on the loading conditions applied to the system, the same situation described above may also occur in the opposite direction. Accordingly, nodes are capable of either remaining rigid or undergoing plastic deformation in both positive and negative directions relative to the graph's axes.

With the use of the concepts of "x" and "y" variables, as well as the associated graph, we will accurately analyze and forecast the conduct of our structural system under the given load conditions, as well as understand how nodes respond to deformation in the simulation process. S represents stress resultant and s represents deformation.

$$\begin{bmatrix} 0 & N^T \\ N & 0 \end{bmatrix} \begin{bmatrix} \dot{x}_* \\ S \end{bmatrix} + \begin{bmatrix} Y_* \\ 0 \end{bmatrix} = \begin{bmatrix} X_* \\ \dot{s} \end{bmatrix} \tag{22}$$

$$y_* \geq 0, y_*^T \dot{x}_* = 0, \quad \dot{x}_* = 0$$

To generate governing relations, integrate the kinetic and kinematic explanations, Equations (16) to (19), with the plasticity relations (22) and the reference system transformations (20) and (21).

3.4 The governing system

The presence of complementarity conditions complicates these equations. Since there is no known exact solution to this type of mathematical problem, it is logical to use a numerical approach to solve it. Consequently, a time marching method is implemented to move the solution forward from one time point, t_n, to the next, t_{n+1}, where n represents consecutive discrete time intervals and t represents the amount of time between them. Consequently, Newmark's time-integration scheme is utilized to express centroidal velocities and accelerations as follows:

$\ddot{\mathbf{u}}_{n+1} = b_0 (\dot{\mathbf{u}}_{n+1} - \dot{\mathbf{u}}_n) - b_1\ddot{\mathbf{u}}_n$ (23)

$\dot{\mathbf{u}}_{n+1} = \mathbf{u}_n + b_2\dot{\mathbf{u}}_n + b_3\ddot{\mathbf{u}}_n + b_4\ddot{\mathbf{u}}_{n+1}$ (24)

in which integration constants (25)

are $b_0 = 1 /\bar{\gamma}\Delta t$, (26)

$b_1 = 1-\bar{\gamma}/\bar{\gamma}$, (27)

$b_2 = \Delta t$, (28)

$b_3 = (0.5 - \bar{\alpha})\Delta t$, (29)

$b_4 = \bar{\alpha}\Delta t^2$, (30)

Appropriate findings are reached based on detailed research (Khan et al., 2013). when α = 0.25 and γ = 0.5. Equations (16) to (19) are combined together at $t = t_{n+1}$, and coupled with the Newmark’s scheme (20) to (30) to give following the governing system:

$$\begin{bmatrix} b_0M_q & A_\pi^TZ - A^T & 0 & 0 \\ A_\pi Z^T - A & 0 & TN & 0 \\ 0 & N^TT^T & 0 & I \end{bmatrix} \begin{bmatrix} \dot{q}_{n+1} \\ X_{n+1} \\ \dot{x}_{*n+1} \\ y_{*n+1} \end{bmatrix} = \begin{bmatrix} -Y_{n+1} \\ -R_x \\ X_* \end{bmatrix} \quad (31)$$

$y_{*n+1} \geq 0$ (32)

$y_{*n+1}^T \, \dot{x}_{*n+1} = 0$ (33)

$\dot{x}_{*n+1} \ \geq 0$ (34)

Variables $\dot{\mathbf{q}}$n+1, $\mathbf{X}n$+1 are un-restricted, right-hand side subvector $\mathbf{Y}n$+1 of (31) is given by:

$$\mathbf{Y_{n+1}} = -\mathbf{A_o^T}\,\boldsymbol{\lambda}_\mathbf{n+1} + \mathbf{A^T_{o(n+1)}} + \mathbf{b_0M_qq_n} - (\mathbf{A_\pi^TZ} - \mathbf{A^T})\mathbf{X_{n+1}} \quad (35)$$

Mass matrix $\mathbf{M_q}$, related with nodal accelerations $\ddot{\mathbf{q}}$, is

$$\mathbf{M_q} = \mathbf{A_d^TmA_d} \quad (36)$$

The mathematical structure of the governing system being approximated is that of a linear complementarity problem (LCP). The variables $[\dot{\mathbf{x}}_*, \mathbf{y}_*]$are restricted to being complementary pairs. Additionally, primary sub-matrix associated with variables$[\dot{\mathbf{q}}, \mathbf{X}]$, is negative semi-definite. Hence, to solve resulting Lemke algorithm was used due to its simplicity and reliability.

3.5 Flow chart

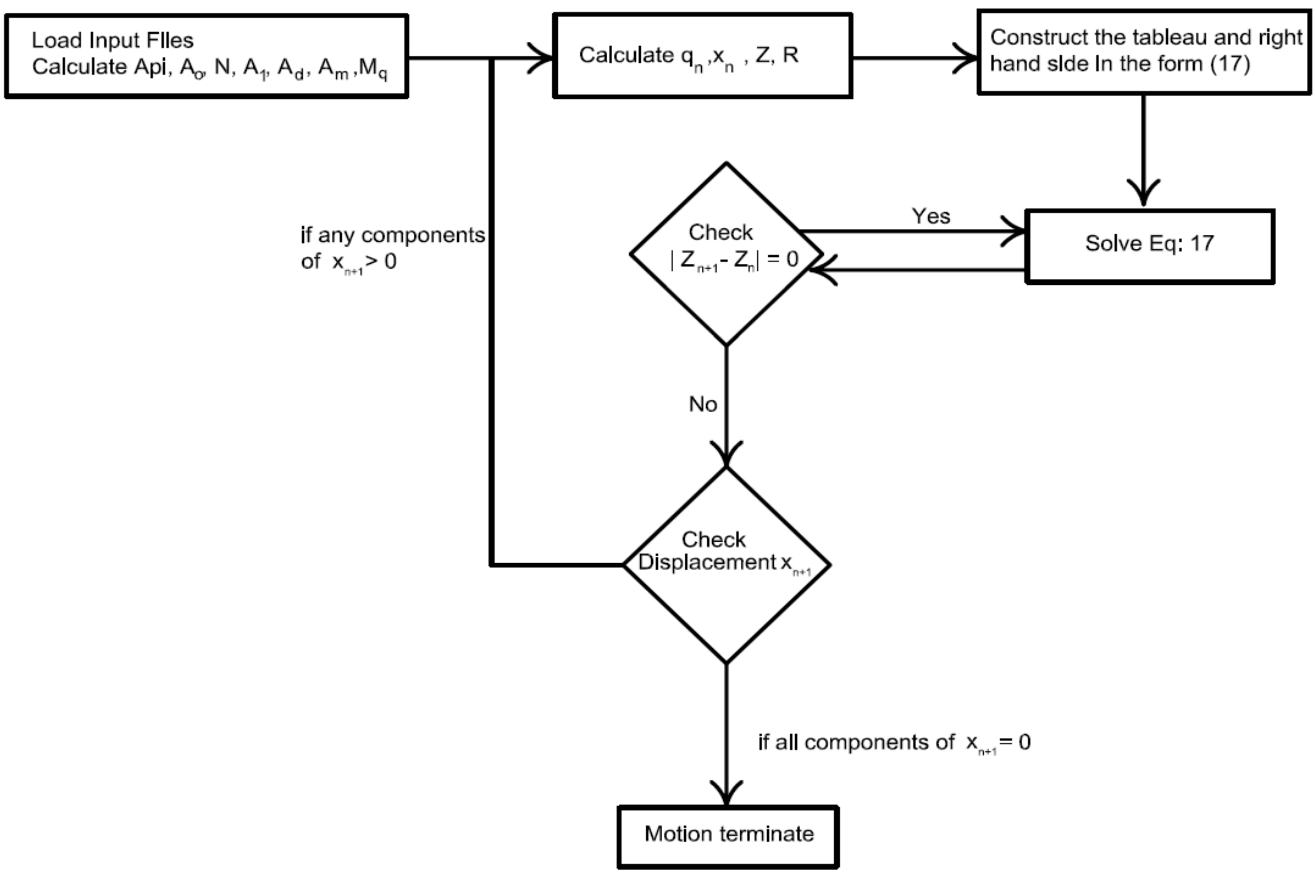

Figure 3.5 Flow Chart

a. As a first step, calculate all the input parameters, including the mass matrix, geometric parameters, flexural capacity, and axial capacity of the line element.

b. At time t = 0, determine the initial deformations, Z (displacement vector), and R (load vector).

c. Create the tabular form of Equation 31

d. Solve the equation using Lemeke's Algorithm

e. Check Z at time t = n+1 (the next time step) to see if it has changed from the earlier time step. This will ensure that analysis is stable and convergent.

f. If the displacement vector x (the motion) at time t = n+1 has all zero components, then it indicates that the motion has terminated.

3.6 Statistical Parameters for Validation

The goal of this part is to assess the accuracy of suggested models of RC beams under impact loading using a set of statistical parameters. The Predicted to Experimental Ratio, Coefficient of Variation, Coefficient of Determination, and Average Absolute Error are the parameters. To evaluate the effectiveness of the models, these parameters are compared to those previously proposed in the literature.

3.6.1 Predicted to Experimental Ratio (PER)

It is commonly used in fields such as physics, chemistry, biology, and engineering to assess the consistency of theoretical predictions with actual experimental results. As used here, "predicted" refers to theoretical or calculated values derived from mathematical models, simulations, or established theories. In general, predicted values are derived from equations or computational methods that describe the behavior of a system under certain conditions.

The term "experimental" refers to values obtained through actual observations and measurements in a controlled laboratory or in the real world. As a result of experiments, these measurements are obtained in order to understand and validate the performance of the system that is being investigated. To calculate the predicted to experimental ratio, the predicted value is divided by the experimental value for a specific quantity or parameter of interest. To assess the degree of agreement among theory and experiment, we can use the following ratio:

Predicted to Experimental Ratio = Predicted Value / Experimental Value

It is generally recognized that a ratio close to 1 indicates that the model or theory used to make the predictions is accurate and reliable under the given circumstances. It is possible that there are discrepancies if the ratio is significantly different from 1. There can be several reasons for such discrepancies, including experimental error, limitations of the model or theory, incomplete understanding of the underlying phenomenon, or unexpected interactions with other variables.

3.6.2 Coefficient of Variation

An assessment of the relative variability or dispersion of a dataset can be made using the Coefficient of Variation (CoV) statistic. A special benefit of this method is that it can be used to compare datasets that have different scales or units of measurement. A CoV is calculated by dividing the standard deviation as

$$\text{Percentage CoV}=\frac{\text{Standard Deviation}}{\text{Mean}}\,x100$$

a. Standard deviation: is the measures of the spread of data points around the mean. A higher SD shows large degree of variability, while a lower value indicates a closer grouping of data points.
b. Mean: The mean, also referred to as the average, represents the arithmetic average of all data points contained in a dataset. A central value is the point around which the data points tend to cluster.

The Coefficient of Variation is expressed as a percentage, which enables easy comparison of the relative variability across datasets.

The Coefficient of Variation can be interpreted as follows:

a. Low CoV values (close to 0%) indicate a low level of relative variability in the dataset. In other words, the individual data points are relatively close to the mean, which suggests that the data are more consistent and less dispersed.
b. An increase in CoV (greater than 0%) indicates that the dataset has a high degree of relative variability. A higher level of variation and dispersion is implied by the data points that are spread further apart from the mean.

It can be concluded that the Coefficient of Variation is a valuable tool in data analysis for comparing datasets with different units and understanding the relative variability of the datasets using different units. It allows researchers and analysts to make meaningful comparisons and draw insights from datasets in many disciplines, including finance, economics, biology, engineering, and social sciences.

3.6.3 Coefficient of Determination (R^2)

R^2, is a fraction of the variation in the dependent variable that can be explained by the independent variable(s). This metric measures how well the independent factors explain the variation in the dependent variable.

This factor ranges from 0 to 1, with the following interpretations:

- $R^2 = 0$: Regression model does not match the data.
- $R^2 = 1$: Represent fit of regression model to the data.

R^2 can be calculated as follows:

$$R^2 = 1 - \frac{SS_{residual}}{SS_{total}}$$

Where:

- $SS_{residual}$ The square root of the difference between the observed and anticipated values of the dependent variable is used to compute it.
- SS_{total} It is determined by taking the square root of the difference between the observed and overall mean value of the dependent variable.

3.6.4 Average Absolute Error

It is a statistical metric used to assess the accuracy of a prediction model or estimator by calculating the average absolute difference between anticipated and actual values. It is particularly useful for evaluating the performance of models that predict continuous numerical values.

The formula to calculate Average Absolute Error is as follows:

$$\text{AAE} = \frac{1}{n} \sum_{i=1}^{n} |y_i - \hat{y}_i|$$

Where,

- y_i is actual/ observed

- $\hat{y_i}$ is predicted value
- n is number of data points

CHAPTER 4: RESULTS & DISCUSSION

4.1 Organization

Various statistical tools are used to validate the proposed models of large displacement solutions (LDS). Statistical analysis is not only applied to the current model, but is also extended to the previously proposed models in order to determine their robustness and efficiency in comparison with the current model. In addition, graphical representations are used to illustrate the variation of results, thereby enhancing understanding

4.2 Large displacement solution validation with experimental data

4.2.1 Experimental Database

A wide-ranging experimental database comprising 102 simply supported RC beams has been collected from various literature sources[28] [29] [30] [31] [32] [33] [34] [35] [36] [37] [38], following a consistent set of inclusion criteria. It should be noted that all the beams in this database have rectangular cross sections and these beams in literature were experience impact loads at the mid span with either flat or spherical contact surfaces.

4.2.2 Key parameters distribution

The purpose of this part is to determine the parameters that have a substantial impact on the performance of RC beams with a basic support. These characteristics comprise several factors such as concrete compressive strength, longitudinal reinforcement, and shear reinforcement. The graph depicts a projectile velocity range of 1 to 16 meters per second, with the most of data lying between 3 and 8 meters per second. Additionally, the impact mass (M) of the tested beams ranges from 100 to 1800 kg with a significant number falling between 300 and 600 kg. Simply supported RC beams are capable of spans between 1000 and 5000 mm, with width and height limits of 100 to 300 mm and 150 to 500 mm, respectively. The longitudinal reinforcement ratio ranges from 0.25% to 3.25%, whereas the shear reinforcement ratio ranges from 0% to 1.4%.

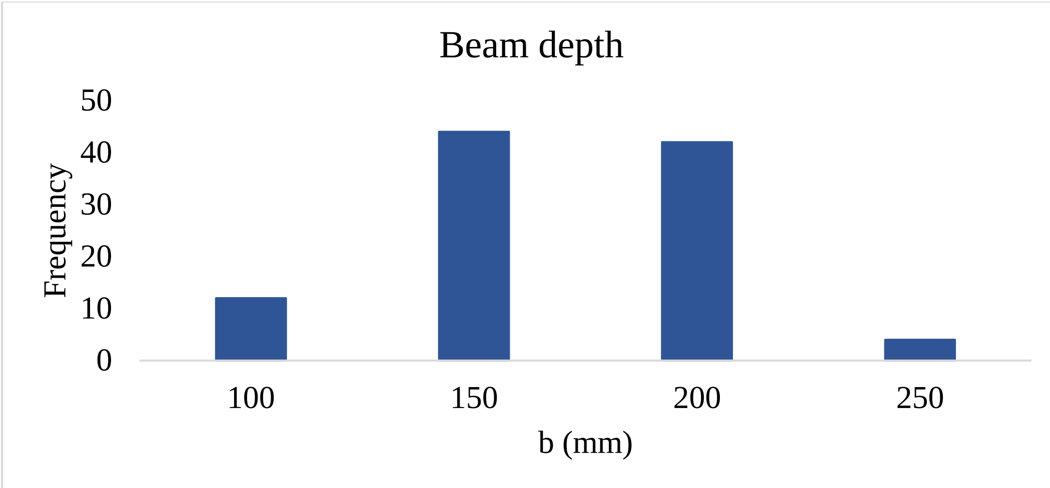

Figure 4.1 Beam depth

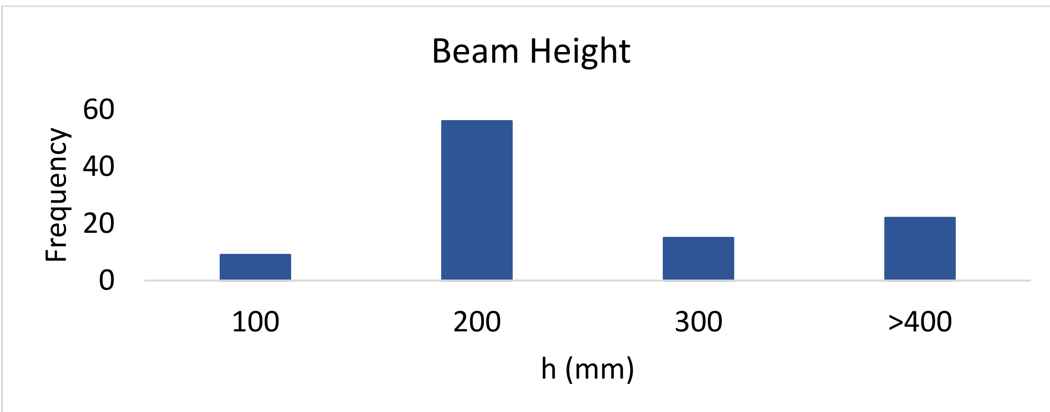

Figure 4.2 Beam Height

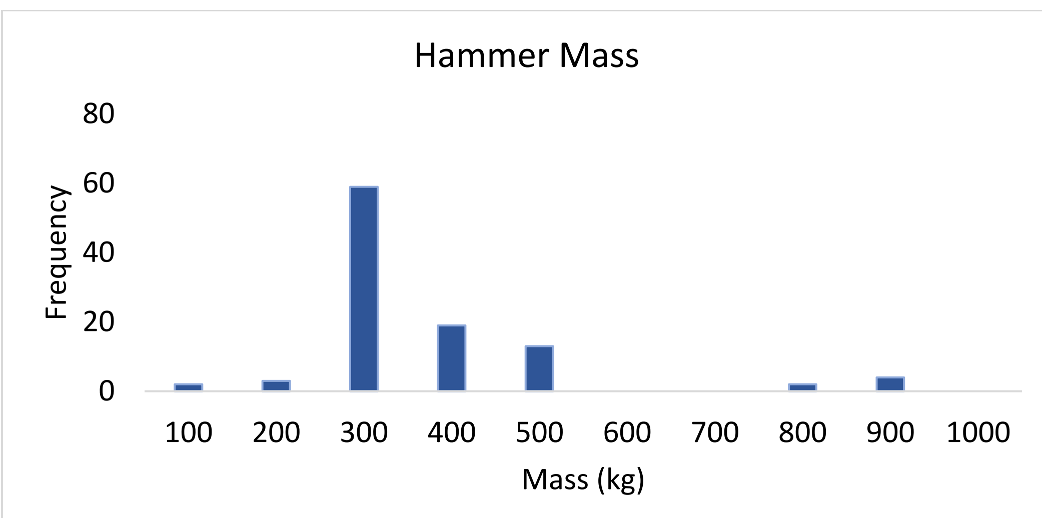

Figure 4.3 Hammer Mass

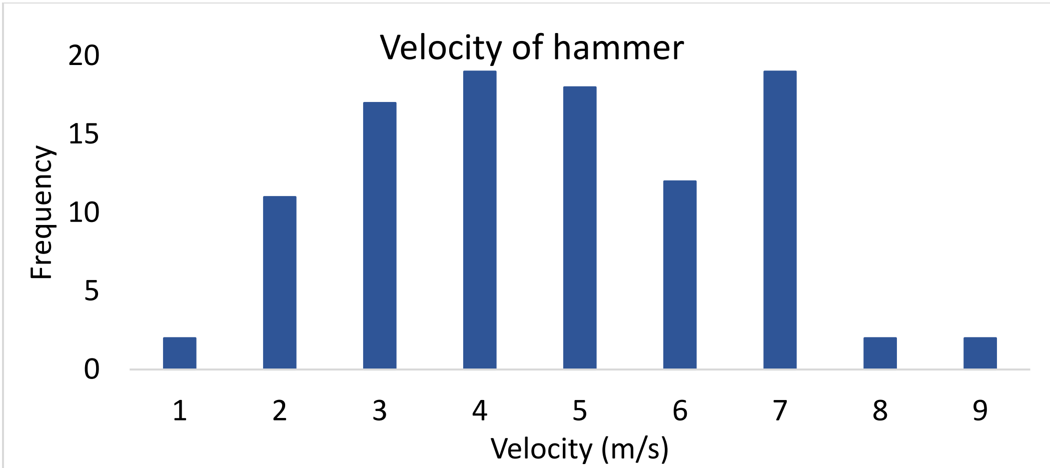

Figure 4.42 Velocity of hammer

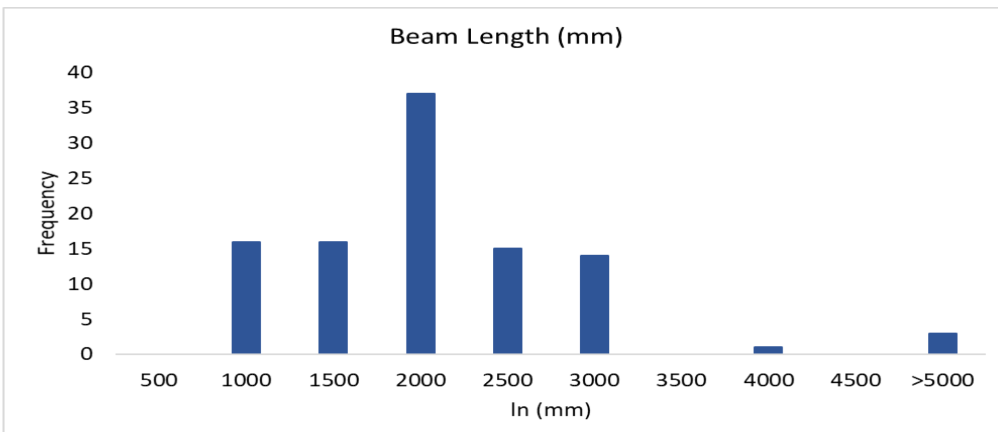

Figure 4.5 Length of beam

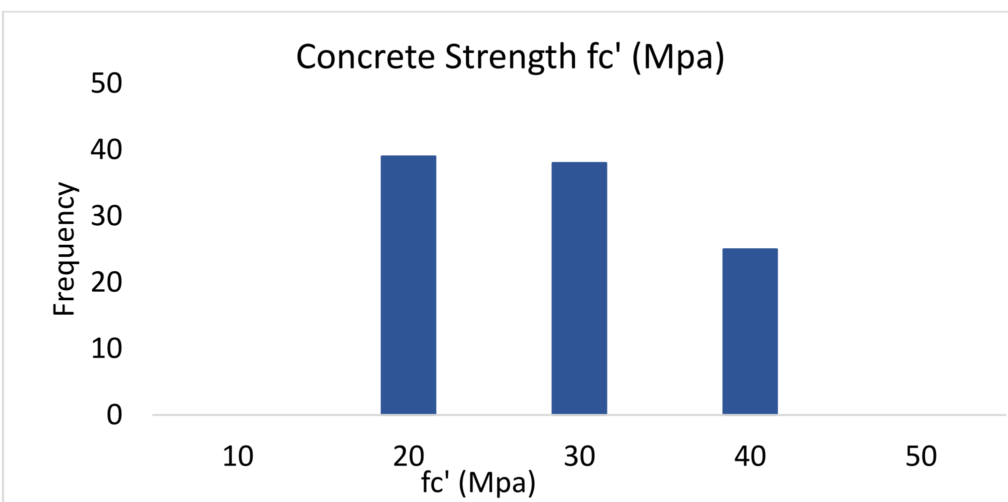

Figure 4.6 Concrete Strength

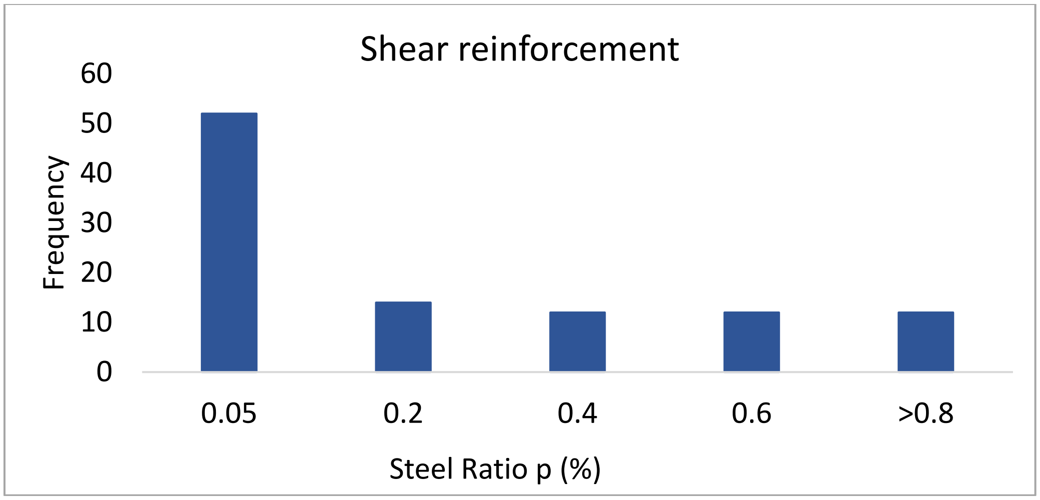

Figure 4.7 Shear reinforcement

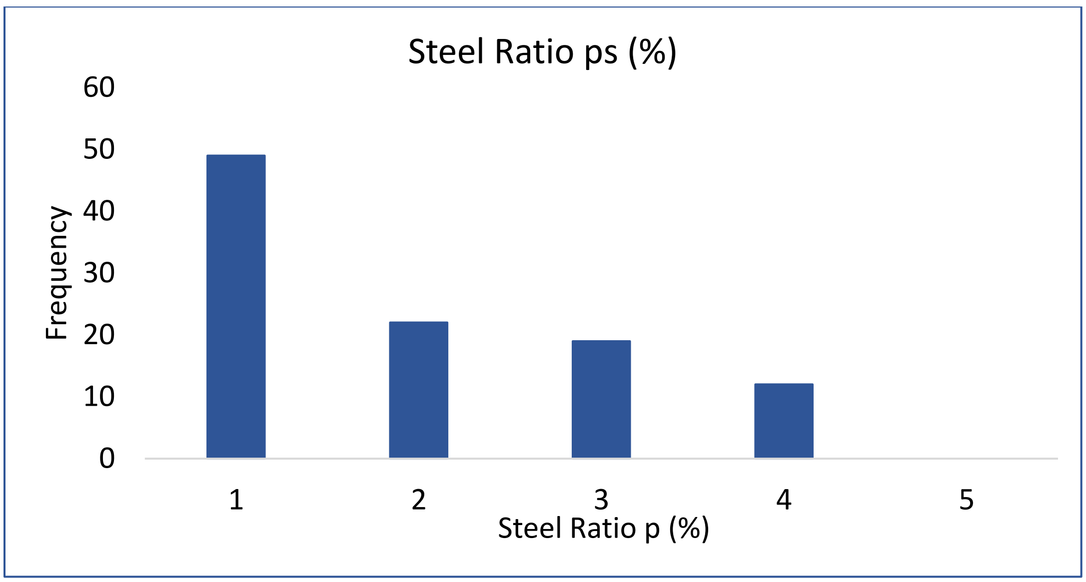

Figure 4.8 Tensile reinforcement ratio

4.2.3. Validation with experimental tested data

The developed large displacement solution (LDS) model is set side by side with the experimental results, as presented in Figure 15. To authenticate this proposed model using the collected dataset, mid span deflection is assessed through coefficient of determination (R^2), a variance-dependent coefficient that approaches 1 (one) for the best predictions. In this case, the correlation factor between the experimental and predicted results is found to be $R^2 = 0.92$, representing a strong predictive capability. Moreover, the line for predicted peak mid-span deflection is $y = 0.96x$, This is perfectly aligned with the 45-degree benchmark, suggesting a strong link between the experimental and projected findings.

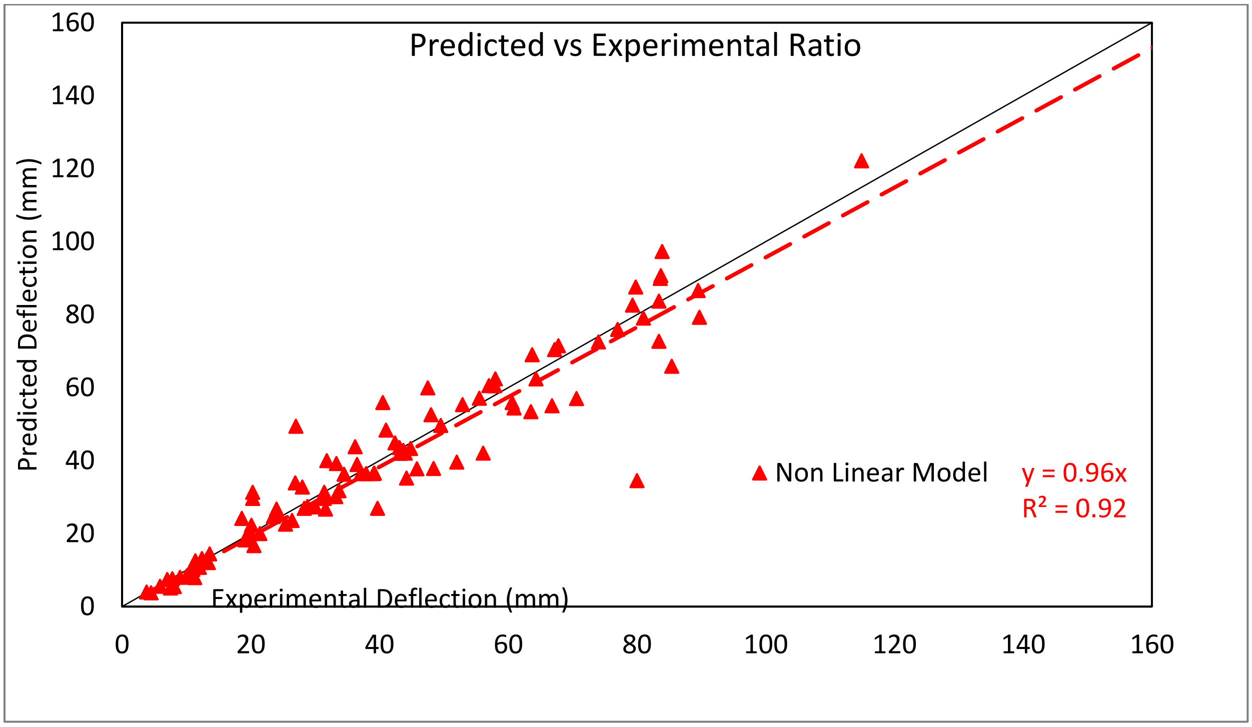

Figure 4.9 Predicted vs Experimental Ratio

The anticipated to experimental deflection ratio is used to evaluate prediction accuracy; a PER close to one indicates greater performance. As a result, the average PER is 0.92, which is quite close to the ideal benchmark of 1. This value shows a coefficient of variation (CoV) of 19.3243%. The figures (16-19) illustrate a sensitivity analysis of various parameters within the proposed large displacement solution formulation. Figure (16) displays that the precise prediction of the maximum mid span deflection of proposed model has a virtual average accuracy of 0.92 within the range of velocity 0.6-1.5. In light of this result, it is clear that the proposed model performs well across a wide range of velocity conditions. In addition, Figure (17) illustrates a comparison between the deflection predicted by the LDS model and the deflection experienced by the experimenter. The LDS model exhibits high accuracy and reliability within the range of 0.6-1.5. Based on the experimental mass ratio and depth of the beam, figures (18) and (19) demonstrate the robust performance of the large displacement solution model. Overall, the large displacement solution method consistently predicts deflection with reasonable accuracy, especially for PERs between 0.6

and 1.5.

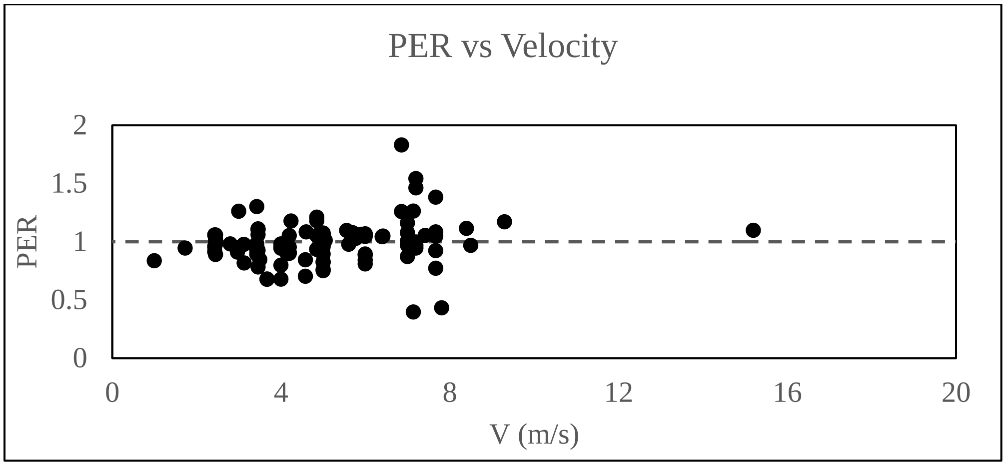

Figure 4.10 PER vs Velocity

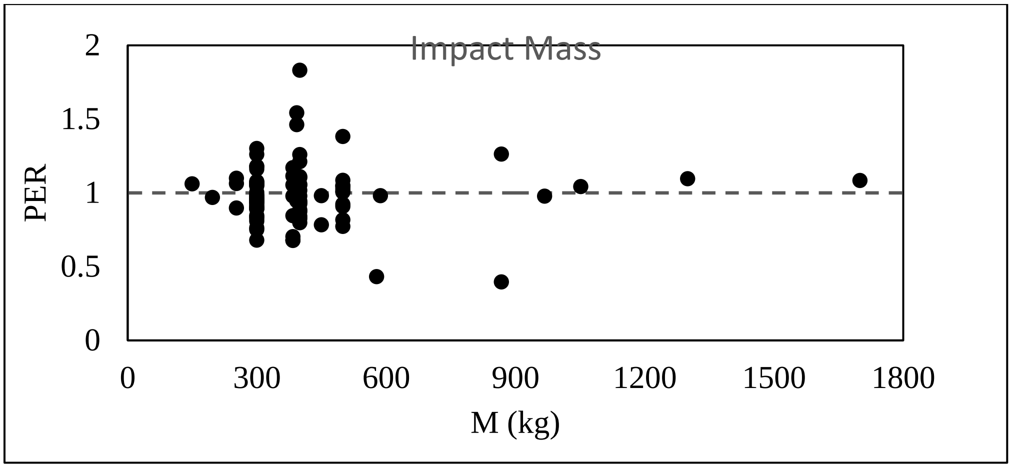

Figure 4.11 Impact Mass

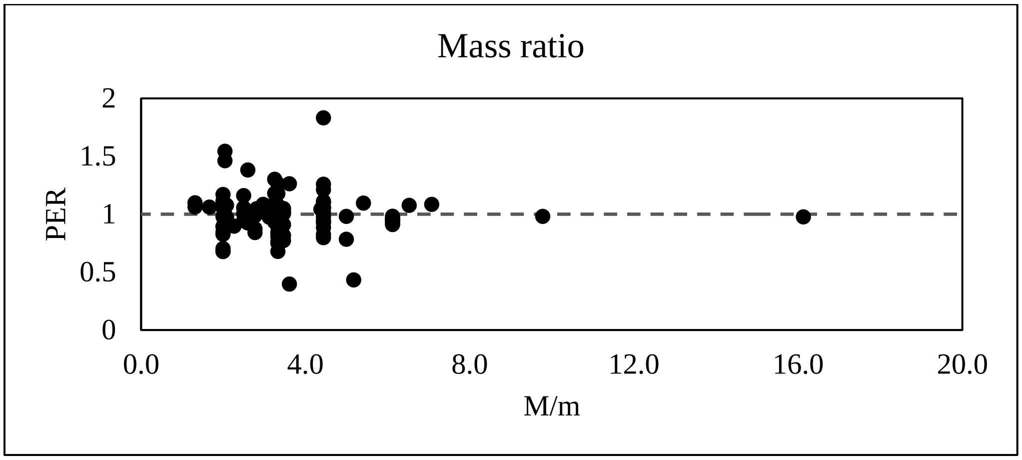

Figure 4.12 Mass Ratio

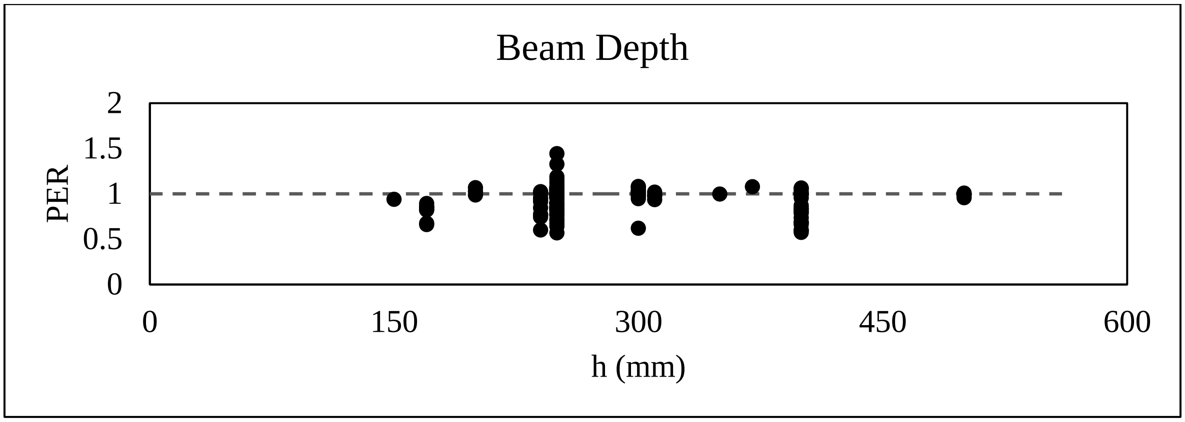

Figure 4.13 Beam Depth

4.3 Validation with the available models

Table 1 presents a comprehensive comparison of various available models, with a focus on statistical parameters. Particularly noteworthy is the high level of reliability of the developed formulation, which outperforms all other models. It is explained that the formulated model has a higher R^2 value compared to the others, a measure of how well it fits the data. Slope of the for the developed formulation is remarkably similar to the slope of the benchmark line. In addition to fitting the data well, the formulated model also closely aligns with the reference or benchmark relationship, making it a particularly robust and accurate representation of the underlying phenomenon.

4.3.1 Adhikary et al. Model

The comparison of Large displacement solution with the Adhikary et al. model is shown in Figure 20. This model is valid to 87 tested beams data. So, only that 87 data were used for LDS (Large displacement Solution) model as well. The R^2 of the LDS model becomes 0.92 with AAE equal to 14.787%. The average PER value becomes 0.989786 with CoV equal to 19.3243%. Similarly, the R^2 of Adhikary etal. formulation is 0.84 having AAE of 18.06%. The mean PER value is 0.85 with coefficient of variation of 34.06% for Adhikary et al.

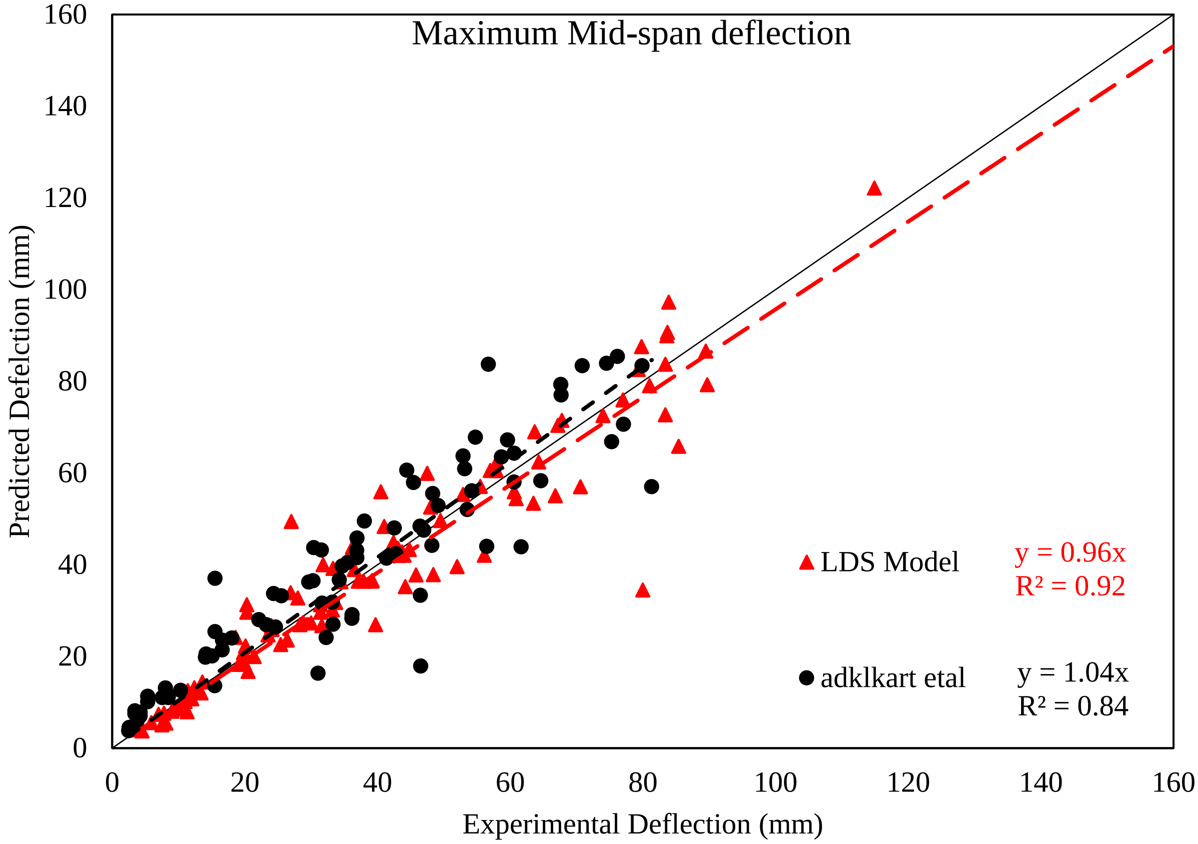

Figure 4.14 LDS model vs Adhikary model

4.3.2 Zhao et al. Model

The comparison of Large displacement Solution (LDS) with the Zhao etal. formulation is shown in Figure 21. This model is valid to 98 tested beams data. The R^2 of the Zhao et al. model is 0.83 with AAE of 26.9%. The mean PER value is 0.91 with coefficient of variation of 24.90%.

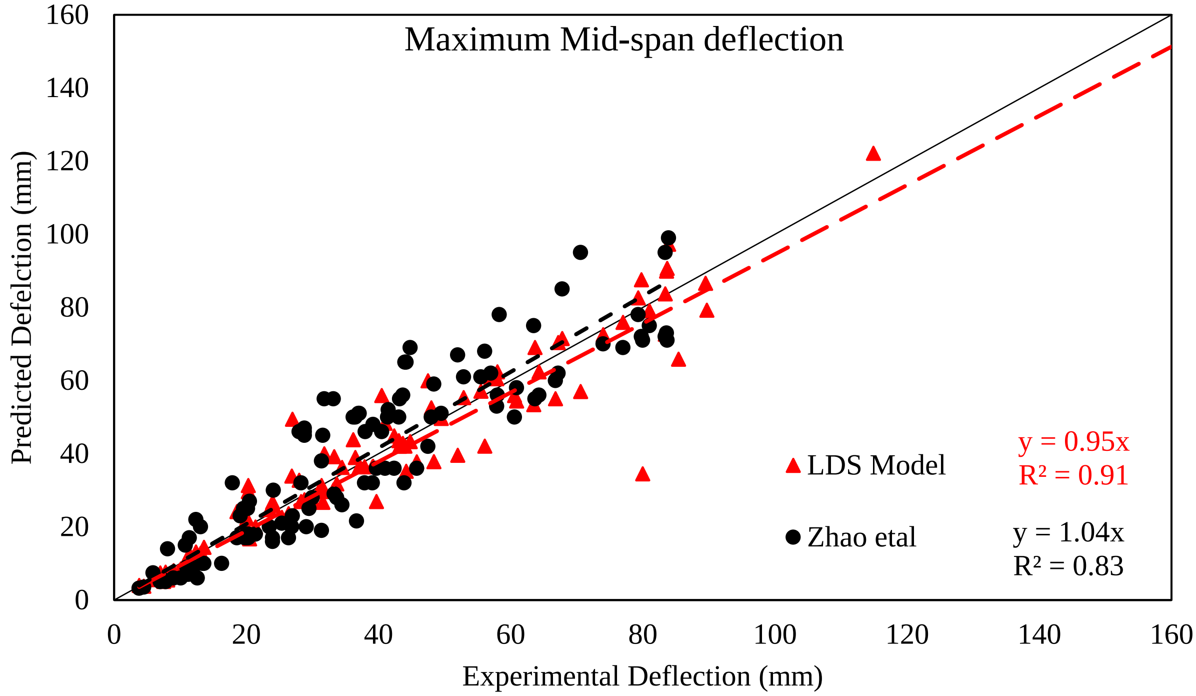

Figure 4.153 LDS model vs Zaho model

4.3.3 Khan et al. Model

The comparison of Large displacement solution with the Khan et al. model is shown in Figure 22. This model is effective to 102 tested beams data. The R^2 of the Khan etal. Formulation is 0.84 with AAE of 18.06% . Mean PER value is 1.21 with coefficient of variation of 30.5%.

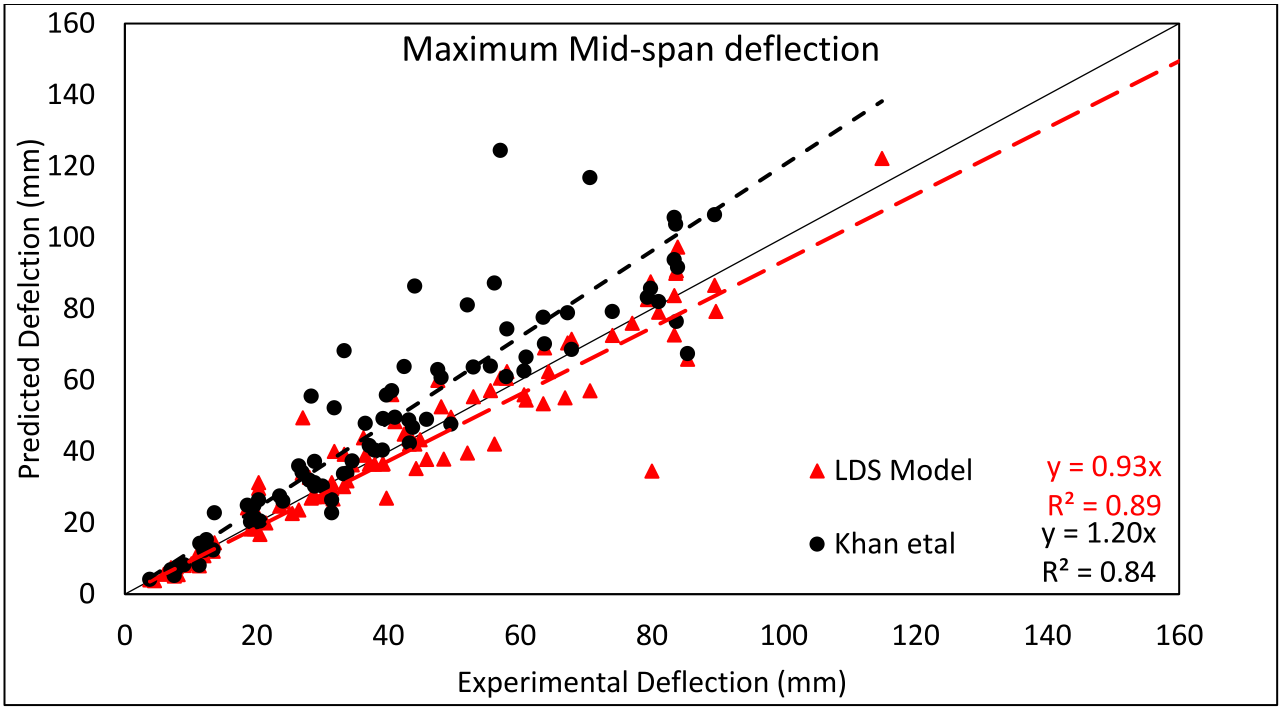

Figure 4.16 LDS model vs Khan etal model

Author	Samples Number	Mean PER	SD	COV (%)	AAE (%)	R^2
Zhao et al.	98	0.91	0.17	18.91	26.91	0.9
Adhikary et al.	87	0.85	0.29	34.06	25.5	0.8
Khan et al.	102	1.21	0.3	24.9	18.06	0.8
Proposed LDS Model	102	0.93	0.19	19.32	14.786	0.9

Table 4.1 Analyzing statistical models to predict mid span deflections under impact loading for reinforced concrete beams

CHAPTER 5: Validating With Khan et al Model

This study represents a noteworthy development over the earlier model proposed by Khan et al[39]., which forms the basis of our research. To further enhance the understanding of the subject matter, we conducted a comprehensive and competitive study based on their work. This was accomplished by conducting a comparative analysis between our enhanced model and the existing. For the purpose of ensuring credibility and validity of our findings, we carefully selected a sample from the previous study conducted by Kishi N, Mikami H[40]. As a result of their study, we were able to evaluate the performance of our enhanced model using well-documented experimental values.

Figure 22 illustrates the section properties used in our research as well as the experimental values from the Kishi N, Mikami H study. As a result of these properties, we are able to make informed evaluations of the enhancements we introduced to the existing model during our comparative analysis.The inclusion of this comprehensive comparative study is intended to contribute significant insight to the field, resulting in a deeper understanding and improved models and applications in the future.

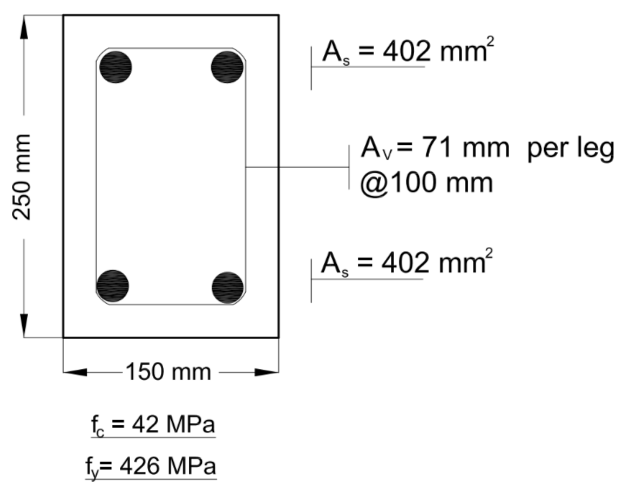

Figure 5.1 Sectional Properties

In the research conducted by Kishi N. and Mikami H., they organize drop hammer test to evaluate the mechanical properties of their samples. In order to conduct the test, the mass of the hammer used for impact, the velocity at which it was dropped onto the samples, as well as the length of the specimens being tested were carefully measured. The accuracy and reliability of their results were dependent on each of these parameters.

They have meticulously documented and presented the data obtained from their drop hammer test in Table 2, including the hammer's mass and velocity, and the length of the sample. Under the impact, the beam-shaped specimens underwent bending or deflection, rather than experiencing any other form of deformation. Accordingly, this flexural pattern aligns perfectly with the design principles of our own model.

Comparing the results of the drop hammer test with the design characteristics of our model allows us to gain valuable insights and make informed decisions regarding the applicability and suitability of our model for practical applications. It further reinforces the credibility of our model and allows it to be confidently applied to similar flexural applications because the observed behavior and the intended application are aligned.

	L (mm)	V (m/s)	M (kg)	Failure Model
Sample	3400	7	300	Flexure

Table 5.1 Detail of Drop Hammer

As part of our comprehensive analysis of the behavior depicted in Figure 24, we plotted the time versus displacement for the sample. Compared to the linear model, the large displacement solution exhibits a significant reduction in value as displacement increases. The change in stiffness of the structural member is one key factor contributing to the decrease in displacement at large deformations. As loads increase, the material's stiffness changes, resulting in a nonlinear response that differs from that predicted by the linear model. Stiffness changes become more pronounced as the deformation intensifies, resulting in a reduction in displacement values. An major component influencing displacement behavior is the creation of extra axial forces inside the structural part. When a material undergoes significant deformations, internal forces arise as a result of structural configuration and loading conditions. Additionally, these axial forces affect the overall response, resulting in a departure from the predictions made by the linear model.

A number of independent studies have investigated and described analogous behavior in their various studies, including Yu and Stronge[41], Bodner and Symonds, as well as Zhang and Yu[42]. The experimental data depicted in Figure 24 originates from a previous study conducted by Kishi N. and Mikami H.

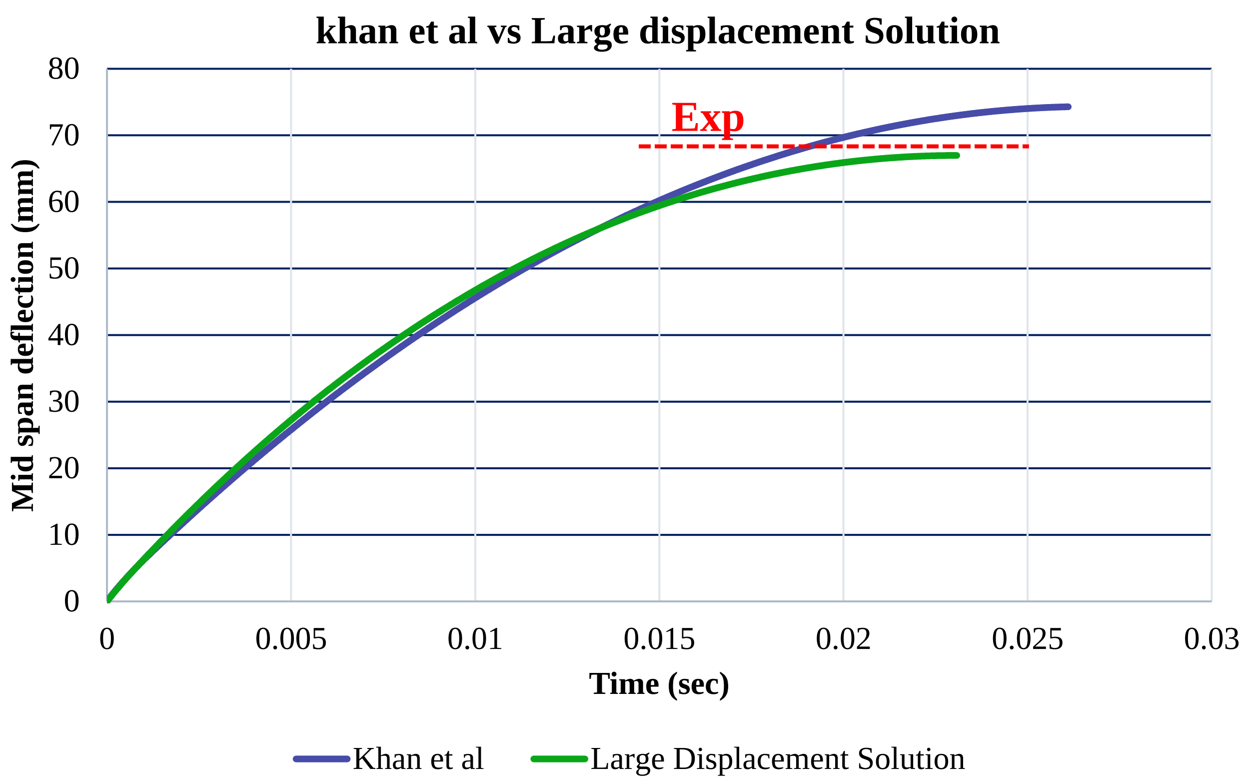

Figure 5.2 khan et al vs LDS model

CHAPTER 6: Conclusion and results

1. The proposed large displacement solution model provides an effective approach for effectively analyzing the behavior of reinforced concrete (RC) beams, especially those that exhibit flexural responses under impact loading conditions. In compared to Khan et al.'s earlier linear model, we may achieve more accurate estimates of the maximum displacement of RC beams subjected to impact load by adopting this new model.
2. The proposed large displacement model has the major advantage of reducing the final maximum mid-span displacement in comparison to the linear model's predictions. With increasing displacement, this effect becomes more pronounced.
3. Significant factor contributing that improve the accuracy of large displacement model is its comprehensive consideration of key factors that affect the structural behavior.

a. Firstly, the model takes into account changes in the stiffness of RC beams during significant deformation. As the stiffness of the material evolves under varying loads, this results in nonlinear response characteristics that are difficult to capture using a traditional linear model.

b. Secondly, the large displacement model incorporates the influence of additional axial forces within RC beams during impact loading. The effects of these forces are significant contributors to the overall structural response as well as being essential to accurately predicting the displacement behavior in a dynamic environment.

4. By adopting this model, RC beam structures under real-world conditions could be made more safe, reliable, and perform better. Additionally, it contributes to the advancement of structural engineering by improving the ability to withstand impact and dynamic loading challenges, while meeting stringent safety standards.

6.1 Recommendation

Following are some of the recommendations based on this study

1. Strain rate should be also incorporated in this large displacement solution model

2. Experimental validation of proposed large displacement model is required

3. Bending shear interaction should also be incorporated in the model

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